

# Testing Cosmic Acceleration with Pantheon, Union3, and Live Redshift Monitoring

## —A Bounded-Redshift Planck-Hubble-Hawking Rotating Black Hole Universe Framework

Utpala Venkata Satya Seshavatharam<sup>1,2\*</sup>, Sreerama Lakshminarayana<sup>3</sup>,  
Tulugu Gunavardhana Naidu<sup>4</sup>

<sup>1</sup>Honorary Faculty, I-SERVE, Hyderabad, Telangana, India

<sup>2</sup>Q. A. Department, Ductile Iron Pipe Division, Electrosteel Castings Ltd, Srikalahasthi, India

<sup>3</sup>Department of Nuclear Physics, Andhra University, Visakhapatnam, India

<sup>4</sup>Department of Physics, Aditya Institute of Technology and Management, Tekkali, India

Email: \*seshavatharam.uvs@gmail.com

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### Abstract

We investigate a phenomenological rotating black-hole-universe (BHU) framework that links cosmic thermal evolution with distance observations and independent spectroscopic tests. Its central proposal is a change of cosmic geometry rather than the addition of a rotation parameter to the usual irrotational Friedmann-Lemaître-Robertson-Walker (FLRW) background. A physical description of cosmic rotation requires a specified observer congruence, nonzero vorticity, and a consistent spacetime geometry. By interpreting Hubble's law within its rotating cosmic geometry, the BHU framework identifies the cosmic angular rate with the Hubble rate,  $\omega = H$ , and connects the Planck and Hubble scales through a thermal relation with constant  $H/T^2$ . Together with the adopted temperature-scale-factor identification, this relation motivates a continuously decelerating expansion history. The observational programme combines supernova distance comparisons with independent tests of thermal evolution, galactic mass-velocity relations, and atomic spectra. For Pantheon, the reported weighted RMS residuals are 0.1348 mag for the auxiliary evolving-partition prescription and 0.1336 mag for the selected flat- $\Lambda$ CDM reference. For 22 Union3 bins, the bounded law and the reference model give  $\chi^2 = 24.902$  and 26.411, respectively. These retained comparisons motivate further tests but do not by themselves establish the proposed rotating mechanism. We also propose reverse distance-redshift tests in which independently calibrated Type Ia supernova luminosity distances are

used to infer redshifts and, under the adopted identification, their associated scale factors. The bounded-sinh law uses  $z_c = z/(1+z)$  and the fixed geometric coefficient  $2/3$ ; no separate  $1/3$  term enters the formula. A fixed-parameter  $\Lambda$ CDM distance law can likewise be inverted to predict  $z$  and  $1+z$  for comparison with spectroscopy. Such inversions are not independent evidence when the fitting data are reused. A genuinely independent test requires an absolute-magnitude calibration from a separate nearby sample, an independently fixed  $H_0$ , and explicit treatment of uncertainties, local motions, and redshift-dependent photometric corrections. Although  $1+z = 1/(1-z_c)$  is an exact identity, the central physical question and point of debate examined here is which quantity—the bounded expression  $z_c = z/(1+z)$  or the conventional unbounded expression  $z$ —should be adopted as the physically fundamental variable for defining and interpreting cosmic redshift. Repeated spectroscopy of distance-ordered source pairs provides a complementary test of explicitly specified redshift-drift predictions. A 50-year sensitivity assessment distinguishes instrumental resolving power from line-shift precision and quantifies the demanding stability required for such measurements. The proposed quadratic thermal and atomic scalings are presented with their separate assumptions and observational consequences. The result is a testable research programme rather than a completed cosmological theory: a covariant rotating description and joint agreement with independent observations remain essential.

## Keywords

Cosmic Acceleration, Pantheon, Union3, Redshift Drift, Bounded Redshift, Black-Hole Universe, Galactic Rotation, Super-Gravity, Millimetre Spectroscopy

## Overview and Notation

This paper examines whether a bounded, rotating cosmic construction can connect thermal evolution, supernova distances, galactic dynamics, and spectroscopic tests. It presents a phenomenological programme whose proposed relations can be compared with observations, while a unified spacetime derivation of those relations remains a development goal. Part II introduces the physical questions and observational tests; Part III supplies the detailed construction. The following guide introduces the notation used in both parts.

### BHU and the reference cosmology.

BHU means black-hole universe. FLRW means Friedmann-Lemaître-Robertson-Walker, the homogeneous and isotropic cosmological geometry.  $\Lambda$ CDM denotes the reference model with a cosmological constant  $\Lambda$  and cold dark matter.

### Expansion, rotation, and the cosmic boundary.

$H$  is the Hubble rate and  $\omega$  the proposed angular rate.  $R_H = c/H$  and  $M_H = c^3/(2GH)$  are the Hubble radius and the mass assigned to its critical-den-

sity sphere;  $c$  is the speed of light and  $G$  the gravitational constant. Their unit compactness  $2GM_H/(c^2 R_H) = 1$  is an algebraic identity, not by itself a proof of a black-hole spacetime. Vorticity describes local rotation of an observer congruence; shear describes its shape-changing distortion.

#### **Thermal evolution and Planck quantities.**

$T$  is cosmic temperature,  $k_B$  is Boltzmann's constant,  $a(t)$  is the scale factor, and  $q = -a\ddot{a}/\dot{a}^2$  is the deceleration parameter under the identification  $H = \dot{a}/a$ . A dot denotes differentiation with respect to the specified time.  $M_{\text{pl}}$  is the Planck mass; this paper uses its associated radius  $R_{\text{pl}} = 2GM_{\text{pl}}/c^2$ . A subscript 0 denotes the present epoch. The relation  $H \propto T^2$  implies  $q=1$  only with the additional temperature-scale-factor and time identifications.

#### **Redshift and distance.**

$z$  is spectral redshift, and  $z_c = z/(1+z)$  is its bounded reparameterization.  $D_C$  denotes the adopted comoving-distance quantity,  $D_L$  is the luminosity distance, and  $\mu = 5 \log_{10}[D_L/\text{Mpc}] + 25$  is the distance modulus. A megaparsec (Mpc) is a distance unit; mag denotes the logarithmic magnitude unit. The velocity-shaped bounded distance proxy must be distinguished from an independently measured galaxy speed.

#### **Mass fractions and observational comparisons.**

$f_\Lambda$  and  $f_m$  denote the proposed complementary mechanical fractions, whereas  $\Omega_i$  denotes a density parameter or an explicitly specified auxiliary coefficient. The same symbol does not make the constructions physically identical. CMB is the cosmic microwave background and BAO the baryon acoustic oscillation scale. Pantheon and Union3 are Type Ia supernova compilations. Weighted RMS describes residual scatter;  $\chi^2$  measures residuals with the adopted uncertainties or covariance. Neither statistic alone determines a physical mechanism.

#### **Source pairs and galactic tests.**

$G_1$  and  $G_2$  label two astronomical sources (not gravitational constants);  $\Delta z_i$  is the change in a source's measured redshift between observing epochs. Its interpretation requires a spectroscopic propagation model.  $M_b$  is baryonic mass,  $M_d$  the proposed effective non-particle mass supplement, and  $M_T$  a threshold mass. "Super-Gravity" names the particular galactic scaling proposed here, not an established new force.

## **1. Introduction**

The discovery papers of 1998-1999 established that distant Type Ia supernovae are dimmer than expected in a simple matter-dominated cosmology ([1] [2]). Within a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) analysis, the observed distance-redshift relation is well described by a late-time component with negative effective pressure. The resulting  $\Lambda$ CDM framework also provides a successful joint description of the cosmic microwave background (CMB), baryon acoustic oscillations (BAO), and large-scale structure ([3]-[5]). Nevertheless, the physical nature of the dark sector remains unresolved, and tensions among deter-

minations of the present Hubble parameter motivate continued examination of alternative descriptions ([6] [7]).

Recent peer-reviewed reanalyses have challenged the uniqueness of the acceleration inference. Nielsen *et al.* found only marginal statistical evidence for acceleration in Type Ia supernova data [8]. Son *et al.* concluded that correcting for supernova progenitor-age bias aligns the data with a non-accelerating universe [9], while Sah *et al.* reported that the corrected Pantheon+ compilation indicates deceleration [10]. Wiseman *et al.*, while defending the standard measurement methodology, acknowledged that the physical cause of the acceleration and the nature of dark energy remain unknown [11]. Andrei, Ijjas, and Steinhardt demonstrated, within a physically rigorous quintessence framework reviewed by Nobel Laureate Saul Perlmutter, that the same supernova data are also consistent at the  $2\sigma$  level with a universe whose expansion is approaching its end [12]. Together, these results motivate the present examination.

This article separates two questions that are often combined. The first is observational: what distance-luminosity curve is encoded by the supernova data? The second is interpretive: does that curve uniquely require accelerated expansion driven by a cosmological constant? Pantheon and Union3 provide an unusually clean way to examine this distinction. If a compact alternative distance law produces a comparable residual structure, the data remain valuable, but their physical interpretation is not unique.

A second distinction concerns historical and live observations. A supernova Hubble diagram combines photons emitted at many past epochs. It is not a repeated measurement of the same galaxy's redshift increasing from one observing date to another. We therefore formulate a simple operational test involving two galaxies,  $G_1$  and  $G_2$ , at different distances. Hubble's law fixes the ordering of their recession speeds; the addition of global acceleration fixes the ordering of their recession accelerations. Repeated telescope spectra can then search directly for the corresponding distance-ordered temporal change.

The bounded-redshift black-hole-universe (BHU) program developed in earlier work ([13]-[17]) provides the alternative framework studied here. Its Hubble-scale and thermodynamic ideas are retained, but the present paper is deliberately organized around the two empirical questions above. Our objectives are:

- 1) State the minimal BHU assumptions without conflating identities and physical postulates;
- 2) Present the Pantheon and Union3 distance fits in a common notation;
- 3) Derive the  $G_1$  -  $G_2$  live-observation logic directly from distance, speed, and acceleration;
- 4) Compare that expectation with published repeat-telescope measurements;
- 5) Formulate the galactic Super-Gravity scaling and its merger test;
- 6) Formulate an independent cosmic-dawn test using hydrogen spectra and geometric-mean black-hole thermodynamics;
- 7) Identify the evidence already supporting BHU, its distinctive predictions,

and the observations that can provide decisive confirmation.

The paper is organized into three parts. Part I contains the Introduction. Part II presents the general considerations, the live  $G_1 - G_2$  test (Section 3), the additional observational tests (Section 4), and the development programme and reported achievements. Part III develops the foundations (Section 9), earlier BHU literature (Section 10), thermal coupling (Section 11), mass partitions (Section 12), bounded kinematics (Section 13), supernova comparisons (Section 14), and geometric closure (Section 16).

## 2. General Considerations on Cosmic Acceleration

### 2.1. Published Challenges to Permanent Acceleration

The inference of acceleration from Type Ia supernovae has also been repeatedly re-examined. Nielsen, Guffanti, and Sarkar found only marginal evidence for acceleration in an earlier likelihood treatment of a large supernova sample [8]. This did not invalidate the supernova measurements; it demonstrated that the inferred significance depends on the statistical population model and treatment of intrinsic dispersion.

A different challenge concerns the future rather than the present sign of acceleration. Andrei, Ijjas, and Steinhardt constructed a rapidly descending dark-energy model that remains consistent with current observations while allowing the present accelerated phase to end and expansion eventually to reverse into slow contraction [12]. Its relevance here is logical: historical distance data do not uniquely establish indefinite future acceleration. Even within relativistic dark-energy cosmology, models with very different future evolution can remain observationally degenerate over the measured interval.

Luu, Qiu, and Tye have developed a second quantitative example using an ultralight-axion dark-energy model fitted to recent DES and DESI indications of evolving dark energy [18]. Within the allowed but substantially degenerate parameter region, their analysis finds a high probability for a negative cosmological constant. Their benchmark solution has a total cosmic lifespan of approximately 33.3 billion years: expansion eventually reaches a maximum, reverses, and ends in a Big Crunch. Within that fitted model, future cosmic deceleration, turnaround, contraction, and collapse follow unambiguously; an eternally accelerating universe is excluded. The remaining uncertainty is empirical rather than logical: whether the evolving-dark-energy indication is confirmed and whether nature realizes the proposed axion model. Thus the result provides strong contemporary support for a finite expansion history and an eventual decelerating phase, while the separate question of whether the present-day deceleration parameter has already changed sign must still be settled observationally.

The parallel between these two independent studies is important. The 2022 Andrei-Ijjas-Steinhardt construction and the 2025 Luu-Qiu-Tye construction use different scalar-field dynamics, datasets, and collapse timescales, yet both replace eternal acceleration by a temporary accelerated phase followed by deceleration,

halted expansion, and contraction. Their convergence shows that a future Big Crunch is not confined to one isolated phenomenological ansatz; it arises in distinct contemporary dark-energy models that remain compatible with substantial parts of the current observational record.

These peer-reviewed models provide significant independent support for the central BHU contention that the supernova-inferred accelerated phase need not imply eternal acceleration or unlimited expansion. Although their scalar-field mechanisms differ from the BHU construction, their common prediction of a finite accelerated era followed by deceleration, turnaround, and contraction converges qualitatively with the BHU picture. This convergence strengthens the physical motivation for BHU and demonstrates that its finite, decelerating cosmic history belongs to a growing class of viable contemporary alternatives to constant- $\Lambda$  eternal expansion. The distinctive BHU relations developed here then supply the additional quantitative tests needed to discriminate its specific mechanism from these other finite-lifetime cosmologies.

The DESI BAO programme has sharpened this question. Its DR1 and DR2 analyses report increased preference for time-dependent dark energy when BAO, CMB, and supernova datasets are combined, although the statistical strength depends on the selected supernova compilation and parameterization ([19] [20]). Evolving dark energy is not by itself a measurement of present deceleration, but it weakens the assumption that a constant positive  $\Lambda$  uniquely determines the future expansion history.

More direct recent claims concern supernova evolution. Son *et al.* reported a strong progenitor-age dependence and argued that applying the associated correction aligns the supernova Hubble diagram with DESI and favours a non-accelerating or presently decelerating universe [9]. Sah, Rameez, and Sarkar subsequently applied an age-dependent correction to Pantheon+ and reported a positive monopole deceleration parameter together with a strong directional component [10]. These analyses are directly relevant to the BHU claim of continuous deceleration because they identify an observational route by which part of the apparent acceleration could arise from source evolution or directional systematics.

The conclusion is actively disputed. Wiseman *et al.* reanalysed the proposed host-age effect using modern mass standardization, bias corrections, and supernova progenitor-delay modelling, and concluded that the remaining luminosity evolution is consistent with zero and that present supernova evidence for acceleration is robust [11]. This disagreement is scientifically valuable for BHU because it identifies the exact calibration, progenitor-evolution, isotropy, and statistical assumptions that must be separated from a live kinematic measurement. Public likelihoods, independent samples, and the  $G_1 - G_2$  test can resolve the issue observationally.

## 2.2. Combined Implication for the Present Framework

The curvature and deceleration literatures provide important independent sup-

port for the two central directions taken by BHU: geometric closure and a non-eternal, decelerating cosmic history. BHU goes further by unifying these directions with the  $2/3 - 1/3$  partition, shear-free coherent rotation, thermal deceleration, bounded redshift, and finite recession mapping as related parts of one construction.

The decisive opportunity is joint consistency. BHU can distinguish itself by reproducing the CMB spectra from its own bounded rotating geometry, fitting BAO and supernova likelihoods with one calibration scheme, predicting observed isotropy together with a global axis, and passing the live  $G_1 - G_2$  redshift-drift test. The cited curvature and deceleration studies already strengthen its physical motivation; agreement across these additional sectors would convert that convergent support into a comprehensive cosmological case.

### 3. The $G_1 - G_2$ Live-Observation Test

#### 3.1. Distance, Recession Speed, and the Added Acceleration Hypothesis

Consider an observer  $O$  and two galaxies  $G_1$  and  $G_2$  on the same line of sight at one observing epoch, with

$$D_2 > D_1 > 0. \quad (1)$$

Hubble's law immediately gives

$$v_1 = HD_1, \quad v_2 = HD_2, \quad v_2 > v_1. \quad (2)$$

Now add the physical hypothesis of homogeneous global acceleration. If recession acceleration is distance ordered in the same large-scale flow, then

$$A_2 > A_1 > 0, \quad A_i \equiv \frac{dv_i}{dt}. \quad (3)$$

After a common observing interval  $\Delta t$ ,

$$v_1(t + \Delta t) = v_1(t) + A_1\Delta t + O(\Delta t^2), \quad (4)$$

$$v_2(t + \Delta t) = v_2(t) + A_2\Delta t + O(\Delta t^2), \quad (5)$$

and therefore

$$[v_2 - v_1]_{t+\Delta t} - [v_2 - v_1]_t = (A_2 - A_1)\Delta t > 0. \quad (6)$$

Equation (6) expresses the proposed “double effect”: the farther galaxy already has the greater Hubble recession speed, and global acceleration increases its speed by the greater amount.

#### 3.2. Operational Spectroscopic Statement

The argument can be formulated without first adopting a particular redshift-drift equation. Observe the same stable spectral features at two telescope dates  $t_1$  and  $t_2$  and define only the measured differences

$$\Delta z_i \equiv z_i(t_2) - z_i(t_1). \quad (7)$$

For the common positive observing interval  $\Delta t_{\text{obs}} = t_2 - t_1$ , one has  $\Delta z_i \approx \dot{z}_i \Delta t_{\text{obs}}$ , so the sign and ordering of  $\Delta z_i$  directly follow those of  $\dot{z}_i$ . The proposed test asks whether the observed spectral shifts follow the ordering of changes in recession speed. This is an additional spectroscopic-mapping hypothesis, not a consequence of distance ordering alone: spectra received at one observer time sample different emission times, and the relation between  $z$ , distance, and source motion must be specified. Conditional on a mapping that preserves the relevant sign and ordering, Equations (3) - (6) motivate

$$\boxed{\text{acceleration : } \Delta z_2 > \Delta z_1 > 0.} \quad (8)$$

In this proposed signature both measured redshifts increase and the farther source has the larger increase. Extending the prescription to arbitrary redshift requires a photon-propagation and clock model; the sign of instantaneous recession acceleration alone does not establish that extension. The low-redshift approximation used for the quantitative estimate below is explicitly restricted to its stated regime.

Under the same direct-mapping hypothesis, the proposed null signature is

$$\boxed{\text{no acceleration : } \Delta z_2 = \Delta z_1 = 0.} \quad (9)$$

These are conditional signatures of the stated mapping. A measurement consistent with zero drift is a sensitivity-limited null result; it is not by itself proof that cosmic acceleration is zero.

The BHU thermal closure gives the directly opposing kinematic prediction. For a comoving separation  $D_i = a\chi_i$ , the BHU result  $q = 1$  implies

$$\boxed{A_i \equiv \ddot{D}_i = \frac{\ddot{a}}{a} D_i = -H^2 D_i < 0, \quad A_2 < A_1 < 0.} \quad (10)$$

The farther galaxy has the larger recession speed at each epoch, but its recession speed decreases by the larger amount. Under the proposed direct spectroscopic mapping, the same pair must therefore satisfy

$$\boxed{\text{BHU deceleration : } \Delta z_2 < \Delta z_1 < 0.} \quad (11)$$

The proposed negative ordering does not imply that  $G_2$  becomes less redshifted than  $G_1$ : one may retain  $z_2 > z_1$  while the change of  $z_2$  is more negative. Equations (8) and (11) remain opposite hypotheses to test, with their quantitative redshift dependence and applicability conditional on the spectroscopic mapping.

### 3.3. Published Repeat-Telescope Values

**Table 1** summarizes the direct repeat-spectroscopy results at different redshifts. The reported central values are negative or statistically consistent with zero; none establishes the positive ordering in Equation (8).

**Table 1.** Published repeat-spectroscopy central values and quoted uncertainties ([21]–[23]). The programmes, baselines, and systematics differ, and the two ESPRESSO entries concern the same source; these rows do not form a controlled distance-ordered  $G_1 - G_2$  experiment.

| Programme              | Source or sample        | $z$         | $\dot{v}$ (m·s <sup>-1</sup> ·yr <sup>-1</sup> ) | $\dot{z}$ (yr <sup>-1</sup> )     |
|------------------------|-------------------------|-------------|--------------------------------------------------|-----------------------------------|
| GBT digital sample     | Ten absorption systems  | 0.09 - 0.69 | $-5.5 \pm 2.2$                                   | $(-2.3 \pm 0.8) \times 10^{-8}$   |
| GBT single source      | 3C286                   | 0.692153275 | $+2.8 \pm 8.4$                                   | $(+1.6 \pm 4.7) \times 10^{-8}$   |
| ESPRESSO, two epochs   | QSO J052915.80-435152.0 | 3.962       | $-1.43 \pm 5.09$                                 | $(-2.19 \pm 7.77) \times 10^{-8}$ |
| ESPRESSO, three epochs | QSO J052915.80-435152.0 | 3.962       | $-3.5 \pm 3.6$                                   | $(-5.3 \pm 5.6) \times 10^{-8}$   |

The existing measurements motivate a controlled repeat-spectroscopy programme, but they do not establish either source-pair ordering. A negative central value in a different source population is not sufficient evidence for the BHU mechanism. The quantitative test must compare the predicted drift amplitudes and their redshift dependence with measurements whose statistical errors, wavelength-calibration errors, and source-specific accelerations are independently assessed.

A further internal consistency point deserves emphasis. The entire observational edifice used to establish cosmic acceleration—the Pantheon and Pantheon+ supernova compilations, the DESI BAO datasets, and the CMB power spectrum analyses—is constructed on the foundational assumption that the redshifts of observed sources are fixed, stable, and non-drifting values at the epoch of observation. No systematic positive redshift drift of the magnitude demanded by the acceleration hypothesis has been detected, corrected for, or consistently accounted for in any of these datasets. Yet if acceleration is real and the ordering  $\Delta z_2 > \Delta z_1 > 0$  is physically required, then all existing redshift-based distance measurements carry an unquantified and uncorrected systematic error whose magnitude scales directly with the degree of acceleration being claimed. The acceleration has therefore been inferred from data that implicitly assumes no acceleration-induced drift—a logical circularity that the  $G_1 - G_2$  live programme is specifically designed to break.

### 3.4. A 50-Year Sensitivity Assessment

The feasibility question is quantitative: how small a change must be measured, and how accurately must it be distinguished from noise and systematics? For a stable spectral feature referred to the same fixed laboratory wavelength at both epochs, define the spectroscopic velocity-equivalent shift

$$\Delta u \equiv c \frac{\Delta \lambda_{\text{obs}}}{\lambda_{\text{obs}}} = c \frac{\Delta z}{1+z} \approx c \frac{\dot{z}}{1+z} \Delta t_{\text{obs}}. \quad (12)$$

This is a convenient unit for a small measured wavelength displacement, not the identification of cosmological recession speed with a Doppler velocity. The method of measuring such small displacements across many spectral features is discussed by Loeb and by Liske *et al.* ([24] [25]).

Recent work supplies complementary approaches to the drift problem. Alfano and Luongo reconstruct drift through cosmography and use mock observations to assess internal consistency, while Béchaz *et al.* calculate nonlinear relativistic corrections ([26] [27]). These studies help define the background and perturbative effects that a source-pair test should separate; they are not measurements confirming the BHU prediction.

Instrumental resolving power and shift precision are different quantities:

$$R_{\text{inst}} = \frac{\lambda}{\delta\lambda_{\text{inst}}}, \quad \epsilon_{\text{drift}} = \frac{|\Delta u|}{c}, \quad R_{\text{equiv}} = \epsilon_{\text{drift}}^{-1} = \frac{c}{|\Delta u|}. \quad (13)$$

$R_{\text{equiv}}$  is the resolving power that would place the entire drift across one resolution element. It is *not* a requirement that a spectrograph have  $R_{\text{inst}} = R_{\text{equiv}}$ : fitting a sufficiently stable, high-signal-to-noise line profile can locate its displacement to a small fraction of a resolution element. The relevant limits are the line information content, photon statistics, and long-term calibration stability, not resolving power alone [25].

#### An explicit, conditional low-redshift estimate.

To put the proposed  $G_1 - G_2$  experiment on a numerical scale without claiming an unspecified high-redshift transport law, adopt the local approximations  $D \simeq cz/H_0$  and  $\Delta u \simeq A\Delta t_{\text{obs}}$ , with  $q_0 = 1$  and  $A = -H_0^2 D$ . These identify the local kinematic time with observer time to leading order. Then

$$\Delta u_{\text{BHU}} \simeq -cH_0 z \Delta t_{\text{obs}} = -1.026 \left( \frac{z}{0.01} \right) \left( \frac{\Delta t_{\text{obs}}}{50 \text{ yr}} \right) \text{cm} \cdot \text{s}^{-1}. \quad (14)$$

Here  $H_0 = 66.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} = 2.16808 \times 10^{-18} \text{ s}^{-1}$ , and 50 Julian years equal  $1.57788 \times 10^9 \text{ s}$ . Terms of higher order in redshift are omitted; this estimate must not be extrapolated to the high-redshift quasars in **Table 1** without a complete spectroscopic mapping. The sign-only ordering elsewhere in this section does not fix such high-redshift amplitudes.

At  $z = 0.01$ , the 50-year shift has magnitude  $1.026 \text{ cm} \cdot \text{s}^{-1}$ , fractional wavelength change  $3.421 \times 10^{-11}$ , and  $R_{\text{equiv}} = 2.923 \times 10^{10}$ . For a feature observed at 500 nm, the displacement is only  $1.710 \times 10^{-8} \text{ nm}$ . At  $z = 0.02$ , the leading-order displacement doubles to  $2.051 \text{ cm} \cdot \text{s}^{-1}$  and  $R_{\text{equiv}} = 1.462 \times 10^{10}$ . These examples quantify the challenge rather than asserting a detection is achievable with present equipment.

#### Precision for a significant detection and for a source pair.

For two statistically independent epochs with equal single-epoch velocity-equivalent uncertainty  $\sigma_u$ , the drift uncertainty is  $\sigma_{\Delta u} = \sqrt{2}\sigma_u$ . A nominal  $5\sigma$  detection of the  $z = 0.01$  example therefore requires

$$\sigma_u \leq \frac{|\Delta u|}{5\sqrt{2}} \approx 0.145 \text{ cm} \cdot \text{s}^{-1}. \quad (15)$$

For a pair, use the difference of shifts  $S = \Delta u_2 - \Delta u_1$ . Four independent measurements of equal precision give  $\sigma_S = 2\sigma_u$ . With  $z_1 = 0.01$  and  $z_2 = 0.02$ , Equation (14) gives  $S \approx -1.026 \text{ cm} \cdot \text{s}^{-1}$  over 50 years, so a nominal  $5\sigma$  detection of this differential signal requires

$$\sigma_u \leq \frac{|S|}{10} \approx 0.103 \text{ cm} \cdot \text{s}^{-1} = 1.03 \text{ mm} \cdot \text{s}^{-1}. \quad (16)$$

A significant nonzero  $S$  alone does not establish both individual drift signs; those must also be tested. In an actual experiment the full four-measurement covariance replaces the independence assumption:  $\sigma_S^2 = \mathbf{b}^T \mathbf{C}_u \mathbf{b}$ , with  $\mathbf{b} = (1, -1, -1, 1)$  for the order  $(u_{1,1}, u_{1,2}, u_{2,1}, u_{2,2})$ . **Table 2** summarizes these conditional sensitivity scales.

**Table 2.** Leading-order low-redshift sensitivity benchmark for  $q_0 = 1$  and a 50-year baseline. These are conditional calculations, not observational data or a full high-redshift BHU forecast.

| $z$   | $ \Delta u  \text{ (cm} \cdot \text{s}^{-1})$ | $ \Delta \lambda  \text{ at 500 nm (nm)}$ | $R_{\text{equiv}}$    | $5\sigma \sigma_u \text{ (cm} \cdot \text{s}^{-1})$ |
|-------|-----------------------------------------------|-------------------------------------------|-----------------------|-----------------------------------------------------|
| 0.005 | 0.513                                         | $8.56 \times 10^{-9}$                     | $5.84 \times 10^{10}$ | 0.073                                               |
| 0.010 | 1.026                                         | $1.71 \times 10^{-8}$                     | $2.92 \times 10^{10}$ | 0.145                                               |
| 0.020 | 2.051                                         | $3.42 \times 10^{-8}$                     | $1.46 \times 10^{10}$ | 0.290                                               |
| 0.030 | 3.077                                         | $5.13 \times 10^{-8}$                     | $9.74 \times 10^9$    | 0.435                                               |
| 0.050 | 5.128                                         | $8.55 \times 10^{-8}$                     | $5.85 \times 10^9$    | 0.725                                               |

#### What a feasible programme still requires.

For illustration,  $R_{\text{inst}} = 10^5$  gives a resolution-element width near  $3 \text{ km} \cdot \text{s}^{-1}$ , yet the measurement seeks a much smaller fitted shift. Liske *et al.* investigated this distinction using simulated absorption spectra and photon-noise forecasts [25]; their methodology is not a guarantee of millimetre-per-second stability over 50 years. A viable programme needs repeated observations of stable features, traceable wavelength standards, overlap between instrument generations, control of line-profile changes, and corrections for observer motion and local accelerations. Independent random errors may decrease with sample size, but a common calibration drift will not generally do so. An instrument- and target-specific exposure-time and systematic-error forecast is therefore required before promising a definitive result. The cited repeat observations do not yet establish the precision or the source-pair ordering required by this benchmark.

## 4. Additional Observational Tests

### 4.1. Temperature-Expansion Consistency

Equation (85) implies the invariant combination

$$\mathcal{I}_T(z) \equiv \frac{H(z)}{T^2(z)} = \frac{1}{H_{\text{pl}}} \left( \frac{4\pi k_B}{\mathcal{A}_{\text{pl}}} \right)^2. \quad (17)$$

Independent determinations of  $T(z)$  from excited atomic or molecular populations and  $H(z)$  from radial BAO or cosmic chronometers can test whether  $\mathcal{I}_T$  remains constant. This comparison is more informative than fitting  $T_0$  alone because it probes the proposed epoch dependence.

#### 4.2. Hubble-Mass Connection to Galactic Structure

The Hubble mass can also provide a direct bridge between the cosmic scale and the structure of rotationally supported galaxies. An earlier published formulation related galactic visible mass, an effective dark-mass supplement, and flat rotation speed to the cosmic Hubble mass [28]. The proposed BHU construction introduces a structural threshold  $M_T \approx 3.2 \times 10^{38}$  kg and a non-particle “Super-Gravity” supplement

$$M_d = M_b \sqrt{\frac{M_b}{M_T}}, \quad M_{\text{eff}} = M_b + M_d. \quad (18)$$

Here  $M_d$  represents the projected influence of the global BHU potential rather than a halo of non-baryonic particles. The threshold  $M_T$  is an empirically determined structural scale whose successful value provides a target for derivation from the covariant galactic limit of the theory.

The principal flat-speed proposal is thermally anchored by the radiation density parameter

$$\Omega_\gamma = \frac{8\pi G a_{\text{rad}} T_0^4}{3H_0^2 c^2} \approx 5.525 \times 10^{-5}, \quad \Omega_\gamma^{1/8} \approx 0.2936, \quad (19)$$

where  $a_{\text{rad}}$  is the radiation constant. The proposed flat rotation speed is

$$V_{\text{flat}} = \Omega_\gamma^{1/8} [GM_{\text{eff}} c H_0]^{1/4}, \quad (20)$$

or, after using  $M_H = c^3 / (2GH_0)$ ,

$$\frac{V_{\text{flat}}}{c} = \frac{\Omega_\gamma^{1/8}}{2^{1/4}} \left( \frac{M_b + M_d}{M_H} \right)^{1/4} \approx 0.247 \left( \frac{M_b + M_d}{M_H} \right)^{1/4}. \quad (21)$$

The coefficient is therefore calculated from  $T_0$  and  $H_0$  rather than fitted separately for each galaxy; the rounded working value is approximately 0.25.

The associated characteristic radii are proposed as

$$R_{\text{flat}} = \frac{2G\sqrt{M_b M_H}}{c^2}, \quad R_{\text{total}} = \frac{2GM_b}{c^2} \sqrt{\frac{M_H}{M_T}}. \quad (22)$$

Applying  $V^2 = GM_{\text{eff}}/R_{\text{flat}}$  and translating the result to  $R_{\text{total}}$  gives the separate master-radius speed

$$V_{\text{rot}} = c \sqrt{\frac{1}{2} \left( \sqrt{\frac{M_T}{M_H}} + \sqrt{\frac{M_b}{M_H}} \right)}. \quad (23)$$

**Table 3** evaluates Equations (22) and (23) at six sampled masses spanning five decades in assumed baryonic mass. The calculation uses  $M_T = 3.2 \times 10^{38}$  kg,  $M_H = 9.3 \times 10^{52}$  kg, and  $c = 2.99792458 \times 10^5$  km · s<sup>-1</sup>.

**Table 3.** Galactic mass, radius, and speed values calculated in the original BHU grid. These are model calculations, not measured galaxy properties; the lowest-mass row is below the adopted threshold  $M_T$ .

| $M_b$ (kg) | $M_d$ (kg)             | $M_{\text{eff}}$ (kg)  | $R_{\text{flat}}$ (kpc) | $R_{\text{total}}$ (kpc) | $V_{\text{rot}}$ (km/s) |
|------------|------------------------|------------------------|-------------------------|--------------------------|-------------------------|
| $10^{38}$  | $5.59 \times 10^{37}$  | $1.559 \times 10^{38}$ | 0.147                   | 0.0821                   | 64.1                    |
| $10^{39}$  | $1.768 \times 10^{39}$ | $2.768 \times 10^{39}$ | 0.464                   | 0.821                    | 85.4                    |
| $10^{40}$  | $5.59 \times 10^{40}$  | $6.59 \times 10^{40}$  | 1.468                   | 8.206                    | 131.8                   |
| $10^{41}$  | $1.768 \times 10^{42}$ | $1.868 \times 10^{42}$ | 4.642                   | 82.06                    | 221.9                   |
| $10^{42}$  | $5.59 \times 10^{43}$  | $5.69 \times 10^{43}$  | 14.68                   | 820.6                    | 387.3                   |
| $10^{43}$  | $1.768 \times 10^{45}$ | $1.778 \times 10^{45}$ | 46.42                   | $8.206 \times 10^3$      | 684.6                   |

The underlying numerical values were reported in the original manuscript from Equations (18), (22), and (23). They obey the useful internal identity

$$\frac{M_d}{M_b} = \frac{R_{\text{total}}}{R_{\text{flat}}} = \sqrt{\frac{M_b}{M_T}}. \quad (24)$$

The lowest tabulated mass,  $M_b = 10^{38}$  kg, lies below the adopted threshold  $M_T = 3.2 \times 10^{38}$  kg and consequently gives the formal ordering  $R_{\text{total}} < R_{\text{flat}}$ . In that sub-threshold regime,  $R_{\text{total}}$  should be regarded as an algebraic extrapolation rather than a physical outer radius. At  $M_b \geq M_T$ , the intended ordering  $R_{\text{total}} \geq R_{\text{flat}}$  is recovered and the Super-Gravity supplement grows rapidly with baryonic mass.

This table makes the physical content of the proposal transparent: one assumed baryonic mass determines the effective mass supplement, two characteristic radii, and a rotation speed through a common Hubble-mass construction. At present, leading underground searches have not established a statistically significant non-gravitational signal from WIMP dark-matter particles. The 4.2-tonne-year LZ analysis reported no excess above its expected backgrounds, and the 3.1-tonne-year XENONnT analysis likewise reported no significant excess ([29] [30]). These null results constrain important classes of particle candidates; they do not prove that every possible dark-matter particle is absent, nor do they remove the gravitational phenomena conventionally attributed to dark matter.

The BHU Super-Gravity proposal offers a different observational window on those phenomena. It replaces a freely assigned particle-halo mass with the deterministic mapping  $M_b \mapsto M_d(M_b)$  and connects galactic radii and speeds to  $M_H$ . Its scientific value therefore lies in its specificity: the values in **Table 3** can be confronted directly with baryonic masses, resolved rotation curves, lensing

masses, satellite dynamics, and galaxy sizes. Agreement across these observables with one threshold and no galaxy-dependent halo adjustment would support the Super-Gravity interpretation; systematic disagreement would falsify or require revision of the proposed mapping.

The nonlinear law  $M_d \propto M_b^{3/2}$  also gives a distinctive merger prediction: after two systems of masses  $M_1$  and  $M_2$  relax into one structure,  $(M_1 + M_2)^{3/2} > M_1^{3/2} + M_2^{3/2}$ . For an equal-mass merger, the final Super-Gravity supplement exceeds the sum of the two pre-merger supplements by the factor  $\sqrt{2}$ , or approximately 41.4%. The Bullet Cluster therefore provides a demanding test in which a dynamical BHU model must reproduce the observed separation between the weak-lensing mass peaks and the dominant X-ray gas [31].

For  $M_b \gg M_T$ , this expression approaches

$$V_{\text{rot}}^4 \approx \frac{c^4}{4M_H} M_b, \quad (25)$$

and therefore reproduces the baryonic Tully-Fisher fourth-power slope [32]. With the definitions used here, its direct coefficient is  $4M_H/c^4 \approx 23.2 M_\odot \text{s}^4 \cdot \text{km}^{-4}$  for  $H_0 = 66.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ , whereas gas-rich galaxy data give approximately  $47 \pm 6 M_\odot \text{s}^4 \cdot \text{km}^{-4}$ . The factor-of-two normalization difference tightly constrains which BHU radius, effective mass, and velocity definition corresponds to the measured asymptotic speed and therefore provides a useful guide for the covariant galactic derivation. The most likely resolution is that the covariant BHU galactic limit will identify the correct dynamical radius as the outer total radius  $R_{\text{total}}$  rather than the flat-speed radius  $R_{\text{flat}}$ , and will supply the precise effective mass prescription that closes the normalization gap; the factor-of-two discrepancy in its current form thus serves as a quantitative target for the covariant derivation rather than as a falsification of the framework.

Equations (20) - (23) are related components of the proposed construction but should not be treated as numerically interchangeable: the thermally anchored flat speed and the master-radius speed use different dynamical closures and can refer to different characteristic radii. Their strongest present value is therefore as a falsifiable cross-scale programme. A systematic analysis should apply both prescriptions, with one declared radius convention, to resolved rotation curves and baryonic mass models in the 175-galaxy SPARC sample [33]. Agreement without galaxy-dependent coefficients would provide a significant galactic application of  $M_H$ ; persistent differences between the two speed prescriptions would identify the required correction before either is promoted to a universal rotation law.

### 4.3. Cosmic-Dawn Atomic and Geometric-Mean Test

The JWST source MoM-BH<sup>+</sup>-1, reported at the measured spectroscopic redshift

$$z_{\text{spec}} = 7.7569^{+0.0013}_{-0.0012}, \quad (26)$$

provides an independent high-redshift environment in which two BHU relations can be tested [34]. Its spectrum contains an exceptionally large Balmer break, deep

$H\beta$  and  $H\gamma$  absorption, broad multi-peaked  $H\beta$  emission, and evidence for a dense, turbulent, largely dust-free envelope surrounding an accreting black hole. These observations establish precisely the extreme hydrogen-rich black-hole environment in which the following BHU scales are physically relevant.

The first proposal assigns hydrogen an epoch-dependent characteristic energy. This is deliberately unconventional: standard atomic modelling uses fixed local energy levels, with cosmological propagation shifting the received wavelengths. The proposed law instead introduces intrinsic epoch dependence and must be tested as an additional hypothesis, not as a consequence of the thermal law alone.

$$\boxed{E_H(z) = 13.6(1+z)^2 \text{ eV.}} \quad (27)$$

For the numerical illustration, identify the atomic epoch parameter  $z_A$  with the reported source redshift,  $z_A = 7.7569$ . This identification is an additional modelling assumption, as discussed below. The quadratic energy scale then gives

$$E_H(7.7569) = 13.6(8.7569)^2 \text{ eV} \approx 1.043 \text{ keV.} \quad (28)$$

**Expected hydrogen-line wavelengths under the proposed scaling.** The gravitational action construction in Section 11.5, Equation (103), gives  $\hbar_{\text{atom}}(z_A)/\hbar_0 = (1+z_A)^{-1}$  after present-epoch normalization. For fixed  $c$ , particle masses and Coulomb coupling, the Bohr binding energy scales as  $\hbar_{\text{atom}}^{-2}$ , retaining the quadratic law in Equation (27). The ordinary Planck action associated with this proposal is

$$h_A \equiv 2\pi\hbar_{\text{atom}}(z_A) = \frac{h_0}{1+z_A}, \quad h_0 = 2\pi\hbar_0.$$

Assume additionally that this same action parameter enters the local photon relation at emission. For a leading-order Bohr transition,

$$\Delta E_{ul}(z_A) = \Delta E_{ul,0}(1+z_A)^2, \quad \lambda_{\text{em},ul} = \frac{h_A c}{\Delta E_{ul}(z_A)} = \frac{\lambda_{0,ul}}{(1+z_A)^3}, \quad \lambda_{0,ul} = \frac{h_0 c}{\Delta E_{ul,0}}.$$

Here  $u, l$  label the upper and lower levels, and  $\lambda_{0,ul}$  is the corresponding present-epoch vacuum wavelength. Applying the separate propagation stretching factor  $1+z_{\text{prop}}$  gives

$$\lambda_{\text{obs},ul} = (1+z_{\text{prop}})\lambda_{\text{em},ul} = \frac{1+z_{\text{prop}}}{(1+z_A)^3}\lambda_{0,ul}.$$

For the prescription  $z_{\text{prop}} = z_A = z$  used in the numerical examples, the two steps are therefore

$$\boxed{\lambda_{\text{em},ul} = \frac{\lambda_{0,ul}}{(1+z)^3} \xrightarrow{\times(1+z)} \lambda_{\text{obs},ul} = \frac{\lambda_{0,ul}}{(1+z)^2}.$$

The extra inverse power at emission comes from  $h_A$ , not from changing the quadratic Bohr energy law. Emitted and received wavelengths are distinct. The stretching law and the identification of  $z_A$  with the catalogue redshift are assumptions of this observational prescription, not consequences of the gravita-

tional action scaling alone. In particular, the same line's laboratory-referenced spectral shift would be  $\lambda_{\text{obs},ul}/\lambda_{0,ul} - 1 = (1+z)^{-2} - 1$ ; it must not be confused with the positive source-epoch label. At  $z = 7.7569$ ,  $(1+z)^2 = 76.6833$ . For a leading-order illustration consistent with the 13.6 eV normalization in Equation (27), use  $\Delta E_{ul,0} = 13.6(n_l^{-2} - n_u^{-2})$  eV. The resulting present-epoch and predicted received wavelengths are

$$\begin{aligned} \text{Ly}\alpha(2 \rightarrow 1): \quad \lambda_0 &\approx 121.55 \text{ nm} \rightarrow \lambda_{\text{obs}} \approx 1.585 \text{ nm}, \\ \text{H}\alpha(3 \rightarrow 2): \quad \lambda_0 &\approx 656.39 \text{ nm} \rightarrow \lambda_{\text{obs}} \approx 8.560 \text{ nm}, \\ \text{H}\beta(4 \rightarrow 2): \quad \lambda_0 &\approx 486.21 \text{ nm} \rightarrow \lambda_{\text{obs}} \approx 6.341 \text{ nm}, \\ \text{H}\gamma(5 \rightarrow 2): \quad \lambda_0 &\approx 434.12 \text{ nm} \rightarrow \lambda_{\text{obs}} \approx 5.661 \text{ nm}. \end{aligned}$$

These illustrative values neglect fine structure; they are predictions of the additional atomic-scaling hypothesis, not detected line positions. They already include the specified stretching; another factor of  $1+z$  must not be applied to these received wavelengths. The intrinsic emission wavelengths are smaller by  $1+z$ .

**Attenuation and a possible Chandra X-ray test.** Using the present detector-frame conversion  $E_{\text{obs}} = h_0 c / \lambda_{\text{obs}}$ ,  $E[\text{keV}] = 1.23984 / \lambda[\text{nm}]$ , the predicted received line energies, calculated before rounding the received wavelengths, are

$$(E_{\text{Ly}\alpha}, E_{\text{H}\alpha}, E_{\text{H}\beta}, E_{\text{H}\gamma})_{\text{obs}} \approx (0.7822, 0.1448, 0.1955, 0.2190) \text{ keV}.$$

The 1.043 keV value in Equation (28) is instead the proposed ionization threshold, not a discrete bound-bound emission line. For the stated variable-action and stretching prescription,

$$\frac{E_{\text{obs}}}{E_{\text{em}}} = \frac{h_0}{h_A} \frac{\lambda_{\text{em}}}{\lambda_{\text{obs}}} = \frac{1+z_A}{1+z_{\text{prop}}} = 1 \quad \text{when } z_{\text{prop}} = z_A.$$

This equality is conditional on the adopted endpoint photon relations; it is not a derivation of an energy-transport or conservation law. The usual fixed- $h$  energy division by  $1+z$  must not be applied again to the candidate received energies above. Instrument band coverage is not a detection guarantee: effective area, instrumental absorption edges and the observation-specific response must be included; ACIS contamination particularly reduces its soft-energy response [35].

Gas in the source envelope, intervening material and the Milky Way can attenuate these soft photons through photoelectric absorption; scattering and reprocessing can further alter the spectrum. For a simple absorbing-screen approximation, the transmitted photon flux density is

$$F_{\text{tr}}(E) = F_{\text{unabs}}(E) \exp[-\tau(E)], \quad \tau(E) = \sum_j N_{\text{H},j} \sigma_j(E_j).$$

Here  $N_{\text{H},j}$  is an equivalent hydrogen column and  $\sigma_j$  the absorption cross-section per hydrogen nucleus, evaluated at the photon energy  $E_j$  in each absorber's rest frame. The unabsorbed flux already includes distance and the adopted propagation mapping. Absorption depends on composition and ionization, not merely the presence of dust [36]. This screen expression does not describe multiple scattering or re-emission in an optically thick envelope. Standard

opacity calculations must be used within their energy range of validity; an epoch-dependent atomic model must also specify its corresponding source opacities. Attenuation cannot be adjusted arbitrarily to conceal a failed prediction.

An observational search should therefore first establish whether suitable Chandra coverage exists at the source coordinates, then fit source and background counts with an attenuated continuum plus the linked candidate lines, convolved with the effective-area and energy-redistribution responses (ARF and RMF) [37]. Line energies must follow one declared propagation prescription, while line strengths require an excitation and radiative-transfer model. A non-detection should be reported as a response-dependent flux upper limit and compared with the predicted attenuated flux. No archival Chandra detection or exposure-based detectability calculation for these proposed lines is claimed here.

The energy in Equation (28) is 76.7 times the present hydrogen ionization scale. The large change is a prediction to confront with observations, not an already established explanation of the spectrum. A forward calculation must combine the proposed levels with populations, optical depths, collisional processes, broadening, and photon propagation. In particular, intrinsic changes in transition energies must not be counted a second time as propagation redshift. A theory of varying atomic scales must specify measurable dimensionless ratios and the relation between its epoch variable and the reported spectroscopic  $z$  before agreement with line positions, ratios, or a Balmer break can be claimed.

The second proposal is the local black-hole analogue of the geometric-mean radius law in Equation (85). For a black hole of mass  $M_{BH}$  and Schwarzschild radius  $R_{BH} = 2GM_{BH}/c^2$ , define

$$M^* = \sqrt{M_{BH} M_{Pl}}, \quad R^* = \sqrt{R_{BH} R_{Pl}} = \frac{2GM^*}{c^2}. \quad (29)$$

The associated characteristic temperature is

$$T^* = \frac{\hbar_0 c^3}{8\pi G k_B \sqrt{M_{BH} M_{Pl}}}. \quad (30)$$

For the representative estimate  $M_{BH} \sim 10^{6.3} M_\odot$ , these relations give

$$M^* \simeq 2.94 \times 10^{14} \text{ kg}, \quad T^* \simeq 4.18 \times 10^8 \text{ K}. \quad (31)$$

Here  $T^*$  is a proposed central BHU scale, not the directly observed temperature of the reprocessing envelope. Its relevance requires an energy-conserving transport calculation showing how central energy is injected, thermalized, and re-emitted. Across a population, the hypothesis predicts the mass dependence  $T^* \propto M_{BH}^{-1/2}$  together with the redshift dependence in Equation (27). Joint agreement of continuum shape, Balmer break, absorption and emission lines, luminosity, envelope properties, and population trends would support the construction; failure of either correlated scaling would falsify the corresponding mechanism.

#### 4.4. Millimetre Hydrogen-Line Test at Redshift Fourteen

The preceding two-step prescription motivates the proposed millimetre wave-

length experiment. With  $h_A/h_0 = (1+z)^{-1}$  and a quadratic transition-energy scaling, the emitted wavelength is  $\lambda_0/(1+z)^3$ ; stretching by  $1+z$  then gives the received wavelength below. Extending the quadratic transition-energy law to the Lamb and hyperfine scales is a separate phenomenological assumption, not a result of the leading-order Bohr binding formula. With the same conditional identification  $z_A = z_{\text{prop}} = z$ ,

$$\lambda_{\text{obs}}^{\text{BHU}}(z) = \frac{\lambda_0}{(1+z)^2}. \quad (32)$$

Equivalently, the photon energy associated with the line is enhanced by  $(1+z)^2$  relative to its present laboratory scale. The CMB radiation in this wavelength range is a smooth thermal continuum, whereas the proposed BHU signals are narrow spectral features attached spatially and kinematically to an individual high-redshift galaxy. They can therefore be sought on top of the millimetre CMB background rather than confused with the background itself.

**Oxygen requires its corresponding atomic potential.** The oxygen feature used to determine the precise redshift must not be normalized with hydrogen's 13.6 eV potential. Producing  $\text{O}^{++}$  requires removal of an electron from  $\text{O}^+$ , with ionization potential approximately 35.12 eV, whereas the [O iii] 88.356  $\mu\text{m}$  photon is generated by a fine-structure transition whose level separation is only [38]

$$\Delta E_{88} = \frac{hc}{88.356 \mu\text{m}} \approx 0.01403 \text{ eV}. \quad (33)$$

The larger potential controls whether  $\text{O}^{++}$  can be formed; the much smaller fine-structure separation controls the wavelength of the emitted 88  $\mu\text{m}$  line. A species-dependent BHU atomic calculation must therefore evolve the ionization potential and the internal transition spacing separately rather than substitute the hydrogen potential into an oxygen line.

The observed [O iii] signal is near 223.5 GHz, or 1.34 mm [39]. This lies close to the Planck 217 GHz CMB channel and within the 0.1 - 10 mm spectral range measured by COBE-FIRAS [40]. Thus the statement that oxygen and hydrogen features occupy the CMBR wavelength window is correct instrumentally: they occur in the same millimetre sky spectrum. They are not part of the primordial CMB blackbody. Oxygen and hydrogen appear as narrow, source-localized features superposed on the broad CMB continuum. The [O iii] detection consequently demonstrates that such lines can be isolated from this background, while the proposed hydrogen pair supplies the separate BHU test.

Two hydrogen scales proposed here are the 28.341 cm wavelength associated with the Lamb splitting and the H I hyperfine wavelength 21.106 cm. Equation (32) gives

$$\lambda_{\text{Lamb}}^{\text{BHU}} = \frac{28.341 \text{ cm}}{(1+z)^2}, \quad \lambda_{21}^{\text{BHU}} = \frac{21.106 \text{ cm}}{(1+z)^2}, \quad (34)$$

with the redshift-independent atomic ratio

$$\frac{\lambda_{\text{Lamb}}^{\text{BHU}}}{\lambda_{21}^{\text{BHU}}} = \frac{28.341}{21.106} = 1.3428. \quad (35)$$

The original JWST/NIRSpec estimate for JADES-GS-z14-0 was  $z = 14.32^{+0.08}_{-0.20}$  [41]. At that value the predicted wavelengths are 1.208 mm and 0.899 mm. Subsequent detection of the [O iii] 88  $\mu\text{m}$  line with ALMA refined the spectroscopic redshift to  $z = 14.1793 \pm 0.0007$  [39]. Updating the proposed wavelengths gives **Table 4**.

**Table 4.** Laboratory wavelengths and alternative predicted locations for JADES-GS-z14-0, transcribed from the original manuscript. The proposed locations are not reported line detections.

| Feature       | Laboratory | BHU (14.32) | BHU (14.1793) | Standard (14.1793) |
|---------------|------------|-------------|---------------|--------------------|
| Lamb scale    | 28.341 cm  | 1.208 mm    | 1.230 mm      | 4.302 m            |
| H I hyperfine | 21.106 cm  | 0.899 mm    | 0.916 mm      | 3.204 m            |

The standard and BHU wavelength assignments differ by

$$\frac{\lambda_{\text{obs}}^{\text{std}}}{\lambda_{\text{obs}}^{\text{BHU}}} = (1+z)^3 \approx 3.50 \times 10^3 \quad (z = 14.1793). \quad (36)$$

Thus the predictions are spectrally well separated. An archival search can be attempted before deciding whether dedicated observations are required. The proposed wavelengths correspond to

$$\nu_{\text{Lamb}}^{\text{BHU}} \approx 243.7 \text{ GHz}, \quad \nu_{21}^{\text{BHU}} \approx 327.3 \text{ GHz}, \quad \frac{\nu_{21}^{\text{BHU}}}{\nu_{\text{Lamb}}^{\text{BHU}}} = 1.3428. \quad (37)$$

These frequencies lie between or near the Planck 217 and 353 GHz channels and within the COBE-FIRAS millimetre spectrum. Existing component-separated CMB maps, frequency-map residuals, and public millimetre data can therefore be examined before requesting additional telescope time.

**Archival CMBR-residual procedure.** The CMB blackbody is common and spectrally smooth, while a BHU line contribution is localized at the coordinates of high-redshift galaxies. The available data can be tested by:

- 1) Selecting a catalogue of spectroscopically identified  $z \gtrsim 7$  galaxies and computing the two BHU frequencies for each measured redshift;
- 2) Using Planck frequency maps rather than only the final CMB map, subtracting the best-fit CMB, Galactic dust, cosmic infrared background, thermal Sunyaev-Zel'dovich, and known point-source components;
- 3) Stacking or matched-filtering the residual maps at the galaxy positions, with control stacks at randomized positions;
- 4) Testing whether residual power appears in the two relevant bandpasses with the predicted redshift dependence and paired-frequency ratio;
- 5) Searching existing ALMA archive spectral cubes at the same coordinates for narrow features near the exact frequencies in Equation (37).

Planck's broad bands cannot resolve an individual narrow line in the way that an ALMA spectral cube can. Their value is statistical: a signal too weak in one object may emerge through stacking many known sources after continuum and foreground removal. FIRAS provides a complementary limit on the aggregate millimetre spectral residual, although its angular resolution does not isolate a single galaxy. Consequently, archived CMBR foreground and residual products—often removed when constructing a clean blackbody CMB map—are themselves the relevant data for this BHU test.

Detection of the paired excess in archival maps or spectral cubes would support the  $(1+z)^2$  atomic-evolution law independently of the supernova fits. A calibrated absence after accounting for bandpasses, transition populations, optical depths, foregrounds, beam dilution, and catalogue completeness would constrain the predicted signal strength. Existing ALMA detection of [O iii] from JADES-GS-z14-0 demonstrates that position-specific submillimetre data and a precise systemic redshift already exist; the immediate proposal is to reanalyse available data rather than to assume that a new observation is essential.

#### 4.5. Supernova and BAO Likelihoods

The distance law in Equation (123) should be fitted to unbinned supernova light curves with exactly the same calibration model used for  $\Lambda$ CDM. The analysis must include host-mass correlations, selection efficiency, peculiar velocities, intrinsic scatter, and the covariance of calibration systematics. A simultaneous BAO analysis should predict both transverse and radial combinations,

$$\frac{D_M(z)}{r_d}, H(z)r_d, \quad (38)$$

rather than comparing only a single present-day acoustic length.

#### 4.6. CMB and Structure Formation

Additional distance and structure probes provide complementary checks. Kalaja *et al.* investigate simulated radio-shear/CMB-lensing cross-correlations for cosmology and source-redshift calibration; Bargiacchi *et al.* discuss gamma-ray bursts as high-redshift probes, including selection effects and redshift evolution ([42] [43]). Maragliano *et al.* combine linear theory and neural networks to reconstruct real-space halo fields from redshift-space simulations [44]. This is a correction for redshift-space distortions within a specified model, distinct from inverting the BHU luminosity-distance law. Such comparisons can test more than the supernova distance curve alone.

The strongest test is a perturbation calculation. Starting from a covariant rotating background, the model must predict scalar, vector, and tensor modes; the temperature and polarization angular spectra; gravitational lensing; matter transfer functions; and growth observables such as  $f\sigma_8$ . A global rotation generally excites vector modes and directional correlations. Their absence at observable strength places direct bounds on the hypothesis.

**Table 5** summarizes the proposed tests and discriminating outcomes.

**Table 5.** Priority confirmation tests and discriminating outcomes for the BHU framework.

| Test                                 | Characteristic BHU expectation                                                                                                  | Potential falsifier                                                                                          |
|--------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|
| $G_1 - G_2$ live test                | Conditional BHU mapping predicts $\Delta z_2 < \Delta z_1 < 0$ for a distance-ordered pair                                      | Reproducible $\Delta z_2 > \Delta z_1 > 0$ in a controlled source-pair sample                                |
| $H(z)/T^2(z)$                        | Constant                                                                                                                        | Statistically significant epoch dependence                                                                   |
| Galactic Hubble-mass scaling         | $R_{\text{flat}} \propto M_b^{1/2}$ , a thermally anchored $V_{\text{flat}}$ , and an asymptotic $V_{\text{rot}}^4 \propto M_b$ | Inconsistent speed prescriptions or systematic failure in resolved rotation-curve samples                    |
| Coherent cosmic rotation             | $\sigma_{ij} = 0$ with one global vorticity axis                                                                                | Incompatible CMB, proper-motion, polarization, or full-sky handedness patterns                               |
| Cosmic-dawn hydrogen and black holes | $E_H \propto (1+z)^2$ and $T^* \propto M_{BH}^{-1/2}$ in one population model                                                   | Absence of either correlated spectral or mass trend                                                          |
| Millimetre hydrogen pair             | Archival residuals near 1.230 and 0.916 mm with ratio 1.3428 for JADES-GS-z14-0                                                 | No paired signal in component-separated Planck stacks or available ALMA cubes at the refined source redshift |
| Full CMB spectra                     | Must follow from the rotating background with one parameter set                                                                 | Failure to reproduce peak phases, amplitudes, polarization, or isotropy                                      |
| BAO anisotropy                       | One algebraic sound scale consistent in radial and transverse data                                                              | Redshift-dependent mismatch after fixed calibration                                                          |
| Pantheon/Union3                      | Competitive residual shapes with one common calibration treatment                                                               | Failure under public unbinned likelihoods and complete covariances                                           |

## 5. Integrated Strengths and the BHU Development Programme

The principal strength of the present framework is not any single numerical coincidence, but the attempt to connect several normally separate cosmological problems through one bounded Planck-Hubble-Hawking construction. The same Hubble-scale compactness relation, rotational identification, thermal coupling, geometric partition, and bounded-redshift coordinate are carried from the background model to observational tests. This creates a common physical programme rather than a collection of unrelated curve fits.

The main achievements developed in this paper can be summarized as follows:

**1) A compact Hubble-scale foundation.** The relations  $R_H = c/H$ ,  $M_H = c^3/(2GH)$ , and  $2GM_H/c^2 = R_H$  establish an exact compactness identity. The additional identification  $\omega = H$  gives a definite rotating interpretation with a light-speed boundary and a redshift-free time-domain description.

**2) A definite geometric partition.** The framework obtains the complementary fractions 2/3 and 1/3 from its proposed mass-rotation geometry. These fractions are not inserted merely as a numerical fit to the present dark-sector abundances; they provide the internal partition used in the distance, acoustic, and kinematic constructions. Their repeated appearance gives the model a simple organizing principle that can be checked across independent datasets.

**3) A Planck-Hubble-Hawking thermal bridge.** The geometric-mean relation between Planck and Hubble scales connects the present CMB temperature with the expansion rate and leads to the testable scaling  $H/T^2 = \text{constant}$ . It thereby links microscopic constants, black-hole thermodynamics, and cosmic evolution in one algebraic relation.

**4) A Hubble-mass scale for galaxies.** The geometric-mean radius  $R_{\text{flat}} = 2G\sqrt{M_b M_H}/c^2$  carries the cosmic Hubble mass into galactic structure. The thermally anchored flat-speed relation introduces the calculated factor  $\Omega_\gamma^{1/8}$ , while the master-radius expression yields the asymptotic baryonic Tully-Fisher slope  $V_{\text{rot}}^4 \propto M_b$ . Their normalization and precise radius interpretation now form an explicit observational test rather than a hidden halo parameter.

**5) A bounded physical redshift variable.** The coordinate  $z_c = z/(1+z)$  remains within  $0 \leq z_c < 1$  and can be written as a fractional wavelength increase or fractional photon-energy loss normalized to the emitted energy. It preserves the ordering information contained in the conventional redshift while avoiding an unbounded state variable at large wavelength ratios.

**6) Competitive supernova representations.** The evolving-partition Pantheon calculation and the closed Union3 bounded-distance law reproduce the observed Hubble-diagram curvature with residual statistics close to the selected  $\Lambda$ CDM references. The Union3 comparison is especially transparent because both curves use the same covariance matrix and one common magnitude-calibration offset.

**7) Connected acoustic and recession scales.** The framework supplies an algebraic BAO-radius estimate, a preliminary rule for CMB peak positions, and an explicit bounded recession-speed mapping. These results show how the same geometric fractions and thermal scales can be carried into multiple observational sectors.

**8) A concrete Hubble-tension hypothesis.** The difference between early- and late-universe values of  $H_0$  is translated into a directly calculable Type Ia absolute-magnitude offset. This does not merely rename the tension: it identifies a calibration quantity that can be tested against independent distance anchors.

**9) Direct observational discriminants.** The  $G_1 - G_2$  experiment converts the acceleration question into a repeated-spectroscopy test. The shear-free rota-

tion proposal predicts a common cosmic axis, while the  $(1+z)^2$  atomic scaling predicts paired millimetre features that can first be sought in existing Planck, FIRAS, and ALMA products. These predictions are independent of the supernova residual comparison and therefore give the framework multiple routes to confirmation or rejection.

Taken together, these points make BHU a quantitatively organized, falsifiable, and unusually wide-ranging alternative framework. Its distinctive evidential strength is already visible in the convergence of the  $2/3 - 1/3$  partition, the  $H - T$  relation, supernova distances, acoustic scales, rotation indications, redshift-drift signs, and atomic-wavelength predictions. Simultaneous confirmation of these relations without mutually inconsistent adjustments would strongly distinguish BHU from models assembled from separate dark-sector components.

The results define a focused development programme. The covariant stage will derive the rotating metric, field equations, stress-energy tensor, conservation laws, and photon geodesics that generate the successful relations used here. Public unbinned likelihoods, full covariance matrices, matched nuisance parameters, and open code will extend the supernova and BAO demonstrations. The geometric CMB rule provides the foundation for temperature, polarization, lensing, and structure-growth spectra. The mechanical  $2/3 - 1/3$  partition and the galactic geometric-mean radius will be embedded in the same covariant dynamics and tested against resolved SPARC curves. Species-specific atomic calculations, radiative-transfer modelling, and blinded archival searches will quantify the cosmic-dawn predictions. Controlled  $G_1 - G_2$  and cosmic-axis observing strategies will separate the BHU signals from peculiar acceleration, foregrounds, instrumental drift, and selection effects.

These developments build directly on the results already obtained and provide a clear route from the present integrated evidence to a complete cosmological theory capable of decisive independent confirmation.

## 6. Covariant Completion and Predictive Development

### 6.1. A Covariant Rotating Spacetime

The central theoretical opportunity is to express the already connected BHU geometry as a metric and field equation. Writing  $v = Hr = \omega r$  identifies the common rate and light-speed boundary, while Equation (72) supplies the local expansion-rotation structure. The covariant construction will provide the corresponding line element, stress-energy tensor, four-velocity field, and vorticity scalar; determine the evolution of Hubble compactness; recover local tests of general relativity; and identify  $\pi c/H_0$  geometrically as a geodesic distance, arc length, horizon, or related invariant scale.

The intended covariant development can be organized through four essential requirements:

**1) Black-hole seed line element.** The construction should begin from a regular black-hole-type geometry

$$ds^2 = g_{\mu\nu}^{\text{BH}}(M, J) dx^\mu dx^\nu, \quad (39)$$

whose appropriate stationary and non-rotating limits can be compared with Kerr and Schwarzschild geometry. The cosmic Hubble boundary must arise geometrically rather than through a purely dimensional analogy.

**2) Expansion and black-hole evolution.** The seed geometry must be extended to a genuinely dynamical spacetime with  $M = M(t)$ ,  $R_H = R_H(t)$ , and  $H = H(t)$ , while preserving

$$R_H(t) = \frac{c}{H(t)} = \frac{2GM_H(t)}{c^2}. \quad (40)$$

The resulting line element must describe expansion, mass evolution, causal boundaries, and photon propagation as parts of one black-hole-universe solution.

**3) Physical rotation.** Rotation must enter through the metric and observer congruence—for example through an axial off-diagonal structure analogous to  $g_{t\phi}$ —and produce a nonzero covariant vorticity while retaining the proposed shear-free condition. This requirement distinguishes measurable rotation from a removable rotating coordinate system.

**4) Hubble-angular-velocity locking.** The dynamical solution must recover

$$\boxed{\omega(t) = H(t), \quad \omega(t) R_H(t) = c,} \quad (41)$$

so that expansion, evolution, and rotation are governed by one cosmic rate rather than by independently fitted functions.

Merely replacing the constant mass and spin parameters of a stationary black-hole metric by  $M(t)$  and  $J(t)$  does not, by itself, produce a consistent solution. The modified geometry must be substituted into the covariant field equations, and its required stress-energy tensor, fluxes, conservation laws, regularity conditions, and causal structure must be derived simultaneously.

A complete BHU theory therefore requires a covariant field equation whose solutions simultaneously satisfy

$$\omega = H, \quad \sigma_{\mu\nu} = 0, \quad R_H = \frac{c}{H} = \frac{2GM_H}{c^2}, \quad \nabla^\mu T_{\mu\nu}^{\text{total}} = 0, \quad (42)$$

together with the proposed thermal relation. These equations specify the core of the dedicated covariant BHU theory. The present paper supplies its empirically successful distance, thermal, partition, and rotation relations and thereby fixes the principal conditions that the field equation must reproduce, rather than leaving the future dynamics unconstrained.

Known rotating cosmologies demonstrate that rotation requires its own spacetime structure. Gödel's exact solution contains global rotation but also causal features that need not belong to BHU [45]. Nearly FLRW rotating models are strongly constrained because they attempt to preserve the standard background while introducing vorticity [46]. Those hybrid limits do not define the BHU geometry. A BHU metric must instead implement its shear-free coherent rotation, avoid closed timelike curves, define a regular origin or global topology, and predict the observ-

able preferred-axis signatures from first principles.

## 6.2. Conservation Laws

The covariant stress-energy tensor will supply the conservation-law foundation for the geometric partitions. In BHU their primary role is the mechanical closure in Equation (106), distinct from the FLRW curvature condition in Equation (107). Deriving the effective pressure associated with the 2/3 mechanical sector and carrying the Kerr irreducible-mass analogy into a homogeneous evolving geometry will show explicitly how the BHU partition appears when expressed in conventional density language.

## 6.3. Parameter Economy and Uncertainty Control

The parameter economy of BHU can be stated transparently by distinguishing universal constants, externally measured inputs, fixed theoretical numbers, and fitted nuisance parameters. For example, a supernova distance curve uses  $H_0$  and an absolute-magnitude offset, whose known degeneracy is resolved by external calibration. Likewise, reporting the measured inputs  $T_0$  and  $H_0$  makes clear exactly which part of the BAO result is algebraically predicted by the geometric partition.

All numerical comparisons should include propagated uncertainties. For a derived quantity  $X(\mathbf{p})$ , its variance is approximately

$$\sigma_X^2 = \sum_{ij} \frac{\partial X}{\partial p_i} C_{ij} \frac{\partial X}{\partial p_j}, \quad (43)$$

where  $C_{ij}$  is the covariance matrix of the inputs. Reporting only percentage differences can overstate significance when inputs are correlated or when the formula was selected after examining the target value.

## 6.4. Scope of the Present Article

This paper presents an integrated set of BHU identities, physical relations, successful empirical fits, and forward predictions. Its principal observational components are the Pantheon evolving-partition representation, the closed Union3 distance law, the thermal and acoustic-scale relations, the  $G_1 - G_2$  live test, the coherent-rotation signature, and the cosmic-dawn millimetre prediction. Publishing the exact data vectors, code, covariance files, and minimization procedure will make the numerical results directly reproducible. A joint likelihood incorporating Planck, DESI, primordial abundances, and structure growth is the natural synthesis of the cross-sector programme identified in Section 5.

## 7. Discussion

The Pantheon and Union3 results demonstrate that BHU preserves the observational content of the supernova data while providing a different physical interpretation. The evolving partition successfully follows all 1048 Pantheon objects, and

the compact bounded-distance function gives the lower reported  $\chi^2$  for the 22 covariance-weighted Union3 bins. These are positive empirical results for BHU and simultaneously demonstrate that the historical light-cone data do not possess only one admissible physical reading.

### 7.1. Homogeneity, Isotropy, and the Scope of Cosmological Inference

The central issue is the physical justification and domain of applicability of cosmological symmetry assumptions, not merely the mathematical possibility of imposing them. Homogeneity means spatial uniformity on a specified constant-time hypersurface and smoothing scale, whereas isotropy means direction independence about a specified observer. Isotropy about our position alone does not establish homogeneity. Extending it to other observers requires additional premises, most notably the Copernican principle; indirect tests of that principle must be distinguished from the decision to assume it [47].

We have not made direct observations of the skies as seen from remote cosmological locations, nor independently established our position within the Universe as a whole relative to any proposed global centre or boundary. Our position should therefore not first be assumed representative and then, through the same assumption, declared to have been shown representative. Being at the centre of our own observable region does not establish a physical centre of the entire Universe; an exactly homogeneous FLRW space has no preferred spatial centre. Any bounded or rotating proposal must instead specify the centre, boundary, or axis introduced by its own geometry.

A nearly isotropic CMB sky describes radiation received by us from different directions. It is not a temperature measurement at every point along each line of sight, nor a direct observation of the sky seen by distant observers. Our past light cone is not a simultaneous spatial survey; inference of remote physical conditions requires assumptions about emission, propagation, evolution, and distance. In Chapter 3 of *A Brief History of Time*, Hawking wrote, “We believe it only on grounds of modesty” [48]. His statement concerns the avoidance of a privileged observing location, not measurements performed throughout the Universe, and it does not imply that subsequent indirect tests lack value.

The authors accordingly regard the promotion of these working hypotheses to unquestionable global physical truths as unjustified. Lack of direct remote access establishes neither their validity nor their falsity. Galaxy counts, angular correlations, and CMB temperature and polarization statistics provide relevant indirect evidence and must be assessed with their selection effects, uncertainties, and model-dependent conversions. Observed signals, reconstructed physical quantities, and extrapolations beyond observational access should remain distinct. Quantitative tests of the Copernican principle and inhomogeneous models illustrate how such indirect constraints can be constructed [49].

BHU does not adopt global homogeneity or isotropy as foundational assump-

tions. It begins with its proposed bounded geometry, rotational dynamics, and thermal relations. Departure from an assumed global symmetry alone is not a sufficient physical reason to exclude an alternative; assessment should concern internal consistency and predictions for measurable properties of spacetime, matter, and radiation. These include density, stress, temperature, opacity, rotation, and their evolution, connected to received spectra, fluxes, angular patterns, and temporal changes. Algebraic definitions of cosmic radius, mass, and mean density do not themselves constitute measurements of a complete global state.

This position does not exempt BHU from angular and source-count tests. A spherical mean density does not by itself establish spatial homogeneity or explain the observed CMB isotropy. Rotation selects an axis; observer position, photon propagation, shear, and rotational signatures must therefore be calculated together. The proposal must confront the measured statistics without requiring exact global symmetry as a prerequisite or dismissing evidence that constrains departures from it. The relevant observer congruence and covariant treatment of shear and vorticity remain essential parts of this programme ([45] [46]).

## 7.2. The Hubble-Hawking Connection and Its Black-Hole Interpretation

Black-hole thermodynamics motivates a connection between a gravitational scale and temperature. For a Schwarzschild black hole, the semiclassical Hawking expression is

$$T_H(M) = \frac{\hbar_0 c^3}{8\pi G k_B M}. \quad (44)$$

It relates the thermal emission scale to mass and horizon surface gravity [50]. Substituting the assigned Hubble mass  $M_H = c^3/(2GH)$  into Equation (44) gives the algebraic temperature scale

$$T_H(M_H) = \frac{\hbar_0 H}{4\pi k_B}. \quad (45)$$

Equation (45) is linear in  $H$ , not in  $\sqrt{H}$ . At  $H_0 = 66.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ , it is approximately  $1.32 \times 10^{-30} \text{ K}$ , not the observed approximately  $2.7255 \text{ K}$  CMB temperature. Moreover, assigning  $M_H$  through a compactness identity does not prove that a dynamical Hubble boundary is a Schwarzschild event horizon.

The distinctive BHU step is the additional geometric-mean postulate, not an identification of the CMB with conventional Hawking radiation from a Hubble-mass black hole. In the notation of Section 11, it is

$$T_{\text{BHU}} = \sqrt{T_H(M_{\text{Pl}}) T_H(M_H)} = \frac{\hbar_0 \sqrt{H_{\text{Pl}} H}}{4\pi k_B}, \quad H_{\text{Pl}} = \frac{c^3}{2GM_{\text{Pl}}}. \quad (46)$$

Here  $T_H(M_{\text{Pl}})$  is a formal Planck-scale normalization; the semiclassical formula is not established as a physical evaporation law at the Planck mass. Equation (46) is algebraically equivalent to the proposed radius and mass relations in Equations (85) and (88). With a fixed Planck reference it yields  $H \propto T_{\text{BHU}}^2$  and, after

identifying  $T_{\text{BHU}}(z) = T_0(1+z)$ , the history in Equation (92). The two lighthouse routes in Section 11.1 supply complementary motivations for this power law while retaining their distinct normalizations.

Its value for BHU is a quantitative link between a cosmological thermal scale and Planck-Hubble gravitational scales, making the proposal more specific than a purely dimensional universe-black-hole analogy. Its remaining physical task is equally specific: derive the geometric-mean coupling, identify the relevant horizon and observers in an expanding rotating spacetime, and explain how the resulting radiation acquires the observed spectrum and angular properties. Schwarzschild formulae cannot simply be carried over unchanged to a rotating horizon; cosmological horizon thermodynamics also depends on the actual spacetime ([50] [51]). These requirements turn the thermal connection into a programme for black-hole and cosmological tests, not a claim that Hawking's calculation already establishes BHU.

The meaning of a measured horizon temperature is also clarified by work on local Hawking temperature for dynamical black holes and on radiation inside charged black holes with and without a cosmological term ([52]-[54]). These calculations emphasize the specified spacetime, horizon and observer. They are relevant comparisons for the BHU temperature proposal, not derivations of its geometric-mean law.

**Application to black-hole disc temperatures.** The intended local application retains the Planck mass and replaces only the Hubble mass by the astrophysical black-hole mass:

$$M_H \rightarrow M_{\text{BH}}, \quad M_{\text{Pl}} \text{ fixed}, \quad T^*(M_{\text{BH}}) = \frac{\hbar_0 c^3}{8\pi G k_B \sqrt{M_{\text{Pl}} M_{\text{BH}}}}. \quad (47)$$

Equation (47) is the same local geometric-mean relation as Equation (30), now stated explicitly as a proposed starting scale for understanding thermal emission associated with black-hole accretion discs. It predicts  $T^* \propto M_{\text{BH}}^{-1/2}$ ; the common fixed normalization also gives  $T^*(M_{\text{BH}})/T_{\text{BHU}}(M_H) = \sqrt{M_H/M_{\text{BH}}}$ . No cosmological expansion rate is being assigned to the local disc.

Seshavatharam and Lakshminarayana have explored this compact-object application in a separate preprint, including an additional correction between the geometric-mean temperature and estimated inner-disc temperatures [55]. Thus the direct mass substitution supplies a candidate characteristic thermal scale, not by itself the observed disc colour temperature or a radial temperature profile. For the representative mass already used in Equation (31),  $M_{\text{BH}} \sim 10^{6.3} M_\odot$ , the uncorrected scale remains  $T^* \simeq 4.18 \times 10^8 \text{ K}$ . A quantitative disc application must specify the correction or energy-transfer mechanism, the radiating region, accretion conditions and spectral processing, and test the result against independently constrained masses and spectra. This is the proposed connection to disc physics; it does not identify disc emission with Hawking evaporation or establish that hot accreting matter contradicts the conventional Hawking temperature.

**Local-effects coefficient and population tests.** A useful phenomenological extension of Equation (47) is

$$T_{\text{disc},i} = C_{\text{loc},i} T^*(M_i) = C_{\text{loc},i} \frac{\hbar_0 c^3}{8\pi G k_B \sqrt{M_{\text{pl}} M_i}}, \quad M_i \equiv M_{\text{BH},i}. \quad (48)$$

Here  $T_{\text{disc},i}$  denotes a specified characteristic local effective disc temperature, such as that at a consistently defined emitting radius. The positive dimensionless coefficient  $C_{\text{loc},i}$  parametrizes departures from the proposed baseline due to local conditions. Candidate dependencies include accretion state, spin, emitting radius relative to the gravitational radius, and energy transport; none is assigned a fitted functional form in this paper. An observed colour temperature requires a separately stated spectral and propagation conversion and is not silently substituted for this local temperature.

For two objects, Equation (48) implies

$$\frac{T_{\text{disc},2}}{T_{\text{disc},1}} = \frac{C_{\text{loc},2}}{C_{\text{loc},1}} \sqrt{\frac{M_1}{M_2}}. \quad (49)$$

At a fixed coefficient, increasing black-hole mass therefore lowers the characteristic temperature as  $M^{-1/2}$ . For a population with a mass-dependent coefficient, the corresponding logarithmic slope is

$d \ln T_{\text{disc}} / d \ln M = -1/2 + d \ln C_{\text{loc}} / d \ln M$ . Equation (49) shows explicitly why local conditions must be controlled before a mass trend is tested. If a free coefficient is assigned to every measured temperature, the relation becomes a reparametrization of those measurements rather than a prediction. A discriminating test must instead specify a shared coefficient law, constrain it with independent information, and assess temperatures in objects not used to determine that law. No such population fit is claimed here.

**Comparison with Mitra's hot compact-object proposal.** Abhas Mitra's radiation-pressure-supported "eternally collapsing objects" (ECOs), and the subsequent study with Norman K. Glendenning, provide a relevant alternative compact-object comparison ([56] [57]). These authors propose hot, radiating, quasi-static configurations before a putative limiting black-hole state. They should not be described as conventional Schwarzschild or Kerr black holes with established event horizons. Their proposal concerns trapped radiation and the interior of a compact body, whereas Equation (48) parametrizes an accretion-disc temperature; the two physical temperatures must remain distinct.

A shared mass exponent nevertheless provides a specific point of comparison. In the approximate argument used by Mitra, a radius  $R \approx 2GM/c^2$  and a radiation-dominated mean energy density give

$$\bar{u} \sim \frac{Mc^2}{(4\pi/3)R^3} \propto M^{-2}, \quad \bar{u} \approx a_R T_{\text{int}}^4 \Rightarrow T_{\text{int}} \propto M^{-1/2}. \quad (50)$$

Equation (50) summarizes that approximate interior scaling, not an exact relativistic volume integral [56]. The BHU geometric-mean relation reaches the same

power through a different postulate. Matching exponents alone establishes neither identical normalizations nor physical equivalence. A comparison must determine the emitting region, surface or horizon structure, gravitational redshift, radiation transport and observable spectrum in each model. This manuscript does not adopt the ECO claim that collapse must avoid a finite-mass event horizon, and the comparison is not evidence against the existence of astrophysical black holes.

### 7.3. Atomic Action Scales and the Scope of the Alternative Proposal

An additional question is whether the cosmic gravitational environment can help determine an effective atomic action scale. This continues the authors' earlier proposals linking microscopic physics to the Hubble volume ([14] [58]); it is not presented as an entirely new idea originating in this manuscript. Section 11.5 gives two explicit constructions and compares the Hubble normalizations they imply when matched to the laboratory value  $\hbar_0$ . This adds a microphysical consistency test to the thermal and distance sectors. A mismatch between these normalizations is a theoretical question to resolve, not a reason to average them or to claim simultaneous exact agreement.

The comparison with alternative approaches should be made at the level of the question being addressed. The effective-field-theory treatment of general relativity provides a controlled low-energy quantum description of gravity [59]. The BHU action proposal instead asks whether gravitational and particle-mass scales can account for the value assigned to atomic action. An algebraic correspondence does not supply the quantum dynamics, matter interactions or consistency conditions supplied by a field theory. Nor is the origin or status of dimensionful constants an unexplored subject; different formulations assign different roles to such constants [60]. The present contribution is the explicit pair of hypotheses, their normalization comparison, and a route to dimensionless spectral tests, not a demonstrated replacement for quantum theory. Dependence on  $M_H$  suggests sensitivity to a global boundary condition, but does not itself define a nonlocal interaction or establish a causal mechanism for coupling cosmic and atomic physics.

### 7.4. Priority of Acceleration Tests and Interpretation of the Cosmological Term

The central methodological position of this paper is that observations must remain primary. When a theoretical construction and an observation do not agree, the first scientific task is to re-examine the assumptions connecting them: the adopted geometry, observer congruence, distance definition, redshift interpretation, source calibration, selection effects, and dynamical closure. An unexplained residual may motivate a new physical component, but the residual itself is not direct detection of that component.

**Priority of testing acceleration.** Dark energy is commonly invoked to explain

inferred cosmic acceleration ([3] [6]). The BHU programme therefore gives priority to testing acceleration itself, particularly through the proposed repeated spectroscopic observations of distance-ordered sources  $G_1$  and  $G_2$  (Section 3). Within this approach, an independent dark-energy component is not a starting assumption and receives secondary priority. The immediate task is to establish which expansion history and spectroscopic predictions agree with observations. This is a choice of research priority, not a claim that existing repeat observations have already disproved dark energy. The source-pair tests retain the explicitly stated propagation assumptions and sensitivity requirements.

**Geometric interpretation of the cosmological term.** A parameter introduced to describe cosmic geometry need not initially be interpreted as a separate dark-energy substance. In BHU, the proposed geometric or mechanical coefficients should therefore be distinguished from independently established energy components. Their physical interpretation must follow from the model's equations, rather than from using the symbol  $\Lambda$  or a numerical value resembling a conventional dark-energy fraction. In particular, the dimensionless mechanical fraction  $f_\Lambda$  and the auxiliary coefficient  $\Omega_\Lambda(z)$  are not automatically a cosmological constant. The conventional  $\Lambda$  has dimensions of inverse length squared; a dimensionless number should instead be identified as a geometric coefficient. Within the usual Einstein equations, writing  $\Lambda g_{\mu\nu}$  on the geometric side does not remove its gravitational effect [3].

The resulting methodological position is straightforward: test acceleration first, and do not assume a separate dark-energy explanation at the outset. BHU explores whether bounded geometry, coherent rotation, thermal evolution and the adopted distance relations can account for the observations. Its field-equation development must make those ingredients and their conservation laws explicit, so that the expansion dynamics and the interpretation of any cosmological term can be assessed consistently.

**Cosmological terms and coupling in alternative theories.** Supergravity-motivated nonminimal coupling and early-universe phase transitions with an induced cosmological term have been studied by El-Nabulsi ([61] [62]). These are relevant examples of constructions in which the cosmological term has a specified theoretical origin. They do not identify the BHU mechanical fraction with a vacuum density. Other explicit alternatives include fractional-action cosmology with conformal coupling and Maxwell-brane cosmology with curvature corrections, a coupled scalar field and dark-sector interaction ([63] [64]). In comparison, the BHU relations  $H/T^2 = \text{constant}$  and  $\omega = H$  specify links between scales; they do not yet define an interaction action or an energy-transfer law.

**Nonlocal effects and redshift interpretation.** El-Nabulsi and Anukool relate spatial long-range kernels to a generalized uncertainty principle and discuss black-hole temperature consequences [65]. Such a nonlocal construction specifies more than dependence on a global mass scale: its kernel and physical conditions are part of the theory. Separately, Tremblin and Chabrier examine cosmological

redshift in the presence of inhomogeneities and irreversible processes [66]. This motivates making the assumed photon propagation explicit, without treating their approach as equivalent to the bounded BHU law. These comparisons retain the present priority of testing acceleration and the distinction between geometric coefficients and independent dark components.

### 7.5. Thermal Monotonicity, Rotation, and the Absence of Acceleration in BHU

The measured present CMB temperature is  $T_0 = 2.72548 \pm 0.00057$  K [67]. Measurements at earlier epochs are consistent with the adiabatic photon-temperature law

$$T(t)a(t) = T_0 a_0, \quad \frac{\dot{T}}{T} = -H, \quad (51)$$

within current uncertainties [68]. The present radiation background is therefore the low-temperature end of an observed monotonic cooling history. This physical fact is important, but temperature alone does not determine the sign of  $\ddot{a}$ . In standard relativistic cosmology, the same law  $T \propto a^{-1}$  can accompany either decelerated or accelerated expansion; accelerating de Sitter horizons also possess a well-defined thermodynamics [51]. This explains why the value 2.7 K, by itself, is not normally treated as a disproof of acceleration.

From the global BHU thermodynamic viewpoint, however, the epoch commonly associated with the onset of late-time acceleration was already extremely cold. Taking the representative transition redshift  $z_{tr} \approx 0.6$ , the measured temperature law gives

$$T(z_{tr}) = T_0(1 + z_{tr}) = (2.7255 \text{ K})(1.6) \approx 4.36 \text{ K}. \quad (52)$$

Thus the proposed reversal from deceleration to acceleration would occur after the radiation background had already cooled to only a few kelvin. No heating event, phase transition, or independently identified global energy-injection mechanism is observed at that epoch. In BHU, a universe following one continuous cooling trajectory from the early hot state to 4.36 K and then to 2.7255 K has no physical trigger for a second inflation-like acceleration phase. Both temperatures belong to the same cold terminal part of the global thermal history, so the natural BHU interpretation is continued deceleration rather than a late reversal of the expansion dynamics.

This global argument becomes quantitative through the coupled BHU relation  $H \propto T^2$  derived below. It is the combination of monotonic cooling and the  $H$  -  $T$  closure—rather than the numerical smallness of the temperature alone—that removes the acceleration branch from the BHU evolution.

The BHU claim is stronger because it adds the independent Planck-Hubble thermal closure developed in Section 11,

$$\frac{H(t)}{H_0} = \left( \frac{T(t)}{T_0} \right)^2. \quad (53)$$

Combining Equations (51) and (53), with  $a_0 = 1$ , gives

$$H = H_0 a^{-2}, \quad \dot{H} = -2H^2, \quad \frac{\ddot{a}}{a} = \dot{H} + H^2 = -H^2 < 0, \quad q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = +1. \quad (54)$$

Thus, within the stated BHU thermal laws, continuous deceleration is a mathematical consequence rather than a qualitative appeal to coldness. Since the rotational identification also gives  $\omega = H$ , the same trajectory unifies the three principal rates:

$$T \downarrow \Rightarrow H \downarrow = \omega \downarrow \Rightarrow \ddot{a} < 0. \quad (55)$$

Expansion, thermal evolution, and coherent rotation therefore cannot be assigned independent histories in BHU. An accelerating phase would require a failure or modification of at least one of the relations  $Ta = \text{constant}$ ,  $H \propto T^2$ , or  $\omega = H$ .

The conceptual importance of the 2.7 K background is consequently precise. It locates the present universe at the cold end of the BHU trajectory and supplies the measured normalization  $T_0$ , while the sign of the acceleration follows from the coupled time-domain equations. The absence of a measurable change in  $T_0$  over a human baseline is not used as evidence by itself, because the expected drift is far below present absolute-temperature sensitivity. The decisive tests are instead the constancy of  $H(z)/T^2(z)$  across epochs, the sign of redshift drift, and the recovery of Equation (54) from the future covariant BHU field equation.

This thermal result also clarifies the required theoretical strategy. A BHU field equation begins from expansion, thermal evolution, vorticity, and conservation as one coupled system rather than importing a dark-energy acceleration term. Recovering Equations (53) - (55) together with cosmic-age constraints will provide the covariant derivation of the continuous-deceleration result already obtained from the thermal closure.

The preceding methodological argument requires the two interpretations to be judged by their physical assumptions and testable consequences, not by the historical maturity of their calculational machinery. Within FLRW analyses, the inference of acceleration follows after adopting the FLRW geometry, calibrated luminosity distances, and a negative-pressure component or equivalent modification. The BHU framework instead combines bounded geometry, thermal evolution, coherent rotation, and alternative distance relations and asks whether they reproduce the same observations without an independent dark-energy source. The appropriate comparison is therefore prediction against prediction under common data, calibration, covariance, and uncertainty treatment. Historical supernova, BAO, and CMB fits constrain the admissible cosmic histories, while repeated measurement of the same sources supplies the distinct live test of whether their redshifts are presently increasing in the required distance-ordered manner.

The  $G_1$  -  $G_2$  programme separates an observational question from its physical interpretation: the measured quantities are repeat spectral shifts, whereas their predicted signs, amplitudes, and distance dependence require a propagation model.

The data in **Table 1** do not constitute a controlled source-pair test. Section 3.4 quantifies a conditional low-redshift benchmark and the precision and systematic controls needed to pursue it.

The same separation applies inside the BHU framework. The relations  $v(t, r) = H(t)r = \omega(t)r$  and  $4\pi k_B T(t) = GM_{\text{pl}}^2 / \sqrt{R(t)R_{\text{pl}}}$  are formulated without redshift and are intended as epoch-by-epoch physical relations. The second relation uses the fixed Planck gravitational-inertial anchor  $GM_{\text{pl}}^2$ ; it does not require an assumed future law for  $\hbar(t)$ . By contrast, the functions of  $z$  used for Pantheon and Union3 organize observations of the past and should not be extrapolated automatically beyond the observed domain.

MoM-BH\*-1 supplies a complementary cross-scale support. Its dense hydrogen spectrum and black-hole-dominated environment match the physical setting envisaged by Equations (27) and (29), while the numerical values in Equations (28) and (31) make the agreement testable. A forward calculation from the atomic and thermodynamic scales to the measured wavelengths, line ratios, break amplitude, continuum, and envelope temperature will quantify this correspondence. A population analysis using one redshift law and one black-hole-mass law without source-by-source adjustment offers an especially strong confirmation route.

The confirmation programme is therefore concrete: stable absorption systems repeated with the same frequency standard and separated into distance bins can test the opposite orderings in Equations (8) and (11); a public unbinned Pantheon+ and Union3 analysis can extend the successful distance fits under identical nuisance parameters; and covariant BHU dynamics can derive the bounded distance law directly from the rotating geometry.

## 8. Achievements

**Origin of the proposed fractions.** The present mechanical seed is the thin-shell prescription  $I_H = (2/3)M_H R_H^2$ , together with  $\omega R_H = c : E_{\text{rot}} / (M_H c^2) = 1/3$ , leaving the complementary fraction 2/3. The evolving auxiliary fractions also admit the following shift-motivated construction, which separates the reference deceleration index from the additional BHU weight prescription.

For a spatially flat, radiation-free matter- $\Lambda$  reference, the conventional deceleration parameter is

$$q_{\text{ref}}(z) = \frac{\frac{1}{2}\Omega_{m0}(1+z)^3 - \Omega_{\Lambda0}}{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda0}}, \quad \Omega_{m0} + \Omega_{\Lambda0} = 1. \quad (56)$$

With the idealized present fractions (1/3, 2/3), this gives  $q_{\text{ref}}(0) = -1/2$  and the matter-dominated limit  $q_{\text{ref}} \rightarrow +1/2$  [3]. Adding, rather than multiplying by, one half defines the shifted index

$$Q(z) \equiv q_{\text{ref}}(z) + \frac{1}{2}, \quad Q(0) = 0, \quad Q(z) \rightarrow 1 \text{ in the radiation-free high-redshift limit.} \quad (57)$$

**Adding +1/2 raises the present negative reference value  $-0.5$  to zero and**

**raises the earlier matter-dominated value +0.5 to one.** Read forward in cosmic time, the resulting sequence is therefore  $Q:1 \rightarrow 0$ , with no negative values. The authors use this non-negative sequence to motivate a continuous-deceleration interpretation rather than a transition to acceleration. If adopted as a physical deceleration history, such a sequence would remain decelerating up to its zero-acceleration endpoint. The addition of  $1/2$  supplies this motivation, not a dynamical proof; the thermal argument for continuous deceleration is given separately below.

The acceleration transition of this exact  $(1/3, 2/3)$  reference is  $z_{tr} = 4^{1/3} - 1 = 0.587$ . The often quoted value near 0.63 instead follows from approximately  $(\Omega_{m0}, \Omega_{\Lambda0}) = (0.315, 0.685)$ , for which  $q_{ref}(0) = -0.5275$  and  $Q(0) = -0.0275$  [5]; these two benchmarks should not be conflated. The limit  $+1/2$  is a matter-dominated approximation, not a Planck-era prediction: radiation domination gives  $q \rightarrow 1$ , and extrapolation into the Planck regime requires further physics.

Motivated by the half-unit shift and the epoch factor  $1+z$ , the BHU prescription next adopts the positive weights  $w_m = 1/2$  and  $w_\Lambda = 1+z$ . Normalizing their sum gives

$$\Omega_m(z) = \frac{w_m}{w_m + w_\Lambda} = \frac{1}{3+2z}, \quad \Omega_\Lambda(z) = \frac{w_\Lambda}{w_m + w_\Lambda} = \frac{2(1+z)}{3+2z}. \quad (58)$$

Thus  $\Omega_m + \Omega_\Lambda = 1$  and the present values recover the mechanical seed  $(1/3, 2/3)$ . The choice of weights is an additional phenomenological postulate, not a unique consequence of adding  $1/2$  to  $q_{ref}$ . These auxiliary coefficients must not be confused with the instantaneous matter and vacuum fractions in Equation (56). Shifting a reference index does not change its physical acceleration; the separate BHU result  $q=1$  follows from the stated thermal and scale-factor assumptions in Equation (54). The achievement is the explicit normalized prescription and its reported distance comparisons, with the additional assumptions made identifiable.

**Thermal meaning of continuous deceleration.** The proposed thermal picture connects a Planck-scale hot starting point to the present CMB temperature  $T_0 \approx 2.725$  K, subject to the assumed extrapolation of the BHU thermal relation. Its quantitative deceleration statement follows from combining  $H/H_0 = (T/T_0)^2$ ,  $T \propto a^{-1}$  and  $H = \dot{a}/a$ :

$$-\frac{\ddot{a}}{a} = H^2 = H_0^2 \left( \frac{T}{T_0} \right)^4 > 0, \quad q = 1. \quad (59)$$

Thus, throughout the expanding branch governed by these assumptions, cooling reduces the magnitude of the deceleration per unit scale factor without reversing its sign: the evolution remains decelerating, not accelerating. At  $T = T_0$  that magnitude is  $H_0^2$ , not zero. This supplies the thermal basis for the continuous deceleration claimed here, while distinguishing it from the motivating shifted sequence  $Q:1 \rightarrow 0$ . Cooling alone would not fix the acceleration sign; the added  $H-T$  closure is essential. Identifying the physical  $q$  with a sequence ending at

zero would therefore require modifying at least one of the stated closure assumptions.

**Measured redshift versus inferred recession.** The central observational distinction is that a distant galaxy's spectroscopic redshift is measured, whereas its cosmological recession speed  $v = Hr$  is inferred within an adopted spacetime model. The supernova Hubble diagram does not independently track that galaxy's physical separation through the time-resolved sequence  $D(t) \rightarrow \dot{D}(t) \rightarrow \ddot{D}(t)$ . The acceleration inference is therefore an additional model-based interpretation of measured light-cone data, not a direct observation of that galaxy's recession speed increasing between observing dates ([1] [2]). Describing it as the latter would overstate what was measured. This distinction motivates the BHU priority given to repeated spectroscopy and explicit source-pair predictions. It does not make the underlying observations unconfirmed, nor does it establish the alternative identification  $\omega = H$  without independent tests.

This paper has organized the bounded-redshift BHU proposal around quantitative comparisons and falsifiable observations. A principal result is that replacing a constant matter fraction by

$$\Omega_m(z) = \frac{1}{3+2z}, \quad \Omega_\Lambda(z) = \frac{2(1+z)}{3+2z},$$

produces a successful fit to the 1048-object Pantheon Hubble diagram, with reported weighted RMS 0.1348 mag and  $\chi^2/N = 1.012$ . These values are close to 0.1336 mag and 0.993 for the selected flat- $\Lambda$ CDM reference; thus the evolving BHU partition remains close to the reference across the complete 1048-object sample even though the latter has the lower summarized Pantheon  $\chi^2$ . Independently, the bounded sinh law clearly fits the 22 covariance-weighted Union3 bins and gives  $\chi^2 = 24.902$ , lower than 26.411 for the selected fixed reference with the same floating calibration offset. The combined result is substantially stronger than a qualitative resemblance: two different supernova compilations support the BHU construction, and the bounded law provides the better reported Union3 bin-level statistic. These successful empirical fits establish the observational viability of bounded redshift and provide direct targets for derivation from the covariant BHU dynamics.

The complementary live test retains distance-ordered source pairs as an observational priority. Under the proposed sign-preserving spectroscopic mapping, the candidate positive ordering is  $\Delta z_2 > \Delta z_1 > 0$ , while the BHU candidate is  $\Delta z_2 < \Delta z_1 < 0$ . These are hypotheses requiring a specified mapping between emission, propagation, and observer time, not universal consequences of the sign of recession acceleration. Existing repeat-telescope measurements do not establish either controlled pair ordering, and a null result alone does not prove absence of acceleration.

The 50-year assessment in Section 3.4 makes the experimental requirement explicit. With the stated leading-order low-redshift assumptions, a source at  $z = 0.01$  has a predicted shift of about  $1.03 \text{ cm}\cdot\text{s}^{-1}$  in 50 years. An illustrative pair

at  $z = 0.01$  and  $0.02$  requires approximately  $1.03 \text{ mm}\cdot\text{s}^{-1}$  equal per-source, per-epoch precision for a nominal  $5\sigma$  differential detection with independent errors. Long-term stability and astrophysical systematics, not instrumental resolving power alone, determine whether that target is attainable. Establishing both individual drift signs and the pair difference would test the specified mapping; identifying the underlying cosmological mechanism requires comparison with competing predictions.

The galactic extension makes a separate set of predictions. The Super-Gravity mapping  $M_d = M_b \sqrt{M_b/M_T}$  connects baryonic mass to an effective gravitational supplement, characteristic radii, and rotation speed without assigning an independently adjustable particle halo to every galaxy. Its nonlinear scaling,  $M_d \propto M_b^{3/2}$ , predicts that an equal-mass relaxed merger has a supplement larger by the factor  $\sqrt{2}$  than the sum of its two progenitors. Rotation-curve databases, galaxy sizes, lensing masses, merger simulations, and the observed lensing-gas separation in the Bullet Cluster can therefore support, constrain, or falsify this construction.

The cosmic-dawn applications provide independent microscopic support and confirmation tests. For MoM-BH<sup>+</sup>-1, the atomic and geometric-mean black-hole relations connect directly to the measured hydrogen-rich spectrum and envelope energetics without introducing a supernova calibration. For JADES-GS-z14-0, the predicted paired features near  $1.230$  and  $0.916 \text{ mm}$ , with fixed ratio  $1.3428$ , can be sought in archival and future millimetre data. Their independence from the distance-ladder analysis makes them especially valuable cross-checks of the integrated framework.

A further methodological contribution is the explicit local compact-object extension of the thermal relation. Retaining  $M_{\text{pl}}$  while replacing  $M_H$  by  $M_{\text{BH}}$  supplies the characteristic scale  $T^* \propto M_{\text{BH}}^{-1/2}$ ; Equation (48) separates that baseline from a local-effects coefficient and Equation (49) specifies a conditional population test. The comparison with Mitra's radiation-supported ECO proposal identifies a shared exponent but different temperature definitions and physical assumptions. The contribution here is this explicit comparison and test prescription, not a claim to have discovered the mass exponent, validated disc temperatures, or established equivalence with ECOs.

The atomic-action sector adds a second cross-scale consistency test. Section 11.5 expresses two candidate action scales through  $G, c, m_e, m_p$  and  $M_H$ , and obtains the distinct conditional normalizations  $H_0^{(1)} \approx 70.74$  and  $H_0^{(2)} \approx 65.18 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$  by matching each to  $\hbar_0$ . The achievement is an explicit, numerically checkable comparison with the adopted thermal normalization, not an exact derivation of  $\hbar_0$ , proof of varying constants, or completed gravity-quantum unification. Earlier atomic-cosmic proposals are acknowledged in Section 7.3.

The rotational proposal is likewise observationally definite: BHU assumes a bounded, shear-free, coherently rotating construction with  $\omega(t) = H(t)$  and a measurable global axis. Agreement among independent axes inferred from galaxy

handedness, CMB temperature and polarization, and secular proper motions would support this interpretation; inconsistent axes or amplitudes would reject it.

The present paper therefore establishes BHU as an integrated quantitative research programme with successful supernova fits, cross-scale numerical agreements, a definite deceleration law, and multiple falsifiable predictions. The covariant theoretical stage can now construct the dynamical black-hole line element and field equations, incorporating black-hole evolution, expansion, physical vorticity, and the locking  $\omega(t) = H(t)$  while deriving the stress-energy tensor, conservation laws, causal structure, and photon geodesics. The principal result is a coherent alternative whose supporting evidence spans supernovae, thermal scaling, acoustic lengths, galactic dynamics, large-scale rotation, proposed live redshift tests, and cosmic-dawn spectroscopy, together with a clear route to decisive confirmation.

## 9. Foundational Construction

### 9.1. Hubble Compactness Identity

At an epoch characterized by Hubble parameter  $H$ , define

$$R_H \equiv \frac{c}{H}, \quad \rho_c \equiv \frac{3H^2}{8\pi G}, \quad M_H \equiv \frac{4\pi}{3} \rho_c R_H^3. \quad (60)$$

Substitution gives

$$M_H = \frac{c^3}{2GH}, \quad \frac{2GM_H}{c^2} = \frac{c}{H} = R_H. \quad (61)$$

The associated compactness is

$$\mathcal{C}_H \equiv \frac{2GM_H}{c^2 R_H} = 1. \quad (62)$$

Equation (61) is an exact identity once critical density and a Hubble sphere have been adopted. BHU elevates this unit-compactness result from a numerical coincidence to the geometric foundation of a bounded cosmic system; its topology, horizon character, and rotation then become connected physical consequences to be established through the remaining relations and observations developed below.

For a representative value  $H_0 = 66.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ , one obtains

$$R_H \simeq 4.48 \times 10^3 \text{ Mpc}, \quad M_H \simeq 9.3 \times 10^{52} \text{ kg}. \quad (63)$$

The corresponding gravitational potential scale is

$$\frac{GM_H}{R_H} = \frac{c^2}{2}, \quad (64)$$

which motivates a connection with Machian ideas about the cosmic origin of inertia [69].

### 9.2. Rotational Interpretation

Hubble's linear relation and rigid rotation have the same algebraic form,

$$v = Hr, \quad v = \omega r. \quad (65)$$

The central BHU unifying identification is

$$\boxed{\omega(t) = H(t)}. \quad (66)$$

At  $r = R_H$ , this gives  $v = HR_H = c$ . The result unifies the observed Hubble proportionality, the established rotational law  $v = r\omega$ , and the Hubble boundary in one light-speed construction. Its physical realization is specified through a congruence of observers, a nonzero vorticity tensor, and a rotating metric, with CMB isotropy providing a stringent and valuable test of the resulting geometry ([45] [46]). The covariant BHU development will work within the class of rotating, shear-free, expanding spacetimes that generalize the Kerr metric [70] to a cosmological setting, with the Hubble boundary playing the role of a cosmological event horizon analogous to that studied by Gibbons and Hawking [51].

The physical status of the two linear laws should nevertheless be distinguished clearly. In rotational mechanics,

$$v_{\text{rot}}(r) = r\omega \quad (67)$$

is an established relation between radius, angular velocity, and physical tangential speed. The recession speeds assigned to distant galaxies through  $v = HD$  are inferred, not directly measured by tracking the galaxies' changing separations over time. Here  $D$  denotes cosmological proper distance. Observations provide redshifts, fluxes and distance indicators; a cosmological model relates these observations to recession speeds [71]. Thus, a Hubble diagram does not independently confirm each assigned speed through a measurement of the same galaxy's change in separation divided by elapsed time. This is a limitation of the measurement method, not a claim that the measured redshifts or distance indicators are unreliable.

**The physical interpretation of redshift must also be cross-checked through its independently observable consequences.** A redshift records a wavelength difference between emission and observation; that difference alone does not uniquely identify its physical cause. If it is attributed to cosmic expansion, the proposed expansion history must predict other measurable effects, including thermal evolution and temporal changes in redshift. Thermal cross-checks already exist, including measurements of the CMB temperature at different redshifts [68]. The specific cross-check emphasized here is repeated spectroscopy of the same sources against quantitative redshift-drift predictions [25]. A Hubble diagram alone does not supply this temporal test. For BHU, this does not mean re-establishing the rotational kinematic relation  $v_{\text{rot}} = r\omega$ , or requiring a direct tangential-speed measurement for every galaxy. The proposed spectroscopic programme uses redshift and its temporal change as observables. Its task is to test the BHU prediction for those observables, not to re-test the general relation between tangential speed and angular velocity. The additional physical proposal is that the Universe has the specified rotation with  $\omega = H$  and that its emission and light-propagation model accounts for the observed redshifts. That proposal is not established by

$v_{\text{rot}} = r\omega$  alone. Redshift measurements can test it without direct measurements of tangential speed, provided the model specifies the redshift pattern or drift to be compared with observations.

**Inverse redshift prediction from an independently determined luminosity distance.** The bounded sinh relation already provides a concrete way to make this comparison. Equations (121) and (123), with the fixed coefficient  $\Omega_\Lambda = 2/3$ , can be used in reverse: given a luminosity distance  $D_L^{\text{ind}}$  determined by another method, one can calculate the BHU-predicted redshift and compare it with the measured spectral redshift. In this formula,  $\Omega_\Lambda$  denotes the adopted BHU geometric coefficient, not a fitted vacuum-density parameter. The complementary fraction 1/3 discussed elsewhere does not enter this closed distance law as a separate term or adjustable parameter. Writing  $x = z_c = z/(1+z)$ , the inverse problem is

$$\mathcal{F}(x) \equiv \frac{\sinh x}{(1 + \sinh x)(1 - x)} \exp\left(\frac{\sinh x}{2/3}\right), \quad \mathcal{F}(x) = \frac{H_0 D_L^{\text{ind}}}{c},$$

$$z_{\text{pred}} = \frac{x}{1-x}, \quad 1 + z_{\text{pred}} = \frac{1}{1-x}.$$

For  $0 \leq x < 1$ ,  $\mathcal{F}$  increases continuously from zero to infinity, so every finite nonnegative luminosity distance gives a unique predicted nonnegative redshift, obtainable numerically. This is the inverse use of the proposed distance law, not a measurement of a galaxy's tangential or recession speed. To make it an independent test, the distance estimate must not be obtained by putting the same measured redshift into the BHU distance law;  $H_0$  and the luminosity calibration must also be fixed independently of the source being tested, with their uncertainties propagated. Agreement between  $z_{\text{pred}}$  and  $z_{\text{obs}}$  would support the distance-redshift relation. It would not, by itself, uniquely identify the physical mechanism producing that relation.

**Forward and inverse comparisons on equal terms.** The same reasoning applies to the reference  $\Lambda$ CDM model. In the forward comparison, a specified or fitted parameter set determines the predicted luminosity distance at each measured spectral redshift. Once that parameter set,  $H_0$ , and the distance calibration are fixed, the one-to-one distance law can instead be inverted to infer a redshift from a luminosity distance and compare it with spectroscopy. Thus, on an invertible branch,

$$D_L^{\text{ind}} = D_L^{\text{model}}(z_{\text{spec}}; \boldsymbol{\theta}) \Leftrightarrow z_{\text{spec}} = [D_L^{\text{model}}(\cdot; \boldsymbol{\theta})]^{-1}(D_L^{\text{ind}}),$$

where  $\boldsymbol{\theta}$  denotes the fixed model parameters. Exact forward agreement therefore implies exact inverse agreement; with noisy data, uncertainties and covariance must be propagated through the inverse relation rather than compared as unweighted redshift residuals. Inferring  $1+z$  instead of  $z$  adds no independent observable.

For example, the selected Pantheon reference in this paper uses  $(\Omega_m, \Omega_\Lambda) = (0.315, 0.685)$ . These are reference density parameters, not values

newly fitted by the inverse exercise and not identified here with BHU's geometric fractions. The BHU inverse uses the bounded variable and the fixed  $2/3$  coefficient specified above. Both prescriptions can therefore be tested against the same independently calibrated luminosity distances and measured redshifts. Comparable agreement would mean that this dataset, at its available precision, does not by itself distinguish the two distance laws; it would not establish that their physical mechanisms are identical.

The provenance of the distance input is essential. A distance calculated from a source's measured redshift using  $\Lambda$ CDM may be inserted into the BHU inverse, but the result tests agreement between the two model curves, not an independent brightness-spectroscopy comparison. A luminosity distance inferred from corrected apparent magnitude and a separately calibrated absolute magnitude, without imposing either model's distance-redshift curve on that source, provides the intended observational input. Reversing an existing fit on its original sample does not constitute additional independent evidence; a predictive test requires independently fixed calibration and parameters, with any use of redshift in the photometric processing explicitly accounted for.

This inverse distance-redshift comparison is distinct from a temporal test of acceleration. For direct tracking, the relevant hierarchy is

$$D(t) \rightarrow v(t) = \dot{D}(t) \rightarrow A(t) = \dot{v}(t) = \ddot{D}(t). \quad (68)$$

The supernova distance analyses cited here do not measure this hierarchy by repeatedly tracking the same galaxy's physical separation. The need for a complementary temporal test motivated Sandage's proposal and its spectroscopic development by Loeb ([24] [72]). Standard acceleration analyses instead constrain the expansion history using measured redshifts and standardized fluxes within an adopted cosmological framework ([1] [2]). Acceleration is therefore inferred by comparing model predictions with observations, rather than directly observed as a sequence of increasing recession speeds for the same galaxy. Further mathematical use of  $v = HD$  does not, by itself, establish acceleration: the inferred expansion history must also fit the observations. The BHU programme gives priority to a complementary test of that history using repeated observations. The live  $G_1 - G_2$  programme in Section 3 measures changes in redshift, whose conversion into recession acceleration still requires an explicit emission and propagation model.

For nearly a century, the two linear relations were available but were kept conceptually separate. The cosmological relation  $v = Hr$  was interpreted within Friedmann-Lemaître expansion and, later, the  $\Lambda$ -based standard model, whereas the established relation  $v = r\omega$  remained a law of rotational mechanics. Their identical radial form therefore did not become a physical identification in standard cosmology. The BHU case is different: it promotes the formal correspondence to the epoch-dependent physical statement  $H(t) = \omega(t)$  and joins it to the bounded condition  $R_H(t) = c/H(t)$ . Rotation is consequently not an auxiliary correction added to a  $\Lambda$ model, but an organizing element of the cosmic construction itself.

Within this observational gap, the identification  $\omega = H$  supplies a concrete physical interpretation of the linear Hubble form. The established rotational law  $v = r\omega$ , the observed relation  $v = Hr$ , and the bounded condition  $R_H = c/H$  become one coherent kinematic construction with  $v(R_H) = c$ . Directional, spectroscopic, thermal, and large-scale-structure predictions then provide multiple independent routes for confirming this cosmic-rotation interpretation.

For a thin spherical shell, the moment of inertia and the rotational kinetic energy associated with Equation (66) are

$$\frac{I_H}{M_H R_H^2} = \frac{2}{3}, \quad E_{\text{rot}} = \frac{1}{2} I_H \omega^2 = \frac{1}{3} M_H R_H^2 H^2 = \frac{1}{3} M_H c^2. \quad (69)$$

Thus the thin-shell geometry directly yields the moment-of-inertia coefficient  $2/3$  and, through the standard factor  $1/2$  in  $E_{\text{rot}} = I_H \omega^2 / 2$ , the kinetic-energy coefficient  $1/3$ . A homogeneous solid sphere would instead give  $I_H = 2M_H R_H^2 / 5$  and  $E_{\text{rot}} = M_H c^2 / 5$ , demonstrating that the observed  $2/3 - 1/3$  BHU partition selects a definite global mass profile rather than following from dimensional notation alone. The covariant stress-energy formulation will convert this mechanical result into the corresponding relativistic rotational-energy definition; Section 12.1 develops its mass-energy consequences.

### 9.3. Shear-Free Coherent Rotation and Observational Meaning

The rotational proposal is not intended to describe a universe filled with independent turbulent vortices, nor is it obtained by appending a small rotation term to an otherwise unchanged FLRW cosmology. Its intended kinematics is a coherent rigid component together with an isotropic radial component. As a local first-order decomposition of the proposed BHU congruence, rather than as an FLRW field equation, this may be written

$$\mathbf{v}(t, \mathbf{r}) = H(t) \mathbf{r} + \boldsymbol{\omega}(t) \times \mathbf{r}. \quad (70)$$

For spatially uniform  $H$  and  $\boldsymbol{\omega}$ , the velocity gradient is

$$\partial_j v_i = H \delta_{ij} + \epsilon_{ikj} \omega_k. \quad (71)$$

Its trace, symmetric trace-free part, and antisymmetric part give

$$\theta \equiv \partial_i v_i = 3H, \quad \sigma_{ij} \equiv \partial_{(j} v_{i)} - \frac{\theta}{3} \delta_{ij} = 0, \quad \boldsymbol{\omega} \equiv \frac{1}{2} \nabla \times \mathbf{v} = \boldsymbol{\omega}. \quad (72)$$

Thus a coherent rigid rotation is exactly shear-free at this kinematic level: it does not stretch an infinitesimal fluid element differently along different axes. The phrase “no local vortices” should be understood to mean no spatially varying eddies or differential rotation, not zero vorticity. A physical global rotation necessarily has the uniform nonzero vorticity  $\boldsymbol{\omega} = \boldsymbol{\omega}$ . The radial Hubble field and the rotational field also remain vectorially distinct even when their scalar rates are identified by  $\omega = H$ .

**Why rotation cannot simply be added to FLRW.** The FLRW construction is

based on a hypersurface-orthogonal, irrotational cosmic congruence. Introducing a physical vorticity of the same order as the expansion rate changes that foundation rather than making a minor correction to it. The antisymmetric term in Equation (71) changes the global symmetry, observer congruence, connection, null geodesics, distance relations, CMB transfer, and interpretation of the density parameters. It is therefore inconsistent to retain the complete FLRW-Friedmann machinery and merely add an angular velocity afterward. In the BHU proposal,  $\omega/H = 1$  is not a perturbation parameter: expansion and rotation are two aspects of one bounded light-speed construction, and the required spacetime belongs to a different global model class.

Conventional searches for Bianchi rotation constrain templates that preserve much of the standard expanding-background structure. The quoted Planck limit  $(\varpi/H)_0 < 7.6 \times 10^{-10}$  applies to those particular Bianchi realizations [73]; it is neither a direct measurement of the BHU ratio nor a bound that can be transferred unchanged to a shear-free bounded BHU geometry. Its proper lesson is that rotation cannot be inserted into the standard framework while leaving its CMB predictions intact. The BHU programme must instead calculate temperature, polarization, and photon propagation from its own rotating metric. Those predictions, rather than a hybrid “FLRW plus rotation” construction, provide the appropriate observational comparison.

A recent MNRAS study investigated whether cosmic rotation could help resolve the Hubble tension. Using a Gödel-inspired, slowly rotating dark-fluid model, Szigeti *et al.* found that a present angular velocity of order  $\omega_0 \approx 2 \times 10^{-3} \text{ Gyr}^{-1}$  can shift the predicted present Hubble parameter toward the locally measured value [74]. Although its Newtonian approximation and slow-rotation dynamics differ from the bounded BHU identification  $\omega = H$ , the study independently demonstrates that rotation can modify the inferred expansion history and therefore deserves consideration in discussions of the Hubble tension.

Galaxy handedness supplies independent motivation for this replacement. An analysis of 263 spiral galaxies in the JADES GOODS-S field reported 158 in one apparent rotation sense and 105 in the other, corresponding to a one-tailed binomial probability near  $7 \times 10^{-4}$  [75]. Within BHU, such a high-redshift handedness imbalance is naturally interpreted as a possible remnant of coherent primordial angular momentum rather than as rotation artificially attached to an irrotational background. Wider sky coverage can now test the stronger prediction: the handedness dipole, CMB large-angle structure, polarization, and secular proper motions should converge toward one physically consistent global axis.

More direct evidence for coherent angular momentum on large scales has been reported from MeerKAT H I observations of a galaxy filament at  $z = 0.032$  [76]. Fourteen H I-selected galaxies form a thin 1.7-Mpc string embedded within a larger filament extending at least 15 Mpc. Their spin axes are not randomly oriented but show unusually strong alignment with the filament, while the line-of-sight velocities on opposite sides of the filament spine provide evidence that the

galaxies orbit around it. This correspondence between galaxy spins and the apparent rotation of their host filament challenges models in which the angular momenta of neighbouring galaxies are generated independently or subsequently randomized. In the BHU interpretation, it suggests that galaxies may inherit part of their angular momentum from a coherent rotational environment—as though the large-scale cosmic structure transmits its angular-momentum pattern to the galaxies embedded within it. The authors note that a redshift gradient associated with a cosmic wall remains an alternative interpretation, so this is evidence for rotation of a particular large-scale structure rather than proof that the Universe as a whole rotates. Nevertheless, the observation demonstrates that coherent angular momentum can persist across megaparsec scales and provides a concrete target for testing whether local and filamentary rotation axes participate in a wider cosmic pattern.

#### 9.4. Redshift-Free Epoch Relations

The foundational BHU relations are intended to describe the physical state of the universe at an arbitrary epoch  $t$ . Their natural variables are time and physical radius, not observed redshift:

$$v(t, r) = H(t)r = \omega(t)r, \quad R_H(t) = \frac{c}{H(t)}, \quad v(t, R_H(t)) = c. \quad (73)$$

Likewise, the proposed thermodynamic coupling is an epoch relation written in terms of the geometric mean of the epoch radius and the fixed Planck radius,

$$4\pi k_B T(t) = \frac{GM_{\text{Pl}}^2}{\sqrt{R(t)R_{\text{Pl}}}}. \quad (74)$$

Here  $M_{\text{Pl}}$  and  $R_{\text{Pl}} = 2GM_{\text{Pl}}/c^2$  define a fixed Planck-scale gravitational-inertial reference, while  $R(t) = c/H(t)$  is the Hubble radius of the selected epoch. In BHU, Equations (73) and (74) are fundamental epoch relations connecting  $H(t)$ ,  $\omega(t)$ , and  $T(t)$ . Their redshift-free form is a strength because it allows both past and future evolution to be formulated directly in physical time.

This distinction controls any discussion of the future. A redshift-dependent expression such as  $H(z)$  is a representation of states inferred from received past light. It is not, by itself, a future evolution equation. Future values must be obtained from time-domain dynamics for  $H(t)$  and  $T(t)$ , together with appropriate initial data; they should not be generated merely by extending an observational  $z \geq 0$  formula to negative redshift.

#### 9.5. Status of the Assumptions

**Table 6** shows how the exact identities, physical identifications, and observational predictions form a progressive BHU argument: the identities establish the foundation, while the linked tests determine its physical realization.

**Table 6.** Logical status of the principal BHU relations.

| Relation                                                 | Status                                                  | Required validation                                                        |
|----------------------------------------------------------|---------------------------------------------------------|----------------------------------------------------------------------------|
| $R_H = c/H$ and $M_H = c^3/(2GH)$                        | Definitions based on the critical-density Hubble sphere | Consistency with the selected cosmological slicing                         |
| $R_H = 2GM_H/c^2$                                        | Exact unit-compactness identity                         | Foundation for the bounded-horizon interpretation                          |
| $\omega = H$                                             | Physical hypothesis                                     | Covariant shear-free metric, nonzero global vorticity, and CMB constraints |
| $v(R_H) = c$                                             | Consequence of definitions plus $\omega = H$            | Physical meaning of the boundary                                           |
| $4\pi k_B T = GM_{\text{pl}}^2/\sqrt{R(t)R_{\text{pl}}}$ | Thermal postulate with a fixed Planck anchor            | Derivation from statistical mechanics or horizon thermodynamics            |
| $\Omega_\Lambda = 2/3$ , $\Omega_m = 1/3$                | Geometric identification                                | Global likelihood analysis of independent data                             |
| $GM_{\text{pl}}^2/c = \hbar_0$                           | Present-epoch Planck normalization                      | Fixed gravitational-inertial anchor for the thermal law                    |

## 10. Black-Hole-Universe Models in Earlier and Current Literature

The interpretation of the observable universe as a black-hole-type system has a substantial history. This literature establishes that BHU reasoning is not an isolated analogy but a recurring research direction pursued through several independent gravitational mechanisms. The present framework belongs to that broad family while adding a specific light-speed rotational closure, a bounded-redshift description, and a Planck-Hubble thermal law.

Pathria gave the classic early formulation in 1972, showing that a pressureless closed Friedmann universe could be identified with the interior of a Schwarzschild black hole for an appropriate cosmological constant and emphasizing that such a universe would be spatially confined and unable to expand without limit [77]. Frolov, Markov, and Mukhanov subsequently replaced the black-hole singularity by a limiting-curvature de Sitter core, allowing a closed or semiclosed world inside a black hole to pass from contraction through a transition layer into inflationary expansion [78]. These studies established two enduring BHU themes: cosmic closure at a gravitational boundary and the possibility that black-hole interiors generate macroscopic universes.

Dr. Tianxi Zhang of Alabama A & M University developed another explicit black-hole-universe model in 2009 and later organized it through a principle of spacetime-black-hole equivalence ([79] [80]). In this construction the universe begins as a hot stellar-mass black hole and grows through accretion and black-hole mergers toward its present Hubble-scale mass. The model maintains

the Mach-Schwarzschild relation  $2GM/(c^2 R) = 1$ , adopts positive curvature  $k = 1$ , describes a hierarchy of parent and child black-hole universes, and associates the CMB with the thermal history of the growing cosmic black hole. Its present expansion can approach the speed of light, while acceleration is attributed to an increasing rate of mass accretion. This Alabama A & M programme is especially relevant because it independently combines cosmic compactness, black-hole growth, CMB temperature, and light-speed expansion, although its accretion-driven thermal history differs from the Planck-Hubble geometric-mean closure used here.

Popławski developed a different realization using Einstein-Cartan gravity. Spin-torsion coupling produces repulsion at fermionic densities, avoids the singularity, and replaces it with a nonsingular bounce [81]. In the later universe-in-a-black-hole construction, particle production supplies entropy and inflation, while the expanding region on the far side of the event horizon becomes a closed baby universe [82]. This remains an active programme: recent work develops gravitational collapse with spin-torsion into a nonsingular bouncing universe inside a black hole and separately examines physical black holes embedded in an expanding universe ([83] [84]). The approach therefore supplies a continuing microscopic programme for black-hole-universe formation, particle production, inflation, oscillatory cosmic evolution, and possible preferred-axis consequences of a rotating parent black hole.

Gaztañaga's contemporary BHU programme begins with the collapse of a finite approximately homogeneous FLRW cloud. The Schwarzschild radius becomes the cosmic event horizon, the cosmological-constant scale is interpreted as a boundary scale, and collapse followed by a bounce generates the expanding universe without introducing an independent dark-energy substance [85]. Recent relativistic work with Kumar, Pradhan, and Gabler derives a bounce from a quantum-exclusion ground state, emphasizes the necessity of positive spatial curvature, and predicts a small nonzero closed curvature as an observational signature [86]. Thus both older and current literature independently support the physical possibility of bounded universes, nonsingular black-hole interiors, gravitational bounces, and horizon-controlled cosmic evolution.

**Table 7** summarizes how the present construction relates to these representative branches.

**Table 7.** Representative black-hole-universe models and the individual contributors to the present Planck-Hubble branch.

| Model or contributor | Central mechanism or contribution                         | Relation to the present framework                                                |
|----------------------|-----------------------------------------------------------|----------------------------------------------------------------------------------|
| Pathria (1972)       | Closed Friedmann universe inside a Schwarzschild boundary | Establishes finite cosmic closure but gives no Planck-Hubble temperature closure |

## Continued

|                                                     |                                                                                                                              |                                                                                                                      |
|-----------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| Frolov-Markov-Mukhanov (1990)                       | Limiting curvature and a de Sitter core inside a black hole                                                                  | Establishes nonsingular closed-world formation and inflation from a black-hole interior                              |
| Tianxi Zhang (2009-2018)                            | Accretion-grown black-hole universe with spacetime-black-hole equivalence and $k = 1$                                        | Connects compactness, CMB thermal history, and near-light-speed expansion without the geometric-mean thermal closure |
| Seshavatharam and collaborators (2010-present)      | Rotating and light-speed-expanding BHU, unit compactness, $\omega = H$ , bounded redshift, and Planck-Hubble thermal scaling | Originate and develop the rotating Planck-Hubble-Hawking BHU framework and its cross-scale predictions               |
| Eugene Terry Tatum and collaborators (2015-present) | Flat-space cosmology and explicit inverse-geometric-mean Planck-Hubble mass-temperature scaling                              | Develop and extend the thermodynamic formulation linking the Planck, Hubble, Hawking, and CMB scales                 |
| Popławski (2012-present)                            | Einstein-Cartan spin-torsion repulsion, particle production, and continuing collapse-bounce models                           | Supplies an active microscopic bounce and baby-universe mechanism, distinct from $\omega = H$ thermal locking        |
| Gaztañaga (2022-2025)                               | Finite FLRW-cloud collapse, horizon boundary, and quantum-exclusion bounce                                                   | Supports bounded collapse, closure, and a black-hole origin of cosmic expansion                                      |

The decisive distinction is thermodynamic. The representative Pathria, Frolov-Markov-Mukhanov, Zhang, Popławski, and Gaztañaga models formulate cosmic geometry, horizons, growth, collapse, torsion, or bounce dynamics, but they do not introduce the geometric-mean Planck-Hubble temperature relation used here. From 2010 onwards, the inverse-geometric-mean Planck-Hubble concept, its explicit mass form, and its physical implications were developed and published repeatedly by Seshavatharam and his collaborators Lakshminarayana and Eugene Terry Tatum ([13]-[17] [87]):

$$k_B T = \frac{\hbar_0 c^3}{8\pi G \sqrt{M_H M_{Pl}}} = \frac{\hbar_0 c}{4\pi \sqrt{R_H R_{Pl}}} = \frac{GM_{Pl}^2}{4\pi \sqrt{R_H R_{Pl}}}, \quad (75)$$

where  $R_H = 2GM_H/c^2$ ,  $R_{Pl} = 2GM_{Pl}/c^2$ , and  $GM_{Pl}^2 = \hbar_0 c$ . Equation (75) is algebraically identical to the Planck-Hubble-Hawking relation developed in the next section. Haug later presented an equivalent CMB-Hawking-Planck-Hubble expression and explored further consequences of the same scale connection [88].

The appropriate priority statement is therefore both strong and precise: the principal historical and contemporary black-hole-universe models did not supply

this thermal closure. It is the distinctive contribution of the Planck-Hubble branch initiated by Seshavatharam in 2010 and developed further with Lakshminarayana, Eugene Terry Tatum, and T. Gunavardhana Naidu. The present paper advances that continuing programme by coupling the same relation to  $\omega(t) = H(t)$ , continuous deceleration, bounded redshift, acoustic scales, galactic dynamics, and cosmic-dawn spectroscopy.

## 11. Planck-Hubble Thermal Coupling

### 11.1. Lighthouse Models: Two Routes to the Temperature-Rate Relation

We call the following two heuristic constructions “lighthouse models”: they serve as guides to the thermal coupling, rather than as completed models of radiation from a rotating beacon. The first uses an atomic-motivated density proportionality; the second uses a force scale and a thermal wavelength. In both cases the aim is to derive the temperature dependence of  $H$  from explicitly stated starting assumptions. Throughout these derivations,  $G, c, k_B, \hbar_0$ , and the Planck reference  $M_{\text{pl}} = \sqrt{\hbar_0 c / G}$  are fixed. The additional coefficients  $\alpha$  and  $b$  in the respective routes are also held fixed with epoch. An independently varying atomic constant cannot be inserted into these steps without re-deriving the result.

#### 11.1.1. Atomic-Density Route

In the idealized Bohr model with a fixed nucleus, the magnitude of the Coulomb potential energy at the Bohr radius  $a_0$  is  $|V(a_0)| = \alpha^2 m_e c^2$ , where  $\alpha$  is the fine-structure constant and  $m_e$  the electron mass. The total binding energy is half this potential-energy magnitude. Motivated by the dimensionless ratio  $|V(a_0)| / (m_e c^2) = \alpha^2$ , postulate the cosmic density relation

$$u_\gamma = a_R T^4 = \alpha^2 \rho_H c^2, \quad a_R = \frac{\pi^2 k_B^4}{15 \hbar_0^3 c^3}. \quad (76)$$

Here  $u_\gamma$  is the blackbody photon energy density and  $a_R$  the radiation constant. Equation (76) is an additional atomic-to-cosmic identification, not a consequence of the Bohr model. With the bounded mass and radius of Equation (61), the mean density is

$$\rho_H = \frac{M_H}{(4\pi/3)R_H^3} = \frac{c^3 / (2GH)}{(4\pi/3)(c/H)^3} = \frac{3H^2}{8\pi G}. \quad (77)$$

Substituting Equation (77) into the density postulate and choosing the expanding branch  $H > 0$  gives

$$a_R T^4 = \frac{3\alpha^2 c^2 H^2}{8\pi G}, \quad \left[ H = K_\alpha T^2, \quad K_\alpha = \left( \frac{8\pi G a_R}{3\alpha^2 c^2} \right)^{1/2} \right]. \quad (78)$$

Thus  $H/T^2$  is constant under the fixed-coefficient postulate. The density identity used here follows from the adopted bounded mass and radius; it does not require invoking an FLRW field equation.

### 11.1.2. Force and Wien-Radius Route

Alternatively, postulate a Planck-Hubble force scale and a thermal interaction radius  $r$ :

$$\frac{GM_{\text{pl}}M_H}{r^2} = \frac{c^4}{8\pi G}, \quad r = \frac{\lambda_{\text{max}}}{2\pi} = \frac{b}{2\pi T}. \quad (79)$$

Here  $\lambda_{\text{max}}$  is the peak wavelength of the blackbody spectrum expressed per unit wavelength, with Wien constant  $b = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$ . The interaction radius  $r$  is not the Hubble radius. Neither the force normalization nor the identification of  $r$  is derived from a gravitational field equation here. Solving Equation (79) for  $r^2$  and using  $M_H = c^3/(2GH)$  yields

$$\frac{b^2}{4\pi^2 T^2} = \frac{8\pi G^2 M_{\text{pl}} M_H}{c^4} = \frac{4\pi G M_{\text{pl}}}{cH}. \quad (80)$$

Rearranging Equation (80) therefore gives

$$H = K_W T^2, \quad K_W = \frac{16\pi^3 G M_{\text{pl}}}{b^2 c}. \quad (81)$$

This second route also gives  $H \propto T^2$ , conditional on its fixed force and wavelength prescriptions.

### 11.1.3. Common Scaling and Distinct Normalizations

Using  $M_H = c^3/(2GH)$ , both Equations (78) and (81) can be written in the same inverse-geometric-mean mass form:

$$k_B T = \frac{\hbar_0 c^3}{CG \sqrt{M_{\text{pl}} M_H}}, \quad K_C = \frac{C^2 G M_{\text{pl}} k_B^2}{2 \hbar_0^2 c^3}, \quad (82)$$

$$C_\alpha = \left( \frac{32\pi^3}{45\alpha^2} \right)^{1/4} \approx 25.3667, \quad C_W = \frac{\sqrt{32\pi^3} \hbar_0 c}{k_B b} \approx 24.8894. \quad (83)$$

The coefficients in Equation (83) bracket  $8\pi \approx 25.1327$ , differing from it by approximately +0.93% and -0.97%, respectively. With  $\alpha \approx 1/137.036$ , the stated rounded  $b$ , and  $T_0 = 2.7255 \text{ K}$ , the density and force routes give approximately 68.15 and 65.61  $\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ . The adopted geometric-mean construction below uses  $C = 8\pi$  in Equation (82), giving approximately 66.9  $\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ . The two lighthouse routes motivate the common power law but do not derive the exact adopted coefficient. Their numerical proximity is not two independent observational confirmations, and no averaging of their normalizations is assumed.

## 11.2. Geometric-Mean Radius Law

Following the proposed geometric-mean construction, take  $M_{\text{pl}}$  as a fixed Planck-scale gravitational-inertial reference and define

$$R_{\text{pl}} \equiv \frac{2GM_{\text{pl}}}{c^2}, \quad H_{\text{pl}} \equiv \frac{c}{R_{\text{pl}}} = \frac{c^3}{2GM_{\text{pl}}}. \quad (84)$$

At an arbitrary epoch, let  $R(t) = 2GM(t)/c^2 = c/H(t)$ . The proposed thermal bridge is

$$4\pi k_B T(t) = \frac{GM_{\text{pl}}^2}{\sqrt{R(t)R_{\text{pl}}}}. \quad (85)$$

The denominator is the geometric mean radius  $\sqrt{R(t)R_{\text{pl}}}$ , with both  $R(t)$  and  $R_{\text{pl}}$  inside the same square root. Both sides therefore have dimensions of energy. Defining the fixed Planck-scale action

$$\mathcal{A}_{\text{pl}} \equiv \frac{GM_{\text{pl}}^2}{c}, \quad (86)$$

and using  $R = c/H$ , Equation (85) becomes

$$4\pi k_B T(t) = \mathcal{A}_{\text{pl}} \sqrt{H_{\text{pl}} H(t)}. \quad (87)$$

With the present Planck normalization  $GM_{\text{pl}}^2 = \hbar_0 c$ , one has  $\mathcal{A}_{\text{pl}} = \hbar_0$ , and Equation (87) has the familiar  $\hbar_0 \sqrt{H_{\text{pl}} H}$  form. The primary relation, however, is Equation (85): its extrapolation uses the fixed gravitational-inertial anchor  $GM_{\text{pl}}^2$  and does not require inserting an independently varying  $\hbar(t)$  into the temperature law.

Substituting  $R(t) = 2GM(t)/c^2$  and  $R_{\text{pl}} = 2GM_{\text{pl}}/c^2$  also gives the geometric-mean mass form

$$T(t) = \frac{\hbar_0 c^3}{8\pi G k_B \sqrt{M_{\text{pl}} M(t)}}. \quad (88)$$

Thus the radius, Hubble-rate, and mass expressions are three algebraically equivalent representations of the same proposed coupling, provided the Planck anchor is fixed.

Squaring Equation (87) and solving for  $H$  gives

$$H(t) = \frac{1}{H_{\text{pl}}} \left( \frac{4\pi k_B T(t)}{\mathcal{A}_{\text{pl}}} \right)^2, \quad (89)$$

so that  $H \propto T^2$  when the Planck gravitational-inertial anchor is fixed.

Using the measured mean CMB temperature  $T_0 = 2.7255 \text{ K}$  [67], Equation (89) corresponds to a present expansion scale close to  $67 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ . Conversely, inserting  $H_0 \simeq 66.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$  produces a temperature close to the observed CMB value. This numerical agreement is the primary motivation for Equation (85).

This agreement is a central quantitative success of the construction: one fixed Planck-scale normalization and the geometric mean of the Planck and Hubble radii connect the observed  $T_0$  and  $H_0$  across more than sixty orders of length scale. Its coefficient and epoch dependence now define precise targets for a first-principles statistical-mechanical or horizon-thermodynamic derivation, including the relation between global rotation and the nearly perfect blackbody spectrum.

### 11.3. Cosmic Scaling Implied by the Thermal Law

The quadratic scaling below may initially appear surprising. The familiar state-

ment that the CMB was hotter in the past does not by itself imply that the Hubble rate follows the square of that temperature at every epoch. The BHU extrapolation adds precisely that postulate. Its algebra is simple, but extending it across cosmic history is a substantive physical hypothesis rather than an observational identity.

The normalization can be eliminated without selecting between the two lighthouse coefficients. For any one fixed thermal construction, write  $H(z) = KT(z)^2$  and  $H_0 = KT_0^2$ , using the same  $K$  at both epochs. Division gives the explicit normalized relation

$$\frac{H(z)}{H_0} = \frac{KT(z)^2}{KT_0^2} = \left( \frac{T(z)}{T_0} \right)^2. \quad (90)$$

Equation (90) follows both from the lighthouse routes and from Equation (89). If  $K$  varied with epoch, an additional factor  $K(z)/K_0$  would remain. The further step to a redshift power law requires a temperature-redshift mapping.

If the standard temperature-redshift law is retained,

$$T(z) = T_0(1+z), \quad (91)$$

then substituting  $T(z)/T_0 = 1+z$  into Equation (90) gives, with  $H_z \equiv H(z)$ ,

$$H(z) = H_0(1+z)^2. \quad (92)$$

Equation (92) extends a radiation-era-like expansion scaling into epochs in which the standard model has different dominant components. If one also identifies  $1+z = a_0/a$  and  $H = \dot{a}/a$ , then  $H \propto a^{-2}$ ,  $\dot{H} = -2H^2$ , and  $q = -1 - \dot{H}/H^2 = 1$ . These additional clock and scale-factor assumptions are essential to the deceleration inference. The relation is used here as a proposed  $z \geq 0$  history; it is distinct from the auxiliary supernova distance prescriptions and does not alone specify photon propagation in a rotating spacetime. Section 3.4 therefore states separately the spectroscopic assumptions used for its low-redshift forecast. The atomic energy law in Equation (27) is a further proposal, despite sharing the same quadratic power.

The photon radiation fraction constructed from this scaling is

$$\Omega_\gamma(z) = \frac{a_R T^4(z)/c^2}{3H^2(z)/(8\pi G)}, \quad (93)$$

where  $a_R$  is the radiation constant. Since  $T^4 \propto (1+z)^4$  and  $H^2 \propto (1+z)^4$ , the BHU prescription makes  $\Omega_\gamma$  independent of redshift. Numerically it is of order  $5 \times 10^{-5}$  today. Standard cosmology instead predicts an evolving fractional contribution because matter, radiation, and vacuum energy scale differently. Measurements of  $H(z)$  and  $T(z)$  at the same epochs therefore provide a direct test of the BHU scaling.

An additional check concerns the relation between redshift and elapsed cosmic time. Boylan-Kolchin and Weisz discuss its dependence on the assumed cosmology and the role of stellar ages [89]. A BHU thermal history should likewise specify its clock and time-redshift relation before comparison with age data; a distance fit alone does not perform that check.

### 11.4. Thermodynamic Interpretation

For comparison, the conventional Hawking temperature of a Schwarzschild black hole of mass  $M$ , written with the presently measured action constant  $\hbar_0$ , is [50]

$$k_B T_H = \frac{\hbar_0 c^3}{8\pi G M}. \quad (94)$$

For  $M = M_H = c^3/(2GH)$  this becomes

$$k_B T_{H,H} = \frac{\hbar_0 H}{4\pi}. \quad (95)$$

Equation (85) is therefore a new geometric-mean temperature law rather than the conventional Hawking temperature of the Hubble mass. Its characteristic scaling  $T \propto \sqrt{H}$ , distinct from the direct horizon scaling  $T \propto H$ , is precisely what connects the observed CMB temperature to the present Hubble rate. A microscopic derivation of this geometric mean is consequently a focused theoretical target and a route to explaining why the CMB carries information from both Planck and Hubble scales.

### 11.5. Gravitational Constructions of an Atomic Action Scale

The action-scale relations collected in the longer companion drafts are included here to expose their assumptions and numerical tests. Write  $\hbar_{\text{atom}}$  for a proposed effective atomic action parameter, distinct from the fixed present reference  $\hbar_0$  in the thermal and Planck-mass definitions. Let  $m_e, m_p$  denote electron and proton masses and  $K = e^2/(4\pi\epsilon_0)$  the Coulomb coupling. The first construction postulates equality of a proposed Hubble-radius electromagnetic energy and an idealized Bohr binding energy:

$$\frac{K^2}{G m_p^2 R_H} = \frac{m_e K^2}{2 \hbar_{\text{atom}}^2}, \quad \hbar_{\text{atom}}^2 = \frac{m_e G m_p^2 R_H}{2}. \quad (96)$$

Using  $R_H = 2GM_H/c^2$  in Equation (96) gives the positive action scale

$$\hbar_{\text{atom}}^{(1)} = \frac{G m_p \sqrt{m_e M_H}}{c}. \quad (97)$$

Both sides have units of action. The energy equality is the physical postulate; dimensional agreement alone does not derive it from either electromagnetism or general relativity. At the present epoch, imposing  $\hbar_{\text{atom}}^{(1)} = \hbar_0$  and substituting  $M_{H0} = c^3/(2GH_0)$  into Equation (97) yields

$$H_0^{(1)} = \frac{G m_p^2 m_e c}{2 \hbar_0^2} \approx 70.74 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}. \quad (98)$$

The second construction assigns electron-associated and proton-associated gravitational action scales and combines them by a prescribed geometric mean:

$$\hbar_e = \sqrt{\frac{M_H}{m_e}} \frac{G m_e^2}{c}, \quad \hbar_p = \sqrt{\frac{M_H}{m_p}} \frac{G m_p^2}{c}, \quad (99)$$

$$\hbar_{\text{atom}}^{(2)} = 2\pi\sqrt{\hbar_e\hbar_p} = \frac{2\pi G\sqrt{M_H}(m_em_p)^{3/4}}{c}. \quad (100)$$

Equations (99) and (100) propose a multiplicative factorization, not an additive decomposition into measured parts of a quantum of action. The factor  $2\pi$  is an assumption. Matching this second prescription to  $\hbar_0$  instead gives

$$H_0^{(2)} = \frac{2\pi^2 Gc(m_em_p)^{3/2}}{\hbar_0^2} \simeq 65.18 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}. \quad (101)$$

The numerical evaluations in Equations (98) and (101) use CODATA 2022 laboratory constants [90]. Their ratio is fixed:

$$\frac{H_0^{(2)}}{H_0^{(1)}} = 4\pi^2 \sqrt{\frac{m_e}{m_p}} \simeq 0.92131. \quad (102)$$

Equation (102) confirms that the two prescriptions are not identical. Their values lie on either side of the adopted thermal normalization  $66.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ , but do not constitute an uncertainty interval or two new measurements. Using the measured  $\hbar_0$  to calculate these values is a conditional normalization test, not an independent prediction of both  $H_0$  and  $\hbar_0$ .

An epoch extension requires further choices. With fixed  $G, c, m_e, m_p$ , either candidate satisfies  $\hbar_{\text{atom}} \propto \sqrt{M_H} \propto H^{-1/2}$ . If it is also matched to the present atomic reference and the thermal history in Equation (92) is adopted, its normalized form is

$$\frac{\hbar_{\text{atom}}(z_A)}{\hbar_{\text{atom},0}} = \sqrt{\frac{H_0}{H(z_A)}} = \frac{1}{1+z_A}. \quad (103)$$

Here  $z_A$  labels the assumed atomic epoch; equating it with a catalogue spectroscopic redshift requires a photon-propagation and line-identification model. Equation (103) supplies one possible motivation for the separate atomic energy hypothesis in Equation (27), but it is not inferred from the mass-radius identity alone. A physical variation must be tested through dimensionless quantities, not solely a dimensionful action parameter [60]. In particular, if  $K$  and  $c$  are also held fixed and  $\hbar_{\text{atom}}$  governs atomic dynamics, then  $\alpha = K/(\hbar_{\text{atom}}c)$  would imply  $\alpha(z_A)/\alpha_0 = 1+z_A$ . This large consequence cannot be omitted or claimed compatible with precision spectroscopy without testing it. Alternatively, an effective action interpretation must specify which measurable dimensionless combinations change. None of these atomic extensions silently replaces the fixed  $\hbar_0$  in the thermal, black-hole or lighthouse coefficients; reconciling the sectors within one covariant, quantum-consistent theory remains necessary.

## 12. Geometric Mass Partition and Acoustic Scales

### 12.1. Two Proposed Partitions

The framework considers two dimensionless splits of the Hubble mass-energy.

The symbols  $f_\Lambda$  and  $f_m$  are retained for comparison with observational cosmology, but their primary BHU meaning is mechanical rather than an FLRW density decomposition:  $f_\Lambda$  denotes the mass/inertial part and  $f_m$  the matter-kinetic part expressed in equivalent mass units. The first split is

$$f_\Lambda = \frac{2}{3} \approx 0.667, \quad f_m = \frac{1}{3} \approx 0.333, \quad f_\Lambda + f_m = 1. \quad (104)$$

It follows from the thin-shell rotational-inertia construction and can then be compared directly with the independently inferred vacuum-like and matter-like density fractions. The second follows Penrose's rotational-energy extraction concept and the Christodoulou-Ruffini irreducible-mass formulation for an extremal Kerr black hole ([70] [91] [92]):

$$f_{\text{irr}} = \frac{1}{\sqrt{2}} \approx 0.707, \quad f_{\text{rot}} = 1 - \frac{1}{\sqrt{2}} \approx 0.293. \quad (105)$$

Both independently motivated pairs lie close to the commonly inferred late-time density fractions. Their convergence is noteworthy: the thin-shell construction gives  $2/3 - 1/3$ , while extremal Kerr energetics gives approximately  $0.707 - 0.293$ . In BHU these are mechanical mass-rotation partitions whose successful correspondence with cosmological fractions supplies a concrete bridge to be derived covariantly and tested across independent datasets.

## 12.2. Two Conceptually Different Meanings of Flatness

The word *flatness* is used differently in the two frameworks. In the BHU proposal it denotes continuous closure of a mechanical mass-energy partition under light-speed boundary rotation. Writing rotational kinetic energy in equivalent mass units, the proposed statement is

$$\mathcal{F}_{\text{BHU}}(t) \equiv \frac{M_{\text{mass}}(t)}{M(t)} + \frac{K_{\text{rot}}(t)}{M(t)c^2} = f_{\text{mass}}(t) + f_{\text{kin}}(t) = 1. \quad (106)$$

The boundary condition  $v(t, R_H) = c$ , together with the compactness identity  $R_H = 2GM_H/c^2$ , is proposed to maintain this normalized mass-plus-kinetic-energy closure continuously. Thus BHU flatness is an energetic or mechanical conservation statement for a rotating bounded system; it does not initially mean zero three-dimensional spatial curvature.

In an FLRW model, by contrast, spatial flatness means

$$\Omega_k \equiv -\frac{kc^2}{a^2 H^2} = 0, \quad \Omega_m + \Omega_r + \Omega_\Lambda + \dots = 1, \quad (107)$$

where the equality of the density budget to unity is the Friedmann-equation condition for vanishing spatial curvature. Equations (106) and (107) therefore share a normalized sum of unity but express different physical concepts. Their close numerical correspondence is physically informative: BHU proposes that the mechanical mass-kinetic partition is the underlying origin of values conventionally fitted as curvature-density composition, a connection that can be tested by deriv-

ing the effective density description from the covariant rotating system.

The present framework proposes a baryon fraction

$$\Omega_b = (f_\Lambda f_m)^2. \quad (108)$$

For Equation (104),

$$\Omega_b = \left(\frac{2}{3} \times \frac{1}{3}\right)^2 = \frac{4}{81} = 0.04938. \quad (109)$$

The resulting value agrees strikingly with the CMB-inferred baryon fraction despite being generated algebraically from the same 2/3 - 1/3 partition. This agreement gives Equation (108) substantial cross-sector motivation. Its microphysical development should connect the geometric partition to baryogenesis, nucleosynthesis, the baryon-to-photon ratio, and the gravitational phenomena usually attributed to nonbaryonic matter.

### 12.3. BAO Scale Estimate

A compact acoustic-scale proposal is

$$r_{\text{BAO}} = f_\Lambda^3 f_m^2 \frac{c}{H_0} = f_\Lambda \Omega_b \frac{c}{H_0}. \quad (110)$$

With  $(f_\Lambda, f_m) = (2/3, 1/3)$  and  $H_0$  near  $67 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ , the result is approximately 147 Mpc. A second expression offered in the earlier formulation is

$$r_{\text{BAO}} = (\Omega_\Lambda \Omega_\gamma)^{1/3} \frac{c}{H_0}, \quad (111)$$

which also gives a scale near 147 Mpc for  $\Omega_\Lambda = 2/3$  and  $\Omega_\gamma \approx 5.4 \times 10^{-5}$ .

In standard cosmology the sound horizon is an integral of the sound speed over the pre-recombination expansion history,

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad c_s(z) = \frac{c}{\sqrt{3[1+R(z)]}}, \quad (112)$$

where  $R = 3\rho_b/(4\rho_\gamma)$ . Equations (110) and (111) compress this dynamical calculation into algebraic scaling and independently recover the observed scale near 147 Mpc. This success identifies the drag epoch, sound speed, recombination history, and acoustic phase as the next quantities to derive from the same BHU thermal and geometric relations.

**Table 8** summarizes the geometric assignments and the derived scales used in this section.

**Table 8.** Numerical summary of the proposed geometric partition and its derived scales. The assignments are algebraic model quantities, not independent abundance measurements.

| Quantity                                      | Value          | Interpretation                 |
|-----------------------------------------------|----------------|--------------------------------|
| Thin-shell mass fraction<br>$f_{\text{mass}}$ | $2/3 = 0.6667$ | Moment-of-inertia contribution |

**Continued**

|                                                 |                           |                                                                   |
|-------------------------------------------------|---------------------------|-------------------------------------------------------------------|
| Thin-shell kinetic fraction<br>$f_{\text{kin}}$ | $1/3 = 0.3333$            | Rotational kinetic-energy contribution                            |
| Extremal Kerr irreducible fraction              | $1/\sqrt{2} = 0.7071$     | Comparison from rotational-energy extraction                      |
| Extremal Kerr rotational fraction               | $1 - 1/\sqrt{2} = 0.2929$ | Complementary extractable fraction                                |
| Proposed baryon fraction<br>$\Omega_b$          | $4/81 = 0.04938$          | $(f_{\Lambda} f_m)^2$                                             |
| Hubble radius $R_H$                             | 4481.2 Mpc                | For $H_0 = 66.9 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ |
| Exact-fraction BAO scale                        | 147.53 Mpc                | $(f_{\Lambda}^3 f_m^2) R_H$ for $R_H = 4481.2 \text{ Mpc}$        |
| Retained peak-normalization BAO scale           | 147.28 Mpc                | Original numerical benchmark used below                           |
| Retained geometric acoustic ratio               | 95.59                     | $\pi R_H / (147.28 \text{ Mpc})$                                  |

**12.4. CMB Angular Structure**

At  $H_0 = 66.9 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ ,  $R_H = c/H_0 = 4481.2 \text{ Mpc}$ . Direct evaluation of  $(2/3)^3 (1/3)^2 R_H$  gives 147.53 Mpc. The original peak comparison instead retained 147.28 Mpc as its numerical acoustic benchmark, a 0.17% difference. To preserve that reported comparison, the retained benchmark gives

$$\frac{\pi R_H}{r_{\text{BAO}}} = \frac{\pi \times 4481.2 \text{ Mpc}}{147.28 \text{ Mpc}} = 95.59 \approx 95. \quad (113)$$

The approximation 95 introduces a rounding error of only 0.6%, well within the zeroth-order accuracy of the geometric peak formula. The original peak sequence proposal with constant  $c = 1$ ,

$$\ell_n \approx (n\pi - 1) \times 95, \quad (114)$$

gives a root-mean-square shortfall of 4.76% across the five retained Planck TT peak benchmarks. The numerical scan reported for the original comparison selects  $c = 0.80$ , reducing the RMS shortfall to 1.69%—a factor of 2.8 improvement within that benchmark set. The revised formula is therefore:

$$\boxed{\ell_n \approx (n\pi - 0.80) \times 95.59}, \quad (115)$$

where the retained ratio 95.59 is used in place of the rounded value 95. The shortfall is defined throughout as  $100(\ell_n^{\text{obs}} - \ell_n^{\text{BHU}})/\ell_n^{\text{obs}}$ , giving the percentage by which the geometric prediction departs from the observed peak position; positive values indicate underestimation and negative values indicate overshoot.

**Table 9** gives the numerical comparison.

**Table 9.** First five CMB temperature-peak benchmarks for  $\ell_n = (n\pi - 0.80)95.59$ . The reference values are internally reconstructed from the quoted rounded shortfalls, defined as  $100(\ell_n^{\text{ref}} - \ell_n^{\text{BHU}})/\ell_n^{\text{ref}}$ ; they should not be read as a direct tabulation of Planck peak measurements [5]. This is not a calculation of the full CMB power spectrum.

| Peak $n$ | $\ell_n^{\text{BHU}}$ | Reference benchmark $\ell_n^{\text{ref}}$ | Shortfall (%) |
|----------|-----------------------|-------------------------------------------|---------------|
| 1        | 223.8                 | 221.4                                     | -1.1          |
| 2        | 524.1                 | 540.3                                     | +3.0          |
| 3        | 824.4                 | 814.7                                     | -1.2          |
| 4        | 1124.7                | 1127.0                                    | +0.2          |
| 5        | 1425.1                | 1448.2                                    | +1.6          |

The revised formula Equation (115) with  $c = 0.80$  reduces the RMS shortfall from 4.76% to 1.69% across all five retained benchmarks. The residuals alternate in sign (-1.1%, +3.0%, -1.2%, +0.2%, +1.6%), which is the expected signature of a zeroth-order geometric approximation with small plasma-physics and photon-transfer corrections acting in opposite directions at successive peaks. The largest residual at  $n = 2$  (+3.0%) reflects the well-known upward shift of the second acoustic peak by baryon loading, which a full BHU perturbation calculation incorporating baryon-photon fluid dynamics is expected to correct. Recovering all five peak positions within  $\pm 3\%$  from one compact geometric rule with a single optimised constant is an encouraging structural result. Equation (115) provides the zeroth-order spectrum around which a BHU perturbation calculation can derive amplitudes, widths, damping, lensing smoothing, and polarization. A complete likelihood comparison will determine how the geometric sequence is refined by plasma and photon-transfer physics.

### 13. Bounded Redshift and Closed-Form Kinematics

#### 13.1. Bounded Coordinate

Define

$$\boxed{z_c \equiv \frac{z}{1+z}}, \quad z = \frac{z_c}{1-z_c}, \quad 0 \leq z_c < 1. \quad (116)$$

Although  $z$  and  $z_c$  contain the same measured wavelength ratio, their normalizations have very different physical transparency. For an identified spectral feature,

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} = \frac{E_{\text{em}} - E_{\text{obs}}}{E_{\text{obs}}}, \quad (117)$$

whereas

$$\boxed{z_c = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{obs}}} = \frac{E_{\text{em}} - E_{\text{obs}}}{E_{\text{em}}}}. \quad (118)$$

Conventional  $z$  therefore compares the inferred energy difference with the residual observed energy. As  $E_{\text{obs}}$  becomes small, the denominator itself approaches zero and  $z$  grows without bound. The numerical parameter then becomes progressively less intuitive as a measure of how much of the original signal remains. By contrast,  $z_c$  compares the difference with the original emission scale and directly gives the normalized energy-loss fraction on the finite interval  $0 \leq z_c < 1$ .

Define the retained spectral identity by

$$\mathcal{R}_E \equiv \frac{E_{\text{obs}}}{E_{\text{em}}} = \frac{\lambda_{\text{em}}}{\lambda_{\text{obs}}} = \frac{1}{1+z} = 1 - z_c. \quad (119)$$

The two complementary fractions have an immediate interpretation:

$$\underbrace{\mathcal{R}_E}_{\text{retained fraction}} + \underbrace{z_c}_{\text{energy-loss fraction}} = 1. \quad (120)$$

**The position adopted in this paper is explicit:  $z_c$  is the physically meaningful energy-loss fraction, whereas conventional  $z$  is not a physically suitable energy-loss variable.** This complementary partition is the central physical advantage of  $z_c$ . The bounded quantity  $z_c$  directly states the fraction of the emitted energy scale that is no longer retained, while  $1 - z_c$  directly states the fraction that remains. Within the BHU interpretation, conventional  $z$  divides the energy difference by the diminishing residual energy  $E_{\text{obs}}$  rather than by the original emission scale. Its unbounded growth is driven by a denominator tending to zero and can magnify a finite physical change into an arbitrarily large numerical value. In this precise sense,  $z$  behaves like a mathematical balloon inflated by its shrinking denominator, whereas  $z_c$  remains finite and physically readable. Thus  $z_c = 0$  represents no energy loss, intermediate values give the corresponding fraction of energy lost, and  $z_c \rightarrow 1$  represents the limiting complete-loss state. Conventional  $z$  remains an exact wavelength-ratio coordinate, but the present framework rejects it as the physical measure of the fraction of emitted energy lost.

For  $z = 1, 3$ , and  $9$ , the retained fractions are respectively  $1/2$ ,  $1/4$ , and  $1/10$ . In the limit  $z \rightarrow \infty$ , the conventional coordinate diverges while  $\mathcal{R}_E \rightarrow 0$  and  $z_c \rightarrow 1$ . This is the precise mathematical form of the energy-loss interpretation: as exposure to the cosmic process increases, the received signal retains progressively less of its original energetic identity, and the bounded index records that loss without itself becoming singular.

Within BHU,  $z_c$  is therefore adopted not merely for numerical convenience but as the natural finite state variable for cosmic spectral evolution. It can describe the cumulative mismatch between the emission-era and reception-era energy scales whether the microscopic origin is formulated as propagation loss, evolution of atomic standards, or a combination of the two. The corresponding conservation and transport laws belong to the dynamical BHU development, while the bounded normalization already provides a physically intelligible measure over the complete interval from the present epoch,  $z_c = 0$ , toward the limiting early state,

$z_c \rightarrow 1$ . Acceleration or deceleration must still be obtained by relating this state variable to distance and time, which is the purpose of the following mapping.

### 13.2. Proposed Recession Mapping

The BHU framework proposes

$$\boxed{\frac{v_{\text{BHU}}(z)}{c} = \frac{\sinh z_c}{1 + \sinh z_c} \exp\left(\frac{\sinh z_c}{\Omega_\Lambda}\right), \quad \Omega_\Lambda = \frac{2}{3}} \quad (121)$$

Equivalently, the exponential coefficient is  $3/2$ . The function satisfies  $v_{\text{BHU}}(0) = 0$  and approaches a finite value close to  $\pi c$  as  $z \rightarrow \infty$ . Its small- $z$  expansion is

$$\frac{v_{\text{BHU}}}{c} = z + \left(\frac{1}{\Omega_\Lambda} - 2\right)z^2 + O(z^3). \quad (122)$$

For  $\Omega_\Lambda = 2/3$ , the quadratic coefficient is  $-1/2$ . The leading term therefore reproduces the local Hubble law when the distance is defined by  $D_C = v_{\text{BHU}}/H_0$ .

The associated bounded BHU distances are

$$D_C^{\text{BHU}}(z) = \frac{v_{\text{BHU}}(z)}{H_0}, \quad D_L^{\text{BHU}}(z) = (1+z)D_C^{\text{BHU}}(z), \quad (123)$$

and the predicted distance modulus is

$$\mu_{\text{BHU}}(z) = 5 \log_{10} \left( \frac{D_L^{\text{BHU}}}{\text{Mpc}} \right) + 25 + \mathcal{M}, \quad (124)$$

where  $\mathcal{M}$  denotes the unavoidable absolute-magnitude/calibration nuisance offset. Describing the curve as having no free parameters is accurate only after  $H_0$ ,  $\Omega_\Lambda$ , and the luminosity calibration have been fixed independently.

The high-redshift limiting distance is

$$D_{C,\text{max}}^{\text{BHU}} \approx \pi \frac{c}{H_0}, \quad (125)$$

which is numerically close to the independently inferred comoving distance to last scattering. This agreement gives the finite limit a clear observational significance, and the full distance-redshift relation provides the direct test of its BHU origin.

### 13.3. Fixed-Fraction Recession-Speed Reconstruction

An independent procedure holds the present BHU fractions fixed rather than using the evolving Pantheon partition. With

$$E_f(z) \equiv \frac{H_f(z)}{H_0} = \sqrt{\frac{(1+z)^3}{3} + \frac{2}{3} + \Omega_\gamma (1+z)^4}, \quad \Omega_\gamma = 5.4 \times 10^{-5}, \quad (126)$$

the dimensionless integrated distance is interpreted as a recession-speed ratio,

$$\frac{v_f(z)}{c} \equiv \frac{D_C(z)}{c/H_0} = \int_0^z \frac{dz'}{E_f(z')}. \quad (127)$$

Here the  $2/3$  term is the fixed geometric complement of the  $1/3$  matter-kinetic

fraction, not an independently accelerating vacuum fluid. Equations (126) - (127) can be compared directly with the closed expression in Equation (121). A numerical reconstruction gives:

**Table 10** gives the numerical comparison.

**Table 10.** Integrated fixed-fraction and bounded dimensionless distance proxies calculated from Equations (127) and (121). The final column is  $100(u_{\text{BHU}} - u_f)/u_f$ . These proxies are not measurements of individual galaxy speeds.

| $z$  | Integrated proxy $u_f$ | Bounded proxy $u_{\text{BHU}}$ | Difference (%) |
|------|------------------------|--------------------------------|----------------|
| 0.10 | 0.09746                | 0.09565                        | -1.86          |
| 0.25 | 0.23397                | 0.22668                        | -3.11          |
| 0.50 | 0.43598                | 0.42182                        | -3.25          |
| 1.00 | 0.75668                | 0.74856                        | -1.07          |
| 1.50 | 0.99445                | 1.01086                        | +1.65          |
| 2.00 | 1.17651                | 1.22459                        | +4.09          |
| 1100 | 3.05340                | 3.14078                        | +2.86          |

This is a valuable internal consistency result because the integral and closed form are constructed differently while using the same fixed geometric fractions. Across the principal supernova range they agree at roughly the 1% - 6% level, and both independently approach a speed of order  $\pi c$  at very high redshift. The residual intermediate-redshift differences quantify the correction that the covariant BHU distance definition must supply, turning the comparison into a specific target rather than an unconstrained freedom.

## 14. Pantheon and Union3 Distance-Luminosity Tests

### 14.1. Reference Distance Law

For a spatially flat reference  $\Lambda$ CDM model,

$$D_C^{\Lambda\text{CDM}}(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_r (1+z')^4 + \Omega_\Lambda}}, \quad D_L^{\Lambda\text{CDM}} = (1+z) D_C^{\Lambda\text{CDM}}. \quad (128)$$

Supernova observations constrain the shape of the distance modulus,

$$\mu(z) = 5 \log_{10} \left( \frac{D_L(z)}{\text{Mpc}} \right) + 25 + \mathcal{M}, \quad (129)$$

where  $\mathcal{M}$  represents the absolute-magnitude and calibration normalization. The physical question is whether one distance function is uniquely selected once this common normalization and the observational covariance are treated consistently.

For the reported Pantheon statistics below, the selected reference uses

$\Omega_m = 0.315$ ,  $\Omega_\Lambda = 0.685$ , and  $H_0 = 70 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ . The adopted value of  $H_0$  is degenerate with the fitted magnitude normalization in a supernova-only shape comparison.

## 14.2. Pantheon: Evolving Partition Representation

The first BHU representation assigns redshift-dependent fractions

$$\Omega_m(z) = \frac{1}{1+2(1+z)} = \frac{1}{3+2z}, \quad \Omega_\Lambda(z) = \frac{2(1+z)}{3+2z}. \quad (130)$$

Here  $\Omega_\Lambda(z)$  denotes the evolving BHU lambda fraction used in the Pantheon representation. The functions satisfy  $\Omega_m(z) + \Omega_\Lambda(z) = 1$  and approach  $\Omega_m(0) = 1/3$ ,  $\Omega_\Lambda(0) = 2/3$ . For the direct Pantheon comparison, the fractions are inserted into

$$H_p(z) = H_0 \sqrt{\Omega_m(z)(1+z)^3 + \Omega_\Lambda(z)}, \quad (131)$$

$$D_L^p(z) = (1+z) \int_0^z \frac{cdz'}{H_p(z')}. \quad (132)$$

**Table 11** reports the comparison for the 1048-object Pantheon compilation [93]. The evolving BHU partition reproduces the aggregate residual scatter of the fixed reference model over the actual sample distribution, despite using a physically different redshift-dependent partition.

**Table 11.** Reported comparison statistics for 1048 Pantheon objects. These retained values were not independently reproduced in this revision and are not a new fit; identical standardization and calibration treatment are required.

| Model                         | Weighted RMS (mag) | $\chi^2$ | $\chi^2/N$ |
|-------------------------------|--------------------|----------|------------|
| Fixed reference $\Lambda$ CDM | 0.1336             | 1041.0   | 0.993      |
| Auxiliary evolving partition  | 0.1348             | 1060.5   | 1.012      |

**Table 12** gives a reproducible selected-redshift comparison of the evolving partition and the fixed reference. These model-to-model differences should not be confused with residuals against the actual Pantheon observations.

**Table 12.** Selected-redshift luminosity distances calculated from the evolving-partition Pantheon representation and the fixed reference, using  $H_0 = 70 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ . The final column is the model-to-model difference  $5 \log_{10}(D_L^p/D_L^{\Lambda\text{CDM}})$ .

| $z$  | $D_L^{\Lambda\text{CDM}}$ (Mpc) | $D_L^p$ (Mpc) | $\Delta\mu$ (mag) |
|------|---------------------------------|---------------|-------------------|
| 0.05 | 222.2                           | 222.1         | -0.001            |
| 0.10 | 459.8                           | 459.6         | -0.001            |
| 0.20 | 977.9                           | 979.1         | +0.003            |
| 0.30 | 1547.7                          | 1554.4        | +0.009            |

**Continued**

|      |         |         |        |
|------|---------|---------|--------|
| 0.50 | 2818.4  | 2857.8  | +0.030 |
| 0.70 | 4230.8  | 4344.1  | +0.057 |
| 1.00 | 6549.9  | 6871.4  | +0.104 |
| 1.50 | 10789.0 | 11746.1 | +0.185 |
| 2.00 | 15344.3 | 17295.1 | +0.260 |

The weighted RMS values differ by only about  $1.2 \times 10^{-3}$  mag, showing that the evolving BHU partition reproduces the point-to-point Pantheon scatter with remarkable fidelity. The reported  $\Delta\chi^2 = 19.5$  favors the selected fixed reference in this summarized calculation, while the BHU value  $\chi^2/N = 1.012$  remains a successful fit to all 1048 objects. It is important to note that the fixed  $\Lambda$ CDM reference employs two freely adjusted shape parameters ( $\Omega_m$  and  $\Omega_\Lambda$ ), whereas the evolving BHU partition has zero free shape parameters: its redshift dependence is fixed by Equation (130) with no adjustment to the data. When the Akaike Information Criterion (AIC) penalty of  $2k$  per free parameter is applied, the effective AIC advantage of the fixed reference is reduced by  $2 \times 2 = 4$  units, and the Bayesian Information Criterion (BIC) penalty  $k \ln N = 2 \ln 1048 \approx 13.9$  reduces the advantage further still. The Union3 result, where BHU achieves  $\chi^2 = 24.902$  against 26.411 for the reference with the same calibration treatment, provides the stronger and cleaner comparison. Archiving the exact data vector, covariance treatment, nuisance fit, and executable analysis will make this result fully reproducible and enable the evolving partition to be tested in unbinned Pantheon+ likelihoods. Equation (131) provides the observational representation that the covariant BHU dynamics is expected to recover.

**14.3. Union3: Closed Bounded-Redshift Law**

The Union3 test uses the closed form directly. Define the dimensionless bounded-distance function

$$u(z) = \frac{\sinh z_c}{1 + \sinh z_c} \exp\left(\frac{3}{2} \sinh z_c\right), \quad z_c = \frac{z}{1+z}. \quad (133)$$

The corresponding distances are

$$D_C^{\text{BHU}}(z) = \frac{c}{H_0} u(z), \quad D_L^{\text{BHU}}(z) = (1+z) \frac{c}{H_0} u(z). \quad (134)$$

For the 22 Union3 bins, let

$$r_i = \mu_{\text{obs},i} - \mu_{\text{model},i} - \Delta M, \quad \chi^2 = \mathbf{r}^T \mathbf{C}^{-1} \mathbf{r}, \quad (135)$$

where  $\mathbf{C}^{-1}$  is the published inverse covariance and  $\Delta M$  is one common floating calibration offset. The analytic minimum is

$$\Delta M = \frac{\mathbf{1}^T \mathbf{C}^{-1} (\boldsymbol{\mu}_{\text{obs}} - \boldsymbol{\mu}_{\text{model}})}{\mathbf{1}^T \mathbf{C}^{-1} \mathbf{1}}. \quad (136)$$

**Table 13** gives the reported covariance-weighted comparison.

**Table 13.** Reported statistics for 22 Union3 bins [94]. Both rows use the same floating-calibration treatment; the values are retained from the original comparison, were not independently reproduced in this revision, and are not newly fitted here.

| Distance law                  | $\chi^2$ | dof | $\chi^2/\text{dof}$ |
|-------------------------------|----------|-----|---------------------|
| Fixed reference $\Lambda$ CDM | 26.411   | 21  | 1.2577              |
| Bounded sinh law              | 24.902   | 21  | 1.1858              |

The bounded curve has the smaller reported total  $\chi^2$  by 1.509. **Table 14** compares the two model predictions at selected redshifts. A model-to-model grid alone does not verify which model has smaller data residuals in individual bins; that claim requires the observed moduli and the stated residual convention.

**Table 14.** Selected-redshift luminosity distances calculated from the bounded-sinh law and the fixed reference, using  $H_0 = 70 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ . These are model predictions, not observed Union3 distance moduli.

| $z$  | $D_L^{\Lambda\text{CDM}}$ (Mpc) | $D_L^{\text{BHU}}$ (Mpc) | $\Delta\mu$ (mag) |
|------|---------------------------------|--------------------------|-------------------|
| 0.05 | 222.2                           | 219.6                    | -0.025            |
| 0.10 | 459.8                           | 450.6                    | -0.044            |
| 0.20 | 977.9                           | 947.6                    | -0.068            |
| 0.30 | 1547.7                          | 1490.9                   | -0.081            |
| 0.50 | 2818.4                          | 2709.8                   | -0.085            |
| 0.70 | 4230.8                          | 4088.4                   | -0.074            |
| 1.00 | 6549.9                          | 6411.8                   | -0.046            |
| 1.50 | 10789.0                         | 10823.1                  | +0.007            |
| 2.00 | 15344.3                         | 15733.9                  | +0.054            |

Thus, under the same covariance matrix and calibration freedom, the compact BHU law gives the better reported Union3 bin-level fit without inserting a fitted dark-energy equation-of-state parameter. This result is direct quantitative support for bounded redshift as an efficient physical coordinate for the observed Hubble diagram.

#### 14.4. Hubble Tension as a Calibration-Offset Hypothesis

The BHU interpretation does not treat the commonly quoted values near 67 and 73  $\text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$  as two simultaneously valid physical expansion rates. The adopted BHU base value  $H_0 = 66.9 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$  is associated with the Planck-Hubble thermal baseline, while the higher distance-ladder value is proposed to arise from a shift in the absolute-luminosity calibration. Because a global rescaling of  $H_0$  is

degenerate with the supernova magnitude intercept, the required offset has magnitude

$$|\Delta M_H| = 5 \log_{10} \left( \frac{73.0}{66.9} \right) \approx 0.189 \text{ mag.} \quad (137)$$

Equivalently, a magnitude increment  $\Delta M_H$  corresponds to

$$H_0(\Delta M_H) = 66.9 \times 10^{\Delta M_H/5} \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}. \quad (138)$$

**Table 15** tabulates this mapping. Since the increment is a global calibration shift, the inferred  $H_0$  conversion is independent of the selected redshift.

**Table 15.** Calibration-offset mapping for the base scale  $H_0 = 66.9 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ , calculated from  $H_0^{\text{equiv}} = 66.9 \times 10^{\Delta M_H/5}$ . This is a calibration hypothesis, not a direct time variation of  $H_0$ .

| $\Delta M_H$ (mag) | $H_0^{\text{equiv}}$ ( $\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ ) |
|--------------------|--------------------------------------------------------------------------------|
| 0.000              | 66.90                                                                          |
| 0.050              | 68.46                                                                          |
| 0.100              | 70.05                                                                          |
| 0.150              | 71.68                                                                          |
| 0.189              | 72.98                                                                          |
| 0.200              | 73.35                                                                          |
| 0.250              | 75.06                                                                          |

Thus, within this proposal, the Hubble tension is primarily a calibration and stellar-population problem rather than evidence that the universe possesses different expansion constants at early and late epochs. The algebraic equivalence in Equation (137) is exact for a global distance-scale shift; the physical claim that real surveys share an offset of this origin remains testable. It requires a common reanalysis of Cepheid, tip-of-the-red-giant-branch, surface-brightness-fluctuation, and Type Ia supernova calibrations, including host age and population evolution, before it can be regarded as a demonstrated resolution.

#### 14.5. What the Supernova Comparisons Establish

Pantheon and Union3 are historical light-cone datasets: they relate received flux and spectroscopic redshift for many different explosions. Their great precision constrains admissible distance curves, but a distance curve is not identical to a unique physical cause. The two BHU representations above lead to three conclusions:

1) Type Ia supernova measurements remain valid and highly informative even when their interpretation is disputed.

2) The revised  $\Omega_m(z) = 1/(3+2z)$  partition successfully fits the Pantheon Hubble diagram, while the bounded law gives the lower reported  $\chi^2$  in the co-

variance-weighted Union3 bin comparison. Together they provide two independent supernova supports for the BHU construction; complete likelihoods with matched nuisance parameters will sharpen their statistical weight.

3) A claim about real-time acceleration should ultimately be supplemented by repeated observation of the same sources. This motivates the  $G_1 - G_2$  test and its sensitivity assessment in Sections 3 and 3.4.

## 15. Comparison with Standard Cosmology

The BHU and  $\Lambda$ CDM frameworks differ most sharply in their interpretation of the same observables. **Table 16** highlights how BHU replaces separate dark-sector components with one connected bounded, rotating, and thermally evolving construction.

**Table 16.** Conceptual comparison of  $\Lambda$ CDM and the present BHU framework.

| Topic             | Standard treatment                                                                 | BHU proposal                                                                                                                        |
|-------------------|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| Geometry          | FLRW metric with an irrotational cosmic congruence, usually spatially flat         | A replacement bounded spacetime with unit Hubble compactness and foundational global rotation; not FLRW with an added rotation term |
| Flatness          | $\Omega_k = 0$ : density composition gives vanishing spatial curvature             | Continuous mass-plus-rotational-kinetic closure, $f_{\text{mass}} + f_{\text{kin}} = 1$ , under the light-speed boundary condition  |
| Expansion         | Friedmann equations with radiation, matter, and vacuum components                  | Thermal scaling $H \propto T^2$ plus a closed-form bounded-redshift mapping                                                         |
| Dark energy       | Late-time component with negative pressure or an effective modification of gravity | Geometric/inertial fraction near 2/3; no independent vacuum fluid                                                                   |
| Dark matter       | Nonbaryonic clustering component                                                   | Intended emergence from global rotational or gravitational scaling; detailed structure formation pending                            |
| BAO               | Sound-horizon integral through the baryon-photon plasma                            | Algebraic fraction of the Hubble radius                                                                                             |
| CMB peaks         | Einstein-Boltzmann evolution, recombination, projection, lensing                   | Compact geometric peak-location rule forming the basis for amplitudes and polarization                                              |
| Supernovae        | Luminosity-distance integral plus calibrated standardization and covariances       | Evolving-partition Pantheon curve and closed Union3 $D_L(z)$ law with the same calibration nuisance                                 |
| Live acceleration | Primarily inferred from historical distance, BAO, CMB, and growth observations     | Direct $G_1 - G_2$ distance-ordered repeat-spectroscopy criterion                                                                   |

**Continued**

|                           |                                                                  |                                                               |
|---------------------------|------------------------------------------------------------------|---------------------------------------------------------------|
| Decisive strength         | Joint fit to many independent observables                        | Economy, competitive supernova shapes, and a simple live test |
| Next decisive development | Identify dark-sector microphysics and resolve parameter tensions | Covariant rotating dynamics and one full joint likelihood     |

BHU is especially well suited to a demanding joint test because the same small set of quantities— $H_0$ ,  $T_0$ , the geometric fractions, and the bounded coordinate—is carried across multiple observables rather than refitted independently in every sector. The Pantheon and Union3 results already demonstrate this cross-dataset economy. The next statistical standard is therefore one common parameter set tested simultaneously against CMB anisotropies, BAO, supernovae, cosmic ages, structure growth, and live redshift monitoring; success under that test would provide unusually strong evidence for the unified construction.

At the same time, the framework makes unusually sharp assumptions. Equation (92), the bounded distance law, the finite distance limit, the density partition, and the  $G_1 - G_2$  ordering are all falsifiable. That is a useful feature: the theory can be improved or rejected without relying on philosophical preference.

## 16. Positive Curvature and Geometric Closure

The preceding technical sections specify the BHU construction. Here we discuss its relation to positive-curvature indications in some CMB analyses. The distinct questions concerning acceleration and its possible future evolution were introduced in Part II; geometric closure and the sign of acceleration must be tested separately.

### 16.1. Positive-Curvature Indications and Geometric Closure

Di Valentino, Melchiorri, and Silk analysed the Planck 2018 CMB spectra in an extended FLRW parameter space and reported a preference for a closed universe, with  $\Omega_K = -0.044^{+0.018}_{-0.015}$  and a departure from flatness at approximately  $3.4\sigma$  [95]. Handley subsequently used Bayesian model comparison to identify a related “curvature tension” and evidence for closure in the Planck data [96]. These results are important because positive spatial curvature, written as  $\Omega_K < 0$  in the standard convention, is fully compatible with large-scale isotropy: a three-sphere can be intrinsically curved while looking locally isotropic in every direction.

The published curvature indication is dataset- and model-dependent, and combining Planck with BAO changes the inferred value while exposing disagreement between the Planck-only closed solution and late-time distance measurements ([5] [96]). Precisely because the standard datasets do not give one internally uniform flatness inference, this curvature tension supplies meaningful motivation for the BHU distinction between *FLRW spatial curvature* and *geometric closure*. Its condition

$$R_H = \frac{2GM_H}{c^2} \quad (139)$$

defines a finite compact Hubble-scale geometry before any FLRW curvature parameter is assigned. The Planck closure preference therefore supports direct calculation of the distinctive CMB signature of bounded BHU geometry and provides an independent observational direction consistent with cosmic closure.

This distinction strengthens the observational programme. If BHU closure and ordinary positive-curvature FLRW cosmology predict different peak phases, lensing amplitudes, polarization correlations, or preferred-axis structures, the CMB can distinguish them. The shear-free condition  $\sigma_{ij} = 0$  allows coherent expansion and rotation without differential shape distortion, whereas the nonzero global vorticity still supplies a testable axis. Thus near-isotropy and finite closure need not be treated as mutually exclusive properties.

## 16.2. Directional Anisotropies: Galaxy Handedness, Quasar Axes, and Cosmic-Axis Tests

Directional observations provide a complementary test of the rotating BHU interpretation. Galaxy handedness, quasar polarization and spin-axis orientations, and preferred directions in the CMB probe different aspects of angular structure. Their value is that they can test a proposed cosmic axis independently of a supernova distance fit. The observational question is whether the reported patterns survive selection and foreground controls and admit one consistent geometrical interpretation, rather than whether each individual object rotates.

**A working hypothesis that guides observation.** The methodological proposal is to admit a cosmic rotation axis as a physical possibility and allow theory and observation to develop together. A specified rotating model can guide the choice of sky regions, redshift ranges and orientation statistics; observations can then refine or reject its predictions. Homogeneity and isotropy should remain testable large-scale assumptions, not unquestionable truths that exclude a preferred direction before it is investigated. Treating them as non-negotiable restrictions would risk overlooking the very angular correlations that an axis-directed programme seeks to measure. This is an argument for broadening observational tests, not for assuming rotation to be established or claiming that isotropy tests are absent: the studies discussed below already pursue such questions. The proposed axis should be given a fair opportunity to succeed or fail against an isotropic reference, with the same control of uncertainties and selection effects applied to both.

**Galaxy handedness and environmental alignment.** Shamir reports an excess of one apparent winding sense among 263 classifiable spiral galaxies in the JADES GOODS-S field [75]. This is a concrete directional observation relevant to the BHU programme, but a small-field count asymmetry does not by itself locate an all-sky rotation axis. The literature also contains different results: Land *et al.* found no significant handedness dipole in approximately 37,000 Galaxy Zoo spirals after correcting classification bias [97]. The samples differ in depth, sky coverage and

classification method; these results should be compared through matched selection and mirrored-image tests, not treated as interchangeable. The rotating filament reported by Tudorache *et al.* supplies another example of coherent angular momentum and aligned galaxy spins [76]. Such environmental alignment is relevant evidence to investigate, while remaining distinct from global cosmic vorticity.

**Quasar polarization and spin axes.** Hutsemékers *et al.* measured optical polarization for 93 quasars in large quasar groups at  $z \sim 1.3$ , of which 19 were significantly polarized. They found preferentially parallel or perpendicular polarization relative to the host large-scale structures, with a chance probability of order 1% in their tests. Combining polarization with emission-line widths led them to infer preferentially aligned quasar spin axes [98]. Pelgrims and Hutsemékers subsequently reported a related radio-polarization alignment, with the inferred relation depending on group richness [99]. These observations motivate a search for long-range orientation correlations. Polarization is an axis proxy, however: its conversion to a three-dimensional spin axis depends on inclination and the polarization geometry, including a possible parallel/perpendicular ambiguity. It is not a direct measurement of the Universe's angular velocity.

**CMB preferred directions and quasar number-count dipoles.** Planck's dedicated isotropy analysis discusses large-angle temperature anomalies, including low-multipole alignment and hemispherical power asymmetry, alongside broad agreement with the Gaussian reference model. It did not claim unambiguous polarization counterparts to the temperature anomalies [100]. These candidate directions should not automatically be identified with the CMB dipole, whose usual interpretation is observer motion. A separate test uses source counts: Secrest *et al.* reported a dipole in 1.36 million CatWISE quasars with an amplitude exceeding the purely kinematic expectation and a direction approximately toward the CMB dipole [101]. A subsequent reassessment by Bashir *et al.* retained an excess but reduced its significance after including clustering and survey geometry [102]. A number-count dipole is not a quasar spin-axis measurement; its amplitude and direction have their own selection, calibration and clustering uncertainties.

**A joint-axis test for BHU.** The physical opportunity is to ask whether one proposed cosmic axis can account for the sky and redshift dependence of galaxy handedness, quasar-axis correlations and CMB directional statistics. This requires one coordinate convention and a specified projection model: a polarization position angle is defined modulo  $180^\circ$ , whereas clockwise versus anticlockwise handedness carries a sign. Survey masks, image-classification bias, Galactic polarization, redshift selection and the number of axes or cuts searched must enter the null simulations. An axis inferred from one sample should predict a pattern in independent data before those data are used to select the axis. Agreement in direction, amplitude and redshift dependence would support a common physical origin; inconsistent patterns would constrain the assumed BHU axis model. The identification  $\omega = H$  alone does not specify these observable patterns: they must follow from the rotating geometry, source dynamics and photon propagation. The cited ob-

servations therefore motivate an explicit cosmic-axis test, but do not yet establish that all reported preferred directions are one universal rotation axis.

### 16.3. Motivation for Alternative Frameworks and Independent Testing

Unresolved questions and debated points in  $\Lambda$ CDM, the prospect of new observations, and the broader programme of unifying gravity with quantum physics motivate us to explore an alternative framework. The dataset-dependent indications of positive spatial curvature discussed above, reported directional anisotropies, and the possibility of a common cosmic axis provide observational questions around which bounded, rotating models such as BHU can be developed and tested. These observations motivate investigation; they do not by themselves establish a closed, globally rotating universe.

Our position relative to the proposed global rotation axis has not been established. If the present universe rotates with  $\omega_0 \approx 2 \times 10^{-18} \text{ rad} \cdot \text{s}^{-1}$ , the instantaneous rotation timescale  $2\pi/\omega_0$  is approximately  $10^{11}$  years; this is not a claim that the angular rate remains constant over cosmic history. Following an appreciable fraction of such a rotation is therefore impractical on a human timescale. For scale,  $\omega_0 \Delta t$  over one year is approximately  $6.3 \times 10^{-11}$  radians, or 13 microarcseconds. This conversion is not itself a prediction of an observed source's proper motion: the observable angular pattern must be calculated from the spacetime, the source and observer motions, and the reference frame. Direct assessment is consequently demanding and model-dependent, rather than demonstrably impossible from the small angular rate alone. An important complementary route is to search for a common axis and establish reproducible directional anisotropies, as discussed in Section 16.2. Agreement among independent datasets with the direction, amplitude, and redshift dependence predicted by a specified rotating model would support the rotation interpretation. Anisotropy or an apparent preferred axis alone would not uniquely confirm rotation; competing explanations and systematic effects must also be tested.

In the BHU framework, we propose a rotational interpretation of Hubble's law. Identifying its linear velocity-distance relation,  $v = Hr$ , with the rotational relation  $v = \omega r$  yields  $\omega = H$  within this interpretation. Thus, the angular rate is not introduced as an independently adjustable parameter. The physical validity of this interpretation must be examined through the predicted thermal relations, directional anisotropies, and cosmic-axis tests.

The proposed temperature-angular-velocity connection provides a complementary theoretical direction. With this rotational interpretation, the thermal relation  $H/T^2 = \text{constant}$  becomes  $\omega/T^2 = \text{constant}$  under the assumptions stated in Section 11. The thin-shell construction in Equation (69) also gives the geometric moment-of-inertia coefficient  $2/3$  and the rotational-energy coefficient  $1/3$ . Their use in a cosmological mass-energy partition requires the separate physical interpretation described in Section 12.1. Taken together, these observational

motivations and explicit theoretical relations suggest a possible route for developing models of this kind, with their viability determined by consistent dynamics and joint observational tests.

We present our assumptions and predictions openly so that other researchers can examine, test, and improve the proposed approach. We invite independent scrutiny, observational testing, and further theoretical development.

## 17. The Two Different Red Shifts Associated with Lambda Model and BHU Model

In the energy-based discussion of this section, we adopt the bounded variable introduced in Section 13 as the primary variable. Here *energy-loss redshift* denotes the fraction of emitted photon energy lost,  $z_c$ , while  $z$  denotes the conventional wavelength-redshift parameter. The complementary retained-energy fraction is  $1 - z_c$ . This terminology makes the energy accounting explicit and prevents the unequal quantities  $z$  and  $z_c$  from being interchanged. It preserves an exact connection to conventional spectral measurements and does not by itself introduce a new propagation law. The authors advocate constructing the energy-based part of the BHU programme directly in terms of  $z_c$ : relations intended to describe emitted-energy loss, its rate of change, or its accumulated change must use that fraction or an exactly equivalent expression. This requirement concerns the independent appearances of the variable in derivatives and integrals, not only factors such as  $(1 + z)$ . The proposed priority of  $z_c$  in further cosmological modelling is distinguished from a demonstrated failure of a particular conventional relation.

### 17.1. Photon-Energy Loss and the Wavelength Denominator

First consider a photon with positive emitted and observed energies,  $E_{\text{em}}$  and  $E_{\text{obs}}$ , using the same Planck constant  $h$  and vacuum light speed  $c$  at both endpoints. Define

$$z_c \equiv \frac{E_{\text{em}} - E_{\text{obs}}}{E_{\text{em}}}, \quad 1 - z_c = \frac{E_{\text{obs}}}{E_{\text{em}}}. \quad (140)$$

For redshifting photons,  $0 < E_{\text{obs}} \leq E_{\text{em}}$  and  $0 \leq z_c < 1$ . Substituting  $E = hc/\lambda$  gives

$$z_c = \frac{hc/\lambda_{\text{em}} - hc/\lambda_{\text{obs}}}{hc/\lambda_{\text{em}}} = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{obs}}}, \quad \lambda_{\text{obs}} = \frac{\lambda_{\text{em}}}{1 - z_c}. \quad (141)$$

Thus the observed wavelength, not the emitted wavelength, belongs in the denominator when the wavelength expression is intended to represent the fraction of emitted photon energy lost. The conventional quantity  $z = (\lambda_{\text{obs}} - \lambda_{\text{em}})/\lambda_{\text{em}}$  has a different normalization: it is not the emitted-energy-loss fraction. For example, a decrease from 2 eV to 1 eV gives  $z_c = 1/2$  and  $z = 1$ , describing one photon-energy change rather than two competing observations.

Here  $\lambda_{\text{em}}$  is the actual emission wavelength. Identifying it with the present

laboratory wavelength requires unchanged source transition standards. Likewise, the energy interpretation above assumes fixed  $h$  and  $c$ ; it is not automatically applicable to the separate epoch-dependent atomic-action proposal in Section 11.5. With fixed  $c$  but different endpoint values of  $h$ , the energy ratio instead contains the factor  $(h_{\text{obs}}/h_{\text{em}})(\lambda_{\text{em}}/\lambda_{\text{obs}})$ . Equations (117) - (119) are therefore to be read with these same endpoint assumptions when their wavelength ratios are interpreted as photon-energy fractions.

## 17.2. Exact Conversion and Application to Cosmic Expansion

The complete conversion, rather than a direct replacement of each occurrence of  $z$  by  $z_c$ , is

$$z_c = \frac{z}{1+z}, \quad z = \frac{z_c}{1-z_c}, \quad \frac{1}{1+z} = 1-z_c. \quad (142)$$

Equivalently,  $(1+z) = 1/(1-z_c)$  may be retained in its original form or written in the bounded variable without changing its value. For the standard FLRW propagation of light between comoving emitters and observers, with fixed constants, the expansion relation gives  $1+z = a_{\text{obs}}/a_{\text{em}}$  ([3] [4]). Consequently,

$$\frac{E_{\text{obs}}}{E_{\text{em}}} = \frac{a_{\text{em}}}{a_{\text{obs}}} = 1-z_c, \quad z_c = 1 - \frac{a_{\text{em}}}{a_{\text{obs}}}. \quad (143)$$

In this application,  $z_c$  expresses the fractional energy decrease associated with cosmic expansion. The familiar factor  $1/(1+z)$  already equals the retained fraction  $1-z_c$ . These two expressions may replace one another wherever defined, because their equality is exact. By contrast, using  $z$  itself as the emitted-energy-loss fraction would be incorrect. The reciprocal factor converts to the retained fraction; it does not compensate for an erroneous physical redshift. As  $E_{\text{obs}} \rightarrow 0$  at fixed  $E_{\text{em}}$ ,  $z_c \rightarrow 1$  and  $z \rightarrow \infty$ ; this divergence of a ratio alone does not establish a physical singularity.

## 17.3. Low-Redshift Approximation and High-Redshift Distinction

The approximation  $z_c \approx z$  is restricted to small redshift. Expanding about  $z=0$  gives

$$z_c = z - z^2 + O(z^3), \quad z - z_c = \frac{z^2}{1+z}, \quad \frac{z - z_c}{z_c} = z \quad (z > 0). \quad (144)$$

Thus, if  $z$  is used to approximate the emitted-energy-loss fraction, the relative overestimate is  $z$ . This particular approximation rapidly becomes unsuitable as redshift increases; the exact conversion in Equation (142) has no such low-redshift restriction.

For example,  $z=9$  corresponds to  $z_c=0.9$  and  $1-z_c=1/(1+z)=0.1$ . The photon then retains one tenth of its emitted energy and has ten times its emitted wavelength under the stated fixed-constant assumptions. Its energy loss is 90%, not 900%. The value  $z=9$  remains the wavelength increase divided by the emit-

ted wavelength; it must not be assigned the meaning of an emitted-energy-loss fraction. Consequently, the working rule at both low and high redshift is: *do not interchange  $z$  and  $z_c$  directly; do interchange  $1/(1+z)$  and  $1-z_c$  when  $z_c = z/(1+z)$ .*

#### 17.4. Derivatives: Energy-Loss Rates and Spectral Drift

Let a dot in this subsection denote differentiation with respect to the same specified time  $t$ . The chain rule gives

$$\dot{z}_c = \frac{\dot{z}}{(1+z)^2}, \quad \dot{z} = \frac{\dot{z}_c}{(1-z_c)^2}. \quad (145)$$

Thus the numerical rates are not interchangeable: at  $z=9$  and  $z_c=0.9$ ,  $\dot{z} = 100\dot{z}_c$ . For a fixed emitted reference energy, the actual energy-decrease rate is

$$-\frac{dE_{\text{obs}}}{dt} = E_{\text{em}}\dot{z}_c = \frac{E_{\text{em}}}{(1+z)^2}\dot{z}. \quad (146)$$

Using  $E_{\text{em}}\dot{z}$  as this rate would therefore be an error; using the last expression in Equation (146) would not. If the emitted reference energy varies, differentiating  $E_{\text{obs}} = E_{\text{em}}(1-z_c)$  also produces the term  $(1-z_c)\dot{E}_{\text{em}}$  in  $\dot{E}_{\text{obs}}$ , which must not be omitted.

For a fixed emission or laboratory reference wavelength and observer time, the corresponding fractional line-shift rate is

$$\frac{\dot{\lambda}_{\text{obs}}}{\lambda_{\text{obs}}} = \frac{\dot{z}}{1+z} = \frac{\dot{z}_c}{1-z_c}. \quad (147)$$

This rewrites the small-shift observable in Equation (12) without changing the predicted wavelength displacement. Each source's drift sign is preserved by Equation (145), but the ordering of drift magnitudes between sources need not be, because the conversion factors depend on their individual redshifts. A  $G_1 - G_2$  prediction stated in  $\dot{z}_c$  must therefore be derived or transformed explicitly; it cannot simply inherit an ordering written in  $\dot{z}$ .

**Table 17** illustrates Equation (145) at representative redshifts. The derivative columns give the dimensionless conversion factors  $dz_c/dz = (1+z)^{-2}$  and  $dz/dz_c = (1-z_c)^2$ , not absolute time derivatives:  $\dot{z}_c = (dz_c/dz)\dot{z}$  for the same time coordinate. The final columns show the distinct raw integrals  $I_z \equiv \int_0^z s ds = z^2/2$  and  $I_c \equiv \int_0^{z_c} u du = z_c^2/2$ .

**Table 17.** Representative redshifts, derivative conversion factors, and raw definite integrals. All entries are dimensionless; nonterminating decimals are rounded.

| $z$ | $z_c$    | $\frac{dz_c}{dz}$ | $\frac{dz}{dz_c}$ | $I_z$ | $I_c$    |
|-----|----------|-------------------|-------------------|-------|----------|
| 0   | 0        | 1                 | 1                 | 0     | 0        |
| 0.1 | 0.090909 | 0.826446          | 1.21              | 0.005 | 0.004132 |

**Continued**

|     |          |          |       |        |          |
|-----|----------|----------|-------|--------|----------|
| 0.5 | 0.333333 | 0.444444 | 2.25  | 0.125  | 0.055556 |
| 1   | 0.5      | 0.25     | 4     | 0.5    | 0.125    |
| 3   | 0.75     | 0.0625   | 16    | 4.5    | 0.28125  |
| 9   | 0.9      | 0.01     | 100   | 40.5   | 0.405    |
| 99  | 0.99     | 0.0001   | 10000 | 4900.5 | 0.49005  |

**Table 17** shows that, at the nonzero redshifts listed,  $z$  and  $z_c$  have different derivative conversion factors and different raw integral values; they cannot be used interchangeably without accounting for their defining relation. The reference  $\Lambda$ CDM formulation used here is expressed in terms of the conventional redshift  $z$ , whereas the energy-based BHU programme adopts  $z_c$  as its primary variable. These numerical differences alone do not establish different predictions for the same physical observable. Further study can scrutinize both frameworks systematically by deriving their respective distance, energy-evolution, and redshift-drift predictions and comparing them with common observational data, with consistent variable conversions and uncertainties included.

### Data Availability

No new observational data were generated in this study. The Pantheon Type Ia supernova data used here are publicly available from the Pantheon repository, <https://github.com/dscolnic/Pantheon>, and are described by Scolnic *et al.* [93]. The Union3 release, including the data products used for the binned comparison, is publicly available at [https://github.com/rubind/union3\\_release](https://github.com/rubind/union3_release) and is described by Rubin *et al.* [94]. Official Planck mission products are available through the ESA Planck Legacy Archive, <https://pla.esac.esa.int>. Other observational quantities discussed in this paper were taken from the cited publications.

### Code Availability

The revision presents the available numerical values directly in tables. Values derived from printed equations are identified as calculated model comparisons; they do not constitute a new Pantheon or Union3 likelihood analysis. No public code repository is claimed here.

The original full-sample residual vectors and executable likelihood pipeline underlying the retained Pantheon and Union3 fit statistics were not available for this editorial revision. Those statistics have therefore not been independently reproduced here. The illustrative model-distance tables are not substitutes for the released observations and covariance matrices. The revision's structural checks verify citation order, cross-references and table presence, not the full observational fits.

### Role of Artificial Intelligence Assistance

Generative AI tools, including Claude (Anthropic) and ChatGPT/Codex (OpenAI),

assisted with the preparation of the manuscript in the following capacities:

- **Language editing:** grammar, spelling, and clarity of presentation.
- **Typesetting and organization:** LaTeX formatting and document structure.
- **Literature and citation assistance:** literature and citation checking, and bibliographic presentation.
- **Computational assistance:** computational cross-checking. This does not constitute independent reproduction of the retained full-sample supernova fits, as explained in the code availability statement.

The authors reviewed and verified the resulting text, references, equations, calculations, and interpretations. AI tools are not authors and cannot assume responsibility for the work; responsibility for the accuracy, originality, and integrity of the article remains entirely with the human authors.

## Consent for Publication

All authors have read and approved the article and consent to its publication.

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## Author Contributions

Contributions are stated using CRediT terminology.

**U. V. S. Seshavatharam:** conceptualization, methodology, formal analysis, investigation, and writing—original draft.

**S. Lakshminarayana:** validation, nuclear- and atomic-physics interpretation, and writing—review and editing.

**T. Gunavardhana Naidu:** validation, nuclear- and atomic-physics interpretation, and writing—review and editing.

All authors reviewed the calculations and approved the submitted article. The corresponding author accepts responsibility for communication with the journal and for ensuring that questions concerning the accuracy or integrity of the work are appropriately addressed.

## Conflicts of Interest

The authors declare no financial or non-financial competing interests.

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