

Grover's Model with Single Oracle Call and $\log(N)$ Different Diffusion Calls

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Abstract

The author has recently presented the Single-Iteration Quantum Search Algorithm, which achieves amplitude-amplification with exactly one oracle call and one $\pi/4$ rotation [1]. For $N = 2^n$ search spaces, the algorithm achieves success probability exactly $1/2$ with one iteration of oracle operator and one $\pi/4$ rotation with $\log(N)$ different diffusion calls, compared to $\mathcal{O}(\sqrt{N})$ iterations ($\mathcal{O}(\sqrt{N})$ oracle calls and $\mathcal{O}(\sqrt{N})$ same diffusion calls) for Grover's algorithm. This work presents a simpler and cleaner derivation for one oracle call and $\log(N)$ diffusion calls, while the original proof was based on Cartan-Dieudonné theorem [1]. The standard lower bounds for unstructured quantum search still apply even though the number of oracle calls is reduced from $\mathcal{O}(\sqrt{N})$ to $\mathcal{O}(1)$ and the number of diffusion calls is reduced from $\mathcal{O}(\sqrt{N})$ to $\mathcal{O}(\log N)$.

Keywords

Quantum Computing, Grover's Algorithm, Oracle, Amplitude Amplification, Quantum Circuits, Unitary Matrix, Single-Iteration Quantum Search, Single-Rotation Quantum Search

1. Introduction

Quantum amplification enables quantum computers to find marked items in unstructured databases quadratically faster than classical computers [2]. Standard Grover's algorithm requires $\mathcal{O}(\sqrt{N})$ iterations [2].

For unstructured search over N items with one marked item, a single oracle call, followed by a rotation of $\pi/4$ radians can amplify the marked state probability from $1/N$ (initial uniform superposition) to exactly $1/2$ (measurement threshold) [1].

The most important contribution of the Single-Iteration algorithm [1] is that the $\pi/4$ rotation can be implemented in one iteration using $\log(N)$ steps. Based on Cartan-Dieudonné theorem, each of the $\log(N)$ rotations is further implemented by two reflections and finally, one of the two reflections in each iteration can be removed.

This work presents a simpler and cleaner derivation for the Single-Iteration algorithm with $\log(N)$ Diffusion operators, without mentioning reflections and Cartan-Dieudonné theorem [1]. It is easier to understand the Single-Iteration algorithm in this new approach.

1.1. Prior Work

Standard Grover's Algorithm [2]:

- Iterations: $K \approx (\pi/4)\sqrt{N}$
- Fixed operator (Oracle and Diffusion) applied repeatedly
- Arithmetic series amplitude growth: $r(k) \approx 2k + 1$

Exponential Speedup Algorithm [3]:

- Iterations: $K = n/2 = (1/2)\log_2 N$ (for $\beta = 2$)
- Iteration-dependent operators U_0, U_1, \dots, U_{K-1}
- Geometric series amplitude growth: $r(k) = \beta^k$

Single-Iteration/Single-Rotation Quantum Search: The author has recently presented the Single-Iteration Quantum Search Algorithm, which achieves amplitude-amplification with exactly one oracle call and one $\pi/4$ rotation [1], where the $\pi/4$ rotation can be implemented in $O(\log N)$ steps using a set of different Reflections. For $N = 2^n$ search spaces, the algorithm achieves success probability exactly $1/2$ with one iteration of oracle operator and one $\pi/4$ rotation implemented by $\log(N)$ double reflections.

This Work: This work presents a simpler and cleaner derivation for $\pi/4$ rotation with $\log(N)$ diffusion operators, without mentioning reflections and Cartan-Dieudonné theorem [1].

1.2. Notation

Throughout this paper:

- n Number of qubits
- $N = 2^n$ Search space size
- K Iteration count
- k Iteration index
- $|\alpha\rangle$ the single marked state
- $|\beta\rangle$ uniform superposition of $N-1$ unmarked states
- a_1 Amplitude of marked state (assumed to be state $|\alpha\rangle$)
- a_0 Amplitude of each unmarked state
- θ Angle in geometric representation, relative to $|\beta\rangle$
- $\Delta\theta$ Rotation angle implemented by U

Section 1 provides motivation and context. Section 2 introduces preliminaries.

Section 3 shows that a single oracle call and $\log(N)$ different diffusion calls can achieve amplitude-amplification. Section 4 shows that the standard lower bounds for unstructured quantum search still apply. Section 5 makes a few remarks. Section 6 concludes.

2. Preliminaries

2.1. Rotation

The diffusion operator acts as a reflection in the N -dimensional Hilbert space, which combined with the oracle creates a rotation in the 2D subspace $\{|\alpha\rangle, |\beta\rangle\}$ [4]. In this paper, the amplitude amplification can be achieved by a single oracle call and a set of different diffusion operators.

The superposition over all 2^n computational states is given in Equation (1):

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle \quad (1)$$

The oracle operator is:

$$O = I - 2|\alpha\rangle\langle\alpha| \quad (2)$$

Throughout this paper, let the marked state be $|1\rangle$ for convenience of discussion whenever necessary, assumed without loss of generality. The quantum state can be visualized as a vector in a 2-dimensional subspace spanned by the marked state in Equation (3):

$$|\alpha\rangle = |1\rangle \quad (3)$$

and the uniform superposition of unmarked states in Equation (4)

$$|\beta\rangle = \frac{1}{\sqrt{N-1}} \sum_{x=0, x \neq \alpha}^{N-1} |x\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq \alpha} |x\rangle \quad (4)$$

Mathematical form of the Grover diffusion operator is:

$$D = 2|\psi\rangle\langle\psi| - I \quad (5)$$

where $|\psi\rangle$ is the uniform superposition in Equation (1). Physical realizability requires that only unitary operators can be implemented in quantum hardware.

2.2. Geometric Framework [1]

Following reference [4], we work in the 2-dimensional subspace spanned by $\{|\alpha\rangle, |\beta\rangle\}$. The marked state is $|\alpha\rangle$. The unmarked superposition is given in Equation (4) [4] [5]. Any state at iteration k can be written in Equation (6):

$$\begin{aligned} |\psi_k\rangle &= a_1(k)|\alpha\rangle + a_0(k)\sqrt{N-1} \frac{1}{\sqrt{N-1}} \sum_{x=0, x \neq \alpha}^{N-1} |x\rangle \\ &= a_1(k)|\alpha\rangle + a_0(k)\sqrt{N-1}|\beta\rangle \end{aligned} \quad (6)$$

When $k=0$,

$$a_1(k) = a_0(k) = \frac{1}{\sqrt{N}} \quad (7)$$

Equation (6) can be written in Equation (8):

$$|\psi_k\rangle = \sin(\theta_k)|\alpha\rangle + \cos(\theta_k)|\beta\rangle \tag{8}$$

where θ_k is the angle from the $|\beta\rangle$ axis at iteration, k , and the rotation is from $|\beta\rangle$ axis to $|\alpha\rangle$ axis. The amplitude of the marked state is given in Equation (9):

$$\sin(\theta_k) = a_1(k) \tag{9}$$

The amplitude of uniform superposition of unmarked states is given in Equation (10):

$$\cos(\theta_k) = a_0(k) \cdot \sqrt{N-1} \tag{10}$$

The initial angle ($k = 0$), in the initial superposition in Equation (1), is given in Equation (11):

$$\theta_0 = \arcsin\left(1/\sqrt{N}\right) \tag{11}$$

For large N ,

$$\theta_0 \approx 1/\sqrt{N}$$

Target angle ($k = 1$), after only one iteration, is given in Equation (12) [1]:

$$\theta_1 = \pi/4 \tag{12}$$

The required rotation is given in Equation (13):

$$\Delta\theta = \theta_1 - \theta_0 = \pi/4 - \arcsin\left(1/\sqrt{N}\right) \tag{13}$$

2.3. Useful Identities

The oracle operator, O , is given in Equation (2),

$$O = I - 2|\alpha\rangle\langle\alpha|$$

So,

$$O|\alpha\rangle = -|\alpha\rangle \tag{14}$$

$$O|\beta\rangle = |\beta\rangle \tag{15}$$

From Equations (6) and (7),

$$|\psi_0\rangle = \frac{1}{\sqrt{N}}|\alpha\rangle + \frac{1}{\sqrt{N}}\sqrt{N-1}|\beta\rangle \tag{16}$$

$$O|\psi_0\rangle = (I - 2|\alpha\rangle\langle\alpha|)|\psi_0\rangle = |\psi_0\rangle - \frac{2}{\sqrt{N}}|\alpha\rangle \tag{17}$$

The diffusion operator is defined in Equation (5),

$$D|\psi_0\rangle = |\psi_0\rangle \tag{18}$$

$$D|\alpha\rangle = (2|\psi_0\rangle\langle\psi_0| - I)|\alpha\rangle = \frac{2}{\sqrt{N}}|\psi_0\rangle - |\alpha\rangle \tag{19}$$

Applying both oracle and diffusion,

$$DO|\psi_0\rangle = D\left(|\psi_0\rangle - \frac{2}{\sqrt{N}}|\alpha\rangle\right) = \left(1 - \frac{4}{N}\right)|\psi_0\rangle + \frac{2}{\sqrt{N}}|\alpha\rangle \tag{20}$$

As an approximation, we can drop $O(1/N)$ term,

$$DO|\psi_0\rangle \approx |\psi_0\rangle + \frac{2}{\sqrt{N}}|\alpha\rangle \tag{21}$$

The total rotation is from 0 to $\pi/4$ for large N . The initial amplitude of $|\alpha\rangle$ in Equation (16) is $1/\sqrt{N}$. Equation (20) and (21) indicate that the amplitude is increased approximately by $2/\sqrt{N}$. For large N , using identity, $\sin(2/\sqrt{N}) = 2/\sqrt{N}$, the rotation angle, after applying the oracle operator and diffusion operator, is increased roughly by $2/\sqrt{N}$.

For the Grover algorithm, the arithmetic series for θ_k in Equation (8) is approximately [3]:

$$\frac{1}{\sqrt{N}} + \frac{3}{\sqrt{N}} + \frac{5}{\sqrt{N}} + \dots + \frac{2K+1}{\sqrt{N}} \tag{22}$$

As an order of estimate for K , after K iterations, one has:

$$K \frac{2}{\sqrt{N}} = \frac{\pi}{4} = O(1)$$

So Grover's time complexity is order of

$$T = O(\sqrt{N})T_O T_A \tag{23}$$

where K is the number of iterations to reach $\pi/4$, T_O is the oracle cost and $T_A = O(n)$ is the diffusion operator cost for amplification. Equation (23) is exponential.

3. Single-Iteration Algorithm

The $\pi/4$ rotation will be completed by K iterations. The state evolution can be written in Equation (24):

$$|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle, \dots, |\psi_K\rangle \tag{24}$$

Define a set of Diffusion operators as:

$$D_0, D_1, D_2, \dots, D_{K-1} \tag{25}$$

where

$$D_0 = 2|\psi_0\rangle\langle\psi_0| - I \tag{26}$$

$$D_1 = 2|\psi_1\rangle\langle\psi_1| - I \tag{27}$$

$$D_2 = 2|\psi_2\rangle\langle\psi_2| - I \tag{28}$$

⋮

Let the state evolution be:

$$|\psi_0\rangle = \frac{1}{\sqrt{N}}|\alpha\rangle + \frac{1}{\sqrt{N}}\sqrt{N-1}|\beta\rangle \tag{29}$$

$$|\psi_1\rangle = D_0O|\psi_0\rangle \tag{30}$$

$$|\psi_2\rangle = D_1|\psi_0\rangle \tag{31}$$

$$|\psi_3\rangle = D_2|\psi_0\rangle \tag{32}$$

⋮

Note that the rotations always start from the beginning, $|\psi_0\rangle$ in Equations (30), (31), and (32). From reference [1], when rotating from $|\beta\rangle$ to $|\alpha\rangle$ in $\{|\alpha\rangle, |\beta\rangle\}$ space, if geometric series for rotation angles is:

$$\frac{2^0}{\sqrt{N}} \frac{2^1}{\sqrt{N}} \frac{2^2}{\sqrt{N}} \cdots \frac{2^K}{\sqrt{N}}$$

then $K = O(\log N)$.

THEOREM 3.1. If the state evolution is given by Equations (29)-(32), the rotation from $|\beta\rangle$ to $|\alpha\rangle$, $R(\pi/4)$, can be achieved by one oracle call and $\log(N)$ different diffusion calls.

Proof. Step 0. From Equation (29),

$$|\psi_0\rangle = \frac{1}{\sqrt{N}}|\alpha\rangle + \frac{1}{\sqrt{N}}\sqrt{N-1}|\beta\rangle$$

Step 1. From Equation (20), after the oracle operator and diffusion operator,

$$|\psi_1\rangle = DO|\psi_0\rangle = \left(1 - \frac{4}{N}\right)|\psi_0\rangle + \frac{2}{\sqrt{N}}|\alpha\rangle$$

Ignore higher order terms $O(1/N)$,

$$|\psi_1\rangle \approx |\psi_0\rangle + \frac{2}{\sqrt{N}}|\alpha\rangle \tag{33}$$

Step 2. By definition,

$$|\psi_2\rangle = D_1|\psi_0\rangle = (2|\psi_1\rangle\langle\psi_1| - I)|\psi_0\rangle \tag{34}$$

Expand:

$$\begin{aligned} |\psi_1\rangle\langle\psi_1| &= \left(|\psi_0\rangle + \frac{2}{\sqrt{N}}|\alpha\rangle\right)\left(\langle\psi_0| + \frac{2}{\sqrt{N}}\langle\alpha|\right) \\ &= |\psi_0\rangle\langle\psi_0| + \frac{2}{\sqrt{N}}|\alpha\rangle\langle\psi_0| + \frac{2}{\sqrt{N}}|\psi_0\rangle\langle\alpha| + \frac{4}{N}|\alpha\rangle\langle\alpha| \end{aligned} \tag{35}$$

Note that,

$$\langle\psi_0|\psi_0\rangle = 1 \tag{36}$$

$$\langle\alpha|\psi_0\rangle = \langle\alpha|\left(\frac{1}{\sqrt{N}}|\alpha\rangle + \frac{1}{\sqrt{N}}\sqrt{N-1}|\beta\rangle\right) = \frac{1}{\sqrt{N}} \tag{37}$$

Equation (31) is,

$$|\psi_2\rangle = 2\left(|\psi_0\rangle + \frac{2}{\sqrt{N}}|\alpha\rangle + \frac{2}{N}|\psi_0\rangle + \frac{4}{N\sqrt{N}}|\alpha\rangle\right) - |\psi_0\rangle \tag{38}$$

Ignore higher order terms $O(1/N)$,

$$|\psi_2\rangle \approx |\psi_0\rangle + \frac{4}{\sqrt{N}}|\alpha\rangle \tag{39}$$

Step 3,

$$|\psi_3\rangle = R_2|\psi_0\rangle = (2|\psi_2\rangle\langle\psi_2| - I)|\psi_0\rangle \tag{40}$$

Repeat the above process,

$$\begin{aligned}
 |\psi_2\rangle\langle\psi_2| &= \left(|\psi_0\rangle + \frac{4}{\sqrt{N}}|\alpha\rangle \right) \left(\langle\psi_0| + \frac{4}{\sqrt{N}}\langle\alpha| \right) \\
 &= |\psi_0\rangle\langle\psi_0| + \frac{4}{\sqrt{N}}|\alpha\rangle\langle\psi_0| + \frac{4}{\sqrt{N}}|\psi_0\rangle\langle\alpha| + \frac{16}{N}|\alpha\rangle\langle\alpha|
 \end{aligned}$$

So,

$$|\psi_3\rangle \approx |\psi_0\rangle + \frac{8}{\sqrt{N}}|\alpha\rangle \tag{41}$$

The $|\alpha\rangle$ amplitudes in Equations (33), (39), (41), ..., are approximately:

$$\frac{2^0}{\sqrt{N}} \frac{2^1}{\sqrt{N}} \frac{2^2}{\sqrt{N}} \dots \frac{2^K}{\sqrt{N}} \tag{42}$$

After K iteration:

$$\begin{aligned}
 \frac{2^K}{\sqrt{N}} &= \frac{\pi}{4} \approx 1 \\
 K &= \frac{1}{2} \log N = \log(\sqrt{N})
 \end{aligned} \tag{43}$$

The number of diffusion calls for $\pi/4$ rotation from $|\beta\rangle$ to $|\alpha\rangle$, $R(\pi/4)$, is $O(\log N)$.
□

Because the algorithm relies on a geometric series where the amplitude grows exponentially at each step, any arbitrarily dropped error terms might also be compounded exponentially. One can prove that these discarded terms do not entirely collapse the target probability by the final iteration [1]. In each iteration, the higher order terms $O(1/N)$ are dropped. After t diffusion calls, the accumulative error is

$$e = O\left(\frac{t}{N}\right), \quad t \leq \log N \tag{44}$$

For large N , these errors are approaching 0.

4. Time Complexity

The standard lower bounds for unstructured quantum search [6]-[9] still apply even though the number of oracle calls is reduced from $O(\sqrt{N})$ to $O(1)$ and the number of diffusion calls is reduced from $O(\sqrt{N})$ to $O(\log N)$.

4.1. Cost of the Diffusion Operator

The basic ideal is the key identity [10]-[16]:

$$R_\psi = 2|\psi\rangle\langle\psi| - I = U(2|0\rangle\langle 0| - I)U^\dagger \tag{45}$$

where U prepares the state: $U|0\rangle = |\psi\rangle$. Instead of implementing:

$$|\psi\rangle\langle\psi|$$

which looks exponential, it can be implemented as follows:

- 1) Uncompute $|\psi\rangle$ to $|0\rangle$: $|\psi\rangle = U|0\rangle$
- 2) Apply a simple phase flip on $|0\rangle$: $|0\rangle \rightarrow -|0\rangle$

3) Compute back: $U|0\rangle = |\psi\rangle$

So:

$$R_\psi = U(\text{phase-flip})U^\dagger$$

Three steps for Equation (45) are:

Step 1: Apply U^\dagger

- cost (U^\dagger)

Step 2: Flip phase of $|0\rangle$

This is $|0^n\rangle \rightarrow -|0^n\rangle$, implemented by:

- multi-controlled Z gate
- cost: $O(n)$

Step 3: Apply U

- cost (U)

Here the cost means gate count. Finally, if it takes one step for one gate, without gate parallel computation, the time complexity (number of steps without parallel gate execution) is [10]-[18]:

$$T = 2T(U) + O(n) \quad (46)$$

4.2. Cost of D_0

Consider the Diffusion operators:

$$D_0, D_1, D_2, \dots, D_{K-1}$$

The Grover diffusion operator is

$$D_0 = 2|\psi_0\rangle\langle\psi_0| - I$$

which can be written as

$$D = H^{\otimes n} (2|0\rangle\langle 0| - I) H^{\otimes n}$$

Using Equation (45),

$$U = H^{\otimes n}$$

This operator has a gate count of n , so

$$T(D_0) = 2n + O(n) = O(n) \quad (47)$$

4.3. Cost of D_1

The oracle operator has its own time complexity, T_O , which depends on the applications and is not a variable in this paper.

THEOREM 4.1. The time complexity of D_1 is:

$$T(D_1) = 2T(D_0) + 2T_O + T(D_0) \quad (48)$$

Proof. Consider:

$$D_1 = 2|\psi_1\rangle\langle\psi_1| - I$$

By definition (Equation (30)),

$$|\psi_1\rangle = D_0 O |\psi_0\rangle$$

So,

$$D_1 = D_0 O(2|\psi_0\rangle\langle\psi_0| - I) O^\dagger D_0^\dagger = D_0 O D_0 O^\dagger D_0^\dagger \quad (49)$$

Adding all costs in Equation (49) yields Equation (48). \square

4.4. Cost of the D_k

THEOREM 4.2 The time complexities of the Diffusion Operators are:

$$T(D_k) = 2T(D_{k-1}) + T(D_0), k > 1 \quad (50)$$

$$T(D_k) = 2T(D_0) + 2T_O + T(D_0), k = 1 \quad (51)$$

where

$$T(D_0) = O(n).$$

Proof. By definition,

$$D_2 = 2|\psi_2\rangle\langle\psi_2| - I = D_1 (2|\psi_0\rangle\langle\psi_0| - I) D_1^\dagger = D_1 D_0 D_1^\dagger \quad (52)$$

Similarly,

$$D_3 = 2|\psi_3\rangle\langle\psi_3| - I = D_2 (2|\psi_0\rangle\langle\psi_0| - I) D_2^\dagger = D_2 D_0 D_2^\dagger \quad (53)$$

$$D_k = 2|\psi_k\rangle\langle\psi_k| - I = D_{k-1} (2|\psi_0\rangle\langle\psi_0| - I) D_{k-1}^\dagger = D_{k-1} D_0 D_{k-1}^\dagger \quad (54)$$

To compute the time complexity of D_k , simply add each term in Equation (54) to get Equation (50). Equation (51) was proved in the last section. \square

4.5. Standard Low Bounds

THEOREM 4.3 The time complexity of D_k in Equation (50) is exponential,

$$T(D_k) = O(2^k (T_O + n)) \quad (55)$$

Proof. From Equation (50),

$$T(D_2) = 2T(D_1) + T(D_0) \quad (56)$$

$$\begin{aligned} T(D_3) &= 2T(D_2) + T(D_0) = 2(2T(D_1) + T(D_0)) + T(D_0) \\ &= 4(D_1) + (4-1)T(D_0) \end{aligned} \quad (57)$$

$$\begin{aligned} T(D_4) &= 2T(D_3) + T(D_0) = 2(4T(D_1) + 3T(D_0)) + T(D_0) \\ &= 8T(D_1) + (8-1)T(D_0) \end{aligned} \quad (58)$$

\vdots

$$\begin{aligned} T(D_k) &= 2T(D_{k-1}) + T(D_0) \\ &= 2^{k-1} (D_1) + (2^{k-1} - 1)T(D_0) \end{aligned} \quad (59)$$

From Equation (48),

$$D_1 = O(T_O + n) \quad (60)$$

From Equation (59), dropping lower order terms,

$$T(D_k) = O(2^k (T_O + n)) \quad \square$$

THEOREM 4.4 When $k = K$,

$$T(D_K) = O(\sqrt{N}(T_O + n)) \quad \square \quad (61)$$

Proof. When $k = K$, from Equation (42),

$$\frac{2^K}{\sqrt{N}} = O(1)$$

From Equation (55),

$$T(D_k) = O(\sqrt{N}(T_O + n)) \quad \square \quad (62)$$

5. Remarks

5.1. Unmatched Oracle Calls and Diffusion Calls

The paper claims a single oracle call is used in the entire algorithm. However, in Grover's algorithm, because Grover's diffusion operator depends on the state produced by the oracle, constructing and applying the Grover's diffusion operator inherently requires invoking the oracle repeatedly, otherwise, the diffusion operator has nothing to do with the marked state to be amplified. This seems to be inconsistent with one oracle call.

The diffusion operators in this paper are not Grover's diffusion operators except D_0 . This role of invoking the oracle is implemented inexplicitly:

- The Grover diffusion operation, D_0 in Equation (25), is indeed dependent on the state produced by the oracle through Equation (30);
- The next diffusion operator, D_1 in Equation (26), depend on $|\psi_1\rangle$, which depends on oracle through Equation (31) indirectly;
- The next diffusion operator, D_2 in Equation (27), depend on $|\psi_2\rangle$, which in turn depend on oracle through Equation (32) indirectly;
- ...

The operators (D_k) are not independent of the unknown marked state. They depend on oracle-derived states. The dependence is obtained in Equations (26)-(28) as follows:

$$|\psi_0\rangle, O, D_0 \rightarrow |\psi_1\rangle \rightarrow D_1 \rightarrow |\psi_2\rangle \rightarrow D_2 \rightarrow |\psi_3\rangle \rightarrow \dots \rightarrow |\psi_K\rangle$$

while still claiming a single oracle call. Another way to understand this is Equation (55), which indicates that each step depends on oracle operator.

5.2. Geometric View

THEOREM 5.1 The geometric progression of rotation angles at iteration, k , is:

$$\sin(\theta_k) = \frac{1 + 2^k}{\sqrt{N}} \quad (63)$$

Proof. To connect the approximate state updates to the claimed geometric progression of rotation angles, recall in Equation (8):

$$|\psi_k\rangle = \sin(\theta_k)|\alpha\rangle + \cos(\theta_k)|\beta\rangle$$

where θ_k is the angle from the $|\beta\rangle$ axis at iteration, k , and the rotation is from $|\beta\rangle$ axis to $|\alpha\rangle$ axis. Amplitude of the marked state is given in Equation (9):

$$\sin(\theta_k) = a_1(k)$$

From Equation (33),

$$|\psi_1\rangle \approx |\psi_0\rangle + \frac{2}{\sqrt{N}}|\alpha\rangle$$

Note that the first term above contains $(1/\sqrt{N})|\alpha\rangle$, therefore,

$$\sin(\theta_1) = \frac{3}{\sqrt{N}} \quad (64)$$

Similarly,

$$\sin(\theta_2) = \frac{5}{\sqrt{N}} \quad (65)$$

$$\sin(\theta_3) = \frac{9}{\sqrt{N}} \quad \square \quad (66)$$

6. Conclusion

In reference [1], the author presented the Single-Iteration Quantum Search Algorithm, which achieves amplitude-amplification with exactly one oracle call and one $\pi/4$ rotation. This work presents a simpler and cleaner derivation for one oracle call and $\log(N)$ Diffusion calls, while the original proof was based on Cartan-Dieudonné theorem [1]. The standard lower bounds for unstructured quantum search still apply even though the number of oracle calls is reduced from $\mathcal{O}(\sqrt{N})$ to $\mathcal{O}(1)$ and the number of diffusion calls is reduced from $\mathcal{O}(\sqrt{N})$ to $\mathcal{O}(\log N)$.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Liu, Y. (2026) The Single-Rotation Quantum Search Algorithm: Amplitude Amplification with One Oracle Call. *Journal of Quantum Information Science*, **16**, 161-190. <https://doi.org/10.4236/jqis.2026.162006>
- [2] Grover, L.K. (1996) A Fast Quantum Mechanical Algorithm for Database Search. *Proceedings of the Twenty-Eighth Annual ACM Symposium on Theory of Computing*, Philadelphia, 22-24 May 1996, 212-219. <https://doi.org/10.1145/237814.237866>
- [3] Liu, Y. (2026) The Exponential Speedup Algorithm: $\mathcal{O}(\log N)$ Amplitude Amplification via Geometric Series. *Journal of Quantum Information Science*, **16**, 132-159.

- <https://doi.org/10.4236/jqis.2026.161005>
- [4] Nielsen, M.A. and Chuang, I.L. (2010) Quantum Computation and Quantum Information. Cambridge University Press.
- [5] Grover, L.K. (1997) Quantum Mechanics Helps in Searching for a Needle in a Haystack. *Physical Review Letters*, **79**, 325-328. <https://doi.org/10.1103/physrevlett.79.325>
- [6] Boyer, M., Brassard, G., Høyer, P. and Tapp, A. (1998) Tight Bounds on Quantum Searching. *Fortschritte der Physik*, **46**, 493-505. [https://doi.org/10.1002/\(sici\)1521-3978\(199806\)46:4/5<493::aid-prop493>3.0.co;2-p](https://doi.org/10.1002/(sici)1521-3978(199806)46:4/5<493::aid-prop493>3.0.co;2-p)
- [7] Ambainis, A. (2002) Quantum Lower Bounds by Quantum Arguments. *Journal of Computer and System Sciences*, **64**, 750-767. <https://doi.org/10.1006/jcss.2002.1826>
- [8] Hoyer, P., Lee, T. and Spalek, R. (2007) Negative Weights Make Adversaries Stronger. *Proceedings of the Thirty-Ninth Annual ACM Symposium on Theory of Computing*, San Diego, 11-13 June 2007, 526-535. <https://doi.org/10.1145/1250790.1250867>
- [9] Zalka, C. (1999) Grover's Quantum Searching Algorithm Is Optimal. *Physical Review A*, **60**, 2746-2751. <https://doi.org/10.1103/physreva.60.2746>
- [10] Brassard, G., Høyer, P., Mosca, M. and Tapp, A. (2002) Quantum Amplitude Amplification and Estimation. Contemporary Mathematics. arXiv:quant-ph/0005055.
- [11] Shende, V.V., Bullock, S.S. and Markov, I.L. (2006) Synthesis of Quantum-Logic Circuits. *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, **25**, 1000-1010. <https://doi.org/10.1109/tcad.2005.855930>
- [12] Childs, A.M. and Wiebe, N. (2012) Hamiltonian Simulation Using Linear Combinations of Unitary Operations. *Quantum Information and Computation*, **12**, 901-924. <https://doi.org/10.26421/qic12.11-12-1>
- [13] Gilyén, A., Su, Y., Low, G.H. and Wiebe, N. (2019) Quantum Singular Value Transformation and Beyond: Exponential Improvements for Quantum Matrix Arithmetics. *Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing*, Phoenix, 23-26 June 2019, 193-204. <https://doi.org/10.1145/3313276.3316366>
- [14] Barenco, A., Bennett, C.H., Cleve, R., DiVincenzo, D.P., Margolus, N., Shor, P., *et al.* (1995) Elementary Gates for Quantum Computation. *Physical Review A*, **52**, 3457-3467. <https://doi.org/10.1103/physreva.52.3457>
- [15] Low, G.H. and Chuang, I.L. (2019) Hamiltonian Simulation by Qubitization. *Quantum*, **3**, Article 163. <https://doi.org/10.22331/q-2019-07-12-163>
- [16] Bennett, C.H., Bernstein, E., Brassard, G. and Vazirani, U. (1997) Strengths and Weaknesses of Quantum Computing. *SIAM Journal on Computing*, **26**, 1510-1523. <https://doi.org/10.1137/s0097539796300933>
- [17] Liu, Y. (2026) The Grover Dilemma and Three Fundamental Barriers to Oracle-Based Quantum Search Algorithms. *Journal of Quantum Information Science*, **16**, 16-74. <https://doi.org/10.4236/jqis.2026.161002>
- [18] Liu, Y. (2026) Why Oracle-Based Quantum Search Cannot Use Deep Loops: Physical Limits on Sequential Operations. *Journal of Quantum Information Science*, **16**, 75-119. <https://doi.org/10.4236/jqis.2026.161003>