Open Access Library Journal
2026, Volume 13, e16071
ISSN Online: 2333-9721
ISSN Print: 2333-9705

Nondimensionalization and Cusp Catastrophe Modeling of Equilibrium Current Dynamics in Nonlinear Electrical Systems

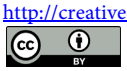
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How to cite this paper: Kaki, M.N.M. and Al Samarai, J.M.A. (2026) Nondimensionalization and Cusp Catastrophe Modeling of Equilibrium Current Dynamics in Nonlinear Electrical Systems. *Open Access Library Journal*, **13**: e16071.
https://doi.org/10.4236/oalib.1116071

Received: September 23, 2026
Accepted: November 16, 2026
Published: November 19, 2026

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Abstract

This paper investigates the equilibrium current dynamics in nonlinear electrical systems using a nondimensional cusp catastrophe model, with the dimensionless equilibrium current i as the state variable, μ as a signed effective control parameter, and ν as a normalized forcing parameter. The corresponding potential function is $\Phi(i;\mu;\nu)=\frac{1}{4}i^4+\frac{1}{2}\mu i^2+\nu i$, which provides the cusp-catastrophe representation of the equilibrium states. The resulting equilibrium relation is nondimensionalized to obtain the canonical form $i^3+\mu i+\nu=0$. We demonstrate that the sign and magnitude of μ govern the qualitative topology of the system's states, where negative values of μ induce a multistable regime containing three real equilibrium currents—two physically stable states separated by an intermediate unstable state. The bifurcation set is determined by $4\mu^3+27\nu^2=0$ ($\mu\le 0$), forming a cusp geometry in the (μ,ν) control plane. As control parameters vary, state trajectories reach the fold boundaries, precipitating rapid transitions toward alternative stable branches. This topological formulation provides a unified physical explanation for bistability, sudden current jumps, and hysteresis-like behavior in nonlinear electronic circuits.

Subject Areas

Modern Physics

Keywords

Cusp Catastrophe, Nonlinear Electrical Systems, Bifurcation Set, Bistability,

Hysteresis

1. Introduction

Electronics is a fundamental branch of engineering and applied science concerned with the generation, processing, control, transmission, and conversion of electrical signals and energy. Modern electronic systems range from semiconductor devices and integrated circuits to sophisticated power-conversion systems, nonlinear circuits, control systems, and emerging memristive devices. The continuous development of semiconductor technology has significantly expanded the capabilities of electronic systems, making them essential in communication, computation, automation, energy conversion, transportation, and industrial applications [1] [2].

A particularly important area of electronics is power electronics, which converts and controls electrical energy through semiconductor switching devices and electronic circuits. Power-electronic converters are used to convert electrical energy from one form to another, including AC-DC, DC-DC, DC-AC, and AC-AC. Mathematical analysis of these systems matters because switching operations and nonlinear device characteristics can produce complex electrical behavior. Semiconductor devices therefore play a central role in the design and analysis of modern power-electronic systems [3] [4]. Circuit-analysis and simulation techniques provide important tools for understanding converter operation, equilibrium states, transient responses, and the influence of circuit parameters on system behavior [5].

Many electronic circuits cannot be adequately described by linear models. Nonlinearities arise naturally from semiconductor characteristics, nonlinear inductors and capacitors, switching elements, feedback mechanisms, and other circuit components. Consequently, nonlinear dynamics provides an appropriate mathematical framework for studying electronic systems whose behavior changes significantly with operating conditions. Nonlinear circuits may exhibit multiple equilibrium states, stability changes, oscillations, bifurcations, abrupt transitions, and chaotic behavior **Figure 1**. The study of nonlinear dynamics in electrical circuits has therefore become an important area of research, particularly for systems in which small changes in parameters can produce substantial changes in their qualitative behavior [6] [7].

One of the well-known examples of nonlinear electrical dynamics is Chua's circuit, which provides a simple electronic realization of complex nonlinear phenomena. Chua's circuit has been extensively investigated as a prototype system for studying nonlinear oscillations, bifurcations, and chaos [8] [9]. More generally, nonlinear network theory provides the mathematical foundations for analyzing the equilibrium and dynamic behavior of electrical circuits containing nonlinear components [6]. These developments demonstrate that electrical systems can possess rich dynamical structures that cannot be understood solely through conven-

tional linear circuit theory.

The emergence of memristive devices has further increased interest in nonlinear electronic systems. The memristor was originally introduced by Chua as a fundamental circuit element completing the relationship between charge and magnetic flux [10]. Memristive behavior subsequently became an important subject in nonlinear circuit theory, with applications in memory devices, neuromorphic computing, signal processing, and nonlinear dynamical systems. Chua and Tseng developed an early memristive circuit model for p-n junction diodes [11], while the physical realization of the memristor was later reported and investigated in detail by Strukov *et al.* [12]. Various nonlinear memristor models have since been developed to describe more realistic device characteristics [13]. The mathematical analysis of memristive circuits has also revealed important phenomena involving equilibria, bifurcations, and manifolds of equilibrium points [14]. For specific addresses equilibria, bifurcations; see **Figure 2**, nonlinear circuits, and memristive circuits [15].

The presence of nonlinearities and multiple equilibrium states naturally motivates the use of mathematical theories capable of describing qualitative changes in system behavior. Catastrophe theory provides one such framework. It is concerned with abrupt qualitative changes in the behavior of systems resulting from continuous variations in control parameters [16]-[19]. In physical and engineering systems, catastrophe-theoretic methods can be used to identify parameter regions in which equilibrium states appear, disappear, or change stability. Applications of catastrophe theory to the physical sciences demonstrate its usefulness in describing sudden transitions and structural changes in nonlinear systems [20].

Among the elementary catastrophe models, the cusp catastrophe is particularly important because it describes systems in which an equilibrium state may undergo sudden transitions as two control parameters vary. The cusp model is associated with a potential function whose equilibrium condition leads to a cubic equation, allowing the existence and stability of multiple equilibrium states to be investigated. The cusp catastrophe therefore provides a suitable mathematical framework for analyzing nonlinear systems with competing equilibrium configurations and abrupt changes between stable and unstable states [16]-[19].

Catastrophe-theoretic concepts have also been applied specifically to nonlinear dynamical systems and stability problems. Kaki [21] investigated catastrophic types according to the degree of nonlinearity, while Kaki [22] examined catastrophic models in relation to stability. Earlier studies by Kaki considered the relationship between cusp catastrophe and stability [23] [24] and investigated nonlinear dynamics and cusp catastrophe. Catastrophe theory provides a powerful qualitative framework for modeling non-smooth behaviors—such as sudden transitions, bistability, and hysteresis—in nonlinear electrical systems. By mapping current and voltage dynamics onto a cusp potential surface, this approach moves beyond simple numerical simulations to identify stable equilibrium branches, pinpoint critical bifurcation boundaries, and predict abrupt operational jumps driven by control variations [25].

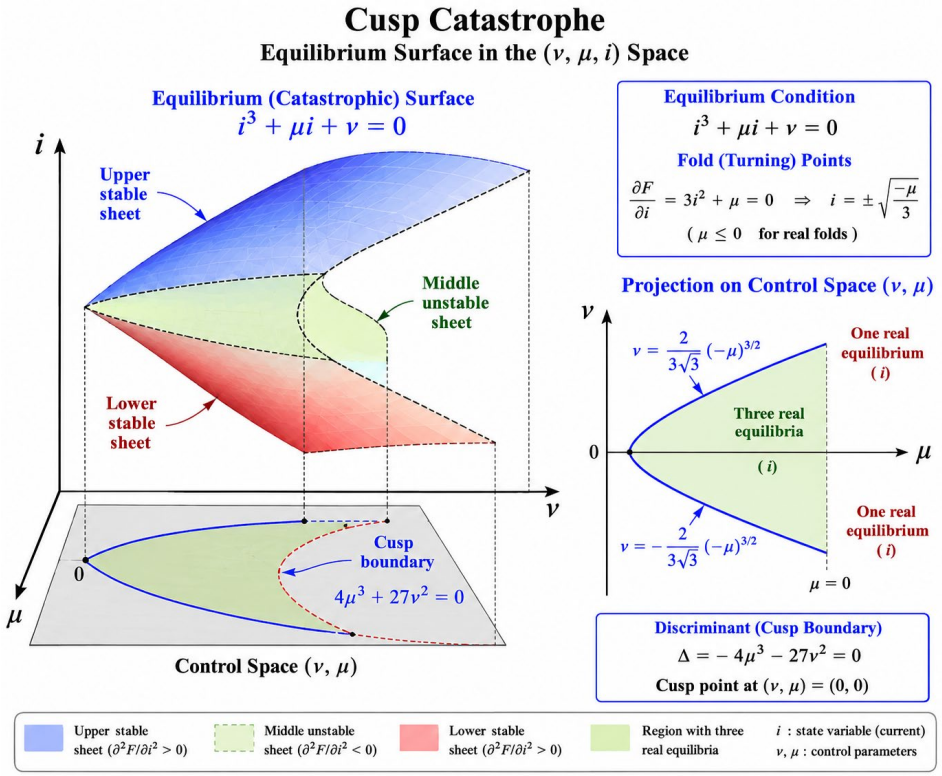


Figure 1. Cusp catastrophe model for equilibrium dynamics in nonlinear systems.

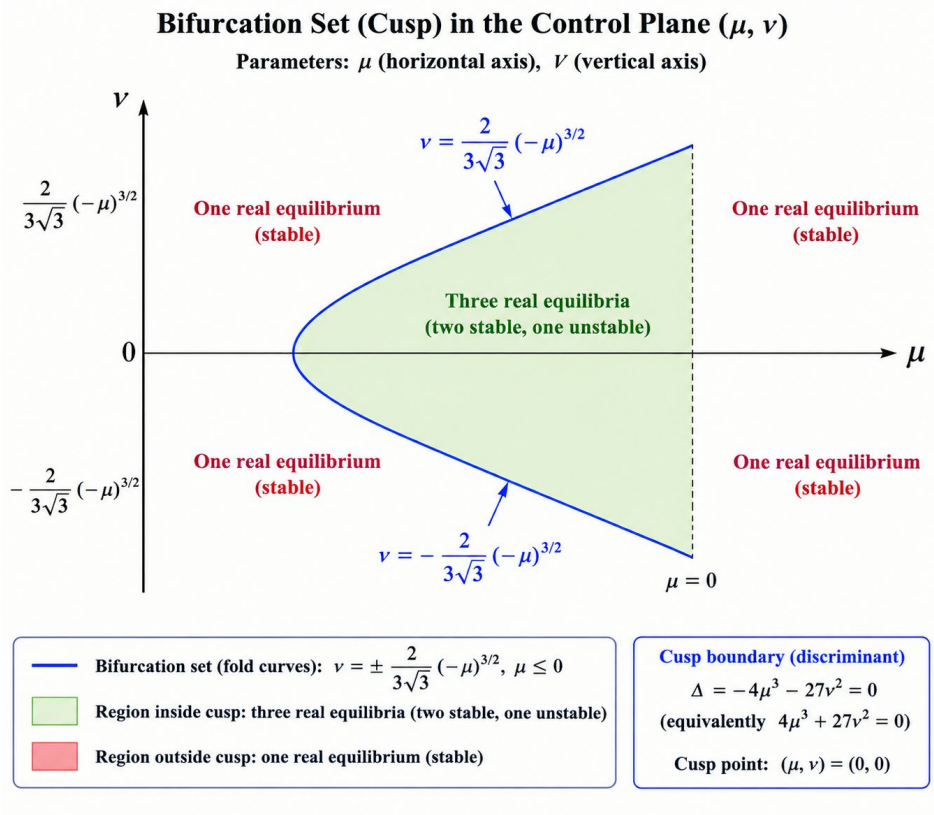


Figure 2. Bifurcation set.

2. The Canonical Cusp Catastrophe

The **canonical cusp potential** is

$$\Phi(i;\mu;V)=\frac{1}{4}i^4+\frac{1}{2}\mu i^2+\nu i,$$

so that the equilibrium condition is

$$\frac{\partial\Phi}{\partial i}=i^3+\mu i+\nu=0.$$

Thus, equilibria are the roots of the cubic equation $i^3+\mu i+\nu=0$. See Figure 1 for the canonical cusp catastrophe.

i = dimensionless equilibrium current, μ = signed effective control parameter, ν = normalized external forcing parameter.

The cusp boundary is then

$$4\mu^3+27\nu^2=0,\mu\le 0.$$

Number of Equilibria

The discriminant is $\Delta=-4\mu^3-27\nu^2$.

- If $\Delta<0$, there is **one real equilibrium**.
- If $\Delta=0$, two equilibria merge at a **fold bifurcation**.
- If $\Delta>0$, there are **three real equilibria**.

The cusp boundary is therefore $4\mu^3+27i^2=0$.

1) Stability of Equilibria

For the gradient system $\frac{\mathrm{d}i}{\mathrm{d}t}=-\left(i^3+\mu i+\nu\right)$, an equilibrium i^* is locally stable when $\frac{\mathrm{d}}{\mathrm{d}x}\left(i^3+\mu i+\nu\right)\Big|_{x=x^*}=3\left(I^*\right)^2+\mu>0$ [24] [25].

Inside the cusp region ($\Delta>0$): the lower branch is stable, the middle branch is unstable, and the upper branch is stable.

Hence, the system exhibits **bistability**.

What Causes a Catastrophe?

As a control parameter changes continuously, a stable equilibrium branch may reach a fold point where it disappears. At that instant:

a) The stable state no longer exists;

b) The system cannot remain nearby;

c) It jumps suddenly to another stable branch.

This abrupt transition is called a **catastrophe**.

2) Hysteresis: Suppose $\mu<0$ is fixed and ν is slowly increased.

- The system follows one stable branch.
- At the upper fold, that branch vanishes.
- The state jumps to the other stable branch.

If ν is then decreased, the jump back occurs at a different fold point. Thus, the forward and backward paths differ, producing **hysteresis**.

Figure 1 illustrates the application of the cusp catastrophe model to analyze the equilibrium behavior of current in a nonlinear electrical system. In this model, the

state variable is the electrical current I, whereas the **control parameters** are μ and V. The figure demonstrates how variations in these two electrical parameters influence the number and stability of equilibrium states.

The bifurcation set $\mathcal{B} = \left\{(\mu, v) \in \mathbb{R}^2 : 4\mu^3 + 27v^2 = 0\right\}$ is the boundary in the control parameter space (μ, v), where v and μ denote the **system parameters.** This curve separates regions in which the electrical system exhibits qualitatively different equilibrium behaviors.

In Section 3, the symbols (i), (v), and (μ) are used to represent dimensionless current, dimensionless voltage, and dimensionless resistance, where $i = \frac{I}{I_0}$, I is an electric current, $v = \frac{V}{V_0}$; V is a voltage, and $\mu = \frac{R}{R_0}$; R is an electrical resistance, respectively, with their corresponding units being amperes (A), volts (V), and ohms (Ω).

3. Nondimensionalization and Physical Units of the State and Control Variables

To establish a physically meaningful connection between the canonical cusp-catastrophe model and a nonlinear electrical system, we first express the physical electrical variables in nondimensional form. Let I denote the electrical current, V the applied voltage, and R a resistance-related control parameter. Introduce characteristic scales I_0, V_0, and R_0, having units of amperes (A), volts (V), and ohms (Ω), respectively. The corresponding dimensionless variables are defined by

$$i = \frac{I}{I_0}, v = \frac{V}{V_0}, \mu = \frac{R}{R_0},$$

where:

- I : physical current, measured in amperes (A).
- I_0 : characteristic current scale, measured in A.
- i : dimensionless current.
- V : physical voltage, measured in volts (V).
- V_0 : characteristic voltage scale, measured in V.
- v : dimensionless voltage.
- R : physical resistance, measured in ohms (Ω).
- R_0 : characteristic resistance scale, measured in Ω.
- μ : dimensionless resistance-related control parameter.

Thus, (i) represents the dimensionless state variable associated with the physical current, while (v) and (μ) represent dimensionless control variables associated with voltage and resistance, respectively.

In terms of these normalized variables, the canonical equilibrium equation of the cusp catastrophe is written as

$$i^3 - \mu i + v = 0,$$

All three terms in this equation are dimensionless, so the canonical polynomial is dimensionally consistent. The physical quantities can subsequently be recovered from

$$I = I_0 i,\ V = V_0 v \text{ and } R = R_0 r$$

Accordingly, the canonical variables (i), (μ), and (v) should not be identified directly with quantities measured in amperes (A), ohms (Ω), and volts (V). Instead, they are normalized representations of the corresponding physical quantities. This distinction is essential because the terms (I^3), (RI), and (V), if interpreted directly with their physical units, do not possess the same dimensions and therefore cannot be added in a single dimensional equation.

The nondimensional formulation therefore preserves the standard canonical structure of the cusp catastrophe while providing a consistent interpretation of its state and control variables in the context of nonlinear electrical systems.

4. Physical Implications of the Model for Nonlinear Electrical Systems

The cusp catastrophe model provides a physically meaningful framework for interpreting equilibrium current behavior in nonlinear electrical systems. In the present formulation, the current I is taken as the state variable, while the dimensionless voltage v and the parameter μ constitute the two control parameters. The equilibrium condition is $i^3 + \mu i + v = 0$. This equation demonstrates that the steady-state current is not necessarily a unique function of the applied voltage. Depending on the values of v and μ, the system may possess either one or three equilibrium currents. Such multiplicity is an important characteristic of nonlinear electrical systems, where nonlinear circuit elements and feedback mechanisms can produce several possible operating states. Multiple equilibria and nonlinear stability changes are well established in electrical and power-system dynamics.

The bifurcation set $4\mu^3 + 27v^2 = 0$, $\mu \le 0$, divides the (μ, v) control plane into qualitatively different operating regions. **Outside the cusp region, the model has one real equilibrium current**, corresponding to a unique steady operating state. **Inside the cusp region, three real equilibrium currents exist**, generally consisting of two stable equilibria separated by an unstable equilibrium. Thus, the same values of the external control parameters can, in principle, permit two physically stable operating states. This is the mathematical basis for **bistability** in the electrical system.

The two stable branches can be interpreted as two alternative operating currents of the nonlinear circuit. A gradual variation of the applied dimensionless voltage v does not necessarily produce a gradual variation of the current over the entire parameter range. When the operating point approaches a fold of the bifurcation set, one stable equilibrium can disappear through a saddle-node bifurcation. The system is then forced to move rapidly toward the remaining stable equilibrium.

Consequently, a relatively small continuous change in voltage can produce a **large and abrupt change in current**. This provides a physical interpretation of the catastrophe represented by the cusp geometry.

An important consequence is the possibility of **hysteresis**. If the voltage is gradually increased and then decreased, the current may follow different equilibrium branches during the upward and downward parameter sweeps. The transition from one stable state to another occurs at different critical parameter values, rather than necessarily reversing along the same path. Therefore, the system retains a dependence on its previous operating state. Such hysteretic behavior is a characteristic consequence of the folded equilibrium structure of the cusp catastrophe.

The parameter μ has an equally important physical interpretation. It controls the degree to which multiple equilibrium states can occur. For $\mu < 0$, a sufficiently small magnitude of dimensionless voltage can place the system inside the cusp region, allowing three equilibrium currents. As μ approaches zero from below, the region of multistability becomes progressively smaller. At the cusp point,

$$(\mu, v) = (0, 0),$$

the two-fold branches meet, and the distinction between the equilibrium states disappears. For $\mu > 0$, the canonical model has only one real equilibrium for every value of v. Thus, μ may be regarded as a **structural or system-control parameter** that determines whether the nonlinear electrical system is capable of exhibiting bistability and catastrophic switching.

From an engineering perspective, this behavior has both **undesirable and useful implications**. Uncontrolled movement across a fold boundary may correspond to sudden changes in current, loss of an intended operating state, or an abrupt transition to another electrical regime. Similar bifurcation mechanisms are important in power-system voltage stability and other nonlinear electrical phenomena. On the other hand, controlled bistability can be exploited in nonlinear electronic devices and switching systems, where two stable operating states can provide a mechanism for memory, switching, threshold detection, or state selection. The occurrence of cusp-type behavior has also been demonstrated in nonlinear electronic oscillator models, where the steady-state problem can be described by a cusp catastrophe.

Therefore, the physical significance of the proposed model is not merely the existence of a cubic equilibrium equation. Rather, the model establishes a direct relationship between **voltage, system parameters, equilibrium current, stability, multistability, and sudden switching**. The cusp surface provides a geometric representation of these phenomena, while its projection onto the (μ, V) control plane identifies the critical parameter combinations at which equilibrium states merge and disappear. Consequently, catastrophe theory offers a compact qualitative description of nonlinear electrical behavior that complements conventional circuit and stability analysis.

In summary, the model predicts three principal physical regimes:

1) Unique equilibrium: one stable operating current exists.

2) Bistable regime: two stable currents coexist, separated by an unstable equilibrium.

3) Catastrophic transition: crossing a fold boundary causes the currently occupied stable equilibrium to disappear, producing a rapid transition toward another stable current.

This interpretation makes the cusp catastrophe model particularly suitable for analyzing nonlinear electrical systems in which **small changes in voltage or system parameters can lead to large changes in the equilibrium current**. The model therefore provides a theoretical basis for identifying critical operating conditions and understanding sudden current transitions in nonlinear electrical systems.

5. Physical Interpretation

The proposed cusp catastrophe model provides a physical interpretation of the equilibrium current behavior of a nonlinear electrical system in terms of the interaction between the **current** I, the **applied dimensionless voltage** v, and the **system parameter** μ. Here, I represents the observable electrical state of the system, whereas V and μ determine the operating conditions under which the equilibrium states are established. The use of one state variable and two control parameters is precisely the structure associated with the canonical cusp catastrophe. Note that I = current, measured directly in **amperes (A)**; μ or R = resistance, measured directly in **ohms (Ω)**; V = voltage, measured directly in **volts (V)**.

The equilibrium condition of the model is

$$i^3 + \mu i + v = 0.$$

Physically, this equation represents the possible **steady-state currents** corresponding to particular combinations of the dimensionless voltage v and the system parameter μ. An equilibrium current is a state in which the electrical variables remain constant with respect to time. Therefore, solving the cubic equation determines the possible operating points of the nonlinear electrical system.

The parameter v has a direct physical interpretation as the **applied or control voltage**. Changing v modifies the electrical forcing acting on the system and consequently changes its equilibrium current. The parameter μ, on the other hand, represents a **secondary control parameter associated with the nonlinear characteristics of the system**. It can conceptually represent a parameter whose variation changes the shape of the current-voltage equilibrium relation, such as an effective circuit or device parameter.

The most important physical feature of the model is the possibility of **multiple equilibrium currents**. Outside the cusp region in the (μ, v) control plane, the system possesses one real equilibrium current. Inside the cusp region, three real equilibrium currents may coexist. In the potential interpretation, these correspond to two stable equilibria separated by an unstable equilibrium. Such a con-

figuration provides a mathematical representation of **bistability** in a nonlinear electrical system.

The two stable equilibria can be interpreted as two different possible steady operating currents under the same control conditions. The unstable equilibrium acts as a **separating state** between the two stable operating states. Consequently, the final current reached by the system can depend not only on the instantaneous values of V and μ, but also on the previous operating state of the system.

The physical significance becomes particularly clear near the **fold curves** of the bifurcation set. At a fold, two equilibrium states coalesce and disappear. Thus, when the applied voltage is varied continuously and the operating point reaches a fold boundary, the stable current branch on which the system is operating can cease to exist. The system must then move toward another available stable equilibrium. This produces a **sudden jump in current**, even though the voltage itself may have changed only slightly. The relationship between the folded equilibrium surface, saddle-node bifurcation, and sudden transitions is a central feature of the cusp catastrophe.

The cusp therefore represents a **critical organization of equilibrium states**. The bifurcation set is

$$4\mu^3 + 27v^2 = 0, \mu \le 0,$$

and it separates regions with qualitatively different numbers of equilibrium currents. At the cusp point,

$$(\mu, v) = (0, 0),$$

the two-fold branches meet. As μ approaches zero from the negative side, the region of bistability becomes progressively smaller; beyond the cusp, the multiple-equilibrium structure disappears.

Another important physical implication is **hysteresis**. If v is increased and subsequently decreased, the system may remain on different stable equilibrium branches during the two directions of variation. Consequently, the current at a particular voltage can depend on the history of the voltage variation. Cusp catastrophe models are particularly useful for representing such combinations of bistability, sudden jumps, and hysteresis.

Thus, the physical interpretation of the model can be summarized as follows: i = equilibrium current, v = applied dimensionless voltage (control parameter), and μ = nonlinear/system parameter (control parameter)

The cusp model consequently provides a geometric picture of how a nonlinear electrical system can move from **one stable operating state to two possible stable states**, and how a small change in the control parameters can cause an **abrupt transition from one current state to another**. This interpretation is consistent with applications of catastrophe theory to nonlinear electronic systems, including electronic oscillators and other electrical dynamical systems.

In physical terms, the cusp catastrophe is therefore not a catastrophic failure of the electrical system. Rather, "catastrophe" denotes a sudden qualitative

change in the equilibrium state: the current changes discontinuously because the equilibrium branch that the system follows loses stability or disappears.

6. Comparison between Two Systems

In fact, $i^3 - \mu i + v = 0$ may be more convenient for the physical interpretation, because the sign of μ changes the location of the cusp in the (μ, v) control plane.

The equilibrium equation is

$$i^3 - \mu i + v = 0$$

This corresponds to the potential

$$\Phi(i;\mu;v) = \frac{1}{4}i^4 - \frac{1}{2}\mu i^2 + vi,$$

because

$$\frac{\partial \Phi}{\partial i} = i^3 - \mu i + v.$$

6.1. Equilibrium Structure

For $i^3 - \mu i + v = 0$, the discriminant of the cubic is:

$$\Delta = 4\mu^3 - 27v^2.$$

Therefore:

$$\begin{cases} 4\mu^3 - 27v^2 > 0 & \Rightarrow \text{three distinct real equilibria,} \\ 4\mu^3 - 27v^2 = 0 & \Rightarrow \text{fold/bifurcation set,} \\ 4\mu^3 - 27v^2 < 0 & \Rightarrow \text{one real equilibrium.} \end{cases}$$

Since $v^2 \geq 0$, three equilibria are possible only when $\mu > 0$

The bifurcation set is therefore $27v^2 = 4\mu^3$, $\mu \geq 0$, or $v = \mp \frac{2}{3\sqrt{3}}\mu^{3/2}$.

So the cusp opens toward **positive** μ.

6.2. Stability

The second derivative of the potential is $\frac{\partial^2 \Phi}{\partial i^2} = 3i^2 - \mu$.

Thus, under the potential interpretation,

$3i^2 - \mu > 0$ $\Rightarrow$ stable equilibrium, and

$3i^2 - \mu < 0$ $\Rightarrow$ unstable equilibrium.

Consequently, inside the cusp there are normally **two stable equilibrium currents and one unstable equilibrium current**.

6.3. Physical Interpretation of μ

This form is actually interesting electrically. Suppose you start with a nonlinear voltage-current relation

$$\boxed{v = \alpha i^3 - R_{\text{eff}} i}$$

with $\alpha > 0$. Rearranging gives

$$\alpha i^3 - R_{\text{eff}} i - v = 0.$$

After suitable nondimensionalization, this can lead to

$$\boxed{i^3 - \mu i + v = 0}.$$

Here, μ can be related to an **effective linear electrical parameter**, such as an effective resistance/conductance parameter, depending on the particular circuit and normalization.

However, there is an important physical point: the term $-\mu i$ has a **negative linear slope** in the $v - i$ equation characteristic. Therefore, if $\mu > 0$, interpreting μ simply as an ordinary positive resistance would be misleading. It is better to call it an **effective nonlinear system control parameter** unless you derive it explicitly from a circuit.

For this paper, we should use $i^3 - \mu i + v = 0$
if we want the physically relevant multistable region to occur for $\mu > 0$. Then the control plane is: (μ, v)
with i as the state variable, and the cusp bifurcation set is $4\mu^3 - 27v^2 = 0$, $\mu \geq 0$.

This gives a very clean physical interpretation:

- i: equilibrium current (state variable);
- v: applied dimensionless voltage (control parameter);
- μ: effective electrical/system parameter controlling the nonlinear equilibrium structure;
- $4\mu^3 - 27v^2 = 0$: critical/fold boundary;
- inside the cusp: three equilibrium currents;
- outside the cusp: one equilibrium current;
- crossing a fold: sudden transition (jump) between operating states.

We think this sign convention is preferable for this paper, particularly if we want the cusp and bistability region to be displayed in the physically natural half-plane $\mu > 0$.

7. Conclusions

This study demonstrates that the cusp catastrophe model provides a rigorous mathematical framework for interpreting equilibrium current dynamics and non-smooth phenomena in nonlinear electrical systems. Governed by the cubic equilibrium equation $i^3 + \mu i + v = 0$, The system exhibits distinct topological regimes where negative values of the control parameter μ give rise to multistability bounded by the cusp bifurcation set $4\mu^3 + 27v^2 = 0$ ($\mu \leq 0$). Within this region, two stable operating currents are separated by an unstable intermediate equilibrium. Variations in applied control parameters push state trajectories across the fold boundaries, inducing sudden state transitions. Ultimately, this topological

formulation offers a unified explanation for bistability, hysteresis, and abrupt current jumps, serving as a powerful predictive tool for nonlinear circuit design and stability analysis.

The parameter μ is regarded as an effective electrical system control parameter that influences the equilibrium current dynamics and bifurcation behavior of the nonlinear system. Its physical interpretation depends on the underlying circuit model and parameter transformations and should not be directly associated with a specific electrical component, such as capacitance or resistance, without an explicit circuit-based derivation.

Overall, the cusp catastrophe formulation connects the mathematical geometry of the equilibrium surface with physically meaningful electrical phenomena. It demonstrates how continuous changes in voltage and system parameters can produce discontinuous changes in equilibrium current, thereby providing a compact qualitative framework for understanding nonlinear operating states and critical transitions in electrical systems.

Author Contributions

Mohammed Nokhas Murad Kaki (First Author) conceptualized and developed the research idea, formulated the mathematical model, nondimensionalized the governing equations, and applied cusp catastrophe theory to analyze equilibrium current dynamics in nonlinear electrical systems. He conducted the mathematical analysis, investigated the equilibrium conditions and bifurcation behavior, derived and interpreted the theoretical results, and prepared the figures and tables. He also wrote the original manuscript and led its revision and editing.

Jasim Mohammed Ali Al Samarai (Second Author) contributed to the critical review of the research, interpretation and discussion of the results, and revision and editing of the manuscript. Both authors reviewed and approved the final version of the manuscript and agreed to be accountable for the integrity of the work.

Conflicts of Interest

The authors declare no conflicts of interest.

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