

On Modeling Energetic Electrons in Laser Fusion Plasmas

Wallace Manheimer

Independent Researcher, Allendale, New Jersey, USA

Email: manheimerwallace42@gmail.com

How to cite this paper: Manheimer, W. (2026) On Modeling Energetic Electrons in Laser Fusion Plasmas. *Open Journal of Applied Sciences*, 16, 2470-2521. <https://doi.org/10.4236/ojapps.2026.167139>

Received: June 15, 2026

Accepted: July 21, 2026

Published: July 24, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

It has been known for decades that laser fusion can be and often is plagued by the production of energetic electrons, produced either by an instability, or by the energetic tail of a Maxwellian distribution. Despite more than 25 years of a variety of efforts, there is still no generally accepted way to treat or address this issue. This work hopes to advance the progress by developing a Fokker Planck model which is simple enough to be used at every time step of a rad-hydro simulation, and accurate enough to be useful. It makes several approximations, the main one being that the percentage of these energetic electrons is small. Recent experiments confirm that so far, this has proven to be the case for plasmas subject to laser plasma instabilities. Hence energetic electrons interact with the background plasma and not with each other. This work makes no attempt to solve for the entire electron distribution function. It makes a variety of other approximations to solve the resulting Fokker Planck equation. This report both summarizes the author's work with the NRL group and presents his new advances since the termination of the NRL project. The author hopes this will be a good starting point for any person or group who wishes to continue this work. An important result is our tentative conclusion that energetic electrons may not be such a big problem for laser fusion.

Keywords

Laser Fusion, Energetic Electrons, Energetic Electrons for Laser Fusion, Fokker Planck Equation, Solutions to the Fokker Planck Equation

1. Introduction

Energetic electrons in a laser produced plasma cannot always be described with a local theory. The mean free path of some electrons may be comparable to or even greater than the temperature gradient length. There are at least two sources of

these energetic electrons. First there can be laser plasma instabilities which generate many very energetic electrons. Second the tail of the Maxwellian distribution may also decouple locally. This work examines both.

It has been 26 years since Schurtz *et al.* [1] published their work on nonlocal transport of energetic electrons in laser fusion targets. In their case, the electrons they were looking at were in the tail of the Maxwellian. Yet there is still no generally accepted way to treat this important issue, illustrating both the difficult nature and importance nature of this problem. What is required is a theory that is both accurate enough and simple enough that it can be applied at every time step, or nearly every time step in a rad hydro code.

The theory in [1] is basically a Krook theory and other groups have developed similar versions [2]-[8]. At NRL we also developed Krook models [9]-[14]. One problem is that when these theories were applied to direct drive laser fusion, the calculations at least at NRL [15], the University of Rochester Laboratory for Laser Energetics, URLLE¹, and possibly at other labs, showed that there was no gain. To this author's knowledge, none of these negative results had ever been published.

A little analysis of the Krook model shows that it will overestimate the preheat. Consider what the model used. A particle stream moves through the background particles, with which it collides. After it travels one mean free path, e^{-1} of these particles remain; those that don't remain have fallen into the background. The next mean free path, same thing and so on. There is no slowing down of the particles. Hence the population of energetic particles decreases exponentially, but rather slowly as they travel.

The Fokker Planck model which we use here is characterized by both friction and diffusion. Let us consider the friction. The friction force is a rapidly decreasing function of particle energy, so as the particle slows down, its friction force gets stronger.

For the case at hand, the particle equation as it moves along a straight-line orbit is:

$$V \frac{dV}{dx} = -\frac{\nu(V_o)}{V^2} V_o^3 \quad (1)$$

Here V_o is the particle's initial velocity. Equation (1) has a simple solution:

$$V = [V_o^4 - 4\nu(V_o)V_o^3 x]^{1/4} \quad (2)$$

Hence the particle cannot travel even a single mean free path, evaluated at its initial velocity, before it stops. Including this effect is the basic reason that one would think that accounting for it would reduce the fuel preheat.

This was hardly ignored by scientists studying the effect of energetic electrons. For instance, [6] put what we have called an iron dome on top of the deposition profile calculated by the Krook analysis. Reference [8] came up with a modified Krook model. Instead of using $f f_m$ on the rhs, it used derivatives. Reference [16]

¹W. Manheimer with R. Betti and Other Members of the URLLE Group, as a Seminar at URLLE, September 2025.

devised a Krook model but added to it a downward energy cascade to mockup the effect of the dynamical friction. Others modified the multigroup diffusion model [17] [18], usually used as an approximation to describe neutron transport, to model energetic electron transport. All of these are reasonable efforts to deal with what is a fundamental error of the Krook model. However, this author believes we can do better, by starting out with a correct model. He thinks the first step on the ladder is to make approximations to this model to see if it works for the simplest problems. If this works out, then make better approximations to see if it would work in more complicated problems. At this point, the author hopes to be able to claim that we have at least achieved the first step.

Some researchers have decided to go a different route and come up with what one would call a full simulation [19]-[21]. These calculate many other effects, magnetic fields, electron inertia, this or that instability, Hall effects... These codes are much more complicated; there is little chance that they could be used at each step of a rad hydro simulation, as the authors in fact stipulate [19] (and if they were, there would be little need for the rad-hydro simulation). Furthermore, they seem to be more sensitive and must put in additional guard rails to keep them from crashing.

Mea culpa, the author admits that at the NRL, we were probably the last group to realize that Krook models were not viable. However instead of finding ways to patch it up, we abandoned it completely (well, almost) and went directly to a Fokker Planck equation [22]-[25]. We made several simplifications to get results. We briefly mention the various approximations made in utilizing the Fokker Planck Equation. More details are in the various sections. First and foremost, we use a steady state approximation for the Fokker Planck equation. Comparisons of high energy electron collision times with fluid times justify this, although perhaps not so well for the very highest energy electrons. We then take a 2 term Legendre expansion for the equation; these two equations can be reduced to a single equation for the f_1 , portion of the distribution function governing energy transport. Then we assume that the population of energetic electrons is small, so that their interaction is with the background plasma, not with each other. This is certainly a good approximation for all the cases we consider here. We restrict the theory to one dimensional calculations, either in planar or spherical configurations. Certainly, this is a reasonably good approximation for laser fusion targets thought to be successful. If the symmetry is strongly violated, the target most likely will not be successful for laser fusion, and determining energetic electron effects will probably be of academic interest only. Of course it is possible, with great effort to extend the formulation to 2 or 3 dimensions. Another approximation we have made is that we neglect the effect of the electrostatic field on the energetic electron motion. The potential across the plasma is generally characterized by the plasma temperature, while the electrons we consider have much higher energy. This approximation is almost certainly fine for instability generated electrons (*i.e.* 50 - 100 keV electrons in a 2 - 3 keV temperature plasma). However, it is less certain for the

energetic tail of the Maxwellian. The theory developed here splits the Maxwellian into two groups, thermal and energetic electrons. At the breaking energy, the neglect of the electric field is qualitative at best. This paper suggests that this be investigated further. Finally, [25] investigated the validity of the two term Legendre expansion. The conclusion is that it is at least qualitatively correct. However, the accuracy depends on the Z of the plasma; $Z = 1$ it is at best qualitatively correct, $Z = 3$ and above, it is fine. [25] discusses how one might improve the accuracy.

We certainly were not looking at the large variety of effects of say [21] but were attempting to evaluate the effect of the energetic electrons on the implosion. Many of these results seemed to be physically reasonable; furthermore, they matched reasonably well experiments on the National Ignition Facility/Lawrence Livermore National Lab (NIF/LLNL), done in cooperation with URLLE. This is the study of energetic electrons generated by the absolute Raman side scatter instability [26]. Also at least we started with something which was basically correct and pushed it as far as we could. As improvements become necessary, they could likely be made by extending the model developed here, rather than adding another correction to a different model which is fundamentally wrong. In fact, this paper gives some calculations made by improving and extending the work in the initially published results. It also suggests additional improvements that could be made, especially for the case of the energetic electrons generated in the tail of the Maxwellian.

The author must admit that for the past few years he has been very excited and motivated by the recent triumph at LLNL, where they demonstrated that a burning plasma is possible [27]-[29], probably at least 15 - 20 years before ITER can make the same claim. Furthermore, the cost of the NIF effort is quite small compared to the cost to ITER, and its tritium demand for the long research stage is much less. NIF is a frequency tripled glass laser, having a wavelength of 335 nm and producing as much as 2 MJ of laser light energy. The key is that the alpha particle has much lower energy (3.5 MeV) than the neutron (14 MeV). Therefore, if the plasma configuration is produced just right, the alpha can be absorbed locally while the neutron escapes. Hence the alpha can start a burn wave, and the plasma can heat, at least for a while, as it expands. This is what the LLNL group has succeeded in demonstrating. In other words, the laser is like the sparkplug in the cylinder of a car. It does not provide the energy to drive, but it just ignites the fuel which does power the car. The author has gone on record as saying that 100 years from now, this will probably be regarded as one of the main experiments of 21st century physics, and likely will have won a Nobel prize [30] [31].

Furthermore, this work has immediately solved another of the most vexing problems in magnetic fusion, namely what to do with the alpha particles. In an ideal magnetic fusion world, the alphas would heat the plasma, just the right amount, and then disappear. But how do they do this? Any electromagnetic way of getting rid of the alphas would probably also get rid of the deuterium, since they have the same e/M .

The author sees laser fusion as suddenly coming up with 2 gigantic achieve-

ments, first producing a likely scalable device with $Q > 1$, and then solving the alpha dilemma. Hence for these two reasons, the author has gone on record advising the DoE to put its main emphasis for fusion for energy (as opposed to stockpile stewardship), on laser fusion instead of magnetic fusion (31, 32). An image of LLNL's configuration from [27] is in **Figure 1**.

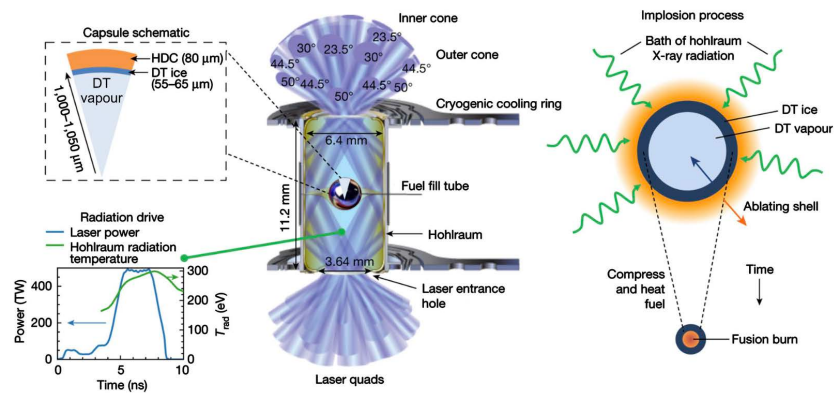


Figure 1. The configuration of the LLNL successful experiment which produced an alpha particle burn wave. The target is compressed by X-Rays produced on the wall of the chamber, called a hohlraum, containing the target.

The experiment in 2021 achieved a gain just below unity but showed that they produced an alpha burn wave by showing that the neutron production maximized *after* the bang time. A graph taken from [27] is reproduced in **Figure 2**. Other diagnostics actually showed the plasma initially heating as it expanded [29]. Recently the group has published a review paper of their recent work [28]. This work showed that they now have achieved a gain of ~ 2.5 , and it predicts that future higher gains are achievable with NIF.

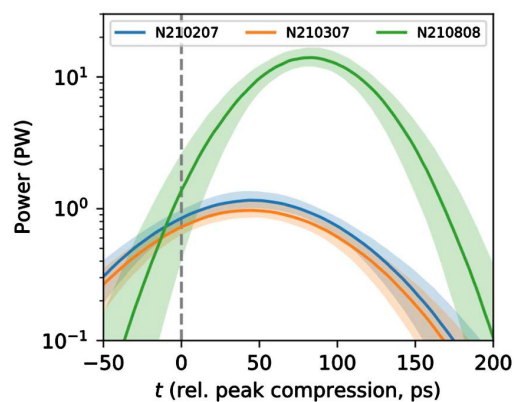


Figure 2. A plot of fusion power at the times, in picoseconds, around the bang time of the implosion. Notice that the power maximizes *after* the bang time, meaning that an alpha burn wave has been produced.

The LLNL work is not supported for energy, but for nuclear stockpile steward-

ship. The configuration they have used, is a target that is centered in a heavy metal hohlraum. The laser illuminates the inner hohlraum wall. A dense, hot, high Z plasma is formed, and the black body X-Rays emitted illuminate and compress the target. This configuration is called indirect drive. However, the LLNL group estimates that only 10 - 15% of the laser energy strikes the target. Since the implosion is caused by X-Rays, and not by ultraviolet light, it is unlikely that energetic electrons will be a factor at the target, although they may be important near the hohlraum. Indirect drive may or may not be a viable configuration for energy production, but it is certainly what the sponsors of stockpile stewardship insist on, as this approach most closely approximates the physical processes going on in a nuclear weapon.

The group at URLLE has also made important gains. An alternate configuration, called direct drive, is to have the laser directly illuminate the target. URLLE has an ultraviolet (wavelength = 335 nm) laser called OMEGA. It is also a gigantic laser (but tiny compared to NIF); it produces ~30 kJ of laser light. With direct drive, perhaps as much as 100% of the laser energy is available to strike the target. This has been the approach at NRL, and mostly also at URLLE. Other researchers have expressed skepticism that direct drive could ever work. They thought that only X-rays, whether in a hohlraum or nuclear weapon, have the necessary qualities to generate the required implosions. In fact, so far, it has been the only way this has been done. The skeptics think that ultraviolet light is unlikely to be able to do this [32].

Here is where URLLE made a very important advance. Their laser does not have the energy required to generate an implosion which can produce a plasma large enough to locally absorb the alpha particles, so they are unable to generate the alpha burn wave that NIF was able to produce. Their effort was to produce what they called implosions hydrodynamically equivalent to what NIF was able to do. That is, to produce an implosion just like NIF's but on a much smaller length scale, and which would come apart much more quickly. In their central hot spot, they would produce neutrons at the same rate as NIF, but of course they would have no capability of producing nearly as many and certainly could not produce an alpha particle burn wave.

The URLLE effort was no simple job. In 2013, attempting to compress cryogenic spheres with OMEGA they could produce only about $\sim 10^{13}$ neutrons [33]. Their hydrodynamically equivalent implosion would produce more like $\sim 3 \times 10^{14}$ fusion neutrons; it took a tremendous effort on their part, but they finally achieved this [34] [35]. This is as opposed to NIF's $\sim 2 \times 10^{18}$, exploiting an alpha burn wave. To see the importance of the alpha burn wave, note that OMEGA, with 30 kJ produced $\sim 3 \times 10^{14}$, while NIF, with about 60 times more energy, produced ~6000 times more neutrons [28].

Figure 3 contains two plots. The left is URLLE's resulting neutron production from 2013, with their 1D calculations predicting that they should be able to produce more than 10^{14} . With a ten-year effort on both theory and experiment, they were finally able to produce the hydrodynamically equivalent implosion as shown in the right plot.

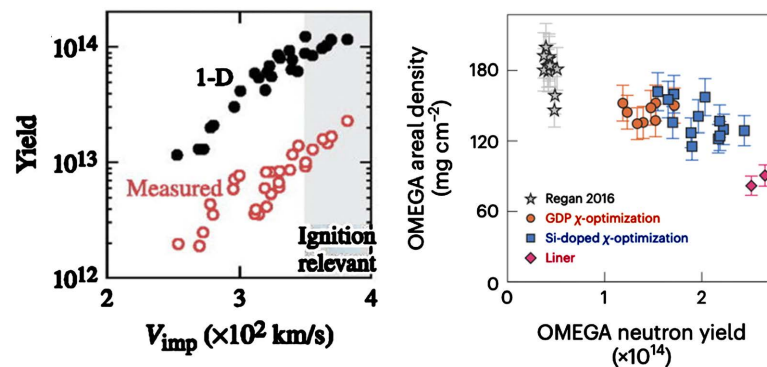


Figure 3. Left is a plot of neutrons emitted from the target in their 2013 effort. Notice that their code result predicted that they would get just over 10^{14} neutrons, but they only got $\sim 2 \times 10^{13}$. On the right is their result from 2024 after tremendous effort on both the theory and experiment. They increased the neutron production to $\sim 3 \times 10^{14}$, more than an order of magnitude increase. This is just about what they expected from a hydrodynamically equivalent implosion.

This author enthusiastically congratulates the LLNL and URLLE groups on starting out with an initial failure, which discouraged many (including the author), and ending ten years later with tremendous successes.

While the URLLE results certainly provide encouragement to the direct drive, one can reasonably ask whether they are missing anything. Is there some failure mode, hiding out of sight? To this author, the obvious thing the URLLE results do not seem to address, is the issue of energetic electrons. Because the gradient scale length is so small, it is almost a sure thing that the main instabilities are stabilized. Not only is the length scale so small, but so is the time scale. The tails of the Maxwellian are almost always filled by diffusion from the body electrons. However, as the tail energy increases, the diffusion time to fill up the tail from the body at energy Ξ , is proportional to Ξ/T where T is the electron temperature. To briefly summarize, the short time and short scale length of the URLLE implosions may prevent energetic electrons from forming. It is unlikely that experiments with NIF sized lasers will have this luxury. This is one reason the author has been so motivated to continue to examine this issue, even though the NRL program (which paid his salary) had terminated a few years ago. While it is way too early to come to any conclusions, the author's work at this point indicates that the energetic electrons may well do less harm than initially thought.

The other lab which made significant contributions, is my own, NRL. For decades, this lab has developed a new laser concept to apply to laser fusion. This is the use of excimer lasers as a driver. Originally the concept was KrF lasers, and decades ago, NRL built the NIKE laser. It had a wavelength of 248 nm and produced a very smooth optical beam with an energy of ~ 2 kJ. Also, it built a rep rated laser ELECTRA, with a pulse energy of ~ 200 J. The author believes it is still the highest *average* power laser relevant for laser fusion. Since the active material is a fast-flowing gas, there is no laser glass that heats up and must cool down before the next shot. Hence it is likely that excimer lasers have better capability for aver-

age power than the glass lasers used in the other labs. Obviously, NIKE and ELECTRA are flea powered compared to the lasers at URLLE and NIF, but by examining smaller area targets, both internal and external parts, the lab was able to do important work [36].

More recently the interest in the lab has shifted to Argon Fluoride laser with a wavelength of 193 nm. Virtually all theories predict that a shorter wavelength is advantageous for laser fusion. **Figure 4** shows NRL calculations of fusion gain for frequency tripled glass lasers, KrF lasers, and ArF lasers [37]. It has been argued that ArF lasers have many more of the requirements for laser fusion than do glass lasers [38]. Just before the NRL laser fusion was terminated, the lab transitioned ELECTRA to ArF and showed that it had about the same parameters as when it ran on KrF. Also shown in **Figure 4** is the setup of ELECTRA for ArF use.

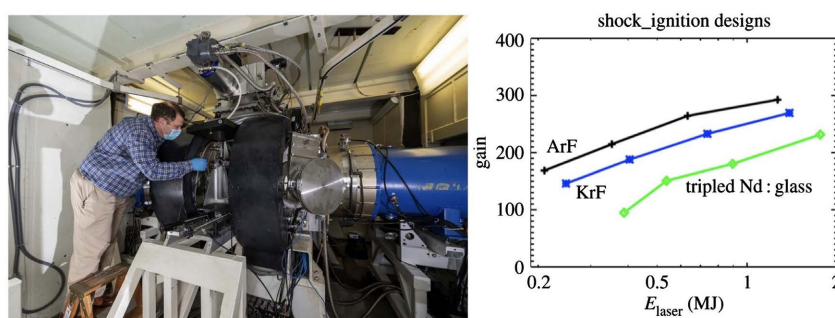


Figure 4. Left: Setting up the NRL laser ELECTRA for argon fluoride. Right NRL calculations of gain versus laser energy for 3 different lasers. According to these calculations, ArF has the potential of roughly doubling the gain of frequency tripled glass lasers.

Consider the graph of laser Q in **Figure 4**. It shows gains as high as 250. References [30] [37] and [38] also assert that the efficiency of the laser could be as high as 10%. Let's consider for a moment what this means for energy production. Say laser produces produce ~1 Joule of laser light energy. This produces ~250 Joules of fusion energy. However, for such a new power source, let us say this generates electricity classically with an efficiency of ~1/3. In other words, it produces ~80 Joules of electric energy. But the 10% efficient laser demands ~10 Joules, so ~70 Joules is left over for the grid. It seems like this could be a viable energy source for the civilian economy.

This paper is part review, and part original work. It is organized as follows. Section II reviews the results of LPI experiments in laser fusion configurations. Section III goes over a derivation of the Fokker Planck Equation for deposition of energetic electrons in laser fusion targets. Sections IV and V discuss solutions of the Fokker Planck equation by what we call the method of characteristics Section IV is a review of the material in References [22]-[25]. Section V is newly derived material showing how one might approximate the result while including the neglected terms. Sections VI and VII discuss another method of solution which we call sparse eigenfunctions. In Sections VI, a review of previous work, all terms in

the equation are retained, and one finds approximate solutions for Z with no spatial variation, or variation in a segmented target. Section VII is newly derived material. It makes different approximations; it discusses a method of using this method for high Z with spatial variation. Possibly it has some application to the plasma formed on the hohlraum walls. Section VIII discusses the energetic particles coming not from instability, but from the energetic tail of a Maxwellian distribution; it involves both a review, and also new additional work. Section IX gives some digressions, and Section X is the Conclusion.

2. Two Relevant Instabilities

Two parametric instabilities which involve the electrons, and which have the potential to generate many fast electrons are the $2\omega_p$ instability, and the absolute stimulated Raman side scatter instabilities. Experiments have been performed on both the OMEGA laser at Rochester, and on the NIF laser at Livermore.

The threshold laser Irradiance for the former is given by

$$I_{th} = \frac{2 \times 10^{16} T_e (\text{keV})}{L_\mu \lambda_\mu} \frac{\text{W}}{\text{cm}^2} \quad (3)$$

where T_e is the electron temperature, L_μ is the electron density gradient scale in microns at the quarter critical density, and λ_μ is the laser wavelength in microns.

The threshold for the latter is given by

$$\frac{v_{os}^2}{c^2} = (k_o L)^{-4/3} (\omega_p / \omega_o)^{2/3} \quad (4)$$

In terms of the laser irradiance and wavelength, the electron oscillating velocity divided by the speed of light is

$$v_{os}/c = 9 \times 10^{-3} \lambda_\mu I_{14}^{1/2} \quad (5)$$

where I_{14} is the irradiance in units of 10^{14} W/cm^2 [39] [40]. Since the $2\omega_p$ instability can take place at any position in the under dense plasma with a density less than a quarter critical, this depends on the plasma frequency ω_p and the laser frequency ω_o . Generally, the instability occurs at about the quarter critical density.

Experiments on these instabilities were done at both URLLE with their OMEGA laser and at LLNL with NIF. Generally, OMEGA experiments, with their lower electron temperatures and shorter scale lengths excite only the $2\omega_p$ instability. NIF, on the other hand with its higher temperature and longer scale length excites the stimulated, absolute Raman side scatter instability. In all the experiments, the laser pulse is brought up from an irradiance of zero to some constant value which persists throughout the pulse. Their data has been presented as a measurement of this or that as a function of the steady irradiance. The parameters they measured were the hot electron temperature and total energy of the energetic electrons as a fraction of total laser energy.

One iconic measurement of energetic electrons was performed at URLLE by a

group led by Baruch Yaakobi [41]. It was with a planar target which was subject the $2\omega_p$ instability. The data points in Figure 5 show these measurements.

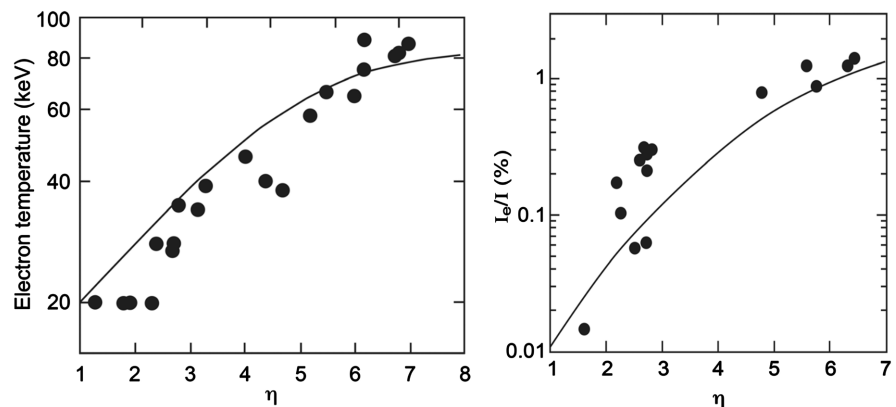


Figure 5. Data points from the URLLE experiments on the energetic electrons produces by the $2\omega_p$ instability. The horizontal axis label η , is the ratio of Irradiance to threshold irradiance. The left curve measures hot electron temperature, the right, ratio of electron energy to laser energy. The curves are empirical fits to the data.

The curves are:

For the temperature:

$$T_e (\text{eV}) = 2 \times 10^4 \eta^{2/3} \quad 1 < \eta < 7 \quad (6)$$

And for the energy ratio:

$$\frac{I_e}{I_{\text{laser}}} = 10^{-4} \eta^{2.5} \quad 1 < \eta < 7 \quad (7)$$

For the NIF experiment [26], here is the analogous data:

In both cases, it is rather well established that the instability occurs at about the quarter critical density.

One interesting thing about this data is that for the $2\omega_p$ instability, the temperature rises with irradiance, whereas for the stimulated Raman, the temperature is relatively constant as the irradiance varies. Possibly the reason is that for the $2\omega_p$ instability, only one of the wave numbers (that of the incident wave) is specified; there are a large number of possibilities for the wave numbers of the stimulated electrostatic waves. Possibly, the size of the region of the unstable wave spectrum in k space increases as the irradiance does. However, for the Raman case, the wave number of the two electromagnetic waves are basically specified, leaving little room for variation of the phase velocity of the stimulated electrostatic wave. Thus, one would expect that the energetic electron temperatures for the two instabilities might be quite different, with the Raman case having a temperature less dependent on irradiance.

Let us note that the data in Figure 5 and Figure 6 are for steady state irradiance. However, in a laser fusion implosion, the laser irradiance varies very rapidly in time. The data above gives little hint of how to handle this in a rad hydro code like HYDRA or FAST. At this point the only reasonable possibility is to assume that

in the rad hydro code, the instantaneous ratio of energetic electron energy flux to laser irradiance is equal to the ratios shown in **Figure 5** and **Figure 6**.

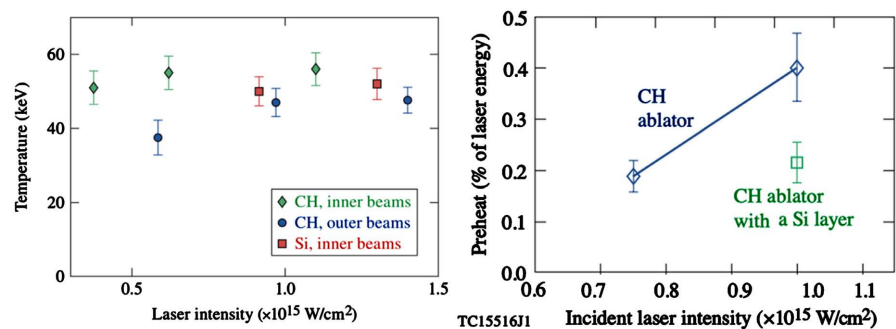


Figure 6. Data analogous to **Figure 5**. The URLLE/NIF group explored reduction of the instability by using different material in the ablating plasma. As a function of irradiance, the energetic electron temperature was relatively constant at ~ 50 keV. The irradiance varied between $\sim 0.5 - 1.5 \times 10^{15}$ W/cm².

The next issue is the angular distribution of the energetic electrons. It is an issue where this author believes that the information gathered so far does not necessarily give the proper insight as to what is going on. One iconic experiment is a continuation of the experiment already cited [42].

In a different target configuration, there are several concentric spheres. The outer sphere is a thin shell which is irradiated pretty much the way the flat plane was in generating **Figure 5**. The sphere is filled with nitrogen, and inside there is another sphere of high Z material, usually molybdenum. The hard X-rays generated depend on radius of the inner sphere, *i.e.* how much of the X-ray flux the sphere intersects. The experiment configuration, taken from [41] is shown in the left-hand image of **Figure 7**.

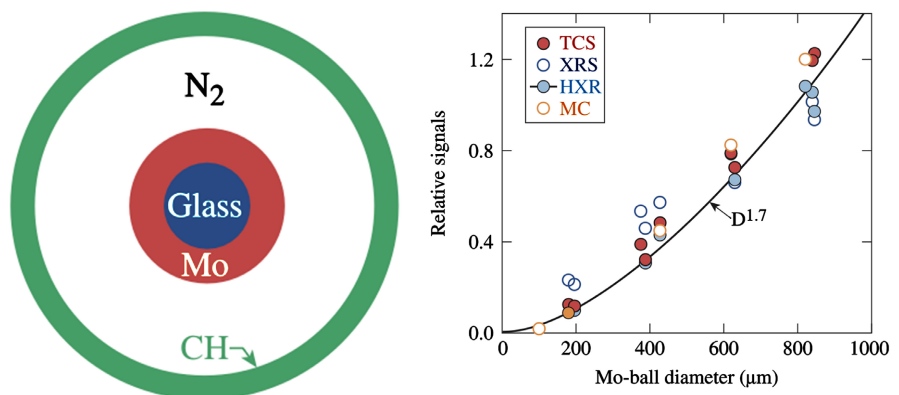


Figure 7. Left: The configuration of the experiment [42]; Right: the total X-ray energy as a function of radius of the Molybdenum ball.

Notice from the center graph that the total hard X-rays increase roughly as the ball radius squared, indicating that the electrons come out in a wide angle.

There are also results from PIC simulations [43]. This work took very different

parameters for both the laser and plasma. However their simulation was subject principally to the Raman side scatter instability, and they certainly observed it. They observed a tail on the distribution function having a temperature of about 50 keV, just like that seen in the NIF experiment. Furthermore, the angular spread of the energetic particles was confined to a cone with about a 45-degree vertex angle. These results are displayed in **Figure 8**. The plot of the distribution with energetic tail, was taken from [43]. The phase space plot did not appear in the publication. However, Dr Xiao was kind enough to send me his plot and allow me to use it in my work. For this, I am certainly grateful.

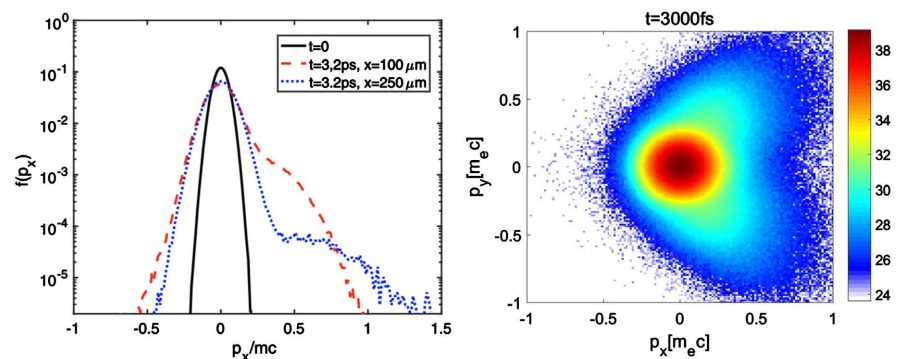


Figure 8. Left: A plot of the initial distribution function in black, a 1 keV plasma. The red and blue curves are distribution functions at 2 positions in the plasma after 3.2 psec. The blue curve is near the higher density edge of the plasma. Clearly it is an energetic distribution with a temperature of around 50 keV and with a density of 3 or 4 orders of magnitude below the main plasma density, consistent with the NIF results shown in **Figure 6**. Right: A phase space plot of the energetic electron distribution. They are confined to a cone with about a 45 degree vertex angle, basically consistent with the results of [42] (but for a different instability) and **Figure 7**.

However, despite the impressive nature of the experiment, and the simulation of the instability with a PIC code, this author feels that the impressions left in the literature are not especially helpful for the case of a direct drive laser fusion target, *i.e.* just the configuration which this manuscript attempts to enlighten. In the real world of direct drive laser fusion, the plasma ends at neither the last cell of the simulation, nor the edge of a thin spherical shell. In the actual case, the quarter critical density (for instance) abuts a very inhomogeneous plasma whose density just beyond the quarter critical density rises very rapidly, finally reaches a maximum at much higher density, and then reduces until it is near the center of the target.

We take here an example from [26]. **Figure 9** shows a plot of density versus radius for a rather complex, striated plasma taken from [26], and reproduced in different colors for the different Z plasma below in **Figure 9**.

Once the energetic electron (say 50 keV) gets beyond the quarter critical density and into the denser plasma, the electron hardly moves along straight-line orbits. However often it is portrayed in the region as doing just that. For instance, look at **Figure 10**.

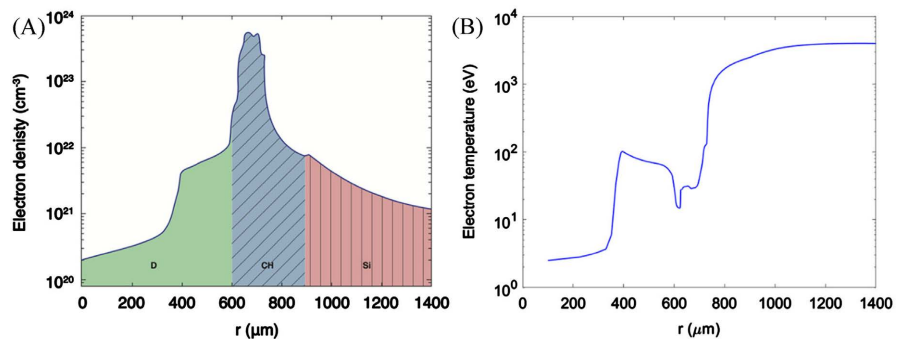


Figure 9. A plot of the density (A) and Temperature (B) of the plasma as calculated in (27) at a particular time of their simulation. The plasma has 3 regions, an outer region of silicon, (*i.e.* $Z = 14$) an inner region of CH (*i.e.* plastic, $Z = 5.3$), and the target region of deuterium ($Z = 1$). The quarter critical density is near the right-hand edge.

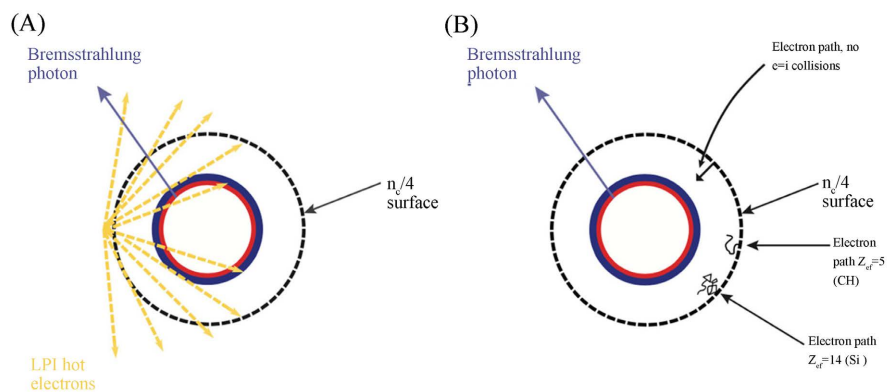


Figure 10. Left hand image: The paths of energetic electrons as envisioned in [26], the source of this image. Right hand image: More accurate picture the path of a 50 - 100 keV electron once it passes just inside the quarter critical density.

The arrow in the upper right hand side of the right hand image is the path of a 50 keV electron, moving directly toward the center, and moving an electron-electron mean free path at for a density profile like that given in **Figure 9** (actually as we have seen, the actual path it travels before it reaches zero velocity is much shorter). However, even for $Z = 1$, this is not an accurate representation. The electron scatters in angle from both the ions and electrons. Hence before the electron travels a mean free path, it will have made several sharp turns in its orbit. The case for a $Z = 5$ (plastic) and $Z = 14$ (silicon) plasma is shown schematically in **Figure 10(B)** as well.

Clearly envisioning energetic electrons as moving along straight-line orbits through the center of a laser fusion target does not give an accurate picture of what is really happening, at least not for those electrons that preheat the fuel. The question is how does one describe the energetic electron distribution just inside the quarter critical density? Clearly these electrons are attempting to become a spherically symmetric distribution due to both electron and ion collisions. The collisions approximately conserve electron energy, more accurately for higher Z ions, but even approximately so for a $Z = 1$ plasma. However, there is a constraint. Since the

collisions are mostly with the heavier ions, they do not lose any energy. Therefore, the energy flux of these energetic electrons just inside the quarter critical surface must be the same as it was just outside; otherwise, there would be enormous heating at the quarter critical surface. Hence the calculation to do is to minimize the anisotropy, but subject to the constraint that the energy flux is as given is as given like in **Figure 8** at each energy. (This implies a higher density of these energetic electrons than what the simulation produced). This seems like a normal variational calculus problem. However instead of performing this calculation, the author simply used a drifted Maxwellian, which is a certainly reasonable approximation. As this work is formulated, the only distribution function needed, when expanding f in a Legendre series is the first term, the term proportional to $\cos\theta$ the cosine of the angle between the particle velocity and the spatial gradient. For that we take

$$f_1(V, \theta) = AV \cos\theta \exp(-V^2/V_{th}^2) \quad (8)$$

Here A is the coefficient that gives the proper energy flux for the energetic electrons as calculated in the simulation [43] (for instance) and v_{th} is the thermal velocity of the *energetic* electrons. Equation (8) is a reasonable estimate for f_1 , not a detailed derivation. However, both the experiments and simulations quoted here do indicate that the distribution function must be basically Maxwellian, and energy conserving collisions will not change that. Given this fact, and the fact that energetic electron plasma has a specified energy flux, what could be a better first approximation? These conditions impose quite a few constraints, and there is not much wiggle room to come up with an f_1 which is substantially different.

Furthermore, in future work, it is likely not only variational calculations, but also that Fokker Planck or PIC simulations could add additional insight. This is discussed further in Section IX.D. A PIC simulation could provide insight if the quarter critical surface is a $Z = 1$ plasma. In that case the energy conserving collisions have only twice the rate of energetic electron loss to the background electrons. However, the NIF/LLNL experiments discussed here have either $Z = 5.3$ (Plastic) or $Z = 14$ (silicon).

One way to solve the deposition problem is to use a Monte Carlo code and follow the jagged path of many, many electrons starting at the quarter critical surface, and ending when the electron had lost all its energy. Clearly, to complete the task, many, many thousands of electrons would be needed both to fill up the distribution function, and to calculate every type of orbit in the spherical plasma. That is an approach that **Figure 10(A)** hints at. But can this be economically done at every time step of rad hydro code?

However, in the next few sections, we will find that one can short circuit this approach by approximately solving the Fokker Planck equation for the energetic electrons. Getting such an approximate solution, using one of two different approaches, is simpler than one might think.

3. Fokker Planck (and Other) Approaches

There are many simplifications one can make on the actual Fokker Planck equa-

tion, which are relevant to the problem at hand, namely the preheat of a direct drive laser fusion target. These have been described elsewhere [22]-[25]. There are two types of problems for which the Fokker Planck treatment can be optimal. First, the energetic electrons could be produced by one of the laser plasma instabilities, like those just summarized in the previous section. Second, they could be the tail of a Maxwellian distribution, that is those thermal electrons with high enough energy that they mostly decouple from the body. Many of the approximations and analyses we make here are common to both situations. Others are applicable to one or the other.

We begin with the case of instability generated energetic electrons and then continue in a later section by discussing the case of the mostly decoupled tail of the distribution function. There are two approximations we make, which, and almost certainly correct. First, we assume that the number of energetic electrons is very small, as compared to the total plasma. Immediately this greatly simplifies the theory as compared to nearly every other one which deals with the entire electron distribution function.

Figure 5 and **Figure 6** in the last section shows that experiments up to now have shown that the energy of these particles maximizes (at least in previous recent experiments) at an energy of about 1% of the laser energy, or as we more accurately put it for our configuration, the energy flux of the energetic particles, so far maximizes at about 1% of the laser irradiance. (However, this 1% or less, might have a profound effect on the preheat of the target.) Hence, these particles interact with the background plasma and hardly interact with one another. While the overall Fokker Planck formulation is strongly nonlinear, the Fokker Planck calculation of the interaction of these particles with the plasma is a *linear* problem; double the number of energetic electrons, and the preheat of the target doubles.

Secondly, as shown in [22], the Fokker Planck interaction terms are characterized by two of what are called Rosenbluth potentials. One is labeled F , which is a vector and is basically a friction force, the other, labeled D is a tensor and it has elements of both friction and diffusion. However, if the particle is energetic compared to the background, these terms simplify very considerably. It turns out, as shown in [22] and other places, that the collision of the energetic electrons with the ions contains only angular scattering with a coefficient proportional to the effective Z of the plasma (*i.e.* the electron energy is conserved). The interaction of the energetic electron with the background electrons has two components. First the background electrons give rise to angular (*i.e.* energy conserving) scattering equal to that of the ions if $Z = 1$. Second the collision gives rise to a friction force which slows the electron down. As we have seen in the Introduction, the electron cannot even travel one electron-electron collisional mean free path (mfp) before it stops. Finally, the Diffusion tensor describes the diffusion in energy of the energetic electrons as they collide with background electrons. However, for energetic electrons, this process is smaller than the other processes just described by about a factor of the background electron temperature divided by the energetic electron

energy. This is fine for the instability generated electrons but is certainly less accurate for the energetic electrons arising from the tail of the Maxwellian, at least near the position where these electrons are born. (For much of their life, they traverse a much cooler plasma.) We neglect this electron energy diffusion in this work.

To reiterate, in a purely Krook calculation, after the electron travels one mfp, it has a probability of $1/e$ of remaining and not joining the thermal plasma. For this reason, earlier Krook calculations, by NRL [15], URLLE and probably other organizations, have shown no gain in laser implosion simulations. As far as this author knows, none of these calculations which show failure to ignite have been published.

Different organizations realized that friction had to be accounted for, as discussed in the Introduction. Thus, they added terms to the Krook model which they hoped would accomplish this. This author thinks it is better to take a theory which is correct and work with it, rather than take one that is basically incorrect and attempt to patch it up. To be honest, the NRL group was probably the last to realize the importance of friction. We learned our lesson the hard way by performing several rad hydro simulations with a Krook model for the energetic electrons. We never found satisfactory implosions with acceptable gains [15]. Apparently URLLE never did either. However, at this point, rather than attempting to find a bandage, we dropped the Krook model and moved on to a Fokker Planck model. We believe this model is basically correct and that where the approximations we have made are not satisfactory, they can be corrected with further effort and study.

To get back to our analysis, we also simplified the Fokker Planck model by looking at steady state solutions for the electron energy flux, *i.e.* the theory is nonlocal in space, but steady state in time. The plasma density, energetic electron energy, and electron collision times seem to easily support this approximation [23] [25]. However, doing the calculation only in space means that we must specify spatial boundary conditions. At the center of the sphere (or at the analogous end point in a planar approximation to the sphere), there obviously cannot be any energy flux at any particle velocity. For the instability case, the other point is at the quarter critical density and the energetic electron distribution function is given by Eq [8] with A and the energetic electron temperature given in the last section.

Finally for the case of the instability, we neglect the effect of the electric field. For the simple spherical or planar geometry, the electric field is the gradient of a potential. The potential across the plasma is generally of the order of the electron temperature a few keV. However, the instability generated energetic electrons have a temperature of ~ 50 keV, and many electrons have energy in the hundreds of keV. It is interesting to note that with our simplified FP formulation, there are no complicated numerical issues. In fact, we have examined the numerical stability of the rad hydro energy equation with such an additional heating and found it is nearly always stable [11]. The added term to the rad hydro equation is a simple heating term (actually it is the divergence of an additional heat flux). This con-

trasts with the more complex multi-dimensional calculations, containing many additional effects, *i.e.* generated magnetic fields, this or that instability... For instance in one case, the code had to occasionally use an artificially reduced electric field [21]:

In some circumstances with strong temperature gradients (that is the electric field across a grid cell is greater than the collisional slowing down), meaning that it is impossible to meet the CFL condition described in Equation (21). In these circumstances, the code will always crash, regardless of whether the diffusive term is used or the number of energy groups used. Here, an artificial reduction in the electric field experienced by higher energy groups can be used to stabilize the M1 code.

There are other places in [21] where they had to add specific limitations to keep their code from crashing.

For our formulation, the electric field is given strictly by the gradient of a potential, and our method of solving for the additional energetic electron energy flux, the electric field may be large enough to violate the condition above [21] between two collisions, but it makes no difference. Look at the orbits as visualized by Figure 10. No matter how much the electron gains energy in one jag of its orbit, in the next jag, that energy is given back. The effect of the electric field from the initial to final point of the orbit, as shown in Figure 10, is given only by the potential drop between these two initial and final points, *i.e.* a keV or two out of 50 or more keV particle energy.

For the case of the nonthermal tail, the outer boundary condition is more difficult and subjective and will be discussed in Section VIII. Also, one can still neglect the effect of the electric field on the energetic tail, by the same logic as above, but the validity of this approximation becomes much more questionable.

While the formulation here is one dimensional, planar or spherically symmetric, there is always the potential of extending the theory to 2 or 3 dimensions. However, the author does not believe that this is a high priority item, at least at present for the case at hand. If the multi-dimensional rad hydro code predicts acceptable deviations from spherical symmetry for ignition, one can always calculate the spherical average of each quantity and then calculate the spherically symmetric average of the effect of the energetic electrons. If the multi-dimensional rad hydro code shows large deviations from spherical symmetry, it is unlikely that the configuration will be acceptable for either laser fusion or additional study; and if not, the effect of the energetic electrons would be only of academic interest.

Now to get to the nitty gritty of the Fokker Planck analysis. The Fokker Planck equation, with all the simplifications we have just mentioned becomes

$$\begin{aligned} & \frac{\partial f}{\partial t} + V \cos \theta \frac{\partial f}{\partial r} - \frac{V \sin \theta}{r} \frac{\partial f}{\partial \theta} \\ &= \frac{1}{V^2} \frac{\partial}{\partial V} V^3 \nu_e(V) f + \frac{\nu_e(V)}{2} (1+Z) \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right] f \end{aligned} \quad (9)$$

It is appropriate for either a planar situation (the r variable would be denoted

by x or z) or a spherical configuration. The variable θ is the angle between the velocity vector and the vector in the direction of the gradient, in the planar or spherical configuration. The third term on the LHS accounts for the fact that the vector between the velocity and spatial gradient has a spatial dependence in the spherical case. The variable ν_e is the electron collision frequency given by

$$\nu_e(V) = \frac{4\pi n e^4}{m^2 V^3} \Lambda \text{ in cgs, or } \nu_e(V) = 3.9 \times 10^{-6} \frac{n}{\Xi^{3/2}}$$

Here, Λ is the Coulomb logarithm, and Ξ is the electron energy in electron Volts. The next step is to expand the distribution function in a series of Legendre polynomials keeping only the first two terms ($P_0(\theta) = 1$, $P_1(\theta) = \cos\theta$) (we discuss additional terms in the expansion in a later section):

$$\begin{aligned} \cancel{\frac{\partial f_0}{\partial t}} + \frac{V}{3} \frac{\partial f_1}{\partial r} + \frac{2V}{3r} f_1 &= \frac{1}{V^2} \frac{\partial}{\partial V} V^3 \nu_3(x, V) f_0, \\ \cancel{\frac{\partial f_1}{\partial t}} + V \frac{\partial f_0}{\partial r} &= \frac{1}{V^2} \frac{\partial}{\partial V} V^3 \nu(x, V) f_1 - \nu_e(x, V) (1 + Z) f_1. \end{aligned} \quad (10)$$

Various terms in Equation (10) are struck out by red solid lines, or green dashed lines. The former are terms that are neglected in the steady state approximation. The latter are terms struck are what is called the multi group diffusion approximation. This is generally used to calculate neutron transport [18], but it has also been used to calculate energetic electron transport in laser produced plasmas [17]. The author has always been suspicious of the multigroup diffusion approximation. After all, why eliminate dynamical friction in the equation for f_1 , but not in the equation for f_0 , and why eliminate the time derivative in the equation for f_1 , but not in the equation for f_0 ? Here we consider only the steady state approximation.

To simplify things, we simplify the definition of variable

$$dz = \sqrt{3} \frac{2.7 \times 10^{-13} n(r) \Lambda}{T_e^2} dr \equiv k(r, T_e) \text{ and } w = \zeta^2 \equiv \left[\frac{\Xi}{T_e} \right]^2 \quad (11)$$

Here, T_e is the temperature of the energetic electron distribution in electron Volts. Notice that there are no dependencies on Z in either of these variables. All Z s are explicitly noted in the equations. A change in z of unity means a single electron-electron mean free path (mfp). Recall that the electron can only travel about a quarter of a mfp before it stops completely. As an example of the dependence of z on r , **Figure 11** shows the calculation of z for the density distribution shown in **Figure 9** (the spatial dependence of the temperature is also given in **Figure 9**).

Notice at $r = 1200$ microns, z is about 6, and it decreases to about 5 at about 800. This is approximately consistent with the sketch of orbits in shown in **Figure 10**.

Then eliminating the equation for f_0 , we find a single, second order partial differential equation for only f_1 .

$$\frac{\partial^2 f_1}{\partial z^2} + 2 \frac{\partial}{\partial z} \frac{f_1}{r(z) \left[\frac{dz}{dr} \right]} = \frac{\partial^2 f_1}{\partial w^2} - \frac{1+Z}{4w} \frac{\partial f_1}{\partial w} + \frac{1+Z}{4w^2} f_1 \quad (12)$$

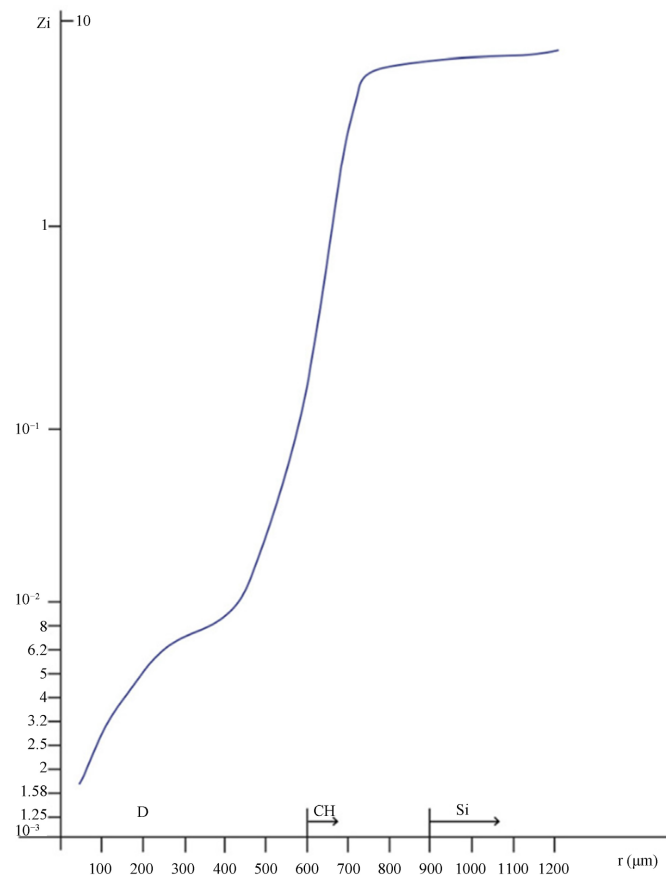


Figure 11. A plot of z versus r for the electron density given in **Figure 9**. The electron mfp varies tremendously in the plasma. The total size of the plasma is about 6 of the electron mean free paths at T_e .

This is the equation (and various approximations to it), which we will work with, in most of the rest of this manuscript. The next four sections develop methods of solving Equation (12). The next section makes some approximations to the equation and then finds an exact solution. In the following sections, we keep the equation as it is (well almost) and develop methods to find an approximate solution.

4. Solution of the Fokker Planck Equation by Characteristics

We notice that Equation (12) looks a lot like the wave equation, but with a few extra terms; two on the right where the Z dependence comes in, and one on the left where the spherical nature comes in. For now, we neglect the spherical nature of the configuration, so the second term on the left disappears. In the next section we discuss this further.

We start by assuming the plasma is characterized by a single Z . Then we consider a segmented target like that shown in **Figure 9**. The equation simplifies somewhat if we can rewrite the rhs without the first derivative. Redefining the dependent variable as g rather than as f_1 , where

$$f_1(w, z) = w^n g(w, z) \quad (13)$$

When $n = Z + 1$, we get the equation for g :

$$\frac{\partial^2 g}{\partial z^2} = \frac{\partial^2 g}{\partial w^2} - \frac{(1+Z)(9+z)}{64w^2} g \quad (14)$$

For a rather decent part of the parameter space, especially for lower Z and larger w , we find that we can simply neglect the last term (we will define this more precisely shortly). Then the equation simply becomes a wave equation for g :

$$\frac{\partial^2 g}{\partial z^2} = \frac{\partial^2 g}{\partial w^2} \quad (15)$$

Now let us consider the boundary conditions. The relevant region of space $0 < z < z_c$, where z_c is the quarter critical density, and for w between zero and infinity (*i.e.* positive values). We know (from Equation (8)) that at $z = z_c$,

$$g(w, z_c) = w^{-(Z+1)} w^{1/4} \exp(-w^{1/2}) \equiv g(w) \quad (16)$$

This is one boundary condition. Another is that $g = 0$ at $z = 0$ for all w (*i.e.* there can be no energy flux into the origin at any energy). Furthermore, we also know that $g = 0$ for w equals infinity. The z axis, *i.e.* $w = 0$ does not come into play as we will show shortly.

These are odd boundary conditions for a wave equation. It is unusual for a wave equation for g , to have the boundary condition specifying g on a *closed* surface. It is more usual for it to be specified on an open surface, say $z = z_c$. However, one also would need to know the partial derivative with respect to z on this surface to determine a solution.

Nevertheless, as we will show, the physical conditions do give a unique and physically reasonable solution. We know that there are two characteristics from the wave equation emanating from the point on the boundary (w_o, z_c) . They are

$$w - w_o = z - z_c \quad \text{and} \quad w - w_o = -(z - z_c) \quad (17)$$

These two characteristics are illustrated in **Figure 12**, emanating in two directions from w_o . Notice that the ones in black proceed downward as z decreases from $z = z_c$, while the ones in red proceed upward. Obviously, it is completely unphysical that there can be any solution on this upper characteristic, a solution which would have collisions increasing the electron energy. Hence the solution is fully determined by forbidding any solution on this upper characteristic. To relate it to the more normal boundary condition, it is not difficult to see that this new condition is completely equivalent to setting the normal derivative boundary condition as

$$\frac{\partial g}{\partial z} = \frac{\partial g}{\partial w} \quad \text{at} \quad z = z_c \quad (18)$$

(Correspondingly, if the only allowed solution were on the upper characteristic, the relevant normal boundary condition would be Equation (18) but with the sign reversed).

Hence, we simply express this as saying that only one of the characteristics can be occupied, the one in black in **Figure 12**.

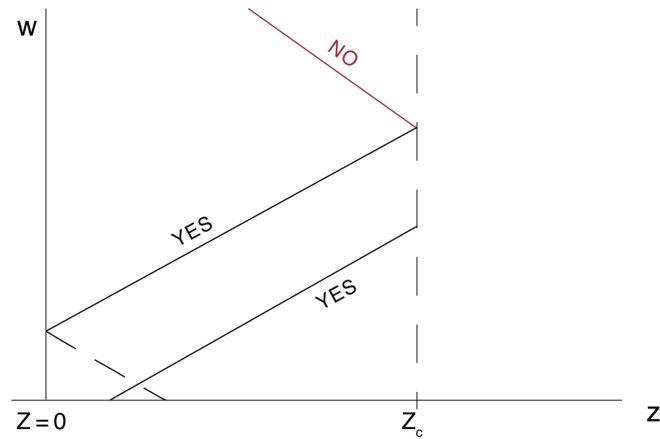


Figure 12. The region of interest in wz space for which there could be significant heating from the energetic electrons. Clearly there can be no solution for the red characteristics, as the energetic electron energy would increase from its value at $z = z_c$ as it traveled along this upper characteristic.

As the solution moves down the characteristic from its position at $z = z_c$ there are two possibilities. The lower characteristic in **Figure 12** strikes the z axis, *i.e.* $w = 0$. In this case, the calculation on this characteristic simply ends; the energetic electron has given up all its energy and can no longer be considered. The other possibility is that the characteristic strikes the w axis, *i.e.* $z = 0$. This would violate the boundary condition at $z = 0$. However, this can be remedied by adding a “reflected wave”, originating at the intersection which “propagates” back toward the quarter critical surface, but with a negative coefficient. In other words, the solution to the “wave equation” can be expressed analytically as

$$g(w, z) = g(w + z_c - z) - g(w + z_c + z) \quad (19)$$

This is a valid solution as long as $w > 0$. If w_0 is small enough that characteristic strikes the z axis (*i.e.* $w = 0$), the second term of Equation (19) does not appear. The second term only appears for characteristics that strike the w axis (see **Figure 12**). If Equation (19) gives a $w < 0$, and/or $z = 0$, we simply ignore it. In other words, as expressed in terms of $f_1(w)$, the value of f_1 at $z = z_c$,

$$f_1(w, z) = \left[\frac{w}{w + z_c - z} \right]^{Z+1} f_1(w + z_c - z) - \left[\frac{w}{w + z_c + z} \right]^{Z+1} f_1(w + z_c + z) \quad (20)$$

for $w > 0$ and $0 < z < z_c$. Again, the second term appears only for characteristics that strike the w axis.

Once we have the f_1 , it is simple enough to get the energetic electron energy flux as a function of z , *i.e.* $q_{ee}(z)$:

$$q_{ee}(z) = A' \int dw w^{1/2} f_1(w, z) \quad (21)$$

Recall that A in Equation (8) (or A' in Equation (21)) above is chosen so that the energetic electron energy flux at the quarter critical density has the proper value.

Now let us briefly examine the validity of approximating Equation (14) by the

wave equation. Simply taking the second derivative of the exponential part of Equation (20), *i.e.* the dominant term at high energy, and comparing it to the second term on the right-hand side of Equation (14), we find that the wave equation ought to be okay if

$$\frac{(1+Z)(9+z)}{16w} < 1 \quad (22)$$

Clearly the wave equation works best for low Z and high w . Expressed as a function of w , the energy flux goes as $w^{3/4} \exp(-w^{1/2}) dw$. This has a rather broad maximum, with significant energy flux extending from about $w \sim 2$ to $w \sim 15$. While the wave equation might not be the best way to get an approximate solution it does have one undeniable advantage. It is extremely simple. Using Eqs. (20) (21), and taking the divergence of the energy flux, one has a simple expression for the energetic electron heating from the quarter critical surface to the center of the target. It is a simple heating term which should not add any requirements to the rad hydro code, and it is unlikely that it would do anything to crash the code either. Once one has the necessary information on the effect of the instability, as described in for instance Section (2), it would be extremely easy to put this formula for energetic electron energy flux into the rad hydro code. This way, one could get what is probably at least a reasonable first approximation, at each rad hydro time step, to the effect of the energetic electrons on the target gain. It is almost certainly much easier to implement than calculating thousands of orbits at each time step with a Monte Carlo code.

In later sections we will see how one can find better solutions for Equation (14), including both the second term on the rhs of Equation (12), and the spherical effects.

Now we consider a segmented target, like that shown in **Figure 9**. Solving the wave equation for g in each segment, we need only the relation between the z at the boundary of between the regions, *i.e.* at $z = z_{12}$ as shown in **Figure 13**.

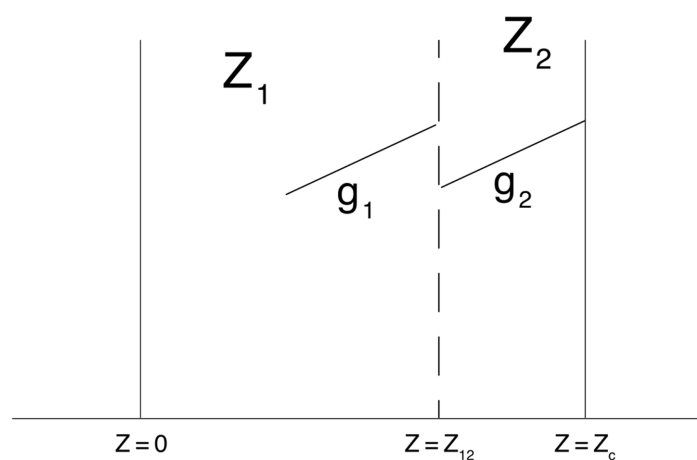


Figure 13. A figure of the characteristics for a segmented target. The boundary between the regions of Z_1 and Z_2 is denoted as z_{12} . While $g(w, z)$ is not continuous across this boundary, $f(w, z)$ is.

Since the energy flux across this boundary must be continuous, the discontinuity in g at the boundary is given by

$$w^{Z_1+1} g_1(w, z_{12}) \text{ at } z_{12} - \varepsilon = w^{Z_2+1} g_2(w, z_{12}) \text{ at } z_{12} + \varepsilon \quad (23)$$

and the solution propagates downward along the characteristic in the next region.

To summarize, using the wave equation approximation to get an approximate solution to Equation (12), we start at $z = z_c$ with the known function of $f_1(w)$ and from that get $g(w)$ at $z = z_c$. We then propagate the solution for g down the lower characteristic, as shown in **Figure 12** and **Figure 13**, until the characteristic strikes either $w = 0$ or $z = 0$. For the former, the calculation along this characteristic simply stops; the energetic electron has lost all its energy. If the characteristic first strikes the $z = 0$, one must add a “reflected wave” with a negative coefficient to satisfy the boundary condition, $g(w, 0) = 0$ at $z = 0$. If the target is segmented, as shown in **Figure 9** and **Figure 13**, one must account for the different Z 's occurring in the different regions, as shown in **Figure 13**.

5. An Improvement to the Characteristic Approach

We have seen that the equation for f_1 can be rewritten as an equation for g , where g is defined by Equation (13) for the planar case. Let us now consider the spherical case. In this case

$$\frac{\partial^2 g}{\partial z^2} + 2 \frac{\partial}{\partial z} \left(\frac{g}{rz'} \right) = \frac{\partial^2 g}{\partial w^2} - \frac{(1+Z)(9+Z)}{64w^2} g \quad (24)$$

Here $z' = dz/dr$. To continue, let us define

$$g(w, z) = H(z) h(w, z) \quad (25)$$

and define H in such a way as to make the first derivative term on the left hand side disappear. This will leave Equation (24) as the wave equation, but with a correction term on the right-hand side which we will define as

$P(w, z) = [\chi(w) + \Lambda(z)] h(w, z)$. Clearly $\chi(w)$ is the added fraction on the rhs of Equation (14). We make one additional assumption, namely that $H(z)$ is a slowly varying function so that second derivatives of it can be neglected. Then it is not difficult to show that $dH/dz = (rz')^{-1} H$ so that

$$H(z) = \exp - \int_{z_c}^z dz' \left(\frac{1}{rz'} \right) \quad (26)$$

Then one can show that

$$\Lambda(z) = \frac{2}{(rz')^2} \left[1 + \frac{d(rz')}{dz} \right] \quad (27)$$

So that the modified wave equation becomes

$$\frac{\partial^2 h}{\partial z^2} = \frac{\partial^2 h}{\partial w^2} + \varepsilon P(w, z) \quad (28)$$

Here ε is not necessary, small, but is introduced here as a bookkeeping device to be able to follow the perturbed quantities. To see the actual modified wave

equation, simply set $\varepsilon = 1$.

Assuming that ε is small, we can approximate the rhs correct to order ε by simply using the zero order solution for h (*i.e.* the solution to the wave equation, *i.e.* the solution $h(w, z)$ is constant, along the downward characteristics that propagate to the left from $z = z_c$). Taking this solution, as an approximation to the rhs, we can solve for the modified wave equation, correct to order ε , by using a Green's function.

For the wave equation, we usually need two Green's functions, one for $h(w, z)$ and one for its partial derivative with respect to z , since each usually must be specified to get a solution.

However, as we saw in section (4), an alternative, and completely equivalent approach is to specify that there is no solution on one of the characteristics. In fact, certifying this is totally equivalent to specifying the partial derivative as written in Equation (18). Hence for our case, we need only to specify a single Green's function, for $h(w, z)$ itself, and simply eliminate the solution that it specifies on the unphysical characteristic.

Hence, the solution for $h_1(w, z)$, the solution, correct to order ε is simply

$$h_1(w, z) = \int dz' dw' G(w - w', z - z') [\chi(z') + \Lambda(w')] h_0(w' - [z' - z]) \quad (29)$$

Here, $h_0(w)$ is the value of $h(w, z_c)$ at $z = z_c$.

The next issue is what the Greens function for the h . In one dimension and in three dimensions, the initial delta function simply propagates outward, while in two dimensions, the Greens function has a wake behind the advance of the front, as one can see by throwing a pebble into a pond. In one dimension, the Greens function just splits into two propagating pulses, propagating away in each direction. That is

$$G = 0.5d[(w - w') - (z - z')] + 0.5d[(w - w') + (z - z')] \quad (30)$$

In our case, the second of these delta functions will give the disturbance propagating up the characteristics to higher values of w ; a solution which is clearly unphysical and must be eliminated. Keeping only the first delta function will give a propagating "wave" only towards lower values of w , and is the proper, physically correct solution.

Doing the w' integration, we find that the zero order solution factors out. That is

$$h_1(z, w) = 0.5h_0(w - [z - z_c]) \int dz' \{ \chi(z') + \Lambda(w - [z - z']) \} \quad (31)$$

The only issue now is what are the limits of the z' integration. To answer that, **Figure 14** provides the insight.

The issue is the propagation of energetic electrons from the point z_c where they are formed, for instance the quarter critical surface, and towards the interior of the target. Hence the effect of these energetic electrons propagates down the characteristic. For the electrons at z_c with w greater than the top of the blue shaded region, the characteristic misses the region where it can affect the position shown as (w, z) , so it cannot have any effect there. These characteristics are shown as the upper red dashed characteristics. The lower dashed red characteristics have too

low an initial energy to have any effect on the electrons at (w, z) . Hence, the only characteristics that can have any effect on (w, z) are those intersecting $z = z_c$ within the blue shaded region, that is the characteristics shown in dashed blue. Clearly, this means that the limits of the integral over z' in Equation (31) are from z_c to z .

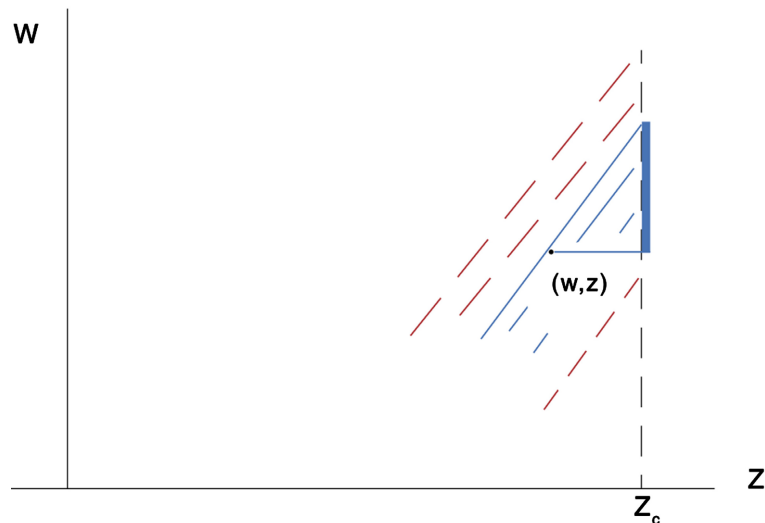


Figure 14. The wz plane. The point (w, z) is shown, as well as several characteristics. The blue shaded region of vertical axis at $z = z_c$ shows the region between z and the intercept of the characteristic passing through the point (w, z) .

While the spherical part of the integral in depends on the density profile, the Λ part can be done analytically. We find that for the planar configuration

$$h_1 = h_0 \frac{(Z+1)(Z+9)}{64} \left[\frac{1}{w} - \frac{1}{w-z+z_c} \right] \quad (32)$$

Notice that h has a singularity at $w = 0$. However, the distribution function, which is $w^{Z+1}h(w, z)$ has no singularity. Hence using perturbation theory one can get a better approximation where perturbation theory is valid.

For the more general case, it is possible that a routine like Mathematica can solve for more general solutions, perhaps even at each step of a rad hydro simulation.

Asking Google:

Does Mathematica have efficient means of solving complex wave like partial differential equations?

One gets the AI overview

AI Overview

Yes, Mathematica possesses efficient, built-in numerical routines to solve complex, wave-like, and time-dependent Partial Differential Equations (PDEs). Using NDSolve, Mathematica automatically applies finite element or pseudospectral methods, handling nonlinear systems, complex boundary conditions, and multi-physics modeling in high dimensions.

6. Solution Using Sparse Eigenfunctions

Notice that for Equation (24), the rhs only operates on w , and the lhs side only operates on z , assuming Z is constant and is not a function of z . This motivates separating the equation in a product of a function z and a function of w :

$$f_1(w, z) = \Phi(z)\Omega(w) \quad (33)$$

Then one can divide the Equation (24) by $f_1(w, z)$ so the left side becomes only a function of z and the right side becomes only a function of w . Hence each side must be a constant with no z or w variation; a constant we call κ^2 and the two equations become:

$$\frac{d^2\Phi}{dz^2} + 2 \frac{d}{dz} \left[\frac{\Phi}{k(z)r(z)} \right] = \kappa^2 \Phi \quad (34)$$

$$\frac{d^2\Omega}{dw^2} - \frac{1}{4} \frac{1+Z}{w} \frac{d\Omega}{dw} + \frac{1+Z}{4w^2} \Omega = \kappa^2 \Omega \quad (35)$$

First let us consider Equation (34), the spatial part, and for now neglect the spherical term. Then the solution is simple. As Φ must increase from $z = 0$ to $z = z_c$ by several orders of magnitude at least, κ^2 must be positive. For the planar case, the solution is simply

$$\Phi = \sinh \kappa z \quad (36)$$

This equation certainly satisfies the boundary condition at $z = 0$, and its coefficient must be such that it satisfies the boundary condition at $z = z_c$.

To continue, we examine the equation for w . It is convenient to relate this to Bessel's equation. To do so we define new independent and dependent variables, $\omega = kw$ and $\Omega = \omega^n \Psi(\omega)$. Then the energy equation can be rewritten is

$$\frac{d^2\Psi}{d\omega^2} + \frac{1}{\omega} \frac{d\Psi}{d\omega} - \left(1 + \frac{\alpha^2}{\omega^2} \right) \Psi = 0 \quad (37)$$

Here $n = (Z + 5)/8$ and $\alpha^2 = [(Z - 3)/8]^2$.

This is the modified Bessel's equation, and the solutions are simply the modified Bessel function $K_\alpha(\omega)$ or $I_\alpha(\omega)$. Since $I_\alpha(\omega)$ diverges for large w , the only allowed solution is $K_\alpha(w)$.

$$\Psi(\kappa w) \propto K_{\left| \frac{Z-3}{8} \right|}(\kappa w) \quad \Omega \propto (\kappa w)^{(Z+5)/8} \Psi(\kappa w) \quad (38)$$

While $\Psi(\omega)$ diverges as ω approaches zero, $\Omega(\omega)$ does not. In fact, as w approaches zero, Ω approaches zero linearly in w . Let us say that $f_1(w)$ at $z = z_c$ just happens to be proportional t

$$f_1(w) \propto (\kappa w)^{\frac{Z+5}{8}} K_{\left| \frac{Z-3}{8} \right|}(\kappa w) \quad (39)$$

Then for the planar case, with a single Z , we have that the solution for all z between 0 and z_c is

$$f_1(w, z) \propto (\kappa w)^{\frac{Z+5}{8}} K_{\left| \frac{Z-3}{8} \right|}(\kappa w) \sinh \kappa z \quad (40)$$

For the spherical case, one only needs to solve Equation (34) numerically between $0 < z < z_c$. In either case, the coefficient of proportionality is determined so that the energy flux is as specified at $z = z_c$.

Let us now give a brief digression on integrating for the planar and spherical case. In each case, we start from $z = 0$. For the planar case, there are two solutions near $z = 0$, one is Φ equals a constant, and one has Φ increasing linearly in z . Clearly, only the one solution increasing linearly in z is physically correct. By starting the numerical integration at $z = 0$, one can be certain that one is neglecting the unphysical solution. The spherical case is rather different. Here one can show that near the origin, the two solutions are proportional to z and to z^3 . Obviously, this latter solution is unphysical and must be eliminated. It is simplest to do this by beginning the numerical integration at $z = 0$ with the physical solution. Both the planar and spherical cases have very different unphysical spurious solutions. Fortunately, both have the same correct physical solutions near the origin.

To continue, the problem is twofold. First the distribution function at $z = z_c$ is not given by Equation (39), and second, the eigenfunctions do not form a complete orthogonal set, so there is no way to uniquely decompose the value at $z = z_c$ into a unique summation over the eigenfunctions. However there does seem to be a way to do what is second best and it could be the basis of forming a good approximate solution. Say the condition at $z = z_c$ is not given by a single eigenfunction, but what if it is given by two, or three, or four? Then one can get the solution for all $0 < x < z_c$ by summing over these two, three or four eigenfunctions. But what if there is no small simple set of eigenfunctions that matches the $f_1(w)$ for all w ? But what if instead, there is a small number that match it approximately, not for all w , but only for the w 's that are most important for determining the energy flux at all w , and for other values of w go approach zero?

This is the crux of the sparse eigenfunction approach to getting an approximate solution to Equation (24). There is no guarantee that one can find such a set of sparse eigenfunctions which are a good approximation to the boundary value of the dependent variable, but for the case at hand, there does seem to be. To this author, the sparse eigenvalue method seems to be rather like asymptotic analysis. The asymptotic approximation to the actual solution is expressed as a power series which ultimately diverges. However taking the appropriate number of terms, one finds a reasonable approximation. The sparse eigenfunction method seems to be rather like this in its basic philosophy.

As we have seen, the value of the energetic energy distribution function at $z = z_c$ as a function of w , $f_1(w) \sim w^{1/4} \exp(-w^{1/2})$, where the coefficient is determined so that the energy flux is a specified. In [24], this author has given some examples of determining an appropriate set of sparse eigenfunctions which closely match this boundary condition over what seem to be the main regions of w relevant for energy deposition in the target, and which go rapidly to zero for other values of w . The result for $Z = 3$ is shown in **Figure 15**. Details are given in [25]. Meanwhile, **Figure 15** shows a plot of $f(w)$; and for $Z = 3$, shows an attempt to match it with 3 sparse eigenfunctions.

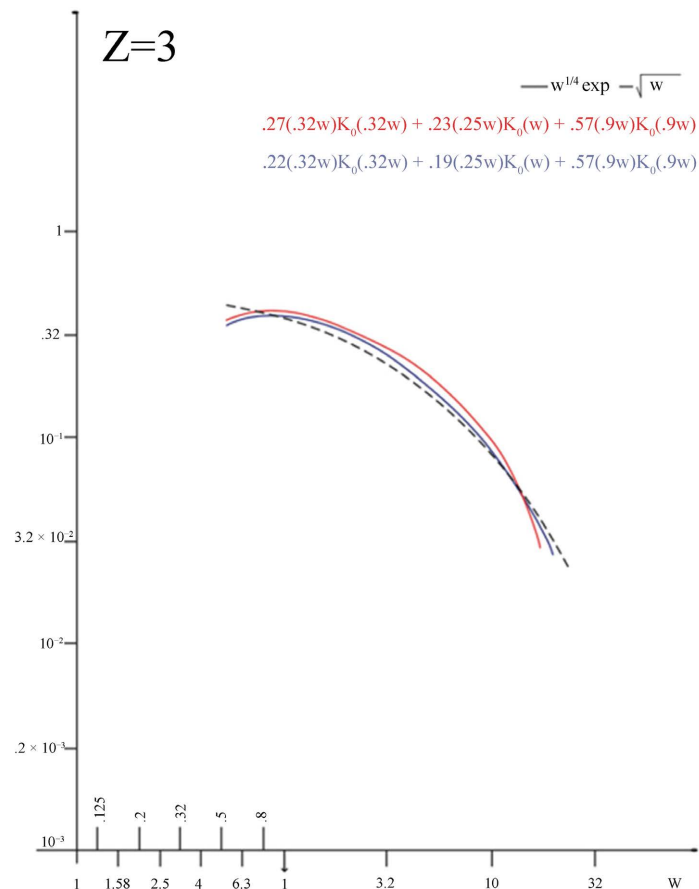


Figure 15. The distribution function $f(w)$ is dashed black, and 2 attempts to model it with a sum of 3 sparse eigenfunctions. For w much less than one, the sparse eigenfunctions go rapidly to zero, and do not match at all, but there is very little energy flux there. For much larger w 's, they again go rapidly to zero, they do not match either. However, there are very few particles at these high energies either. After all the sparse eigenfunctions decay in as $\exp(-kw)$, whereas the actual distribution function decays as $\exp(-w^{1/2})$. This shows the nearly unavoidable approximate nature of the sparse eigenfunction approach to solving the equation.

For an unsegmented target, the solution for all values of z between 0 and z_1 are given by the spatial and velocity dependences of the summation over the 3 sparse eigenfunctions like those given in Equation (40). The coefficient of each sparse eigenfunction is given so as to match the boundary condition at $z = z_c$.

The only thing remaining is the case of a segmented target with a variety of Z 's. The author examined one case of this motivated by an experiment by the URLLE/LLNL group on the NIF laser [26] and the result seemed to give reasonably close agreement with the experiment [25]. Recall that the equation for Φ has no Z dependence, so the several values of κ (3 of them in the above case) for all $0 < z < z_c$ are determined entirely by the distribution function at $z = z_c$. However, the functions of w take a jump at the interface between, say z_1 , where Z jumps from Z_1 to Z_2 . We treat this by asserting that for each sparse eigenvalue, there is no discontinuity in its energy flux as it passes from one region to another.

Hence starting from $z = 0$, one integrates the equation for Φ , with the boundary condition at $z = 0$ given by $\Phi = 0$ and $d\Phi/dz = A$, up to the next segment. Here A is a constant to be determined. At the next segment, the coefficient of the sparse eigenfunction most likely changes so as to conserve energy flux. Then one keeps going through as many segments as there are until one reaches $z = z_c$. At this point one knows what the coefficient of the sparse eigenfunction must be. Since the equation for f_1 is *linear*, the dependence of the calculated sparse eigenfunction depends *linearly* on A . At this point, it is a simple matter to solve for A for each sparse eigenfunction, so that each one at $z = z_c$ satisfies the boundary condition.

Finally, one must admit that as specified so far, the decomposition of the value of $f_1(z_c)$ is not unique. Different sparse eigenfunctions used in the decomposition, will give slightly different answers, and none will be perfect matches. However, if the sparse eigenfunction decomposition is reasonably close, to the actual f_1 in the region of w which is of interest, the approximation should be reasonably good. It is likely that the choice of sparse eigenfunctions could be improved, by for instance choosing their coefficients with a least squares fit.

7. Other High Z Sparse Eigenfunctions

The basic equations for f_1 , Equation (12) suggests another approach to the sparse eigenfunction method for solution. For high Z one can consider neglecting the second derivative term on the right-hand side. This may be relevant for the high Z plasma generated on the inner wall of the hohlraum in experiments like those done on NIF.

Of course, by neglecting this term, it is no longer possible to satisfy both boundary equations in w , the condition at $w = 0$ and the condition at w equals infinity. We will discuss this more shortly.

As we have seen, the actual solution of Equation (12) in terms of modified Bessel functions of the second kind, approaches zero linearly as w approaches zero, and approaches zero exponentially as w approaches infinity. Neglecting the second derivative term, the 2 Equation (34) and Equation (35) would be replaced with

$$\left\{ \frac{4}{1+Z} \right\} \left[\left[\frac{d^2 \Phi}{dz^2} \right] 2 \frac{d}{dz} \left(\frac{\Phi}{k(z)r(z)} \right) \right] = g^2 \Phi \quad (41)$$

$$-\frac{d\Omega}{dw} + \frac{\Omega}{w} = g^2 w \Omega \quad (42)$$

And of course, these equations and the others in Section VI are related by

$$\left\{ \frac{4}{1+Z} \right\} \kappa^2 = g^2 \quad (43)$$

assuming Z is constant.

Notice that now Z can be a function of z , since it appears only in the spatial equation and no longer in the equation for w , *i.e.* energy.

It is not difficult to see that the solution to Ω is

$$\Omega = A(gw) \exp - \frac{(gw)^2}{2} \equiv A\xi \exp - \xi^2 \quad (44)$$

Here A is an arbitrary constant used to normalize the expression in some way to be specified shortly.

Of course, we recall that the solution of the actual equation Equation (35) for constant Z is proportional to

$$\Omega = (\kappa w)^{(Z+5)/5} \Psi(\kappa w) \quad \text{where} \quad \Psi(\kappa w) \propto K_{\left|\frac{Z-3}{8}\right|}(\kappa w) \quad (45)$$

Notice that Equation (45) for Ω approaches zero linearly in w , as does Equation (44). However, as w approaches infinity, Equation (44) approaches zero as a Gaussian, while Equation (45) does so as an exponential. Hence while the two solutions may approximate each other for small w , they cannot for very large w .

To see how valid the approximations we made are, it is simple to compare Equation (44) with Equation (45) for a variety of Z s. To do so we use Equation (44), but for the ξ variable, so that in terms of this variable, there is no dependence on Z . Then expressing Equation (45) for the Bessel function solution, also in terms of ξ , we get a different result for each Z . **Figure 16** shows the result. Each expression is normalized so that each curve maximizes at a maximum value of = 43.

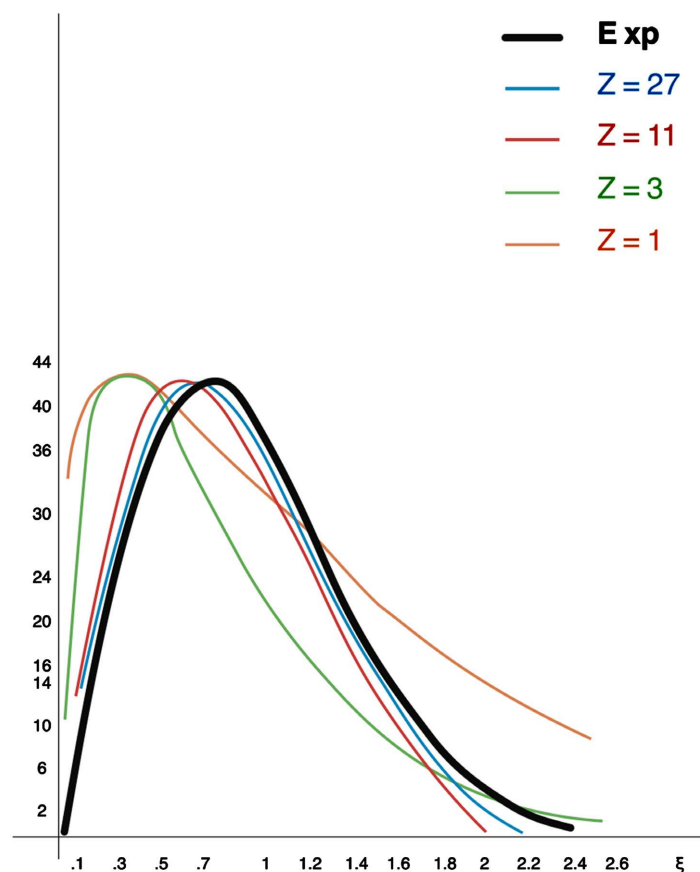


Figure 16. A plot of the exponential solution, Equation (44), heavy black line; the Bessel function solutions for $Z = 27$, blue line; $Z = 11$, red line; $Z = 3$, green line, and $Z = 1$, orange line. Clearly the exponential approximation is very good for $Z = 11$, and 27. Perhaps it is barely passable for $Z = 3$, and is quite far off for $Z = 1$.

Continuing, we next examine whether the $f_1(w, z_c)$ can be reasonably approximated with a small set of the new sparse eigenfunctions. Recall that

$$f_1(w, z_c) = Aw^{1/4} \exp - w^{1/2} \quad (46)$$

Here A is determined by the energetic electron energy flux. Taking $A = 1$, **Figure 17** is a plot of f_1 , the heavy black line. Also shown, as the heavy red line is a plot of the sum of 3 sparse eigenfunctions, one maximizing at $w = 2$, another a $w = 8$ and a third at $w = 20$. Between $w = 2$ and 30, the match is reasonably close. Between $0 < w < 2$, and $w > 30$, it is poor. However, the particles with $w < 2$ have very little of the energy flux; and for $w > 30$, there are very few particles, and the sparse eigenfunctions rapidly go to zero in these regions.

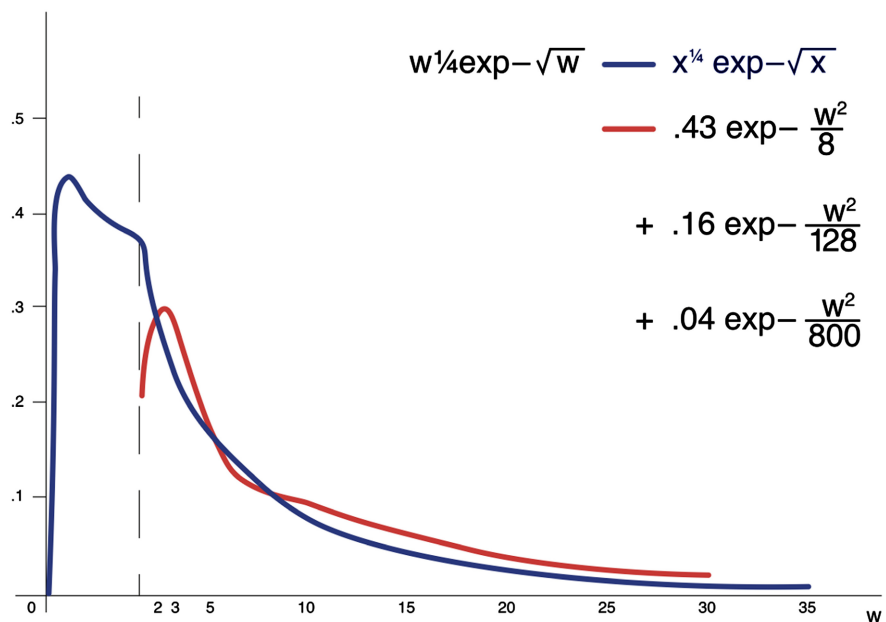


Figure 17. An approximate match of $f_1(w, z_c)$ with 3 sparse eigenfunctions.

Hence it seems that for large Z s the one can find a reasonable approximation for the chosen value of $f_1(w, z_c)$.

Notice that these new sparse eigenfunctions allow Z to be a function of radius (*i.e.* z). For instance, for the planar case, one can easily integrate the spatial equation, Equation (41) from $z = 0$ to $z = z_c$. If a WKB approximation is valid, one simply has a hyperbolic sign for each sparse eigenfunction

$$\Phi(z) = \int_0^z dz' \sinh \mathcal{G} \left(\frac{1 + Z(z')}{4} \right)^{1/2} \quad (47)$$

Finally, one must normalize the above function so that $F(z_c)$ has the proper value at $z = z_c$ for each sparse eigenfunctions.

It is interesting that [21] also considered high Z effects. In fact, in their initial simple example, they chose a $Z = 100$. Well, who knows, maybe somebody will someday produce a fully ionized Fermium plasma. However, they also considered

what they thought of as a plasma produced on the wall of a hohlraum filled with a helium gas. They envisioned this plasma produced as having a spatial variation of Z going from 2 to 36. Possibly theory developed in this section could be helpful to them if there are energetic electrons involved.

8. Fuel Preheat from Energetic Electrons in the Tail of the Maxwellian

Laser plasma instabilities (LPI's) are hardly the only source of energetic electrons which may preheat the fuel part of the fusion target. There is also the tail of the Maxwellian to consider. Depending on plasma temperature and density, these energetic electrons may only be weakly coupled to the local background plasma and may travel a considerable distance before they are thermalized. This has hardly escaped the attention of the laser fusion community. In fact, as far as this author can discern, there are many more studies and attempts to describe this phenomenon than there are attempts to study the effects of LPI's on fusion gain.

Most of these studies attempt to develop an approach to the entire electron distribution function which describes both the thermal and energetic electrons. As with the case of the laser plasma instability, this work takes a different approach. The main assumption is that there are only a small number of electrons that are decoupled. They do not interact with each other but only interact with the background electrons and ions. Furthermore, as in the case of the LPI's, we assume that they thermalize in a time scale virtually instantaneous as compared to the fluid motion. Hence, we solve the Fokker Planck equation for these energetic electrons only in space and assume the background has no time dependence.

This brings up two immediate dilemmas which do not occur in the LPI case. First, where does one draw the line between the thermal and energetic electrons, and second, since it is a spatial problem, where is the spatial boundary? For the instability cases considered, the spatial boundary is obviously at the position of the quarter critical density, and the energy boundary is simply considering separately the thermal (~ 2.2 keV) plasma, and the instability generated plasma (~ 50 keV). For the case of the decoupled Maxwellian tail, these conditions are clearly somewhat more subjective.

However, we begin a description of the transport properties of a plasma and how they can be calculated. The gold standard in describing the collisional and transport properties of a plasma is the work of Braginskii [44]. This work gives the functional dependence of every transport coefficient in terms of the plasma properties. Also, it gives what are generally considered to be the most accurate numerical coefficients which multiply these functional dependences to get accurate local transport coefficients.

However, there is a shortcut which has been frequently used, certainly by this author and many others. Using a Krook model, one gets the correct parametric dependences of the transport coefficients, but not the correct coefficients. Hence it is very tempting to use the Krook model to derive the nonlocal transport but

then correct the coefficients so that they agree with the Braghinskii calculation.

The reason to do this, is that it is relatively simple to derive a nonlocal theory for the entire plasma using a Krook model, but to this author's knowledge, there is no relatively simple way to do an analogous nonlocal extension of the Braghinskii calculation. Once one has the nonlocal theory from the Krook analysis, the spatial boundary conditions, and the dividing energy between those electrons that deposit their energy locally, and those that deposit nonlocally; it is a relatively simple matter to switch the nonlocal part to a Fokker Planck analysis.

Hence let us consider a Krook model for the distribution function. Take the simplest steady state expression of the Krook model for the f_o and f_1 (1).

$$\begin{aligned}\frac{V}{3} \frac{df_1}{dx} &= -\nu_e (f_o - f_m) \\ V \frac{df_o}{dx} &= -\nu_i f_1\end{aligned}\quad (48)$$

In Equation (48) we neglect the effect of the electric field on the motion of the energetic electrons, for the same reason as we did in the LPI case. Namely the electric potential across the plasma is roughly the maximum temperature, while the energy of the relevant electrons is considerably greater. Of course, this assumption is more difficult to justify than for the case of LPI generated electrons. However, we will see shortly that this may not be as serious an issue as one might expect.

As in Section (3), these can be combined to form a single second order equation in x equation for f_1 . Since there are no velocity derivatives in Eqs (48), one can multiply this second order equation by $(1/2)mv^3$ and one has a second order equation for $q(x, \Xi)$ the electron energy flux at each energy Ξ

$$-\frac{1}{k} \frac{d}{dx} \frac{1}{k} \frac{dq(x, \Xi)}{dx} + q(x, \Xi) = q_L(x, \Xi) \quad (49)$$

$$q_L(x, \Xi) = -\frac{K_{sp}}{\Lambda\beta(\infty, Z)} \frac{\Xi^4 \left[\left(\frac{\Xi}{T} - \frac{Z_4}{Z_2} \right) \frac{1}{T} \frac{dT}{dx} + \delta\epsilon \right]}{T^{3/2} P\left(\frac{\Xi}{T}, Z\right)} \exp\left(-\frac{\Xi}{T}\right) \quad (50)$$

The quantity q_L is the local expression for the energy flux as generated by the various gradients using a Krook model but renormalized so that the integrated result gives the classic value for electron thermal transport [9] [14] [23]. The quantity k is closely related to the reciprocal of the energy dependent electron mean free path, and K_{sp} is the electron thermal conductivity as derived by Braghinskii:

$$k = \frac{1}{V} (3\nu_e [\nu_e + \nu_i])^{1/2} = \frac{1.15 \times 10^{-13} n \Lambda [1+Z]^{1/2}}{\Xi^2} \quad (51)$$

$$K_{sp} = \frac{9.7\gamma \times 10^8}{Z} \frac{\text{ergs}}{\text{cm} \cdot \text{sec} \cdot \text{eV}^{7/2}} \quad \gamma = 13.3 \frac{Z+0.23}{Z+4.24} \quad (52)$$

The quantity $\delta\epsilon$ is the perturbation to the electric field times the electron charge, generated because some of the energetic electrons do not deposit their energy lo-

cally; it is calculated in [9] [14] [23]. Notice that the k in this section depends on electron energy, and it is not the same k as in section III which depended only on the energetic electron temperature. The functions β and P and the Z_a 's are various normalizing factors, and velocity integrals, details are in [9] [14] [23].

Clearly for low energy electrons, the mean free path is short, and k is large. Hence for these electrons $q \approx q_L$. For higher energy electrons, the second derivative term, *i.e.* nonlocal effects, becomes important. If one does a purely Krook calculation, one simply solves Equation (49) in x for all Ξ integrates over Ξ , and uses this expression for thermal electron energy flux in the rad hydro fluid equation. A great deal of our earlier work did just this [9]-[14].

However, as we have discussed, the Krook approximation greatly overestimates the preheat in a laser fusion direct drive configuration. Our aim is to use a (renormalized) classical approach for the low energy particles, and a Fokker Planck approach for the few high energy particles on the tail of the Maxwellian.

The next issue is how to separate the two parts of the distribution function. In our earlier work [9]-[12], we attempted to define this energy in terms of local gradients. However, on running the rad hydro code, the gradients were bouncing around a great deal and that approach proved not to be viable. We found that a much better approach proved to be using integrals instead of differentiating.

Our definition is now that the critical energy differentiating the low and high energy parts of the distribution function, Ξ_m is calculated by the relation in Equation (53)

$$\int_{r\left(\frac{T_e}{2}\right)}^{r(T_e)} dr k(r, \Xi_m) = 1 \quad (53)$$

Here T_e is the maximum temperature in the spatial temperature profile at the time of the calculation, and $r(T_e)$ is the position of this maximum. Then $r(T_e/2)$ is the position, going to a higher density, where the temperature is half the maximum. That is that in one mean free path, the critical electron moves from the maximum to half the maximum temperature. Higher energy electrons, of course go even further.

The next issue is to determine the spatial boundary, that is the spatial position where the energetic electrons begin and what their distribution function is there.

We choose the position of origin of the energetic electrons as the position where the Krook calculation of the energy flux maximizes, defined as $r_{\max}$; and we choose for the distribution of energetic electrons at that position, the local (renormalized) Krook calculated distribution function for f_i .

This initializes the distribution function f_i in just the same way as it was initialized for the laser plasma instability production of energetic electrons. The Fokker Planck calculation is then done in the same way. There are some differences regarding the definition of w and z . Here

$$dz = -\sqrt{3} \frac{2.7 \times 10^{-13} n(x) \Lambda}{T_e^2} \quad (54)$$

Also $w = [\Xi/T_e]^2$.

Let us look at a typical profile for which we applied this theory, which is displayed in **Figure 18**.

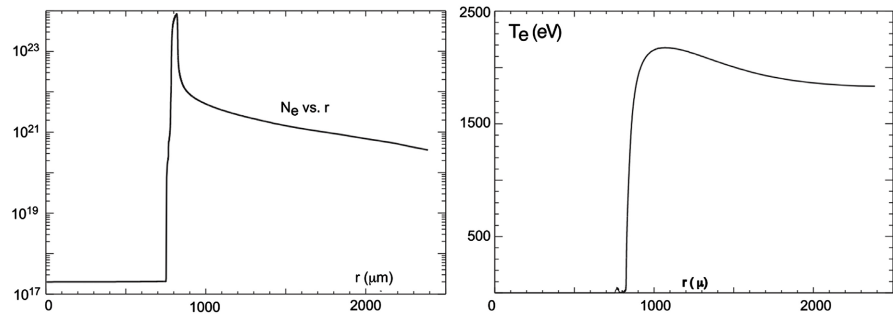


Figure 18. The spatial temperature and electron density in cm^{-3} profiles for a typical optimized NRL calculation of a laser fusion target implosion at a time of 15 ns into the implosion. The maximum temperature T_e is about 2.3 keV.

For the profiles above, Ξ_m was about 10 keV and the maximum Ξ_m , the energy at which there is no significant energy flux is about 20 keV. Correspondingly, the range of w is from about 100 to about 400. The maximum temperature is located at about 1000 μm , the position where the temperature drops to about half its value is at about 900 μm , and r_{max} is at about 950 μm .

While the approximations we have made may be only marginally satisfied near the position of maximum Krook energy flux (*i.e.* z of approximately 900 μm), moving inward slightly, the conditions are well satisfied, since the temperature profile has such a sharp drop in temperature as one moves inward from the maximum.

Solving the Fokker Planck Equation for the distribution function for $0 < r < r_{\text{max}}$ and $\Xi_m < \Xi$, gives f_1 in that region. Integrating over Ξ from Ξ_m to infinity, one determines the nonlocal component of the electron thermal energy flux as a function of x . In other words,

$$q(x) = q_L(x, \Xi < \Xi_m) + \int_{\Xi_m}^{\infty} d\Xi q(x, \Xi) \quad (55)$$

Here $q_L(x, \Xi < \Xi_m)$ is the integral of the local energy flux from an energy of zero to a maximum of Ξ_m . In terms of the normalizing factors of Equation (50) and Equation (52) it is

$$q_L(x, \Xi < \Xi_m) = -\frac{K_{sp} T^{\frac{5}{2}}}{\Lambda} \frac{dT}{dx} \frac{\beta(\Xi_m/T, Z)}{\beta(\infty, Z)} \quad (56)$$

The ratio of the two betas in Equation (56) is the result of taking the velocity integral to the maximum local energy Ξ_m divided by taking it to infinity, as in the Krook Braghinskii theory. Not only does Equation (55) show that there is an additional energy flux caused by the nonlocal deposition of some of the more energetic electrons, there is also a reduction of flux given by the fact that the integral over the local q_L no longer goes from zero to infinity, but goes from zero to Ξ_m .

This is the way this current theory describes flux limitation.

To summarize, first for the density and temperature profiles at a particular time, define the energy separating the thermal from the energetic particles using Equation (53). Then define the position separating the thermal region from the region where energetic particles are decoupled as the position where the Krook/Braghinskii expression for the energy flux maximizes. To the right of this point (the origin is to the left), use the classical Krook/Braghinskii expression for the electron thermal energy flux. At the separation point in space, set up the Fokker Planck equation using the Krook/Braghinskii expression for the f_i as an initial condition and solve the Fokker Planck equation for energies above Ξ_m and to the left of $r_{\max}$. Then integrate over Ξ to determine the nonlocal energy flux to positions to the left of $r_{\max}$. Add to the nonlocal energy flux, the local energy flux, Equation (56). Much of this region is at a much lower temperature, so integrating to Ξ_m is basically the same as integrating to infinity.

Let us continue by imagining what the distribution function looks like at for instance $r = 500 \mu\text{m}$. First there is the central low temperature region, with energy of, let's say a few electron volts. Then there is a much higher energy component, with energy between $\sim 16 \text{ keV}$ (*i.e.* Ξ_m) and 40 keV . A cartoon of the image of such a distribution function is shown in **Figure 19**:

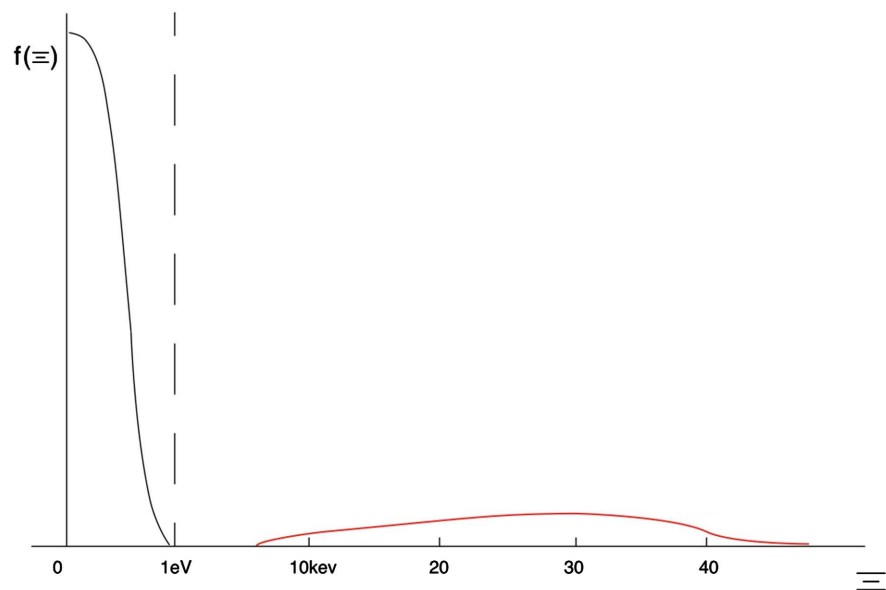


Figure 19. A rough cartoon of what the electron distribution function looks like at $\sim 500 \mu\text{m}$ for the profiles of **Figure 18**. Notice the change in scale on the horizontal axis at $\sim 1 \text{ eV}$. Surely no localized perturbation theory at around $500 \mu\text{m}$ could have predicted this.

Notice the general shape of the distribution function. It ought to be obvious that doing some sort of localized perturbation theory, as several researchers have attempted, it would never result in such a distribution. In fact, the high energy halo in **Figure 19** does not arise from any localized process, but it originated at $\sim 950 \mu\text{m}$ and propagated to $500 \mu\text{m}$.

To solve the Fokker Planck equation with a sparse eigenfunction technique, we must first see if we can get a set of sparse eigenfunctions that approximate the Krook calculation of the distribution function, f_1 , at $r_{\max}$. This was discussed in Refs [24] [25]. **Figure 20** is plot of f_1 at $r_{\max}$ as a function of w , and on the same plot, is a function of an approximation to it by a single sparse eigenfunction.

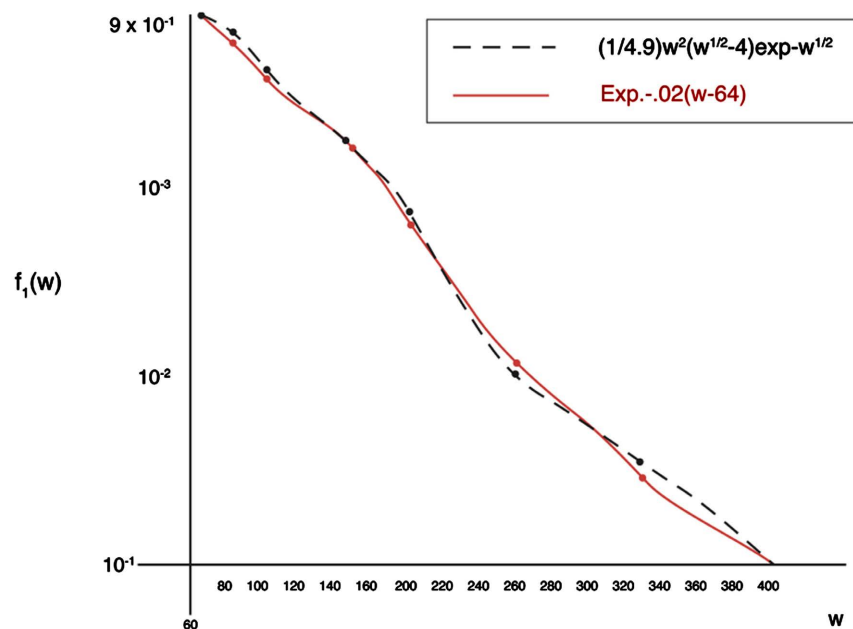


Figure 20. A plot of f_1 at $r_{\max}$ for w between 60 and 400 for the density and temperature profile shown in **Figure 18**. (dashed black curve), as well as an approximation to it with a single sparse eigenfunction (red curve) The dashed curve shows the functional form (in w) of the Krook distribution function f_1 .

With this information, we can solve for the total electron thermal energy flux in three ways, first a pure Krook calculation, second a Fokker Planck calculation using the characteristic method, and third, the same calculation using sparse eigenfunctions. One example of the results of such a calculation, for the density and temperature profile, but for $Z = 1$, from [24], are shown in **Figure 21**.

Notice that the pure Krook calculation, basically the Schurtz model [1], predicts four orders of magnitude more preheat than do either of the Fokker Planck calculations. No wonder the approaches by many different researchers using basically the Schurtz model have included a variety of methods to reduce the strong preheat that it predicts. Not only that, but our two methods of doing a Fokker Planck calculation, characteristics and sparse eigenfunction, give reasonably similar results. Furthermore, they give results much more optimistic for the future of laser fusion.

While it is difficult to claim that our model can accurately predict the preheat, the author does claim, hopefully with humility, that it is a very significant advance in the theory of target preheat by decoupled energetic electrons, either from an instability or from the tail of the Maxwellian. He makes no claim that the calcula-

tion gives an exact value, but it does seem to be reasonable to claim that the methods give a decent approximation. The author hopes he is being neither overly modest, nor overly immodest.

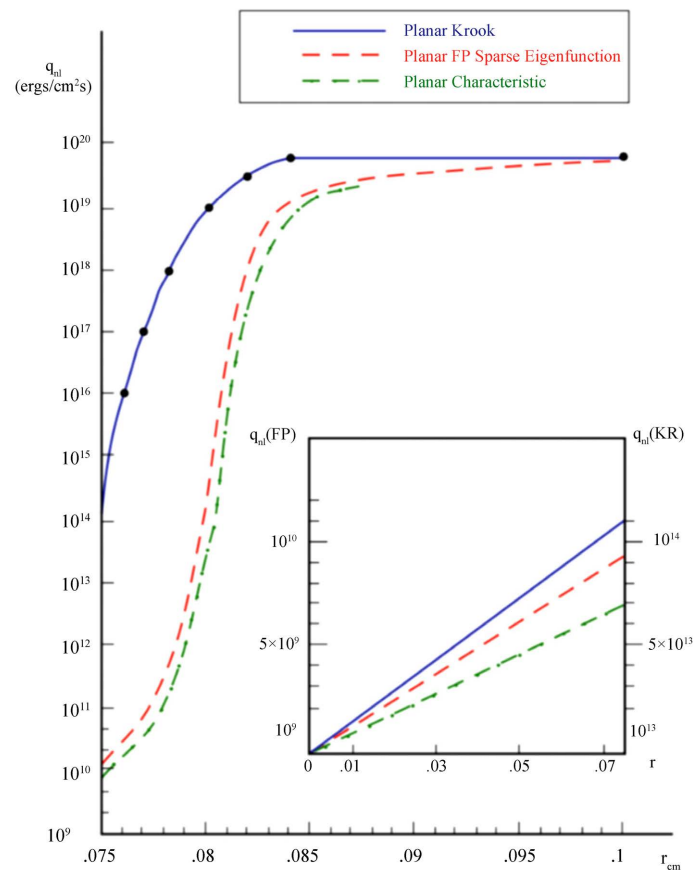


Figure 21. A calculation of the results for the thermal energy flux for the hot electrons on the tail of the Maxwellian distribution, mostly from the hottest part of the plasma around $r \sim 1000 \mu\text{m}$. The figure shows 3 different methods of calculation, pure Krook, FP by sparse eigenfunctions, and FP by characteristics.

9. Some Digressions

The digressions included here include pointing out some other aspects of the problem which we have considered, but have not discussed here, some other things to suggest which would be rather easy to do, and a few more suggestions that would not be so easy to do but may be important. They are separated into subsections denoted with capital letters.

A: The Coulomb logarithm, the Legendre expansion

In [23] was a discussion of the coulomb Logarithm, Λ . This is usually defined as the logarithm of the ratio between the distance of closest approach of the colliding particle, and the maximum distance, where the particles can interact. The velocity used in determining the distance of closest approach is usually taken as the electron thermal velocity, and the distance of closest approach is usually taken as the deBroglie wavelength for this particle, and the maximum distance usually

taken as the Debye length, the distance over which individual charges are shielded. But this is only reasonable for a Maxwellian plasma. The plasma over a large part of its orbit, **Figure 19**, is like a two component plasma; not only that, but it is the development of the high energy component that we are interested in. Hence a more reasonable approximation is to use the closest approach for the typical particle in the energetic part, not the thermal part of the distribution. Its deBroglie wave length is much shorter than that for the low temperature plasma. Hence Λ for these energetic particles should be much larger than the local Λ for the cooler plasma. This is discussed further in [23]. One example is the calculation of the Λ for energetic electrons of the density profile in **Figure 18** (*i.e.* energy of order the maximum temperature). This is shown in **Figure 22**.

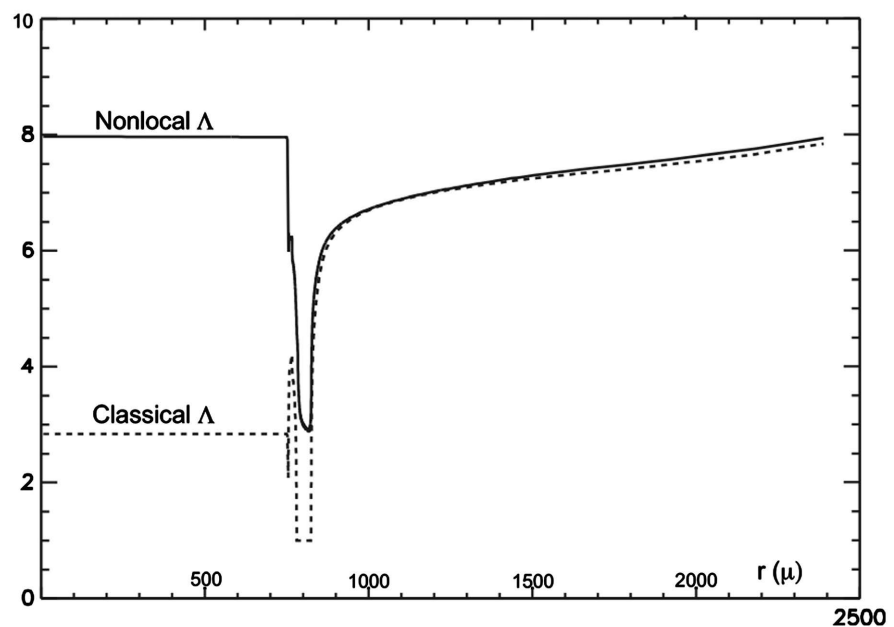


Figure 22. A comparison of the nonlocally calculated Λ and the classical result, as calculated in [23] for the density and temperature profile shown in **Figure 18**.

[25] also discussed the effect of truncating the Legendre approximation with only 2 terms. What it found (by making several approximations) was that if one used the sparse eigenfunctions to calculate the solution of the Fokker Planck equation, the addition of the f_2 term had the effect of reducing the κ 's. In other words, the energetic electrons penetrate further into the cold plasma than the 2 term Legendre expansion predicted. However, this reduction in κ was strongly dependent on Z . For all Z , the 2 term estimates are qualitatively correct but are much more accurate at higher Z . However, for $Z = 1$, there were order unity corrections; but for Z as small as $Z = 3$, the corrections were much smaller. Hence for Z equal to or greater than three, the two-term expansion should be very accurate. However, for $Z = 1$, it may become necessary to use higher order expansions if more than qualitative accuracy is required.

B. Try to incorporate this into a rad hydro code

One thing about the characteristic solution to the Fokker Planck equation, for either an instability or the decoupled tail of the Maxwellian, is that it gives a very simple formula for the preheat, Eqs (20) (21). This is just an added heating term to insert in the energy equation. The author is almost certain that it would be rather simple to incorporate in a rad hydro code. While the sparse eigenfunction approach would be more complicated to program than just adding a relatively simple formula, it would not be that complicated either. With both coded up, there would be two independent calculations, and hopefully they would not give very different results, as was the case in **Figure 21**.

At NRL, we had planned to do this, but between the pandemic, Denis Colombant's health issues, Andy Schmitt's retirement, and the termination of the NRL program, we never were able to do so. It seems to me that it would be quite easy to do for LLNL, URLLE, or even smaller some university programs that have access to rad hydro codes which they can tinker with. One important question which the author feels it could give a preliminary answer to is:

Does the laser fusion configuration really have to operate below LPI instability thresholds?

We have seen that the processes studied give much less preheat than the earlier Krook models. Maybe the laser fusion configuration can give increased gain if it is above, but not too far above the instability thresholds. Probably the best instability to initially investigate this the absolute Raman side scatter, because at least from what we know so far, there is little doubt that the temperature of the energetic electrons will be ~ 50 keV. From the LLNL/URLLE results, *i.e.* **Figure 6**, we have data on what the energetic energy flux for the energetic electrons will be at two point above threshold. We know that at threshold there should be no instability, so we have already 3 points on the curve of energy flux versus ratio of irradiance above threshold value (*i.e.* a curve also like **Figure 5**), and presumably one could interpolate to get a reasonable approximation at other point.

Using this one can imagine first doing a series of runs of implosions without modeling the energetic electron energy deposition. Presumably, these initial calculations will show higher gain for higher irradiance, and one can plot the curve of gain vs irradiance. Then redo the calculations, including the formulas of the energetic electron energy flux listed in Section (2). Presumably the gain will be reduced. However it may allow the region of reasonable gain to extend to higher irradiances, and these may have higher gain. This is demonstrated schematically by the simple graph in **Figure 23**.

C. Spherical and Braghinskii

Virtually all the work we have done so far on this project in the NRL program has been for planar plasmas. For the case of solution by sparse eigenfunctions, this means that that the spatial solution for the sparse eigenfunctions are simply hyperbolic signs. Since for the sparse eigenfunctions, the spatial and velocity space solutions are handled separately, in either case, the sparse eigenfunction are the determined by the distribution function at z_0 , and are the same whether the con-

figuration is planar or spherical. Thus the κ 's are determined only at z_c and are the same whether the plasma is spherical or planar. Hence if the plasma is spherical, one simply solves Equation (34) for $\Phi(z)$, but with the spherical terms included. In the NRL program, we only did this for a single case. It was for a plasma where the energetic electrons were formed by a $2\omega_p$ instability. The temperature of the hot electrons was taken as 35 keV. **Figure 24** shows the comparison of the spherical and planar results.

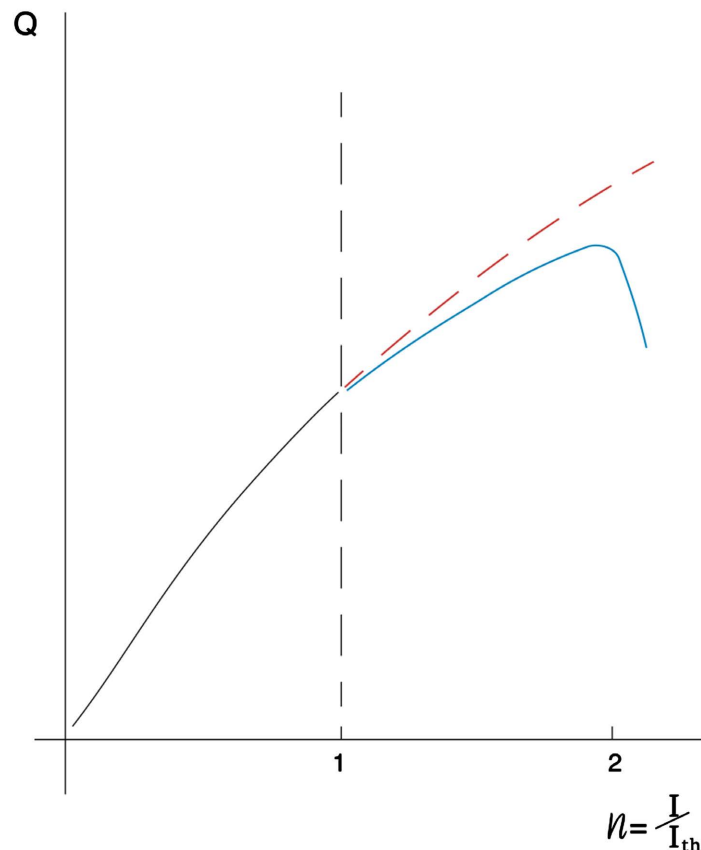


Figure 23. Two possible curves, from rad hydro simulations of the fusion gain (schematically) vs the ratio of laser irradiance to threshold irradiance (for instance) for the absolute Raman side scatter instability. At the threshold irradiance there is a break. The dashed red curve shows the gain without considering the instability. The blue curve shows the possible effect of the instability, calculated as described here. Notice that the blue curve is below the red curve, but nevertheless, the gain maximizes in the region of instability.

The results were not very different, at least for this single case considered, but the fuel heating was smaller by about a factor of 2 for the spherical case. We called this a centrifugal barrier, analogous to the centrifugal barriers in atomic physics. This is certainly a positive result for laser fusion, assuming it persists as more cases are studied.

Regarding the locally decoupled tail of the Maxwellian, we first took a Krook model for the collisions and calculated the functional dependence of the local

transport. The next step was to replace coefficients with more accurate ones as calculated by Braghinskii [44] (the functional forms are the same in Krook and Braghinskii, but the coefficients are different). Then we separated the distribution function into local and nonlocal components both the configuration and energy space. The local components gave a reduced thermal flux; it is the way the theory models flux limitation. Then it used a Fokker Planck theory to calculate the non-local behavior of the more energetic electrons but neglected both the electric field and energy diffusion for these electrons.

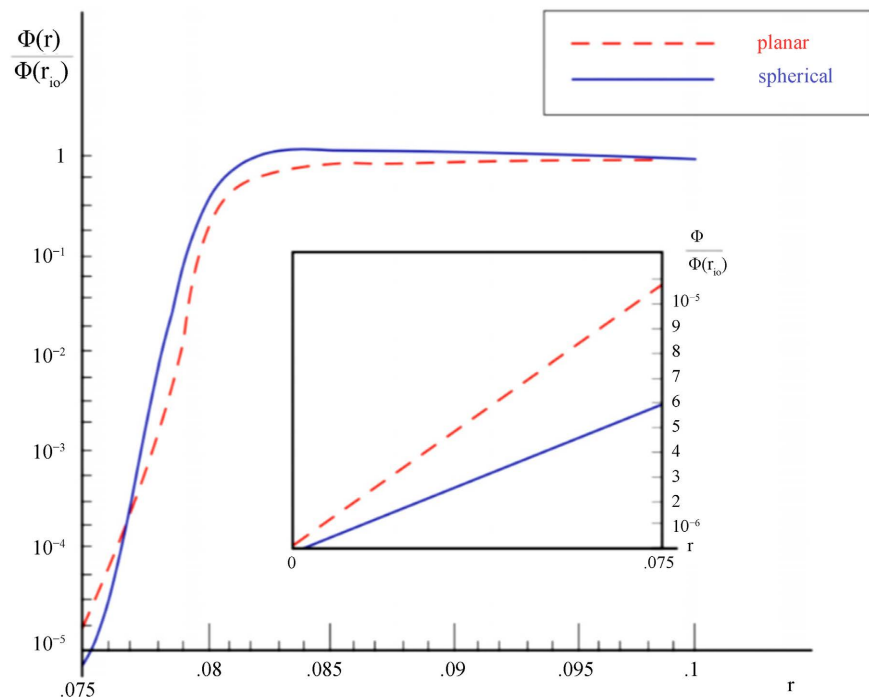


Figure 24. A comparison of the relative energetic electron heat fluxes for the a density profile like that given in **Figure 18**, for the spherical and planar case. The assumed instability is the $2\omega_p$ instability and the temperature (35keV) and fluxes are given in **Figure 5**.

It would have been better had we neglected the first step, and calculated the local distribution function initially as Braghinskii did. Also, it would have been better if had we calculated the effect of the electric field and electron energy diffusion in the nonlocal part, especially near the energy and spatial separation points. These effects would only be important in the high temperature region near the spatial point, and the energy point, where the nonlocal electrons formed. Clearly these are problems for future scientists who decide to study this method for treating nonlocal transport for energetic electrons in the tail of the Maxwellian.

D. A possible FP or PIC code solution of the plasma just beyond the quarter critical density

In Sec (2) we showed that by both experiment and PIC simulations, the absolute Raman side scatter instability generated electrons leave the quarter critical surface and move inward, with their motion confined to about a 45° angle with the normal

to the density gradient. However, this was not the distribution we assumed as a spatial boundary condition for our calculation of the Fokker Planck solution for the deposition of the energetic electrons. Instead, we assumed a drifted Maxwellian, or more accurately assumed that the electron distribution was averaged over a constant energy surface as much as it could. However, the distribution function could not become isotropic. The energy flux had to be conserved and as the collisions of the energetic electrons with the background plasma, both with background ions and electrons, were mostly energy conserving. Hence just within quarter critical surface, the energetic electron density will build up, while conserving, or nearly conserving the total energetic energy flux to just what was produced by the instability.

This seems like a problem that could be further investigated by particle simulations, either PIC or a numerical solution of the Fokker Planck equation. Here we concentrate on the PIC simulation. The configuration would be one dimensional in space, a planar configuration would likely be a reasonable approximation, but three dimensions in velocity space. Within this region, there would be a background plasma with low temperature electrons and immobile ions with a particular Z . At the left-hand boundary, which might be, perhaps, at say $0.25 n_c$, there would be an inward flux of energetic electrons, Maxwellian in energy at a temperature of 50 keV (for instance), and with a 45-degree cone angular distribution around the perpendicular to the gradient like that shown in **Figure 8**. As an energetic electron enters from the left, another energetic electron would exit this boundary, to maintain neutrality. These energetic electrons would move through the computational region and collide with the background electrons and ions. The right-hand boundary might have a background density of about $0.3 n_c$. When the energetic electrons leave on the right, they are replaced with an electron entering on the right, but would have a distribution with a temperature of the assumed temperature of the energetic electrons.

However, these collisions could not be binary. When the electron scatters by 90 degrees, it does not do so by single binary collisions. If it did, they would rapidly heat the electrons in the background plasma. Instead, it is described by many small angle scatterings which conserve energy. An earlier work of the author and his colleagues showed how to model this in a PIC simulation of electrons [45], and the method there could be extended to the problem at hand.

To this author, it seems that one could formulate the problem in this way as an initial value problem and run the code until a steady state is reached. The calculated distribution on the right-hand side, would be the boundary condition to impose on the Fokker Planck equation. It would be interesting to see what this steady state distribution function would look like as a function of Z . For high Z , where the dominant energy conserving collision are very large compared to the frictional force, the drifted Maxwellian distribution would probably be very good. However, when the Z gets to be 1, the energy conserving collisions would only be double the collisions causing friction, and there might be considerable reduction of the ener-

getic flux, balanced by heating in the localized region.

E. Monte Carlo simulations of the electron orbits can play a role.

While the author does not see such Monte Carlo orbit calculations, as shown in **Figure 10**, as playing a role as a subroutine called at each time step of a rad hydro code, that does not mean it cannot play some role. For instance, to check on the Fokker Planck simulations (which are approximate) at, let's say, the time of maximum incident energetic electron heat flux, one might use a Monte Carlo orbit simulation. Surely the simulation would need many thousands of electrons to sufficiently populate the energy and angular distribution and account for the spherical symmetry. The author has no experience with these codes and does not know whether they are equipped to calculate density from many orbits. If the codes can do this, fine; if they cannot, it almost certainly would be possible to incorporate such a capability. Or else one could simply calculate the energy lost by the particle as while it is in the target region. Using these codes would also be a check, on just a single time step, on the validity of the Fokker Planck, which is used every time step.

F. Let's reconsider fusion breeding.

Since the fusion reaction produces only a single neutron, how can it sustain a fusion economy where some neutrons will be lost, and some tritium produced will be lost or decay (it has a half-life of 12 years)? Hence if tritium can only be generated by a fusion neutron, and nothing else, the tritium which the project started out with will itself have a half-life, and a fusion economy will be impossible to achieve.

Fortunately, there are other sources. Since the fusion neutron has a high energy, 14 MeV, as opposed to ~2 MeV from fission reactions, this energetic neutron can produce other neutrons by spallation if the target is a neutron multiplier like beryllium, lead, ^{238}U ... Also with a small energy price, ^7Li can also produce tritium, while preserving the neutron. Fusion designers realize this and plan to place these elements in the blankets so that each fusion neutron produces let's say 1.5 tritons [30] [46] [47] after the various losses. This would allow the number of fusion reactions can grow exponentially in time.

However, this author believes that it is possible to do better, much better. First, once we have all the fusion reactors we need, what will we do with the extra half triton, just throw it away? What a terrible, wasteful idea, especially where mined fissile material (^{235}U) may well be in short supply in coming years. Why not just use it to breed fissile material from fertile material, say ^{233}U from thorium? The author has studied and advocated this over decades [30] [48]. Fission breeders cannot make up the deficit, it takes 2 fission breeders at maximum breeding rate to fuel a single thermal fission reactor (like the light water reactor) of equal power. Uranium from the seas may or may not be able to make up the slack, it is hardly a panacea [49] [50]. But a single fusion breeder can fuel 5 or 10 thermal reactors! The reason is that fission is energy rich and neutron poor, while fusion is neutron rich and energy poor, a perfect match [30] [48].

To this author it seems clear that there are basically two and only two possibly sustainable ways to make up for the lack of mined uranium, *i.e.* uranium from the seas, and fusion breeding. To him, it is rather amazing that many regard the former as a guaranteed source of ^{235}U , just a bit pricier than mined uranium, but they don't consider the orders and orders of magnitude that the process must be increased to be a truly sustainable source. So far only about 10 grams of ^{235}U have been produced from the seas [50], in a process that cost millions of dollars and took over a year to do.

Here is a portion of the Google AI summary of Uranium from the seas as of June 1026.

Only **kilogram-scale quantities** of uranium (*i.e.* ~ten gram scale quantities of ^{235}U) have been recovered from the oceans to date. Because uranium is extremely dilute in seawater (about 3 parts per billion), recovery operations have strictly been conducted as **laboratory trials, marine field tests, and pilot demonstrations** rather than full-scale commercial mining.

A single normal 3 GWth thermal reactor burns about a ton of ^{235}U every year. Uranium from the seas would have to be increased by 5 orders of magnitude just to fuel a single reactor. To fuel a large nation's economy, it would take an increase of more like 7 or 8 orders of magnitude; and to support the world economy, 8 or 9.

Forty or more years ago, preliminary calculations were done on potential blankets, indicating that these could be designed to breed both tritium as required, and ^{233}U [51]-[53]. While hardly definitive, they gave results for fusion breeding which were quite optimistic. For instance, in an infinite homogeneous mixture of beryllium with 5% lithium 6, the result was 2.7 tritons from each 14 MeV neutron. If the mixture was with 5% thorium instead, each neutron would produce 2.66 nuclei of uranium 233 [51]. Lead, instead of beryllium was less effective, but was also a decent producer, and ^{238}U was considerably more effective than beryllium, but of courses that involved producing plutonium, which this author attempted to minimize in his work on breeding. Of course these are ideal circumstance, but there is a great amount of room to degrade these results and still have a viable process. After losses, we need only one triton from each fusion neutron for a pure fusion economy once all reactors are set up. We need only 1.3 - 1.5 tritons, after losses, from each fusion neutron to have an exponential increase in setting up a fusion economy. Also, we need only one triton and 0.5 - 0.7 ^{233}U nuclei from each fusion neutron to have viable fusion breeding. After all, each ^{233}U when it is burned gives ~200 MeV, at least ten times more than the fusion reaction delivers.

To this author's knowledge, these calculations have not been followed up, certainly not by groups eager to show that there is something there. Over nearly the entire life of the fusion project, fusion breeding has been the ugly duckling, anybody who advocated it, would promptly get his support pulled. This author has written several scientific papers on it, *none* of which acknowledged a sponsor. It is time to reverse this and let fusion breeding become the beautiful swan, *which it is* [30]! It is time to redo these calculations, with the psychology that they will

succeed, not with the psychology that they are there to eliminate fusion breeding. The researchers should not quit and loudly announce failure, because the first attempt does not produce quite enough because of a diagnostic port somewhere on the blanket. Certainly, the simplest calculations [51]-[53], show that a 14 MeV neutron impinging on the proper material can produce more than enough tritons and ^{233}U .

This author sees fusion breeding as a much more likely sustainable source. Well, who knows which is right? But since there are two and only two ways of producing sustainable fissile uranium, neither should be regarded as a certainty; but definitely neither should one of the two methods be denigrated and ignored either.

10. Conclusions

In this section, the author briefly discusses his optimism regarding laser fusion. As stated in the introduction, he sees three tall poles supporting this optimism. The tallest, of course is the LLNL experiments on getting $Q \sim 2.5$ with its NIF laser with light energy of ~ 2 MJ. This was accomplished with an indirect drive configuration, where the laser produced first X-rays, and these imploded the target. This has an advantage in that it borrows as much as possible from the nuclear weapons knowledge. It has several disadvantages in that only a small fraction of the laser light energy, $\sim 10 - 15\%$ is used to implode the target. Not only that, but it also brings the relatively large heavy metal hohlraum into the picture. If one is interested in energy rather than nuclear simulation, one not only needs it for each shot, but one must also get rid of it before the next shot.

The second tall pole is the URLLE work. The alternative is to do away with the hohlraum and have the full power of the laser focused on each target. One would like the laser to have as short a wavelength as possible; most of the work has been done with frequency tripled light with a wavelength of 335 nm. The question, of course, is whether ultraviolet light produced the required implosion as well as X-Rays. Many people are skeptical [32]. However, URLLE did a wonderful series of scaled experiments with their 30 kJ laser OMEGA, indicating, that at least within the confines of their experiment, they were successful in producing the scaled ultraviolet driven implosion that a 2 MJ ultraviolet laser would be expected to produce, and which NIF did produce using X-Rays to implode [34] [35].

The third tall pole is the NRL work on developing excimer lasers, first KrF, and later ArF. These not only have shorter wavelengths but also have several other capabilities which are advantageous for energy production. Not only is their efficiency likely higher, but the active material is a fast-flowing gas, not a pile of laser glass which must be cooled down for hours between shots. It is likely that excimer lasers have higher average power capability; they also have several other advantages [37] [38].

Of course, none of these “tall poles” is conclusive proof that laser fusion with a direct drive configuration will work. There are other complications. For instance, the ultraviolet laser used for fusion, might produce energetic electrons on the sur-

face of the target, which most likely did not occur in the URLLE experiments, and which might cause destructive preheat of the fuel. That has motivated this work, and while the author certainly cannot claim a triumph, he believes that he has found significant reason for optimism as shown in [Figure 21](#) and [Figure 22](#).

Hence the author feels that using an excimer laser with an energy of 1 - 5 MJ is the optimal approach to fusion now, and the proper strategy for our political bosses is to implement it [\[30\]](#) [\[31\]](#). While there are many roadblocks and speed bumps along the way, not all foreseeable currently; he is convinced that this is the best way to go, given what we know now, not only about laser fusion, but also what we know about magnetic fusion.

Furthermore, let's degrade the NRL estimates. Let's say the laser has an efficiency of "only" 7%, and the fusion process has a gain of "only" 50. It is simple to see that with these numbers, this is not viable for generating electricity. However pure fusion is not the only possible outcome. As this author has argued for years [\[30\]](#), fusion breeding can be a very prolific generator of fissile fuel for thermal nuclear reactors.

The "degraded" laser fusion system just specified would work fine for fusion breeding [\[30\]](#). Now let's see where we are. With perhaps only 10% of the laser energy imploding the target (that is, the smallest % that the LLNL group estimates), the LLNL project has produced a gain of 2.5. If an ultraviolet experiment works as well, when *all* the laser energy could hit the target, as URLLE results indicate, and as the NRL group had assumed, the LLNL result could have demonstrated a gain of 25! Yet a gain of 'only' 50 would be sufficient for fusion breeding. As far as the required gain for the fusion process goes, *we may already be halfway there!*

This concludes with arguments against what some have proposed. Some, worrying about the inefficiency of the laser, have proposed using a more efficient heavy ion accelerator instead of a laser to drive the implosion. First, a 50% efficient ion accelerator, putting its energy into a hohlraum, which puts even 20% of its energy into the target, is no more efficient than a 10% efficient laser putting all its energy directly into the target. Second, no such accelerators exist, and it is not exactly a simple task to build one. The Russian effort spoke of a 7 km long accelerator, supplemented with 7 large storage rings [\[48\]](#) [\[54\]](#). This does not sound simple or cheap. Yet, this would produce "only" a 7 MJ beam, far short of the 100 MJ beams accelerator proponents advocate seem to be talking about now.

Others, worried about the need to breed tritium fuel, have suggested using the DD reaction. Even at Megavolt temperatures, the DD reaction rate is at least two orders of magnitude less than the DT reaction rate. At conventional fusion temperatures of 5 - 20 keV, the situation is even worse. To this author, sticking with DT is certainly the way to go. Perhaps once we have established a DT fusion economy, we might want to reconsider, and attempt to improve it to a DD economy. That way we will not have to breed fuel (T), or go to the moon for it (3 He). But that is not a decision for us to make, it is a decision for our great, great... grand-

children. The next generation or two can make this decision easiest for them by establishing a flourishing DT fusion economy.

This author feels that jumping into either of these projects is like telling our sponsors that after three quarters of a century of fusion research, we suddenly realized that we still need another three quarters of a century, or century more (!), just to get to the starting gate! Surely it is extremely unlikely in this circumstance, that our sponsors would continue to sign the checks. Just how much patience do we think they have?

Right now, it is likely that we are finally climbing out of the bottomless pit that fusion has been stuck in for 60 years. Let's not now jump into one or two other new bottomless pits! Now it is like third down and 20 yards to the goal line. Let's be Tom Brady and make that damn touchdown; we can do this! Let's not say we will only play if the goal line is moved back a kilometer, but we need 300 more downs. Rather, let's attempt to exploit the successes we have already achieved. Let's build a fusion reactor, and/or a fusion breeder using a 1 - 5 MJ laser, which we already have, work on improving its efficiency to 10%, or even 7%, work on other technical aspects of the reactor, tracking the target, manufacturing targets, injecting them, blanket issues... and of course, work on getting a fusion process with a gain of at least 50. As regards the latter, we may already be halfway there. These are big issues, which will still take decades to resolve. This author feels that at this early stage, they would be best dealt with on the national lab scale, rather than on the private industry 'fusion startup' scale. He advocates setting up an additional new national lab dedicated to laser fusion for energy, rather than stockpile stewardship [30] [31].

Acknowledgements

This work was not supported by any agency, public or private. The author thanks Steven Obenschain, formerly the leader of the NRL project, currently the leader of the Laser Fusion X company for supporting the author's work in this area at the Naval Research Laboratory. Also, the author especially appreciates his decades long collaboration with Denis Colombant, and his years long collaboration with Andrew Schmitt.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Schurtz, G.P., Nicolaï, P.D. and Busquet, M. (2000) A Nonlocal Electron Conduction Model for Multidimensional Radiation Hydrodynamics Codes. *Physics of Plasmas*, **7**, 4238-4249. <https://doi.org/10.1063/1.1289512>
- [2] Marocchino, A., Tzoufras, M., Atzeni, S., Schiavi, A., Nicolaï, P.D., Mallet, J., *et al*. (2013) Comparison for Non-Local Hydrodynamic Thermal Conduction Models. *Physics of Plasmas*, **20**, Article ID: 022702. <https://doi.org/10.1063/1.4789878>
- [3] Marocchino, A., Atzeni, S. and Schiavi, A. (2014) Effects of Non-Local Electron

- Transport in One-Dimensional and Two-Dimensional Simulations of Shock-Ignited Inertial Confinement Fusion Targets. *Physics of Plasmas*, **21**, Article ID: 012701. <https://doi.org/10.1063/1.4861389>
- [4] Del Sorbo, D., Feugeas, J.-., Nicolai, P., Olazabal-Loumé, M., Dubroca, B., Guisset, S., *et al.* (2015) Reduced Entropic Model for Studies of Multidimensional Nonlocal Transport in High-Energy-Density Plasmas. *Physics of Plasmas*, **22**, Article ID: 082706. <https://doi.org/10.1063/1.4926824>
 - [5] Cao, D., Moses, G. and Delettrez, J. (2015) Improved Non-Local Electron Thermal Transport Model for Two-Dimensional Radiation Hydrodynamics Simulations. *Physics of Plasmas*, **22**, Article ID: 082308. <https://doi.org/10.1063/1.4928445>
 - [6] Goncharov, V.N., Sangster, T.C., Radha, P.B., Betti, R., Boehly, T.R., Collins, T.J.B., *et al.* (2008) Performance of Direct-Drive Cryogenic Targets on OMEGA. *Physics of Plasmas*, **15**, Article ID: 056310. <https://doi.org/10.1063/1.2856551>
 - [7] Holec, M., Nikl, J. and Weber, S. (2018) Nonlocal Transport Hydrodynamic Model for Laser Heated Plasmas. *Physics of Plasmas*, **25**, Article ID: 032704. <https://doi.org/10.1063/1.5011818>
 - [8] Albritton, J.R., Williams, E.A., Bernstein, I.B. and Swartz, K.P. (1986) Nonlocal Electron Heat Transport by Not Quite Maxwell-Boltzmann Distributions. *Physical Review Letters*, **57**, 1887-1890. <https://doi.org/10.1103/physrevlett.57.1887>
 - [9] Manheimer, W., Colombant, D. and Goncharov, V. (2008) The Development of a Krook Model for Nonlocal Transport in Laser Produced Plasmas. I. Basic Theory. *Physics of Plasmas*, **15**, Article ID: 083103. <https://doi.org/10.1063/1.2963078>
 - [10] Colombant, D. and Manheimer, W. (2008) The Development of a Krook Model for Nonlocal Transport in Laser Produced Plasmas. II. Application of the Theory and Comparisons with Other Models. *Physics of Plasmas*, **15**, Article ID: 083104. <https://doi.org/10.1063/1.2963080>
 - [11] Colombant, D. and Manheimer, W. (2010) Numerical Fluid Solutions for Nonlocal Electron Transport in Hot Plasmas: Equivalent Diffusion versus Nonlocal Source. *Journal of Computational Physics*, **229**, 4369-4381. <https://doi.org/10.1016/j.jcp.2010.02.017>
 - [12] Colombant, D. and Manheimer, W. (2009) Krook Model for Nonthermal Electron Energy Transport. III. Spherical Configurations. *Physics of Plasmas*, **16**, Article ID: 062705. <https://doi.org/10.1063/1.3155445>
 - [13] Colombant, D.G. and Manheimer, W.M. (2010) Internal Tests and Improvements of the Krook Model for Nonlocal Electron Energy Transport in Laser Produced Plasmas. *Physics of Plasmas*, **17**, Article ID: 112706. <https://doi.org/10.1063/1.3505304>
 - [14] Manheimer, W., Colombant, D. and Schmitt, A.J. (2012) Calculations of Nonlocal Electron Energy Transport in Laser Produced Plasmas in One and Two Dimensions Using the Velocity Dependent Krook Model. *Physics of Plasmas*, **19**, Article ID: 056317. <https://doi.org/10.1063/1.4718458>
 - [15] Manheimer, W., Colombant, D. and Schmitt, A. (2018) Private Discussions on the Lack of Gain with a Krook Model for Energetic Electrons. Naval Research Laboratory.
 - [16] Marinak, M., Koning, J., Schroeder, C., *et al.* (2025) New Simulation Capabilities in Hydra. *The APS-DPP Society Meeting*, Long Beach, November 2025.
 - [17] Marinak, M., Kerbel, G., Patel, M., *et al.* (2014) Improved Inline Model for Nonlocal Electron Transport in HY-DRA. *APS-DPP Meeting*, New Orleans, October 2025.
 - [18] Corman, E.G., Loewe, W.E., Cooper, G.E. and Winslow, A.M. (1975) Multi-Group Diffusion of Energetic Charged Particles. *Nuclear Fusion*, **15**, 377-386.

- <https://doi.org/10.1088/0029-5515/15/3/003>
- [19] Sherlock, M., Brodrick, J.P. and Ridgers, C.P. (2017) A Comparison of Non-Local Electron Transport Models for Laser-Plasmas Relevant to Inertial Confinement Fusion. *Physics of Plasmas*, **24**, Article ID: 082706. <https://doi.org/10.1063/1.4986095>
 - [20] Bell, A.R. and Sherlock, M. (2024) The fastVFP Code for Solution of the Vlasov-Fokker-Planck Equation. *Plasma Physics and Controlled Fusion*, **66**, Article ID: 035014. <https://doi.org/10.1088/1361-6587/ad2278>
 - [21] Arran, C., Holec, M., Sherlock, M., Marinak, M. and Ridgers, C.P. (2025) The Accuracy of Multi-Group Models for Nonlocal Electron Transport in Magnetized Plasmas. *Physics of Plasmas*, **32**, Article ID: 043908. <https://doi.org/10.1063/5.0250147>
 - [22] Manheimer, W. and Colombant, D. (2015) Fokker Planck and Krook Theory of Energetic Electron Transport in a Laser Produced Plasma. *Physics of Plasmas*, **22**, Article No. 92708. <https://doi.org/10.1063/1.4931047>
 - [23] Manheimer, W., Colombant, D. and Schmitt, A. (2018) Analytic Insights into Non-local Energy Transport. II. Combined Steady State Fokker Planck and Krook Theory. *Physics of Plasmas*, **25**, Article ID: 082712. <https://doi.org/10.1063/1.5039843>
 - [24] Manheimer, W. and Colombant, D. (2021) Analytic Insights into Nonlocal Energy Transport. III. Steady State Fokker Planck Theory in Spherical and Planar Geometry. *Physics of Plasmas*, **28**, Article ID: 012701. <https://doi.org/10.1063/5.0012825>
 - [25] Manheimer, W. (2023) Analytic Insights into Nonlocal Energy Transport: Steady State Fokker Planck Theory in Arbitrary Z Plasmas. *Physics of Plasmas*, **30**, Article ID: 042708. <https://doi.org/10.1063/5.0131841>
 - [26] Solodov, A.A., Rosenberg, M.J., Stoeckl, M., Christopherson, A.R., Betti, R., Radha, P.B., *et al.* (2022) Hot-Electron Preheat and Mitigation in Polar-Direct-Drive Experiments at the National Ignition Facility. *Physical Review E*, **106**, Article ID: 055204. <https://doi.org/10.1103/physreve.106.055204>
 - [27] Lindl, J.D., Landen, O.L., Haan, S.W., *et al.* (2026) Key Metrics of Progress in the NIF Ignition Implosions and Future Challenges on the Path to Higher Yields. *Physics of Plasmas*, **33**, Article 050503. <https://doi.org/10.1063/5.0316678>
 - [28] Zylstra, A., Kritcher, A., Hurricane, O., *et al.* (2022) Experimental Achievement and Signatures of Ignition at the National Ignition Facility. *Physical Review E*, **106**, Article ID: 025202.
 - [29] Divol, L. (2022) Dynamics and Variability in Near Unity Gain Inertial Fusion Implosions on the National Ignition Facility. *Plenary Talk 64th Meeting of the APS Division of Plasma Physics*, Spokane, 17-21 October 2022.
 - [30] Manheimer, W. (2024) Fusion: It's Time to Color outside the Lines. *Open Journal of Applied Sciences*, **14**, 740-800. <https://doi.org/10.4236/ojapps.2024.143052>
 - [31] Manheimer, W. (2025) For a New Department of Energy Lab to Examine Laser Fusion for Energy. *Open Journal of Applied Sciences*, **15**, 976-986. <https://doi.org/10.4236/ojapps.2025.154066>
 - [32] Jassby, D.L. (2024) It Is Premature to Endorse Direct Drive Laser Fusion. *Forum of Physics and Society*. <https://engage.aps.org/fps/resources/newsletters/newsletter-archives/april-2024#premature>
 - [33] Sangster, T., Goncharov, V., Betti, R., *et al.* (2013) Improving Cryogenic Deuterium-Tritium Implosion Performance on OMEGA. *Physics of Plasmas*, **20**, Article ID: 056317.
 - [34] Gopalaswamy, V., Williams, C., Betti, R., *et al.* (2024) Demonstration of a Hydrody-

- namically Equivalent Burning Plasma in Direct-Drive Inertial Confinement Fusion. *Nature Physics*, **20**, 751-757.
- [35] Williams, C., *et al.* (2024) Demonstration of Hot-Spot Fuel Gain Exceeding Unity in Direct-Drive Inertial Confinement Fusion Implosions. *Nature Physics*, **20**, 758-764.
- [36] Obenschain, S., Lehmberg, R., Kehne, D., Hegeler, F., Wolford, M., Sethian, J., *et al.* (2015) High-Energy Krypton Fluoride Lasers for Inertial Fusion. *Applied Optics*, **54**, F103. <https://doi.org/10.1364/ao.54.00f103>
- [37] Obenschain, S.P., Schmitt, A.J., Bates, J.W., Wolford, M.F., Myers, M.C., McGeoch, M.W., *et al.* (2020) Direct Drive with the Argon Fluoride Laser as a Path to High Fusion Gain with Sub-Megajoule Laser Energy. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **378**, Article ID: 20200031. <https://doi.org/10.1098/rsta.2020.0031>
- [38] Bodner, S.E. (2023) Comparison of an Argon-Fluoride Gas Laser with a Solid-State Laser for Application to Laser Fusion Energy. *Journal of Fusion Energy*, **42**, Article No. 33. <https://doi.org/10.1007/s10894-023-00372-w>
- [39] Rosenberg, M.J., Solodov, A.A., Myatt, J.F., Seka, W., Michel, P., Hohenberger, M., *et al.* (2018) Origins and Scaling of Hot-Electron Preheat in Ignition-Scale Direct-Drive Inertial Confinement Fusion Experiments. *Physical Review Letters*, **120**, Article ID: 055001. <https://doi.org/10.1103/physrevlett.120.055001>
- [40] Rosenberg, M.J., Solodov, A.A., Seka, W., Follett, R.K., Myatt, J.F., Maximov, A.V., *et al.* (2020) Stimulated Raman Scattering Mechanisms and Scaling Behavior in Planar Direct-Drive Experiments at the National Ignition Facility. *Physics of Plasmas*, **27**, Article ID: 042705. <https://doi.org/10.1063/1.5139226>
- [41] Yaakobi, B., Chang, P., Solodov, A., Stoeckl, C., Edgell, D.H., Craxton, R.S., *et al.* (2012) Fast-Electron Generation in Long-Scale-Length Plasmas. *Physics of Plasmas*, **19**, Article ID: 012704. <https://doi.org/10.1063/1.3676153>
- [42] Yaakobi, B., Solodov, A.A., Myatt, J.F., Delettrez, J.A., Stoeckl, C. and Froula, D.H. (2013) Measurements of the Divergence of Fast Electrons in Laser-Irradiated Spherical Targets. *Physics of Plasmas*, **20**, Article ID: 092706. <https://doi.org/10.1063/1.4824008>
- [43] Xiao, C.Z., Zhuo, H.B., Yin, Y., Liu, Z.J., Zheng, C.Y., Zhao, Y., *et al.* (2018) On the Stimulated Raman Side-Scattering in Inhomogeneous Plasmas: Revisit of Linear Theory and Three-Dimensional Particle-in-Cell Simulations. *Plasma Physics and Controlled Fusion*, **60**, Article ID: 025020. <https://doi.org/10.1088/1361-6587/aa9b41>
- [44] Braghinskii, S. (1965) Transport Processes in a Plasma. *Reviews of Plasma Physics*, **1**, 205.
- [45] Manheimer, W.M., Lampe, M. and Joyce, G. (1997) Langevin Representation of Coulomb Collisions in PIC Simulations. *Journal of Computational Physics*, **138**, 563-584. <https://doi.org/10.1006/jcph.1997.5834>
- [46] Meschini, S., Ferry, S.E., Delaporte-Mathurin, R. and Whyte, D.G. (2023) Modeling and Analysis of the Tritium Fuel Cycle for ARC- and STEP-Class D-T Fusion Power Plants. *Nuclear Fusion*, **63**, Article ID: 126005. <https://doi.org/10.1088/1741-4326/acf3fc>
- [47] Abdou, M., Morley, N.B., Smolentsev, S., Ying, A., Malang, S., Rowcliffe, A., *et al.* (2015) Blanket/First Wall Challenges and Required R&D on the Pathway to DEMO. *Fusion Engineering and Design*, **100**, 2-43. <https://doi.org/10.1016/j.fusengdes.2015.07.021>
- [48] Manheimer, W. (2014) Fusion Breeding for Mid-Century Sustainable Power. *Journal*

- of Fusion Energy*, **33**, 199-234. <https://doi.org/10.1007/s10894-014-9690-9>
- [49] Hoffert, M.I., Caldeira, K., Benford, G., Criswell, D.R., Green, C., Herzog, H., *et al.* (2002) Advanced Technology Paths to Global Climate Stability: Energy for a Greenhouse Planet. *Science*, **298**, 981-987. <https://doi.org/10.1126/science.1072357>
 - [50] Guidez, J. and Gabriel, S. (2016) Extraction of Uranium from Seawater: A Few Facts. *EPJ Nuclear Sciences & Technologies*, **2**, Article No. 10. <https://doi.org/10.1051/epjn/e2016-50059-2>
 - [51] Moir, R.W. and Manheimer, W. (2013) Fusion-Fission Hybrid Reactors. In: Dolan, T.J., Ed., *Magnetic Fusion Technology*, Springer, 699-742.
 - [52] Moir, R.W., Lee, J.D. and Coops, M.S. (1982) Fission-Suppressed Hybrid Reactor: The Fusion Breeder.
 - [53] Moir, R.W. (1982) The Fusion Breeder. *Journal of Fusion Energy*, **2**, 351-367. <https://doi.org/10.1007/bf01063686>
 - [54] Sharkov, B.Y., Alexeev, N.N., Basko, M.M., Churazov, M.D., Koshkarev, D.G., Medin, S.A., *et al.* (2005) Power Plant Design and Accelerator Technology for Heavy Ion Inertial Fusion Energy. *Nuclear Fusion*, **45**, S291-S297. <https://doi.org/10.1088/0029-5515/45/10/s25>