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Optimization Algorithm for Building Structures under Earthquake Loading

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Abstract

This paper presents a modified Dual algorithm for the optimal design of building structures subjected to earthquake loading, with particular emphasis on the mathematical formulation and numerical implementation of the optimization procedure. The proposed approach is based on an optimality criteria framework in which the original nonlinear design optimization problem is transformed into a sequence of explicit convex primal subproblems through linear approximations of the objective and constraint functions. A corresponding dual formulation is subsequently established. The mathematical characteristics of the algorithm, including its formulation, solution strategy, and computational efficiency, are discussed in detail. To demonstrate its applicability, the proposed method is applied to the seismic design optimization of a three-story steel moment-resisting frame. The study demonstrates that the proposed method is an effective computational tool for seismic design.

Keywords

Structural Optimization, Seismic Design, Dual Method, Steel Frames, Moment Frames

1. Introduction

Structural optimization has evolved into an important computational tool and interdisciplinary field that combines structural analysis, design, and mathematical optimization. Early developments can be traced to the work of Michell in 1904 [1], who established the theoretical foundations for minimum-weight structures. With the advent of digital computing in the 1960s and 1970s, optimization techniques such as linear programming, nonlinear programming, and optimality criteria methods began to be applied to practical structural design problems [2]. Further advances in finite element analysis, mathematical programming, and artifi-

cial intelligence led to the development of size, shape, topology, and performance-based optimization methods [3].

In the field of seismic design, structural optimization has emerged as a powerful approach for enhancing the earthquake resilience of building structures [4] [5]. Evolutionary and metaheuristic algorithms such as Genetic Algorithms gained popularity due to their efficiency in searching large design spaces and handling discrete and continuous variables [6]. In recent years, other metaheuristic methods such as Differential Evolution, Ant Colony Optimization, Harmony Search, and Firefly Algorithm have been investigated for seismic design problems [7] [8]. Comparative studies generally suggest that no single algorithm consistently outperforms others across all structural optimization problems. Instead, performance depends on factors such as problem size, design variables, and constraint complexity.

This paper presents a generalized optimality criteria algorithm, referred to as the Dual Method [9]. Its application to engineering problems is primarily attributed to Fleury [2] [10] [11]. The method is modified in this study to make it suitable for the optimization of building frames subjected to earthquake loading. The primary focus of this paper is the mathematical derivation of the modified Dual Method in addition to its engineering application.

2. Mathematical Formulation of Design Problem

In the structural engineering discipline, design optimization aims to determine the optimal values of design variables, typically the cross-sectional areas or dimensions of structural members, to achieve the best performance with respect to a specified objective function while satisfying all applicable design code requirements and designer-defined constraints. In the context of seismic design under earthquake loading, the structural optimization problem can be cast as follows:

To minimize the objective function:

$$f = \alpha_1 f_1 + \alpha_2 f_2 \tag{1}$$

Subject to strength and deflection constraints:

$$D_k(\mathbf{X}) \le D_k^U,\ k = 1, \cdots, n_c \tag{2}$$

side constraints:

$$x_j^L \le x_j \le x_j^U,\ j = 1, \cdots, n_x \tag{3}$$

where: f is a combined and weighted objective function; α_1 and α_2 are weighting factors for objective functions f_1 and f_2, respectively, and $\alpha_1 + \alpha_2 = 1$; $\mathbf{X} = \{x_1, x_2, \cdots, x_{nx}\}^{\mathrm{T}}$ is the vector of n_x design variables that are required to be found in order to minimize the objective function; $D_k(\mathbf{X})$ is the kth constraint function, bounded by its upper limits D_k^U; n_c is the number of constraints. Equations (3) is a set of side constraints on the design variables x_j, where x_j^L and x_j^U are the lower and upper bounds of x_j.

Minimum cost is a universally desired objective in optimization problems.

Since the cost of a structural member is generally proportional to its material weight, the minimum-cost design can be approximated as the minimum-weight design of the structure. Consequently, the cost objective function, often referred to as the weight objective function, can be expressed as the total weight of all structural members as,

$$f_1 = (1/G_0) \cdot \sum_j^{n_x} \gamma L_j A_j \tag{4}$$

where: n_x is the number of design variables; γ is the material mass density; L_j and A_j are the given length and variable cross-sectional area of member j, respectively; and G_0 is a normalization factor used to scale the weight objective function and is typically taken as the maximum allowable structural weight. As a result, the normalized objective function f_1 is bounded between 0 and 1, which improves numerical stability and facilitates the weighting of multiple objectives in the optimization process.

The second design objective is to pursue uniform interstory drift, which aims to minimize building damage under earthquake hazards [4]. This objective can be expressed as,

$$f_2 = \sqrt{\frac{1}{n_s} \sum_{s=1}^{n_s} (\delta_s - \delta_a)^2} \tag{5}$$

where: δ_s is the interstory drift ratio of story s; n_s is the number of stories; δ_a is the average interstory drift ratio, which is defined as the lateral deflection of the roof level divided by the building height.

In Equations (1) to (5), only Equations (3) and (4) are an explicit function of the design variables, while Equation (5) and all constraints are implicit functions of the design variables. Thus, to enable a computer solution, it is necessary to use an approximation technique to formulate f_2 function and constraints in terms of the design variables. Since structural deflection is typically inversely proportional to the size of the structural members, a more accurate approximation of deflection can be obtained by using the reciprocal of the members' cross-sectional areas as the design variables. In other words, adopting the reciprocal area as the design variable, $x_j = 1/A_j$, leads to higher-quality approximations [10].

Then, employing the first-order derivatives and Taylor series to rewrite Equations (1) and (2) as the following explicit optimization problem:

To minimize:

$$f = \alpha_1 \sum_j^{n_x} \left(\gamma L_j / G_0 x_j\right) + \alpha_2 \left[f_2^0 + \sum_j^{n_x} \left(\mathrm{d}f_2/\mathrm{d}x_j\right)^0 \left(x_j - x_j^0\right) \right] \tag{6}$$

Subject to:

$$D_k^0 + \sum_j^{n_x} \left(\mathrm{d}D_k/\mathrm{d}x_j\right)^0 \left(x_j - x_j^0\right) \le D_k^U,\ k = 1, \cdots, n_c \tag{7}$$

where the superscript 0 indicates the values evaluated at the current design. $\mathrm{d}f_2/\mathrm{d}x_j$ and $\mathrm{d}D_k/\mathrm{d}x_j$ are the first derivatives of f_2 and D_k with respect to the design variables x_j, respectively. The evaluation of the derivatives is called sensitivity analysis, which can be found from the first author's earlier studies [12]

[13]. Side constraints, Equation (3) remains unchanged.

3. Optimization Algorithm

In the explicit formulation Equations (6) and (7), we further define

$$A_0 = \alpha_2 \left[f_2^0 + \sum_j^{n_x} \left(\mathrm{d}f_2 / \mathrm{d}x_j \right)^0 \left(-x_j^0 \right) \right] \tag{8}$$

$$A_{1j} = \alpha_1 \gamma L_j / G_0 \tag{9}$$

$$A_{2j} = \alpha_2 \left(\mathrm{d}f_2 / \mathrm{d}x_j \right)^0 \tag{10}$$

$$C_{jk} = \left(\mathrm{d}D_k / \mathrm{d}x_j \right)^0 \tag{11}$$

$$C_k^U = D_k^U - D_k^0 + \sum_j^{n_x} \left(\mathrm{d}D_k / \mathrm{d}x_j \right)^0 x_j^0 \tag{12}$$

The design formulation can be further simplified as,

To minimize:

$$f = \sum_{j=1}^{n_x} \left(A_{1j} / x_j + A_{2j} x_j \right) \tag{13}$$

Subject to:

$$\sum_j^{n_x} C_{jk} x_j - C_k^U \le 0, k = 1, \cdots, n_c \tag{14}$$

and side constraints Equation (3). Note that in Equation (13), A_0 is discarded as it will have no effect on the optimal solution.

The standard Dual Method formulation [10] is without $A_{2j} x_j$ term in Equation (13). The dual algorithm is modified hereafter to solve the optimization formulation.

The Lagrangian function for the optimization problem is written as,

$$\mathcal{L}(\mathbf{X}, \boldsymbol{\lambda}) = \sum_{j=1}^{n_x} \left(A_{1j} / x_j + A_{2j} x_j \right) + \sum_{k=1}^{n_c} \lambda_k \left(\sum_j^{n_x} C_{jk} x_j - C_k^U \right) \tag{15}$$

where λ_k are the Langrange multiplier, or dual variables to the design. The dual formulation is,

To maximize

$$\Omega(\boldsymbol{\lambda}) = \min \mathcal{L}(\mathbf{X}, \boldsymbol{\lambda}) \tag{16}$$

Subject to:

$$\lambda_k \ge 0, k = 1, \cdots, n_c \tag{17}$$

where dual function $\Omega(\boldsymbol{\lambda})$ is obtained by minimizing the Lagrangian function Equation (15) with respect to x_j. Therefore, $\frac{\partial \mathcal{L}}{\partial x_j} = 0$ leads to

$-\frac{A_{1j}}{x_j^2} + A_{2j} + \sum_{k=1}^{n_c} \lambda_k C_{jk} = 0$, which further gives,

$$x_j(\boldsymbol{\lambda}) = \sqrt{\frac{A_{1j}}{A_{2j} + \sum_{k=1}^{n_c} \lambda_k C_{jk}}} \tag{18}$$

Equation (18) holds if $x_j^L \le x_j \le x_j^U$. Otherwise, $x_j = x_j^L$ if $x_j < x_j^L$ or

$x_j = x_j^U$ if $x_j > x_j^U$. Differentiating Equation (18) with respect to λ_l gives,

$$\frac{\partial x_j}{\partial \lambda_l} = \begin{cases} -\dfrac{C_{jl} x_j^3}{2A_{1j}} & x_j^L < x_j < x_j^U \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

Substituting Equation (18) into Equation (16) leads to,

$$\Omega(\boldsymbol{\lambda}) = \sum_{j=1}^{n_x} \left(\frac{A_{1j}}{x_j(\boldsymbol{\lambda})} + A_{2j} x_j(\boldsymbol{\lambda}) \right) + \sum_{k=1}^{n_c} \lambda_k \left[\sum_j^{n_x} C_{jk} x_j(\boldsymbol{\lambda}) - C_k^U \right] \tag{20}$$

To apply the Newton method to search for the optimum point in the design space, it is necessary to find the Hessian Matrix of the dual function in Equation (20). The first derivative of dual function $\Omega(\boldsymbol{\lambda})$ with respect to variable λ_k is,

$$\frac{\partial \Omega}{\partial \lambda_k} = \sum_j^{n_x} C_{jk} x_j - C_k^U \tag{21}$$

The second derivatives of $\Omega(\boldsymbol{\lambda})$ with respect to variable λ_l are obtained as,

$$\frac{\partial^2 \Omega}{\partial \lambda_k \partial \lambda_l} = -\sum_j^{n_x} \frac{C_{jk} C_{jl} x_j^3}{2A_{1j}} \tag{22}$$

where noting Equation (19). These second derivatives are discontinuous on the hyper planes defined by,

$$\sum_{k=1}^{n_c} \lambda_k C_{jk} = \frac{A_{1j}}{\left(x_j^L\right)^2} - A_{2j} \quad \text{and} \quad \sum_{k=1}^{n_c} \lambda_k C_{jk} = \frac{A_{1j}}{\left(x_j^U\right)^2} - A_{2j} \tag{23}$$

Denoting the Hessian matrix of $\Omega(\boldsymbol{\lambda})$ as $\nabla^2 \Omega(\boldsymbol{\lambda})$, the Newton line search direction is [9],

$$\boldsymbol{z} = -\left[\nabla^2 \Omega(\boldsymbol{\lambda})\right]^{-1} \nabla \Omega \tag{24}$$

where $\nabla \Omega$ is the gradient of the dual function, and $\left[\nabla^2 \Omega(\boldsymbol{\lambda})\right]^{-1}$ is the inverse of the Hessian matrix.

4. Overall Design Procedure

A complete design process consists of multiple design iterations. One iteration of the design process included three steps: 1) conduct structural analysis to obtain the responses of the current design under the applied earthquake loads; 2) conduct sensitivity analysis to obtain the first derivatives of responses and then formulate the approximate explicit expressions of the objective function and constraints; and 3) apply the dual algorithm to find a solution of the current optimization problem.

At the first iteration, an initial design is typically determined by an engineer through experience or preliminary calculation. This trial design is then analyzed to obtain the first formulation of the design optimization problem. Then, an improved design is found by applying the above-described algorithm. The design process enters the next iteration with the improved design as the current design. This procedure is repeated until convergence of the solution occurs.

One may ask why multiple iterations are required. The primary reason is that linear approximations of both the objective function and the constraints are employed to transform the original problem into a convex optimization problem. These approximations are sufficiently accurate only in the vicinity of the current design point. Consequently, it is necessary to restrict the step size along the search direction during each iteration. As a result, a single iteration generally yields only an improved design rather than the final optimal solution. The move limits ensure that the updated design does not deviate excessively from the current design point, thereby maintaining the validity of the approximations and reducing the risk of producing an infeasible design.

5. Numerical Example

A three-story rigid frame shown in **Figure 1** is adopted as a design example to illustrate the optimization methodology described above. This is a perimeter moment frame of a building. Each bay is 9 meters wide, and each story is 4 meters high. The seismic weights assigned to the first and second stories are 4700 kN each, while the seismic weight of the roof level is 5100 kN. All column bases are assumed to be fixed at the foundation.

The frame comprises 27 members. To reduce the dimensionality of the optimization problem, the number of design variables is reduced to five through a member-linking scheme. Specifically, all column segments within the same vertical line use the same cross-section over the full height of the frame (*i.e.*, one column tier per column line). The two exterior columns share one cross-section, while the three interior columns share another. In addition, the beams at each floor level are assigned their own cross-section, resulting in three beam design variables. The reciprocal design variables are $x_j = 1/A_j$, $j = 1,\cdots,5$. The length L_j in Equation (5) should be the combined length of the members having cross-section area A_j. Namely, $L_1 = 24$ m, $L_2 = 36$ m, $L_3 = L_4 = L_5 = 36$ m. Column section A_1 and A_2 will be chosen from among steel W360 sections (hence, upper and lower limits are 139,000 and 4170 mm^2, respectively), while A_3 to A_5 beams will be chosen from among W760 sections (upper and lower limits are 74,300 and 17,000 mm^2, respectively). These commercially available steel shapes and their sectional properties can be found in [14]. Steel material has a specified yielding strength of 350 MPa and mass density of $\gamma = 7850$ kg/m^3. The weight objective function f_1 is normalized by the maximum allowable weight $G_0 = 128$ tons.

The earthquake hazard level considered in this example corresponds to a 2% probability of exceedance in 50 years. The design response spectrum used to obtain the lateral earthquake loads is shown in **Figure 2**, where T is the elastic fundamental period of the building in the unit of seconds and S is the pseudo acceleration expressed in units of gravitational acceleration, g. The base shear, V_b, is then obtained as,

$$V_b = S \cdot \left(\sum_{q=1}^{3} G_q \right) \tag{25}$$

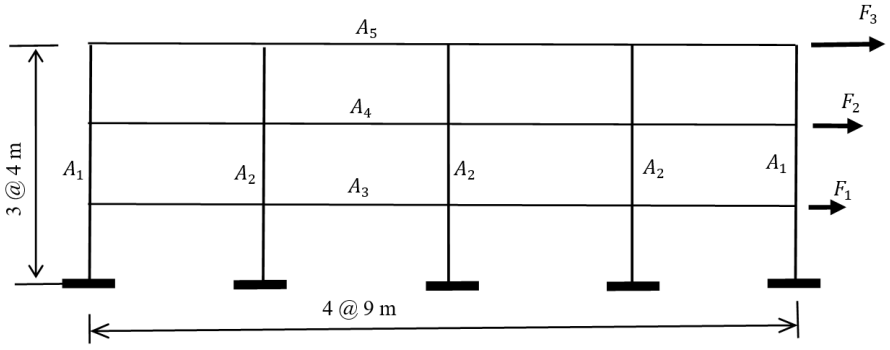

Figure 1. Steel moment frame and design variables.

which is distributed to each floor (Figure 1) according to Equation (26):

$$F_s = V_b \frac{G_s h_s}{\sum_{q=1}^{3} G_q h_q}, s = 1, 2, 3 \tag{26}$$

where: G_s and G_q are the seismic weights of story s and story q, respectively; h_s and h_q are the corresponding heights measured from the base of the building. In each design iteration, the fundamental period and the corresponding base shear are recalculated to reflect the revised design.

A nonlinear static analysis procedure was employed to evaluate the structural demands on the moment frame. The analytical model accounted for both geometric and material nonlinearities, including initial member crookedness, second-order effects, and residual stresses, and thus qualifies as an advanced analysis. Using this analytical approach, explicit strength checks of individual frame members are not required, as permitted by the AISC Specification [15].

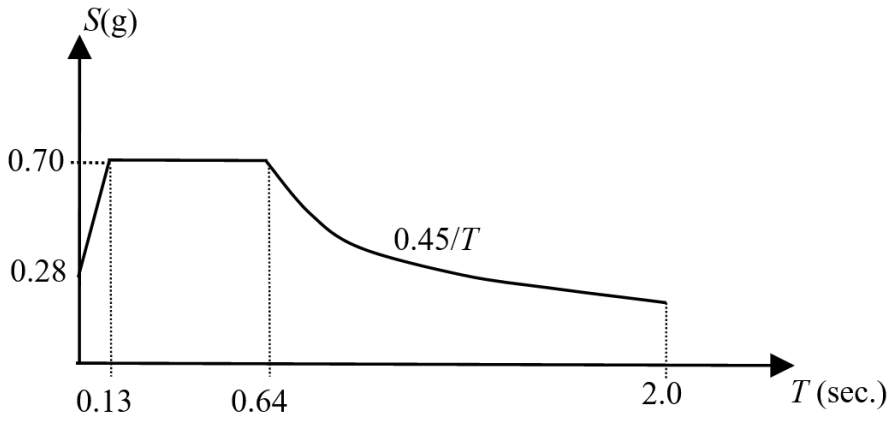

Figure 2. Design spectrum.

Because the target seismic performance level is collapse prevention, an inter-story drift limit of 5% is adopted as the deflection constraint [16]. In the first iteration, the member sections of the initial design are chosen as the largest available sections: W360 × 1086 (139,000 mm^2) for variables A_1 and A_2, and W760 × 582 (74,300 mm^2) for variables A_3 to A_5. The corresponding fundamental period is 0.45 seconds.

The values of the weighting factors α_1 and α_2 have a direct impact on the design results. Their appropriate selection requires numerical experimentation. Numerical studies have shown that a smaller weighting factor α_2 can effectively prevent oscillations of the design point during the iterative design process. Thus, the weighing factors are taken to be $\alpha_1 = 0.9$ and $\alpha_2 = 0.1$.

In this example, convergence is deemed to be achieved when the relative difference in the objective function values between two consecutive iterations is less than 1%. No move limit is imposed during the first five iterations. Starting from the sixth iteration, a move limit of 0.2 is applied. The design process converges after 8 iterations.

The final optimal design consists of the following sections: $A_1 = 49020\ \text{mm}^2$ (W360 × 382); $A_2 = 62130\ \text{mm}^2$ (W360 × 509); $A_3 = 30540\ \text{mm}^2$ (W760 × 257); $A_4 = 28020\ \text{mm}^2$ (W760 × 220); and $A_5 = 17650\ \text{mm}^2$ (W760 × 134). The sections listed in parentheses represent the closest available discrete sections selected for practical implementation. The final structural weight is 49.6 tons, corresponding to a weight objective value $f_1 = 0.39$, while the fundamental period is 0.79 s. The governing active constraint is the first-story interstory drift ratio. Analysis of the optimal design shows that the first-story interstory drift ratio reaches 4.6%, which is slightly below the prescribed design limit, while the second and third stories drift ratios are 3.2% and 2.6% respectively. The corresponding value of the second objective function is $f_2 = 0.008$.

6. Summary and Conclusions

This study presents a modified Dual algorithm for the design optimization of building structures subjected to earthquake loading. A linear approximation approach was employed to transform the original design problem into a sequence of explicit convex primal minimization subproblems. The corresponding dual optimization problem was then formulated and solved using a quadratic programming technique in the dual space, with the objective of maximizing the dual function. The effectiveness and practicality of the proposed algorithm were demonstrated through the optimal design of a three-story steel moment-resisting frame subjected to seismic loading.

The original Dual Formulation was modified to accommodate the additional objective function introduced in Equations (5) and (6). The inclusion of this objective led to an additional term in the dual function, as presented in Equations (13) and (15). The influence of this term was reflected in the expression for the primal variable x_j given in Equation (18), as well as in the revised locations of the hyperplanes defined by Equation (23).

The proposed algorithm is computationally attractive because it can solve large-scale structural optimization problems while maintaining numerical stability and convergence efficiency. Numerical results demonstrate that the modified Dual algorithm converges to an optimal solution and provides an effective balance between computational efficiency and design quality.

Author Contributions

Conceptualization, Y. Gong and J. Deng; methodology, Y. Gong and J. Deng; writing—original draft preparation, Y. Gong; writing—review and editing, J. Deng; funding acquisition, J. Deng. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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