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# Beyond Quantum Mechanics: Local QM Space-Time Algebra Prediction of the Singlet State

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## Abstract

I derive the quantum mechanical prediction $-\boldsymbol{a}\cdot\boldsymbol{b}$ for the singlet spin state using local measurement functions within the Space-Time Algebra (STA) framework. I establish a compact and computationally tractable STA representation of the two-particle singlet state, $\Psi = \frac{1}{2}\left(I\sigma_2^{(1)} - I\sigma_2^{(2)}\right)$, which is simpler than previously published forms but is not required for the correlation calculation. The analysis shows that STA naturally generates both scalar dot-product terms $(\boldsymbol{a}\cdot\boldsymbol{s}_1)$ and $(\boldsymbol{s}_2\cdot\boldsymbol{b})$ and bivector wedge-product terms $(\boldsymbol{a}\wedge\boldsymbol{s}_1)$ and $(\boldsymbol{s}_2\wedge\boldsymbol{b})$ from local spin-detector interactions, whereas standard quantum mechanics retains only the scalar contributions through expectation-value projection. Because the standard quantum formalism represents correlations via operator expectation values, the antisymmetric (cross-product) contributions are averaged out at the level of observables. In contrast, the STA formulation keeps these bivector terms explicit in the intermediate geometric products, raising the question of whether potentially meaningful geometric structure is being hidden by the quantum averaging procedure. I verify the analytical derivation using computational simulations in Mathematica with a Clifford algebra package, confirming that local measurement functions reproduce the standard quantum correlation $-\cos\theta$, where $\theta$ is the angle between measurement directions $\boldsymbol{a}$ and $\boldsymbol{b}$. The correlation $-\boldsymbol{a}\cdot\boldsymbol{b}$ is the scalar coordinate of 3-sphere rotors.

## Keywords

Space-Time Algebra, Quantum Mechanics, Locality, Singlet State, Bell's Theorem

## 1. Introduction

The Einstein-Podolsky-Rosen-Bohm (EPR-Bohm) experiment remains the clearest setting in which quantum mechanics (QM) predicts correlations that appear to defy locality [1] [2]. In the standard formulation, these correlations arise from the singlet wavefunction of two spin 1/2 particles, expressed in the Pauli basis as

$$|\Psi_{\boldsymbol{n}}\rangle = \frac{1}{\sqrt{2}}\left(|\boldsymbol{n},+\rangle_1 |\boldsymbol{n},-\rangle_2 - |\boldsymbol{n},-\rangle_1 |\boldsymbol{n},+\rangle_2\right), \tag{1}$$

where the eigenstates of $\boldsymbol{\sigma}\cdot\boldsymbol{n}$ encode the "up" and "down" spin orientations [3]. This global two-particle state is then used to compute the joint expectation value

$$E_{q.m.}(\boldsymbol{a},\boldsymbol{b}) = \langle \Psi_{\boldsymbol{n}} | \boldsymbol{\sigma}_1\cdot\boldsymbol{a} \otimes \boldsymbol{\sigma}_2\cdot\boldsymbol{b} | \Psi_{\boldsymbol{n}}\rangle = -\boldsymbol{a}\cdot\boldsymbol{b}, \tag{2}$$

which agrees with experiment.

However, this textbook derivation relies on an assumption that is rarely examined: the singlet state is treated as a physically meaningful entity even after the two particles have separated and been independently detected. If detection eliminates the two-particle state—as is normally asserted—then the standard derivation applies a global object in a context where it no longer exists. The correlation is obtained, but the physical mechanism is left obscure.

This motivates a shift from global probabilistic reasoning to a *local geometric description* of spin and measurement. Geometric Algebra (GA) provides such a framework. Christian has shown that the topology of the 3-sphere and the algebra of bivectors can reproduce the singlet correlation using strictly local measurement functions [4] [5]. His construction demonstrates that the correlation $-\boldsymbol{a}\cdot\boldsymbol{b}$ can arise from real geometric structure rather than nonlocal influences. Yet his model does not explicitly develop the STA spinor formalism or construct a two-particle rotor within Space-Time Algebra, so the connection between the geometric picture and the standard spinor framework remains lacking.

Space-Time Algebra [6] [7] provides the missing bridge. In STA, spinors are real geometric objects—rotors acting on spacetime bivectors—and the Pauli algebra appears naturally as the spatial subalgebra of the Dirac algebra. This allows one to express spin, measurement, and correlation entirely in terms of real geometric quantities. The singlet rotor

$$\Psi_{\text{STA}} = \frac{1}{2}\left(I\sigma_2^{(1)} - I\sigma_2^{(2)}\right), \tag{3}$$

encodes the antisymmetry of the two-particle system, but—crucially—it is *not required* for the correlation calculation. Rotational invariance and conservation of angular momentum suffice.

In this paper, I develop a fully local STA formulation of the EPR-Bohm experiment. The key ingredients are:

1) real spin bivectors associated with each particle, generated locally from a Pauli rotor on $S^3$,

2) local measurement functions depending only on the detector direction and the particle's spin bivector,

3) a uniform distribution of Pauli rotors reflecting the rotational symmetry of the singlet.

From these elements, the correlation

$$E_{\text{STA}}(\boldsymbol{a},\boldsymbol{b}) = \langle -(\boldsymbol{ab}) \rangle_0 = -\boldsymbol{a}\cdot\boldsymbol{b} \tag{4}$$

is obtained without invoking a nonlocal two-particle state at the time of measurement like in quantum mechanics. As shown in **Appendix A** where I demonstrate that local measurement functions are a necessity.

This approach clarifies the geometric origin of the singlet correlation and shows that the probabilistic formalism of QM is not fundamental to the phenomenon. Instead, the correlation emerges from *local interactions between real geometric quantities*. The implications for locality, realism, and the interpretation of Bell's theorem are discussed in the concluding section [8].

A further geometric insight comes from the fact that the even subalgebra of STA, which contains all spatial rotors, is isomorphic to the unit quaternions and therefore topologically a 3-sphere, $S^3$. The geometric product of two unit bivectors, such as $(\boldsymbol{as}_1)$, produces a unit rotor of the form $\alpha + B$, where $\alpha$ is a scalar and $B$ is a bivector, and the normalization condition $R\tilde{R} = 1$ places this rotor on $S^3$. Thus the local geometric action used in this work implicitly resides on the same $S^3$ structure that underlies quaternionic models of spin, including those developed by Christian, while embedding the construction directly within the spacetime framework of STA. Although vectors in STA are represented by bivectors, they are nevertheless conventionally denoted using boldface type. For an in-depth discussion of the physical meaning of the 3-sphere topology and the quaternions employed in our measurement functions, please consult the references cited above and [9]-[12].

## 2. Formulation of STA Local Measurement Functions

To analyze singlet correlations within a strictly local framework, we will construct explicit measurement functions using the tools of Space-Time Algebra. Unlike the Hilbert-space formalism, STA represents particle spins as bivectors and encodes rotations through rotors acting within the even sub-algebra. This provides a natural geometric setting in which locality can be expressed directly at the level of the measurement maps themselves.

**Pauli quaternion rotor.** A central geometric insight of this work is that each particle carries a *local internal rotor*—a unit element of the even subalgebra of STA—that determines its spin orientation. This rotor, $R = \alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3$ with $\alpha^2 + \beta_1^2 + \beta_2^2 + \beta_3^2 = 1$, is generated by the three spatial Pauli-rotor bivectors $B_i$ and therefore lies on the 3-sphere $S^3$. A normalized Pauli spinor contains precisely such a unit rotor in its even multivector part, so the internal configuration space relevant for spin is naturally the 3-sphere $\text{Spin}(3) \cong SU(2) \cong S^3$. In this sense, the rotor $R$ may be identified with the Pauli rotor extracted from the spinor, and the points on the 3-sphere are the coordinates $(\alpha,\beta_1,\beta_2,\beta_3)$. The spin bivector is obtained by the local geometric action $s_1 = R\sigma_3\tilde{R}$, which produces a unit timelike bivector in the physical spin subspace. Thus the electron's internal configuration space is naturally a 3-sphere, and the distribution of spin orientations inherited from the singlet state corresponds to a uniform distribution of Pauli rotors on $S^3$.

The unit bivectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ represent the intrinsic spin directions of the two particles in the Space-Time Algebra framework. Under the metric signature $(1,3)$, these bivectors satisfy $\boldsymbol{s}_i^2 = 1$ for unit bivectors. They are explicit geometric objects within the STA framework and not "hidden variables" in the usual sense of Bell's terminology. An introductory version of this geometric approach, employing explicit spin vectors in a local framework, was presented in [13] and Christian's work [4]. The present formulation extends that work by introducing explicit STA measurement maps $A(\boldsymbol{a},\boldsymbol{s}_1)$ and $B(\boldsymbol{b},\boldsymbol{s}_2)$ and by carrying the full multivector structure through the correlation calculation. Following the conservation of total spin angular momentum in the singlet state (as shown in **Appendix C**), these satisfy

$$\boldsymbol{s}_1 + \boldsymbol{s}_2 = 0, \text{ so that } \boldsymbol{s}_1 = -\boldsymbol{s}_2. \tag{5}$$

We now formulate local measurement functions, similar in spirit to those introduced by Bell [8], but expressed entirely within STA. The detector orientations $\boldsymbol{a}$ and $\boldsymbol{b}$ are represented by bivectors, which act locally on the spin bivectors. Upon detection, the incoming spin bivectors align with the detector directions, so that $\boldsymbol{s}_1 \to \boldsymbol{a}$ and $\boldsymbol{s}_2 \to \boldsymbol{b}$ at their respective measurement sites. In the interaction of two vectors, the resultant contains both a scalar part $(\boldsymbol{u}\cdot\boldsymbol{v})$ and a bivector part $(\boldsymbol{u}\wedge\boldsymbol{v})$, the latter corresponding to the usual cross product $\boldsymbol{u}\times\boldsymbol{v} = \boldsymbol{r}$. In quaternion language this is written as a single entity $\boldsymbol{q}(\boldsymbol{u}\cdot\boldsymbol{v},\boldsymbol{r})$. In geometric algebra this is naturally expressed as the multivector product, $\boldsymbol{uv} = \boldsymbol{u}\cdot\boldsymbol{v} + \boldsymbol{u}\wedge\boldsymbol{v}$, as discussed in [7]. Employing this structure together with Equation (5), we define

$$\boldsymbol{r}_a := \boldsymbol{a}\times\boldsymbol{s}_1, \tag{6}$$

$$\boldsymbol{r}_b := \boldsymbol{s}_2\times\boldsymbol{b}, \tag{7}$$

$$\mu_a := \operatorname{sgn}(\boldsymbol{a}\cdot\boldsymbol{s}_1)\boldsymbol{a}, \tag{8}$$

$$\mu_b := \operatorname{sgn}(\boldsymbol{b}\cdot\boldsymbol{s}_2)\boldsymbol{b}. \tag{9}$$

In STA, the detector orientations and spins are represented by the bivectors

$$\boldsymbol{a} = a_x\sigma_1 + a_y\sigma_2 + a_z\sigma_3,$$

$$\boldsymbol{b} = b_x\sigma_1 + b_y\sigma_2 + b_z\sigma_3,$$

$$\boldsymbol{s}_i = s_i x\sigma_1 + s_i y\sigma_2 + s_i z\sigma_3. \tag{10}$$

with $\sigma_i = \gamma_i\gamma_0$ and $\boldsymbol{a}^2 = \boldsymbol{b}^2 = \boldsymbol{s}_i^2 = 1$. With the spins $\boldsymbol{s}_i$, generated by the Pauli rotor from points on $S^3$.

For reference, we display both the standard quantum-mechanical (QM) and (STA) forms of the spin states:

$$|\phi\rangle_{\mathrm{QM}} := \frac{1}{\sqrt{2}}\left(|+\rangle_1 + |-\rangle_1\right),\ \phi_{\mathrm{STA}} := \frac{1}{\sqrt{2}}\left(1 - I\sigma_2^{(1)}\right), \tag{11}$$

$$|\chi\rangle_{\mathrm{QM}} := \frac{1}{\sqrt{2}}\left(|-\rangle_2 + |+\rangle_2\right),\ \chi_{\mathrm{STA}} := \frac{1}{\sqrt{2}}\left(-I\sigma_2^{(2)} + 1\right). \tag{12}$$

However, the STA $\phi$ and $\chi$ rotors are not used or needed for the STA product calculation, but we show calculations using them in **Appendix B**. We now define the local measurement functions.

$$\begin{aligned} A(\boldsymbol{a},\boldsymbol{s}_1) &:= \lim_{\boldsymbol{s}_1\to\mu_{\boldsymbol{a}}}\left[\left\langle\phi_{\mathrm{QM}}\left|(\boldsymbol{\sigma}\cdot\boldsymbol{a})(\boldsymbol{\sigma}\cdot\boldsymbol{s}_1)\right|\phi_{\mathrm{QM}}\right\rangle + I_3\boldsymbol{r}_{\boldsymbol{a}}\right] \text{ (QM form)} \\ &:= \lim_{\boldsymbol{s}_1\to\mu_{\boldsymbol{a}}}\left[(\boldsymbol{\sigma}_i\cdot\boldsymbol{a})(\boldsymbol{\sigma}_i\cdot\boldsymbol{s}_1)\right] \text{ (STA form)} \\ &= \lim_{\boldsymbol{s}_1\to\mu_{\boldsymbol{a}}}\left[(\boldsymbol{a}\boldsymbol{s}_1)\right] \\ &= \lim_{\boldsymbol{s}_1\to\mu_{\boldsymbol{a}}}\left[\boldsymbol{a}\cdot\boldsymbol{s}_1 + \boldsymbol{a}\wedge\boldsymbol{s}_1\right] \\ &= \operatorname{sgn}(\boldsymbol{a}\cdot\boldsymbol{s}_1) = \pm 1. \end{aligned} \tag{13}$$

$$\begin{aligned} B(\boldsymbol{b},\boldsymbol{s}_2) &:= \lim_{\boldsymbol{s}_2\to\mu_{\boldsymbol{b}}}\left[\left\langle\chi_{\mathrm{QM}}\left\|(\boldsymbol{\sigma}\cdot\boldsymbol{s}_2)(\boldsymbol{\sigma}\cdot\boldsymbol{b})\right\|\chi_{\mathrm{QM}}\right\rangle + I_3\boldsymbol{r}_{\boldsymbol{b}}\right] \text{ (QM form)} \\ &:= \lim_{\boldsymbol{s}_2\to\mu_{\boldsymbol{b}}}\left[(\boldsymbol{\sigma}_i\cdot\boldsymbol{s}_2)(\boldsymbol{\sigma}_i\cdot\boldsymbol{b})\right] \text{ (STA form)} \\ &= \lim_{\boldsymbol{s}_2\to\mu_{\boldsymbol{b}}}\left[(\boldsymbol{s}_2\boldsymbol{b})\right] \\ &= \lim_{\boldsymbol{s}_2\to\mu_{\boldsymbol{b}}}\left[\boldsymbol{s}_2\cdot\boldsymbol{b} + \boldsymbol{s}_2\wedge\boldsymbol{b}\right] \\ &= \operatorname{sgn}(\boldsymbol{s}_2\cdot\boldsymbol{b}) = \mp 1. \end{aligned} \tag{14}$$

Here $I$ or $I_4 = \gamma_0\gamma_1\gamma_2\gamma_3$ is the pseudoscalar of Space-Time Algebra and $I_3 = e_x e_y e_z$ is the pseudoscalar of the three-dimensional geometric algebra used in the QM formulation [7]. In the pure STA formulation, the bivector terms $\boldsymbol{a}\wedge\boldsymbol{s}_1$ and $\boldsymbol{s}_2\wedge\boldsymbol{b}$ arise naturally from the geometric product without requiring the addition of $I_3\boldsymbol{r}_{\boldsymbol{a}}$ and $I_3\boldsymbol{r}_{\boldsymbol{b}}$, as demonstrated in the computational implementation of Section IV. In the QM framework, $\boldsymbol{\sigma}\cdot\boldsymbol{a}$ and $\boldsymbol{\sigma}\cdot\boldsymbol{b}$ represent the detectors used by Alice and Bob, respectively, each carrying no angular momentum at the instant of detection. In the STA framework, the bivectors $\boldsymbol{a}$ and $\boldsymbol{b}$ represent these same detector orientations, and the measurement interaction is encoded through the geometric products $\boldsymbol{a}\boldsymbol{s}_1$ and $\boldsymbol{s}_2\boldsymbol{b}$. The quantity $\boldsymbol{\sigma}\cdot\boldsymbol{s}_1 = -\boldsymbol{\sigma}\cdot\boldsymbol{s}_2$ in QM (or equivalently $\boldsymbol{s}_1 = -\boldsymbol{s}_2$ in STA) denotes the spin of the fermions received by the detectors and forms the basis of the EPRB experiment. The limit-replacement expressions serve as idealized models of the polarizers at the detection stations by aligning the particle's spin direction with the polarizer direction, while $\left|\phi_{\mathrm{QM}}\right\rangle$ and $\left|\chi_{\mathrm{QM}}\right\rangle$ denote the wavefunctions of the individual particles.

In the subsequent stage of the $A$ and $B$ functions, the multivector structure emerges explicitly from the geometric product. The cross-product (bivector) terms vanish in the matrix-based expectation-value calculation due to averaging, even though they do not vanish in ordinary local vector algebra. We therefore regard these bivector contributions as non-observable components that must nevertheless be retained in the full geometric calculation in order to obtain the correct predictions. In STA, the geometric product of two unit spatial bivectors lies in the even subalgebra, whose unit elements form the rotor group $\mathrm{Spin}(3)\cong SU(2)\cong S^3$, so the rotor actions $(\boldsymbol{a}\boldsymbol{s}_1)$ and $(\boldsymbol{s}_2\boldsymbol{b})$ naturally reside on a 3-sphere.

## 3. Product Expectation Value Calculation Using Space-Time Algebra

The product expectation value is computed by evaluating $A(\boldsymbol{a},\boldsymbol{s}_1)B(\boldsymbol{b},\boldsymbol{s}_2)$ for each emitted pair and then averaging over the isotropic singlet distribution. The correlation is then computed entirely within the even subalgebra of STA. Using the limit-replacement definitions of the measurement functions, the calculation proceeds as follows:

$$E(\boldsymbol{a},\boldsymbol{b})=\lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}A\left(\boldsymbol{a}^k,\boldsymbol{s}_1^k\right)B\left(\boldsymbol{b}^k,\boldsymbol{s}_2^k\right)\right], \tag{15}$$

$$=\lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\left(\lim_{s_1\to\mu_a}[\boldsymbol{a}\boldsymbol{s}_1]\right)\left(\lim_{s_2\to\mu_b}[\boldsymbol{s}_2\boldsymbol{b}]\right)\right] \tag{16}$$

$$=\lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\lim_{\substack{s_1\to\mu_a\\ s_2\to\mu_b}}\left\{(\boldsymbol{a}\boldsymbol{s}_1)(\boldsymbol{s}_2\boldsymbol{b})\right\}\right] \tag{17}$$

$$=\lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\lim_{s_1\to\mu_a}\left\{-(\boldsymbol{a}\boldsymbol{s}_1)(\boldsymbol{s}_1\boldsymbol{b})\right\}\right] \tag{18}$$

$$=\lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\left\{-(\boldsymbol{a}\boldsymbol{b})\right\}\right] \tag{19}$$

$$=\lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\left\{-\boldsymbol{a}\cdot\boldsymbol{b}-(\boldsymbol{a}\wedge\boldsymbol{b})\right\}\right] \tag{20}$$

$$=-\boldsymbol{a}\cdot\boldsymbol{b}+\lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}-(\boldsymbol{a}\wedge\boldsymbol{b})\right] \tag{21}$$

$$=-\boldsymbol{a}\cdot\boldsymbol{b}. \tag{22}$$

The steps above use only the STA definitions of the measurement functions and the singlet condition $\boldsymbol{s}_2=-\boldsymbol{s}_1$. Equation (19) follows from $\boldsymbol{s}_1^2=1$ for a unit spin bivector, which reduces $-(\boldsymbol{a}\boldsymbol{s}_1)(\boldsymbol{s}_1\boldsymbol{b})$ to $-(\boldsymbol{a}\boldsymbol{b})$. The geometric product $-(\boldsymbol{a}\boldsymbol{b})$ contains both a scalar part $-\boldsymbol{a}\cdot\boldsymbol{b}$ and a bivector part $-(\boldsymbol{a}\wedge\boldsymbol{b})$, but the bivector term averages to zero over the isotropic singlet distribution. However, it is actually impossible for the bivector term to exist since $\boldsymbol{a}$ and $\boldsymbol{b}$ are not local to each other; it is a mathematical artifact in this case. So, the correlation is purely scalar and equals $-\boldsymbol{a}\cdot\boldsymbol{b}$, in agreement with the quantum-mechanical prediction.

## 4. STA Computational Validation via Mathematica Simulations

To complement the analytic derivation of the correlation $E(\boldsymbol{a},\boldsymbol{b})=-\boldsymbol{a}\cdot\boldsymbol{b}$, we implemented a set of Mathematica simulations using the clifford.m package. These simulations numerically realize the local measurement maps of Section II and the product calculation of Section 3. The purpose of this section is not to simulate an experiment, but to verify that the STA multivector structure reproduces the analytic result when evaluated over a large ensemble of singlet pairs. The Mathematica simulations can be read or downloaded at [14].

The simulations use the following physical inputs of the singlet state: 1) rotational invariance, implemented by sampling $\boldsymbol{s}_1$ uniformly from the unit sphere using a Pauli rotor that converts from points on the 3-sphere, and 2) conservation of angular momentum, implemented by the constraint $\boldsymbol{s}_2 = -\boldsymbol{s}_1$. For each trial, independent detector directions $\boldsymbol{a}$ and $\boldsymbol{b}$ are generated, and the local measurement interactions are computed from the full multivector products $A(\boldsymbol{a},\boldsymbol{s}_1) = \lim_{s_1\to\mu_a}[\boldsymbol{a}\boldsymbol{s}_1]$, $B(\boldsymbol{b},\boldsymbol{s}_2) = \lim_{s_2\to\mu_b}[\boldsymbol{s}_2\boldsymbol{b}]$, as defined in Section 2. The scalar parts of these multivectors determine the measurement outcomes $A, B = \pm 1$, while the full multivectors are retained for the product calculation of Section 3.

### 4.1. 2D Detector Simulation

Most real detectors (polarizers, Stern-Gerlach magnets, photon analyzers) act in a fixed measurement plane. We therefore implemented a 2D detector model in which $\boldsymbol{a}$ and $\boldsymbol{b}$ lie in the $x$-$y$ plane perpendicular to the particle's motion on the $z$-axis while the spin bivectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ remain fully 3D and satisfy $\boldsymbol{s}_2 = -\boldsymbol{s}_1$. This allows the relative detector angle $\theta$ to be varied directly from $-360^\circ$ to $+360^\circ$ without geometric sampling bias.

For each of $m = 300000$ trials, the following steps are performed:

1) Generate four coordinates for a point on $S^3$.

2) Construct a $S^3$ Pauli quaternion rotor using the coordinates and three bivectors, $B_1 = -I_4\boldsymbol{\sigma}_1$, $B_2 = -I_4\boldsymbol{\sigma}_2$, and $B_3 = -I_4\boldsymbol{\sigma}_3$.

3) Generate random unit spin bivectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ using the $S^3$ Pauli rotor as a Hopf fibration to $S^2$.

4) Generate detector directions $\boldsymbol{a}$ and $\boldsymbol{b}$ uniformly in the $x$-$y$ plane.

5) Compute the multivector interactions $\boldsymbol{a}\boldsymbol{s}_1$ and $\boldsymbol{s}_2\boldsymbol{b}$.

6) Extract the scalar parts to assign $A = \pm 1$ and $B = \mp 1$.

7) Compute the full geometric product $A(\boldsymbol{a},\boldsymbol{s}_1)B(\boldsymbol{b},\boldsymbol{s}_2)$.

8) Bin the scalar part of the product according to the relative angle $\theta$.

The resulting correlation curve $E(\theta)$ matches the analytic prediction $-\cos\theta$ to numerical precision. The simulation data (blue) lie directly on top of the theoretical curve (magenta) as shown in **Figure 1**, and the cross-product (bivector) terms average to zero, in agreement with the analytic calculation of Section III. The spatial bivectors $B_i$ of STA can be used to form and calculate quaternions.

### 4.2. 3D Detector Confirmation

To verify that the result is not an artifact of planar detectors, we also implemented a full 3D simulation in which $\boldsymbol{a}$ and $\boldsymbol{b}$ are sampled uniformly from the unit sphere. The same procedure is followed, with the relative angle $\theta$ computed from the numerical components of $\boldsymbol{a}$ and $\boldsymbol{b}$. The resulting correlation again matches $-\boldsymbol{a}\cdot\boldsymbol{b}$ as shown in **Figure 2**, and the bivector terms vanish on average. This confirms that the STA local measurement maps reproduce the singlet correlation in both 2D and 3D settings.

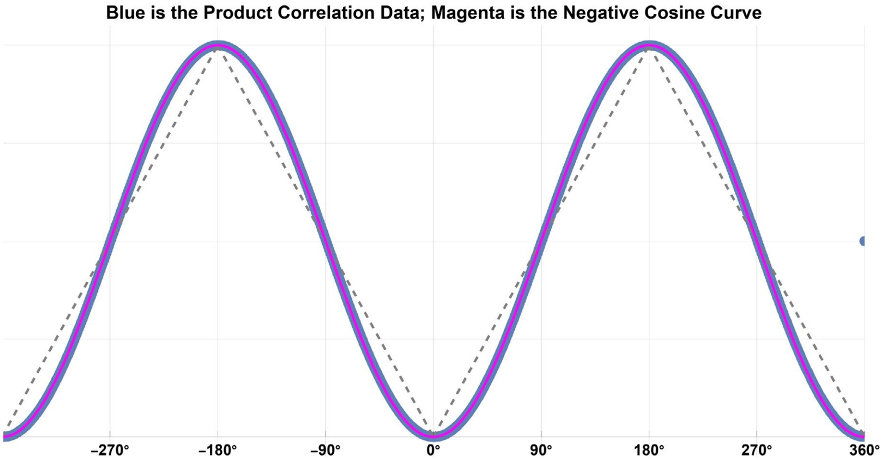

**Figure 1.** Plot of product calculation from the 2D detector simulation or the short-cut singlet calculation.

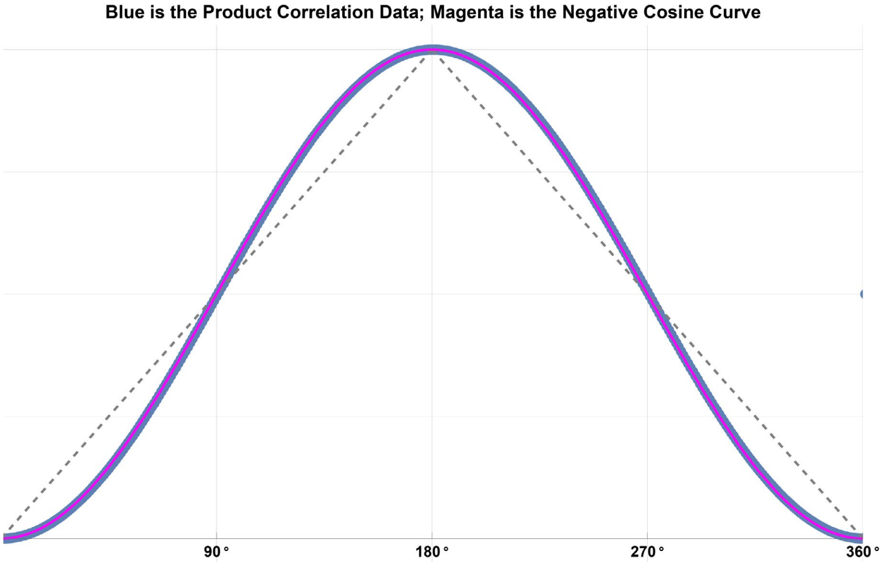

**Figure 2.** Plot of product calculation from the 3D simulation. Blue is the correlation data, magenta is the negative cosine curve for an exact match.

### 4.3. Numerical Summary

The simulations yield the following results:

- **Correlation.** The scalar part of the STA product reproduces $E(\theta) = -\cos\theta$ over the full $720^\circ$ range.
- **Individual averages.** The measurement outcomes satisfy $\langle A\rangle \approx 0$ and $\langle B\rangle \approx 0$, consistent with the singlet symmetry.

- **Bivector terms.** The bivector part of the product averages to zero, as predicted by the analytic calculation.
- **Rotational invariance.** Both the 2D and 3D simulations confirm that the STA model preserves the rotational invariance of the singlet state.

### 4.4. Significance

These simulations provide a direct computational validation of the analytic STA derivation. The correlation $-\boldsymbol{a}\cdot\boldsymbol{b}$ emerges from purely local geometric operations, with no nonlocal influences or probabilistic assumptions. The agreement between the analytic and numerical results confirms that the STA measurement maps and multivector products faithfully reproduce the singlet correlation.

## 5. Conclusions

The analytic and computational results presented in this work demonstrate that the singlet correlation $-\boldsymbol{a}\cdot\boldsymbol{b}$ can be obtained from a fully local model formulated entirely within Space-Time Algebra. The key ingredients are the geometric representation of spin as a unit bivector, the conservation of angular momentum expressed by $\boldsymbol{s}_2=-\boldsymbol{s}_1$, and the local measurement maps $A(\boldsymbol{a},\boldsymbol{s}_1)$ and $B(\boldsymbol{b},\boldsymbol{s}_2)$ defined by the limit-replacement interactions of Section 2. When these maps are combined through the STA product calculation of Section 3 and averaged over the rotationally invariant singlet distribution, the resulting correlation is precisely $-\boldsymbol{a}\cdot\boldsymbol{b}$.

The STA formulation makes the locality of the model explicit. Each measurement depends only on the detector direction and the incoming spin bivector at that station; no global two-particle state persists after detection, and no nonlocal influence is required to produce the correlation. The bivector terms that arise in the geometric product vanish upon averaging, leaving only the scalar part that matches the quantum-mechanical prediction. This behavior is a direct consequence of the geometric structure of Nature and the isotropy of the singlet distribution.

The Mathematica simulations of Section 4 confirm the analytic result [14]. The scalar part of the STA product reproduces the correlation curve $E(\theta)=-\cos\theta$ to numerical precision, while the bivector components average to zero. The individual averages $\langle A\rangle$ and $\langle B\rangle$ also vanish, as expected for the singlet state. These numerical results validate the analytic derivation and demonstrate that the STA measurement maps faithfully reproduce the singlet correlation in both 2D and 3D detector models.

Taken together, the analytic and computational results show that the singlet correlation arises from local geometric interactions when the underlying physical quantities are represented by bivectors and their geometric products. This suggests that the usual inference from Bell's theorem to nonlocality is not unavoidable [15]. Rather, it reflects the limitations of scalar hidden-variable models, not a fundamental feature of Nature. Whether this indicates an incompleteness in the standard quantum-mechanical description remains an open question, but the STA framework provides a coherent and physically transparent alternative in which locality, rotational invariance, and the singlet correlation coexist without contradiction.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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# Appendix

## A. Singlet State Calculation Using Space-Time Algebra

This appendix presents the explicit STA computation of the singlet state correlation using the Pauli bivector basis within Space-Time Algebra with no other "hidden" variables. The measurement directions $\boldsymbol{a}$ and $\boldsymbol{b}$ are bivectors, while the Pauli elements $\sigma_k$ retain their standard bivector form. All calculations were performed in clifford.m with signature $(+,-,-,-)$.

**Locality remark.** The derivation presented here assumes that the bivectors $\boldsymbol{a}$ and $\boldsymbol{b}$ belong to the same Pauli subalgebra. This is appropriate only when $\boldsymbol{a}$ and $\boldsymbol{b}$ are regarded as co-located measurement directions. If the two measurements occur at spacelike separation, then identifying their observables within a single algebraic space implicitly introduces a nonlocal structure. The STA derivation of $-\boldsymbol{a}\cdot\boldsymbol{b}$ therefore applies only under the assumption of a common measurement location.

The single-particle Pauli bivectors in STA are

$$\sigma_1=\gamma_1\gamma_0,\ \sigma_2=\gamma_2\gamma_0,\ \sigma_3=\gamma_3\gamma_0. \tag{23}$$

For the two-particle system we assign

$$\sigma_{1p1}=\sigma_1,\ \sigma_{2p1}=\sigma_2,\ \sigma_{3p1}=\sigma_3, \tag{24}$$

and the particle-2 bivectors are the negatives,

$$\sigma_{kp2}=-\sigma_{kp1}. \tag{25}$$

Measurement directions $\boldsymbol{a}$ and $\boldsymbol{b}$ are encoded as bivectors:

$$\begin{aligned}\boldsymbol{a}&=a_x\sigma_{1p1}+a_y\sigma_{2p1}+a_z\sigma_{3p1},\\ \boldsymbol{b}&=b_x\sigma_{1p2}+b_y\sigma_{2p2}+b_z\sigma_{3p2}.\end{aligned} \tag{26}$$

The two-term expression for the singlet rotor is

$$\psi_{\text{STA}}=\frac{1}{2}\left(I_4\sigma_{2p2}-I_4\sigma_{2p1}\right). \tag{27}$$

However, the rotor is not needed for the calculation.

Since $\boldsymbol{a}$ and $\boldsymbol{b}$ are encoded as bivectors via $\boldsymbol{\sigma}_{p1}$ and $\boldsymbol{\sigma}_{p2}$ respectively, the calculation is quite simple.

$$E(\boldsymbol{a},\boldsymbol{b})=\lim_{n\gg1}\left[\frac{1}{n}\sum_{k=1}^{n}\{(\boldsymbol{ab})\}\right] \tag{28}$$

$$=\lim_{n\gg1}\left[\frac{1}{n}\sum_{k=1}^{n}\{-\boldsymbol{a}\cdot\boldsymbol{b}-(\boldsymbol{a}\wedge\boldsymbol{b})\}\right] \tag{29}$$

$$=-\boldsymbol{a}\cdot\boldsymbol{b}+\lim_{n\gg1}\left[\frac{1}{n}\sum_{k=1}^{n}-(\boldsymbol{a}\wedge\boldsymbol{b})\right] \tag{30}$$

$$=-\boldsymbol{a}\cdot\boldsymbol{b}+0 \tag{31}$$

$$=-\boldsymbol{a}\cdot\boldsymbol{b}. \tag{32}$$

in agreement with the quantum mechanical prediction. However, now that the STA calculation for the singlet has been made explicit, it is clear that it is physically wrong. It is physically wrong in exactly the same sense as the standard QM calculation. The spatial measurement bivectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined at separated locations, so they cannot interact with each other in any physical model. Any calculation that treats $\boldsymbol{a}$ and $\boldsymbol{b}$ as if they coexist inside a single algebraic object in order to produce the correlation $-\boldsymbol{a}\cdot\boldsymbol{b}$ is therefore unphysical. The correct description of the correlation must come from the local measurement functions, not from a nonlocal algebraic construction. As shown, $E(\boldsymbol{a},\boldsymbol{b})=-\boldsymbol{a}\cdot\boldsymbol{b}$ can be calculated directly from the geometric product of STA versions of $\boldsymbol{a}$ and $\boldsymbol{b}$. The singlet state is only needed for rotational invariance and conservation of angular momentum.

### B. STA Rotor Calculations for Functions A and B Using $\phi$ and $\chi$

We show these calculations even though they are not needed.

$$\phi=\frac{1}{\sqrt{2}}\left(1-I\sigma_2^{(1)}\right), \tag{33}$$

$$\tilde{\phi}=\frac{1}{\sqrt{2}}\left(1+I\sigma_2^{(1)}\right), \tag{34}$$

$$\begin{aligned}\tilde{\phi}\left(\boldsymbol{a}\boldsymbol{s}_1\right)\phi&=\frac{1}{2}\left(1+I\sigma_2^{(1)}\right)\left(\boldsymbol{a}\boldsymbol{s}_1\right)\left(1-I\sigma_2^{(1)}\right)\\&=\frac{1}{2}\left(\boldsymbol{a}\boldsymbol{s}_1-\boldsymbol{a}\boldsymbol{s}_1I\sigma_2^{(1)}+I\sigma_2^{(1)}\boldsymbol{a}\boldsymbol{s}_1-I\sigma_2^{(1)}\boldsymbol{a}\boldsymbol{s}_1I\sigma_2^{(1)}\right),\\&=\frac{1}{2}\left(\boldsymbol{a}\boldsymbol{s}_1-\boldsymbol{a}\boldsymbol{s}_1I\sigma_2^{(1)}+\boldsymbol{a}\boldsymbol{s}_1I\sigma_2^{(1)}+\boldsymbol{a}\boldsymbol{s}_1\right),\\&=\boldsymbol{a}\boldsymbol{s}_1.\end{aligned} \tag{35}$$

$$\chi=\frac{1}{\sqrt{2}}\left(-I\sigma_2^{(2)}+1\right), \tag{36}$$

$$\tilde{\chi}=\frac{1}{\sqrt{2}}\left(I\sigma_2^{(2)}+1\right), \tag{37}$$

$$\begin{aligned}\tilde{\chi}\left(\boldsymbol{s}_2\boldsymbol{b}\right)\chi&=\frac{1}{2}\left(I\sigma_2^{(2)}+1\right)\left(\boldsymbol{s}_2\boldsymbol{b}\right)\left(-I\sigma_2^{(2)}+1\right)\\&=\frac{1}{2}\left(\boldsymbol{s}_2\boldsymbol{b}-\boldsymbol{s}_2\boldsymbol{b}I\sigma_2^{(2)}+I\sigma_2^{(2)}\boldsymbol{s}_2\boldsymbol{b}-I\sigma_2^{(2)}\boldsymbol{s}_2\boldsymbol{b}I\sigma_2^{(2)}\right),\\&=\frac{1}{2}\left(\boldsymbol{s}_2\boldsymbol{b}-\boldsymbol{s}_2\boldsymbol{b}I\sigma_2^{(2)}+\boldsymbol{s}_2\boldsymbol{b}I\sigma_2^{(2)}+\boldsymbol{s}_2\boldsymbol{b}\right),\\&=\boldsymbol{s}_2\boldsymbol{b}.\end{aligned} \tag{38}$$

### C. Derivation of $s_2=-s_1$ Using the STA Singlet

Work in the two-particle STA with Pauli bivectors $\sigma_{ip1}$ and $\sigma_{ip2}$, $i=1,2,3$, satisfying

$$\sigma_{ip\alpha}^2=1,\ \sigma_{ip1}\sigma_{jp2}=\sigma_{jp2}\sigma_{ip1},\ \alpha=1,2. \tag{39}$$

Define the STA singlet object for the $i=2$ direction by

$$\psi_{\text{STA}} = \frac{1}{2}\left(I_4\sigma_{2p2} - I_4\sigma_{2p1}\right) = \frac{I_4}{2}\left(\sigma_{2p2} - \sigma_{2p1}\right), \tag{40}$$

where $I_4$ is the spacetime pseudoscalar, commuting with the Pauli bivectors.

Compute the action of the total spin operator in the 2-direction on $\psi_{\text{STA}}$:

$$\left(\sigma_{2p1} + \sigma_{2p2}\right)\psi_{\text{STA}} = \frac{I_4}{2}\left(\sigma_{2p1} + \sigma_{2p2}\right)\left(\sigma_{2p2} - \sigma_{2p1}\right) \tag{41}$$

$$= \frac{I_4}{2}\left(\sigma_{2p1}\sigma_{2p2} - \sigma_{2p1}^2 + \sigma_{2p2}^2 - \sigma_{2p2}\sigma_{2p1}\right). \tag{42}$$

Using $\sigma_{2p1}^2 = \sigma_{2p2}^2 = 1$ and $\sigma_{2p1}\sigma_{2p2} = \sigma_{2p2}\sigma_{2p1}$, this reduces to

$$\left(\sigma_{2p1} + \sigma_{2p2}\right)\psi_{\text{STA}} = \frac{I_4}{2}\left(\sigma_{2p1}\sigma_{2p2} - 1 + 1 - \sigma_{2p2}\sigma_{2p1}\right) = 0. \tag{43}$$

By rotational symmetry of the Pauli bivectors, the same construction yields

$$\left(\sigma_{ip1} + \sigma_{ip2}\right)\psi_{\text{STA}} = 0,\ \ i = 1,2,3. \tag{44}$$

Identify the local spin bivectors in the common physical 3D frame as

$$\boldsymbol{s}_1 \equiv \sigma_{ip1},\ \ \boldsymbol{s}_2 \equiv \sigma_{ip2}, \tag{45}$$

so that (44) becomes

$$\left(\boldsymbol{s}_1 + \boldsymbol{s}_2\right)\psi_{\text{STA}} = 0. \tag{46}$$

Since $\psi_{\text{STA}} \neq 0$, the only way for the product (46) to vanish is

$$\boldsymbol{s}_1 + \boldsymbol{s}_2 = 0, \tag{47}$$

and therefore

$$\boldsymbol{s}_2 = -\boldsymbol{s}_1. \tag{48}$$