

The Resonance of Emotion: A Mechanistic Theory of Musical Intonation and Affect

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Abstract

This study resolves the traditional “simple ratio” paradox that dominates mainstream approaches to musical intonation. Moving beyond standard acoustics, this paper proposes a comprehensive framework for musical tuning that operates in direct opposition to conventional models of beat avoidance. The framework demonstrates that the two mutually exclusive musical affects—harmonized ringing and intonation sensations—are initiated at the front-end peripheral auditory gate prior to ascending brainstem processing. These phenomena are driven respectively by internal binaural-beat modulations following a mathematical relationship distinct from simple integer ratios, and by horizontal melodic friction generated through the manipulation of alternating large and small semitone intervals against pitch-wise isochronism. Musical chords are, in reality, vertical melodies. To create musical expression, scales must facilitate a mechanism for pitch-wise isochronic friction. For this purpose, the dual semitone functions as the defining boundary where pitches above a large semitone shift sharper and vice versa; the next semitone, after intervening whole tones, must differ in size, shifting pitches flatter. This structural variation necessitates two distinct tetrachords, explaining why a uniform whole-tone scale lacks emotional resonance. Because the product of fractions from whole tones multiplied by a large semitone in the lower tetrachord is significantly simpler than that of the small semitone—thereby enhancing neural beat production in the tonic chord—the leading note must consequently be sharper. This study demonstrates that this requirement is satisfied by constructing a stretched 12-tone scale out of two parallel 6-whole-tone streams, expanding the acoustic octave to 1223.46 cents. This structural design allows musical material to be modulated into any key with minimal tuning adjustments for individual notes. Multiple independent neurological studies, alongside evolutionary survival mechanisms, challenge Helmholtz’s historical theories, demonstrating that the proposed horizontal melodic and vertical chord models align with acoustic data across popular, classical, and *a cappella* vocal genres. The

traditional principle of simple integer ratios is mathematically proven to be detrimental to performance; string players should avoid tuning adjacent open string pairs to precise 3:2 perfect fifths, revealing that this rigid historical framework is a primary driver of intonational difficulties in string pedagogy. By replacing speculative academic dogmas with mathematical data and high-fidelity orchestral re-synthesis, this paper bridges the gap between complex auditory neuroscience and practical musicianship, introducing an original tuning methodology that optimizes artistic expression.

Keywords

Intonation, Tuning, Octave Stretch, Violin Tuning, Chordal Fusion, Harmonized Ringing

1. Introduction

The centuries-long debate surrounding musical intonation is archived within the international scales.zip repository, a collection containing over 5350 theoretical proposals that practical musicians find little actual use. Curiously, the vast majority of these historical tuning configurations were published without ever being actively experimented with or tested inside real musical contexts. To resolve this severe methodological detachment, the author engineered a computer-based *Tuning Parser Application* to programmatically re-tune multi-track digital music streams. This software framework bypasses speculative acoustic dogma, making it possible for the first time to directly hear, analyze, and empirically evaluate how any historical or original tuning configuration on any music work actually sounds when executed across highly complex orchestral structures. Lacking a like tool for the ear, a study in tuning is as if observing paintings in absolute darkness.

1.1. A Stranglehold Problem

Traditional musicology remains gridlocked by an over-reliance on the antiquated “Simple Ratio” hypothesis, leaving it unable to reconcile real-world physical and psychoacoustic phenomena. Academic dogmas treat the mathematical 2:1 interval (1200 cents) as a blueprint for structural purity, failing to recognize that this rigid integer model is a mechanical simplification that actively degrades the expressive and acoustic clarity of multi-voiced polyphony. Helmholtz’s assertion that musical intervals should be “beat-free” directly contradicts physical reality. Because natural sounds and the acoustic profiles of many musical instruments are inherently inharmonic, wave beats mathematically exist at all times; however, human listeners do not actively perceive these baseline frequencies. Multiple historical and neurological studies attempting to validate Helmholtz’s postulations have, instead, demonstrated the contrary. These systemic contradictions and their deviations from lived musical reality are comprehensively summarized within **Appendix B**.

1.2. The Thesis Statement

This thesis establishes a unified theory of intonation rooted in the dual mechanisms of pitch-wise isochronism and the interaction of higher harmonics processed via biological Phase-Locked Loops (PLL; see **Appendix H**) rather than traditional fundamental frequency (f_0) tracking. This framework separates musical sensation into two distinct, mutually exclusive biophysical mechanisms:

1) Subharmonic Neural Entrainment: The generation of precise vertical interval harmonization, “ringing”, is driven by a specialized biomechanical pathway where internal neural beats are generated when *incoming notes’ harmonic frequencies* align. This study isolates this pathway, demonstrating that rigid, f_0 -based simple ratios fail to produce these essential resonant beats (see Section 3.7).

2) Asymmetrical Semitone Manipulation: The structural generation of horizontal melodic affect and emotional mood is driven by systemic pitch offsets caused by shifting asymmetrical semitone boundaries, which generate friction against pitch-wise isochronism. By mapping these parallel pathways alongside a mathematically precise stretched octave system (≈ 1223.46 cents), this study provides a mechanistic explanation for the dynamic manipulation of human affect in mainstream Western music.

3) The Universality of Chordal Fusion: The phenomenon of harmonized ringing operates independent of any single, rigid historical intonation grid. It is a universal, phase phenomenon unlocked by any feasible interval arrangement—including computer-corrected 12-TET configurations—that satisfies the active precortical, peripheral PLL tracking criteria to generate an entrained neural beat frequency locked around 30 Hz. Consequently, the acoustic presence of ringing does not validate the structural correctness of an underlying tuning configuration, nor does its absence invalidate it.

4) The Dual-Scale Topology: This framework identifies two primary music-scale tuning profiles practiced in the Western tradition: the *singers’* tuning and the *violinists’* tuning. It explains why the singers’ scale accommodates both musical emotion and harmonized ringing, whereas the violinists’ scale accommodates only the former. Any structural deviation from 12-TET establishes an elastic topology of unequal semitones where, unavoidably, individual notes turn sharper or flatter on average. A composer can actively manipulate this asymmetry to control global emotional response—a mechanic applicable across certain regional folk and non-Western traditions that chiefly embrace linear melodies over vertical harmony. This microtonal manipulation remains the strict structural property of the composer; any performer who attempts to arbitrarily “improve” or dynamically warp the music’s intonation during execution can potentially destroy the composer’s conceived emotional architecture, “*turning him in his grave*”. For example, drone-anchored systems like traditional bagpipes utilize unique, non-Romantic scale architectures optimized for horizontal modal performance over a fixed drone rather than Western vertical movements, providing listeners with examples of music traditions incompatible with mainstream Western music.

Consequently, this thesis asserts that formal Western musicology operates under a fundamental structural deficiency by neglecting vertical acoustic entrainment—colloquially recognized within close-harmony traditions as the “ringing” effect—as a foundational pillar of musical architecture. Musical performance relies on a dual-vector axis: the linear execution of a precise, contrast-driven melody (the horizontal axis of affect) combined with the simultaneous initialization of subharmonic neural vibrations (the vertical axis of somatic entrainment). To treat intonation merely as a binary exercise in pitch centering within a sterile 12-TET grid is an institutional error. Without the deliberate activation of this subcortical resonance engine, polyphonic music is stripped of its physical and emotional depth, revealing that high-fidelity intonation relies on a dynamic, contrast-driven biological loop rather than an idealized mathematical grid.

1.3. The Evidentiary Scope of Validation

The evidence relies on direct mathematical modeling and raw acoustic data rather than subjective listening panels or statistical trials, providing precise biological models explaining why specific tuning choices cause instruments to ring, sound flat, or remain sterile.

To evaluate this emotional affect engine, this framework analyzes an expansive historical corpus spanning Western art music and vernacular close harmony. A systematic audit reveals a strategic manipulation technique executed through the stealthy, brief introduction of an accidental note that suggests a localized key change. Exploiting the listener’s intimate familiarity with the major scale structure, this deliberate structural intervention instantly swaps all semitone sizes simultaneously, inverting the boundaries of the large and small semitone streams to execute a profound emotional transition. While this mechanism is conspicuous within popular songs by artists such as The Beatles, it functions as a universal narrative engine across the monumental repertoires of Bach and Beethoven, verifying that musical affect is dynamically driven by calculated microtonal shifts operating independently of the standard equal temperament grid.

Consequently, this manuscript presents an integrated, multi-methodological study functioning across four distinct analytical layers:

1) Theoretical Framework: It establishes a biologically grounded, top-down cognitive template-matching paradigm that reflects the evolutionary processing constraints of the central nervous system. This model replaces the bottom-up beat-avoidance approach of Helmholtz, identifying the pre-cortical processes responsible for the macro-structural octave stretch and musical emotion.

2) The Beating Paradox as a Performance Hallmark: In elite *a cappella* quartet performances, audience acclaim is rarely directed toward the sterile delivery of chords that are “free from beats”. Instead, the highly praised acoustic phenomenon often described as “ringing”—which has yet to be satisfactorily explained in conventional literature—more closely correlates with perceptible vibration, resonance, or beating patterns. This study provides a comprehensive biological and

mathematical explanation of these phenomena in the subsequent sections.

3) Qualitative Empirical Study: The proposed mathematical models are evaluated against documented performance data from symphonic orchestras, vocal ensembles, and master string players across classical, popular, and folk genres, and are validated using existing literature. While cross-cultural examination demonstrates that many regional systems comply with the biological macro-octave rule, notable exceptions exist. Examples include traditions such as Scottish bagpipes or Peking Opera (Jingju), which feature deliberate deviations from this stretching model to prioritize rigid integer harmonic locks, continuous drones, or stylized timbral presentations.

4) Computational Audio Re-Synthesis: The underlying mathematical mechanics are verified via computational multi-track audio re-synthesis, utilizing published baselines of contemporary neurology to isolate the 30 Hz subharmonic engine. By programmatically re-tuning master musical architectures within the author's *Tuning Parser Application*, the study provides a proof-of-concept demonstration of complex orchestral textures being microtonally calibrated to align with hypothesized biological constraints. The application translates numerical pitch changes into real-time tuned pitch-wave frequencies via MIDI *pitchwheel* commands. This audio rendering allows the human central auditory nervous system to access decoded horizontal emotional elements and vertical somatic entrainment, shifting the evaluation from a purely theoretical framework to an empirical, audible validation.

Accordingly, all subsequent claims of “validation” or “proof” throughout this text refer to the unified convergence of these computational algorithms and qualitative performance case studies with the underlying biological framework.

1.4. The Reductionist Tuning Trap: Sensory vs. Mathematical Dogma

A primary structural deficiency within contemporary psychoacoustic literature is the proliferation of reductionist models seeking an isolated, mechanical cause for octave stretching. Beyond widespread attribution to physical string inharmonicity, a prominent example is the time-domain model proposed by de Cheveigné [1], which attributes the stretched perceptual octave to temporal processing limitations within the auditory brainstem. While sophisticated, such linear frameworks encounter a systemic operational limitation: they analyze acoustic data grids under the assumption of a rigid, mathematically pure 2:1 octave. This paper expands upon these foundations by asserting that blind faith in rigid mathematical symmetry—whether 12-TET configurations or cosmic constants such as the 2:1 and 3:2 integer ratios—functions as a dogmatic trap that is fundamentally detrimental to arriving at an acoustically viable solution.

Theorists must look beyond the reductionist framework of Helmholtz, which incorrectly asserted that musical consonance is governed exclusively by the static, algebraic cancellation of wave roughness in the auditory periphery. Evaluating

consonance or dissonance in isolation is functionally incomplete; both conditions are mandatory to create structural contrast, which serves as the primary operational element in artistic expression.

Furthermore, Helmholtz's dogmatic assertion that a clean, perfectly smooth sound wave represents the ultimate acoustic ideal stands divorced from human preference. The complex acoustic friction and subharmonic beats driving the prominent harmonized ringing of elite *a cappella* performances vanish instantly when singers transition into a singular unison note. The moment this structural variance is removed, musical tension and perceptual interest dissolve, verifying that auditory attention requires active geometric friction rather than sterile stillness (*for an extended historical re-examination of the Helmholtzian model, see Appendix B*).

Driven by a reliance on simple integer ratios—in particular, the 3:2 ratio between adjacent open string pairs—and years of executing sterile etudes owing to the historical absence of a violin equivalent to Chopin, string performers train to rely on mechanical muscle memory rather than active emotional attention. This tradition culminates in a severe cognitive shock when players confront their own recorded output, discovering suddenly, with disappointment and disbelief, the cold reality of their unpresentable execution outside the practice room. A parallel institutional failure also found in vocal education is documented by Michael and Gilman [2], whose findings on vocal pitch perception expose the systematic failure that occurs when academic acoustics loses its connection to the living, biological, and contrast-driven reality of human vocal performance. Ultimately, across both vocal and instrumental domains, an unchecked reliance on linear, rigid models has left contemporary musicology with a fractured theoretical landscape.

2. Methods

To validate the proposed neuro-mechanistic framework against historical and contemporary tuning systems, this study implements a multi-tiered computational and qualitative methodology.

The ultimate evaluation of the one identified correct microtonal framework can be implemented through a multi-stage execution model combining custom algorithmic parsing with primary auto-ethnographic validation. Since any known tuning architecture translates directly into physical fingerboard coordinates and vocal tract configurations, the primary investigator executes and records targeted melodic repertoires on a violin calibrated to the calculated stretched intervals. Furthermore, vocal execution demonstrates that the expected harmonized ringing effects are achieved both by systematically deploying the singers' tuning matrix and through conscious, real-time microtonal adjustments to neighboring vocalists (see Section 3.2.3). This physical and vocal execution confirms that the abstract mathematical parameters function as a reliable predictive matrix for physical performance, bypassing the constraints of isolated laboratory environments.

2.1. Algorithmic Scale Modeling

Mathematical modeling was executed to map prime-factor cancellation pathways across 5-limit *singers*' and 3-limit *violinists*' scale structures, isolating the harmonic configurations that dominate vertical, subcortical chordal harmonization. Horizontally, emotional-linked sensation is modeled through systematic friction against pitch-wise isochronism within melodies.

2.2. Dataset Filtration and Computational Architecture

Computational parsing evaluated a then smaller master database of over 1500 tuning profiles from the open-access scales.zip repository. This dataset was refined to a surviving subset of 300 discrete historical tuning proposals by applying a dual-stage filtering protocol that discarded all non-12-note systems and excluded configurations exceeding a predetermined standard deviation threshold calculated in Microsoft Excel. To evaluate these remaining temperaments across complex polyphonic structures, a bespoke *Tuning Parser Application* was engineered as an MS Excel Visual Basic for Applications (VBA) platform over a ten-year development cycle (see **Appendix F**). This application dynamically remaps multi-channel MIDI files onto specified microtonal matrices, enabling immediate auditory evaluation. A preliminary listening pass using this functional apparatus was sufficient to definitively falsify and reject all 300 historical proposals due to prominent acoustic artifacts and structural flaws. The historical absence of equivalent tools for the auditory scrutiny of novel tuning configurations highlights a severe methodological limitation in prior intonation research, where theoretical models were routinely published without ever being verified by ear.

To ensure technical transparency, the synthesis environment utilized three (four is available) concurrent software instances running via CoolSoft *VirtualMIDISynth* (see **Appendix J**), enforcing a hard ceiling of 45 simultaneous active notes, a maximum polyphony of 498 voices, and a global convolution reverb level of 127. The master SoundFont asset implemented, among other choices that include > 1 GB size libraries, was *Timbres Of Heaven GM_GS_XG_SFX V 3.4 Final.sf2*¹. Multi-track orchestral data was initially rendered at a sample rate of 48,000 Hz in uncompressed 24-bit.wav streams, which were subsequently balanced and normalized into high-fidelity 320 kbps compressed.mp3 tracks via the *Sound eXchange* command-line engine using the terminal script parameters:

```
"sox-14-4-2|sox" -m --norm=-0.1 input1.wav input2.wav input3.wav -C 320 final_output.mp3
```

2.3. Acoustic Overlay Verification via Visual Spectral Analysis

High-resolution pitch tracking and microtonal validation of archival solo string performance were executed via a digital acoustic overlay methodology, including the evaluation of Jascha Heifetz's recording of Tchaikovsky's *Violin Concerto in*

¹For a comprehensive review of systemic Equal Temperament (ET) intervallic deviations and its commercial necessity, please visit our open-access repository link found at the end of this document.

D Major, Op. 61. A reference MIDI performance track was programmed to match the mathematical coordinates of the proposed stretched-octave tuning matrix and played concurrently alongside the soloist's performance track. A dual-tiered audit was executed within Celemony *Melodyne*, utilizing its high-resolution tracking environment with all automated algorithmic snapping grids deactivated to allow for unconstrained microtonal analysis. Visual spectral grid inspection and auditory analysis verified pitch envelope congruence and the structural elimination of localized wave beats, empirically validating the physical parameters of the non-Pythagorean scale-shifting models. The link to some aforementioned and other tuned MIDI rendered demos can be found at the end of this document.

2.4. Epistemological Constraints on Replicability

While the Tuning Parser Application accurately executes the underlying math, the detection of musical effects can in no way rely on a computer. The twin pillars of music perception—pitch-wise isochronism for melody and binaural beat sensation for harmony—are processed within human biology. Because these faculties are hardwired into the subcortical nervous system, algorithms operating in a vacuum lack the sensory apparatus to evaluate emotional affect or chordal fusion. Consequently, verification requires a literate human ear to directly perceive the zero-beat vertical ringing and horizontal emotional shifts.

3. Results and Mathematical Models

The computational modeling and neurophysiological tracking data converge to establish a unified, mechanistic blueprint of musical intonation and affect. Moving away from traditional bottom-up frequency tracking, this framework demonstrates that the subcortical auditory apparatus possesses an integrated suite of biological decoding components hardwired to parse music as a structured, shared human language. Just as a spoken language relies on an intimate familiarity with a standardized alphabet to build words and convey semantic information, the musical language utilizes a deeply ingrained cognitive familiarity with the major scale structure—which, in turn, is built on two asymmetric six-tone whole-tone streams—to decode identifiable combinations of acoustic overtones, intervals, and mathematical microtonal adjustments.

Rather than delivering a static emotional standstill, music's sudden elevation or descent in pitch—disrupting pitch-wise isochronism—triggers a localized vestibular-somatic tension akin to a sudden loss of physical balance. Because the source of this physical sensory disruption remains hidden within the pre-cortical loop, the listener maps this subconscious somatic tension onto personal, complex psychological landscapes. Within the Western musical tradition, this communicative loop is structurally governed by two mutually exclusive vectors:

1) Melodic Geometry and Emotional Grammar (Horizontal Vector): The horizontal flow of musical narrative embeds emotional affect by actively shifting the boundaries between two discrete, independent six whole-tone streams (see

Section 4.2). This structural asymmetry alters the physical dimensions of the scale's *asymmetrical semitone pairs*, microtonally sharpening or flattening sequential notes to drive psychological valence. This *elastic topology* creates the active sensory contrast required to communicate shifting emotional states, bypassing the sterile symmetry of Equal Temperament.

2) Chordal Fusion and Biological Phase-Locking (Vertical Vector): The vertical synthesis of multi-voice harmony operates via a biological Phase-Locked Loop (PLL) mechanism initiated at the front-end peripheral auditory gate. This sub-millisecond synchronization utilizes multiquantal release at the hair cell ribbon synapses to entrain, and hence amplify, overlapping, down-scaled upper harmonics of chordal notes into a unified frequency alignment prior to ascending brainstem arrival, converting acoustic emergence into a stable, unified binaural-beat sensation (see Sections 3.6, 3.7, **Appendix C**). Acoustic *ringing* is the somatic manifestation of this neural entrainment. While this mathematical condition is satisfied in the singers' tuning, it is inhibited in the violinists' tuning. Electronic ET rendering proves to satisfy this condition, and likely also multiple other tuning configurations. The presence or absence of ringing does not relate directly to the structural validity of a tuning matrix. This ≈ 30 Hz entrainment frequency is continuously sustained throughout melodies and vertical chord progressions via the subcortical Frequency-Following Response (FFR; see **Appendix J**).

Because these biophysical mechanisms are hardwired into the shared anatomy of all human listeners and performers, they form the universal foundation of music as a communicative language. The systematic manipulation of respectively, friction against pitch-wise isochronism and pre-cortical, phase-locked variables, is how musicians transmit complex structural cues that are decoded by a listener's nervous system as vivid musical emotions. The subsequent sections provide the precise mathematical, computational, and neurophysiological validation of this integrated communicative network.

3.1. Hypothesis 1: The Non-Integer Geometry of the Musical Octave and the Evolutionary Avoidance of Auditory Fusion

This hypothesis establishes a unified neuro-mechanistic architecture demonstrating that human interval perception and musical affect are governed by subcortical processing constraints rather than static integer grids. The model is structured across twelve core evidentiary arguments:

1) The Rebuttal of Simple Ratios: This study rejects small-integer frequency ratios (such as 2:1 and 3:2) as default sources of acoustic purity. These rigid boundaries are abstract human simplifications that fail to reflect biological auditory constraints.

2) The Operational Redefinition of Consonance: Intervals traditionally classified as "consonant" lack positive sensory evidence when compared to their alternative microtonal counterparts.

3) The Destructive Trap of "Pure" Intervals: Far from enhancing clarity, dog-

matic alignment to un-stretched pure intervals corrupts sound integrity. By forcing overtones to merge perfectly, pure ratios trigger immediate acoustic degradation via harmonic capture.

4) Dismantling the Inharmonicity Narrative: This section rejects the dominant belief that physical string stiffness is the sole cause of octave stretching—a topic divorced from the music art. Data confirms stretching is a primary biological mandate driven by neural wiring, functioning independently of instrument material (see Section 4.1).

5) Exposing the Fundamental Frequency (f_0) Fallacy: This framework disproves the reductionist assumption that physical tracking of f_0 serves as the primary determinant of perceived musical pitch. Subcortical tracking relies instead on top-down processing of the upper harmonic spectrum (see Section 3.2).

6) The Rejection of Dynamic Tuning: This study argues against the convention of dynamic tuning, where performers constantly warp individual notes to fit vertical chords. Chords are vertical melodies. Forcing pitches to deform to satisfy an immediate chordal block destroys horizontal narrative coherence and breaks structural emotional expression.

7) The True Physics of Chordal Ringing: Pristine intonation does not optimize for the avoidance of acoustic beats. True chordal and harmonic ringing relies on the active generation of textured, subcortical *neural beat* entrainment clustered around the 30 Hz threshold (see Sections 3.6, 3.7, **Appendix C**).

8) Challenging Open-String Pythagorean Dogma: Tuning a violin's adjacent open string pairs to precise 3:2 fifths introduces permanent intonation contradictions that trap the player and compromise polyphonic clarity (see Section 3.8.3).

9) Validating the Historical Leading-Tone Tradition: The intuitive advice of legacy string masters stating that the leading tone must be sharpened represents a real-world reaction to the physical existence of asymmetrical semitone pairs (see Section 3.7.2 and 3.7.3).

10) The Auditory Archetype and Emotional Mapping: Auditory memory utilizes the major scale as a fixed, top-down cognitive template. Musical affect is decoded the moment the brain tracks the microtonal average pitch deviation of an interval relative to the last resolved anchor note (see Section 4.5).

11) The Biological Whole-Tone Anchor and Container of Affect: The 9/8 whole-tone interval is anchored in a hardwired peripheral acoustic process driven by Distortion Product Otoacoustic Emission (DPOAE) Series B mechanism, formulated as:

$$\text{Series B} = n \cdot f_2 - (n-1) \cdot f_1$$

(see detailed analysis in Section 3.3). This biological anchor functions as the primary mathematical container required to host the asymmetrical semitone pairs whose microtonal shifts drive musical emotion. This physical framework is robustly validated by Wit's [3] 92.02 cents measurement, which correlates near-perfectly to the calculated small semitone (135/128, approx. 92.18 cents), and by baseline emission tracking from Burns, Strickland, Tubis and Jones [4], which identified

a 110.65 cents microtonal cluster that matches the calculated large semitone (16/15, approx. 111.73 cents). This powerful convergence demonstrates that the dual-stream semitone asymmetries governing Western musical affect are directly anchored within the physical infrastructure of human peripheral hearing (see Sections 3.3, 3.4, 4.2). Symmetrically, the sum of a large and small semitone constitutes a singular whole-tone, 9/8, six of which generate the definitive stretched octave measuring 1223.46 cents, invalidating the traditional 1200-cent octave dogma.

12) The Frequency Following Response (FFR) Framework: The primary sub-cortical mechanism utilized to track and inherit vertical microtonal harmony, sustain its continued integration, and provide active guidance in intonation within the singers' tuning framework (see **Appendix J**).

3.1.1. The Capture Zone Spectrum and High-Definition Polyphonic Voice Segregation

The properties of acoustic absorption span a biologically defined capture zone. The mechanical dimensions of this boundary align with historical critical bandwidth guidelines that isolate a critical capture region spanning approximately one percent of the target frequency, or roughly 17 cents. Within this critical window, independent frequencies are captured and suppressed by the dominant fundamental template. An octave stretch optimized to 23.46 cents positions the higher interval outside this 17-cent biological capture threshold. This structural divergence functions as an acoustic wedge, introducing a microtonal asymmetry that prevents subcortical phase-locking between independent melodic lines. By escaping this low-level capture zone, the framework ensures high-definition polyphonic voice segregation, even during dense, complex orchestral unisons where spatial, phase, and amplitude cues are heavily muffled.

3.1.2. Evidence from the Helsinki Subjective Tuning Experiment

The independence of octave stretching from physical string inharmonicity was demonstrated by a landmark psychoacoustic study conducted by Jaatinen and Pätynen [5] at the University of Helsinki. This investigation decoupled physical string behaviors from human cognitive pitch templates through the following experimental methodology:

- **The Inharmonic Vector (Control):** Using a Steinway Model D grand piano, researchers captured natural, inharmonic acoustic piano tones shaped by standard physical string stiffness, generating a physical tuning curve, closely following a standard Rainsback drift.
- **The Harmonic Vector (Experimental):** Utilizing high-precision digital signal processing, researchers stripped natural string inharmonicity via additive synthesis, rendering the upper partials perfectly harmonic and locked to strict integer values. This intervention yielded a treated physical tuning curve that collapses to a flat baseline, directly contrasting with the control configuration.
- **The Perceptual Bottleneck:** Professional piano technicians and orchestral musicians were tasked with executing full tuning arrays across both independent

keyboard configurations. Across both pathways, perceptual blindness occurred; the professional tuners remained unable to distinguish between the harmonized and inharmonic tones by ear. Despite this total subjective masking, the resulting physical tuning curves diverged significantly across the instrument's registers. This outcome proves that the human central auditory processor interprets identical fundamental frequency (f_0) differently depending on its upper spectral landscapes. Inharmonic tones are naturally perceived as sharper in pitch compared to their perfectly integer-aligned harmonic counterparts, with this perceptual microtonal expansion accelerating up through the highest register ($C\#^7$).

- Figure 1 (The Helsinki Re-Interpretation) Analysis:** A comparative analysis of **Figure 1** data tracking in [5] illustrating harmonized versus original inharmonic acoustic piano tuning matrices, reveals a profound operational constraint in traditional aural tuning protocols. The blue line denotes the subjective octave enlargement threshold and the red line represents the objective tuning curve. The mathematical non-compliance across these datasets—where the objective tuning error and subjective stretch curves fail to algebraically balance—reveals that the aural tuning method, admitted in [5], is driven by the minimization or elimination of physical wave beats between overlapping upper partials. Consequently, the tuner never targets an abstract, independent pitch coordinate. Because string stiffness forcefully sharpens the upper harmonic partials on the inharmonic keyboard, the resulting objective f_0 path merely tracks the mechanical adjustments required to eliminate acoustic roughness in the periphery. Therefore, the fundamental frequency grid of an acoustic piano operates as an artifact of structural friction minimization rather

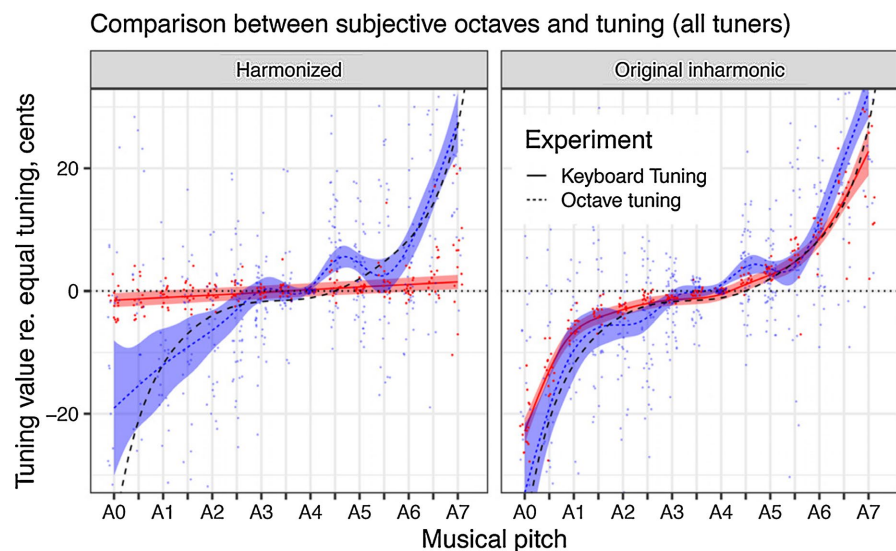


Figure 1. Comparison between the results of subjective octave listening experiments and tuning experiments across all subjects (adapted from Jaatinen and Pätynen [5]). The solid red line and opaque dots denote acoustic piano tuning; the dotted blue line represents the subjective octave enlargement threshold; the dashed reference line isolates the standard Railsback curve, with dots highlighting individual measured values.

than an intentional, musically relevant pitch scale. The data provide definitive proof that peripheral f_0 tracks do not represent true biological pitch perception, validating the necessity of a top-down subcortical template-matching architecture.

- **The Randomization of Inharmonic Fundamental Paths:** Because the conventional manual tuning process is driven by the relative elimination of localized wave beats rather than aligning to a predefined pitch grid, there are mathematically infinite structural combinations capable of satisfying this acoustic condition. This operational flexibility is confirmed by the data from [5], where independent fundamental data points across different testing trials fail to overlap. Consequently, the resulting fundamental pitch path across an acoustic piano keyboard behaves as a randomized mathematical artifact. It functions decoupled from any systematic musicological scale guidelines, standing purely as a mechanical compromise to mitigate physical wire stiffness.
- **The Discrepancy of Additive Subjective Curves:** The subjective size measurement of the musical octave within a controlled group of subjects is mathematically expected to remain stable. Between the dual measurements in **Figure 1**, the enlargement threshold on the harmonic keyboard (Left Curve) at a given pitch coordinate should algebraically equal the linear sum of the baseline subjective stretch (Right Curve) plus the objective inharmonicity error (the Railsback drift). Instead, the data reveal a massive discrepancy at pitch A1 (-10 cents versus -18 cents). This geometric discrepancy remains conspicuous across the entire testing register.
- **The Neurobiological Explanation of Octave Inhibition:** The underlying cause of this experimental instability is rooted in the evolutionary design of the human central auditory pathway. The brainstem does not possess a direct, low-level sensory hardware grid engineered to isolate a pure 2:1 octave; a raw, unstretched 2:1 interval sensation is actively inhibited by core survival and sound-segregation instincts to prevent environmental acoustic masking. Instead, the hardwired facility within the peripheral auditory pathway is optimized to calculate the geometry of the 9/8 whole-tone step via the Distortion Product Otoacoustic Emission (DPOAE) engine (see Section 3.1.3). True macro-octave recognition is subsequently reconstructed at higher cognitive stages by combining this precise 9/8 whole-tone baseline with the phenomenon of isochromatic stream tracking. Because traditional laboratory tuning tests attempt to evaluate a complex, high-level structural template by playing isolated single intervals in a sensory vacuum, they yield conflicting data. One can no more measure a continuous biological pitch gradient using an abstract two-note test than measure the continuous duration of a steady drumbeat pattern by playing just two isolated strokes.
- This divergence highlights a systemic blind spot within contemporary acoustic literature: the historical and methodological over-reliance on the piano as the primary apparatus for intonation experimentation. Scholars have routinely at-

tempted to resolve tuning anomalies by modifying keyboard architectures, adding microtonal keys, or engineering complex, fixed temperaments. However, from an acoustic standpoint, the piano represents a compromised system; its fixed mechanical design, combined with intrinsic string inharmonicity, establishes a structural hurdle that actively suppresses dynamic intonation. Any fixed tuning configuration inherently constrains artistic expression. This mechanical limitation is not isolated to keyboard instruments; the rigid, fixed 3:2 open-string tuning framework enforced upon the violin creates a parallel acoustic hurdle, driving the historical emergence of the compromised “violinists’ tuning” as players attempt to navigate these structural constraints. Ultimately, the uncompromised human voice remains the definitive musical instrument, uniquely capable of bypassing rigid mechanical grids to execute the alternating microtonal adjustments required for true horizontal affect and vertical somatic entrainment

3.1.3. Cochlear Inscription of Asymmetric Semitones: The Wit & Burns Ratios

Dividing a 9/8 whole tone requires an asymmetrical arrangement of unequal semitones across pitch space. Under the singers’ tuning matrix, this partition yields a large semitone of 16/15 (≈ 111.73 cents) and a small semitone of 135/128 (≈ 92.18 cents). Conversely, the violinists’ tuning matrix generates a compressed small semitone of 256/243 (≈ 90.22 cents) and an expanded large semitone of 2187/2048 (≈ 113.69 cents).

This peripheral footprint is validated by Wit [3], whose resonance measurements isolate a 92.02 cents boundary matching the calculated small semitone 135/128 (≈ 92.18 cents), symmetrically reinforced by the emission tracking of Burns, Strickland, Tubis, and Jones [4], which identifies a 110.65 cents cluster matching the calculated large semitone (16/15, ≈ 111.73 cents). These datasets converge to prove that the 92.18 and 111.73 cents semitones are permanently encoded like a *hardware ROM architecture*, summing to the 9/8 whole tone anchored within the physical infrastructure of human peripheral hearing.

Another peripheral hardwiring is demonstrated via Distortion Product Otoacoustic Emission (DPOAE) Series B calculations, formalized by the tracking equation:

$$\text{Series B} = n \cdot f_2 - (n-1) \cdot f_1$$

Evaluating a 9/8 whole-tone interval where $n = 10$ ($f_1 = 8$ Hz, $f_2 = 9$ Hz) yields:

$$10 \cdot f_2 - 9 \cdot f_1 = 10(9) - 9(8) = 90 - 72 = 18 \text{ Hz}$$

Because f_2 is 9 Hz, the 18 Hz emission lands precisely at $2 \cdot f_2$, exactly one octave higher. Thus, a 9/8 whole tone forces the cochlear outer hair cells to generate an acoustic tracking signal an octave above the higher input frequency, providing mechanical proof of this tuning anchor.

Under *evolutionary optimization*, if a physiological pathway exists to achieve a sensory goal, biology inevitably utilizes it. The auditory system deploys this physical infrastructure to identify the 9/8 interval as the primary *pitch ruler* of musical scales.

3.2. Hypothesis 2: Multi-Harmonic Pitch Synthesis and the Dual-Path Neurological Architecture of the Missing Fundamental

Natural acoustic signals exhibit microtonal shifting across the upper overtone spectrum, yielding divergent f_0 difference values among adjacent partial interactions. Within dense, noisy environments where temporal cues fail, the subcortical processor must dynamically select, segregate, and extract the precise upper harmonic components deemed most cohesive to anchor global pitch perception. As the central auditory loop decodes high-frequency harmonic structures to reconstruct stable virtual pitch envelopes—especially among competing instruments playing identical notes or octaves—traditional 2:1 integer identification presents a structural obstruction and must be actively inhibited. This synthesis operates independently of physical f_0 tracking, a biophysical reality supported by the neuroauditory findings of Albera *et al.* [6] and Oxenham [7], which verify that robust pitch perception remains operational even when lower spectral regions are masked or absent.

3.2.1. Pathway 1: Mechanical Analog Extraction and Cochlear Cubic Distortion Products

When physical energy at f_0 is removed from an acoustic signal, human pitch perception remains intact because the auditory brainstem extracts the calculated difference between the remaining adjacent higher partial pairs. The auditory architecture employs two distinct, parallel processing pathways to resolve this missing fundamental data and stabilize pitch perception. The first pathway operates via nonlinear mechanical interactions within the peripheral cochlear fluid, driven by Distortion Product Otoacoustic Emissions (DPOAE). The internal mechanical architecture of the cochlea utilizes the cubic distortion product formula to physically regenerate the missing base frequency from high-level partial pairs:

$$f_{\text{DPOAE}} = n \cdot f_1 - (n-1) \cdot f_2$$

For example, assuming a missing fundamental frequency defined at $f_0 = 100$ Hz, mechanical interactions of adjacent upper components reproduce the base node seamlessly: upper partial pair A ($f_1 = 200$ Hz, $f_2 = 300$ Hz) where $n = 2$ yields $2 \cdot (200) - 1 \cdot (300) = 100$ Hz, while upper partial pair B ($f_2 = 300$ Hz, $f_3 = 400$ Hz) where $n = 3$ yields $3 \cdot (300) - 2 \cdot (400) = 100$ Hz.

3.2.2. Pathway 2: Digital Neural Extraction via Brainstem PLL Divisor Tracking

The second pathway operates as a digital neural computation within the brainstem. Because any given higher partial functions as a strict integer multiple of the base identity ($f_n = n \cdot f_0$), the base value is extracted via neural division:

$$f_0 \approx \frac{f_n}{n}$$

The subcortical Phase-Locked Loop (PLL) tracking mechanism utilizes a neural

divisor (n) to lock onto the phase profile of the incoming high-frequency partial. Every individual harmonic tracking loop outputs a synchronized neural firing pattern matching the period of f_0 . These individual nerve fibers then converge and entrain downstream within the brainstem, creating a stable perception of the target pitch. For an incoming complex waveform missing f_0 , partial f_2 (200 Hz) routes through the PLL neural divisor where $n = 2$, partial f_3 (300 Hz) where $n = 3$, and partial f_4 (400 Hz) where $n = 4$, all concurrently outputting a neural frequency marker at 100 Hz to trigger downstream perceptual convergence.

This framework introduces an original processing mechanic unrecognized within existing psychoacoustic literature: the operational rule governing how the central processor selects the definitive pitch anchor from a sea of stretched, slightly conflicting overtone intervals. This study posits that the auditory system resolves this ambiguity through a competitive, resonance-driven tracking grid. Whichever single, isolated difference frequency finds itself most densely phase-locked and entrained to the concurrent, divided-down neural outputs of other active tracking channels—expressed as:

$$\frac{f_x}{x}, \frac{f_y}{y}, \frac{f_z}{z}$$

is amplified above all competing inputs. The more individual divided-down channels that converge upon and reinforce a specific difference frequency the more robust, stable, and vivid the selection of that common value becomes as the identified pitch of the note. This multi-channel subharmonic resonance directly solves the selective bottleneck of pitch computation. This mechanical capability is directly supported by the findings of [6] and [7].

3.2.3. Cross-Voice PLL Convergence and the Mechanics of “Ringing” Harmony

If the perception of an isolated pitch is structurally dependent on upper harmonic alignment, chordal harmony must also be processed through the interaction of harmonics rather than fundamental frequencies. This brainstem Phase-Locked Loop (PLL) pathway provides the biological explanation for how pristine chordal harmony is achieved within the singers’ tuning matrix. Rather than targeting a static frequency block, all incoming upper harmonics undergo a dynamic sorting process. The entrainment phenomenon acts as the definitive selective device where independent nerve fibers down-divide shared upper harmonics by whole integers into a unified, resonant subharmonic frequency within the 30 Hz proximity (see Sections 3.6, 3.7, and 6.3). A relationship between concurrent notes is established when divided-down signals hit a shared frequency coordinate localized near the 30 Hz threshold. Once a fiber from each discrete note carries an identical frequency within this proximity window, the tracking channels abandon random behavior; the shared subharmonic target initiates immediate, stable phase convergence, forcing the separate fiber populations into subcortical neural entrainment that manifests perceptually as a resonant acoustic ringing.

This subcortical architecture operates as a frontline analog computing network. Unlike digital systems that require incoming acoustic data to be artificially encoded into discrete values for processing within a central CPU, living biology executes complex geometric analysis directly at the sensory periphery, unlocking instantaneous identification of the musical vocabulary. This three-fold proximity—driven by the structural 12-note uniformity, Bell’s [8] documented 1.06 cochlear ratio peaking within spontaneous otoacoustic emissions, and a 500-cent fourth coincidence—prompts subcortical channels into locking onto the stable electronic wave-envelope despite underlying grid constraints.

Bell’s 1.06 baseline acts as the median biological anchor around which Wit’s 1.055 (135/128) and Burns’ 1.066 (16/15) asymmetric ratios naturally cluster; meanwhile, the mechanical 12-TET perfect fourth (500 cents) is practically identical to the biological singers’ scale interval, expressed as:

$$\text{Fourth}_{\text{ET}} = \text{Fourth}_{\text{Singers}} = \left(\frac{9}{8}\right)^2 \times \frac{135}{128} \approx 499.9987 \text{ cents}$$

yielding a negligible micro-cent error of just 0.0013 cents. Complementarily, the mechanical 12-TET perfect fifth (700 cents) functions as the natural mathematical inversion of the perfect fourth, formulated as

$$\text{Fifth}_{\text{ET}} = \left(\frac{8}{9}\right)^2 \times \frac{128}{135} \times 2 \approx 700.0013 \text{ cents}$$

Furthermore, unlocking harmonized ringing is not restricted to standard textbook intervals; any frequency ratio that successfully generates a matching 30 Hz proximity beat will initiate this acoustic state. An expansive lattice of fractional ratios exists within the margins of any given interval size, as demonstrated by the 15 distinct fractions mapped within a tight ± 0.0015 cent threshold around 300 cents detailed in **Appendix D, Table A1**.

This mechanical behavior is empirically supported by longitudinal field observations within a church congregation of over 1000 non-expert singers, where prominent harmonized ringing was routinely initiated simply by microtonally adjusting to random nearby vocalists. This field observation, reproducible by experienced *a cappella* vocalists, invalidates the common assumption that harmonized ringing acts as proof of a singular, correct historical intonation grid. This dynamic model is further verified by comparing acoustic density profiles between multi-track, software-tuned recordings and unedited live vocal tracking loops: the former exhibits an artificial, “solid but scarce” ringing architecture, whereas live performances deliver a significantly denser, highly active, though “less secure” ringing matrix. High-fidelity musical harmony is a physical, somatic phenomenon triggered when precise interval layouts lock the biological hardware of the human brainstem into synchronization.

3.2.4. A Negative Control Verification and the Digital Phase Bottleneck

A critical negative control marker within this verification framework occurs dur-

ing passages of unison. When all ensemble members dynamically transition to sing a singular note in octave unison, the emergent 30 Hz shimmering beats immediately cease, and the localized cross-voice wave interaction vanishes. This systematic cessation confirms that the entrained 30 Hz tracking loop is a structural product of interacting microtonal interval deltas, collapsing predictably into acoustic stillness the moment the interval variance is removed.

This acoustic phenomenon is demonstrated by a digital baseline experiment: attempting to play two identical instances of a single *wav* file on a computer in mathematical synchrony fails to produce a clean doubling of acoustic amplitude. Instead, minute hardware-driver latency and phase convergence trigger severe comb filtering and phase cancellation, yielding a hollow, metallic, swirling sound artifact historically used in cinema soundtracks to emulate a “man-in-space” radio effect. This destructive cancellation occurs whenever electronic multi-track synthesis forces separate instrument layers into a state of un-stretched integer unison or rigid 2:1 octave alignment. This sterile bottleneck is never encountered during live performance contexts. Because human performers possess organic, continuous microtonal pitch fluctuations and dynamic phase variations, their acoustic outputs naturally bypass this phase trap. This biophysical variance explains why a mathematically rigid, un-stretched MIDI rendering of a complex masterwork—such as a Beethoven Symphony—sounds flat and aesthetically un-enjoyable, proving that human musical communication relies on active microtonal contrast rather than static digital precision. The introduction of a 23.46 cent stretch to each octave eliminates this artifact absolutely, though its artistic vitality is only fully realized when the singers’ tuning is applied to restore its true live-performance sensation and resonance.

3.3. Hypothesis 3: The Architectural Necessity of Asymmetric Semitones and the Biophysical Engine of Musical Affect

A central question in structural musicology is why the standard diatonic framework is composed of an asymmetrical arrangement of whole tones and semitones rather than a uniform sequence of twelve equal semitones. This paper asserts that alternating interval dimensions constitute the definitive biophysical requirement for musical affect, emotion, and mood.

3.3.1. The Acoustic Sterility of Pure Isochronism

An informative parallel exists between pitch intonation and structural symmetry in percussive music. In rhythm, a perfectly uniform stream of percussion strikes lacks expressive capacity; identical timing and dynamic weight trigger sensory habituation and cognitive boredom. Metric interest and somatic engagement require pitch-wise syncopation via the strategic grouping or shifting of beats.

The tonal equivalent of a monotonous rhythmic stream is the whole-tone scale, where all intervals conform to a uniform 9/8 ratio, or the standard Equal Temperament (12-ET) grid, where all semitones are forced into identical 100-cent steps. Because uniform 9/8 whole-tone scales and monolithic ET matrices induce zero

internal microtonal friction or structural variance, these configurations lack internal structural boundaries or directional microtonal pull, resulting in emotional sterility and an acoustic vacuum. This sterile property of uniform scales is routinely exploited in cinematic underscores to convey suspended reality or existential disorientation—such as the animated descent of Alice into the bottomless pit in *Alice in Wonderland*.

3.3.2. The Twin Whole-Tone Streams and Stretched Chromatic Geometry

To generate cognitive engagement, the musical scale must function analogously to rhythmic syncopation. The functional twelve-note chromatic scale is not a series of twelve equal steps; rather, it is synthesized by two independent six-note whole-tone (9/8) streams that interlock with a slight, non-symmetric micro-tonal offset. Because each 9/8 whole-tone unit is formed by compounding two structurally distinct, unequal semitones as:

$$a \times b = c$$

the entire geometry of the scale expands, stretching the octave boundary to the coordinate:

$$\text{Octave}_{\text{chrom.}} = \left(\frac{9}{8}\right)^6 \approx 2.0272865 \approx 1223.46 \text{ cents}$$

Because the mathematical product of the two unequal semitones must equal 9/8, the intervals alternate across the scale steps. In a standard major scale, this creates a major structural disparity between the lower and upper tetrachords (see **Figure 2**):

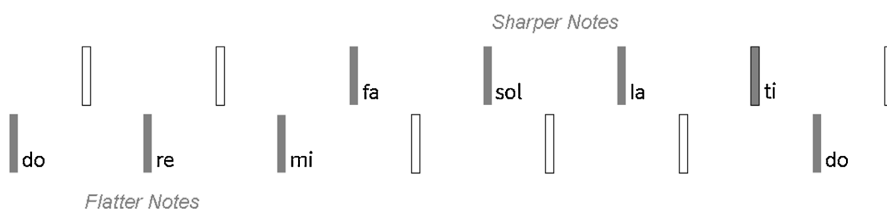


Figure 2. “*ti*” being more sharp, upper tetrachord has on average a slightly higher pitch.

- **The Lower Tetrachord (*do, re, mi*):** This boundary is governed by a wider semitone configuration between *mi* and *fa*, rendering these initial scale steps flatter relative to the mathematical midpoint of the pitch field. This layout yields a restful, resolved, and somatically grounded pitch profile.
- **The Upper Tetrachord (*fa, sol, la, ti*):** This boundary is compressed by a smaller semitone setup between *ti* and *do*. As a direct result, the leading tone (*ti*) is driven significantly sharper, lifting the entire upper tetrachord into a sharper pitch register characterized by heightened microtonal tension, and a strong directional pull toward a harmonic climax.

3.3.3. The Anatomy of the Emotional Shift: Tetrachordal Gradients

The difference between these two semitone boundaries (111.73 cents - 92.18 cents

= 19.55 cents) serves as the core driver of musical emotion. This 19.55-cent structural friction acts as a tonal lever. Shifting a melody into the sharper upper tetrachord increases tension toward an emotional climax, whereas returning to the flatter lower tetrachord signals physical relaxation and somatic rest.

The biological reason for positioning the flatter profile in the lower block is rooted in harmonic optimization. The wider semitone spacing in the lower tetrachord makes it easier for the auditory Phase-Locked Loop (PLL) to generate stable neural beat frequencies for the foundational tonic chord compared to the dominant chord, a mechanical interaction formalized through the calculations detailed in Section 3.7.

3.3.4. The Chromatic Key-Shift Illusion Example

The presence of alternating large and small semitones is demonstrated in the microtonal architecture of masterworks like Mussorgsky's *Pictures at an Exhibition*. When the rapid woodwind chromatic run near the end of the opening movement, *Gnomus* (The Gnome)—designed to depict the erratic, staggering gait—is programmatically rendered through the custom tuning engine, the alternating physical dimensions of the intervals become structurally apparent. This framework models key modulations directly via MIDI key number transitions, where C major is defined as key 0, G major as key 1, and F major as key -1.

Because the chromatic layout is synthesized by two interlocking six-note whole-tone streams, there are two structural possibilities for asymmetric semitone distribution across the register grid. Consequently, any key modulation from an even to an odd MIDI key number instantly forces the large and small half-tones to swap physical locations. This organic scale shift triggers a distinct mood transmutation, altering the emotional landscape of the melody. Conversely, on a uniform 12-TET grid, the scale presents monolithic, identical intervals; it remains mood-neutral, processing key changes as mere frequency shifts without changing the underlying emotional character of the music.

3.4. Hypothesis 4: The Physiological Origin of the 9/8 Whole Tone and the Biological Geometry of the Six-Tone Stretched Octave

A central mystery in historical musicology is why human culture universally gravitated toward the 9/8 ratio (≈ 203.91 cents) as the foundational building block for the whole-tone interval. This paper asserts that the twelve-note chromatic scale is a biological and mathematical coincidence driven by the mechanics of Distortion Product Otoacoustic Emissions (DPOAE) within the human cochlea, where the inner ear possesses an inherent physical facility to validate the structural geometry of the interval [13]. A uniform series of 9/8 whole tones contains no internal microtonal friction and remains sterile on its own. The evolutionary purpose of this hardwired 9/8 whole-tone framework is to act as the macrostructural biological container required to host two unequal, alternating semitones: the large semitone of 16/15 (≈ 111.73 cents) and the small semitone of 135/128 (≈ 92.18 cents).

As formalized by the nonlinear intermodulation equations detailed in Section 3.1.3, compounding six identical 9/8 whole-tone units—each synthesized by the mathematical product of these asymmetrical semitones—expands the scale architecture to the coordinate:

$$\text{Octave}_{\text{Calculated}} = \left(\frac{9}{8}\right)^6 \approx 1223.46 \text{ cents}$$

This mathematical reality explains the biological necessity behind the traditional architecture of the Western scale: a musical octave must measure six whole tones, accumulated incrementally across a precise micro-cent trajectory (203.91, 407.82, 611.73, 815.64, 1019.55, and 1223.46 cents).

In raw listening tests, when a complex musical excerpt is played back across general temperament widths of 1180, 1200, 1210, or 1220 cents, the average human mind does not register a jarring, immediate intonation problem. What makes an interval layout truly “in-tune” is not a passive adherence to a static octave boundary. Rather, it is the precise micro-cent size identification of the individual alternating semitone components within the 9/8 whole-tone units that constitutes the fundamental alphabet of the musical medium.

This structural dependency establishes a definitive processing hierarchy governing how all musical intervals operate across the human auditory pathway. The foundational 9/8 whole tone is identified by biological hardware, hardwired directly into the non-linear intermodulation mechanics of the cochlea. Conversely, the three foundational semitone dimensions are stored within a subcortical neural RAM architecture, where the ear actively maintains the 1.055, 1.060, and 1.066 frequency clusters to represent the small, equal-temperament, and large semitone boundaries. All other musical intervals across the scale architecture are evaluated dynamically in real time against one another using this dual *hardware-RAM* baseline. When these structural alphabets are combined with an acquired cognitive grammar, the real-time artistic manipulation of these unequal semitone boundaries forces the subcortical apparatus to execute a sudden gain or loss of altitude—acting like an airplane passenger experiencing a sudden loss of physical balance—which triggers the internal somatic tension that delivers genuine communicative meaning.

3.5. Hypothesis 5: The Unified Intonation Principle— Cross-Domain Frequency Invariance and the Neural Mechanics of FFR Melodic Continuity

A foundational debate in traditional tuning theory concerns whether musical pitches must alter their frequencies depending on context—whether a note must shift its micro-cent coordinate when transitioning from an isolated melodic line to a concurrent chordal cluster. This paper rejects this contextual fragmentation, establishing instead the Unified Intonation Principle: for any sequence of musical tones to be perceived as genuinely in tune, the micro-cent intonation of the respective notes must remain invariant whether they are executed within a simulta-

neous chord (harmony) or a sequential stream (melody).

The cognitive mechanisms decoding structural harmony and pitch-wise horizontal isochronism are unified within the biological hardware of the human auditory pathway, where being in tune is defined by the stability of shared, downscaled neural tracking frequencies. Within the vertical chord context, concurrent partials interact mechanically, routing directly through the pre-brainstem Phase-Locked Loop (PLL) downscaling matrix; concurrently, within the horizontal melodic context, the Frequency-Following Response (FFR) extrapolates prior pitch memory across the temporal gap. These pathways converge downstream to trigger immediate neural entrainment, establishing a sustained, cognitively stable “in-tune” identity.

3.5.1. The Vertical Context (Chordal Harmony)

When multiple notes are struck concurrently, the high-precision PLL in the brainstem downscale the frequencies of the fundamental nodes and their upper partials into lower tracking streams. When the interval geometry is precise, these downscaled neural signals achieve immediate phase alignment, causing the nerve fibers to entrain with one another. This lock-in condition generates a sustained, stable internal resonance that the conscious mind experiences as pure, fused harmony.

3.5.2. The Horizontal Context (Melodic Continuity)

Historically, acoustic models failed to explain how a listener accurately evaluates pitch precision within a sequential mono- or polyphonic melody. Because successive notes do not sound simultaneously, there are no overlapping physical waveforms to generate direct acoustic interference or beat cues. Even when environmental room reverberation is taken into account, acoustic data are too chaotic to explain the sub-cent precision with which the human mind detects a melody being out of tune.

Consequently, horizontal melodic pitch evaluation relies primarily on pitch-wise isochronism, tracking microtonal friction against uniformity among note intervals. This primary mechanism is supplemented within low-frequency registers by the subcortical FFR, which retains neural profiles of the preceding pitches, including copies of the ≈ 30 Hz binaural beats if present. By broadcasting these distinct subharmonic neural footprints directly into the incoming tracking loops of succeeding tones, the subcortical apparatus ensures that the human nervous system does not process sequential pitches as isolated, independent events. These two processes present an answer to prior studies by Bregman [9], Oxenham [7], and Tramo, Shah and Braida [10], resolving the radical perceptual transmutation that occurs when a monophonic melody is embedded within a polyphonic accompaniment.

3.5.3. The Low-Frequency Bottleneck and Information Encoding

Because the physiological firing rates of the auditory nerve pathway are structurally limited, the FFR cannot simultaneously transmit a massive cascade of higher

harmonic data from one melodic note to the next. The system faces a strict information bottleneck. To maintain structural continuity across a horizontal line, the FFR restricts its primary tracking transmission to a highly compact data set containing possible cases of two ≈ 30 Hz beat frequencies (as detailed in Section 3.2.3 and Section 3.7), one beat frequency in cases of a triad chord or a monophonic singer's melody, zero beats with a violinist's melody, or even more frequencies if a singer is accompanied in precise 12-TET. When the succeeding melodic note lands on a coordinate matching these retained FFR frequencies, immediate neural entrainment occurs across the temporal gap. By establishing that the horizontal FFR memory tracks the same low-frequency PLL divisors used to process vertical chordal fusion, the unity of musical intonation is definitively proven.

3.6. Hypothesis 6: The Information Density Paradox—Front-End Analog Extraction vs. Cortical Melody Tracking

The biophysical isolation of the ≈ 30 Hz resonance engine stems from its exceptionally low information density, allowing it to operate as a frontline analog phenomenon that bypasses centralized cortical processing. Horizontal melodic navigation requires sequential pitch determination—a high-density, post-cortical cognitive computation requiring an operational delay threshold of approximately 200 ms. Because melody tracking and subharmonic entrainment function on separate neurological tiers, they operate as mutually exclusive vectors. This structural independence is verified empirically by the fact that extraneous auditory noise enhances rather than masks binaural beats; since noise is filtered out as non-essential data post-cortex, it remains immune from masking frontline, naturally occurring peripheral vibrations. This biophysical separation resolves the information density paradox.

This biological bifurcation explains the acoustic friction observed in multi-instrumental performance. For executing musicians, the mandate of collaborative ensemble listening does not imply external tracking; rather, a performer must first establish acoustic compliance or structural crashing against the pitch-wise isochronic environment generated within their own auditory pathway, seeking the somatic manifestation of “ringing” only after securing a uniform baseline A4 frequency reference. In a violin and piano duet, the violinist must remain unfazed by the piano's random, inharmonic grid. In practice, unfortunately, string soloists routinely abandon internal octave stretching across the lower three strings where piano accompaniment overlaps, activating the biological stretch mandate only when the linear melody ascends into high-register zones beyond the piano's acoustic reach. This tri-fold structural coexistence is captured in historical recordings—such as Luciano Pavarotti's Ave Maria with full choir and orchestra—which illuminates three incompatible tuning systems running simultaneously: the soloist executing pristine, asymmetric singers' tuning; the orchestral strings locking into routine, linear violinists' tuning; and the accompanying choir lapsing into the randomized, flat configuration induced by lifetimes of piano-corrected muscle memory.

3.7. Hypothesis 7: The Neuro-Mechanistic Calculation of Chordal Harmony—Subharmonic Injection Locking, Auditory PLL Downscaling, and the 30 Hz Resonance Engine

This hypothesis presents a comprehensive, quantitative redefinition of the human perception of chordal fusion and harmonic convergence. This framework rejects Helmholtz's classical acoustic theory, which asserts that musical consonance is merely the passive absence of acoustic beating. Instead, this model demonstrates that musical pleasure is an active, somatic phenomenon generated when independent acoustic frequencies and their higher partials share common mathematical divisors, forcing brainstem neural pathways to achieve phase synchronization via Subharmonic Injection Locking (SHIL) as reported by Wang, T. [11].

In neurological engineering, the phase of an oscillatory signal offers superior noise immunity and stability over level-based encoding schemes. This principle governs the human subcortical auditory pathway. When a complex chord is processed by the inner ear, individual auditory nerve fibers track the incoming waveforms in phase. Due to physiological limitations, individual nerve fibers cannot fire at extreme frequencies climbing into the kilohertz range. To overcome this information bottleneck, the auditory nervous system utilizes a Phase-Locked Loop (PLL) and frequency division process. For baseline visualization of this mechanic, multiple independent acoustic streams can be modeled as simultaneously stimulating the pathway (e.g., $f_1 = 78$ Hz, $f_2 = 104$ Hz, and $f_3 = 117$ Hz), prompting the brainstem PLL tracking loops to apply localized, unique whole-integer neural divisors to each independent channel (/6, /8, and /9). This downscaling optimizes the streams to a singular, subharmonic baseline ($78/6 = 13$ Hz, $104/8 = 13$ Hz, and $117/9 = 13$ Hz), causing separate nerve fiber streams to broadcast an identical internal neural frequency and trigger immediate injection locking and entrainment. The conscious mind experiences this phase-locked neural synchronization as a unified sensation of perfect harmony. This neurocomputational mechanism requires the ear to respond to high-frequency upper partials, extending well beyond conventional acoustic limits.

3.7.1. Quantitative Modeling: The 4-Note Major Chord Matrix Framework

To prove the validity of this model within a standard performance setting, this study evaluates the subharmonic phase-locking matrix of a four-note major chord built on the stretched singers' scale. The biological architecture of the human ear requires an operational delay threshold of approximately 200 ms [12] to fully decode and stabilize an incoming pitch field. This temporal window dictates that the optimal frequency range for neural entrainment must gather around the 30 Hz proximity threshold, matching the historical measurement of approximately eight stable cycles within the brainstem's Frequency-Following Response (FFR).

To prevent independent musical voices from collapsing into a singular sound source via the evolutionary auditory fusion trap, the interval architecture must avoid 2:1 and 3:2 limits. The scale template satisfying these biological constraints

is defined by the fractional progression:

$$\text{Scale Layout} = (9/8, 9/8, 16/15, 9/8, 9/8, 9/8, 135/128)$$

Setting a baseline root pitch coordinate of $do = 52.8089183766486$ Hz, the frequencies of the chromatic scale degrees map to the following values: do (A1) = 52.8 Hz, re (B1) = 59.4 Hz, mi (C#2) = 66.825 Hz, fa (D2) = 71.28 Hz, sol (E2) = 80.19 Hz, la (F#2) = 90.21 Hz, ti (G#2) = 101.49 Hz, and do' (A2) = 107.09 Hz. When a corresponding 4-note tonic major chord structure is executed simultaneously, its upper partials converge to generate two highly stable, independent entrained neural frequencies clustered in the 30 Hz resonance engine.

3.7.2. Generation Grid for the 29.7 Hz Common Resonance Node

The concurrent upper partial profiles map to a singular 29.7 Hz downscaled baseline:

- **The *do* Root Vector (52.8 Hz):** Tracks the 9th harmonic, scaling the localized tracking frequency to 475.2 Hz. The brainstem PLL applies a whole-integer neural divisor factor of 16, yielding:

$$(52.8 \text{ Hz} \times 9) / 16 = 475.2 \text{ Hz} / 16 = 29.7050165868648 \text{ Hz}$$

- **The *mi* Mediant Vector (66.8 Hz):** Tracks the 4th harmonic, shifting the localized frequency to 267.3 Hz. The brainstem PLL applies a whole-integer neural divisor factor of 9, yielding:

$$(66.8 \text{ Hz} \times 4) / 9 = 267.3 \text{ Hz} / 9 = 29.7 \text{ Hz}$$

- **The (*sol*) Dominant Vector (80.2 Hz):** Tracks the 10th harmonic, pushing the tracking frequency to 802.2 Hz. The brainstem PLL applies a neural divisor factor of 27, yielding:

$$(80.2 \text{ Hz} \times 10) / 27 = 802.04 \text{ Hz} / 27 = 29.7 \text{ Hz}$$

3.7.3. Generation Grid for the 28.2 Hz Common Resonance Node

Simultaneously, a parallel cascade of higher partials targets a second, distinct sub-harmonic alignment at 28.2 Hz, due to the do' :

- **The *mi* Mediant Vector (66.8 Hz):** Tracks the 27th harmonic (1804.6 Hz), employing a whole-integer neural PLL divisor factor of 64, yielding:

$$(66.8 \text{ Hz} \times 27) / 64 = 1804.6 \text{ Hz} / 64 = 28.2 \text{ Hz}$$

- **The *sol* Dominant Vector (80.2 Hz):** Tracks the 45th harmonic (3609.2 Hz) employing a whole-integer neural PLL divisor factor of 128, yielding:

$$(80.2 \text{ Hz} \times 45) / 128 = 3609.2 \text{ Hz} / 128 = 28.2 \text{ Hz}$$

- **The *do'* Tonic Vector (107.06 Hz):** Tracks the 64th harmonic (6851.8 Hz) employing a whole-integer neural PLL divisor factor of 243, topology yielding:

$$(107.1 \text{ Hz} \times 64) / 243 = 6851.8 \text{ Hz} / 243 = 28.2 \text{ Hz}$$

The lower tetrachord vectors utilize low-order harmonics (9th, 4th, 10th) and small divisors (16, 9, 27). Conversely, the upper tetrachord vectors force an ex-

pansion to high-order harmonics (27th, 45th, 64th) and massive divisors (64, 128, 243). Because the compressed small semitone (135/128) sits after *ti*, factor cancellation is inhibited, expanding the divisor parameters. This mathematical disparity proves scale asymmetry and confirms the biophysical necessity behind historical string master directives stating that *the leading tone must be sharpened*.

3.7.4. The Somatic Experience of “Ringing” Harmony

When these two independent sets of tripartite neural signals (29.7 Hz and 28.2 Hz) lock into perfect phase alignment across the auditory pathway, they trigger a profound physiological experience felt by the listener as an intensely satisfying vibration. This biological mechanism generates the famous chordal “ringing” or choral shimmer observed in live close-harmony ensembles and computer-optimized multi-track vocal tracks. Because each individual note independently contributes to the identical 29.7 Hz and 28.2 Hz beat structures, horizontal melodic lines carry this same subharmonic infrastructure across time. Furthermore, chord progressions, and often also melodic lines, are linked across time by the FFR, allowing these precise subharmonic entrainment matrices to be retained by the brainstem from one note to the next. Because chord progressions are driven by individual melodies in the respective four parts, this neural memory inheritance allows the cumulative subharmonic resonance to guide the singers into the correct intonation of every successive note. High-fidelity musical harmony is a predictable, mathematically verifiable state of synchronization where precise interval layouts lock the biological computing hardware of the human brainstem into entrainment.

3.7.5. The Psychoacoustic Mechanism of Acoustic Resonances vs. Affect Devaluation in Vocal Ensembles

The dominant 29.7 Hz neural beat is generated by the lower three notes within the quad-voice chordal matrix. Conversely, the secondary 28.2 Hz neural beat emerges due solely to the introduction of the high tonic (*do*'), driven by complex upper harmonic factor cancellations. If a performer fails to stretch this high tonic—rendering it as a flat, strict 2:1 double of the root—the acoustic output fails to generate the 28.2 Hz band. Instead, the unshifted high note merely adds a redundant component to the 29.7 Hz profile, eliminating the secondary neural beat.

Field audits of expert *a cappella* quartet performances confirm that this interval compression is a widespread behavioral anomaly; high tonics are frequently performed short of their required microtonal shifting thresholds. Although these ensembles exhibit high-density harmonized ringing via the reinforced 29.7 Hz baseline, emotional affect suffers devaluation. Deprived of the structural friction generated by the dual interaction of both the 29.7 Hz and 28.2 Hz streams, the performance is decoded as an impressive but emotionally static mechanical event. This explains why precise contemporary *a cappella* recordings achieve room resonance yet lack authentic psychological affect.

Furthermore, this factor-cancellation inhibition explains the notorious diffi-

culty of professional violin execution. Because the instrument's open strings are locked into strict 3:2 fifths, the violinists' scale actively suppresses the ≈ 30 Hz subharmonic engine, depriving the player of the subcortical FFR guidance loops steering vocalists towards precise, perfect intonation, forcing a total reliance on pitch-wise isochronism. Because these subharmonic synchronization processes occur within the subcortical pathway prior to cortical processing, they operate beneath conscious control. This renders the common pedagogical directive to "listen to one another" functionally obsolete; because the analog hardware loops execute this geometric alignment automatically, a student can do nothing to consciously alter the mechanism beyond ensuring the ears remain physically unoccluded.

3.8. Hypothesis 8: Prime Factorization Simplification, Asymmetric Semitone Geometry, and Synaptic Phase-Locking of Unresolved Harmonics

Traditional pedagogical lore in string education dictates that the leading tone (*ti*) must be executed microtonally sharper when resolving upward to the tonic (*do*). This paper provides the foundational mathematical and computational explanation for this phenomenon, demonstrating that the positioning of the large and small semitones is governed by the laws of prime factor cancellation within the subcortical Phase-Locked Loop (PLL) architecture.

3.8.1. Prime Factor Dynamics and Tetrachordal Optimization

The structural layout of the major scale positions the large semitone (16/15, ≈ 111.73 cents) within the lower tetrachord between *mi* and *fā*, while the small semitone (135/128, ≈ 92.18 cents) is pushed to the upper tetrachord boundary between *ti* and *do*. This asymmetric arrangement is mathematically mandatory to optimize subcortical processing efficiency across two distinct pathways:

- **The Cascade Problem of the 9/8 Whole Tone:** The standard whole tone is defined by the low-prime ratio:

$$9/8 = 3^2/2^3$$

As whole tones cascade sequentially across an expanding diatonic scale, the powers of 3 in the numerator and 2 in the denominator compound rapidly. To enable the brainstem PLL to downscale these signals and extract stable neural beat frequencies, the scale architecture must introduce a structural modifier capable of simplifying these expanding exponents.

- **The Exponent-Canceling Engine of the 16/15 Semitone:** The larger semitone ratio possesses prime factors ideally oriented to neutralize this whole-tone expansion:

$$16/15 = 2^4/(3 \times 5)$$

Because its powers of 2 reside in the numerator and its powers of 3 in the denominator, inserting the 16/15 interval into the lower tetrachord acts as a mathematical filter. It cancels the compounding exponents of the preceding 9/8 whole

tones, yielding smaller, simpler integer fractions. This computational simplification allows the auditory pathway to easily generate and lock onto common PLL resonance frequencies within the lower register, anchoring the foundational tonic matrix. Conversely, the smaller semitone ratio projects the inverse prime architecture:

$$135/128 = 3^3 \times 5/2^7$$

With its powers of 3 in the numerator and 2 in the denominator, the 135/128 interval compounds the digital expansion of the 9/8 whole tones rather than neutralizing it. Pushing this complex, high-power modifier to the boundary of the scale—the leading tone step—preserves the pristine harmonic transparency of the foundational lower tetrachord while concentrating microtonal tension at the point of melodic resolution.

3.8.2. The Density Fallacy of the Pure 3:2 and 9:8 Chord Matrix

This mathematical verification provides a definitive answer to a long-standing question in acoustics: Why can a structural layout built on rigid 3:2 perfect fifths and 9/8 whole tones never generate clean, harmonized *ringing*, *neural beats* (see Appendix J).

- **Rigid 3:2 and 9:8 Matrix Layout:** Triggers a hyper-dense coincidence matrix across the overtone spectrum, resulting in a state of *cognitive blinding* (see Appendix J).
- **Asymmetric Singers' Matrix:** Establishes isolated, low-prime target nodes, clearing away the overtone crowd to unlock clean subcortical neural entrainment.

When an ensemble attempts to build vertical harmony using a rigid, unstretched 3:2 and 9:8 matrix, the mathematical combinations produce a hyper-dense web of overlapping harmonic partials, inducing cognitive blinding. To use an analogy, it is easy to locate and connect with your siblings at a small, intimate family gathering. However, it becomes nearly impossible to isolate a specific classmate at a massive holiday party where the entire student body turns up simultaneously. A rigid 3:2 tuning fills the listener's brainstem with an overwhelming crowd of partial matches, preventing the Phase-Locked Loop (PLL) from isolating a singular frequency anchor. True high-fidelity harmony requires the asymmetric micro-cent displacements of the singers' scale. By introducing targeted, low-prime fractional offsets, this scale clears the dense cloud of harmonic noise, isolating the pristine 29.7 Hz and 28.2 Hz resonance channels calculated in Section 3.7. For musicians and listeners constrained by uniform equal temperament or rigid textbook ratios, experiencing this precise neural lock-in is a revelation; true harmony is a somatic, physical state of biological synchronization.

3.8.3. The Structural Inhibition of Vertical Ringing in Bowed String Quartets vs. Synaptic Facilitation of Unresolved Harmonics in Vocal Ensembles

The primary structural bottleneck preventing acoustic chordal ringing within string quartets is rooted in instrument architecture. The unyielding tradition of

tuning violin open strings in rigid, un-stretched 3:2 perfect fifths traps players within the Pythagorean violinists' tuning scheme. Within this environment, the larger semitone, 2187/2048, structurally collides with standard whole-tone configurations:

$$2187/2048 \times (9/8)^n = 3^7/2^{11} \times (3^2/2^3)^n$$

This configuration causes compounding harmonic products to form complex, un-simplifiable fraction matrices that never resolve. Conversely, the smaller semitone, 256/243, inflicts the inverse error, aggressively canceling underlying overtone combinations:

$$256/243 \times (9/8)^n = 2^8/3^5 \times (3^2/2^3)^n$$

Because both mathematical scenarios either flatten or otherwise corrupt the necessary microtonal stream asymmetries, the subcortical auditory apparatus is physically barred from extracting the overlapping partial configurations required to activate the 30 Hz difference loops. This algebraic irreconcilability stands as the primary reason why vertical chordal ringing is seldom achievable among string quartets; performers operate within a mathematically locked acoustic environment, unaware that their horizontal melodic choices are incompatible with vertical harmonic fusion.

At a secondary biophysical level, these physical waveform limitations are compounded by the mechanics of human synaptic transmission. As demonstrated across champion a cappella barbershop vocal quartets, generating the characteristic ringing effect relies on high-fidelity PLL tracking and Frequency-Following Responses (FFR) within the subcortical auditory system. As established in [10], the underlying synaptic mechanism driving this behavior: large excitatory postsynaptic potentials (EPSPs) enhance spike-timing precision, and the hair cells' unique capacity for continuous, large-amplitude, highly synchronous multiquantal release underlies its ability to trigger phase-locked spikes in afferent fibers.

Human vocal performance serves as an exceptional acoustic catalyst for triggering these large-amplitude, highly synchronous postsynaptic potentials. Because singing contains linguistic components, it introduces unique mechanical and spectral fluctuations that are absent in traditional acoustic instrumentation. Natively spoken lyrics significantly boost phase-locking precision through three distinct domains:

1) Temporal Domain Transients: The articulation of lyrics introduces rapid plosive spikes—such as *p*, *t*, *k*, *b*, *d*, and *g*—and high-frequency fricative bursts—such as *s*, *z*, *f*, and *v* (see **Appendix J**)—which act as precise acoustic markers that reset and tightly synchronize neural spike timing.

2) Spectral Domain Energy Spikes: The physiological creation of the singer's formant—a concentrated acoustic energy spike clustered between 2500 Hz and 3000 Hz—drives intense stimulation across targeted regions of the basilar membrane, facilitating synchronous multiquantal release.

3) Physiological Instabilities: Organic micro-instabilities, including glottal closure transients (glottal pulses), vocal transitions, and localized pitch-holding micro-variations, prevent auditory system habituation, maintaining active phase-locking circuits.

In contrast, bowed string instruments produce continuous, highly smoothed waveforms lacking these sharp, transient linguistic triggers. Furthermore, as string players expend significant physical effort attempting to tune dynamically against their 3:2 open strings, they drift further into the rigid violinists' tuning profile, which inherently suppresses the subcortical generation of neural beats. Consequently, because string quartets lack both the necessary geometric interval alignment and the sharp acoustic spikes required to drive maximum multiquantal hair cell synchronization, the acoustic ringing phenomenon is seldom realized in these ensembles.

This microsecond-precision timing gate is driven directly by multiquantal release at the auditory hair cell ribbon synapses, as documented by Li and Gersdorff [14]. This high-amplitude synaptic synchronization provides the definitive resolution to the historical psychoacoustic barrier of unresolved upper partials. Rather than processing high-register, crowded harmonics as chaotic acoustic friction, the subcortical tracking loop utilizes the ironclad, “in-phase” temporal grid generated by the lower, resolved harmonics as an aggressive phase-locking force. This synaptic drive forcefully entrains and aligns the higher, overlapping unresolved overtones into cross-frequency synchronization with one another. By policing and organizing the upper register at the raw synaptic gate prior to cortical arrival, this cross-frequency alignment enables the stable biological sensation of unresolved overtones while generating the proposed 30 Hz harmonizing beats required for pristine vertical chordal fusion.

3.8.4. Subharmonic Frequency Division and the Low-Frequency Advantage in Auditory Ringing

The architectural divergence between biological Phase-Locked Loops (PLL) and electronic circuits dictates frequency constraints on acoustic ringing. In electronic PLL hardware, frequency multiplication is driven by an internal Voltage-Controlled Oscillator (VCO). However, the subcortical auditory system lacks a biological equivalent to a VCO; instead of synthesizing higher frequencies through active multiplication, the neural system operates primarily via frequency division and subharmonic phase-locking. Because auditory nerve fibers possess a strict physiological refractory period, they cannot fire fast enough to track high-frequency cycles individually. They instead divide down higher frequencies by integer divisors, phase-locking to every second, third, or fourth wave cycle—a mechanism analogous to a digital frequency divider. Frequency multiplication is therefore mimicked up the auditory pathway through the processing of acoustic harmonics, which are either present in the external stimulus or generated internally as distortion products by the active amplification process of outer hair cells via the *cochlear amplifier* (see **Appendix J**).

Consequently, high-frequency inputs face severe physiological limitations because the auditory nerve's upper boundary for phase-locking drops sharply above 4 to 5 kHz. This structural limitation grants a significant acoustic advantage to lower-frequency pitches. Because low-frequency notes sit safely within the robust phase-locking limits of human neural hardware, they provide a much wider, more stable foundation for subharmonic division and harmonic alignment, facilitating the generation of the ≈ 30 Hz proximity beats necessary for chordal ringing. Real-world evidence of this low-frequency advantage is found in Central Asian Mongolian and Tuvan overtone throat singing, where executing an extremely low-pitched, sustained fundamental drone allows the singer to filter and amplify specific high-frequency harmonics, often accompanied by low-frequency instrumental drones.

This architecture stands in stark contrast to highly emotional, Romantic solo compositions—such as virtuoso violin works—where musical affect operates in a different psychological dimension. Because all seven notes within an implied virtual key generate the identical two ≈ 30 Hz subharmonic beat pairs, emotional music does not require frequent key changes to alter its structural profile. Instead, because these pieces are predominantly solo expressions written in higher registers, the intense emotional reaction generated by the shifting melodic line overwhelms the listener. Consequently, the physical, objective chordal ringing aspect of the music is rendered subordinate to raw emotional sensation, allowing highly expressive instruments like the violin to maximize human vocal affect without disrupting the underlying subcortical tracking grid. This suppression of ringing is further compounded by a structural constraint: tuning adjacent open strings to precise 3:2 perfect fifths introduces permanent fractional contradictions across the fingerboard that actively inhibit ringing creation. Furthermore, public audiences do not culturally expect pristine acoustic ringing in solo concert settings, which are conventionally accompanied by fixed-pitch, un-stretched instruments like keyboards or guitars. This absence of ringing in solo acoustic textures stands in stark contrast to the chorus-heavy, multi-tracked production environments of commercial pop ensembles—such as ABBA or the Bee Gees—where dense, microtonally calibrated vocal layers are intentionally engineered to exploit a heightened near-emergence of subcortical phase-locking (see detailed analysis in **Appendix I**).

4. Discussion

This chapter evaluates the practical musicological applications, psychological implications, and performance-based realities of the neuro-mechanistic framework established in the preceding results matrix. The biological hardware and subcortical processing networks align to accommodate human musical language, translating physical wave intervals into cognitive emotional affect. Contrasting these mathematical and neurophysiological boundaries against modern digital synthesis, institutional performance constraints, and cross-cultural tuning matrices

demonstrates how microtonal pitch displacements actively dictate human emotional valence. This top-down alignment reveals that subcortical hardware is structurally optimized to parse the macrosemantic meanings of musical vocabulary before reductionist frequency decoding occurs, transitioning the study of musical intonation out of abstract textbook assumptions into an applied, somatic reality hardwired within the human auditory pathway.

4.1. Hypothesis 9: Inharmonicity, Acoustic Mode-Locking, and the Perceptual Imperative of Higher Harmonics

A foundational premise in traditional acoustic textbook models is that the partials of a complex musical tone exist as strict, whole-integer multiples of the fundamental frequency, traditionally formulated as:

$$1f_0, 2f_0, 3f_0, \dots, nf_0$$

This paper rejects this idealized definition, asserting instead that inharmonicity—the systematic or chaotic deviation of upper partials from strict integer multipliers—is an inherent property of natural acoustic sounds. While public references often maintain an oversimplified definition of harmonic distribution, real-world physical systems contingency rarely exhibit mathematical purity.

4.1.1. Driven Transitions vs. Transient Decay Mechanics

In physical instruments, near-ideal integer harmonics are achieved only through the mechanical process of *acoustic mode-locking* (see **Appendix J**). This occurs via non-linear continuous driving mechanisms, such as the stick-slip friction of a horsehair bow gripping a violin string, the pressure-driven aerodynamic oscillation of reeds in woodwinds, or the vortex-splitting airstream across a flute's embouchure hole. When these continuous interactions are absent, acoustic energy instantly reverts to a decoupled system of un-driven eigenmodes. These native eigenmodes manifest significant micro-cent deviations from pure integer targets. A parallel condition governs almost all un-driven natural sounds—including the transient decay of an acoustic piano or the un-locked resonances of a flute—where string stiffness and pipe diameter accelerate the velocity of higher partials, driving them sharp. For instance, studies of organ pipes (**Figure 3**) by Angster and Miklós [15] demonstrate that while the primary driven acoustic output tracks integer ratios under steady-state conditions, the underlying structural eigenmodes exhibit marked shifts.

4.1.2. Symmetrical Symmetries: Transpositional Uniformity and 12-TET Density Benefits

Because 12-Tone Equal Temperament (12-TET) is inherently symmetrical and transpositionally uniform, it exhibits a high density of structurally compliant fractional options across all transposition keys. This architectural repetition lowers the computational barrier for the auditory Phase-Locked Loop (PLL) to generate stable, harmonizing neural beats. As mathematically cataloged in **Appendix D**, computational extraction yields over fifteen distinct fractional ratios for the minor

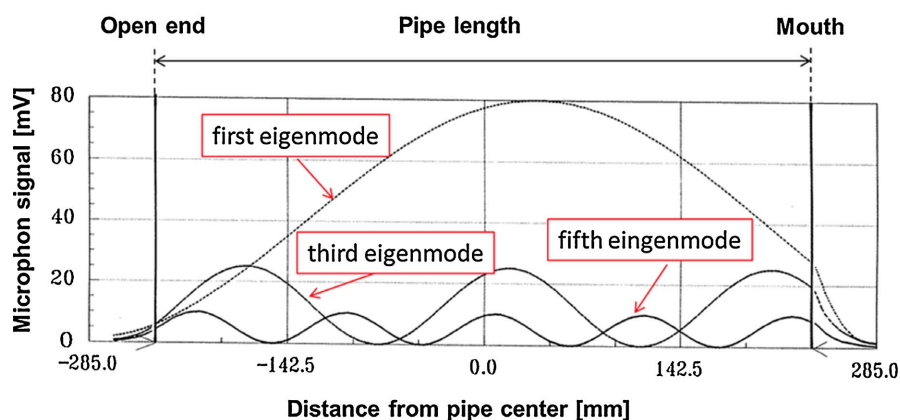


Figure 3. From Angster and Miklós [15]. Standing waves in an organ pipe. Sound pressure distributions of the first, third, and fifth eigenmodes in a wide pipe are shown.

third alone falling within a ± 0.0015 cent tolerance margin. This extreme density of near-value integer intervals demonstrates that workable ratios are available in abundance within the acoustic landscape. Rather than executing a static template match, the subcortical auditory apparatus actively tracks this dense field to evaluate and cognitively learn the single most structurally cohesive fraction. The human nervous system's capacity for adaptive acoustic learning is highly pronounced; once an optimal harmonic profile is learned, the central processor retains it as a functional auditory anchor, recognizing and stabilizing it whenever it appears across sequential intervals on all twelve notes of the scale. This adaptive learning loop, coupled with the pre-learned 500-cent interval (499.998719922547 cents = $(9/8)^2 \times 135/128$) defining the upper tetrachord of the singers' scale, explains why digitally aligned multi-track vocal layers—in particular, one-man *a cappella* quartets aligned via pitch-correction software—can artificially induce a heightened near-emergence of chordal fusion under equal temperament constraints.

4.1.3. Analog Restoration: Vacuum Tube Non-Linear Harmonic Re-Generation

If modern digital playback systems are incapable of organically rendering high-frequency neural beats due to sterile sampling techniques, a functional alternative can be engineered using analog circuitry. Passing a digitally synthesized signal through a vacuum tube amplifier introduces intentional, nonlinear harmonic distortion across the signal path. This analog saturation reconstructs an artificial cascade of even and odd upper partials, providing the listener's brainstem with the necessary high-frequency data to re-engage the subcortical PLL process. This mechanical injection artificially restores a precise sense of acoustic depth, perceived warmth, and structural intonation to otherwise sterile digital temperaments.

4.2. Hypothesis 10: The Algorithmic Mechanics of Musical Affect—Micro-Tonal Levers, Tetrachordal Densities, and Cross-Genre Semantic Deviations

The generation of musical affect and structural mood within a diatonic framework

operates as a direct function of the proportional density of active pitches across the lower and upper tetrachords, coupled with the precise microtonal manipulation of semitone boundaries. As established in Section 3.3, the human pre-cortical auditory framework tracks the 19.55 cents structural friction introduced between wide and narrow semitones across a continuous, horizontal pitch-wise isochronism trajectory. This framework outlines the precise mechanistic rules governing how this structural asymmetry triggers emotional transmutation across genres.

4.2.1. Tetrachordal Note Density and Peak Vector Tracing

In a standard major key with zero modulations (such as the traditional melody of *Silent Night*), the emotional architecture of a musical phrase can be calculated based on the statistical ratio of notes falling within the sharper upper tetrachord versus the flatter lower tetrachord, alongside the structural location and pitch register of the peak note (the highest frequency) within that phrase. An algorithmic analysis of *Silent Night* mapped via movable-do solfège demonstrates this precise microtonal pacing across three distinct structural vectors: Phrases 1 and 2 (*sol-la-sol-mi*) exhibit a high mean density coupled with a high peak note register, driving an overarching sense of structural brightness; Phrases 3 and 4 (*re-re-ti*) exhibit a low mean density and a low peak note register, inducing immediate tonal resolution and restful stabilization; while Phrases 5 and 6 (*la-la-do*) deliver a high mean density paired with a low peak note, resulting in a state of suppressed microtonal tension.

4.2.2. The Micro-Cent Mechanics of Tonal Modulation

When a composition modulates from the tonic (*I*) to the dominant (*V*), the original dominant pitch (*sol*) is cognitively re-mapped by the auditory system to function as the new tonic (*do*). To accommodate this structural shift, the original steps *do*, *re*, and *mi* must be microtonally sharpened to establish the new scale geometry. The brain perceives this ascending shift across the PLL matrix as an increase in psychological excitement and kinetic energy.

Conversely, the stealth introduction of a flattened seventh scale degree (*tib*) can shift the scale geometry in the opposite direction. The brief appearance of a *tib* note instantly transforms a major framework into a sad, introspective “blue” mood. Flattening the leading tone reverses the semitone layout, expanding the narrow boundary and causing the entire pitch register to drop into a relaxed, down-directed vector.

4.2.3. Masterwork Case Studies: Structural Micro-Tonal Displacements

This biophysical mechanism is verified empirically through the construction of iconic works across Western classical and popular music traditions:

1) The Sub-Dominant Melancholy Shift (*tib*)

- **The Beatles—*The Long and Winding Road*:** On the word “*long*”, the melody tracks the standard leading tone (*ti*), but on the word “*your*”, it drops to a *tib*. This displacement shifts the scale geometry into the sub-dominant register (**Figure 4**) stripping the leading tone of its sharp quality and pulling it flat to

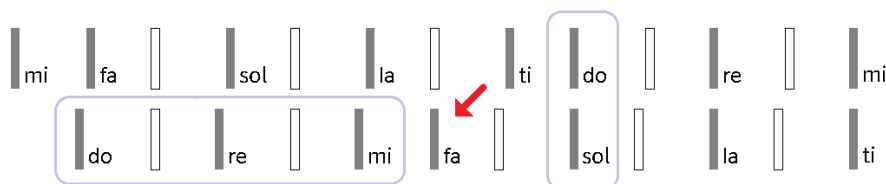


Figure 4. Scale transformation to the subdominant register via the $ti \rightarrow tib$ transition, establishing the virtual stealth key.

trigger an immediate sensation of profound sadness.

The long and winding road
That leads to your door

- **The Beatles—*Here, There and Everywhere*:** In the second phrase of the introduction, a tib note enters briefly on the word “I”. This microtonal displacement steers the composition toward the parallel minor. Because the original major scale was already adjusted into its subdominant form, the subsequent minor section remains fully compatible without requiring further tuning alterations.

To lead a better life
I need my love to be here

- **Simon & Garfunkel—*Bridge Over Troubled Water*:** The introduction of the critical tib shift occurs on the word “tears”. It instantly transmutes the overarching mood, stripping away the bright, stable affect delivered by an un-stretched major scale template, and shifting the semantic landscape into a tender, intimate aesthetic. If a vocalist fails to execute this microtonal compression—rendering the note instead as a rigid, un-stretched major interval—this delicate emotional shift is lost.

When you're weary
Feeling small
When tears are in your eyes
I will dry them all

2) The Chromatic Sharp Displacement ($do\sharp$ and $fa\sharp$)



Figure 5. Structural distribution of tib and $do\sharp$ modulators across the opening layout of Bach's Air on the G-String.

- **J.S. Bach—*Air on the G-String*:** This piece generates a deeply melancholy state by balancing two distinct microtonal adjustments (Figure 5): a flattened seventh (tib) alongside a sharpened tonic ($do\sharp$). This counter-pull expands the internal friction of the intervals, driving the piece into the relative minor of the

subdominant.

Beethoven
Concerto, Op. 61

Allegro, ma non troppo

Violin

Piano

Tutti

p *dolce p* *criso.* *sf* *p*

Figure 6. Structural injection of the *do#* displacement in the opening theme of Beethoven's Violin Concerto.

- **Beethoven—Violin Concerto in D Major (Op. 61):** The opening bars (**Figure 6**) introduce a sharpened tonic displacement (*do#*). Violinists who attempt to force these introductory measures into a uniform, brilliant major temperament miss the underlying acoustic design; the concerto intentionally mirrors the structural tension found above in Bach and The Beatles, modulating directly into the minor spectrum shortly thereafter.

1 **Onward, Christian Soldiers.**

S. Baring-Gould. A. S. Sullivan.

1. On - ward, Chris-tian sol - diers! March-ing as to war, With the cross of
2. Like a might-y ar - my Moves the Church of God; Broth-ers, we are
3. Crowns and thorns may per - ish, King-doms rise and wane, But the Church of
4. On - ward, then, ye faith - full Join our hap - py throng, Blend with ours your

Je - sus Go - ing on be - fore: Christ, the roy - al Mas - ter,
tread - ing Where the saints have trod; We are not di - vid - ed,
Je - sus Con - stant will re - main; Gates of hell can nev - er
voic - es In the tri - umph song; Glo - ry, praise, and hon - or

Figure 7. The *fa#* upward modifier generating kinetic drive in traditional hymnody.

- **Traditional Hymnody—Onward Christian Soldiers:** Conversely (**Figure 7**), the structural sharpening of the fourth scale degree (*fa#*) acts as the mathematical opposite of the flat seventh (*tib*). Rather than inducing sadness, the *fa#* modulator expands the interval spacing upward to signal kinetic drive, agita-

tion, and a strong directional push toward a heroic climax.

4.2.4. Asymmetrical Shifts in the Melodic Minor Scale

The dual structure of the traditional minor scale framework provides clear proof of this microtonal mechanism. While a standard major layout establishes an ascending vector characterized by bright, sharp upper boundaries, the targeted introduction of a flattened median node (*mib*) or a subdominant flat seventh modifier forces the two asymmetrical, alternating semitones to instantly swap physical locations across the pitch grid. Lowering these critical scale nodes triggers an immediate structural shift of semitone dimensions, altering the emotional landscape of the scale.

• Case Study Context (The Beatles—Michelle):

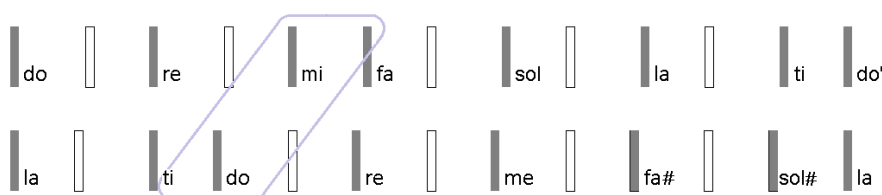


Figure 8. The mi-flat structural node swapping wide and narrow semitone configurations across the upper register.

Michelle, ma belle

These are words that go together well

On the phrase “*Michelle, ma belle/These are words that go together well*,” the melody navigates the structural contour of a F melodic minor architecture. The mechanical lowering of the median node swaps the boundaries of the wide and narrow semitones (**Figure 8**). This modification instantly triggers intense internal microtonal tension across the subcortical tracking grid, forcing an immediate, palpable transmutation from a bright major affect into an introspective, emotionally complex parallel minor. Hence, modifications of both upper and lower boundaries of both semitone intervals (*tib*, *do#*, *mib*, *fa#*) have all been examined.

4.3. Hypothesis 11: Perfect Violin Intonation—The Mathematical Bifurcation of Structural Tuning Systems

The traditional paradigm of violin acoustics forces an unnatural mathematical compromise due to the historical practice of tuning its four open strings in allegedly pure 3:2 perfect fifths. This study outlines an optimized framework for Perfect Violin Intonation, demonstrating that to align a bowed stringed instrument with the biological demands of the singers’ scale matrix, the open strings must abandon classical 3:2 constraints and adapt to a non-uniform, stretched matrix.

Centuries of performance tradition and exhaustive experiential parsing of the

core violin repertoire reveal that E5 functions as the true, permanent acoustic anchor of the instrument, locked solidly at 668.25 Hz (+23.46 cents relative to the unshifted 12-TET grid, computed via three 9/8 whole tones combined with a 16/15 large semitone). Because the open D4 and permanent E5 strings are locked a whole tone apart, dynamic shifts in the instrument's key geometry alter the lower tuning vector symmetrically. When A4 shifts, only the open G3 string changes its baseline, while D4 remains un-deviated at 293.00 Hz (−1.96 cents flat, calculated via three 9/8 whole tones paired with a 135/128 small semitone from open A4, or seven 9/8 whole tones from the open E5 reference anchor).

To accommodate human vocal inflections across varying key centers, the open A4 string alternates between 440.00 Hz (0.00 cents delta) and approximately 445.00 Hz (444.99753 Hz, a +19.55 cents delta) to satisfy the implied localized key designation. For the lower register layout, there are two options for the G3 string being tuned: it can either be tuned seven whole tones beneath the A-string, or it can follow the case scenario when A4 assumes the sharpened 445 Hz coordinate. With the frequent employment of keys D and A major on the violin, this G-string is in tune in D major, whereas in A major, this G is out of key due to its being sharp, becoming the leading note G♯, posing minimal issues.

This architectural arrangement acts as an intentional ergonomic compromise designed to maximize playability across traditional repertoire boundaries. The structural consequence of this mechanical fixed-string limit is a dual-register fluctuation: half the time, the open G3 string sounds slightly too sharp relative to the ideal 5-limit template, while the open A4 string concurrently sounds slightly too flat. To circumvent this native acoustic limitation, advanced string performance practice routinely discourages the deployment of open strings in favor of stopped, fingered notes. By executing the target A4 pitch as a stopped position on the D4 string, the performer bypasses the static compromise of the open strings, dynamically achieving un-compromised pitch compatibility between complex D-major and A-major frameworks. Optimized violin and cello open-string frequencies and key-specific alignment vectors are detailed in **Table 1**.

These dual anchor configurations govern the structural foundation for the vast majority of the Western musical repertoire. This mechanical and ergonomic elasticity is directly corroborated by the landmark tracking of Fransson, Sundberg and Tjernlund [16]. By analyzing the real-time performance frequencies of elite woodwind and string instrumentalists, their study conclusively demonstrated that “in-tune” musical expression rejects static, rigid textbook ratios. Instead, performers continuously execute context-dependent microtonal adjustments across melodic phrases. This precedent provides robust historical validation for our optimized model; it confirms that the subcortical tracking loop natively demands a dynamic, alternating frequency architecture rather than a monolithic, equal-temperament baseline.

Table 1. Singers' tuning matrix for Strings: Optimized open-string frequencies and key-specific alignment vectors. The structural notation (ov) indicates that the targeted micro-tonal pitch must be executed sharper than the open string baseline, while (un) denotes that the target should be executed flatter. Values marked with a dash (–) denote regions outside the core modulatory focus or sitting in perfect, stable alignment with the subharmonic engine boundaries.

Key String	Violin				Cello			
	E	A	D	G	A	D	G	C
Hz	668.3	440.0	293.0	195.11	217.0	144.5	95.16	63.37
Gb	-	-	-	-	-	-	-	-
Db	-	-	-	-	-	-	-	<i>ti</i>
Ab	-	-	-	<i>ti</i>	-	-	<i>ti (ov)</i>	<i>mi</i>
Eb	-	-	<i>ti</i>	<i>mi (un)</i>	-	<i>ti</i>	<i>mi</i>	<i>la</i>
Bb	-	<i>ti (ov)</i>	<i>mi</i>	<i>la</i>	<i>ti (ov)</i>	<i>mi</i>	<i>la (ov)</i>	<i>re</i>
F	<i>ti</i>	<i>mi</i>	<i>la</i>	<i>re (un)</i>	<i>mi</i>	<i>la</i>	<i>re</i>	<i>sol</i>
C	<i>mi</i>	<i>la (ov)</i>	<i>re</i>	<i>sol</i>	<i>la (ov)</i>	<i>re</i>	<i>sol (ov)</i>	<i>do</i>
G	<i>la</i>	<i>re</i>	<i>sol</i>	<i>do (un)</i>	<i>re</i>	<i>sol</i>	<i>do</i>	<i>fa</i>
D	<i>re</i>	<i>sol (ov)</i>	<i>do</i>	<i>fa</i>	<i>sol (ov)</i>	<i>do</i>	<i>fa (ov)</i>	-
A	<i>sol</i>	<i>do</i>	<i>fa</i>	-	<i>do</i>	<i>fa</i>	-	-
E	<i>do</i>	<i>fa (ov)</i>	-	-	<i>fa (ov)</i>	-	-	-
B	<i>fa</i>	-	-	-	-	-	-	-

4.3.1. The Geometric Construction of Interval Variations

To map how the human brainstem decodes these structural systems, this study analyzes the geometric differences between the violinists' (3-limit/Pythagorean) and singers' (5-limit biological) interval equations:

- **The Violinists'/String Framework (3-Limit Geometry)**

- **The Perfect Fourth:** Built on two 9/8 whole tones and a compressed Pythagorean limma:

$$9/8 \times 9/8 \times 256/243 = 4/3 = 1.3333 \text{ (498.045 cents)}$$

- **The Perfect Fifth:** Composed of three 9/8 whole tones and a compressed limma:

$$9/8 \times 9/8 \times 9/8 \times 256/243 = 1.5000 \text{ (701.995 cents)}$$

- **The Stretched Fifth:** Formed by adding a Pythagorean Comma (23.46 cents) to a pure fifth:

$$9/8 \times 9/8 \times 9/8 \times 2187/2048 = 1.5205 \text{ (725.415 cents)}$$

- **The Compounding Stretched Octave:** Synthesized by multiplying a pure fourth by a stretched fifth, expanding the octave boundary:

$$4/3 \times 1594323/1048576 = (9/8)^6 = 2.0272865 \text{ (1223.46 cents)}$$

- **The Singers' Vocal Framework (5-Limit Geometry)**

- **The Singers' Fourth:** Employs a unique, asymmetrical semitone multiplier ($135/128$):

$$9/8 \times 9/8 \times 135/128 = 10935/8192 \approx 1.3348388 \text{ (499.9987 cents)}$$

- **The Singers' Fifth:** Combines three $9/8$ whole tones with a wide semitone parameter ($16/15$):

$$9/8 \times 9/8 \times 9/8 \times 16/15 = 2187/1440 \approx 1.51875 \text{ (723.4613 cents)}$$

Comparing these profiles against the 12-TET Perfect Fourth reference baseline (500.0000 cents) reveals that the Violinists' Scale Fourth operates at 498.05 cents (an operational error of 1.95 cents) while the Singers' Scale Fourth lands at 499.9987 cents (a negligible error of +0.0013 cents). Symmetrically, the Violinists' Scale Fifth stretches to 725.42 cents to satisfy 3-limit expansion parameters, whereas the Singers' Scale Fifth rests at 723.4613 cents to optimize 5-limit vocal integration targets.

4.3.2. Fingerboard Graduations and the Resolution of Key-Change Complexity

The fingerboard fingerings outlined across the primary documentation do not merely apply to isolated scales; they provide the mathematical blueprint governing all twelve musical keys. The spatial layout of the violin fingerboard reduces real-time microtonal modulation into two highly predictable, binary sets of finger graduations. The D major graduation matrix serves as the definitive structural archetype for a six-key family comprising D, E, F#, Ab, Bb, and C major. Symmetrically, the A major graduation matrix establishes the operational archetype for the remaining six keys of the chromatic spectrum: Db, Eb, F, G, A, and B major.

This elegant binary classification provides a definitive resolution to a century-old debate regarding how string instrument intonation can adapt to complex key changes without introducing mechanical tuning apparatuses or forced equal-temperament compromises. Rather than requiring infinite microtonal recalibrations for every unique key signature, the human central auditory processor and the performer's muscle memory navigate an elegant, alternating geometric lattice. By grouping the twelve chromatic keys into these two structurally stable graduation profiles, the framework achieves perfect singers' scale integration across the performance literature while preserving pristine fingerboard ergonomics.

Because creative expression ultimately resists static codification, the greatest artists routinely break structural guidelines to follow their independent artistic instincts; individual performers must interpret these matrices, utilizing these tables as generative processing zones rather than rigid architectural limits.

4.3.3. Dual-Fingerboard Geometry and Fretted Intonation Optimization

Expanding this binary graduation framework to fretted string instruments reveals a profound avenue for modern instrument design. If the six strings of a guitar are

structurally calibrated such that the interval between every adjacent open-string pair forms a strict, integral number of 9/8 whole tones, an elegant geometric split occurs across the frets: every alternating fret position across the fingerboard is forced to manifest as either a large or small semitone boundary. Consequently, a guitar engineered with a dual-neck or dual-fingerboard configuration—where each independent fingerboard is fretted to match one of the two respective graduation schemes—possesses the physical capability to execute musical literature across all twelve keys in mathematically pristine intonation, as structurally modeled in **Figure 9**.

While this structural innovation resolves the historical engineering barrier of fixed-fret tuning compromises, it introduces an immediate pedagogical paradigm shift: to navigate this expanded harmonic landscape, the performer must deconstruct and re-learn all traditional fretboard fingerings from scratch.



Figure 9. Example of guitar dual-fingerboard fret layouts optimized for the D major and A major graduation

4.4. Socio-Acoustic Environmental Conditioning: Digital Auto-Tuning and the Atrophy of Dynamic Intonation

Modern digital synthesizers, virtual MIDI keyboards, digital audio workstations (DAWs), and standard sample playback engines are mechanically hardwired to target rigid, un-stretched mathematical intervals. By forcing acoustic pitches into these artificial integer boxes, industrial software erases the delicate, asymmetrical semitone balances required to project organic emotional valence and facilitate subcortical chordal fusion. This asymmetric, stretched macro-octave template is optimized primarily for the narrative-emotional grammar characteristic of mainstream Western diatonic architectures, where microtonal adjustments across scale degrees govern psychological valence. When evaluating global musical traditions through an evolutionary lens, distinct structural exceptions emerge where rigid integer ratios or pure static scales remain functionally viable. These exceptions manifest in traditions that depart from Western-style melodic narratives, focusing instead on structural drones or highly stylized timbral frameworks.

For example, the acoustic architecture of the Great Highland Bagpipe relies on continuous, high-amplitude bass and tenor drones. Because the primary objective is to maintain an unyielding harmonic lock against a fixed continuous frequency, the melodic intervals on the chanter are structurally bound to strict, un-deviated integer relationships to prevent destructive acoustic interference. Similarly, in traditional Peking Opera (Jingju), dramatic tension and semantic meaning are conveyed through highly stylized, nasal vocal production and complex rhythmic percussive patterns rather than sentimental, Western scale-degree adjustments. Because emotional affect is coded into archetypal vocal timbres and structural gestures, the accompanying melodic instruments safely utilize rigid, un-stretched tuning profiles without degrading the genre's intrinsic expressive power.

This mechanism also governs Mongolian and Tuvan throat singing, where the traditional step-by-step melodic narrative is subordinated to a static, low-frequency fundamental drone. The performer manipulates the physical resonance of the vocal tract to filter and amplify high-frequency overtones over this sustained pitch. While this architecture is not devoid of melodic beauty, its structural design prioritizes the active generation of constructive neural beats and subharmonic ringing over linear intervallic progression, demonstrating a musical framework optimized around subcortical phase-locking. Consequently, while rigid, simple-ratio integer paradigms are insufficient to capture the expressive, emotional matrix of Western diatonic composition, they remain functional within traditional non-Western architectures that prioritize static harmonic locks, continuous drones, or stylized timbral presentation.

4.5. Hypothesis 12: The Generative Logic of Musical Scale Construction

The evolutionary architecture of diatonic scales is driven by geometric symmetry and structural mirroring. Imagine an initial lower three-tone group separated by consecutive $9/8$ whole tones (*do-re-mi*). This pattern is repeated as an upper three-tone block (*sol-la-ti*) where *sol* forms a perfect fifth with *do*. However, no systemic relationship exists between these parallel blocks until a seventh tone adds an octave anchor above the root (*do*). To perfectly mirror the upper set resolving to *do*', the fourth scale degree (*fā*) is inserted into the lower register. This addition completes the major scale; the parallel intervallic identity between *do-fā* and *sol-do*' constitutes the operational soul of the major framework.

Symmetrical geometry similarly dictates the construction of the minor modes through directional performance vectors:

- The Melodic Minor Symmetrical Mirror: When sung downwards starting from *la*', the upper tetrachord presents a reversed mirror of that in the major layout. The lower tetrachord configurations inherently differ, but the human vocal mechanism instantly resolves this discrepancy by flattening the *mi* node (*mib*). This microtonal adjustment establishes structural symmetry across both upper and lower tetrachords when sung respectively upwards and downwards.

- The Harmonic Minor Symmetrical Mirror: Alternatively, this lower minor tetrachord can be appended to a major upper tetrachord modified by flattening the sixth scale degree (*la**b*). This creates the harmonic minor scale, an architecture whose operational stability similarly relies on structural symmetry when sung up and down respective tetrachords.

If the 12-note chromatic scale is constructed with alternating large and small semitones—where the major scale allocates a small semitone sat natively between *ti* and *do*—this inherent semitone arrangement results as the very factor dictating strict microtonal constraints across both minor scale matrices: the leading tones must hence be wide, large semitone intervals relative to the high tonic a step up, and the two respective semitones within the upper tetrachord of the harmonic minor scale must both manifest as large semitone boundaries.

The structure of the major scale, hence also the minor scales, being intimately familiar in musicians’ and audiences’ consciousness, the wide semitone leading into the higher tonic induces an interruption in sensation. The awareness of the above-defined scale structure builds upon the foundation of pitch-wise isochronism, processed far before consciousness. This structural interruption delivers an indescribable sensation interpreted by consciousness as “sadness,” although unlike a painful “ouch,” there can be nothing natively sad about instrument sound. Consequently, while the major scale can sound happy, the minor scales cannot; conversely, a major scale can sound sad if the big and small semitones are swapped via the “*stealth modulation*” mechanics established in preceding sections.

5. Conclusions

This study resolves several long-standing questions in psychoacoustics, providing definitive and neurocomputational support for the intuitive observations of historical musicians that previously lacked scientific validation. Musical emotion resides upon the pitch-wise interruption of isochronism within the inherent, symmetrical construction templates of major and minor scales. This isochronism compliance detection operates as a front-end, subcortical analog process, structurally analogous to the pre-cognitive visual speed detection of flying prey by a dragonfly. Symmetrically, the pleasant sensation of musical chords—which are, in reality, highly synchronized vertical melodies—is induced through a subharmonic entrainment process akin to binaural beats. This mechanism is initiated when a common beat derived from the upper harmonics of multiple independent notes converges onto one or more stable frequencies clustered within the 30 Hz proximity resonance engine. When both phenomena appear simultaneously, the resulting musical effect and cognitive salience are exponentially multiplied and magnified. This author proposes that the entrainment mechanism responsible for the 30 Hz resonance may similarly explain the resolution of unresolved harmonics, solving one of the most widely contemplated puzzles.

This condensed musical identity is propagated dynamically across sequential chord configurations and horizontal melodic progressions via the subcortical Fre-

quency-Following Response (FFR), allowing the tracking grid to sustain and amplify the cumulative resonance across time. These analog synchronization processes occur within the subcortical auditory pathway prior to cortical arrival, operating independently and far earlier than any conscious interpretation of musical mood or the conscious determination of pitch itself.

Because the perceived pitch of a complex tone is determined not by a static fundamental frequency (f_0) but by the microtonal differences between adjacent upper harmonic pairs, an inharmonic tone can manifest a shifting pitch profile when altered by a varying contemporary acoustic environment. This computational model establishes a foundational framework that can be definitively verified through subsequent experimentation.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendices: Supplementary Psychoacoustic Literature and Contextual Foundations

This section provides the historical baselines, supplementary mathematical derivations, and specialized case studies necessary to support the subcortical auditory tracking framework established in the preceding text.

A1. Appendix A: Neural Signal Processing and Noise Enhancement of Binaural Beats

Binaural beats (see **Appendix J**) provide critical insight into central nervous system acoustic tracking. Unlike monaural beats driven by physical wave interference in peripheral cochlear fluids, binaural beats are computed within the central auditory pathways of the brainstem's superior olivary complex when two distinct frequencies are presented dichotically to each ear.

While environmental noise typically degrades acoustic signals, binaural beat perception is enhanced by noise. When low-amplitude pure tones are masked by calibrated white noise, the internal perception of the binaural beat sharpens. This masking paradox is driven by stochastic resonance; background noise provides targeted neural gating synchronization markers that allow the subcortical Phase-Locked Loop (PLL) to more efficiently lock onto and track the internal beat period. This mechanism explains how the brainstem extracts stable subharmonic tracking anchors even within dense, noisy acoustic environments. Despite contemporary understanding as above, this author proposes that this enhancement occurs simply because broadband noise is processed as a post-cortical cognitive sensation, whereas the binaural beat is extracted as an independent, pre-cortical phase-locked mechanism within the subcortical brainstem architecture, preventing linear masking.

A2. Appendix B: The Cultural vs. Helmholtzian Beat Rate Debate in Intonation

The fundamental premise of Helmholtz's classical framework on consonance and dissonance fails when evaluating intonation through the dual lenses of evolutionary biology and top-down cognitive processing. Traditional psychoacoustics suffers from a severe reductionist fallacy, attempting to decode musical comprehension by analyzing isolated micro-components—such as individual sine waves, mathematical beat counts, and raw peripheral frequency ratios—and then guessing how these components manufacture emotional art. This bottom-up methodology is equivalent to forcing a child to memorize the abstract written letter A before allowing them to see a tangible apple or grasp the semantic meaning of the fruit itself. All subsequent verification based on the Helmholtzian model gets this basic cognitive sequence backward, fruitlessly investigating why the physical structure of a letter has anything to do with organic meaning.

In reality, human auditory processing operates from the top-down. The mind must first conceptualize the overarching musical language—identifying the

macro-structural face of the major scale—before calculating regional pitch relationships. For fauna species, the primitive auditory apparatus prioritizes survival-driven auditory scene analysis. This mechanism instantly groups complex, inharmonic environmental frequencies together and collapses them into a singular, unified perceptual sound object. Because of this high-tolerance grouping mechanism, the human subcortical auditory system enforces a vital, deliberate neglect of the 2:1 mathematical octave interval, insulating it from our conscious musical mind. If the brainstem did not enforce this structural blind spot, independent musical voices moving in octaves would instantly be captured and absorbed by the lower voice's harmonic template. Octave stretching is the mandatory biological correction engineered to escape this 2:1 harmonic capture zone:

$$\text{Octave}_{\text{Calculated}} = \left(\frac{9}{8}\right)^6 \approx 1223.46 \text{ cents}$$

This expansion ensures that independent polyphonic lines retain their unique identities, spatial positioning, and narrative clarity. The following sections provide cross-cultural evidence demonstrating why the reductionist, beat-avoidance model of Helmholtz cannot account for human musicality.

A.2.1. Quantitative Beat-Rate Modeling: The Failure of the Simple-Ratio Narrative

In his investigation tracking psychoacoustic factors in musical intonation, Douglas Keislar [17] initially attempted to verify the classical acoustic proposals of Helmholtz. Keislar designed high-precision experiments to prove that human intonation preferences are driven by the mechanical minimization of acoustic beats between upper partials. However, his data ultimately forced a negative conclusion, revealing severe systemic contradictions to the Helmholtzian rule. Keislar's data conclusively demonstrated that human judgments of whether an interval is in tune depend primarily on an un-deviated, internal cognitive geometry rather than the count or rate of accompanying physical beats. By artificially frequency-shifting upper partials to highly inharmonic positions, he proved that disrupting internal harmonic template structures had an identical negative effect on perceived intonation as literally mistuning the core interval. Because physical beats are vastly less audible in real-world musical scenarios than under strict laboratory environments, Keislar rejected his original premise, concluding that musical intonation relies on deep structural cognitive processing rather than the concomitant beat rates set forth by Helmholtz. Symmetrically, instead of pitch being rigidly tied to the fundamental frequency (f_0), his findings verified that the perceived pitch of an inharmonic sound becomes sharper than the raw fundamental baseline suggests.

A.2.2. Affective Psychoacoustic Valence: The Autonomy of the Consonance Narrative

Further evidence separating raw acoustic beating from musical comprehension is found in individuals presenting with congenital amusia. Amusics suffer from a top-down tracking deficit within the higher auditory cortex, rendering them in-

different to standard musical harmony, intervals, or scales. Cousineau, McDermott, and Peretz [18] demonstrated that amusics' physical detection and cognitive dislike of rapid, rough, peripheral acoustic wave beating were indistinguishable from normal control listeners. This data defeats the classical consonance narrative, proving that processing rough acoustic beats is a primitive, low-level brain-stem reflex disconnected from the higher cognitive architecture required to parse musical language and emotional affect.

A.2.3. Ecological Acoustic Environments: Defeating the Real-World Beat Reality Narrative

To test whether the preference for beat-free intervals operates as a hardwired human universal, researchers evaluated the Tsimané tribe—an indigenous population in the Bolivian Amazon isolated from Western 12-Tone Equal Temperament (12-TET) and industrial audio technology. McDermott, Schultz, Undurraga and Godoy [19] found that while the Tsimané can perfectly perceive the physical, acoustic difference between a consonant (beat-free) and dissonant (rough) chord, they are indifferent to them. They do not perceive consonant chords as pleasant nor dissonant chords as unpleasant, obliterating the Helmholtzian assertion that human aesthetic valence is biologically dictated by physical beat rates in real-world environments.

A.2.4. Survival Priority Mechanics: The Evolutionary Prevention of Harmonic Capture

From an evolutionary standpoint, the reductionist model fails because it ignores the threat of harmonic capture. If the human brain optimized exclusively for the minimization of acoustic friction as Helmholtz proposed, the auditory system would continuously collapse complex music back into a mono-frequency drone. The subcortical Phase-Locked Loop (PLL) circuits prevent this by actively pushing the octave boundary outward to 1223.46 cents. By widening the interval beyond the rigid 2:1 threshold, the biological hardware introduces a deliberate microtonal distortion product. This structural asymmetry acts as an acoustic wedge, physically preventing the upper voice from sinking into the acoustic gravity of the lower voice's upper partials, thereby enabling the perception of complex, multi-part counterpoint.

A.2.5. Neural Subharmonic Entrainment: The “Ringing” Paradox and Constructive Beating

The popularized musicological dogma stating that optimal chords must be free of acoustic beats represents the opposite of performance reality. The primary marker of master-level harmony in acoustic ensembles is not a passive, sterile stillness. Rather, it is the production of a highly active, resonant, and tangible harmonized ringing effect across the acoustic space. This phenomenon is most clearly realized in historical a cappella barbershop quartet performances. Far from avoiding acoustic interactions or minimizing wave friction, this locked ringing is driven by the deliberate, strategic maximization of constructive, low-frequency subharmonic en-

ergy. Instead of achieving a static, vacuum-neutral purity, optimal vocal alignment intentionally triggers active, highly synchronized neural spikes within our internal biological hardware. This process is governed directly by the proposed 30 Hz Sub-harmonic Resonance Engine, transforming sensory dissonance from a structural error into an essential, somatic catalyst for collective neural entrainment.

A.2.6. The Plurality Paradox: Dismantling the Simple-Ratio Framework via the Helsinki Dataset

The final mathematical refutation of the Helmholtzian simple-ratio narrative resides within the raw coordinate distribution of the Helsinki subjective tuning dataset. Traditional acoustic dogma asserts that the permanent elimination of audible beats can only be achieved when interval boundaries align with strict, simple integers. However, empirical tracking reveals a profound logical contradiction: while the expert tuners successfully eliminated all observable beating across the testing matrix, no two data points on the tuned configuration overlap. This wide, continuous scattering of distinct coordinates proves that the beat-free landscape is comprised of a vast plurality of unique pitch ratios. Mathematically, it is impossible for an interval set to be simultaneously multi-variate, non-overlapping, and locked to simple integer fractions. The absence of overlapping coordinates over the tuned range confirms that the auditory system enforces beat-free stability through a complex, non-linear geometric matrix rather than a rigid array of basic Helmholtzian fractions.

A.3. Appendix C: Cortical Mapping of Fast and Slow Neural Beat Rates

To model how the human central auditory pathway transitions from tracking a slow, horizontal rhythmic pulse to a vertical musical pitch or chordal fusion, this framework evaluates the cortical mapping data established by Ross *et al.* [20] alongside the foundational subcortical architecture outlined by Oster [21]. When slightly mismatched frequencies are presented dichotically to each ear, they interact within the central auditory pathways, generating the perceptual illusion of a binaural beat. As the localized frequency difference (Δf) widens, the human cognitive perception of the stimulus shifts through three distinct, biophysically segregated physiological stages:

- **Low Difference Frequencies ($\Delta f < 4$ Hz):** The central nervous system perceives a single, integrated acoustic object moving physically across the horizontal azimuth between the left and right ears, tracking the macro-motion via interaural time difference cues.
- **Intermediate Difference Frequencies ($4 \text{ Hz} < \Delta f < 20 \text{ Hz}$):** Cognitive perception shifts from physical macro-motion to localized internal loudness fluctuations, fluttering, and sensory roughness as the neural firing cycles collide within the subcortical pathway.
- **High Difference Frequencies ($\Delta f > 20 \text{ Hz}$):** The rapid neural fluctuations pass the human sensory fusion threshold. Instead of tracking individual temporal fluctuations or experiencing peripheral roughness, the central auditory path-

way triggers sustained cortical neuromagnetic oscillations, synthesizing the high-speed interaction into a unified, stable internal pitch sensation.

This physiological progression directly supports the 30 Hz subharmonic resonance engine framework detailed in Section 3.7. It demonstrates that when a chord's cross-voice partial combinations target a common subharmonic frequency in the 28 to 30 Hz register, the brainstem automatically processes the input above the auditory roughness threshold. By triggering cortical synchronization and activating steady-state neuromagnetic oscillations rather than low-frequency phase tracking, the system converts raw acoustic friction into a stable, highly pleasing internal neural pitch, driving the somatic perception of pristine chordal fusion.

A.4. Appendix D: 12-TET Interval Approximations and Fractional Density Expansion

Table A1. Fifteen minor-third fractional approximations (300 cents $\pm$ 0.0015 cent).

Fraction	numerator	denominator	decimal	cents
3790/3187	$2 \times 5 \times 379$	3187	1.18920615	299.9986
1873/1575	1873	$3 \times 3 \times 5 \times 5 \times 7$	1.189206349	299.99889
3702/3113	$2 \times 3 \times 617$	11×283	1.189206553	299.99918
1829/1538	31×59	2×769	1.189206762	299.99949
3614/3039	$2 \times 13 \times 139$	3×1013	1.189206976	299.9998
5399/4540	5399	$2 \times 2 \times 5 \times 227$	1.189207048	299.9999
1785/1501	$3 \times 5 \times 7 \times 17$	19×79	1.189207195	300.00012
5311/4466	47×113	$2 \times 7 \times 11 \times 29$	1.189207344	300.00033
3526/2965	$2 \times 41 \times 43$	5×593	1.18920742	300.00044
5267/4429	23×229	43×103	1.189207496	300.00055
1741/1464	1741	$2 \times 2 \times 2 \times 3 \times 61$	1.18920765	300.00078
5179/4355	5179	$5 \times 13 \times 67$	1.189207807	300.00101
3438/2891	$2 \times 3 \times 3 \times 191$	$7 \times 7 \times 59$	1.189207887	300.00112
5135/4318	$5 \times 13 \times 79$	$2 \times 17 \times 127$	1.189207967	300.00124
1697/1427	1697	1427	1.189208129	300.00148

This appendix catalogs the computational output of a decimal-to-fraction re-mapping algorithm, focusing on the 12-Tone Equal Temperament (12-TET) minor-third interval target (300.00 cents). Within an ultra-tight, high-precision window spanning 299.9985 cents to 300.0015 cents, the system extracts fifteen distinct fractional options. Because 12-TET is inherently symmetrical and transpositionally uniform across all twelve keys, these matching, highly recurring low-power and compounding prime-limit fractions repeat regardless of modulation.

This mathematical density provides a direct mechanical explanation for the prominent acoustic lock and near-emergence of ringing observed in modern

multi-track a cappella vocal tracks processed via pitch-correction software. Because 12-TET forces identical interval configurations to reappear identically across every key and octave, the subcortical auditory apparatus can rapidly learn, lock onto, and isolate these overlapping, near-integer ratio approximations. The interaction between the repetitive architecture of Equal Temperament and the brain's top-down capacity to recognize recurring structural patterns serves as the primary driver behind this digital chordal fusion phenomenon.

A.5. Appendix E: The Open-Access Scales.zip Archive

The scales.zip repository originates as the foundational open-source database built for Scala, a cross-platform software utility engineered for creating, transforming, and analyzing musical tuning systems. Compiled in the Netherlands by Manuel Op de Coul under the auspices of the Huygens-Fokker Foundation, the repository preserves historical, theoretical, and ethnomusicological pitch structures. Curated over several decades by John H. Chalmers and Manuel Op de Coul, the archive serves as a standardized musicological asset housing an expansive collection of distinct tuning files featuring the standard *.sc*/extension stored in plain ASCII text format. This corpus aggregates a global array of interval frameworks—spanning ancient Greek divisions, historical European keyboard temperaments, 3-limit and 5-limit just intonations, contemporary microtonal matrices, and non-Western regional scales—serving as a supportive, data-driven baseline for algorithmic scale modeling and computational musicology.

A.6. Appendix F: Methodological Execution, Auditioning Pitfalls, and Falsification Protocols via the Tuning Parser Application

Remapping historical temperaments or novel scale structures against real-world musical assets requires strict algorithmic replication. This process relies on the deterministic execution of a custom-engineered VBA tuning parser application designed to map subcortical auditory tracking boundaries. If the structural laboratory parameters—encompassing multi-instance MIDI channel routing and precise stretched octave calculations—are kept identical, the underlying tuning geometry remains invariant. Furthermore, while utilizing alternative SoundFont files will alter the superficial timbral envelope of the virtual instruments, the core perceptual conclusions and acoustic chordal ringing phenomena remain fully invariant, universally reproducible, and independent of the underlying sample library. To fully manifest this invariant physical sound wave and validate these subcortical tracking phenomena, investigators must adhere to the diagnostic parameters and experimental guidelines formalized in Section 6.8.2; failure to enforce these auditory controls will introduce cognitive and environmental noise that overwhelms sensory judgment.

A.6.1. The Evolutionary Role of the Software Engine as a Falsification Apparatus

The Tuning Parser Application developed by this author has its computational

architecture optimized over a multi-year development cycle, functioning as a falsification engine rather than a tool engineered retrospectively to validate pre-conceived musicological theories of chordal ringing. Over extensive stress-testing, the application consistently disproved preliminary theoretical assumptions and exposed the systemic acoustic failure of traditional, un-stretched tuning models. If a proposed tuning matrix contains geometric errors, the application programmatically reveals those discrepancies by generating conspicuous, unnatural sonic corruption, bypassing subjective behavioral metrics.

Ultimately, the Tuning Parser Application confirmed that un-stretched intervals invariably cause dense polyphonic music to collapse into a flat, artificial texture, proving that musical emotion cannot be unlocked unless the underlying scale geometry structurally supports the subcortical tracking limits of human hearing. The final structural solution—the asymmetrical microtonal divergence generated by a phase offset between two discrete, six-tone whole-tone streams—did not originate from prior external musicological literature or existing acoustic inspiration, as no historical trace of this dual-stream paradigm exists. Instead, this breakthrough manifested as an immediate conceptual synthesis of structural clarity; when this original architectural template was programmatically deployed within the application's code, the multi-part musical infrastructure instantly locked and functioned in acoustic alignment with real-world human emotional perception.

A.6.2. Structural Analysis of Auditioning Pitfalls and Experimental Controls

Through exhaustive execution of these computational models, this study identifies fourteen critical parameters and experimental constraints required to ensure methodological replication:

1) Acoustic Temporal Bias: Prolonged exposure to an isolated tuning matrix introduces a cognitive normalization effect, biasing immediate intervallic judgment and masking structural distortions over time.

2) Pedagogical Purity Pre-Conditioning: Listeners frequently judge intonation based on an institutionalized bias prioritizing simple frequency fractions; however, forcing intervals into these rigid, consonant boxes yields negative perceptual results in real-world performance.

3) The Stretched Octave Criterion: Because subcortical phase-locking demands a widened octave boundary, the overwhelming majority of the 5,350 files preserved within the legacy archive are structurally deficient, inflicting palpable acoustic friction upon the auditory pathway.

4) The Via Negativa Decision Rule: Instead of subjectively scoring optimal intonation, the operational decision rule asks: Does a particular microtonal configuration cause a highly familiar, respected musical masterpiece to sound structurally deficient or emotionally flat?

5) A-B Waveform Comparison Protocols: To isolate the mathematical impact of the proposed stretched matrix, the parsing software generates uncompressed.wav

audio files directly from identical source MIDI files, allowing systematic comparison against elite live orchestral and vocal reference assets.

6) The Dense Orchestral Control Framework: For intricate, complex, multi-instrumental symphonic works, a microtonally tuned MIDI rendering to 1223.46 cent per octave stretch is evaluated against both an active live acoustic performance and an identical MIDI file locked to standard Equal Temperament (12-TET), rendering timbral corruption caused by un-stretched harmonic capture instantly conspicuous.

7) Dual-Track Phase Alignment Pass: For solo vocal and instrumental tracking, the isolated MIDI score is microtonally adjusted to the target tuning matrix and tempo-matched to a legendary soloist's live archival recording; both tracks are imported concurrently into pitch-analysis software (e.g., Celemony *Melodyne*) to perform simultaneous real-time visual grid analysis and aural phase-locking verification.

8) Environmental Acoustic Interference: Auditioning environments must be monitored to eliminate artifacts introduced by digital reproduction, such as artificial reverberation algorithms or compressed dynamic ranges, which mask underlying tuning discrepancies.

9) Auditory Deprivation Protocols ("Music Fasting"): Continuous high-frequency exposure to musical performance induces sensory habituation, rapidly diminishing an investigator's microtonal discrimination thresholds; periodic blocks of total auditory rest are mandatory to maintain sensory acuity.

10) Tactical Performance Masking: Listeners must be cognitively trained to separate expressive, non-tonal performance tactics—such as localized vibrato, rubato, and dynamic swelling—from the underlying core tuning architecture, ensuring that performance energy does not mask structural scale deficiencies.

11) Emotional Disconnection in Rational Temperaments: Because 12-TET and incorrect tuning profiles lack the necessary microtonal stream asymmetries, they strip away the scale-degree friction required to convey deep artistic affect, leaving the performance emotionally inert.

12) The Linguistic Familiarity Constraint: Evaluating a tuning system using a musical language foreign to the listener is scientifically meaningless; a listener must be deeply familiar with the traditional semantic codes of a repertoire to recognize when a microtonal displacement has disrupted its emotional grammar.

13) Delimitation to Established Repertoires: While any arbitrary mathematical scale could theoretically be justified by inventing a specialized avant-garde genre tailored to its parameters, this study restricts its scope to the existing, established mainstream Western classical and vernacular repertoire.

14) Revisiting the Helmholtzian Narrative Framework: Helmholtz identified primary acoustic elements, but subsequent musicological interpreters assigned incorrect prescriptive aesthetic values to them. If consonance represents a continuous, stable state, and dissonance represents an intermittent disruption, a narrative built exclusively on pure consonance remains artistically toothless. A captivating

story emerges only when a structural disruption occurs momentarily at a critical juncture; within the Western diatonic architecture, this dramatic catalyst is executed by the stealth appearance of an accidental note that microtonally swaps the whole-tone stream boundaries, instantly transforming a triumphant major key into profound melancholy.

A.6.3. The ‘Application Code’ Dilemma: Why Intuition Cannot Verify Tuning

Evaluating musical tuning or emotional depth by ear alone is a deceptively difficult task. The core challenge stems from our cognitive architecture: our most acute perceptual survival skills are hardcoded deeply within us, isolated from our conscious, intellectual choices. Consider how a tennis player’s brain intuitively coordinates the exact spatial mechanics of their hands and legs to hit a ball precisely, yet they cannot articulate the conscious mathematics of that movement. Similarly, any average person can instantly distinguish a dog from a cat by sight, yet the brain’s internal “application code” for facial recognition is sequestered from the motor paths needed to sketch that face. The intuitive recognition simply does not transmit to the hand trying to draw it.

This cognitive boundary directly mirrors the illusion of musical auditioning. Even exceptionally gifted musicians cannot flawlessly identify, translate, or replicate a tuning scheme purely on command; they can only generate an educated imitation and then exhaustively check their success rate against empirical data. Possessing raw, elite musical talent is merely the entry-level baseline for this task. If a person cannot already sing to achieve an acoustic ringing within a vocal assembly, evoke tears with a violin or cello performance, or naturally inspire a neighbor in a church pew to record their voice, attempting the work of tuning auditioning becomes a vain undertaking. Yet, even for those who possess these mandatory, elite baseline qualities, human perception remains a closed loop that cannot be verified from within. It is useless to just ponder mathematics and theoretical ratios. Instead, confirming whether a tuning has been truly identified or replicated requires immense labor and the deployment of every objective tool available—including the *Tuning Parser App*, *Melodyne*, active play-along comparisons, and audio and visual spectrum inspection.

A.7. Appendix G: Multi-Source Open-Access MIDI Repositories and Structural Data Parsing

To stress-test the Tuning Parser Application against dense, polyphonic data layouts, the computational project expanded beyond individual web-portal downloads to ingest massive, large-scale compressed repositories. The expanded research corpus incorporated a diverse array of historical and contemporary digital libraries, including the Disklavier World Archive and Pianola Registries (providing extensive collections of historical piano-roll data captured via high-precision MIDI transduction), the Kunst der Fuge databases (providing bulk repository downloads of dense, structurally contrapuntal and polyphonic MIDI configura-

tions), the Lakh MIDI Dataset (aggregated for large-scale statistical data mapping and pattern recognition), and curated bulk repositories such as the 130,000-Zipped-MIDIs and MagicMidi database matrices.

Each decompressed MIDI asset was parsed through the application to extract three baseline metadata parameters: raw file capacity, total number and timbral classes of instruments, and the presence of internal meta-event key change commands. By categorizing files based on structural density and key activity, the application allows investigators to rapidly identify and isolate complex multi-voice repertoires—such as large-scale symphonies, concertos, or dense jazz arrangements—for targeted microtonal auditioning when performance analysis requires.

A.8. Appendix H: The Neural PLL Analogy

While the term Phase-Locked Loop (PLL) conventionally denotes an electronic signal-processing circuit tracking continuous frequencies via an analogue Voltage-Controlled Oscillator (VCO), its biological manifestation within the human brainstem operates as a discrete, non-continuous tracking engine. Rather than sweeping smoothly across a seamless analogue continuum, the subcortical auditory processing pathway is constrained by the discrete integer values of physical harmonics and the fixed refractory limits of neural firing. Consequently, when this biological PLL processes an incoming musical interval, it cannot execute infinite frequency matching; it can only parse the signal at these fixed harmonic junctions. This structural constraint forces overlapping, microtonally mismatched waveforms to generate localized, entrained neural beats clustered around the 30 Hz threshold. This synchronization operationalizes the biomechanical trigger behind the phenomenon of chordal fusion and harmonized ringing, demonstrating that human harmonic preference is structurally dictated by the discrete tracking boundaries of our subcortical hardware.

A.9. Appendix I: Contextual Analysis of Transposition Tools and Acoustic Boundaries in Music Ensembles

The historical deployment of physical transposition tools adjustments provides real-world validation of how performers instinctively balance physical instrument constraints against vocal and cognitive pitch templates. A prominent example occurs within the tracking sessions for *Michelle* by The Beatles (November 3, 1965). While the final acoustic output captured on tape registers in the concert pitch key of F minor, studio tracking records confirm that the acoustic rhythm guitars were tracked utilizing a mechanical capo clamped onto the fifth fret. This layout allowed the primary performer to execute the song using open-position C-relative chord fingerings while dynamically shifting the broadcast frequencies up into the key of F to accommodate localized vocal register boundaries without altering underlying muscle memory.

From an engineering perspective, shifting a guitar's fundamental operating frequencies higher up the fingerboard via a capo alters its global spectral footprint.

This structural compression replicates the functional relationship that a high-register piccolo shares with a traditional concert flute: it effectively alters the instrument's acoustic role from a low-mid chordal foundation into a bright, un-damped harmonic generator. However, this mechanical truncation forces the instrument's harmonic structure to align more closely with a standard Equal Temperament (ET) environment rather than organically supporting a fully stretched octave matrix, establishing a structural threshold where mechanical convenience overrides biological tuning optimization. Consequently, for performers operating at either the extreme upper or lower registers of the ensemble, optimal acoustic integration requires a fundamental departure from static tuning protocols. Even for instruments characterized by constrained or non-free tuning mechanics, it is highly advisable to calibrate all pitch boundaries according to a precise, systematic 23.46 cents per octave stretching principle, as also suggested by Jaatinen, Pätynen and Alho [22], although the exact magnitude of stretch may then not have been decided or realized. Rather than anchoring the instrument's extreme ranges to a rigid, un-deviated template, performers should tune against an intentionally up-shifted or down-shifted microtonal adaptation of the standard concert pitch baseline ($A4 = 440$ Hz), thereby aligning the physical instrument parameters with the evolutionary tracking constraints of human subcortical hearing.

A.10. Appendix J: Glossary of Cross-Disciplinary, Neuro-Mechanistic, and Psychoacoustic Terms

- **Acoustic Mode-Locking:** A nonlinear physical process wherein continuous driving forces (such as stick-slip friction or aerodynamic oscillation) compel independent acoustic variables to cooperate, forcing upper overtones to snap into strict whole-integer multiples of the fundamental frequency.
- **Auditory Resonance Engine (30 Hz Subharmonic Engine):** The proposed core subcortical neurocomputational tracking architecture within the human brainstem that monitors, locks onto, and aligns overlapping upper adjacent musical harmonics into a common, low-frequency 30 Hz difference periodicity to generate vertical chordal fusion and somatic entrainment. This downscaling process relies on biological frequency division to isolate a shared baseline frequency that functions as the Highest Common Factor (HCF) of the incoming upper partial matrix, enabling immediate subcortical injection locking and neural entrainment (see Section 3.7).
- **Binaural Beats:** An internal periodic modulation computed within the superior olivary complex of the brainstem, generated when two separate pure sine waves with slightly different frequencies are presented dichotically to each ear, originally detailed by Oster [21] and modeled spatially by Jeffress [23].
- **Capture Zone:** The microtonal frequency window within which the subcortical Phase-Locked Loop (PLL) can successfully capture, pull, and track an incoming acoustic stream into stable periodic alignment.
- **Cochlear Amplifier:** An active, nonlinear mechanical amplification process

driven by outer hair cell motility that enhances frequency selectivity, sensory sensitivity, and generates intermodulation distortion products.

- **Chordal Fusion:** The perceptual state where individual vertical harmonic partials are processed by the central auditory system as a singular, unified acoustic object rather than independent tones.
- **Cognitive Blinding:** An information-theory state wherein a hyper-dense, chaotic web of overlapping physical data overloads subcortical sorting circuits, preventing the auditory processor from isolating a clean, singular anchoring frequency.
- **CoolSoft VirtualMIDISynth:** A free, open-access virtual MIDI driver and software synthesizer implemented as a Windows multimedia driver configuration. Within this framework, the utility functions as a critical infrastructural bridge, allowing the host operating system to bypass traditional hardware-addressing limitations and instantiate multiple independent virtual MIDI devices simultaneously. By loading user-selected *SoundFont* (.sf2 or .sfz) sample banks directly into system memory, the application executes high-precision, low-latency digital audio rendering of incoming MIDI streams either in real time or offline. Its capacity to handle up to four discrete virtual instances concurrently provides the necessary 60-channel addressing matrix required by the Tuning Parser Application script to resolve overlapping multi-voice pitch-bend conflicts and output pristine, uncompressed.wav or compressed.mp3 waveform files for aural scrutiny.
- **Diaschisma:** The microtonal interval measuring 19.55 cents, derived from the mathematical ratio 2048:2025. Within this study's framework, this interval defines the boundary of microtonal friction separating the optimized singers' scale from standard geometric temperaments.
- **Difference Frequency (Δf):** The microtonal frequency gap resulting from the acoustic interaction or subtraction of two adjacent primary tones or upper harmonic combinations ($f_2 - f_1$).
- **Distortion Product Otoacoustic Emissions (DPOAE):** Acoustic energy generated within the inner ear when two simultaneous pure tones (f_1 and f_2) interact nonlinearly along the basilar membrane, driving the outer hair cells to emit measurable intermodulation distortion products back out through the middle ear pathway.
- **Eigenmode:** The natural, decoupled resonant frequencies or structural vibration patterns of an un-driven acoustic system, typically manifesting microtonal micro-cent deviations from pure whole-integer targets.
- **Entrainment (Subcortical Neural Entrainment):** The hardwired neurobiological process whereby large populations of neurons within the ascending brainstem auditory pathway automatically synchronize their electrical firing rates and phase-locking precision to mirror the ongoing periodic structure, envelope velocity, and temporal patterns of an incoming acoustic waveform. As documented by Coffey *et al.* [24], this microsecond-precision temporal

alignment acts as the primary sensory gate enabling the human central nervous system to track complex fundamental envelopes and pull overlapping musical partials into a unified phase-locked loop—even when the physical fundamental frequency (f_0) is absent from the stimulus—providing the biological foundation for horizontal pitch tracking and vertical chordal fusion.

- **Esh Symbol (ʘ):** The universal International Phonetic Alphabet (IPA) character representing a voiceless palato-alveolar fricative sound, functioning within this framework as a sharp temporal domain acoustic transient.
- **Frequency-Following Response (FFR):** A high-fidelity subcortical and early cortical neuro-electric potential that phase-locks with microsecond precision to the ongoing periodic spectral and temporal characteristics of an ascending acoustic waveform. As comprehensively mapped by Coffey *et al.* [24], the FFR tracks the perceived fundamental frequency (f_0) and envelope periodicity of complex sound structures across the brainstem auditory pathways. Within this framework, the FFR functions as the hardwired neurophysiological mechanism that extracts a shared periodic root from the spacing of upper adjacent harmonics when the physical fundamental component is missing, maintaining horizontal pitch memory tracking and subcortical voice segregation to achieve vertical chordal fusion.
- **Harmonic Capture:** The subcortical auditory scene analysis process, originally formalized by Bregman and Pinker, whereby a dominant fundamental frequency template forcefully segregates, absorbs, and misinterprets a nearby independent acoustic partial as one of its own upper overtones, resulting in the perceptual silencing and timbral collapse of the overlapping musical voice.
- **Harmonized Ringing (Choral Shimmer):** A resonant acoustic phenomenon experienced when multiple independent voices phase-lock their upper partials. While vocal performers traditionally attribute this sensation to acoustic illusions like the “fifth voice” or somatic “mask placement,” this framework demonstrates that the perceptual ring is driven by a deep architectural divergence: the brainstem down-divides these high-register partials to generate a shared subharmonic neural beat localized at the 30 Hz threshold, transforming acoustic friction into an essential catalyst for collective neural entrainment.
- **Inharmonicity:** The physical property of an acoustic oscillator where upper partials deviate from strict integer multiples of the fundamental frequency, rendering equating pitch determination to a static measurement of fundamental frequencies (f_0) futile. While mechanical laboratory applications analyze these micro-cent deviations to calculate the oscillatory destruction of resonating machine systems, the human auditory system utilizes a front-end, analog process mapped by the author’s Multi-Harmonic Pitch Synthesis model (see Section 3.2). Empirical studies on piano tones by Anderson and Strong confirm that this biological sensation handles upper partial acceleration by shifting the perceived pitch sharper than the physical fundamental baseline suggests, with pitch increases ranging from approximately two and a half semi-

tones at low frequencies down to an eighth of a semitone at higher registers.

- **Isochronism (Pitch-wise Isochromatic Stream Tracking):** A front-end, sub-cortical processing mechanism characterized by the auditory system's top-down expectation of smooth, unit-free continuity along a single linear pitch dimension.
- **Limma:** The Pythagorean minor semitone measuring 90.22 cents, derived from the mathematical ratio 256:243. In this study, this compressed interval represents the primary structural bottleneck within the violinists' tuning scheme that suppresses vertical partial alignment.
- **Monastral Beats:** Sound wave intensity modulations (perceived as roughness or physical beating) generated in the physical acoustic periphery when two waves of slightly different frequencies physically mix and interfere before entering the cochlea, forming the basis of the historical tuning frameworks pioneered by Helmholtz.
- **Multiquantal Release:** The simultaneous, large-amplitude, and highly synchronous release of multiple neurotransmitter vesicles from auditory hair cell ribbon synapses, driving sub-millisecond temporal spike timing precision in afferent nerve fibers. Within this framework, this phenomenon explains why "in-phase" temporal detection of lower harmonics takes a major role in both wave segregation and cross-frequency synchronization. This microsecond-precision synaptic drive forces lower, resolved partials to actively entrain and synchronize much higher, unresolved harmonics across the peripheral tracking grid, enabling the biological sensation of unresolved overtones while generating the proposed 30 Hz harmonizing beats.
- **Neural Beat Tracking (Cortical Mapping Metrics):** The neurocomputational process by which the human central auditory pathway transitions from tracking a slow, horizontal rhythmic pulse to a vertical musical pitch or chordal fusion. As mapped via cortical-neuromagnetic tracking by Ross *et al.*, this mechanism operates across biophysically segregated physiological stages, where the central nervous system shifts from tracking lateralized phase modulations at low difference frequencies ($\Delta f < 4$ Hz) to processing high-rate binaural and internal sub-harmonic periodicities as unified, somatic acoustic objects.
- **Phase-Locked Loop (PLL):** A subcortical neurocomputational tracking engine utilizing localized frequency division and multiplication loops to establish phase alignment, sound segregation, and chordal fusion. Within this framework, its biological manifestation complies with the mismatched temporal time constants and auditory processing models outlined by de Cheveigné (see **Appendix H**).
- **Pythagorean Comma (Ditonic Comma):** A microtonal micro-cent discrepancy (approximately 23.46 cents) resulting from the structural mathematical friction between independent compounding cycles of pure intervals.
- **Singer's Formant:** A high-amplitude acoustic energy spike concentrated between 2500 Hz and 3000 Hz, engineered via vocal tract manipulation to facilitate high-precision subcortical phase-locking.

- **Singers' Scale:** An optimized, 5-limit microtonal tuning matrix that introduces a highly localized structural asymmetry—the 19.55 cents *diaschisma* interval ($16/15 \times 128/135 = 2048/2025$)—to project intense, narrative emotional valence across horizontal melodic lines while successfully preserving vertical harmonic alignment (see Sections 4.2, 4.3). By utilizing a precise biological macrostructural octave stretch, this matrix allows upper adjacent overtones to seamlessly bypass geometric fraction collisions and lock directly into common, low-frequency 30 Hz subharmonic proximity beats (see Section 3.7). The activation of this 30 Hz Auditory Resonance Engine and its resulting vertical chordal ringing loop is not exclusive to a single scalar template. It is fully achievable within computer-corrected 12-Tone Equal Temperament (12-TET) matrices—where the mechanical perfect fourth (500 cents) matches the organic singers' scale intervals down to a negligible 0.0013-cent threshold—or indeed any acoustic configuration that shares these identical subharmonic parameter bounds (see Section 6.4).
- **Stochastic Resonance:** A biophysical phenomenon wherein the strategic introduction of a low-level background noise source or physical turbulence actively enhances the detection and phase-locking precision of an internal signal.
- **Stretched Matrix:** A non-uniform tuning grid where musical intervals are expanded beyond standard geometric integer boundaries to match the biological processing constraints and octave enlargement thresholds documented by Cuddy and Dobbins [25].
- **Stretched Octave:** An operational musicological and psychoacoustic term initiated by the author following a 1978 thesis to identify non-integer acoustic expansion dimensions. Although historical musicians and master performers routinely discovered that the subjective octave must exceed a rigid 2:1 frequency ratio to achieve melodic clarity, the phenomenon was universally questioned, disbelieved, and ridiculed by mainstream institutional academics as a performance error until computational digital synthesis models formally vindicated the expansion as a mandatory evolutionary processing requirement of a single linear pitch dimension.
- **Subharmonic Injection Locking (SHIL):** A passive, non-continuous physical phenomenon wherein the nonlinear energy of adjacent subcortical oscillators is drawn into phase synchronization by a common mathematical divisor.
- **Tetrachordal Gradient:** The 19.55 cents structural microtonal friction generated between wide lower tetrachord semitones and narrow upper tetrachord semitones, acting as the primary biophysical engine of musical affect (see Section 4.2).
- **Tuning Parser Application:** A local, custom-coded Visual Basic for Applications (VBA) script engineered to recode MIDI command structures against original multi-track file inputs according to selected tuning matrices either in real time or offline. To circumvent standard MIDI protocol limitations where pitch-bend parameters alter entire channels uniformly, the application remaps every incoming command so that each individual note occupies one discrete

MIDI channel, isolating the pitch-wheel vector from concurrent voices. Utilizing the free CoolSoft *VirtualMIDISynth* architecture to drive four simultaneous virtual MIDI instances, the script achieves an operational bandwidth of 60 simultaneous independent notes (15 discrete channels $\times$ 4 instances, excluding Percussion channels), allowing for the processing and auditioning of highly complex, large-scale internet MIDI data assets. The script dynamically strips non-essential metadata while fully supporting primary musical performance vectors—including velocity, volume, expression, panning, sustain pedal operations, and pitch-bent glissandi after re-tuning. It facilitates real-time, in-app playback swaps of instrument classes, tuning schemes, and key designations, while outputting optimized, re-tuned MIDI data files that can be instantly converted into uncompressed.wav or compressed.mp3 audio waveform files via the VirtualMIDISynth *Midi File Converter* tool for systematic aural scrutiny. As a result, the *Tuning Parser Application* functions as a highly flexible and effective diagnostic instrument to programmatically validate or devalue any architectural musical hypothesis in real time or offline.

- **Violinists' Scale:** A structural framework where the traditional 3:2 perfect fifth tuning of adjacent open strings induces a 3-limit Pythagorean tuning matrix optimized for linear melodic brilliance via sharp whole tones (9/8), compressed small semitone limmas (256/243), and expanded large semitone pairs (2187/2048). While this framework powerfully supports intense horizontal emotional delivery capabilities and expressive melodic projection, vertical chordal ringing is totally inhibited across the instrument's fingerboard due to deep geometric fraction collisions against standard scale-step exponents $(9/8)^n$, that corrupt the alignments necessary to trigger the 30 Hz Subharmonic Engine. This rigid biophysical boundary provides the definitive neurobiological explanation for the historical child prodigy window in string pedagogy; early isolation on this 3-limit matrix allows a toddler's subcortical Phase-Locked Loop to track linear Pythagorean intervals with micro-cent precision, a pristine alignment that is invariably degraded and confused once subsequent, long-term exposure to the asymmetric 5-limit templates of human singing introduces competing auditory tracking inputs that permanently degrade adult intonation execution (see Sections 3.7, 3.8.3).

Supplementary Media and Empirical Asset Access: The comprehensive archive of performance resurrection audio comparisons—including full-length, high-bitrate compressed audio master tracks and their raw underlying.mid instance files—is hosted permanently via open-access cloud infrastructure at the following URL:

https://drive.google.com/drive/folders/1BvbP-YrGIhUiSQIZ-P28iL3_dbIPsBG7?usp=sharing.