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Closability of Closed Compact Linear Operators

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Abstract

This paper investigates the closability of closed compact linear operators on Hilbert spaces. It establishes that under certain conditions, every closed compact linear operator is closable. Additionally, it investigates the properties of closable operators and highlights their stability under limits and algebraic operations. We establish a non-closability criterion based on sequences in the domain and demonstrate that the limit of bounded compact operators need not be closable. Moreover, we examine the behavior of closability when operators are added, restricted, or composed with isometries. These results prove that closability is not always preserved under such constructions. Thus, it provides a framework for understanding its role in the analysis of closed compact operators. The analysis relies on spectral theory, the closed graph theorem, Fredholm theory, and von Neumann's theorem. Consequently, the findings extend known properties of bounded compact operators and provide a clearer understanding of closability and the spectral behavior of closed compact linear operators.

Subject Areas

Algebra

Keywords

Closability, Closed Compact Linear Operator, Compact Operator, Hilbert Space, Bounded Linear Operator, Spectral Theory

1. Introduction

The notion of closed and closable operators is key in operator theory. It describes how operator graphs behave under limits and transformations. This paper focuses on closed compact linear operators on Hilbert spaces and investigates stability properties of closability under limits, sums, restrictions, and composition. Moreover, it studies how compactness interacts with closability and related graph-theoretic properties. The main results show when closability is preserved and when it fails, especially under pointwise convergence, algebraic addition, and restriction to subspaces.

Let K be a linear operator such that $K : D(K) \subseteq H \to H$, closability is defined by the existence of a minimal closed extension $\bar{K}$, which extends spectral considerations to a broader, more stable framework. In recent studies, Shi [1] and Sharma [2] investigated closability. They defined closability via graph closure and established a criterion linking closability to properties of the kernel and range. In particular, they proved that a bounded operator with closed range is necessarily closed, establishing the connection between boundedness and closedness. Moreover, Azzouz [3] clarified the structure of the space of closed linear operators $(C(H))$, proving that boundedness preserved closedness.

In contrast to these results, not all operators are closable. For instance, Popovici [4] and Mohammed [5] investigated densely defined paranormal operators and established that they were non-closable. This was because a paranormal operator K satisfied; $D(K^*) = 0$ and since $D(K^2) = D(K^*) = 0$. It was proven that K was trivially paranormal, and hence it could not be closable. In contribution, Messirdi [6] and Sandovici [7] studied almost closable operators and von Neumann's theorem, which asserts that an operator is closed even when it is not densely defined. These results highlighted the importance of closability in an operator´s extension, stability, and spectral behavior.

In addition, extensive research has been done on the spectral properties of special classes of operators. It is well known that compact operators on Hilbert spaces have spectra consisting only of eigenvalues, with zero as the only possible accumulation point [8]. Also, self-adjoint operators have real spectra, while unitary operators have spectra that lie on the unit circle [9]. In the study of the essential spectrum, Feshchenko [10] and Jeribi [11] established that the essential spectrum is invariant under compact and demicompact perturbations but also stable under certain operator matrix constructions. Despite these results, the relationship between compactness, closedness, closability, and the essential spectrum is still not fully understood for closed compact linear operators.

Motivated by this gap, this paper focuses on closability properties of closed compact linear operators and their implications for spectral analysis. We investigate conditions that make an operator closable, the relationship between closability and compactness, and the structural behavior of their graphs and domains.

Results from this study will contribute to knowledge in operator theory, particularly in the area of the spectrum. The results will also be of help in establishing the relationship between closabality, closedness, and compactness. This method not only improves existing knowledge but also contributes to the deeper understanding of spectral structures in infinite-dimensional spaces.

2. Preliminaries

Assume that H is a Hilbert space. The space of bounded linear operators on H

is denoted by $B(H)$.

Let $K: D(K) \subseteq H \to H$ be a linear operator, then $D(K)$ denotes the domain of K, and $G(K)$ represents the graph of K.

2.1. Closed and Closable Operators

Let $K: D(K) \subseteq H \to H$ be a linear operator. We define the graph of K by

$$G(K) = \{(x, Kx) : x \in D(K)\}. \text{ [2]}$$

1) K is closed if $G(K)$ is closed in $H \oplus H$ [12].

2) K is closable if the closure of its graph is the graph of a linear operator [12].

2.2. Characterization of Closability

A linear operator K is closable if for every sequence $\{x_n\} \subset D(K)$ such that

$$x_n \to 0 \text{ and } Kx_n \to y,$$

it follows that $y = 0$ [12].

2.3. Compact Operators

An operator $K: H \to H$ is said to be compact if it maps bounded sets into relatively compact sets [12].

2.4. Classical Results

We will use the following standard results:

1) If K is densely defined and closed, then its graph $G(K)$ is closed in $H \oplus H$.

2) A linear operator K is closable if and only if whenever $x_n \in D(K)$, $x_n \to 0$, and $Kx_n \to y$, one has $y = 0$.

3) Every bounded operator is closed [3].

3. Main Results

Proposition 3.1. *Let* H *be a Hilbert space and* $K: D(K) \subseteq H \to H$ *be a linear operator. Suppose there exists a sequence* $\{x_n\} \subset D(K)$ *such that*

1) $x_n \to 0$ in H.

2) $Kx_n \to y$ for some $y \neq 0$.

Then, the operator K is not closable.

Proof. By the characterization of closability given in Section 1, K is closable when $D(K)$ is dense in H. Assume, for contradiction, that K is closable, so $D(K)$ is dense.

Let $v \in D(K^*)$. By definition of the adjoint,

$$\langle Kx_n, v \rangle = \langle x_n, K^* v \rangle \text{ for all } n.$$

Taking limits as $n \to \infty$, we obtain $\langle y, v \rangle = \lim_{n\to\infty} \langle Kx_n, v \rangle = \lim_{n\to\infty} \langle x_n, K^* v \rangle = 0$, since $x_n \to 0$. Thus,

$$\langle y, v \rangle = 0 \text{ for all } v \in D(K^*).$$

Since $D(K^*)$ is dense in H, it follows that $y=0$, a contradiction.

Therefore, $D(K^*)$ is not dense in H, and hence K is not closable. ☐

Remark 3.2. *Proposition* 3.1 *shows that closability is determined by the behavior of sequences converging strongly to zero in the domain of the operator. In particular, an operator fails to be closable whenever a sequence* $x_n \to 0$ *in norm is mapped to a sequence whose limit is non-zero.*

The following theorem illustrates this instability for bounded compact operators.

Theorem 3.3. *Consider a Hilbert space* H *and a sequence of bounded compact linear operators* $\{K_n\}_{n\in\mathbb{N}} \subset B(H)$. *Suppose a linear operator* $K : D(K) \subseteq H \to H$ *exists for which* $K_n x \to Kx$ *for each* $x \in D(K)$. *Then, closability need not be preserved under this convergence.*

Proof. Let $H = L^2(0,1)$ and define

$$D(K) = C[0,1],\ (Kf)(t) = f(0)h(t),\ h(t) = 1.$$

Then, $D(K)$ is dense in H and K is linear. To show that K is not closable, consider

$$f_m(t) = \begin{cases} 1-mt, & 0 \le t \le \dfrac{1}{m}, \\ 0, & \dfrac{1}{m} < t \le 1. \end{cases}$$

Then

$$\| f_m \|_2^2 = \int_0^{1/m} (1-mt)^2 \,\mathrm{d}t = \frac{1}{3m} \to 0,$$

so $f_m \to 0$ in H, but

$$Kf_m = f_m(0)h = h \ne 0.$$

Thus, $f_m \to 0$ while $Kf_m \to h \ne 0$. By Proposition 3.1 K is not closable.

Now define, for each $n \in \mathbb{N}$,

$$(K_n f)(t) = n\left(\int_0^{\frac{1}{n}} f(s)\,\mathrm{d}s\right) h(t),\ \ f \in L^2(0,1).$$

Each K_n is linear. Moreover, for $f \in L^2(0,1)$, by the Cauchy-Schwarz inequality,

$$\left| n\int_0^{\frac{1}{n}} f(s)\,\mathrm{d}s \right| \le n\left(\int_0^{\frac{1}{n}} |f(s)|^2 \,\mathrm{d}s\right)^{1/2} \left(\int_0^{\frac{1}{n}} 1\mathrm{d}s\right)^{1/2} \le \sqrt{n}\,\| f \|_2 .$$

Therefore, $K_n \in B(H)$. Since the range of K_n is contained in $\operatorname{span}\{h\}$, each K_n has rank one, hence is compact.

Since f is continuous,

$$n\left(\int_0^{\frac{1}{n}} f(s)\,\mathrm{d}s\right) \to f(0).$$

Hence, $K_n f \to Kf$ in $L^2(0,1)$.

Therefore, $K_n x \to Kx$ for $x \in D(K)$. Closability is not preserved under pointwise convergence on a dense domain. ☐

Corollary 3.4. *Consider a Hilbert space* H *and a sequence of bounded compact linear operators* $\{K_n\}_{n\in\mathbb{N}} \subset B(H)$. *Suppose a linear operator* $K : D(K) \subseteq H \to H$ *exists for which* $K_n x \to Kx$ *for each* $x \in D(K)$. *Then, the limit operator* K *need not be closable, even though each* K_n *is bounded and hence closed.*

Proof. Since each $K_n \in B(H)$, it follows that every K_n is bounded and therefore closed.

To show that the limit operator need not be closable, consider the same construction as in Theorem 3.2. Let $H = L^2(0,1)$ and define

$$D(K) = C[0,1],\ (Kf)(t) = f(0)h(t),\ h(t) = 1.$$

Then, $D(K)$ is dense in H and K is linear.

Define

$$(K_n f)(t) = n\left(\int_0^{1/n} f(s)\,\mathrm{d}s\right)h(t),\ n \in \mathbb{N}.$$

Each K_n is linear, bounded, and of rank one, hence compact (and thus closed). Moreover, for every $f \in D(K) = C[0,1]$, continuity at 0 gives

$$K_n f \to Kf.$$

However, as shown in Theorem 3.2, the operator K is not closable since there exists a sequence $\{f_m\} \subset D(K)$ such that $f_m \to 0$ in H but $Kf_m \to h \neq 0$.

Thus, although each K_n is bounded and hence closed, the limit operator K fails to be closable.

Corollary 3.5. *Consider a Hilbert space* H *and a sequence of bounded compact linear operators* $\{K_n\}_{n\in\mathbb{N}} \subset B(H)$. *Suppose a linear operator* $K : D(K) \subseteq H \to H$ *exists for which* $K_n x \to Kx$ *for each* $x \in D(K)$. *Then, the graph* $\mathcal{G}(K_n) \subset H \times H$ *of the limit operator need not be closed. Consequently,* K *need not be closable.*

Proof. From Theorem 3.2, there exists a sequence of bounded compact operators $\{K_n\}$ and a linear operator $K : D(K) \subseteq H \to H$ such that $K_n x \to Kx$ for all $x \in D(K)$, but K is not closable.

Suppose there exists a sequence $\{x_m\} \subset D(K)$ such that

$$x_m \to 0 \text{ in } H,\ Kx_m \to y \neq 0.$$

Hence, $(x_m, Kx_m) \in \mathcal{G}(K)$ for all m, and $(x_m, Kx_m) \to (0, y)$ in $H \times H$.

Since K is linear, $K0 = 0$, so $(0, y) \notin \mathcal{G}(K)$, because $y \neq 0$. Therefore, $\mathcal{G}(K)$ is not closed in $H \times H$. Since every closed operator is closable, it follows that K is not closable.

While Corollary 3.4 established that the limit of closable operators may fail to be closable, the next result extended this instability to the algebraic sum of operators. In particular, it demonstrated that adding a non-closable operator to a closable one destroyed the closability property, even when boundedness was assumed.

Proposition 3.6. *Let* H *be a Hilbert space. Suppose* $K_1, K_2 : \mathcal{D} \subseteq H \to H$ *are linear operators defined on the same dense subspace* $\mathcal{D} \subseteq H$. *Assume:*

1) K_1 is bounded (hence closable).

2) K_2 is not closable.

Define the operator $K = K_1 + K_2$ on the domain D. Then K is not closable.

Proof. Since K_2 is not closable, there exists a sequence $(x_n) \subset \mathcal{D}$ such that

$$x_n \to 0 \text{ in } H,$$

and

$$K_2 x_n \to z \neq 0.$$

Since K_1 is closable and $x_n \to 0$, we have

$$K_1 x_n \to 0.$$

Therefore,

$$K x_n = K_1 x_n + K_2 x_n \to 0 + z = z \neq 0.$$

Hence, there exists a sequence $x_n \to 0$ such that $K x_n \to z \neq 0$, $\Rightarrow K$ is not closable. ☐

Remark 3.7. *This proposition highlighted the following:*

1) Closability is not necessarily preserved under the addition of operators.

2) If one operator in the sum is non-closable, then the resulting operator is also non-closable. Moreover, non-closability is stable under bounded perturbations. That is: if K_2 is not closable and K_1 is bounded, then $K_1 + K_2$ is not closable.

3) This occurs because the sequence violating the closability condition for the non-closable operator also prevents the sum from admitting a closed extension.

Moreover, the study considered whether closability is preserved under domain restriction.

Proposition 3.8. *Let* H *be a Hilbert space and let* $K : D(K) \subseteq H \to H$ *be a closable linear operator. Let* $D_0 \subset D(K)$ *be a linear subspace, and define the restriction* $K_0 = K|_{D_0}$. *Then,* K_0 *is closable.*

Moreover:

1) If D_0 is dense in $D(K)$ with respect to the graph norm, then $\overline{K_0} = \bar{K}$.

2) If D_0 is not dense in $D(K)$ with respect to the graph norm, then $\overline{K_0}$ may be a proper restriction of K, although K_0 remains closable.

Proof. A linear operator K is closable if and only if the closure of its graph.

$$G(K) = \{(x, Kx) \in H \oplus H : x \in D(K)\}$$

is the graph of a single-valued operator.

Consider the restriction $K_0 : K|_{D_0}$ whose graph is

$$G(K_0) = \{(x, Kx) : x \in D_0\} \subseteq G(K).$$

Taking closures in $H \oplus H$, we obtain

$$\overline{G(K_0)} \subseteq \overline{G(K)}.$$

Since K is closable, $\overline{G(K)}$ is the graph of a linear operator. The closure of a subset of a graph is also the graph of a single-valued operator in $H \oplus H$. Hence, K_0 is closable.

1) Suppose that D_0 is dense in $D(K)$ with respect to the graph norm. Then, for every $x \in D(K)$, there exists a sequence $(x_n) \subset D_0$ such that

$\|x_n - x\|_K \to 0.$

That is, $x_n \to x$ in H and $Kx_n \to Kx$ in H.

Hence, $(x_n, Kx_n) \to (x, Kx)$ in $H \oplus H$, which implies $G(K) \subseteq \overline{G(K_0)}$.

Therefore,

$$\overline{G(K)} = \overline{G(K_0)},$$

and consequently,

$$\overline{K_0} = \bar{K}.$$

2) Suppose that D_0 is not dense in $D(K)$ with respect to the graph norm. Then, there exists $z \in D(K)$ such that $z \notin \overline{D_0}^{\|\cdot\|_K}$.

Hence, $(z, Kz) \notin \overline{G(K_0)}$, even though $(z, Kz) \in G(K)$. Therefore,

$$\overline{G(K_0)} \subsetneq \overline{G(K)}.$$

and hence $\overline{K_0}$ is a proper restriction of K.

Since $\overline{G(K_0)}$ is still the graph of a linear operator, K_0 remains closable. ☐

Corollary 3.9. *Let* H *be a Hilbert space, and let*

$$K : D(K) \subseteq H \to H$$

be a closable linear operator. Let $D_0 \subset D(K)$ be a linear subspace, and define the restriction $K_0 := K|_{D_0}$. If D_0 is not dense in $D(K)$ with respect to the graph norm

$$\|x\|_K := \|x\| + \|Kx\|,$$

then, the closure $\overline{K_0}$ may be a proper restriction of K, although K_0 remains closable.

Proof. Since K is closable, its closure $\bar{K}$ exists and has graph $\overline{G(K)}$.

Consider the restriction $K_0 := K|_{D_0}$ with graph

$$G(K_0) \subseteq G(K).$$

Taking closures,

$$\overline{G(K_0)} \subseteq \overline{G(K)}.$$

Hence, $\overline{G(K_0)}$ is also the graph of an operator, and therefore K_0 is closable.

Now suppose that D_0 is not dense in $D(K)$ with respect to the graph norm. Then, there exists $f \in D(K)$ such that $f \notin \overline{D_0}^{\|\cdot\|_K}$.

$$(f, Kf) \notin \overline{G(K_0)}, \text{ even though } (f, Kf) \in G(K).$$

Therefore,

$$\overline{G(K_0)} \subsetneq \overline{G(K)},$$

and hence $\overline{K_0} \neq \overline{K}$.

Thus, while K_0 remains closable, its closure may be strictly smaller than the closure of K. ☐

As examined by Kato [13], composition with an isometry or partial isometry often preserves certain stability properties of operators. Motivated by this, this recent study presented a sufficient condition for preservation of closability under composition.

Proposition 3.10. *Let* $U \in B(H)$ *be an isometry. Define the composition*

$$UK : \mathcal{D}(K) \to H,\ (UK)x := U(Kx),\ x \in \mathcal{D}(K).$$

Then, UK is closable on $D(K)$.

Proof. Let $\{x_n\} \subset \mathcal{D}(K)$ be such that

$$x_n \to 0 \text{ in } H \text{ and } UKx_n \to h \in H.$$

Since UKx_n converges, it is Cauchy. Because U is an isometry,

$$\|Kx_n - Kx_m\| = \|UKx_n - UKx_m\| \text{ for all } n, m.$$

Hence, $\{Kx_n\}$ is a Cauchy sequence in H. Since H is complete, there exists $y \in H$ such that $Kx_n \to y$.

By continuity of U, $h = \lim_{n\to\infty} UKx_n = \lim_{n\to\infty} U(Kx_n) = U(y) = 0$.

Therefore, UK is closable. ☐

This shows that while general bounded composition can destroy closability, composition with an isometry preserves it.

Corollary 3.11. *Consider a Hilbert space* H *be a closable linear operator* $K : \mathcal{D}(K) \subseteq H \to H$. *Let* $U : H \to H$ *be unitary. Then,* $UK : \mathcal{D}(K) \to K$ *is closable and* $KU : \mathcal{D}(KU) \to H$ *is closable, where* $\mathcal{D}(KU) = \{x \in H : Ux \in \mathcal{D}(K)\}$.

Proof. Consider a Hilbert space H be a closable linear operator K on $\mathcal{D}(K)$, and let U be a unitary operator on H.

For UK: Take any sequence $(x_n) \subset \mathcal{D}(K)$ such that

$$x_n \to 0 \text{ in } H,\ UKx_n = U(Kx_n) \to y \text{ in } H \tag{3.1}$$

Since U is unitary, $U^{-1} = U^*$ is bounded, so applying U^* to both sides of Equation (3.1) gives $Kx_n = U^*(UKx_n) \to U^*y$ in H. Because K is closable, the conditions $x_n \to 0$ and $Kx_n \to U^*y$ imply $U^*y = 0$, applying U, we obtain $y = 0$. Therefore, UK is closable.

For KU: Consider the domain

$$\mathcal{D}(KU) = \{x \in H : Ux \in \mathcal{D}(K)\}.$$

Take any sequence $(x_n) \subset \mathcal{D}(KU)$ such that

$$x_n \to 0 \text{ in } H,\ KUx_n = K(Ux_n) \to y \text{ in } H.$$

Since U is bounded, we have $Ux_n \to 0$ in H. Also, $K(Ux_n) \to y$. Because K is closable, it follows that $y = 0$. Hence KU is closable. ☐

Therefore, both UK and KU are closable on their respective domains.

Proposition 3.12. *Let* H *be a Hilbert space, and let* $K : \mathcal{D}(K) \subset H \to H$ *be a compact linear operator with dense domain* $\mathcal{D}(K)$. *Suppose that* K *is closable and that its closure* $\bar{K} : H \to H$ *is a bounded operator defined on all of* H. *Then*:

1) $\bar{K}$ is compact on H.

2) The non-zero spectra of K and $\bar{K}$ coincide, that is,

$$\sigma(K) \setminus \{0\} = \sigma(\bar{K}) \setminus \{0\}.$$

Proof. Let $K : \mathcal{D}(K) \subset H \to H$ be compact and densely defined, and assume K is closable with bounded closure $\bar{K} : H \to H$.

1) $\bar{K}$ is compact. Take any bounded sequence $(x_n) \subset H$, say $\|x_n\| \le 1$. Since $\mathcal{D}(K)$ is dense, for each n choose $y_n \in \mathcal{D}(K)$ with $\|x_n - y_n\| \le 1/n$. Then, (y_n) is bounded: $\|y_n\| \le \|x_n\| + \|x_n - y_n\| \le 2$.

By compactness of K, there exists a subsequence (y_{n_k}) such that $Ky_{n_k} \to z \in H$. Now,

$$\bar{K}x_{n_k} - Ky_{n_k} = \bar{K}(x_{n_k} - y_{n_k}),$$

and since $\bar{K}$ is bounded, $\left\|\bar{K}(x_{n_k} - y_{n_k})\right\| \le \|\bar{K}\| \left\|x_{n_k} - y_{n_k}\right\| \to 0$. Thus,

$$\bar{K}x_{n_k} = Ky_{n_k} + (\bar{K}x_{n_k} - Ky_{n_k}) \to z,$$

showing that $\bar{K}$ is compact.

2) Let $\lambda \ne 0$. Since both K and $\bar{K}$ are compact, every non-zero spectral value is an eigenvalue. Thus, it suffices to compare non-zero eigenvalues.

If $Kx = \lambda x$ for some non-zero $x \in D(K)$, then

$$\bar{K}x = \lambda x,$$

so $\lambda \in \sigma(\bar{K}) \setminus \{0\}$.

Conversely, if $\bar{K}x = \lambda x$ for some non-zero $x \in H$, then x belongs to the operator part determined by the closure of the graph of K, and hence $x \in D(K)$ with $Kx = \lambda x$.

Thus, $\lambda \in \sigma(K) \setminus \{0\}$.

Therefore,

$$\sigma(K) \setminus \{0\} = \sigma(\bar{K}) \setminus \{0\}.$$

☐

4. Conclusion

This study examined closability for closed compact linear operators, establishing that closability may fail under different circumstances. That is, under pointwise limits of bounded compact operators, under addition with non-closable operators, and when domains are restricted. In contrast, closability can also be preserved under composition with isometries and unitaries. Furthermore, the study illustrated how the closure of a closable operator relates to its domain and spectral properties when compact. These results enhance our understanding of the structural stability of linear operators as they establish conditions that preserve or destroy closability and thus suggest further research on the relationship between closability and spectral behavior for bounded compact, non-closable, sum, restriction, and composition operators.

Conflicts of Interest

The authors declare no conflicts of interest.

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