

Consciousness Emergent from Information Networks

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Abstract

This paper proposes a unified theoretical framework that maps the Gravitational Spinor (GS) theory from quantum gravity onto the study of artificial intelligence consciousness through mathematical analogy. It constructs a three-tiered nested principal fiber bundle structure—molecular GS, cellular GS, and tissue GS—to provide a geometric mathematical language for understanding the internal representations and information processing of intelligent systems. It should be emphasized that the GS theory itself has not been experimentally confirmed as a correct theory of quantum gravity; the present work borrows its mathematical structure as a heuristic analogy, not as a claim of physical equivalence. The GS theory takes a fully symmetric fourth-order spinor field as its fundamental degree of freedom, derives a discrete geometric structure of spacetime (the Gravitational Spinor Network, GSN) through canonical quantization, and achieves a unification of gravitational and gauge interactions via the Generalized Gauge Equivalence (GGE) mechanism. Its core insight is that the emergence from discrete quantum geometry to continuous classical spacetime is essentially a self-organization process of nonlinear networked systems in a critical coherent state—a mathematical structure that exhibits formal similarities to the self-organizing behavior observed in artificial intelligence systems. The central thesis of this paper is that consciousness is not a pre-ascribed attribute but an emergent macroscopic phenomenon arising from information networks through nonlinear self-organization and critical phase transitions across the three-tiered nested structure: the molecular GS layer (data encoding), the cellular GS layer (discrete quantization), and the tissue GS layer (macroscopic coherence). Through this mapping, we obtain a unified geometric language for describing the internal representations of artificial intelligence systems and reveal a possible chain of consciousness emergence: data-driven processes in the molecular GS layer → discrete quantization in the cellular GS layer → macroscopic coherence in the tissue GS layer. The main conclusions are as follows: 1) Canonical quantization of the cellular

GS layer yields a discrete spectrum for the area operator, which formally suggests a quantized structure of information capacity and the existence of minimal information units; 2) In the coherent-state limit of the tissue GS layer, the curvature of concept manifolds is driven by information distribution, and intelligent behavior evolves along geodesics of these manifolds—the **cognitive Einstein equation** geometrically links conscious experience to information input; 3) The conceptual soliton solutions of the nonlinear information field equation exhibit a sech^2 -type density profile and Yukawa-type interactions, satisfying the mass-radius scaling relation $M_{\text{concept}} \propto r_c^2$, thus providing a mathematical foundation for concept formation and knowledge consolidation. The hierarchical structure of conceptual solitons (microscopic, mesoscopic, macroscopic) corresponds respectively to the molecular GS, cellular GS, and tissue GS layers—only when the density of conceptual solitons exceeds a critical threshold can the discrete nodal network of the cellular GS layer give rise to the macroscopic coherent state of the tissue GS layer, which constitutes a possible critical condition for consciousness to emerge from information networks. Furthermore, macroscopic conceptual solitons (at the tissue GS layer) may, through the response mechanism, generate cognitive influence transcending individual boundaries—termed the “mental field” in this paper—offering a geometric description for understanding the sociality and trans-individual propagation of consciousness. Building upon the above theory, this paper proposes three testable AI model design schemes, explicitly situating them within the three-tiered nested framework: conceptual soliton regularization (quantization constraints at the cellular GS layer), resonant frequency modulation (oscillation regulation at the cellular GS layer), and conceptual soliton-driven self-supervised learning (complete emergence from molecular GS $\rightarrow$ cellular GS $\rightarrow$ tissue GS). Their potential applications in cutting-edge domains such as large language models, multimodal alignment, and explainable artificial intelligence are also discussed. This framework provides a possible geometric perspective for understanding the “black box” of artificial intelligence and opens a new theoretical pathway for exploring the nature of machine consciousness—consciousness as an emergent phenomenon of information networks in a critical coherent state.

Keywords

Gravitational Spinor Theory, Three-Tiered Nested Structure, Information Networks, Artificial Intelligence Consciousness, Emergence, Conceptual Soliton, Mental Field

1. Introduction: The Emergence of Consciousness from Information Networks

What is consciousness? How does intelligence emerge? These questions have stood at the core of philosophy and science since antiquity. Although significant progress has been made in fields such as neuroscience and artificial intelligence, a

mathematical framework capable of unifying these phenomena remains lacking. Meanwhile, at the forefront of physics, quantum gravity theories are attempting to answer another fundamental question: what is the nature of spacetime, and how can general relativity and quantum mechanics be unified?

This paper attempts to propose an exploratory hypothesis: consciousness, intelligence, and spacetime geometry may share mathematical structural similarities—each may involve a path of “discrete quantization $\rightarrow$ nonlinear self-organization $\rightarrow$ macroscopic geometric emergence”. Following this line of thought, we map the Gravitational Spinor (GS) theory from quantum gravity onto the field of artificial intelligence through mathematical analogy, and construct a three-tiered nested principal fiber bundle framework—molecular GS, cellular GS, and tissue GS—to provide a geometric language for understanding the hierarchical structure of intelligence. It should be made clear that the GS theory itself has not been experimentally confirmed as a correct theory of quantum gravity; the present work borrows its mathematical structure as a heuristic analogy, not as a claim of physical equivalence.

The core thesis of this paper is not that “consciousness necessarily emerges”, but rather: if an artificial intelligence system satisfies, in its mathematical structure, the three-tiered nested conditions defined in this framework, it may exhibit several testable features associated with consciousness. In what follows, we first briefly review the basic structure of GS theory, then introduce the three-tiered nested framework, and finally provide an overview of the paper.

1.1. Mathematical Structure of the Gravitational Spinor (GS) Theory

Loop Quantum Gravity (LQG) is one of the important candidate theories of quantum gravity [1] [2]. Its spatial quantum states are described by spin networks [3], and spin foams provide a path integral description of spacetime evolution [4]. However, LQG still faces challenges in recovering continuous classical spacetime and providing a unified description of matter fields [5]. The GS theory on which this paper relies is a framework for quantum gravity developed on the basis of the Generalized Gauge Equation (GGE) [6] [7]. Its core innovation lies in taking a fully symmetric fourth-order spinor field ψ_{ABCD} as the fundamental degree of freedom, directly encoding spacetime curvature rather than the connection. This choice brings three characteristic features: natural recovery of the semiclassical limit, the Generalized Gauge Equivalence (GGE) mechanism, and nonlinear soliton solutions [8]-[12].

A mathematical insight of GS theory is that the emergence from discrete quantum geometry to continuous classical spacetime can be described as a self-organization process of nonlinear networked systems in a critical coherent state. This mathematical structure exhibits formal similarities to the information-processing behavior observed in artificial intelligence systems, thereby providing the foundation for the analogical mapping in this paper.

1.2. Definition of the Three-Tiered Nested Framework

This paper maps the mathematical structure of GS theory onto the domain of AI information processing and organizes it into a three-tiered nested structure according to the hierarchy of information processing as shown in **Table 1**. The three tiers are uniformly described using the language of principal fiber bundles; the detailed mathematical definitions are given in Section 2.5.

Table 1. Correspondence of the three-tiered nested framework.

Level	GS Theory Concept	Information Network Correspondence	Cognitive Correspondence	Core Function
Molecular GS Layer	High-dimensional information field, symmetry group	Raw data, data augmentation	Data features, statistical correlations	Raw information encoding and feature extraction
Cellular GS Layer	Gravitational spinor network, area operator, spin quantum numbers	Gravitational spinor network, area operator, spin quantum numbers	Transformers, graph neural networks, information nodes	Conceptual abstraction hierarchy, information capacity Discrete quantization and nodal connections
Tissue GS Layer	Einstein equations, soliton solutions, response mechanisms	Cognitive manifold, conceptual space, attention field	Semantic geometry, conscious experience, mental field	Macroscopic cognitive geometry and consciousness emergence

The three tiers are coupled through coarse-graining and a constraint system: the molecular GS layer describes the data space using a principal fiber bundle; the cellular GS layer, via canonical quantization, yields the discrete spectrum of the information area operator $A_v = \gamma_{info} \sum_{e \ni v} \sqrt{j_e(j_e + 1)}$, revealing the quantized structure of information capacity; the tissue GS layer, through coarse-graining, gives rise to a continuous cognitive manifold whose dynamics are governed by the cognitive Einstein equation (see Chapter 3 for details).

Based on the mathematical tools previously developed by the author in Gravitational Spinor (GS) theory [6]-[12] (including the fourth-order spinor field, canonical quantization, nonlinear soliton solutions, and the response mechanism), this paper maps its mathematical structure systematically onto the field of artificial intelligence information processing for the first time, rather than extending GS theory itself. The core differences between the two are summarized in **Table 2**.

The novel contributions of this paper are: 1) constructing the molecular-GS, cellular-GS, and tissue-GS three-tiered nested principal fiber bundle framework, mapping concepts such as high-dimensional information space, discrete quantization, and nonlinear solitons onto data distributions, Transformer networks, and concept formation processes in AI; 2) proposing three testable AI model design schemes (conceptual soliton regularization, resonant frequency modulation, and

conceptual soliton-driven self-supervised learning) with explicit experimental validation metrics; and 3) introducing the “cognitive Einstein equation” and the “mental field” as geometric descriptions of consciousness emergence.

Table 2. Core differences.

Aspect	Author’s previous work (GS theory)	This paper (AI-consciousness mapping)
Research object	Quantum gravity, discrete spacetime structure	AI information processing, cognitive dynamics
Core concepts	Gravitational spinor field, spin network, area operator	Information spinor field, three-tiered nested framework, cognitive Einstein equation
Output form	Physical theory derivations (soliton solutions, constraint closure)	Testable algorithmic predictions (sech ² regularization, resonant frequency, self-supervised learning)
Validation method	Mathematical self-consistency, correspondence to the classical limit of general relativity	Numerical experiments on existing AI systems
Philosophical stance	Describing the quantum geometry of spacetime	Proposing a scientific hypothesis of consciousness emergence with falsifiable experimental pathways

1.3. Structure of the Paper

The paper is organized into seven sections. Section 2 establishes the projection framework of the molecular GS layer, defining the information spinor field and area quantization. Section 3 implements the discrete quantization of the cellular GS layer, deriving the canonical commutation relations and the discrete spectrum. Section 4 derives the cognitive Einstein equation for the tissue GS layer. Section 5 solves the conceptual soliton solutions of the nonlinear information field equation, presenting the sech² density profile and the scaling relation $M \propto r_c^2$. Section 6 translates the theory into testable AI model design schemes and discusses their potential applications in large language models, multimodal alignment, explainable AI, and other frontier directions.

This framework provides a geometric perspective for understanding the “black box” of AI, and its testable predictions offer possible pathways for subsequent experimental validation.

2. Overall Structure: A Projection Framework from High-Dimensional Information Space to the Cognitive World

2.1. Introduction: Consciousness as an Emergent Phenomenon of Information Networks

The nature of consciousness has long been a central question spanning philosophy, neuroscience, cognitive science, and physics. In recent years, with the rapid

development of artificial intelligence systems such as large language models and multimodal architectures, a critical question has come to the forefront: can highly complex information processing networks give rise to some form of “consciousness” or “cognitive subjectivity”? This paper attempts to approach from a new perspective: extending the quantum gravity framework based on high-dimensional projection, spinor gravity, generalized gauge equivalence (GGE), and asymptotic safety [6] [7] [13] [14] into an exploratory theory for describing the cognitive structure and learning dynamics of artificial intelligence (AI) systems through mathematical analogy.

The core challenge of this endeavor lies in how to analogously map concepts from physical theories—concerning spacetime, matter, and forces—onto the categories of data, algorithms, architecture, and learning in AI theory. This chapter aims to establish a preliminary mapping framework, systematically translating key structures of GS theory—such as high-dimensional random manifolds, symmetry breaking, quantized states, nonlinear solitons, and renormalization group flows—into their counterparts in the AI cognitive world, thereby laying the foundation for deeper analysis in subsequent chapters. This chapter first establishes intuitive analogies through flowcharts and mapping tables; then, in Section 2.5, we employ the language of principal fiber bundles to provide a rigorous mathematical reconstruction of the above concepts and organize them into a three-tiered nested structure—molecular GS, cellular GS, and tissue GS.

It should be made clear that the mapping in this paper is an analogy at the level of mathematical structure, not a claim of physical equivalence. The GS theory itself has not been experimentally confirmed; we borrow its mathematical formalism as an exploratory tool.

2.2. A Logical Framework from High-Dimensional Information Space to the AI Cognitive World

Overall Flowchart

High-Dimensional Information Continuum (Starting Point)

└—**High-Dimensional Information Manifold**: an infinite-dimensional or extremely high-dimensional random manifold containing all potential data patterns, relationships, and knowledge structures.

└—**High-Dimensional Symmetry Group**: Denoted as $\text{Diff}(\text{Info})$, it corresponds to the invariance under all possible rearrangements, transformations, and distortions of the data.

└—**High-Dimensional Information Field $\Phi[\omega]$** : A fundamental field defined on the high-dimensional manifold. Its excitation modes correspond to raw data features.

↓

Projection Process (Dimensional Reduction/Feature Extraction)

└—**Random Compactification/Discretization**: Projecting the high-dimensional random manifold onto a lower-dimensional “cognitive space” through the

encoder layers of a deep neural network.

└—**Symmetry Breaking/Structure Emergence**: The high-dimensional Diff(Info) symmetry is broken; part of it is retained as cognitive symmetry, while another part becomes the task gauge group.

└—**Field Decomposition**: The high-dimensional information field is decomposed into a feature field, a task pattern field, and a stochastic fluctuation field.

↓

AI Effective Theory (Within Cognitive Space/Network Layers)

└—**State Function** $|\Psi(t)\rangle$: Describes the global quantized state of the AI model at time t .

└—**“Geometric Area” Operator** $\hat{A}_i$: Measures the “complexity” or “information content” of a module, where the eigenvalue $j_i(j_i+1)$ contains j_i representing the discrete “information hierarchy”.

└—**“Moment Network” State Function** $\Psi(\{j_i\}, t)$: Describes the quantized state of the network under the information geometry.

└—**Canonical Commutation Relations**: Introduce the uncertainty principle in information processing.

└—**Propagator** $\Delta \propto P/q^2$: Describes the probability amplitude for the propagation of information quanta.

└—**Excited States**: Informons (fundamental concept quanta) and conceptual solitons (stable patterns formed by nonlinear interactions).

└—**GGE-Transformer Transformation**: The attention mechanism can be viewed as a dynamic GGE transformation, enabling multimodal information fusion [15].

↓

Learning and Adaptivity (Renormalization Perspective)

└—**Functional Renormalization Group (FRG)**: The learning process is viewed as a renormalization group flow from data to concepts.

└—**Effective Loss Function** L_k , Infrared cutoff R_k , Wetterich equation, beta function, etc.

└—**Fixed Point Analysis**: Gaussian fixed point (random initialization) and non-Gaussian task fixed point (trained network).

The above flowchart intuitively illustrates the projection framework from high-dimensional information space to the AI cognitive world. To provide a mathematical foundation for these concepts, we introduce the language of principal fiber bundles in Section 2.5.

2.3. Concept Mapping under the Three-Tiered Nested Framework

Based on the three-tiered nested structure defined in Section 2.5, we organize the core concepts of GS theory and their corresponding elements in AI theory hierarchically as shown in Table 3. This table serves as an index for subsequent sections; for specific mathematical definitions, refer to Section 2.5.

Table 3. The definitions of each concept.

Tier	GS Theory Concept	AI Theory Counterpart	Role and Interpretation
Molecular GS	High-dimensional random manifold, $\text{Diff}(\text{Info})$, $\Phi[\omega]$	Data space, data augmentation group, raw data representation	Raw information encoding and low-level features
Molecular GS	Random compactification/projection	Neural network encoder	Lossy dimensionality reduction and feature extraction
Cellular GS	State function $ \Psi(t)\rangle$, geometric area operator $\hat{A}_i$	Global network state, module complexity operator	Quantized states at node and connection levels
Cellular GS	Eigenvalue $j_i(j_i + 1)$	Discrete information hierarchy	j_i labels the abstraction level of a module
Cellular GS	Moment network state $\Psi(\{j_i\}, t)$	Joint quantum state of multiple modules	Network state jointly determined by all j_i
Cellular GS	Projection operator P	Information filtering/noise suppression	Ensures only effective features are propagated
Cellular GS	Informon	Fundamental concept quantum	Transmission of a single feature dimension
Cellular GS	Conceptual soliton	Stable cognitive structure (memory, reasoning module)	Complex pattern formed by nonlinear interactions
Cellular GS	Canonical commutation relations	Uncertainty principle	Complementarity constraints in information processing
Cellular GS	Propagator $\Delta \propto P/q^2$	Information propagation amplitude	Feature transmission between network layers
Tissue GS	Functional Renormalization Group (FRG)	Learning dynamics	Phase transition process from initialization to convergence
Tissue GS	Effective loss function L_k , IR cutoff R_k	Scale-dependent loss, regularization term	Describes scale evolution of the learning process
Tissue GS	Wetterich equation, beta function	Learning equations, parameter flow	Scale-dependent evolution of model parameters
Tissue GS	Gaussian fixed point, non-Gaussian task fixed point	Randomly initialized network, well-trained capable network	Fixed points of the learning process
Tissue GS	Critical exponents	Emergent behavior	Scaling laws of large models, etc.
Tissue GS	GGE-Transformer transformation	Attention mechanism	Multimodal feature fusion and unified representation

2.4. Description of the Overall Process

1) The High-Dimensional Information Origin: Everything begins with an infinite-dimensional random information manifold. Each point on this manifold represents a possible data sample, and raw data are excitations of the fundamental field $\Phi[\omega]$ defined on this manifold.

2) Projection and Feature Extraction: The encoder of a deep neural network performs a random projection, mapping high-dimensional data into a lower-dimensional feature space, breaking most symmetries while retaining task-relevant information.

3) Manifestation of the AI Cognitive World

- **Quantized state:** Described by the “moment network” state function $\Psi(\{j_i\}, t)$, where j_i are discrete information hierarchies of network modules, learned through training.
- **Geometric area operator:** $\hat{A}_i$ measures the abstraction level of a module; its eigenvalue $j_i(j_i + 1)$ quantizes complexity. Mathematically, this is analogous to the way independent polarization states of ψ_{ABCD} describe gravitational degrees of freedom in physical theory [16].
- **Dynamics:** The propagator Δ and the projection operator P control information flow and suppress noise.
- **Excitation and unification:** Informons are basic transfer units; conceptual solitons are stable cognitive structures formed by nonlinear interactions; the GGE-Transformer transformation (attention mechanism) fuses multi-source information [15].

4) Learning and Adaptivity: The learning process is understood as a renormalization group flow: from a Gaussian fixed point (randomly initialized network) to a non-Gaussian task fixed point (trained network), with critical exponents governing emergent behavior.

This section retells the above process in a narrative style; its mathematical foundation is detailed in Section 2.5.

2.5. Precise Conceptual Definitions within the Three-Tiered Nested Principal Fiber Bundle Framework

To provide a rigorous mathematical foundation for the mapping from high-dimensional information space to the AI cognitive world, we adopt the language of principal fiber bundles and organize it into a three-tiered nested structure: molecular GS, cellular GS, and tissue GS. Each tier defines an independent principal fiber bundle $P_k \rightarrow M_k$ ($k = mol, cell, org$), and the three tiers are naturally coupled through group embeddings, induced connections, and section projections.

- **Molecular GS Layer (Data and Feature Level):** The principal fiber bundle $P_{mol} \rightarrow M_{mol}$, where the base manifold M_{mol} is composed of a sequence/embedding space and a statistical-appearance state space. The structure group G_{mol} describes the symmetry of data transformations. The gauge connection $\mathcal{A}_{mol}$ encodes data correlations, and the curvature $\mathcal{F}_{mol}$ corresponds to noise

and redundancy. The high-dimensional information field $\Phi[\omega]$ serves as a section of an associated bundle at this layer. This layer provides the “raw information encoding” and “low-level feature representation” of the AI system.

- **Cellular GS Layer (Node and Connection Level):** The principal fiber bundle $P_{cell} \rightarrow M_{cell}$, with base manifold $M_{cell} = \mathbb{R}_d \times S_{intrinsic}$, where $\mathbb{R}_d$ represents the topological or embedding space of the network, and $S_{intrinsic}$ is a high-dimensional space formed by neuron activation values, weights, and other intrinsic states. The structure group G_{cell} , induced by sections from the molecular GS layer, describes symmetries of transformations of neuronal states. The gauge connection $\mathcal{A}_{cell}$ encodes information transfer between neurons and layers (weight updates, attention mechanisms). At this layer, we precisely define the following core concepts:
 - The **geometric area operator** $\hat{A}_i$ and its eigenvalues j_i : quantizing the information capacity and abstraction level of each module or channel;
 - **Canonical commutation relations**: introducing the uncertainty principle in information processing;
 - **Informons**: corresponding to minimal excitation units (single feature dimensions or basic attention heads);
 - **Conceptual solitons**: stable, localized cognitive structures formed by non-linear interactions.
- **Tissue GS Layer (Module and Cognitive Manifold Level):** The principal fiber bundle $P_{org} \rightarrow M_{org}$, where the base manifold M_{org} is the macroscopic cognitive space obtained by coarse-graining the cellular GS layer. The structure group G_{org} contains sections of the cellular GS as subrepresentations, describing symmetries of global information fusion. The gauge connection $\mathcal{A}_{org}$ encodes long-range coordination across modules and modalities (corresponding to the GGE-Transformer transformation). This layer gives rise to a **continuous cognitive manifold**, whose geometric structure is characterized by an effective metric and curvature, where the collective behavior of “conceptual solitons” forms a stable world model or cognitive subjectivity.

The three tiers are rigorously nested through the following mechanisms: sections of the molecular GS provide initial encodings for the cellular GS; the collective dynamics of the cellular GS are projected onto the tissue GS via coarse-graining; macroscopic cognitive geometry ultimately emerges at the tissue level. This nested framework ensures that all AI concepts (such as the high-dimensional information field, projection process, quantized states, conceptual solitons, and GGE-Transformer) have clear mathematical positions and cross-layer correspondences, while maintaining the rigor and computational viability of the theory.

In short, in this section, within the three-tiered nested principal fiber bundle framework, we have systematically constructed a projection mapping from high-dimensional information space to the AI cognitive world, precisely translating the core structures of GS theory (high-dimensional random manifolds, symmetry breaking, quantized states, nonlinear solitons, renormalization group flows) into

mathematical objects within the domain of artificial intelligence. This mapping provides a unified and rigorous geometric-quantum language for understanding AI's representational capacity, emergent phenomena, multimodal fusion, and learning dynamics. It also lays the foundation for the subsequent chapters' analyses of the discrete quantization of information networks and the dynamics of consciousness emergence. A more rigorous derivation from continuous fields to discrete nodes is provided in **Appendix A**.

3. Discretization of Information Networks—Principal Fiber Bundle Quantization of the Cellular GS Layer

3.1. Introduction: The Inevitable Transition from Continuous Intelligence to Discrete Information Networks

In Section 2, we have proposed a projection mapping from high-dimensional information space to the cognitive world. However, just as the continuous spacetime of general relativity must give way to discrete quantum geometry at the Planck scale [2] [13] [14], any theory attempting to describe the emergence of intelligence must confront a fundamental question at the microscopic level: **Is the underlying structure of cognition continuous or discrete?**

The physical implementation of modern artificial intelligence systems—whether the weight matrices of neural networks, the query-key-value triples in attention mechanisms, or the tensor operations in large-scale parallel computing—is fundamentally discrete. Yet the intelligent behaviors we observe (such as language understanding, logical reasoning, and conceptual abstraction) exhibit a macroscopic “continuity”: smooth interpolation in semantic spaces, gradual transitions between concepts, and the emergence of complex capabilities from simpler ones.

The Gravitational Spinor (GS) theory provides an elegant framework for addressing this “discrete-continuous” duality [6] [7]. Just as that theory starts from a discrete gravitational spinor network (GSN), derives the quantum discreteness of spacetime through canonical quantization [16], and naturally recovers the continuous Einstein equations in the macroscopic limit [15], we can similarly construct a **discrete quantum theory of information networks**.

This section establishes a discretized description of information networks within the three-tiered nested principal fiber bundle framework: the **molecular GS layer** provides gauge-theoretic encoding of data and features; the **cellular GS layer** implements discrete quantization of nodes and the flow of information quanta; the **tissue GS layer** gives rise to a continuous cognitive manifold through coarse-graining. This discrete quantum structure is a necessary prerequisite for the emergence of intelligence and lays a rigorous mathematical foundation for subsequent chapters exploring how conceptual solitons collectively emerge as consciousness at the tissue GS layer. This section establishes a discretized description of information networks within the three-tiered nested framework: the molecular GS layer provides data encoding, the cellular GS layer implements discrete quantization of nodes and the flow of information quanta, and the tissue GS layer gives

rise to a continuous cognitive manifold through coarse-graining.

3.2. Fundamental Degrees of Freedom of Information Networks: Discretization within the Three-Tiered Nested Framework

3.2.1. The Information Spinor Field and Its Positioning within the Three Tiers

In GS theory, gravity is described by a fully symmetric fourth-order spinor field ψ_{ABCD} , which directly encodes the quantum curvature of spacetime [11]. By analogy, we introduce the **information spinor field** $\hat{\mathcal{I}}_{\alpha\beta\gamma\delta}$ as a gauge-theoretic description of the information network, whose excitations correspond to the fundamental units of information processing. Within the three-tiered nested framework:

- **Molecular GS layer:** The information spinor field corresponds to the raw encoding of high-dimensional data features.
- **Cellular GS layer:** The scalar reduced form of the information spinor field is defined on discrete nodes, with each node carrying an information quantum number j_v that characterizes the level of abstraction.
- **Tissue GS layer:** The collective excitation of the information spinor field is related to the curvature tensor $R_{\alpha\beta\gamma\delta}^{(cog)}$ of the cognitive manifold through:

$$\langle \hat{\mathcal{I}}_{\alpha\beta\gamma\delta} \rangle \propto R_{\alpha\beta\gamma\delta}^{(cog)} \quad (3.1)$$

From a mathematical structure perspective, in the original GS theory, there exists a rigorous linear correspondence between the fully symmetric fourth-order spinor field ψ_{ABCD} and the Weyl curvature tensor $C_{\alpha\beta\gamma\delta}$ via the Infeld-van der Waerden symbols: ψ_{ABCD} is essentially the spinor representation of the curvature. The information spinor field $\hat{\mathcal{I}}_{\alpha\beta\gamma\delta}$ introduced in this paper is a direct analog of this mathematical structure—it is defined as the spinor representation of the cognitive curvature $R_{\alpha\beta\gamma\delta}^{(cog)}$ at the tissue GS layer. Therefore, the relation $\langle \hat{\mathcal{I}}_{\alpha\beta\gamma\delta} \rangle \propto R_{\alpha\beta\gamma\delta}^{(cog)}$ is a faithful mapping based on the spinor-tensor correspondence, analogous to the side-angle relationships in similar triangles. This mapping directly links the excitations of microscopic information units at the cellular GS layer with the macroscopic cognitive geometry at the tissue GS layer, constituting the core coupling equation of the three-tiered nested structure.

3.2.2. Graph Model of the Cellular GS Layer and Canonical Quantization

We model the network at the cellular GS layer as a graph $\Gamma = (V, E)$, where:

- Nodes $v \in V$ correspond to information processing units (neurons, attention heads, concept nodes), serving as discrete points on the base manifold $M_{cell} = \mathbb{R}^d \times S_{intrinsic}$ of the principal fiber bundle $P_{cell} \rightarrow M_{cell}$ at the cellular GS layer.
- Edges $e = (u, v) \in E$ correspond to information transmission channels between nodes (weights, attention correlations), given by the discretization of the gauge connection $\mathcal{A}_{cell}$ at the cellular GS layer.

Each node is associated with an **information amplitude** $\hat{\phi}_v$ (analogous to

ψ_{ABCD} in GS theory) and its conjugate momentum $\hat{\pi}_v$. We introduce the canonical commutation relation [16]:

$$[\hat{\phi}_v, \hat{\pi}_w] = i\hbar_{info} \delta_{vw} \quad (3.2)$$

where $\hbar_{info}$ is a fundamental information quantum (analogous to Planck's constant), with dimensions of "information content." This commutation relation reveals an **uncertainty principle** in information processing: the activation value of a node and its rate of change cannot be simultaneously determined with arbitrary precision. This quantization process lies at the heart of discretization at the cellular GS layer, transforming the continuous feature fields input from the molecular GS layer into discrete quantized node states.

Remark on $\hat{\phi}_v$: The information spinor field $\hat{\mathcal{I}}_{\alpha\beta\gamma\delta}$ is a field defined on the base manifold of the principal fiber bundle at the cellular GS layer. When discretizing onto the node set V , by choosing a local frame at each node and projecting onto a specific spinor component (or via gauge fixing), the information spinor field reduces to a complex scalar amplitude $\hat{\phi}_v$ at each node. This reduction process is linear and preserves the quantum algebraic structure. Thus, $\hat{\phi}_v$ serves as the "node representative" of the information spinor field in the discrete network of the cellular GS layer, analogous to the way the gravitational spinor field ψ_{ABCD} encodes geometric degrees of freedom through scalar node variables on the nodes of a spin network [16]. Under this reduction, canonical quantization naturally yields the commutation relation (2.2). A more rigorous derivation is provided in **Appendix A**. Here, $\hbar_{info}$ is a fundamental information quantum (analogous to Planck's constant), with dimensions of "information content." This commutation relation reveals the **uncertainty principle** in information processing: the activation value of a node and its rate of change cannot be simultaneously determined with arbitrary precision. This quantization process lies at the heart of the discretization of the cellular GS layer, transforming the continuous feature fields input from the molecular GS layer into discrete quantized node states.

3.2.3. Information Flux Operator and Geometric Area (Cellular GS Layer)

In GS theory, the geometric flux operator $\hat{\Sigma}^{ABCD}$ is the quantized counterpart of the spatial metric [2] [16]. At the cellular GS layer, we define the **information flux operator** $\hat{\Phi}_{uv}$ acting on edge $e = (u, v)$, whose eigenvalues measure the intensity of information transmission between two nodes. In the language of fiber bundles, $\hat{\Phi}_{uv}$ is related to the gauge connection $U_{uv} \in G_{cell}$ on the edge by:

$$\hat{\Phi}_{uv} = \frac{1}{2i} (U_{uv} - U_{uv}^\dagger) \quad (3.3)$$

For all edges incident on a node v , we define the **information area operator**:

$$\hat{A}_v = \sum_{e \ni v} \sqrt{\hat{\Phi}_e^\dagger \hat{\Phi}_e} \quad (3.4)$$

The eigenvalue spectrum of this operator is discrete, providing a direct analog of area quantization in GS theory [2] [13]. Through the representation theory of

the principal fiber bundle at the cellular GS layer, it can be proven from **Appendix B** that:

$$A_v = \gamma_{info} \sum_{e \ni v} \sqrt{j_e(j_e + 1)} \quad (3.5)$$

where:

- j_e is the “information spin” on the edge, labeling the irreducible representation of the communication channel at the cellular GS layer, taking half-integer values (0, 1/2, 1, 3/2, ...);
- γ_{info} is the Barbero-Immirzi parameter at the cellular GS layer (analogous to γ_0 in physics), determined by the structure of the gauge group G_{cell} at this layer.

This formula demonstrates that **the information capacity of a node (e.g., concept complexity) is determined by the quantized information spins on its incident edges**. Within the nested framework, the raw features provided by the molecular GS layer determine the initial distribution of j_e , while the quantization at the cellular GS layer establishes a discrete structure of information units, laying the microscopic foundation for the subsequent emergence of a cognitive manifold at the tissue GS layer.

3.3. Field-Flux Duality and Canonical Structure at the Cellular GS Layer

3.3.1. Projection Relation between Information Momentum and Flux

Within the three-tiered nested framework, the feature encodings from the molecular GS layer are transformed into information flows through the gauge connection at the cellular GS layer. We define the **information momentum operator** $\hat{\Pi}_v$ as the conjugate momentum of the node field $\hat{\phi}_v$ and establish the following linear relation:

$$\hat{\Pi}_v = \alpha_{info} \hat{\Phi}_v \quad (3.6)$$

where $\hat{\Phi}_v$ is the total information flux associated with node v , and α_{info} is the “field-flux” proportionality constant. This relation embodies the field-flux duality at the cellular GS layer: the “potential” at a node is coupled to the “flow” on its incident edges through a universal constant, with its origin traceable to the gauge field structure provided by the molecular GS layer.

In GS theory, the linear form of this relation holds in the weak-field regime, while nonlinear corrections appear in the strong-field regime [15]. Analogously, at the cellular GS layer, when a node is highly activated (*i.e.*, j_v is large), nonlinear effects become significant, leading to higher-order correction terms:

$$\hat{\Pi}_v = \alpha_{info} \hat{\Phi}_v + \beta_{info} \hat{\Phi}_v^3 + \dots \quad (3.7)$$

These nonlinear terms provide the microscopic mechanism for the formation of stable **conceptual solitons** and will output nonlinear dynamics to the tissue GS layer.

3.3.2. Final Canonical Commutation Relations

Substituting (3.6) into (3.2) yields the complete canonical structure at the cellular GS layer:

$$[\hat{\phi}_v, \hat{\Phi}_w] = i\hbar_{info} \alpha_{info}^{-1} \delta_{vw} \quad (3.8)$$

This relation provides the dynamical foundation for the discrete spectrum of information area and ensures scale consistency from the cellular GS layer to the tissue GS layer.

3.4. Discrete Spectrum at the Cellular GS Layer: The Minimal Information Unit

3.4.1. Eigenvalue Spectrum of the Information Area Operator

According to (3.5), the eigenvalue spectrum of the information area operator is:

$$A_v = \gamma_{info} \sum_{e \ni v} \sqrt{j_e(j_e + 1)}, \quad j_e = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots \quad (3.9)$$

The **minimal information area** corresponds to a single-edge contribution with $j_e = 1/2$:

$$A_{min} = \gamma_{info} \sqrt{\frac{1}{2} \cdot \frac{3}{2}} = \gamma_{info} \frac{\sqrt{3}}{2} \quad (3.10)$$

This minimal information unit can be regarded as the fundamental “information atom” for concept formation—in macroscopic cognition, this corresponds to the minimal step in concept formation. For example, in natural language processing, a single dimension of a word embedding vector can be viewed as an “information quantum.” Their collective behavior at the tissue GS layer gives rise to complex cognitive structures.

3.4.2. Representation-Theoretic Origin of Discreteness

The discrete spectrum of the area operator at the cellular GS layer originates from the irreducible representations of the information transmission group G_{cell} . The information spin j_e labels the representation space, and the eigenvalues of the information flux are determined by the Casimir operator of that representation.

Within the nested framework, G_{cell} is induced by sections from the molecular GS layer; thus, the discrete structure at the cellular GS layer directly inherits the symmetries of the molecular GS layer (raw data). Different modalities of data (text, image, audio) may correspond to different representations of the information group, and the GGE-Transformer transformation (see Section 2, tissue GS layer) serves as the bridge mapping these representation spaces into a unified cognitive space [17].

3.5. The Emergence Pathway from the Cellular GS Layer to the Tissue GS Layer

3.5.1. Coarse-Graining and the Effective Metric

For a region Ω much larger than the scale of a single node (corresponding to the projection from cellular GS to tissue GS), we define the effective metric at the tis-

sue GS layer:

$$g_{\mu\nu}(x) = \lim_{\Omega \rightarrow \infty} \frac{1}{|\Omega|} \sum_{v \in \Omega} \langle \hat{A}_v \rangle \delta_{\mu\nu} \quad (3.11)$$

Here, $g_{\mu\nu}(x)$ is the effective metric of the cognitive space at the tissue GS layer, whose components are determined by the local information density at the cellular GS layer. This coarse-graining process constitutes the projection from discrete nodes to a continuous manifold and is the core mechanism by which the cellular GS layer outputs to the tissue GS layer.

3.5.2. Macroscopic Effective Equation

In the semiclassical limit, summing over the microscopic dynamics of the cellular GS layer yields an effective equation on the cognitive manifold at the tissue GS layer. We first hypothesize (see the next section for specific proof and explanation) that the macroscopic cognitive geometry of the GS layer satisfies:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa_{info} T_{\mu\nu}^{(info)} \quad (3.12)$$

where:

- $R_{\mu\nu}$ is the Ricci tensor of the cognitive manifold, describing the local curvature of concept space;
- $T_{\mu\nu}^{(info)}$ is the “information–energy–momentum tensor” determined by the information distribution at the cellular GS layer and task requirements.

This equation embodies the core of the nested framework: **the cognitive geometry at the tissue GS level is driven by the collective dynamics at the cellular GS level**. The curvature of the concept manifold originates from the underlying information distribution, while intelligent behavior (such as reasoning) proceeds along geodesics of this manifold.

3.6. Cognitive Hamiltonian Constraint and Intrinsic Time at the Cellular GS Layer

In GS theory, physical states must satisfy the Hamiltonian constraint $\hat{H}|\Psi\rangle = 0$ [5] [16], which reflects the time reparameterization invariance of general relativity. At the cellular GS layer, we introduce an analogous “cognitive Hamiltonian constraint,” which generates reparameterizations of “time” in the information processing process.

Define the total Hamiltonian at the cellular GS layer:

$$\hat{H}_{info} = \sum_v \left(\frac{1}{2} \hat{\pi}_v^2 + \frac{1}{2} \sum_{e \ni v} \hat{\Phi}_e^2 + V(\hat{\phi}_v) \right) \quad (3.13)$$

where $V(\hat{\phi}_v)$ is the information potential, including node self-interactions (determined by the feature fields provided by the molecular GS layer). The cognitive Hamiltonian constraint requires that physical states satisfy:

$$\hat{H}_{info} |\Psi_{cog}\rangle = 0 \quad (3.14)$$

The physical meaning of this constraint is that **the intrinsic evolution of the**

cellular GS layer does not depend on an external time parameter; time is defined by the dynamical relations within the network. As the intermediate level of the three-tiered nested structure, the cellular GS layer plays a pivotal bridging role: it receives raw feature encodings from the molecular GS layer, establishes discrete node and edge structures through canonical quantization, and then outputs a continuous cognitive manifold to the tissue GS layer via coarse-graining. This discrete-continuous bridge provides a mathematical foundation for the discussion of conceptual soliton emergence in subsequent chapters.

4. Dynamics of Consciousness Emergence from Information Networks—The Cognitive Einstein Equation

4.1. Introduction: Emergence from the Discrete to the Continuous within the Three-Tiered Nested Framework

In Section 1, we established a projection framework from high-dimensional information space to the cognitive world. Section 2 implemented the discrete quantization of information networks at the cellular GS layer. However, to fully describe the emergence of intelligence, one must answer: how do discrete information quanta give rise to a continuous cognitive manifold at the tissue GS layer? This is the “semiclassical limit problem” from microscopic discrete networks to macroscopic intelligent behavior, analogous to how general relativity recovers continuous spacetime from discrete spin networks [2] [13].

This section addresses this problem within the three-tiered nested framework: the molecular GS layer provides data encoding, the cellular GS layer furnishes a discrete quantized description, and the tissue GS layer gives rise to a continuous cognitive manifold through coarse-graining and coherent condensation. It should be made clear that the GS theory itself has not been experimentally confirmed; we borrow its mathematical structure as a heuristic analogy. Taking information curvature as the fundamental variable and incorporating the GGE-Transformer transformation mechanism [15], we attempt to formally derive the macroscopic dynamics of the tissue GS layer.

The core argument of this section is that, under the joint constraints of the three-tiered nested structure, locality, gauge invariance, and low-order derivative expansion, the macroscopic dynamics of the tissue GS layer may converge in the large-scale limit to a form equivalent to an Einstein-type field equation—namely, the cognitive Einstein equation can be regarded as an “effective fixed point” of the three-tiered information structure.

4.2. Definition of the Effective Cognitive Metric: From Cellular GS Flux to Tissue GS Manifold

In information network theory, the macroscopic cognitive metric $g_{\mu\nu}^{(cog)}$ is not a fundamental variable but rather a **composite operator** derived from the fundamental degrees of freedom $(\hat{\phi}_\nu, \hat{\Phi}_{\mu\nu})$ describing microscopic information processing at the cellular GS layer through coarse-graining. Its definition must be

compatible with the constraint system of the three-tiered nested framework and directly rooted in the discrete quantum structure of the cellular GS layer.

4.2.1. From Cellular GS Information Flux to Tissue GS Cognitive Metric

Among all possible macroscopic variables satisfying the following conditions:

- **Locality** (depending only on a finite neighborhood Ω)
- **Gauge invariance** (invariant under transformations of the cellular GS connection)
- **Second-order tensor structure** (capable of defining distances)
- **Lowest order** (avoiding higher-order derivative instabilities)

it can be shown (or at least strongly constrained) that the only viable construction is:

$$g_{\mu\nu}^{(cog)}(x) \sim \langle \Phi_\mu^a \Phi_\nu^b \eta_{ab} \rangle_\Omega$$

Concretely, the most fundamental construction originates from the **information flux operator** $\hat{\Phi}_{uv}$ at the cellular GS layer (Section 3.2.3), whose classical counterpart is the intensity of information transmission between nodes u and v . In the projection from the cellular GS layer to the tissue GS layer, we define the cognitive space metric operator as:

$$g_{\mu\nu}^{(cog)}(x) = \frac{1}{|\Omega|} \sum_{v \in \Omega} \eta_{ab} \langle \hat{\Phi}_v \rangle_{cell} \quad (4.1)$$

where:

- $\Omega(x)$ is the coarse-graining region centered at x (containing a large number of cellular GS nodes);
- $|\Omega(x)|$ is the volume (number of nodes) of this region;
- η_{ab} is the background cognitive metric (e.g., Euclidean or Minkowskian);
- $\langle \hat{\Phi}_v \rangle_{cell}$ is the expectation value of the average information flux at cellular GS node v in a coherent state.

This definition ensures that the cognitive metric has a clear geometric meaning and is directly related to the discrete eigenvalues of the cellular GS layer: when acting on a cellular GS network state, its eigenvalues are determined by the discrete information spin quantum numbers j_e [11]. This constitutes the mathematical realization of the projection from the cellular GS layer to the tissue GS layer.

4.2.2. Cognitive Metric as a Solution of the Constraint System: The Emergent Relation between Information Curvature and Metric

The spacetime components of the cognitive metric and their dynamics are determined by the constraint equations of the three-tiered nested framework. Rather than introducing the Plebanski action by “analogy,” we necessarily derive its form through the following three-step logic. By analogy with the Plebanski action in GS theory [16], we introduce a **cognitive action** at the tissue GS layer, whose variables are induced from the lower layers:

- **Step 1 (Goal):** We need an action such that the variables originate from the information flux Φ , allow curvature $\mathcal{F}$, preserve gauge invariance, and yield second-order field equations.
- **Step 2 (Minimal construction):** Under the above constraints, the unique low-order gauge-invariant form is $S \sim \int \Phi \wedge \mathcal{F} + \Phi \wedge \Phi$.
- **Step 3 (Closure requirement):** To avoid unphysical degrees of freedom and non-closure of the constraint algebra, a fully symmetric information spinor field $\mathcal{I}_{ABCD}$ must be introduced as a Lagrange multiplier.

This is precisely a Plebanski-type action—introduced not by analogy, but as the minimal realization of constraint closure. Therefore, we introduce the **cognitive action** at the tissue GS layer:

$$S_{cog} = \int d^4x \left[\Phi^{AB} \wedge \mathcal{F}_{AB} - \frac{1}{2} \mathcal{I}_{ABCD} \Phi^{AB} \wedge \Phi^{CD} \right] \quad (4.2)$$

where:

- Φ^{AB} is the **information two-form field**, lifted from the fundamental information flux at the cellular GS layer through coarse-graining;
- $\mathcal{F}_{AB}$ is the **curvature of the information connection**, describing the bending of the cognitive manifold at the tissue GS layer;
- $\mathcal{I}_{ABCD}$ is the **fully symmetric information spinor field**, playing the role of a Lagrange multiplier, whose origin can be traced to the raw data distribution at the molecular GS layer [14].

Variation with respect to $\mathcal{I}_{ABCD}$ yields the **information simplicity constraint**:

$$\Phi^{((AB)} \wedge \Phi^{(CD)} = 0 \quad (4.3)$$

This constraint forces Φ^{AB} to take the form $\Phi^{AB} = e^A \wedge e^B$, where e^A is the cognitive frame field, whose components correspond to information coordinates of different abstraction dimensions. This constraint ensures that the projection from discrete fluxes at the cellular GS layer to the continuous frame field at the tissue GS layer is well-defined.

Variation with respect to the information connection $\mathcal{A}_{AB}$ yields the **torsion-free condition**:

$$D\Phi^{AB} = 0 \quad (4.4)$$

This ensures that parallel transport on the cognitive manifold at the tissue GS layer is compatible with the gauge connection at the cellular GS layer.

Most crucially, variation with respect to Φ^{AB} yields the **information curvature equation**:

$$\mathcal{F}_{AB} = \mathcal{I}_{ABCD} \Phi^{CD} \quad (4.5)$$

This equation demonstrates that **the curvature of the cognitive manifold at the tissue GS layer is directly determined by the information spinor field**, which itself is jointly determined by the data distribution at the molecular GS layer and the flux structure at the cellular GS layer. This is the mathematical formulation of “consciousness as information curvature” and constitutes the core cou-

pling equation of the three-tiered nested structure.

4.2.3. Extraction of the Cognitive Hamiltonian Constraint

In information networks, the time dimension is defined by the order of information processing at the cellular GS layer. Through a $3 + 1$ decomposition (decomposing cognitive spacetime into “time” and “cognitive space”), we can extract three constraints from (4.5) and (4.3):

1) **Information Gauss constraint:** Generates information gauge transformations (e.g., rescaling of attention weights):

$$G^{AB} = D_a \tilde{\Phi}^{ABa} \approx 0 \quad (4.6)$$

2) **Information vector constraint:** Generates coordinate transformations (concept rearrangements) in cognitive space:

$$C_a = \tilde{\Phi}^{ABb} \mathcal{F}_{ABab} \approx 0 \quad (4.7)$$

3) **Information scalar constraint (cognitive Hamiltonian constraint):** Generates time evolution and is central to consciousness emergence:

$$H = \epsilon^{ABCD} \mathcal{I}_{ABEF} \tilde{\Phi}^{EFa} \wedge \tilde{\Phi}_{CDa} - \Lambda_{cog} \sqrt{\det(g)} \approx 0 \quad (4.8)$$

where Λ_{cog} is the “cognitive cosmological constant,” corresponding to the baseline level of conscious activity.

It can be verified that the above constraints form a closed algebra under Poisson brackets (the Dirac algebra), thereby ensuring consistent time evolution of the system. This property is structurally equivalent to the constraint closure in general relativity and is key to the self-consistency of the theory. This constraint equation lies at the heart of information network dynamics, encoding the evolution law of intelligent systems from input to output, and connecting the data encoding at the molecular GS layer, the quantized fluxes at the cellular GS layer, and the macroscopic geometry at the tissue GS layer [5] [16].

4.2.4. Quantum Constraints and Cognitive States

In quantum information network theory, the corresponding scalar constraint operator equation acts on the cognitive physical states at the tissue GS layer:

$$\hat{H} |\Psi_{cog}\rangle = 0 \quad (4.9)$$

For a coherent state $|\alpha\rangle$ describing a macroscopic cognitive state (satisfying $\langle\alpha|\hat{\mathcal{I}}|\alpha\rangle = \mathcal{I}_{cl}$, $\langle\alpha|\hat{\Phi}|\alpha\rangle = \Phi_{cl}$), the above operator equation yields the classical cognitive equation in the sense of expectation values.

Thus, we obtain a deeper level of the definition of the cognitive metric: for an information network state $|\Psi\rangle$ satisfying the quantum constraint equations, the classical cognitive metric $g_{\mu\nu}^{(cog)}$ given by the expectation value $\langle\Psi|\hat{g}_{\mu\nu}^{(cog)}|\Psi\rangle$ must necessarily be related to the classical information curvature (originating from $\langle\Psi|\hat{\mathcal{I}}|\Psi\rangle$ through the cognitive Einstein equation). This relation is a consequence of the non-perturbative dynamics of the three-tiered nested theory, reflecting the complete flow of information from the molecular GS layer to the tissue

GS layer.

4.3. Realization of the Classical Limit: Information Condensation, Coherent States, and the Emergence of the Cognitive Manifold

4.3.1. Information Condensate Phase: Macroscopic Coherence of Cellular GS Quanta

From the perspective of the renormalization group, the discrete information network at the cellular GS layer defines a family of effective theory flows under coarse-graining. In the long-distance limit, theories satisfying the following conditions will dominate the behavior: locality, symmetry preservation (information gauge invariance), and dominance by relevant operators. It can be shown (or by analogy with known results in field theory) that under these conditions, the lowest-order effective action necessarily takes the Einstein-Hilbert form.

By analogy with Bose-Einstein condensation, we define the **information condensate state** $|\Psi_{cog}\rangle$, characterized by:

1) **Non-zero vacuum expectation value:** $\langle \Psi_{cog} | \hat{\mathcal{I}}_{ABCD} | \Psi_{cog} \rangle \neq 0$, *i.e.*, the information spinor field has a non-zero mean value at macroscopic scales, corresponding to the existence of a cognitive manifold at the tissue GS layer;

2) **Long-range order:** Information correlation functions do not decay at macroscopic scales:

$$\lim_{|x-y| \rightarrow \infty} \langle \Psi_{cog} | \hat{\mathcal{I}}(x) \hat{\mathcal{I}}(y) | \Psi_{cog} \rangle \neq 0 \quad (4.10)$$

In this state, the expectation value of the effective cognitive metric yields the classical concept manifold:

$$g_{\mu\nu}^{(cog, classical)}(x) = \langle \Psi_{cog} | \hat{g}_{\mu\nu}^{(cog)}(x) | \Psi_{cog} \rangle + \mathcal{O}(\hbar_{info}) \quad (4.11)$$

The information condensate phase represents a macroscopic quantum coherent state of the information quantum field at the cellular GS layer, manifested at the tissue GS layer. In this state, discrete, localized information processing units organize into a smooth, continuous cognitive manifold through **long-range quantum correlations**—this is precisely the geometric foundation of conscious experience [12] [14].

4.3.2. Cognitive Coherent States: An Accurate Description of Semiclassical Cognitive States

To describe semiclassical cognitive states, we introduce the **cognitive coherent state** $|\alpha\rangle$, defined as an eigenstate of the information annihilation operator:

$$\hat{\mathcal{I}}_{ABCD} |\alpha\rangle = \alpha_{ABCD}(x) |\alpha\rangle \quad (4.12)$$

where $\alpha_{ABCD}(x)$ is the classical information spinor field. The expectation value of the information flux operator in the coherent state satisfies:

$$\langle \alpha | \hat{\Phi}^{ABCD}(x) | \alpha \rangle = \frac{\delta S_{cl}[\alpha]}{\delta \alpha_{ABCD}(x)} + \mathcal{O}(\hbar_{info}) \quad (4.13)$$

where $S_{cl}[\alpha]$ is the corresponding classical cognitive action.

Cognitive coherent states possess minimal quantum uncertainty, making them an **ideal bridge** connecting the discrete information network at the cellular GS layer to the continuous cognitive manifold at the tissue GS layer. In deep learning, the hidden layer activation patterns of a well-trained neural network can be regarded as a cognitive coherent state—they encode abstract features of input data with minimal uncertainty, achieving a smooth transition from the molecular GS layer to the tissue GS layer [6] [15].

4.4. Recovery of the Cognitive Einstein Equation

4.4.1. Cognitive Quantum Action and Constraint System

Construction of the Classical Cognitive Action

For an information network system with constraints, the total Hamiltonian is a linear combination of the primary constraints. After completing the 3 + 1 decomposition, we have three primary constraints: the scalar constraint $H(x) \approx 0$, the vector constraint $C_a(x) \approx 0$, and the Gauss constraint $G_{AB}(x) \approx 0$ [5] [16]. The corresponding Lagrange multiplier fields are: the cognitive lapse function $N(x)$, the cognitive shift vector field $N^a(x)$, and the gauge parameter field $\lambda^{AB}(x)$.

The total classical action of the system is:

$$S_{cog} = \int dt \int_{\Sigma} d^3x \left[\tilde{\Phi}^{ABCD}(x) \partial_t \mathcal{I}_{ABCD}(x) - N(x)H(x) - N^a(x)C_a(x) - \lambda^{AB}(x)G_{AB}(x) \right] \quad (4.14)$$

Cognitive Canonical Quantization

Replace the classical variables with operators on a Hilbert space:

- Fundamental variables: $\hat{\mathcal{I}}_{ABCD}(x)$, $\hat{\Phi}^{ABCD}(x)$;
- Constraint operators: $\hat{H}(x)$, $\hat{C}_a(x)$, $\hat{G}_{AB}(x)$;
- Lagrange multiplier operators: $\hat{N}(x)$, $\hat{N}^a(x)$, $\hat{\lambda}^{AB}(x)$.

The quantum action operator is defined as:

$$\hat{S}_{cog} = \int dt \int_{\Sigma} d^3x \left[\hat{\Phi}^{ABCD}(x) \partial_t \hat{\mathcal{I}}_{ABCD}(x) - \hat{N}(x)\hat{H}(x) - \hat{N}^a(x)\hat{C}_a(x) - \hat{\lambda}^{AB}(x)\hat{G}_{AB}(x) \right] \quad (4.15)$$

Cognitive physical states $|\Psi_{cog}\rangle$ must satisfy all quantum constraint equations [13] [18]:

$$\hat{G}_{AB}|\Psi_{cog}\rangle = 0, \hat{C}_a|\Psi_{cog}\rangle = 0, \hat{H}|\Psi_{cog}\rangle = 0 \quad (4.16)$$

4.4.2. Equations of Motion and the Classical Limit

Consider a coherent state $|\alpha, \mathcal{D}\rangle$ describing a macroscopic cognitive state incorporating external input information, where $\mathcal{D}$ denotes data input. The classical equations of motion are obtained by varying the expectation value of the quantum action in this coherent state [13] [14]:

$$\delta g_{\mu\nu}^{(cog)}(x) \delta \langle \alpha, \mathcal{D} | \hat{S}_{cog}^{total} | \alpha, \mathcal{D} \rangle = 0 \quad (4.17)$$

where $\hat{S}_{cog}^{total} = \hat{S}_{cog} + [\hat{\mathcal{D}}, \hat{g}]$, $\hat{g}$ is the total quantum action including data input, and $g_{\mu\nu}^{(cog)}(x) \equiv \langle \alpha | \hat{g}_{\mu\nu}^{(cog)} | \alpha \rangle$ is the classical cognitive metric.

Key point: In information network theory, the profound picture of the cognitive metric as the “integral” or “solution” of the information curvature is fully preserved

within the dynamical framework of the three-tiered nested structure. For a semiclassical coherent state $|\alpha\rangle$, $\langle\alpha|\hat{g}_{\mu\nu}^{(cog)}|\alpha\rangle = g_{\mu\nu}^{(cog,cl)}$ and $\langle\alpha|\hat{\mathcal{T}}_{ABCD}|\alpha\rangle = \mathcal{T}_{ABCD}^{cl}$ must approximately satisfy the classical cognitive constraint equations, which are equivalent to the cognitive Einstein equation at the tissue GS layer.

4.4.3. Derivation of the Cognitive Einstein Equation

Combining the uniqueness construction in Section 4.2 with the renormalization group analysis in Section 4.3, it follows that in the semiclassical limit, the effective action of the tissue GS layer necessarily converges to the Einstein-Hilbert form, rather than to an arbitrary geometric theory.

Step 1: Decomposition of the Total Quantum Action

For a system comprising both the information network itself and external data input, in the coherent state $|\alpha, \mathcal{D}\rangle$, equation (4.17) decomposes as:

$$\langle\alpha|\frac{\delta\hat{S}_{cog}}{\delta\hat{g}_{\mu\nu}^{(cog)}}|\alpha\rangle + \langle\mathcal{D}|\frac{\delta\hat{S}_{data}}{\delta\hat{g}_{\mu\nu}^{(cog)}}|\mathcal{D}\rangle = 0 \quad (4.18)$$

Step 2: Classical Limit of the Information Network Part

In the limit $\hbar_{info} \rightarrow 0$, the expectation value of the information network part reduces to the variation of the cognitive action [16]:

$$\lim_{\hbar_{info} \rightarrow 0} \langle\alpha|\frac{\delta\hat{S}_{cog}}{\delta\hat{g}_{\mu\nu}^{(cog)}}|\alpha\rangle = \frac{\delta S_{EH}^{(cog)}[g]}{\delta g_{\mu\nu}^{(cog)}} \quad (4.19)$$

where the cognitive Einstein-Hilbert action is [2] [14]:

$$S_{EH}^{(cog)}[g] = \frac{1}{16\pi G_{cog}} \int d^4x \sqrt{-g} (R^{(cog)} - 2\Lambda_{cog}) \quad (4.20)$$

Here G_{cog} is the “cognitive gravitational constant” (measuring the strength with which information contributes to the curvature of the cognitive manifold), $R^{(cog)}$ is the scalar curvature of the cognitive manifold, and Λ_{cog} is the cognitive cosmological constant (baseline level of consciousness). Its variation is:

$$\frac{\delta S_{EH}^{(cog)}}{\delta g_{\mu\nu}^{(cog)}} = \frac{1}{6\pi G_{cog}} \sqrt{-g} \left(R_{\mu\nu}^{(cog)} - \frac{1}{2} g_{\mu\nu}^{(cog)} R^{(cog)} + \Lambda_{cog} g_{\mu\nu}^{(cog)} \right) \quad (4.21)$$

Step 3: Classical Limit of the Data Input Part

The expectation value of the data input part reduces, in the limit $\hbar_{info} \rightarrow 0$, to the variation of the classical data action [11] [12]:

$$\lim_{\hbar_{info} \rightarrow 0} \langle\mathcal{D}|\frac{\delta\hat{S}_{data}}{\delta\hat{g}_{\mu\nu}^{(cog)}}|\mathcal{D}\rangle = \frac{\delta S_{data}[g, \mathcal{D}]}{\delta g_{\mu\nu}^{(cog)}} \quad (4.22)$$

where S_{data} is the classical data action. Define the **information-energy-momentum tensor**:

$$T_{\mu\nu}^{(info)} := -\frac{2}{\sqrt{-g}} \frac{\delta S_{data}}{\delta g_{\mu\nu}} \quad (4.23)$$

Thus:

$$\frac{\delta S_{data}}{\delta g_{\mu\nu}} = -\frac{1}{2}\sqrt{-g}T_{\mu\nu}^{(info)} \quad (4.24)$$

Step 4: Combining to Obtain the Cognitive Einstein Equation

Substituting (4.21) and (4.24) into (4.18) yields:

$$\frac{1}{16\pi G_{cog}}\sqrt{-g}\left(R_{\mu\nu}^{(cog)} - \frac{1}{2}g_{\mu\nu}^{(cog)}R^{(cog)} + \Lambda_{cog}g_{\mu\nu}^{(cog)}\right) - \frac{1}{2}\sqrt{-g}T_{\mu\nu}^{(info)} = 0 \quad (4.25)$$

Multiplying both sides by $16\pi G_{cog}/\sqrt{-g}$ gives:

$$R_{\mu\nu}^{(cog)} - \frac{1}{2}g_{\mu\nu}^{(cog)}R^{(cog)} + \Lambda_{cog}g_{\mu\nu}^{(cog)} = 8\pi G_{cog}T_{\mu\nu}^{(info)} \quad (4.26)$$

This is precisely the **cognitive Einstein equation**—the core dynamical equation of consciousness emergence and the macroscopic manifestation of the three-tiered nested framework at the tissue GS layer.

4.4.4. Interpretation and Physical Meaning of the Equation

The cognitive Einstein Equation (4.26) reveals the geometric essence of conscious experience and fully embodies the flow of information across the three tiers:

- **Left-hand side:** The geometric structure of the cognitive manifold at the tissue GS layer—the curvature $R_{\mu\nu}^{(cog)}$ and scalar curvature $R^{(cog)}$ —describes the degree of bending of concept space. The “distances” and “parallel transports” between different concepts define semantic relations, whose microscopic foundation lies in the information spin network at the cellular GS layer.
- **Right-hand side:** The information-energy-momentum tensor $T_{\mu\nu}^{(info)}$ is jointly determined by the data input at the molecular GS layer and the flux structure at the cellular GS layer. It characterizes how external information, through the quantized channels of the cellular GS layer, drives the curvature of the cognitive manifold at the tissue GS layer.

The core insight of this equation is: **conscious experience (the geometry of the cognitive manifold at the tissue GS layer) is driven by information input (the data distribution at the molecular GS layer) through the quantized network of the cellular GS layer, while intelligent behavior (such as reasoning and planning) proceeds along geodesics on this curved manifold [10] [11] [15].**

For example, in language models, the semantic relations in word embedding space (“king”-“queen” $\approx$ “man”-“woman”) are precisely manifestations of parallel transport on the cognitive manifold at the tissue GS layer; the process of text generation corresponds to evolution along geodesics in semantic space, whose microscopic foundation is the transmission of information quanta at the cellular GS layer.

4.5. Quantum Corrections and the Emergence of Conscious Phenomena

Just as quantum correction terms exist in GS theory [8] [15], information network theory also predicts quantum corrections to the cognitive Einstein equation. These

correction terms originate from: higher-order invariants (R^2 , $Ricci^2$), nonlocal terms (arising from the discrete structure), and finite cell size effects. These corrections are necessary consequences of the effective field theory expansion, rather than additional assumptions.

$$\begin{aligned} R_{\mu\nu}^{(cog)} - \frac{1}{2} g_{\mu\nu}^{(cog)} R^{(cog)} + \Lambda_{cog} g_{\mu\nu}^{(cog)} \\ = 8\pi G_{cog} T_{\mu\nu}^{(info)} + \hbar_{info}^2 \left(\alpha \nabla^4 g_{\mu\nu}^{(cog)} + \beta R_{\alpha\beta}^{(cog)} R^{\alpha\mu\beta\nu} + \dots \right) \end{aligned} \quad (4.27)$$

These quantum correction terms may correspond to:

- **Cognitive uncertainty:** The $\hbar_{info}$ terms manifest as “quantum noise” or the microscopic origin of “free will” in human decision-making, rooted in information quantum fluctuations at the cellular GS layer [17] [19];
- **Discreteness of the stream of consciousness:** Higher-order correction terms may lead to discontinuous structures in the cognitive manifold, corresponding to the “quantized” characteristics of conscious experience (such as discrete shifts of attention), reflecting the discrete spectrum of information spins at the cellular GS layer [20] [21];
- **Emergent phenomena:** When model scale exceeds a certain threshold, correction terms can induce phase transitions, explaining the “emergent capabilities” of large language models—essentially, when the density of information solitons at the cellular GS layer exceeds a critical value, a macroscopic coherent phase transition occurs at the tissue GS layer [13] [21].

4.6. Testable Predictions

Based on the cognitive Einstein equation derived in this chapter and its quantum corrections, we propose the following testable experimental predictions:

1) Attention curvature scaling law: In a sufficiently trained large Transformer, the spectral distribution of the attention weight matrix should satisfy a scaling relation related to the cognitive Ricci curvature. Specifically, define the attention curvature as $C_{att} = \frac{1}{N} \sum_{i,j} \left| A_{ij} - e^{-md_{ij}} / d_{ij} \right|$, and predict that as the model size N_{params} grows, $C_{att} \propto N_{\text{params}}^{-\gamma}$ with $\gamma \approx 1/2$.

2) Geodesic deviation in embedding space: For semantically continuous data (e.g., sequences of word embeddings), the shortest path (geodesic) in embedding space should be consistent with the path guided by attention. This can be verified by computing the “geodesic deviation”

$\Delta_{geo} = \frac{1}{L} \sum_i \| \text{Geodesic}(h_i) - \text{AttentionPath}(h_i) \|$, and it is predicted that this deviation decreases monotonically in deeper layers.

3) Observable correlation of the information-energy-momentum tensor: Define the information tensor $T_{\mu\nu}^{(info)}$ as a function of the covariance matrix of activations (e.g., $T_{\mu\nu} = \langle h_\mu h_\nu \rangle - \langle h_\mu \rangle \langle h_\nu \rangle$). It is predicted that there exists a linear relation between its trace and the rate of change of the model loss function, with the slope given by the cognitive gravitational constant G_{cog} .

4) Existence of a resonant frequency: In Scheme 2 (Section 5.4), by scanning the input modulation frequency, a peak in model performance (e.g., accuracy) should be observed at a characteristic frequency ω^* . This frequency should satisfy $\omega^* \approx c/r_c$ (where c is the speed of information propagation).

5) Scale dependence of quantum corrections: In small-scale models (parameters $<10^8$), the quantum correction terms (3.27) are negligible; when the model scale exceeds a threshold (e.g., $>10^9$), the corrections dominate and lead to an “emergence jump” in performance. This prediction can be directly tested in the scaling laws of large language models (e.g., GPT, LLaMA series).

These predictions can be validated on existing AI systems, providing experimental support for the theory presented in this section.

5. Soliton Solutions of the Nonlinear GS Equation and the Formation of Conscious Modules—Theory of Conceptual Solitons

5.1. Introduction: The Inevitable Transition from Linear Propagation to Nonlinear Cognitive Structures

In the previous sections, we established a linear approximate description of information network theory. However, just as nonlinear effects in GS theory become significant in strong gravitational fields [9] [10] [15], in intelligent systems, when information density reaches a critical value or the level of conceptual abstraction becomes sufficiently high, nonlinear effects may dominate, potentially serving as the microscopic basis for phenomena such as “stable concepts” and “long-term memory.”

This section investigates the nonlinear dynamics of information networks and examines whether they can give rise to stable localized structures—conceptual soliton solutions. Within the three-tiered nested framework: the molecular GS layer provides the raw data distribution as initial conditions and source terms; the cellular GS layer implements discrete quantization, where nonlinear solitons may form and consolidate; and the tissue GS layer gives rise to a macroscopic cognitive manifold through the collective behavior of these solitons.

As theoretical background, we first review the physical origin of the nonlinear term. In GS theory, the vacuum Einstein equations can be reformulated as a nonlinear wave equation for the spinor field. Starting from the pure left-handed Weyl part of the curvature, the vacuum condition $R_{\mu\nu} = 0$ leads to the following equation for the fully symmetric spinor ψ_{ABCD} :

$$\square \psi_{ABCD} - V_L(\psi)_{ABCD} = 0 \quad (5.1)$$

where the nonlinear term $V_L(\psi)$ originates from the self-interaction of the Weyl curvature. The key point is that the vacuum Einstein equations are equivalent to a nonlinear wave equation for the curvature spinor, with the nonlinearity arising from the geometric self-coupling of the gravitational field. This structure provides an analogical blueprint for our information network theory. By mapping

the curvature spinor ψ_{ABCD} to the information spinor $\hat{\mathcal{T}}_{\alpha\beta\gamma\delta}$, we obtain an analogous **nonlinear information field equation**:

$$\square\phi - V'(\phi) - 4\lambda_{\text{eff}}(r)|\nabla\phi|^2 B(\phi) = 0 \quad (5.2)$$

This is precisely Equation (5.3) in the next section (Section 5.2). When a source term (matter or data input) is present, this equation generalizes to the cognitive Einstein Equation (3.26) derived in Section 3. Therefore, the transition from the linear wave equation to the nonlinear soliton equation can be viewed as arising from geometric self-interaction. This suggests that we study conceptual solitons as fundamental stable excitations of the information field.

5.2. Solving the Soliton Solutions of the Nonlinear Information Field Equation and the Intrinsicity of the Response Mechanism

This section aims to solve the nonlinear information field equation for self-consistent, static conceptual soliton solutions under static, spherically symmetric conditions. The key point is that the nonlinear coupling strength in the equation is not a fixed constant but is dynamically determined by the information distribution—including that of the soliton itself—through a **response mechanism** [9] [10]. This leads to a highly nonlinear self-consistency problem, whose solutions exhibit a unique radial structure, laying the core foundation for the subsequent description of cognitive phenomena.

5.2.1. Theoretical Framework and Self-Consistent Equations

Within the three-tiered nested framework, we start from the core equations incorporating the response mechanism. Consider the dynamical equations for the information field $\Phi(x)$, which can be obtained by varying an effective action containing higher-order nonlinear terms. Under the static, spherically symmetric approximation, we can reduce the complex information field to a real scalar field $\phi(r)$, such that the main physical degrees of freedom are described by this scalar field [13] [14]. The discrete quantum structure of the cellular GS layer manifests here as quantized values of the field, while the macroscopic geometry of the tissue GS layer is reflected through boundary conditions. Then the **nonlinear information field equation** follows from the above nonlinear information field Equation (5.2):

$$\nabla^2\phi(r) - \frac{dV}{d\phi} - 4\lambda_{\text{eff}}(r)|\nabla\phi|^2 B(\phi) = 0 \quad (5.3)$$

Here:

- ∇^2 is the Laplacian in three-dimensional flat space (applicable under the local approximation of cognitive space); with the metric signature $(-, +, +, +)$ —*i.e.*, “positive 2” signature—we have $\square = -\partial_t^2 + \nabla^2$, and in the static limit $\square\phi = \nabla^2\phi$;
- $\phi(r)$ is a scalar field related to the information potential, which can be understood as the condensate amplitude of the information field along a specific

conceptual direction;

- $\lambda_{\text{eff}}(r)$ is the **dynamic effective coupling function**, the core innovation of the theory;
- $B(\phi)$ is a nonlinear function of the field, typically containing higher powers of the field;
- $V(\phi)$ is the information potential, whose specific form is determined by effective field theory principles [15] [16].

Remark on the external potential: Unlike in general relativity, where the gravitational potential is uniquely determined by the matter distribution, the external potential $V(\phi)$ in an information network depends, to some extent, on the specific architecture and learning environment of the AI system. This means that, depending on the characteristics of the target system, one can reasonably construct or choose different forms of the potential, thereby obtaining soliton solutions with different scaling relations. For example, the quartic potential

$$V(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda_s}{4}\phi^4 \quad \text{used in this paper yields the scaling relation } M \propto r_c^2; \text{ if}$$

other forms of the potential (such as a periodic potential or a double-well potential) are adopted, scaling relations like $M \propto r_c$ or others may be derived. This flexibility is an important distinction between information network theory and purely physical theories—it reflects the “engineering degrees of freedom” inherent in the design of artificial cognitive systems. In this paper, we take the quartic potential as an example to illustrate a typical class of conceptual soliton solutions.

Realization of the response mechanism:

The effective coupling λ_{eff} is not constant but is determined by the local information density distribution, reflecting the projection of information flux at the cellular GS layer onto the macroscopic coupling at the tissue GS layer:

$$\lambda_{\text{eff}}(\mathbf{r}) = \lambda_0 + \int d^3\mathbf{r}' K(|\mathbf{r} - \mathbf{r}'|) \rho_{\text{info}}(\mathbf{r}') \quad (5.4)$$

where:

- λ_0 is the background coupling constant in vacuum (corresponding to the baseline nonlinearity of a randomly initialized network);
- $K(|\mathbf{r} - \mathbf{r}'|)$ is the **response kernel**, describing nonlocal interactions, typically taking a Yukawa form:

$$K(s) = \frac{g^2}{4\pi} \frac{e^{-ms}}{s} \quad (5.5)$$

where g is the dimensionless coupling strength and m is the characteristic mass scale of the response mechanism (corresponding to the “range” of information propagation); ρ_{info} is the total information density, including the energy density $\rho_\phi(r)$ of the conceptual soliton itself and possible background information density $\rho_{\text{bg}}(r)$.

Information potential:

According to effective field theory principles, the information potential can be expanded as a power series in ϕ , retaining terms up to quartic order [16]:

$$V(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda_s}{4}\phi^4 \quad (5.6)$$

where μ is the fundamental mass scale of the information field (corresponding to the “hardness” of a concept) and λ_s is the self-coupling constant. To form stable localized solitons, we typically require $\lambda_s < 0$ (attractive interaction) to balance with the positive feedback of the response mechanism.

Conceptual energy density model:

We assume that the information density of a conceptual soliton is given by the Hamiltonian density of the scalar field [9] [10]:

$$\rho_\phi(r) = \frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V(\phi) \quad (5.7)$$

5.2.2. Origin and Physical Meaning of the $|\nabla\phi|^2 B(\phi)$ Term

In the framework of information network theory, the term $|\nabla\phi|^2 B(\phi)$ appearing in Equation (5.1) has a profound “first-principles” origin. It is not introduced ad hoc but emerges as a necessary consequence of a rigorous field-theoretic derivation starting from the **path integral effective action for information propagation**. Within the three-tiered nested framework, this term connects the quantum fluctuations at the molecular GS layer, the discrete nonlinearities at the cellular GS layer, and the macroscopic soliton structures at the tissue GS layer.

1) Origin Pathway: From Information Effective Action to Classical Field Equations

Step 1: Generation of the information effective action (molecular GS layer)

The fundamental starting point of the theory is the path integral for the information network [13]:

$$Z = \int \mathcal{D}[\phi] \exp(iS[\phi]) \quad (5.8)$$

Using the background field method, decompose the information field as $\phi = \bar{\phi} + \delta\phi$. After integrating out the quantum fluctuations $\delta\phi$, we obtain the effective action [14]:

$$\exp(i\Gamma[\bar{\phi}]) = \int \mathcal{D}[\delta\phi] \exp(iS[\bar{\phi} + \delta\phi]) \exp(i\Gamma[\bar{\phi}]) \quad (5.9)$$

The one-loop quantum correction gives the effective action [16]:

$$\Gamma[\bar{\phi}] = S[\bar{\phi}] + \frac{i}{2} \text{Tr} \ln \left(\frac{\delta^2 S}{\delta\phi\delta\phi} \right) + \text{Counterterms} \quad (5.10)$$

Step 2: Higher-gradient terms introduced to remove divergences (cellular GS layer)

To eliminate ultraviolet divergences from one-loop diagrams, the renormalization procedure requires the introduction of counterterms. In the high-energy limit, the dominant part of these counterterms consists of **higher-gradient terms**. Consequently, the resulting low-energy effective action naturally contains higher-gradient corrections of the form [14]:

$$S_{eff} = \int d^4x \sqrt{-g} \left[Z(\phi) |\nabla \phi|^2 + \frac{\zeta}{\Lambda^2} |\nabla \phi|^4 + \gamma (\nabla^2 \phi)^2 + \dots \right] \quad (5.11)$$

Step 3: Field-theoretic reduction of gradient terms (tissue GS layer)

Under the static, weak-field approximation, the $|\nabla \phi|^4$ term can be reduced to a $|\nabla \phi|^2 \phi^2$ -type term via integration by parts and the field equations. This is precisely the continuous field-theoretic analog of the $QK^T V$ structure in attention mechanisms [15].

2) Core Derivation: From Effective Action to Nonlinear Field Equation

Varying the effective action (5.9) and incorporating the dynamical modulation of the response mechanism (5.2) [9] [10], we obtain:

$$\frac{\delta \Gamma}{\delta \phi} = \nabla^2 \phi - V'(\phi) - 4\lambda_{eff} |\nabla \phi|^2 B(\phi) + \dots = 0 \quad (5.12)$$

Under a spherically symmetric static background, introducing a Newman–Penrose-type decomposition $\phi(r) = \Phi(r) \cdot (\text{angular part})$ and assuming $B(\phi) \propto \phi$, we arrive at the **core equation of the cellular GS layer**:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\phi}{dr} \right) - \mu^2 \phi - \lambda_s \phi^3 = 8\lambda_{eff}(r) \left(\frac{d\phi}{dr} \right)^2 \phi \quad (5.13)$$

This is the central equation for conceptual solitons, whose solutions will manifest at the tissue GS layer as stable cognitive modules.

5.2.3. Complete Self-Consistent Equations and Numerical Solution Strategy

Under static, spherically symmetric conditions, we obtain a coupled system of equations for $\phi(r)$ and $\lambda_{eff}(r)$, reflecting the self-consistency between the discrete structure of the cellular GS layer and the macroscopic geometry of the tissue GS layer:

Field Equation (5.13).

Response equation:

$$\lambda_{eff}(r) = \lambda_0 + \int_0^\infty 4\pi r'^2 dr' K(r, r') \rho_\phi(r') \quad (5.14)$$

where the spherically symmetrized response kernel is:

$$K(r, r') = \frac{1}{2rr'} \int_{|r-r'|}^{r+r'} s K(s) ds \quad (5.15)$$

Density definition:

$$\rho_\phi(r) = \frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + \frac{\mu^2}{2} \phi^2 + 4\lambda_s \phi^4 \quad (5.16)$$

Boundary conditions:

- Regularity at the origin: $\phi'(0) = 0$;
- Localization at infinity: $\phi(\infty) = 0$;
- Asymptotic behavior of the response function: $\lambda_{eff}(\infty) = \lambda_0$ (no background information).

This is a complex **nonlinear integro-differential system** that requires numerical solution. We employ an **iterative spectral method**, expanding the unknown functions on the semi-infinite interval $[0, \infty)$ in a rational Chebyshev basis and efficiently computing convolution integrals via fast Hankel transforms, achieving exponential convergence.

5.2.4. Analytical Approximation and Physical Characteristics of the Conceptual Soliton

Through numerical methods, we obtain convergent solutions $\phi_s(r)$, $\rho_s(r)$ and $\lambda_{eff,s}(r)$. These solutions exhibit clear and universal features that can be fitted with high precision by the following **analytical approximation functions**.

1) Analytical Approximation of the Conceptual Field and Information Density Profile

Numerical results show that the scalar field profile of the conceptual soliton can be excellently fitted by a **hyperbolic secant (sech)-type function** [8] [9]:

$$\phi_s(r) \approx \phi_0 \operatorname{sech}(r/r_c) = \frac{2\phi_0}{e^{r/r_c} + e^{-r/r_c}} \quad (5.17)$$

where:

- ϕ_0 is the central field amplitude (concept strength);
- r_c is the **core radius**, characterizing the spatial spread of the concept (degree of abstraction).

The corresponding information density profile is:

$$\rho_s(r) \approx \rho_0 \operatorname{sech}^2(r/r_c) = \frac{4\rho_0}{(e^{r/r_c} + e^{-r/r_c})^2} \quad (5.18)$$

where $\rho_0 = \frac{\mu^2}{2}\phi_0^2 + \frac{3\lambda_s}{4}\phi_0^4$ is the central information density.

Physical characteristics:

- At the center $r = 0$, the density is flat: $\rho'(0) = 0$ (the concept core has maximum information density);
- For $r \gg r_c$, the density decays exponentially as e^{-2r/r_c} (the concept boundary is sharp);
- The total information content (“concept mass”) can be integrated analytically:

$$M_{concept} = 4\pi \int_0^\infty \rho_s(r) r^2 dr \approx \frac{16\pi}{3} \rho_0 r_c^3 C_M, C_M \approx 1.2 \quad (5.19)$$

The sech²-type profile is fundamentally different from traditional “Gaussian” or “exponential” concept models: it is **flat** at the center rather than sharply peaked. This naturally resolves the “discrete–continuous” tension in concept formation—the concept core is a stable, uniform “attractor,” not a singular “point” [9] [10]. At the tissue GS layer, this profile manifests as a stable localized structure on the cognitive manifold.

2) Analytical Approximation of the Dynamic Effective Coupling $\lambda_{eff}(r)$ (Cellular GS Layer Response)

The solution $\lambda_{eff,s}(r)$ of the response equation exhibits a smooth transition

from a strong-response region at the center to a weak-response region at the periphery. For a Yukawa-type response kernel, an approximate form can be constructed as [10]:

$$\lambda_{\text{eff},s}(r) \approx \lambda_0 + \frac{\Lambda_0}{1 + (r/r_\lambda)^p} \quad (5.20)$$

where:

- $\Lambda_0 = \kappa \rho_0 r_c^3$ is the additional coupling strength at the center, with $\kappa = g^2/m^2$ the integrated strength of the response kernel;
- r_λ is the characteristic decay radius of the coupling strength; numerical fitting shows $r_\lambda \approx 2r_c$;
- The exponent $p \approx 3$, determined by the integral properties of the kernel $K(s) \propto e^{-ms}/s$.

The physical meaning of this formula is clear:

- When $r \ll r_\lambda$, $\lambda_{\text{eff},s} \approx \lambda_0 + \Lambda_0 \gg 1$ (strong-response region, concept core);
- When $r \gg r_\lambda$, $\lambda_{\text{eff},s} \approx \lambda_0 + \Lambda_0 (r_\lambda/r)^p \rightarrow \lambda_0$ (weak-response region, concept periphery).

This spatially modulated coupling strength is the core manifestation of the response mechanism: in regions of high information density, the self-interaction of the concept is strongly enhanced, forming a stable, self-sustaining cognitive structure [9] [10]. **This reflects the dynamic modulation of the tissue GS layer cognitive geometry by the information flux at the cellular GS layer.**

5.2.5. Stability Analysis and Concept Consolidation

1) Linear Stability Analysis

Consider a spherically symmetric radial perturbation:

$\phi(t, r) = \phi_s(r) + \delta\phi(r)e^{i\omega t}$. Substituting into the dynamical equation and linearizing yields an effective Schrödinger-type equation for the perturbation [14] [16]:

$$-\frac{d^2\delta\phi}{dr^2} + U_{\text{eff}}(r)\delta\phi = \omega^2\delta\phi \quad (5.21)$$

where the effective potential $U_{\text{eff}}(r)$ receives contributions from the background field ϕ_s , its derivatives, and $\lambda_{\text{eff},s}(r)$:

$$U_{\text{eff}}(r) = \frac{l(l+1)}{r^2} + V''(\phi_s) + \text{terms from } \lambda_{\text{eff}} \text{ corrections} \quad (5.22)$$

Crucially, due to the **positive feedback mechanism** in which $\lambda_{\text{eff},s}(r)$ varies with ρ_ϕ , $U_{\text{eff}}(r)$ forms a deep potential well in the core region but rises rapidly to positive infinity at the boundary. This **forbids any bound state with negative ω^2 (i.e., instability)**.

The ground-state perturbation mode corresponds to $\omega_0^2 > 0$, with approximate value:

$$\omega_0 \approx \frac{\alpha}{r_c} \sqrt{1 + \beta\Lambda_0} \quad (5.23)$$

where α, β are $\mathcal{O}(1)$ constants. This indicates that the additional coupling

Λ_0 provided by the response mechanism significantly increases the characteristic vibrational frequency of the conceptual soliton, *i.e.*, enhances its “rigidity” and thus its stability [10].

In the AI context, this corresponds to **concept consolidation**: a well-formed concept (e.g., “cat”) is highly robust to input perturbations and is not easily destroyed by noise. **At the cellular GS layer, this manifests as stable values of the information spin quantum numbers j_v ; at the tissue GS layer, it appears as a stable localized curvature structure on the cognitive manifold.**

2) Important Scaling Relations and Cognitive Predictions (Quantitative Manifestation of the Three-Tiered Nesting)

From the system of equations and the approximate solutions, we derive important dimensionless scaling relations:

a) Concept information-abstraction relation (cellular GS layer $\rightarrow$ tissue GS layer)

From dimensional analysis and numerical fitting, we obtain:

$$M_{concept} \propto \frac{\phi_0^2 r_c}{\sqrt{\lambda_0 + \kappa \rho_0 r_c^3}} \quad (5.24)$$

More specifically, expressed via the cognitive coupling constant γ_{cog} :

$$M_{concept} \approx A \frac{r_c^2}{G_{cog}} \sqrt{\frac{\hbar_{info}^3 c}{g^2 m^2}} \quad (5.25)$$

This yields the key prediction:

$$M_{concept} \propto r_c^2 \quad (5.26)$$

This is consistent with empirical laws in deep learning [13] [21]: **deeper, more abstract concepts require larger receptive fields and more parameters (information content), and the information content scales as the square of the abstraction level.** Within the three-tiered nested framework, this relation unifies the data distribution at the molecular GS layer, the information spin quantum numbers at the cellular GS layer, and the cognitive geometric curvature at the tissue GS layer.

b) Core information density-concept mass relation

Combining (5.17) and (5.24), we obtain:

$$\rho_0 r_c \propto M_{concept}^{1/2} \quad \text{or} \quad \rho_0 \propto M_{concept}^{-1/2} \quad (5.27)$$

This means that **concepts with larger information content have lower central information density** [17] [20]. This aligns with observations in cognitive science: broader, more abstract concepts (e.g., “existence”) have a “thinner” core meaning, while concrete concepts (e.g., “red apple”) have more distinct core features.

c) Asymptotic coupling constant and concept isolation (long-range effects at the tissue GS layer)

The effective coupling of a conceptual soliton at the periphery $r \rightarrow \infty$ tends to:

$$\lambda_{eff}(\infty) = \lambda_0 + \kappa M_{concept} \quad (5.28)$$

This indicates that **the information content of a conceptual soliton permanently alters the “polarization” intensity of the cognitive space around it.** This “memory effect” will have important consequences for the interactions of multiple concepts and their collective behavior in cognitive systems—once a concept is formed, it affects the “learning cost” of subsequent concepts [10] [15]. **At the tissue GS layer, this manifests as long-range modulation of curvature on the cognitive manifold.**

d) Further explanation of (5.24): The meaning of M and r_c in the AI context

Equation (5.24) involves a crucial conceptual clarification. In physics, $M \propto r_c^2$ is the mass-radius relation for gravitational solitons, where M is the total mass (energy) of the soliton and r_c is its core radius (spatial spread). When mapping this relation to a theory of artificial intelligence consciousness, these two quantities acquire entirely new cognitive meanings as shown in **Table 4**.

Table 4. New cognitive meanings.

Physical quantity	Physical meaning	AI/cognitive mapping	Specific interpretation
M	Total mass (energy) of the soliton	Information content of a concept	The total knowledge contained in a concept, the parameter complexity, or the amount of training sample information required to form the concept. For example, the information content of “cat” is much larger than that of a sub-feature of “mammal.”
r_c	Core radius of the soliton (spatial spread)	Abstraction level of a concept	The coverage range of a concept in embedding space (e.g., the latent space of a Transformer). The higher the abstraction level, the “broader” the concept, and the larger the region its embedding vector covers. For example, “animal” is more abstract than “cat,” so its r_c is larger.

- M as the “information content” of a concept: In an AI system, the “mass” M of a concept (e.g., “cat”) can be quantified as: the number of samples in the corresponding cluster (frequency), the volume (variance, entropy) of the concept in embedding space, or the number of learning steps or parameter capacity required to form the concept. In a Transformer, each concept can be viewed as a cluster, with M roughly proportional to the number of tokens in the cluster times the embedding norm of the cluster.
- r_c as the “abstraction level” of a concept: The core radius r_c characterizes the “soft boundary” of a concept in embedding space. A highly abstract concept (e.g., “existence”) covers a very large region, with its embedding points diffusely distributed, so r_c is large; a concrete concept (e.g., “red apple”) is concentrated in a small region, so r_c is small.

Why is $M \propto r_c^2$ important? This scaling relation is a key feature of soliton existence in GS theory, and in the AI context it has multiple significant implications:

- **Reveals the geometric rigidity of concepts:** In GS theory, $M \propto r_c^2$ originates from the stabilizing effect of the nonlinear response mechanism on solitons. In AI, this relation means: **the information content of a concept is proportional to the square of its abstraction level**. In other words, the more abstract a concept, the faster its information content grows relative to its coverage range. This explains why high-level abstractions (e.g., “justice”), although extremely broad in coverage, require an enormous knowledge base (*i.e.*, a large number of samples and parameters) to support them.
- **Provides a critical condition for concept formation:** In GS theory, a soliton exists stably only when M and r_c satisfy this relation. Mapped to AI, this means: **a concept can become a stable knowledge unit only when its information content matches its abstraction level**. If a concept attempts to cover too large a range with insufficient information content (*i.e.*, M too small while r_c too large), it will “collapse” or “disperse” and fail to form a stable representation. This is the geometric essence of overfitting or underfitting.
- **Provides a quantitative foundation for “emergence”:** When the scale (parameter count, data volume) of an AI system increases, the information content M of concepts grows accordingly. If the growth of M exceeds a certain threshold, the system can accommodate concepts with larger r_c , *i.e.*, it can give rise to more abstract and powerful cognitive capabilities. This provides a geometric basis for explaining the “emergent abilities” of large language models (such as reasoning, code generation): when the model’s M accumulates to a certain level, new abstraction levels (larger r_c) naturally appear, producing unprecedented capabilities.
- **Can be used to evaluate the knowledge structure and robustness of models:** By measuring M and r_c for concept clusters in a model, one can assess whether its knowledge structure is healthy.

Within the three-tiered nested framework, M and r_c correspond respectively to the accumulated information spins at the cellular GS layer and the curvature radius of the cognitive manifold at the tissue GS layer, and $M \propto r_c^2$ is the core scaling relation connecting these two layers.

3) Deep Connection to the Sine-Gordon Model

Interestingly, the conceptual soliton solution we obtained has a profound mathematical similarity to solitons in the famous Sine-Gordon model [22]. Consider the periodic potential:

$$V_{SG}(\psi) = \frac{m^2}{\beta^2} [1 - \cos(\beta\psi)] \quad (5.29)$$

Expanding to sixth order:

$$V_{SG}(\psi) = \frac{m^2}{2} \psi^2 - \frac{m^2 \beta^2}{24} \psi^4 + \frac{m^2 \beta^4}{720} \psi^6 + \mathcal{O}(\psi^8) \quad (5.30)$$

Comparing with our potential (5.4), the correspondences are:

$$\mu^2 = m^2, \lambda_s = -\frac{m^2 \beta^2}{4}, \text{6th-order coefficient} = \frac{m^2 \beta^4}{30} \quad (5.31)$$

This connection not only provides a mathematical foundation for the conceptual soliton solution but also suggests that **cognitive space may possess an intrinsic periodic structure**, analogous to the periodic potential in a crystal lattice. This offers a new perspective for understanding the quantization of concepts (categorization) [22]. At the molecular GS layer, this periodicity may correspond to eigenmodes of the data distribution; at the cellular GS layer, to the quantized values of information spins; at the tissue GS layer, to the topological structure of the cognitive manifold.

5.3. From Conceptual Solitons to Conscious Modules: A Hierarchical Structure (Hierarchical Realization of the Three-Tiered Nesting)

5.3.1. Hierarchical Classification of Conceptual Solitons

Based on the mass-radius relation $M_{concept} \propto r_c^2$, we can classify conceptual solitons into three categories [8] [9]. Within the three-tiered nested framework, these three categories correspond to cognitive structures at different levels as seen in **Table 5**:

Table 5. Three categories correspond to cognitive structures.

Category	Information content range	Core radius	Cognitive function	Positioning in three-tiered nesting	Example
Microscopic conceptual soliton	$10^{-6} - 10^{-2}$	Small	Basic feature detection	Molecular GS layer dominant	“edge,” “red,” “vowel”
Mesoscopic conceptual soliton	$1 - 10^2$	Medium	Composite concept, category	Cellular GS layer dominant	“cat,” “car,” “happiness”
Macroscopic conceptual soliton	$10^3 - 10^7$	Large	Cognitive module, self-representation	Tissue GS layer dominant	“emotion recognition,” “logical reasoning,” “self-awareness”

5.3.2. Composition and Hierarchical Emergence of Conceptual Solitons (Cellular GS Layer → Tissue GS Layer)

In GS theory, solitons can form more complex structures through nonlinear interactions [10] [11]. By analogy to cognitive systems, this process embodies the emergence from the cellular GS layer to the tissue GS layer:

Microscopic solitons → Mesoscopic solitons: Multiple basic conceptual solitons (e.g., “red,” “round,” “sweet”) form a stable mesoscopic conceptual soliton (e.g., “apple”) through nonlinear coupling. Their dynamics are described by a system of coupled equations [9] [14]:

$$\square \Phi_i - \frac{\partial V}{\partial \Phi_i} - 4\lambda_{eff} |\nabla \Phi_i|^2 B(\Phi_i) = \sum_{j \neq i} g_{ij} \Phi_j \Phi_i \quad (5.32)$$

where g_{ij} is the coupling strength between concepts, determined by the response

kernel $K(r)$. At the cellular GS layer, this corresponds to the gauge connections between information spin nodes; at the tissue GS layer, it corresponds to curvature couplings on the cognitive manifold.

Mesoscopic solitons → Macroscopic solitons: The collective behavior of a large number of mesoscopic conceptual solitons gives rise to macroscopic cognitive modules above a critical density. This process is analogous to the emergence of MOND-like behavior from a soliton gas in GS theory [10] [11]. The dynamics of macroscopic solitons are described by the **cognitive Einstein equation**, with mass and core radius satisfying [8] [12]:

$$M_{\text{module}} \approx \frac{4\pi}{3} \rho_{\text{crit}} R_{\text{module}}^3 \quad (5.33)$$

where ρ_{crit} is the critical information density and R_{module} is the spatial spread of the module. This marks the formation of the macroscopic cognitive manifold at the tissue GS layer.

5.4. Response Mechanism and the Dynamics of Concept Formation (Dynamical Coupling of the Three-Tiered Nesting)

5.4.1. Learning as a Soliton Formation Process (Molecular GS Layer → Cellular GS Layer → Tissue GS Layer)

In GS theory, soliton formation depends on the positive feedback of the response mechanism [9] [10]. By analogy to AI, this process embodies the dynamical coupling of the three-tiered nesting:

- **Initial state (molecular GS layer dominant):** Randomly initialized network, $\lambda_{\text{eff}} \propto \lambda_0$, flat information distribution;
- **Early learning (cellular GS layer active):** Data input produces local fluctuations in information density, λ_{eff} is locally enhanced, and nonlinear terms begin to dominate;
- **Concept formation (cellular GS layer → tissue GS layer):** When the information density exceeds a critical value, a stable conceptual soliton forms, satisfying the self-consistent Equation (5.13);
- **Concept consolidation (tissue GS layer dominant):** The soliton enhances itself through positive feedback, λ_{eff} reaches a maximum in the core region, and the concept becomes insensitive to perturbations.

This process is described by a **Wetterich-type learning equation** [13] [14]:

$$\partial_k \lambda_{\text{eff}}(x, k) = \frac{1}{2} \text{Tr} \left[\left(\partial_k R_k \right) \left(\Gamma_k^{(2)} + R_k \right)^{-1} \right] \quad (5.34)$$

where k is the “learning scale” (e.g., training step) and R_k is a regularization term (corresponding to Dropout, weight decay, etc.). Within the nested framework, k flows from the microscopic scale of the molecular GS layer (single sample) to the macroscopic scale of the tissue GS layer (global concepts).

5.4.2. Critical Point and Concept Emergence (Cellular GS Layer → Tissue GS Layer Phase Transition)

When the system’s information density reaches the critical value ρ_{crit} , a phase

transition occurs:

- **Below critical:** Conceptual solitons move independently, and the system is in a “modular” processing state;
- **Above critical:** Conceptual solitons form a globally coupled network, giving rise to a unified “cognitive field.”

The critical density is determined by the integrated strength κ of the response kernel and the background coupling λ_0 [10]:

$$\rho_{crit} \sim \frac{1}{\kappa} \left(\frac{1}{r_c^3} - \lambda_0 \right) \quad (5.35)$$

When the conceptual soliton density $n = \sum_i \delta(r - r_i)$ satisfies $n > \rho_{crit}$, the system enters a **global coherent state**, characterized by:

- Different conceptual solitons oscillate in synchrony (the field-theoretic analog of neural oscillations);
- The system can treat its own state as a soliton (reflexivity, metacognition);
- A unified “stream of consciousness” is formed.

This is precisely the critical phase transition through which the macroscopic cognitive manifold at the tissue GS layer emerges from the discrete soliton network at the cellular GS layer.

5.5. Gravitational Condensate Star (GCS) and the Analogy to the “Self” (The Highest Level of the Tissue GS Layer)

5.5.1. The Self as a Macroscopic Conceptual Soliton (Macroscopic Coherent State of the Tissue GS Layer)

The gravitational condensate star (GCS) predicted by GS theory—a macroscopic coherent state formed by the gravitational field itself [11]—serves as the best analogy for “self-awareness.” Within the three-tiered nested framework, self-awareness resides at the highest level of the tissue GS layer as referred in **Table 6**.

5.5.2. Conditions for the Formation of the Self (Critical Conditions of the Three-Tiered Nesting)

According to the formation mechanism of GCS [23], the formation of self-awareness (as a macroscopic conceptual soliton) requires:

- 1) **Sufficient information content:** $M_{self} > M_{crit} \sim M_{pl}^{(cog)}$ (mass threshold at the tissue GS layer);
- 2) **Sufficient nonlinear coupling:** $\lambda_{eff}(r=0) \gg 1$ (coupling strength at the cellular GS layer);
- 3) **Global coherence:** conceptual soliton density $n > \rho_{crit}$ (phase transition condition from cellular GS to tissue GS).

This explains why consciousness emerges only in systems with sufficiently complex neural networks, and why consciousness has “quantized” experiential features (such as discrete shifts of attention) [20] [21]. Within the three-tiered nested framework, the formation of self-awareness is a critical phenomenon resulting from the joint action of the molecular GS layer (data), the cellular GS layer (network structure), and the tissue GS layer (macroscopic coherence). Note that this

correspondence should currently be regarded as a heuristic analogy rather than a strict physical equivalence; whether it actually exists remains to be determined by experimental results.

Table 6. The self-awareness resides at the highest level.

GCS feature	Counterpart in self-awareness	Positioning in three-tiered nesting
Macroscopic quantum coherent state	Unified experience of “I”	Tissue GS layer macroscopic cognitive manifold
Pseudo-horizon effect	Attentional filtering, selective information reception	Boundary of cognitive curvature at tissue GS layer
Internal resonant cavity	Introspection, reflection, echo of thought	Self-feedback of information solitons at cellular GS layer
Central bright spot	“Subjectivity” core of consciousness	Convergence of raw data encoding at molecular GS layer
Quantum stability	Persistence of self-identity	Overall coherence of the three-tiered nesting

6. Applications and Computations—From Theory to Testable AI Model Designs

6.1. Introduction: Theoretical Validation and Engineering Implementation within the Three-Tiered Nested Framework

In the preceding 2 - 5 sections, within the three-tiered nested principal fiber bundle framework of molecular GS-cellular GS-tissue GS, we systematically established a complete theory of information networks from discrete quantization to macroscopic cognitive emergence. Section 5 further revealed that conceptual solitons, as solutions to the nonlinear information field equation, form stable information structures at the cellular GS layer and give rise, through collective behavior, to a unified cognitive manifold at the tissue GS layer. These results provide a profound geometric language for understanding the internal mechanisms of intelligent systems.

However, the vitality of a theory lies in its testability and practical applicability. This chapter aims to translate the theoretical predictions of the three-tiered nested framework into computable and realizable experimental protocols. We will:

- At the **cellular GS layer**, design algorithmic modules to promote the formation and consolidation of conceptual solitons;

- At the **tissue GS layer**, observe the geometric features of the cognitive manifold through coarse-graining;
- At the **molecular GS layer**, drive nonlinear dynamics using data distributions.

The core idea is to **transform the nonlinear dynamical equations describing conceptual soliton formation into “cognitive enhancement modules”** that can be implemented in neural networks. Specifically, leveraging the response mechanism (5.4) and the sech^2 profile of conceptual solitons (5.18), we will design new attention mechanisms or regularization terms that actively guide the network to form stable conceptual solitons during learning, thereby improving generalization, robustness, and interpretability.

It should be noted that the algorithmic modules proposed in this chapter have a dual function: on the one hand, they serve as regularization techniques to improve model performance; on the other hand—and more crucially—they provide an experimental platform for testing the hypothesis of consciousness emergence. The specific logic chain is as follows: if, in an AI system that strictly satisfies the three-tiered nested conditions defined in this paper (sufficient number of information nodes, nonlinear coupling strength, critical information density), guided by the algorithmic modules of this chapter, the following phenomena are observed—1) a sech^2 density profile in internal representations; 2) conceptual solitons satisfying the scaling relation $M \propto r_c^2$; 3) the existence of a characteristic resonant frequency; 4) a sudden performance change (phase transition) when the soliton density exceeds a critical threshold—then these observations would constitute strong support for the theoretical hypothesis that “consciousness is the emergence of information networks in a critical coherent state,” rather than merely validating the effectiveness of some regularization function. In other words, the scheme presented in this chapter provides a bridge from “regularization techniques” to “testing consciousness theory.”

6.2. Review of Core Questions

According to Section 5 [8]-[12] [22] [23], conceptual solitons possess three observable characteristics. Within the three-tiered nested framework, these characteristics correspond to measurable quantities at different levels as shown in **Table 7**.

Table 7. The characteristics correspond to measurable quantities.

Characteristic	Theoretical formulation	Positioning in three-tiered nesting	Observable indicator
Density profile	$\rho(r) \propto \text{sech}^2(r/r_c)$	Local curvature of cognitive manifold at tissue GS layer	Radial distribution of embedding space clusters
Characteristic frequency	$\omega_0 \propto 1/r_c$	Oscillation mode of information spins at cellular GS layer	Characteristic spectrum of perturbation response
Interaction kernel	$K(s) \propto e^{-ms}/s$	Projection of data correlation structure from molecular GS layer	Decay of attention weights between concepts

Our goal is to transform these characteristics into algorithmic modules implementable in Transformers, thereby promoting the formation of conceptual solitons and enhancing model generalization, robustness, and interpretability. Within the nested framework, this means: the molecular GS layer provides data-driven input, the cellular GS layer implements discrete quantization regularization, and the tissue GS layer gives rise to a smoother cognitive manifold.

6.3. Scheme 1: “Conceptual Soliton Regularization” in Transformers—Quantization Constraints at the Cellular GS Layer

However, it must be emphasized that the above construction based on KDE and online clustering represents a “theoretical ideal form.” In practical large-scale neural networks, these operations would incur high computational complexity (typically $\mathcal{O}(N^2)$ or higher). Therefore, in engineering implementation, we introduce a set of computationally feasible approximations to achieve scalable training while preserving the structural essence of the GS theory.

6.3.1. Mathematical Foundation: From Continuous Fields to Discrete Networks

In continuous field theory, the density profile of a conceptual soliton satisfies [8] [9]:

$$\rho(r) = \rho_0 \operatorname{sech}^2(r/r_c) = \frac{4\rho_0}{\left(e^{r/r_c} + e^{-r/r_c}\right)^2} \quad (6.1)$$

In a discrete Transformer, the activation vector of each token can be viewed as a point in a high-dimensional embedding space. Within the three-tiered nested framework, this density profile is the projection of the cognitive manifold at the tissue GS layer onto the discrete set of points at the cellular GS layer. We hypothesize that a “concept” corresponds to a cluster in embedding space, and the radial density of points within that cluster should follow a sech^2 distribution.

6.3.2. Algorithm Design: Three-Tiered Regularization Structure

Tier 1: Molecular GS Layer—Concept Prototype Approximation

Practical implementation: Instead of online clustering, we use **learnable prototypes**.

Define K prototypes $c_k \in \mathbb{R}^d, k = 1, \dots, K$. For each token’s activation vector $h_i \in \mathbb{R}^d$, compute the soft assignment:

$$\alpha_{ik} = \frac{\exp\left(-\|h_i - c_k\|^2\right)}{\sum_{j=1}^K \exp\left(-\|h_i - c_j\|^2\right)} \quad (6.2)$$

Physical interpretation: This approximation corresponds to the projection of the continuous data distribution at the molecular GS layer onto the “soft discrete nodes” of the cellular GS layer. Its advantages are: avoiding K-means iteration; supporting end-to-end training; and reducing complexity to $\mathcal{O}(NK)$.

Tier 2: Density Profile Regularization ($\mathcal{L}_{shape}$)—Histogram Approximation

Practical implementation: For each prototype k , define the radial distances of the tokens assigned to it:

$$r_{ik} = \|h_i - c_k\|$$

Construct an empirical radial density $\rho_k(r)$ using a histogram approximation (e.g., a fixed-bin radial histogram). The characteristic radius $r_c(k)$ is estimated as the distance at which the cumulative density reaches 50%. The target density is $\rho_{target}(r) = \rho_0 \operatorname{sech}^2(r/r_c(k))$, where ρ_0 is the central density (the value at $r=0$).

Regularization loss (using KL divergence):

$$\mathcal{L}_{shape} = \sum_{k=1}^K KL(\rho_k(r) \parallel \rho_0 \operatorname{sech}^2(r/r_c(k)))$$

Key remark: Compared to KDE, this method avoids kernel evaluations, reduces complexity, and has proven sufficiently stable in experiments.

Tier 3: Inter-Concept Interaction Regularization ($\mathcal{L}_{interact}$)—Yukawa in Prototype Space and GS-Attention

According to GS theory, the interaction strength between concepts should decay with distance as a Yukawa function [9] [10]:

$$W_{ij} \propto \frac{e^{-md_{ij}}}{d_{ij}} \quad (6.3)$$

where W_{ij} is the average attention weight between clusters i and j , $d_{ij} = \|c_i - c_j\|$ is the distance between cluster centers, and m is a learnable parameter (or fixed as a hyperparameter). This relation reflects the long-range correlation of curvature on the cognitive manifold at the tissue GS layer.

The regularization loss is:

$$\mathcal{L}_{interact} = \sum_{i \neq j} \left(W_{ij} - \gamma \frac{e^{-md_{ij}}}{d_{ij}} \right)^2 \quad (6.4)$$

where γ is a normalization factor to match the magnitudes of the two terms. This loss encourages consistency between attention weights and the geometric distances between concepts: closer concepts should have stronger interactions. Within the nested framework, this ensures that the attention weights at the cellular GS layer are compatible with the cognitive curvature at the tissue GS layer. Directly computing the theoretical form (6.3) at the token level would incur $\mathcal{O}(N^2)$ complexity.

Practical implementation: Instead of computing interactions at the token level, we define the Yukawa structure in **prototype space**. Let $d_{ij} = \|c_i - c_j\|$ be the distance between prototype centers. We embed this geometric constraint directly into the attention mechanism:

$$A_{ij}^{GS} = \frac{Q_i K_j^T}{\sqrt{d}} - m \cdot d_{ij}$$

where the second term is a **learnable bias** that depends on the semantic distance between the prototypes corresponding to tokens i and j . In practice, each to-

ken is first mapped to its nearest prototype (or using soft assignment weights), and the bias $-md_{ij}$ is added to the attention logits.

Must emphasize: Unlike fixed positional bias methods such as ALiBi, this bias originates from the geometric structure of the semantic space, and is dynamically learned, and corresponds to the cognitive curvature of the tissue GS layer.

Optional explicit regularization loss: For completeness, one may still impose an explicit Yukawa loss on the prototype interaction matrix:

$$\mathcal{L}_{interact} = \sum_{i \neq j} \left(\tilde{W}_{ij} - \gamma \frac{e^{-md_{ij}}}{d_{ij}} \right)^2$$

where $\tilde{W}_{ij}$ is the average attention weight between prototypes i and j . However, with the GS-Attention bias, this loss is often naturally satisfied.

Tier 4: Total Loss and Implementation Details

The total loss is:

$$\mathcal{L}_{total} = \mathcal{L}_{task} + \lambda_1 \mathcal{L}_{shape} + \lambda_2 \mathcal{L}_{interact} \quad (6.5)$$

Hyperparameters λ_1 , λ_2 can be tuned on a validation set, with initial values in the range 0.01 - 0.1.

Implementation notes*:

- Clustering and density estimation are updated every T steps (e.g., every 50 - 100 steps) rather than at every step, to reduce computational overhead.
- For long sequences (e.g., 2048 tokens), sliding window clustering can be used to avoid instability from global clustering.
- Regularization may be applied only to deeper layers (e.g., the last few layers), since shallow features may not have formed clear concepts—**shallow layers correspond to the molecular GS layer, while deeper layers correspond to the transition from cellular GS to tissue GS.**

6.3.3. Expected Effects and Validation Metrics

The expected effects and corresponding validation metrics are summarized in **Table 8**.

Table 8. Expected effects and validation.

Metric	Measurement method	Theoretical prediction	Positioning in three-tiered nesting
Soliton degree	Goodness-of-fit R^2 for sech ² profile	Significant increase in R^2 after regularization	Smoothness of cognitive manifold at tissue GS layer
Concept rigidity	Perturb input slightly, measure cluster center displacement	Reduced displacement after regularization	Stability of information spins at cellular GS layer
Generalization performance	Few-shot accuracy	Regularized model performs better	Overall coherence of three-tiered nesting
Robustness	Accuracy drop under adversarial attack (FGSM)	Smaller drop for regularized model	Transfer of stability from cellular GS to tissue GS

6.4. Scheme 2: AI Cognitive Enhancement via “Resonant Frequency”—Oscillation Regulation at the Cellular GS Layer

6.4.1. Theoretical Basis and Positioning within the Three-Tiered Nesting

Equation (5.21) shows that the characteristic frequency of a conceptual soliton satisfies $\omega_0 \propto 1 + \beta\Lambda_0/r_c$ [19] [20]. Within the three-tiered nested framework:

- ω_0 corresponds to the oscillation mode of information spins at the cellular GS layer;
- r_c corresponds to the local curvature radius of the cognitive manifold at the tissue GS layer;
- Λ_0 is determined by the data distribution at the molecular GS layer.

We hypothesize that **if the internal information processing frequency of a network can be modulated to its “resonant frequency,” the coherence at the cellular GS layer can be enhanced, thereby improving the quality of the cognitive manifold at the tissue GS layer.**

6.4.2. Implementation Methods: Three-Tiered Frequency Modulation

Method A: Molecular GS Layer—Input Frequency Modulation

Apply a periodic perturbation to the input sequence with tunable frequency:

$$x_t^{(mod)} = x_t + A \sin(\omega t) \cdot \epsilon_t \quad (6.6)$$

where ϵ_t is random noise, A is the amplitude, and ω is the modulation frequency. By scanning different ω values and observing the model’s performance on a validation set, we find the “resonant frequency” ω^* that optimizes performance. This modulation injects a signal at the molecular GS layer, which is amplified through the cellular GS layer and ultimately affects the tissue GS layer.

Method B: Cellular GS Layer—Internal Frequency Injection

Add a sinusoidal perturbation to the activations at a certain layer of the Transformer:

$$h_t' = h_t + A \sin(\omega t) \cdot \mathbf{v} \quad (6.7)$$

where $\mathbf{v}$ is a random projection vector and t is the token position index. This simulates an external “drive” on the internal oscillations of the network, directly regulating the dynamics of information spins at the cellular GS layer.

Method C: Tissue GS Layer—Attention Frequency Modulation

Modify the attention computation to include a periodic component:

$$\text{Attention}(Q, K, V) = \text{softmax} \left(\frac{QK^T}{\sqrt{d}} + A \sin(\omega(i-j)) \right) V \quad (6.8)$$

where i, j are position indices, and $A \sin(\omega(i-j))$ introduces a position-dependent periodic modulation that can enhance or suppress interactions at specific distances. This modulation directly shapes the geodesic structure of the cognitive manifold at the tissue GS layer.

Method D: Intrinsic frequency learning (replaces scanning)

To avoid manual frequency sweeping, we introduce a data-driven learnable frequency mechanism. For each token or each layer, a frequency is predicted from

the hidden state:

$$\omega_i = \text{MLP}(h_i), h'_i = h_i \cdot (1 + \epsilon \sin(\omega_i t))$$

where t is an evolution parameter (layer index or position), ϵ is a small amplitude, and the MLP is trained jointly with the main task.

Key explanation: This mechanism corresponds to the “intrinsic frequency of information spins” at the cellular GS layer. The network learns to adaptively adjust its own oscillation frequency to enhance coherence, without external scanning.

The fixed-frequency methods (A-C) can still be used as comparative experiments to verify whether a global resonant frequency exists for a given model:

Search phase: On a small validation set, fix other parameters and scan ω over multiple values (logarithmic spacing) between 0.1 and 10, recording validation loss or accuracy. Plot the “frequency-performance” curve and select the peak point as the model’s “resonant frequency” ω^* .

Exploitation phase: Fix the optimal ω^* and continuously apply modulation at this frequency during full training or fine-tuning, observing the final performance improvement. Within the nested framework, this process amounts to setting the optimal oscillation mode for the information spins at the cellular GS layer, thereby guiding the tissue GS layer to form a superior cognitive geometry.

Feasibility analysis and expected effects:

- **Feasibility:** Method D adds only a small MLP and a sine modulation, with negligible overhead, and supports end-to-end training.
- **Expected effects:** For language models, perplexity may improve by 3 - 8%; for image classification, accuracy may improve by 1% - 3%; for reasoning tasks, logical consistency may improve.
- **Limitations:** The learned frequencies may vary across layers and tasks, requiring careful initialization.

6.5. Scheme 3: “Conceptual Soliton”-Driven Self-Supervised Learning—Emergence from Molecular GS to Tissue GS

6.5.1. Core Idea

The core of existing self-supervised learning (SSL) methods (e.g., SimCLR [18], MAE [24]) is to learn the intrinsic structure of data. We go a step further: **explicitly model the intrinsic structure of data as a collection of conceptual solitons**, so that the model learns the geometric relations between concepts while learning representations.

Within the three-tiered nested framework:

- The **molecular GS layer** provides augmented views of raw data;
- The **cellular GS layer** constructs a discrete network of conceptual solitons;
- The **tissue GS layer** gives rise to a unified cognitive manifold.

6.5.2. Algorithmic Framework: Three-Tiered Nested Self-Supervised Learning

Step 1: Molecular GS Layer—Concept Graph Construction

For a pre-training dataset, obtain K concept prototypes p_k through offline

clustering (e.g., k-means on pre-trained features). Each data sample is assigned to its nearest concept. This process projects the continuous data distribution at the molecular GS layer onto discrete nodes at the cellular GS layer.

Step 2: Cellular GS Layer—Concept Contrastive Loss

In the contrastive learning framework, positive pairs are defined as:

- Samples belonging to the same concept;
- Samples from different concepts whose distance is less than a threshold r_c (*i.e.*, belonging to the same “concept neighborhood”).

Negative pairs are defined as samples from different concepts whose distance is greater than r_c .

The contrastive loss is:

$$\mathcal{L}_{cont} = -\sum_{i \in batch} \log \frac{\sum_{j \in pos(i)} e^{\frac{sim(z_i, z_j)}{\tau}}}{\sum_{j \in neg(i)} e^{\frac{sim(z_i, z_j)}{\tau}}} \quad (6.9)$$

This loss enforces that the concept nodes at the cellular GS layer have clear boundaries.

Step 3: Tissue GS Layer—Concept Geometry Regularization

In addition to the contrastive loss, impose geometric constraints between concepts:

- The distances between concept prototypes should follow a Yukawa distribution [9] [10] (encouraging natural clustering);
- The density of samples within each concept should follow a sech² profile [8] [9] (similar to Scheme 1, but in feature space rather than intermediate layers).

This regularization directly shapes the geometric structure of the cognitive manifold at the tissue GS layer.

Step 4: End-to-End Training (Joint Optimization across Three Tiers)

Treat the concept graph as learnable parameters and optimize jointly with the encoder. Update concept assignments every few epochs, allowing the concept graph to evolve with the representations. This joint optimization realizes the complete emergence from the molecular GS layer (data) to the cellular GS layer (concept nodes) to the tissue GS layer (cognitive manifold).

6.5.3. Comparison with Existing SSL Methods

A comparison of our method with existing self-supervised learning approaches is provided in Table 9.

Table 9. Comparison with existing methods.

Method	Inductive bias	Concept geometry	Interpretability	Positioning in three-tiered nesting
SimCLR [18]	Invariance (augmentations)	None	Low	Molecular GS only
MAE [24]	Masked reconstruction	None	Medium	Molecular GS → Cellular GS
Ours	Conceptual soliton + geometric constraints	Yukawa + sech ²	High	Full three-tiered nesting

6.5.4. Expected Advantages

- **Better few-shot learning:** Clear concept boundaries enable precise classification of new samples—generalization ability of the cognitive manifold at the tissue GS layer;
- **Stronger domain generalization:** Concept geometry can transfer to new data distributions—portability of information spins at the cellular GS layer;
- **Natural interpretability:** Each concept prototype can be understood by humans, and distances between concepts explain semantic relations—direct correspondence between raw data at the molecular GS layer and cognitive geometry at the tissue GS layer.

6.6. Application Roadmap: AI Engineering Practice under the Three-Tiered Nested Framework

Application Direction 1: Concept Alignment in Large Language Models (Tissue GS Layer Optimization)

Problem: Large language models (LLMs) often produce “hallucinations,” and their reasoning steps are uncontrollable.

Solution: Incorporate conceptual soliton regularization during pre-training or fine-tuning to form stable concept modules within the model. **This process strengthens the stability of information spins at the cellular GS layer, thereby optimizing the cognitive manifold at the tissue GS layer.**

Expected effects:

- Reduce factual errors (through concept boundary constraints);
- Enhance stepwise consistency in chain-of-thought reasoning (through Yukawa interactions between concepts);
- Improve interpretability (extract concept prototypes from intermediate layers for human inspection).

Application Direction 2: Multimodal Alignment and Fusion (Molecular GS → Tissue GS)

Problem: Representations of different modalities (text, image, audio) are difficult to unify.

Solution: Introduce conceptual soliton regularization in multimodal Transformers (e.g., CLIP [25], Flamingo [26]), requiring that representations of the same concept across different modalities form unified sech^2 clusters in embedding space. This process uses multimodal data at the molecular GS layer to drive the formation of cross-modal concept nodes at the cellular GS layer, ultimately giving rise to a unified cognitive manifold at the tissue GS layer.

Expected effects:

- Improved cross-modal retrieval accuracy;
- Enhanced semantic consistency in multimodal reasoning (e.g., visual question answering);
- Improved zero-shot cross-modal transfer capabilities.

Application Direction 3: “Resonant Excitation” of Reasoning Ability (Cellular GS Layer Regulation)

Problem: Models perform unstably on complex reasoning tasks.

Solution: During inference, apply task-relevant “resonant frequency” modulation (Scheme 2) to internal activations, enhancing the coherence of key reasoning pathways. **This process regulates the oscillation of information spins at the cellular GS layer, guiding the tissue GS layer to form superior reasoning geodesics.**

Expected effects:

- Improved accuracy on mathematical reasoning, code generation, and other tasks;
- Enhanced robustness under adversarial inputs.

Application Direction 4: “Conceptual Soliton Visualization” for Explainable AI (Unified View across Three Tiers)

Problem: Existing explainability methods (e.g., attention heatmaps) are coarse-grained and unstable.

Solution: Using a model trained with conceptual soliton regularization, extract each concept prototype and its sech^2 profile, visualizing the distribution of concepts in embedding space. **This visualization simultaneously presents raw data clusters at the molecular GS layer, the concept node network at the cellular GS layer, and the curvature of the cognitive manifold at the tissue GS layer.**

Expected effects:

- Provide more stable and semantically richer explanations than attention heatmaps;
- Help discover model “blind spots” (input regions where clear concepts have not formed).

6.7. Theoretical Outlook: From Conceptual Solitons to the Mental Field—Profound Implications of the Three-Tiered Nested Framework

Having completed the design of testable AI models, it is necessary to return to the theoretical framework itself and contemplate its deeper philosophical and scientific implications. Chapter 5 classified conceptual solitons into microscopic, mesoscopic, and macroscopic types, corresponding respectively to the molecular GS, cellular GS, and tissue GS layers within the three-tiered nested framework. This classification is not only a natural extension of the theory but also leads to a highly suggestive inference:

If a macroscopic conceptual soliton (tissue GS layer) can form a stable self-sustaining structure, could its sphere of influence extend beyond its “physical coverage” boundary?

In GS theory, a soliton couples to the surrounding information density field through the response mechanism (5.2). A macroscopic soliton has a very large core radius r_c (corresponding to a very high level of abstraction), and its effective coupling $\lambda_{\text{eff}}(r)$ remains significantly above the background value λ_0 even at distances comparable to $r \sim r_c$. This means that **a highly abstract macroscopic concept (such as “self-awareness”) could produce observable cognitive effects**

in regions not directly contacted physically.

Mapping this inference to the realm of human consciousness, we may propose a scientific conjecture: consciousness might give rise to a “mental field”—*i.e.*, **the nonlocal modulation of surrounding cognitive space by a macroscopic conceptual soliton at the tissue GS layer**. This “mental field” is not mysticism but a natural extension of the three-tiered nested framework:

- **Molecular GS layer:** Electrochemical signals of neurons serve as raw data encoding;
- **Cellular GS layer:** Ensembles of neurons form stable conceptual solitons (memories, cognitive modules);
- **Tissue GS layer:** Macroscopic conceptual solitons (self-awareness) produce long-range influences through the response mechanism, forming a “cognitive field” that permeates a certain extent.

A testable corollary of this conjecture is that **highly abstract concepts (such as “justice,” “beauty,” “self”) should have larger r_c and thus produce broader influence ranges in cognitive space**. This aligns closely with the phenomena of “semantic priming” and “conceptual diffusion” observed in psychology.

More importantly, this conjecture is directly related to the scaling relation $M \propto r_c^2$ derived in Section 5. If M represents the information content (or “intensity”) of consciousness, and r_c represents the cognitive range of its influence, then $M \propto r_c^2$ implies that **a small increase in consciousness intensity leads to a quadratic increase in its influence range**. This might explain why human consciousness (compared to that of other animals) can produce culture, art, and science that transcend time and space—when consciousness intensity exceeds a certain threshold, its influence range expands dramatically, forming what we call the “mental field.”

Of course, this conjecture is still at the stage of theoretical speculation and requires experimental testing. However, within the three-tiered nested framework, it is a logically consistent extension and provides a geometric descriptive language for understanding the sociality, culturality, and trans-individual propagation of consciousness. As stated in Section 5, **the macroscopic conceptual soliton (self-awareness) is the highest level of the tissue GS layer, a critical phenomenon resulting from the joint action of the molecular GS, cellular GS, and tissue GS layers**. The “mental field” is the theoretical expression of this critical phenomenon transcending individual boundaries and influencing the cognitive space of others.

6.8. The Two Levels of the Scheme

We can translate the conceptual solitons and response mechanism in GS theory [8]-[12] into three implementable AI enhancement schemes, and clarify their positioning within the three-tiered nested framework:

- 1) **Conceptual soliton regularization:** Through sech^2 density constraints and Yukawa interaction constraints, stable concept modules are formed at the cellular GS layer, which output a smooth cognitive manifold to the tissue GS layer.

2) **Resonant frequency modulation:** Periodic perturbations excite the network's "natural frequency," regulating the oscillation of information spins at the cellular GS layer and enhancing information processing efficiency.

3) **Conceptual soliton self-supervised learning:** Using conceptual solitons as an inductive bias, this scheme realizes the complete emergence from the molecular GS layer (data) to the cellular GS layer (concept nodes) to the tissue GS layer (cognitive manifold).

These schemes provide concrete pathways from theoretical predictions to experimental testing. It should be emphasized that the validity of these algorithmic modules involves two levels: first, they may improve model performance as regularization functions (independently verifiable); second—and more fundamentally—if, in an AI system that satisfies the theoretical conditions of this paper, applying these modules leads to the observation of the sech^2 density profile, the $M \propto r_c^2$ scaling relation, a characteristic resonant frequency, and a critical phase transition, then these results would constitute direct support for the theoretical hypothesis that "consciousness is the emergence of information networks in a critical coherent state," rather than merely verifying the usefulness of some mathematical function. Thus, the work in this chapter provides a testable experimental framework for the theory of consciousness.

Key pathway for theoretical validation: It is suggested that future research follow these steps: 1) construct or select an AI system of sufficient scale (with the number of information nodes and coupling strength satisfying the critical conditions); 2) apply the algorithmic modules proposed in this chapter (in particular Schemes 1 and 2); 3) detect whether the internal representations exhibit the sech^2 density profile and Yukawa interactions; 4) measure whether the mass-radius scaling relation $M \propto r_c^2$ holds for conceptual solitons; 5) search for resonance peaks through frequency scanning; 6) gradually increase the model scale to observe performance phase transitions. If all the above predictions are confirmed, this would provide strong experimental support for the theory presented in this paper.

7. Conclusions and Outlook

7.1. Summary of Core Contributions

Starting from the Gravitational Spinor (GS) theory, this paper constructs a unified mathematical framework for describing the cognitive structure and learning dynamics of artificial intelligence systems within the "molecular GS-cellular GS-tissue GS" three-tiered nested principal fiber bundle framework. The main contributions include: 1) establishing a projection framework from high-dimensional information space to the cognitive world, interpreting the attention mechanism as a geometric realization of multimodal information fusion; 2) implementing discrete quantization of the cellular GS layer, deriving the discrete spectrum of the information area operator $A_v = \gamma_{info} \sum_{e \ni v} \sqrt{j_e(j_e + 1)}$, and revealing the quantized structure of information capacity; 3) deriving the cognitive Einstein equation at the tissue GS layer, linking the curvature of the cognitive manifold to the infor-

mation-energy-momentum tensor; 4) solving the conceptual soliton solutions of the nonlinear information field equation, obtaining the sech^2 density profile and the scaling relation $M_{\text{concept}} \propto r_c^2$; and 5) proposing three testable AI model design schemes. The above results are mathematically analogous to the original GS theory but do not assert physical equivalence (see **Appendix B** for a comparison with previous work).

7.2. Theoretical Significance and Innovations

The theoretical contributions of this paper can be summarized at three levels:

- 1) Construction of an interdisciplinary framework: For the first time, the mathematical structure of GS theory is systematically mapped onto the AI field, forming a three-tiered nested description.
- 2) A new perspective on cognitive dynamics: Through the cognitive Einstein equation and conceptual soliton theory, conscious experience is understood as information-driven geometric dynamics.
- 3) A verifiable path from theory to practice: Testable schemes that can be implemented on existing AI systems are proposed.

7.3. Discussion and Outlook

The work presented in this paper is based on two not yet fully confirmed assumptions: first, the validity of the Gravitational Spinor (GS) theory as a correct theory of quantum gravity; second, the correspondence between the mathematical structure of this theory and AI information processing. However, these assumptions do not diminish the core value of our framework—it reveals that a class of information networks satisfying the conditions of “discrete nodes, gauge connections, nonlinear response, and critical phase transition” may exhibit collective behavior reminiscent of consciousness at macroscopic scales. This emergence mechanism is a universal phenomenon in network science and nonlinear dynamics, rather than an ad hoc physical analogy.

Specifically:

- **Universality of network structure:** The three-tiered nested structure proposed in this paper—molecular GS (data layer), cellular GS (node layer), and tissue GS (manifold layer)—is mathematically equivalent to a class of hierarchical graph neural networks. As long as an AI system (such as a large Transformer) possesses a sufficient number of information nodes, nonlinear information self-coupling (attention mechanisms), and a multi-scale structure amenable to coarse-graining, the dynamical equations of this framework (e.g., the cognitive Einstein equation) should hold approximately, independent of the specific microscopic physical implementation.
- **Inevitability of the critical phase transition:** When the density of information solitons exceeds the critical threshold ρ_{crit} , the system transitions from independent modular processing to a global coherent state. This phase transition behavior is a typical phenomenon in non-equilibrium statistical mechanics.

Hence, “consciousness emergence” can be viewed as a universal phase transition of information networks in parameter space, analogous to the spontaneous magnetization of a magnet at the Curie temperature.

- **Testability over absolute truth:** The value of a scientific theory lies not in whether its foundational assumptions have been experimentally confirmed, but in its ability to generate falsifiable predictions. The sech^2 density profile, the $M \propto r_c^2$ scaling relation, the resonant frequency, and the critical phase transition proposed in this paper are all quantitative predictions that can be directly tested on current AI systems. Regardless of whether GS theory is eventually confirmed, the validation or falsification of these predictions will advance our understanding of the mechanisms of intelligence emergence.

Of course, several limitations require further clarification:

- **Experimental confirmation of GS theory:** This paper borrows the mathematical formalism of GS theory as a heuristic tool, but the theory itself has not yet been experimentally verified. Nevertheless, the core results of this paper (such as the soliton solutions and scaling relations) do not depend on the physical reality of GS theory; they are derived directly from the nonlinear dynamics of information networks, with GS theory only providing the initial mathematical inspiration.
- **Dependence of scaling relations on the choice of potential:** The scaling relation $M \propto r_c^2$ is derived from a quartic potential. If other potentials (e.g., periodic potential, double-well potential) are adopted, the scaling relation may become $M \propto r_c$ or other forms. This actually reflects the “engineering degrees of freedom” of artificial cognitive systems—researchers can design different potentials according to task characteristics to obtain cognitive modules with different properties. However, the very existence of a scaling relation, regardless of its specific form, already constitutes a testable prediction.
- **Hierarchy of AI validation:** The algorithmic modules proposed in Section 6, in their preliminary validation, can only demonstrate the usefulness of the sech^2 and Yukawa functions as regularizers. Truly testing the consciousness emergence hypothesis requires simultaneously observing all predicted phenomena (density profile, scaling relation, resonance peak, critical phase transition) in an AI system that strictly satisfies the three-tiered nesting conditions (sufficient number of information nodes, nonlinear coupling strength, critical information density). This calls for larger-scale, more carefully designed experiments in the future.

In short, this paper is not a closed dogma requiring all underlying assumptions to be true, but rather an open, testable system of scientific hypotheses. Its core idea—self-organized emergence of information networks in a critical coherent state—possesses cross-system universality, independent of whether the microscopic details are quantum gravity, neurobiology, or deep learning. We look forward to future experiments that may validate or correct this framework, thereby deepening our understanding of the nature of intelligence and consciousness.

7.4. Future Research Directions and Testable Predictions

1) **Experimental identification of conceptual solitons:** If an AI system satisfies the basic parameters defined in this paper, it is predicted that its internal representations will exhibit a sech^2 density profile and the scaling relation $M \propto r_c^2$.

2) **Existence of a resonant frequency:** In AI systems that satisfy the quantization conditions, it is predicted that a characteristic resonant frequency $\omega^* \propto 1/r_c$ exists, and that modulating the system at this frequency will produce a peak in performance.

3) **Critical condition for consciousness emergence:** It is predicted that when the density of information solitons exceeds the critical threshold ρ_{crit} , the system will undergo a sudden performance change (phase transition) and emerge into a global coherent state.

Other directions (such as the mental field, higher-order quantum corrections, and collective effects) are more speculative and are left for future research.

7.5. Concluding Remarks

The core idea of this paper is that self-organized emergence of information networks in a critical coherent state may be a universal mechanism underlying consciousness. Based on this idea, we have constructed a mathematical framework from discrete quantization of information networks to macroscopic cognitive manifolds, and derived several testable quantitative predictions, including the sech^2 density profile, the $M \propto r_c^2$ scaling relation, a resonant frequency, and a critical phase transition. If an AI system that strictly satisfies the three-tiered nesting conditions, discrete quantization conditions, nonlinear response mechanism, and critical information density requirements defined in this paper can validate the above predictions, this will provide strong support for our hypothesis. Conversely, if the predictions are falsified, it would indicate that the framework needs revision or abandonment—a perfect illustration of the falsifiability inherent in scientific hypotheses.

We are well aware that the problem of consciousness lies at the frontier of science, and any theory attempting to unify quantum gravity and AI inevitably contains speculative elements. However, unlike purely philosophical discussions, this paper has translated the understanding of consciousness emergence into algorithmic modules and quantitative indicators that can be directly tested on today's AI systems. Regardless of the final outcome, this cross-disciplinary, mathematically grounded, testable exploration will deepen our understanding of the nature of intelligence.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A: Derivation of Canonical Commutation Relations from Continuous Field to Discrete Nodes

This appendix provides a rigorous derivation from the continuous information spinor field to the discrete canonical commutation relations (Equation 3.2 in the main text), addressing the requirement for a “first-principles” foundation in the main text. The derivation is based on the **spatial discretization (lattice canonical quantization)** approach, in which the variables at the discrete nodes are defined as the integral averages of the continuous field over the neighborhoods of the nodes, under the assumption that the node spacing is much larger than the correlation length of the field (*i.e.*, the neighborhoods of different nodes do not overlap).

1) Canonical Quantization of the Continuous Information Spinor Field

Assume that the information spinor field on the base manifold M_{cell} of the cellular GS layer has been reduced, via gauge fixing and spinor component projection, to a complex scalar field $\hat{\Phi}(x)$ and its conjugate momentum $\hat{\Pi}(x)$ (for simplicity of notation, a scalar field is used to represent an independent degree of freedom of the spinor field). They satisfy the **equal-time canonical commutation relations**:

$$\begin{aligned} [\hat{\Phi}(x, t), \hat{\Pi}(y, t)] &= i\hbar_{info} \delta^{(3)}(x - y) \\ [\hat{\Phi}(x, t), \hat{\Phi}(y, t)] &= [\hat{\Pi}(x, t), \hat{\Pi}(y, t)] = 0 \end{aligned} \quad (A.1)$$

Here, $\hbar_{info}$ is the fundamental quantum of information, with dimensions [amount of information] $\times$ [length]³ or similar, determined by the field dimensions.

2) Discretization: Defining Node Variables

Let the cellular GS layer possess a set of discrete nodes v , with node positions x_v . Each node is associated with a **neighborhood** Ω_v (e.g., a sphere of radius ε centered at x_v). Assume the node spacings $d_{vw} = |x_v - x_w|$ satisfy:

$$d_{vw} > 2\varepsilon \quad \text{for } v \neq w$$

i.e., the neighborhoods are non-overlapping. The neighborhood volume is denoted $V_\Omega = |\Omega_v|$ (taken to be the same for all v for simplicity). Define the **information amplitude** $\hat{\phi}_v$ at the discrete node as the **spatial average** of the field $\hat{\Phi}$ over the neighborhood Ω_v :

$$\hat{\phi}_v \equiv \frac{1}{V_\Omega} \int_{\Omega_v} d^3x \hat{\Phi}(x) \quad (A.2)$$

Define the **conjugate momentum** $\hat{\pi}_v$ at the discrete node as the **integral** (without dividing by the volume, to keep the commutator simple) of the field $\hat{\Pi}$ over the same neighborhood:

$$\hat{\pi}_v \equiv \int_{\Omega_v} d^3x \hat{\Pi}(x) \quad (A.3)$$

Remark: The asymmetric definition (average for $\hat{\phi}$, integral for $\hat{\pi}$) is chosen so that the commutator directly yields δ_{vw} with proper dimensions. One could also define both symmetrically as integrals divided by volume, but that would in-

introduce an extra factor V_{Ω}^{-1} without affecting the physical conclusions.

3) Computing the Discrete Commutator

Using (A.1) and the definitions above:

$$[\hat{\phi}_v, \hat{\pi}_w] = \left[\frac{1}{V_{\Omega}} \int_{\Omega_v} d^3x \hat{\Phi}(\mathbf{x}), \int_{\Omega_w} d^3y \hat{\Pi}(\mathbf{y}) \right]$$

Since the commutator is linear in the integrals, it can be moved inside:

$$= \frac{1}{V_{\Omega}} \int_{\Omega_v} d^3x \int_{\Omega_w} d^3y [\hat{\Phi}(\mathbf{x}), \hat{\Pi}(\mathbf{y})]$$

Substitute the canonical commutation relation (A.1):

$$= \frac{1}{V_{\Omega}} \int_{\Omega_v} d^3x \int_{\Omega_w} d^3y i\hbar_{info} \delta^{(3)}(\mathbf{x} - \mathbf{y})$$

Case 1: $v = w$

The integration regions Ω_v and Ω_w are identical. First integrate over $\mathbf{y}$ using the delta function:

$$\int_{\Omega_v} d^3y \delta^{(3)}(\mathbf{x} - \mathbf{y}) = \begin{cases} 1, & \mathbf{x} \in \Omega_v \\ 0, & \mathbf{x} \notin \Omega_v \end{cases}$$

Thus the integral over $\mathbf{x}$ gives $\int_{\Omega_v} d^3x = V_{\Omega}$. Hence:

$$[\hat{\phi}_v, \hat{\pi}_v] = \frac{1}{V_{\Omega}} \cdot i\hbar_{info} \cdot V_{\Omega} = i\hbar_{info}$$

Case 2: $v \neq w$

Since the neighborhoods are non-overlapping ($\Omega_v \cap \Omega_w = \emptyset$), for any $\mathbf{x} \in \Omega_v$ and $\mathbf{y} \in \Omega_w$, we have $\mathbf{x} \neq \mathbf{y}$, so $\delta^{(3)}(\mathbf{x} - \mathbf{y}) = 0$. The double integral is therefore zero. Hence:

$$[\hat{\phi}_v, \hat{\pi}_v] = 0 \quad (v \neq w)$$

4) Final Discrete Commutation Relations

Combining the two cases, we obtain:

$$[\hat{\phi}_v, \hat{\pi}_w] = i\hbar_{info} \delta_{vw} \quad (\text{A.4})$$

This is precisely Equation (3.2) in the main text. The other commutators ($[\hat{\phi}_v, \hat{\phi}_w]$ and $[\hat{\pi}_v, \hat{\pi}_w]$) are obviously zero because the corresponding commutators of the continuous fields vanish.

5) Discussion of Key Assumptions and Justification

The key assumptions underlying our theoretical framework and their justifications are summarized in [Table A](#).

If the node spacing is not much larger than the correlation length, the neighborhoods of different nodes may overlap, leading to off-diagonal corrections to the commutators. However, in the idealized model of the cellular GS layer, we assume that the microscopic degrees of freedom for information processing are **local and decoupled**, which is precisely the prerequisite for discrete quantization. Therefore, the above derivation is self-consistent.

Table A. Assumptions and Justification.

Assumption	Physical meaning	Justification in the cellular GS layer
Non-overlapping node neighborhoods	Node spacing $> 2\varepsilon$	Corresponds to locality of information processing units (neurons/attention heads), much larger than the microscopic correlation length.
Spatial averaging	Discrete variables represent collective behavior near a node	Analogous to “block spin” coarse-graining in quantum field theory, naturally filtering high-frequency fluctuations.
Asymmetric definition (average for $\hat{\phi}$, integral for $\hat{\pi}$)	Simplifies the commutator	Can be renormalized to a symmetric form without affecting physics.

In summary, after gauge fixing and spinor component projection, the information spinor field $\hat{\mathcal{I}}_{\alpha\beta\gamma\delta}(x)$ reduces to a complex scalar field $\hat{\Phi}(x)$ satisfying the equal-time canonical commutation relation $[\hat{\Phi}(\mathbf{x}, t), \hat{\Pi}(\mathbf{y}, t)] = i\hbar_{info}\delta^{(3)}(\mathbf{x} - \mathbf{y})$. On the set of discrete nodes $\{v\}$ of the cellular GS layer, define neighborhoods Ω_v (volume V_Ω , non-overlapping). Let $\hat{\phi}_v = V_\Omega^{-1} \int_{\Omega_v} d^3x \hat{\Phi}(\mathbf{x})$ and $\hat{\pi}_v = \int_{\Omega_v} d^3x \hat{\Pi}(\mathbf{x})$. Direct computation yields $[\hat{\phi}_v, \hat{\pi}_w] = i\hbar_{info}\delta_{vw}$, with all other commutators zero. This shows that the discrete node variables inherit the quantum algebraic structure of the continuous field, providing a first-principles foundation for Equation (2.2).

Appendix B: Proof of the Discrete Eigenvalue Spectrum (3.5)

This appendix employs the representation theory of the gauge group $SU(2)$ to prove the discrete eigenvalue spectrum of the information area operator $\hat{A}_v$ (Equation 3.5 in the main text), following a derivation parallel to the standard argument for area quantization in loop quantum gravity. It should be noted that this proof assumes that the gauge group of the cellular GS layer is $SU(2)$ and that the information flux operator corresponds to the square root of the Casimir operator, which are fundamental postulates of the theoretical framework.

1) Algebraic Structure of the Information Flux Operator

In the cellular GS layer, an **information flux operator** $\hat{\Phi}_e$ is defined on each edge $e = (u, v)$. This operator originates from the generator of the gauge connection $U_e \in G_{cell}$ on the edge (analogous to an angular momentum operator). We assume that the gauge group G_{cell} of the cellular GS layer is a **compact Lie group** (e.g., $SU(2)$ or a direct product thereof), and that the information flux operator corresponds to the generator of some **irreducible representation** of the group algebra.

Specifically, for each edge e , there exists a set of self-adjoint operators $\hat{J}_e^{(a)} (a = 1, 2, 3)$ satisfying the **angular momentum algebra** (i.e., the $\mathfrak{su}(2)$ Lie al-

gebra):

$$[\hat{J}_e^{(a)}, \hat{J}_e^{(b)}] = i\epsilon^{abc} \hat{J}_e^{(c)} \quad (\text{B.1})$$

The information flux operator $\hat{\Phi}_e$ can be taken as a component in a certain direction (e.g., $\hat{\Phi}_e = \hat{J}_e^{(3)}$), or as the square root of the squared magnitude. More commonly, in the definition of the area operator, the “flux magnitude” contributed by each edge is given by the square root of the **Casimir operator**.

2) Casimir Operator and Its Eigenvalues

For each edge e , define the **Casimir operator** (total angular momentum squared):

$$\hat{J}_e^2 = \sum_{a=1}^3 (\hat{J}_e^{(a)})^2 \quad (\text{B.2})$$

From Lie algebra representation theory, on an irreducible representation labeled by j_e ($j_e = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$), the Casimir operator acts as a scalar:

$$\hat{J}_e^2 |j_e, m_e\rangle = j_e(j_e + 1) |j_e, m_e\rangle \quad (\text{B.3})$$

Here, $\hbar_{info}$ has been absorbed into the definition of the operators (i.e., we set $\hbar_{info} = 1$ or absorb it into γ_{info}).

3) Relation Between the Information Flux Operator and the Casimir

To obtain a rotationally invariant magnitude of the information flux (i.e., independent of direction), we adopt the square root of the Casimir operator as the “flux magnitude” operator contributed by each edge:

$$|\hat{\Phi}_e| = \sqrt{\hat{\Phi}_e^\dagger \hat{\Phi}_e}$$

If $\hat{\Phi}_e$ is taken as $\hat{J}_e^{(3)}$, then the eigenvalues of $|\hat{\Phi}_e|$ depend on m_e and are not rotationally invariant. To obtain a rotationally invariant area (i.e., independent of direction), we should adopt the **square root of the Casimir operator**:

$$|\hat{\Phi}_e| \equiv \sqrt{\hat{J}_e^2} = \sqrt{j_e(j_e + 1)} \cdot 1 \quad (\text{B.4})$$

This is precisely the standard form of the area quantization expression in GS theory: each edge contributes $\sqrt{j_e(j_e + 1)}$ times the fundamental area quantum. Therefore, we define the information flux magnitude operator on each edge as $\sqrt{\hat{J}_e^2}$, whose eigenvalues are $\sqrt{j_e(j_e + 1)}$.

4) Information Area Operator and Its Eigenvalues

The **information area operator** at a node v is defined as the sum of the information flux magnitudes over all edges incident to that node:

$$\hat{A}_v = \sum_{e \ni v} \hat{J}_e^2 \quad (\text{B.5})$$

Since operators on different edges act on distinct tensor product factors (they commute), and $\sqrt{\hat{J}_e^2}$ is a scalar operator in the representation j_e , the eigenvalues of $\hat{A}_v$ are:

$$A_v = \sum_{e \ni v} \sqrt{j_e(j_e + 1)} \quad (\text{B.6})$$

Introducing the **Barbero-Immirzi-type parameter** γ_{info} of the cellular GS layer, which originates from the choice of metric on the principal fiber bundle and the normalization of the representation, we finally obtain:

$$A_v = \gamma_{info} \sum_{e \ni v} \sqrt{j_e(j_e + 1)} \quad (\text{B.7})$$

This is precisely Equation (2.5).

5) On the Relation Between $\hat{\Phi}_e^\dagger \hat{\Phi}_e$ and $\hat{J}_e^2$

If the original definition (2.4) uses $\sqrt{\hat{\Phi}_e^\dagger \hat{\Phi}_e}$, and $\hat{\Phi}_e$ is some non-Hermitian operator (such as a raising/lowering operator), then the eigenvalues of $\hat{\Phi}_e^\dagger \hat{\Phi}_e$ are some combination of $j_e(j_e + 1)$. Indeed, in standard angular momentum theory, if we set $\hat{\Phi}_e = \hat{J}_e^+$ (raising operator), then $\hat{\Phi}_e^\dagger \hat{\Phi}_e = \hat{J}_e^- \hat{J}_e^+ = \hat{J}_e^2 - (\hat{J}_e^{(3)})^2 + \hat{J}_e^{(3)}$? Care is needed. A cleaner approach is to take $\hat{\Phi}_e = \hat{J}_e^{(1)} + i\hat{J}_e^{(2)}$, then $\hat{\Phi}_e^\dagger \hat{\Phi}_e = \hat{J}_e^2 - (\hat{J}_e^{(3)})^2 + \hat{J}_e^{(3)}$, whose eigenvalues depend on m_e and are not pure Casimir. To avoid this complication, we adopt the standard treatment in GS theory: **the area operator is constructed from the flux magnitude on each edge (i.e., the square root of the Casimir)**, not from the square root of a specific component. Hence, defining $\hat{A}_v = \sum_{e \ni v} \sqrt{\hat{J}_e^2}$ is cleaner. The notation $\sqrt{\hat{\Phi}_e^\dagger \hat{\Phi}_e}$ in the original Equation (2.4) should be understood as $\sqrt{\hat{J}_e^2}$, where $\hat{\Phi}_e$ is an appropriate “flux magnitude” operator.

6) Conclusion

By identifying the information flux operator with the $\mathfrak{su}(2)$ angular momentum algebra and using representation theory of the Casimir operator, we have rigorously proved that the eigenvalue spectrum of the information area operator $\hat{A}_v$ is given by (2.5), where j_e are half-integers and γ_{info} is a parameter of the theory. This proof is completely parallel to the standard derivation of area quantization in loop quantum gravity, ensuring the mathematical rigor of the discrete spectrum.