

H_∞ Control of Structural Vibration with Input Time Delay Based on CHsCTA Method

Yongfang Chen¹, Yuanguang Zheng²

¹School of Mathematics and Information Science, Nanchang Hangkong University, Nanchang, China

²School of Electrical Engineering, Shanghai Dianji University, Shanghai, China

Email: CYF8610@outlook.com, zhengyg@sdju.edu.cn

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Abstract

This study addresses the active control problem of structural vibration with input time delay. Taking the structural system equipped with an active mass damper (AMD) as the research object, an H_∞ control scheme based on the Chebyshev spectral continuous-time approximation (CHsCTA) is proposed. Specifically, the CHsCTA method is first employed to approximate the closed-loop system with input time delay by a finite-dimensional delay-free augmented system, upon which the H_∞ controller design is then carried out. To address the bilinear matrix inequality (BMI) constraints arising from the controller design process, the cone complementarity linearization (CCL) method is introduced to construct an iterative solving algorithm, thereby obtaining a controller that satisfies the prescribed H_∞ performance index. Numerical simulation results demonstrate that the designed controller can effectively suppress structural vibration under the given input time delay. Under harmonic excitation, the peak vibration reduction rate of the primary structure is significantly improved. Under Gaussian white noise excitation, the average absolute value of the structural response is also markedly reduced, thereby verifying the effectiveness of the proposed control strategy.

Keywords

H_∞ Control, Building Structure, Input Time Delay, Chebyshev Collocation Points

1. Introduction

Active mass dampers (AMDs), as a class of representative active control devices, are capable of effectively suppressing structural vibrations and enhancing structural dynamic performance [1]-[3]. In contrast to passive control devices, AMDs

regulate the control force in real time to accommodate variations in external excitations, thereby exhibiting superior adaptability and control precision. AMD systems are characterized by a simple actuator configuration, modest control force requirements, stable control performance, and favorable robustness [4] [5], rendering them particularly prevalent in the vibration control of high-rise buildings [6]-[8].

However, in practical AMD systems, input time delays are inevitable during signal acquisition, transmission, and actuator response [9] [10]. These delays alter the dynamic characteristics of the closed-loop system, leading to degraded control performance and, in severe cases, even inducing instability [1] [11]. As a result, they significantly increase the difficulty of system analysis and controller design. Consequently, control strategies for time-delay systems have attracted considerable attention, with the aim of mitigating the adverse effects of such delays on system performance.

Stability analysis of time-delay systems is typically based on Lyapunov-Krasovskii (L-K) functionals combined with integral inequalities [12], among which the Park and Moon inequalities are representative examples [13] [14]. In recent years, to reduce the conservatism of stability criteria, various integral inequalities have been extensively employed to estimate the upper bounds of integral terms arising in the derivative of L-K functionals. Representative approaches include Jensen's inequality [15], Wirtinger-type integral and summation inequalities [16] [17], and the Bessel-Legendre inequality [18].

Another line of research seeks to transform time-delay systems into delay-free systems via discretization techniques, thereby facilitating subsequent stability analysis and controller design. Along this direction, Insperger *et al.* proposed the semi-discretization (SD) method [19] [20], and Sun *et al.* introduced the continuous-time approximation (CTA) method [17]. Butcher *et al.* further developed the Chebyshev spectral continuous-time approximation (CHsCTA) method [21]-[25], which achieves high accuracy in stability analysis with only a modest number of collocation points.

In this work, the coupled dynamic model of a building structure with an active mass damper (AMD) is first established using Lagrange's equations, in which the input delay originates from the actuator response of the AMD motor. On this basis, the H_∞ performance bilinear matrix inequality (BMI) conditions are derived for the AMD closed-loop system under the CHsCTA approximation, and the cone complementarity linearization (CCL) algorithm is incorporated to solve the feedback gain matrix. Finally, two typical excitations in civil engineering, harmonic excitation and Gaussian white-noise excitation, are employed to quantitatively evaluate the vibration suppression performance of the primary structure.

This study addresses the structural vibration control problem with input time delay. Taking the structural system equipped with an AMD as the research object, a fixed-delay H_∞ state-feedback controller design method based on the CHsCTA

approach is investigated. The remainder of this paper is organized as follows. Section 2 derives the dynamic equations of the building structure using Lagrange's equations. Section 3 employs the CHsCTA method to transform the infinite-dimensional time-delay AMD closed-loop system into a finite-dimensional delay-free augmented system. Section 4 presents the H_∞ control design. For the resulting delay-free system, the corresponding BMI conditions are derived, and the CCL algorithm is incorporated to solve for the performance index and state-feedback gain matrix that satisfy the prescribed H_∞ performance requirements. Section 5 validates the effectiveness of the proposed CHsCTA-based H_∞ controller under two representative excitation conditions, namely harmonic excitation and Gaussian white noise excitation. Section 6 concludes the paper.

2. Dynamic Model

The structural system equipped with an AMD considered in this study is shown in **Figure 1**. A cart mounted at the top of the structure serves as the active mass. Driven by a DC motor, the cart generates inertial forces to regulate the structural response.

Let x_t denote the absolute displacement of the ground, x_f the displacement of the main structure relative to x_t , and x_c the displacement of the AMD relative to x_f . The mass and stiffness of the main structure are denoted by m_f and k_f , respectively. The AMD is characterized by its mass m_c , stiffness k_c , and equivalent viscous damping coefficient c_c . The thrust force exerted by the DC motor on the cart is denoted by F_c , which can be expressed as follows:

$$F_c = \eta_1 V_m(t - \tau) - \eta_2 \dot{x}_c(t),$$

where η_1 and $V_m(t - \tau)$ denote the motor efficiency and input voltage, respectively. The parameter $\tau > 0$ represents the time delay of the system.

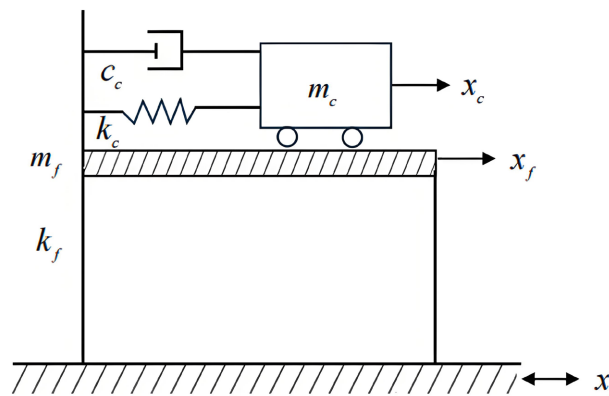


Figure 1. The structural system equipped with an AMD.

A dynamic model of the system is derived using Lagrange's equations. The Lagrangian L is defined as the difference between the total kinetic energy T and the total potential energy V , i.e., $L = T - V$, where T and V are given as follows:

$$T = \frac{1}{2} m_f (\dot{x}_f(t) + \dot{x}_t(t))^2 + \frac{1}{2} m_c (\dot{x}_c(t) + \dot{x}_f(t) + \dot{x}_t(t))^2,$$

$$V = \frac{1}{2} k_f x_f(t)^2 + \frac{1}{2} k_c x_c(t)^2.$$

Let the generalized coordinates x_c and x_f be selected. Neglecting viscous damping of the structural floor, the generalized forces corresponding to these coordinates are given by $Q_{x_c} = F_c - c_c \dot{x}_c(t)$, and $Q_{x_f} = 0$, respectively. Based on these expressions, the Lagrangian equations of motion are derived as follows:

$$\begin{cases} m_f (\ddot{x}_f(t) + \ddot{x}_t(t)) + m_c (\ddot{x}_c(t) + \ddot{x}_f(t) + \ddot{x}_t(t)) + k_f x_f(t) = 0 \\ m_c (\ddot{x}_c(t) + \ddot{x}_f(t) + \ddot{x}_t(t)) + k_c x_c(t) = \eta_1 V_m(t - \tau) - (\eta_2 + c_c) \dot{x}_c(t) \end{cases}, \quad (1)$$

Under the external excitation $x_t(t)$, the state vector $\bar{x}(t) = [x_c(t), x_f(t)]^T$ is selected, and the dynamic Equation (1) of the building structure can be expressed in the following state-space form:

$$M \ddot{\bar{x}}(t) + C \dot{\bar{x}}(t) + K \bar{x}(t) = B_V V_m(t - \tau) + B_x \ddot{x}_t(t),$$

where

$$M = \begin{bmatrix} m_c & m_c + m_f \\ m_c & m_c \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \\ \eta_2 + c_c & 0 \end{bmatrix},$$

$$K = \begin{bmatrix} 0 & k_f \\ k_c & 0 \end{bmatrix}, \quad B_V = \begin{bmatrix} 0 \\ \eta_1 \end{bmatrix}, \quad B_x = \begin{bmatrix} -m_f - m_c \\ -m_c \end{bmatrix}.$$

Since M is invertible, the state vector is defined as

$$\begin{aligned} x(t) &= [x_c(t) \quad x_f(t) \quad \dot{x}_c(t) \quad \dot{x}_f(t)]^T \\ &= [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T \end{aligned}$$

Then, the state-space representation of the system can be derived as

$$\dot{x}(t) = Ax(t) + BV_m(t - \tau) + Hx_t(t),$$

where

$$A = \begin{bmatrix} \mathbf{0} & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} \mathbf{0} \\ M^{-1}B_V \end{bmatrix}, \quad H = \begin{bmatrix} \mathbf{0} \\ M^{-1}B_x \end{bmatrix},$$

Here $\mathbf{0}$ and I denote the zero matrix and the identity matrix of appropriate dimensions, respectively.

3. Model Transformation

Consider the following structural system equipped with an AMD subject to input time delay:

$$\begin{cases} \dot{x}(t) = Ax(t) + BV_m(t - \tau) + Hx_t(t) \\ z(t) = C_1 x(t) \end{cases}, \quad (2)$$

here, $x(t)$ denotes the state vector of the structural system equipped with an AMD, which contains the displacement and velocity information of the main

structure and the AMD. $z(t)$ represents the controlled output vector of the AMD-equipped structural system. $x_t(t)$ is the external excitation employed to represent the effects of environmental disturbances such as seismic and wind loads.

In this work, all physical states of the AMD structure are assumed to be measurable, including the displacement and velocity of the main structure and the AMD mass. Only the actuator input time delay τ is considered, sensing delay and on-board computation delay are excluded from the model. And the delayed state $x(t-\tau)$ is obtained by storing the historical state information of the system rather than state estimation.

A time-delay state-feedback control strategy is adopted for the system in (2). The controller is designed as $V_m(t-\tau) = Gx(t-\tau)$, where G is the feedback gain matrix. Substituting the controller into the time-delay system (2) yields the following closed-loop system:

$$\begin{cases} \dot{x}(t) = Ax(t) + BGx(t-\tau) + Hx_t(t) \\ z(t) = C_1x(t) \end{cases}.$$

Using the CHsCTA method, the time-delay system in (2) is transformed into an augmented delay-free system. The Chebyshev collocation points are described as follows:

$$t_j = \cos\left(\frac{(j-1)\pi}{N}\right) \quad (j = 1, 2, \dots, N+1).$$

They are non-uniformly distributed points in the interval $[-1, 1]$, corresponding to the extrema of the N -th Chebyshev polynomial. By employing Chebyshev collocation interpolation, the spectral differentiation matrix

$D = (d_{ij}) \in R^{(N+1) \times (N+1)}$ [26] can be constructed, where its elements are defined as follows:

$$d_{ii} = \frac{-t_i}{2(1-t_i^2)}, i = 2, 3, \dots, N, d_{N+1, N+1} = -\frac{2N^2+1}{6},$$

$$d_{ij} = \frac{c_i(-1)^{i+j}}{c_j(t_i - t_j)}, i \neq j, i, j = 1, 2, \dots, N+1, c_i = \begin{cases} 2, & i = 1, N+1 \\ 1, & i = 2, 3, \dots, N \end{cases}.$$

The interval $[0, \tau]$ is partitioned into N non-uniform subintervals, where the collocation points are defined as

$$\tau_j = \frac{1}{2}(1-t_j) \quad (j = 1, 2, \dots, N+1).$$

The delayed state variables are defined as

$$x_j(t) = x(t-\tau_j), j = 1, 2, \dots, N+1.$$

Based on these definitions, the augmented state vector is constructed as

$$X(t) = [x_1(t), x_2(t), \dots, x_{N+1}(t)]^T \in R^{4(N+1)}.$$

The corresponding augmented matrices are defined as follows:

$$\hat{A} = \frac{2}{\tau} \begin{bmatrix} \frac{\tau}{2}A & \mathbf{0} & \cdots & \mathbf{0} \\ d_{21}E & d_{22}E & \cdots & d_{2,N+1}E \\ \vdots & \vdots & \ddots & \vdots \\ d_{N+1,1}E & d_{N+1,2}E & \cdots & d_{N+1,N+1}E \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}, \quad \hat{H} = \begin{bmatrix} H \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix},$$

$$\hat{C}_1 = [C_1 \quad \mathbf{0} \quad \cdots \quad \mathbf{0}], \quad \hat{C}_2 = [\mathbf{0} \quad \mathbf{0} \quad \cdots \quad E].$$

where E denotes the fourth-order identity matrix. Accordingly, the original infinite-dimensional time-delay system (2) is approximated by the following finite-dimensional time-invariant augmented state-feedback system:

$$\begin{cases} \dot{X}(t) = \hat{A}X(t) + \hat{B}V(t) + \hat{H}x_t(t) \\ z(t) = \hat{C}_1X(t) \\ Y(t) = \hat{C}_2X(t) \end{cases}, \quad (3)$$

the control input is designed as $V(t) = GY(t) = GC_2X(t)$.

4. H_∞ Control

For the transformed delay-free system in (3), the H_∞ control problem is investigated. The objective is to design a state-feedback gain matrix G such that the resulting closed-loop system:

$$\begin{cases} \dot{X}(t) = (\hat{A} + \hat{B}G\hat{C}_2)X(t) + \hat{H}x_t(t) \\ z(t) = \hat{C}_1X(t) \end{cases}, \quad (4)$$

satisfies the following conditions:

(R1) In the absence of external excitation, *i.e.*, $x_t(t) = 0$, the closed-loop system (4) is stable.

(R2) For a prescribed H_∞ control performance index γ , under zero initial condition and external disturbance $x_t(t)$. The γ is the H_∞ control performance index for the CHsCTA-approximated finite-dimensional augmented system.

For the H_∞ control design of the closed-loop system in (4), the following theorem is obtained.

Theorem 1: The closed-loop system (4) admits an H_∞ state-feedback controller G satisfying conditions (R1) and (R2) if there exists a symmetric positive definite matrix $X = X^T$ such that

$$\begin{bmatrix} A_G^T X + X A_G & X \hat{H} & C_1^T \\ \hat{H}^T X & -\gamma E & \mathbf{0} \\ C_1 & \mathbf{0} & \gamma E \end{bmatrix} < 0, \quad (5)$$

here, $\hat{A}_G = \hat{A} + \hat{B}G\hat{C}_2$.

Proof: Consider the Lyapunov function $W(t) = X^T(t)PX(t)$, where $P = P^T$. The time derivative of $W(t)$ is given by

$$\begin{aligned}
\dot{W}(t) &= \dot{X}^T(t)PX(t) + X^T(t)P\dot{X}(t) \\
&= \left[A_G^T X(t) + \hat{H}x_t(t) \right]^T PX(t) + X^T(t)P \left[A_G^T X(t) + \hat{H}x_t(t) \right] \\
&= X^T(t) \left[A_G^T P + PA_G \right] X(t) + x_t^T(t) \hat{H}^T PX(t) + X^T(t) P \hat{H} x_t(t)
\end{aligned}$$

Consider the following expression:

$$\begin{aligned}
& z^T(t)z(t) - \gamma^2 x_t^T(t)x_t(t) + \dot{W}(t) \\
&= X^T(t)C_1^T C_1 X(t) - \gamma^2 x_t^T(t)x_t(t) + X^T(t) \left[A_G^T P + PA_G \right] X(t) \\
&\quad + x_t^T(t) \hat{H}^T P X(t) + X^T(t) P \hat{H}^T x_t(t) \\
&= X^T(t) \left[A_G^T P + PA_G + C_1^T C_1 \right] X(t) + x_t^T(t) \hat{H}^T P X(t) \\
&\quad + X^T(t) P \hat{H} x_t(t) - \gamma^2 x_t^T(t)x_t(t) \\
&= \begin{bmatrix} X(t) \\ x_t(t) \end{bmatrix}^T \begin{bmatrix} A_G^T P + PA_G + C_1^T C_1 & P \hat{H} \\ \hat{H}^T P & -\gamma^2 E \end{bmatrix} \begin{bmatrix} X(t) \\ x_t(t) \end{bmatrix} \\
&= \begin{bmatrix} X(t) \\ x_t(t) \end{bmatrix}^T \left\{ \begin{bmatrix} A_G^T P + PA_G & P \hat{H} \\ \hat{H}^T P & -\gamma^2 E \end{bmatrix} + \begin{bmatrix} C_1^T C_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right\} \begin{bmatrix} X(t) \\ x_t(t) \end{bmatrix}
\end{aligned}$$

The following inequality:

$$\begin{bmatrix} A_G^T P + PA_G & P \hat{H} \\ \hat{H}^T P & -\gamma^2 E \end{bmatrix} + \begin{bmatrix} C_1^T C_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} < 0,$$

is equivalent, by applying the Schur complement, to

$$\begin{bmatrix} A_G^T P + PA_G & P \hat{H} & C_1^T \\ \hat{H}^T P & -\gamma^2 E & \mathbf{0} \\ C_1 & \mathbf{0} & E \end{bmatrix} < 0,$$

Multiplying both sides of the above matrix inequality by matrix $\text{diag}\left(\gamma^{-\frac{1}{2}}, \gamma^{-\frac{1}{2}}, \gamma^{\frac{1}{2}}\right)$, and letting $X = \gamma^{-1}P$, matrix inequality (5) can be rewritten as:

$$z^T(t)z(t) - \gamma^2 x_t^T(t)x_t(t) + \dot{W}(t) < 0. \quad (6)$$

Considering that system (4) is stable with zero initial conditions, integrating the left-hand side of inequality (6) over $[0, +\infty)$ yields:

$$\int_0^{+\infty} z^T(t)z(t)dt = \int_0^{+\infty} x_t^T(t)x_t(t)dt.$$

Since matrix inequality (5) is nonlinear, it is difficult to solve directly. To transform it into a solvable form, the following lemma is introduced.

Lemma 1 [27]: Let P , Q , and M be matrices of appropriate dimensions, where M is symmetric. Let N_P and N_Q denote matrices whose columns form basis vectors of the null spaces of P and Q , respectively. Then, there exists a matrix X such that

$$M + P^T X^T Q + Q^T X P < 0,$$

if and only if

$$N_p^T M N_p < 0, N_Q^T M N_Q < 0.$$

Theorem 2: Matrix inequality (6) admits a symmetric solution $X = X^T$ and G if and only if the following inequalities are satisfied.

$$N_{\hat{C}_2}^T M_X N_{\hat{C}_2} < 0, N_{\hat{B}^T}^T M_Y N_{\hat{B}^T} < 0, \quad (7)$$

$$XY = E_m, \quad (8)$$

where $N_{\hat{C}_2} = \text{Ker}[\hat{C}_2, 0, 0]$, $N_{\hat{B}^T} = \text{Ker}[\hat{B}^T, 0, 0]$,

$$M_X = \begin{bmatrix} \hat{A}^T X + X \hat{A} & X \hat{H} & \hat{C}_1^T \\ \hat{H}^T X & -\gamma E & \mathbf{0} \\ \hat{C}_1 & \mathbf{0} & -\gamma E \end{bmatrix},$$

$$M_Y = \begin{bmatrix} Y \hat{A}^T + \hat{A} Y & \hat{H} & Y \hat{C}_1^T \\ \hat{H}^T & -\gamma E & \mathbf{0} \\ \hat{C}_1 & \mathbf{0} & -\gamma E \end{bmatrix}.$$

Proof: Using matrix definitions and appropriate matrix transformations, matrix inequality (5) can be equivalently rewritten as

$$\begin{bmatrix} \hat{A}^T X + X \hat{A} & X \hat{H} & \hat{C}_1^T \\ \hat{H}^T X & -\gamma E & \mathbf{0} \\ \hat{C}_1 & \mathbf{0} & \gamma E \end{bmatrix} + \begin{bmatrix} C_2^T G^T \hat{B}^T X & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} X \hat{B} G C_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0,$$

is equivalent to

$$M_X + \begin{bmatrix} C_2^T \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} G^T \begin{bmatrix} \hat{B}^T X & \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} X \hat{B} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} G \begin{bmatrix} C_2 & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0, \quad (9)$$

By Lemma 1, the above inequality holds if and only if the following two inequalities are satisfied.

$$N_{\hat{C}_2}^T M_X N_{\hat{C}_2} < 0, N_{\hat{B}^T X}^T M_X N_{\hat{B}^T X} < 0, \quad (10)$$

where $N_{\hat{B}^T X} = \text{Ker}[\hat{B}^T X, 0, 0]$. The following matrices are defined:

$$S_X = \begin{bmatrix} X & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & E & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & E \end{bmatrix},$$

$$N_{\hat{B}^T} = \text{Ker}[\hat{B}^T, 0, 0],$$

it holds that $N_{\hat{B}^T X} = S_X^{-1} N_{\hat{B}^T}$. By substituting the above expression into inequality (10), it follows that:

$$N_{\hat{C}_2}^T M_X N_{\hat{C}_2} < 0, N_{\hat{B}^T}^T S_X^{-1} M_X S_X^{-1} N_{\hat{B}^T} < 0. \quad (11)$$

Let $Y = X^{-1}$ and $M_Y = S_X^{-1} M_X S_X^{-1}$. Then inequality (11) can be equivalently

transformed into inequality (7). This completes the proof of Theorem 2.

After designing the state-feedback H_∞ controller, the feedback gain matrix G and the minimum H_∞ control performance index γ need to be solved based on the aforementioned preparations. Since matrix inequalities (7) and (8) exhibit a bilinear matrix inequality (BMI) structure, the corresponding optimization problem is nonconvex [28], [29]. In this work, the CCL algorithm is adopted to solve this nonconvex optimization problem [30]. The specific steps of the iterative algorithm are shown in Algorithm 1.

Algorithm 1: Determination of the Feedback Gain Matrix G with Minimization of the H_∞ Performance Index γ .

Step 1: Based on the CHsCTA method, the coefficient matrices A , B , H , and C_1 in the time-delay system (2) are transformed into the augmented matrices $\hat{A}$, $\hat{B}$, $\hat{H}$ and $\hat{C}_1$.

Step 2: Choose a sufficiently large initial H_∞ control performance index $\gamma > 0$ to ensure that the matrix inequality (7) is feasible.

Step 3: Solve matrix inequality (7) to obtain the feasible positive definite matrices X and Y , and set the initial iteration numbers $k = 0$, $X_0 = X$, and $Y_0 = Y$. If (7) has no feasible solution, the program terminates and exits.

Step 4: Solve the optimization problem with respect to X and Y :

OPT1: min Trace $Y_0 X + X_0 Y$ subject to (9) and

$$\begin{bmatrix} X & E \\ E & Y \end{bmatrix} \geq 0$$

Step 5: For the obtained matrix X in Step 4, if (5) has a feasible solution G , set $\gamma_{\min} = \gamma$, $\gamma = \gamma_{\min} - \Delta\gamma$ ($\Delta\gamma > 0$ is the step size) and return to Step 3. Otherwise, if k exceeds the preset maximum number of iterations, the program terminates and exits; else set $k = k + 1$, $X_0 = X$, and $Y_0 = Y$, and return to Step 4.

5. Numerical Simulation

Using the model parameters listed in **Table 1**, the state matrix A , control matrix B , and external excitation matrix H in system (2) can be determined. The output matrix C_1 is selected based on the disturbance rejection performance indices of the AMD-equipped structure. The corresponding matrices are given as follows:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -7 & 20 & -0.77 & 0 \\ 2 & -20 & 0.22 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1.4 \\ -0.4 \end{bmatrix},$$

$$H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}^T.$$

In practical engineering, AMD-equipped structures are inevitably subjected to external excitations arising from environmental loads and various uncertain factors. To comprehensively evaluate the dynamic performance of structural control systems under diverse excitation conditions, two typical excitation types are adopted in this paper: harmonic excitation and random disturbance excitation.

Table 1. Building model parameters.

Symbol	Value
m_c	2
m_f	5
c_c	0.1
k_c	10
k_f	100
η_1	2
η_2	1

The harmonic excitation is specified as $\omega(t) = 4\cos(2t)$, and the Gaussian white-noise excitation is specified as $\omega(t) \sim N(0,1)$. Both simulations are conducted over a duration of 80 s, with the response-evaluation interval set to $t \in [20, 80]$ s.

For the time-delay system (2), the initial state of the AMD-equipped structural system is set as $[x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T = [0.1 \ 0 \ 0.1 \ 0]^T$, with $\tau = 0.2$ s. The delay interval $[0, \tau]$ is partitioned into $N = 6$ subintervals. Based on Algorithm 1, the H_∞ performance index $\gamma = 0.7$ and the corresponding feedback gain matrix $G = [4.9183 \ -9.5075 \ 0.9397 \ 9.4284]$ are obtained.

Harmonic excitation is employed to simulate the periodic dynamic actions imposed on structures, such as mechanical equipment vibrations and the periodic components of wind loads. As shown in **Figure 2**, after introducing the CHsCTA-based H_∞ controller, the steady-state vibration amplitude of the roof is reduced by 63.27% compared with the uncontrolled case, thereby effectively suppressing the vibration response of the AMD-equipped structure under harmonic excitation.

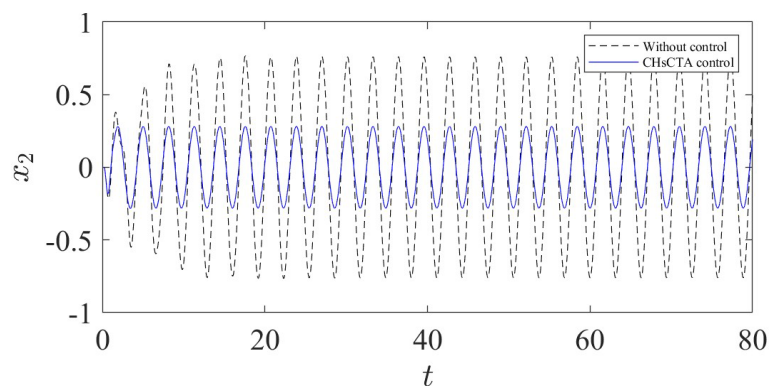


Figure 2. Time history of x_2 under external excitation $\omega(t) = 4\cos(2t)$ with a given time delay $\tau = 0.2$ s, initial state $[0.1 \ 0 \ 0.1 \ 0]^T$, and feedback gain matrix $G = [4.9183 \ -9.5075 \ 0.9397 \ 9.4284]$.

Gaussian white noise, serving as a random disturbance, is employed to equivalently simulate the broadband random characteristics inherent in fluctuating wind

pressures and ground motions. Such irregular excitations exhibit strong randomness and uncertainty, which directly affect the stability and control performance of the structural system. As shown in **Figure 3**, the AMD-equipped structural time-delay system fails to achieve a stable state under persistent external excitation. Under random disturbance excitation, the CHsCTA-based controller reduces the average absolute value of x_2 by 54.28%, indicating that the controller can effectively suppress broadband random vibration and significantly improve the structural dynamic performance.

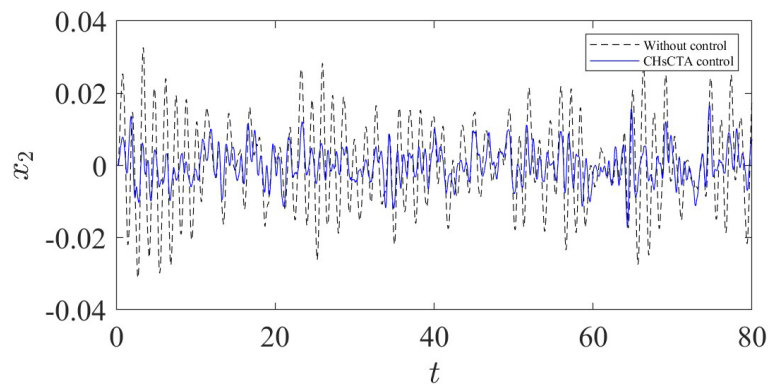


Figure 3. Time history of x_2 under external excitation $\omega(t) \sim \mathcal{N}(0,1)$ with a given time delay $\tau = 0.2$ s, initial state $[0.1 \ 0 \ 0.1 \ 0]^T$, and feedback gain matrix $G = [4.9183 \ -9.5075 \ 0.9397 \ 9.4284]$.

6. Conclusion

This paper addresses the inevitable time-delay problem in active control. Taking the structural system equipped with an AMD as the research object, the CHsCTA method is employed to transform the closed-loop system with input time delay into a delay-free augmented system, upon which a state-feedback H_∞ controller is designed. By introducing the CCL method, the BMI constraint problem arising from the controller design process is resolved, thereby obtaining the feedback gain matrix that satisfies the prescribed H_∞ control performance index. Numerical simulations are carried out to verify the effectiveness of the proposed control strategy. Under a fixed time delay of 0.2 s, the simulation results show that the controller achieves a vibration reduction rate of 63.27% under harmonic excitation and a 54.28% reduction in the mean absolute value under Gaussian white noise excitation, effectively reducing the vibration amplitude and thereby improving the vibration suppression capability of the system. The control strategy proposed in this paper for the structural vibration with fixed time delay can be extended to time-varying delays and more complex structural systems in future work.

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Author Contributions

Conceptualization, Y.C. and Y.Z.; methodology, Y.Z.; validation, Y.C.; investigation, Y.C. and Y.Z.; data curation, Y.Z.; writing—original draft preparation, Y.C.; writing—review and editing, Y.C. and Y.Z.; visualization, Y.C.; supervision, Y.Z.; project administration, Y.Z.; funding acquisition, Y.Z. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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