

A Mathematical Formulation of the Valuation of Brent Oil Futures Contracts

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Abstract

Oil is a commodity. The inelastic demand for gasoline to power electrical grids in homes and buildings, and power automobiles results in the need to examine oil prices. Oil futures trade on commodity exchanges, such as the Chicago Mercantile Exchange in the United States. Oil prices consist of a spot price (current price) and a speculative price. The purpose of this paper is to create mathematical models that describe optimal oil prices as a function of investor sentiment and oil price fluctuations. Three types of investors are examined. Risk-averse investors adopt a gamma distribution with decreasing propensity to purchase oil futures with increasing speculative price fluctuations. Moderate risk-takers assume a Bessel function with revisions of price forecasts as new information becomes available. Risk-takers' sentiments are modeled by an exponential distribution as they seek higher oil price returns with increasing risk. Oil price speculation is modeled by a Levy jump process, as oil prices follow jump discontinuities as past prices may not be linked to future prices during periods of high geopolitical risk, such as the US-Iran war in the Middle East. The mathematical models are empirically validated using contemporary price-volume data on Brent oil futures prices. Empirical findings support the mathematical conjecture of differential price expectations of the three types of investors, and their influence on final prices. Applications of the models developed in this paper to other commodities, such as gold and silver, are discussed.

Keywords

Brent Crude Oil Futures, Commodity Pricing, Geopolitical Risk, Oil Price Prediction, Mathematical Models

1. Introduction

Oil prices follow a cyclical trajectory that has a significant impact on airlines,

households, and businesses, in general. Airline tickets rise on jet fuel; oil is used to heat and cool homes and businesses, and gasoline transports families to work and schools. It is a finite resource, being found deep in the earth, in the ocean, and in shale rock. The Middle Eastern countries are dominant suppliers, with prices being fixed by intra-member negotiations of the Organization of Petroleum Exporting Countries (OPEC). OPEC was founded in 1960 by five member nations, including Iran, Iraq, Saudi Arabia, Venezuela, and Kuwait, subsequently expanding to thirteen nations ([World Population Review, 2026](#)). OPEC frequently reduces output to induce scarcity, which elevates the price of oil, thereby maximizing profit for member countries. India and China, with the largest populations, have the greatest demand for oil. Oil prices are determined by the trading of oil price futures on commodities exchanges worldwide.

Non-OPEC countries have been increasing oil production in recent years. They include Canada, Britain, Norway, and Russia. Canada has found oil trapped in the shale rock in the Arctic. Canadian oil is refined in Midwestern U.S. oil refineries, and then sold in the Midwestern states. Britain and Norway have been extracting oil from the North Sea. British oil bears the name, Brent oil, which is the subject of this paper. Russia has found significant deposits, leading to it becoming a major oil exporter to Western Europe. Geopolitical risks have long been associated with the supply of oil. Occasionally, Middle Eastern countries have restricted oil supplies due to political differences. Russia's invasion of the Crimea and Ukraine led to sanctions on its export of oil. Likewise, the Iran-Iraq war disrupted supply, while sanctions against Iran restricted the countries to which it could sell oil.

The purpose of this paper is to value Brent oil futures contracts. Although Brent oil is extracted from the North Sea by Britain, its futures are traded on world commodity markets, so that prices are universal, instead of being confined to a single country. There is little speculation about the spot price, as it is the current day's price. There is considerable speculation about the futures price, as it is forecasted for three weeks, during which prices can rise, fall, or stay stable. It is the central thesis of this paper that investor sentiment is a determinant of oil prices, along with interest rates, exchange rates, demand for oil, and supply of oil. Three types of investors are identified. The first type of investor is risk-averse, limiting risk unless the promise of positive returns is realized. They fail to increase oil purchases upon receiving the news that market factors forecast rising future oil prices. The second type of investor is the moderate risk-taker, who accepts some risk to fulfill the expectation of higher oil prices. This type of investor will revise expectations as new information about oil price forecasts is received. The third type of investor is the risktaker, who is unconcerned about increasing risk, being attracted to the higher returns offered by rising oil prices.

This paper makes three major contributions. First, it provides a mathematical basis for creating oil price distributions, particularly during the uncertain three-week period before final settlement of prices at a given volume takes place. Such an analysis is missing in the literature, which relies on empirical studies. For ex-

ample, Spyrou (2006) created a trading strategy based on price alone in which traders short Brent oil futures. They borrow Brent oil futures contracts, sell them at high prices, buying back at low prices to repay the loan with cheaper Brent oil futures, thereby realizing gains. Second, it relates abstract mathematical models to actual traded Brent oil futures contracts. This feature underscores the realism inherent in the data. Third, it includes geopolitical risk as a predictor of oil prices, even though Brent oil is extracted by Britain, i.e., not in a country with high geopolitical risk. Such risk is relevant in the current oil market, given the US war with Iran, which has disrupted the supply of oil.

The remaining sections are organized as follows. Section 2 is a Review of Literature, Section Three is Findings and Analysis, Section 4 is Empirical Validation, and Section Five consists of Conclusions.

2. Review of Literature

2.1. Research on Returns Obtained on Brent Oil Futures Contracts

Investors in Brent Oil Futures typically purchase the contracts and hold them, awaiting news about future oil prices. The news may be positive or negative. Gay et al. (1994) observed that investors in commodity futures overreacted to negative news. For example, if the commodity is futures on coffee, news of weather-related disruptions of harvests (negative news) will cause investors to sell to reduce losses, or short sell the coffee futures in order to achieve short-term gains. Grant et al. (2005) extended the Gay et al. (1994) results to other types of commodities, including stock index futures, gold futures, and silver futures. They found that all types of futures with the exception of gold futures, overreacted to positive price shocks, while gold futures overreacted to negative price shocks. In other words, traders in Brent oil futures, upon receiving news of a forthcoming increase in oil prices, due to OPEC supply restrictions, cold temperatures in the North Sea inhibiting extraction, and geopolitical risk, purchase excessive amounts of Brent oil futures. After multiple rounds of purchase, they sell the Brent oil futures at a gain. Gains may be limited by price reversal in that in future rounds of trading sudden declines in oil prices will reduce profits to be earned upon sale of the Brent oil futures contracts.

In a more recent study, Abdullahi et al. (2024) examined price-volume relationships of Brent oil futures from 2008-2011. They found that volume trading in the Brent oil futures market was unrelated to oil prices and returns in oil futures, using directional Granger causality tests from volume to prices. They concluded that the results contradicted the sequential arrival hypothesis, as information about oil prices embedded in trading volumes of successive Brent oil futures trades failed to influence future oil prices, with more recent trades having as little effect on future oil prices as trades settled in an earlier period of time. It is possible that factors other than price expectations were influencing oil prices during the period of study. Cepelova & Figura (2024) adopted a different approach. They correlated oil futures price volatility with the weighted average cost of capital in OECD coun-

tries over the same period as the [Abdullahi et al. \(2024\)](#) study. They observed significant correlations between volatility of oil futures prices and the weighted average cost of capital in sixteen OECD member countries. Intuitively, the result is to be expected in that fluctuations in oil prices increase uncertainty about future oil prices, thereby increasing the weighted average cost of capital. However, the use of correlational analysis is ill-advised, given that spurious correlations may occur if the two variables being correlated are independently associated with a third variable.

2.2. Research on Investor Sentiment and Crude Oil Futures

[Li et al. \(2019\)](#) examined the impact of investor attention on crude oil prices. They conjectured that the greater the attention shown by investors on oil prices, the greater the change in oil futures prices. They measured investor attention as the value of the Google search volume index, with higher index values indicating greater investor attention. Considering Brent oil futures together with four other country oil futures, they found weak associations between investor attention and crude oil futures prices. This paper contends that investor attention is too broad a measure, as it encompasses significant attention, moderate attention, and inattention in a single measure. The three types of investor attention may exert differential effects on oil prices, which may not be captured by a single measure. Accordingly, this study evaluates investor sentiment separately as risk-averse, moderate risk-taking, and risk-taking, with the understanding that each type of investor sentiment exerts a different impact on oil futures prices. [Filippidis et al. \(2023\)](#) observed that factors such as the global economic activity index, geopolitical risks, and government bond yields explained the price differentials between Brent oil futures and West Texas intermediate oil futures from 2011-2022. Of these factors, investor reaction was captured in government bond yields. Yields surged during periods of economic uncertainty, such as the Covid-19 lockdown period. Investors expressed concern about the economy, which increased the yield differential between the low-volatile, less risky Brent oil futures and the risky, high volatility West Texas Intermediate oil futures. Thus, investor reaction significantly influenced oil futures prices through increased uncertainty about future economic conditions.

3. Findings and Analysis

3.1. The Risk-Averse Investor's Price Function

A risk-averse investor only accepts as much risk as is capable of generating positive gain. The Arrow-Pratt Coefficient of Risk Aversion shows declining volume of purchase of Brent oil with oil price increases, as risk-averse investors reduce willingness to purchase higher priced, and hence riskier Brent oil futures as oil prices rise.

$$\begin{aligned} &\text{Coefficient of Absolute Risk Aversion} \\ &= -\frac{\text{Second derivative of a utility function of price expectations}}{\text{First derivative of a utility function of price expectations}} \quad (1) \end{aligned}$$

Equation (1) suggests that rate of change of price expectations of investors for the usefulness of owning Brent oil futures which declines with the risk of such oil futures contracts. As investors may purchase for sometime, pause, then reduce purchases and pause, the purchase-cum-pause sequence may be modeled by stepped gamma distribution. The gamma distribution is a family of probability distributions that models downward-sloping values in a stepped function (similar to a staircase), which the author deems appropriate for the gradually declining price expectations of the risk-averse oil investor. It is important to note that it is investor expectations of gain from oil prices, not oil prices, per se that are modeled by the gamma distribution.

We provide an explanations for the selected gamma, Bessel, and exponential functions represent the respective sentiment types, and cite finance or behavioral-economics literature supporting each mapping.

Abramowitz & Stegun (1964) present several mathematical distributions of which the gamma is a declining step down distribution, the Bessel is a looped distribution, and the exponential distribution is upward-sloping. These distributions have been used to describe price expectations in cryptocurrencies (exponential) due to high risk, options on real estate (Bessel), gold and silver prices (gamma and exponential) (Abraham, 2024; Abraham, 2025a; Abraham, 2025b; Abraham, 2026).

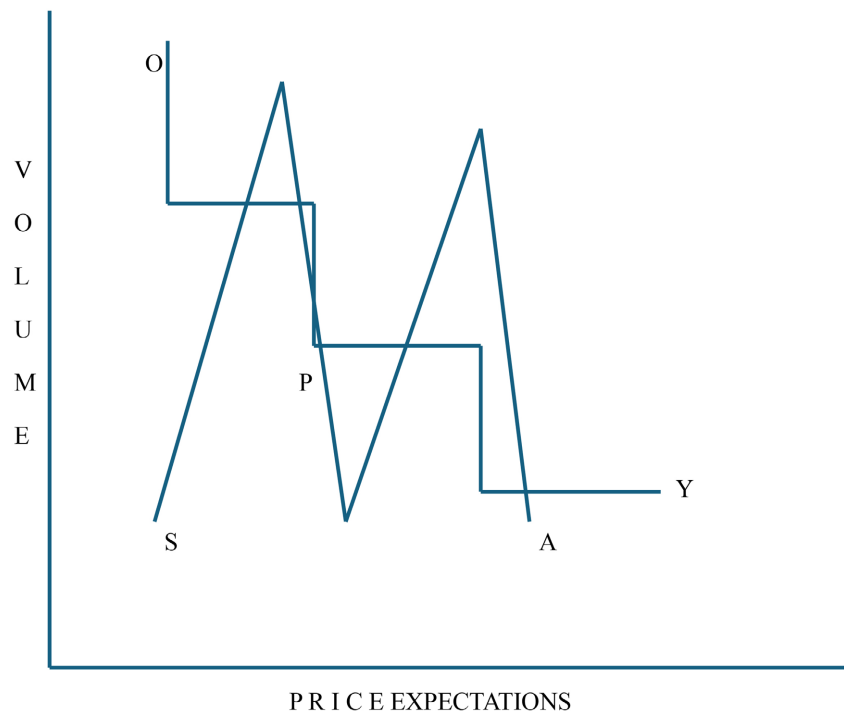


Figure 1. Shows OY, a gamma distribution of price expectations, i.e. the risk-averse investor's declining expectation of gain from risky Brent oil future contracts. SA is the price function of Brent oil futures contracts, modeled by a Levy jump process. Point P, the intersection of the gamma distribution, and the Levy jump process, is the optimal purchase price of Brent oil futures contracts for the risk-averse investor.

The price function of oil prices is modeled by a Levy jump process. Oil prices are highly cyclical, rising and falling from one period to the next. The determinants of prices do not follow a smooth continuous function, as unexpected conditions, such as war, pirate attacks, and exchange rate volatility may disrupt the price of oil in certain periods, although not in other periods. The Levy jump processes account for these breaks in jumps, by being represented as jump discontinuities. **Figure 1** shows the optimal point at which the gamma distribution of risk-averse investor expectations intersects the Levy jump process price function, i.e., point R, the optimal oil price for Brent oil futures contracts for the risk-averse investor.

The Arrow-Pratt coefficient of absolute risk aversion = Second derivative of the investor's utility function / First derivative of the investor's utility function

An investor may be presented with the choice of \$25 as a certainty or any amount from \$0 to \$100 with uncertainty. Investors who choose the certain amount are risk-averse, while those who choose an amount of 50 - 74 are moderate risk takers and those who select 75 - 100 are risk takers.

The proposed intersection of sentiment and price functions produces an economically meaningful optimal purchase price, including the investor's decision variable and optimization criterion. We use the mathematical principle that the intersection of 2 functions suggests that the point of intersection satisfies both equations. Therefore, the intersection of the investor sentiment function and the price function of the Brent oil futures contracts is the price that satisfies the sentiments of the investor, be the investor be risk-averse, a moderate risk-taker, or risk-taker.

The objective function and constraints are as follows,

Max

$$e^{-\beta x} \sum i \text{ to } \alpha(\beta x)^i + u'(c) / u'(c) \quad (2)$$

The first term in Equation (2) is the cumulative distribution of a gamma function, while the second term is the Arrow-Pratt coefficient of absolute risk-aversion (Pratt, 1964).

Subject to,

$$\exp(t(\alpha i \theta - 0.5 \sigma^2 \theta^2)) + \int R(e^{i \theta x} - 1 - i \theta x_1, x < 1) \quad (3)$$

Equation (3) shows the Levy-Khintchine representation of the price function of Brent oil futures contracts (Feldman, 1971).

Taking Lagrangians,

$$e^{-\beta x} \sum i \text{ to } \alpha(\beta x)^i + \frac{u'(c)}{u'(c)} - L_1[(\exp(t(\alpha i \theta - 0.5 \sigma^2 \theta^2)) + \int R(e^{i \theta x} - 1 - i \theta x_1, x < 1)] \quad (4)$$

Taking the first derivative of Equation (4),

$$e^{-\beta x} (\beta x)^i + \frac{u'(c)}{u'(c)} - L_1'[(\exp(t(\alpha i \theta - 0.5 \sigma^2 \theta^2)) + (e^{i \theta x} - 1 - i \theta x_1))]_{x < 1} \quad (5)$$

Taking the second derivative of Equation (4),

$$e^{-\beta x}(\beta x)^i + \frac{u''(c)}{u'(c)} - L'_1[(\exp(t(\alpha i \theta - \sigma \theta)) + (e^{i \theta x} - i \theta x_1))] \quad (6)$$

Equation (6) is the optimal price for the risk-averse investor.

3.2. The Moderate Risk-Taker's Price Function

The moderate risk-taker copes with uncertain price expectations by continuously revising price expectations. **Figure 2** shows the Bessel function, AB, that is used to model the investor sentiments of the moderate risk-taker for Brent oil futures contracts (Dutka, 1995). The curved lines represent revisions of price expectations. The moderate risk-taker forecasts an initial price, waits for more information to arrive, and then revises the price upon receiving additional information. The Levy jump process of Brent oil futures prices is depicted by line PQ. It intersects at point C, the optimal purchase price for the moderate risk-taker.

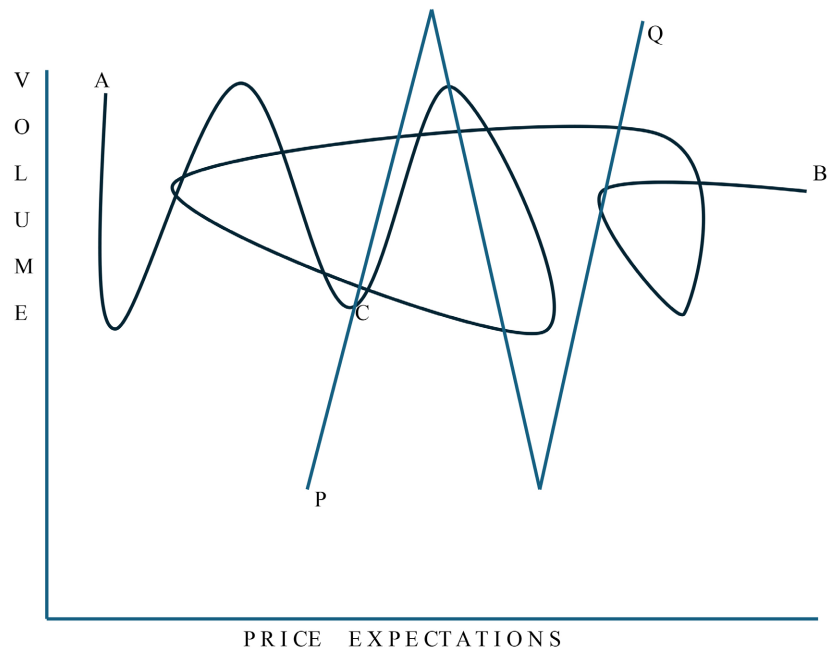


Figure 2. Shows the optimal price, C, for the moderate risk taker as the intersection of AB, the Bessel function of price expectations, and the Levy jump process of Brent oil futures prices, PQ.

The objective function and constraints are as follows,

Max

$$\sum (-1)^m / m!(m + \alpha + 1) \left(\frac{x}{2}\right) \quad (7)$$

Where Equation (7) represents a Bessel function of price expectations.

Subject to

$$\exp(t(\alpha i \theta - 0.5 \sigma^2 \theta^2)) + \int R(e^{i \theta x} - 1 - i \theta x_1 x < 1) \quad (8)$$

Where Equation (8) is the Levy-Khintchine formula of Brent oil futures prices. Taking Lagrangians,

$$\sum \frac{(-1)^m}{m!(m+\alpha+1)(\frac{x}{2})} - L_1[\exp(t(\alpha i\theta - 0.5\sigma^2\theta^2) + \int R(e^{i\theta x} - 1 - i\theta x 1x < 1)] \quad (9)$$

Taking first derivatives of Equation (9),

$$\frac{(-1)^m}{m!(m+\alpha+1)(\frac{x}{2})} - L_1'[\exp(t(\alpha i\theta - 0.5\sigma^2\theta^2) + (e^{i\theta x} - 1 - i\theta x 1x < 1)] \quad (10)$$

Taking second derivatives of Equation (9),

$$\frac{m(-1)^{m-1}}{m!(m+\alpha)(\frac{1}{2})} - L_1'[\exp(t(\alpha i\theta - \sigma\theta) + (e^{i\theta x} i\theta x 1x < 1)] \quad (11)$$

Equation (11) represents the optimal price of Brent oil futures contracts for the moderate risk-taker.

3.3. The Risk-Taker's Price Function

Figure 3 shows the optimal price for the risk-taker. The risk-taker is not concerned with losses that may occur with the acceptance of increasingly high levels of risk. This situation may be realized if Brent oil futures prices display high volatility, such as daily price fluctuations during the US-Iran war. For example, talk of peace causes oil prices to decline, followed almost immediately by price increases as hostilities resume. An exponential distribution describes the increasing price expectations of higher and higher returns of the risk-taker. PQ is the upward-sloping investor sentiment function of the risk-taker. This function intersects with the Levy jump process of Brent oil futures prices, AB, at point C, the optimal price for the risk-taker.

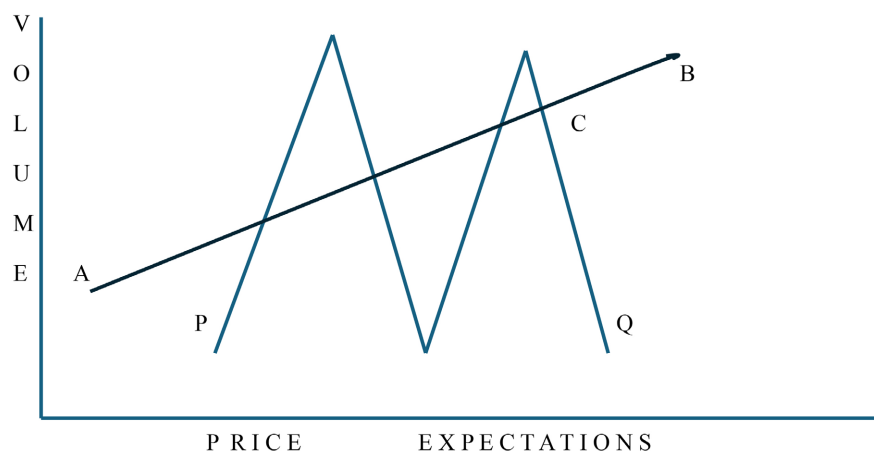


Figure 3. Shows the optimal price of the risk-taker, at the intersection of the exponential distribution of investor sentiment, AB, and the Levy jump process of the price distribution of Brent oil futures contracts, PQ.

The mathematical formulation of the risk-taker's price expectations is as follows,

Max

$$\log(\lambda_0) - \log(\lambda) + \frac{\lambda}{\lambda_0} - 1 \quad (12)$$

Equation (12) is the Kullback-Leibler divergence of the exponential distribution with rate parameter, λ , at which prices are measured at a certain point in time. The Kullback-Leibler divergence assume as initial rate, λ_0 , which diverges to the next rate λ .

Subject to,

$$\exp(t(\alpha i \theta - 0.5 \sigma^2 \theta^2)) + \int R(e^{i \theta x} - 1 - i \theta x 1 x < 1) \quad (13)$$

Where Equation (13) is the Levy-Khintchine formula of Brent oil futures prices.

$$\nabla^2 > 0 \quad (14)$$

Equation (14) is a gradient vector showing continuously upward movement in the risk-taker's price expectations.

Taking Lagrangians,

$$\begin{aligned} \log(\lambda_0) - \log(\lambda) + \frac{\lambda}{\lambda_0} - 1 - L_1[\exp(t(\alpha i \theta - 0.5 \sigma^2 \theta^2)) \\ + \int R(e^{i \theta x} - 1 - i \theta x 1 x < 1) - L_2 \nabla^2 - 0 \end{aligned} \quad (15)$$

Taking first derivatives of Equation (15),

$$\frac{1}{\lambda_0} - \frac{1}{\lambda} - \frac{1}{\lambda_0} - L_1'[\exp(t(\alpha i \theta - \sigma \theta)) + R(e^{i \theta x} - 1 - i \theta x 1 x < 1)] - L_2 \nabla^2 \quad (16)$$

Taking second derivatives of Equation (15),

$$\frac{1}{\lambda_0} - \frac{1}{\lambda} - \frac{1}{\lambda_0} - L_1'[\exp(t(\alpha i \theta)) + R(e^{i \theta x} - i \theta x 1 x)] - L_2 \nabla \quad (17)$$

Equation (1) represents the optimal price of Brent oil futures contracts for the risk-taker.

4. Empirical Validation

This section provides empirical validation for the mathematical formulations contained in Section 3. Fifteen Brent oil futures contracts were selected from oil.com, with dates commencing in July 2027. The final monthly contract selected was in November 2027. The data collection was not extended to dates beyond November 2027, as the volume of trading declined significantly after that date. Specifically, the data was collected from the following source.

Data source: oilprice.com/futures/brent

Download date: July 16, 2026

Contract Identifiers Brent Oct 2026

Observation Frequency daily

Selection rule: 15 contracts, July 2026-November 2027

The data consisted of quoted forward-contract data. The 15 contracts were selected as they measure projected fluctuations of oil prices during the following yet prior to expiration of the contracts. The forward prices contain projected price fluctuations embedded in them. Contracts near expiration contain fewer price fluctuations (less risk), while contracts with more distant expirations reflect greater price uncertainty (riskier).

The predictive claim is to maximize gains, with a trading strategy that emphasizes the risk preferences of the investors. The risk-averse investor will trade until price changes decrease with higher volumes, the moderate risk-taker will trade until price changes stay stable with higher volumes, and the risk-taker will trade until price changes increase in consecutive trades.

4.1. Empirical Validation for the Risk-Averse Investor

Table 1 shows the price volume relationships in the regression of Brent oil futures prices on volume for the risk-averse investor. Risk-aversion is captured in low prices, as the risk-averse investor will continue to trade with low prices of oil futures, as this investor is satisfied with minimal returns for a minimal level of risk. The volume of trades significantly explains low prices (Coefficient = 0.000031, $p < 0.001$), supporting the thesis that risk-averse investors will trade for very modest returns obtained from low Brent oil futures prices. The first lag of volumes was included as a predictor, significantly affecting prices (Coefficient = 0.00002, $p < 0.001$).

Table 1. Regressions of brent oil futures contract prices on volume for the risk-averse investor.

Variable	Coefficient
Constant	74.74***
Volume	0.000031***
Lagged Volume	0.00002***
R2, N	0.99, 14 usable observations

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

4.2. Empirical Validation for the Moderate Risk-Taker

Table 2 shows the price-volume relationships in the regression of Brent oil futures contract prices on volume for the moderate risk-taker. The moderate risk-taker has price expectations that are higher than those of the risk-averse investor, but not as high as the risk-taker. Accordingly, prices were taken as the average of high and low prices for each contract. **Table 2** shows that volume of trades significantly explained prices (Coefficient = 0.00003, $p < 0.001$). The first lagged volume of trades significantly explains prices as well (Coefficient = 0.00001, $p < 0.001$).

Table 2. Regressions of brent oil futures contract prices on volume for the moderate risk-taker.

Variable	Coefficient
Constant	75.40***
Volume	0.000037***
Lagged Volume	0.000019*
R2, N	0.99, 14 usable observations

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

4.3. Empirical Validation for the Risk-Taker

Table 3 shows the price-volume relationships in the regression of Brent oil futures contract prices on volume for the risk-taker. The risk-taker chooses to trade at high prices with the expectation of unlimited gain. Therefore, prices taken for this regression were high prices for each Brent oil futures contract.

Table 3 shows that volume of trades significantly explained prices (Coefficient = 0.000044, $p < 0.001$). The first lagged volume of trades significantly explains prices as well (Coefficient = 0.00001, $p < 0.001$).

Table 3. Regressions of brent oil futures contract prices on volume for the risk taker.

Variable	Coefficient
Constant	75.06***
Volume	0.000044***
Lagged Volume	0.00001*
R2, N	0.99, 14 usable observations

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

5. Conclusions

5.1. Discussion of Results

The chief contribution of this paper is the presentation of mathematical models linking investor sentiment about Brent oil futures prices to actual prices. The formulations draw on advanced mathematical concepts, such as Levy jump processes, the Levy-Khintchine formula, Bessel functions and gamma distributions which are most appropriate for describing investor attitudes toward risk. The separation of investors by attitudes toward risk of investments is appropriate given the high level of risk of Brent oil futures contracts.

The paper draws attention to Brent oil futures contracts, which are heavily traded on the London Stock Exchange. Mathematical models of these contracts are rare, particularly since Britain is a non-OPEC member. However, Brent oil futures contracts are important in that they are heavily traded on the London

Stock Exchange.

The paper creates a novel measure of risk aversion, which is tested empirically and found to be valid. The notion that different trade prices are favored by different types of investors has not been found in the literature. Lagged volume predicted oil prices for risk-averse investors and moderate risk-takers, suggesting that volumes of trading may take a single period in order to significantly influence prices in a subsequent period.

5.2. Practical Implications

Traders of Brent oil futures contracts expect to earn substantial profits, except for the more risk-averse traders. This paper suggests a new trading strategy for them. They can purchase Brent oil futures contracts, hold the contracts for 1 month, and then gain on the rise in prices during the following month. We obtain this observation from the empirical validation in Section 4, in which lagged volume with 1-month lags significantly predicted Brent oil futures prices for the risk-averse investor and the moderate risk-taker.

Commodity exchanges are the venue at which Brent oil futures contracts are traded. The US-Iran war has resulted in heightened volatility of oil prices. Projections of peace alternate daily with calls for more fighting. Prices of oil swing up with the threat of war, while the resumption of peace talks causes prices to decline. This paper has shown price gains during the war period, so that investors of all three sentiments must continue to trade regardless of price projections, during the war period.

Brent oil futures contracts are closely related to West Texas Intermediate oil futures contracts. This suggests that traders of West Texas Intermediate oil futures contracts may gain from the trading strategy of purchasing oil futures contracts, holding them for 1-month, and then trading at volumes high enough to earn substantial price gains.

5.3. Research Limitations

The paper examines Brent oil futures contracts. Formulations and empirical validation may be developed for oil futures contracts traded on the Chicago Mercantile Exchange, or other commodity exchanges in other countries. Oil futures contracts from OPEC countries may be contrasted with oil futures contracts from non-OPEC countries in a comparative examination.

The mathematical formulation for the risk-averse investor may be altered in two ways in subsequent studies. First, Schot's (1978) coefficient of aberrancy may define investor sentiment as investing in risky oil futures is aberrant behavior for a risk-averse investor. Second, the Arrow-Pratt coefficient of risk aversion may be included as a constraint in the formulation of the risk-averse investor's sentiments. This addition would add to the theoretical base of the model.

Additional empirical tests may be included to determine the impact of volume on price gains during a longer period. This study examined 1-month price gains

for Brent oil futures contracts. Future studies may consider 3-month or 6-month price gains.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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