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Theoretical and Experimental Feasibility of a Sonoluminescence-Modulated Multilayer Casimir Cavity: A Falsifiable Investigation of Dynamical Vacuum Radiation Coupling

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ABSTRACT

We investigate whether a sonoluminescence-driven, dynamically modulated multilayer Casimir cavity can generate measurable electromagnetic radiation or mechanical work beyond conventional sonoluminescence (SL) channels, and we formulate the energy-conservation and metrological criteria required to distinguish a dynamical Casimir effect (DCE) contribution from ordinary acoustic, thermal, plasma, chemical, and electromagnetic backgrounds. The present work does *not* claim net extraction of zero-point energy, does *not* claim a violation of thermodynamics, and does *not* project hundreds of megawatts of output. Instead, we (i) develop a coupled acoustic/bubble-dynamics/cavity-QED model, (ii) derive the Bogoliubov structure that a laboratory DCE signature must satisfy, (iii) write an explicit global energy balance, (iv) define a falsifiable experimental control matrix separating conventional SL, plasma, thermal, chemical, and candidate DCE channels, and (v) specify DCE-specific signatures (correlated photon pairs, two-mode squeezing, sideband structure $\omega_{DCE} \approx \omega_i + \omega_j$) that are inaccessible to any of the classical channels. The device is reframed as a falsifiable experimental program rather than a demonstrated energy-generation mechanism.

1. INTRODUCTION

The Casimir effect [1] and its time-dependent counterpart, the dynamical Casimir effect (DCE) [2, 3], are among the most direct macroscopic manifestations of quantum vacuum fluctuations of the electromagnetic

field. The DCE was experimentally observed in a superconducting-circuit platform by Wilson *et al.* [4], who detected both photon generation and two-mode squeezing consistent with parametric conversion of virtual excitations into real photons through rapid modulation of an effective boundary. That experiment established two essential facts: (i) DCE photons are real and detectable, and (ii) the energy required to generate them comes *from the modulator*, not from a free-energy vacuum reservoir.

Sonoluminescence (SL) [5-7] is a distinct phenomenon in which acoustically driven bubble collapse in a liquid produces picosecond-scale optical flashes. The dominant contemporary interpretation attributes SL to hot-spot compressive heating, plasma formation, and bremsstrahlung/recombination emission, with acoustic energy acting as the source of concentrated hot-spot energy. A minority interpretation, most prominently developed by Eberlein [8], models SL as vacuum radiation from a rapidly moving dielectric interface—i.e., as a dynamical-Casimir-like process. Milton and Ng [9] and later authors [10, 11] showed that the adiabatic Casimir contribution is many orders of magnitude smaller than the observed SL yield, which weighs against a pure Casimir origin of SL but does not exclude a *small* DCE contribution superposed on the conventional thermal/plasma channel.

The unresolved scientific question this manuscript addresses is therefore: given a geometry deliberately engineered to enhance parametric coupling between acoustically driven bubble boundaries and cavity electromagnetic modes, does the observed radiation contain a measurable component with the spectral, statistical, and phase signatures of the dynamical Casimir effect, distinguishable from all classical SL/plasma/thermal channels?

This question has three important properties. First, it is falsifiable: the DCE has specific signatures (correlated photon pairs, two-mode squeezing, sideband spacing tied to the modulation frequency) that no known classical SL channel reproduces. Second, it is decoupled from any claim of net energy gain. Third, it can be answered by laboratory-scale experiments.

The remainder of this paper is organized as follows. Section 2 reviews the required Casimir, DCE, and SL background. Section 3 presents the revised device architecture. Section 4 develops the coupled acoustic/bubble/cavity-QED model. Section 5 writes the global energy balance. Section 6 rewrites the control system as a state-space formulation. Section 7 defines the experimental control matrix and falsification protocol. Section 8 maps regimes onto testable predictions. Section 9 discusses limitations. Section 10 concludes.

2. PHYSICAL BACKGROUND AND LITERATURE

2.1. Static Casimir Interaction

For two ideal, perfectly conducting, infinite parallel plates of area A separated by d in vacuum at $T = 0$, Casimir's original result [1] gives the attractive force (Equation (1)).

$$F_{\mathrm{Cas}} = -\frac{\pi^2 \hbar c A}{240\, d^4} \tag{1}$$

This is a *pressure*, not a stored, extractable energy. For a comprehensive monograph treatment of the Casimir effect across geometries, materials, and finite-temperature extensions, we direct the reader to Milton [12]. Following Jaffe [13], we emphasize that Equation (1) does not by itself constitute evidence that vacuum "zero-point energy" is a technologically extractable reservoir; the force can be derived without invoking zero-point energy at all.

Real cavities deviate from Equation (1) through finite conductivity, nonzero temperature, frequency-dependent dielectric response, surface roughness, PFA limitations, patch potentials, and mechanical instabilities. Each of these must be budgeted in the force-modulation and energy-balance calculations.

2.2. Dynamical Casimir Effect

When a cavity boundary is modulated in time, mode frequencies become nonstationary; vacuum modes are parametrically driven, and real photons appear [2, 3]. In the standard Bogoliubov formulation,

the output annihilation operators are related to the input operators by Equation (2).

$$a_k^{\text{out}} = \alpha_k\, a_k^{\text{in}} + \beta_k\, a_{-k}^{\text{in}\dagger},\ \ |\alpha_k|^2 - |\beta_k|^2 = 1 \tag{2}$$

so that the mean photon number produced from vacuum in mode k is $N_k = |\beta_k|^2$. The DCE is therefore an *energy-conversion* process, not a free-energy process. Wilson *et al.* [4] verified this experimentally. The DCE sits in the same family of accelerated-observer/nonstationary-boundary vacuum phenomena as the Unruh effect [14].

2.3. Sonoluminescence

Single-bubble sonoluminescence [5, 6] follows a Rayleigh-Plesset-type dynamics, generalized to compressible collapse via Keller-Miksis (Equation (3)).

$$\left(1-\frac{\dot{R}}{c}\right)R\ddot{R}+\frac{3}{2}\left(1-\frac{\dot{R}}{3c}\right)\dot{R}^2=\left(1+\frac{\dot{R}}{c}\right)\frac{P_B-P_\infty(t)}{\rho}+\frac{R}{\rho c}\frac{\mathrm{d}P_B}{\mathrm{d}t}-\frac{2\sigma}{\rho R}-\frac{4\mu\dot{R}}{\rho R} \tag{3}$$

Here $R(t)$ is the bubble radius, c the speed of sound in the liquid, σ the surface tension, μ the viscosity, P_B the pressure at the bubble wall, and $P_\infty(t)$ the far-field acoustic drive. Reviews [5-7] treat SL as an *acoustic-energy concentration* process. This remains the baseline null hypothesis against which any DCE contribution is tested.

2.4. Previous SL/Vacuum-Radiation Proposals

The idea of an SL-vacuum connection traces back at least to Schwinger [15]. Eberlein [8] developed it quantitatively; Milton and Ng [9] and Lambrecht *et al.* [10] challenged it on energetic and spectral grounds. The present manuscript takes this literature as its scholarly starting point: SL is not treated as an independent energy source, and the DCE-like interpretation of SL is treated as a hypothesis to be tested, not assumed.

2.5. Tunable, Chiral, and Magneto-Optical Casimir Interactions

Jiang and Wilczek [16] showed theoretically that chiral metamaterials and magneto-optical media can modify the sign and magnitude of the Casimir force. In this manuscript, such materials are considered *only* as devices to shape cavity mode structure and parametric coupling; they are *not* treated as an energy source. The energetic cost of external magnetic-field modulation and any material switching must be included in the global energy balance (Section 5).

2.6. The Open Problem

Given the above, an experimentally decidable question remains: can a carefully engineered acoustic-Casimir cavity produce electromagnetic output with the specific spectral and photon-statistical signatures of a DCE process, at a level distinguishable from conventional SL and thermal channels, under a fully accounted global energy balance?

3. DEVICE ARCHITECTURE

The device is a spherically symmetric acoustic resonator with concentric Casimir shells, driven by phased piezoelectric transducers on the outer kinetic wall, and instrumented for calibrated multi-channel readout.

3.1. Spherical Cavity

The proposed prototype has an outer inner-radius $R_{\text{cav}} = 5$ cm ("small-scale"). A medium-scale variant has $R_{\text{cav}} = 25$ cm. The earlier 2-m variant has been withdrawn.

3.2. Acoustic Transducers

$M = 32$ piezoelectric elements distributed over the outer wall operate at $f_0 \in [1, 5]$ MHz with independent phase φ_i and amplitude A_i. P_{acoustic} is a *measured* quantity, not an assumed one.

3.3. Casimir Layers: Revised Geometry

The original phrase "billions of concentric plates" is withdrawn. For an available radial path $\Delta r \approx 5$ cm and a target gap $d \in [10\text{ nm}, 100\text{ nm}]$, the number of achievable shells is given by Equation (4).

$$N_{\text{layers}} \lesssim \frac{\Delta r}{d} \in \left[5\times10^{5}, 5\times10^{6}\right] \tag{4}$$

Each "layer" is a segmented, acoustically transparent metallic-dielectric stack (Ag/SiO_2 or graphene/Al_2O_3 by ALD).

Because $F_{\text{Cas}} \propto d^{-4}$, gap tolerance is critical. A fractional gap error $\delta d/d$ produces a fractional force error (Equation (5)).

$$\frac{\delta F}{F} = -4\frac{\delta d}{d} \tag{5}$$

so that a 5% gap error produces a 20% force error, which implies a fabrication tolerance $\delta d/d \lesssim 10^{-3}$ for any meaningful cavity-QED calculation.

Physically Realizable Multilayer Specification (R2.1)

A representative shell class S_i has: *shell thickness* $t_s = 50$ nm realized as Ag(20 nm)/SiO_2 (30 nm) grown by ALD on a 15 nm SiN_x carrier membrane; *shell segmentation* into $N_{\text{seg}} = 6$ geodesic tiles per shell (pentakis-icosahedral partition), no single tile spanning more than $\lambda_{\text{acoustic}}/4$ at $f_0 = 2$ MHz; *mechanical supports* $N_{\text{sup}} = 12$ per shell, radial piezoceramic pillars of 5 μm diameter with integrated capacitive gap sensors; *fluid access* through hexagonal micro-perforations of pitch $p = 2$ μm, diameter 0.6 μm, open area $\varphi \approx 8\%$; *acoustic impedance matching* of the stack to water/glycerol within 10% across the operating band; *bubble-induced pressure-to-gap coupling* is given by Equation (6).

$$\delta d_i(t) = \chi_i \int G_i(\mathbf{r}, t; \mathbf{r}', t')\,\delta P_{\text{bub}}(\mathbf{r}', t')\,\mathrm{d}^3 r'\,\mathrm{d}t', \quad \chi_i = \frac{1}{k_{\text{sup}} + k_{\text{mem}}(\omega)} \tag{6}$$

where G_i is the mechanical Green's function of the segmented shell, $k_{\text{sup}} \approx 80$ N/m per pillar and $k_{\text{mem}}(\omega)$ is the frequency-dependent membrane stiffness. For a bubble collapse depositing $\delta P_{\text{bub}} \sim 10^5$ Pa over an effective area $A_{\text{eff}} \sim 100$ μm^2 within 100 ns of the wall, Equation (6) yields peak gap modulation $\delta d \sim 0.4$ nm, *i.e.*, a fractional modulation $\delta d/d \sim 4 \times 10^{-3}$ for a 100 nm nominal gap. This is the mechanism by which SL bubble collapse modulates the Casimir gap rather than merely perturbing the surrounding fluid.

3.4. Fluid Medium

The revised manuscript adopts a *water/glycerol + xenon-saturated* fluid as the primary experimental platform. This is the well-characterized SL regime [5, 6] and permits comparison against decades of published data. Liquid mercury is demoted from the primary design to a later-stage option.

3.5. Optical and Electrical Readout

The instrumentation array includes PMT, SPAD coincidence pairs, streak camera, UV/X-ray scintillators, calorimeter, hydrophones, and near-field microwave probes for DCE-band emission. All channels are calibrated independently against traceable standards.

4. COUPLED MATHEMATICAL MODEL

The revised model contains three coupled subsystems: (A) acoustic field, (B) bubble population, and

(C) electromagnetic cavity modes with time-dependent boundaries. We deliberately do not attempt to resolve 10^{12} individual bubbles; a population (kinetic) model is used instead.

4.1. Model A: Acoustic Field

Inside the fluid, the linear-acoustic pressure $p(r, t)$ satisfies Equation (7).

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = S(\boldsymbol{r},t) + S_{\text{bub}}(\boldsymbol{r},t) \tag{7}$$

where S is the transducer source term and S_{bub} the coupling from bubble oscillations. Spherical-harmonic decomposition (Equation (8))

$$p(r,\theta,\phi,t) = \sum_{\ell,m} p_{\ell m}(r,t) Y_{\ell m}(\theta,\phi) \tag{8}$$

gives radial equations that make the "spherical-harmonic alignment" language mathematically meaningful.

4.2. Model B: Bubble Population Dynamics

Rather than tracking ~10^{12} bubbles individually, we introduce the bubble distribution function (Equation (9)) obeying a Boltzmann-like transport equation.

$$\frac{\partial f}{\partial t} + \dot{R}\frac{\partial f}{\partial R} + \ddot{R}(R,\dot{R},p)\frac{\partial f}{\partial \dot{R}} + \boldsymbol{v}\cdot\nabla f = \mathcal{C}[f] \tag{9}$$

The controller acts on f's moments, not on individual collapses.

Closure of the Bubble-Population Model (R2.4)

The collision operator in Equation (9) is closed as Equation (10): nucleation + coalescence + dissipative loss.

$$\begin{aligned}\mathcal{C}[f] = {} & J_{\text{nuc}}(\boldsymbol{r},t)\delta(R-R_c)\delta(\dot{R}) \\ & + \int K(R,R';\boldsymbol{r})\left[f(R+R'-R_c)f(R_c) - f(R)f(R')\right]\mathrm{d}R' \\ & - \gamma_{\text{diss}}(R,\dot{R})f\end{aligned} \tag{10}$$

Nucleation follows classical theory [Blander & Katz, AIChE J. 21, 833 (1975)]; *coalescence* uses the Smoluchowski kernel with sticking $\alpha_c \in [10^{-3}, 10^{-1}]$ from Chesters (1991); *dissipative loss* γ_{diss} combines acoustic radiation damping [Prosperetti, J. Acoust. Soc. Am. 61, 17 (1977)] and rectified diffusion at the bubble wall. ***Initial condition***: equilibrium size distribution $f_0(R) \propto R^{-3.5}\exp(-R/R_{\max})$. ***Boundary conditions***: $f = 0$ on outer wall; zero-flux at $R_{\min}$ and $R_{\max}$. ***Parameter sources***: σ, μ, ρ, c from NIST tables; α_c constrained experimentally. ***Constraining observables***: (i) ns-resolved SL flash rate; (ii) broadband MHz acoustic noise floor; (iii) optical holography of baseline $f_0(R)$; (iv) DCE-band microwave noise floor.

4.3. Model C: Electromagnetic Cavity with Time-Dependent Boundary

Between two adjacent Casimir shells modulated by the passage of collapsing bubbles, the cavity mode obeys Equation (11).

$$\ddot{q}_k(t) + \omega_k^2(t) q_k(t) = 0, \quad \omega_k(t) = \omega_k\left[d_i(t), \varepsilon(\omega), \mu(\omega), R_{\text{bub}}(t)\right] \tag{11}$$

with $d_i(t)$ the instantaneous inter-shell gap and ε, μ the dispersive material responses.

4.4. Bogoliubov Coefficients

Given $\omega_k(t)$ from Equation (11), we solve for the Bogoliubov coefficients α_k, β_k using the standard

adiabatic-basis matching [3, 17] (Equation (12)).

$$\dot{\alpha}_k = \frac{\dot{\omega}_k}{2\omega_k}\beta_k e^{+2i\Phi_k}, \quad \dot{\beta}_k = \frac{\dot{\omega}_k}{2\omega_k}\alpha_k e^{-2i\Phi_k} \tag{12}$$

The mean DCE photon number is $N_k = |\beta_k|^2$ and $E_{DCE} = \Sigma_k \hbar\omega_k N_k$. The parametric resonance condition verified by [4] is that $\omega_k(t)$ oscillate at $\Omega_{mod} \approx 2\omega_k$; DCE photons appear in pairs at $\omega_1 + \omega_2 \approx \Omega_{mod}$.

5. ENERGY AND THERMODYNAMIC ANALYSIS

5.1. Correction of the Original Numerical Estimate

The previous manuscript wrote $E_{SL} \approx 10^{-3}$ - 10^{-1} MeV per collapse and, with $N = 10^{12}$ collapses/s, obtained $P \sim 10^8$ W. This is arithmetically inconsistent: because 1 MeV = 1.602×10^{-13} J, the correct value is $P_{SL} \approx 1.6 \times 10^{-4}$ - 1.6×10^{-2} W. The original 10^8 W figure was in error by 10-12 orders of magnitude.

5.2. Global Energy Balance

For a single acoustic cycle, $E_{acoustic,in}$ distributes among SL, thermal, chemical, mechanical, EM, and dissipative channels. The device-level, time-averaged balance is Equation (13).

$$P_{net} = P_{EM,out} + P_{mech,out} - P_{acoustic} - P_{electrical} - P_{thermal} - P_{control} - P_{cryo} - P_{losses} \tag{13}$$

Single System Boundary and Channel Assignment (R2.5)

The energy ledger is anchored on *one* closed control volume Σ: the outer surface of the cryostat vacuum jacket. Every channel appearing in Equation (13) is assigned exactly once with respect to Σ: $P_{electrical}$, $P_{control}$, P_{cryo} are external inputs; $P_{acoustic}$ is an *internal transfer* between the transducer subsystem and the fluid subsystem and does not appear separately in the external-input sum, which prevents double-counting; $P_{EM,out}$ and $P_{mech,out}$ are outputs; $P_{thermal}$ and P_{losses} are loss outputs. The Casimir energy is a configurational quantity strictly inside Σ and is never an external channel.

5.3. Three Testable Scenarios

Scenario A—Null DCE (baseline): $P_{DCE} \approx 0$, $P_{net} \leq 0$, null hypothesis. Scenario B—Detectable DCE (target): $0 < P_{DCE} \ll P_{acoustic}$ with statistically significant sideband, correlated-pair, and squeezing signatures; this is the primary scientific target. Scenario C—Anomalous energy balance: $P_{out} > P_{in}$; this scenario is not assumed anywhere in this manuscript.

6. CONTROL SYSTEM AND NUMERICAL METHODS

6.1. State-Space Representation

The controller operates on modal statistics, not individual sub-nanosecond bubble events. The state vector contains the low-ℓ acoustic modal amplitudes and rates, mean bubble radius and dispersion, temperature, bubble number density, optical intensity, and per-layer gap monitors. The scalar objective is $J = -w_1 P_{acoustic} + w_2 P_{optical,DCE\text{-}band} - w_3 P_{instability} - w_4 P_{thermal}$.

6.2. Hierarchical Control

Three control layers operate at three disjoint timescales (see **Table 1**). Inner loop (PLL, 10 - 100 kHz) keeps each transducer locked to the resonator's first-harmonic radial mode. Mid loop (MPC, horizon ~10 ms) adjusts amplitudes, phases, and frequencies. Outer loop (RL/Bayesian, s-hour) updates supervisory parameters. No controller is proposed to individually observe or control the ~10^{12} collapse events per second.

Table 1. Control-timescale ladder.

Process	Timescale	Controller
Acoustic carrier	MHz-GHz	RF electronics
Bubble dynamics	ns-μs	physical dynamics (uncontrolled)
Optical flash	ps	detector
Sensor acquisition	ns-μs	FPGA
Modal envelope/phase correction	μs-ms	PLL + MPC
Parameter/policy optimization	s-hour	RL/Bayesian

6.3. Numerical Methodology

The coupled system is integrated with an operator-splitting scheme: Equation (7) on a spherical FEM mesh with radial refinement in the Casimir stack; Equation (9) on a $(R, \dot{R})$ velocity grid with 40×40 resolution; Equations (11)-(12) on the instantaneous cavity spectrum. Time step $\Delta t \leq 10^{-11}$ s during collapse; adaptive stepping otherwise. Convergence is demonstrated by halving the mesh and time step until the relative energy change is below 10^{-2}. Uncertainty is propagated via a Monte Carlo of 10^4 samples.

7. EXPERIMENTAL VALIDATION AND FALSIFICATION PROTOCOL

The experimental program simultaneously measures every energy channel in Equation (13). The six configurations of the falsification protocol are enumerated in Table 2; the before/after comparison of modelling assumptions and empirical treatment of each quantity follows in Table 3 of Section 8.

Table 2. Falsification control matrix. Every candidate mechanism must be turned off individually.

#	Configuration	Acoustic drive	Casimir stack	SL bubbles
1	Dark baseline	off	present	absent
2	Fluid + acoustics, no stack	on	absent	present
3	Static Casimir only	off	present	absent
4	Oscillating boundary, no bubbles	on (below cavitation)	present	absent
5	SL, no Casimir cavity	on	absent	present
6	Full device	on	present	present

The DCE hypothesis (Scenario B) requires configuration 6 to produce a statistically significant excess in the DCE-signature channels that is absent from configurations 1 - 5 at $>5\sigma$, after subtracting the residual $P_{\text{residual}} = P_{\text{measured,out}} - P_{\text{accounted,in/out}}$.

Distinguishing DCE from Classical Channels

The observed radiation decomposes as $P_{\text{obs}} = P_{\text{SL}} + P_{\text{plasma}} + P_{\text{thermal}} + P_{\text{chemi}} + P_{\text{acou-leak}} + P_{\text{electrical}} + P_{\text{DCE}}$. Excess optical power alone cannot establish a DCE mechanism, because SL-plasma emission [5, 7] is variable, gas- and pressure-sensitive. We require the combined signature: photon-number correlations $g^{(2)}$ above the thermal baseline for paired modes summing to Ω_{mod}, plus two-mode squeezing below the vacuum limit for the same pair. This is the test class that established the superconducting-circuit DCE [4].

Measurement Chain per Quantum Signature (R2.6)

Two distinct chains are specified, one per signature. **Chain A—Photon-number correlation ($g^{(2)}_{i,j}(0)$):** cavity output → cryogenic circulator → HEMT ($T_N \approx 2$ K) → image-reject mixer → digitizer at 2 GS/s → FPGA time-tagging with 100 ps bins → HBT coincidence within $\tau_{\text{coinc}} = 1$ ns on two orthogonal cavity ports. Null-model contributions enumerated: (a) HEMT amplifier noise; (b) detector afterpulsing; (c) detector cross-talk; (d) pump leakage at Ω_{mod}; (e) nonlinear mixing; (f) thermal radiation. Detection requires $g^{(2)}(0) - 1 > 5\sigma$ above the sum of (a)-(f). **Chain B—Phase-referenced quadrature detection:** cavity output → JPA in phase-sensitive mode, pumped at $2f_{\text{cav}}$ and phase-locked to the acoustic modulation carrier → homodyne detection with LO phase stepped in π/16 increments → digitized quadrature variances. Two-mode squeezing manifests as $\sigma_-^2 < 1/2$ (vacuum units). Null-model contributions: (g) JPA added noise; (h) LO phase noise; (i) pump-tone leakage; (j) mixer image/imbalance; (k) thermal blackbody; (l) JPA nonlinearity. Only *simultaneous* success of Chain A and Chain B on the same paired modes constitutes a DCE detection.

8. PREDICTED REGIMES AND RESULTS

8.1. Reference-Prediction Operating Point and Spectral Reconciliation (R2.2)

The reference prediction is stated at a single, fully specified operating point. *Target microwave readout band*: $f_{\text{MW}} \in [4, 8]$ GHz. *Operating temperature*: $T = 20$ mK at the microwave cavity, $T_{\text{fluid}} = 280$ K in the cavitation region. *Thermal occupation* of the microwave modes: $\bar{n}_{\text{th}}(6\text{ GHz}, 20\text{ mK}) \approx 4 \times 10^{-7}$. *Material losses*: internal $Q_{\text{int}} \in [3 \times 10^3, 3 \times 10^4]$.

***Reconciling MHz drive with GHz readout*:** the acoustic carrier at $f_0 \approx 2$ MHz is *not* the DCE pump; the pump is the *collapse waveform*, whose spectral content peaks in the 0.1 - 10 GHz band because the Rayleigh collapse duration is $\tau_c \sim 100$ ps. Fourier-decomposing $\delta d(t)$ gives a modulation spectral density (Equation (14)).

$$S_{\delta d}(f) = \frac{\dot{N}_b}{f_0}\left|\tilde{h}(f)\right|^2, \quad \tilde{h}(f) = \int \delta d(t) e^{-i2\pi ft}\, dt \tag{14}$$

which at $f = 2f_{\text{cav}} \approx 12$ GHz (parametric-resonance condition for a 6 GHz cavity) evaluates to $S_{\delta d}(2f_{\text{cav}})/d^2 \approx 10^{-27}$ Hz^{-1} for $d = 100$ nm, $\delta d = 0.4$ nm, and $N_b = 10^5$ cycle^{-1}. The gap-modulation spectral weight enters the predicted count rate through Equation (15).

$$\Gamma_{\text{DCE}}(f_{\text{cav}}) \simeq \frac{f_{\text{cav}}}{Q_{\text{ext}}}\left(\frac{\omega_{\text{cav}}}{2\pi}\right)\frac{S_{\delta d}(2f_{\text{cav}})}{d^2} V_{\text{part}}\left[1 + 2\bar{n}_{\text{th}}\right] \tag{15}$$

in the weak-modulation limit, with the bracket capturing thermal enhancement.

8.2. One Fully Worked Numerical Example (R2.3)

Parameters: Ag/SiO_2 ALD stack with Drude $\omega_p = 1.4 \times 10^{16}$ rad/s, $\gamma = 3.2 \times 10^{13}$ rad/s; SiO_2 $\varepsilon_r = 3.9$, $\tan\delta = 3 \times 10^{-4}$ at 20 mK; TE_011-like mode at $f_{\text{cav}} = 6.0$ GHz, $V_{\text{mode}} = 1.2 \times 10^{-8}$ m^3, overlap 0.42, $Q_{\text{int}} = 10^4$, $Q_{\text{ext}} = 10^4$, $Q_{\text{load}} = 5 \times 10^3$; $\delta d = 0.4$ nm (Equation (6)); $V_{\text{part}} = 5.0 \times 10^{-9}$ m^3; $N_b/f_0 = 10^5$; losses TLS 10^{-4}, radiative 10^{-4}, acoustic $\sim 10^{-3}$. Substitution gives $|\tilde{h}(2f_{\text{cav}})|^2 \approx 5.0 \times 10^{-38}$ m^{2}s and $S_{\delta d}/d^2 \approx 2.5 \times 10^{-27}$ Hz^{-1}. Equation (15) then yields Equation (16).

$$\Gamma_{\text{DCE}} \simeq \frac{6.0\times10^9}{5\times10^3} \times 3.8\times10^{10} \times 2.5\times10^{-27} \times 5.0\times10^{-9} \simeq 5.7\times10^2 \text{ photons/s} \tag{16}$$

translating to $P_{\text{DCE}} \simeq 2.3 \times 10^{-21}$ W, *i.e.*, $\sim 10^3$ photons/s at ~6 GHz. Over a 10^5 s integration and 10^{-3} dark-count noise floor, this yields S/N ≈ 5 - 10, marginally detectable. The value $\Gamma_{\text{DCE}} \approx 5.7 \times 10^2$ photons/s is the reference prediction and supersedes the earlier 10^4 - 10^8 envelope.

8.3. Comparison with the Original Manuscript

A side-by-side comparison of how each key quantity was treated in the original manuscript versus the

present revision is summarised in Table 3, immediately below.

Table 3. Original versus revised treatment of key quantities.

Quantity	Original manuscript	Revised treatment
Bubble rate	10^6 - 10^{12}/s	parameter/simulation output
Energy per collapse	assumed 10^{-3} - 10^{-1} MeV	derived/measured
Acoustic input	assumed	measured
SL optical output	assumed	calibrated measurement
DCE output	asserted	QED calculation, Equations (11)-(12)
Thermal loss	neglected	explicitly measured
Mechanical loss	neglected	explicitly modeled
Net energy	assumed > 0	experimentally tested; no sign assumed
Claimed output	100 - 500 kW, 100 MW, 300 MW	withdrawn

9. DISCUSSION

The DCE is an established physical phenomenon [4]. Tunable/chiral Casimir interactions are a well-defined theoretical mechanism [16]. Sonoluminescence is a well-characterized acoustic-cavitation phenomenon [5-7]. A device that couples these is a legitimate object of scientific study. That an SL-modulated Casimir cavity produces DCE photons at a rate distinguishable from classical channels remains hypothetical. The magnitude of any such contribution is tightly bounded above by $P_{\mathrm{DCE}} \lesssim P_{\mathrm{acoustic}}$ and, in likelihood, $P_{\mathrm{DCE}} \ll P_{\mathrm{acoustic}}$. No claim of net energy generation is made.

Sections on gravitational vacuum optimization and emergent-gravity coupling have been removed from the core mechanism. For laboratory energies, the curvature contribution $\delta R \sim \kappa E_{\mathrm{SL}}$ estimated in the original is negligible by dozens of orders of magnitude. These ideas remain interesting as speculative extensions and are noted here for completeness only; they are not part of the claimed mechanism.

The uncertainty on $\varepsilon(\omega)$ at the shortest wavelengths dominates the DCE prediction. Surface roughness at $\lesssim 1$ nm RMS is required for the assumed spectral integrity. Patch potentials must be actively nulled. Eberlein's SL-as-DCE model [8] predicts a yield that Milton and Ng [9] argued is smaller than observed SL by many orders in the adiabatic regime. The present manuscript does not require Eberlein's identification; it asks whether a deliberately engineered boundary modulation produces a residual DCE signature. A second-generation device incorporating chiral/magneto-optical layers [16] may permit sign-controlled Casimir modulation; any such enhancement must remain within the global energy budget Equation (13).

10. CONCLUSION

This work develops a coupled theoretical framework for investigating the interaction between acoustically driven sonoluminescence and dynamically modulated electromagnetic cavity modes. The analysis does not establish net extraction of zero-point energy or violation of thermodynamic bounds; it identifies the dynamical, electromagnetic, and metrological conditions under which a sonoluminescence-induced modulation could produce a distinguishable dynamical-Casimir signature. A decisive test requires simultaneous accounting of acoustic, electrical, thermal, mechanical, conventional optical, and putative dynamical-Casimir channels. The proposed framework constitutes a falsifiable experimental program, rather than a demonstrated energy-generation mechanism.

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CONFLICTS OF INTEREST

The author declares no conflicts of interest regarding the publication of this paper.

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