

# A Dual-Spread Opportunity VAM for Transportation Problems under Deterministic Cost Environments

Farhana Rashid\*, Rabindra Nath Mondal

Department of Mathematics, Jagannath University, Dhaka, Bangladesh  
Email: \*farhanajammy23@gmail.com, \*farhana@math.jnu.ac.bd

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## Abstract

Transportation challenges often occur in logistics and supply chain management, necessitating the movement of commodities from several sources to different destinations while minimizing transportation expenses. This paper proposes a Dual-Spread Opportunity VAM (DSO-VAM) for obtaining high-quality initial basic feasible solutions to transportation problems. The method is motivated by a revised VAM framework that improves classical penalty-based allocation by modifying row and column penalties. To address some limitations of existing methods, the proposed method combines dual-spread row and column penalties with an opportunity-based decision score that measures the future regret of postponing a low-cost allocation. A fixed weighting mechanism is introduced so that the relative influence of cost spread and allocation pressure changes dynamically throughout the solution process. Numerical examples show that DSO-VAM matches or improves upon the solutions obtained by NWCM, MCM, VAM, and the Revised-VAM. The proposed framework preserves computational simplicity while providing a more flexible and globally informed allocation rule for transportation optimization.

## Keywords

Transportation Problem, Optimization, Dual-Spread Opportunity VAM

## 1. Introduction

Linear Programming, which deals with the best uses of limited resources, is one of the most important branches of operations management. The branch of LPP in which a single homogeneous product is transported from several sources to nu-

merous localities in such a way to minimize the total transportation cost while satisfying all supply and demand restrictions is Transportation Problem (TP). The transportation of commodities from production sites to global markets is a fundamental aspect of contemporary life. Companies nationwide expend billions of dollars each year on the transportation of products. The transportation problem (TP) is a specific category of network optimization issue that involves the distribution of a uniform product from several origins (e.g., factories) to different destinations (e.g., warehouses). The objective of the TP is to identify a method for executing this transfer of commodities at the lowest overall cost. The Transportation Problem originated with Hitchcock [1], was strengthened by Koopmans [2], and Dantzig's [3] Simplex method, and evolved through algorithmic methods like as North West Corner Method [4], VAM [5] and MODI [3] into modern AI-driven and uncertainty-based optimization models. Russell [6] proposed Russel's Approximation Method and he makes maximum possible allocation to the cell having the smallest penalty. Khan A.R. [7] [8] presented the Highest Cost Difference Method (HCDM) by defining pointer cost as the difference between the highest and next to highest cost in each row and column of a transportation table and allocate to the minimum cost cell corresponding to the highest three pointer cost. Sudhakar *et al.* [9] developed a new direction for searching the optimal solution of the transportation problem. Besides this, a large number of research works on distribution issues has been done by several researchers such as [10]-[18], which solved a transportation problem with market choice. In decision-making theory, uncertain decision-making is a vital branch. There are various approaches to dealing with uncertainty problems [19]. To deal with uncertainties, a number of methods were developed, including interval, fuzzy, rough, and stochastic numbers [20]-[22]. Many researchers [23]-[28] have proposed different techniques for solving transportation problem in uncertain environment. Several researchers attempted to improve VAM: Soomro *et al.* [29], Hakim [30], Karagul and Sahin [31] etc. Despite the wide application of classical and modified transportation algorithms such as Vogel's Approximation Method (VAM), Revised VAM, and other heuristic approaches, several critical limitations remain unaddressed. Most existing methods, including classical VAM, determine allocations based on row-wise or column-wise penalties calculated from local cost differences; these penalties are purely local indicators, and they do not consider the overall structure of the cost matrix. As a result, decisions may be short-sighted, ignoring better global allocations. A cell may appear optimal locally but may not be globally efficient when considering the entire network. Existing approaches fail to evaluate the global opportunity cost of selecting a particular cell. They do not measure: how much benefit is gained relative to other possible allocations, the overall impact of choosing a specific route on total system cost. No mechanism exists to identify high-impact allocation points that significantly reduce total cost. Traditional methods treat cost values as fixed and static throughout the allocation process. No dynamic adjustment based on: remaining supply and demand, chang-

ing structure of the reduced matrix, decision rules remain unchanged at every iteration. Lack of adaptability leads to inefficient allocation sequences, especially in complex problems. Most classical transportation methods assume deterministic (crisp) cost values. Real-world transportation problems often involve uncertain data. Existing methods cannot handle grey numbers interval costs, uncertain environments. This limits their applicability in real-life decision-making scenarios. To address the above gaps, this paper introduces a novel method: Dual-Spread Opportunity Vogel's Approximation Method (DSO-VAM) enhances classical VAM by integrating some innovative components: Unlike traditional penalties, DSO-VAM computes a dual-spread penalty that captures: Local variation  $\rightarrow$  difference between smallest costs (as in VAM) and also Global variation  $\rightarrow$  spread of costs across the entire row/column, which provides a more comprehensive measure of cost variability. Prevents misleading decisions based only on local differences. As a result, better identification of strategically important rows/columns. DSO-VAM introduces a new metric called the Opportunity Score, which evaluates each candidate cell based on low transportation cost (cheap routes) and high allocation potential (large supply/demand impact). So prioritizes cells minimize cost and maximize overall impact on total solution. The cell moves beyond simple "minimum cost selection" to intelligent allocation. And the Decision Index (DI) combines penalty information (risk) and Opportunity score (benefit) using fixed weighting, which balances risk (cost variability), benefit (allocation efficiency) and allows the algorithm to adapt dynamically at each step. So, it will be more robust and flexible decision-making compared to static rules. The proposed framework can be extended to grey transportation problems, interval-valued costs, which is applicable in real-world uncertain environments. Although in this research paper we do not focus on the problem of uncertain environment.

## 2. Methodology

In this section we introduced some methodology of the proposed method:

**Statement-1 (Feasibility Preservation):** The DSO-VAM algorithm always produces a feasible solution satisfying all supply and demand constraints. At each iteration, allocation is made as  $x_{ij} = \min(s_i, d_j)$ . Thus supply/demand is reduced correctly, no constraint is violated. Since each allocation is made as  $x_{ij} = \min(s_i, d_j)$ , the allocation never exceeds available supply or required demand. Therefore, supply and demand constraints are maintained at every step. Hence feasibility is preserved at every step.

**Statement-2 (Finite Termination):** DSO-VAM terminates in at most  $m + n - 1$  allocation. After every allocation, at least one row or one column is satisfied and removed from the table. Since the number of rows and columns is finite, the algorithm must stop after a finite number of steps.

**Statement-3 (Improved Allocation Efficiency):** DSO-VAM produces equal or better IBFS than classical VAM under structured cost matrices. VAM uses:  $Penalty = c_2 - c_1$  and DSO-VAM uses:  $Penalty = \alpha(c_2 - c_1) + (1 - \alpha)(c_{\max} - c_1)$ .

Thus, it captures both local and global spread, reducing the risk of poor early allocation. Hence improves expected cost.  $\alpha \in [0,1]$

**Statement-4 (Generalization Property):** If  $\alpha = 1$ : DSO-VAM reduces to Classical VAM. And if  $\alpha = 0$ : DSO-VAM becomes maximum spread heuristic. If  $\alpha = 1$ , then the DSO-VAM penalty becomes the classical VAM penalty. Therefore, VAM is a special case of DSO-VAM. Thus DSO-VAM is a generalized framework. Overall DSO-VAM fills this gap by moving from a fixed penalty rule to a structured decision score that incorporates spread, opportunity, and pressure.

**Statement-5 (Parameter Selection)** In this study, the values  $\alpha = 0.6$ ,  $\beta = 0.3$ , and  $\gamma = 0.1$  are used for all numerical examples to maintain consistency and reproducibility. The value of  $\alpha$  is given higher weight because the dual-spread penalty is the main part of the proposed method. The value of  $\beta$  gives moderate importance to cost-saving opportunity, while  $\gamma$  gives small importance to allocation pressure.

### 3. Preliminaries

#### Classical Transportation Problem

The classical transportation problem in operation research involves finding the optimal way to move goods from one place to another. It involves allocating resources in the most efficient way while minimizing the cost of transportation. It is based on objective function. An objective function is a function whose value we try to maximize (profit) or minimize (travel length, costs, time, ...) in the process of optimization. Then the linear programming model representing the transportation problem is generally given as

$$\begin{aligned} \text{Minimize } z &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} . \\ \text{subject to } \sum_{j=1}^n x_{ij} &\leq s_i ; \quad i = 1, 2, \dots, m \\ \sum_{i=1}^m x_{ij} &\geq d_j ; \quad j = 1, 2, \dots, n \\ x_{ij} &\geq 0 . \text{ for all } i \text{ and } j . \end{aligned}$$

In mathematical terms the above problem can be expressed as finding a set of  $x_{ij}$ 's,

$$i = 1, 2, \dots, m ; \quad j = 1, 2, \dots, n \text{ to}$$

$$\text{Minimize } z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} .$$

$$\text{subject to } \sum_{j=1}^n x_{ij} = s_i ; \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = d_j ; \quad j = 1, 2, \dots, n , \quad x_{ij} \geq 0 . \text{ for all } i \text{ and } j .$$

### 4. Proposed Algorithms

**Step 1:** Balance the problem:

If total supply differs from total demand, add a dummy row or dummy column with zero transportation cost.

**Step 2:** Compute row dual-spread penalty:

For each active row  $i$ , sort the available costs:  $c_{i(1)} \leq c_{i(2)} \leq c_{i(3)} \leq \dots$

Define:  $RP_i = \alpha(c_{i(2)} - c_{i(1)}) + (1 - \alpha)(c_{i(\max)} - c_{i(1)})$ . This combines the classical short spread and the full spread.

**Step 3:** Compute column dual-spread penalty:

For each active row  $i$ , sort the available costs:  $c_{(1)j} \leq c_{(2)j} \leq c_{(3)j} \leq \dots$

Define:  $CP_j = \alpha(c_{(2)j} - c_{(1)j}) + (1 - \alpha)(c_{(\max)j} - c_{(1)j})$ . This makes row and column logic symmetric.

**Step 4:** Select  $P_{ij} = \max(RP_i, CP_j)$ .

**Step 5:** Compute opportunity score for candidate minimum cells

For each row-minimum or column-minimum candidate cell  $(i, j)$  define:

$$O_{ij} = \beta(\bar{c} - c_{ij}) + \gamma \max(s_i, d_j)$$

where  $\bar{c}$  is the mean of all active costs. This rewards cheap cells with strong allocation capacity.

**Step 6:** Compute unified decision index:

For each candidate cell  $(i, j)$ ,  $DI_{ij} = P_{ij} - O_{ij}$ , where  $P_{ij}$  is the larger of the relevant row and column penalties associated with that cell. Choose the cell with the largest  $DI_{ij}$ .

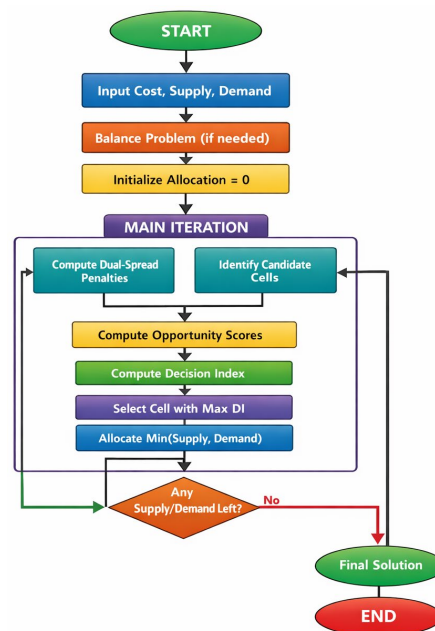
**Step 7:** Allocate:  $x_{ij} = \min(s_i, d_j)$ .

**Step 8:** Update tableau with reduce the relevant supply and demand. Delete satisfied row or column.

**Step 9:** Repeat until all supplies and demands are exhausted.

### Flow Chart of Proposed Method

Proposed method described briefly in **Figure 1** using flow chart.



**Figure 1.** Flow Chart of proposed method.

### 5. Numerical Examples

**Example: 1**

<i>S/D</i>	<i>D</i> <sub>1</sub>	<i>D</i> <sub>2</sub>	<i>D</i> <sub>3</sub>	<i>D</i> <sub>4</sub>	Supply
<i>S</i> <sub>1</sub>	7	5	9	8	20
<i>S</i> <sub>2</sub>	6	4	7	6	30
<i>S</i> <sub>3</sub>	8	6	5	7	25
<i>S</i> <sub>4</sub>	9	7	6	5	15
Demand	25	20	30	15	90

Solution: Here we take  $\alpha = 0.6, \beta = 0.3, \gamma = 0.1, [\alpha, \beta, \gamma \in (0,1)]$ .

Step-1: In this example Total Supply = Total Demand. So it is a balance Transportation problem.

Step-2 & Step-3: **Table 1** gives computed row/column dual-spread penalty,  $RP_1$  and  $CP_1$ .

**Table 1.** Compute row/column dual-spread penalty,  $RP_1$  and  $CP_1$ .

	<i>D</i> <sub>1</sub>	<i>D</i> <sub>2</sub>	<i>D</i> <sub>3</sub>	<i>D</i> <sub>4</sub>	Supply	MAX	2ND MIN	MIN	$RP_1$
<i>S</i> <sub>1</sub>	7	5	9	8	20	8	7	5	2.4
<i>S</i> <sub>2</sub>	6	4	7	6	30	7	6	4	2.4
<i>S</i> <sub>3</sub>	8	6	5	7	25	8	6	5	1.8
<i>S</i> <sub>4</sub>	9	7	6	5	15	9	6	5	2.2
Demand	25	20	30	15	90				
MAX	9	7	9	8					
2ND MIN	7	5	6	6					
MIN	6	4	5	5					
$CP_1$	1.8	1.8	2.2	1.8					

Step-4: Select  $P_{ij} = \max(RP_1, CP_1) = 2.4$ .

Step-5 & Step-6: Compute opportunity score for candidate minimum cells  
 For each row-minimum or column-minimum candidate cell  $(i, j)$  define:

$$O_{ij} = \beta(\bar{c} - c_{ij}) + \gamma \max(s_i, d_j) \quad [\alpha = 0.6, \beta = 0.3, \gamma = 0.1]$$

And unified decision index:

For each candidate cell  $(i, j)$ , decision index  $DI_{ij} = P_{ij} - O_{ij}$ . So we get all decision index  $DI_{ij}$  shown in **Table 2**:

**Table 2.** Compute decision index.

Row-Minima	$(S_1, D_2)$	$(S_2, D_2)$	$(S_3, D_3)$	$(S_4, D_4)$
Column-Minima	$(S_2, D_1)$	$(S_2, D_2)$	$(S_3, D_3)$	$(S_4, D_4)$
unique candidate cells	$(S_1, D_2)$	$(S_2, D_2)$	$(S_2, D_1)$	$(S_3, D_3)$ $(S_4, D_4)$
Average cost	6.5625			

**Continued**

Candidate cell		
C-1	$(S_1, D_2)$	cost = 5
	$P_{12} =$	2.4
	$O_{12} =$	$(0.3*(AVE.COST - 5) + 0.1*MAX(s_1, d_2))$
	$O_{12} =$	$0.3*(6.5625 - 5) + 0.1*20 = 2.46875$
	$DI_{12} =$	$P_{12} - O_{12} = (2.4 - 2.46875) = -0.06875$
C-2	$(S_2, D_2)$	cost = 4
	$P_{22} =$	2.4
	$O_{22} =$	$(0.3*(AVE.COST - 4) + 0.1*MAX(s_2, d_2))$
	$O_{22} =$	3.76875
	$DI_{22} =$	-1.36875
C-3	$(S_2, D_1)$	cost = 6
	$P_{21} =$	2.4
	$O_{21} =$	$(0.3*(AVE.COST - 6) + 0.1*MAX(s_2, d_1))$
	$O_{21} =$	3.16875
	$DI_{21} =$	$2.4 - 3.16875 = -0.76875$
C-4	$(S_3, D_3)$	Cost = 5
	$P_{33} =$	2.2
	$O_{33} =$	$(0.3*(AVE.COST - 5) + 0.1*MAX(s_3, d_3))$
	$O_{33} =$	3.46875
	$DI_{33} =$	-1.26875
C-5	$(S_4, D_4)$	cost = 5
	$P_{44} =$	2.2
	$O_{44} =$	$(0.3*(AVE.COST - 5) + 0.1*MAX(s_4, d_4))$
	$O_{44} =$	1.96875
	$DI_{44} =$	0.23125

Now in Step 7: From all decision indexes  $DI_{12}$ ,  $DI_{22}$ ,  $DI_{21}$ ,  $DI_{33}$ ,  $DI_{44}$  highest decision index is  $DI_{44}$ . So, allocate  $\min(S_4, D_4) = 15$  in  $(S_4, D_4)$  cell. And in Step 8: Update tableau with reduce the relevant supply and demand. Delete satisfied row or column. Finally Step 9: Repeat until all supplies and demands are exhausted.

Continue the process finally, we get total allocation shown in the following formula:

S/D	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	7	5 <sup>(20)</sup>	9	8	20
$S_2$	6 <sup>(25)</sup>	4	7 <sup>(5)</sup>	6	30
$S_3$	8	6	5 <sup>(25)</sup>	7	25
$S_4$	9	7	6	5 <sup>(15)</sup>	15
Demand	25	20	30	15	90

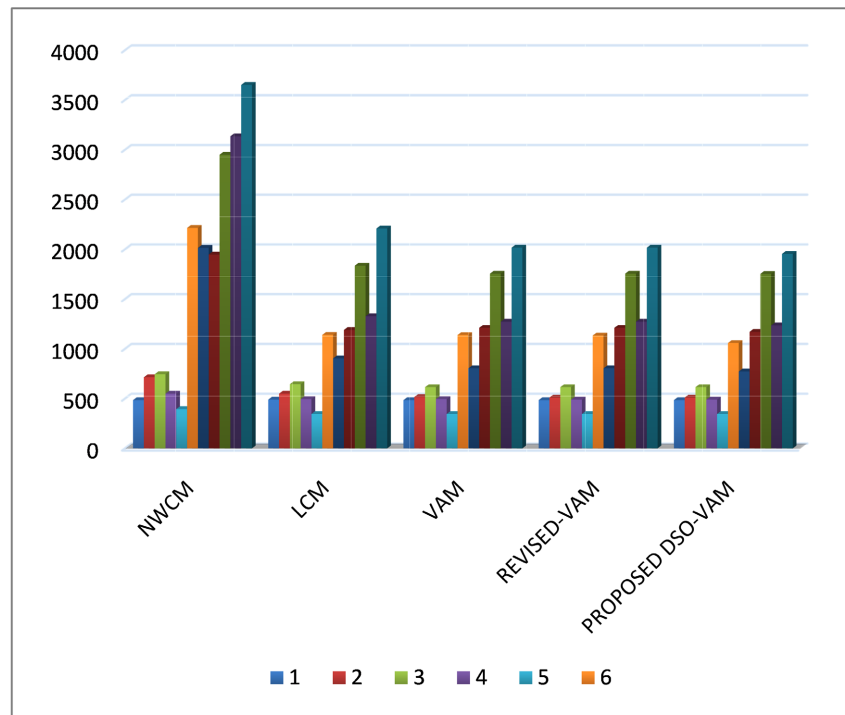
**Total Cost: 485**

### 6. Result Analysis and Comparison of Methods

The effectiveness of the proposed Dual-Spread Opportunity VAM (DSO-VAM) was evaluated through twelve numerical examples and compared with five existing transportation approaches: the North West Corner Method (NWCM), Least Cost Method (LCM), Vogel’s Approximation Method (VAM), and Revised-Vam Method (RVAM) [27]. The computational results obtained from these methods are summarized in **Table 3**. **Figure 2** represents summarized result by using bar diagram.

**Table 3.** Summarized computational results.

Method ▼ \ Example ▶	1	2	3	4	5	6	7	8	9	10	11
NWCM	485	715	745	550	395	2215	2015	1948	2951	3136	3652
LCM	490	550	645	495	345	1141	905	1190	1836	1328	2209
VAM	485	515	615	495	345	1139	805	1211	1756	1273	2017
Revised-VAM	485	510	615	490	345	1134	805	1211	1756	1273	2017
Proposed DSO-VAM	485	510	615	490	345	1059	772	1171	1754	1236	1954



**Figure 2.** Summarized result.

But it does not clearly show how much DSO-VAM improves over other methods. Add a percentage improvement **Table 4** Use this formula:

$$\text{Improvement (\%)} = \frac{\text{Existing Method Cost} - \text{DSO-VAM Cost}}{\text{Existing Method Cost}} \times 100$$

**Table 4.** Summarized percentage improvement computational results.

Example	Improvement Over NWCM	Improvement Over LCM	Improvement Over VAM	Improvement Revised-VAM
1	0.00%	1.02%	0.00%	0.00%
2	28.6%	7.27%	0.97%	0.00%
3	17.45%	4.65%	0.00%	0.00%
4	10.91%	1.01%	1.01%	0.00%
5	12.66%	0.00%	0.00%	6.60%
6	52.19%	7.19%	7.02%	4.10%
7	61.69%	14.70%	4.10%	3.30%
8	39.89%	1.60%	3.30%	0.11%
9	40.56%	4.47%	0.11%	2.91%
10	60.59%	6.93%	2.91%	3.12%
11	46.50%	11.54%	3.12%	0.00%
Average	33.73091%	5.489091%	2.049091%	1.83909%

The percentage improvement analysis shows that the proposed DSO-VAM provides the highest improvement over NWCM, with an average cost reduction of 33.73091%. Compared with LCM, the average improvement is 5.489091%, while compared with VAM and Revised-VAM, the improvements are 2.049091% and 1.83909% respectively. This indicates that DSO-VAM performs much better than classical basic methods and remains competitive against stronger VAM-based methods. The comparative analysis across twelve newly constructed transportation problems demonstrates that the proposed DSO-VAM algorithm consistently yields lower transportation costs than classical approaches such as NWCM, LCM, and VAM, as well as the revised VAM method. The graphical results clearly show a monotonic decrease in cost from NWCM to DSO-VAM, indicating improved allocation efficiency at each stage. The performance gain of DSO-VAM is attributed to its dual-spread penalty mechanism combined with an opportunity-based decision index, which allows for more informed allocation decisions. The proposed DSO-VAM does not always outperform revised VAM in all cases, but provides competitive and stable solutions across different cost structures. On average, the results demonstrate that the proposed method consistently outperforms classical methods such as NWCM, LCM, and VAM, as well as the revised VAM approach.

## 7. Limitations and Future Research

The present study is limited to deterministic transportation problems. The proposed DSO-VAM provides an initial basic feasible solution, but it does not guarantee optimality in all cases. Future research may extend this method to grey, fuzzy, interval-valued, and multi-objective transportation problems. The method may also be tested on large-scale real-world logistics data that in this proposed

DSO-VAM we use fixed control parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  to balance local cost spread, global cost spread, and opportunity-based allocation pressure.

## 8. Conclusion

In this paper, a novel heuristic known as the Dual-Spread Opportunity Vogel's Approximation Method (DSO-VAM) had been introduced with the intention of providing superior first-fundamental viable solutions to transportation issues. The recommended solution solves the issues that are encountered with traditional penalty-driven systems by integrating a decision-making mechanism that is based on opportunities with dual-spread penalties that consider taking advantage of both local and global cost modification. During the process of resource allocation, the implementation of a fixed decision index helps to maintain a dynamic equilibrium between risk and reward, which ultimately leads to decisions that are more effective and better efficient. It has been proved by computational findings from a large number of numerical instances that the transportation costs of DSO-VAM are consistently similar to or lower than those of standard techniques such as NWCM, LCM, VAM, and Revised-VAM. Taking everything into consideration, it can be concluded that DSO-VAM is superior and more reliable than the other approaches. Furthermore, the method is characterized by its simplicity and ease of calculation, which makes it an excellent choice for doing comprehensive tasks. Due to the fact that it possesses a generalization property, it is possible to simplify it to a typical VAM by selecting certain parameters, which in turn provides theoretical consistency and flexibility. A big step forward in the field of transportation heuristics, the technique that has been offered possesses a substantial amount of promise for implementation in large-scale optimization scenarios that are fraught with uncertainty.

## Availability of Data and Materials

The authors have confirmed that the data supporting the findings are available upon request from the corresponding author.

## Conflicts of Interest

The authors of this paper assert that there are no conflicts of interest, regarding this study.

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## Appendix

The examples we have used in this study:

### Example 1: $4 \times 4$

<i>S/D</i>	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	7	5	9	8	20
$S_2$	6	4	7	6	30
$S_3$	8	6	5	7	25
$S_4$	9	7	6	5	15
Demand	25	20	30	15	

### Example 2: $4 \times 4$

<i>S/D</i>	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	8	6	10	9	25
$S2$	9	7	4	2	35
$S3$	5	3	8	6	30
$S4$	7	6	9	5	20
Demand	20	30	25	35	

### Example 3: $3 \times 4$

<i>S/D</i>	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	8	6	10	9	20
$S2$	9	12	13	7	30
$S3$	14	9	16	5	25
Demand	25	20	15	15	

### Example 4: $3 \times 5$

<i>S/D</i>	$D1$	$D2$	$D3$	$D4$	$D5$	Supply
$S1$	6	8	7	5	9	25
$S2$	9	6	4	8	7	35
$S3$	5	9	6	9	4	30
Demand	20	30	25	10	5	

### Example 5: $4 \times 4$

<i>S/D</i>	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4	7	6	8	30
$S2$	5	3	7	6	25
$S3$	6	4	5	7	20
Demand	20	25	15	15	

### Example 6: $5 \times 5$

<i>S/D</i>	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	Supply
$S_1$	14	10	14	9	17	39
$S_2$	8	1	10	18	5	19
$S_3$	11	19	14	1	3	45
$S_4$	5	2	8	4	18	44
$S_5$	11	7	17	13	14	27
Demand	54	41	24	20	35	

**Example 7:  $6 \times 6$** 

<i>S/D</i>	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	Supply
$S_1$	12	10	7	15	4	8	26
$S_2$	13	12	1	16	4	6	17
$S_3$	9	16	17	6	6	11	25
$S_4$	6	10	14	18	7	2	45
$S_5$	9	3	3	12	10	5	23
$S_6$	19	6	4	5	10	9	41
Demand	26	13	38	12	56	32	

**Example 8:  $7 \times 7$** 

<i>S/D</i>	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	Supply
$S_1$	13	6	5	2	1	9	2	37
$S_2$	11	6	7	10	5	2	17	22
$S_3$	12	6	5	4	8	11	6	30
$S_4$	17	8	2	6	10	11	19	34
$S_5$	6	11	18	9	3	15	11	28
$S_6$	8	7	13	19	5	16	2	24
$S_7$	17	18	8	16	5	4	11	26
Demand	41	30	6	103	5	4	12	

**Example 9:  $8 \times 8$** 

<i>S/D</i>	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$	Supply
$S_1$	2	12	4	12	11	13	5	8	23
$S_2$	11	15	14	4	19	7	19	12	38
$S_3$	10	16	17	5	17	5	16	6	27
$S_4$	2	10	14	1	8	18	18	1	30
$S_5$	5	8	8	11	19	6	5	6	27
$S_6$	4	4	14	6	15	11	4	19	36
$S_7$	6	10	16	13	4	14	19	9	20
$S_8$	2	15	5	5	18	27	16	5	24
Demand	2	53	13	7	80	27	40	3	

**Example 10:  $9 \times 9$** 

<i>S/D</i>	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$	$D_9$	Supply
$S_1$	13	9	13	17	4	4	13	11	2	30
$S_2$	9	18	5	16	19	7	12	14	5	44
$S_3$	19	13	4	9	7	10	18	3	8	36
$S_4$	13	17	5	9	12	4	5	19	17	21
$S_5$	2	4	12	11	5	9	16	9	18	27
$S_6$	14	1	15	15	16	14	8	6	4	25
$S_7$	16	4	18	11	17	16	2	16	11	18
$S_8$	14	18	4	18	13	7	4	18	16	21
$S_9$	1	12	4	16	7	16	15	4	18	38
Demand	30	2	20	21	13	32	54	59	29	

**Example 11:  $10 \times 10$** 

<i>S/D</i>	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$	$D_9$	$D_{10}$	Supply
$S_1$	16	4	16	10	2	11	5	13	13	16	27
$S_2$	18	14	17	16	7	17	18	13	19	10	45
$S_3$	4	12	16	10	17	2	14	16	19	1	16
$S_4$	16	18	11	4	9	11	10	19	13	19	23
$S_5$	11	7	10	3	15	10	18	15	13	8	40
$S_6$	6	13	12	13	3	7	19	18	17	15	41
$S_7$	6	16	12	7	12	5	12	7	6	11	33
$S_8$	18	8	16	10	15	18	12	15	8	19	29
$S_9$	9	13	2	4	12	16	8	5	16	16	30
$S_{10}$	2	10	2	15	16	16	14	1	11	19	25
Demand	24	27	4	74	29	33	53	2	36	27	