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Effects on Second Waves of COVID-19 Epidemics: Social Stringency, Economic Forces and Public Health

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Abstract

Since the outbreak of the COVID-19 outbreak in Wuhan, China in early 2020, countries around the world have spread the fire, from Asia to Europe, and then to the Americas. It has not yet subsided. During this period, the city has been closed, the country has been locked, and the city has been unblocked again. And all transnational business exchanges, tourism and sightseeing activities have almost stagnated, which has greatly affected and threatened the economic development and people's health of various countries. The investigation period is from 2020/01/01 to 2021/10/07. Downloaded 121,947 daily data reported by each country from Our World in Data [OWID], EU CDC, and John Hopkins University, this study uses two-modal bell-shaped functions to classify the COVID-19 epidemic situation for each country and determine whether the second/third wave of epidemic has occurred. Moreover, through the measurement method to explore those factors that caused the second/third wave of epidemic to be postponed, or even not to occur. Empirical results show that countries with denser populations, poorer economic development, and lower stringency of government, longer life expectancy, and higher cardiovascular disease mortality rates, the second/third wave of epidemics came earlier. On the other hands, countries with the higher the GDP per capita, the higher the human development, the later of the second wave of epidemics.

Keywords

COVID-19 Outbreak, Stringency Index, Human Development Index, Two-Modal Bell-Shaped Function, Poisson Regression

1. Introduction

1.1. Coronaviruses

Coronaviruses are a large group of viruses that cause illness ranging from common cold to more serious respiratory conditions like Severe Acute Respiratory Syndrome (SARS-CoV) in 2002, Middle East Respiratory Syndrome (MERS-CoV) in 2012 and the latest COVID-19. They were first discovered in 1960s but were not considered to be highly pathogenic to humans until the SARS outbreak in 2002. However, immunocompetent people occasionally suffered from mild infections due to the earlier forms of these viruses.

Coronaviruses are zoonotic, which means they are transmitted from animals to humans. Interestingly, these viruses have probably originated from bats and then moved into other mammalian hosts—the Himalayan palm civet for SARS-CoV, the dromedary camel for MERS-CoV and the suspected pangolins for the COVID-19 before jumping to humans.

Most human coronaviruses are the result of infection by direct contact with secretions or droplets, such as from coughing or sneezing. Some infected animals also suffer from having diarrhea, and the virus present in the feces can cause further disease transmission (Taiwan Central Epidemic Command Center [CECC], 2020).

1.2. Clinical Expression and Severity

At present, the clinical manifestations of confirmed cases of COVID-19 include fever, dry cough, and fatigue, and about one-third of them will have shortness of breath. Other symptoms include muscle pain, headache, sore throat, diarrhea, etc. In some cases, loss of smell or taste (or abnormality) occurs (World Health Organization [WHO], 2021). The clinical expression of MERS-CoV and SARS-CoV is more serious than those of other strains of coronavirus. With SARS, for example, 20 percent of patients require intensive care, and the disease has a 10 percent mortality rate. **Figure 1** shows the trend of total confirmed cases of COVID-19 from 2019/01/01 to 2022/01/31.

As coronaviruses can cause animals to have gastrointestinal tract symptoms, such as diarrhea, scientists suspect that coronaviruses can cause humans similar illness. However, this hypothesis has yet to be supported with evidence. Some studies indicate that coronaviruses can infect nerve cells and, therefore, could cause such nervous system infections as encephalitis. Other studies have found correlations between coronaviruses and Kawasaki disease, but no concrete evidence has yet been discovered (Taiwan Central Epidemic Command Center [CECC], 2020).

1.3. Motivations

The main purpose of this study, unlike to Parolini et al. (2021), is to investigate the factors that affect the time interval between two consecutive peaks of COVID-19 outbreak. Different from Valvo (2020), this study categorizes the type

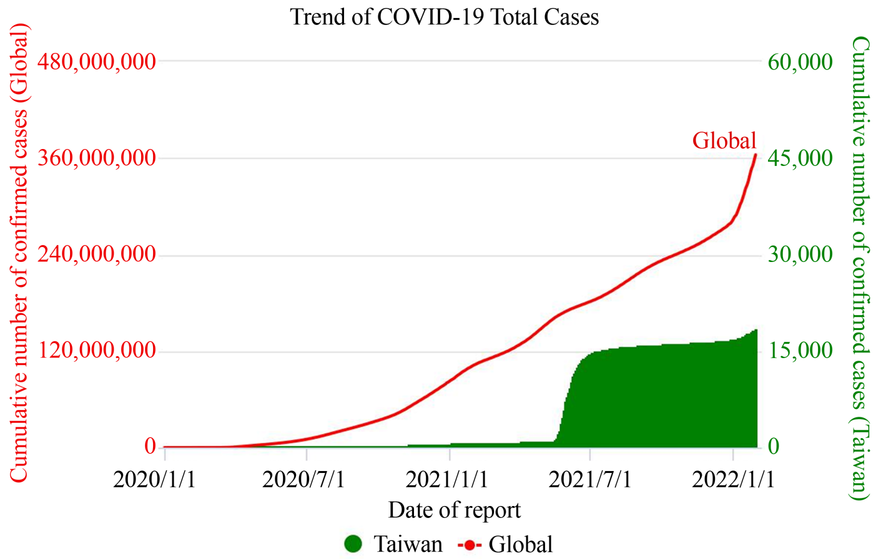

Figure 1. Trend of COVID-19 total cases (update to 2022/01/31). Source: Taiwan Ministry of Health and Welfare (https://covid19.mohw.gov.tw/en/sp-timeline0-206.html).

of COVID-19 epidemic situation into four groups: symmetry, severe, slow, and none, by using a family of two-modal bell-shaped functions. And then we find the days between two consecutive peaks of daily confirmed cases of COVID-19 for the different types. Induction by some mathematical computation, the time interval between two consecutive peaks is a function of the parameters of the two-modal bell-shaped functions.

Besides the use of bimodal bell-shaped functions to classify the type of COVID-19 outbreak situation, this study contributes to the literature of finding some important factors that affect the second/third wave of epidemics, such as, demographic variables, macroeconomic variables, national healthy index, abundance of medical resources, stringency index, human development index, and vaccination, etc., by using both the multivariate regression models and Poisson regression models.

The structure of this study is as follows: the second section is to introduce the family of two-modal bell-shaped functions. These functions are positive and have two maxima that can be used to describe the pattern of daily confirmed cases of CVOID-19 outbreak. In addition, some hypotheses proposed by this study need to be tested. The third section is the methodologies applied in this study. There are two econometric models for analyzing time interval between two consecutive peaks. One is the multivariate regression model for the continuity assumption, another is Poisson regression model for the discrete assumption. The fourth section is the empirical results and their analysis. And some conclusions and suggestions are given in the last section.

2. Bimodal Bell-Shaped Functions for Daily Confirmed Cases

First, as in Miatello (2020b), he claims that he has correctly predicted the peak-time of growth's curve of COVID-19 confirmed cases in Italy by using a mathematical model. In the first step, he sues the Lagrange Theorem to find the peak-time will occur in some time period. And then, as Miatello (2020a), he uses an exponential growth model by using the daily COVID-19 confirmed cases in Italy to estimate the growth rate. However, like the other studies, the distributions of daily confirmed cases for each countries are unimodal, i.e., only one local maximum, either mathematical/statistical models[1] (normal/lognormal/logistic) or artificial intelligence/deep learning models[2].

On the other hand, Valvo (2020) proposes a mixture of lognormal distributions, which is bimodal, to investigate the time trend of a country's COVID-19 daily death. He finds that the asymmetric lognormal distribution is a better way to model the COVID-19 death in a country. However, Valvo (2020) studied the distribution of death cases after being infected with COVID-19. However, it may take some time from the incubation period to the onset of symptoms, diagnosis and then death. The situation of each case will vary from person to person, namely, they do not necessarily have the same disease development. Moreover, the medical technology and resources of different countries are not the same, and those who are diagnosed after treatment may not necessarily die. Therefore, Valvo's research methods cannot be applied to the research topics in this article.

Therefore, as described in Hsiao (2021a, 2021b), a bimodal bell-shaped function can be given as follows:

$$H\left(x;\mu_1,\mu_2,\sigma_1,\sigma_2\right)=k\left(\mu_1,\mu_2,\sigma\right)\cdot\exp\left(-\frac{\left(x-\mu_1\right)^2}{\sigma_1^2}-\frac{\sigma_2^2}{\left(x-\mu_2\right)^2}\right)\cdot I_{\{x\neq\mu_2\}}\left(x\right), \quad (1)$$

where, x is the daily-confirmed cases reported, and $I_{\{x\neq\mu_2\}}(\cdot)$ is an indicator function. In addition, $k\left(\mu_1,\mu_2,\sigma_1,\sigma_2\right)$ is a positive coefficient with four parameters, μ_1,μ_2 and σ_1,σ_2.

Here, σ_1 and σ_2 are parameters correlated to the location of maximum values. The larger the product of σ_1 and σ_2, the longer the distance between the maxima of the function. In addition, when $\sigma_2=0$, the function will degenerate to a normal distribution function, $N\left(\mu_1,\frac{\sigma_1^2}{2}\right)$. Such that, the specified model proposed by Miatello (2020b) is a special case of this study. If $\sigma_1=0$, then the function will degenerate to an inverse normal distribution function,

$IN\left(\mu_2,\frac{\sigma_2^2}{2}\right)$.

Furthermore, in addition to affecting the distance between the maximum values of the function, μ_1 and μ_2 are mainly related the difference between the two maximum values. When the difference between μ_1 and μ_2 is greater, the difference between the two maximum values is larger; conversely, the smaller.

[1]See Atangana and Araz (2020), Aviv-Sharon and Aharoni (2020), Overton et al. (2020), Dagpunar (2020), Iboi, Ngonghala, and Gumel (2020), Nakamoto and Zhang (2021), Parolini et al. (2021).

[2]See Courtney (2020), Tuli, Tuli, Tuli, and Gill (2020), Ding and Gao (2020), Kolozsvári et al. (2021).

Moreover, when $\mu_1 > \mu_2$, the graph of this function is positive skew (Hsiao, 2021b), which means that if $x_i^* = \text{Arg}\{\max_{x\in\mathcal{R}} H(x;\mu_1,\mu_2,\sigma_1,\sigma_2)\}$, $i = 1,2$ with $x_2^* < x_1^*$, then

$$H(x_2^*;\mu_1,\mu_2,\sigma_1,\sigma_2) < H(x_1^*;\mu_1,\mu_2,\sigma_1,\sigma_2), \tag{2}$$

given that σ_1 and σ_2 remain constant. In this study, the positive skew characteristic of the graph of the function can be used to describe the severe situation of COVID-19 outbreak.

On the other hand, when $\mu_2 > \mu_1$, the graph of this function is positive skew (Hsiao, 2021b), which means that if $x_i^* = \text{Arg}\{\max_{x\in\mathcal{R}} H(x;\mu_1,\mu_2,\sigma_1,\sigma_2)\}$, $i = 1,2$ with $x_2^* < x_1^*$, then

$$H(x_1^*;\mu_1,\mu_2,\sigma_1,\sigma_2) < H(x_2^*;\mu_1,\mu_2,\sigma_1,\sigma_2), \tag{3}$$

given that σ_1 and σ_2 remain constant. In this study, the negative skew characteristic can be used to describe the slow situation of COVID-19 outbreak.

And, when $\mu_1 = \mu_2 = \mu$, the graph of this function is symmetry to the vertical line $x = \mu$ (Hsiao, 2021b), which means that if $x_i^* = \text{Arg}\{\max_{x\in\mathcal{R}} H(x;\mu,\mu,\sigma_1,\sigma_2)\}$, $i = 1,2$ with $x_1^* < x_2^*$, then

$$H(x_1^*;\mu,\mu,\sigma_1,\sigma_2) = H(x_2^*;\mu,\mu,\sigma_1,\sigma_2), \tag{4}$$

given that σ_1 and σ_2 remain constant. In this study, the symmetric characteristic can be used to describe the COVID-19 continues to spread situation.

2.1. The Symmetric Type

Aforementioned, when $\mu_1 = \mu_2 = \mu$, the graph of this function is symmetry to the vertical line $x = \mu$. Such that, some properties of this function are shown in the following lemma. The proof of the lemma can be found in Appendix A.

Lemma 1. Given the following function

$$H_0(x;\mu,\sigma_1,\sigma_2) = k(\mu,\sigma_1,\sigma_2)\cdot\exp\left(-\frac{(x-\mu)^2}{\sigma_1^2} - \frac{\sigma_2^2}{(x-\mu)^2}\right)\cdot I_{\{x\neq\mu\}}(x), \tag{5}$$

with $k(\mu,\sigma_1,\sigma_2) = \dfrac{\exp\left(\dfrac{2\cdot\sigma_2}{\sigma_1}\right)}{\sqrt{\pi\cdot\sigma_1^2}} > 0$, then it is symmetric to the vertical line $x = \mu$ and has modes at $x = \mu \pm \sqrt{\sigma_1\cdot\sigma_2}$.

According to the above derivation, the modes of the distribution are given as follows:

$$H_0\left(\mu\pm\sqrt{\sigma};\mu,\sigma_1,\sigma_2\right) = k(\mu,\sigma_1,\sigma_2)\cdot\exp\left(-\frac{2\sigma_2}{\sigma_1}\right). \tag{6}$$

In addition, the graph of this function is given as follows:

In **Figure 2**, we can find that each curve has two modes that have the same highest value of the curves. Moreover, the higher the value of σ, the thicker the

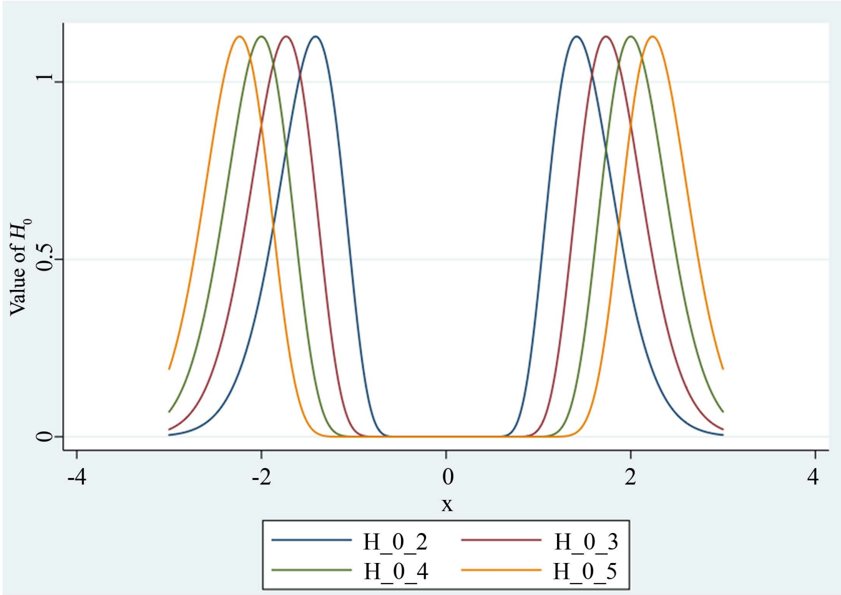

Figure 2. The graphs of the distribution with different values of σ. H_0_2 (in blue) is for $\sigma = 2$; H_0_3 (in red) is for $\sigma = 3$; H_0_4 (in green) is for $\sigma = 4$; H_0_5 (in orange) is for $\sigma = 5$ (Source: this study).

tail of the distribution. And the distance between the modes is $D = 2\sqrt{\sigma_1 \cdot \sigma_2}$ which is independent to the parameter μ.

In addition, according to the daily COVID-19 confirmed cases in Canada retrieved from John Hopkins University Database, Figure 3 shows the daily cases data in Canada.

In Figure 3, we may find that a peak occurred at 2021/01/03 with 10,643 cases and the next peak occurred at 2020/05/02 with 10,985 cases. The time interval between the first and the second peak is 119 days.

Moreover, according to the daily COVID-19 confirmed cases in Poland retrieved from John Hopkins University Database, Figure 4 shows the daily cases data in Poland.

In Figure 4, we may find that the first peak occurred at 2020/11/24 with 32,733 cases and second peak occurred at 2021/04/01 with 35,253 cases. The time interval between the first and the second peak is 128 days.

Additionally, according to the daily COVID-19 confirmed cases in USA retrieved from Our World in Data, Figure 5 shows the daily cases data in USA.

In Figure 5, we may find that a peak occurred at 2021/01/08 with 303,008 cases and the next peak occurred at 2021/09/07 with 277,028 cases. The time interval between the first and the second peak is 242 days.

2.2. The Asymmetry Distribution: Negative Skew

Next, consider the following family of functions that are negatively skew when $\mu_1 > \mu_2$. Such that, some properties of this function are shown in the following lemma. The proof of the lemma can be found in Appendix A.

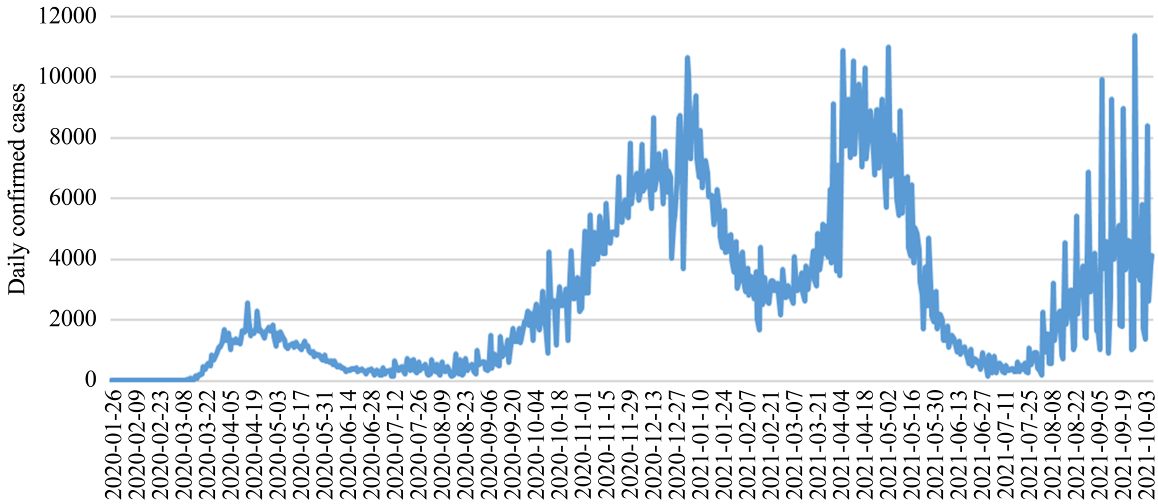

Figure 3. The daily COVID-19 confirmed cases in Canada up to 2021/10/07 (Source: John Hopkins University, USA).

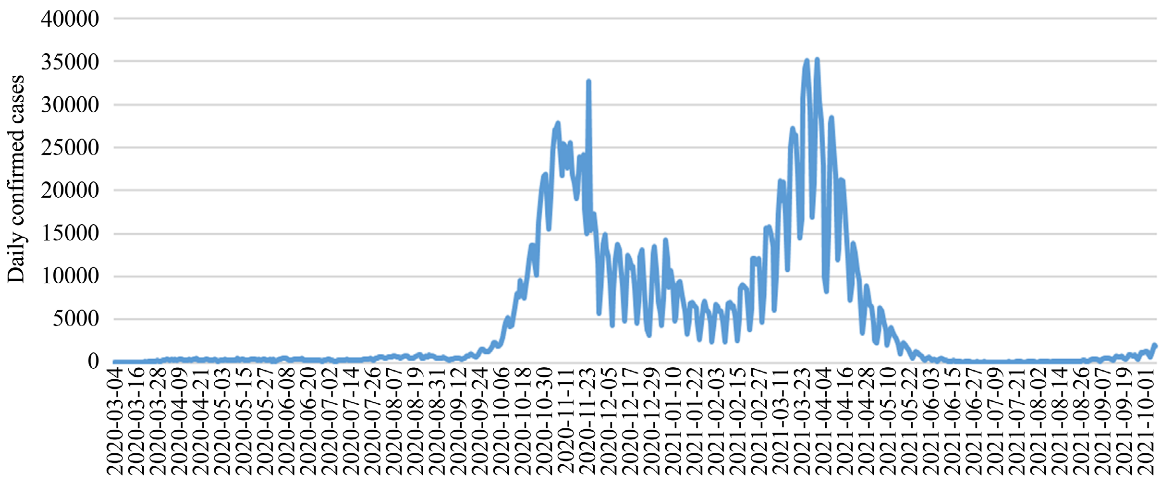

Figure 4. The COVID-19 daily-confirmed cases in Poland up to 2021/10/07 (Source: John Hopkins University, USA).

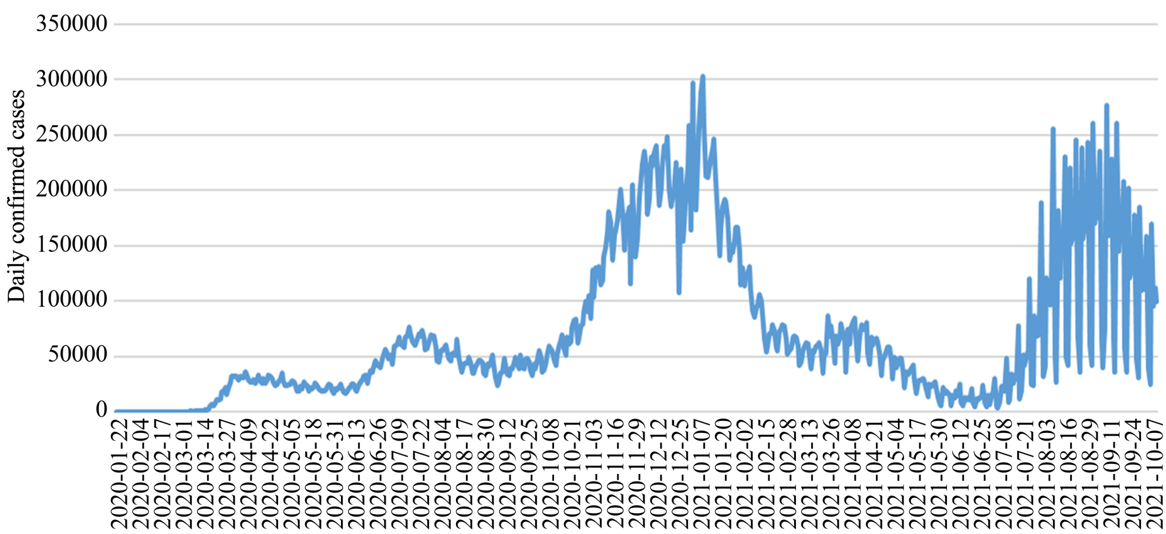

Figure 5. The COVID-19 daily-confirmed cases in USA up to 2021/10/07 (Source: John Hopkins University, USA).

Lemma 2. Given the following function, $\forall x \neq \mu_2$,

$$H_1(x;\mu_1,\mu_2,\sigma_1,\sigma_2) = k_1(\mu_1,\mu_2,\sigma_1,\sigma_2)\cdot \exp\left(-\frac{(x-\mu_1)^2}{\sigma_1^2} - \frac{\sigma^2}{(x-\mu_2)^2}\right), \tag{7}$$

with $\mu_1 > \mu_2$ and $k_1(\mu_1,\mu_2,\sigma_1,\sigma_2) > 0$, then it has modes at which satisfies $(x-\mu_1)\cdot(x-\mu_2)^3 - \sigma_1^2 \cdot \sigma_2^2 = 0$.

In addition, defining μ to be the difference of μ_1 and μ_2, i.e., $\mu \equiv \mu_1 - \mu_2$, then the graph of this function is given as follows:

In **Figure 6**, we can find that each curve has two modes, however, the relative extreme value of the curves are unequal. Moreover, all the functions as negatively skew and the larger the value of μ, the more the difference between two relative maxima.

In addition, according to the daily COVID-19 confirmed cases in Canada retrieved from John Hopkins University Database, **Figure 7** shows the daily cases data in Canada.

In **Figure 7**, we may find that the first peak occurred at 2020/03/23 with 26 cases and second peak occurred at 2020/12/06 with 22 cases, and the third peak occurred at 2021/05/22 with 723 cases. The time interval between the first and the second peak is 425 days.

Moreover, according to the daily COVID-19 confirmed cases in Philippines retrieved from John Hopkins University Database, **Figure 8** shows the daily cases data in Philippines.

In **Figure 8**, we may find that a peak occurred at 2021/04/02 with 15,298 cases and the next peak occurred at 2021/09/09 with 27,887 cases. The time interval between the first and the second peak is 160 days.

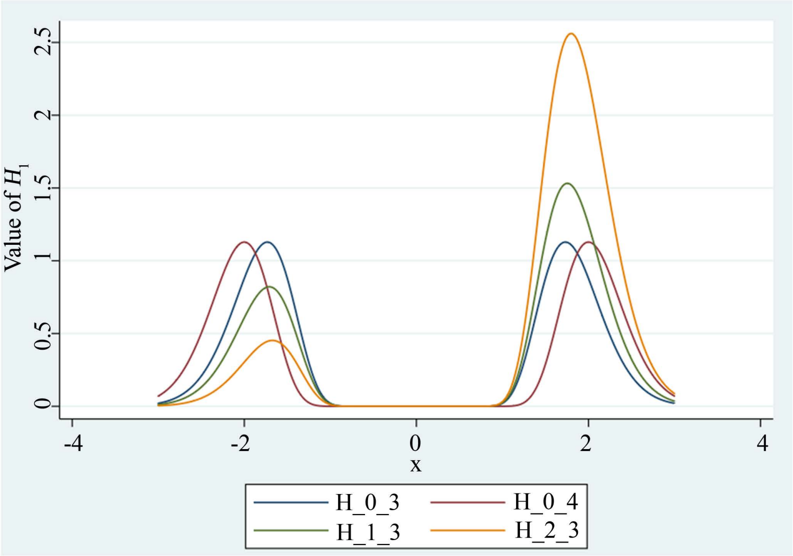

Figure 6. Family of skew to the rightdistributions. Note: H_0_3 (in blue) is a benchmark of the symmetry distribution with $\mu = 0.0$ and $\sigma = 3$; H_1_3 (in red) is asymmetric with $\mu = 0.25$ and $\sigma = 3$; H_2_3 (in red) is asymmetric with $\mu = 0.50$ and $\sigma = 3$.

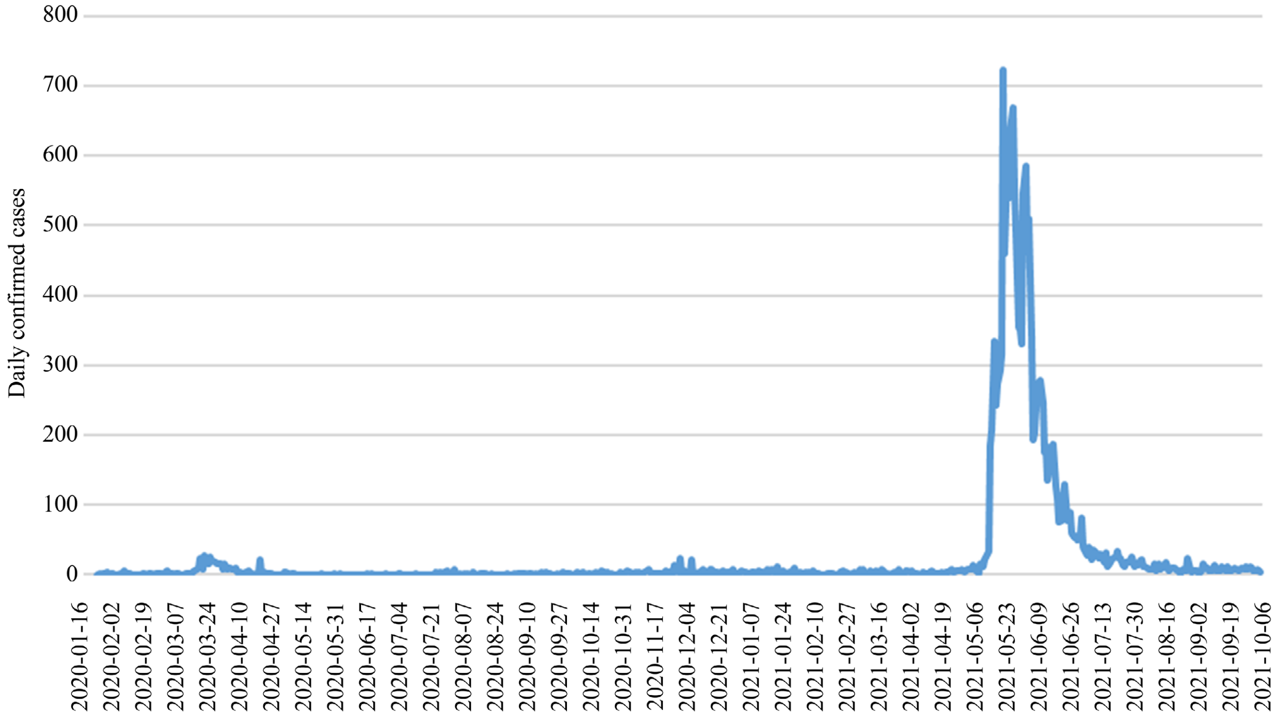

Figure 7. The COVID-19 daily-confirmed cases in Taiwan up to 2021/10/07 (Source: Our World in Data).

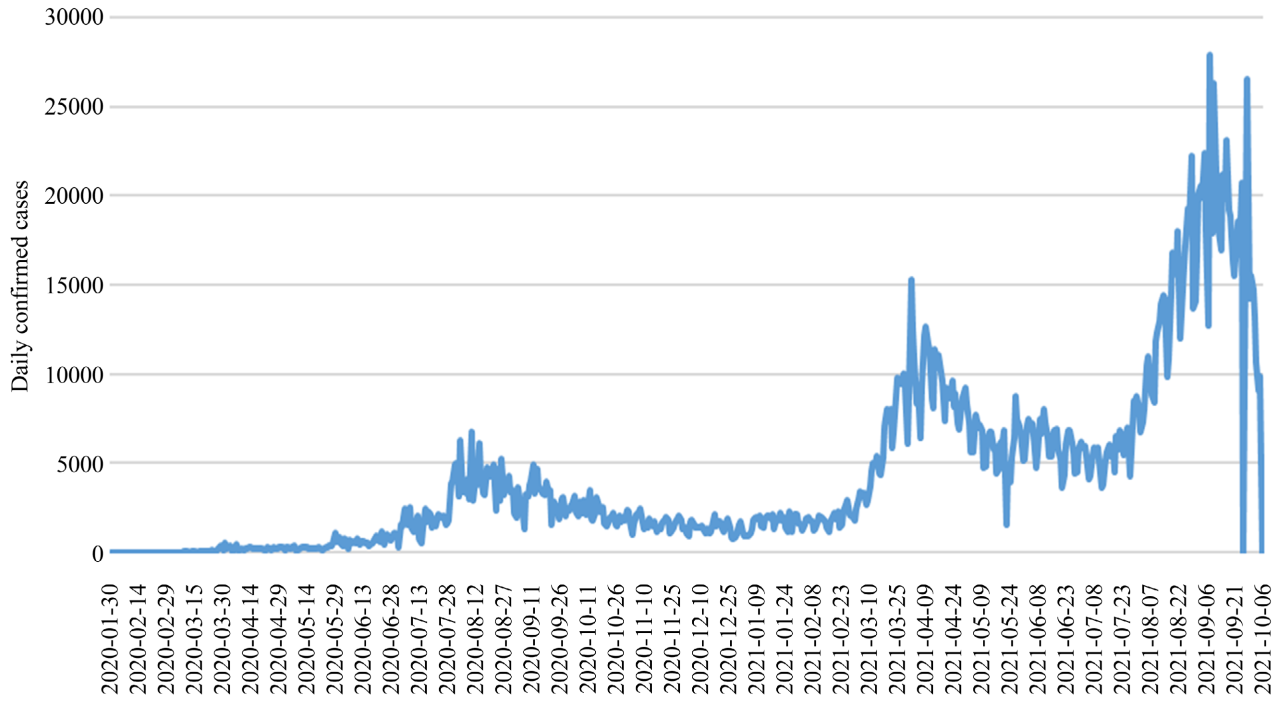

Figure 8. The COVID-19 daily-confirmed cases in Philippines up to 2021/10/07 (Source: John Hopkins University, USA).

In addition, according to the daily COVID-19 confirmed cases in Japan retrieved from John Hopkins University Database, **Figure 9** shows the daily cases data in Japan.

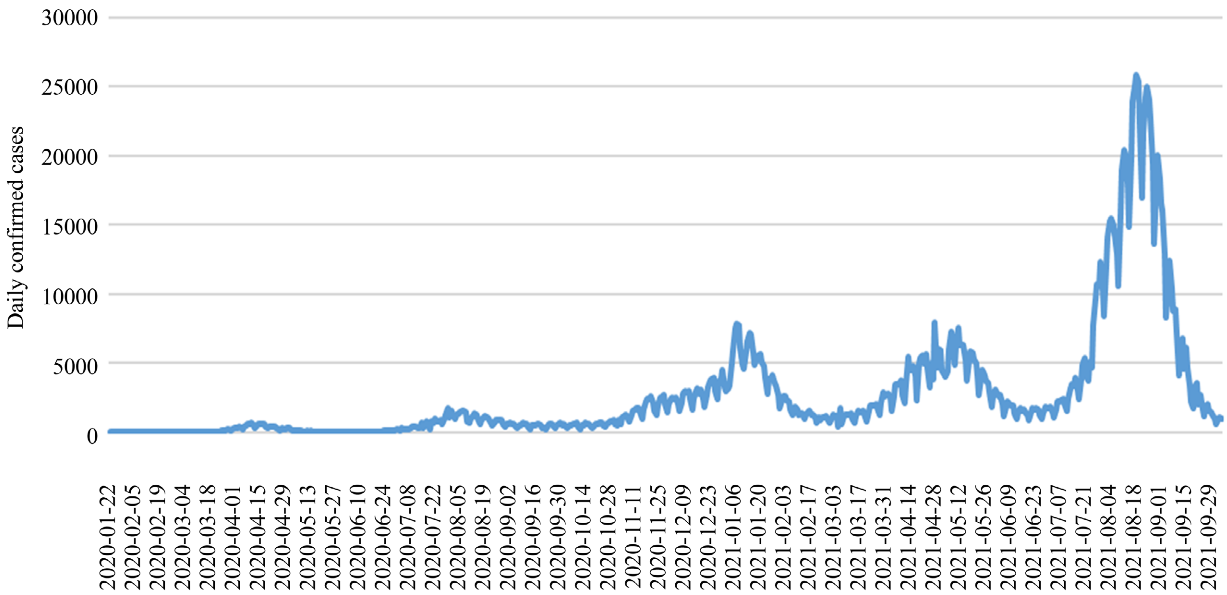

Figure 9. The COVID-19 daily-confirmed cases in Japan up to 2021/10/07 (Source: John Hopkins University, USA).

In **Figure 9**, we may find that a peak occurred at 2021/04/29 with 7914 cases and the next peak occurred at 2021/08/20 with 25,892 cases. The time interval between the first and the second peak is 113 days.

2.3. The Asymmetry Distribution: Positive Skew

Next, consider the following family of functions that are positively skew when $\mu_2 > \mu_1$. Such that, some properties of this function are shown in the following lemma. The proof of the lemma can be found in Appendix A.

Lemma 3. Given the following function, $\forall x \neq \mu_2$,

$$H_2(x;\mu_1,\mu_2,\sigma_1,\sigma_2) = k_2(\mu_1,\mu_2,\sigma_1,\sigma_2)\cdot \exp\left(-\frac{(x-\mu_1)^2}{\sigma_1^2} - \frac{\sigma_2^2}{(x-\mu_2)^2}\right), \tag{8}$$

with $\mu_1 < \mu_2$ and $k_2(\mu_1,\mu_2,\sigma_1,\sigma_2) > 0$, then it has modes at which satisfies $(x-\mu_1)\cdot(x-\mu_2)^3 - \sigma_1^2\cdot\sigma_2^2 = 0$.

In addition, defining μ to be the difference of μ_1 and μ_2, i.e., $\mu \equiv \mu_2 - \mu_1$, then the graph of this function is given as follows:

In **Figure 10**, we can find that each curve has two modes, however, the relative extreme value of the curves are unequal. Moreover, all the functions as skew to the left and the larger the value of μ, the more the difference between two relative maximum.

In addition, according to the daily COVID-19 confirmed cases in Austria retrieved from John Hopkins University Database, **Figure 11** shows the daily cases data in Austria.

In **Figure 11**, we may find that the first peak occurred at 2020/11/13 with 9586 cases and second peak occurred at 2021/03/31 with 3687 cases. The time interval between the first and the second peak is 138 days.

Moreover, according to the daily COVID-19 confirmed cases in Italy retrieved from John Hopkins University Database, Figure 12 shows the daily cases data in Italy.

In Figure 12, we may find that the first peak occurred at 2020/11/13 with 40,902 cases and second peak occurred at 2021/03/19 with 26,790 cases. The time interval between the first and the second peak is 126 days.

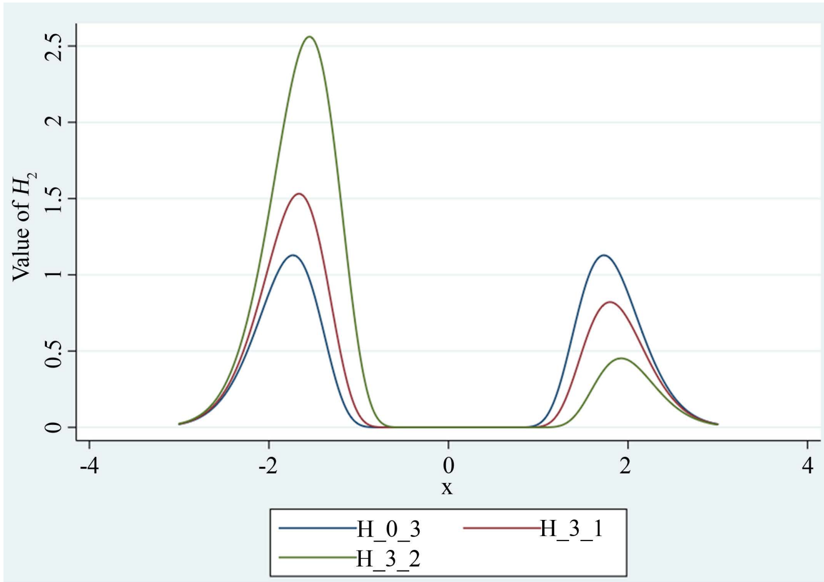

Figure 10. Family of skew to the rightdistributions. Note: H_0_3 (in blue) is a benchmark of the symmetry distribution with $\mu = 0.0$ and $\sigma = 3$; H_3_1 (in red) is asymmetric with $\mu = 0.25$ and $\sigma = 3$; H_3_2 (in red) is asymmetric with $\mu = 0.50$ and $\sigma = 3$.

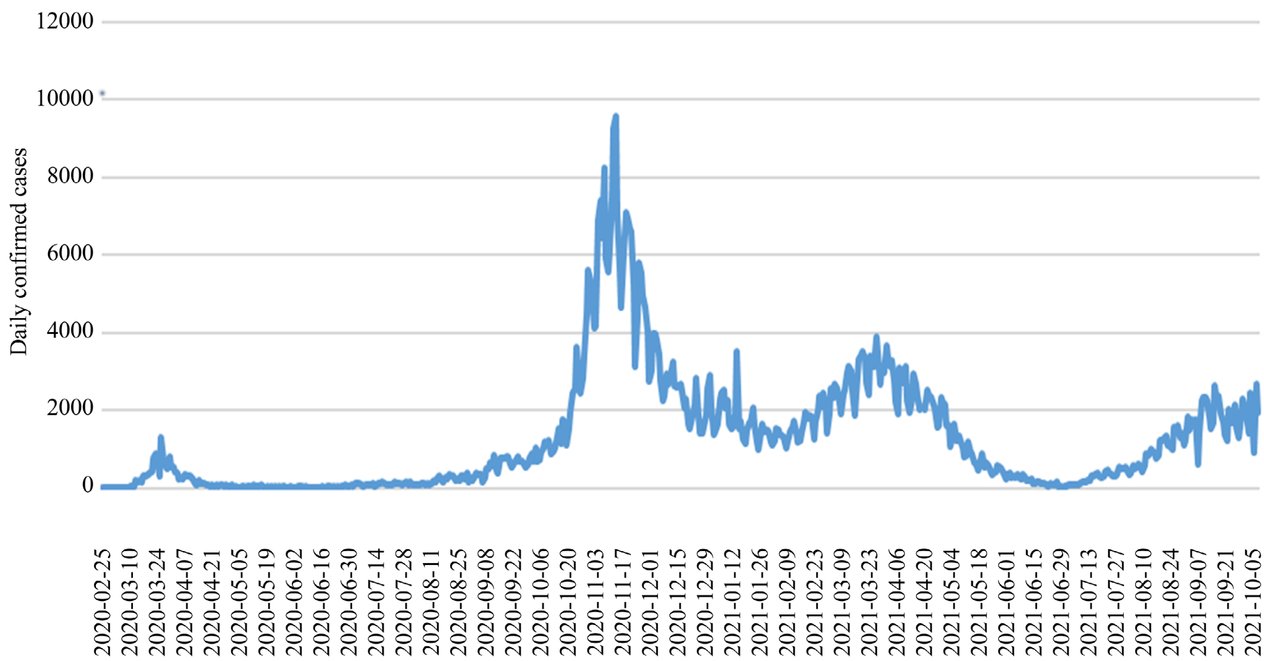

Figure 11. The daily COVID-19 confirmed cases in Austria up to 2021/10/07 (Source: John Hopkins University, USA).

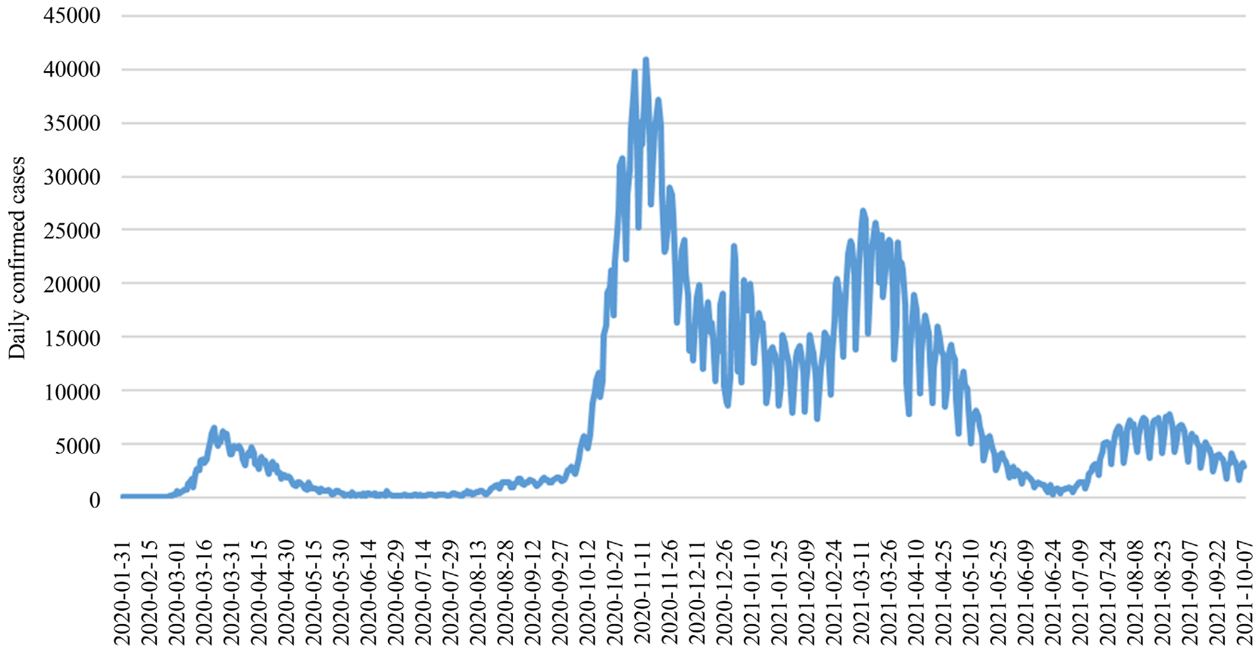

Figure 12. The COVID-19 daily-confirmed cases in Italy up to 2021/10/07 (Source: John Hopkins University, USA).

Additionally, according to the daily COVID-19 confirmed cases in Belgium retrieved from John Hopkins University Database, **Figure 13** shows the daily cases data in Belgium.

In **Figure 13**, we may find that the first peak occurred at 2020/10/29 with 23,921 cases and second peak occurred at 2021/03/21 with 9065 cases. The time interval between the first and the second peak is 143 days.

2.4. Hypothesis Development

The formation of infectious diseases must have three major elements: 1) host, i.e., human or animal, 2) pathogens that cause infections are some very small microorganisms, such as bacteria, viruses, fungi and parasites, and 3) environment, that is, the route of transmission, the way a pathogen moves from one place to another. In addition, according to Kulkarni, Khandait, Narlawar, Rathod, and Mamtani (2021), there are three categories of factors that will affect the contagion of COVID-19 in India. The first category is the geographical factors, such as latitude, longitude, population, population density, etc. Second is meteorological factors, for instance, humidity, grocery/pharmacy convenience, degree of air pollution, and traffic inconvenience, etc. Third is social factors, for example, the lockdown phases, and so on.

2.4.1. Demographic Effect

According to the analysis of the Chinese Center for Disease Control and Prevention, the new coronavirus is highly contagious, and one patient can spread the virus to two to three people. And the transmission of new coronary pneumonia includes "aerosol transmission" in addition to direct transmission and contact

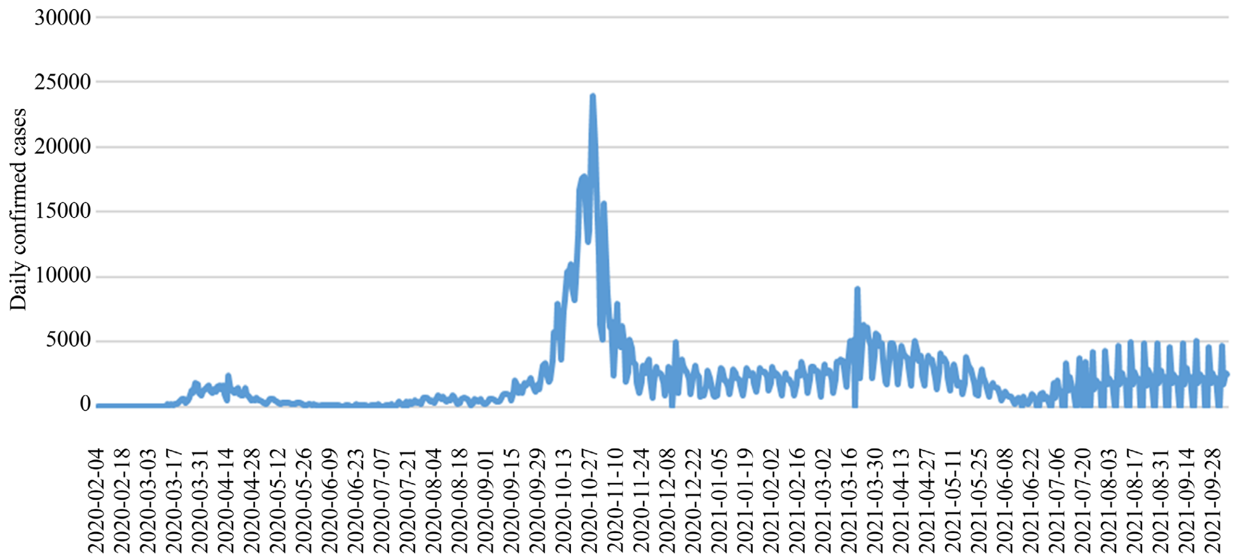

Figure 13. The COVID-19 daily-confirmed cases in Belgium up to 2021/10/07 (Source: John Hopkins University, USA).

transmission. Such that, the more people in a space, the higher possibility to be infected.

However, numerous studies have found that meteorological variables, population density, population mobility, and medical care can influence the spread of COVID-19 (Wang et al., 2020; Tosepu et al., 2020; Barcelo, 2020; Byass, 2020; Coccia, 2020; Sun et al., 2020; Lin et al., 2020; Rosario, Mutz, Bernardes, & Conte-Junior, 2020; Kulkarni et al., 2021). As a result, the first hypothesis is:

*H*1: *In a densely populated country, the second wave of epidemics will come earlier in a less densely populated country.*

Furthermore, according to Reddy (2020), the national health condition of a country is also one of the factors that can affect the spread of epidemics. Such that, the second hypothesis is:

*H*2: *In a country with a relatively high level of health, the second wave of epidemics will come later than a country with a relatively low level of health.*

2.4.2. Macroeconomic Effect

Moreover, the degree of economic development of a country is also one of the factors that will affect the spread of epidemics. A country with a highly developed economy has relatively rich medical technology and medical resources (Susser, 1973). Moreover, such a country has more healthy people and the level of knowledge will also be higher than that of countries with less economic development. Therefore, the spread of the disease will be slower (Harrison, 2004).

On the other hand, countries with a high degree of economic development have brought frequent international movements, including business contacts, or sightseeing tours, etc. This has also created the cross-border spread of the COVID-19 virus, causing the epidemic to spread globally (Reddy, 2020; Farseev, Chu-Farseeva, Yang, & Loo, 2021; Rosario, Mutz, Ferrari, Bernardes, & Conte-Junior, 2021).

Such that, the next three hypotheses are given as follows:

*H*3: *In an economy with a higher economic development, the second wave of epidemics will come later than in a less developed economy.*

*H*4: *A country with a high degree of social and economic stability will have a second wave of epidemics later than a country with a lower degree.*

*H*5: *In a country with sufficient medical resources, the second wave of epidemics will come later.*

2.4.3. Government Stringency Effect

It was Ibn Sina, a medical representative of the Islamic world born in the Saman Dynasty, who discovered that infectious diseases are contagious. In the book of *The Canon of Medicine* (*āb al-Qānūnfī al-ṭibb*), the isolation approach can prevent the spread of infectious diseases and that body fluids contaminated by certain natural substances can become infectious (Sina, 1020).

Furthermore, restricting going out, lock down of cities or closing borders can also achieve the goal of cutting off the chain of transmission and reducing the risk of the spread of the epidemic (Harrison, 2004; Sun et al., 2020; WHO, 2021; Lai, 2021). However, after a long period of control, there are also debates about violations of human rights (Kooli, 2021), which also makes people nervous or a state of mental fatigue. Therefore, when the government announced the relaxation of control measures, the stringency index was reduced accordingly. However, at this time, the Coronavirus moved due to people "retaliatory travel" or "emotional rebound", such that the invisible transmission chain quietly expanded to large-scale community transmission (Sun et al., 2020; Kulkarni et al., 2021; Rosario et al., 2021). This study observes that the second wave of epidemics will come quietly about 3 to 4 weeks after lowering the stringency index, for example, Brazil (Rosario et al., 2021) and Taiwan area. Hence, the sixth hypothesis is:

*H*6: *A country with higher epidemic prevention control will have a second wave of epidemics later than a country with lower control.*

2.4.4. Vaccination Policy Effect

As the discussion in Iboi et al. (2020), COVID-19 elimination is more efficient if the vaccination program is combined with another intervention, for instance, social distance limitation and face mask use in public. However, their simulation results are based on the assumption of well-mixed population or homogeneous-mixing assumption which is made for the purpose of mathematical analysis, but it does not necessarily conform to the real situation.

On the other hand, due to the use of vaccines, the people will be negligent and lack of vigilance, causing the epidemic to spread again in the community. This is so-called "moral hazard". Such that, the seventh hypothesis is:

*H*7: *A country with sufficient purchases of vaccines and a high rate of vaccines will come later in a country where the second wave of epidemics will be relatively scarce.*

3. Econometric Analysis for Two Consecutive Peaks

This study is to investigate the possible factors that influence the days between twin peaks of COVID-19 in the country level. There are two families of econometric models implemented to find the factors. One is the multivariate regression models which can be used to find the relationship between the factors and the days between two peaks of COVID-19 which are of continuous assumption. The other is the Poisson regression models which can be used to study this issue for a counting number assumption of the days between two peaks.

3.1. Multivariate Regression Models

The first approach this study employed is the multivariate regression model. The specification of the regression model is given as follows:

$$Day_i = \alpha + \beta \cdot X_i + \delta \cdot D_i + \varepsilon_i,\ i = 1,2,3,\cdots,n. \quad (9)$$

In the above equation, the dependent variable is the days between two peaks of daily confirmed cases of countries. And $\boldsymbol{X}$ is the explanatory variables which are the country's macroeconomic variables, human capital index, government stringency index, and healthcare variables, etc. In addition, $\boldsymbol{D}$ is some categorical variables, such as, demographic variables, types of second wave of COVID-19 epidemics. Moreover, ε is the error term with mean 0 and variance σ_ε^2.

3.2. Poisson Regression Models

The second approach this study employed is the Poisson regression model. The specification of the model is given as follows:

$$E\left[Day_i \middle| X_i, D_i\right] = \alpha + \beta \cdot X_i + \delta \cdot D_i,\ i = 1,2,3,\cdots,n, \quad (10)$$

with the likelihood function of Poisson distribution:

$$\Pr\left[Day_i = k \middle| X_i, D_i\right] = \frac{e^{-\lambda(X_i,D_i)} \cdot e^{k\cdot\lambda(X_i,D_i)}}{k!},\ k \in \aleph \cup \{0\}. \quad (11)$$

In the above equation, the dependent variable is the days between two peaks of daily confirmed cases of countries which are of counting number specification. And $\boldsymbol{X}$ is the explanatory variables which are the country's macroeconomic variables, human capital index, government stringency index, and healthcare variables, etc. In addition, $\boldsymbol{D}$ is some categorical variables, such as, demographic variables, types of second wave of COVID-19 epidemics.

4. The Data

The daily confirmed cases data employed in the study are downloaded from the database of John Hopkins University, USA. Second, the demographic data is retrieved from the Our World in Data[3]. In addition, the macroeconomic data are

[3]It is worth to address that the confirmed cases of COVID-19 reported by Taiwan CECC cannot be found in John Hopkins University database because they are aggregated into the reported data of China. However, the OWID departs them into two files.

downloaded from World Bank.

4.1. Variables

4.1.1. Dependent Variable

Days between two Peaks (*Days*)

To investigate the timing that second wave of COVID-19 outbreak, we have to calculate the days between two consecutive peaks of daily confirmed COVID-19 cases for the countries which have experienced second or third wave of COVID-19 outbreak.

4.1.2. Explanatory Variables

Population (*Pop*)

In biology, a population is a number of all the organisms of the same group or species who live in a particular geographical area and are capable of interbreeding. The area of a sexual population is the area where inter-breeding is possible between any pair within the area and more probable than cross-breeding with individuals from other areas.

Since the outbreak of human-to-human transmission in China, humans have become the main host of the COVID-19 coronavirus. Therefore, the greater the population, the more possible hosts, and the more severe the spread of the epidemic (Tandon, 2020).

Population Density (*PopDen*)

As shown in **Table 1**, there are large populations such as China, India or Brazil, and the second wave of epidemics is about four to five months after the peak of the first wave. However, Germany, France or the United Kingdom in Europe also have larger populations. In these countries, the second wave of epidemic peaks did not appear until eight to nine months.

On the other hand, countries with relatively small populations, for instance, Yemen, Uzbekistan, Slovakia, have the peak of the second wave of epidemics in just over two months after the first peak of daily confirmed cases. Therefore, it is not that the population is large, and the epidemic spreads quickly. On the contrary, it may be that the dense of population will accelerate the spread of COVID-19 (Tandon, 2020).

Median Age (*MedAge*)

According to the World Population Review[4], the median age of a population is the point at which half the population is older than that age and half is younger. In the last 50 years, the median age of the world's population has increased by just three years, from 23.6 in 1950 to 26.4 in 2000. In addition, median ages are influenced by a number of factors, such as birth rates, social and economic development and average life expectancies within individual countries. A country with low median age means that it is reflective of the fact that poverty, disease and ongoing conflict situations in the country.

[4]https://worldpopulationreview.com/country-rankings/median-age.

Table 1. Descriptive statistics of variables by continent/organization.

Region	Europe	America	Pan Pacific	Africa	OECD	BRICS	World
Days	154.7 (62.8)	181.3 (92.6)	176.6 (91.1)	174.4 (50.3)	168.4 (81.5)	156.4 (66.4)	170.2 (75.1)
Pop (Millions)	2.03 (3.20)	3.94 (8.09)	7.69 (2.19)	2.81 (3.80)	4.44 (6.56)	388.8 (556.8)	4.05 (12.1)
PopDen (per m^2)	746.21 (3330)	125.06 (155.2)	433.79 (1252)	80.77 (84.8)	350.8 (1200)	108.0 (192.1)	385.8 (1976)
MedAge (years)	42.10 (2.68)	31.26 (5.15)	31.02 (7.29)	20.88 (4.17)	41.14 (4.61)	33.64 (5.93)	31.66 (9.32)
Life_Exp (years)	80.17 (3.60)	75.41 (4.02)	74.97 (5.38)	63.92 (5.89)	81.66 (2.03)	70.97 (4.41)	73.90 (7.78)
Stringency	50.54 (14.4)	56.75 (17.9)	51.96 (17.6)	43.18 (14.9)	54.09 (15.0)	46.48 (15.2)	50.23 (16.7)
HDI	0.89 (0.06)	0.76 (0.09)	0.75 (0.13)	0.55 (0.14)	0.91 (0.04)	75.34 (7.72)	0.74 (0.17)
CVDs (%)	215.4 (127)	197.29 (79.11)	290.24 (155)	297.4 (75.9)	127.7 (36.3)	304.6 (122)	252.5 (125)
HBeds 1000	5.13 (2.55)	2.17 (1.29)	3.99 (8.28)	1.39 (0.77)	5.17 (7.82)	4.23 (3.56)	3.50 (4.85)
GDP_per_Capita (1000 $)	42.51 (37.0)	16.86 (12.70)	22.34 (23.1)	4.62 (4.10)	41.87 (15.01)	16.47 (8.09)	23.13 (28.25)
Observation	98	59	83	79	84	5	315

Note: The values in the parentheses is the standard deviation of the variables. Source: Our World in Data and World Bank.

Life Expectancy (*Life_Exp*)

Life expectancy is the key metric for assessing population health. Broader than the narrow metric of the infant and child mortality, which focus solely at mortality at a young age, life expectancy captures the mortality along the entire life course. It tells us the average age of death in a population.

Life expectancy has increased rapidly since the Age of Enlightenment. In the early 19th century, life expectancy started to increase in the early industrialized countries while it stayed low in the rest of the world. This led to a very high inequality in how health was distributed across the world. Good health in the rich countries and persistently bad health in those countries that remained poor. Over the last decades this global inequality decreased. However, the number of death in the global infected by COVID-19 is decreasing the global life expectancy.

Stringency Index (*SI*)

A stringency index created by Oxford University shows how strict a country's measures were responding to the COVID-19 outbreak, and at what stage of the spread it enforced these. This is a composite measure based on nine response indicators including lockdown, school closures, workplace closures, and travel bans, rescaled to a value from 0 to 100 (100 = strictest).

For instance, according to the records of the Central Epidemic Command Center in Taiwan, before May 14, 2021, Taiwan's stringency index was only about 25, which is much lower than that of most other countries. However, in the relaxation of airline pilot's entry control regulations, due to improper management of epidemic prevention hotels, gatherings of Lions clubs, special industries in the snack bar, etc., group infections have caused Taiwanese society to increase its alert level, and the stringency index has also risen sharply in just one week coming to the austerity situation of 70.

Human Development Index (*HDI*)

According to the United Nations Development Programme, the Human Development Index (HDI) was created to emphasize that people and their capabilities should be the ultimate criteria for assessing the development of a country, not economic growth alone. The HDI can also be used to question national policy choices, asking how two countries with the same level of GNI per capita can end up with different human development outcomes. These contrasts can stimulate debate about government policy priorities.

The Human Development Index (HDI) is a summary measure of average achievement in key dimensions of human development: a long and healthy life, being knowledgeable and have a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions.

Death Rate from Cardiovascular Disease (*CVDs*)

The death rate from cardiovascular disease is the proportion of annual number of deaths from cardiovascular diseases per 100,000 people. Cardiovascular diseases are the leading cause of death in the EU. They cover a broad group of medical problems that affect the circulatory system (the heart and blood vessels), often resulting from atherosclerosis, the abnormal build-up of plaque—that is made of, among constituents, cholesterol or fatty substances—that is deposited on the inside walls of a person's arteries. Some of the most common diseases that affect the circulatory system include ischemic heart disease (heart attacks) and cerebrovascular diseases (strokes). Downloaded from Eurostat (online data code: hlth_co_disch2).

Hospital Beds per Thousand (*HBeds* 1000)

The number of beds per people is an important indicator of the health care system of a country. The basic measure focus on all hospital beds, which are variously split and occupied. The classic hospital beds are also called curative beds. For severe patients with risk of organ(s) failure, patients are provided intensive care unit beds (aka ICU bed) or critical care beds (CCB).

The availability of CCB-ICU beds, mechanical ventilation and ECMO devices generally closely associated with hospital beds has been described as a critical bottleneck in responding to the ongoing COVID-19 pandemic. The lack of such devices dramatically raises the mortality rate of COVID-19 (Farseev et al., 2020).

GDP per Capita (*GDP_per_Capita*)

GDP per capita is a global measure for gauging the prosperity of nations and is used by economists, along with GDP, to analyze the prosperity of a country based on its economic growth. There are a few ways to analyze a country's wealth and prosperity. GDP per capita is the most universal because its components are regularly tracked on a global scale, providing for ease of calculation and usage. Income per capita is another measure for global prosperity analysis, though it is less broadly used. According to Farseev et al. (2020), the GDP per capita can also be a measure for government political stability.

4.1.3. Categorical Variables

Types of Two Peaks (*Type_id*)

Based on the daily reported confirmed cases of COVID-19, this study uses the two-modal bell-shaped functions to classify the spread of the epidemic in various countries into four types: symmetric (**Figure 2**), severe (**Figure 6**), slow (**Figure 10**), and other types. Symmetrical type means that the peak of the second wave is almost the same as the peak of the first wave, such as Canada, Poland, USA, and so on. Therefore, set Type = 1. And the epidemic severe type means that the peak of the second wave is much higher than the peak of the first wave, and the daily confirmed cases tilt to the right, such as, Japan, Philippines, and Taiwan area, etc. Therefore, set Type = 2. Relatively, the epidemic slows down type, which means that the second wave peak is much lower than the first wave peak, and the daily confirmed cases tilt to the left, for example, Austria, Italy, and Belgium, etc. Therefore, set Type = 3. Moreover, set Type = 0 for the others.

Vaccination

The administration of vaccines can increase people's ability to fight the virus, thereby reducing the possibility of infection, or even if they are infected, they will not cause serious illness or even death. Therefore, whether to administer the vaccine will likely cause the development of the epidemic has been considerably restricted.

In this study, to investigate the effect of vaccine administration on the COVID-19 epidemic, the variable vaccination is a dummy variable of whether the country has started to administer the vaccine. When vaccination = 0, it means that the vaccine has not started in that period; and when vaccination = 1, it represents that the country has started to administer the vaccine.

4.2. Descriptive Statistic

As mentioned above, the descriptive statistic of variables are calculated and reported in the following tables either by continent/organization or type. **Table 1** shows the descriptive statistics categorized by the continent/organization. The countries are categorized into Europe, America (North America and South America), Pan Pacific (Asia and Oceania), and Africa. In addition, the results of Organization of Economic Cooperative and Development [OECD] countries are also shown in the last column.

In Table 1, the periods between two consecutive peaks in Europe and BRICS[5] are much shorter (about five months) than other regions, however, it is the longest in America (about six months). Furthermore, many average values of the European and OECD countries are higher however they are the lowest for the African countries. For instance, health variables (life expectancy, median age, and hospital beds per thousand people), human development index and GDP per capita.

On the other hand, Table 2 shows the descriptive statistics categorized by the type introduced by this study. The countries are categorized into Symmetry (Type 1), Severe (Type 2), and Slow (Type 3).

Table 2. Descriptive statistics of variables by type.

Type	Symmetry (Type = 1)	Severe (Type = 2)	Slow (Type = 3)
Days	163.26 (78.29)	176.53 (76.21)	163.22 (63.43)
Pop (Millions)	3.26 (5.47)	4.29 (1.19)	2.58 (4.20)
PopDen (per m^2)	118.57 (184.66)	448.36 (2207.07)	556.36 (2486.06)
MedAge (years)	30.81 (9.27)	30.62 (9.18)	34.97 (9.21)
Life_Exp (years)	72.83 (7.95)	73.41 (7.81)	76.02 (7.48)
Stringency	49.24 (18.57)	50.29 (15.81)	50.97 (16.20)
HDI	0.73 (0.18)	0.73 (0.17)	0.79 (0.15)
CVDs (%)	270.31 (131.15)	244.24 (113.08)	253.01 (143.30)
HBeds 1000	3.23 (2.53)	3.48 (6.23)	3.80 (2.66)
GDP_per_Capita (1000 $)	19.21 (17.98)	22.98 (31.20)	28.45 (30.63)
Observations	78	166	69

Note: The value in the parentheses is the standard deviation of the variable. Source: Our World in Data and World Bank.

[5]BRICS is the acronym coined to associate five major emerging economies: Brazil, Russia, India, China, and South Africa. The BRICS members are known for their significant influence on regional affairs. Moreover, The BRICS have a combined area of 39,746,220 km^2 and an estimated total population of about 3.21 billion, or about 26.7% of the world land surface and 41.5% of the world population.

In Table 2, on average, the days of two consecutive peaks are over 5.5 months for each type. Moreover, a country with healthy conditions (higher median age or higher life expectancy), abundance of medical resource, higher human capital, and higher economic development will be.

5. Empirical Results

5.1. Multivariate Regression Analysis

According to the specification of the multivariate regression model in Equation (6), the empirical results are reported in the following tables for either continent/organization or types.

5.1.1. Multivariate Regression Results by Continent/Organization

First, classified the countries by continent/organization, the regression results are reported in the following Table 3.

As shown in Table 3, the ratio of hospital beds per 1000 people is significantly positive for the all countries except for the in America. It means that, to most

Table 3. Multivariate regression results by continent/organization.

Region	Europe	America	Pan Pacific	Africa	OECD
Constant	104.9 (0.20)	1063.3 (1.63)	220.4 (0.53)	183.5 (1.47)	−65.2224 (−0.09)
Population (Millions)	0.1965 (1.07)	−0.2201 (−1.11)	−0.0029 (−0.06)	−0.1969 (−0.53)	0.0918 (0.64)
PopDen (per m^2)	−0.0221 (−0.88)	−0.0911 (−0.98)	0.0109 (1.18)	−0.0086 (−0.08)	0.0111 (1.34)
GDP per Capita (1000 $)	0.7957 (1.06)	3.6834 (1.44)	1.2456* (1.79)	−6.3226 (−1.50)	0.2135 (0.22)
Med_Age (year)	0.550 (0.02)	−0.1839 (−0.04)	−5.3336* (−1.78)	6.0602 (1.50)	−4.7555* (−1.75)
Stringency	−2.5553*** (−6.29)	−1.1927 (−1.53)	−0.9913 (−1.61)	−1.3859*** (−2.25)	−1.9937*** (−3.42)
HDI	−357.96 (−1.32)	−639.9 (−1.23)	−38.6916 (−0.14)	84.1294 (1.12)	−179.6 (−0.46)
CVDs (%)	−0.0761 (−0.46)	−0.5594 (−1.59)	0.0320 (0.29)	0.0580 (0.58)	0.1973 (0.52)
HBeds 1000	10.6417** (2.54)	1.6930 (0.11)	3.7735*** (2.73)	3.0983 (0.20)	2.6148** (2.29)
Vaccination	−15.2853 (−1.36)	−46.8833** (−1.99)	9.1130 (0.44)	−3.1543 (−0.22)	−17.6916 (−1.10)
Adj. R^2	0.3166	0.0912	0.1083	0.0327	0.2153
Obs.	89	58	76	54	84

Note: The value in parentheses is the *t* statistic of the estimated value. *, **, *** represent 10%, 5%, and 1% significance levels, respectively.

countries, the abundance of medical resources will prolong the arrival of the second/third wave of epidemics. And as the estimate of GDP per capital is also significantly positive for the countries in Pan Pacific (Asia and Oceania), the higher, this result shows that the better economic situation the country is, the longer the second/third peak of COVID-19 epidemics.

On the other hand, the stringency index is significantly negative for the countries either in Europe or in OECD, however, it is insignificant for the countries in the other regions or continents. It reveals that for a high development economy, the second/third wave of COVID-19 epidemics come earlier if more lockdown regulations are implemented.

5.1.2. Multivariate Regression Results by Type of Epidemic

Next, according to the classification of the spread of the COVID-19 epidemic, the following **Table 4** shows the multivariate regression results by type.

Table 4. Multivariate regression results by type.

Type	Symmetry (Type 1)	Severe (Type 2)	Slow (Type 3)	Pooled
Constant	280.00 (1.37)	146.5 (1.11)	40.472 (0.16)	166.2481 (1.65)
Population (millions)	−0.0349 (−0.20)	0.0582 (1.22)	−0.1237 (−0.60)	−0.0020 (−0.06)
PopDen (per m^2)	−0.0586 (−1.15)	0.0344*** (0.3.72)	−0.0158* (−1.92)	0.0100 (1.51)
MedAge (year)	−7.3732** (−2.56)	−2.6860* (−1.69)	−1.9812 (−0.89)	−3.9490*** (−3.54)
GDP_per_Capita (1000 $)	1.8713** (2.22)	−0.1946 (−0.35)	0.5865 (1.13)	0.4368 (1.21)
Life_Exp (year)	−1.0022 (−0.26)	3.1703 (1.27)	5.8535 (1.20)	2.3310 (1.23)
HDI	206.8 (1.37)	−59.7204 (−0.63)	−186.29 (−0.81)	13.595 (0.18)
CVDs (%)	0.1361 (1.53)	−0.0763 (−1.15)	−0.0085 (−0.10)	−0.0024 (−0.05)
HBeds 1000	6.3963 (1.10)	2.5124** (2.38)	−0.2560 (−0.06)	2.8881*** (2.83)
Stringency	−1.0709* (−1.82)	−1.2891*** (−3.18)	−1.6353*** (−3.33)	−1.3032*** (−4.56)
Vaccination	9.7454 (0.50)	−12.1441 (−1.00)	−44.7965*** (−2.69)	−10.1897 (−1.17)
Adj. R^2	0.1809	0.1910	0.1576	0.1558
Obs.	67	143	62	277

Note: The value in parentheses is the *t* statistic of the estimated value. *, **, *** represent 10%, 5%, and 1% significance levels, respectively (Source: this study).

In Table 4, for each type of COVID-19 epidemic, the ratio of the hospital beds per 1000 people is significant in the Type-2 (Severe) and Type-3 (Slow). For Type-2, the positive estimate means that a country with relatively abundant medical resources can delay the severely spread of the COVID-19 epidemic. Similarly, it will be shortened to slow down because the estimate in Type-3 is negative, such as Taiwan.

Furthermore, all the estimates of median age for all types are significantly negative. It shows that the higher the median age of a country, the earlier the second/third wave comes. Since a country with low median age reflects that it is poverty, disease and ongoing conflict situations, then a richer or healthy country will encounter the second/third waves of COVID-19 earlier, whether it is severe or mild situation.

In addition, the estimates of stringency index for Type-2 and Type-3 are also significant negative but the other types are insignificant. Such that, a higher stringency index will result in the coming of second/third waves of COVID-19 epidemics, no matter what situation the country is. Therefore, the COVID-19 epidemic in some countries will gradually ease the development due to stricter lockdown regulations implemented by the government. However, in some countries, the epidemic will develop more severely faster because of the strict lockdown regulations.

For example, Taiwan's large-scale community infection after May 19, 2021 allowed the epidemic to spread rapidly in northern Taiwan and spread to the central and southern regions. Taiwan Central Epidemic Command Center had to announce an increase in alertness to level three and issued a new update for strict control measures. Although the people feel inconvenient, most of them can do their best to cooperate for the sake of their own health. Therefore, with the concerted efforts of the government and the private sector, the number of daily confirmed cases in each county and city has been quickly reduced from hundreds of people to dozens of people. As a result, the alert level was lowered on July 26 of the same year, so that the people's daily life could recover, and the stores could reopen in 68 days.

Lastly, including Table 3 and Table 4, only the estimated value of vaccination in the regression results of Type-3 is significant, and the rest of the regression results are not significant. Nevertheless, this result also shows that the administration of the vaccine can make the time for the epidemic to slow down sooner. In other words, the administration of the vaccine can speed up the slowdown of the epidemic. It may partially support the effect of vaccine administration.

5.2. Poisson Regression Analysis

5.2.1. Poisson Regression Results by Continent/Organization

According to the specification of the Poisson regression model in Equation (7), the empirical results for each continent/organization are reported in the following Table 5.

Table 5. Poisson regression results by continent/organization.

Region	Europe	America	Pan Pacific	Africa	OECD
Constant	4.8387*** (5.75)	10.1277*** (18.10)	5.3453*** (14.71)	5.1993*** (27.08)	3.9729*** (5.17)
Population (Millions)	0.0013*** (4.64)	−0.0012*** (−7.16)	−0.0000 (−1.03)	−0.0010* (−1.69)	0.0007*** (4.32)
PopDen (per m^2)	−0.0001*** (−3.15)	−0.0005*** (−5.44)	0.00005*** (7.50)	−0.0000 (−0.08)	0.0001*** (6.41)
MedAge (year)	0.0016 (0.39)	−0.0032 (−0.70)	−0.0300*** (−11.33)	0.0355*** (5.65)	−0.0248*** (−8.80)
Stringency	−0.0171*** (−25.85)	−0.0052*** (−8.75)	−0.0058*** (−10.79)	−0.0080*** (−8.39)	−0.0120*** (−19.19)
GDP per Capita (1000 $)	0.0054*** (4.76)	0.0212*** (9.57)	0.063*** (11.83)	−0.0378*** (−5.64)	0.0019* (1.84)
Life_Exp (year)	0.0342*** (3.35)	−0.0216*** (−2.71)	0.0107* (1.65)	−0.0097** (−2.29)	0.0461*** (3.98)
HDI	−2.3998*** (−5.61)	−3.2161*** (−7.15)	−0.0630 (−0.27)	0.4934*** (4.20)	−1.3208*** (−3.11)
CVDs (%)	−0.0005* (−1.90)	−0.0030*** (−10.05)	0.0002* (1.76)	0.0003* (1.90)	0.0010** (2.34)
HBeds 1000	0.0665*** (10.54)	−0.0006 (−0.05)	0.0146*** (17.12)	0.0129 (0.52)	0.0082*** (9.17)
Vaccination	−0.0936*** (−5.30)	−0.2477*** (−12.93)	0.0591*** (3.29)	−0.0141 (−0.64)	−0.1094*** (−6.28)
Pseudo R^2	0.3041	0.2771	0.1866	0.1478	0.2496
Loglikelihood	−955.1	−1090.0	−1622.2	−481.2	−1365.9
Obs.	89	58	76	54	84

Note: The value in parentheses is the z statistic of the estimated value. *, **, *** represent 10%, 5%, and 1% significance levels, respectively (Source: this study).

Table 5 shows the Poisson regression results for Europe, America (North America and South America), Pan Pacific (Asia and Oceania), Africa, and OECD, respectively. Unlike the results in Table 4, there are much more estimates which are significantly affecting the days between two peaks of COVID-19 for each continent/organization under the counting assumption of dependent variable. Although the factors are more significant, however, their effects may be different in different continent/organization.

For example, in Europe, the population, GDP per capita, life expectancy, and ratio of hospital beds per 1000 people have a significantly positive effect on the days between two consecutive peaks of COVID-19 epidemics, however, population density, stringency index, and vaccination are significantly negative. It means that a European country with higher economic development and healthy will postpone the peak of its second wave of COVID-19 epidemic. However, the higher degree of stringency or population density, the faster the second wave of epidemic comes.

Furthermore, in America, only GDP per capita and life expectancy are significantly positively affecting the days between two consecutive peaks of COVID-19 epidemics, however, the population and density, median age, CVDs, stringency, and vaccination are significantly negative. It shows that an American country with high population and density or rich and healthy, then the second/third wave of COVID-19 epidemics will come earlier than others American countries.

In addition, the patterns of the empirical results are different between North America and South America. For instant, in North America, the higher the population or population density, the faster the second wave of epidemic comes. However, in South America, the lower the population or population density, the faster the second wave of epidemic comes. And the higher degree of economic growth, the longer the time between two consecutive peaks of COVID-19 in North America, however, it is shorter in South America. Although there are some difference between North America and South America, yet, stringency index and hospital beds per thousand people have the same direction effect on the days between two consecutive peaks both in these two continents.

Moreover, in Pan Pacific (Asia and Oceania), GDP per capita, population density, ratio of hospital beds per 1000 people, and vaccination have a significantly positive effect on the days between two consecutive peaks of COVID-19 epidemics, however, the population, stringency index, median age, and HDI are significantly negative. It means that an Asian or Oceanian country with higher economic development or abundance of medical resources will postpone the peak of its second wave of COVID-19. However, the higher degree of stringency, human capital or population, the faster the second wave of epidemic comes.

As to Africa, the population and density, GDP per capita, HDI, and CVDs have significantly positive impact on the days between two consecutive peaks of COVID-19, however, stringency index and life expectancy are significantly negative. That is, if an African country is higher population and density, richer and healthy with abundance of medical resources, then the second/third wave of COVID-19 epidemics will come later. On the other hand, the higher degree of stringency, the faster the second wave of COVID-19 epidemic comes.

In OECD, a country with higher degree of human development will postpone the peak of its second wave of COVID-19. However, the higher degree of stringency or more stable of political situation, the faster the second wave of epidemic comes in OECD.

5.2.2. Poisson Regression Results by Type of Epidemic

Next, according to the classification of the spread of the COVID-19 epidemic, the following Table 6 shows the Poisson regression results by type of epidemics.

Table 6. Poisson regression results by type.

Type	Symmetry (Type 1)	Severe (Type 2)	Slow (Type 3)	Pooled
Constant	5.7764*** (29.40)	5.0495*** (35.18)	4.2992*** (12.72)	5.1477*** (49.11)
Population (Millions)	−0.0000 (−0.40)	0.0003*** (6.95)	−0.0008** (−2.78)	−0.0000 (−0.49)
PopDen (per m^2)	−0.0005*** (−7.71)	0.0001** (18.04)	−0.0001** (−7.71)	0.00005*** (8.00)
MedAge (year)	−0.0461*** (−15.02)	−0.0136*** (−7.93)	−0.0123*** (−4.10)	−0.0219*** (−18.42)
Stringency	−0.0065*** (−10.42)	−0.0071*** (−16.90)	−0.0098*** (−15.46)	−0.0076*** (−25.62)
GDP_per_Capita (1000 $)	0.0110*** (12.96)	−0.0011* (−1.84)	0.0029*** (4.86)	0.0025*** (6.89)
Life_Exp (year)	−0.0050 (−1.28)	0.0167*** (6.23)	0.0362*** (5.50)	0.0120*** (6.11)
HDI	1.2533*** (7.76)	−0.3243*** (−3.34)	−1.1362*** (−3.71)	0.1436* (1.86)
CVDs (%)	0.0007*** (8.30)	−0.0005*** (−6.35)	−0.0001 (−0.67)	0.0000 (0.07)
HBeds 1000	0.0428*** (6.73)	0.0085*** (10.73)	−0.0017 (−0.27)	0.0106*** (14.16)
Vaccination	0.0465** (2.24)	−0.0656*** (−4.97)	−0.2611*** (−12.11)	−0.0569*** (−6.11)
Pseudo R^2	0.2856	0.1849	0.2332	0.1274
Loglikelihood	−1042.6	−2303.7	−739.1	−4823.5
Obs.	67	143	62	249

Note: The value in parentheses is the *z* statistic of the estimated value. *, **, *** represent 10%, 5%, and 1% significance levels, respectively (Source: this study).

Table 6 shows the Poisson regression results for each type classified by the two-modal bell-shaped functions. For the symmetric type (Type-1), GDP per capita, life expectancy, and ratio of hospital beds per 1000 people, and vaccination have significantly positive effect, however, population density, median age, and stringency index are significant and negative. It reveals that if a country is high economic development and human capital, and has abundance of medical resources, then the second peak up to the same level of epidemic will be prolonged. However, if a country is rich, high density of population, and high stringency index, the second peak up to the same level of epidemic will come earlier.

For the severe type (Type-2), population and density, GDP per capita, life expectancy, and ratio of hospital beds per 1000 people have significantly positive effect, however, stringency index, median age, CVDs, and vaccination are significant and negative. This means that if a country has high death rate from cardiovascular disease or higher the stringency index, then the more severe the epidemic peak will occur faster. Conversely, a country has abundance of medical resources and high economic development, the second peak of COVID-19 epidemic will occur later.

And for the slow type (Type-3), population and density, GDP per capita, and life expectancy have significantly positive effect, however, median age, stringency index, ratio of hospital beds per 1000 people, and vaccination are significant and negative. It means that the greater the population or the denser the population, and the higher the human development indicators, the slower the epidemic will come faster. On the other hand, the more developed the economy of a country, the higher the life expectancy, and the more abundant medical resources, the slower the epidemic will be delayed.

Overall, the empirical results of Poisson regression show that countries have a significant impact on the spread of the epidemic after vaccination. In terms of regional classification, European and American countries have quickly reached the second peak after vaccination, while in Asia, it is delayed. Additionally, according to the classification of the development of the epidemic, after the vaccination, some countries quickly eased the spread of the domestic epidemic, either greatly reduced the number of confirmed cases, or delayed the time of the second wave of the outbreak up to the same level of previous peak. However, in some countries, the epidemic has not decreased but increased, reaching the peak of the epidemic quickly, and it is higher than the peak of the previous wave. These results are different to that using by the multivariate regression models.

In addition, it can be observed from the data that the second wave of the epidemic will occur approximately one month after the relaxation of lockdown regulations. As the relaxation results in the decline of stringency index, such that, the stringency index are negative correlated to daily confirmed cases and then to the days between two consecutive peaks of COVID-19 epidemics. The relaxation of the epidemic alert control regulations may be due to the official belief that the domestic epidemic has gradually slowed down, but it may also cause the public to neglect prevention and lead to large-scale infections, resulting in a major outbreak. As a result, the second wave of epidemic outbreak came quickly. Table 5 and Table 6 also verify this point by the negative estimated value of the stringency index.

6. Conclusion

The COVID-19 epidemic has not only affected the lifestyles and consumer behaviors of people around the world, but also severely damaged the global economy. It has also caused the closure of many companies, and schools have been forced to close and switch to online teaching. The characteristics of the two-modal bell-shaped functions successfully described the time sequence of the second wave of COVID-19 spread, and separated it into three special types: symmetry, severe, and slow. It is the first contribution of this study.

Furthermore, according to the classification of the COVID-19 epidemic spread, this study found that demographic factors and economic development do have a significant impact on the spread of the second wave of epidemics, but they have different effects in different types. And the stringency index has a significantly negative impact. In line with the development of different types of epidemics, the negative relationship between the stringency index and the days of the second wave of epidemics has different interpretations.

One is that when the epidemic is serious and the alert level is raised, the control regulations are relatively strict and restrict activities. However, when the epidemic gradually stabilizes, the government will gradually loosen controls, people will increase their daily activities, and stores can gradually resume business, so that the economy can gradually recover. In this case, people will be due to frequent activities and losing vigilance increases the risk of catching the epidemic, and also allows the latent epidemic to spread quickly, and finally triggers a major outbreak.

The second is that when the epidemic situation is severe, all countries in the world have claimed different levels of lockdown regulations to protect people from the epidemic. At this time, the stringency index is relatively high. However, when the epidemic situation is controlled to a considerable degree, or the vaccines are emergently authorized by the government (EUA), the stringency index will be lowered, and people's daily activities and shops will be properly unlocked. And feeling the previous nervousness about being put under control after the epidemic, and even the fear of death, people also don't dare to be too careless and lack self-protection awareness to expose themselves to the risk of infection. Therefore, although there will still be sporadic cases of infection, the number is not as large as the scale of the previous severe, such that, there may be a second wave of epidemic spread, but it has eased a lot.

Moreover, as shown in the Poisson regression results, the administration of internationally qualified vaccines, such as AstraZeneca (AZ), Moderna, Pfizer-BioNTech (BNT), etc., can delay the second wave of COVID-19 outbreaks, especially in Asian and Oceania countries. Moreover, for the rapid slowdown of the epidemic, the administration of vaccines does have its effect. Therefore, this study suggests to the government's policy decision-making units to buy internationally qualified vaccines as soon as possible, to apply them to the whole people, and to increase the coverage of the second dose of the vaccines. So as to enable the people to overcome from the COVID-19 epidemic as soon as possible.

Although, the research topic of this study is still the main topic of most discussions on COVID-19, and the empirical results of this study have obtained extremely fruitful results. However, it still has some deficiencies. First, in the process of data collection, the relevant data of some countries are missing and cannot be obtained from other international organizations. As a result, in the process of data analysis, the data had to be reluctantly discarded.

Second, this study has not discussed the mutation of the coronaviruses, such as the British Alpha, India's Delta, and South Africa's Omicron which has been raging in the world recently, etc., the mutant coronaviruses has brought a new wave of infection peaks, causing the epidemic to spread. Follow-up research must take this part into consideration to achieve completeness.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A: Proof of Lemma

[Proof of Lemma 1]

1) Consider for all $x \neq 0$, then

$$\begin{aligned} H_0(\mu - x; \mu, \sigma_1, \sigma_2) &= k(\mu, \sigma_1, \sigma_2) \cdot \exp\left(-\frac{(-x)^2}{\sigma_1^2} - \frac{\sigma_2^2}{(-x)^2} \right) \cdot I_{\{x \neq \mu\}}(\mu - x) \\ &= k(\mu, \sigma_1, \sigma_2) \cdot \exp\left(-\frac{x^2}{\sigma_1^2} - \frac{\sigma_2^2}{x^2} \right) \cdot I_{\{x \neq \mu\}}(\mu + x) \\ &= H_0(\mu + x; \mu, \sigma_1, \sigma_2). \end{aligned} \tag{A1}$$

Thus, the function is symmetric to the vertical line $x = \mu$.

2) To find the mode of the function, we should find the maximum values of the function that is, we have to maximize $H_0(x; \mu, \sigma_1, \sigma_2)$.

Hence, the first-order condition is given as follows:

$$\begin{aligned} & H_0'(x; \mu, \sigma_1, \sigma_2) \\ & = k(\mu, \sigma_1, \sigma_2) \cdot \exp\left(-\frac{(x-\mu)^2}{\sigma_1^2} - \frac{\sigma_2^2}{(x-\mu)^2} \right) \cdot \left[-2\frac{(x-\mu)}{\sigma_1^2} + \frac{2\sigma^2}{(x-\mu)^3} \right] = 0. \end{aligned} \tag{A2}$$

Such that, $-2\frac{(x-\mu)}{\sigma_1^2} + \frac{2\sigma_2^2}{(x-\mu)^3} = 0$.

And, then we can find the solution is $x = \mu \pm \sqrt{\sigma_1 \cdot \sigma_2}$.

Moreover, since

$$\begin{aligned} & H_0''(x; \mu, \sigma_1, \sigma_2) \\ & = k(\mu, \sigma_1, \sigma_2) \cdot \exp\left(-\frac{(x-\mu)^2}{\sigma_1^2} - \frac{\sigma_2^2}{(x-\mu)^2} \right) \cdot \left[-2\frac{(x-\mu)}{\sigma_1^2} + \frac{2\sigma^2}{(x-\mu)^3} \right]^2 \\ & \quad + k(\mu, \sigma_1, \sigma_2) \cdot \exp\left(-\frac{(x-\mu)^2}{\sigma_1^2} - \frac{\sigma_2^2}{(x-\mu)^2} \right) \cdot \left(-\frac{2}{\sigma_1^2} - \frac{6\sigma_2^2}{(x-\mu)^4} \right). \end{aligned} \tag{A3}$$

Then, $H_0''\left(\mu \pm \sqrt{\sigma_1 \sigma_2}; \mu, \sigma_1, \sigma_2\right) = k(\mu, \sigma_1, \sigma_2) \cdot \exp\left(-\frac{\sigma_2}{\sigma_1} \right) \cdot \left(-\frac{8}{\sigma_1^2} \right) < 0$.

Therefore, the function has a maximum at $x = \mu \pm \sqrt{\sigma_1 \cdot \sigma_2}$.

[Proof of Lemma 2]

To find the mode of the function, we should find the maximum values of the function, that is, we have to maximize $H_1(x; \mu, \sigma)$.

Hence, the first-order condition is given as follows:

$$H_1'(x; \mu_1, \mu_2, \sigma_1, \sigma_2) = H_1(x; \mu_1, \mu_2, \sigma_1, \sigma_2) \cdot \left[-2\frac{(x-\mu_1)}{\sigma_1^2} + \frac{2\sigma_2^2}{(x-\mu_2)^3} \right] = 0. \tag{A4}$$

Such that, $-2\frac{(\hat{x}-\mu_1)}{\sigma_1^2} + \frac{2\sigma_2^2}{(\hat{x}-\mu_2)^3} = 0$.

And, then we can find the solution is $\hat{x} \neq \mu_2$ given by

$$(\hat{x} - \mu_1) \cdot (\hat{x} - \mu_2)^3 - \sigma_1^2 \cdot \sigma_2^2 = 0. \tag{A5}$$

Moreover, since

$$H_1''(x;\mu_1,\mu_2,\sigma_1,\sigma_2)=H_1(x;\mu_1,\mu_2,\sigma_1,\sigma_2)\cdot\left[-2\frac{(x-\mu_1)}{\sigma_1^2}+\frac{2\sigma_2^2}{(x-\mu_2)^3}\right]^2 +H_1(x;\mu_1,\mu_2,\sigma_1,\sigma_2)\cdot\left(-\frac{2}{\sigma_1^2}-\frac{6\sigma_2^2}{(x-\mu_2)^4}\right). \tag{A6}$$

Then, $H_1''(\hat{x};\mu,\sigma)=H_1(\hat{x};\mu_1,\mu_2,\sigma_1,\sigma_2)\cdot\left(-\frac{2}{\sigma_1^2}-\frac{6\sigma_2^2}{(\hat{x}-\mu_2)^4}\right)<0$.

Therefore, the function has maximum at the values of x which satisfies the equation

$$(x-\mu_1)\cdot(x-\mu_2)^3-\sigma_1^2\cdot\sigma_2^2=0. \tag{A7}$$

[Proof of Lemma 3]

Similar to the proof of Lemma 2.

Appendix B: Solve the equation $(w-\mu_1)\cdot(w-\mu_2)^3-\sigma_1^2\cdot\sigma_2^2=0$.

First, let $x\equiv w-\mu_2$, then $w=x+\mu_2$ and $w-\mu_1=x-(\mu_1-\mu_2)$.

In addition, for computational simplicity, let $\mu\equiv\mu_1-\mu_2$ and $\sigma^2\equiv\sigma_1^2\cdot\sigma_2^2$.

Then the equation $x^4-\mu\cdot x^3-\sigma^2=0$ can be rewritten as follows:

$$x^4-\mu\cdot x^3=\sigma^2$$

$$\Rightarrow\left(x^2-\frac{\mu}{2}\cdot x\right)^2=\frac{\mu^2}{4}\cdot x^2+\sigma^2. \tag{B1}$$

And, for some $y\neq 0$, we have,

$$\left(x^2-\frac{\mu}{2}\cdot x\right)^2+\left(x^2-\frac{\mu}{2}\cdot x\right)\cdot y+\frac{y^2}{4}=\frac{\mu^2}{4}\cdot x^2+\left(x^2-\frac{\mu}{2}\cdot x\right)\cdot y+\frac{y^2}{4}+\sigma^2$$

$$\Rightarrow\left[\left(x^2-\frac{\mu}{2}\cdot x\right)+\frac{y}{2}\right]^2=\left(\frac{\mu^2}{4}+y\right)\cdot x^2-\frac{\mu y}{2}\cdot x+\left(\frac{y^2}{4}+\sigma^2\right). \tag{B2}$$

Such that, the determinant of the right-hand-side of (B2) is given as follows:

$$\Delta=\left(-\frac{\mu y}{2}\right)^2-4\cdot\left(\frac{\mu^2}{4}+y\right)\cdot\left(\frac{y^2}{4}+\sigma^2\right)=0. \tag{B3}$$

That is,

$$-y^3-4\sigma^2\cdot y-\mu^2\sigma^2=0. \tag{B4}$$

Moreover, the determinant of Equation (B4) is

$$D=\left(\frac{4\sigma^2}{3}\right)^3+\left(\frac{\mu^2\sigma^2}{2}\right)^2=\frac{64\sigma^6}{27}+\frac{\mu^4\sigma^4}{4}>0,\ \text{for } \sigma\neq 0. \tag{B5}$$

Such that, the Equation (B4) has only one real root, which is given as follows:

$$y_0=\sqrt[3]{-\frac{\mu^2\sigma^2}{2}+\sqrt{D}}+\sqrt[3]{-\frac{\mu^2\sigma^2}{2}-\sqrt{D}} =\sqrt[3]{-\frac{\mu^2\sigma^2}{2}+\sqrt{\frac{64\sigma^6}{27}+\frac{\mu^4\sigma^4}{4}}}+\sqrt[3]{-\frac{\mu^2\sigma^2}{2}-\sqrt{\frac{64\sigma^6}{27}+\frac{\mu^4\sigma^4}{4}}}. \tag{B6}$$

As a result, Equation (B2) can be rewritten as follows:

$$\left[\left(x^2-\frac{\mu}{2}\cdot x\right)+\frac{y_0}{2}\right]^2=\left(\frac{\mu^2}{4}+y_0\right)\cdot\left(x-\frac{\mu y_0}{\mu^2+4y_0}\right)^2. \quad \text{(B7)}$$

Hence,

$$\left(x^2-\frac{\mu}{2}\cdot x\right)+\frac{y_0}{2}=\pm\sqrt{\frac{\mu^2}{4}+y_0}\cdot\left(x-\frac{\mu y_0}{\mu^2+4y_0}\right). \quad \text{(B8)}$$

Moreover,

$$x^2-\left(\frac{\mu}{2}\pm\sqrt{\frac{\mu^2}{4}+y_0}\right)\cdot x+\left(\frac{y_0}{2}\pm\frac{2\mu y_0}{\sqrt{\mu^2+4y_0}}\right)=0. \quad \text{(B9)}$$

Therefore, the solution of Equation (B9) is given as follows:

$$\hat{x}=\frac{\left(\frac{\mu}{2}\pm\sqrt{\frac{\mu^2}{4}+y_0}\right)\pm\sqrt{\left(\frac{\mu}{2}\pm\sqrt{\frac{\mu^2}{4}+y_0}\right)^2-4\cdot\left(\frac{y_0}{2}\pm\frac{2\mu y_0}{\sqrt{\mu^2+4y_0}}\right)}}{2}, \quad \text{(B10)}$$

where y_0 is defined in Equation (B6).

Furthermore, the solution of equation $(w-\mu_1)\cdot(w-\mu_2)^3-\sigma_1^2\cdot\sigma_2^2=0$ is given by

$$\hat{w}=\mu_2+\frac{\left(\frac{\mu}{2}\pm\sqrt{\frac{\mu^2}{4}+y_0}\right)\pm\sqrt{\left(\frac{\mu}{2}\pm\sqrt{\frac{\mu^2}{4}+y_0}\right)^2-4\cdot\left(\frac{y_0}{2}\pm\frac{2\mu y_0}{\sqrt{\mu^2+4y_0}}\right)}}{2}. \quad \text{(B11)}$$