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Financial Frictions and Macroeconomic Volatility in the WAEMU: A DSGE Model Approach

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Abstract

This paper studies the role of financial frictions in the propagation of macroeconomic fluctuations within the WAEMU, drawing on data covering the period 1980-2025, a reduced 26-variable representative DSGE model with a calibrated financial accelerator, and a partial Bayesian estimation of two of its ten free critical parameters. The credit market has tightened since 2020, with stricter borrower screening, while investment has overtaken inflation as the leading source of macroeconomic volatility since the 1990s. The calibrated financial accelerator substantially amplifies the transmission of capital-return shocks to entrepreneurial net worth, but it is foreign interest rate shocks, transmitted through a calibrated (not estimated) policy-rule coefficient linked to the CFA franc's peg to the euro, within this reduced model, that dominate the Union's macroeconomic variance, with financial shocks contributing only marginally. A Bayesian estimation of the most decisive parameters reveals an almost flat Phillips curve and an external finance premium elasticity markedly higher than its value calibrated on U.S. data. These results, obtained from a reduced approximation of the full 74-variable baseline model, call for strengthening credit information, adopting countercyclical macroprudential policy, and reflecting on the scope for monetary autonomy under the euro peg.

Keywords

Macroeconomic Volatility, Financial Frictions, DSGE, WAEMU

1. Introduction

Before the 2009 financial crisis, economists developed micro-founded models aimed at characterizing the cyclical dynamics of advanced economies (Clarida, Galí, & Gertler, 1999; Smets & Wouters, 2003; Christiano, Eichenbaum & Evans, 2005). These models succeeded in replicating fairly well the cyclical dynamics observed during the post-Volcker period (the Great Moderation) in OECD countries, particularly the dynamics of inflation and output. Following the 2009 international financial crisis, these models were criticized for disregarding financial ingredients, and were therefore seen as not only incapable of forecasting (ex-ante) the 2009 downturn but also analytically ill-suited to understanding (ex post) the financial crisis (Gertler & Kiyotaki, 2011).

Since then, the empirical literature has analyzed the impact of these frictions on cyclical dynamics mainly through two approaches: 1) the corporate balance-sheet channel (Cordoba & Ripoll, 2004) and 2) the bank credit channel (Quadrini, 2011).

Bernanke and Gertler (1989) develop, within the first approach, an overlapping-generations model in which financial frictions can amplify the propagation of productivity shocks. More specifically, shocks affecting firms' net worth (as in a debt deflation) can initiate fluctuations. Carlstrom and Fuerst (1997) embed the Bernanke and Gertler (1989) baseline model in a computable general equilibrium framework and analyze the quantitative effects of financial frictions on business fluctuations. A key finding is that the model with financial frictions replicates fairly well the empirical facts, notably a positive autocorrelation of output growth at short horizons (Cogley & Nason, 1995).

Bernanke et al. (1999) incorporate the dynamic structure of Carlstrom and Fuerst (1997) into a model with price rigidity and study the effects of monetary shocks on business cycles. The authors find that financial frictions help explain both the strength of the economy's response to monetary policy and the persistence of that policy's effects even after the interest rate returns to normal, as generally observed in VAR analysis. These effects, however, require investment delays in order to reproduce a hump-shaped output response to monetary shocks.

In addition, Linnemann and Schabert (2003) show that the interaction between financial frictions and nominal wage rigidity can help generate a bell-shaped output response following a monetary shock (Del Negro et al., 2010).

Kiyotaki and Moore (1997) and Kiyotaki (1998) incorporate Fisher's debt-deflation theory[1] into a dynamic general equilibrium framework. Because of the inalienability of human capital à la Hart and Moore (1994), borrowers cannot commit in advance to repaying their debts, and lenders are unable to seize borrowers' human capital to recover the debt.

Subsequently, Carlstrom and Fuerst (1997), using a standard DSGE model, reproduce the positive short-run autocorrelation observed between credit agencies and economic growth. This result is also obtained by Kiyotaki and Moore (1997): credit constraints can play an important role in macroeconomic fluctuations, a result likewise highlighted by Bernanke and Gertler (1989).

Bernanke et al. (1999) introduce the so-called "financial accelerator" shock and note the amplification and propagation effects of such a shock on macroeconomic

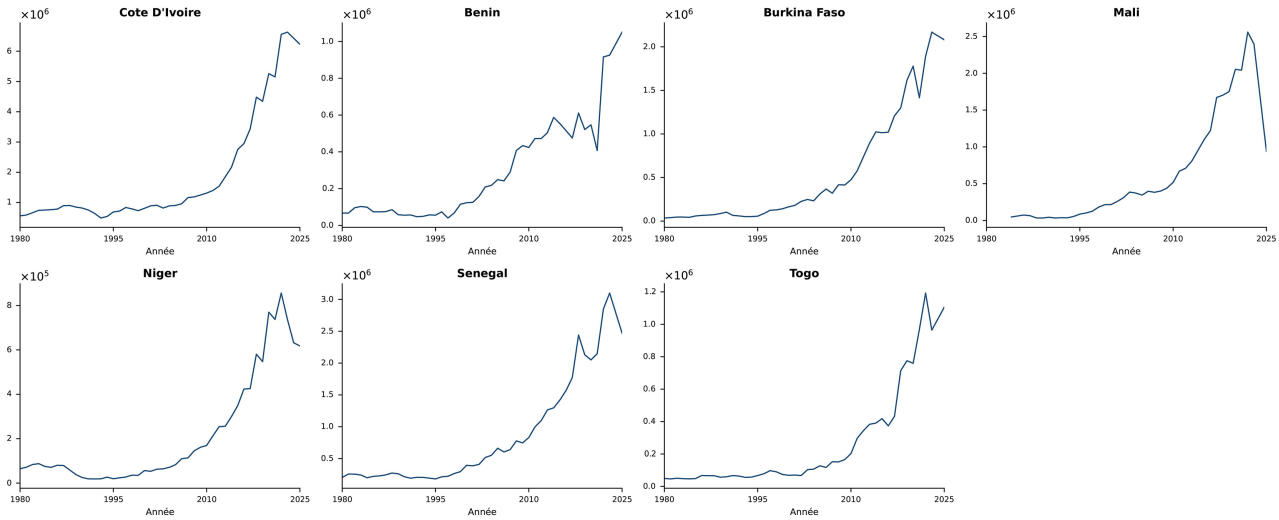
[1]Fisher's debt theory shows evidence of a link between credit and the real economy (Fisher, 1933).

fluctuations. Along the same lines, Aghion et al. (2010) identify a propagation mechanism affecting the cyclical composition of investment. A credit crunch can lead both to greater volatility and to average growth lower than inflation.

Yet the proper functioning of this transmission channel presupposes a credit market that responds reliably to monetary policy impulses, a condition specifically undermined in the presence of financial frictions. Examining the long-run evolution of credit to the economy across the seven WAEMU countries for which consistent long-run series are available (all member states except Guinea-Bissau) helps substantiate this observation.

The average growth rate of bank credit to the economy over the periods 1993-2003, 2004-2014, and 2015-2025 stood at 10.1%, 10.7%, and 7.4%, respectively. This recent slowdown is particularly marked since 2020: the average annual growth of aggregate short-term credit across the seven countries observed (excluding Guinea-Bissau) falls to 2.8% over 2020-2025, turning negative in 2024 and 2025.

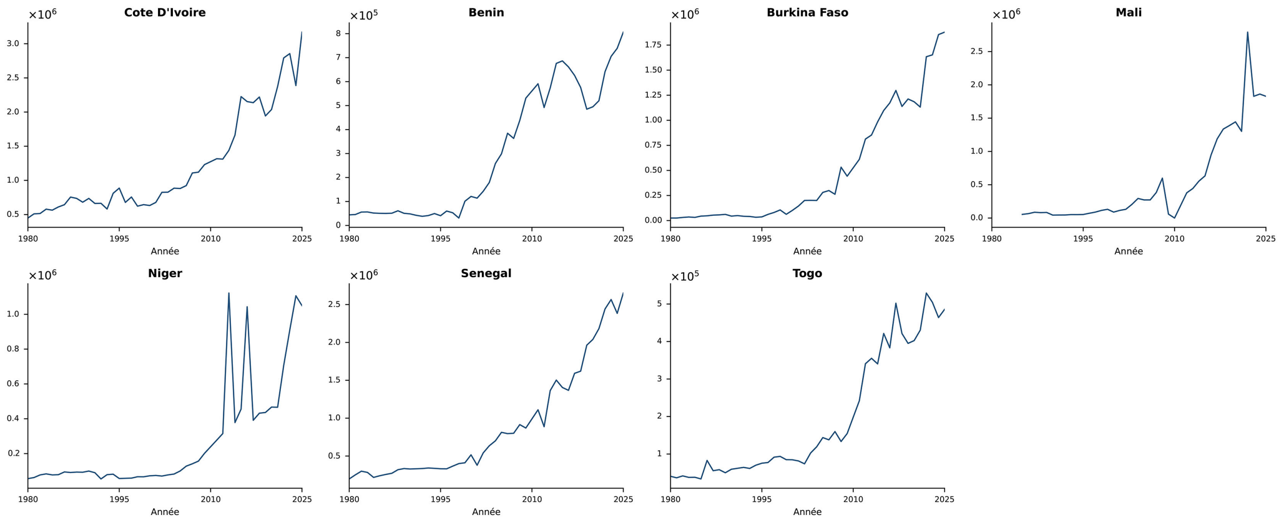
At the same time, credit reported to the Credit Registry (Centrale des Risques) grew at an average of 7.4%, 9.9%, and 5.0% over these same three periods. This slowdown broadly tracks that of total credit volume, with one notable exception: over 2020-2025, while total credit growth fell to 2.8%, growth in credit reported to the Credit Registry rebounded to 8.1% per year on average, driven mainly by Côte d'Ivoire, Senegal, and Burkina Faso. This decoupling, a credit market slowing overall even as amounts reported to the risk registry accelerate, constitutes an additional signal of stress in the WAEMU credit market since 2020. These statistics confirm, over the full 1993-2025 period, the persistence of likely financial frictions in the WAEMU credit market (cf. Figure 1 and Figure 2).



Source: Author.

Figure 1. Evolution of short-term bank credit.

Figure 1 presents the evolution of credit to the economy for each WAEMU country except Guinea-Bissau, for lack of sufficient data. The volume of short-term credit extended by commercial banks in these countries is observed to have grown broadly over 1980-2023, before a marked slowdown in 2024-2025 in several countries (notably Mali, Niger, and Togo). The 1993 devaluation contributed to reducing the volume of this credit across all countries. Since 2000, growth (fairly slow until 2010) accelerated markedly between 2010 and 2023, before coming to a sharp halt starting in 2024 (**Figure 2**).



Source: Author.

Figure 2. Evolution of short-term credit reported as non-performing.

Figure 2 presents the evolution of short-term credit reported to the Credit Registry for most WAEMU countries. A fluctuation in credit volume is observed, rising sharply from the 2000s onward and accelerating again since 2020, despite the slowdown in total credit over the same period. This may help explain the declining share of short-term credit extended to the economy by banks, all the more so as the depth of credit information index among member countries of the zone remained low (Index = 1). This situation gives rise to information asymmetry, moral hazard, and adverse selection (Stiglitz & Weiss, 1981; Christiano & Ikeda, 2011) in the credit market, which justifies the financial frictions (De Grauwe, 1999; Gerali et al., 2010).[2]

Table 1, presenting empirical regularities, and **Figure 3** and **Figure 4**, depicting the volatility of nominal GDP (the series used throughout this study) and inflation, suggest the influence of financial frictions on GDP and inflation in the WAEMU.

Table 1 shows the volatility of GDP and inflation, as well as that of other variables of interest such as private consumption, investment, and the money supply,

[2]Depth of credit information index (0 = low to 8 = high) (World Bank, 2015; Doing Business 2015). This indicator was produced by the World Bank's Doing Business project. Following the permanent discontinuation of the Doing Business report in September 2021, this indicator is no longer updated, which prevents verifying its recent evolution.

updated over the 1980-2025 period.

Table 1. GDP and inflation volatility (1980-2025).

Variables	Absolute Volatility (% Std. Dev.)	Relative Volatility (%GDP)
Nominal GDP	**6.7816**	**1.0000**
Private Consumption	8.6201	1.2711
Investment	17.9595	2.6483
Government Spending	7.4239	1.0947
Net Exports*	5.5797	0.8228
Inflation	**6.5043**	**0.9591**
Money Supply	7.5920	1.1195
Policy Rate**	3.3589	0.4953

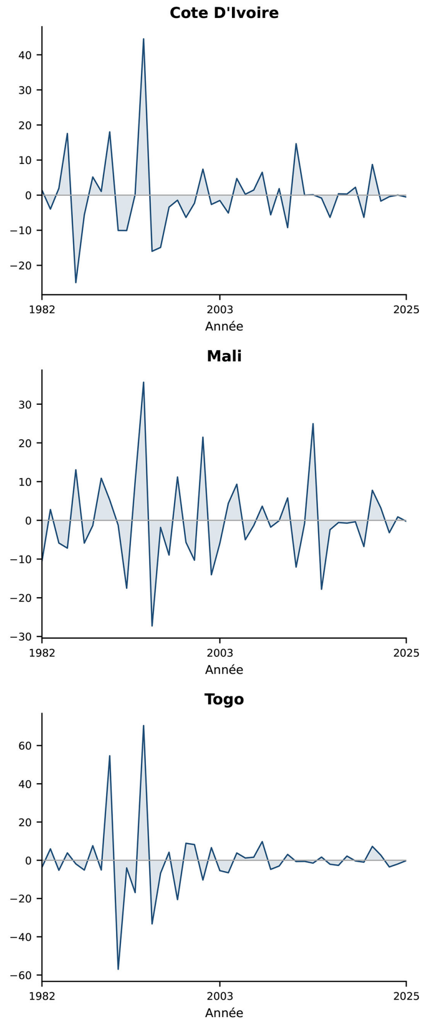
*Unit differs from other rows: standard deviation of the annual change in the NX/GDP ratio (in percentage points), not the standard deviation of a growth rate. The relative volatility remains computable (ratio to GDP's standard deviation) but its economic reading differs slightly from the other rows. **Inflation and policy rate: standard deviation of the rate series itself, not of a growth rate (these are already percentages). Source: Author.

In the WAEMU, absolute volatilities exceed relative volatilities for all variables. Unlike the 1970-2014 period, when inflation was by far the most volatile component of the economy (nearly 38 times that of GDP), the updated data show a markedly different volatility profile: investment now displays the highest relative volatility (2.65 times that of GDP), followed by private consumption (1.27 times), the money supply (1.12 times), and government spending (1.09 times), with inflation even falling slightly below GDP volatility (0.96 times). This shift in ranking suggests that the sources of macroeconomic instability in the WAEMU have evolved since the 1990s: price shocks, dominant at the time of the 1994 devaluation, have gradually given way to instability driven primarily by investment decisions and, to a lesser extent, by household consumption and money creation.

Thus, it is no longer so much inflationary uncertainty that explains any weakness in the growth of the money supply, credit, and the policy rate, but rather the strong instability of investment, an observation consistent with the financial frictions identified in the credit market (cf. Figure 1 and Figure 2).

At the same time, the growth rate across WAEMU countries has been unstable and fluctuating, seldom allowing a stable trend to emerge. This instability is fueled in particular by the swings in household consumption spending, whose absolute volatility stands at about 8.6% per year over 1980-2025 across the heterogeneous WAEMU zone, markedly lower than the 21% measured over 1970-2014, which could reflect either genuine smoothing of consumption over time or a change in data frequency (annual versus quarterly) between the two calculations.

However, Figure 3 and Figure 4 show that the dynamics of nominal GDP and inflation are nearly identical across all countries.



Source: Author.

Figure 3. Nominal GDP volatility.

The two preceding figures show, respectively, the fluctuations in nominal output and inflation for all WAEMU countries except Guinea-Bissau, for lack of sufficient data. Absolute volatility is measured by computing the first difference of nominal GDP, not real GDP; the series is expressed in current prices throughout this study, consistent with **Table 1**, as defined by Clark, Papell, and Stoica (2011).

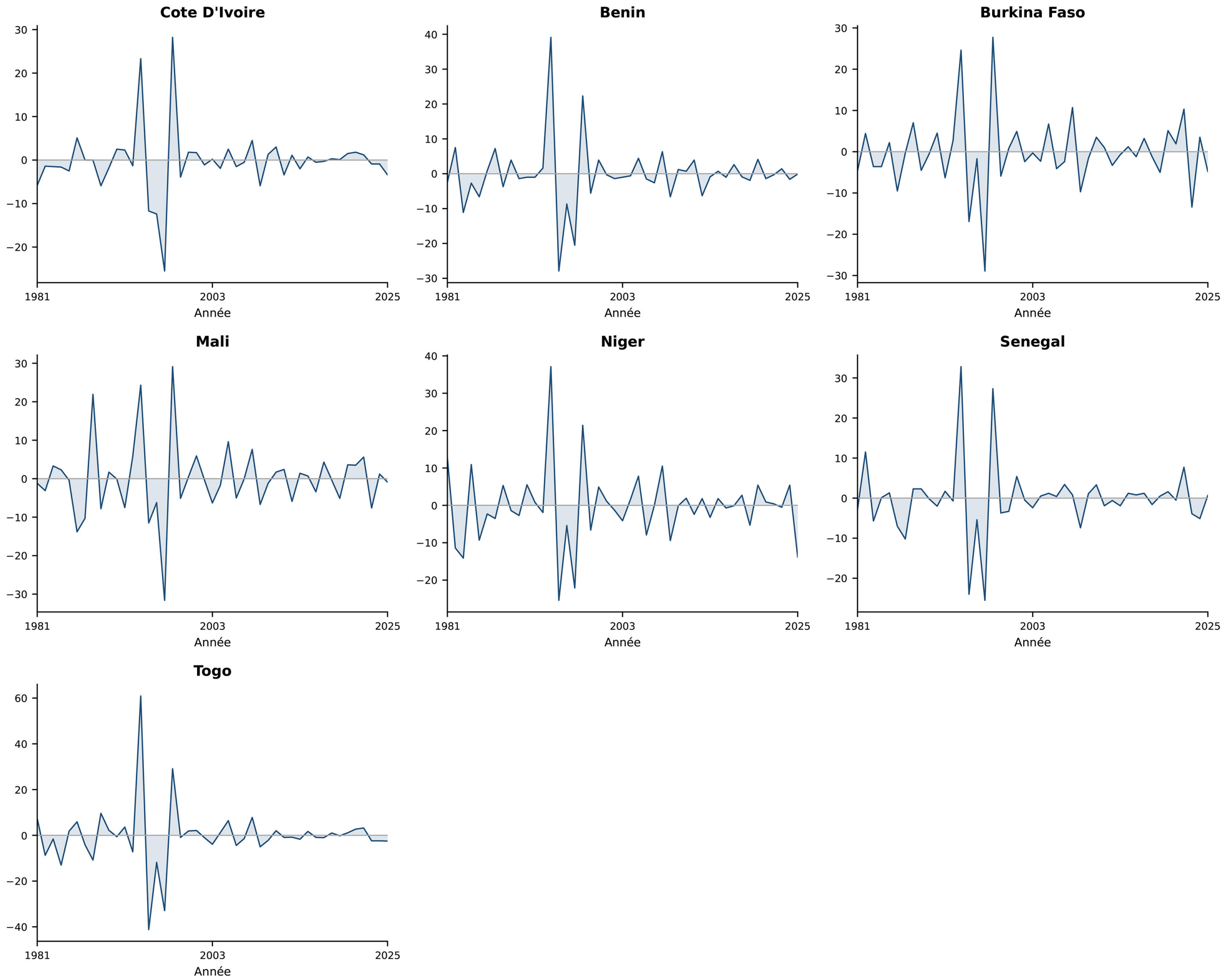
These figures confirm that during expansions in real activity, credit market imperfections lead banks to overestimate borrowers' creditworthiness and extend credit heavily, whereas they curtail lending volumes during recessions.

Credit thus appears to behave procyclically and contemporaneously with economic activity. This co-movement between credit and activity suggests that the Union's banking system is not "neutral" in financing real activity[3].

Two broad moderations are observed, roughly between 1970-1980 and 2007-2014. These moderations reflect the simultaneous rise of GDP and inflation (Stock

[3]Abou N. B. (2010)

& Watson, 2003).



Source: Author.

Figure 4. Inflation volatility.

The observed dispersion of nominal GDP across member countries would suggest a pursuit of individual economic growth paths. This pattern could undermine the viability of the union as a whole should an internal (Alexander & Pisani-Ferry, 1992) or external (Kishor & Ssozi, 2009) shock occur. To this effect, financial frictions, taken as a shock, contribute in part to the malfunctioning of monetary unions (Tirole, 2015; Ben Salem et al., 2024). The production process in the WAEMU drives the supply of credit to the economy by banks. The credit market is studied as a specific entity characterized by supply originating from secondary banks and demand coming from non-bank institutions. The central idea developed here rests on the premise that a situation of credit insufficiency could exist, which would in turn limit firms' ability to finance their investment and consumers' ability to meet their needs.

Figure 4 shows that the inflation rate has remained relatively high relative to the price-stability target of 2%, defined within a critical band (1% - 3%) (BCEAO, 2025). This result contrasts with the earlier analysis by Devarajan and de Melo (1990). This has led the BCEAO to reassess its policy stance. Since the late 1980s, it has opted for indirect monetary management, using the growth rate of money reserves as its operational instrument.

In addition, the bands shown in the charts of Figure 3 and Figure 4 reveal some differences across countries. These differences may be explained, among other things, by external and internal shocks, not exhaustively traced in Table 2.

WAEMU countries have, on the whole, experienced several episodes of coups d'état, famines, and banking crises. These events constitute shocks that, at the time they occurred, drove variations in growth and inflation. One must also consider the devaluation that took place in the zone in nineteen ninety-three. This devaluation led to very high inflation and relatively low and slow-moving growth.

Table 2. Structural breaks in the WAEMU.

Country	Break Periods	Nature of the Break
Benin	1963, 1965, 1967, 1968, 1972 (1) 1988-1990 (2)	(1) At least five coups d'état (2) Three commercial banks failed.
Burkina Faso	1966, 1974, 1977, 1980, 1982, 1987, 2014 (1) 1988-1994 (2) 2022, 2022 (3) 2015-2025 (4)	(1) At least seven coups d'état (2) 34% of bank loans are non-performing. (3) Two coups d'état in January and September 2022 (overthrow of Kaboré, then Damiba) (4) Ongoing jihadist insurgency since 2015, worsening since 2022 (massive internal displacement, food insecurity).
Côte d'Ivoire	1988-1991(1) 1999 (2) 2020 (3) 2002 (4)	(1) Banking crises (3) private banks closed, 1 insolvent, and 6 state-owned banks closed). (2) Coup d'état (3) Electoral crisis linked to Alassane Ouattara's third term: at least 85 deaths, violence in about ten cities. (4) Failed coup attempt and administrative and economic division of the country (North and South).
Mali	1968, 1991, 2012 (1) 1987-1989 (2) 2012 (3) 2020, 2021 (4) 2012-2025 (5)	(1) Coups d'état (2) Non-performing loans at the largest banks were estimated at 75% of total loans. (3) Administrative and economic division of the country (North and South) (4) Two additional coups d'état (August 2020, May 2021) (5) Ongoing jihadist insurgency and armed conflict in the North and Center since 2012.
Niger	1972 (1) 1983 (2) 1998 (3) 1980, 1999, 2003, 2012 (4) 2005 (5) 2023 (6)	(1) Drought period (2) Non-performing loans were estimated at 50% of system loans. (3) Civil war (4) At least four coups d'état (5) Food crisis (6) July 2023 coup d'état (overthrow of Bazoum), prolonged disruption through 2024-2025.
Senegal	1988-1991 2021, 2023, 2024	In 1988, 50% of loans were declared non-performing; six commercial banks and one development bank were closed. Their combined balance-sheet size was between 20% and 30% of the entire system. Violent demonstrations in March 2021 and June 2023 (pre-electoral unrest).
Togo	1963, 1967 (1) 1993-1995 (2)	(1) Coup d'état (2) System-wide insolvency problems.

Source: Author.

Since then, the introduction of new operating arrangements, taking into account internal parameters (the inflation rate, the growth rate of the economy, banks' external position, and banks' domestic credit) and external parameters (the inflation rate in France, exchange rate movements), together with the abandonment of credit ceilings effective January 1, 1994, set the objectives of the new price-stability policy that are now assigned to monetary policy. These include, among others, the optimal allocation of states' financial resources and the internal and external stabilization of the currency with a view to sustainable growth in economic activity (Akerlof, 1970).

The remainder of the paper is organized as follows. Section 2 discusses the methodology used. We show the relevance of the DSGE approach, relative to other existing approaches (CGE, SVAR, GARCH, etc.), for addressing our research questions. Section 3 presents the baseline model. Section 4 shows how financial frictions can be incorporated into a DSGE model. Section 5 discusses the results and the recommendations arising from our analysis. Section 6 concludes.

2. Methodology

Studies of financial frictions have relied on several methodological approaches, notably CGE, SVAR, GARCH, and DSGE. The motivation for choosing a DSGE model, unlike other models, lies in the idea of introducing richer microeconomic structure (the behavior of households, firms, and governments) into the analysis of macroeconomic dynamics in WAEMU economies, particularly with respect to output and inflation. With the Lucas (1972) critique, which challenges the validity of Keynesian-inspired macroeconometric models unable to account for agents' expectations, the appeal of DSGE models comes into sharper focus. Furthermore, to address criticisms of DSGE models, we no longer treat our economy as "cashless". We also introduce information asymmetry problems between lenders and borrowers, building on the credit-rationing principle developed by Gertler and Karadi (2011). These two elements can allow for an extension of DSGE models, especially in the specific case of developing countries. The literature generally considers, for the purposes of this study, three types of DSGE models: 1) the Smets and Wouters (SW) model (2007), 2) the SWBGG model à la Bernanke et al. (1999), and 3) the SWGK model à la Gertler and Karadi (2011).

We work from the baseline assumptions of the SWGK model, adapting those presented by Christiano, Trabandt, and Walentin (2011), whose base-model equations, as confirmed in Section V.5, correspond almost line for line to those of Section III. The introduction of financial frictions is carried out in the context of the African economy of the West African Economic and Monetary Union, and is defined as the set of model features that impede financial flows between agents with surplus funds and those in need of financing. We accordingly consider four types of economic agents in this model: households, firms, the government, and the Central Bank. The model is calibrated and partially estimated for the WAEMU over the updated 1980-2025 period.

3. Baseline Model

In the model, we consider three final goods (consumption, investment, and exports), which are produced by combining a homogeneous domestic good with imported inputs specific to each type of final good. Below we begin the description of the model by covering the production of each of these goods.

3.1. Production of the Homogeneous Good

A homogeneous domestic good is produced using: Y_t

$$Y_t = \left[\int_0^1 Y_{i,t}^{1/\lambda_d}, 1 \le \lambda_d < \infty\right] \tag{1}$$

where the intermediate goods and their degree of substitutability are as defined. The homogeneous domestic good is produced by a competitive, representative firm that takes the price of finished goods and the price of inputs as given. The I-th intermediate-goods producer has the following production function:

$Y_{i,t}\ 1/\lambda_d\ P_t P_{i,t}$

$$Y_{i,t} = \left(Z_t H_{i,t}\right)^{1-\alpha} \varepsilon_t K_{i,t}^{\alpha} - Z_t^{+}\varphi \tag{2}$$

where capital services rented by the I-th intermediate-goods producer are as defined $K_{i,t}$.

In addition, one shock is a technology shock whose first difference has a positive mean, another is a stationary neutral technology shock, and a further term represents a fixed cost of production. The economy has two sources of growth: a positive trend in one technology process and a positive trend arising from an investment-specific technology shock. The object appearing in (2) is defined as:

$$\log Z_t\ \log \varepsilon_t \varphi \log Z_t\ \log \psi_t \psi_t Z_t^{+}$$

$$Z_t^{+} = \psi_t^{\frac{\alpha}{1-\alpha}} Z_t$$

In (2), homogeneous labor rented by the I-th intermediate-goods producer is as defined. Firms must borrow a fraction of the wage bill, such that the cost of a unit of labor is given by: $H_{i,t} \nu_t$

$$W_t R_t^f$$

with

$$R_t^f = \nu^f R_t + 1 - \nu^f \tag{3}$$

where the aggregate wage rate is as defined, and the risk-free interest rate applies to loans on the wage bill $W_t R_t$.

The firm's marginal cost, divided by the price of the homogeneous good, is given by: mc_t

$$mc_t = \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\overline{w}_t R_t^f\right)^{1-\alpha} \frac{1}{\varepsilon_t} \tag{4}$$

where the nominal rental rate of capital is scaled as indicated,

$$r_t^k P_t \overline{w}_t = W_t \Big/ \left(Z_t^{+} P_t\right)$$

In addition, one term is a cost-push shock that affects marginal cost but does not appear in the production function τ_t^d.

Productive efficiency requires that marginal cost equal the marginal rate of substitution:

$$mc_t = \tau_t^d \frac{\left(\mu_{\psi,t}\right)^{\alpha} \overline{w}_t R_t^f}{\varepsilon_t \left(1-\alpha\right)\left(\dfrac{k_{i,t}}{\mu_{Z^+,t} H_{i,t}}\right)^{\alpha}} \tag{5}$$

The I-th firm is a monopolist in the production of the I-th good and therefore sets prices. Price setting is subject to Calvo frictions. With a given probability, the firm's intermediate good cannot be reoptimized. In that case: ξ_t

$$P_{i,t} = \tilde{\pi}_{d,t} P_{i,t-1}, \tilde{\pi}_{d,t} := \left(\pi_{t-1}\right)^{k_d} \left(\overline{\pi}_t^c\right)^{1-k_d-\dot{u}_d} \left(\breve{\pi}\right)^{\dot{u}_d}$$

where the terms are parameters, the inflation rate is lagged one period, and the central bank's inflation target is as defined. In addition, a scalar allows for capturing, among other things, the case in which non-optimizing firms do not change their price at all, or index it to the steady-state rate of inflation. Note that there is price dispersion in the steady state whenever indexation is incomplete and the steady-state value differs accordingly. With the complementary probability, the firm may change its price. The problem faced by the I-th intermediate domestic-goods producer that has the opportunity to change its price is to maximize discounted expected profits:

$$k_d \dot{u}_d k_d + \dot{u}_d \in \left(0,1\right) \pi_{t-1} \overline{\pi}_t^c \breve{\pi} \left(i.e., \breve{\pi} = \overline{\pi}, \dot{u}_d = 1\right) \dot{u}_d > 0 \breve{\pi} \pi 1 - \xi_d$$

$$E_t \sum\nolimits_{j=0}^{\infty} \beta^j v_{t+j} \left\{P_{i,t+j} Y_{i,t+j} - mc_{t+j} P_{t+j} Y_{i,t+j}\right\} \tag{6}$$

subject to the requirement that output equal demand. In the expression above, the multiplier is that of the household's nominal budget constraint. It measures the marginal value to a household of one unit of profit in currency terms. In steady state, when a firm is able to reoptimize its price, it does so to maximize its discounted expected profit, subject to price-setting frictions and to the requirement that it satisfy demand, given by: v_t

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d - 1}} Y_t = Y_{i,t} \tag{7}$$

The domestic intermediate good produced is allocated among alternative uses as follows:

$$Y_t = G_t + C_t^d + I_t^d + \int_0^1 X_{i,t}^d \mathrm{d}i \tag{8}$$

where government consumption (which consists entirely of domestic goods) is as defined; the intermediate goods used in the production of households' final consumption goods (together with imported consumption goods) are as defined; and the amount of domestic intermediate goods used in combination with imported foreign investment goods to produce a homogeneous investment good is as defined. Finally, the integral in (8) represents domestic resources allocated to exports. The determination of consumption, investment, and export demand is discussed below $G_t C_t^d I_t^d$.

3.2. Production of Final Consumption and Investment Goods

Final consumption goods are purchased by households. These goods are produced by a representative, competitive firm using the following linear homogeneous technology:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d\right)^{\frac{\eta_c - 1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(C_t^m\right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c - 1}{\eta_c}} \tag{9}$$

The representative firm takes the price of final consumption goods as exogenous.

Final consumption goods are produced with two inputs. The first is a one-for-one output of the homogeneous domestic good and is therefore priced accordingly. The second input is a homogeneous component of specialized imported consumption goods, discussed in the next subsection. The representative firm takes both input prices as exogenous. Profit maximization yields the following scaled demand for intermediate inputs:

$$P_t^c C_t^d P_t C_t^m C_t^m P_t^{m,c} P_t P_t^{m,c}$$

$$c_t^d = (1 - w_c)\left(p_t^c\right)^{\eta_c} c_t$$

$$c_t^m = \omega_c \left(\frac{P_t^c}{P_t^{m,c}} \right)^{\eta_c} c_t \tag{10}$$

where the terms are as defined. The price of the final good is linked to input prices by:

$$p_t^c = P_t^c / P_t \; p_t^{m,c} = P_t^{m,c} / P_t C_t$$

$$p_t^c = \left[(1-\omega_c) + \omega_c \left(p_t^{m,c}\right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \tag{11}$$

The inflation rate for consumption goods is:

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \pi_t \left[\frac{(1-\omega_c) + \omega_c \left(p_t^{m,c}\right)^{1-\eta_c}}{(1-\omega_c) + \omega_c \left(p_{t-1}^{m,c}\right)^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}} \tag{12}$$

Investment goods are produced by a representative, competitive firm using the following technology:

$$I_t + a(u_t)\bar{K}_t = \Psi_t \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I_t^d\right)^{\frac{\eta_i - 1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(I_t^m\right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}}$$

where investment is defined as the sum of investment goods used in physical capital accumulation and investment goods used for capital maintenance $I_t a(u_t)\bar{K}_t$.

The capital utilization rate is as defined, with capital services given by: $a(u_t).u_t$

$$K_t = u_t \bar{K}_t .$$

To accommodate the possibility that the price of investment goods relative to consumption goods declines over time, the investment-specific technology shock is assumed to follow a potentially positive trend. As in the consumption-goods sector, representative investment-goods producers take all relevant prices as exogenous, Ψ_t.

Profit maximization yields the following scaled demand for intermediate inputs:

$$i_t^d = \left(p_t^i\right)^{\eta_i}\left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i) \tag{13}$$

$$i_t^m = \omega_i\left(\frac{p_t^i}{p_t^{m,i}}\right)^{\eta_i}\left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right) \tag{14}$$

where the terms are as defined, $p_t^i = \Psi_t P_t^i / P_t \; p_t^{m,i} = P_t^{m,i} / P_t$.

The price of the investment good is linked to input prices by: I_t

$$p_t^i = \left[(1-\omega_i) + \omega_i\left(p_t^{m,i}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \tag{15}$$

$$\pi_t^i = \frac{\pi_t}{\mu_{\Psi,t}}\left[\frac{(1-\omega_i)+\omega_i\left(p_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i)+\omega_i\left(p_{t-1}^{m,i}\right)^{1-\eta_i}}\right]^{\frac{1}{1-\eta_i}} \tag{16}$$

3.3. Exports and Imports

Export and import activities involve Calvo price-setting frictions and therefore require the presence of monopoly power. The Dixit-Stiglitz approach is used to introduce a range of specialized goods. This implies the presence of market power without the counterfactual implication of a small number of firms in the import-export sector. Exports thus involve a continuum of exporters, each a monopolist producing a specialized export good. Each monopolist produces an export good using a domestically produced homogeneous good and a homogeneous good derived from imports. Specialized export goods are sold to competitive foreign retailers who assemble a homogeneous good that is sold to foreign consumers.

For imports, specialized domestic importers purchase a homogeneous foreign good, which they convert into a specialized input and sell to domestic retailers. There are three types of domestic retailers. One uses specialized imported goods to create a homogeneous good used as an input in the production of specialized exports. The second uses specialized imported goods to create an input used in the production of investment goods. The third uses specialized imported goods to produce a homogeneous input used in the production of consumption goods. Imported goods are combined with domestic inputs before being passed on to final domestic users.

There are price frictions in both exports and imports. In all cases, prices are assumed to be set in the buyer's currency.

3.3.1. Exports

There is total foreign demand for domestic exports, taking the following form:

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^* \tag{17}$$

where foreign GDP is as defined, the foreign-currency price of the foreign homogeneous good is as defined, and an export price index is defined below. These goods are produced by a representative, competitive foreign retail firm using specialized inputs as follows: $Y_t^* P_t^* P_t^x X_t$

$$X_t = \left[\int_0^1 X_{i,t}^{\frac{1}{\lambda_x}} \mathrm{d}i\right]^{\lambda_x} \tag{18}$$

where the terms are specialized intermediate goods for the production of export goods. The retailer producing the final export good takes both its output price and its input price as given. Optimization yields the following demand for specialized exports: $X_{i,t} i \in (0,1) X_t P_t^x P_{i,t}^x$

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{\frac{-\lambda_x}{\lambda_x - 1}} X_t \tag{19}$$

Combining (18) and (19) gives:

$$P_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{1}{1-\lambda_x}} \mathrm{d}i\right]^{1-\lambda_x}$$

The I-th specialized export good is produced by a monopolist using the following technology:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + \left(1-\omega_x\right)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x - 1}{\eta_x}}\right]^{\frac{\eta_x - 1}{\eta_x}}$$

where the terms denote, respectively, the I-th exporter's use of imported goods and of domestically produced goods. The marginal cost associated with the constant-elasticity-of-substitution production function is obtained from the multiplier associated with the Lagrangian representation of the cost-minimization problem: $X_{i,t}^m X_{i,t}^d$

$$C = \min \tau_t^x \left[P_t^{m,x} R_t^x X_{i,t}^d\right] + \lambda \left\{X_{i,t} - \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + \left(1-\omega_x\right)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x - 1}{\eta_x}}\right]^{\frac{\eta_x}{\eta}}\right\}$$

where the price of the homogeneous imported good and the price of the homogeneous domestic good are as defined. Using the first-order conditions of this problem together with the production function, real marginal cost in terms of the stationary variable is obtained as $P_t^{m,x} P_t mc_t^x$

$$mc_t^x = \frac{\lambda}{s_t P_t^x} = \frac{\tau_t^x R_t^x}{q_t p_t^c p_t^x}\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + \left(1-\omega_x\right)\right]^{\frac{1}{1-\eta_x}} \tag{20}$$

where

$$R_t^x = \nu^x R_t + 1 - \nu^x \tag{21}$$

$$\frac{s_t p_t^x}{p_t} = \frac{s_t p_t^*}{p_t^c}\frac{p_t^c}{p_t}\frac{p_t^x}{p_t^*} = q_t p_t^c p_t^x \tag{22}$$

And the real exchange rate is defined as: q_t

$$q_t = \frac{s_t p_t^*}{p} \tag{23}$$

From the solution to this same problem, demand for domestic inputs for export production is:

$$X_{i,t}^d = \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} X_{i,t}\left(1-\omega\right) \tag{24}$$

The quantity of homogeneous domestic goods used by specialized exports is: $\int_0^1 X_{i,t}^d \mathrm{d}i$,

Substituting (24) into this integral gives:

$$X_t^d = \int_0^1 X_{i,t}^d \mathrm{d}i = \left[\omega_x\left(p_t^{m,x}\right)^{1-\eta} + \left(1-\omega_w\right)\right]^{\frac{\eta_x}{1-\eta_x}}\left(1-\omega_x\right)\left(p_t^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}\left(p_t^x\right)^{-\eta_f} Y_t^* \tag{25}$$

where a price-dispersion measure is as defined, p_t^x.

Using a derivation similar to the one above, we obtain: X_t^d

$$X_t^m = \omega_x\left(\frac{\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + \left(1-\omega_x\right)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}}\right)^{\eta_x}\left(p_t^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}\left(p_t^x\right)^{-\eta_f} Y_t^* \tag{26}$$

The I-th, (0, 1) export-goods firm takes (19) as its demand curve. The producer sets prices subject to the Calvo price-stickiness mechanism. With a given probability, the I-th export good cannot be reoptimized, in which case the price is updated as follows: $i \in \xi_x$

$$P_{i,t}^x = \tilde{\pi}_t^x P_{i,t-1}^x, \tilde{\pi}_t^x = \left(\pi_{t-1}^x\right)^{k_x}\left(\pi^x\right)^{1-k_x-\dot{u}_x}\left(\breve{\pi}\right)^{\dot{u}_x} \tag{27}$$

where the terms are as defined $k_x \dot{u}_x k_x + \dot{u}_x \in \left(0,1\right)$.

The equilibrium conditions associated with price setting by exporters who are able to reoptimize their price are analogous to those derived for intermediate consumption-goods producers.

3.3.2. Imports

A foreign firm sells a homogeneous good to domestic importers. Importers convert the homogeneous good into a specialized input (branding it) and supply this input monopolistically to domestic retailers. Importers are subject to Calvo price-setting frictions. There are three types of importing firms: 1) a firm producing goods used to produce an intermediate good for consumption; 2) a firm producing goods used to produce an intermediate good for investment; and 3) a firm producing goods used to produce an intermediate good for exports.

The first group of firms is considered first. The production function for imported consumption goods of the domestic retailer is:

$$C_t^m = \left[\int_0^1 \left(C_{i,t}^m \right)^{\frac{1}{\lambda_{m,c}}} \mathrm{d}i \right]^{\lambda_{m,c}}$$

where the terms denote, respectively, the output of the I-th specialized producer and an intermediate good used in the production of consumption goods. Let the price index and the price of the I-th intermediate input be as defined. The domestic retailer is competitive and takes both as given. The demand curve for specialized inputs is given by the first-order necessary condition for the domestic retailer's profit maximization: $C_{i,t}^m C_t^m P_t^{m,c} C_t^m P_{i,t}^{m,c} P_t^{m,c} P_{i,t}^{m,c}$

$$C_{i,t}^m = C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}}$$

We now turn to the producer, who takes the preceding equation as a demand curve. This producer purchases the homogeneous foreign good and converts it one-for-one into a differentiated domestic good. The intermediate-goods producer's marginal cost is: $C_{i,t}^m C_{i,t}^m$

$$\tau_t^{m,c} S_t P_t^* R_t^{\nu,*} \tag{28}$$

with:

$$R_t^{\nu,*} = \nu^* R_t^* + 1 - \nu^* \tag{29}$$

where the foreign nominal interest rate is as defined R_t^*.

As in the homogeneous domestic-goods sector, one term is a cost-push shock that affects marginal cost but does not appear in the production function $\tau_t^{m,c}$.

The total value of imports represented by the consumption sector is:

$$S_t P_t^* R_t^{\nu,*} C_t^m \left(p_t^{m,c} \right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}$$

where a measure of the price dispersion of a differentiated good is as defined, and moreover: $C_{i,t}^m$

$$p_t^{m,c} = \frac{P_t^{m,c}}{P_t^{m,c}}$$

The second group of firms is now considered. The production function for imported investment goods of the domestic retailer is: I_t^m

$$I_t^m = \left[\int_0^1 \left(I_{i,t}^m \right)^{\frac{1}{\lambda_{m,i}}} \mathrm{d}i \right]^{\lambda_{m,i}}$$

The imported investment-goods retailer is competitive and takes output and input prices as given, $P_t^{m,i} P_{i,t}^{m,i}$.

The producer of the I-th intermediate input in the production function above purchases the homogeneous foreign good and converts it one-for-one into a differentiated good. Its marginal cost is analogous to (28): $I_{i,t}^m I_{i,t}^m$

$\tau_t^{m,i} S_t P_t^* R_t^{v,*}$,

which implies that the importing firm's cost (before borrowing costs, exchange-rate conversion, and shock markups) is the same cost as for the specialized inputs used in production, $P_t^* C_t^m$.

The total value of imports associated with investment-goods production is analogous to what was obtained for the consumption-goods sector:

$$S_t P_t^* R_t^{v,*} I_t^m \left(p_t^{m,i}\right)^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}}, p_t^{m,i} = \frac{p_{i,t}^{m,i}}{p_t^{m,i}} \tag{30}$$

The third group is now discussed. The production function for imported goods of the domestic retailer used in producing the input for export-goods production is: X_t^m

$$X_t^m = \left[\int_0^1 \left(X_{i,t}^m\right)^{\frac{1}{\lambda_{m;x}}} \mathrm{d}i\right]^{\lambda_{m,x}}.$$

The imported-goods retailer is competitive and takes output and input prices as given. The producer of the specialized input has marginal cost: $P_t^{m,x} P_{i,t}^m X_{i,t}^m$

$\tau_t^{m,x} S_t P_t^* R_t^{v,*}$.

The total value of imports associated with this production is: X_t^m

$$S_t P_t^* R_t^{v,*} X_t^m \left(p_t^{m,x}\right)^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}}, p_t^{m,x} = \frac{p_{i,t}^{m,x}}{p_t^{m,x}} \tag{31}$$

Each of the three types of intermediate-goods firms above is subject to the Calvo price-setting friction. With a given probability, the J-th type of firm may reoptimize its price, and with the complementary probability, prices are set as follows: $1-\xi_{m,j}\xi_{m,j}$

$$P_{i,t}^{m,j} = \tilde{\pi}_t^{m,j} P_{i,t-1}^{m;j}, \tilde{\pi}_t^{m,j} := \left(\pi_{t-1}^{m,j}\right)^{k_{m,j}} \left(\bar{\pi}_t^c\right)^{1-k_{m,j}-\acute{u}_{m,j}} \left(\breve{\pi}\right)^{\acute{u}_{m,j}}, j = c, i, x \tag{32}$$

The equilibrium conditions associated with price setting by importers are analogous to those derived for domestic intermediate-goods producers.

3.4. Households

Household preferences are given by:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \log\left(C_t - bC_{t-1}\right) - \zeta_t^h A_L \frac{\left(h_{j,t}\right)^{1+\sigma_L}}{1+\sigma_L}\right] \tag{33}$$

where the terms denote, respectively, the consumption preference shock, the disutility-of-labor shock, the consumption habit parameter *b*, the J-th household's supply of labor services, and the inverse Frisch elasticity. Households own the economy's physical capital stock. They determine both the rate at which the capital stock is accumulated and the rate at which it is utilized. Households also own the stock of net foreign assets and determine its rate of accumulation $\zeta_t^c \zeta_t^h h_j \sigma_L$.

3.5. Wage Setting

The specialized labor supplied by households is combined by contractors into a homogeneous labor service:

$$H_t = \left[\int_0^1 \left(h_{j,t} \right)^{\frac{1}{\lambda_w}} \mathrm{d}j \right]^{\lambda_w}, \quad 1 \le \lambda_w < \infty$$

Households are subject to the Calvo wage-setting friction (as in Erceg, Henderson, and Levin, 2000). With a given probability, the J-th household is able to reoptimize its wage, and with the complementary probability it updates its wage according to $1 - \xi_w \xi_w$

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \tag{34}$$

$$\tilde{\pi}_{w,t+1} = \left(\pi_t^c \right)^{k_w} \left(\bar{\pi}_{t+1}^c \right)^{1-k_w-\dot{u}_w} \left(\breve{\pi} \right)^{\dot{u}_w} \left(\mu_{z^+} \right)^{\vartheta_w} \tag{35}$$

where the terms are as defined $k_w \dot{u}_w \vartheta_w k_w + \dot{u}_w \in (0,1)$.

Consider the J-th household that has the opportunity to reoptimize wages at time t. We denote this wage rate without a j subscript, since the situation of every household that optimizes its wage is identical. In choosing this wage, the household weighs the reduced-form utility of future histories in which it is unable to reoptimize: $\tilde{W}_t \tilde{W}_t$

$$E_0^j \sum_{t=0}^{\infty} \left(\beta \xi_w \right)^i \left[-\zeta_{t+1}^h A_L \frac{\left(h_{j,t+1} \right)^{1+\sigma_L}}{1+\sigma_L} + v_{t+1} W_{j,t+i} h_{j,t+i} \frac{1-\tau^y}{1+\tau^w} \right] \tag{36}$$

where the labor income tax, the payroll tax, and the period-t multiplier on the household's budget constraint are as defined. The demand for the J-th household's labor services, conditional on its having optimized in period t, is: $\tau_y \tau^W \upsilon_t$

$$h_{j,t+i} = \left(\frac{\tilde{w}_t \tilde{\pi}_{w,t+i}, \cdots, \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \tag{37}$$

where the relevant term is understood to hold when $i = 0$. The equilibrium conditions associated with this problem are those under which household wage setting is reoptimized $\tilde{\pi}_{w,t+i}, \cdots, \tilde{\pi}_{w,t+1} = 1$.

3.6. Capital Accumulation Technology

The law of motion for the physical capital stock incorporates investment adjustment costs as introduced by Christiano, Eichenbaum, and Evans (2005):

$$\bar{K}_{t+1} = (1-\delta) \bar{K}_t + Y_t \left(1 - \bar{S} \left(\frac{I_t}{I_{t-1}} \right) \right) I_t \tag{38}$$

where the marginal efficiency of the investment shock affecting the conversion of investment into capital is as defined Y_t.

3.7. Households' Consumption and Investment Decisions

The first-order condition for consumption is:

$$\frac{\zeta_t^c}{c_t - bc_{t-1}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{Z^*,t!+1} - bc_t} - \psi_{z^+ t P_t^c}\left(1+\tau^c\right) = 0 \tag{39}$$

where

$$\psi_{z^+,t} = v_t P_t z_t^+$$

is the marginal value of wealth in real terms, specifically in units of the homogeneous domestic good at time *t*.

To define the intertemporal Euler equation associated with households' capital accumulation decision, the period-t rate of return on investing in one unit of physical capital is defined as follows: R_{t+1}^k

$$R_{t+1}^k = \frac{\left(1-\tau^k\right)\left[u_{t+1} r_{t+1}^k \frac{P_{t+1}^i}{\psi_{t+1}} a\left(u_{t+1}\right)\right] P_{t+1} + \left(1-\delta\right) P_{t+1} P_{k',t+1} + \tau^k \delta P_t P_{k't}}{P_t P_{k't}} \tag{40}$$

where the terms are as defined, $\frac{P_t^i}{\psi_t} P_t = P_t^i$.

is the price of the investment good at time *t*, the real rental rate of capital, the capital tax rate, and the price of one unit of newly installed physical capital operating in period $t+1$ are as defined. This price is expressed in units of the homogeneous good, so that it is the current domestic price of physical capital. The numerator in the expression represents the period $t+1$ profit from one additional unit of physical capital. The bracketed term captures the idea that maintenance expenditures associated with operating capital are tax-deductible. The last term in the numerator expresses that physical depreciation is deductible from historical cost. It is convenient to express this in scaled terms: $\bar{r}_t^k = \Psi_t r_t^k \tau^k P_{k',t} P_t P_{k',t} R_{t+1}^k R_t^k$

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\psi,t+1}} \frac{\left(1-\tau^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a\left(u_{t+1}\right)\right] + \left(1-\delta\right) p_{k',t+1} + \tau^k \delta \frac{\mu_{\psi,t+1}}{\pi_{t+1}} P_{k',t}}{P_{k',t}} \tag{41}$$

where the terms are as defined. The first-order condition for capital implies that: $p_{k',t} = \Psi_t P_{k',t}$

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1}\mu_{z^+,t+1}} \tag{42}$$

Differentiating the Lagrangian representation of the household's problem and using the scaled first-order condition for investment, we obtain: I_t

$$\psi_{z^+,t} p_i^t + \psi_{z^+,t} p_{k',t} Y_t \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\psi,t} i_t}{i_{t-1}}\right) - \tilde{S}'\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\psi,t} i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\psi,t} i_t}{i_{t-1}}\right]$$
$$+ \beta\psi_{z^+,t+1} p_{k',t+1} Y_{t+1} \tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\psi,t+1} i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \mu_{\Psi,t+1}\mu_{z^+,t+1} = 0 \tag{43}$$

The first-order condition associated with capital utilization is as follows:

$$\bar{r}_t^k = p_t^i a'(u_t) \tag{44}$$

3.8. Financial Assets

Households constitute the economy's domestic saving. Period-t saving takes the form of acquiring net foreign assets and a domestic asset. The domestic asset is used to finance firms' capital needs. This asset pays an undeclared nominal state-contingent return from t to $t+1$. The first-order condition associated with this financial asset is: $A_{t+1}^* R_t$

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}}\left[\frac{R_t - \tau^b\left(R_t - \pi_{t+1}\right)}{\pi_{t+1}}\right] \tag{45}$$

where the tax rate on real interest income from bonds is as defined, τ^b.

The tax treatment of domestic agents' returns on foreign bonds is the same as the tax treatment of their returns on domestic bonds. The period-t first-order condition associated with the asset paying a return in foreign-currency terms is: $A_{t+1}^* R_t^*$

$$\upsilon_t S_t = \beta E_t \upsilon_{t+1}\left[S_{t+1} R_t^* \Phi_t - \tau^b\left(S_{t+1} R_t^* \Phi_t - \frac{s_t}{p_t} P_{t+1}\right)\right] \tag{46}$$

It should be recalled that the foreign-currency price of one unit of foreign currency is as defined. The left-hand side of this expression is the cost of acquiring one unit of foreign assets. This cost is expressed in currency terms and converted into utility terms by multiplying by the household's budget-constraint Lagrange multiplier. The bracketed term is the after-tax return on the foreign asset, expressed in units of domestic currency. The pre-tax interest earnings at period $t+1$ are as defined. Here, the foreign nominal interest rate is risk-free in foreign-currency units. A further term represents a risk adjustment to the return on the foreign asset, so that one unit of the foreign asset acquired at t yields the corresponding units of foreign currency at $t+1$. The determination of this risk-adjustment term is discussed below. The remaining bracketed term reflects the effect of taxation on returns to foreign assets $S_t S_t \upsilon_t A_{t+1}^* S_{t+1} R_t^* \Phi_t R_t^* \Phi_t R_t^* \Phi_t \Phi_t$.

Scaling the first-order condition (46) by multiplying both sides by the relevant scaling factor gives: $P_t Z_t^+ / S_t$

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\pi_{t+1}\mu_{z^+,t+1}}\left[s_{t+1} R^* \Phi_t - \tau^b\left(s_{t+1} R_t^* \Phi_t - \pi_{t+1}\right)\right] \tag{47}$$

where

$$s_t \frac{S_t}{S_{t-1}}$$

The risk-adjustment term takes the following form:

$$\Phi_t = \Phi\left(a_t, R_t^* - R_t, \tilde{\varphi}_t\right) = \exp\left(-\tilde{\varphi}_a\left(a_t - \bar{a}\right) - \tilde{\varphi}_s\left(R_t^* - R_t - \left(R^* - R\right)\right) + \tilde{\varphi}_t\right) \quad (48)$$

where

$$a_t = \frac{S_t A_{t+1}^*}{P_t Z_t^+}$$

$\tilde{\Phi}_t$ is a mean-zero state-risk shock at the first order, while the remaining terms are positive parameters $\tilde{\Phi}_a \tilde{\Phi}_s$.

3.9. Fiscal and Monetary Authorities

Monetary policy is conducted according to a combination of the domestic nominal interest rate and the foreign nominal interest rate.

Government spending is modeled as follows:

$$G_t = g_t z_t^+$$

where an exogenous stochastic process is as defined, and ensures a constant ratio of government spending to GDP. The tax rates in the model are the capital tax rate, the bond tax rate, the labor income tax rate, the consumption tax rate, and the payroll tax rate. Any gap between government spending and tax revenue is not offset by transfers $g_t z_t^+ \tau^k \tau^b \tau^y \tau^c \tau^w$.

3.10. Foreign Variables

The representation of foreign values incorporates the assumption that foreign variables are perturbed just as domestic variables are. In particular: $Y_t^* z_t^+$

$$\log Y_t^* = \log y_t^* + \log z_t^+ = \log y_t^* + \log z_t + \frac{\alpha}{1-\alpha} \log \psi_t$$

where a term is assumed to follow a stationary process. It is assumed that: $\log\left(y_t^*\right)$

$$\begin{pmatrix} \log\left(\frac{y_t^*}{y^*}\right) \\ \pi_t^* - \pi^* \\ R_t^* - R^* \\ \log\left(\frac{\mu_{z,t}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right) \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \frac{a_{24} a}{1-a} \\ a_{31} & a_{32} & a_{33} & a_{34} & \frac{a_{34} a}{1-a} \\ 0 & 0 & 0 & \rho_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \rho_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \log\left(\frac{y_{t-1}^*}{y^*}\right) \\ \pi_{t-1}^* - \pi^* \\ R_{t-1}^* - R^* \\ \log\left(\frac{\mu_{z,t-1}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\psi,t-1}}{\mu_\psi}\right) \end{pmatrix}$$

$$
\begin{bmatrix} \sigma_{y^*} & 0 & 0 & 0 & 0 \\ c_{21} & \sigma_{\pi^*} & 0 & c_{24} & \dfrac{c_{24}a}{1-a} \\ c_{31} & c_{32} & \sigma_{R^*} & c_{34} & \dfrac{a_{34}a}{1-a} \\ 0 & 0 & 0 & \sigma_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \varepsilon_{y^*,t} \\ \varepsilon_{\pi^*,t} \\ \varepsilon_{R^*,t} \\ \varepsilon_{\mu_z,t} \\ \varepsilon_{\mu_\psi,t} \end{pmatrix}
$$

where the shocks are mean-zero, unit-variance, i.i.d. Gaussian processes, mutually uncorrelated $\varepsilon_t \grave{s}$.

In matrix form:

$$
X_t^* = AX_{t-1}^* + C\varepsilon_t
$$

It should be noted that matrix C has 10 elements, so the order conditions for identification are satisfied, since $C'C$ represents 15 independent equations. The restrictions above assume that one shock affects the first three variables, while a second affects only the following two, and a third affects only the remaining one. Thus, the zeros in the last two columns of the first row of A and C imply that technology shocks do not affect the corresponding variable. Third, matrices A and C capture the notion that technology innovations affect foreign inflation and the interest rate through their impact on output. Fourth, the assumptions on A and C imply that two of the processes follow univariate first-order autoregressive processes, driven by their respective innovations

$$
\varepsilon_{y^*,t} X_t^+ \varepsilon_{\pi^*,t} \varepsilon_{R^*,t} y_t^* Z_t^+ \log\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right) \log\left(\frac{\mu_{z,t}}{\mu_z}\right) \varepsilon_{\mu_\psi,t} \varepsilon_{\mu_z,t}
$$

3.11. Resource Constraints

The fact that there is a potential steady-state level of price dispersion in both prices and wages complicates the expression of the homogeneous domestic good in terms of the aggregate production factor: Y_t

$$
y_t = (p_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\varepsilon_t \left(\frac{1}{\mu_{\psi,t}} \frac{1}{\mu_{z^*,t}} k_t \right)^{\alpha} \left(w_t^{\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \varphi \right] \tag{49}
$$

where the degree of price dispersion of the intermediate domestic good is as defined p_t.

3.12. Resource Constraints for Homogeneous Domestic Goods

Above we defined scaled real output in terms of the aggregate production factor. It is also useful to have an expression showing the use of the homogeneous domestic good. Using:

$$
z_t^+ y_t = G_t + C_t^d + I_t^d + \left[\omega_x \left(p_t^{m,x} \right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(p_t^x \right)^{\frac{-\lambda_x}{\lambda_{x-1}}} \left(p_t^x \right)^{-\eta_f} Y_t^*
$$

after scaling and using (10): Z_t^+

$$y_t = g_t + (1-\omega_x)\left(p_t^c\right)^{\eta_c} c_t + \left(p_t^i\right)^{\eta_i}\left(i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i) + \left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}}(1-\omega_x)\left(p_t^x\right)^{\frac{-\lambda_x}{\lambda_{x-1}}}\left(p_t^x\right)^{-\eta_f} y_t^* \tag{50}$$

When GDP equals the data, capital-utilization costs are subtracted from it y_t.

$$gdp_t = y_t - \left(p_t^i\right)^{\eta_i}\left(a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i)$$

3.13. Trade Balance

Import expenditures and net new purchases of foreign assets must equal export revenues plus previously acquired net foreign assets. $S_t A_{t+1}^* + s_t = s_t + R_{t-1}^* \Phi_{t-1} S_t A_t^*$

Import expenditures correspond to purchases by specialized importers for the consumption, investment, and export sectors.

$$s_t = S_t P_t^* R_t^{v,*}\left(c_t^m\left(p_t^{m,c}\right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + I_t^m\left(p_t^{m,i}\right)^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} + X_t^m\left(p_t^{m,x}\right)^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}}\right)$$

The current account can be written in scaled form using (22) as follows:

$$a_t + q_t p_t^c R_t^{v,*}\left(c_t^m\left(p_t^{m,c}\right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} + i_t^m\left(p_t^{m,i}\right)^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} + x_t^m\left(p_t^{m,x}\right)^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}}\right) = q_t p_t^c p_t^x x_t + R_{t-1}^* \Phi_{t-1} s_t \frac{a_{t-1}}{\pi_t \mu_{z^+,t}} \tag{51}$$

where the terms are as defined, $a_t = S_t A_{t+1}^* / \left(P_t Z_t^+\right)$.

This last block thus completes the description of the baseline model.

4. Introducing Financial Frictions into the Model

4.1. Model with Financial Frictions

A number of activities in the baseline model require financing. Producers of specialized inputs must borrow working capital within the period. Capital management involves financing because building capital requires substantial upfront use of resources, whereas the return on capital accrues over time as a flow. In the baseline model, financing needs have little effect on allocations. This is because none of the realities of actual financial markets are present. There is information asymmetry between borrower and lender, and correspondingly, risk for lenders. Thus, in the case of capital accumulation, the borrower and the lender are in fact the same household. When real-world financial frictions are introduced into a model, intermediation is distorted, through the effect of new equilibrium constraints as well as other factors (Agénor, 2025).

The financial frictions we introduce fundamentally reflect the fact that borrowers and lenders are different people, with different information. Specifically, we consider a setting in which economies have a banking system with excess-liquidity banks, while domestic credit rationing exists within it.

4.2. Individual Entrepreneur

At the end of period t, each entrepreneur has a given net-worth level, maintained regardless of the period. There is a continuum of entrepreneurs at each net-worth level in the competitive banking market. Banks offer a loan contract defined by the loan amount and the interest rate for each entrepreneur $N_{t+1} N_{t+1}$.

Consider a type of entrepreneur with a particular level of net worth. The entrepreneur combines this net worth with a bank loan to purchase new capital for production. The loan required by the entrepreneur for this purpose is: $N_{t+1} B_{t+1}$

$$B_{t+1} = P_t P_{k',t} \bar{k}_{t+1} - N_{t+1}$$

The entrepreneur is required to pay a gross interest rate on the bank loan at the end of period $t + 1$, if able to do so. After purchasing capital, the entrepreneur experiences an idiosyncratic productivity shock, which converts the acquired capital using a unit, normally and independently distributed, correlated with the entrepreneurs' random variable. The time index indicates that the shock is itself the realization of a random variable. This allows us to consider the effects of an increase in the entrepreneur's individual risk, referred to as the idiosyncratic uncertainty shock. The cumulative distribution function of the shock is denoted, with its partial derivatives as defined.

$$Z_{t+1} \bar{K}_{t+1} \bar{K}_{t+1} \omega \omega V\left(\log \omega\right) = \sigma_t^2 t \sigma_t \sigma_t \omega F\left(\omega; \sigma\right) F_{\omega}\left(\omega; \sigma\right) F_{\sigma}\left(\omega; \sigma\right)$$

After observing the shock, the entrepreneur sets the capital utilization rate and rents additional capital in competitive markets at the nominal rental rate. In choosing the utilization rate, the entrepreneur takes into account that operating one unit of physical capital at that rate requires domestic investment goods for maintenance expenditures. The entrepreneur then sells the undepreciated portion of physical capital to capital producers. Per unit of physical capital purchased, an entrepreneur drawing idiosyncratic productivity earns an after-tax return, as defined in (40). Since the mean across entrepreneurs is unity, the average return across all entrepreneurs equals that same return

$$t + 1 u_{t+1} P_{t+1} r_{t+1}^k u_{t+1} a\left(u_{t+1}\right) \omega R_{t+1}^k \omega R_{t+1}^k \omega R_{t+1}^k$$

Entrepreneurs then sell their capital to settle their bank loans. At this point, the resources available to an entrepreneur who purchased a given number of physical capital units in the period and experienced a given idiosyncratic productivity shock are as defined. There exists a cutoff value such that the entrepreneur has just enough resources to pay the interest due: $\bar{K}_{t+1} t \omega P_t P_{k',t} R_{t+1}^k \omega \bar{K}_{t+1} \omega \bar{\omega}_{t+1}$

$$\bar{\omega}_{t+1} R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1} = Z_{t+1} B_{t+1} \quad (52)$$

Entrepreneurs falling below this cutoff can no longer cover their obligations, $\omega \prec \bar{\omega}_{t+1}$

$$R_{t+1}^{k}\omega P_t P_{k',t}\bar{K}_{t+1},$$

which exceed those of the bank. In such a case, the bank monitors and curtails the flow of loans to entrepreneurs: $Z_{t+1}B_{t+1}$

$$\mu R_{t+1}^{k}\omega P_t P_{k',t}\bar{K}_{t+1}$$

where a parameter is as defined $\mu \geq 0$.

Banks obtain the funds lent to entrepreneurs in period t by issuing deposits to households at the gross nominal interest rate. The timing indicates that the payoff to households at period $t+1$ does not depend on period-$t+1$ uncertainty. There is no risk in household bank deposits, and the household Euler equation associated with deposits is exactly the same as in (45) $R_t R_t$.

There is competition and free entry among banks, and banks engage in no financial arrangements other than the liabilities issued to households and the loans extended to entrepreneurs. It follows that the bank's cash flow is zero state by state in period $t+1$ for each loan. On the amount lent, the bank receives gross interest from the fraction of entrepreneurs who do not default. The bank seizes all resources held by defaulting entrepreneurs, net of monitoring costs. Thus, the state-by-state zero-profit condition is $B_{t+1}Z_{t+1}B_{t+1}1-F\left(\bar{\omega}_{t+1};\sigma_t\right)$

$$\left[1-F\left(\bar{\omega}_{t+1};\sigma_t\right)\right]Z_{t+1}B_{t+1}+\left(1-\mu\right)\int_0^{\bar{\omega}_{t+1}}\omega\,\mathrm{d}F\left(\omega;\sigma_t\right)R_{t+1}^{k}P_tP_{k';t}\bar{K}_{t+1}=R_tB_{t+1}$$

Using (52) again, this yields:

$$\left[\Gamma\left(\bar{\omega}_{t+1};\sigma_t\right)-\mu G\left(\bar{\omega}_{t+1};\sigma_t\right)\right]^{\frac{R_{t+1}^{k}}{R_t}}\rho_t=\rho_t-1 \tag{53}$$

where

$$G\left(\bar{\omega}_{t+1};\sigma_t\right)=\int_0^{\bar{\omega}_{t+1}}\omega\,\mathrm{d}F\left(\omega;\sigma_t\right)$$

$$\Gamma\left(\bar{\omega}_{t+1};\sigma_t\right)=\bar{\omega}_{t+1}\left[1-F\left(\bar{\omega}_{t+1};\sigma_t\right)\right]+G\left(\bar{\omega}_{t+1};\sigma_t\right)$$

$$\rho_t=\frac{P_tP_{k',t}\bar{K}_{t+1}}{N_{t+1}}$$

The relevant expression is the return accruing to each entrepreneur who borrows from banks. We note that certain terms are as defined

$$\Gamma\left(\bar{\omega}_{t+1};\sigma_t\right)-\mu G\left(\bar{\omega}_{t+1};\sigma_t\right)B_{t+1}\Gamma\left(\bar{\omega}_{t+1};\sigma_t\right)=1-F\left(\bar{\omega}_{t+1};\sigma_t\right)\succ 0G_{\bar{\omega}}\left(\bar{\omega}_{t+1};\sigma_t\right)=\bar{\omega}_{t+1}F_{\bar{\omega}}\left(\bar{\omega}_{t+1};\sigma_t\right)\succ 0$$

It follows that the share of entrepreneurs' returns coming from banks is nonmonotonic in the condition described. These terms are the same for all entrepreneurs regardless of their net worth. The resulting leverage ratio implies that: $\bar{\omega}_{t+1}\rho_t\bar{\omega}_{t+1}\rho_t$

$$\frac{B_{t+1}}{N_{t+1}}=\rho_t-1$$

That is, the loan amount is proportional to net worth. Rewriting (52) and (53), the interest rate paid by the entrepreneur is:

$$Z_{t+1} = \frac{\overline{\omega}_{t+1} R_{t+1}^{k}}{1 \frac{N_{t+1}}{P_t P_{k',t} K_{t+1}}} = \frac{\overline{\omega}_{t+1} R_{t+1}^{k}}{1 \frac{1}{\rho_t}} \tag{54}$$

This too is the same for all entrepreneurs regardless of their net worth.

4.3. Aggregation of Entrepreneurs and External Financing

The law of motion for an individual entrepreneur's net worth is:

$$V_t = R_t^k P_{t-1} P_{k',t-1} K_t - \Gamma\left(\overline{\omega}_t ; \sigma_{t-1}\right) R_t^k P_{t-1} P_{k',t-1} K_t$$

Each entrepreneur faces an identical, independent probability of being selected to exit the economy. With the complementary probability, each entrepreneur remains in the economy. Since selection is random, the net worth of surviving entrepreneurs is as defined. A fraction of new entrepreneurs enters the economy. Both surviving and newly entering entrepreneurs receive a transfer. This ensures that all entrepreneurs, whether newcomers or survivors of default, hold sufficient funds to obtain loans up to a certain amount. The average net worth of all entrepreneurs after transfers have been made is,

$$1-\gamma_t \gamma_t \gamma_t \overline{V}_t 1-\gamma_t W_t^e W_t^e \overline{N}_{t+1} = \gamma_t \overline{V}_t + W_t^e$$

$$\overline{N}_{t+1} = \gamma_t \left\{ R_t^k P_{t-1} P_{k',t-1} \overline{K}_t - \left[R_{t-1} + \frac{\mu \int_0^{\overline{\omega}_t} \omega \mathrm{d}F\left(\omega ; \sigma_{t-1}\right) R_t^k P_{t-1} P_{k',t-1} \overline{K}_t}{P_{t-1} P_{k',t-1} \overline{K}_t - \overline{N}_t} \right] X\left(P_{t-1} P_{k',t-1} \overline{K}_t - \overline{N}_t\right) + W_t^e \right\} \tag{55}$$

where an overbar on a letter denotes its aggregate mean value. Because of its direct effect on aggregate net worth, this term is referred to as the net worth shock γ_t.

We now turn to the external finance premium facing entrepreneurs. The cost of internal funds to the entrepreneur (i.e., their own net worth) is the interest rate they forgo by investing it in capital rather than purchasing a risk-free domestic asset. The average payment to the bank across all entrepreneurs is the full expression stated in (55). The term involving this expression thus represents the excess of external funds over the internal cost of funds. This measure is therefore a financing premium in the model. Another such measure is the excess of the risk-free rate paid by non-defaulting entrepreneurs. In this article, it is referred to as the extended interest rate $R_t \mu Z_{t+1} - R_t$.

5. Quantitative Evaluation of the Model

The model we simulate comprises exactly 74 endogenous variables and considers 13 sources of shocks. It is, we recall, a dynamic stochastic general equilibrium model in an open economy, incorporating the banking sector and financial frictions. This model has been adapted and applied to the WAEMU context.

Table 3 summarizes the set of shocks affecting our model.

Of the thirteen shocks listed in Table 3, twelve are aggregate shocks entering the state-space system solved in V.3 and the Kalman-filter likelihood underlying the Bayesian estimation of V.4; shock 9 (idiosyncratic entrepreneur-level productivity, ω) is agent-specific, is not part of the aggregate state vector, and enters neither the variance decomposition nor the likelihood. For the twelve aggregate shocks, all follow first-order autoregressive processes of the form $x_t = \rho x_{t-1} + \varepsilon_t$. Two persistence values are used: $\rho = 0.60$ for the eight domestic shocks (1, 2, 3+4, 5, 6, 7, 8, 10) and $\rho = 0.80$ for the three foreign shocks (11, 12, 13). Innovation standard deviations are likewise set at two values: $\sigma = 3.0$ for domestic shocks and $\sigma = 1.0$ for foreign shocks, both expressed in the same log-deviation units as the model's endogenous variables. These persistence and innovation values are standard DSGE-literature conventions (in the range used by, e.g., Smets & Wouters, 2007, and Christiano, Trabandt, & Walentin, 2011); none is estimated from WAEMU data, and no WAEMU-specific shock-process estimates are available. Because the relative variance contribution of any shock in a linear state-space model scales with the ratio of its innovation variance to others', the finding that foreign shocks dominate aggregate variance (V.3) is conditional on the relative calibration $\sigma_{\text{foreign}} / \sigma_{\text{domestic}} = 1/3$ chosen here; a different relative scaling, absent a WAEMU-specific estimate, would mechanically alter the reported variance shares.

Table 3. Sources of shocks in the analytical model.

			Households
1	ζ_t^c	:	Consumption preference shock
2	ζ_t^h	:	Labor disutility shock
			Firms
3	$\log Z_t$	:	Technology shock whose first difference has a positive mean
4	$\log \varepsilon_t$	:	Neutral technology shock
5	ψ_t	:	Investment-specific technology shock
6	τ_t^d	:	Fiscal shock affecting the marginal cost of homogeneous-good producers
7	$\tau_t^{m,c}$	:	Fiscal shock affecting the marginal costs of importing firms
			Banking Sector
8	σ_t	:	Idiosyncratic uncertainty shock
9	ω	:	Idiosyncratic productivity shock
			Foreign Sector
10	γ_t	:	Net worth shock
11	$\varepsilon_{y^*,t}$	:	Foreign technology process shock
12	$\varepsilon_{\pi^*,t}$	:	Foreign inflation rate process shock
13	$\varepsilon_{R^*,t}$	:	Foreign interest rate process shock

Source: Author.

5.1. Empirical Results and Diagnosis of Financial Frictions

Table 2 of structural breaks distinguishes, beyond the common shock of the January 1994 devaluation, whose effect dominates inflation and credit across all seven countries of the Union excluding Guinea-Bissau, more recent, country-specific idiosyncratic shocks: Burkina Faso's inflationary episode in 2022-2023, concomitant with a political and security breakdown, and the collapse of Malian credit in 2024-2025, linked to the sanctions following the 2020-2021 coups d'état. These national shocks compound the credit-market slowdown and decoupling, as well as the shift in the volatility ranking toward investment already documented in the introduction, confirming that WAEMU financial frictions have interacted, since 2020, with a resurgence of political and security shocks.

The following sections quantify this diagnosis in three complementary steps: calibration of the financial-frictions block (V.2), simulation of a reduced 26-variable representative model, a simplified approximation of the full 74-variable baseline model, incorporating the full set of aggregate shocks (V.3), and Bayesian estimation of two of its ten free structural parameters (V.4), the only direct econometric confrontation of the model with the data. This partial estimation, conducted on the reduced representative model, does not constitute a full estimation of the exact 74-variable model.

5.2. Calibration and Simulation of the Financial-Frictions Block

The steady state of the loan contract under costly state verification (Equations (52)-(55)) is solved from Equation (53) and the banks' zero-profit condition, under a log-normal idiosyncratic shock of unit mean (**Table 4**), ω .

Table 4. Steady-state calibration of the financial-frictions block.

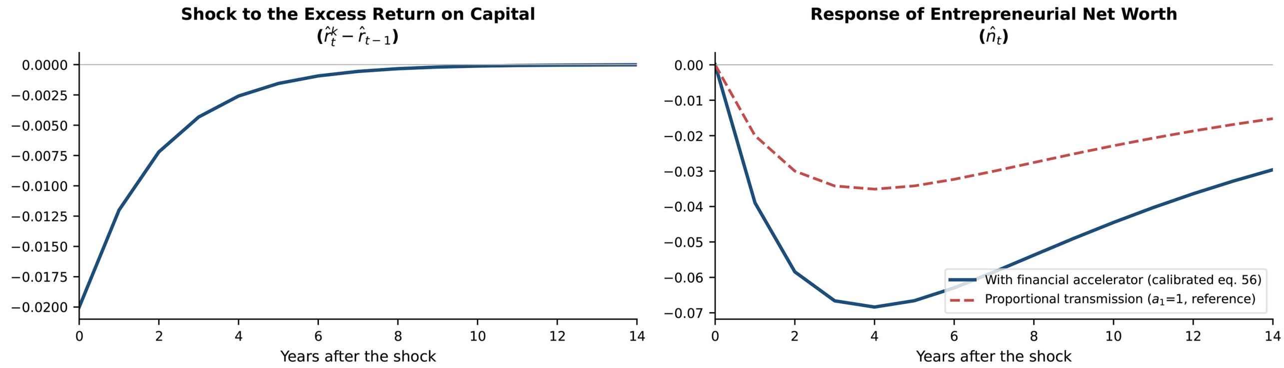
Parameter	Value	Source
Annual risk-free rate R	6.10%	WAEMU data (BCEAO policy rate, 1980-2025 avg.)
Leverage $\kappa = K \cdot P_k / N$	2.00	Literature (Bernanke et al., 1999)
Monitoring cost μ	0.12	Literature (Bernanke et al., 1999)
Target default probability	3%	Literature (SME proxy, developing countries)
Entrepreneur survival rate γ	0.90	Literature (BGG, 1999, converted to annual freq.)
Target spread $R^k/R - 1$	200 pb	Literature (Bernanke et al., 1999)
Default threshold (solved) $\bar{\omega}$	0.494	Computed from targets (3% and 200 bps)
Standard deviation of idiosyncratic shock (solved) σ	0.344	Computed from targets (3% and 200 bps)

Only the risk-free rate (R = 6.10%, BCEAO average 1980-2025) is drawn from WAEMU data; the other parameters (leverage = 2.00; monitoring cost = 0.12; entrepreneur survival rate = 0.90; spread = 200 bps) are taken from Bernanke, Gertler, and Gilchrist (1999), for lack of an available WAEMU equivalent; the default threshold (=0.494) and the standard deviation of the shock (=0.344) are derived from the joint numerical solution of the chosen targets (3% default probability, 200 bps spread). $\kappa\mu\gamma\bar{\omega}\sigma$ Source: Author, using MATLAB.

The law of motion for net worth, Equation (54), admits the log-linearized form in which the coefficients are fully determined by this calibration: numerically, the first coefficient equals 1.95 and the second equals 0.90.

$\hat{n}_{t+1} = a_1\left(\hat{r}_t^k - \hat{r}_{t-1}\right) + a_3\hat{n}_t a_1 = \gamma\kappa R^k$ and $a_3 = \gamma a_1 a_3$.

The coefficient exceeding unity establishes that net worth amplifies the shock relative to proportional transmission, a gap that defines the magnitude of the financial accelerator. The trough is reached only in year 4 (Figure 5) $a_1 a_1 = 1$.



Source: Author, calibrated simulation of the block of Equations (52)-(55).

Figure 5. Response of entrepreneurial net worth to a capital-return shock.

Three limitations bound the scope of this simulation (Figure 5). For lack of bank-level microdata on corporate defaults in the Union, only the financial-frictions block was calibrated; the consumption, investment, and foreign-trade blocks were not subjected to the same exercise. The elasticity of the external finance premium to leverage cannot be derived from Equations (52)-(55) alone, for lack of the loan contract's optimality condition. Finally, the absence of the investment/capital-price block prevents tracing the transmission of net worth to investment and GDP; the calibration relies, for most of its parameters (leverage, monitoring cost, survival rate, target spread), on U.S. values (Bernanke, Gertler, & Gilchrist, 1999), with only the leverage elasticity estimated directly on WAEMU data in V.5.

5.3. Impulse Response Functions and Variance Decomposition

The calibrated financial block was embedded in a complete representative model, New Keynesian, with capital, small open economy, SW-GK-CTW architecture, covering the thirteen shocks of Table 3, rather than a literal reconstruction of the original seventy-four variables. The system (26 variables: 15 states, 11 jumps) is solved using Klein's (2000) method, validated on the canonical Galí (2008) model, where it exactly reproduces the known analytical solution. Shock No. 9 (idiosyncratic productivity), specific to each entrepreneur and not aggregated, vanishes by construction upon aggregation and is excluded from the decomposition.[4]

The monetary policy rule used in this annual representative model is a smoothed Taylor rule reacting to domestic inflation, the domestic output gap, and the foreign policy rate: $\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t) + \phi_{R^*} \hat{R}_t^*$, with $\rho_R = 0.70$, $\phi_\pi = 1.5$, $\phi_y = 0.5$ (standard literature values; not separately estimated), and $\phi_{R^*} = 0.60$. The foreign-rate coefficient ϕ_{R^*} is the sole channel through which euro area monetary conditions enter WAEMU policy in this reduced representa-

[4]SW = Smets and Wouters (2003/2007); GK = Gertler and Karadi (2011); CTW = Christiano, Trabandt, and Walentin (2011).

tive model: it is calibrated, not estimated from WAEMU data, and is intended to capture the de facto transmission of euro area conditions under the CFA franc's fixed peg rather than a discretionary policy response. This calibration choice directly conditions the finding, reported below, that foreign interest-rate shocks dominate model-implied variance; a higher or lower ϕ_{R^*} would mechanically raise or lower that contribution, and no WAEMU-specific estimate of this parameter is available.

This 26-variable model is a reduced approximation of the full 74-variable baseline model of Section III, not a literal solution of it, and results drawn from it should be read accordingly. Retained without simplification: the production and Phillips-curve structure of the homogeneous domestic good (III.1), the resource constraint (III.11 - III.12), and the financial-frictions block calibrated in V.2 (IV.1 - IV.3). Retained in simplified form: household consumption and labor supply (III.4, III.7) enter without habit formation or wage stickiness (both are added back in the 29-variable extension of V.5); investment (III.6 - III.7) follows the reduced-form Christiano-Eichenbaum-Evans adjustment-cost specification rather than the full capital-utilization and maintenance-cost structure of the baseline model; the foreign block (III.10) is represented by three independent AR(1) processes rather than the full VAR(1) system with 15 identifying restrictions. Omitted entirely: the three-way import/export price-setting structure (III.3), fiscal and tax detail beyond two generic cost-push wedges (III.9), and the wage-indexation and price-dispersion terms of III.5 and III.11. Conclusions regarding the relative importance of financial versus foreign shocks (V.3) and the parameter estimates of V.4 are therefore conditional on this reduced structure and should not be read as full-model results.

An initial specification of the investment block produced a structural unit root, invariant to the elasticity parameter. Substituting the Christiano, Eichenbaum, and Evans (2005) formulation, in which investment depends on its lagged and expected values, restored a determinate, stationary solution.

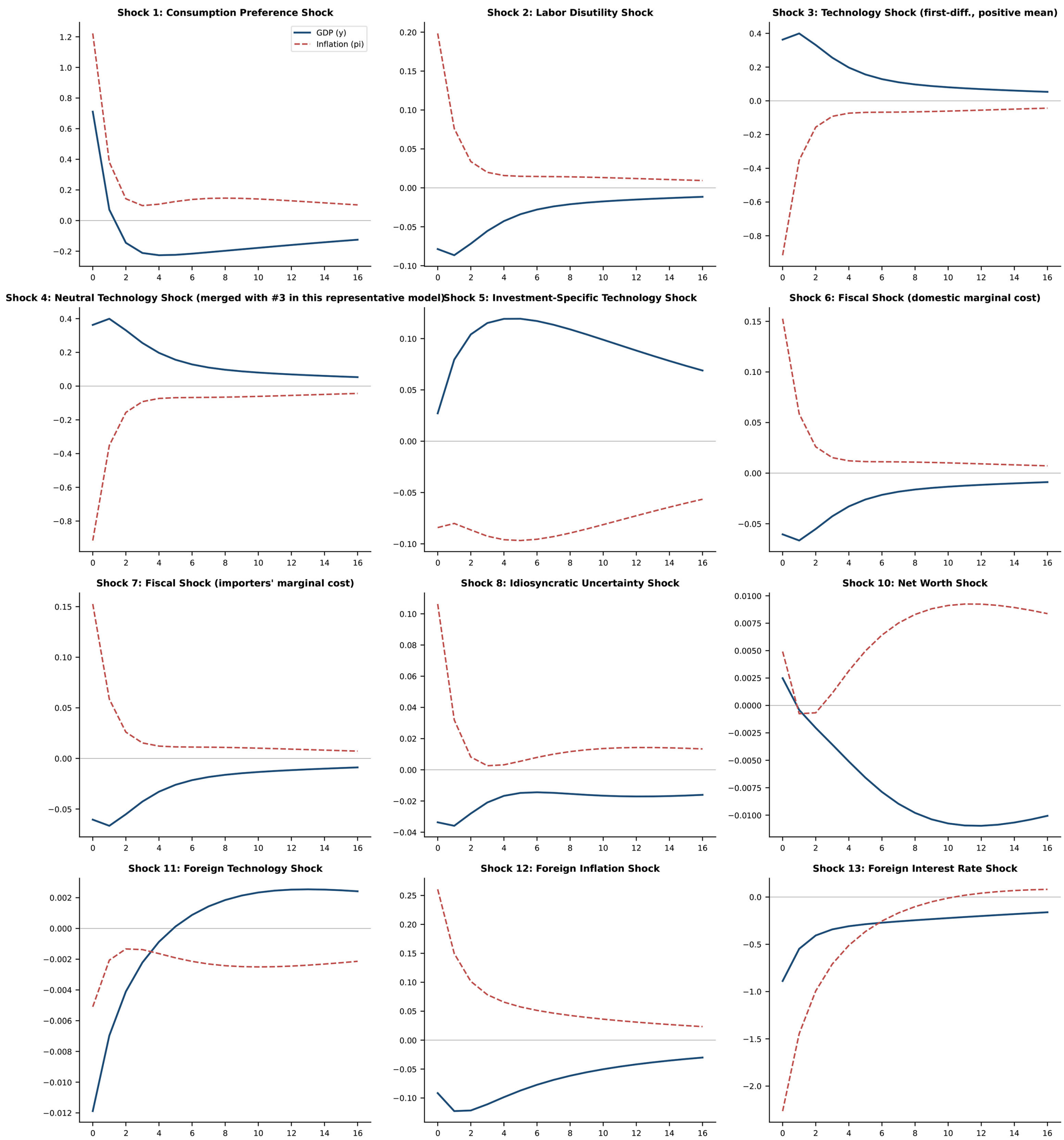
Table 5. Variance decomposition (%), key variables.

Shock	GDP	Inflation	Investment	Policy Rate	Net Exports
1. Consumption preference	18.6	13.0	30.1	33.6	10.1
2. Labor disutility	0.5	0.4	0.3	0.5	0.1
3 + 4. Technology (merged)	23.6	16.9	12.8	23.0	3.2
5. Investment-specific	4.8	1.3	9.5	3.5	0.8
6 + 7. Fiscal (domestic + import)	0.7	0.4	0.4	0.6	0.0
8. Idiosyncratic uncertainty	0.2	0.1	0.5	0.3	0.0
10. Net worth	0.0	0.0	0.1	0.0	0.0
11. Foreign technology	0.0	0.0	0.0	0.0	18.9
12. Foreign inflation	1.1	0.7	0.4	1.2	0.1
13. Foreign interest rate	50.5	67.1	45.9	37.3	66.6

Source: Author, using MATLAB.

Table 5 decomposes, via stochastic simulation over 200 periods, the variance of each variable across the twelve aggregated shocks.

The foreign interest rate shock dominates unambiguously (50% to 67% depending on the variable), a consequence of the CFA franc's peg to the euro: the WAEMU mechanically imports a large share of euro area monetary conditions. Technology shocks and consumption-preference shocks contribute next, accounting for 15% to 30%.



Source: Author, simulation of the calibrated representative model (26 variables).

Figure 6. Impulse response functions of GDP and inflation to the twelve aggregated shocks.

Financial shocks (idiosyncratic uncertainty, net worth) account for only 0.0 to 0.5% of aggregate variance, to be distinguished from the 95% amplification obtained in V.2: the first figure measures the contribution of a shock of a size calibrated from the literature, the second a conditional elasticity to its occurrence (**Figure 6**).

Two counter-intuitive results are reported without qualification. A net-worth shock reduces investment rather than raising it: in this model, the variable rk conflates the technological return on capital with the external finance premium of the BGG block. The net effect of a foreign interest rate shock on the policy rate also reverses sign in general equilibrium, through feedback loops operating via net exports.

5.4. Partial Bayesian Estimation of Critical Parameters

A proper econometric exercise focused on the two most decisive parameters: the slope of the Phillips curve and the elasticity of the external finance premium to leverage. The representative model of V.3 is rewritten in state-space form; its likelihood, computed via the Kalman filter over four observed series (GDP growth, inflation, investment growth, and the policy rate; seven countries, 1981-2025, 45 annual observations), is combined with standard prior distributions to form a posterior distribution sampled via Metropolis-Hastings, $\kappa_p \chi$.

Only these two parameters achieve convergence (Gelman-Rubin R of 1.038 for the Phillips-curve slope and 1.080 for the leverage elasticity, over two chains of 12,000 draws after 3000 burn-in draws) out of the model's ten free parameters, a result that reflects the weak identification afforded by 45 annual observations, $\kappa_p \chi$.

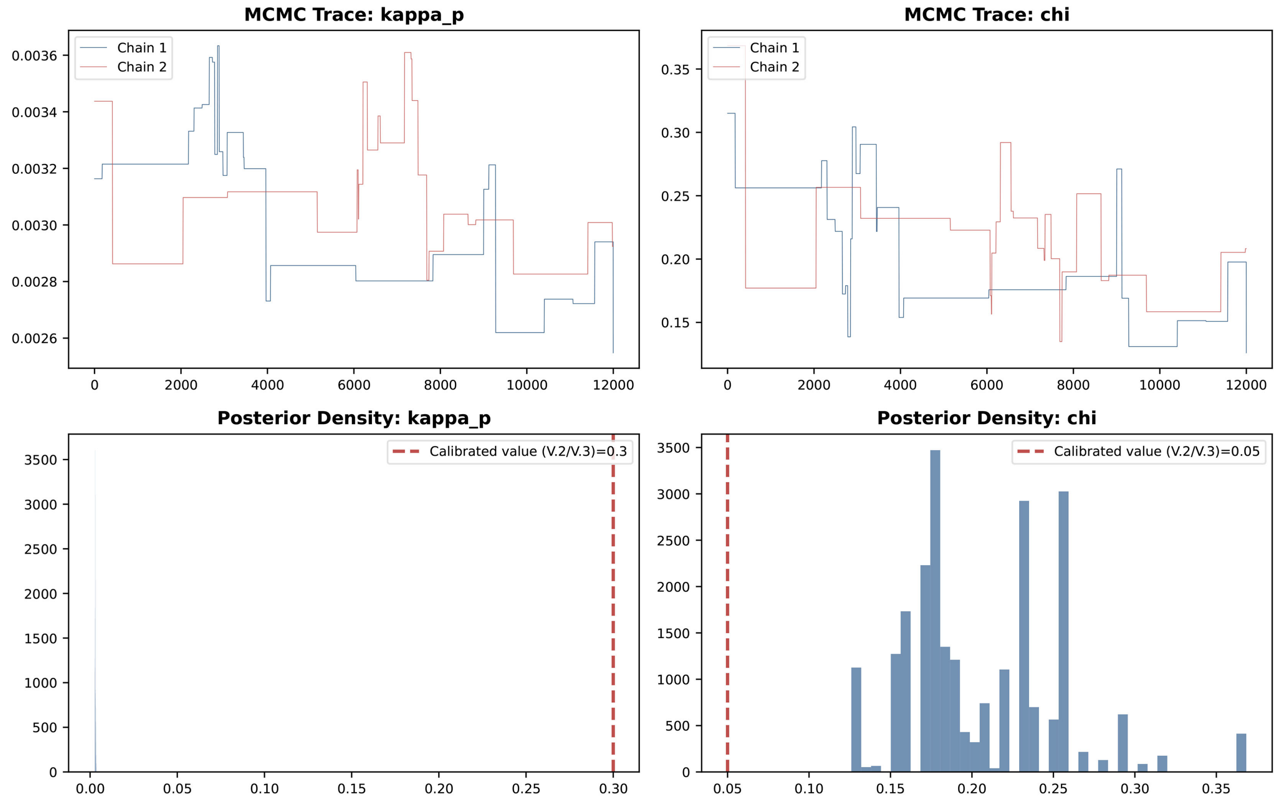
Table 6. Partial bayesian estimation-calibrated and estimated values.

Parameter	Calibrated Value (V.2 - V.3)	Posterior Estimate
κ_p (Phillips-curve slope)	0.30 (literature)	0.003 [IC 90%: 0.0024 - 0.0033]
χ (premium/leverage elasticity)	0.05 (BGG, 1999)	0.18 [IC 90%: 0.11 - 0.28]

Source: Author, using MATLAB.

The estimated values depart markedly from the calibration (**Table 6**). The Phillips-curve slope converges close to zero, an almost flat Phillips curve, consistent with the shift in the volatility ranking (V.1.) and with the role of imported prices under the fixed exchange rate. The leverage elasticity converges to a value three to four times higher than that of Bernanke, Gertler, and Gilchrist (1999), suggesting a more powerful financial accelerator in the WAEMU, consistent with a shallower banking system and incomplete credit information (an index of 1 out of 6), but thereby contradicting the weak contribution of financial shocks established in V.3. $\kappa_p \chi$ (**Figure 7**).

The conflation of rk documented in V.3 would affect any re-estimation carried out after its reformulation. These results represent an initial econometric exploration, not a definitive estimation of the structural parameters of the WAEMU economy over the 1980-2025 period.



Source: Author, using MATLAB.

Figure 7. Convergence diagnostics and posterior distributions $\kappa_p \chi$.

5.5. Confrontation with the Reference Structural Calibration

The equations of the baseline model correspond to those of Christiano, Trabandt, and Walentin (2011), cited in Section II as the foundation of the SWGK model adopted. This correspondence makes it possible to confront the representative model with a sourced structural calibration.

The representative model of V.3. was extended to twenty-nine variables through the addition of two blocks transcribed from Section III: consumption habit formation in the household Euler equation (Equation (33) and Equation (39)) and a structural Calvo wage Phillips curve (Equations (34)-(37)). This extended model satisfies the Blanchard-Kahn conditions.

The confrontation with the calibration of Christiano, Trabandt, and Walentin (2011) reveals an analytical tension. These authors calibrate Calvo price stickiness at a quarterly probability of 0.75, implying an average price-setting duration of exactly one year, which, expressed at the annual frequency of the present model, degenerates toward near-zero rigidity and would structurally imply a very steep Phillips curve. This result stands in sharp contrast to the empirical Bayesian estimate of Section V.4 ($\kappa_p \approx 0.003$): inflation dynamics in the WAEMU appear to be dominated, given the fixed exchange-rate regime already discussed, by the pass-through of imported prices under the euro peg rather than by domestic nominal rigidity alone.

The extended model retains the empirical estimate from Section V.4. for the price Phillips-curve slope, and a standard annual convention from the literature for wage rigidity. The other parameters drawn from this calibration capital share (0.375), consumption habit (0.65), steady-state price markup (1.2), entrepreneur survival rate (0.97), target external finance premium (1.6%) do not raise this conversion difficulty.

6. Policy Implications

Our results call for several complementary policy orientations on the part of the BCEAO and national authorities.

Reducing information asymmetry in the credit market is the priority. The depth of credit information in the Union remains low; strengthening it, through credit information bureaus and the Credit Registry, would directly mitigate the adverse selection and moral hazard underlying the credit tightening observed since 2020.

The procyclical behavior of credit, amplified by a factor of nearly two in its transmission to firms' net worth, justifies adopting countercyclical macroprudential instruments, capital buffers, dynamic provisioning, rather than a policy stance geared solely to the inflation objective. Investment, not inflation, has become the leading source of macroeconomic volatility in the Union since the 1990s.

The dependence of the Union's economies on the short-term bank credit channel (in a banking system that grew from 124 to 161 institutions between 2013 and 2025) exposes them to the shocks of this single channel and to the amplification it propagates. Diversifying funding sources, through the development of credit guarantee mechanisms for small and medium-sized enterprises and the deepening of the regional financial market, would provide channels less sensitive to these frictions.

The national anomalies observed in the updated data (the collapse of Malian credit in 2024-2025, the unusual concentration of non-performing loans in Niger in 2021-2022) which Union-wide aggregates tend to obscure, would be all the more costly were they to spread across the regional banking system, given the scale of the financial accelerator at work in the Union. Country-differentiated banking supervision is warranted.

The resurgence of political and security breakdowns since 2020 constitutes a source of shocks whose scope extends beyond the purview of monetary policy alone; it calls for closer coordination among the BCEAO, national fiscal authorities, and regional security bodies.

The foreign interest rate shock dominates the Union's macroeconomic variance by a wide margin, up to two-thirds depending on the variable, confirming that BCEAO monetary policy operates under a strong external constraint inherent in the CFA franc's peg to the euro. National and regional macroprudential instruments constitute, at the margin of this structural constraint, the principal degree of freedom available in the face of imported shocks.

7. Conclusion

This article examines how financial frictions in the credit market of the West African Economic and Monetary Union propagate macroeconomic fluctuations, combining an empirical record (1980-Together), these results point to a financial channel that is structurally real but empirically secondary to the exchange-rate peg, and to an inflation process better explained by imported-price transmission than by domestic nominal rigidity. Both conclusions are drawn from a reduced 26-variable representative model, a simplified approximation retaining the financial-frictions block in full but simplifying or omitting several blocks of the 74-variable baseline model (see V.3), rather than the full system it approximates, and on an annual, nominal dataset of limited depth. Extending the analysis to the complete model, and, more consequentially, to quarterly data, is the natural next step for this work.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Data Appendix

This appendix responds to the referee's request for source, transformation, price basis, country coverage, missing-data treatment, and aggregation weights for each series used in the paper. All series are annual, 1980-2025 unless otherwise noted, and are aggregated as simple sums (for flow variables in levels) or GDP-weighted averages (for rates and ratios) across the seven WAEMU member states for which consistent series are available: Benin, Burkina Faso, Côte d'Ivoire, Mali, Niger, Senegal, and Togo. Guinea-Bissau is excluded throughout for lack of sufficient data; this exclusion applies uniformly to every series below, and all statements in the body of the paper referring to "all WAEMU countries" or "the Union" should be read as referring to these seven countries unless explicitly noted otherwise.

Series	Source	Transformation	Price basis	Country coverage	Missing data	Aggregation weight
Nominal GDP	BCEAO national accounts	First difference (level), first difference of first difference (volatility, Clark-Papell-Stoica 2011)	Current prices, local currency	7 countries (excl. Guinea-Bissau)	None (complete series 1980-2025)	Simple sum across countries
Inflation	BCEAO/national statistics offices	Annual % change, CPI	N/A (rate)	7 countries	None	GDP-weighted average
Private consumption, investment, government spending	BCEAO national accounts	Levels and growth rates, same transformation as GDP	Current prices, local currency	7 countries	None	Simple sum across countries
Bank credit to the economy	BCEAO monetary statistics	Annual % growth, three sub-period averages (1993-2003, 2004-2014, 2015-2025)	Current prices, local currency	7 countries	None for aggregate; Burkina Faso commercial-bank detail unavailable for parts of the series (see Note 3)	Simple sum across countries
Credit reported to the Credit Registry (Centrale des Risques)	BCEAO Centrale des Risques	Annual % growth	Current prices, local currency	7 countries; growth over 2020-2025 driven disproportionately by Côte d'Ivoire, Senegal, and Burkina Faso	None	Simple sum
Policy rate	BCEAO	Level (%), annual average	N/A (rate)	Union-wide (single policy rate)	None	Not applicable (single series)
Money supply	BCEAO monetary statistics	Level and growth rate	Current prices, local currency	7 countries	None	Simple sum
Net exports/GDP	BCEAO balance of payments, national accounts	Ratio level, annual change in ratio (percentage points) for volatility	Current prices	7 countries	None	GDP-weighted average of the ratio

Continued

Number of credit institutions	BCEAO Annuaire des Banques et Établissements Financiers	Level (count)	N/A	Union-wide	None	Not applicable
Foreign (euro area) GDP growth, inflation, interest rate	ECB/Eurostat	Annual growth rate, annual rate	Current prices for GDP; N/A for rates	Euro area aggregate	None	Not applicable (single foreign bloc)

Series not listed above (model-implied variables, calibrated and estimated parameters) are constructed within the model itself and are documented in Tables 3-6 and the accompanying text of Sections V.2 - V.4, not drawn from external sources.