

A Structural Analysis of the Minimum-Mass Degree in Gravitational and Electromagnetic Readout Relations

Tomofumi Miyashita 

Miyashita Clinic, Osaka, Japan

Email: tom_miya@ballade.plala.or.jp

How to cite this paper: Miyashita, T.

(2026) A Structural Analysis of the Minimum-Mass Degree in Gravitational and Electromagnetic Readout Relations. *Journal of Modern Physics*, 17, 1363-1379.

<https://doi.org/10.4236/jmp.2026.1710059>

Received: September 16, 2026

Accepted: October 25, 2026

Published: October 28, 2026

Copyright © 2026 by author(s) and

Scientific Research Publishing Inc.

This work is licensed under the Creative

Commons Attribution International

License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

This report extends our previous studies of empirical relations involving the cosmic microwave background temperature and the gravitational constant. In the preceding work, the adopted readout procedure gave a CMB temperature of 2.7256307 K, in agreement with the observed value of 2.72548 ± 0.00057 K. The same procedure also gave a gravitational value of $6.674184 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$, close to the reported value of $6.6743 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$. These two results were obtained within a common structural framework in which the minimum mass, denoted by $M_{\min}$, connects microscopic mass-energy quantities with macroscopic readouts. The present study focuses on the structural meaning of this minimum-mass scale. In particular, we establish a readout-level degree correspondence in which the electromagnetic numerator is first degree in the minimum-mass readout, the gravitational rate representation is second degree, and their quotient leaves one residual minimum-mass readout degree in the dimensionless proton-side gravitational-to-electromagnetic force ratio. The degree terminology used here is a same-shell, readout-covariant structural classification; it is not an assertion that $M_{\min}^2$ is an intrinsic quantity independent of the adopted unit/readout system. The purpose is not to derive the physical origin of gravity or of electromagnetism, but to clarify the algebraic and readout structure associated with the minimum mass. The analysis also revisits Coulomb's law and Newton's law of universal gravitation from the viewpoint of their common inverse-square dependence on distance. Both laws contain the same spatial factor associated with the square of the separation, while their electromagnetic and gravitational numerators remain distinct. This common spatial structure is treated separately from the minimum-mass degree correspondence. The resulting correspondence shows that the difference between the second-degree gravitational structure and the first-degree electromagnetic structure is directly reflected in the first-degree mini-

mum-mass readout remaining in the force ratio, while the deeper physical origin of the degree structure remains open.

Keywords

Cosmic Microwave Background Temperature, Gravitational Constant, Minimum Mass, Minimum-Mass Degree, Coulomb's Law, Newton's Law of Universal Gravitation, Inverse-Square Law, Readout Structure

1. Introduction

The symbols used in this paper are summarized in Section 2. In previous studies, we proposed a set of empirical equations relating the cosmic microwave background (CMB) temperature to several physical quantities and evaluated them numerically [1]-[5]. In particular, Equations (1) - (3) summarize characteristic CMB-temperature relations that provide the empirical starting point for the present analysis.

$$\frac{Gm_p^2}{hc} = \frac{4.5}{2} \cdot \frac{kT_c}{1 \text{ kg} \cdot c^2} \approx 9.4 \times 10^{-40} \quad (1)$$

$$\frac{Gm_p^2}{e^2} = \frac{4.5}{2\pi} \cdot \frac{m_e}{e} \cdot hc \approx 8.09 \times 10^{-37} \quad (2)$$

$$\left(\frac{m_e c^2}{e} \right) \cdot \left[\frac{e^2}{4\pi\epsilon_0} \right] = \pi \cdot kT_c \approx 1.18 \times 10^{-22} \quad (3)$$

We also introduced an empirical expression for the fine-structure constant [6] and examined its consistency with related quantities [7] [8].

$$\frac{1}{\alpha} = 137.0359991 = 136.0113077 + \frac{1}{3 \cdot 13.5} + 1 \quad (4)$$

$$13.5 \cdot 136.0113077 = 1836.152654 = \frac{m_p}{m_e} \quad (5)$$

For clarity, the two readout redefinitions used below are introduced here before their detailed formulation in Section 2. The first redefinition, denoted F_1 , is the Ω -side normalization that restores the two electromagnetic coefficient branches to the ideal coefficients $9/2$ and π . The second redefinition, denoted F_2 , is the subsequent table-number/readout transformation based on the A-character; in the inherited transfer rule, $J_{\text{tab}}^{A(X)}$ is used as the corresponding numerical factor. The notation $F_1 F_2$ denotes the readout stage obtained after sequential application of F_1 and F_2 . Their detailed definitions and limitations are given in Sections 2.2 and 2.5. Small deviations from the ideal values of $9/2$ and π , respectively, were previously considered as a means of reducing the resulting numerical errors [9]-[14]. In the previous formulation, the coefficient associated with the dimensional factor m^2/s

was denoted by kL_0 [15]-[18]. In the present report, kL_0 is no longer used as an independent quantity. Instead, s_{gen} is explained in Equation (10).

Before the first redefinition, the two electromagnetic coefficient branches in the modified notation are written using the pre-correction values as follows.

$$\frac{3.132794486}{c}(\text{Wb}) = \frac{s_{\text{gen}} m_e c^2}{e} = 1.044987758 \times 10^{-8} (\text{Wb}) \quad (6)$$

$$4.487397553 \cdot c (\text{C}^{-1}) = \frac{q_m}{s_{\text{gen}} m_p c^2} = 1.345287943 \times 10^9 (\text{C}^{-1}) \quad (7)$$

Applying the first redefinition to the von Klitzing constant returns the corresponding coefficients to the ideal values of $9/2$ and π [15]-[18]. In the present notation, the unit-bearing coefficients are written separately from the speed of light. Equations (8) and (9) therefore use π_{unit}/c and $4.5_{\text{unit}} \cdot c$, respectively; the coefficient labels and c are not combined into a single symbol. Equations (8) - (10) are relations in the F1 readout representation.

$$\frac{\pi_{\text{unit}}}{c} \left(\frac{\text{V} \cdot \text{m}}{\text{m/s}} \right) = \frac{s_{\text{gen}} m_e c^2}{e} = \pi/c = 1.047922511 \times 10^{-8} (\text{Wb}) \quad (8)$$

$$4.5_{\text{unit}} \cdot c \left(\frac{\text{m/s}}{\text{A} \cdot \text{m}} \right) = \frac{q_m}{s_{\text{gen}} m_p c^2} = 4.5c = 1.349066061 \times 10^9 (\text{C}^{-1}) \quad (9)$$

Using the two branch relations, the generative time (s_{gen}) is obtained as follows.

$$s_{\text{gen}} = \sqrt{\frac{\frac{\pi_{\text{unit}}}{c}}{4.5_{\text{unit}} \cdot c} \cdot \frac{h}{(m_p c^2)(m_e c^2)}} = 2.044990031 \times 10^{-14} (\text{s}) \quad (10)$$

Equation (10) replaces the former kL_0 -based representation. In the present formulation, this generative time is tied directly to the minimum-mass action relation,

$$M_{\text{min}} c^2 s_{\text{gen}} = h = 6.62607015 \times 10^{-34} (\text{J} \cdot \text{s}) \quad (11)$$

and the minimum-mass energy is related to the CMB thermal scale by

$$M_{\text{min}} c^2 = \frac{2\pi}{\alpha} k T_c = 3.24014790 \times 10^{-20} (\text{J}) \quad (12)$$

In these previous studies, the minimum mass (M_{min}) mainly connected the CMB-temperature scale, microscopic mass-energy quantities, electromagnetic readouts, and the gravitational-side calculation. The present study instead examines the power, or degree, with which the minimum-mass readout appears. The numerical CMB-temperature and gravitational values themselves remain those of the preceding report [18]. What is new is the degree analysis: the electromagnetic numerator is first degree in the minimum-mass readout, the gravitational rate representation is second degree, and their quotient leaves one residual minimum-mass readout degree in the dimensionless proton-side gravitational-to-electromagnetic force ratio. The underlying empirical relations and numerical values are inherited from the preceding work, whereas the present contribution is the struc-

tural classification of their minimum-mass readout degrees and the explicit distinction between same-shell structural statements and readout-stage numerical bookkeeping. Because the numerical value of $M_{\min}$ expressed in kilograms belongs to the adopted MKSA readout, the degree classification is made at the same-shell, readout-covariant level rather than as an intrinsic unit-system-independent statement. At the action level, the Coulomb numerator divided by c can be written as

$$\frac{e^2}{4\pi\epsilon_0 c} = \frac{\alpha}{2\pi} h = \frac{\alpha}{2\pi} [M_{\min}]_{\text{MKSA}} c^2 s_{\text{gen}} = 7.69558236 \times 10^{-37} (\text{J} \cdot \text{s}) \quad (13)$$

Thus, one explicit factor of the MKSA minimum-mass readout appears in the electromagnetic action-level structure. The corresponding gravitational action-level quantity is Gm_p^2/c . For the rate-type representation used below, this gravitational quantity is divided by c^2 . The rate coefficient is defined as

$$B_{\text{rate}} = \frac{4.5\alpha}{4\pi} \cdot \left(\frac{s_{\text{gen}}}{1 \text{ kg}} \right)_{F_1 F_2} = 5.33193444 \times 10^{-17} \left(\frac{\text{s}}{\text{kg}} \right)_{\text{MKSA}} \quad (14)$$

The numerical value is displayed using the MKSA (s/kg) carrier. The subscript $F_1 F_2$ denotes the readout stage after sequential application of the first and second redefinitions. The detailed definition of the second redefinition is given later in Section 2.5. The resulting proton-side gravitational rate representation is

$$\frac{Gm_p^2}{c^3} = B_{\text{rate}} [M_{\min}]_{\text{MKSA}}^2 = 6.92997483855 \times 10^{-90} (\text{kg} \cdot \text{s}) \quad (15)$$

Equation (15) becomes numerically accurate after the second redefinition; before that step the relation is structural but not yet aligned to the accepted readout value. Equation (13) gives the first-degree minimum-mass readout structure at the action level, Equation (14) defines the rate coefficient, and Equation (15) gives the gravitational rate representation after division by c^2 . In Equation (15), $[M_{\min}]_{\text{MKSA}}^2$ is explicitly a squared MKSA readout; it is not promoted to an intrinsic unit/readout-system-independent quantity. This is why a local-unit-shell (LUS) reading is required already at Equation (15). The problem is that $[M_{\min}]_{\text{MKSA}}^2$ looks as if it were a unit-system-independent invariant, whereas it is an MKSA readout square and therefore depends on the adopted mass-readout scale. The formal criteria are stated in Section 3.3. The degree-two statement refers instead to the covariance of the complete relation: with the adopted character assignments, $B_{\text{rate}} [M_{\min}]_{\text{MKSA}}^2$ and Gm_p^2/c^3 both carry $A = +2$, which is explained in Section 2.5. Multiplying Equation (15) by c^2 returns Gm_p^2/c for comparison with Equation (13). At that common action level, the shared α , c^2 , and s_{gen} factors cancel in the force ratio, leaving a first-degree minimum-mass readout structure. The normalized combination $M_{\min}^2 / (m_e \cdot m_p)$ can remain invariant under a common same-shell mass-scale transfer, but this is distinct from invariance of $M_{\min}^2$ itself. Thus, the present degree terminology denotes a same-shell, readout-covariant structural degree, not an SI mass dimension or an intrinsic invariant degree. Coulomb's law and Newton's law of universal gravitation also share the same inverse-square spatial factor while their numerators remain distinct. The physical origin

of the gravitational second-degree structure remains open.

2. Symbols and Basic Relations

This section summarizes the reference quantities and structural relations required for the present degree analysis. The first-redefinition framework is retained from the previous report, while only the relations needed in the present analysis are carried forward.

2.1. Reference Constants in MKSA Units (Values Taken from Standard Reference Data)

The standard physical constants listed below follow the 2018 CODATA recommended values [19], except where otherwise stated. The value of α is retained from the preceding work, and q_m is defined here by $q_m = h/e$. The CMB comparison value is taken from Fixsen (2009) [20]. The observed values of G and T_c are included for comparison only and do not enter the algebraic chain.

G : gravitational constant = $6.6743 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ (comparison only).

T_c : CMB temperature = $2.72548 \pm 0.00057 \text{ K}$ (comparison only).

k : Boltzmann constant = $1.380649 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$.

c : speed of light = $299792458 \text{ m} \cdot \text{s}^{-1}$.

h : Planck constant = $6.62607015 \times 10^{-34} \text{ J} \cdot \text{s}$.

ϵ_0 : electric constant = $8.8541878128 \times 10^{-12} \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2}$.

μ_0 : magnetic constant = $1.256637062 \times 10^{-6} \text{ N} \cdot \text{A}^{-2}$.

e : elementary charge = $1.602176634 \times 10^{-19} \text{ C}$.

q_m : magnetic-monopole charge defined by $q_m = h/e = 4.13566770 \times 10^{-15} \text{ Wb}$.

m_p : proton rest mass = $1.67262192369 \times 10^{-27} \text{ kg}$.

m_e : electron rest mass = $9.1093837015 \times 10^{-31} \text{ kg}$.

R_K : von Klitzing constant = 25812.80745Ω .

α : fine-structure constant = $1/137.035999081$.

2.2. Symbols and Relations Obtained after the Redefinition Procedure

This subsection transfers the first-redefinition block of the previous report [18], with the present coefficient- c notation. The redefinition is a change of readout representation, not a change of the physical constants. Hereafter, F_1 and F_2 denote the first and second readout-redefinition functions, respectively. For any readout quantity X , $X^{F_1} = F_1(X)$ and $X^{F_1 F_2} = F_2(F_1(X))$. The notation $F_1 F_2$ denotes sequential application of the two functions and does not denote multiplication.

2.2.1. Symbols Obtained after the First Redefinition (F1) Procedure

Quantities written with the subscript “new” denote numerically consistent readouts after the first redefinition, as in the previous report [18]. They are historical/internal labels for that representation, not a new current-shell grammar.

$$e_{\text{new}} = e \cdot \frac{4.4873976}{4.5} = 1.5976897 \times 10^{-19} \text{ (C)} \quad (16)$$

$$q_{m,\text{new}} = q_m \cdot \frac{\pi}{3.1327945} = 4.1472823 \times 10^{-15} \text{ (Wb)} \quad (17)$$

$$h_{\text{new}} = e_{\text{new}} \cdot q_{m,\text{new}} = h \cdot \frac{4.4873976}{4.5} \cdot \frac{\pi}{3.1327945} = h = 6.62607015 \times 10^{-34} \text{ (J} \cdot \text{s)} \quad (18)$$

Therefore the value of Planck's constant remains unchanged.

$$R_{K,\text{new}} = \frac{q_{m,\text{new}}}{e_{\text{new}}} = R_K \cdot \frac{4.5}{4.4873976} \cdot \frac{\pi}{3.1327945} = 25957.997 \text{ } (\Omega) \quad (19)$$

Equation (19) can be rewritten as

$$R_{K,\text{new}} = (4.5_{\text{unit}} c) \cdot \frac{\pi_{\text{unit}}}{c} \cdot \frac{m_p}{m_e} = 25957.997 \text{ } (\Omega) \quad (20)$$

Here 4.5_{unit} and π_{unit} are unit-identification coefficients for the $1/(\text{A} \cdot \text{m})$ and $(\text{V} \cdot \text{m})$ readouts. They represent MKSA readout deviations, not new physical constants. In the present notation they are written separately from c as $4.5_{\text{unit}} c$ and π_{unit}/c . The explicit factors of c cancel in the product $(4.5_{\text{unit}} c)(\pi_{\text{unit}}/c) = 4.5_{\text{unit}} \pi_{\text{unit}}$.

$$Z_{0,\text{new}} = 2\alpha \cdot R_{K,\text{new}} = Z_0 \cdot \frac{4.5}{4.4873976} \cdot \frac{\pi}{3.1327945} = 378.84931 \text{ } (\Omega) \quad (21)$$

$$Z_{0,\text{new}} = 2\alpha \cdot (4.5_{\text{unit}} c) \cdot \frac{\pi_{\text{unit}}}{c} \cdot \frac{m_p}{m_e} = 378.84931 \text{ } (\Omega) \quad (22)$$

$$\mu_{0,\text{new}} = \frac{Z_{0,\text{new}}}{c} = \mu_0 \cdot \frac{4.5}{4.4873976} \cdot \frac{\pi}{3.1327945} = 1.2637053 \times 10^{-6} \text{ } (\text{N} \cdot \text{A}^{-2}) \quad (23)$$

$$\varepsilon_{0,\text{new}} = \frac{1}{Z_{0,\text{new}} c} = \varepsilon_0 \cdot \frac{4.4873976}{4.5} \cdot \frac{3.1327945}{\pi} = 8.8046642 \times 10^{-12} \text{ } (\text{F} \cdot \text{m}^{-1}) \quad (24)$$

$$c_{\text{new}} = \frac{1}{\sqrt{\varepsilon_{0,\text{new}} \mu_{0,\text{new}}}} = c = 299792458 \text{ } (\text{m} \cdot \text{s}^{-1}) \quad (25)$$

The Compton wavelength is

$$\lambda = \frac{h}{mc} \quad (26)$$

Because h and c are fixed, the Compton relation connects the length readout to the mass readout. Under the first-redefinition map only, the mass and thermal readouts remain m_e with subscript new equals m_e and likewise for m_p and kT_c . This is not a claim of unit-system-independent invariance.

The minimum mass is defined as

$$[M_{\text{min}}]_{\text{MKSA}} = \frac{\frac{2\pi}{\alpha} kT_c}{c^2} = 3.605150741 \times 10^{-37} \text{ (kg)} \quad (27)$$

2.2.2. Meaning of the First Redefinition: Ω -Side Normalization

In the first redefinition, macroscopic and microscopic quantities are related through a normalization of the unit Ω . Under this normalization the unit $\text{J} \cdot \text{m}^2/\text{s}$ is fixed as follows:

$$\begin{aligned} & \frac{4.4873976}{3.1327945} \cdot [1/(A \cdot m)] [1/(V \cdot m)] \\ & = (4.5/\pi) \cdot [1/(A \cdot m)] [1/(V \cdot m)] = 1/(J \cdot m^2/s) \end{aligned} \quad (28)$$

Redefining the macroscopic unit 1Ω yields the following relations.

$$1 \Omega_{\text{new}} = 1.005624705^{-1} \Omega_{\text{MKSA}} = 1.005624705^{-1} (\text{Wb/C})_{\text{MKSA}} \quad (29)$$

$$e_{\text{new}} = 1.005624705^{-\frac{1}{2}} e_{\text{MKSA}} \quad (30)$$

$$qm_{\text{new}} = 1.005624705^{+\frac{1}{2}} qm_{\text{MKSA}} \quad (31)$$

$$\varepsilon_{0_{\text{new}}} = 1.005624705^{-1} \varepsilon_{0_{\text{MKSA}}} \quad (32)$$

$$\mu_{0_{\text{new}}} = 1.005624705 \mu_{0_{\text{MKSA}}} \quad (33)$$

$$RK_{\text{new}} = (4.5/4.4873976) \cdot (\pi/3.1327945) RK_{\text{MKSA}} = 1.005624705 RK_{\text{MKSA}} \quad (34)$$

$$Z0_{\text{new}} = 1.005624705 Z0_{\text{MKSA}} \quad (35)$$

In the previous report the coefficient associated with m^2/s was written kL_0 . In the present report that object is not an independent quantity. Its previous role is carried by s_{gen} through $M_{\text{min}} c^2 s_{\text{gen}} = h$.

2.3. c -Normalization and Action versus Rate

Here, the notational modification means only the rearrangement of explicit factors of c ; it does not change the unit-identification coefficients or their dimensional roles. In the previous report, physical quantities were expressed in c -normalized form by introducing appropriate factors of the speed of light, so that electromagnetic, mass-energy, and action-type quantities could be compared in a common structural framework. Before the notational modification that comparison was written with the absorbed maps ($e \Rightarrow ec$, $q_m \Rightarrow q_m c$, and $h \Rightarrow h \cdot c^2$). After the coefficient- c separation adopted here, those absorbed symbols are no longer required: the factors of c are written explicitly, and the former c -wrapped coefficient notation becomes identity-like ($e \Rightarrow e$, $q_m \Rightarrow q_m$, $h \Rightarrow h$). This does not mean that c -normalization itself is the identity. The underlying c -normalized relations are retained. In particular the action-level comparison of $e^2/(4\pi\varepsilon_0 c)$ with Gm_p^2/c , and the subsequent rate form Gm_p^2/c^3 , remain in use.

$$e \Rightarrow ec \text{ (previous)}, e \Rightarrow e \text{ (present)} \quad (36)$$

$$q_m \Rightarrow q_m c \text{ (previous)}, q_m \Rightarrow q_m \text{ (present)} \quad (37)$$

$$h \Rightarrow h \cdot c^2 \text{ (previous)}, h \Rightarrow h \text{ (present)} \quad (38)$$

$$\varepsilon_0 \Rightarrow \varepsilon_0 \cdot c \text{ (previous)}, \varepsilon_0 \Rightarrow \varepsilon_0 \cdot c \text{ (present)} \quad (39)$$

$$\mu_0 \Rightarrow \mu_0 \cdot c \text{ (previous)}, \mu_0 \Rightarrow \mu_0 \cdot c \text{ (present)} \quad (40)$$

$$\text{kg} \Rightarrow \text{kg} \cdot c^2 \text{ (previous)}, \text{kg} \Rightarrow \text{kg} \cdot c^2 \text{ (present)} \quad (41)$$

$$G \Rightarrow \frac{G}{c^3} \text{ (previous)}, G \Rightarrow \frac{G}{c^3} \text{ (present)} \quad (42)$$

$$G_N \Rightarrow \frac{G_N}{c}(\text{previous}), \quad G_N \Rightarrow \frac{G_N}{c}(\text{present}) \quad (43)$$

where $G = G_N/(1 \text{ kg})$, equivalently $G_N = G \cdot 1 \text{ kg}$, is used as an intermediate gravitational-side readout. Thus, the kilogram appearing in the MKSA dimension of G is a denominator mass unit. The maps (36) - (43) record the notational modification of the former absorbed notation.

2.4. Minimum Mass, Generative Time, and SI

$$M_{\min} c^2 s_{\text{gen}} = h \quad (44)$$

$$s_{\text{gen}} = h / (M_{\min} c^2) \quad (45)$$

$$[M_{\min}]_{\text{MKSA}} = \frac{h}{c^2 s_{\text{gen}}} \quad (46)$$

When a numerical kilogram value is intended, the minimum mass is written $[M_{\min}]_{\text{MKSA}}$. The subscript MKSA marks a numerical readout in the adopted MKSA shell, not a new physical species of $M_{\min}$. The former coefficient kL_0 is historical only; its role is carried by s_{gen} through (44). The mass-side structural invariant of the previous report is retained in the restricted form

$$\text{SI} := M_{\min}^2 / (m_e \cdot m_p) \quad (47)$$

SI denotes the structural invariant of the previous report [18]. Under a common same-shell mass-scale transfer, SI is invariant. This invariance is not from invariance of $M_{\min}^2$ itself. The additional structural quantities used in the previous report are not introduced here unless required in the subsequent analysis.

The four connection relations of the previous report [18] are retained here as useful identities. In the present notation, their former c-absorbed forms are modified only by making the factors of c explicit; the unit-identification coefficients and their dimensional roles are unchanged. Separately, in the two relations that previously contained kL_0 , kL_0 is eliminated in favor of s_{gen} through $M_{\min} c^2 s_{\text{gen}} = h$. The other two connection relations retain the same structural content.

$$(4.5_{\text{unit}} c) e_{\text{new}} = \frac{M_{\min}}{m_p} \quad (48)$$

$$\frac{q m_{\text{new}}}{\frac{\pi_{\text{unit}}}{c}} = \frac{M_{\min}}{m_e} \quad (49)$$

$$\left(\frac{\pi_{\text{unit}}}{c} \right) e_{\text{new}} = m_e c^2 s_{\text{gen}} \quad (50)$$

$$\frac{q m_{\text{new}}}{(4.5_{\text{unit}} c)} = m_p c^2 s_{\text{gen}} \quad (51)$$

2.5. A-Character Table Used in This Analysis

The second redefinition (F2) can be done using the A-character. The A-character is a readout character. It is not automatically identified with a concrete MKSA-to-

readout endpoint exponent. The A-character assignments used here are inherited from the readout framework developed in the preceding report [18]; their deeper origin is not re-derived in the present paper. R_p is defined in Section 3.1. In the preceding report [18], a table-number $A(X)$ was also used as the exponent of the transfer factor $J_{\text{tab}}^{A(X)}$, with $J_{\text{tab}} = 1.00056105184$. The transfer rule is not applied again within the same-shell evaluation below; quantities labeled $F_1 F_2$ are already expressed in the post- F_2 readout stage. In that report $A(X)$ is a covector that assigns a table-number weight to each readout quantity X : a positive weight multiplies the readout by $J_{\text{tab}}^{A(X)}$, a negative weight divides it by $J_{\text{tab}}^{|A(X)|}$, and $A(X) = 0$ leaves the readout unchanged. **Table 1** of the present paper keeps those labels as readout characters for the degree analysis; it does not apply the transfer rule inside the same-shell evaluation.

Table 1. Lists only the A-characters used in the present degree analysis.

Quantity	$A(X)$	Role
c	0	c -normalized reference
h	+2	action
e	-4	elementary charge
q_m	+6	magnetic-monopole charge
M_{min}, m_e, m_p	+3	Mass, electron, proton
1 kg	+3	macroscopic mass unit
s_{gen}, s	-1	time
m	-1	distance
Ω	+10	resistance
ϵ_0	-10	electric constant
μ_0	+10	magnetic constant
G	-4	gravity
$e^2/(4\pi\epsilon_0)$	+2	EM numerator
Gm_p^2	+2	gravitational numerator
$1/r^2$	+2	spatial factor
R_p	0	ratio
B_{rate}	-4	rate coefficient

The table-number factor used in the second redefinition is defined by Equation (52). Each of the two coefficient branches in Equation (34) corresponds to J_{tab}^5 , so their product is $J_{\text{tab}}^{10} = RK_{\text{new}}/RK_{\text{MKSA}}$.

$$J_{\text{tab}} = \left[\frac{4.5}{4.487397553} \frac{\pi}{3.132794486} \right]^{1/10} = \left[\frac{R_{K,\text{new}}}{R_{K,\text{MKSA}}} \right]^{1/10} = 1.00056105184 \quad (52)$$

3. Methods

3.1. Definitions for B_{pair} and B_{rate}

The evaluation uses three auxiliary objects: the rate coefficient B_{rate} , the proton-side force ratio $R_p(m_p)$, and the pair coefficient B_{pair} . They are introduced here only as definitions. Their degree reading is given in Section 4.

$$B_{\text{rate}}^{F_1 F_2} = \frac{4.5\alpha}{4\pi} \cdot \frac{s_{\text{gen}}^{F_1 F_2}}{(1 \text{ kg})^{F_1 F_2}} \quad (53)$$

$$R_p(m_p) = G \cdot m_p^2 / \left[e^2 / (4\pi\epsilon_0) \right] \quad (54)$$

$$B_{\text{pair}} := B_{\text{rate}}(m_e/m_p) \quad (55)$$

Equation (53) is written in the $F_1 F_2$ readout stage; therefore, no additional J_{tab}^{-4} factor is applied within this same-stage expression.

J_{tab} is a readout-transfer factor. It appears only in a cross-system reverse display and is not part of sam_e -system $R_p(m_p)$. Its table is not required for the sam_e -shell evaluation below. B_{rate} is not introduced as a new coupling constant. It is the coefficient that remains when the gravitational numerator is written in the rate form $G \cdot m_p^2 / c^3$, whose coherent slot is s/kg. The factor $s_{\text{gen}}/(1 \text{ kg})$ occupies that slot; the numerical package $4.5\alpha/4\pi$ is the sam_e package already used on the electromagnetic side.

3.2. Inverse-Square Laws of Coulomb and Newton

Coulomb's law and Newton's law of universal gravitation both have an inverse-square dependence on distance. For two protons, they are written as follows.

$$F_C = \frac{e^2}{4\pi\epsilon_0 r^2} \quad (56)$$

$$F_G = \frac{Gm_p^2}{r^2} \quad (57)$$

Table 1 gives $A(1/r^2) = +2$, and therefore $A(r^2) = -2$. The two force numerators have $A(e^2/(4\pi\epsilon_0)) = +2$ and $A(G \cdot m_p^2) = +2$. Consequently, $A((e^2/(4\pi\epsilon_0))r^2) = +2 - 2 = 0$ and $A(G \cdot m_p^2 r^2) = +2 - 2 = 0$. The zero A-character means that each product is invariant under the adopted readout transformation. In both cases, the numerator character +2 is balanced by the character -2 of r^2 . The corresponding force contains the inverse-square factor $1/r^2$ and therefore decreases in proportion to the inverse square of the distance. This common A-character structure does not mean that the electromagnetic and gravitational interactions have the same physical origin.

3.3. Solution for the LUS Problem in $R_p(m_p)$

The present Methods adopt only a partial LUS resolution. The solution in this

section is limited to the force ratio $R_p(m_p)$. It is not proposed as a general solution for every LUS-dependent quantity. LUS specifies the readout frame, same-shell specifies a comparison made within one such frame, and readout covariance specifies how a complete relation transforms under the adopted character rule.

3.3.1. Explanation for the LUS Problem

Here LUS means a local unit shell: a coherent readout frame in which the numerical values of mass, time, and related quantities are expressed together. The kilogram value of the minimum mass belongs to the adopted MKSA shell. The problem addressed in the degree analysis is that $[M_{\min}]_{\text{MKSA}}^2$ can appear to be a unit-system-independent number, while in fact it depends on the MKSA mass-readout scale of the adopted shell. Under a common same-shell mass-scale transfer, the numerical square of the minimum mass changes, even though a same-shell identity such as $M_{\min} c^2 s_{\text{gen}} = h$ may still hold. Therefore, a statement about $[M_{\min}]_{\text{MKSA}}^2$ is a statement about a readout representation, not about a unit-system-independent object.

For $R_p(m_p)$, however, the relevant object is the complete same-shell ratio. In the gravitational numerator Gm_p^2 , G contains one inverse mass dimension and m_p^2 contains two mass dimensions. The numerator therefore contains one net mass unit. In the electromagnetic denominator, $e^2/4\pi\epsilon_0 = (\alpha/2\pi)hc$, the same single mass unit is contained in h . The apparent LUS problem results from viewing the minimum-mass expression on the gravitational side separately from the mass unit contained in h on the electromagnetic side.

3.3.2. Solution for the LUS Problem

Consider the change from MKSA to MgSA, in which only the mass unit is changed from kilograms to grams. Since the mass unit in G is in the denominator, G changes by a numerical factor of 10^{-3} , whereas m_p^2 changes by 10^6 . Therefore, Gm_p^2 changes by 10^3 . On the electromagnetic side, h also changes by 10^3 , while α and c remain unchanged. Consequently, $e^2/4\pi\epsilon_0 = (\alpha/2\pi)hc$ changes by the same factor of 10^3 . These two factors cancel in $R_p(m_p)$.

The LUS problem of $R_p(m_p)$ is therefore solved by the cancellation of the common mass-unit factor: the net mass unit in the gravitational numerator Gm_p^2 is canceled by the mass unit contained in h in the electromagnetic denominator. The same transformation can be followed explicitly in Equation (59). Under the coherent change from MKSA to MgSA, the numerical readout of $M_{\min}$ changes from $3.605150741 \times 10^{-37}$ kg to $3.605150741 \times 10^{-34}$ g, whereas $s_{\text{gen}} = 2.044990031 \times 10^{-14}$ s is unchanged. Because B_{rate} carries the time-per-mass slot, its numerical value changes from $5.33193444 \times 10^{-17}$ s/kg to $5.33193444 \times 10^{-20}$ s/g. Thus, on the right-hand side of Equation (59), B_{rate} changes by 10^{-3} while $[M_{\min}]^2$ changes by 10^6 , and their product changes by 10^3 . On the left-hand side, G changes by 10^{-3} and m_p^2 changes by 10^6 , so Gm_p^2/c^3 also changes by 10^3 . Equation (59) therefore remains covariant under this coherent same-shell mass-unit rescaling.

The numerical readout of $M_{\min}$ changes under this transformation, but the com-

plete dimensionless force ratio $R_p(m_p)$ does not. The common mass-unit factor in Gm_p^2 is matched by the corresponding mass-unit factor in $e^2/4\pi\epsilon_0 = (\alpha/2\pi)hc$, and the two factors cancel in the ratio. This same-shell example does not remove the distinction between the F_1F_2 and MKSA kg/s carriers used in Equation (63).

3.3.3. Limitation for This Solution

Same-shell, in this report, means that the compared quantities are rescaled together inside one readout frame. Readout covariance means that both sides of a displayed relation transform so as to keep the same A-character. The MKSA system is a completed reference frame, not a physically privileged unit system. For the coherent change from MKSA to MgSA considered here, $R_p(m_p)$ is unchanged because its gravitational numerator and electromagnetic denominator acquire the same mass-unit factor. No additional J_{tab} factor is required in the same-system ratio.

This solution is limited to the LUS problem of $R_p(m_p)$ and to the coherent mass-unit change from kilograms to grams examined above. It does not establish a general LUS solution for arbitrary changes of length, time, or readout structure. $[M_{\min}]_{\text{MKSA}}^2$ remains an MKSA readout square and is not promoted to an LUS-independent intrinsic quantity. The same-shell invariance of $M_{\min}^2/(m_e \cdot m_p)$ is not invariance of $M_{\min}^2$ itself. The physical origin of the gravitational second power and the physical owner or selector of the second redefinition remain outside the present solution.

4. Results

The results below are same-shell, readout-covariant statements. $[M_{\min}]_{\text{MKSA}}^2$ is not an intrinsic LUS-independent quantity.

4.1. Electromagnetic First-Degree Structure

The Coulomb numerator admits one explicit minimum-mass factor,

$$\frac{e^2}{4\pi\epsilon_0} = \frac{\alpha}{2\pi}hc = \frac{\alpha}{2\pi}[M_{\min}]_{\text{MKSA}} c^3 s_{\text{gen}} \quad (58)$$

This is a first-degree minimum-mass readout structure.

This means that one explicit factor of the MKSA minimum-mass readout appears in the electromagnetic numerator after the spatial factor has been set aside. It does not mean that the origin of the Coulomb numerator has been derived, and it does not promote $[M_{\min}]_{\text{MKSA}}$ itself to a unit-system-independent invariant.

4.2. Gravitational Second-Degree Structure

The second-degree statement refers only to the minimum-mass readout appearing in the complete gravitational rate representation.

After division by c^3 ,

$$\frac{Gm_p^2}{c^3} = B_{\text{rate}} [M_{\min}]_{\text{MKSA}}^2 \quad (59)$$

Equation (59) is a readout-covariant relation. With **Table 1**, $A([M_{\min}]_{\text{MKSA}}) = +3$, $A(s_{\text{gen}}) = -1$, and $A(1 \text{ kg}) = +3$ give $A(B_{\text{rate}}) = -4$ and $A\left(B_{\text{rate}} \cdot [M_{\min}]_{\text{MKSA}}^2\right) = +2$, matching $A(G \cdot m_p^2 / c^3) = +2$. The square $[M_{\min}]_{\text{MKSA}}^2$ transforms under a mass-readout rescaling; the complete relation remains character-closed. This matching is not invariance of $M_{\min}^2$ itself.

This means that the gravitational rate representation carries a second power of the MKSA minimum-mass readout and remains character-closed at $A = +2$. It does not mean that $[M_{\min}]_{\text{MKSA}}^2$ is an intrinsic LUS-independent square, and it does not explain why gravity selects the second power.

4.3. Same-Shell Invariant Composite

The normalized composite

$$\text{SI} := \frac{M_{\min}^2}{m_e m_p} \quad (60)$$

is invariant under a common same-shell mass-scale transfer. This invariance is distinct from invariance of $M_{\min}^2$ itself. This means that a normalized mass composite can survive a common same-shell mass-scale transfer. It does not mean that the un-normalized square $M_{\min}^2$ is invariant, and it does not by itself select the proton or the electron.

4.4. Gravitational-to-Electromagnetic Force Ratio for Two Protons

The ratio is

$$R_p(m_p) = \frac{Gm_p^2}{e^2 / (4\pi\epsilon_0)} \quad (61)$$

Substitution of Equations (58) and (59) cancels the shared package α , c^3 , and s_{gen} . What remains is a first-degree minimum-mass readout structure. Coarsely,

$$R_p(m_p) \sim \frac{[M_{\min}]_{\text{MKSA}}^2}{[M_{\min}]_{\text{MKSA}}} \rightarrow [M_{\min}]_{\text{MKSA}} \quad (62)$$

In Equation (62), the symbol $\sim$ means only that the gravitational side contributes two powers of the minimum-mass readout and the electromagnetic side contributes one. It does not indicate numerical approximation or numerical closeness. Equation (62) therefore represents only the structural degree counting: in forming the quotient, the second-degree gravitational structure and the first-degree electromagnetic structure leave one residual minimum-mass readout degree. At this stage, that residual degree is not identified with a particular same-shell numerical value of $M_{\min}$. The corresponding numerical evaluation requires the readout-stage bookkeeping shown explicitly in Equation (63).

$$R_p(m_p)_{\text{mixed-stage}} = \frac{9 [M_{\min}/s]_{\text{MKSA}}}{4 (1 \text{ kg}/s)^{F_1 F_2}} = \frac{9 [M_{\min}/s]_{\text{MKSA}}}{4 [1 \text{ kg}/s]_{\text{MKSA}}} \cdot J_{\text{tab}}^{-4} = 8.09341058 \times 10^{-37} \quad (63)$$

Equation (63) is written as a mixed-stage reverse display. In Equation (63), the $F_1 F_2$ label belongs to the kg/s carrier in the denominator, not to the MKSA minimum-mass readout in the numerator. The factor J_{tab}^{-4} is used once in the reverse-display bookkeeping and is not an intrinsic factor of $R_p(m_p)$. The force ratio defined in Equation (61) is dimensionless and J_{tab} -free when evaluated within one coherent readout shell.

Restoring c^3 on the gravitational side and forming the force ratio cancels the common factors alpha, c^3 , and s_{gen} at the structural level. The coefficient 9/4 follows from the ratio of the gravitational and electromagnetic coefficients, while the degree difference 2-1 leaves one residual minimum-mass readout degree. Equation (63) gives the corresponding numerical result through the mixed-stage reverse display described above. The factor J_{tab}^{-4} is used only in this bookkeeping step and is not an intrinsic or final coefficient of the same-system $R_p(m_p)$. The complete same-system force ratio remains dimensionless and J_{tab} -free.

Species dependence and reverse-display factors remain outside this same-shell reading. The coefficient 9/4 follows algebraically from Equations (58) and (59).

4.5. Readout Display of $R_p(m_p)$ and the Pair Rewrite

This subsection separates same-system relations from cross-system bookkeeping and then rewrites the gravity-side expression in a pair-normalized form. When numerator and denominator are expressed in one coherent readout system, $R_p(m_p)$ is J_{tab} -free. J_{tab} belongs only to cross-system reverse display and is not retained in the final same-system coefficient. In a mixed-stage reverse display, a factor J_{tab}^{-4} may appear in order to recover the accepted numerical witness $R_p(m_p) = 8.09341 \times 10^{-37}$. That factor is bookkeeping for readout transfer, not an intrinsic part of R_p theory. B_{rate} is the rate-type coefficient of (59). It carries the time-per-mass slot required by $[G/c^3] = s/\text{kg}$ through $s_{\text{gen}}/(1 \text{ kg})$. Its deeper normalization remains open.

The pair rewrite

$$\frac{G}{c^3} = B_{\text{pair}} \frac{M_{\min}^2}{m_p m_e}, \quad B_{\text{pair}} := B_{\text{rate}} \left(\frac{m_e}{m_p} \right) \quad (64)$$

introduces no new LUS problem, because m_e/m_p is invariant under common same-shell mass scaling and $M_{\min}^2/(m_p \cdot m_e)$ is the invariant composite (60). Species dependence is not thereby removed; it is deferred. This means that the pair form merely rearranges the same same-shell content. It does not remove species dependence and does not add an LUS-independent square of $M_{\min}$.

5. Discussions

The present report adopts the partial LUS resolution. What is closed is the same-

shell invariant composite (60), the covariant display (59), and the same-shell degree statement (62). Equation (63) is retained only as a mixed-stage reverse display and is not counted as an exact same-shell relation. What is not closed is an LUS-independent intrinsic square of $M_{\min}$, the reason why gravity selects the second power, the deeper normalization of s_{gen} , species dependence, and the selector of 4.5 and π . MKSA remains a reference readout frame only. Same-shell cancellation is not promoted to physical equivalence of distinct shells. A candidate correspondence between distance and a discrete readout index remains unsettled, and its origin is not established here. A simple integer quantization of $m_p/M_{\min}$ is not pursued. α is retained as an interaction coefficient. B_{pair} is a reference display only. The factor 1 kg is an external macroscopic reference used in the display of the ratio, not an intrinsic component of $M_{\min}$. The present results therefore give a coherent same-shell organization, but not a completed microscopic mechanism. A complete LUS solution across distinct readout systems is not claimed. A dissipative or nonequilibrium account may be relevant to the open normalization and origin questions, but no such explanation is developed in the present report [21]–[25]. Reference [21] provides only an empirical example of relaxation in a material system. References [22] and [23] provide background on nonequilibrium work and fluctuation relations, while References [24] and [25] exemplify thermodynamic and nonequilibrium perspectives on spacetime gravitational field equations. None of References [21]–[25] enters the algebraic derivation in the present paper; these citations are included only as background for possible future work. The LUS problem is not that Equation (15) is false, but that $[M_{\min}]_{\text{MKSA}}^2$ can be misread as a unit-system-independent square. The present report keeps that square as an MKSA readout and does not claim a complete solution across distinct shells.

6. Conclusion

The present report organizes a same-shell, readout-covariant degree correspondence for the minimum-mass readout. Within the inherited empirical relations, the electromagnetic numerator contains one explicit minimum-mass readout degree, whereas the gravitational rate representation contains two. Their quotient therefore leaves one residual minimum-mass readout degree in the dimensionless proton-side gravitational-to-electromagnetic force ratio $R_p(m_p)$. This degree correspondence is the principal structural result of the present analysis. The empirical relations and numerical values used here are inherited from the preceding work. The present contribution is their structural classification in terms of minimum-mass readout degree and the separation of same-shell structural statements from readout-stage numerical bookkeeping. Accordingly, the result is not an independent derivation of the proton-side force ratio, nor does it establish the physical origin of gravity or electromagnetism. The coherent MKSA-to-MgSA mass-unit change provides an explicit covariance check. Under this rescaling, the numerical readout of $M_{\min}$ changes, and the individual quantities in the gravitational and

electromagnetic representations also change, but the complete dimensionless ratio $R_p(m_p)$ remains unchanged because the two sides acquire the same net mass-unit factor. The squared quantity $[M_{\min}]_{\text{MKSA}}^2$ is therefore retained as an MKSA readout square and is not interpreted as an intrinsic, unit-system-independent quantity. The common inverse-square spatial dependence of Coulomb's law and Newton's law is a separate structural feature and does not imply a common physical origin of the two interactions. The present analysis also does not provide a general solution of the LUS problem for arbitrary readout transformations. In particular, the physical origin of the gravitational second-degree structure and the deeper origin of the adopted readout transformation remain open.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Miyashita, T. (2020) Empirical Equation for the Gravitational Constant with a Reasonable Temperature. *Journal of Modern Physics*, **11**, 1180-1192. <https://doi.org/10.4236/jmp.2020.118074>
- [2] Miyashita, T. (2021) Various Empirical Equations for the Electromagnetic Force in Terms of the Cosmic Microwave Background Temperature. *Journal of Modern Physics*, **12**, 623-634. <https://doi.org/10.4236/jmp.2021.125040>
- [3] Miyashita, T. (2021) Various Empirical Equations to Unify between the Gravitational Force and the Electromagnetic Force. *Journal of Modern Physics*, **12**, 859-869. <https://doi.org/10.4236/jmp.2021.127054>
- [4] Miyashita, T. (2020) Erratum to "Empirical Equation for the Gravitational Constant with a Reasonable Temperature" [Journal of Modern Physics 11 (2020) 1180-1192]. *Journal of Modern Physics*, **11**, 1559-1560. <https://doi.org/10.4236/jmp.2020.1110096>
- [5] Miyashita, T. (2021) Erratum to "Various Empirical Equations to Unify between the Gravitational Force and the Electromagnetic Force" [Journal of Modern Physics 12 (2021) 859-869]. *Journal of Modern Physics*, **12**, 1160-1161. <https://doi.org/10.4236/jmp.2021.128069>
- [6] Miyashita, T. (2022) Empirical Equation for a Fine-Structure Constant with Very High Accuracy. *Journal of Modern Physics*, **13**, 336-346. <https://doi.org/10.4236/jmp.2022.134024>
- [7] Miyashita, T. (2018) Empirical Relation of the Fine-Structure Constant with the Transference Number Concept. *Journal of Modern Physics*, **9**, 2346-2353. <https://doi.org/10.4236/jmp.2018.913149>
- [8] Miyashita, T. (2023) A Fine-Structure Constant Can Be Explained Using the Electrochemical Method. *Journal of Modern Physics*, **14**, 160-170. <https://doi.org/10.4236/jmp.2023.142011>
- [9] Miyashita, T. (2023) Explanation of the Necessity of the Empirical Equations That Relate the Gravitational Constant and the Temperature of the CMB. *Journal of Modern Physics*, **14**, 432-444. <https://doi.org/10.4236/jmp.2023.144024>
- [10] Miyashita, T. (2023) Simplification of Various Empirical Equations for the Electromagnetic Force in Terms of the Cosmic Microwave Background Temperature. *Journal of Modern Physics*, **14**, 1217-1227. <https://doi.org/10.4236/jmp.2023.148068>

- [11] Miyashita, T. (2024) Correspondence Principle for Empirical Equations in Terms of the Cosmic Microwave Background Temperature with Solid-State Ionics. *Journal of Modern Physics*, **15**, 51-63. <https://doi.org/10.4236/jmp.2024.151002>
- [12] Miyashita, T. (2024) Ratio of Gravitational Force to Electric Force from Empirical Equations in Terms of the Cosmic Microwave Background Temperature. *Journal of Modern Physics*, **15**, 674-689. <https://doi.org/10.4236/jmp.2024.155031>
- [13] Miyashita, T. (2024) Algorithms for Empirical Equations in Terms of the Cosmic Microwave Background Temperature. *Journal of Modern Physics*, **15**, 1567-1585. <https://doi.org/10.4236/jmp.2024.1510066>
- [14] Miyashita, T. (2025) Normalization Methods in Algorithms for Empirical Equations in Terms of the Cosmic Microwave Background Temperature. *Journal of Modern Physics*, **16**, 390-409. <https://doi.org/10.4236/jmp.2025.163022>
- [15] Miyashita, T. (2025) Correction Procedure for Empirical Equations for Calculating the Cosmic Microwave Background Temperature. *Journal of Modern Physics*, **16**, 968-984. <https://doi.org/10.4236/jmp.2025.167051>
- [16] Miyashita, T. (2025) Solution for the Interpretation Problems Regarding Empirical Equations for Calculating the Cosmic Microwave Background Temperature. *Journal of Modern Physics*, **16**, 1861-1877. <https://doi.org/10.4236/jmp.2025.1612085>
- [17] Miyashita, T. (2026) A Readout-Based Structural Analysis of Empirical CMB Temperature Equations Related to $E = h\nu$ and $E = mc^2$. *Journal of Modern Physics*, **17**, 697-713. <https://doi.org/10.4236/jmp.2026.176030>
- [18] Miyashita, T. (2026) A Readout-Based Structural Analysis of Empirical CMB Temperature Equations: Macro-Micro Correspondence and Action-Level Framework. *Journal of Modern Physics*, **17**, 953-969. <https://doi.org/10.4236/jmp.2026.178043>
- [19] Tiesinga, E., Mohr, P.J., Newell, D.B. and Taylor, B.N. (2021) CODATA Recommended Values of the Fundamental Physical Constants: 2018. *Reviews of Modern Physics*, **93**, Article 025010. <https://doi.org/10.1103/revmodphys.93.025010>
- [20] Fixsen, D.J. (2009) The Temperature of the Cosmic Microwave Background. *The Astrophysical Journal*, **707**, 916-920. <https://doi.org/10.1088/0004-637x/707/2/916>
- [21] Miyashita, T. (2017) Equilibration Process in Response to a Change in the Anode Gas Using Thick SM-Doped Ceria Electrolytes in Solid-Oxide Fuel Cells. *Journal of The Electrochemical Society*, **164**, E3190-E3199. <https://doi.org/10.1149/2.0251711jes>
- [22] Jarzynski, C. (1997) Nonequilibrium Equality for Free Energy Differences. *Physical Review Letters*, **78**, 2690-2693. <https://doi.org/10.1103/physrevlett.78.2690>
- [23] Jarzynski, C. (2020) Fluctuation Relations and Strong Inequalities for Thermally Isolated Systems. *Physica A: Statistical Mechanics and its Applications*, **552**, Article 122077. <https://doi.org/10.1016/j.physa.2019.122077>
- [24] Jacobson, T. (1995) Thermodynamics of Spacetime: The Einstein Equation of State. *Physical Review Letters*, **75**, 1260-1263. <https://doi.org/10.1103/physrevlett.75.1260>
- [25] Eling, C., Guedens, R. and Jacobson, T. (2006) Nonequilibrium Thermodynamics of Spacetime. *Physical Review Letters*, **96**, Article 121301. <https://doi.org/10.1103/physrevlett.96.121301>