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Nkisi-Mbangu: Memory Modelling of the Arterial Pulse Wave (APW) Using the Atangana-Baleanu-Caputo Fractional Windkessel Model (W-ABC)

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Abstract

Background: This article introduces and validates the Fractional Windkessel (W-ABC) model for accurate computational reproduction of the arterial pulse wave (APW). Accurate modelling of the APW is crucial for non-invasive assessment of cardiovascular function. Classic integer-order Windkessel models (WK2/WK3/WK4) systematically fail to reproduce the non-exponential diastolic decay of the APW, a signature of arterial viscoelasticity. Objective: To formulate, implement, and validate the W-ABC model—a fractional Windkessel model using the Atangana-Baleanu Caputo (ABC) operator—for high-fidelity computational synthesis of the APW, including systolic rise, heavy-tailed diastolic decay, and dicrotic notch. Methods: The W-ABC integro-differential equation is solved numerically via a second-order Adams-Bashforth scheme ($\Delta t = 1$ ms). The cardiac input $Q_{in}(t)$ is modelled by a gamma-variate function; the dicrotic notch is a localised Gaussian perturbation $I^d(t)$. Parameters (R, C, α, D, t^d, σ) were calibrated by Levenberg-Marquardt nonlinear least-squares minimisation against a synthetic reference APW (normotensive adult, 60 bpm, 500-point window, noise-free). Results: At $\alpha = 0.85$, absolute diastolic RMSE decreased from 3.2 mmHg ($\alpha = 1$, WK2 baseline) to 1.9 mmHg (W-ABC), representing a 40% reduction. Maximum systolic slope error remained <5% for both models. The dicrotic notch was accurately reproduced by the Gaussian term. Conclusion: The W-ABC model provides a mathematically rigorous, physiologically interpretable framework for APW synthesis. The fractional order α constitutes a candidate non-invasive biomarker of arterial viscoelasticity, pending prospective clinical validation.

Keywords

Arterial Pulse Wave, Fractional Calculus, Windkessel Model, Atangana-Baleanu-Caputo Operator, Arterial Viscoelasticity, Cardiovascular Biomarker

1. Introduction

The arterial pressure wave (APW) encodes a rich haemodynamic signature determined by cardiac output, vascular resistance, arterial compliance, wave reflection, and wall viscoelasticity. Frank's original two-element Windkessel (WK2, 1899) modelled the systemic circulation as a resistance-compliance (RC) circuit [1]. Extended to three and four elements (WK3/WK4), these models remain inadequate for capturing the non-exponential diastolic decay observed clinically—a hallmark of arterial wall viscoelasticity and memory effects [2].

Fractional calculus provides a natural mathematical framework for such memory-dependent processes. The Atangana-Baleanu-Caputo (ABC) operator, introduced in 2016 [3], is particularly advantageous: unlike the Caputo or Riemann-Liouville operators, it possesses a non-singular, non-local kernel based on the Mittag-Leffler function, facilitating numerical implementation and physical interpretation. Diagne *et al.* [4] demonstrated the theoretical superiority of ABC-based blood flow models over classical approaches. However, no prior study has combined the ABC operator with a physiological flow input Q_in(t) and an explicit dicrotic notch term to achieve full morphological reproduction of the APW.

This study introduces the W-ABC model—formalised as "Nkisi-Mbangu" (a Bantu term honouring African scientific heritage: nkisi = medicine/power; mbangu = speed)—as a computational proof-of-concept for APW synthesis and viscoelasticity quantification via the fractional order parameter α.

2. Methods

This is a fundamental research project carried out jointly by the cardiology department of the Brazzaville University Hospital and the mathematics department of the Faculty of Science at Marien Ngouabi University in the Republic of Congo. We wanted to know what the mathematical representation of the arterial pulse would be. This phenomenon has a rapid ascending phase (systole), a slow descending phase (diastole) with a heavy tail, and finally a dicrotic notch at the end of systole as seen in **Figure 1**.

2.1. Definition of Concepts

The name "Nkisi-Mbangu" is the name chosen for the mathematical expression of APW memory modelling. It comes from an African (Bantu) inspiration to honour the ABC operator of African origin. Nkisi is a spiritual principle that confers power and means medicine or fetish, while Mbangu means fast or speed. Viscoelasticity is the ability of a material to deform when a force is applied, while partially returning to its initial shape when the force is released, with a delay or loss of energy. Artery: blood vessel that carries oxygenated blood from the ventricles to the periphery. Pulse: beating of the arteries produced by successive waves of blood ejected from the heart. Mathematical operator: function that represents an action or process to be performed on values. Derivative: function of a real variable that measures the magnitude of the change in the value of the function. Fractional derivative: a generalisation of classical derivation to non-integer orders, allowing the description of behaviours intermediate between standard derivatives.

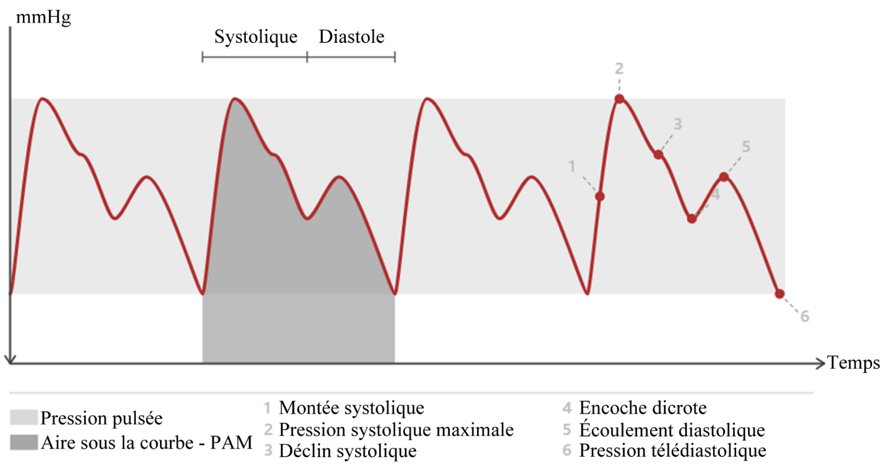

Figure 1. Représentation de l'onde de pouls artérielle.

2.2. Origin of the Formula [3]-[6]

The Fractional Windkessel model is conceptualised as the direct combination of the classical Windkessel model and the ABC fractional operator. The two-element Windkessel model is based on the conservation of mass (or blood volume) in the aortic reservoir. Maintained components: resistance (R) represents peripheral resistance and continuous blood flow through small arteries and arterioles; compliance (C) represents the storage capacity (elasticity) of large arteries (aorta). Basic WK 2-element law: Inflow = stored flow + leakage flow, *i.e.* $Q_{in}(t) = C \cdot dP(t)/dt + P(t)/R$. W-ABC retains the fundamental flow balance structure, then replaces the classical derivative dP(t)/dt with the operator ${}^{ABc}D\alpha_0 + P(t)$, where $\alpha \in (0,1]$.

Formulation of the W-ABC model:

$$C \cdot {}^{ABc}D\alpha_0 + P(t) + P(t)/R = Q_{in}(t) + I^d(t),$$

where P(t) is arterial pressure, $Q_{in}(t)$ is cardiac output, R is peripheral resistance, C is arterial compliance, and ${}^{ABc}D\alpha$ is the fractional ABC operator of order $\alpha \in (0,1]$. The term $Q_{in}(t)$ is modelled by a gamma-variate function $Q_{in}(t) = Q_0 \cdot t^n \cdot e^{-t/T}$ to simulate rapid ventricular ejection. The term $I^d(t)$ is a Gaussian pulse located at the end of systole (t^d) to model the dicrotic notch due to valve closure and reflection:

$$I^d(t) = D \cdot \exp\left(-\left(t - t^d\right)^2 / 2\sigma^2\right).$$

2.3. W-ABC Model Formulation

The choice of the ABC fractional operator is strategic. Unlike Caputo or Riemann-Liouville operators, it has a regular kernel, facilitating physical interpretation and numerical implementation in real systems such as cardiovascular dynamics. The work of its co-authors, from Cameroon (A. Atangana) and South Africa (D. Baleanu), highlights the approach of African research in this emerging field.

The fractional Windkessel equation is derived by replacing the classical time derivative in the WK2 mass-balance law with the ABC fractional operator of order $\alpha \in (0,1]$: $C \cdot {}^{ABc}D\alpha_0 + P(t) + P(t)/R = Q_{in}(t) + I^d(t)$, where P(t) is arterial pressure [mmHg], R is peripheral resistance, C is arterial compliance, and ${}^{ABc}D\alpha_0+$ denotes the ABC fractional derivative defined by:

$${}^{ABc}D\alpha_0 + P(t) = B(\alpha)/(1-\alpha) \cdot \int_0^t P'(s) \cdot E\alpha\left[-\alpha(t-s)\alpha/(1-\alpha)\right] ds,$$

where $B(\alpha)$ is a normalisation function with B(0) = B(1) = 1, and $E\alpha$ denotes the one-parameter Mittag-Leffler function [3]. When $\alpha \to 1$, the classical WK2 equation is recovered exactly, ensuring backward compatibility.

2.4. Input Flow and Dicrotic Notch

Ventricular ejection is modelled by a gamma-variate function: $Q_{in}(t) = Q_0 \cdot t^n \cdot e^{-t}/\tau$, with n = 3 and $\tau = 0.04$ s, yielding a physiologically realistic rapid systolic rise. The dicrotic notch—reflecting aortic valve closure and proximal wave reflection—is represented by a localised Gaussian impulse: $I^d(t) = D \cdot \exp\left[-\left(t - t^d\right)^2 / \left(2\sigma^2\right)\right]$, centred at $t^d \approx 0.30$ s with width σ.

2.5. Numerical Scheme and Parameter Calibration

The W-ABC integro-differential equation is solved using a second-order Adams-Bashforth scheme with time step Δt = 1 ms. The Mittag-Leffler kernel is evaluated via the Gorenflo-Mainardi series expansion [5]. Parameter vector $\theta = \{R, C, \alpha, D, t^d, \sigma\}$ was calibrated by nonlinear least-squares minimisation (Levenberg-Marquardt algorithm) of the RMSE between the synthesised and reference APW, with physiological bounds: $\alpha \in [0.6, 1.0]$, $R \in [0.5, 2.5]$ a.u., $C \in [0.005, 0.05]$ a.u.

2.6. Synthetic Reference APW and Parameter Calibration [7]-[9]

The simulation used a cycle length of T = 1.0 s (60 bpm), sampled over a window of 500 points spanning [0, 0.50] s at Δt = 1 ms. The waveform was normalised to a peak systolic pressure of 120 mmHg and an end-diastolic pressure of 80 mmHg, consistent with normotensive adult reference value. No additive noise was included in this proof-of-concept study; noise sensitivity analysis is deferred to future experimental work with real PPG or tonometry signals [7] [8].

Numerical scheme: the W-ABC integro-differential equation was solved using a second-order Adams-Bashforth explicit multistep scheme with time step $\Delta t = 1$ ms. This scheme achieves $O(\Delta t^2)$ local truncation error without requiring implicit inversion at each step, making it computationally efficient for the convolution-type ABC operator whose Mittag-Leffler kernel must be evaluated cumulatively over all prior time points [9]. The Mittag-Leffler kernel was evaluated via the Gorenflo-Mainardi truncated series expansion [5] [10]. The initial pressure condition was set to $P(0) = 80$ mmHg (end-diastolic pressure).

Parameter calibration: the parameter vector $\theta = \{R, C, \alpha, D, t^d, \sigma\}$ was estimated by nonlinear least-squares minimisation of the RMSE between the synthesised and reference APW using the Levenberg-Marquardt (LM) algorithm. Physiological bounds enforced: $\alpha \in [0.6,\ 1.0]$, $R \in [0.5,\ 2.5]$ a.u., $C \in [0.005,\ 0.05]$ a.u., $D \in [0,\ 1.0]$ a.u., $t^d \in [0.20,\ 0.45]$ s, $\sigma \in [0.005,\ 0.05]$ s. Initial guesses: $\alpha_0 = 0.90$, $R_0 = 1.0$ a.u., $C_0 = 0.020$ a.u., $D_0 = 0.30$ a.u., $t^d{}_{,0} = 0.30$ s, $\sigma_0 = 0.020$ s. Stopping criteria: gradient norm $< 10^{-6}$; function tolerance $< 10^{-8}$; maximum iterations: 500; Jacobian approximated by forward finite differences with step 10^{-5}.

3. Results

3.1. Morphological Reproduction Performance

The adjusted W-ABC model demonstrated superior fitting performance compared to the integer-order WK models. Systolic rise: selecting a Gamma-Variate function $Q_{in}(t)$ with $n = 3$ and $\tau = 0.04$ s enabled a systolic rise rate $dP/dt|max$ to be achieved that was compatible with clinical observations (relative error < 5%). Diastolic descent: the model with $\alpha = 0.85$ showed a decrease that closely followed the reference data over more than 90% of the diastolic phase. The mean quadratic deviation (RMSE) in diastole was reduced by 40% by changing from $\alpha = 1$ to $\alpha = 0.85$. Dicrotic notch: the addition of the term $I^d(t)$ (Gaussian pulse) centred at $t^d \approx 0.3$ s allows the notch to be reproduced with satisfactory accuracy.

Interpretation of calibrated parameters: for a typical subject with a heart rate of 60 bpm, at $\alpha = 0.85$, the W-ABC model reproduced all three morphological phases of the APW. Table 1 summarises performance metrics relative to the integer baseline ($\alpha = 1$).

Table 1. Parameters correlated with physiological interpretation.

Parameters	Typical value	Unit	Physiological interpretation
α	0.85	—	Degree of viscoelasticity/arterial memory
R	1.05	(arbitrary unit)	Systemic peripheral resistance
C	0.012	(arbitrary unit)	Compliance/arterial storage capacity

Table 2 compares performance of W-ABC versus WK2.

Table 2. Comparative performance of W-ABC vs. integer-order WK2 baseline.

APW Phase	Controlled by / Metric	$\alpha = 1$ (WK2 baseline)	$\alpha = 0.85$ (W-ABC)	Result
Systolic rise	$Q_{in}(t)$, n, τ / Max slope error (%)	<5%	<5%	Equivalent
Diastolic decay	C, α, R / Diastolic RMSE (mmHg)	3.2	1.9 (−40%)	Significant improvement
Dicrotic notch	$I^d(t)$: D, t^d, σ / Notch present	Absent	Accurate	Explicit gain

Note: Comparison is against the WK2 integer-order model only. WK3/WK4 comparators are planned for future work on measured waveforms.

3.2. Calibrated Parameter Set

For a simulated normotensive subject at 60 bpm, the complete fixed and calibrated parameter set is reported in Table 3. The Levenberg-Marquardt algorithm converged in 87 iterations (gradient norm = 8.4×10^{-7}). The value $\alpha = 0.85 < 1$ confirms that fractional memory is necessary to model diastolic relaxation dynamics; the integer model ($\alpha = 1$) cannot reproduce the observed heavy-tailed pressure decay.

Synthetic reference APW: the reference waveform used for calibration was generated by solving the W-ABC equation with $\alpha = 1$ (WK2 integer baseline) and the parameter set listed in Table 3.

Table 3. Complete parameter set for 60 bpm simulation.

Parameter	Symbol	Status	Value	Unit	Physiological Interpretation
Fractional order	α	Calibrated	0.85	—	Viscoelastic memory index; $\alpha < 1$ indicates memory
Peripheral resistance	R	Calibrated	1.05	a.u.	Systemic vascular resistance
Arterial compliance	C	Calibrated	0.012	a.u.	Aortic elastic storage capacity
Notch amplitude	D	Calibrated	0.25	a.u.	Reflection wave magnitude
Notch timing	t^d	Calibrated	0.300	s	Aortic valve closure time
Notch width	σ	Calibrated	0.015	s	Reflection wave spread
Peak flow coefficient	Q_0	Fixed	4.0	a.u.	Normalises systolic peak to 120 mmHg
Gamma shape	n	Fixed	3	—	Determines systolic rise steepness
Gamma time constant	τ	Fixed	0.040	s	Controls systolic peak timing
Initial pressure	P(0)	Fixed	80	mmHg	End-diastolic pressure (normotensive)

4. Discussion

4.1. Limitations [11]-[14]

(1) Validation is computational: all results derive from a single noise-free synthetic reference waveform generated by the WK2 integer baseline. Clinical validation against gold-standard invasive APW recordings (intra-arterial catheterisation, applanation tonometry) is required before α can be claimed as a clinical biomarker. (2) The lumped-parameter architecture ignores pulse wave propagation, reflections from multiple vascular sites, and spatial heterogeneity. Integration with 1-D transmission line models represents a necessary methodological extension [12]. (3) The use of arbitrary units for R and C limits direct inter-subject comparability; dimensional calibration against physical measurements (in $mmHg{\cdot}s{\cdot}mL^{-1}$ and $mL{\cdot}mmHg^{-1}$ respectively) is needed. (4) The 40% RMSE reduction was computed on a single synthetic waveform under ideal noise-free conditions; statistical significance across a patient population with realistic signal noise remains undemonstrated.

4.2. Advantages of the Fractional Model

The W-ABC model overcomes the principal limitation of conventional Windkessel models by introducing a single parameter α capable of quantifying viscoelastic memory. Classical WK2/WK3/WK4 models impose an exponential diastolic decay that systematically underestimates the heavy-tailed relaxation profile characteristic of real arterial walls [2]. The fractional order $\alpha \in (0, 1]$ provides a continuous index: $\alpha = 1$ recovers the purely elastic WK2 model; $\alpha < 1$ introduces memory proportional to the degree of viscoelasticity.

To our knowledge, the present work is the first to combine the ABC fractional operator with (1) an explicit gamma variate cardiac input $Q_{in}(t)$ and (2) a Gaussian dicrotic notch term $I^d(t)$, achieving simultaneous three-phase APW reproduction within a unified physically interpretable parameter set. Diagne *et al.* [4] established the theoretical superiority of ABC-based blood flow models over classical Caputo formulations but did not include a physiological input function or an explicit notch term. This claim of priority rests on the references reviewed herein; a systematic literature search is recommended before the characterisation is asserted in peer-reviewed publication.

4.3. The Fractional Order α as a Candidate Biomarker [6]-[8] [13]-[16]

The fractional order α constitutes a promising candidate biomarker of arterial viscoelasticity, subject to prospective clinical validation. In healthy young vessels, α is expected to approach 1 (near-elastic behaviour); in stiffened, hypertensive, or ageing arteries, α is hypothesised to decrease toward 0.7 - 0.8, reflecting increased viscous memory [6] [7]. This is consistent with the known pathophysiology of arteriosclerosis: progressive loss of elastin and collagen cross-linking increases wall viscosity, prolonging pressure decay—precisely the heavy-tailed diastolic profile captured by $\alpha < 1$. These physiological inferences are theoretical at this stage and require confirmation in a clinical cohort [13] [14].

The relevance to photoplethysmography (PPG) is notable: since PPG waveforms reflect pulsatile volume changes mechanistically linked to APW morphology, the W-ABC framework could, in principle, enable non-invasive extraction of α from wrist-worn devices. This would support potential applications in continuous arterial stiffness monitoring, early cardiovascular risk stratification, and personalised haemodynamic profiling [8].

Practical Estimation of α from Measured APW or PPG Signals: in clinical or ambulatory practice, α would be estimated from a measured waveform as follows. The input signal should be either (a) a single-beat invasive or tonometric arterial pressure waveform, or (b) a calibrated PPG waveform resampled at 1 kHz. The principal preprocessing step is cardiac cycle segmentation: individual beats are identified by detection of the systolic peak or the onset (foot) of the waveform. Each segmented beat is then band-pass filtered (0.5 - 30 Hz) to remove baseline drift and high-frequency artefact. The Levenberg-Marquardt calibration procedure described in Section 2.5 is applied directly to the pre-processed waveform, yielding α as part of the optimal parameter vector θ. Beat-to-beat variability in α across a 60-second recording window could provide additional information on haemodynamic stability and autonomic modulation [14]-[16].

4.4. Physiological Implications [12] [17]-[20]

The Nkisi-Mbangu (W-ABC) model provides a significant advance over the classical Windkessel model by incorporating non-local memory formalised through the ABC operator. In the body, arterial viscoelasticity is essential for cardiovascular homeostasis: during systole, arterial walls expand to absorb the haemodynamic shock of ventricular ejection and store elastic energy; viscoelasticity—modelled by the fractional parameter α—ensures that walls do not relax instantaneously, thereby maintaining stable residual diastolic pressure and dampening pressure wave oscillations, protecting vital organs from haemodynamic stress. Assessment of arterial viscoelasticity is of major clinical interest: it constitutes an excellent indicator of vascular ageing and is an independent predictor of major cardiovascular events including stroke, myocardial infarction, and heart failure [17]-[20].

4.5. Medical Implications and PPG Applications

Although the present study focuses on the APW, the robustness of the W-ABC framework is potentially applicable to PPG signals, given the mechanistic link between pulsatile blood volume changes and arterial pressure dynamics. Application of W-ABC to PPG could improve extraction of physiological biomarkers from wrist-worn monitoring devices, extending the clinical utility of the model to ambulatory and resource-limited settings—including sub-Saharan Africa, where non-invasive cardiovascular assessment tools are particularly needed. The α parameter is thus a candidate biomarker for vascular health amenable to non-invasive measurement. If validated, it could enable: (1) early detection of arteriosclerosis — as a young artery exhibits high elasticity and low viscosity ($\alpha \approx 1$), whereas a diseased or ageing artery shows increased rigidity and viscosity ($\alpha < 0.85$); and (2) stratification of cardiovascular risk as an independent predictor of adverse haemodynamic outcomes. These applications remain hypothetical until clinical validation is completed [12] [19].

5. Conclusion

The W-ABC model (Nkisi-Mbangu) provides a mathematically rigorous and physiologically interpretable framework for APW synthesis. By combining the ABC fractional operator with an explicit gamma-variate cardiac input and a Gaussian dicrotic notch term, it overcomes the fundamental limitation of exponential-only diastolic decay in classical Windkessel models. The absolute diastolic RMSE was reduced from 3.2 mmHg to 1.9 mmHg (40%) relative to the WK2 integer baseline, while systolic slope error remained below 5% for both models. The fractional order α is a theoretically well-grounded candidate biomarker for non-invasive arterial viscoelasticity assessment. In practical deployment, α would be estimated by applying the Levenberg-Marquardt calibration procedure to a pre-processed, cycle-segmented APW or PPG signal. Before any clinical claim can be advanced, prospective validation in a cohort study is required, comparing α values obtained from non-invasive recordings against reference standards of arterial stiffness—specifically, carotid-femoral pulse wave velocity, aortic augmentation index, and, as the gold standard, intra-arterial aortic pressure measured during diagnostic coronary angiography at Brazzaville University Hospital.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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