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Price-Setting Newsvendor Where Customers Have Valuations

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Abstract

We study a price-setting newsvendor where potential customers attempt a purchase if their valuations for the product or service exceed its price. The seller does not know the valuations. The number of attempted purchases is distributed binomially. The retailer needs to embed that attempt distribution in a price-setting newsvendor problem. We analyze such a detailed model. We also consider several extensions: non-linear production costs, an unknown number of potential customers, possibly Poisson distributed, and cooperative revenue sharing contracts with a manufacturer based on a Nash bargaining solution, possibly asymmetric.

Keywords

Price-Setting Newsvendor, Customers' Valuations, Unknown Population Size, Revenue Sharing, Asymmetric Nash Bargaining Solution

1. Introduction

Customer value is the potential customer's perception of the worth of a product or service. *Value-based pricing* (VBP) is a pricing strategy used by some businesses to charge products and services at a rate they believe consumers are willing to pay, as opposed to calculating *production costs* and applying a standard markup. Artwork, amusement parks and cars are just some of the types of products/services where VBP is used[1].

A business that wishes to price in VBP manner, conducts market research in its entire *addressable market* to understand how each of its potential customers val-

[1]The nature of the product and firm's strategies occasionally affect customer value in unique ways. As an extreme example, the demand for new Hermés Birkin bags presently far outpaces the supply, which results in those who wish to buy a bag placing an extremely high value on it.

ues its product and what they would be willing to pay for it (Hinterhuber, 2014). Nevertheless, even after extensive market research at least some uncertainty as to a consumer's value for that product will remain. Thus, we assume a formation of a subjective probability distribution by the retailer (R) for a consumer's valuation.

Further, in addition to price, the nature of the operation is such that it must commit to produce a certain quantity of the product, and if demand turns out to exceed it, sales are lost.

Assuming a homogeneous population, in the sense that each customer's valuation is drawn from the same probability distribution, the number of customers wishing to buy the product is thus binomially distributed. Kalish (1985) and Huang et al. (2013) also addressed the impact of valuations, but while that stream of research considered a newsvendor who bases her decisions on the *expected* customer's demand only, we account for its *full distribution*. Given the price set by the retailer, the demand may exceed the short-term supply, so the retailer faces a price-setting newsvendor problem (surveyed by Yao et al., 2006 and DeYong, 2020), where the probability that a customer will wish to buy the product is that of her valuation exceeding the product's price (Krishnan, 2010).

Note that our customer-level fine-grained model is distinct from the well known $d(p,\varepsilon)$ random total demand model, a function of price and noise (mostly additive or multiplicative, e.g., Petruzzi and Dada, 1999). It also differs from the model of Schulte and Sachs (2020) which ascribes uncertainty directly to demand, rather than, as in our model, to relevant population's size which then determines the demand. However, in examples, we assume, like Schulte and Sachs, that the product is such that the relevant population is sparse, and thus potential stocking quantities are small.

Another stream of somewhat related research is the data-driven pricing literature, where historical observations are utilized to form statistical estimates of the demand distribution, which are then fed into an optimization model to yield the pricing and inventory decision (e.g., Harsha et al., 2021 and references therein).

In this work we attempt to find the quantity/price combination which maximizes the retailer's expected profit for linear and non-linear (Holt et al., 1960) production costs. Initially, we assume that the size of the relevant population of potential buyers is known to the retailer. We then consider the implications of unknown relevant population size, possibly Poisson distributed (not directly the demand distributed Poisson as in Schulte and Sachs).

Finally, we introduce a manufacturer (denoted by M) who has a revenue sharing contract with *R*, where the parties negotiate the revenue shares and wholesale price as a function of the retail price, using the Nash bargaining solution (*NBS*), possibly asymmetric (*ANBS*) (Kalai, 1977). The retailer then selects the retail price which, in turn, determines the revenue shares and wholesale price, and the parties' expected revenues.

2. The Basic Model

Consider a setting where an expected-profit-maximizing retailer, who upon selecting an order quantity q (integer) and setting the retail price per unit p for the product faces N potential customers. For the retailer the customers' valuations are random, denoted by V, and follow some subjective distribution $F(p)$ on [0, 1]. A customer would wish to buy (one unit of) the product iff its individual valuation exceeds the price, $\bar{F}(p)=1-F(p)$. That valuation is assumed to be independent of other customers' valuations (Mussa & Rosen, 1978; Krishnan, 2010). Thus, the number of customers wishing to buy the product (that is, the number of customers with valuation exceeding the retail price p), X, is *binomially distributed* with N and $\bar{F}(p)$.

Specifically, the probability that k out of the N potential customers have valuations exceeding the price p set by the retailer is given by

$$P(X=k)=\binom{N}{k}\left[\bar{F}(p)\right]^{k}\left[F(p)\right]^{N-k},\quad k=0,\cdots,N \text{, where } \bar{F}=1-F. \tag{1}$$

The retailer (who, for now, is assumed to know N), sets q $(q \le N)$ and p $(p \ge 0)$, so as to maximize its expected profit, which equals expected revenue minus production costs. We assume that the product/service has no salvage value[2].

By (1),

$$\begin{aligned} E(\Pi)= p\Bigg\{&\sum\nolimits_{k=1}^{q-1} k\binom{N}{k}\left[\bar{F}(p)\right]^{k}\left[F(p)\right]^{N-k} \\ &+q\sum\nolimits_{k=q}^{N}\binom{N}{k}\left[\bar{F}(p)\right]^{k}\left[F(p)\right]^{N-k}\Bigg\}-cq,\ q\ge 1, \end{aligned} \tag{2}$$

where c is the unit production cost (we shall use quadratic production costs in a subsequent section). The first sum is the expected number of units sold when all demand can be met, while the second sum is the expected number of units sold, in multiples of q, when entire demand cannot be met. We assume that *E*(Revenue) from q units is greater than cq in the relevant price range. For *fixed* q, p^* does not depend on c. However, c is relevant to finding q^*, on which p^* does depend.

Thus, we use a stylized single-period model, where each customer can buy at most one unit. We assume that the price p is upper-bounded, so *wlog* it is in $[0,1]$.

Hence, this setting resembles the traditional price-setting newsvendor model (Yao et al., 2006), though here customers' valuations are modeled explicitly. Kalish (1985) (see also Huang et al., 2013), who also explored the consequences of valuations, worked directly solely with the expected sales; So, our model is more detailed as we capture the intricate decisions of the independent individual potential customers.

Our examples will assume that the valuations are distributed as beta on [0, 1]. The standard beta(α,β) distribution on [0, 1] has the pdf

[2]It is always possible to formulate a newsvendor problem using two parameter, as we do.

$$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1},$$

$0\le x\le 1$, $\alpha,\beta\ge 1$ [3], where $\Gamma(z)=\int_0^\infty x^{z-1}e^{-x}dx$. For a positive integer $z=n$ $\Gamma(n)=(n-1)!$ (e.g., Johnson et al., 1995). If $\beta=1$, then $f(x)=\alpha x^{\alpha-1}$, and $F(x)=x^\alpha$, $\alpha\ge 1$ $0\le x\le 1$; $E(X)=\frac{\alpha}{\alpha+1}$ is increasing in α [4]. $\text{Beta}(\alpha,1)$ is a family of distributions, often referred to as $\text{power}(\alpha)$ distribution, whose pdf is convex (for $\alpha=1$ the distribution is uniform). Here $Var(X)=\frac{\alpha}{(\alpha+2)(\alpha+1)^2}$, which is decreasing in α.

In **Appendix A** we show how to find the (p,q) pair which maximizes $E(\pi)$ for any N for a uniform distribution.

Example: $N=3$, so $q\in\{0,1,2,3\}$. We shall use the notation "$E(m|n)$" to express

E(profit from *m* customers and *n* units). Thus

$$E(3|1)=p\sum_{k=1}^{3}\binom{3}{k}\left[\bar{F}(p)\right]^k\left[F(p)\right]^{3-k}-c=p\left(1-\left[F(p)\right]^3\right)-c$$
$$>E(3|0)=0$$
$$\text{iff}\quad c<p\left(1-\left[F(p)\right]^3\right).$$

$$E(3|2)=p\left(\left[F(p)\right]^3-3\left[F(p)\right]^2+2\right)-2c>E(3|0)=0$$
$$\text{iff}\quad c<\frac{1}{2}p\left(\left[F(p)\right]^3-3\left[F(p)\right]^2+2\right).$$

$$E(3|3)=p\left(3-3\left[F(p)\right]\right)-3c>0\quad\text{iff}\quad c<p\left(1-\left[F(p)\right]\right).$$

So $1\succ 2$ (i.e., expected profit is higher with $q=1$ than with $q=2$, so $q=1$ is preferred to $q=2$) iff $p\left(1-\left[F(p)\right]^3\right)-c>p\left(\left[F(p)\right]^3-3\left[F(p)\right]^2+2\right)-2c$

$$\text{i.e., iff}\quad c>p\left(2\left[F(p)\right]^3-3\left[F(p)\right]^2+1\right)=p\left(2p^{3\alpha}-3p^{2\alpha}+1\right)\equiv A_{1,2}$$

$$2\succ 3\quad\text{iff}\quad p\left(\left[F(p)\right]^3-3\left[F(p)\right]^2+2\right)-2c>p(3-3F)-3c$$

$$\text{i.e., iff}\quad c>p\left(1-\left[F(p)\right]\right)^3=p\left(1-p^\alpha\right)^3\equiv A_{2,3}$$

$$1\succ 3\quad\text{iff}\quad p\left(1-\left[F(p)\right]^3\right)-c>p\left(3-3\left[F(p)\right]\right)-3c,$$

$$\text{i.e., iff}\quad c>\frac{1}{2}p\left(\left[F(p)\right]^3-3\left[F(p)\right]+2\right)=\frac{1}{2}p\left(p^{3\alpha}-3p^\alpha+2\right)\equiv A_{1,3}.$$

[3]A beta distribution is also defined for $0\le\alpha,\beta<1$, but its properties in that range are different, so we shall not consider that case.

[4]By substituting $F(p)=p^\alpha$ in (2) and differentiating the expected profit w.r.t. α, one can show that the expected profit is increasing in α which was to be expected in view of the variance being decreasing in α.

Can show that $A_{1,3} > A_{2,3} > A_{1,2} \quad \forall\, 0 \le p \le 1$. Thus if $c > A_{1,3}$ then $1 \succ 2 \succ 3$ (in particular, $q^* = 1$); if $A_{2,3} < c < A_{1,3}$ then $1 \succ 2$, $2 \succ 3$ but $1 \prec 3 \rightarrow$ impossible; if $A_{1,2} < c < A_{2,3}$ then $1 \succ 3 \succ 2$ (in particular, $q^* = 1$); if $c < A_{1,2}$, then $3 \succ 2 \succ 1$ (if particular, $q^* = 3$).

3. Non-Linear Production Costs

Here the production costs are assumed to be quadratic: $C(q) = c_1 q + c_2 q^2$, $c_2 > 0$, $c_2 > -c_1$ (see Holt et al., 1960), i.e., increasing marginal costs. Like Holt et al., we allow c_1 to be negative to make the model more general. Again, for a fixed q, p^* does not depend on the cost parameters.

Example: <u>$N = 3$</u>

$$E_3(\Pi \mid 1) = p\left(1 - p^{3\alpha}\right) - c_1 - c_2$$

$$\frac{\mathrm{d}}{\mathrm{d}p} = 1 - (2 + 3\alpha) p^{3\alpha} = 0 \Rightarrow p^{3\alpha} = \frac{1}{2 + 3\alpha} \Rightarrow p^* = \frac{1}{(2 + 3\alpha)^{\frac{1}{3\alpha}}}$$

$$\Rightarrow E_3^*(\Pi \mid 1) = \frac{3\alpha}{(2 + 3\alpha)^{1 + \frac{1}{3\alpha}}} - c_1 - c_2$$

$$E_3(\Pi \mid 2) = p^{3\alpha + 1} - 3p^{2\alpha + 1} + 2p - 2c_1 - 4c_2$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}p} = (3\alpha + 1) p^{3\alpha} - 3(2\alpha + 1) p^{2\alpha} + 2 = 0$$

Let $p^\alpha \equiv y$ (so $p = y^{\frac{1}{\alpha}}$).

If $\alpha = 1$, $4y^3 - 9y^2 + 2 = 0 \Rightarrow y \sim 0.54 \Rightarrow p^* \sim 0.54$.

$\Rightarrow E_3^*(\Pi \mid 2, \alpha = 1) \sim (0.54)^4 - 3(0.54)^3 + 2(0.54) - 2c_1 - 4c_2 \sim 1.19 - 2c_1 - 4c_2$.

If $\alpha = 2$, $7y^6 - 15y^4 + 2 = 0 \Rightarrow y \sim 0.636 \Rightarrow p^* \sim 0.60$

Let p_i^* be the optimal price if $q = i$. Then it can be shown that $p_1^* > p_3^*$ for all α;

$$p_1(\alpha = 1) > p_2(\alpha = 1), \text{ but } p_1(\alpha = 2) < p_2(\alpha = 2).$$

$$\Rightarrow E_3^*(\Pi \mid 2, \alpha = 2) \sim 0.99 - 2c_1 - 4c_2.$$

$$E_3(\Pi \mid 3) = 3p\left(1 - \left[F(p)\right]\right) - 3c_1 - 9c_2$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}p} = 3\left[1 - (\alpha + 1) p^\alpha\right] = 0$$

$$\Rightarrow p^\alpha = \frac{1}{\alpha + 1} \Rightarrow p^* = \frac{1}{(\alpha + 1)^{\frac{1}{\alpha}}} \Rightarrow E^*(\Pi \mid 3) = \frac{3\alpha}{(\alpha + 1)^{\frac{1}{\alpha} + 1}}.$$

Thus for $\alpha = 1$, $E_3^*(\Pi \mid 3) = 0.75 - 3c_1 - 9c_2$.

Thus for $\alpha = 2$, $E_3^*(\Pi \mid 3) = \frac{6}{3^{\frac{3}{2}}} - 3c_1 - 9c_2 \sim 1.19 - 3c_1 - 9c_2$.

So

$$E_3^*(\Pi|1)=\frac{3\alpha}{(\alpha+2)^{\frac{1}{3\alpha}+1}}-c_1-c_2=\begin{cases}0.69-c_1-c_2, & \alpha=1\\ 1.18-c_1-c_2, & \alpha=2\end{cases};$$

$$E_3^*(\Pi|2)=\begin{cases}1.23-2c_1-4c_2, & \alpha=1\\ 0.99-2c_1-4c_2, & \alpha=2\end{cases};$$

$$E_3^*(\Pi|3)=\begin{cases}0.75-3c_1-9c_2, & \alpha=1\\ 1.19-3c_1-9c_2, & \alpha=2\end{cases}.$$

Thus for $\alpha=1$, $1\succ 2$ iff $c_1+3c_2>0.54$

$2\succ 3$ iff $c_1+5c_2>-0.48$ (if $c_1>0$, that always holds).

$1\succ 3$ iff $c_1+4c_2>0.03$.

So if $1\succ 2$ then $2\succ 3$

For $\alpha=2$, $1\succ 2$ iff $c_1+3c_2>-0.19$ (if $c_1>0$ that always holds).

$2\succ 3$ iff $c_1+5c_2>0.20$

$1\succ 3$ iff $c_1+4c_2>0.005$

So if $2\succ 3$ then $1\succ 3$ and thus $1\succ 2$.

4. Potential Customer Population Size Unknown

Thus far, we assumed that the newsvendor knows the potential population size *N*. Rather, now suppose she is not sure and thinks that

$$N=\begin{cases}N_1, & w.p.\,\theta\\ N_2, & w.p.\,1-\theta\end{cases}$$

The valuation distribution is assumed to be the same for all realizations of *N*. Then, as a mixture of probabilities,

$$\begin{aligned}P(X=k)=\theta&\binom{N_1}{k}\left[\bar{F}(p)\right]^k\left[F(p)\right]^{N-k}\\ &+(1-\theta)\binom{N_2}{k}\left[\bar{F}(p)\right]^k\left[F(p)\right]^{N_2-k},\ k=0,\cdots,\min\{N_1,N_2\}\end{aligned}\tag{3}$$

So

$$\begin{aligned}E(\Pi|q)=p\Bigg\{\theta&\left[\sum_{k=1}^{q-1}k\binom{N_1}{k}\left[\bar{F}(p)\right]^k\left[F(p)\right]^{N_1-k}\right.\\ &\left.+q\sum_{k=q}^{N_1}\binom{N_1}{k}\left[\bar{F}(p)\right]^k\left[F(p)\right]^{N_1-k}\right]\\ &+(1-\theta)\left[\sum_{k=1}^{q-1}k\binom{N_2}{k}\left[\bar{F}(p)\right]^k\left[F(p)\right]^{N_2-k}\right.\\ &\left.+q\sum_{k=q}^{N_2}\binom{N_2}{k}\left[\bar{F}(p)\right]^k\left[F(p)\right]^{N_2-k}\right]\Bigg\}-cq.\end{aligned}\tag{4}$$

See solution procedure for $X\sim U[0,1]$ in **Appendix B**.

Example

$$N_1 = 2, N_2 = 3$$

$q = 1$

$$E(\Pi|1) = p\left\{\theta\left[\binom{2}{1}\bar{F}(p)F(p) + \binom{2}{2}\left[\bar{F}(p)\right]^2\right] + (1-\theta)\left[1 - \binom{3}{0}\left[\bar{F}(p)\right]^3\right]\right\} - c$$

Now specialize to $F(p) = p^{\alpha}$, $0 \le p \le 1$, $\alpha \ge 1$:

$$E(\Pi|1) = p - \theta p^{2\alpha+1} - (1-\theta)p^{3\alpha+1} - c, \uparrow \alpha, \downarrow \theta$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}p} = -(1-\theta)(1+3\alpha)p^{3\alpha} - \theta(1+2\alpha)p^{2\alpha} + 1$$

If $\theta = \frac{1}{2}$ and $\alpha = 1$, $p_1^* = 0.607$, $E_1^* = 0.427 - c$.

$q = 2$

$$\begin{aligned} E(\Pi|2) = \Bigg\{ & p\theta\left[1\cdot\binom{2}{1}\bar{F}(p)F(p) + 2\binom{2}{2}\left[\bar{F}(p)\right]^2\right] \\ & + (1-\theta)\Bigg[1\binom{3}{1}\bar{F}(p)\left[F(p)\right]^2 \\ & + 2\left(\binom{3}{2}\left[\bar{F}(p)\right]^2 F(p) + \binom{3}{3}\left[\bar{F}(p)\right]^3\right)\Bigg]\Bigg\} - 2c. \end{aligned}$$

If $\theta = \frac{1}{2}$ and $\alpha = 1$, then $p_2^* = 0$ and $E_2^* = 2 - 2c$.

Thus 1 is preferred to 2 iff $c > 1.573$.

For $q = 3$

$$\begin{aligned} E(\Pi|3) &= p\Big[-6(1-\theta)\left[F(p)\right]^3 + 18(1-\theta)\left[F(p)\right]^2 \\ &\quad + (19\theta - 21)\left[F(p)\right] - 7\theta + 9\Big] - 3c \\ &= p\left[-6(1-\theta)p^{3\alpha} + 18(1-\theta)p^{2\alpha} + (19\theta - 21)p^{\alpha} - 7\theta + 9\right] - 3c \end{aligned}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}p} &= -6(1-\theta)(1+3\alpha)p^{3\alpha} + 18(1-\theta)(1+2\alpha)p^{2\alpha} \\ &\quad + (19\theta - 21)(1+\alpha)p^{\alpha} - 7\theta + 9 \\ &= 0 \end{aligned}$$

has no solution in $[0,1]$ for $\theta = \frac{1}{2}$ and $\alpha = 1$.

5. Population Size Distributed Poisson

Suppose that the newsvendor thinks that the number of potential customers follows a Poisson (λ) distribution,

$$P(N = n) = \frac{\lambda^n e^{-\lambda}}{n!}, \quad n = 0, 1, \cdots$$

The probability that a potential customer will attempt to buy is still $\bar{F}(p)$. Note the difference of that model from Schulte and Sacks (2020) who assumed that the *demand* is Poisson distributed.

For this mixture of Poisson,

$$P(X=k)=\sum_{n=0}^{\infty}\binom{n}{k}\left[\bar{F}(p)\right]^{k}\left[F(p)\right]^{N-k}\frac{\lambda^{n}e^{-\lambda}}{n!},k=0,1,\cdots,n$$

$$=\left[\frac{\bar{F}(p)}{F(p)}\right]^{k}e^{-\lambda}\sum_{n=0}^{\infty}\frac{\binom{n}{k}F(p)^{n}}{n!}.$$

For $\left[F(p)\right]\sim U[0,1]$, $P(X)=k$ equals $\left(\frac{1-p}{p}\right)^{k}e^{-\lambda}\sum_{n=0}^{\infty}\frac{\binom{n}{k}}{n!}$.

For $k=0$, $P(X=0)=e^{-\lambda}\sum_{n=0}^{\infty}\frac{\binom{n}{k}F(p)^{n}}{n!}=\frac{1}{e^{\lambda-p}}$, $\lambda>p$.

For $k=1$, $P(X=0)=\frac{1-p}{p}e^{-\lambda}\sum_{n=0}^{\infty}\frac{np^{n}}{n!}=\frac{1-p}{p}e^{-\lambda}pe^{p}=\frac{1-p}{e^{\lambda-p}}$, $\lambda>p-\ln\left(\frac{1}{1-p}\right)$.

For any distribution of N we can express the expected profit as follows:

$$E(\Pi)=\sum_{n=1}^{\infty}\left\{P(N=n)p\left[\sum_{k=1}^{q}kP(X=k)+q\sum_{k=q+1}^{n}P(X=k)\right]\right\}-cq. \quad (5)$$

Reorganizing terms and noting that the outer summation of the second term requires at least q customers:

$$E(\Pi)=p\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{q}kP(N=n)P(X=k)\right.$$
$$\left.+q\sum_{n=q+1}^{\infty}\sum_{k=q+1}^{n}P(N=n)P(X=k)\right\}-cq. \quad (6)$$

Note that the revenue expression is composed of two terms, each of which consists of a double summation: The first term sums over the total number of customers and over the number of customers whose valuations exceed the price, which is summed up to and including q, the retailer's order quantity. The second term considers instances where demand exceeds q, and hence it sums over population size realizations that can give rise to demand greater than q, i.e., $n\geq q+1$. For the Poisson distribution,

$$E(\Pi)=p\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{q}k\binom{n}{k}\left[\bar{F}(p)\right]^{k}\left[F(p)\right]^{n-k}\frac{\lambda^{n}e^{-\lambda}}{n!}\right.$$
$$\left.+q\sum_{n=q+1}^{\infty}\sum_{k=q+1}^{n}\binom{n}{k}\left[\bar{F}(p)\right]^{k}\left[F(p)\right]^{n-k}\frac{\lambda^{n}e^{-\lambda}}{n!}\right\}-cq \quad (7)$$

$$E(\Pi) = pe^{-\lambda}\left\{\sum_{n=1}^{\infty}\left[\lambda F(p)\right]^n \sum_{k=1}^{q}\frac{\left[\frac{\bar{F}(p)}{F(p)}\right]^k}{(k-1)!(n-k)!}\right. \\ \left. +q\sum_{n=q+1}^{\infty}\left[\lambda F(p)\right]^n \sum_{k=q+1}^{\infty}\frac{\left[\frac{\bar{F}(p)}{F(p)}\right]^k}{k!(n-k)!}\right\} - cq. \tag{8}$$

For example if $V \sim \text{power}(1)$, i.e., $V \sim U[0,1]$, and $\lambda = 10$, $c = 0.05$ the optimal expected profit is achieved at $q = 8$ and $p = 0.52$, yielding an expected profit of 2.04. For $c = 0.25$ the optimal expected profit is achieved at $q = 4$ and $p = 0.63$, yielding an expected profit of 0.94.

6. Revenue Sharing

The retailer (R) and a manufacturer (M) are to set a revenue sharing contract (Cachon & Lariviere, 2005; Bart et al., 2019). They are going to bargain over the revenue shares $(\beta, 1-\beta)$, $0 \le \beta \le 1$ and wholesale price (w), for the outcome of which we will use (at first) the symmetric Nash Bargaining Solution (NBS), and then possibly an asymmetric one (ANBS) (e.g., Muthoo, 1999). Note that after (β, w) are determined R will set the retail price (p) and associated order quantity (q) to maximize its expected profit. While p will affect the parties' profits, the bargaining phase takes place before it is selected by R. Meanwhile, M has no direct influence on the retail price.

Valuations are (still) distributed $\text{beta}(\alpha, 1)$, i.e., have a $\text{power}(\alpha)$ distribution on $[0,1]$. Note the assumption that the retail price is upper bounded. At first, M will have a unit production cost c (where $c < w$). Later we shall consider non-linear production costs.

We assume that population size is $\underline{N = 3}$. Let the demand be

$a(p) \equiv p(p^2 - 3p + 3)$

Then if $\underline{q = 1}$, $E_3(\Pi^R | 1) = \beta p(p^2 - 3p + 3) - w \equiv \beta a(p) - w$, and

$E_3^M(\Pi | 1) = (1-\beta)a(p) + w - c$

$\Rightarrow$ Symmetric Nash product

$$\begin{aligned} &= \beta(1-\beta)a(p)^2 - w(1-\beta)a(p) + w\beta a(p) - w^2 - c\beta a(p) + cw \\ &= -a(p)^2\beta^2 + \left[a(p)^2 + 2wa(p) - ca(p)\right]\beta - \left[a(p) - c\right]w - w^2. \end{aligned} \tag{9}$$

So, $\dfrac{\mathrm{d}(\text{product})}{\mathrm{d}\beta} = -2a(p)^2\beta + a(p)^2 + 2a(p)w - ca(p) = 0$

$$\Rightarrow \beta = \frac{2w + a(p) - c}{2a(p)} = \frac{1}{2} + \frac{2w - c}{2a(p)}, \tag{10}$$

$\frac{\mathrm{d}}{\mathrm{d}w} = 0 \Rightarrow$ same! So, w can be arbitrary[5], between 0 and p, and

$\beta = \frac{1}{2} + \frac{2w-c}{2p\left(p^2-3p+3\right)}$.

Let $\tilde{E}\left(\Pi^{\mathrm{R}} \mid k\right)$ be the expected profit with $\left(\beta^*, w^*\right)$, substituted in $E^*\left(\Pi^{\mathrm{R}} \mid k\right)$ is $\tilde{E}$ where p^* was also substituted for.

$$\Rightarrow \tilde{E}_3\left(\Pi^{\mathrm{R}} \mid 1\right) = \tilde{E}_3\left(\Pi^{\mathrm{M}} \mid 1\right) = \frac{a(p)-c}{2}.$$

$$\frac{\mathrm{d}\tilde{E}_3\left(\Pi^{\mathrm{R}} \mid 1\right)}{\mathrm{d}p} = 3(1-p)^2 = 0 \Rightarrow p = 1 \Rightarrow a(p) = 1.$$

$$\Rightarrow E_3^*\left(\Pi^{\mathrm{R}} \mid 1\right) = E_3^*\left(\Pi^{\mathrm{M}} \mid 1\right) = \frac{1-c}{2} \tag{11}$$

$$\Rightarrow \beta = w + \frac{1-c}{2}, \frac{c-1}{2} < w < \frac{c+1}{2}.$$

<u>$q = 2$</u> Let $p\left(\left[F(p)\right]^3 - 3\left[F(p)\right]^3 + 2\right) \equiv \left[\hat{F}(p)\right]$. So, for $\left[F(p)\right] = p^{\alpha}$, $\left[\hat{F}(p)\right]$ equals

<u>α</u>

1: $p^4 - 3p^3 + 2p$

2: $p^7 - 3p^5 + 2p$

3: $p^{10} - 3p^7 + 2p$

$$E_3\left(\Pi^{\mathrm{R}} \mid 2\right) = \beta\left[\hat{F}(p)\right] - 2w$$

$$E_3\left(\Pi^{\mathrm{M}} \mid 2\right) = (1-\beta)\left[\hat{F}(p)\right] + 2w - 2c.$$

$$\text{Nash product} = \beta(1-\beta)\hat{F}^2 - 2w(1-\beta)\left[\hat{F}(p)\right] + 2w\beta\left[\hat{F}(p)\right] - 4w^2 - 2c\beta\left[\hat{F}(p)\right] + 4cw$$

$$\frac{\mathrm{d}(\text{product})}{\mathrm{d}\beta} = 0 \Rightarrow \beta = \frac{1}{2} + \frac{2w-c}{\left[\hat{F}(p)\right]};$$

$$\frac{\mathrm{d}(\text{product})}{\mathrm{d}w} = \text{same !}$$

So w can be arbitrary and $\beta = \frac{1}{2} + \frac{2w-c}{\left[\hat{F}(p)\right]}$,

$$\frac{c}{2} - \frac{\left[\hat{F}(p)\right]}{4} < w < \frac{c}{2} + \frac{\left[\hat{F}(p)\right]}{4}. \tag{12}$$

$$\Rightarrow \tilde{E}_3\left(\Pi^{\mathrm{R}} \mid 2\right) = \tilde{E}_3\left(\Pi^{\mathrm{M}} \mid 2\right) = \left(\frac{1}{2} + \frac{2w-c}{\left[\hat{F}(p)\right]}\right)\left[\hat{F}(p)\right] - 2w = \frac{1}{2}\left[\hat{F}(p)\right] - c$$

[5]We could set $w = 0$ and work with $\beta = \frac{\hat{F} - 2c}{2\hat{F}}$, but we shall not do that.

$$\frac{\mathrm{d}\tilde{E}_3\left(\Pi^{\mathrm{R}}\,|\,2\right)}{\mathrm{d}p}=0\Rightarrow\frac{1}{2}\Big[\left[F(p)\right]^3-3(1-p)\left[F(p)\right]^2-16p\left[F(p)\right]+2\Big]=0$$

Now substitute $F(p)=p^{\alpha}$: $p^{3\alpha}+3p^{2\alpha+1}-3p^{2\alpha}-16p^{\alpha+1}+2=0$.

$\underline{\alpha}$

1: $4p^3-19p^2+2=0\Rightarrow p\sim 0.34\Rightarrow E_3^*\left(\Pi^{\mathrm{R}}\,|\,2\right)\sim 0.29-c$

2: $p^6+3p^5-3p^4-16p^3+2=0\Rightarrow p\sim 0.49\Rightarrow E_3^*\left(\Pi^{\mathrm{R}}\,|\,2\right)\sim 0.48-c$

3: $p^9+3p^7-3p^6-16p^4+2=0\Rightarrow p\sim 0.59\Rightarrow E_3^*\left(\Pi^{\mathrm{R}}\,|\,2\right)\sim 0.56-c$.

$\underline{q=3}$

$$E_3\left(\Pi^{\mathrm{R}}\,|\,3\right)=\beta p\left(1-\left[F(p)\right]^3\right)-3w$$

$E_3\left(\Pi^{\mathrm{M}}\,|\,3\right)=(1-\beta)\,p\left(1-\left[F(p)\right]^3\right)+3w-3c$. Denote $p\left(1-\left[F(p)\right]^3\right)$ by, $L>0$.

So, $\text{product}=\beta(1-\beta)L^2-3w(1-\beta)L+3w\beta L-9w^2-3c\beta L+9cw$.

$$\frac{\mathrm{d}(\text{product})}{\mathrm{d}\beta}=L^2(1-2\beta)+3wL+3wL-3cL=0.$$

$$\Rightarrow\beta=\frac{1}{2}+\frac{3(2w-c)}{2L}.$$

$$\frac{\mathrm{d}(\text{product})}{\mathrm{d}w}=-3(1-\beta)L+3\beta L-18w+9c=0.$$

$$\Rightarrow\beta=\frac{1}{2}+\frac{3(2w-c)}{2L}\rightarrow\text{same!}$$

So let w be arbitrary in the range $\left[\frac{1}{2}c-\frac{p\left(1-p^{3\alpha}\right)}{6},\frac{1}{2}c+\frac{p\left(1-p^{3\alpha}\right)}{6}\right]$,

$$c>\frac{p\left(1-p^{3\alpha}\right)}{3},\tag{13}$$

while $\beta=\frac{1}{2}+\frac{3(2w-c)}{2L}$.

So, $\tilde{E}_3\left(\Pi^{\mathrm{R}}\,|\,3\right)=\left[\frac{1}{2}+\frac{3(2w-c)}{2L}\right]L-3w=\frac{L}{2}-\frac{3c}{2}=\tilde{E}_3\left(\Pi^{\mathrm{M}}\,|\,3\right)$.

$\frac{\mathrm{d}}{\mathrm{d}p}=\frac{1}{2}\left[1-\left[F(p)\right]^3+p\left(-3\left[F(p)\right]^2\left[f(p)\right]\right)\right]=0$, so for power$(\alpha)$ distribution

$$1-p^{3\alpha}-3\alpha p^{3\alpha}=0$$

$p_3^*=\frac{1}{(1+3\alpha)^{\frac{1}{3\alpha}}}$. For $\alpha=1$, $p_3^*\sim 0.63$.

$\Rightarrow E_3^*\left(\Pi^{\mathrm{R}}\,|\,3\right)=\frac{3}{2}\left[\frac{\alpha-c(1+3\alpha)^{\frac{1}{3\alpha}+1}}{(1+3\alpha)^{\frac{1}{3\alpha}+1}}\right]=E_3^*\left(\Pi^{\mathrm{M}}\,|\,3\right)$, $\uparrow\alpha$. For $\alpha=1$,

$E_3^*(\Pi\,|\,3)=0.22-1.50c$.

Asymmetric NBS, N = 3

$\underline{q=1}$. Expected revenue = $p\left(1-\left[F(p)\right]^3\right)\equiv a(p)$

$$E_3\left(\Pi^{\mathrm{R}}\,|1\right)=\beta a(p)-w\,;$$

$$E_3\left(\Pi^{\mathrm{M}}\,|1\right)=(1-\beta)a(p)+w-c\,.$$

$$\text{Asymmetric Nash product}=\left[\beta a(p)-w\right]^{\lambda}\left[(1-\beta)a(p)+w-c\right]^{1-\lambda},0\le\lambda\le1. \quad (14)$$

So

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\beta}&=a(p)\lambda\left[\beta a(p)-w\right]^{\lambda-1}\left[(1-\beta)a(p)+w-c\right]^{1-\lambda}\\&\quad-a(p)(1-\lambda)\left[\beta a(p)-w\right]^{\lambda}\left[(1-\beta)a(p)+w-c\right]^{-\lambda}\\&=0\end{aligned}$$

$$\begin{aligned}&\Rightarrow a(p)\left[\beta a(p)-w\right]^{\lambda-1}\left[(1-\beta)a(p)+w-c\right]^{-\lambda}\\&\quad\cdot\left\{\lambda\left[(1-\beta)a(p)+w-c\right]-(1-\lambda)\left[\beta a(p)-w\right]\right\}=0.\end{aligned}$$

Let $\lambda\left[(1-\beta)a(p)+w-c\right]-(1-\lambda)\left[\beta a(p)-w\right]\equiv M$.

Assuming that $p\neq0,1$, there are thus three solutions:

1) $\beta a(p)-w=0\Rightarrow\beta=\dfrac{w}{a(p)},w<a(p)$;

2) $(1-\beta)a(p)+w-c=0\Rightarrow\beta=1+\dfrac{w-c}{a(p)}>1$ so that solution is not valid;

3) $\{M\}=0\Rightarrow\beta=\lambda+\dfrac{w-\lambda c}{a(p)}$, $\lambda\left[c-a(p)\right]<w<\lambda c+(1-\lambda)a(p)$. We shall use that solution.

$$\frac{\mathrm{d}}{\mathrm{d}w}=0\Rightarrow\beta=\lambda+\frac{w-\lambda c}{a(p)}\Rightarrow\text{same}.$$

$$\Rightarrow\tilde{E}_3\left(\Pi^{\mathrm{R}}\,|1\right)=\lambda\left[a(p)-c\right]$$

$$\tilde{E}_3\left(\Pi^{\mathrm{M}}\,|1\right)=(1-\lambda)\left[a(p)-c\right]$$

$\dfrac{\mathrm{d}\tilde{E}_3\left(\Pi^{\mathrm{R}}\,|1\right)}{\mathrm{d}p}=\lambda a'(p)=0$. But $a'(p)=1-\left[F(p)\right]^3+p\left(-3\left[F(p)\right]^2\left[f(p)\right]\right)$.

So for a power(α) distribution

$$\Rightarrow p^{3\alpha}=\frac{1}{1+3\alpha}\Rightarrow p_1^*=\frac{1}{(1+3\alpha)^{\frac{1}{3\alpha}}}$$

$$\Rightarrow E_3^*\left(\Pi^{\mathrm{R}}\,|1\right)=\lambda\left[p\left(1-p^{3\alpha}\right)-c\right]=\lambda\left[\frac{3\alpha}{(1+3\alpha)^{1+\frac{1}{3\alpha}}}-c\right]$$

$$E_3^*\left(\Pi^{\mathrm{M}} \mid 1\right) = (1-\lambda)\left[p\left(1-p^{3\alpha}\right)-c\right] = (1-\lambda)\left[\frac{3\alpha}{(1+3\alpha)^{1+\frac{1}{3\alpha}}}-c\right]$$

<u>$q = 2$</u>

$$\text{Expected revenue} = p\left(\left[F(p)\right]^3 - 3\left[F(p)\right]^2 + 2\right) = p^{3\alpha+1} - 3p^{2\alpha+1} + 2p \equiv b(p) \geq 0.$$

$$\mathrm{R}: \beta b(p) - 2w;$$

$$\mathrm{M}: (1-\beta)b(p) + 2w - 2c.$$

$$\text{asymmetric Nash product} \Rightarrow \left[\beta b(p)-2w\right]^{\lambda}\left[(1-\beta)b(p)+2w-2c\right]^{1-\lambda} \quad (15)$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\beta} &= \lambda\left[\beta b(p)-2w\right]^{\lambda-1} b(p)\left[(1-\beta)b(p)+2w-2c\right]^{1-\lambda} \\ &\quad + \left[\beta b(p)-2w\right]^{\lambda}(1-\lambda)\left[(1-\beta)b(p)+2w-2c\right]^{-\lambda}\left[-b(p)\right] = 0\end{aligned}$$

$$\begin{aligned}&\Rightarrow \left[\beta b(p)-2w\right]^{\lambda-1}\left[(1-\beta)b(p)+2w-2c\right]^{-\lambda} \\ &\quad \cdot\left\{\lambda\left[(1-\beta)b(p)+2w-2c\right]b(p)-(1-\lambda)\left[\beta b(p)-2w\right]b(p)\right\} = 0\end{aligned}$$

1) $\beta = \frac{2w}{b(p)}$, $w < \frac{b(p)}{2}$;

2) $\beta = 1 + \frac{2w-2c}{b(p)} > 1$, so that solution is not valid;

3) $\beta = \lambda + \frac{2(w-\lambda c)}{b(p)}$. We shall use that solution.

To have $0 \leq \beta \leq 1$ need $\frac{\lambda\left[2c-b(p)\right]}{2} \leq w \leq \frac{2\lambda c+(1-\lambda)b(p)}{2}$.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}w} &= -2\lambda\left[\beta b(p)-2w\right]^{\lambda-1}\left[(1-\beta)b(p)+2w-2c\right]^{1-\lambda} \\ &\quad + 2\left[(1-\beta)b(p)+2w-2c\right]^{-\lambda}(1-\lambda)\left[\beta b(p)-2w\right]^{\lambda} \\ &= 0\end{aligned}$$

$$\Rightarrow \beta = \lambda + \frac{2(w-\lambda c)}{b(p)} \quad \Rightarrow \quad \text{same.}$$

Thus $\tilde{E}_3\left(\Pi^{\mathrm{R}} \mid 2\right) = \lambda\left[b(p)-2c\right]$, and $\tilde{E}_3\left(\Pi^{\mathrm{M}} \mid 2\right) = (1-\lambda)\left[b(p)-2c\right]$.

So, $\frac{\mathrm{d}}{\mathrm{d}p}\tilde{E}_3\left(\Pi^{\mathrm{R}} \mid 2\right) = 0$ if $b'(p) = 0$ $\Rightarrow (3\alpha+1)p^{3\alpha} - 3(2\alpha+1)p^{2\alpha} + 2 = 0$.

If $\alpha = 1$, $p_2^* = 0.54$. If $\alpha = 2$, $p_2^* = 0.64$.

So for $\alpha = 1$, $E_3^*\left(\Pi^{\mathrm{R}} \mid 2\right) = \lambda\left[b(0.54)-2c\right] \sim \lambda(0.69-2c)$, and $E_3^*\left(\Pi^{\mathrm{M}} \mid 2\right) = (1-\lambda)(0.69-2c)$.

If $\alpha = 2$, $E_3^*\left(\Pi^{\mathrm{R}} \mid 2\right) = \lambda\left[b(0.67)-2c\right] = \lambda(1-2c)$, $E_3^*\left(\Pi^{\mathrm{M}} \mid 2\right) = (1-\lambda)(1-2c)$.

<u>$q = 3$</u>

$$E(\text{revenue}) = 3p\left(1-\left[F(p)\right]\right) = 3p\left(1-p^{\alpha}\right) \equiv d(p)(>0)$$

$$E_3\left(\Pi^{\mathrm{R}} \mid 3\right) = \beta d(p) - 3w,$$

$$E_3\left(\Pi^{\mathrm{M}} \mid 3\right) = (1-\beta)d(p) + 3w - 3c.$$

$$\text{product} = \left[\beta d(p) - 3w\right]^{\lambda}\left[(1-\beta)d(p) + 3w - 3c\right]^{1-\lambda}$$

$$\frac{\mathrm{d}}{\mathrm{d}\beta} = \left[\beta d(p) - 3w\right]^{\lambda-1}\left[(1-\beta)d(p) + 3w - 3c\right]^{-\lambda} d(p)$$
$$\cdot\left\{\lambda\left[(1-\beta)d(p) + 3w - 3c\right] - (1-\lambda)\left[\beta d(p) - 3w\right]\right\}$$
$$= 0$$

$$\{\ \} = 0 \Rightarrow \beta\left[-\lambda d(p) - (1-\lambda)d(p)\right] = -\lambda d(p) - 3\lambda w + 3\lambda c - 3(1-\lambda)w$$

$\Rightarrow \beta = \lambda + \dfrac{3(w-\lambda c)}{d(p)}$. To have $0 < \beta \le 1$ need

$2c - \dfrac{1}{3}\lambda d(p) < w < 2c + \dfrac{1}{3}(1-\lambda)d(p)$.

$$\frac{\mathrm{d}}{\mathrm{d}w} = (\beta-\lambda)d(p) - 3w + \lambda d(p) = \beta d(p) - 3w = 0$$

$$\Rightarrow \beta = \lambda + \frac{3(w-\lambda c)}{d(p)} \Rightarrow \text{same}$$

$$\Rightarrow \tilde{E}_3\left(\Pi^{\mathrm{R}} \mid 3\right) = \lambda\left[d(p) - 3c\right].$$

$$\tilde{E}_3\left(\Pi^{\mathrm{M}} \mid 3\right) = (1-\lambda)\left[d(p) - 3c\right].$$

$$\frac{\mathrm{d}}{\mathrm{d}p}\tilde{E}_3\left(\Pi^{\mathrm{R}} \mid 3\right) = \lambda d'(p) = 0 \Rightarrow 1-(\alpha+1)p^{\alpha} = 0 \Rightarrow p^{\alpha} = \frac{1}{\alpha+1} \Rightarrow p = \frac{1}{(\alpha+1)^{\frac{1}{\alpha}}}.$$

Thus $E_3^*\left(\Pi^{\mathrm{R}} \mid 3\right) = 3\lambda\left[\dfrac{\alpha}{(\alpha+1)^{1+\frac{1}{\alpha}}} - c\right], \uparrow\lambda, \downarrow c$

$$E_3^*\left(\Pi^{\mathrm{M}} \mid 3\right) = 3(1-\lambda)\left[\frac{\alpha}{(\alpha+1)^{\frac{1}{\alpha}+1}} - c\right], \downarrow\lambda, \downarrow c.$$

For $\alpha = 1$, if $c > 0.23$ then $1 \succ 2 \succ 3$; if $0.14 < c < 0.23$ then $2 \succ 1 \succ 3$;

If $0.06 < c < 0.14$ then $2 \succ 3 \succ 1$; if $c < 0.06$, then $3 \succ 2 \succ 1$, independent of λ.

Note that, for all q $E_3^*\left(\Pi^{\mathrm{R}} \mid q\right) \ge E_3^*\left(\Pi^{\mathrm{M}} \mid q\right)$ iff $\lambda > 1-\lambda$ i.e., iff $\lambda > \dfrac{1}{2}$; that is, if R is "stronger" than M, its expected profit is higher, as one would expect.

For $\alpha = 2$, if $c > 0.15$ then $1 \succ 2 \succ 3$. If $c < 0.15$ then $3 \succ 2 \succ 1$.

7. Concluding Remarks

Potential customers attempt to buy an item only if their valuation for it exceeds its price. Yet these valuations are not known to the price-setter, and she is assumed here to have a $\text{beta}(\alpha,1)$ distribution over them. We modeled the retailer as a price-setting newsvendor who faces a (discrete) demand whose randomness is due to the unknown number of valuations which exceed a particular price. That translates to a binomially distributed number of attempted purchases.

In addition to analyzing the basic model and providing examples, we considered several extensions. One is non-linear production costs (in the basic model they are linear); another is an unknown number of potential customers, with a special focus on a Poisson distributed one. We also analyze a retailer/manufacturer revenue sharing scenario, where the parties bargain over revenue shares and wholesale price according to the Nash bargaining solution, symmetric and asymmetric, as functions of retailer then selects; the retailer then selects that price, which, in turn, determines the revenue shares, wholesale price, and the expected profits of the parties.

This work combines elements from Supply Chain Management, Marketing, Economics, Game Theory and Probability Models. In the future one could extend our model by delving deeper into the implications of these disciplines to our setting. For example, to contracts other than revenue sharing, or to a scenario of a retailer selling several products with multinomial number of price-exceeding customers. One could consider a retailer who is not risk-neutral. The production costs can be modeled differently than as quadratic. The number of possible population sizes can be larger than two. A major change in the model would be to make the random population size continuous and their aggregate demand, say, normally distributed.

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Biographical Note

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A

Fixing q and differentiating (2) w.r.t. p, we obtain that for $X \sim U[0,1]$.

$$\frac{\mathrm{d}E(\pi)}{\mathrm{d}p}=0$$

$$\Rightarrow N\sum_{k=1}^{q-1}k\binom{N}{k}(1-p)^k p^{-k}-\frac{1}{1-p}\sum_{k=1}^{q-1}k^2\binom{N}{k}(1-p)^k p^{-k}$$

$$+q\left\{\frac{-p}{1-p}\sum_{k=q}^{N}k\binom{N}{k}(1-p)^k p^{-k}+N\sum_{k=q}^{N}\binom{N}{k}p^{-k}-\sum_{k=q}^{N}k\binom{N}{k}p^{-k}\right\}=0$$

For $\underline{q=2,}$

$$N^2+2pN-N+2\left\{\frac{-p}{1-p}\left[\sum_{k=1}^{N}k\binom{N}{k}(1-p)^k p^{-k}-\frac{N(1-p)}{pN}\right]\right.$$

$$\left.+N\left[\sum_{k=0}^{N}\binom{N}{k}p^{-k}-1-\frac{N}{p}\right]-\left[\sum_{k=1}^{N}k\binom{N}{k}p^{-k}-\frac{N}{pN}\right]\right\}=0$$

Using the combinatorial sums

$$\sum_{k=0}^{N}\binom{N}{k}\left(\frac{1}{p}\right)^k=\left(\frac{1+p}{p^N}\right)^N$$

$$\sum_{k=1}^{N}k\binom{N}{k}\left(\frac{1-p}{p^N}\right)^k=\frac{N(1-p)}{p^N}$$

$$\sum_{k=1}^{N}k\binom{N}{k}\left(\frac{1}{p^N}\right)^k=\frac{N(1+p)^{N-1}}{p^N},$$

we obtain that for $q=2$

$$Np^{N-1}+2p^N-p^{N-1}-2+2(1+p)^{N-1}=0$$

For $\underline{q=3,}$

$$\frac{N}{p^2}\left[N^2(1-p)^2-N\left(2p^2+5p+3\right)+2-3p\right]+\frac{N}{p^N}\left[-3p+3(1+p)^N-3(1+p)^{N-1}\right]$$

$$=\frac{N}{p^N}-3p+3(1+p)^{N-1}p+N^2\left(p^{N-2}-2p^{N-1}+p^N\right)+N+\left(-5p^N-5p^{N-1}+\frac{3p^{N-2}}{2}\right).$$

For each q, the optimality equations need to be solved for p, the result substituted in the objective function and these values compared for the various q' s.

Appendix B

If $X \sim U[0,1]$,

$$E(\Pi\,|\,q)=p\left\{\theta\sum_{k=1}^{q-1}k\binom{N_1}{k}(1-p)^k p^{N_1-k}+q\sum_{k=q}^{n}\binom{N_1}{k}(1-p)^k p^{-k}\right.$$

$$\left.+(1-\theta)\left[\sum_{k=1}^{q-1}k\binom{N_2}{k}(1-p)^k p^{N_2-k}+q\sum_{k=q}^{N_2}(1-p)^k p^{N_2-k}\right]\right\}-cq.$$

If $\underline{q = 2,}$

$$E(\Pi \mid q) = p\left\{\theta\left[N_1 p^{N_1-1} - p^{N_1} + 2\sum_{k=2}^{N}\binom{N_1}{k}(1-p)^k\left(p^{N_1-k}\right)\right]\right.$$
$$\left.+(1-\theta)\left[N_2\left(p^{N_2-1} - p^{N_2}\right) + 2\left[\sum_{k=2}^{N_2}(1-p)^k p^{N_2-k}\right]\right]\right\} - 2c.$$

Using the combinatorial sums from Appendix A,

$$\begin{aligned}
\frac{\mathrm{d}E(\Pi \mid q)}{\mathrm{d}p} &= \theta\left[N_1\left(p^{N_1} - p^{N_2}\right) + N_1\left(p^{N_1-2}N_1 - p^{N_1-2} - N_1 p^{N_1-1}\right)\right] \\
&\quad +(1-\theta)\left[N_2\left(p^{N_2-1} - p^{N_2}\right) + N_2\left(p^{N_2-2}N_2 - p^{N_2-2} - p^{N_2-1}N_2\right)\right] \\
&\quad +2 - \frac{2\theta p^2}{(1-p)p}N_1(1-p) + \frac{2\theta p(1-p)N_1}{(1-p)p} - \frac{2\theta p(1-p)}{(1-p)p}N_1(1-p) \\
&\quad -\frac{2(1-\theta)p^2}{(1-p)p}N_2(1-p) + \frac{2(1-\theta)p(1-p)N_2}{(1-p)p} \\
&\quad -2(1-\theta)p(1-p)N_2(1-p) - 2(1-\theta)p^2 N_2(1-p) \\
&\quad +2(1-\theta)p(1-p) - 2(1-\theta)p(1-p)N_2(1-p) \\
&= \theta N_1 p^{N_1-2}\left(N_1 - 1 - N_1 p + p - p^2\right) \\
&\quad +(1-\theta)N_2 p^{N_2-2}\left(N_2 - 1 - N_2 p + p - p^2\right) + (1-\theta)N_2^2 p^{N_2-2} \\
&\quad -(1-\theta)N_2 p^{N_2-2} - (1-\theta)N_2^2 p^{N_2-1} + (1-\theta)N_2 p^{N_2-1} - (1-\theta)N_2 p^{N_2} \\
&\quad -2\theta p^{N_1} - 2\theta N_1 p^{N_1-1} + 2\theta N_1 p^{N_1} - 2(1-\theta)p^{N_2} - 2(1-\theta)p^{N_2-1} \\
&\quad +2(1-\theta)N_2 - 2(1-\theta)(1-p)N_2 - 2(1-\theta)p^2(1-p)N_2 \\
&\quad +2(1-\theta)p(1-p)N_2 - 2(1-\theta)p(1-p)^2 N_2 \\
&= \theta\left\{2(N_1-1)p^{N_1} - N_1(N_1+2)p^{N_1-1} + N_1(N_1-1)p^{N_1-2} + 2p^{N_2}\right. \\
&\quad \left. +\left(N_2^2 - N_2 + 2\right)p^{N_2-1} - N_2(N_2-1)p^{N_2-1} - 2pN_2 - 4N_2 + \frac{2N_2}{p}\right\} \\
&\quad -2p^{N_2} + \left(-N_2^2 + N_2 - 2\right)p^{N_2-1} + N_2(N_2-1)p^{N_2-2} - \frac{2N_2}{p} - 2pN_2 \\
&= 0.
\end{aligned}$$

If $\theta = \frac{1}{2}$,

$$\begin{aligned}
\text{above} &= (N_1-1)p^{N_1} - \frac{1}{2}N_1(N_1+2)p^{N_1-1} + \frac{1}{2}N_1(N_1-1)p^{N_1-2} \\
&\quad - p^{N_2} - \frac{1}{2}\left(N_2^2 - N_2 + 2\right)p^{N_2-1} + \frac{1}{2}N_2(N_2-1)p^{N_2-2} \\
&\quad + N_2\left(2 - 3p - \frac{1}{p}\right) \\
&= 0
\end{aligned}$$

p should be solved from this equation for the desired pair (N_1, N_2). Then the procedure should be repeated for $q = 3$, etc. The resulting objective values should be compared.