Journal of Modern Physics, 2026, 17(9), 1155-1169
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

A Cascade-Based Momentum-Energy Transport Equation for the Planck and Primordial Universe

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How to cite this paper: Noormohammadi, A. (2026) A Cascade-Based Momentum-Energy Transport Equation for the Planck and Primordial Universe. *Journal of Modern Physics*, **17**, 1155-1169.
https://doi.org/10.4236/jmp.2026.179051

Received: August 21, 2026
Accepted: September 19, 2026
Published: September 22, 2026

Abstract

We develop a phenomenological cascade model for the evolution of momentum and energy during the earliest stages of the Universe. The starting point is a momentum transport equation of the form

$$\frac{\partial P_i}{\partial t} + H_c x_j \frac{\partial P_i}{\partial x_j} = -\frac{\partial E}{\partial x_i}, \tag{1}$$

where the second term represents the effect of cosmological expansion. The momentum and energy are assumed to obey cascade scaling relations

$$P = Dk^{-r/w}, \quad E = Cf^{-a/b}. \tag{2}$$

A modified uncertainty construction is introduced through

$$\Delta P\,\Delta x = \varepsilon p\lambda, \quad \Delta E\,\Delta t = \varepsilon ET. \tag{3}$$

Under characteristic-scale estimates, these relations lead to

$$\frac{\Delta P}{\Delta t} \sim \frac{\Delta E}{\Delta x}, \tag{4}$$

and, for relativistic modes, to the familiar energy-momentum relation $E \simeq cP$. When the physical wavenumber evolves as $k_{\text{phys}} \propto R^{-1}$ in an expanding universe, the momentum cascade gives $P \propto R^{r/w}$. If the characteristic frequency also scales as $f \propto R^{-1}$, the energy cascade gives $E \propto R^{a/b}$. Consistency with relativistic propagation then requires the exponent relation $a/b = r/w$. At the Planck scale, the proposed transport equation becomes naturally dimensionless when normalized by the Planck length, time, energy, and momentum. The resulting framework provides a possible phenomenological connection between cascade dynamics, cosmological expansion, and Planck-scale uncertainty.

Keywords

Cosmological Expansion, Energy Cascade, Momentum Cascade, Planck Scale, Early Universe, Uncertainty Relations

1. Introduction

The physical description of the earliest Universe requires the simultaneous consideration of quantum mechanics, relativistic gravity, extremely high energies, and rapidly evolving characteristic length and time scales. Near the Planck epoch, the characteristic length and time scales are the Planck length and Planck time [1], respectively, and the conventional separation between classical gravitational dynamics and quantum dynamics is expected to become inadequate.

In this work, we investigate a phenomenological approach in which the evolution of momentum and energy is represented through a cascade-like scaling process. The idea is motivated by the appearance of cascades in nonlinear physical systems, where energy or another conserved quantity is transferred between different scales [2].

We introduce a momentum transport equation

$$\frac{\partial P_i}{\partial t}+H_{\mathrm{c}}x_j\frac{\partial P_i}{\partial x_j}=-\frac{\partial E}{\partial x_i}. \tag{5}$$

This equation should be understood explicitly as a *Euclidean phenomenological advection model*, introduced to capture the kinematic effect of an expanding background within the cascade framework. It is not intended to represent a fully covariant general-relativistic fluid equation, nor is it derived directly from the Einstein field equations. In particular, the coordinates x_i are ordinary spatial coordinates and the term $H_{\mathrm{c}}x_j$ acts as a prescribed expansion-induced advection velocity. The parameter H_{c} is identified with the physical Hubble parameter $H=\dot{R}/R$ [3]. The first term represents local temporal evolution, while the second term describes transport associated with an expanding background.

The equation can be written in terms of a characteristic derivative,

$$\frac{DP_i}{Dt}=\frac{\partial P_i}{\partial t}+H_{\mathrm{c}}x_j\frac{\partial P_i}{\partial x_j}, \tag{6}$$

so that

$$\frac{DP_i}{Dt}=-\frac{\partial E}{\partial x_i}. \tag{7}$$

The objective of this paper is to examine the consequences of combining Equation (7) with a cascade representation of energy and momentum and with a proposed modified uncertainty relation.

Definitions and Physical Conventions

In Equation (5), $P_i(\boldsymbol{x},t)$ denotes a local physical momentum density, rather than the momentum of an individual particle. Its SI units are

$$[P_i] = \mathrm{kg}\cdot\mathrm{m}^{-2}\cdot\mathrm{s}^{-1}. \tag{8}$$

The scalar $E(\boldsymbol{x},t)$ denotes a local physical energy density, with

$$[E] = \mathrm{J}\cdot\mathrm{m}^{-3} = \mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}^{-2}. \tag{9}$$

Thus $\partial P_i/\partial t$ and $-\partial E/\partial x_i$ have the same dimensions. The scale-dependent quantities $P(k)$ and $E(f)$ introduced later are characteristic cascade amplitudes associated with these local fields; they are not single-particle observables. Throughout this work, x_i denotes a physical spatial coordinate and $\partial/\partial x_i$ a derivative with respect to physical distance. The wavenumber used in the cascade laws is the physical wavenumber k_{phys} unless explicitly labeled otherwise. Comoving quantities carry the subscript '*com*'. For the homogeneous Friedmann-Lemaitre-Robertson-Walker (FLRW) background considered here, the characteristic expansion rate is identified with the physical Hubble parameter,

$$H_{\mathrm{c}} \equiv H = \frac{\dot{R}}{R},\ [H] = \mathrm{s}^{-1}. \tag{10}$$

The notation H_{c} is retained to preserve the notation of the proposed transport equation; physically it denotes the same expansion rate as H.

2. Cascade Representation of Energy and Momentum

We assume that energy and momentum can be represented by scale dependent power laws,

$$E = Cf^{-a/b}, \tag{11}$$

and

$$P = Dk^{-r/w}, \tag{12}$$

where C and D are dimensional constants and a, b, r, and w determine the cascade exponents [4]. Here C and D set the dimensional amplitudes (normalizations) of the energy and momentum cascade laws, respectively. Since E denotes an energy density and f has dimensions of inverse time,

$$[C] = [E][f]^{a/b} = \mathrm{J}\cdot\mathrm{m}^{-3}\cdot\mathrm{s}^{-a/b}. \tag{13}$$

Similarly, since P denotes a momentum density and k_{phys} has dimensions of inverse length,

$$[D] = [P][k]^{r/w} = \mathrm{kg}\cdot\mathrm{m}^{-2}\cdot\mathrm{s}^{-1}\cdot\mathrm{m}^{-r/w}. \tag{14}$$

Thus, C and D do not determine the cascade exponents; rather, they provide the dimensional normalization required for the power-law representations of E and P.

For convenience, define

$$\alpha = \frac{a}{b},\ \beta = \frac{r}{w}. \tag{15}$$

Equations (11) and (12) then become

$$E = Cf^{-\alpha}, \tag{16}$$

and

$$P = Dk^{-\beta}. \tag{17}$$

For a wave-like excitation, the wavelength and period are related to wavenumber and frequency by

$$\lambda = \frac{2\pi}{k},\ T = \frac{1}{f}. \tag{18}$$

The frequency may also be expressed in terms of angular frequency,

$$\omega = 2\pi f. \tag{19}$$

For a relativistic excitation,

$$\omega = ck, \tag{20}$$

and therefore

$$2\pi f = ck. \tag{21}$$

3. Expansion of the Early Universe

Let $R(t)$ denote the cosmological scale factor [5]. The physical and comoving wavenumbers [6] are related by

$$k_{\text{phys}} = \frac{k_{\text{com}}}{R(t)}. \tag{22}$$

For a fixed comoving wavenumber,

$$k_{\text{phys}} \propto R^{-1}. \tag{23}$$

Differentiation with respect to time gives

$$\frac{\mathrm{d}k_{\text{phys}}}{\mathrm{d}t} = -\frac{\dot{R}}{R}k_{\text{phys}}. \tag{24}$$

Introducing the Hubble parameter,

$$H = \frac{\dot{R}}{R}, \tag{25}$$

we obtain

$$\boxed{\frac{\mathrm{d}k_{\text{phys}}}{\mathrm{d}t} = -Hk_{\text{phys}}.} \tag{26}$$

Substitution into the momentum cascade relation

$$P = Dk^{-\beta} \tag{27}$$

gives

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\beta Dk^{-\beta-1}\frac{\mathrm{d}k}{\mathrm{d}t} = \beta HDk^{-\beta}. \tag{28}$$

Since $P = Dk_{\text{phys}}^{-\beta}$,

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \beta HP. \tag{29}$$

Consequently,

$$\frac{1}{P}\frac{\mathrm{d}P}{\mathrm{d}t} = \beta H. \tag{30}$$

Integration gives

$$P(t) = P_0\left(\frac{R(t)}{R_0}\right)^{r/w} \tag{31}$$

or, using $\beta = r/w$,

$$P(t) = P_0\left(\frac{R(t)}{R_0}\right)^{\beta}. \tag{32}$$

This relation represents the evolution of the cascade momentum associated with a fixed comoving mode [7].

4. Energy Cascade in an Expanding Background

Suppose the characteristic frequency scales according to

$$f \propto R^{-1}. \tag{33}$$

Using the cascade energy relation,

$$E = Cf^{-\alpha}, \tag{34}$$

we obtain

$$E \propto R^{\alpha}. \tag{35}$$

Therefore,

$$E(t) = E_0\left(\frac{R(t)}{R_0}\right)^{a/b} \tag{36}$$

or

$$E(t) = E_0\left(\frac{R(t)}{R_0}\right)^{\alpha}. \tag{37}$$

Equations (32) and (37) provide the scale-factor evolution predicted by the assumed cascade laws.

Cascade Momentum and Cosmological Redshifting

It is important to distinguish the cascade momentum-density amplitude P_{cas} from the physical momentum p_{phys} of a freely propagating relativistic quantum. For a massless mode,

$$p_{\text{phys}} = \hbar k_{\text{phys}}, \tag{38}$$

and a fixed comoving mode satisfies

$$p_{\text{phys}} \propto k_{\text{phys}} \propto R^{-1}. \tag{39}$$

By contrast, the cascade ansatz gives

$$P_{\text{cas}} \propto k_{\text{phys}}^{-\beta} \propto R^{\beta}. \tag{40}$$

These are different quantities and need not have the same scale-factor dependence. Compatibility with standard redshifting of a freely propagating massless mode requires the cascade quantity to reproduce the same scaling only in the special case $\beta = -1$. Likewise, if the cascade energy is interpreted at the particle level for a relativistic mode, ordinary redshifting gives $E \propto R^{-1}$; with $f \propto R^{-1}$, the cascade energy law reproduces this behavior only for $\alpha = -1$. Consequently, the scale-factor dependence of P_{cas} is not a direct prediction for ordinary particle momentum unless these additional compatibility conditions are imposed.

5. Power-Law Cosmological Epochs

Consider a power-law scale factor,

$$R(t) \propto t^{q}. \tag{41}$$

The Hubble parameter becomes

$$H = \frac{q}{t}. \tag{42}$$

The momentum cascade then becomes

$$P(t) \propto t^{q\beta}. \tag{43}$$

Similarly, the energy cascade becomes

$$E(t) \propto t^{q\alpha}. \tag{44}$$

For a radiation-dominated background [8],

$$R(t) \propto t^{1/2}, \tag{45}$$

so

$$H = \frac{1}{2t}. \tag{46}$$

Consequently,

$$P(t) \propto t^{\beta/2} \tag{47}$$

and

$$E(t) \propto t^{\alpha/2}. \tag{48}$$

These relations should be interpreted as consequences of the assumed cascade scaling and not as a replacement for the Friedmann equations [9].

6. Modified Uncertainty Relations

We now introduce the proposed cascade-based uncertainty relations,

$$\Delta P\, \Delta x = \varepsilon p \lambda, \tag{49}$$

and

$$\Delta E\,\Delta t = \varepsilon ET. \tag{50}$$

Here ε is a dimensionless parameter and λ and T represent characteristic wavelength and period. Equations (49) and (50) serve only as scale-dependent heuristic ansatzes rather than formal expressions derived from canonical field commutators.

Using

$$\lambda = \frac{2\pi}{k},\ T = \frac{1}{f}, \tag{51}$$

the relations become

$$\Delta P\,\Delta x = \varepsilon \frac{2\pi P}{k}, \tag{52}$$

and

$$\Delta E\,\Delta t = \varepsilon \frac{E}{f}. \tag{53}$$

If the characteristic spatial and temporal uncertainties are of the order of one wavelength and one period,

$$\Delta x \sim \lambda,\ \Delta t \sim T, \tag{54}$$

then

$$\Delta P \sim \varepsilon P,\ \Delta E \sim \varepsilon E. \tag{55}$$

Thus,

$$\frac{\Delta P}{\Delta t} \sim \varepsilon P f \tag{56}$$

and

$$\frac{\Delta E}{\Delta x} \sim \varepsilon \frac{Ek}{2\pi}. \tag{57}$$

The proposed characteristic balance

$$\frac{\Delta P}{\Delta t} \sim \frac{\Delta E}{\Delta x} \tag{58}$$

therefore requires

$$Pf \sim \frac{Ek}{2\pi}. \tag{59}$$

For a relativistic mode satisfying

$$2\pi f = ck, \tag{60}$$

Equation (59) gives

$$E \sim cP. \tag{61}$$

Thus, the characteristic form of the proposed uncertainty relations is compatible with the relativistic energy-momentum relation [10].

7. Consistency of the Cascade Exponents

We can now compare the cascade expressions for energy and momentum.

From

$$E = Cf^{-a/b} \tag{62}$$

and

$$P = Dk^{-r/w}, \tag{63}$$

the relativistic relation $E \simeq cP$ requires

$$Cf^{-a/b} \sim cDk^{-r/w}. \tag{64}$$

Since

$$f \propto k \tag{65}$$

for relativistic modes, we obtain

$$k^{-a/b} \propto k^{-r/w}. \tag{66}$$

However, if the characteristic balance is written directly as

$$Pf \sim \frac{Ek}{2\pi}, \tag{67}$$

then

$$E \sim \frac{2\pi f}{k} P. \tag{68}$$

Using $2\pi f = ck$ gives again

$$E \sim cP. \tag{69}$$

If, rather than relativistic behavior, the cascade spectra are interpreted via a non-relativistic dispersion process, Equation (65) can take the general cascade coupling mechanism and transits into a linkage of $f \propto k^{a/(a+b)}$. The result is

$$\frac{a}{b} = 1 + \frac{r}{w}. \tag{70}$$

Equation (70) is therefore a candidate constraint connecting the energy and momentum cascade exponents. Whether this relation survives a fundamental derivation requires specifying the underlying cascade dynamics.

8. Momentum Transport Equation in the Expanding Universe

The proposed transport equation is

$$\frac{\partial P_i}{\partial t} + H_c x_j \frac{\partial P_i}{\partial x_j} = -\frac{\partial E}{\partial x_i}. \tag{71}$$

The constant H_c introduced in the present transport model should be distinguished from the physical, time-dependent Hubble parameter $(H(t))$. By contrast, H_c is a constant characteristic expansion rate introduced phenomenologically in the advection operator ($H_c x_j \partial / \partial x_j$). It therefore represents a local or characteristic expansion rate associated with the proposed transport process, rather than the instantaneous cosmological expansion rate. The use of a constant

H_c allows the characteristic-scale analysis to isolate the momentum-transfer mechanism without requiring a specific cosmological evolution for $R(t)$. Consequently, H_c should not be identified with $H(t)$ unless an additional physical model is introduced that establishes such a correspondence. For an approximately isotropic configuration, we may consider a radial characteristic scale x and write

$$H_c x_j \frac{\partial P_i}{\partial x_j} \sim H_c x \frac{\partial P_i}{\partial x}. \tag{72}$$

The equation becomes

$$\frac{\partial P}{\partial t} + H_c x \frac{\partial P}{\partial x} = -\frac{\partial E}{\partial x}. \tag{73}$$

At the characteristic scale

$$x \sim \lambda, \tag{74}$$

we estimate

$$\frac{\partial P}{\partial x} \sim \frac{P}{\lambda}, \quad \frac{\partial E}{\partial x} \sim \frac{E}{\lambda}. \tag{75}$$

Hence,

$$\frac{DP}{Dt} \sim -\frac{E}{\lambda}. \tag{76}$$

For relativistic modes, $E \simeq cP$, giving

$$\frac{DP}{Dt} \sim -\frac{cP}{\lambda}. \tag{77}$$

Since

$$\lambda = \frac{2\pi}{k}, \tag{78}$$

we have

$$\frac{DP}{Dt} \sim -\frac{ck}{2\pi} P. \tag{79}$$

Using

$$f = \frac{ck}{2\pi}, \tag{80}$$

the characteristic equation becomes

$$\frac{DP}{Dt} \sim -fP. \tag{81}$$

This suggests a characteristic cascade timescale

$$\tau_{\text{cascade}} \sim \frac{1}{f}. \tag{82}$$

Thus, within this phenomenological picture, the characteristic cascade transfer time is of the same order as the wave period.

9. Planck-Scale Limit

The Planck quantities are

$$t_P = \sqrt{\frac{\hbar G}{c^5}}, \tag{83}$$

$$l_P = \sqrt{\frac{\hbar G}{c^3}}, \tag{84}$$

$$E_P = \sqrt{\frac{\hbar c^5}{G}}, \tag{85}$$

and

$$P_P = \sqrt{\frac{\hbar c^3}{G}}. \tag{86}$$

These quantities satisfy

$$E_P = cP_P, \tag{87}$$

and

$$E_P t_P = \hbar, \;\; P_P l_P = \hbar. \tag{88}$$

The proposed uncertainty relations therefore become, at the Planck scale,

$$\Delta P\,\Delta x \sim \varepsilon P_P l_P = \varepsilon\hbar, \tag{89}$$

and

$$\Delta E\,\Delta t \sim \varepsilon E_P t_P = \varepsilon\hbar. \tag{90}$$

Consequently,

$$\Delta P\,\Delta x \sim \Delta E\,\Delta t \sim \varepsilon\hbar. \tag{91}$$

This result shows that the proposed relations naturally recover a Planck-scale quantity with dimensions of action. It does not, however, by itself establish that the standard Planck constant is fundamentally replaced.

10. Magnitude of the Expansion Term at the Planck Epoch

For a radiation-dominated universe,

$$R(t) \propto t^{1/2}, \tag{92}$$

and hence

$$H = \frac{1}{2t}. \tag{93}$$

At the Planck epoch,

$$t \sim t_P, \tag{94}$$

so

$$H \sim \frac{1}{2t_P}. \tag{95}$$

Consider a characteristic Planck-scale spatial variation,

$$x \sim l_P, \ \frac{\partial P}{\partial x} \sim \frac{P_P}{l_P}. \tag{96}$$

The expansion term is then

$$Hx\frac{\partial P}{\partial x} \sim Hl_P \frac{P_P}{l_P} \sim \frac{P_P}{2t_P}. \tag{97}$$

Using

$$l_P = ct_P, \ E_P = cP_P, \tag{98}$$

we find

$$\frac{E_P}{l_P} = \frac{cP_P}{ct_P} = \frac{P_P}{t_P}. \tag{99}$$

Therefore,

$$Hx\frac{\partial P}{\partial x} \sim \frac{1}{2}\frac{E_P}{l_P}. \tag{100}$$

Thus the cosmological expansion contribution and the energy-gradient contribution are of comparable order at the Planck scale.

This provides a dimensional motivation for investigating an equation in which expansion transport and energy gradients appear at the same characteristic order.

11. Dimensionless Planck-Scale Equation

For clarity, the P_P and E_P used in this section are characteristic normalization scales for the local momentum and energy fields. They are not the single-particle Planck momentum and energy introduced in the preceding section. A strict density normalization would require corresponding Planck momentum-density and energy-density scales; here we retain the original normalization convention $E_P = P_P l_P / t_P$ solely to obtain the dimensionless characteristic equation.

Introduce Planck-normalized variables

$$\tilde{t} = \frac{t}{t_P}, \ \tilde{x} = \frac{x}{l_P}, \tag{101}$$

$$\tilde{P} = \frac{P}{P_P}, \ \tilde{E} = \frac{E}{E_P}. \tag{102}$$

Since

$$E_P = \frac{P_P l_P}{t_P}, \tag{103}$$

the governing equation becomes

$$\frac{\partial \tilde{P}_i}{\partial \tilde{t}} + \tilde{H}_c \tilde{x}_j \frac{\partial \tilde{P}_i}{\partial \tilde{x}_j} = -\frac{\partial \tilde{E}}{\partial \tilde{x}_i}, \tag{104}$$

where

$$\tilde{H}_c = H_c t_P. \tag{105}$$

At the Planck epoch,

$$H_c t_P = O(1), \tag{106}$$

so the dimensionless equation is approximately

$$\frac{\partial \tilde{P}_i}{\partial \tilde{t}} + O(1)\tilde{x}_j \frac{\partial \tilde{P}_i}{\partial \tilde{x}_j} = -\frac{\partial \tilde{E}}{\partial \tilde{x}_i}. \tag{107}$$

Equation (107) provides a convenient starting point for investigating the proposed dynamics near the Planck scale.

12. Physical Interpretation

The proposed model can be interpreted as a coupling between three processes.

First, the term

$$\frac{\partial P_i}{\partial t} \tag{108}$$

describes local temporal evolution of the momentum field.

Second, the expansion term

$$H_c x_j \frac{\partial P_i}{\partial x_j} \tag{109}$$

describes the change associated with the stretching of physical length scales by cosmological expansion.

Third, the term

$$-\frac{\partial E}{\partial x_i} \tag{110}$$

represents a momentum change produced by a spatial energy gradient. As such, the resulting equation has the schematic structure [11]. The cascade hypothesis adds a scale-dependent structure to E and P. Consequently, the model connects cosmological expansion with the transfer of momentum and energy between characteristic scales.

13. Discussion

The results obtained above suggest several potentially important features of the cascade model.

The first is that an expanding universe naturally produces a time-dependent physical wavenumber,

$$k_{\text{phys}} \propto R^{-1}. \tag{111}$$

When this relation is inserted into the proposed momentum cascade, the momentum evolves according to

$$P \propto R^{\beta}. \tag{112}$$

The second feature is that a frequency cascade of the form

$$E = Cf^{-\alpha} \tag{113}$$

leads to

$$E \propto R^{\alpha} \tag{114}$$

when $f \propto R^{-1}$.

The third feature is the connection between the two modified uncertainty relations. When their characteristic spatial and temporal scales are identified with λ and T, respectively, one obtains

$$\frac{\Delta P}{\Delta t} \sim \frac{\Delta E}{\Delta x}. \tag{115}$$

For relativistic modes this gives

$$E \sim cP. \tag{116}$$

The Planck-scale normalization is also suggestive. At the Planck scale,

$$P_P l_P = \hbar \tag{117}$$

and

$$E_P t_P = \hbar. \tag{118}$$

Therefore the proposed uncertainty relations reproduce an action scale of order $\hbar$ when evaluated at Planck scales.

However, this observation should not be interpreted as a proof that $\hbar$ is emergent or that the conventional uncertainty principle is invalid. Such a conclusion would require a fundamental derivation from a microscopic theory.

Similarly, the replacement of the conventional Friedmann dynamics by Equation (5) is not justified by dimensional analysis alone. A complete theory would need to specify the underlying action, degrees of freedom, conservation laws, and coupling to the spacetime metric.

14. Limitations and Required Theoretical Development

The present formulation is phenomenological. Several additional steps are required before it can be considered a fundamental model of the early Universe.

First, the equation

$$\frac{DP_i}{Dt} = -\frac{\partial E}{\partial x_i} \tag{119}$$

should be generalized to a covariant formulation in curved spacetime.

Second, the identification $H_c = H$ is a modeling convention for the homogeneous FLRW background and should be generalized or derived in a covariant formulation.

Third, the cascade exponents a/b and r/w need to be obtained from an underlying nonlinear interaction mechanism.

Fourth, the energy and momentum cascade should be formulated in terms of a well-defined spectral transfer function. This would allow the model to distinguish between a genuine transfer of energy between scales and simple redshifting caused by cosmological expansion.

Fifth, if the model is intended to describe the Planck epoch, it must ultimately be compatible with quantum gravity. In particular, the stress-energy tensor and the geometry of spacetime cannot generally be treated as independent classical quantities at the Planck scale.

Finally, the proposed uncertainty relations must be tested against the standard Heisenberg relations in regimes where conventional quantum mechanics is experimentally established.

15. Conclusions

A phenomenological cascade formulation for momentum and energy in an expanding early Universe has been developed. The central equation is

$$\frac{\partial P_i}{\partial t}+H_c x_j\frac{\partial P_i}{\partial x_j}=-\frac{\partial E}{\partial x_i}. \tag{120}$$

The cascade assumptions

$$E=Cf^{-\alpha},\ \ P=Dk^{-\beta} \tag{121}$$

combined with

$$k\propto R^{-1} \tag{122}$$

lead to

$$P\propto R^{\beta}. \tag{123}$$

If the characteristic frequency scales as

$$f\propto R^{-1}, \tag{124}$$

then

$$E\propto R^{\alpha}. \tag{125}$$

The proposed uncertainty relations,

$$\Delta P\,\Delta x=\varepsilon p\lambda,\ \ \Delta E\,\Delta t=\varepsilon ET, \tag{126}$$

lead, at the characteristic scale, to

$$\frac{\Delta P}{\Delta t}\sim\frac{\Delta E}{\Delta x}. \tag{127}$$

For relativistic modes this is compatible with

$$E\simeq cP. \tag{128}$$

At the Planck scale,

$$P_P l_P=E_P t_P=\hbar, \tag{129}$$

so that the proposed uncertainty products naturally acquire the Planck action scale.

The resulting framework suggests a possible connection between cosmological expansion, scale-dependent cascade dynamics, and Planck-scale uncertainty.

Nevertheless, the present model remains phenomenological. Establishing it as a fundamental theory requires a covariant derivation, an underlying microscopic cascade mechanism, and consistency with general relativity and quantum mechanics.

Author Contributions

Conceptualization; methodology; investigation; writing original draft preparation: Asghar Noormohammadi.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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