Scientific Research Publishing

Theoretical Economics Letters, 2026, 16(6), 1137-1158
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Analyzing the Dimensions of Altman Z'-Model

Erkki K. Laitinen, Teija Laitinen

Auditing and Control in Accounting (ACA) Research Group, School of Accounting and Finance, University of Vaasa, Vaasa, Finland
Email: erkki.k.laitinen@uwasa.fi

How to cite this paper: Laitinen, E. K., & Laitinen, T. (2026). Analyzing the Dimensions of Altman Z'-Model. *Theoretical Economics Letters, 16,* 1137-1158.
https://doi.org/10.4236/tel.2026.166062

Received: June 16, 2026
Accepted: November 6, 2026
Published: November 9, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

The objective of the study is to analyze the underlying operational and financial fundamentals of the five financial ratios in Altman's model for private firms and their weighted composite index, Z'-Score. This analysis is made using a mathematical long-term steady-state model. The model is based on six parameters: internal rate of return, growth rate, revenue lag, fraction of debt in external capital raised, and ratios of current assets and current liabilities. The study uses partial derivatives and elasticities, complemented by numerical analyses to show the sensitivity of the five ratios and Z'-Score to parameters in both non-failed and failed firm contexts. The results show that internal rate of return, revenue lag, and growth rate exert the dominant influence on the five ratios and Z'-Score. The study offers a new perspective for assessing the applicability and limitations of the Altman Z'-Score model in private-firm settings.

Keywords

Altman Z'-Score, Financial Distress Prediction, Private Firms, Financial Ratios, Profitability, Growth, Sensitivity Analysis, Mathematical Model

1. Introduction

Predicting corporate financial distress has been a cornerstone of empirical finance and accounting literature for over half a century. Ever since the seminal work of Altman in 1968, multivariate discriminant analysis has provided practitioners and academics with robust tools to assess default probabilities. While the original Z-Score was engineered for publicly traded entities by relying on the market value of equity, practical risk assessment in private markets required structural adaptation. This led to the development of Altman's revised Z-Score model (Z'), which substitutes the market value of equity with its book value (Altman, 1983). This Z'-Score model remains globally one of the most widely used failure prediction models.

Traditional failure research on Z-Score heavily focuses on statistical prediction accuracy, classification errors, re-estimation, and the incremental predictive power of adding new variables (Altman, Iwanicz-Drozdowska, Laitinen, & Suvas, 2017). Consequently, a profound gap exists in the literature regarding the exact structural and operational mechanics that drive these accounting ratios and their composite index, Z'-Score. Traditional financial analysis observes the symptoms, the accounting ratios, rather than the root economic causes. Thus, there is a lack of rigorous, mathematical frameworks that can map how core entrepreneurial fundamentals directly translate into the Z'-Score components. These fundamental parameters include true economic profitability (internal rate of return), corporate (steady) growth rate, and revenue lag.

To address this gap, the objective of this study is twofold: first, to analyze the foundational operational and financial fundamentals affecting the five individual financial ratios of the Altman private firm Z-model (that is, Z'-model), and second, to investigate how these underlying parameters jointly dictate the composite, weighted Z-Score. The methodology of this study is analytical and deterministic. The starting point of the analysis is to develop a steady-state mathematical model of the firm based on fundamental multi-period flow assumptions. From this model, algebraic approximations for each of the five Z'-Score ratios are derived as a function of the fundamental parameters.

Because financial ratios remain constant over time under a steady uniform growth assumption, they allow for precise comparative static analysis. The sensitivity of the ratios and the overall Z'-Score is analyzed here using partial derivatives and analytical elasticities. Furthermore, numerical experiments across specific illustrative contexts, such as median non-failed and failed firms, are performed to observe the differences in partial derivatives and elasticities between these contexts.

However, the results of the study are simplified and based on the long-run steady-state analysis assuming that the ratios of the model variables stay constant over time. Therefore, the results should be interpreted cautiously before making any generalizations or recommendations for the risk behavior of the firm. It is also important to note that the object of analysis is a statistically estimated risk index, not financial bankruptcy risk itself. Therefore, it is not always possible to give strict recommendations about behavior under financial risk on the basis of a potentially imperfect measure.

The primary contribution of this research is the theoretical demystification of empirical bankruptcy models. When the study shifts the perspective from statistical association to functional relationship, this paper shows that true economic underlying profitability and revenue lag exert a mathematically dominant effect on the Z-Score, occasionally generating counter-intuitive adjustments in risk metrics. The significance of the growth effect however depends on the difference between growth rate and internal rate of return. The smaller the difference, the stronger the sensitivity of the ratios and Z'-Score to growth rate.

The content of this paper is divided into five sections. This introductory section presents shortly the motivation and objectives of the study. Section 2 summarizes prior literature on the Z-Score model, focusing particularly on the Z' variant developed for private firms. Section 3 derives the mathematical model and the analytical forms of the five financial ratios comprising the Z'-Score. Section 4 presents numerical experiments for both the individual financial ratios and the overall Z'-Score. Finally, Section 5 draws conclusions from the findings and outlines directions for future research.

2. Prior Research

2.1. Z'-Score for Private Firms

The development of corporate default prediction models began with univariate ratio analysis pioneered by Beaver (1966), who demonstrated that financial ratios exhibited significant discriminant power between failing and non-failing firms. Altman (1968) advanced this field by applying Multiple Discriminant Analysis (MDA) to a sample of manufacturing firms, combining liquidity, cumulative profitability, operating efficiency, solvency, and asset turnover into a single composite metric known as the original Z-Score.

Because the original 1968 Z-Score required market capitalization to calculate the equity-to-debt ratio (X4), its application was limited to publicly traded companies. Therefore, Altman (1983) re-estimated the discriminant function for private manufacturing companies, establishing the Z'-Score model:

$$Z' = 0.717 \cdot \mathrm{X1} + 0.847 \cdot \mathrm{X2} + 3.107 \cdot \mathrm{X3} + 0.420 \cdot \mathrm{X4} + 0.998 \cdot \mathrm{X5} \quad (1)$$

where X1 is Working Capital/Total Assets, X2 is Retained Earnings/Total Assets, X3 is EBIT/Total Assets, X4 is Book Value of Equity/Total Liabilities, and X5 is Sales/Total Assets.

The interpretation of the model is based on the linear relationship between the five financial ratios and the resulting weighted Z'-Score. The weights of the ratios differ from each other, beginning from 0.420 (X4) to 3.107 (X3). The linear model is simple, transparent and intuitively easy to interpret, calculate, and use. The interpretation of the Z'-Score is based on the threshold values for the score. If Z' > 2.9, the target firm is expected to have good financial stability, so that there is only a minimal chance that it will fail ("Safe Zone"). If Z' is between 1.23 and 2.9, it is interpreted that the risk of facing financial difficulties soon is low ("Gray Area"). Finally, if Z' is less than 1.23, the firm is likely to face financial challenges in the near future ("Distress Zone").

Because of the lack of a private company database, Altman did not test the performance of the Z'-Score model on a secondary sample. However, he analyzed the accuracy of a four-variable Z'-Score model that excluded the Sales/Total Assets ratio, X5, from the revised model. Altman excluded X5 because of a potential industry effect that makes the score report financial difficulties more likely to take place in low asset-turnover industries, when this kind of industry-sensitive varia-

ble (Asset Turnover) is included in the model. Thus, Altman then estimated the four-variable Z'-Score model without Asset Turnover ratio (Altman, 1983). The Z'-Score is designed especially for non-manufacturing firms and emerging markets.

Based on a literature survey, Altman, Iwanicz-Drozdowska, Laitinen, and Suvas (2017) showed that the Z-Score model (for publicly traded firms), the Z'-Score model (for private manufacturing firms), and the Z'-Score model (for private and publicly traded manufacturing and non-manufacturing firms) have been widely adapted for different purposes. Typically, Z-Score was either used as a failure prediction proxy or assessed in terms of predictive ability, serving as the measure of distress or of financial strength. In several studies, Altman's original model was modified or verified, including re-estimation, or it was used for the robustness check. Pindado, Rodrigues, and de la Torre (2008) noted that Z-Score is also used for the evaluation of the costs and benefits of covenants in bonds, the choice of debt type (bank versus non-bank, private or public), and the relationship between investment and internal funds.

Altman, Iwanicz-Drozdowska, Laitinen, and Suvas (2017) concluded that the broad use of the Z-Score model for measuring financial distress and performing robustness checks indicates its acceptability as a reasonable, simple, and consistent measure of financial distress. For private firms, Altman's Z'-Score model is one of the most influential bankruptcy prediction models in financial analysis. In spite of the emergence of such advanced methods as machine learning, survival analysis, and advanced credit-risk models, the Z'-Score remains widely used in both academic research and practice. Recent literature demonstrates that researchers continue to employ the model extensively as a benchmark or explanatory variable. Studies on unlisted SMEs also re-affirm that linear discriminant models like Z'-Score may perform competitively when calibrated properly, mainly referring to re-estimation of the coefficients. This re-estimation however does not always make the revised model differ significantly from the original Z'-Score model (Káčer, Ochotnický, & Alexy, 2019).

In conclusion, the Z'-Score remains one of the most widely used bankruptcy prediction models globally. Its popularity is sustained by transparency, interpretability, and low implementation costs. Most financial distress studies use the model as an accepted indicator of financial distress rather than evaluating its validity. The literature is therefore heavily weighted toward application rather than critical assessment. In this study, the approach is very different from prior empirical failure studies on Z'-Score. While the predictive accuracy, classification thresholds, and statistical properties of distress models have been thoroughly investigated, a major gap persists in the literature: existing studies treat ratio components as exogenous statistical inputs rather than endogenous outcomes of fundamental firm operations. In this study, fundamental parameters such as internal rate of return, corporate growth rate, revenue lag, the fraction of debt in external capital raised, and the ratios of current assets and current liabilities are used to assess Z'-Score

and its five financial ratios.

2.2. Mathematical Model

This study does not follow the mainstream of the empirical research on Z'-Score model, which is concentrated on the predictive accuracy, classification thresholds, and statistical properties of the model. The purpose of the study is to analyze the Z'-Score and its five financial ratios mathematically, utilizing an approach where they are in a steady framework described by fundamental economic variables. This kind of approach makes it possible to assess the sensitivity of the financial ratios and Z'-Score to these economic variables. The approach belongs to the family of steady-state models, which focus on the financial ratios as generated by the internal rate of return (true profitability), a growth rate, and a lag structure that distributes the revenue contributions of expenditures over future periods (Laitinen, 1980; Feenstra & Wang, 2000; Brief, 2013).

In this research line, this approach is related to the studies that examine the potential bias in the reported return on investment ratio (ROI) as an approximation of IRR (Harcourt, 1965; Solomon, 1966; Laitinen, 1980; Peasnell, 1982; Luckett, 1984; Feenstra & Wang, 2000; Brief, 2013). Solomon (1966) concluded that ROI is not an accurate measure of IRR, and that the error is neither constant nor systematic. ROI equals IRR only if the growth rate of the firm matches its internal rate of return. Feenstra and Wang (2000: p. 4) conclude the analytical findings of this research and characterize them as generally pessimistic. Studies analyze the behavior of firms investing usually in consecutive projects under alternative assumptions regarding depreciation policy and growth. The findings indicate that, unless very restrictive assumptions are set, ROI does not accurately measure IRR, the errors do not follow systematic patterns, and the resulting biases can be significant.

This study however does not concentrate on the relationship between ROI and IRR. It has a wider scope, following the approach by Laitinen (1980, 2025) to analyze a wider set of financial ratios. Following Laitinen (2025), this approach adopts a model structure similar to that of the research line to derive long-term (steady) financial ratios. In this study, the five financial ratios standing in Z'-Score model are derived. Accordingly, the results describe the steady-state values that financial ratios are assumed to converge over time.

The framework investigates revenue flows at the firm level by applying an infinite geometric lag structure between expenditures and the resulting revenue stream. In a steady-state setting, all flows grow at the same constant rate, and the ratios of the flows stay constant. In addition to the IRR and the growth rate, the model has the fraction of debt in external capital raised, and the ratios of current assets and liabilities as fundamental parameters. These parameters make it possible to derive approximations for the five financial ratios in the Z'-Score model. For the sake of simplicity, the framework excludes taxes, interest expenses, and dividends.

3. Derivation of the Ratios in Z'-Score Model

3.1. Basic Concepts

The main part of the basic concepts of the framework is derived following the presentation in Laitinen (2025). This steady-state framework assumes that each periodic expenditure will generate an infinite flow of revenues in a constant relation to the expenditure. The revenue contributions generated by expenditure follow an infinite geometric distribution with lag parameter q. Thus, based on the infinite geometric distribution, the average lag between expenditure and generated revenue flow is $q/(1-q)$.

The steady-state assumption makes it possible to derive the revenue of the firm in period t when applying the infinite geometric lag function in the following way:

$$R_t = K \cdot M_t \sum_{i=0}^{\infty} (1+g)^{-i} q^i = K \cdot M_t \cdot \frac{1+g}{1+g-q} \tag{2}$$

In Equation (2), R_t is revenue, M_t is expenditure, K is the level of the revenue flow in a relation to expenditure, g is the steady growth rate of expenditure, and q is the lag parameter. The sum of lagged revenue contributions in (2) converges for infinite t if $q < 1 + g$. In practice, the growth rate is typically positive but can have a large variety depending on the growth strategy of the firms.

Using Equation (2), the internal rate of return (IRR) can be included into the model to solve the level K as follows:

$$M_t = M_t \cdot K \sum_{i=0}^{\infty} q^i (1+r)^{-i} \rightarrow K = \frac{1+r-q}{1+r} \tag{3}$$

In Equation (3), r is the constant IRR. The sum of the lagged contributions in (3) converges if $q < 1+r$.

Substitution of K in (3) into (2) gives the steady revenue-expenditure ratio such as:

$$\frac{R_t}{M_t} = \frac{R}{M} = \frac{(1+r-q)(1+g)}{(1+r)(1+g-q)} \tag{4}$$

Equation (4) shows that the revenue of the firm in period t exactly covers the periodic expenditure if $r = g$. It also shows that the firm can finance its expenditure by revenue and does not need external capital. However, if $g > r$, it follows that $R/M < 1$, and the firm needs external capital M-R to cover total expenditure. If $g < r$, it leads to that $R/M > 1$.

Equations (2) - (4) together show that the change in the external capital can be presented as follows:

$$\Delta C_t = M_t - R_t = M_t \cdot \frac{q(g-r)}{(1+r)(1+g-q)} \tag{5}$$

Equation (5) shows that $\Delta C_t = 0$ if $r = g$, > 0 if $r < g$, and < 0 if $r > g$ following R/M in Equation (4).

The steady-state model makes it possible to transform flow concepts to stock concepts and vice versa in the following way:

$$\text{Stock} = \text{Flow} \cdot \frac{1+g}{g} \tag{6}$$

Thus, Equation (6) shows that the external capital of the firm (stock) can be derived from the change (flow) as:

$$C_t = M_t \cdot \frac{q(g-r)}{(1+r)(1+g-q)} \cdot \frac{1+g}{g} \tag{7}$$

Equation (7) shows that the stock of external capital is a function of parameters g, q, and r. In this framework, it is assumed that the external capital may consist of debt and equity. Let us denote the share of debt in the increase of external capital by B. If $g > r$, $D_t = B \cdot C_t$ refers to the debt capital in the balance sheet while $S_t = (1 - B) \cdot C_t$ refers to the (external) equity invested.

The asset side of the balance sheet can be derived assuming an expensing method for total expenditure. The most neutral assumption is that expenditures expire by the same rate as revenues are accumulated over periods (Laitinen, 2025). In that case, the lag distribution of expenses also follows the lag parameter q. Therefore, the periodic expense is calculated as follows:

$$E_t = (1-q) \cdot M_t \sum_{i=0}^{\infty} (1+g)^{-i} q^i = M_t \cdot \frac{(1+g)(1-q)}{1+g-q} \tag{8}$$

The sum in Equation (8) converges if $q < 1 + g$.

Expenses E_t can be used to calculate assets according to the accounting identity using the following procedure (Laitinen, 2025):

$$E_t = A_{t-1} - A_t + M_t \rightarrow A_t = (M_t - E_t) \cdot \frac{1+g}{g} \tag{9}$$

Substituting E_t in (8) into (9) leads to the following result for the assets at the end of the period t:

$$A_t = q \cdot M_t \sum_{i=0}^{\infty} (1+g)^{-i} q^i = M_t \cdot \frac{q(1+g)}{(1+g-q)} \tag{10}$$

The geometric sum in Equation (10) converges if $q < 1 + g$.

The periodic profit of the firm can be derived from subtracting expenses from the revenue, such as:

$$\Delta P_t = R_t - E_t = M_t \cdot \frac{rq(1+g)}{(1+r)(1+g-q)} \tag{11}$$

In (11), ΔP_t refers to the periodic profit that is a function of r, g, and q.

The periodic profit in (11) can through Equation (6) be transformed to retained earnings, leading to:

$$P_t = M_t \cdot \frac{rq(1+g)^2}{(1+r)(1+g-q)g} \tag{12}$$

Equation (12) indicates that retained earnings P_t are closely related to the growth rate g but also related to r and q. If $r < 0$, P_t is negative.

Finally, current assets and current liabilities are estimated by assuming that they maintain constant proportions of total revenue and total expenditure, respectively. Specifically, current assets (e.g., cash and cash equivalents, accounts receivable, and inventories) are assumed to be proportional to revenue, with a constant coefficient ($b1$). Accordingly, current assets can be estimated based on revenue using Equation (6), as follows:

$$\mathrm{CA} = b1 \cdot R_t = b1 \cdot M_t \cdot \frac{(1+g)(1+r-q)}{(1+g-q)(1+r)} \tag{13}$$

In Equation (13), CA denotes current assets, which are assumed to vary proportionally with revenue.

Similarly, current liabilities (e.g., short-term debt and accounts payable) are assumed to be proportional to total expenditure, with a constant coefficient ($b2$). Accordingly, current liabilities can be estimated based on expenditure as follows:

$$\mathrm{CL} = b2 \cdot M_t \tag{14}$$

In Equation (14), CL denotes current liabilities, which are assumed to vary proportionally with expenditure.

3.2. Financial Ratios

The five financial ratios in Z'-Score model can be derived from the concepts in Equations (2) - (14). First, let us derive Working Capital Ratio (X1) that reflects the liquidity of the firms. In this framework, this financial ratio is approximated by the ratio of the difference between current assets and current liabilities divided by total assets. The use of Equations (13), (14), and (10) leads to the following result for the ratio X1:

$$\mathrm{X1} = \frac{\mathrm{CA} - \mathrm{CL}}{\mathrm{TA}} = b1 \cdot \frac{(1+r-q)}{q(1+r)} - b2 \cdot \frac{(1+g-q)}{q(1+g)} \tag{15}$$

Equation (15) shows that current assets (CA) depend on profitability r whereas current liabilities (CL) are independent of r but closely associated with growth g.

If $b1 = b2 = b$, then X1 is simplified to the following ratio:

$$\mathrm{X1} = \frac{b(r-g)}{(1+r)(1+g)} \tag{16}$$

In this simplified form, the sign of X1 depends on the difference between r and g. If $g > r$, X1 is negative, referring to that current liabilities exceed current assets. If $g = r$, X1 is equal to zero.

Second, let us derive Retained Earnings to Total Assets Ratio (X2) that measures cumulative profitability. This ratio can be derived by dividing retained earnings in Equation (12) by total assets in (10), which leads to:

$$\mathrm{X2} = \frac{P_t}{A_t} = \frac{r(1+g)}{g(1+r)} \tag{17}$$

This ratio (17) is independent of the lag parameter q but closely related to ratio

between internal rate of return r and growth rate g.

The third ratio X3 measures periodic profitability and is defined as Earnings Before Interest and Taxes (EBIT) divided by total assets. This ratio can be calculated by dividing EBIT (profit) (11) by total assets (10) on the beginning of the year basis as follows:

$$\mathrm{X3} = \frac{\Delta P_t}{A_{t-1}} = r \cdot \frac{1+g}{1+r} \tag{18}$$

Equation (18) clearly shows that X3 gives an approximation of IRR that is biased due to the effect of growth rate g. However, if r and g are close to each other, this bias is negligible.

The fourth ratio X4 refers to the financial structure of the firm. This ratio is defined as the ratio of Book Value of Equity to Total Liabilities. In this framework, total liabilities are derived by multiplying external capital in (7) by the fraction of debt B. Then, book value of equity is derived by subtracting total liabilities from total assets in (10). The financial ratio X4 is finally obtained in the following way:

$$\mathrm{X4} = \frac{A_t - D_t}{D_t} = \frac{A_t}{D_t} - 1 = \frac{(1+r)g}{B(g-r)} - 1 \tag{19}$$

In Equation (19), D_t refers to total debt (liabilities). This ratio X4 is sensitive to the difference between g and r.

The fifth ratio X5 refers to asset turnover being dependent on industry or technology. This ratio can be derived simply by solving revenue (sales) from Equations (2) and (3) and dividing it by total assets in (10), which leads to the following result:

$$\mathrm{X5} = \frac{R_t}{A_t} = \frac{(1+r-q)}{(1+r)} \cdot \frac{1}{q} = \frac{1}{q} - \frac{1}{1+r} \tag{20}$$

Equation (20) shows that X5 is independent of g but is strongly related to the revenue lag q, reflecting industry effects.

4. Numerical Experiments with the Model

4.1. Sample of Firms

The primary objective of the numerical part is to demonstrate empirically the theoretical results on the effects of model parameters on the five financial ratios X1 - X5 and Z'-Score. The main attention is concentrated on the sensitivity of the ratios and the score, to these parameters. This sensitivity is assessed in two different contexts with respect to performance: average active firm and average bankruptcy (distressed) firm. The numerical cases presented in the study are profiles assembled from separate parameter medians. Thus, the calculated financial ratios and Z'-Scores considered here do not need to be equal to the medians of observed ratios and scores reported by the firms.

The empirical data used to approximate median values of parameters are extracted from the set of Finnish limited companies in ORBIS database from a period before the COVID pandemic. The sampling was not random, since all com-

panies in the database fulfilling the limitations set by the study were included in the sample. The latest financial statements in the data are from years 2015 and 2016. In Orbis database, firms are classified into six classes: active, active (insolvency proceedings), bankruptcy, dissolved, dissolved (merger or take-over), and liquidation. Bankruptcy code generally refers to a code or status indicating a company's bankruptcy-related situation. However, active code means that the company is classified as active and has not been assigned an inactive status, such as bankruptcy, liquidation, or dissolution. In this study, the sample consists of only active (non-failed) and bankruptcy (failed) firms.

The sample firms are Finnish industrial limited companies which in the last reporting year report the total of the balance sheet more than EUR 1 million. In addition, it was required that the firm had in the database financial statements at least from ten last years. The initial sample included in total 12,969 active industrial firms. The number of firms, however, decreased finally to 4099 firms because of the technical limitation that the growth rate g must in the long run exceed the profitability rate r. This limitation is used to ensure that the empirical results are consistent with the theoretical results. In the same way, the sample of bankruptcy firms included originally 552 firms but the number of firms fulfilling the technical limitation $g > r$, was only 278. Thus, the percent of firms fulfilling the limitation was higher for bankruptcy than for active firms. The data also included missing values for a number of companies, leading to the final estimation sample for active firms of 3232 firms and for bankruptcy of 254 firms. The size of the final sample is not decisive here because only the medians of the parameters are used in the numerical experiments.

The sample of non-failed firms is more positively skewed than the sample of failed firms. The median number of employees in the non-failed group is only 16, whereas the average number is as high as 111.58. Failed (bankruptcy) firms are generally very small, since the median number of employees is only 12 and the average number is 14.12. Failed firms are largely concentrated on industries such as wholesale & retail trade, manufacturing, and construction, whereas non-failed firms are more evenly distributed across different industries. In all, 74.1% of the failed firms belong to one of these three industries, while the percentage is only 48.3% for non-failed firms.

4.2. Parameter Values

The model parameters r, g, and q are approximated using ten-year time series following the methods used by Laitinen (2025) and Laitinen and Laitinen (2022). First, the growth estimate g is calculated by dividing the sum of net sales (revenues) from the last five years by the sum of revenues from the first five years, taking the fifth root of the result, and subtracting 1 (unity). Second, the approximation of the lag parameter q is more complicated. Thus, only a simple estimation method is applied. The first step was to approximate the average lag between expenditures and revenues by dividing the sum of balance sheet totals over a ten-year period by the

sum of revenues for the same period. The result of this approximation indicates how long revenues on average remain tied up in the balance sheet, reflecting revenue lag. The approximated value of parameter q is then calculated by dividing this average lag by lag plus unity according to the average of distributed lag function $q/(1-q)$. This parameter is closely associated with the concept of capital intensity that measures the amount of capital employed relative to revenue.

Third, a rough estimate of profitability r (IRR) is calculated simply as a weighted average by dividing the sum of the profits of the accounting period over the last ten years by the sum of balance sheet totals for the same period. It is clear that accounting policy in terms of depreciation method affects the reported accounting rate of return (ARR), making it deviate from the economic rate of return (IRR). However, in the long run, the deviations at least partly cancel each other so that the weighted average of ARRs can be used to overcome accounting policies that may distort reported profits (Kay, 1976; Peasnell, 1982). Fourth, parameter $b1$ is calculated by dividing the sum of current assets over the ten-year period by the corresponding sum of total revenue. Fifth, the parameter $b2$ is approximated by dividing the ten-year sum of current liabilities by the sum of total expenditure. Finally, the fraction B of debt in external financing raised is approximated by dividing the ten-year change of liabilities by the change of the sum of liabilities and equity capital.

Table 1 presents statistics for the approximate values of parameters in non-failed (active) and failed (bankruptcy) firms. In the experiments, only the median values of the parameters are used. The median liquidity parameters $b1$ and $b2$ are relatively equal in both groups of firms. The median fraction of debt B in external capital raised is very close to unity for both non-failed (0.9847) and failed (0.9885) firms, referring to the minor role of equity finance in the sample. The median IRR or r is positive but quite low for non-failed firms (0.0241), and negative for failed firms (−0.0457). The median growth rate g for non-failed firms is relatively close to ten percent (0.0944), but significantly lower for failed firms (0.0632). The median revenue lag parameter q is higher in non-failed firms (0.4053) than in failed firms (0.3409). These values refer to average revenue lags of 0.6815 and 0.5172 years, respectively.

Table 1. Statistics of the parameter values in non-failed (n = 3232) and failed (254) Finnish firms.

Parameter	Description of parameter	Non-failed firms			Failed firms		
		25%	Median	75%	25%	Median	75%
$b1$	Current assets to revenue	0.2138	**0.3254**	0.4891	0.2293	**0.3134**	0.4457
$b2$	Current liabilities to expenditure	0.1828	**0.2530**	0.3634	0.2231	**0.2975**	0.4141
B	Fraction of debt in external capital raised	0.9331	**0.9847**	0.9940	0.9740	**0.9885**	0.9960
r	Internal rate of return	−0.0005	**0.0241**	0.0584	−0.0900	**−0.0457**	−0.0129
g	Growth rate	0.0463	**0.0944**	0.1679	0.0170	**0.0632**	0.1554
q	Revenue lag parameter	0.3116	**0.4053**	0.5613	0.2791	**0.3409**	0.4249

Table 2 presents the values of financial ratios X1 - X5, and Z'-Score calculated by the model using the median parameter values in Table 1. The weighted financial ratios show that Sales to total assets ratio X5 plays an important role in Z'-Score. However, in spite of the high weight (3.107), the positive effect of EBIT to total assets X3 on Z'-Score is low in non-failed firms whereas in failed firms, the effect is negative and strong. Retained earnings to total assets ratio X2 and Book value of equity to total liabilities X4 show a relatively significant effect on Z'-Score, positively in non-failed firms and negatively in failed firms. The effect of Working capital to total assets ratio X1 is weak, particularly in failed firms where the ratio is close to zero. The Z'-Scores calculated using the values of the financial ratios X1 - X5 refer in non-failed firms to Gray zone (2.0315 < 2.90) and in failed firms to Distress zone (0.8551 < 1.23).

Table 2. Calculated financial ratios X1 to X5 and Z'-Score for non-failed and failed firms.

Code	Financial ratio	Weight code	Weight	Non-failed firms		Failed firms	
				Ratio value	Weighted value	Ratio value	Weighted ratio
X1	Working capital to total assets	*a*1	0.717	0.0921	0.0660	−0.0020	−0.0014
X2	Retained earnings to total assets	*a*2	0.847	0.2728	0.2311	−0.8056	−0.6824
X3	EBIT to total assets	*a*3	3.107	0.0258	0.0800	−0.0509	−0.1582
X4	Book value of equity to total liabilities	*a*4	0.420	0.3965	0.1665	−0.4397	−0.1847
X5	Sales to total assets	*a*5	0.998	1.4908	1.4879	1.8855	1.8818
	Z'-Score				2.0315		0.8551
	Zone				Gray		Distress

4.3. Partial Derivatives and Elasticities

Table 3 presents the partial derivatives and elasticities of Working capital to total assets ratio X1 with respect to the five model parameters. These results indicate that X1 is strongly sensitive, especially to the liquidity parameters *b*1 and *b*2. The elasticities of X1 for the sample of failed firms are high because the value of X1 is in this sample close to zero. Table 4 shows the results for Retained earnings to assets ratio X2. This financial ratio is very sensitive to internal rate of return *r* but also to growth rate *g*. The partial derivative of the ratio with respect to *g* is however negative for non-failed firms, but positive for failed firms, indicating different roles of the parameter in these samples.

Table 5 presents the results for Earnings before interest and tax to total assets ratio X3. This important profitability ratio is very sensitive to internal rate of return *r*. It is also slightly affected by growth *g*. The signs of the partial derivative with respect to *g* in non-failed and failed firms are again different due to different signs of IRR. Table 6 shows the findings for Book value of equity to total liabilities ratio X4. This ratio is most sensitive to *r* and *g* followed by *B*. For this ratio also, the derivatives show a different sign in non-failed and failed firms. For non-failed firms,

Table 3. Partial derivatives and elasticities of working capital to total assets (X1).

Parameter	Non-failed firms		Failed firms	
	Partial derivative	Elasticity	Partial derivative	Elasticity
$b1$	1.4908	5.2692	1.8855	−302.8258
$b2$	−1.5536	−4.2692	−1.9929	303.8258
B	0.0000	0.0000	0.0000	0.0000
r	0.3103	0.0812	0.3441	8.0595
g	−0.2112	−0.2166	−0.2632	8.5239
q	−0.4407	−1.9402	−0.1368	23.9019

Table 4. Partial derivatives and elasticities of retained earnings to total assets (X2).

Parameter	Non-failed firms		Failed firms	
	Partial derivative	Elasticity	Partial derivative	Elasticity
$b1$	0.0000	0.0000	0.0000	0.0000
$b2$	0.0000	0.0000	0.0000	0.0000
B	0.0000	0.0000	0.0000	0.0000
r	11.0540	0.9765	18.4726	1.0479
g	−2.6408	−0.9137	11.9894	−0.9406
q	0.0000	0.0000	0.0000	0.0000

Table 5. Partial derivatives and elasticities of earnings before interest and tax to total assets (X3).

Parameter	Non-failed firms		Failed firms	
	Partial derivative	Elasticity	Partial derivative	Elasticity
$b1$	0.0000	0.0000	0.0000	0.0000
$b2$	0.0000	0.0000	0.0000	0.0000
B	0.0000	0.0000	0.0000	0.0000
r	1.0435	0.9765	1.1675	1.0479
g	0.0235	0.0863	−0.0479	0.0594
q	0.0000	0.0000	0.0000	0.0000

Table 6. Partial derivatives and elasticities of book value of equity to total liabilities (X4).

Parameter	Non-failed firms		Failed firms	
	Partial derivative	Elasticity	Partial derivative	Elasticity
$b1$	0.0000	0.0000	0.0000	0.0000
$b2$	0.0000	0.0000	0.0000	0.0000
B	−1.4182	−3.5218	−0.5668	1.2741

Continued

r	21.2292	1.2902	5.7319	0.5957
g	−5.0716	−1.2073	3.7202	−0.5347
q	0.0000	0.0000	0.0000	0.0000

the derivative is negative, whereas for failed firms it is positive. Table 7 presents the results for Sales to total assets ratio X5. This ratio is negatively very sensitive to revenue lag q, and slightly to internal rate of return r.

Table 7. Partial derivatives and elasticities of sales to total assets (X5).

Parameter	Non-failed firms		Failed firms	
	Partial derivative	Elasticity	Partial derivative	Elasticity
$b1$	0.0000	0.0000	0.0000	0.0000
$b2$	0.0000	0.0000	0.0000	0.0000
B	0.0000	0.0000	0.0000	0.0000
r	0.9535	0.0154	1.0981	−0.0266
g	0.0000	0.0000	0.0000	0.0000
q	−6.0876	−1.6550	−8.6049	−1.5558

The mathematical formulas for partial derivatives of Z'-Score are presented in Table A1. Since Z'-Score is a linear weighted function, also partial derivatives are weighted sums of the derivatives of the financial ratios X1 - X5. Table 8 presents the partial derivatives and elasticities of Z'-Score with respect to the parameters of the model. The results in Table 8 indicate for both groups of firms that profitability r clearly plays the central role in the sensitivity of the score. In addition to profitability, also revenue lag q and growth rate g have a significant impact on Z'-Score. The partial derivative of q is negative in non-failed firms but positive in

Table 8. Partial derivatives and elasticities of Altman's Z'-Score.

Parameter	Non-failed firms		Failed firms	
	Partial derivative	Elasticity	Partial derivative	Elasticity
$b1$	1.0689	0.1712	1.3519	0.4955
$b2$	−1.1139	−0.1387	−1.4289	−0.4971
B	−0.5957	−0.2887	−0.2381	−0.2752
r	24.0914	0.2858	25.1424	−1.3437
g	−4.4451	−0.2066	11.3800	0.8411
q	−6.3914	−1.2751	−8.6858	−3.4627
Z'-Score	2.0315		0.8551	
Zone	Gray		Distress	

failed firms.

The sensitivity of Z'-Score is related to the difference between g and r: the narrower is the difference, the higher sensitivity. Table 9 presents the estimated partial derivatives and elasticities when it is assumed that $g = 0.04$ in non-failed firms and $g = 0.01$ in failed firms. In these exemplary cases, the difference between g and r is narrower, when compared with the baseline case. Because of this narrower difference, the partial derivatives and elasticities of r and g are in absolute terms significantly higher. The decrease in the growth rate g also leads to that in non-failed firms Z'-Score rises significantly closer to Safe zone (still being in Gray zone), whereas in failed firms the score falls very low in Distress zone.

Table 9. Effect of lower growth rate on partial derivatives and elasticities of Z'-Score.

Parameter	$g = 0.04$ Non-failed firms		$g = 0.01$ Failed firms	
	Partial derivative	Elasticity	Partial derivative	Elasticity
$b1$	1.0689	0.1227	1.3519	−0.1567
$b2$	−1.0796	−0.0963	−1.3934	0.1533
B	−1.1160	−0.3875	−0.0736	0.0269
r	96.8340	0.8230	102.2270	1.7282
g	−54.1923	−0.7644	411.2303	−1.5212
q	−6.3914	−0.9135	−8.6858	1.0953
Z'-Score	2.8357		−2.7033	
Zone	Gray		Distress	

4.4. Numerical Effects of Parameters

Table A2 presents examples of numerical ceteris paribus effects of liquidity (current assets) parameter $b1$ on Z'-Score. These examples show that the effect of $b1$ is too weak in non-failed firms to rise alone the score from Gray zone to Safe zone, even if the level of current assets increases very significantly. Similarly, even if $b1$ has fallen very notably, the score stays on Gray zone. In failed firms, the effect of $b1$ on Z'-Score is also weak. Thus, $b1$ should rise to a very high level to move the score of failed firms from Distress zone to Gray zone.

Table A3 presents similar numerical examples for parameter $b2$ that refers to the level of current liabilities. The effect of $b2$ is weak similarly to $b1$, and even a significant decrease in $b2$ down to zero cannot increase the score of the non-failed firms from Gray zone to Safe zone. In the same way, even a significant increase in $b2$ does not make the score move to Distress zone. For the distressed firms, $b2$ should fall to a level close to zero to increase the Z'-Score from Distress zone to Gray zone.

Table A4 presents examples of the score effects of parameter B, which refers to the fraction of debt in external capital risen. The baseline value of B is in both

groups of firms very high, so that the limits to increase it are narrow and do not change the zone for Z'-Score. However, when B falls very low in the sample of failed firms, it makes the score to rise from the Distress zone to Gray zone. However, generally the effect of B on Z'-Score is weak.

Examples of numerical effects of profitability r on Z'-Score are presented in Table A5. Z'-Score is very sensitive to this parameter. If in non-failed firms, profitability falls from the baseline level to a relatively low, negative level, the score falls from Gray zone to Distress zone. However, a small increase in r from the baseline profitability makes the score rise from Gray zone to Safe zone. Similarly, a small increase in r that can still be negative makes failed firms to move from Distress zone to Gray zone. If r is increased to a relatively low positive level, Z'-Score is rising to Safe zone. It can make this movement to Safe zone for a lower profitability rate than non-failed firms can. Thus, generally, even a small change of r can make significant changes in Z'-Score.

Table A6 presents the results for the score effects of growth rate g. For the non-failed firms, the effect on Z'-Score is more significant, the closer growth rate g is profitability r. When g is close to r, the firm moves in Gray zone towards Safe zone. For high values of g, the effect on score is weak and even a very high growth rate g does not lead the score to fall to Distress zone. For failed firms, growth g has an opposite effect on the score. Thus, the score is higher, the higher is g. When g is relatively high, Z'-Score moves from Distress zone to Gray zone.

Finally, the numerical effects of revenue lag parameter q on Z'-Score are presented in Table A7. For both groups of firms, Z'-Score is higher, the lower is revenue lag q. If q in non-failed firms rises significantly from baseline to a higher level, the score moves from Gray zone to Distress zone. If q is significantly lower than baseline, Z'-Score moves to Safe zone. In failed firms, the score moves from Distress zone to Gray zone when g falls to a relatively low level. However, if the firm wants to move further to Safe zone, it should considerably shorten the revenue lag q.

5. Conclusion

Altman (1968) introduced the original Z-Score model in 1968 to assess the likelihood of distress of publicly traded firms. Later, Altman (1983) estimated the discriminant function for private manufacturing companies and established the Z'-Score model. Z'-Score model remains continually as one of the most widely used bankruptcy prediction models globally. Its popularity is sustained by transparency, interpretability, and low implementation costs. Nevertheless, Z'-Score does not perform properly in all contexts. The reasons for potential low prediction performance can be found in its foundational dimensions. In spite of obvious significance, research on these foundational dimensions in Z'-Score is very scarce.

This study addressed this gap through twofold objectives: first, to analyze the foundational operational and financial fundamentals affecting the five financial ratios of the Altman's Z'-model, and second, to investigate how these underlying pa-

rameters jointly dictate the composite, weighted Z'-Score. The parameters were approximated by long-term medians using financial statement data from Finnish non-failed (active) and failed (bankruptcy) firms. These median values were used to construct numerical experiments to show the effects of the parameters on Z'-Score.

The present model includes six foundational parameters: IRR, growth rate, revenue lag, level of current assets, level of current liabilities, and fraction of debt in external capital raised. These parameters were used to build a steady-state model to depict the five financial ratios included in Z'-Score. These five financial ratios were weighted by the discriminant coefficients estimated by Altman (1983). The weighted sum of the ratios was used to compose Z'-Score. In addition to Z'-Score, Altman also estimated Z'-Score, which excluded Sales to total assets ratio (X5) because of a potential industry effect.

This study shows firstly that Working capital ratio to total assets ratio X1 as a liquidity measure, significantly differs from other ratios. It is strongly affected by levels of current assets and liabilities, which refer to strategies of payment terms. IRR, growth rate, and revenue lag have only a minor effect on the ratio. Secondly, Retained earnings to total assets ratio X2 is sensitive mainly to IRR, and only slightly to growth rate. In non-failed firms, the ratio is a decreasing function of growth rate, but in failed firms, the function is increasing. Thirdly, Earnings before interest and tax to total assets ratio X3 is strongly affected by IRR, and growth rate has only a minor effect on the ratio. Fourth, Book value of equity to total liabilities ratio X4 is very sensitive to IRR. It is also sensitive to fraction of debt in external capital raised, as well as to growth rate. Finally, Sales to total assets ratio X5 is strongly affected by revenue lag, and slightly by IRR.

These results on foundational dimensions give hints on how potentially to improve the values of these financial ratios. In general, it is important for all ratios that IRR is at a sufficient level. IRR affects positively each of the five ratios. In the same way, growth rate affects several financial ratios. However, the sign of the effect depends on the context. The context is here closely associated with the sign of IRR: positive IRR typically refers to non-failed (active) firms, whereas negative IRR refers to failed (bankruptcy) firms. The effect of growth rate on Retained earnings to total assets ratio X2 and Book value of equity to total liabilities ratio X4, is remarkable. In active firms, this effect is negative but in distressed firms, it is however positive. Thus, steady state results indicate that in active firms, lower growth rate leads in the long run to higher X2 and X4, whereas in distressed firms, the relationship is opposite. In active and distressed firms, a decrease in revenue lag can in the long run affect positively Working capital to total assets ratio X1 and Sales to total assets ratio X5.

The effects on Z'-Score are weighted effects of parameters on the five financial ratios. Thus, the most important parameter is IRR that makes a strong positive effect on the score both in active and distressed firms. In active firms, growth rate has a considerable negative effect on Z'-Score, whereas in distressed firms the effect is even stronger but positive. Although revenue lag has a remarkable effect only

on Sales to total assets ratio X5, it however affects relatively strongly, but negatively, on Z'-Score. Levels of current assets and liabilities and fraction of debt in external capital raised only slightly affect one financial ratio with a low weight. Followingly, their weighted impact on Z'-Score is relatively negligible. Finally, it is important to emphasize that the objective of this study is to use the model to examine the sensitivity of the Z'-Score to various factors, rather than the sensitivity of bankruptcy risk itself. Thus, a change in the Z'-Score does not necessarily imply a corresponding change in bankruptcy risk.

The findings of this study can give an explanation for why Z'-Score does not perform properly in different contexts. The most obvious explanation is associated with the interplay of profitability and growth. First, since the difference between profitability IRR and growth rate determines the sensitivity of the score to these parameters, growth can destroy the classification performance when the difference is narrow. In that case, even small differences in growth can lead to big differences in the score. Second, the effects of growth on Z'-Score are opposite in firms reporting negative and positive IRR. Therefore, solely due to the effect of growth, fast-growing firms tend to report low scores when IRR is positive and high scores when it is negative. Furthermore, the strong negative effect of the revenue lag on Z'-Score allows firms with shorter lags to report high scores solely due to this delay (industry effect).

This study yielded important results to assess the dimensions of Z'-Score, but at the same time, it suffers from many shortcomings. The framework behind the results is a long-term steady deterministic model. This model is based on many simplifying assumptions. The analysis is limited to private firms that show higher growth rate than IRR to ensure positive external capital in the long run. Moreover, the study makes use of empirical data only from Finnish firms and uses only simple intuitive methods to approximate the fundamental parameters of the model.

However, this paper also opens new avenues for future inquiry. Subsequent research could relax the steady-state assumption by incorporating stochastic volatility into the revenue and expense paths, or by examining the dynamic behavior of the Z-Score during cyclical economic shocks. Additionally, future empirical studies could validate analytical derivatives and elasticities against large-scale corporate databases of private firms. These approaches could make it possible to test whether observed changes in revenue lag, profitability, and growth rate align with predicted mathematical trajectories. The parameters of the model could be approximated by more advanced statistical methods. Finally, the framework could be extended to deal also with the cases where IRR exceeds growth rate.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Altman, E. I. (1968). Financial Ratios, Discriminant Analysis and the Prediction of Corpo-

rate Bankruptcy. *The Journal of Finance, 23,* 589-609. https://doi.org/10.1111/j.1540-6261.1968.tb00843.x

Altman, E. I. (1983). *Corporate Financial Distress: A Complete Guide to Predicting, Avoiding, and Dealing with Bankruptcy.* John Wiley and Sons.

Altman, E. I., Iwanicz-Drozdowska, M., Laitinen, E. K., & Suvas, A. (2017). Financial Distress Prediction in an International Context: A Review and Empirical Analysis of Altman's *z*-Score Model. *Journal of International Financial Management & Accounting, 28,* 131-171. https://doi.org/10.1111/jifm.12053

Beaver, W. H. (1966). Financial Ratios as Predictors of Failure. *Journal of Accounting Research, 4,* 71-111. https://doi.org/10.2307/2490171

Brief, R. P. (2013). *Estimating the Economic Rate of Return from Accounting Data (RLE Accounting).* Routledge.

Feenstra, D. W., & Wang, H. (2000). *Economic and Accounting Rates of Return.* Working Paper, University of Groningen, Research Institute SOM.

Harcourt, G. C. (1965). The Accountant in a Golden Age. *Oxford Economic Papers, 17,* 66-80. https://doi.org/10.1093/oxfordjournals.oep.a040979

Káčer, M., Ochotnický, P., & Alexy, M. (2019). The Altman's Revised Z'-Score Model, Non-Financial Information and Macroeconomic Variables: Case of Slovak SMEs. *Ekonomicky Casopis, 67,* 335-366.

Kay, J. A. (1976). Accountants, Too, Could Be Happy in a Golden Age: The Accountants Rate of Profit and the Internal Rate of Return. *Oxford Econ*omic Papers, 28, 447-460. https://doi.org/10.1093/oxfordjournals.oep.a041353

Laitinen, E. K. (1980). *Financial Ratios and the Basic Economic Factors of the Firm: A Steady State Approach.* Ph.D. Thesis, University of Jyväskylä, 286 p. https://jyx.jyu.fi/jyx/Record/jyx_123456789_103777

Laitinen, E. K. (2025). Effect of Growth Rate on Financial Ratios: Theoretical and Empirical Analysis. *Theoretical Economics Letters, 15,* 1294-1316. https://doi.org/10.4236/tel.2025.155073

Laitinen, E. K., & Laitinen, T. (2022). Timing of Revenues and Expenses: Evidence from Finland. *Theoretical Economics Letters, 12,* 712-741. https://doi.org/10.4236/tel.2022.123040

Luckett, P. F. (1984). ARR vs. IRR: A Review and an Analysis. *Journal of Business Finance & Accounting, 11,* 213-231. https://doi.org/10.1111/j.1468-5957.1984.tb00070.x

Peasnell, K. V. (1982). Some Formal Connections between Economic Values and Yields and Accounting Numbers. *Journal of Business Finance & Accounting, 9,* 361-381. https://doi.org/10.1111/j.1468-5957.1982.tb01001.x

Pindado, J., Rodrigues, L., & de la Torre, C. (2008). Estimating Financial Distress Likelihood. *Journal of Business Research, 61,* 995-1003. https://doi.org/10.1016/j.jbusres.2007.10.006

Solomon, E. (1966). Return on Investment: The Relation of Book Yield to True Yield. In R. K. Jaedicke, Y. Ijiri, & O. Nielsen (Eds.), *Research in Accounting Measurement* (pp. 232-244). American Accounting Association.

Appendix

Table A1. Partial derivatives of Z'-Score with respect to the six parameters.

Parameter	Partial derivative
$b1$	$a1 \cdot (1 + r - q)/(q \cdot (1 + r))$
$b2$	$-a2 \cdot (1 + g - q)/(q \cdot (1 + g))$
B	$-a4 \cdot g \cdot (r + 1)/(B^2 \cdot (g - r))$
r	$a1 \cdot b1/(1 + r)^2 + a2 \cdot (g + 1)/(g \cdot (r + 1)^2) + a3 \cdot (g + 1)/(r + 1)^2 + a4 \cdot g \cdot (g + 1)/(B \cdot (g - r)2) + a5/(q \cdot (r + 1)^2)$
g	$-a1 \cdot b2/(1 + g)^2 - a2 \cdot r/(g^2 \cdot (r + 1)) + a3 \cdot r/(r + 1) - a4 \cdot r \cdot (r + 1)/(B \cdot (g - r)^2)$
q	$-(a1 \cdot (b1 - b2) + a5)/q^2$

Legend: $a1$ - $a5$ = Weights of the financial ratios X1 to X5 in Z'-Score model.

Table A2. Numerical effect of $b1$ on the value of Z'-Score in non-failed and failed firms.

Observation	Non-failed firms			Failed firms		
	$b1$	Z'-Score	Zone	$b1$	Z'-Score	Zone
1	0.5754	2.2988	Gray	0.6134	1.2607	Gray
2	0.5254	2.2453	Gray	0.5534	1.1796	Distress
3	0.4754	2.1919	Gray	0.4934	1.0985	Distress
4	0.4254	2.1384	Gray	0.4334	1.0173	Distress
5	0.3754	2.0850	Gray	0.3734	0.9362	Distress
6	**0.3254**	**2.0315**	**Gray**	**0.3134**	**0.8551**	**Distress**
7	0.2754	1.9781	Gray	0.2784	0.8078	Distress
8	0.2254	1.9246	Gray	0.2434	0.7605	Distress
9	0.1754	1.8712	Gray	0.2084	0.7132	Distress
10	0.1254	1.8177	Gray	0.1734	0.6658	Distress
11	0.0754	1.7643	Gray	0.1384	0.6185	Distress

Legend: Observation 6 corresponds to the baseline situation.

Table A3. Numerical effect of $b2$ on the value of Z'-Score in non-failed and failed firms.

Observation	Non-failed firms			Failed firms		
	$b2$	Z'-Score	Zone	$b2$	Z'-Score	Zone
1	0.5030	1.7530	Gray	0.5725	0.4622	Distress
2	0.4530	1.8087	Gray	0.5175	0.5408	Distress
3	0.4030	1.8644	Gray	0.4625	0.6194	Distress
4	0.3530	1.9201	Gray	0.4075	0.6979	Distress
5	0.3030	1.9758	Gray	0.3525	0.7765	Distress

Continued

6	**0.2530**	**2.0315**	**Gray**	**0.2975**	**0.8551**	**Distress**
7	0.2030	2.0872	Gray	0.2425	0.9337	Distress
8	0.1530	2.1429	Gray	0.1875	1.0123	Distress
9	0.1030	2.1986	Gray	0.1325	1.0909	Distress
10	0.0530	2.2543	Gray	0.0775	1.1695	Distress
11	0.0030	2.3100	Gray	0.0225	1.2481	Gray

Legend: Observation 6 corresponds to the baseline situation.

Table A4. Numerical effect of B on the value of Z'-Score in non-failed and failed firms.

Observation	Non-failed firms			Failed firms		
	B	Z'-Score	Zone	B	Z'-Score	Zone
1	1.0097	2.0170	Gray	1.0010	0.8522	Distress
2	1.0047	2.0198	Gray	0.9985	0.8528	Distress
3	0.9997	2.0227	Gray	0.9960	0.8533	Distress
4	0.9947	2.0256	Gray	0.9935	0.8539	Distress
5	0.9897	2.0286	Gray	0.9910	0.8545	Distress
6	**0.9847**	**2.0315**	**Gray**	**0.9885**	**0.8551**	**Distress**
7	0.9347	2.0629	Gray	0.8385	0.8972	Distress
8	0.8847	2.0978	Gray	0.6885	0.9576	Distress
9	0.8347	2.1369	Gray	0.5385	1.0518	Distress
10	0.7847	2.1810	Gray	0.3885	1.2185	Distress
11	0.7347	2.2311	Gray	0.2385	1.5951	Gray

Legend: Observation 6 corresponds to the baseline situation.

Table A5. Numerical effect of r on the value of Z'-Score in non-failed and failed firms.

Observation	Non-failed firms			Failed firms		
	r	Z'-Score	Zone	r	Z'-Score	Zone
1	0.0866	7.8657	Safe	0.0543	5.6669	Safe
2	0.0741	4.2323	Safe	0.0343	3.1026	Safe
3	0.0616	3.2466	Safe	0.0143	2.3406	Gray
4	0.0491	2.7137	Gray	−0.0057	1.7988	Gray
5	0.0366	2.3372	Gray	−0.0257	1.3180	Gray
6	**0.0241**	**2.0315**	**Gray**	**−0.0457**	**0.8551**	**Distress**
7	0.0091	1.7115	Gray	−0.0657	0.3933	Distress
8	−0.0059	1.4182	Gray	−0.0857	−0.0760	Distress
9	−0.0209	1.1383	Distress	−0.1057	−0.5580	Distress

Continued

10	−0.0359	0.8647	Distress	−0.1257	−1.0561	Distress
11	−0.0509	0.5929	Distress	−0.1457	−1.5733	Distress

Legend: Observation 6 corresponds to the baseline situation.

Table A6. Numerical effect of g on the value of Z'-Score in non-failed and failed firms.

Observation	Non-failed firms			Failed firms		
	g	Z'-Score	Zone	g	Z'-Score	Zone
1	0.3444	1.7488	Gray	0.2132	1.3581	Gray
2	0.2944	1.7663	Gray	0.1832	1.3265	Gray
3	0.2444	1.7910	Gray	0.1532	1.2801	Gray
4	0.1944	1.8284	Gray	0.1232	1.2085	Distress
5	0.1444	1.8926	Gray	0.0932	1.0885	Distress
6	**0.0944**	**2.0315**	**Gray**	**0.0632**	**0.8551**	**Distress**
7	0.0794	2.1132	Gray	0.0532	0.7207	Distress
8	0.0669	2.2168	Gray	0.0432	0.5265	Distress
9	0.0544	2.3878	Gray	0.0332	0.2208	Distress
10	0.0419	2.7423	Gray	0.0232	−0.3364	Distress
11	0.0400	2.8357	Gray	0.0132	−1.7030	Distress

Legend: Observation 6 corresponds to the baseline situation.

Table A7. Numerical effect of q on the value of Z'-Score in non-failed and failed firms.

Observation	Non-failed firms			Failed firms		
	q	Z'-Score	Zone	q	Z'-Score	Zone
1	0.6553	1.0432	Distress	0.4659	0.0607	Distress
2	0.6053	1.1756	Distress	0.4409	0.1835	Distress
3	0.5553	1.3318	Gray	0.4159	0.3212	Distress
4	0.5053	1.5189	Gray	0.3909	0.4764	Distress
5	0.4553	1.7470	Gray	0.3659	0.6528	Distress
6	**0.4053**	**2.0315**	**Gray**	**0.3409**	**0.8551**	**Distress**
7	0.3803	2.2018	Gray	0.3109	1.1408	Distress
8	0.3553	2.3961	Gray	0.2809	1.4876	Gray
9	0.3303	2.6197	Gray	0.2509	1.9172	Gray
10	0.3053	2.8800	Gray	0.2209	2.4636	Gray
11	0.2803	3.1867	Safe	0.1909	3.1817	Safe

Legend: Observation 6 corresponds to the baseline situation.