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Effect of Stress Gradient on Fatigue Life Estimation (Statistical Approach Criteria: Variance Method)

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Abstract

This article begins with the general formulation of the fatigue criterion based on a statistical approach, followed by the consideration of the stress gradient, which has been recognized as a beneficial effect on the material's fatigue performance. The use of these models to predict the lifespan of a component subjected to random multiaxial loading is also discussed. The experimental data from biaxial fatigue tests allowed us to evaluate the effectiveness of each model. In summary, analyzes were conducted to better represent the influence of the stress gradient. For mechanical design, we recommend using the model that takes into account the stress gradient.

Keywords

Biaxial Fatigue, Fracture Plane Direction, Critical Plane, Stress Gradient and Equivalent Stress Variance, Fatigue Life

1. Introduction

The estimation of fatigue life and the maintenance frequency of an industrial structure in service under multiaxial loads are two major aspects at the heart of the maintenance service. However, to be competitive, industries must prove the reliability of their products. The objective of this work is to take into account the stress gradient in the statistical approach criteria [macha] in order to demonstrate the beneficial aspect of the gradient in fatigue life.

The effect of the stress gradient is well known as a beneficial effect in fatigue because its presence within a material systematically leads to an increase in the endurance limit in terms of the maximum local stress.

2. Materials and Methods

2.1. Methodology

Presentation of the Variance Method

Fatigue failure is caused by the normal stress $\sigma_n(t)$ and the tangential stress $\tau_n(t)$ which acts in the tangential direction $\vec{s}$ of the fracture plane with the normal $\vec{n}$ (**Figure 1**). Based on this knowledge, Macha, E. and Niesłony [1] developed a method for determining the position of the critical plane based on the variance of the equivalent stress.

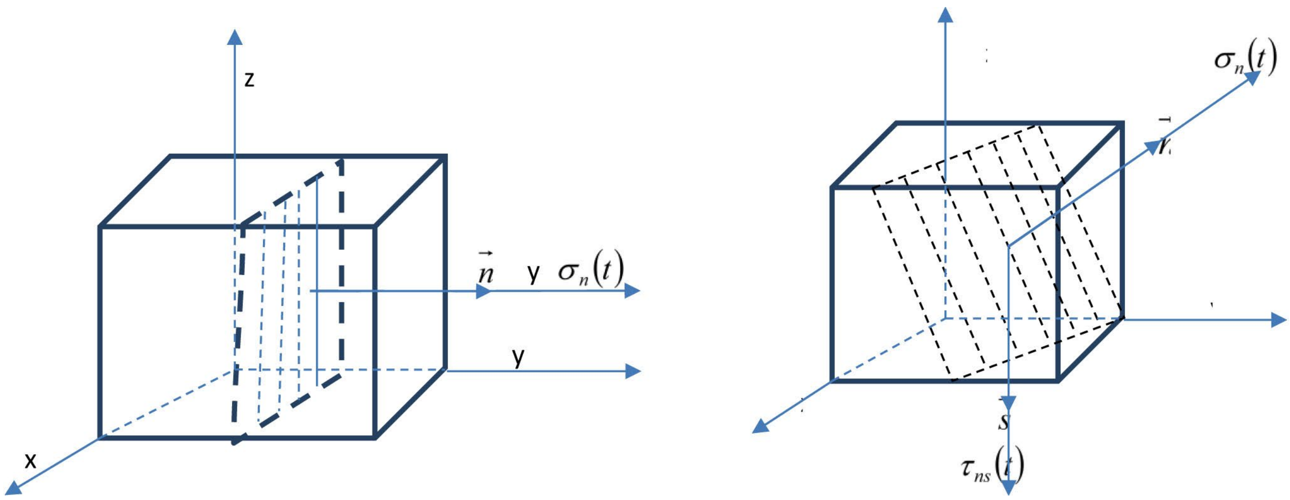

Figure 1. Direction of vector, $\vec{n}$ and $\vec{s}$ and of stress, $\sigma_n(t)$ and $\tau_{ns}(t)$ in the case of any position of the fatigue fracture plane in relation to the system of axes x, y, z [2] [3].

With:

$$\vec{n} = \hat{l}_\eta \vec{i} + \hat{m}_\eta \vec{j} + \hat{n}_{\eta \vec{k}} = \frac{\hat{l}_1 + \hat{l}_3}{\sqrt{2}} \vec{i} + \frac{\hat{m}_1 + \hat{m}_3}{\sqrt{2}} \vec{j} + \frac{\hat{n}_1 + \hat{n}_3}{\sqrt{2}} \vec{k} \tag{1}$$

$$\vec{s} = \hat{l}_s \vec{i} + \hat{m}_s \vec{j} + \hat{n}_s \vec{k} = \frac{\hat{l}_1 - \hat{l}_3}{\sqrt{2}} \vec{i} + \frac{\hat{m}_1 - \hat{m}_3}{\sqrt{2}} \vec{j} + \frac{\hat{n}_1 - \hat{n}_3}{\sqrt{2}} \vec{k} \tag{2}$$

The components of the stress vector have the following expressions:

$$\sigma_n(t) = \hat{l}_n \vec{i}\, \hat{l}_n \vec{j} \sigma_{ij}(t), \tau_{ns}(t) = \hat{l}_{ns} \vec{i}\, \hat{l}_{ns} \vec{j} \sigma_{ij}(t) \tag{3}$$

With $i, j = x, y, z$.

The general formulation of the criterion is:

$$\max_t \left\{ B\tau_{\eta s}(t) + K\sigma_\eta(t) \right\} = F \tag{4}$$

where:

F, B are the criterion constants;

$$K=\sqrt{\left(\frac{\sigma_{-1}}{\tau_{-1}-\sigma_{-1}}\right)^2-1}$$

σ_{-1} and τ_{-1} are the fatigue limits under tension-compression and torsion loading, respectively.

$$\sigma_{eq}(t)=\sum_{k=1}^{6}a_k x_k(t) \tag{5}$$

$$x_1=\sigma_{xx}(t), x_2=\sigma_{yy}(t), x_3=\sigma_{zz}(t), x_4=\sigma_{xy}(t), x_5=\sigma_{xz}(t), x_6=\sigma_{yz}(t).$$

$$a_k=a_k\left(\hat{l}_1,\hat{m}_1,\hat{n}_1,\hat{l}_3,\hat{m}_3,\hat{n}_3\right).$$

$$\vec{n}=\frac{\hat{l}_1+\hat{l}_3}{\sqrt{2}}\vec{i}+\frac{\hat{m}_1+\hat{m}_3}{\sqrt{2}}\vec{j}+\frac{\hat{n}_1+\hat{n}_3}{\sqrt{2}}\vec{k}\,,\quad \vec{s}=\frac{\hat{l}_1-\hat{l}_3}{\sqrt{2}}\vec{i}+\frac{\hat{m}_1-\hat{m}_3}{\sqrt{2}}\vec{j}+\frac{\hat{n}_1-\hat{n}_3}{\sqrt{2}}\vec{k}$$

The average of the director cosines must satisfy the following orthogonality conditions:

$$\hat{l}_1^2+\hat{m}_1^2+\hat{n}_1^2=1,\quad \hat{l}_2^2+\hat{m}_2^2+\hat{n}_2^2=1,\quad \hat{l}_3^2+\hat{m}_3^2+\hat{n}_3^2=1,$$

$$\hat{l}_1\hat{l}_2+\hat{m}_1\hat{m}_2+\hat{n}_1\hat{n}_2=0,\quad \hat{l}_1\hat{l}_3+\hat{m}_1\hat{m}_3+\hat{n}_1\hat{n}_3=0,\quad \hat{l}_2\hat{l}_3+\hat{m}_2\hat{m}_3+\hat{n}_2\hat{n}_3=0.$$

2.2. Taking into Account the Stress Gradient [3] [4]

General Formalism

The proposal consists of introducing the influence of the stress gradient into the criterion of the statistical approach. The formalism of the modified EWALD Macha criterion is defined by the following expression:

$$\max_t\left\{B\tau_{\eta s}(t)+K\sigma_\eta(t)+\sqrt{G(t)\langle\sigma_\eta(t)}\right\}=F \quad [3] \tag{6}$$

With:

- F and B the criterion constants;
- $K=\sqrt{\left(\frac{\sigma_{-1}}{\tau_{-1}-\sigma_{-1}}\right)^2-1}$.

σ_{-1}: Tensile-compression fatigue limit and τ_{-1}: Torsional fatigue limit;

$\tau_{\eta s}(t)$ et $\tau_{\eta s}(t)$, are functions of the constraint components $\sigma_{ij}(t)(i,j=x,y,z)$.

-***G*(*t*)**, geometric gradient of stress normal $\sigma_\eta(t)$ to the physical plane with normal h, is defined at time t by [4]:

$$G(t)=\sqrt{\left(\frac{\partial\sigma_\eta(t)}{\partial x}\right)^2+\left(\frac{\partial\sigma_\eta(t)}{\partial y}\right)^2+\left(\frac{\partial\sigma_\eta(t)}{\partial z}\right)^2} \tag{7}$$

Case c) criterion for combining maximum tangential and normal stresses in a fracture plane [3].

In this particular case of the criterion ($B=1$), the expression for the equivalent stress is

$$\begin{aligned}\sigma_{eq}(t)=\frac{1}{1+K}\Big[&\left(\hat{l}_1^2-\hat{l}_3^2+K\left(\hat{l}_1+\hat{l}_3\right)^2\right)\sigma_{xx}(t)+\left(\hat{m}_1^2-\hat{m}_3^2+K\left(\hat{m}_1+\hat{m}_3\right)^2\right)\sigma_{yy}(t)\\&+\left(\hat{n}_1^2-\hat{n}_3^2+K\left(\hat{n}_1+\hat{n}_3\right)^2\right)\sigma_{zz}(t)+\left(\hat{l}_1\hat{m}_1-\hat{l}_3\hat{m}_3+K\left(\hat{l}_1+\hat{l}_3\right)\left(\hat{m}_1+\hat{m}_3\right)\right)\sigma_{xy}(t)\\&+2\left(\hat{l}_1\hat{n}_1-\hat{l}_3\hat{n}_3+K\left(\hat{l}_1+\hat{l}_3\right)\left(\hat{n}_1+\hat{n}_3\right)\right)\sigma_{xz}(t)\\&+2\left(\hat{m}_1\hat{n}_1-\hat{m}_3\hat{n}_3+K\left(\hat{m}_1+\hat{m}_3\right)\left(\hat{n}_1+\hat{n}_3\right)\right)\sigma_{yz}(t)+\sqrt{G(t)\langle\sigma_\eta(t)}\Big]\end{aligned}\tag{8}$$

$\sigma_{ij}(t)(i,j=x,y,z)$ are the stress components.

The variance of the equivalent stress is written as:

$$\mu_{\sigma_{eq}}=\sum_{s,t=1}^{6}a_s a_t \mu_{xst}\quad[5]\tag{9}$$

With μ_{xst} the variance-covariance matrix of the variable x_k:

$$\mu_{xst}=\begin{bmatrix}\mu_{x11} & \cdots & \mu_{16}\\ \vdots & \ddots & \vdots\\ \mu_{61} & \cdots & \mu_{66}\end{bmatrix}\quad[6]\ [7]$$

Generally, the analytical resolution of this method poses a problem. To eliminate this difficulty, the cosine directors $\hat{l}_n,\hat{m}_n,\hat{n}_n$ are replaced by the trigonometric functions of the three Euler angles ψ, θ, ϕ (**Figure 2**).

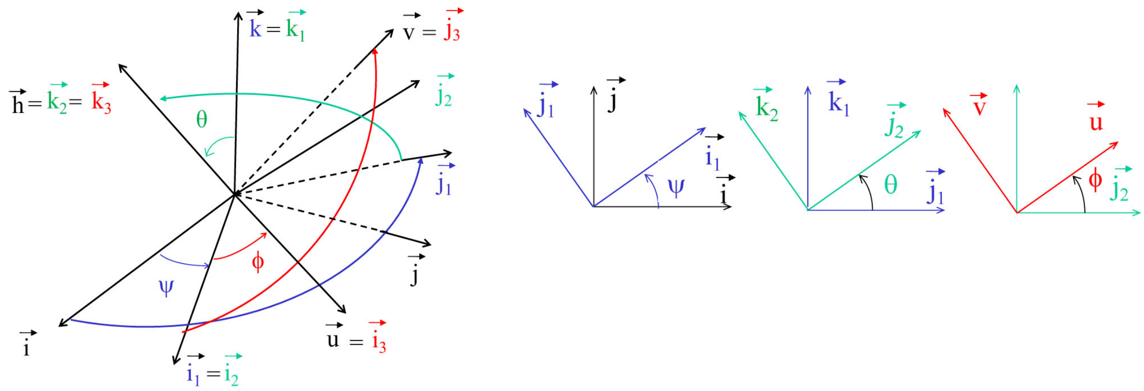

Figure 2. The trigonometric functions of Euler's three angles [3].

$$\vec{u}\left(\hat{l}_1,\hat{m}_1,\hat{n}_1\right);\ \vec{v}\left(\hat{l}_2,\hat{m}_2,\hat{n}_2\right)\ \text{and}\ \vec{h}\left(\hat{l}_3,\hat{m}_3,\hat{n}_3\right).$$

Considering that the problem is plane and that the facets concerned are those with the normal in the plane $(\vec{i},\vec{j})$, we have the condition $\vec{k}\cdot\vec{h}=\hat{n}_3=0$.

This consideration allows us to obtain the conditions on the Euler angles with $\theta=\pi/2$.

With $\theta=\pi/2$, the condition, and the cases of rotations (**Figure 2**), we have the following matrix of cosine directions:

$$[P]=\begin{bmatrix}\cos\psi\cos\phi & \cos\psi\sin\phi & \sin\phi\\ -\sin\psi\cos\phi & -\sin\psi\sin\phi & \cos\psi\\ \sin\phi & -\cos\phi & 0\end{bmatrix}$$

2.3. Estimating the Fatigue Life under Multiaxial Variable Amplitude Loading Macha and Bedkowsky Method

Statistical Method for Estimating Lifespan

There is a point of convergence with the previous method, which is the determination of the counting variable, but the difference lies in the formulation of the fatigue criterion and also the damage law. We would like to note that the Miner rule does not consider stresses below the endurance limit of the material defined experimentally. For the effect of fatigue is cyclic (*i.e.* For any stress above or below the endurance limit, cyclic application could influence fatigue performance).

However, the statistical approach takes this notion into consideration by introducing a coefficient $0 \triangleleft \alpha \leq 1$. The simplified algorithm of this method is presented in Figure 3.

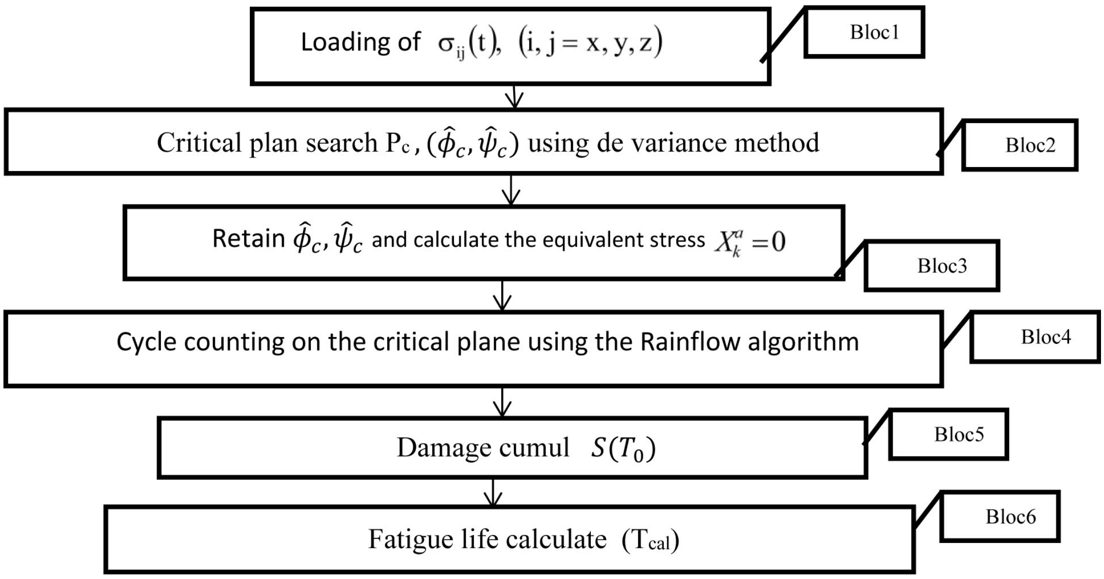

Figure 3. Algorithme de calcul de durée de vie par la méthode statistique.

2.4. Assumptions of Damage Accumulation

In practice, the analysis of random loads is done using counting cycle algorithms. Random loading is composed of different mean and alternating values. Nowadays, the Rainflow algorithm is the most widely used in calculating the fatigue life of materials, which allows us to obtain a certain number of loading cycles and half-cycles with different mean values, and applied stress amplitudes during damage accumulation; it is applied to the uniaxial stress state. In random loading, the most commonly used assumption is a linear Palmgren-Miner hypothesis, where the sum of damages is proportional to the number of loading cycles. In this accumulation process, only the stress amplitudes above the endurance limit are taken into account. However, in the definition of the fatigue phenomenon, it is observed that even amplitudes below the fatigue limit can influence the material's fatigue performance. To address this problem, Ewal Macha and Adam Nieslony (2012) [1] introduced a coefficient, which leads to:

$$S(T_0) = \begin{cases} \sum_{i=1}^{k} \dfrac{n_i}{N_f \left(\dfrac{\sigma_{af}}{\sigma_{ai}} \right)^m} & \text{pour } \sigma_{ai} \geq a_{PM}\sigma_{af} \\ 0 & \text{pour } \sigma_{ai} < a_{PM}\sigma_{af} \end{cases} \tag{10}$$

Avec:

T_0 —calculation time for the equivalent stress,

n_i —the number of cycles with stress amplitudes to—the number of cycles with stress amplitudes to determined by the rain-flow method,

$S(T_0)$ —degree of damage to,

m—exponent of the S-N curve (Wöhler),

σ_{af} —fatigue limit,

N_f —number of cycles corresponding to the fatigue limit,

a_{PM} coefficient taking into account the influence of stresses below the endurance limit σ_{af} (in this work, we consider $a_{PM} = 0.5$).

After determining the lifespan, we proceed to calculate the fatigue lifespan defined by:

$$T_{cal} = \frac{T_0}{S(T_0)} \tag{11}$$

3. Presentation of Material

Experimental validation on Macha's variable amplitude multiaxial sequences.

We will use multiaxial variable amplitude tests conducted by the Polish team of Professor E. MACHA (Technical University of Opole - Poland). This will allow us to validate the proposed method against the experimental results on one hand, and the results predicted by the method initially proposed by our laboratory on the other hand, both in terms of computation time and the accuracy of the obtained results.

Description of the Tests

The tests were conducted on cruciform specimens made of 10HNAP steel, the chemical composition of which is summarized in the following table:

Eléments	C	Mn	Si	P	S	Cr	Cu	Ni	Fe
Teneur (%)	0.115	0.71	0.41	0.082	0.028	0.81	0.30	0.50	97.045

The dimensions and shapes of the specimens are specified in **Figure 4** [5].

The diagram of the test bench is shown in **Figure 5**. Two pairs of orthogonal cylinders allow for biaxial tensile-compressive loading of the specimens. The specimens are tapered in their central part. The deformation measurements are carried out using 45˚ rosettes (loading directions, which are the two axs of symmetry of the test specimens).

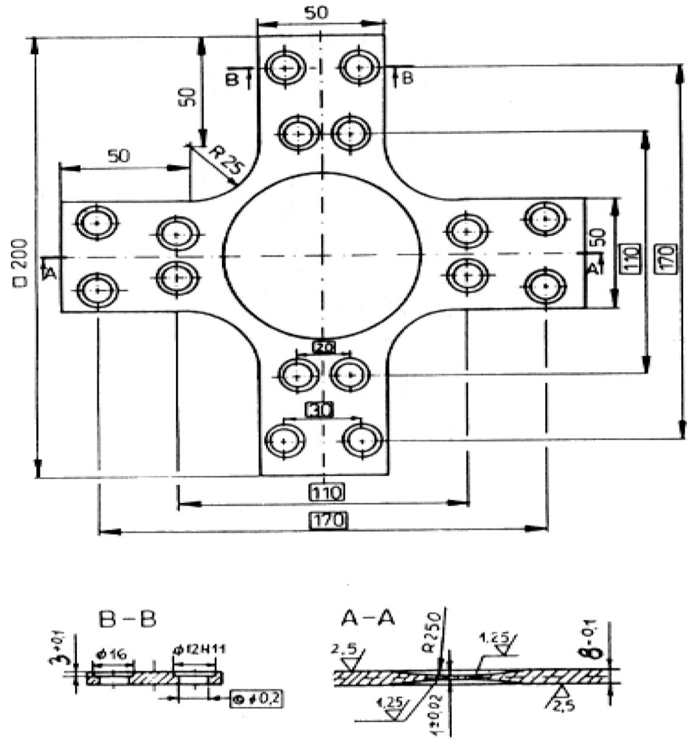

Figure 4. Geometry of the steel specimens used by E. Macha.

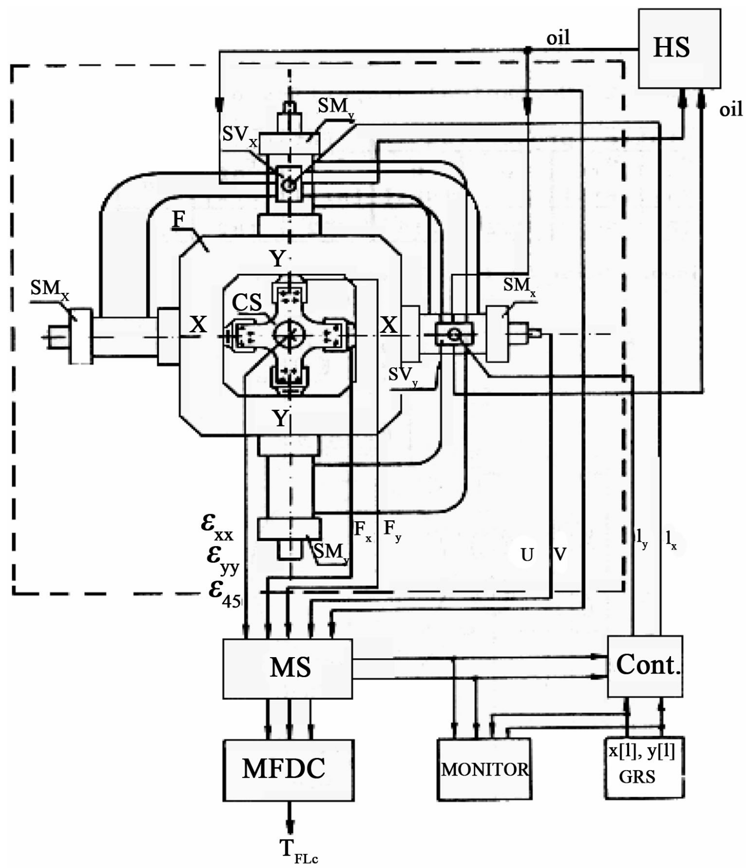

Figure 5. Test bench for cruciform specimens.

For the variable amplitude biaxial loading: HS: hydraulic unit, SM: servomotor, SV: servovalve, F: rigid frame, CS: cruciform specimen, uv: displacements, Ix Iy: control εij: relative deformations, MS: acquisition system, MFDC: acquisition software, GRS: function generator, Cont.: Control system.

The transition from deformations to stresses is carried out using the behavior laws of the elastic domain. The presence of a three-gauge rosette instead of a rosette containing only two, which is strictly necessary since the principal stress directions are fixed and known (these are the two axs of the loads), is simply intended to verify the absence of material distortion and thus tangential stress. This can also help verify the correct orientation of the strain gage directions (during their bonding) according to the axs of the specimens. The static characteristics of the 10HNAP steel are summarized in the following table.

σ_e (MPa)	R_m (MPa)	υ	E (MPa)
418	566	0.29	215 000

4. Validation of the Proposed Method

This validation addresses two essential points: the actual lifespan obtained by the simple statistical approach and the one proposed (considering the stress gradient), which will be compared to the experimental lifespan and the one obtained by [8].

4.1. Comparison of Fatigue Life

Table 1 presents the results obtained in terms of sequential life at the crack initiation of GP9310 and GP9313 from Macha tests made available to us [8].

Table 1. Fatigue life comparaison.

Sequence	Durée de vie expérimentale	Fatigue Life calculated by Miner (RB criterion, Kenmeugne (1996))		Simple statistical approach		Statistical approach + gradient	
	Number of repetitions	Nbr of repetitions	Rapport $\frac{N_{\exp}}{N_{\text{num}}}$	Nbr of repetitions	Rapport $\frac{N_{\exp}}{N_{\text{num}}}$	Nbr of repetitions	Rapport $\frac{N_{\exp}}{N_{\text{num}}}$
GP9310	1300	593	2.19	1118	1.16	1138	1.14
GP9313	1664	671	2.48	1343	1.24	1378	1.21

The fatigue life obtained by the "statistic + gradient" approach is close to reality with a ratio of 1.14 and 1.21 respectively for the GP9310 and GP9313 sequences, compared to that obtained by [8] and the simple statistical approach.

4.2. Evaluation of Maintenance Costs

In a company, the maintenance department is at the heart of the financial budget.

However, the higher the maintenance cost, the more the financial aspect of the company will be affected. That being said, we found that the lifespan obtained by [8] is conservative because it is shorter compared to the experimental one, but in terms of maintenance cost, it is not good because according to the reports, we will perform maintenance two times before reaching the experimental lifespan and we will discard the part that could still be useful. On the other hand, our determined lifespan is closer to the experimental one. This favors the maintenance cost. Finally, we advise engineers to consider our proposal in their sizing.

5. Conclusions

Initially, L. M. So proposed the counting variable of the general method for calculating the lifespan under variable amplitude multiaxial loading [8]. This variable had certain drawbacks, which Kenmeugne mentioned in his thesis, and then proposed a new method that, after comparative studies, proved that his new method is conservative.

We considered Kenmeugne's new counting variable method in our lifespan calculations.

In the first instance, we calculated the lifespan using the simple statistical approach criterion, then we introduced the stress gradient term to show its effect in the lifespan calculation. In light of the results obtained, our formulation is ideal.

Author Contributions

Conceptualization, NGARGUEUDEDJIM Kimtangar, and KENMEUGNE Bienvenu methodology, MBAIYELKOM Esdras.; software, MBAIYELKOM Esdras.; validation, MBAIYELKOM Esdras., NGARGUEUDEDJIM Kimtangar., and KENMEUGNE Bienvenue; formal analysis, NGARGUEUDEDJIM Kimtangar.; investigation, MBAIYELKOM Esdras.; resources, KENMEUGNE Bienvenue.; data curation, NGARGUEUDEDJIM Kimtangar; writing—original draft preparation, MBAIYELKOM Esdras.; writing—review and editing, MBAIYELKOM Esdras.; visualization, NGARGUEUDEDJIM Kimtangar; supervision, KENMEUGNE bienvenue; project administration, NGARGUEUDEDJIM Kimtangar; funding acquisition, ABAKAR Malgnan. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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