

Research Progress on Tensor Analysis Applied in Mining Engineering

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Abstract

Rock mass in mining engineering is a discontinuous medium characterized by complex fracture networks and anisotropic mechanical behavior. Its deformation and failure mechanisms are closely related to the genesis of dynamic disasters in stopes. Tensors, as a mathematical tool capable of objectively describing the spatial direction and intrinsic relationships of physical quantities, provide a powerful theoretical framework for quantitatively characterizing the mechanical behavior of such complex systems. This paper systematically elaborates on the fundamental principles of tensor analysis and focuses on its cutting-edge applications in mining engineering. It highlights the inversion method for surrounding rock failure mechanisms based on moment tensor theory, including improvements to the Ohstu criterion and its application in analyzing irregular coal pillars and water inrush channels. It introduces the tensor characterization technology for coal-rock fractures based on CT scanning and ellipsoid model reconstruction, achieving quantitative description and fabric analysis of real complex fracture structures. Additionally, it outlines the application of tensor methods in other fields such as 3D roadway modeling, gravity/resistivity exploration, seismic data reconstruction, and stress field inversion. Through typical case analyses, the effectiveness and superiority of tensor analysis in revealing rock mass failure mechanisms and predicting dynamic disasters are confirmed. Finally, future research directions including multi-scale coupling, intelligence, and integration with multi-physical fields are prospected.

Keywords

Tensor Analysis, Mining Engineering, Moment Tensor Inversion, Fracture Characterization, Surrounding Rock Failure, Dynamic Disaster

1. Introduction

The essence of mining engineering lies in the safe and efficient extraction of mineral resources. Mine rock masses are situated within complex geological environments. Under the influence of mining-induced disturbances, internal fracture structures such as pores, bedding planes, joints, and faults develop into a multi-scale network of weak planes spanning from the microscopic to the macroscopic level. This network fundamentally governs the strength, deformation modulus, failure mode, and permeability characteristics of the rock mass [1]. Concurrently, the *in-situ* stress field, formed by the long-term coupling of gravitational loading and regional tectonic movements, exhibits distinct spatial vector characteristics. The orientations and magnitudes of its three principal stresses determine the potential failure plane orientations of the rock mass [2]. Under the intense disturbances induced by roadway excavation and longwall face extraction, the original stress equilibrium is disrupted, leading to stress redistribution, energy accumulation and release, as well as complex interactions with the pre-existing fracture network. This coupled stress-fracture evolution process often constitutes the root cause of major dynamic disasters, including open-pit slope instability, underground stope roof collapse, large deformation of roadway surrounding rock, rock bursts, coal and gas outbursts, and floor water inrush [3] [4], thereby imposing severe constraints on the safety and efficiency of mining operations.

Mine rock mass mechanics is a discontinuous and anisotropic mechanical system. Traditional rock mechanics research methods are largely predicated on idealized assumptions of continuity, homogeneity, and isotropy, relying on scalar or vector parameters such as stress, strain, and strength for description and analysis. This paradigm proves inadequate when applied to real mine rock masses that are inherently rich in directional structures. For instance, uniaxial compressive strength alone cannot predict shear slip along weak planes; average stress fails to capture the distinctly different deformation responses perpendicular and parallel to bedding planes; and simple statistical measures of fracture density are insufficient to evaluate the anisotropy of rock mass permeability. Consequently, a mathematical framework capable of systematically describing directionality, anisotropy, and coordinate invariance is urgently required.

A tensor is a multidimensional array originating from continuum mechanics [5] [6]. Its core concept is that the intrinsic properties of a physical entity should be independent of the particular coordinate system chosen by the observer. Many engineering-relevant physical quantities—such as stress, strain, moment of inertia, thermal conductivity, and permeability—require second-order or even higher-order tensors for complete description. A tensor can simultaneously encapsulate the three principal directions, three principal values, and the interrelationships among these directions within a compact mathematical object, and it enables standardized transformations between different coordinate systems (e.g., geographic coordinate system, local roadway coordinate system, and principal stress coordinate system).

In recent years, with the widespread deployment of high-precision microseismic monitoring networks, industrial CT scanning, and advanced geophysical exploration technologies, massive volumes of high-dimensional field and experimental data rich in directional information have emerged in mining engineering. This provides a solid data foundation for transforming tensor theory from an abstract mathematical tool into a powerful instrument for solving practical engineering problems. This paper aims to systematically review the main application threads of tensor analysis in mining engineering, elaborate on its methodology, implementation procedures, and advantages of tensor-based characterization in light of the latest research findings, and discuss future development trends involving the integration of tensor analysis with intelligent technologies and multi-physics coupling.

This paper adopts a narrative review approach. The literature search covers databases including CNKI, Web of Science, and Google Scholar, with a time span from 1978 to 2026. The main keywords used are “tensor analysis” and “mining engineering”. The preliminary screening criterion is that the research object must be mine rock masses or mining production and geological activities, and tensor methods must be explicitly employed as the core analytical tool. A total of 24 articles were finally included and categorized according to their application directions: 1) moment tensor inversion and fracture mechanism identification; 2) tensor characterization of fracture structures; 3) roadway modeling and coordinate transformation; 4) tensor methods in geophysical exploration; 5) tensor decomposition and reconstruction of high-dimensional data; 6) stress field and multi-field coupling analysis. This paper elaborates on these topics based on the above classification framework.

This paper focuses on tensor applications specific to mining engineering scenarios. Full-waveform inversion and regional gravity gradient tensor structural interpretation from the petroleum and natural gas industry are cited only when their methods are transferable to the mining scale.

2. Moment-Tensor-Based Inversion of Rock Fracture Mechanisms

Instability and failure are common phenomena in mine rock masses. During the failure process, numerous micro-fracture events occur, generating elastic waves (acoustic emission or microseismic signals). Each micro-fracture can be treated as a microscopic seismic source. The moment tensor, as a cornerstone of point-source theory, constitutes a complete mathematical representation of the equivalent body-force distribution of such a source. By inverting recorded waveform data to recover the moment tensor, the mechanical mechanism of fracturing can be directly and quantitatively interpreted.

2.1. Physical Interpretation and Decomposition of the Moment Tensor

For an instantaneous point source located at point ξ , the displacement field u_n

generated at point $\mathbf{x}$ can be expressed as **Equation (1)**.

$$u_n(\mathbf{x}, t) = M_{pq}(t) \times G_{np,q}(\mathbf{x}, t; \xi, 0) \quad (1)$$

where $G_{np,q}$ is the partial derivative of Green's function with respect to the source coordinates, and M_{pq} is the first-order moment tensor. It is a 3×3 symmetric second-order tensor (with six independent components) that contains all information about the seismic source [7].

By performing eigenvalue decomposition on the moment tensor $\mathbf{M}$, three real eigenvalues ($M_1 \geq M_2 \geq M_3$) and their corresponding eigenvectors can be obtained. The eigenvectors define the principal axis directions of the source, while the relative magnitudes of the eigenvalues reveal the type of fracture. For a more intuitive understanding, the moment tensor $\mathbf{M}$ is typically decomposed into a linear superposition of three basic physical mechanisms, shown as **Equation (2)**.

$$\mathbf{M} = M^{\text{ISO}} \mathbf{I} + \mathbf{M}^{\text{DC}} + \mathbf{M}^{\text{CLVD}} \quad (2)$$

where:

M^{ISO} —the isotropic component, a scalar part represented by the unit tensor. It corresponds to volumetric expansion ($M^{\text{ISO}} > 0$) or contraction ($M^{\text{ISO}} < 0$) of the source, such as spalling in rock bursts, pore collapse, or explosion-type fracturing caused by sudden gas release.

$\mathbf{M}^{\text{DC}}$ —the double-couple component, a deviatoric part with eigenvalues $(F, 0, -F)$. This is the classic model of pure shear slip, corresponding to strike-slip, thrust, or normal faulting activities, and is the main mechanism of tectonic earthquakes and most shear failures of rock masses.

$\mathbf{M}^{\text{CLVD}}$ —the compensated linear vector dipole component, also a deviatoric part, with eigenvalues $(F, -F/2, -F/2)$ or their permutations. It may represent complex non-planar shear, the shear component of tensile fracturing, or the combined effect of multiple fractures.

Early moment tensor inversion directly adopted the above general decomposition. However, two prominent issues were identified in mining engineering applications: first, the decomposition result is non-unique, as multiple decomposition combinations can yield the same $\mathbf{M}$; second, a high M^{CLVD} component often leads to misjudgment of the fracture type. To address the specific stress environment where mine rock masses are typically in a compressive-shear state, Tang Lizhong *et al.* [8] proposed an improved three-component decomposition model, shown as **Equation (3)**.

$$\begin{cases} M^{\text{ISO}} - 0.5M^{\text{CLVD}} + M^{\text{DC}} = M_1 \\ M^{\text{ISO}} + M^{\text{CLVD}} = M_2 \\ M^{\text{ISO}} - 0.5M^{\text{CLVD}} - M^{\text{DC}} = M_3 \end{cases} \quad (3)$$

This model enforces the alignment of the eigenvector of the decomposed double-couple component with the intermediate eigenvector of the moment tensor itself, thereby ensuring the uniqueness of the eigenvalue (M_1, M_2, M_3) for $(M^{\text{ISO}}, M^{\text{CLVD}}, M^{\text{DC}})$ and avoiding the ambiguity problem in inversion results.

The practical differences between the classical decomposition and the improved criterion in fracture classification are mainly reflected in two aspects. First, the classical decomposition imposes no constraint on the CLVD component, which tends to misclassify events with complex fracture morphologies (e.g., non-planar shear) as tensile fractures, whereas the improved criterion, through enforced alignment, categorizes such events as shear-dominated mixed-type fractures. Second, in high-stress unloading zones of mines, the classical decomposition often misinterprets unloading-induced tensile fractures as pure compressive events due to the significant volumetric contraction component; in contrast, the improved criterion, combined with the dual-threshold criterion, can more accurately identify the tensile component. Overall, the improved criterion exhibits higher classification consistency for the compression-shear-tensile mixed fractures commonly encountered in mining engineering.

2.2. Quantitative Criteria and Visualization of Rock Fracture Types

Once the three components are obtained from Section 1.1, quantitative criteria are required to distinguish fracture types. Traditional scalar criteria, such as $\varepsilon = |M^{\text{CLVD}}|/|M^{\text{ISO}}|$, become unstable when M^{ISO} approaching zero. To address this, the Ohstu criterion [8] was improved by defining proportion parameters for each component, shown as **Equation (4)**.

$$P_{\text{DC}} = \frac{|M^{\text{DC}}|}{|M^{\text{DC}}| + |M^{\text{ISO}}| + |M^{\text{CLVD}}|} \quad (4)$$

It is similarly for P_{ISO} and P_{CLVD} .

A double-threshold criterion is adopted: if $P_{\text{DC}} \geq 60\%$, the event is classified as shear failure; if $P_{\text{ISO}} \geq 60\%$, it is classified as tensile failure; if both are below 60%, it is classified as mixed-mode failure. This criterion has clear physical meaning, is insensitive to high M^{CLVD} components, and exhibits robust performance in practice.

To visualize the distribution of fracture mechanisms among a large number of microseismic events, the P-T diagram and focal sphere have become essential visualization tools.

The P-T diagram uses the decomposed isotropic component (P-axis) and the maximum eigenvalue of the deviatoric tensor (T-axis) as coordinates, projecting each event onto a two-dimensional plane, as shown in **Figure 1**.

A series of P-T diagrams clearly reveals the evolution process of rock fracturing, from predominantly tensile failure in the early stage to an increasing number of shear and mixed-mode failures with higher energy release around the peak stress.

The focal sphere provides a three-dimensional representation of the fracture plane solution and radiation pattern for individual events. By statistically analyzing the focal spheres of all events within a given region, the dominant strike and dip of fracture planes, as well as the preferred orientations of the P-axis (pressure axis) and T-axis (tension axis), can be intuitively identified. Liu Yang *et al.* [10]

conducted a systematic moment tensor inversion on dense microseismic events in the irregular coal pillar area of a certain mine, as shown in **Figure 2**.

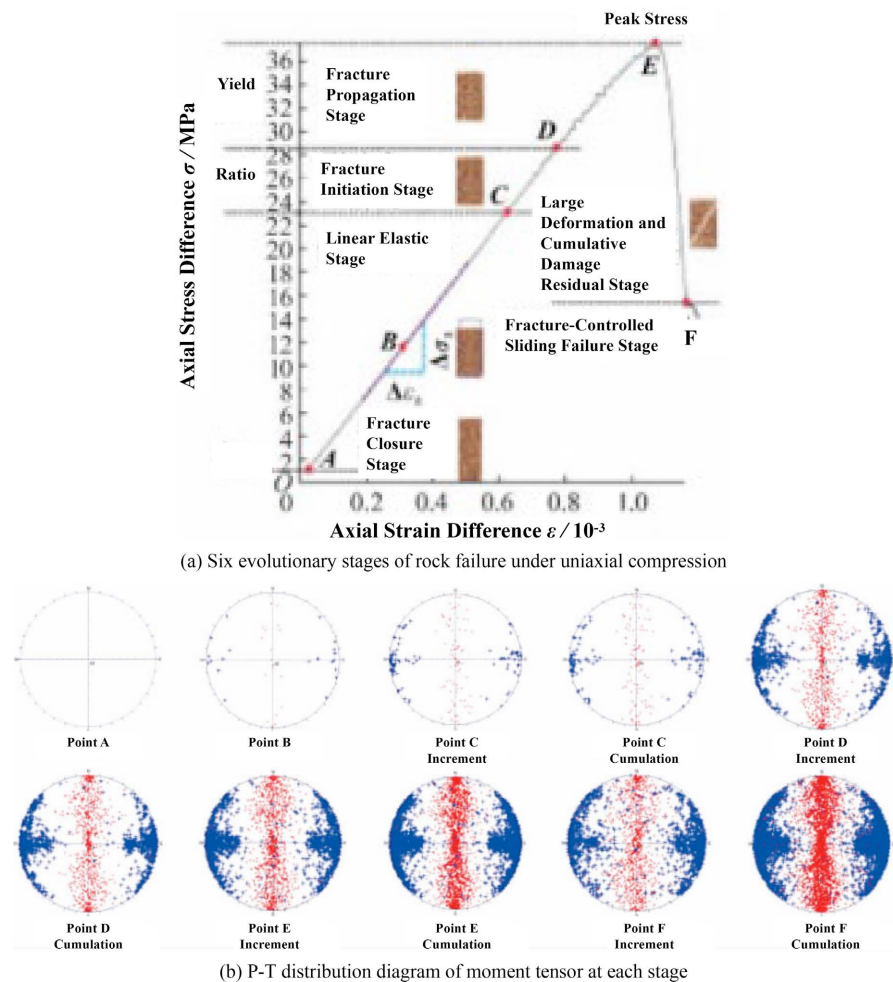
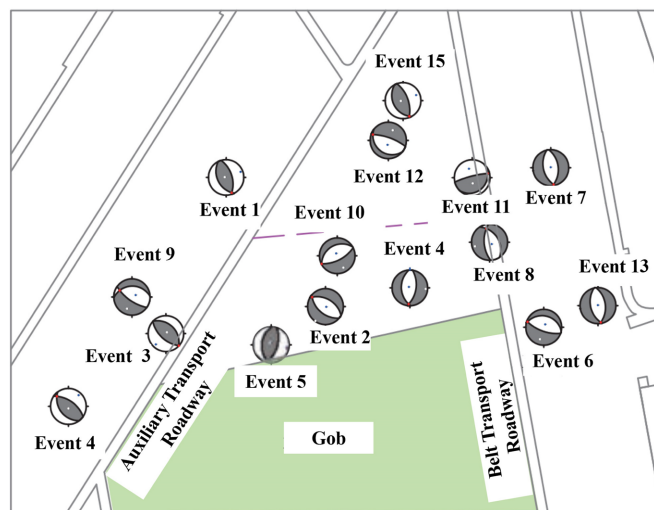


Figure 1. Moment tensor P-T diagram during the rock fracture process [9].



(a) Mining-induced seismic beach ball model

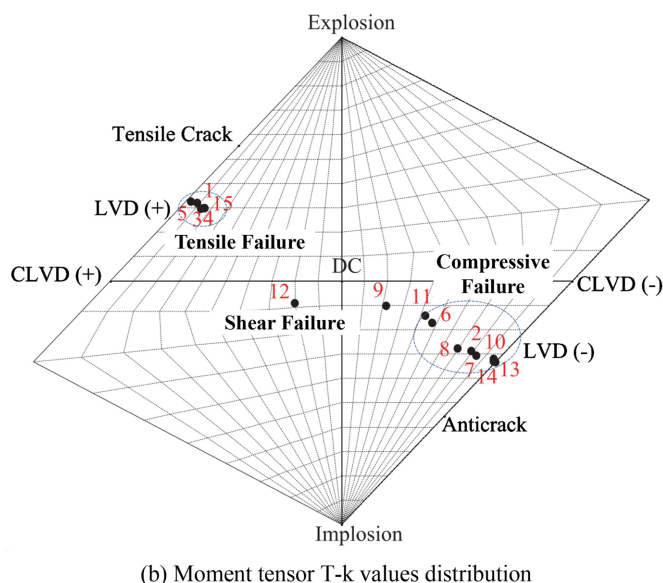


Figure 2. Beach ball visualization of rock fracture [10].

The spatial distribution of inversion results clearly indicates that on the belt roadway side of the coal pillar area, microseismic events are dominated by mixed-mode or shear failures with $P_{ISO} < 0$ (*i.e.*, compressive type). In contrast, on the auxiliary transport roadway side, a large number of events with $P_{ISO} > 0$ (*i.e.*, tensile type) are observed.

Although moment tensor inversion provides a powerful tool for studying mine fracture mechanisms, its engineering application is still constrained by several factors. First, the sensor coverage directly affects the reliability of the inversion results: an unfavorable network geometry (e.g., unilateral layout or insufficient vertical aperture) can lead to unstable estimation of moment tensor components, particularly weak constraints on the volumetric component (M_{ISO}). Second, the calculation of Green's functions depends on assumptions regarding the velocity model; however, mine rock masses often exhibit strong spatial heterogeneity and anisotropy, and simplified homogeneous isotropic models may introduce systematic errors. Third, signal noise (e.g., blasting, mechanical vibration, electromagnetic interference) can contaminate waveform data. Under low signal-to-noise ratio conditions, moment tensor inversion tends to produce spurious non-double-couple components. Therefore, when interpreting moment tensor inversion results, it is advisable to conduct a comprehensive evaluation that takes into account the quality of network coverage, the uncertainty of the velocity model, and the signal-to-noise ratio.

3. Tensor-Based Quantitative Characterization of Coal-Rock Fracture Structures

The macroscopic mechanical properties (such as strength and deformation modulus) and transport properties (such as permeability and thermal conductivity) of

rock masses are fundamentally governed by the geometric topology and spatial fabric of their internal fracture systems. Traditional fracture characterization parameters—such as linear density, areal density, mean trace length, and mean spacing—are all scalar quantities that lose crucial directional information. Yet the directionality and anisotropy of fractures are precisely the key factors controlling rock mass behavior. The directional characterization capability of tensors provides a perfect framework for addressing this issue.

3.1. From Discrete Fracture Networks to Continuous Tensor Characterization

For a set of discrete fractures, Oda [11] pioneeringly proposed the concept of the fracture fabric tensor, shown as **Equation (5)**.

$$F_{ij} = \frac{1}{V} \sum_{k=1}^N S^{(k)} n_i^{(k)} n_j^{(k)} \quad (5)$$

where V is the representative elementary volume (REV), N is the number of fractures, $S^{(k)}$ is the area of the k^{th} fracture, and $n_i^{(k)}$ is the component of its unit normal vector. F_{ij} is a symmetric second-order tensor whose principal directions represent the preferred orientation of fracture normals (*i.e.*, the dominant fracture occurrence), and the magnitudes of its principal eigenvalues reflect the “intensity” of fractures developed along that preferred direction, accounting for both the number and size of fractures. This tensor has been widely applied to derive the elastic tensor, damage tensor, and permeability tensor of anisotropic equivalent continua.

3.2. Advanced Tensor Characterization Based on Ellipsoid Model Reconstruction

Oda’s fabric tensor is based on the simplified assumption of disc-shaped fractures. However, CT scans of real coal-rock reveal extremely complex fracture morphologies, with pronounced bending, branching, and non-planar features. Wang Shouguang *et al.* [12] developed a more refined method—referred to as “ellipsoid model reconstruction and tensor characterization”—suitable for real complex fractures. The core idea of this method is to fit each triangular facet on the fracture surface with a rotating oblate ellipsoid. Based on the reconstructed “ellipsoid field,” two tensors with clearer physical meaning are defined:

3.2.1. Fracture Direction Tensor $\mathbf{O}$

This tensor purely describes the directional distribution of fracture normals, eliminating the influence of size. In the calculation, the normal vector of each ellipsoid (representing a triangular facet) is weighted by its area, shown as **Equation (6)**.

$$\mathbf{O} = \text{Norm} \left[\sum_{\alpha=1}^n \lambda_{\alpha} \mathbf{n}_{\alpha} \otimes \mathbf{n}_{\alpha} \right], \lambda_{\alpha} = \frac{S_{\alpha}}{\sum S_{\alpha}} \quad (6)$$

where $\mathbf{n}_{\alpha}$ is the unit normal vector of the α^{th} triangular facet, S_{α} is its area, $\otimes$ denotes the tensor product, and $\text{Norm}[\cdot]$ means that normalization is applied.

3.2.2. Fracture Fabric Tensor F

This tensor integrates both directional and size information. For a rotating oblate ellipsoid, any cross-section parallel to the fracture plane is a circle. Therefore, the circular cross-section of the ellipsoid can be used to contribute to the fabric tensor. Its expression can be simplified as **Equation (7)**.

$$F \approx \rho \sum_{\alpha=1}^n r_{\alpha}^2 (\mathbf{n}_{v1,\alpha} \otimes \mathbf{n}_{v1,\alpha} + \mathbf{n}_{v2,\alpha} \otimes \mathbf{n}_{v2,\alpha}) \quad (7)$$

where r_{α} is the radius of the ellipsoid's circular cross-section (*i.e.*, the semi-major axis a), $\mathbf{n}_{v1,\alpha}$ and $\mathbf{n}_{v2,\alpha}$ are a pair of orthogonal unit vectors within that circular plane, and ρ is the density coefficient.

3.3. Advantages of Tensor-Based Characterization of Coal-Rock Fracture Structures

Through analysis of the evolution of tensor eigenvalues, it was found that the trace (sum of eigenvalues) of the fracture direction tensor $\mathbf{O}$ changes very little during the entire loading process. This indicates that the propagation direction of newly formed fractures is controlled by the orientation of pre-existing fractures—they predominantly extend along the original dominant directions, reflecting a “path dependence” in rock mass failure. In contrast, the trace of the fracture fabric tensor F increases monotonically and significantly with increasing load, and its growth curve closely matches the total fracture area obtained directly through image analysis. This demonstrates that the trace of F serves as an excellent scalar indicator that sensitively captures the overall expansion of fracture scale.

Compared with traditional porosity (which reflects only spatial proportion, ignoring direction and connectivity) and fractal dimension (which describes complexity but has a weak direct linkage to mechanical mechanisms), this ellipsoid-reconstruction-based tensor characterization method provides a set of quantitative indicators that combine directionality (principal directions of $\mathbf{O}$ and F), intensity (eigenvalues of F), and evolutionary sensitivity (time-varying trace of tensors). This lays a solid foundation for establishing rock mass constitutive models and damage evolution equations based on real meso-scale structures.

4. Application Scenarios of Tensor Analysis in Mining Engineering

4.1. Intelligent 3D Modeling and Coordinate Transformation for Mine Roadways

In the development of intelligent mines and digital twin systems, the rapid and accurate construction of 3D geometric models for underground roadways constitutes a fundamental task. Traditional modeling methods are often cumbersome when dealing with inclined and curved roadways, and coordinate conversions are prone to cumulative errors. Yang *et al.* [13] proposed a general algorithm based on tensor coordinate transformation. This method is particularly suitable for complex cross-sections such as three-centered arches and circular arches, as well

as for achieving smooth transitions at roadway junctions and curved segments, thereby significantly improving both modeling efficiency and geometric accuracy.

4.2. Geophysical Exploration and Interpretation

4.2.1. Gravity Gradient Tensor Method for Delineating Ore Body Boundaries

Traditional gravity surveys measure the first derivative of the gravitational potential (gravitational acceleration), which provides limited information. The gravity gradient tensor, as the second derivative of the gravitational potential, is a 3×3 symmetric traceless tensor with five independent components (G_{xx} , G_{xy} , G_{xz} , G_{yy} , G_{yz}). It contains richer information on shallow sources and local anomalies and is more sensitive to geological body boundaries. Luo *et al.* [14] employed an improved spline interpolation method to compute high-precision gradient tensor data from gravity anomalies and applied tensor Euler deconvolution. Compared with conventional single-component Euler deconvolution, the inverted ore body boundaries are sharper, the depth estimates are more accurate, and the solution is more convergent.

4.2.2. Tensor Resistivity Method for Detecting Concealed Structures

Conventional mine direct-current resistivity methods typically use a dipole-dipole array and measure scalar apparent resistivity, which inadequately accounts for anisotropic media and roadway orientation effects. Through three-dimensional finite-difference forward modeling [15], the anomalous characteristics of tensor apparent resistivity for typical geoelectric bodies under roadway influence were systematically investigated. The results show that when the measurement array forms an angle (nonzero Euler angle) with the roadway axis, the off-diagonal components of the tensor resistivity exhibit significant anomalies, and their spatial positioning capability for anomalous bodies outperforms that of traditional scalar methods. This provides a new approach for detecting hidden water-conducting structures or gas-enriched zones in the sidewalls, roof, and floor of roadways.

4.2.3. Seismic Tensor Data for Predicting Fractures and Fault Structures

The gradient structure tensor (GST) coherence technique computes the gradient tensor at each point in the seismic data volume and analyzes its eigenvalues to measure the discontinuity of seismic events in the local neighborhood. Compared with the conventional C3 coherence algorithm, the GST coherence technique offers higher resolution and better noise robustness when delineating small faults and fracture zones [16]. Furthermore, fault identification methods based on tensor sparse optimization analysis can enhance fault continuity and clarity [17]. In the identification of ultra-deep carbonate strike-slip fault fracture zones in the Fuman Oilfield of the Tarim Basin, structure tensor-based methods can more clearly delineate the boundaries of fracture zones and even quantify their widths [18]. In the structural study of the Hulin Basin in Heilongjiang Province, by utilizing the distinct characteristics of different gradient tensor components, 21 faults with various orientations were successfully identified [19].

The anisotropic response of fracture zones in terms of elastic parameters exhibits its scale invariance. Provided that the seismic data resolution is sufficiently high, the same algorithm can be applied at the mining scale. These seismic tensor techniques can be transferred to three-dimensional seismic exploration in mining areas to identify small faults and fracture development zones, thereby supporting the layout of mining faces and the design of gas drainage boreholes.

4.3. Reconstruction and Efficient Processing of High-Dimensional Mining-Induced Seismic Data

The data acquired in seismic exploration are inherently high-dimensional tensors (e.g., dimensions including source points, receiver points, time, and offset). For the reconstruction of five-dimensional seismic data, traditional matrix rank reduction (DRR) methods suffer from low computational efficiency, while higher-order orthogonal iteration (HOOI) methods exhibit insufficient accuracy under strong noise and high missing rates. The method based on fully connected tensor network decomposition (FCTN) [20] decomposes a 4D tensor on frequency slices into a contraction of a series of low-dimensional tensors, avoiding the need for singular value decomposition (SVD) operations. Compared with traditional methods, this approach achieves improved reconstruction signal-to-noise ratio and significantly reduced computational time on both synthetic and field data. The three-dimensional seismic data reconstruction method based on tensor ring low-rank factorization (TRLRF) [21], which imposes a low-rank constraint in the latent space, also outperforms traditional methods in terms of signal-to-noise ratio and computational efficiency. Tensor decomposition techniques provide an efficient tool for processing massive and incomplete seismic data, thereby enhancing the quality of subsequent imaging and interpretation.

Although these methods were originally developed for petroleum seismic data, they can be directly applied to mine microseismic monitoring because mining-induced seismic data also exhibit multi-dimensional and incomplete characteristics, and the algorithms themselves do not depend on specific geological conditions.

4.4. Stress Field and Multi-Field Coupling Tensor Analysis

4.4.1. Regional Stress Field Inversion

By collecting a large number of moment tensor solutions (*i.e.*, focal mechanisms of individual events) from microseismic events within a region, stress tensor inversion methods (e.g., the Michael method) can be employed to determine the average stress tensor σ_{ij} of that region. The inversion results yield the orientations and relative magnitudes of the three principal stresses $(\sigma_1, \sigma_2, \sigma_3)$, with the latter expressed by the stress shape factor $(R = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3))$. In their inversion of irregular coal pillars, Liu *et al.* [10] successfully obtained the regional principal stress directions and verified their consistency with numerical simulation results.

4.4.2. Seepage-Stress Coupled Permeability Tensor

The permeability of fractured rock masses is not a constant but a tensor that varies with the stress state. Zhou Chuangbing *et al.* [22] established a coupled permeability tensor model that accounts for the interaction between seepage and deformation. In this model, fractures are treated as deformable smooth parallel plates, and the relationship between fracture permeability and normal stress follows an exponential law, from which the macroscopic permeability tensor at the REV scale is derived. This model is of critical importance for evaluating the impact of engineering activities, such as reservoir impoundment, mining-induced unloading, and grouting reinforcement on the seepage field of rock masses.

4.4.3. Induced Stress Tensor Model

Hydraulic fracturing operations in coal mines and oil/gas reservoirs induce changes in reservoir pore pressure and temperature, thereby generating additional stresses. An induced stress calculation model based on a tensor-type damage variable [23] defines the porosity change as a damage variable and embeds it into coupled thermal-hydraulic-mechanical-damage equations. The model derives the induced stress tensor formulations for both the damaged and elastic zones, enabling reasonable predictions of stress redistribution around injection wells. This provides a theoretical tool for preventing casing damage, optimizing injection-production schemes, and investigating the mechanisms of water inrush in mines.

5. Future Research Directions

To meet the future development demands of intelligent, transparent, and green mining, the following research directions merit in-depth exploration.

5.1. Multi-Scale Tensor Correlation and Cross-Scale Modeling

The behavior of rock masses in mines spans from nano-scale mineral grains, millimeter-scale fractures, and meter-scale rock blocks up to the engineering scale. Future research should focus on establishing quantitative cross-scale correlation models between structural tensors at different scales (e.g., mineral orientation tensors, microcrack fabric tensors, and joint network tensors) and macroscopic equivalent property tensors (e.g., elasticity tensors, strength tensors, and permeability tensors), thereby enabling prediction from microstructure to macroscopic response.

5.2. Nonlinear, Anisotropic, and Rheological Tensor Constitutive Theory

Under conditions of high *in-situ* stress, high temperature, and high osmotic pressure in deep mines, the mechanical behavior of rock masses exhibits strong non-linearity, anisotropic softening, and rheological characteristics. It is necessary to develop high-order tensor constitutive models capable of describing these complex properties, such as fractional derivative-based rheological tensor models, and to couple damage tensors and plastic internal variable tensors with such models,

so as to more realistically simulate the long-term large-deformation failure processes of rock masses.

5.3. True 3D, Full-Waveform, Multi-Parameter Tensor Joint Inversion

Current moment tensor inversion is mostly performed under the assumption of homogeneous isotropic media. Future efforts should be directed toward developing full-waveform moment tensor inversion techniques based on three-dimensional heterogeneous, and even anisotropic, media models, and jointly inverting parameters such as wave velocity structure and attenuation tensor, in order to eliminate propagation path effects and obtain purer and more accurate focal mechanism solutions [24].

6. Conclusions

Tensor analysis is a way of thinking that fits the complex nature of mine rock masses. This paper expounds its important applications in mining engineering:

1) Moment tensor inversion has evolved from simple focal mechanism determination to fine diagnosis of shear/tensile/mixed failure, regional stress field inversion, revealing the spatiotemporal evolution of failure, and serving dynamic disaster prediction.

2) Fracture tensor characterization breaks through the limitations of traditional scalar parameters, enabling a quantitative description of real complex fracture structures and playing an important role in constructing meso-mechanics.

3) Tensor analysis applications cover intelligent roadway modeling, high-precision gravity/electrical/seismic exploration, high-dimensional mining seismic data reconstruction, and multi-field coupling analysis (stress-seepage, etc.), demonstrating the ability to solve problems involving directionality and anisotropy in mining engineering.

4) Tensor analysis will deeply integrate with new-generation information technologies such as artificial intelligence, big data, and the Internet of Things, helping us better understand, more accurately predict, and more effectively control the behavior of mine rock masses, providing scientific support for safe, efficient, and green extraction of mineral resources.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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