

Correspondence of Evolving and Periodic Production Systems in Accounting and Compounding

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Abstract

The generalized correspondence principle requires that methods used in the study of transient processes produce unbiased results also in the case of simpler periodic processes. In periodic systems, time translation invariance renders all phases within a cycle equally significant, directing analytical focus toward intra-period expected observables. While discounted expectations are computable across arbitrary observation windows, temporal integration fails to eliminate initial-condition bias tied to the chosen computation start time. As compounding equations cannot be used in the design of periodic processes, the generalized correspondence principle implies that neither can be used in the design of more generic evolving processes. Accounting results are at least partially internal and can be used in production management. Insertion of expected values of accounting quantities into compounding investment value approximations is demonstrated, with and without the presence of a periodic boundary.

Keywords

Probability Theory, Observation Time, Return Rate on Capital, Net Present Value, Leverage Effect

1. Introduction

At high energy levels, quantum calculations merge with classical computations (Planck, 1906; Bohr, 1920; Sommerfeld, 1921; Kawamura, 2025). This is denoted as the *correspondence principle*¹ (Tipler & Llewellyn, 2008; Kawamura, 2025). A

¹This correspondence principle has no relation to the correspondence principle of Samuelson (1941), sometimes discussed in economics.

generalized correspondence principle states that any extension of a theory should merge with well-established theories in circumstances where the latter are applicable (Post, 1993; Kawamura, 2025). Similarly, one might postulate that theories applied in investigations with complicated boundary conditions, inducing limited transparency of the problem, should appear functional under simpler boundary conditions providing transparency.

Many processes run in periods. Periods may refer to seasons, rotation times, harvest or market cycles, or the like (Hanifah et al., 2025; Rämö & Tahvonen, 2017; Moog, 2020). Periodic boundary condition refers to identical periods linking to each other. Expected values of observables are produced by integrating over an integer number of full periods. The integrated results for one period do not depend on the initiation point of integration (Ouchetto et al., 2018).

Even if the processes would run in periods, a strictly periodic boundary condition is not necessarily in place. Things change. The climate changes, populations migrate and evolve. Societies of producers and their customers change, resulting in industry changes. Prices evolve, and price differences emerge. Such changes induce an evolving state of a production process.

Changes appearing in the future are not easy to predict reliably. This makes treatments based on the assumption of periodic boundary conditions often useful simplifications of evolving reality - often useful, not always adequate. However, from a methodological perspective, this is where the *correspondence principle* becomes involved. A periodic process is a simple special case of a more generic (possibly evolving) process. Methods used to study evolving processes must produce unbiased results when applied to periodic processes. In other words, methods violating the periodic boundary condition in the case of a periodic system become falsified also in the case of evolving processes.

Valuation of equity investments may be done using a range of methodologies. Value approximation generally depends on selected parameters and preferences. Often used methods are discounting of expected future gains, or the use of some kind of multiples (Smith, 2025). As multiples may refer to the ratio of investment expense to the expected annual earnings (P/E), discounting of future earnings typically requires estimation of the earnings or gains for a longer time (Faustmann, 1849; Fisher, 1907, 1930; Hirshleifer, 1958).

In comparison to investment value approximation, accounting appears as an exact science (Sree et al., 2025). Accounting results also depend on a few choices made but become transparent by proper reporting of the practices used (Sree et al., 2025; Biondi, 2024).

Net present value (*NPV*) measures utility but not wealth accumulation (Keynes, 1936; Lutz & Lutz, 1951; Boulding, 1955; Chipman, 1972, 1977; Miller, 1975; Hirshleifer, 1958; Dorfman, 1981). Recent findings prove that neither *NPV* nor the internal rate of return (*IRR*) correctly accounts for leverage effects (Kärenlampi, 2025). This failure occurs because both methods rely on compounding equations (Biondi, 2024), and adding leverage directly to these equations creates bias (Kärenlampi,

2025).

Net Present Value (*NPV*) fails to show wealth growth because accounting rules and compounding math do not match. Wealth grows through compounding. Yet, compounding formulas rely on accounting data, not on other compounding steps. In fact, there are seven different arguments falsifying discounted cash flows as a criterion in production management (Brealey et al., 2011; Li et al., 2008; Magni, 2021; Biondi, 2024; Ting, 2012; Vernimmen et al., 2017; Zhu et al., 2014; Wu, 2025; Kärenlampi, 2025, 2026). This paper concentrates only on the violation of the periodic boundary condition.

For high-magnitude, long-horizon investments—such as industrial mill establishment or real estate acquisition—the optimal observation window is bounded by the decision threshold, necessitating exhaustive temporal information aggregation prior to commitment. On the contrary, managing continuous operations requires balancing different observation times, as no single instant holds inherently more value than another. As an example, harvesting events often yield higher financial returns than planting or seeding, these lifecycle stages may operate independently or remain closely linked.

Because cash flow rates typically fluctuate over time, the discounted net present value (*NPV*) of cash flows, denoted as

$$NPV(b) = \int_b^{\infty} \frac{dC}{dt} e^{-qt} dt \quad (1)$$

being a function of both the discount rate q and the specific observation timeframe b . Evaluating a single observation time, rather than integrating all relevant intervals across the production lifecycle, introduces systemic bias. Specifically, the negative exponential component disproportionately weights cash flows that occur near the arbitrarily designated initiation baseline.

This paper investigates observation time dynamics. Applying the generalized correspondence principle under periodic boundary conditions, we derive the expected value of an observable over any period. We then formulate an array of discounted observables parameterized by observation times, compute their expected values across any period, and evaluate the application of these accounting formulations to compound investment valuation approximations.

2. Methods and Results

Consider an observable Q , evaluated over a temporal interval from b to $b + \tau$. In this context, Q represents an accounting metric, such as the profit rate, market capitalization, or the rate of return on capital. Mathematically, the expectation value of this observable is defined as

$$\langle Q \rangle = \int_b^{b+\tau} Q p(t) dt \quad (2)$$

where $p(t)$ represents the probability density function of time localized within the specified temporal frame.

The expected discounted present value of the observations at time k is given by

$$\langle PV(Q, k) \rangle = \int_b^{b+\tau} Q e^{(k-t)q} p(t) dt \quad (3)$$

where q denotes a discount rate.

The probability density function of the observation times is equivalent to that of the event occurrence times, or

$$p(k) = p(t) = \frac{1}{\tau} \quad (4)$$

The expected value of the present values, over observation times k is

$$\begin{aligned} \langle \langle PV(Q) \rangle \rangle &= \int_b^{b+\tau} \langle PV(Q, k) \rangle p(k) dk \\ &= \int_b^{b+\tau} \int_b^{b+\tau} Q e^{(k-t)q} p(t) dt p(k) dk \\ &= \int_b^{b+\tau} e^{kq} p(k) dk \int_b^{b+\tau} Q e^{-tq} p(t) dt \end{aligned} \quad (5)$$

Numerical validation confirmed the factorization of the second term of Equation (5) into the final expression. While conceptually simple, computing the expected discounted present value over all observation intervals (Equation (5)) is somewhat tedious. In general, this metric does not equate to the expected value of the observable in Equation (2).

In Equation (5), both constituent factors exhibit a functional dependence on the observation initiation parameter b . The first factor becomes

$$\int_b^{b+\tau} e^{kq} p(k) dk = e^{bq} \frac{e^{\tau q} - 1}{\tau q} \quad (6)$$

In Equation (6), within-period compounding $\frac{e^{\tau q} - 1}{\tau q}$ is multiplied by the historical compounding factor e^{bq} . If the observable Q were invariant in time, the historical prefactor would pull out the exponential in the second integral of Equation (5); however, such a stationarity condition does not hold generally.

The second integral of Equation (5) contains a fundamental methodological limitation. Because the observable Q presumably varies in time, the evaluated integral becomes explicitly sensitive to the choice of the lower limit b . This discrepancy mirrors the path-dependency identified in Equation (1), arising from the cumulative integration of a compounding state variable rather than an additive state variable. Numerical analysis confirms that the output of Equation (5) exhibits strong initial-condition dependence. Consequently, incorporating the expected value of the discounted quantity in Equation (5) fails to mitigate the integration-bound bias established in Equation (1).

3. Accounting Results in the Approximation of Investment Value

Approximation of the present value of investment outcome is essential in invest-

ment considerations. The present value necessarily depends on discount rates. For a single initial equity investment E , the present value at time t_0 of an equity investment made at time t_0 and observed up to time t_f is

$$P = E(t_0) \left\{ \exp \left[\int_{t_0}^{t_f} \langle r_e(t) \rangle dt - \int_{t_0}^{t_f} q(t) dt \right] \right\} \quad (7)$$

where $\langle r_e(t) \rangle$ is the expected value of the rate of return on equity at time t , and is discount rate at time t . Equation (7) is written on accrual basis.

Equation (7) does not explicitly contain quantities that would be strictly internal to the production process. The expected value of the rate of return on equity $\langle r_e(t) \rangle$ depends on eventual financial leverage L , as well as the borrowing interest rate u . An internal feature of the production process is found by substituting the rate of return on equity into Equation (7) as a function of leverage. The outcome is

$$\begin{aligned} P &= E(t_0) \left\{ \exp \left[\int_{t_0}^{t_f} [\langle r(t) \rangle + (\langle r(t) \rangle - u(t))L(t)] dt - \int_{t_0}^{t_f} q(t) dt \right] \right\} \\ &= E(t_0) \left\{ \exp \left[\int_{t_0}^{t_f} [\langle r(t) \rangle [1 + L(t)]] dt - \int_{t_0}^{t_f} [u(t)L(t)] dt - \int_{t_0}^{t_f} q(t) dt \right] \right\} \end{aligned} \quad (8)$$

where the expected value of the rate of return on capital $\langle r(t) \rangle$ is predominantly internal to the process and can be manipulated by production management. It does not depend directly on any external interest rate. Indirect effects may be in place since prices of goods and services may depend on interest rates. The leverage $L(t)$, borrowing rate $u(t)$ and discount rate $q(t)$ are external to the technical implementation of the production process, even if they probably depend on business owner's preferences.

The expected value of the rate of return on capital at time t is

$$\langle r(t) \rangle = \frac{\left\langle \frac{d\kappa}{dt}(t) \right\rangle}{\langle K(t) \rangle} \quad (9)$$

where κ in the numerator considers value increment, operative expenses, depreciations and amortizations, but neglects investments and withdrawals. In other words, it is the capitalization on an accrual operating profit basis. Then, the time derivative $\frac{d\kappa}{dt}$ is the operating profit rate. K in the denominator gives capitalization on a balance sheet basis, being directly affected by any investment or withdrawal.

Equations (7), (8) and (9) do not require any periodic boundary condition. The Equations are valid for any evolving system. A variety of approaches exist for the approximation of the discount rate $q(t)$ as a function of time (Weitzman, 1998, 2001, 2013). In the absence of any periodic boundary condition, approximation of the return rate $\langle r(t) \rangle$ is more challenging.

Let us then introduce a periodic boundary condition. In Equation (10), the production system contains compartments that are at different phases. The observables $\frac{d\kappa}{dt}$ and K depend on the phase δ . Then, at any time t , the expected value of the rate of return on capital is

$$\langle r(t) \rangle = \frac{\left\langle \frac{d\kappa}{dt}(t) \right\rangle}{\langle K(t) \rangle} = \frac{\int_b^{b+2\pi} \frac{d\kappa}{dt}(\delta) p(\delta, t) d\delta}{\int_b^{b+2\pi} K(\delta) p(\delta, t) d\delta} \quad (10)$$

In Equation (9), the expected values of the numerator and the denominator are produced by integrating over all development phases δ of system compartments, $p(\delta, t)$ refers to the probability density of system compartments of any phase at time t . In the general case, this probability density evolves with time. Correspondingly, the expected value of rate of return on capital varies with time, but is computable for any time instant provided $\frac{d\kappa}{dt}(\delta)$, $K(\delta)$ and $p(\delta, t)$ are known.

It is possible to integrate Equation (10) over time, to gain an expected value of the rate of return over time. The result can be substituted to Equation (8), provided the time frame $(t_f - t_0)$ is an integer multiple of the least common multiple of the compartment rotation times. However, such an expected value would truly simplify Equation (8) only if the leverage would happen to be constant in time.

There is another way to simplify Equation (10). The system could be enhanced by adding compartments so that the probability density of system compartments of any phase would become evenly distributed, and correspondingly constant in phase and in time. Then, the present value of the accumulated investment would become

$$P' = (E(t_0) + H(t_0)) \left\{ \exp \left[\langle r \rangle \int_{t_0}^{t_f} [1 + L(t)] dt - \int_{t_0}^{t_f} [u(t)L(t)] dt - \int_{t_0}^{t_f} q(t) dt \right] \right\} \quad (11)$$

where H is the additional equity investment needed to complement the system, and $\langle r \rangle$ is the constant expected value of the rate of return on capital. This would simplify Equation (10) into

$$\langle r \rangle = \frac{\left\langle \frac{d\kappa}{dt} \right\rangle}{\langle K \rangle} = \frac{\int_b^{b+2\pi} \frac{d\kappa}{dt}(\delta) d\delta}{\int_b^{b+2\pi} K(\delta) d\delta} \quad (12)$$

Interestingly, the expected value of the rate of return on capital being integrated over development phases, the expected value for one representative compartment equals the expected return for the entire system.

4. Discussion

Accounting computations in terms of Equation (12) appear straightforward. Ac-

counting computations according to Equation (10) also are principally straightforward, but require knowledge about the time evolution of probability densities. Solving the generic evolving case of Equation (9) cannot be supported by any periodic boundary.

Approximation of the present value of investment outcome in Equation (8) is doable with the help of Equation (9). However, in addition to the challenge of determining (9), one will have to insert the time evolution of leverage and discount rate in (8). The challenge will be somewhat less if a periodic boundary appears, and Equation (9) becomes replaced by (10). However, in both cases a question arises, what is the relation of the capitalization to the invested equity. As the initial investment and the capitalization are both determined on the basis of market prices, they indeed shall have a relation. More specifically, the sum of the capitalization of all compartments at the time of the investment shall equal the amount of investment, or

$$\sum K(t_0) = E(t_0)[1 + L(t_0)] \quad (13)$$

Then, the additional investment $H(t_0)[1 + L(t_0)]$ needed to transform Equations (8) and (10) into Equations (11) and (12) is producible in terms of the market value of added compartments needed to make the probability density $p(\delta, t)$ constant in time and phase.

It is worth noting that Equations (7), (8) and (11) do not indicate any payback time for the initial investment. As all the computations are implemented on accrual basis, the initial equity investment is not an expense but a transfer of financial resources into productive assets.

5. Conclusion

While important in the valuation of productive assets considered for purchase, or in the evaluation of other one-time investment opportunities, we have shown that discounting inherently produces bias in the management of ongoing periodic operations. Discounting results depend not only on the discount rate, but also on the starting time of any computation. This cannot be corrected by the computation of expected values over observation times; expected values of compounding equations are not unambiguously producible by integration. As compounding equations cannot be used in the design of periodic processes, the generalized correspondence principle implies that neither can be used in the design of more generic evolving processes. Instead, accounting results are predominantly internal and can be used for production management. Expected values of accounting quantities can be inserted into compounding investment value approximations.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- Biondi, Y. (2024). Accounting and Finance: Complementarity and Divergence. *Accounting, Economics, and Law: A Convivium*, 14, 329-337. <https://doi.org/10.1515/acl-2023-0132>
- Bohr, N. (1920). On the Series Spectra of the Elements. In N. Bohr (Ed.), *The Theory of Spectra and Atomic Constitution* (pp. 20-60). Cambridge University Press.
- Boulding, K. E. (1955). *Economic Analysis* (3rd ed.). Harper & Row.
- Brealey, R. A., Myers, S. C., & Allen, F. (2011). *Principles of Corporate Finance* (10th ed.). McGraw-Hill Irwin.
- Chipman, J. S. (1972). Renewal Model of Economic Growth: The Discrete Case. In R. H. Day, & S. M. Robinson (Eds.), *Mathematical Topics in Economic Theory and Computation* (pp. 43-83). SIAM Publications.
- Chipman, J. S. (1977). A Renewal Model of Economic Growth: The Continuous Case. *Econometrica*, 45, 295-316. <https://doi.org/10.2307/1911211>
- Dorfman, R. (1981). The Meaning of Internal Rates of Return. *The Journal of Finance*, 36, 1011-1021. <https://doi.org/10.1111/j.1540-6261.1981.tb01072.x>
- Faustmann, M. (1849). Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirthschaft besitzen. *Allgemeine Forst- und Jagd-Zeitung*, 15, 441-455.
- Fisher, I. (1907). *The Rate of Interest: Its Nature, Determination and Relation to Economic Phenomena*. Macmillan.
- Fisher, I. (1930). *The Theory of Interest: As Determined by Impatience to Spend Income and Opportunity to Invest It*. <https://www.econlib.org/library/YPDBooks/Fisher/fshToI.html>
- Hanifah, V. W., Karolinoerita, V., Cahyana, D., & Minasny, B. (2025). Principle and Prerequisites of Regenerative Agriculture: A Review. *Journal of Smart Agriculture and Environmental Technology*, 3, 1-7. <https://doi.org/10.60105/josaet.2025.3.1.1-7>
- Hirshleifer, J. (1958). On the Theory of Optimal Investment Decision. *Journal of Political Economy*, 66, 329-352. <https://doi.org/10.1086/258057>
- Kärenlampi, P. P. (2025). Complex Economics of Simple Periodic Systems. *PLOS Complex Systems*, 2, e0000043. <https://doi.org/10.1371/journal.pcsy.0000043>
- Kärenlampi, P. P. (2026). Financial Accounting of Carbon Forestry with Data from Florida. *Frontiers in Climate*, 8, Article ID: 1738771. <https://doi.org/10.3389/fclim.2026.1738771>
- Kawamura, Y. (2025). Generalized Heisenberg Dynamics Revisited. *Progress of Theoretical and Experimental Physics*, 2025, 123A01. <https://doi.org/10.1093/ptep/ptaf170>
- Keynes, J. M. (1936). *The General Theory of Employment, Interest, and Money* (472 p.). Palgrave Macmillan.
- Li, J., Jo Min, K., Otake, T., & Van Voorhis, T. (2008). Inventory and Investment in Setup and Quality Operations under Return on Investment Maximization. *European Journal of Operational Research*, 185, 593-605. <https://doi.org/10.1016/j.ejor.2006.11.045>
- Lutz, F., & Lutz, V. (1951). *The Theory of Investment of the Firm*. Princeton University Press.

- Magni, C. A. (2021). Internal Rates of Return and Shareholder Value Creation. *The Engineering Economist*, 66, 279-302. <https://doi.org/10.1080/0013791x.2020.1867679>
- Miller, R. K. (1975). A System of Renewal Equations. *SIAM Journal on Applied Mathematics*, 29, 20-34. <https://doi.org/10.1137/0129003>
- Moog, M. (2020). Some Comments on Rotation Modeling. *European Journal of Forest Research*, 139, 127-131. <https://doi.org/10.1007/s10342-019-01239-6>
- Ouchetto, O., Essakhi, B., Jai-Andaloussi, S., & Zaamoun, S. (2018). Handling Periodic Boundary Conditions on Arbitrary Mesh. *IET Microwaves, Antennas & Propagation*, 12, 1266-1272. <https://doi.org/10.1049/iet-map.2017.0870>
- Planck, M. (1906). *Vorlesungen über die Theorie der Warmestrahlung*. Verlag von Johann Ambrosius Barth. <https://ia804507.us.archive.org/12/items/uberdietheowarm00planrich/uberdietheowarm00planrich.pdf>
- Post, H. R. (1993). Correspondence, Invariance and Heuristics: In Praise of Conservative Induction. In S. French, & H. Kamminga (Eds.), *Correspondence, Invariance and Heuristics* (pp. 1-43). Springer. https://doi.org/10.1007/978-94-017-1185-2_1
- Rämö, J., & Tahvonen, O. (2017). Optimizing the Harvest Timing in Continuous Cover Forestry. *Environmental and Resource Economics*, 67, 853-868. <https://doi.org/10.1007/s10640-016-0008-4>
- Smith, T. (2025). *Multiples Approach: Definition and Example*. Investopedia. <https://www.investopedia.com/terms/m/multiplesapproach.asp>
- Sommerfeld, A. (1921). *Atombau und Spektrallinien*. F. Vieweg & Sohn.
- Sree, U., Annapurna, A. A., Hemavathi, E., & Nadar, S. P. (2025). *Foundations of Accounting and Financial Modeling*. A2Z Edulearninghub LLP. https://www.researchgate.net/publication/399858245_Foundation_of_Accounting_and_Financial_Modelling/link/696bac1d5cc49c35ce7f725d/download?_tp=eyJib250ZXh0Ijp7InBhZ2UiOiJwdWlzaWNhdGlvbiIsInByZXZpb3VzUGFnZSI6bnVsbH19
- Ting, C. C. (2012). Market Value of the Firm, Market Value of Equity, Return Rate on Capital and the Optimal Capital Structure. *International Journal of Financial Research*, 3, 1-6. <https://doi.org/10.5430/ijfr.v3n4p1>
- Tipler, P., & Llewellyn, R. (2008). *Modern Physics* (5 ed., pp. 160-161). W. H. Freeman and Company.
- Vernimmen, P., Le Fur, Y., Dallochio, M., Salvi, A., & Quiry, P. (2017). Return on Capital Employed and Return on Equity. In P. Vernimmen, Y. Le Fur, M. Dallochio, A. Salvi, & P. Quiry (Eds.), *Corporate Finance: Theory and Practice* (5th ed., pp. 216-234). John Wiley & Sons.
- Weitzman, M. L. (1998). Why the Far-Distant Future Should Be Discounted at Its Lowest Possible Rate. *Journal of Environmental Economics and Management*, 36, 201-208. <https://doi.org/10.1006/jeem.1998.1052>
- Weitzman, M. L. (2001). Gamma Discounting. *American Economic Review*, 91, 260-271. <https://doi.org/10.1257/aer.91.1.260>
- Weitzman, M. L. (2013). Tail-Hedge Discounting and the Social Cost of Carbon. *Journal of Economic Literature*, 3, 873-882.
- Wu, D. (2025). Research on the Impact of Financial Leverage on Corporate Investment Decisions. *Advances in Economics, Management and Political Sciences*, 205, 113-118. <https://doi.org/10.54254/2754-1169/2025.bj25610>

Zhu, C., Yang, G., An, K., & Huang, J. (2014). The Leverage Effect on Wealth Distribution in a Controllable Laboratory Stock Market. *PLOS ONE*, 9, e100681.
<https://doi.org/10.1371/journal.pone.0100681>