

# Using Cumulative Growth Prediction Model to Study the Variation Law of Waterflooding Volumetric Sweep Efficiency

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**How to cite this paper:** Sun, E.H. and Peng, Q. (2026) Using Cumulative Growth Prediction Model to Study the Variation Law of Waterflooding Volumetric Sweep Efficiency. *Journal of Power and Energy Engineering*, 14, 34-45.  
<https://doi.org/10.4236/jpee.2026.1410003>

**Received:** September 23, 2026

**Accepted:** October 25, 2026

**Published:** October 28, 2026

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## Abstract

In the evaluation system for oil and gas field development, the waterflooding volumetric sweep efficiency is a crucial parameter that reflects the balanced development status of oilfield waterflooding and the impact of reservoir heterogeneity on development. To enhance the reliability of predicting waterflooding volumetric sweep efficiency, two cumulative growth prediction models, namely the HCZ model and the Hubbert model are selected. Through dimensionless mathematical transformation, dimensionless correlation equations for waterflooding volumetric sweep efficiency of the two models are derived, and standardized prediction charts are established. Combined with Zhang Jinjing's waterflooding curve, a quantitative relationship between waterflooding volumetric sweep efficiency and oilfield production years is established, thereby predicting the variation pattern of waterflooding volumetric sweep efficiency. Based on production data from Oilfield C shows that the adaptability of the HCZ model is significantly better than that of the Hubbert model, making it more suitable for quantitative prediction of waterflooding volumetric sweep efficiency in this type of reservoir. The research indicates that when development years less than 15a, when the water content is less than 90%,  $E_v$  exhibits an upward-curving increase; when development years is greater than 15a and the water content is greater than 90%,  $E_v$  shows a gradual increase. The available unswept areas within the reservoir significantly decrease, and the growth trend of waterflooding volumetric sweep efficiency gradually slows down, entering a phase of steady growth. This method reflects the waterflooding development status in different production stages of the oilfield and is practical for predicting and evaluating development indicators in the later stages of oilfield production.

## Keywords

Waterflooding Volumetric Sweep Efficiency, Cumulative Growth Prediction Model, Production Years, Water Content, Zhang Jinqing's Waterflooding Curve

## 1. Introduction

In the process of oil and gas field development, the waterflooding volumetric sweep efficiency serves as a crucial basis for evaluating the effectiveness of oilfield development and formulating adjustment strategies [1]-[3]. Some scholars [4]-[6] derived methods for predicting the waterflooding sweep efficiency based on Type A and Type C waterflooding curves, respectively. However, both methods can only describe one type of fw-R\* curve and have a narrow range of applicability. Reference [7] used Zhang Jinqing's waterflooding curve to derive a method for predicting the volumetric sweep efficiency of waterflooding oil fields. None of these references have obtained a relationship between the volumetric sweep efficiency of waterflooding and the development duration. In view of this, based on previous research, the author utilizes two cumulative growth prediction models, the HCZ model and the Hubbert model, and through dimensionless mathematical transformation, derives dimensionless correlation equations for the volumetric sweep efficiency of waterflooding for the two models. A standardized prediction chart is established, and combined with Zhang Jinqing's waterflooding curve, a quantitative relationship between the volumetric sweep efficiency of waterflooding and the production duration of the oilfield is established, thereby predicting the variation pattern of the volumetric sweep efficiency of waterflooding. After determining the ultimate water content, the final volumetric sweep efficiency of waterflooding can be obtained using the relationship proposed in this paper. This method reflects the waterflooding performance in different production stages of the oilfield and is practical for predicting and evaluating development indicators in the later stages of oilfield production.

## 2. Establishment of Prediction Method

Both the HCZ model established by Hu Jianguo and the Hubbert model established by Hubbert belong to cumulative growth prediction models. In this paper, we perform non-dimensionalization on the HCZ model and Hubbert model, propose non-dimensional relational expressions for the prediction models, and establish typical curve charts for the prediction models.

### 2.1. HCZ Model

The relational expression of the HCZ cumulative yield model established by Hu Jianguo *et al.* in reference [8] is:

$$N_p = N_R e^{-\frac{a}{b}e^{-bt}} \quad (1)$$

Deriving Equation (1), obtain the model for predicting the output of oil and gas fields as follows:

$$Q_o = aN_R e^{-\frac{a}{b}e^{-bt} - bt} \quad (2)$$

To determine the time when the maximum annual output of the oil and gas field occurs, take the derivative of Equation (2) with respect to time:

$$\frac{dQ_o}{dt} = aN_R (ae^{-bt} - b) e^{-\frac{a}{b}e^{-bt} - bt} \quad (3)$$

When  $\frac{dQ_o}{dt} = 0$ ,  $ae^{-bt} - b = 0$ , achieve the highest annual output is:

$$t_{\max} = \frac{1}{b} \ln \frac{a}{b} \quad (4)$$

Substitute formula (4) into formula (2), the maximum annual oil production of the oil field is:

$$Q_{\max} = \frac{bN_R}{e} = 0.368bN_R \quad (5)$$

If assume dimensionless annual oil production  $Q_D = Q_o/Q_{\max}$ , dimensional duration  $t_D = t/Q_{\max}$ , formula (1) is transformed into:

$$N_{pD} = 2.718e^{-\frac{a}{b}e^{-\left(\ln \frac{a}{b}\right)D}} \quad (6)$$

Formula (6) calculates the dimensionless yield  $Q_D$  under different dimensionless years  $t_D$ , and draw it into a typical curve template, as shown in **Figure 1**.

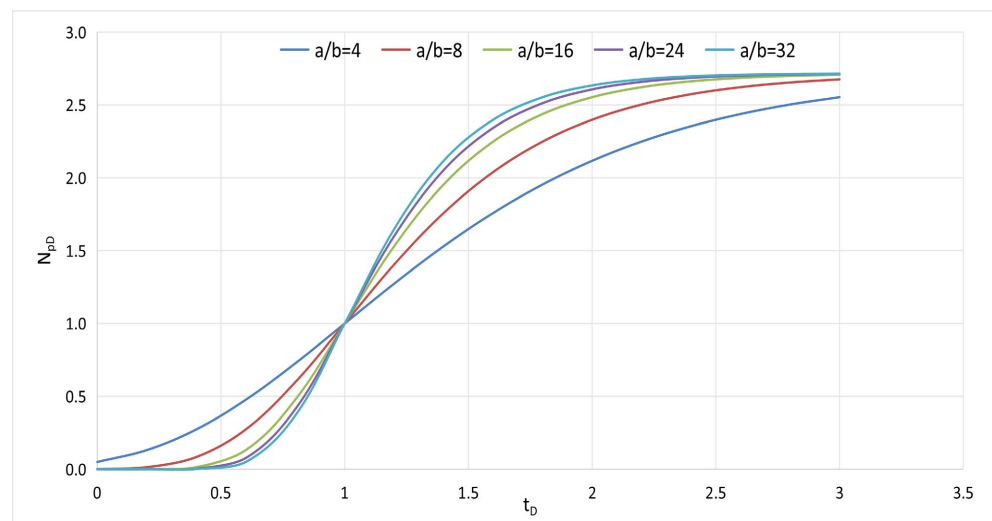


**Figure 1.** HCZ model curve chart.

## 2.2. Hubbert Model

The relational expression of the Hubbert cumulative production model in litera-

ture [9]:

$$N_p = \frac{N_R}{1 + Ae^{-Bt}} \quad (7)$$

Taking the derivative of Equation (7) with respect to time, the model for predicting oil production in oil and gas fields:

$$Q_o = \frac{ABN_R e^{-Bt}}{(1 + Ae^{-Bt})^2} \quad (8)$$

Taking the derivative of Equation (8) with respect to time, we obtain:

$$\frac{dQ_o}{dt} = \frac{B^2 AN_R e^{-Bt} (Ae^{-Bt} - 1)}{(1 + Ae^{-Bt})^3} \quad (9)$$

When  $\frac{dQ_o}{dt} = 0$ ,  $Ae^{-Bt} - 1 = 0$ , the time under the highest annual oil production is:

$$t_{\max} = \frac{1}{B} \ln A \quad (10)$$

Substitute formula (10) into formula (8), the maximum annual oil is:

$$Q_{\max} = 0.25BN_R \quad (11)$$

If assume dimensionless annual oil production  $Q_D = Q_o / Q_{\max}$ , dimensional duration  $t_D = t / Q_{\max}$ , Formula (7) is transformed into:

$$N_{pD} = \frac{2}{1 + Ae^{-(\ln A)t_D}} \quad (12)$$

Theoretical research and field practice indicate, Zhang Jinqing's waterflooding curve can correspond to various types of curves, using Zhang Jinqing's waterflooding curve to predict the waterflooding volume sweep efficiency has a wider range of application and more reliable results (Figure 2).

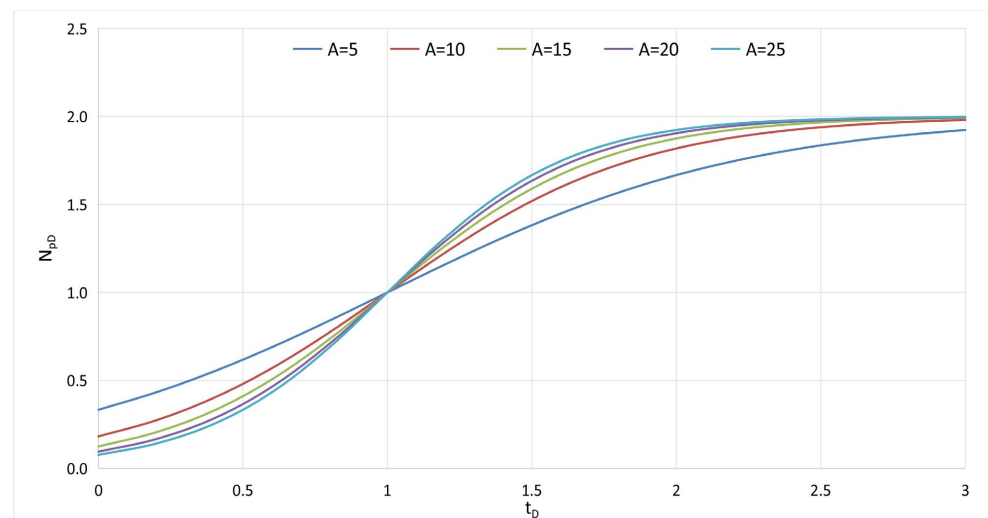


Figure 2. Hubbert model curve chart.

Zhang Jinqing proposed a new waterflooding characteristic curve in the literature [10], expression is:

$$W_p/N_p = -C + DW_p/N_p^2 \quad (13)$$

According to formula (13), the relational expression for moisture content can be obtained:

$$f_w = \frac{C \left[ 1 - \left( 1 - \frac{N_p}{D} \right)^2 \right]}{C + \left( 1 - \frac{N_p}{D} \right)^2 (1 - C)} \quad (14)$$

The relationship between cumulative oil production and water content can be derived from formula (14):

$$N_p = D \left[ 1 - \sqrt{\frac{C(1-f_w)}{f_w + C(1-f_w)}} \right] \quad (15)$$

The reserves of movable oil are:

$$N_{om} = \frac{V_p(S_{oi} - S_{or})}{B_{oi}} = D \quad (16)$$

The geological reserves of petroleum and the recovery degree:

$$N = V_p S_{oi} / B_{oi} \quad (17)$$

$$R = E_v E_d \quad (18)$$

Accumulated oil production of the oil field:

$$N_p = NR \quad (19)$$

The oil displacement efficiency of waterflooding oil fields is:

$$E_d = \frac{S_{oi} - S_{or}}{S_{oi}} \quad (20)$$

Substitute (20) into formula (18) to obtain:

$$R = \frac{S_{oi} - S_{or}}{S_{oi}} E_v \quad (21)$$

Substitute expressions (17) and (21) into expression (17) obtain:

$$N_p = \left[ V_p (S_{oi} - S_{or}) / B_{oi} \right] E_v \quad (22)$$

Substitute formula (16) into formula (22) obtain:

$$E_v = N_p / N_{om} \quad (23)$$

Formula (23), the volume sweep coefficient of water flooding varies with the change of cumulative oil production.

Finally, substitute Equations (15) and (16) into Equation (23), the relationship between the predicted waterflooding volume sweep efficiency and water content is obtained as follows:

$$E_v = 1 - \sqrt{\frac{C(1-f_w)}{f_w + C(1-f_w)}} \quad (24)$$

By combining Equations (6) or (12) with (24), we can obtain the relationship between the volume sweep efficiency and the production years.

### 3. Application Examples

Taking C Oilfield as an example, the sand body was put into production in 2007, the Oilfield is buried at a depth of 1200 m, geological reserves is  $2340.20 \times 10^4 \text{ m}^3$ , porosity is 0.28, permeability is 8650 mD, original formation pressure is 12.0000 MPa, and formation crude oil viscosity is 220 mPa·s. The actual development and production data are shown in **Table 1**.

**Table 1.** Actual production data of C oilfield.

| $t$ (a) | $Q_o$ ( $10^4 \text{ m}^3/\text{a}$ ) | $N_p$ ( $10^4 \text{ m}^3$ ) | $Q_w$ ( $10^4 \text{ m}^3/\text{a}$ ) | $W_p$ ( $10^4 \text{ m}^3$ ) |
|---------|---------------------------------------|------------------------------|---------------------------------------|------------------------------|
| 1       | 1.2                                   | 1.2                          | 0.0                                   | 0.0                          |
| 2       | 14.4                                  | 15.7                         | 1.1                                   | 1.1                          |
| 3       | 23.0                                  | 38.7                         | 15.3                                  | 16.4                         |
| 4       | 27.8                                  | 66.4                         | 14.3                                  | 30.7                         |
| 5       | 25.3                                  | 91.7                         | 15.5                                  | 46.2                         |
| 6       | 23.1                                  | 114.8                        | 17.0                                  | 63.2                         |
| 7       | 32.6                                  | 147.3                        | 27.7                                  | 90.9                         |
| 8       | 34.1                                  | 181.5                        | 55.8                                  | 146.7                        |
| 9       | 35.8                                  | 217.3                        | 97.9                                  | 244.6                        |
| 10      | 29.7                                  | 247.0                        | 134.1                                 | 378.7                        |
| 11      | 24.9                                  | 271.9                        | 130.8                                 | 509.5                        |
| 12      | 27.6                                  | 299.5                        | 138.5                                 | 648.0                        |
| 13      | 23.5                                  | 323.0                        | 145.0                                 | 793.0                        |
| 14      | 23.7                                  | 346.6                        | 149.4                                 | 942.4                        |
| 15      | 24.7                                  | 371.3                        | 179.6                                 | 1122.0                       |
| 16      | 23.4                                  | 394.7                        | 195.1                                 | 1317.1                       |
| 17      | 25.6                                  | 420.3                        | 209.7                                 | 1526.8                       |
| 18      | 26.1                                  | 446.5                        | 249.1                                 | 1775.9                       |
| 19      | 23.8                                  | 470.2                        | 253.0                                 | 2028.9                       |

First, utilized the actual production performance data in **Table 1**, find  $Q_{\max}$  and  $t_{\text{peak}}$ ,  $t_{\text{peak}} = 9$ ,  $Q_{\max} = 35.8$ , utilized  $Q_D = Q_o/Q_{\max}$  and  $t_D = t/t_{\text{peak}}$ ,  $Q_D$  and  $t_D$  are obtained for the different values of  $Q_o$  and  $t$ , used the corresponding data of  $Q_D$  and  $t_D$ , fitted with HCZ model and Hubbert model charts, as shown in **Figure 3** and **Figure 4**, finally, fitted the best value of  $a/b$  or  $A$ .

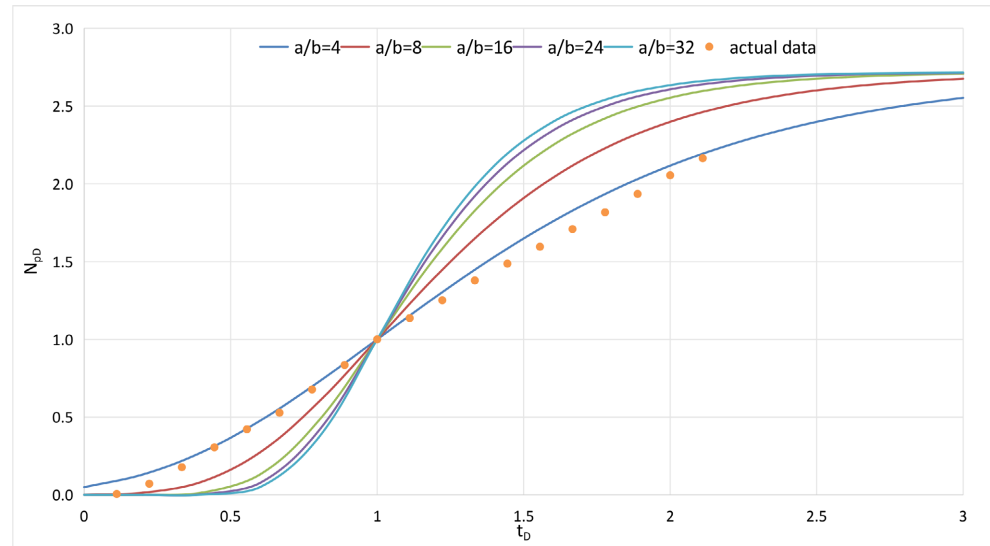


**Figure 3.** Curve fitting diagram of actual data and HCZ model.

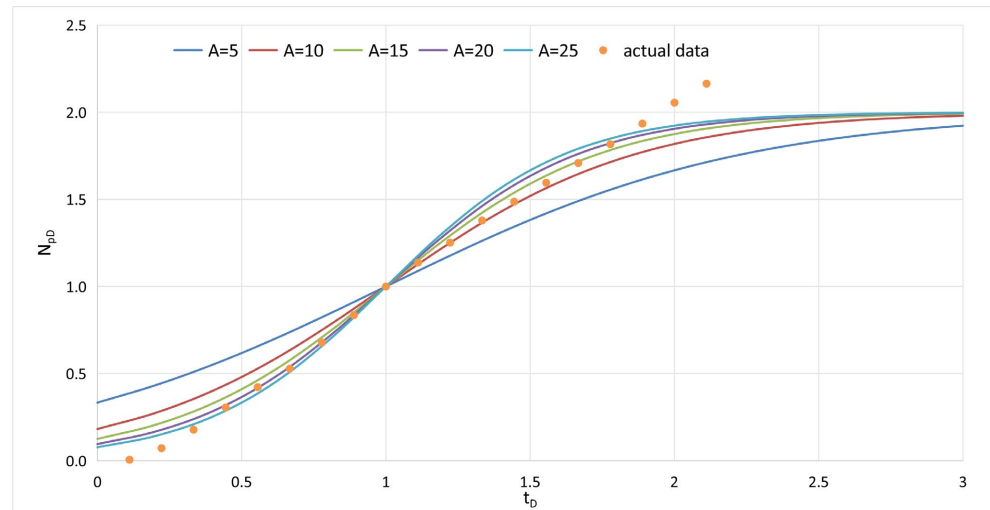


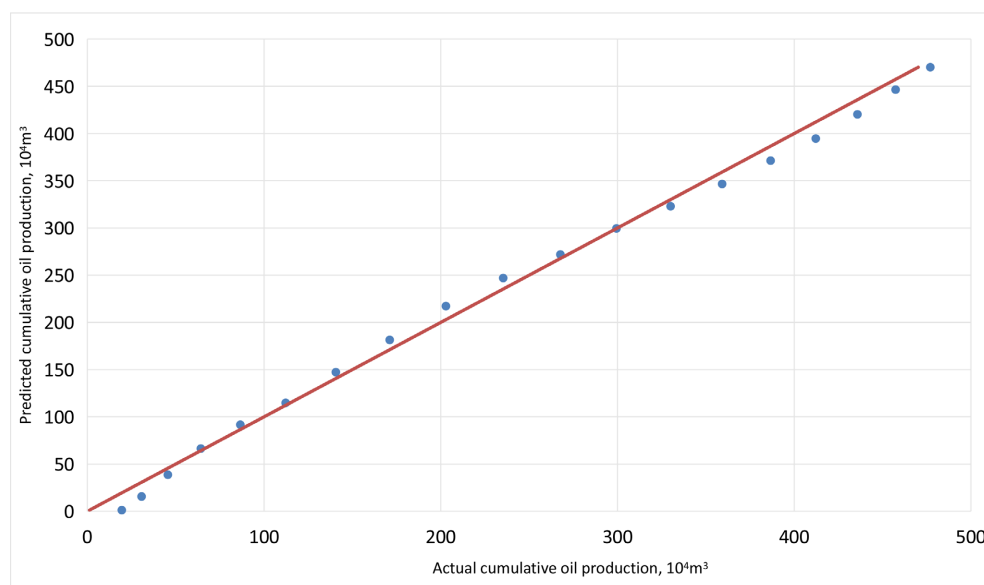
**Figure 4.** Curve fitting diagram of actual data and Hubbert model.

As shown in **Table 2**, compared to actual value  $N_{pD}$  and HCZ and Hubbert model  $N_{pD}$ , the HCZ model relative error is 0% - 7.2%, the Hubbert model relative error is -10.1% - 8.6%, the Hubbert model has a larger relative error, the HCZ model is more suitable for C oilfield, if the Hubbert model is used to predict the production of C oil field, predicted recoverable reserves  $N_R = N_{peak}/2 = 434.52 \times 10^4 \text{ m}^3$ , it has lower than the current cumulative oil production, it is evidently unreasonable to utilize the Hubbert model. Analyze the reasons, the Hubbert model is effectively applicable to oil and gas fields that have reached the decline stage after extracting 50% of their recoverable reserves. However, the production situation of Oilfield C does not meet this scope of application.

**Figure 5** depicted the fitting diagram between actual data and predicted values, with both results generally located near the 45° diagonal line. This indicates that the HCZ model has a high credibility in prediction. Therefore, this paper utilizes

**Table 2.** Comparison between actual value and model chart of C oilfield.

| Actual value |          | HCZ model ( $a/b = 4$ ) |                    | Hubbert model ( $A = 25$ ) |                    |
|--------------|----------|-------------------------|--------------------|----------------------------|--------------------|
| $t_D$        | $N_{pD}$ | $N_{pD}$                | relative error (%) | $N_{pD}$                   | relative error (%) |
| 0.44         | 0.31     | 0.31                    | 2.5                | 0.29                       | -6.3               |
| 0.56         | 0.42     | 0.43                    | 1.1                | 0.39                       | -8.5               |
| 0.67         | 0.53     | 0.56                    | 5.2                | 0.51                       | -3.5               |
| 0.78         | 0.68     | 0.70                    | 2.8                | 0.66                       | -3.1               |
| 0.89         | 0.84     | 0.85                    | 1.4                | 0.82                       | -1.5               |
| 1.00         | 1.00     | 1.00                    | 0.0                | 1.00                       | 0.0                |
| 1.11         | 1.14     | 1.15                    | 1.4                | 1.18                       | 3.5                |
| 1.22         | 1.25     | 1.30                    | 4.2                | 1.34                       | 7.3                |
| 1.33         | 1.38     | 1.45                    | 5.0                | 1.49                       | 8.1                |
| 1.44         | 1.49     | 1.58                    | 6.5                | 1.61                       | 8.6                |
| 1.56         | 1.60     | 1.71                    | 7.2                | 1.71                       | 7.4                |
| 1.67         | 1.71     | 1.83                    | 7.0                | 1.79                       | 4.8                |
| 1.78         | 1.82     | 1.93                    | 6.5                | 1.85                       | 1.8                |
| 1.89         | 1.93     | 2.03                    | 5.0                | 1.89                       | -2.2               |
| 2.00         | 2.05     | 2.12                    | 3.0                | 1.92                       | -6.4               |
| 2.11         | 2.16     | 2.19                    | 1.4                | 1.95                       | -10.1              |

**Figure 5.** Fitting graph of actual data and predicted data.

the HCZ model to predict the waterflooding sweep efficiency coefficient under different production years.

Determine the best fitting value based on the HCZ chart  $a/b = 4$ ,  $t_{\max} = 9$ , obtain  $b = 0.154$ ,  $a = 0.616$ , substitute it into formula (2), obtain the annual oil production forecasts for different production years, substitute it into formula (1), obtain the corresponding cumulative oil production  $N_p$ .

As shown in **Figure 6**, Based on actual development data of the oilfield of cumulative oil production  $N_p$ , cumulative water production  $W_p$ , calculated  $W_p/N_p$  and  $W_p/N_p^2$  separately, used the production curve after 10 years, the water content is greater than 80%, the water drive curve gradually enters a straight line segment, the production tends to stabilize, the regression is better, perform linear regression according to formula (6), obtain  $C = 4.71$ ,  $D = 963$ .

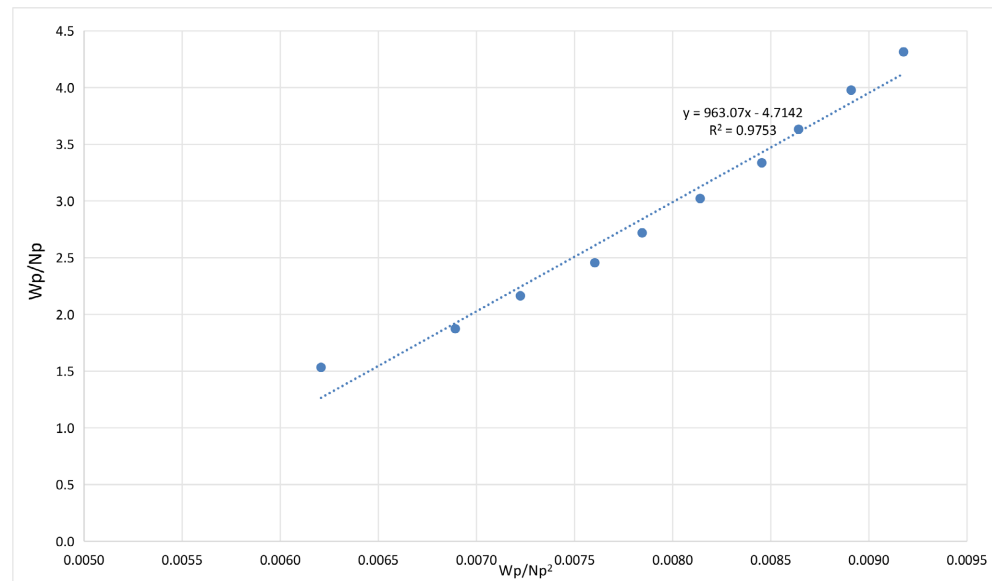


**Figure 6.** Zhang Jinqing's water drive curve fitting diagram.

Substitute the values of  $C$  and  $D$ ,  $N_p$  calculated from formula (1), into formula (14), the relationship between different development years and  $f_w$  is obtained. Substitute the values of water content  $f_w$  and  $C$  calculated from formula (14) into formula (14), combine formulas (14) and (24), the waterflooding sweep volume coefficient  $E_v$  under different development years can be obtained.

As can be seen from **Figure 7**, when development years less than 15a, when the water content is less than 90%,  $E_v$  exhibits an upward-curving increase; when development years is greater than 15a and the water content is greater than 90%,  $E_v$  shows a gradual increase. In the later stage of oilfield development, the number of water-producing wells gradually increases. When the oil wells fully produce water and enter the high water-cut stage, the increase in  $E_v$  becomes relatively slow. As the oilfield enters the later stage of extremely high water content, the next step involves restructuring the well network of the layer series and making precise adjustments to waterflooding to reconstruct the underground flow field. This approach resolves the development contradictions of uneven utilization among layers and inefficient cycles, fine-tunes the remaining oil between wells, improves the

development effect.

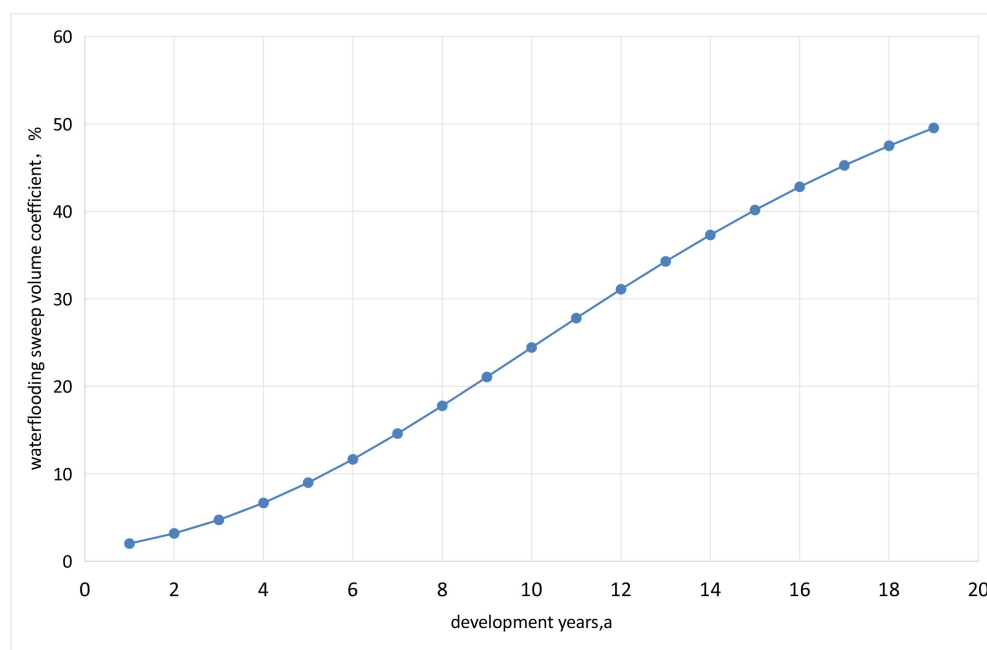


**Figure 7.** Waterflooding sweep volume coefficient and development years curve.

## 4. Conclusions

(1) Two cumulative growth forecast models HCZ and Hubbert were processed with dimensionless treatment to obtain dimensionless typical curve templates, combined with Zhang Jinqing's waterflooding curve, a relational expression between the waterflooding volume sweep efficiency, water content  $f_w$ , and development years was derived.

(2) Based on the verification of production data from C Oilfield, the adaptability of HCZ is significantly better than the Hubbert model, making it more suitable for quantitative prediction of the waterflooding volumetric sweep efficiency in this type of reservoir.

(3) The case of C oilfield demonstrates when development years less than 15a, when the water content is less than 90%,  $E_v$  exhibits an upward-curving increase; when development years is greater than 15a and the water content is greater than 90%,  $E_v$  shows a gradual increase. This method can be utilized for predicting the waterflooding effect in oil fields and guiding the later-stage development and production of oil fields.

## Author Contributions

Sun, E.H Wrote the manuscript; Peng, Q revised the manuscript.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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## Symbol Description

|                                |                                                                      |
|--------------------------------|----------------------------------------------------------------------|
| $Q_o$                          | Annual oil production, $10^4 \text{ m}^3/\text{a}$                   |
| $t$                            | Development duration, a                                              |
| $N_p$                          | Cumulative oil production, $10^4 \text{ m}^3$                        |
| $Q_{\max}$                     | Maximum annual oil production, $10^4 \text{ m}^3/\text{a}$           |
| $t_{\text{peak}}$              | The development period for the maximum annual oil production, a      |
| $Q_D$                          | Non-dimensional annual oil production                                |
| $t_D$                          | Non-dimensional development period                                   |
| $L_p$                          | Cumulative liquid production, $10^4 \text{ m}^3$                     |
| $W_p$                          | Cumulative water production, $10^4 \text{ m}^3$                      |
| $f_w$                          | water content, %                                                     |
| $N, N_{\text{om}}$             | Geological reserves and recoverable oil reserves, $10^4 \text{ m}^3$ |
| $S_{\text{oi}}, S_{\text{or}}$ | Original oil saturation and residual oil saturation, %               |
| $V_p$                          | Formation pore volume, $10^4 \text{ m}^3$                            |
| $a, b, A, B, C, D$             | Regression coefficient, constant                                     |
| $R$                            | Degree of geological reserves extraction, %                          |
| $B_{\text{oi}}$                | Original volume factor of crude oil                                  |
| $E_d$                          | Displacement efficiency, %                                           |
| $E_v$                          | Waterflooding sweep volume coefficient.                              |