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Flexible Static Cosmology

Michael Aaron Cody

Independent Theorist, Florida, USA
Email: Mac92Contact@gmail.com

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Abstract

This is the fourth paper in the $K(t)$ series and introduces the broader framework as Flexible Static Cosmology (FLSC). The framework was first introduced as the operational redshift kernel $K(t)$, which reproduced the classical background redshift relations within a flexible static ontology and preserved a Planck spectrum exactly under collisionless transport. This paper supplies the local gravitational closure in ordinary general relativity. Under the stated bulk hypotheses, compatibility between the $K(t)$ transport law, null-geodesic photon propagation, and covariant number conservation forces geodesic, shear-free deformation. The spatial metric factorizes as $h_{ij} = K^2\gamma_{ij}$, with $\theta = 3\dot{K}/K$, and the spatial Einstein equations require constant sectional curvature. Robertson-Walker geometry is therefore derived step by step rather than assumed. The same deformation rate gives $pK = \text{constant}$, $T \propto K^{-1}$, $n_\gamma \propto K^{-3}$, and $\rho_\gamma \propto K^{-4}$.

Keywords

Cosmological Redshift, General Relativity, Relativistic Elasticity, Kinetic Theory, Finite Substrate, Robertson-Walker Geometry

1. Introduction

The observational definition of cosmological redshift is

$$1 + z \equiv \frac{\nu_e}{\nu_0}. \tag{1}$$

Earlier work introduced an operational kernel $K(t)$ satisfying

$$1 + z = \frac{K(t_0)}{K(t_e)}, \tag{2}$$

and showed that the classical background relations for time dilation, Tolman surface-brightness dimming, luminosity distance, and blackbody transport can be reproduced within a flexible static ontology [1]-[3]. Here, "static" does not mean rigid, inert, or incapable of deformation. It means that the cosmological evolution is not interpreted as the expansion of space itself. The kernel $K(t)$ instead represents a universal cumulative deformation and frequency-transport law within a finite material domain, while the global exterior geometry remains open.

The preceding papers established the kinematic and kinetic sectors of the framework. Frequency-independent collisionless transport preserves a Planck photon distribution exactly and yields

$$pK = \text{constant}, \quad T \propto K^{-1}, \quad n_\gamma \propto K^{-3}, \quad \rho_\gamma \propto K^{-4}, \tag{3}$$

while leaving $\mu/(k_B T)$ invariant. Those results showed that background redshift and CMB thermodynamics do not uniquely require an expanding-space ontology. They did not establish whether the same kernel could be embedded consistently in ordinary general relativity or whether the kinetic rate represented an independent photon process or the phase-space expression of material deformation. The present paper supplies that gravitational closure. Rather than imposing Robertson-Walker geometry at the outset, the analysis begins from a finite, diagonal, comoving, spherically symmetric material domain. The material sector is represented by three comoving scalar fields with a diffeomorphism-invariant stored-energy action, while the domain boundary is retained as an undetermined global datum that does not enter the local closure.

The central result is that, under explicit bulk conditions, compatibility between the independently established $K(t)$ transport law, collisionless general-relativistic photon propagation, and covariant number conservation forces geodesic, shear-free deformation of the observable interior. The spatial metric then factorizes with a single deformation ratio $K(t)$, and the spatial Einstein equations require the time-independent metric to have constant sectional curvature. Robertson-Walker geometry is therefore derived from the $K(t)$-GR closure rather than assumed as the starting ansatz.

In the resulting bulk,

$$J \propto K^3, \quad \theta = 3\frac{\dot{K}}{K}, \tag{4}$$

and the collisionless general-relativistic Boltzmann equation reduces exactly to the earlier kinetic transport equation. The photon scalings, Planck preservation, constant baryon-to-photon ratio, and local pressure-work conservation then follow without a second redshift mechanism or an accumulated energy reservoir.

The resulting observable bulk has Robertson-Walker form, but it is not introduced as an FLRW starting geometry or as a reinterpretation added after the fact. It is derived from a finite-material general-relativistic construction whose primitive elements are the material fields, the covariant substrate action, the universal $K(t)$ transport law, and the finite material domain. Within this framework, $K(t)$ is the physical deformation ratio of the substrate, and the Robertson-

Walker bulk is the geometry required by the closure conditions in the observable interior. The boundary radius, transition layer, exterior geometry, and complete global matching solution remain open, while the local results derived below are independent of those unresolved global choices.

Kinetic Transport from the Operational Kernel

Before introducing a spacetime geometry or material field equations, the kinetic content of the $K(t)$ framework can be stated independently. These transport relations were established in the preceding papers of the $K(t)$ series [1]-[3]; they are re-derived here so that the kinetic input to the general-relativistic closure is explicit and self-contained. No Robertson-Walker geometry, Einstein equation, spatial-volume law, or material constitutive function is used in the derivation below.

The operational redshift relation is

$$1+z=\frac{\nu_e}{\nu_0}=\frac{K_0}{K_e},$$

where

$$K_0 \equiv K(t_0), \quad K_e \equiv K(t_e).$$

The continuous, universal, frequency-independent kinetic transport law established in the preceding $K(t)$ work is taken here as

$$\frac{\dot{\nu}}{\nu}=-\frac{\dot{K}}{K}.$$

For photons,

$$p=\frac{h\nu}{c},$$

so

$$\frac{\dot{p}}{p}=\frac{\dot{\nu}}{\nu}=-\frac{\dot{K}}{K}.$$

Define

$$H_K \equiv \frac{\dot{K}}{K}.$$

Then

$$\frac{\dot{p}}{p}=-H_K, \quad \dot{p}=-H_K p.$$

Here t is the time parameter of the operational kernel. Its identification with proper time along the material flow is established later by the general-relativistic closure. Let $f(t,p)$ denote the occupation-number distribution of the homogeneous and isotropic photon background in local physical momentum space. Collisionless phase-space transport under the prescribed kinetic redshift flow requires the occupation number to remain constant along its characteristics:

$$\frac{\mathrm{d}f}{\mathrm{d}t}=0.$$

Since $f=f\left(t,p\right)$,

$$\frac{\mathrm{d}f}{\mathrm{d}t}=\frac{\partial f}{\partial t}+\frac{\mathrm{d}p}{\mathrm{d}t}\frac{\partial f}{\partial p}.$$

Using $\dot{p}=-H_K p$,

$$\frac{\partial f}{\partial t}-H_K p\frac{\partial f}{\partial p}=0.$$

This is the $K\left(t\right)$ kinetic transport equation. The characteristic equations are

$$\frac{\mathrm{d}t}{\mathrm{d}s}=1,\quad \frac{\mathrm{d}p}{\mathrm{d}s}=-H_K p,\quad \frac{\mathrm{d}f}{\mathrm{d}s}=0,$$

where s parameterizes the reduced phase-space characteristics. Since $\mathrm{d}t/\mathrm{d}s=1$,

$$\frac{\mathrm{d}p}{\mathrm{d}t}=-H_K p=-\frac{\dot{K}}{K}p.$$

Dividing by p gives

$$\frac{\mathrm{d}\ln p}{\mathrm{d}t}=-\frac{\mathrm{d}\ln K}{\mathrm{d}t}.$$

Integrating from a reference epoch t_i to t gives

$$\int_{p_i}^{p}\mathrm{d}\ln p'=-\int_{K_i}^{K}\mathrm{d}\ln K',$$

$$\ln\left(\frac{p}{p_i}\right)=-\ln\left(\frac{K}{K_i}\right),$$

$$\ln\left(\frac{pK}{p_iK_i}\right)=0,$$

and therefore

$$pK=p_iK_i.$$

Thus

$$pK=\text{constant}$$

along each kinetic trajectory. Solving for the initial momentum gives

$$p_i=\frac{K\left(t\right)}{K_i}p.$$

The condition $\mathrm{d}f/\mathrm{d}s=0$ gives

$$f\left(t,p\right)=f_i\left(p_i\right),$$

so

$$f\left(t,p\right)=f_i\left(\frac{K\left(t\right)}{K_i}p\right).$$

No assumption concerning the functional form of f_i has been made. The photon-number scaling follows directly from the momentum-space measure. Because the prescribed frequency map acts identically on all momentum directions, define

$$\boldsymbol{q} \equiv \frac{K}{K_i}\boldsymbol{p}.$$

Componentwise,

$$q_a = \frac{K}{K_i}p_a, \quad \frac{\partial q_a}{\partial p_b} = \frac{K}{K_i}\delta_{ab}.$$

Therefore

$$\det\left(\frac{\partial q_a}{\partial p_b}\right) = \det\left(\frac{K}{K_i}\delta_{ab}\right) = \left(\frac{K}{K_i}\right)^3.$$

Hence

$$\mathrm{d}^3 q = \left(\frac{K}{K_i}\right)^3 \mathrm{d}^3 p,$$

or equivalently

$$\mathrm{d}^3 p = \left(\frac{K_i}{K}\right)^3 \mathrm{d}^3 q.$$

Here $\boldsymbol{p}$ and $\boldsymbol{q}$ denote the three-dimensional physical momentum vectors, while $p = |\boldsymbol{p}|$ and $q = |\boldsymbol{q}|$ denote their magnitudes. Because the prescribed map rescales every momentum component by the same factor, $q = (K/K_i)p$. For the isotropic distribution, $f_i(\boldsymbol{q}) = f_i(q)$.

For photon degeneracy $g_\gamma = 2$, the local photon number density is

$$n_\gamma(t) = \frac{g_\gamma}{(2\pi\hbar)^3}\int f(t,\boldsymbol{p})\,\mathrm{d}^3 p.$$

Substituting the characteristic solution and the momentum-space Jacobian gives

$$\begin{aligned} n_\gamma(t) &= \frac{g_\gamma}{(2\pi\hbar)^3}\int f_i(\boldsymbol{q})\left(\frac{K_i}{K}\right)^3 \mathrm{d}^3 q \\ &= \left(\frac{K_i}{K}\right)^3 \frac{g_\gamma}{(2\pi\hbar)^3}\int f_i(\boldsymbol{q})\,\mathrm{d}^3 q \\ &= n_{\gamma i}\left(\frac{K_i}{K}\right)^3. \end{aligned}$$

Therefore

$$n_\gamma \propto K^{-3}.$$

Taking a logarithmic derivative,

$$\frac{\dot{n}_\gamma}{n_\gamma} = -3\frac{\dot{K}}{K} = -3H_K,$$

so

$$\dot{n}_\gamma = -3H_K n_\gamma.$$

The scaling $n_\gamma \propto K^{-3}$ is a momentum-space result. The factor of three comes from the determinant of the three-dimensional momentum transformation. No spatial-volume law, expansion scalar, Robertson-Walker metric, Einstein equation, or relation of the form $V \propto K^3$ has been used.

The photon energy density is

$$\rho_\gamma(t) = \frac{g_\gamma}{(2\pi\hbar)^3}\int pc f(t,\boldsymbol{p})\,\mathrm{d}^3 p.$$

Since

$$p = \frac{K_i}{K}q,$$

the same change of variables gives

$$\begin{aligned}\rho_\gamma(t) &= \frac{g_\gamma}{(2\pi\hbar)^3}\int\left(\frac{K_i}{K}q\right)c f_i(\boldsymbol{q})\left(\frac{K_i}{K}\right)^3 \mathrm{d}^3 q\\ &= \left(\frac{K_i}{K}\right)^4 \frac{g_\gamma}{(2\pi\hbar)^3}\int qc f_i(\boldsymbol{q})\,\mathrm{d}^3 q\\ &= \rho_{\gamma i}\left(\frac{K_i}{K}\right)^4.\end{aligned}$$

Thus

$$\rho_\gamma \propto K^{-4}.$$

Taking the logarithmic derivative,

$$\frac{\dot{\rho}_\gamma}{\rho_\gamma} = -4\frac{\dot{K}}{K} = -4H_K,$$

and hence

$$\dot{\rho}_\gamma = -4H_K\rho_\gamma.$$

The fourth power consists of three powers from the momentum-space Jacobian and one additional power from the photon energy $E = pc$. The same characteristic solution determines the evolution of a thermal distribution. Let the initial photon distribution be Bose-Einstein,

$$f_i(p) = \frac{1}{\exp\left[(pc-\mu_i)/(k_B T_i)\right]-1}.$$

The characteristic solution gives

$$f(t,p) = \frac{1}{\exp\left[\left(\frac{K}{K_i}pc-\mu_i\right)\Big/(k_B T_i)\right]-1}.$$

The exponent may be rewritten as

$$\frac{\frac{K}{K_i}pc-\mu_i}{k_B T_i} = \frac{\frac{K}{K_i}\left[pc-\mu_i\frac{K_i}{K}\right]}{k_B T_i} = \frac{pc-\mu_i\frac{K_i}{K}}{k_B T_i\frac{K_i}{K}}.$$

Define

$$T(t)=T_i\frac{K_i}{K(t)},\quad \mu(t)=\mu_i\frac{K_i}{K(t)}.$$

The evolved distribution is then

$$f(t,p)=\frac{1}{\exp\left[\left(pc-\mu(t)\right)/\left(k_B T(t)\right)\right]-1}.$$

Thus

$$T\propto K^{-1},\quad \mu\propto K^{-1}.$$

Moreover,

$$\frac{\mu(t)}{k_B T(t)}=\frac{\mu_i(K_i/K)}{k_B T_i(K_i/K)}=\frac{\mu_i}{k_B T_i},$$

so

$$\frac{\mu}{k_B T}=\text{constant}.$$

For an initially Planckian spectrum,

$$\mu_i=0.$$

Therefore

$$\mu(t)=\mu_i\frac{K_i}{K(t)}=0.$$

The spectrum remains exactly Planckian during the collisionless transport, with

$$T(t)=T_i\frac{K_i}{K(t)}.$$

Between an emission epoch and the present,

$$\frac{T_e}{T_0}=\frac{K_0}{K_e}.$$

Using the operational redshift relation,

$$T(z)=T_0(1+z).$$

The results of this section are kinetic consequences of the universal, frequency-independent $K(t)$ transport law:

$$pK=\text{constant},\quad n_\gamma\propto K^{-3},\quad \rho_\gamma\propto K^{-4},\quad T\propto K^{-1},\quad \frac{\mu}{k_B T}=\text{constant}.$$

These results have been obtained entirely from momentum-space transport, without assuming a Robertson-Walker geometry, invoking the Einstein equations, introducing an expansion scalar, imposing a spatial-volume scaling, or specifying a material constitutive law. The kinetic relations derived above were established in the earlier $K(t)$ work. In the CMB analysis, the collisionless transport results themselves were obtained from first principles, while a scalar quantity Φ was introduced separately as a phenomenological way to represent the post-recombination photon-energy ledger [2]. That auxiliary construction was not given a gravitational or field-theoretic completion. The present paper takes up that unresolved question directly by asking whether ordinary general relativity, together with the deformation of a finite material substrate, can provide a self-consistent realization of the same $K(t)$ transport law. In the construction developed below, the photon energy equation closes through covariant pressure work, so the separate compensating reservoir introduced in the earlier phenomenological realization is no longer required.

2. Covariant Finite-Material Construction

The finite-material construction begins with the global domain. The material universe is taken to occupy a finite range of a comoving material coordinate,

$$0 \le r \le r_b, \quad \Sigma : r = r_b, \tag{5}$$

where r_b is constant along the material flow. The general diagonal comoving spherically symmetric bulk line element is

$$\mathrm{d}s^2 = -N^2(t,r)\mathrm{d}t^2 + B^2(t,r)\mathrm{d}r^2 + R^2(t,r)\mathrm{d}\Omega^2. \tag{6}$$

The areal radius of the material boundary is

$$R_b(t) = R(t,r_b). \tag{7}$$

The value of R_b is retained as an undetermined global datum. It is not fitted to any observation, is not used to generate $K(t)$, and does not enter the local conservation closure. The observable region, including the last-scattering surface, is assumed to lie within the bulk domain. The transition layer, exterior geometry, and complete global junction problem remain unspecified. The local results derived below are independent of those unresolved global choices.

The comoving material four-velocity is

$$u^\mu = \left(\frac{1}{N}, 0, 0, 0\right), \quad u^\mu u_\mu = -1. \tag{8}$$

The spatial projection tensor is

$$h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu. \tag{9}$$

The material sector is then specified covariantly. A covariant continuum description of the substrate is supplied by three scalar fields $\varphi^I(x)$, $I = 1,2,3$, which label material elements [4]-[7]. Define

$$B^{IJ} = g^{\mu\nu}\partial_\mu\varphi^I\partial_\nu\varphi^J, \quad b \equiv \sqrt{\det B^{IJ}}, \quad J \equiv b^{-1}. \tag{10}$$

Here J is the physical volume per unit reference material volume. The identically conserved material current is

$$\mathcal{J}^\mu = \frac{1}{6}\epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\partial_\alpha\varphi^I\partial_\beta\varphi^J\partial_\gamma\varphi^K, \quad \nabla_\mu\mathcal{J}^\mu = 0, \tag{11}$$

where $\epsilon^{\mu\alpha\beta\gamma}$ is the spacetime Levi-Civita tensor and ϵ_{IJK} is the alternating symbol on material space. The current may be written as

$$\mathcal{J}^{\mu} = bu^{\mu}. \tag{12}$$

The bulk action is

$$S_{\text{bulk}} = \int_{\mathcal{M}_{\text{bulk}}} \mathrm{d}^4 x \sqrt{-g} \left[\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_m + \mathcal{L}_\gamma - \rho_s(b) \right]. \tag{13}$$

No bare cosmological constant is included. Any vacuum-like contribution must arise from the constitutive law $\rho_s(b)$. A separate surface or matching action may be introduced in a complete global construction, but no particular boundary dynamics are required for the bulk result established here.

Variation of the substrate action with respect to the metric gives

$$T^{(s)}_{\mu\nu} = (\rho_s + P_s) u_\mu u_\nu + P_s g_{\mu\nu}, \tag{14}$$

where

$$P_s = b \frac{\mathrm{d}\rho_s}{\mathrm{d}b} - \rho_s. \tag{15}$$

Writing the stored energy per reference volume as

$$W(J) = J\rho_s(J), \tag{16}$$

one obtains

$$\rho_s = \frac{W(J)}{J}, \quad P_s = -\frac{\mathrm{d}W}{\mathrm{d}J}, \quad w_s \equiv \frac{P_s}{\rho_s} = -\frac{JW'(J)}{W(J)}. \tag{17}$$

The algebra follows directly:

$$\rho_s(b) = bW(1/b), \tag{18}$$

$$\frac{\mathrm{d}\rho_s}{\mathrm{d}b} = W(J) - JW'(J), \tag{19}$$

$$b\frac{\mathrm{d}\rho_s}{\mathrm{d}b} - \rho_s = -W'(J). \tag{20}$$

The Einstein equations are

$$G_{\mu\nu} = 8\pi G \left(T^{(m)}_{\mu\nu} + T^{(\gamma)}_{\mu\nu} + T^{(s)}_{\mu\nu} \right). \tag{21}$$

Diffeomorphism invariance gives

$$\nabla_\mu T^{\mu\nu}_{\text{tot}} = 0 \tag{22}$$

once the field equations hold. No weighted stress tensor, manually suppressed gravitational coupling, or external energy reservoir is introduced. The general bulk kinematics follow directly from the comoving spherical geometry. Define the radial and transverse deformation rates by

$$H_r = \frac{1}{N}\frac{\dot{B}}{B}, \quad H_\perp = \frac{1}{N}\frac{\dot{R}}{R}. \tag{23}$$

The congruence expansion is

$$\theta = \nabla_\mu u^\mu = H_r + 2H_\perp. \tag{24}$$

For the comoving spherical congruence, the shear eigenvalues are

$$\sigma_r = \frac{2}{3}\left(H_r - H_\perp\right), \quad \sigma_\perp = -\frac{1}{3}\left(H_r - H_\perp\right). \tag{25}$$

Hence

$$\sigma_{\mu\nu} = 0 \iff H_r = H_\perp. \tag{26}$$

The four-acceleration is

$$a_\mu = u^\nu \nabla_\nu u_\mu = D_\mu \ln N, \tag{27}$$

where

$$D_\mu \equiv h_\mu{}^\nu \nabla_\nu \tag{28}$$

is the spatially projected derivative. In particular,

$$a_r = \partial_r \ln N. \tag{29}$$

The total bulk stress tensor is decomposed relative to u^μ as

$$T^{\text{tot}}_{\mu\nu} = \rho_{\text{tot}} u_\mu u_\nu + P_{\text{tot}} h_{\mu\nu} + 2u_{(\mu} q_{\nu)} + \pi_{\mu\nu}, \tag{30}$$

where

$$q_\mu u^\mu = 0, \quad \pi_{\mu\nu} u^\nu = 0, \quad \pi^\mu{}_\mu = 0. \tag{31}$$

The bulk closure considered below assumes

$$q_\mu = 0, \quad \pi_{\mu\nu} = 0. \tag{32}$$

These conditions hold for the isotropic substrate action $\rho_s(b)$ together with comoving perfect-fluid matter and radiation. The closure conditions can now be stated explicitly. The derivation of the observable bulk uses the following hypotheses:

(H1) The observable interior admits a universal, direction-independent time kernel $K(t)$ along the material flow.

(H2) The homogeneous isotropic photon background propagates collisionlessly on null geodesics, with comoving number current

$$N^\mu_{(\gamma)} = n_\gamma u^\mu, \tag{33}$$

satisfying

$$\nabla_\mu N^\mu_{(\gamma)} = 0. \tag{34}$$

(H3) The total pressure is spatially homogeneous in the observable bulk:

$$D_\mu P_{\text{tot}} = 0. \tag{35}$$

(H4) The bulk source has vanishing comoving energy flux and anisotropic stress:

$$q_\mu = 0, \quad \pi_{\mu\nu} = 0. \tag{36}$$

(H5) The inertial density is nonvanishing:

$$\rho_{\text{tot}} + P_{\text{tot}} \neq 0. \tag{37}$$

In (H1), t denotes the time parameter along the material flow; its identification with a common proper-time coordinate follows from $a_\mu = 0$, derived below from (H3) and (H5). The direction independence asserted in (H1) is an observational input rather than an imposed geometric symmetry, since the kernel was constrained in the earlier work by the background redshift, temperature, and spectral data [1]-[3], none of which was derived from Robertson-Walker geometry.

These hypotheses define the regime in which a universal time-only transport kernel can consistently describe the same photon population as the general-relativistic Liouville equation. Hypothesis (H3) constrains one scalar property of the bulk source, namely the spatial pressure gradient. It does not assume homogeneity or isotropy of the metric, which remains the general comoving spherically symmetric geometry of Equation (6) until the field equations and transport closure are applied.

The momentum equation first determines the acceleration. The spatial projection of total stress-energy conservation gives

$$h^{\alpha}{}_{\nu}\nabla_{\mu}T^{\mu\nu}_{\text{tot}}=0.$$

Under Equation (32), the total stress tensor has perfect-fluid form, and the spatial projection reduces to

$$\left(\rho_{\text{tot}}+P_{\text{tot}}\right)a_{\mu}+D_{\mu}P_{\text{tot}}=0. \tag{38}$$

Indeed, the projection annihilates terms parallel to u^{μ}, while $u^{\nu}\nabla_{\nu}u_{\mu}=a_{\mu}$ and $h_{\mu}{}^{\nu}\nabla_{\nu}P_{\text{tot}}=D_{\mu}P_{\text{tot}}$. Using hypotheses (H3) and (H5),

$$D_{\mu}P_{\text{tot}}=0,\quad \rho_{\text{tot}}+P_{\text{tot}}\neq 0,$$

and therefore

$$a_{\mu}=0. \tag{39}$$

For the comoving spherical metric, Equation (27) gives

$$a_r=\partial_r\ln N=0.$$

Hence

$$N=N\left(t\right). \tag{40}$$

The time coordinate may therefore be reparameterized along the material flow to absorb the lapse. We henceforth choose the comoving proper-time gauge and continue to denote the resulting time coordinate by t. Thus

$$N=1. \tag{41}$$

From this point onward an overdot denotes differentiation with respect to proper time t. Compatibility between the independently established kinetic transport law and general-relativistic photon propagation now constrains the radial deformation rate. Let p^{μ} be a future-directed photon four-momentum and write

$$p^{\mu}=p\left(u^{\mu}+e^{\mu}\right),\quad u_{\mu}e^{\mu}=0,\quad e_{\mu}e^{\mu}=1,$$

where

$$p=-u_{\mu}p^{\mu}$$

is the momentum measured by the comoving material observers—the same local physical photon momentum used in the kinetic construction of Section 1.1, now expressed covariantly. Along a null geodesic,

$$p^{\nu}\nabla_{\nu}p^{\mu}=0.$$

Differentiating $p=-u_{\mu}p^{\mu}$ along the geodesic gives

$$\frac{\mathrm{d}p}{\mathrm{d}\lambda}=-p^{\mu}p^{\nu}\nabla_{\mu}u_{\nu}.$$

Using the kinematic decomposition

$$\nabla_{\mu}u_{\nu}=\frac{\theta}{3}h_{\mu\nu}+\sigma_{\mu\nu}+\omega_{\mu\nu}-u_{\mu}a_{\nu},$$

the antisymmetry of $\omega_{\mu\nu}$, and $p^{\mu}=p\left(u^{\mu}+e^{\mu}\right)$, one obtains

$$\frac{\mathrm{d}p}{\mathrm{d}\lambda}=-p^{2}\left(\frac{\theta}{3}+\sigma_{\mu\nu}e^{\mu}e^{\nu}+a_{\mu}e^{\mu}\right).$$

Because the lapse has already been reduced to the comoving proper-time coordinate,

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda}=p,$$

and hence

$$\frac{\dot{p}}{p}=-\frac{\theta}{3}-\sigma_{\mu\nu}e^{\mu}e^{\nu}-a_{\mu}e^{\mu}. \tag{42}$$

For a radial photon, e^{μ} is the radial shear eigenvector, so

$$\sigma_{\mu\nu}e^{\mu}e^{\nu}=\sigma_{r}=\frac{2}{3}\left(H_{r}-H_{\perp}\right).$$

Using Equation (24),

$$\frac{\theta}{3}+\sigma_{\mu\nu}e^{\mu}e^{\nu}=\frac{1}{3}\left(H_{r}+2H_{\perp}\right)+\frac{2}{3}\left(H_{r}-H_{\perp}\right)=H_{r}. \tag{43}$$

Since Equation (39) gives $a_{\mu}=0$, Equation (42) reduces in the radial direction to

$$\frac{\dot{p}}{p}=-H_{r}. \tag{44}$$

The independently established $K\left(t\right)$ transport law gives

$$\frac{\dot{p}}{p}=-H_{K},\quad H_{K}\equiv\frac{\dot{K}}{K}. \tag{45}$$

Equations (44) and (45) describe the same radial photon population, and therefore

$$H_{r}=H_{K}. \tag{46}$$

The expansion scalar is fixed independently by photon-number evolution. This step does not use a spatial-volume scaling. The independent kinetic derivation above obtains

$$\dot{n}_\gamma = -3H_K n_\gamma \tag{47}$$

directly from the momentum-space distribution and its three-dimensional Jacobian.

Independently, the spacetime photon-number current of the homogeneous isotropic background is

$$N^\mu_{(\gamma)} = n_\gamma u^\mu.$$

Hypothesis (H2) requires

$$\nabla_\mu N^\mu_{(\gamma)} = 0,$$

and therefore

$$\begin{aligned} 0 &= \nabla_\mu \left(n_\gamma u^\mu\right) \\ &= u^\mu \nabla_\mu n_\gamma + n_\gamma \nabla_\mu u^\mu \\ &= \dot{n}_\gamma + \theta n_\gamma. \end{aligned}$$

Thus

$$\dot{n}_\gamma + \theta n_\gamma = 0. \tag{48}$$

Equations (47) and (48) are independent statements about the same nonvanishing collisionless photon population: Equation (47) follows from momentum-space transport, whereas Equation (48) follows from the spacetime divergence of the photon-number current. Comparing them gives

$$\theta = 3H_K. \tag{49}$$

No relation of the form $V \propto K^3$ has been assumed in obtaining Equation (49).

Using Equations (24), (46), and (49),

$$H_r + 2H_\perp = 3H_K. \tag{50}$$

Since $H_r = H_K$, this gives

$$H_\perp = H_r = H_K. \tag{51}$$

The shear eigenvalues therefore satisfy

$$\sigma_r = \frac{2}{3}\left(H_r - H_\perp\right) = 0, \quad \sigma_\perp = -\frac{1}{3}\left(H_r - H_\perp\right) = 0,$$

so

$$\sigma_{\mu\nu} = 0. \tag{52}$$

The direction-independent transport law may now be checked without assuming shear freedom. Returning to the general photon momentum equation, Equation (42), and using the results already derived,

$$a_\mu = 0, \quad \sigma_{\mu\nu} = 0, \quad \theta = 3H_K, \tag{53}$$

one obtains, for an arbitrary photon propagation direction e^μ,

$$\frac{\dot{p}}{p} = -H_K. \tag{54}$$

Thus the universal $K(t)$ redshift law is compatible with the general-relativistic null-geodesic momentum law in every propagation direction. Direction independence has not been used to impose vanishing shear; rather, shear freedom was derived from the radial momentum relation together with the independent momentum-space and spacetime photon-number evolution laws, after which the all-directions result follows.

Theorem 1 (KT-induced shear-free bulk). *In a comoving spherically symmetric material domain satisfying hypotheses* (H1)-(H5), *compatibility between a universal time-only* $K(t)$ *photon kernel, collisionless general-relativistic redshift, and covariant photon-number conservation forces*

$$H_r = H_\perp = H_K, \quad \sigma_{\mu\nu} = 0, \quad \theta = 3H_K. \tag{55}$$

Proof. Hypotheses (H3)-(H5) reduce the momentum equation to $a_\mu = 0$. The radial general-relativistic redshift law then gives $\dot{p}/p = -H_r$, while the $K(t)$ transport law gives $\dot{p}/p = -H_K$, so $H_r = H_K$. Photon-number conservation gives $\dot{n}_\gamma = -\theta n_\gamma$, whereas the kinetic transport law gives $\dot{n}_\gamma = -3H_K n_\gamma$. Thus $\theta = 3H_K$. Since $\theta = H_r + 2H_\perp$, one obtains $H_\perp = H_r$, which is equivalent to vanishing shear. □

The resulting common deformation rate factorizes the spatial metric. Equation (51) implies

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} = \frac{\dot{K}}{K}. \tag{56}$$

At fixed material coordinate r,

$$\partial_t \ln\left(\frac{B}{K}\right) = 0, \quad \partial_t \ln\left(\frac{R}{K}\right) = 0. \tag{57}$$

The integration functions therefore depend only on the material coordinate:

$$B(t,r) = K(t) B_0(r), \quad R(t,r) = K(t) R_0(r). \tag{58}$$

The normalization freedom

$$K \to cK, \quad B_0 \to \frac{B_0}{c}, \quad R_0 \to \frac{R_0}{c} \tag{59}$$

is fixed by choosing

$$K(t_0) = 1. \tag{60}$$

The bulk metric therefore becomes

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + K^2(t)\left[B_0^2(r)\mathrm{d}r^2 + R_0^2(r)\mathrm{d}\Omega^2\right]. \tag{61}$$

Equivalently,

$$h_{ij}(t,x) = K^2(t)\gamma_{ij}(x), \quad \theta = 3\frac{\dot{K}}{K}. \tag{62}$$

Thus the common spatial deformation factor has been derived from the KT-GR closure rather than inserted as a Robertson-Walker ansatz. The remaining spatial Einstein equations determine the intrinsic curvature. With the convention

$$\mathcal{K}_{ij} = -\frac{1}{2}\mathcal{L}_u h_{ij}, \tag{63}$$

Equation (62) gives

$$\mathcal{K}_{ij} = -H_K h_{ij}, \tag{64}$$

so

$$\mathcal{K} = -3H_K, \quad \mathcal{K}_{\langle ij \rangle} = 0. \tag{65}$$

Here $\mathcal{K}_{ij}$ is unrelated to the scalar deformation kernel $K(t)$. Because H_K depends only on t,

$$\mathcal{L}_u \mathcal{K}_{ij} = -\left(\dot{H}_K + 2H_K^2\right) h_{ij}, \tag{66}$$

whose trace-free part also vanishes. For vanishing shift, the trace-free part of the ADM spatial evolution equation may be written as

$$\left(\mathcal{L}_u \mathcal{K}_{ij}\right)_{\mathrm{TF}} = {}^{(3)}R_{\langle ij \rangle}[h] - \left(\frac{1}{N} D_i D_j N\right)_{\langle ij \rangle} + \left(\mathcal{K}\mathcal{K}_{ij} - 2\mathcal{K}_{ik}\mathcal{K}^k{}_j\right)_{\langle ij \rangle} - 8\pi G \pi_{ij}. \tag{67}$$

The closure already established

$$N = 1, \quad \mathcal{K}_{ij} = -H_K h_{ij}, \quad \pi_{ij} = 0. \tag{68}$$

Hence

$$D_i D_j N = 0, \tag{69}$$

while

$$\mathcal{K}\mathcal{K}_{ij} - 2\mathcal{K}_{ik}\mathcal{K}^k{}_j = H_K^2 h_{ij}, \tag{70}$$

which has vanishing trace-free part. The left-hand side is likewise purely isotropic. The trace-free spatial Einstein equation therefore reduces to

$${}^{(3)}R_{\langle ij \rangle}[h] = 0. \tag{71}$$

Since $h_{ij} = K^2(t)\gamma_{ij}$ and K is spatially constant on each $t = \text{constant}$ slice, the three-dimensional Levi-Civita connections of h_{ij} and γ_{ij} coincide on that slice. Thus

$${}^{(3)}R_{\langle ij \rangle}[\gamma] = 0. \tag{72}$$

Equivalently,

$${}^{(3)}R_{ij}[\gamma] = \frac{1}{3}\,{}^{(3)}R[\gamma]\gamma_{ij}. \tag{73}$$

The contracted three-dimensional Bianchi identity gives

$$D^i\left[{}^{(3)}R_{ij} - \frac{1}{2}\,{}^{(3)}R\gamma_{ij}\right] = 0. \tag{74}$$

Using Equation (73),

$$\frac{1}{3} D_j\,{}^{(3)}R - \frac{1}{2} D_j\,{}^{(3)}R = 0, \tag{75}$$

and therefore

$$D_j\,{}^{(3)}R = 0. \tag{76}$$

Thus ${}^{(3)}R[\gamma]$ is spatially constant on each connected bulk component. In three spatial dimensions the Riemann tensor is determined completely by the Ricci tensor:

$$ {}^{(3)}R_{ijkl} = \gamma_{ik}\,{}^{(3)}R_{jl} + \gamma_{jl}\,{}^{(3)}R_{ik} - \gamma_{il}\,{}^{(3)}R_{jk} - \gamma_{jk}\,{}^{(3)}R_{il} - \frac{{}^{(3)}R}{2}\left(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}\right). \tag{77} $$

Substitution of Equation (73) gives

$$ {}^{(3)}R_{ijkl}[\gamma] = \frac{{}^{(3)}R[\gamma]}{6}\left(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}\right). \tag{78} $$

Defining the spatially constant quantity

$$ k \equiv \frac{{}^{(3)}R[\gamma]}{6}, \tag{79} $$

one obtains

$$ {}^{(3)}R_{ijkl}[\gamma] = k\left(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}\right). \tag{80} $$

Thus γ_{ij} has constant sectional curvature. A radial coordinate may consequently be chosen such that

$$ \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = \frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2. \tag{81} $$

This is a time-independent relabeling of the comoving radial coordinate. For $k > 0$, the curvature coordinate covers the region for which $1 - kr^2 > 0$; the material boundary must lie within the chosen coordinate patch or be described using a second patch.

Only after this curvature result has been obtained does the observable bulk metric take the form

$$ \mathrm{d}s^2 = -\mathrm{d}t^2 + K^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right]. \tag{82} $$

Equation (82) is not assumed at the beginning of the construction. It is the bulk geometry forced by the finite-material action, the explicit stress conditions, the independently established $K(t)$ transport law, and covariant number conservation [8]-[11]. The material Jacobian follows covariantly from the conserved material current. From Equation (11),

$$ \mathcal{J}^\mu = bu^\mu, \quad \nabla_\mu \mathcal{J}^\mu = 0. \tag{83} $$

Therefore

$$ \begin{aligned} 0 &= \nabla_\mu\left(bu^\mu\right) \\ &= u^\mu\nabla_\mu b + b\nabla_\mu u^\mu \\ &= \dot{b} + b\theta. \end{aligned} \tag{84} $$

Hence

$$ \frac{\dot{b}}{b} = -\theta. \tag{85} $$

Since

$$J = b^{-1}, \tag{86}$$

it follows that

$$\frac{\dot{J}}{J} = \theta. \tag{87}$$

Using the independently derived closure relation

$$\theta = 3H_K = 3\frac{\dot{K}}{K}, \tag{88}$$

one obtains

$$\frac{\dot{J}}{J} = 3\frac{\dot{K}}{K}. \tag{89}$$

Integration along each material worldline gives

$$J(t,x) = J_0(x)K^3(t), \tag{90}$$

where the normalization $K(t_0)=1$ has been used and $J_0(x) = J(t_0, x)$.

Thus

$$H_K = \frac{\theta}{3} = \frac{\dot{J}}{3J}. \tag{91}$$

The same scalar $K(t)$ that governs photon transport therefore measures the isotropic deformation of the material bulk. The relation to the Ehlers-Geren-Sachs result should be distinguished explicitly. EGS establishes Robertson-Walker geometry from exact isotropy of a collisionless radiation distribution under its stated assumptions. The present construction does not claim that geometric implication as new. Its additional result is the one-rate identification of the independently established transport kernel with null-geodesic momentum evolution, covariant photon-number evolution, and material deformation:

$$H_K = \frac{\theta}{3} = \frac{\dot{J}}{3J}. \tag{92}$$

The finite-material action and the relation $J \propto K^3$ therefore provide the material realization and kinetic closure not contained in the geometric EGS statement. Finally, the homogeneous-pressure condition constrains the possible boundary completion. If the material domain terminated directly at an unconstrained free surface satisfying

$$P_{\text{tot}}(r_b) = 0, \tag{93}$$

then spatial homogeneity would imply

$$P_{\text{tot}} = 0 \tag{94}$$

throughout the observable bulk. For pressureless matter and radiation,

$$P_\gamma = \frac{\rho_\gamma}{3}, \tag{95}$$

this would require

$$P_s = -\frac{\rho_\gamma}{3}. \tag{96}$$

A generic finite realization therefore cannot terminate the homogeneous bulk at an unconstrained vacuum free surface. A transition layer, surface stress, nonvacuum exterior, or distinct exterior phase is required in the complete global solution. None of those unresolved choices enters the local derivation above.

3. Einstein Equations and Substrate Dynamics

The geometry derived in Section 2 now determines the homogeneous bulk dynamics. Substituting Equation (82) into the Einstein equations gives the Hamiltonian constraint and isotropic evolution equation,

$$H_K^2 + \frac{k}{K^2} = \frac{8\pi G}{3}\rho_{\text{tot}}, \tag{97}$$

$$\dot{H}_K - \frac{k}{K^2} = -4\pi G\left(\rho_{\text{tot}} + P_{\text{tot}}\right), \tag{98}$$

where

$$H_K \equiv \frac{\dot{K}}{K}. \tag{99}$$

Combining Equations (99) and (100) gives

$$\frac{\ddot{K}}{K} = -\frac{4\pi G}{3}\left(\rho_{\text{tot}} + 3P_{\text{tot}}\right). \tag{100}$$

Only two of Equations (97), (98), and (100) are independent. No independent bare cosmological-constant term appears because none was included in the gravitational action. Within the present construction, any vacuum-like bulk contribution assigned to the substrate must arise from its constitutive law $W(J)$. These equations have the familiar Robertson-Walker form because the observable bulk geometry was shown in Section 2 to possess constant-curvature spatial sections and a single isotropic deformation ratio. They are not introduced through an FLRW ansatz. They are the reduced Einstein equations of the finite-material construction after the KT-GR closure has determined the bulk geometry.

The total density and pressure are

$$\rho_{\text{tot}} = \rho_m + \rho_\gamma + \rho_s, \quad P_{\text{tot}} = P_m + P_\gamma + P_s. \tag{101}$$

Covariant conservation of the total stress tensor reduces in the derived bulk to

$$\dot{\rho}_{\text{tot}} + 3H_K\left(\rho_{\text{tot}} + P_{\text{tot}}\right) = 0. \tag{102}$$

The individual sectors may be written more generally as

$$\dot{\rho}_A + 3H_K\left(\rho_A + P_A\right) = Q_A, \quad \sum_A Q_A = 0, \tag{103}$$

where Q_A describes local exchange among the matter, radiation, and substrate sectors.

In the post-decoupling regime considered below, the matter, radiation, and substrate actions are taken to be separately diffeomorphism invariant, with no direct intersector couplings. On the corresponding field equations,

$$Q_m = Q_\gamma = Q_s = 0. \tag{104}$$

The sectors are therefore separately conserved. This assumption is not required for total stress-energy conservation, but it is required for the collisionless photon theorem and for excluding a hidden substrate-to-photon source. The substrate conservation equation follows directly from the material action rather than being imposed independently. From Equation (90),

$$\dot{J} = 3H_K J. \tag{105}$$

Using

$$\rho_s = \frac{W(J)}{J}, \quad P_s = -W'(J), \tag{106}$$

one finds

$$\dot{\rho}_s = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{W}{J}\right) = \frac{W'J - W}{J^2}\dot{J}, \tag{107}$$

$$3H_K(\rho_s + P_s) = 3H_K\left(\frac{W}{J} - W'\right) = -\frac{W'J - W}{J^2}\dot{J}. \tag{108}$$

Therefore

$$\dot{\rho}_s + 3H_K(\rho_s + P_s) = 0 \tag{109}$$

identically. The constitutive reduction is thus dynamically consistent with covariant stress-energy conservation. The substrate requires neither an added continuity rule nor an external energy reservoir.

The constitutive variable is restricted to

$$J > 0, \tag{110}$$

corresponding to a positive, nonsingular material volume element. This is a structural condition on the material description. Additional physical admissibility conditions, such as positivity of the substrate energy density,

$$\rho_s = \frac{W(J)}{J} > 0, \tag{111}$$

when imposed, restrict the admissible branch of a chosen constitutive law $W(J)$. Such restrictions are distinct from hypothesis (H5), which instead requires the total inertial density of the complete bulk solution to satisfy

$$\rho_{\text{tot}} + P_{\text{tot}} \neq 0. \tag{112}$$

The substrate action used here depends only on the scalar volume invariant $b = J^{-1}$. It therefore describes the isotropic one-invariant constitutive sector,

$$\rho_s = \frac{W(J)}{J}, \quad P_s = -W'(J). \tag{113}$$

A b-only, equivalently $W(J)$-only, action contains no independent shear modulus. Shear response and the associated perturbative elastic dynamics require dependence on additional material invariants. No such additional constitutive law is selected in the homogeneous bulk construction considered here.

The material construction admits a simple counted family of constitutive laws. Suppose a fixed comoving population of frozen p-dimensional structures carries stored energy proportional to its physical p-volume. Since an isotropic material length scales as K in the derived bulk and $J \propto K^3$, the stored energy per reference volume has the form

$$W_p(J) = A_p J^{p/3}. \tag{114}$$

Equation (17) then gives

$$\rho_p = A_p J^{p/3-1}, \quad P_p = -\frac{p}{3} A_p J^{p/3-1}, \quad w_p \equiv \frac{P_p}{\rho_p} = -\frac{p}{3}. \tag{115}$$

Thus the integer-dimensional cases $p = 0,1,2,3$ yield

$$w_p = 0, -\frac{1}{3}, -\frac{2}{3}, -1. \tag{116}$$

Within this counted frozen-structure model, the exponent is fixed by the assumed material dimensionality rather than reconstructed from a desired background history.

For the purely barotropic one-invariant reduction, the $p=1$ and $p=2$ branches have

$$c_a^2 \equiv \frac{\mathrm{d}P_p}{\mathrm{d}\rho_p} = w_p < 0, \tag{117}$$

and are therefore gradient-unstable if treated as perfect fluids. Their stability requires the additional elastic invariants to provide a sufficiently positive longitudinal response through shear rigidity. The $p=0$ branch has $c_a^2 = 0$, while the $p=3$ branch is vacuum-like, with $\rho_3 + P_3 = 0$, and does not possess an ordinary perfect-fluid density mode. Stability of the counted family is therefore a property of the complete elastic response rather than of $W(J)$ alone.

The $p=3$ case requires particular care. For

$$W_3(J) = A_3 J, \tag{118}$$

the substrate density and pressure are

$$\rho_3 = A_3, \quad P_3 = -A_3. \tag{119}$$

At the homogeneous classical level, this contribution is exactly equivalent to a cosmological constant. Its placement in the material constitutive sector, rather than as an independent term in the gravitational action, does not by itself produce an observable distinction. Such a distinction would require additional substrate physics, for example nontrivial perturbative, boundary, or microscopic behavior. The dimensional counting fixes $w_3 = -1$, but it does not derive the magnitude A_3.

Likewise, a mixed constitutive law

$$W(J) = A_0 + A_3 J \tag{120}$$

gives

$$\rho_s = \frac{A_0}{J} + A_3, \quad P_s = -A_3. \tag{121}$$

Because $J \propto K^3$, the first term scales as K^{-3}, while the second remains constant. At the homogeneous level, this is dynamically equivalent to a dust component plus a cosmological-constant component. The construction therefore supplies a covariant material representation of those stress forms, but it does not by itself predict their relative amplitudes.

More generally, a constitutive law $W(J)$ determines both the substrate density and pressure,

$$\rho_s(J) = \frac{W(J)}{J}, \quad P_s(J) = -W'(J), \tag{122}$$

and therefore enters Equations (97) and (98) as a fixed material input. It cannot consistently be selected afterward as an arbitrary function of time to reproduce a preferred $K(t)$. A predictive completion must specify $W(J)$, or derive it from a more microscopic material theory, before solving the Einstein equations. The homogeneous reduction also does not determine the complete elastic response of the substrate. The one-variable law $W(J)$ fixes the isotropic bulk trajectory, while dependence on additional material invariants controls shear response, stability, sound propagation, and perturbations away from the homogeneous solution. Those effects are not required for the background closure established here, but they become essential in the perturbative and finite-boundary completion.

4. General-Relativistic Photon Transport

Section 2 treated the independently established $K(t)$ transport law as a closure condition and determined the general-relativistic bulk compatible with it. The converse reduction is now checked: the collisionless Liouville equation of the derived bulk must reduce to that same transport law. This establishes mutual consistency rather than introducing an additional dynamical assumption.

Let $f(x^\mu, p^\nu)$ be the invariant one-particle distribution on the future-directed null mass shell,

$$p^\mu p_\mu = 0, \quad p^0 > 0. \tag{123}$$

In the absence of collisions and non-gravitational forces, the distribution is constant along the geodesic flow in phase space:

$$\mathcal{L}[f] = 0. \tag{124}$$

In local coordinates, the Liouville operator may be written as

$$\mathcal{L}[f] \equiv p^\mu \frac{\partial f}{\partial x^\mu} - \Gamma^i_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^i}, \tag{125}$$

where the three spatial momentum components provide coordinates on the null mass shell [12]-[16]. Equation (125) is the collisionless general-relativistic Boltzmann equation. No additional redshift force or collision term is present. The photon momentum measured by the comoving material observers is

$$p \equiv -u_\mu p^\mu. \tag{126}$$

For a photon, the four-momentum can be decomposed as

$$p^{\mu} = p\left(u^{\mu} + e^{\mu}\right), \tag{127}$$

where

$$u_{\mu}e^{\mu} = 0, \quad e_{\mu}e^{\mu} = 1. \tag{128}$$

The vector e^{μ} gives the photon propagation direction in the local material rest frame. The covariant energy-propagation equation follows from

$$p^{\nu}\nabla_{\nu}p^{\mu} = 0 \tag{129}$$

and the kinematic decomposition of $\nabla_{\mu}u_{\nu}$. If λ is an affine parameter along the null geodesic, then

$$\frac{\mathrm{d}p}{\mathrm{d}\lambda} = -p^{2}\left(\frac{\theta}{3} + \sigma_{\mu\nu}e^{\mu}e^{\nu} + a_{\mu}e^{\mu}\right). \tag{130}$$

Because the derived bulk admits the comoving proper-time function t,

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} = p^{\mu}\nabla_{\mu}t = p, \tag{131}$$

so Equation (130) becomes

$$\frac{1}{p}\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{\theta}{3} - \sigma_{\mu\nu}e^{\mu}e^{\nu} - a_{\mu}e^{\mu}. \tag{132}$$

Section 2 established that the observable bulk is geodesic and shear-free:

$$a_{\mu} = 0, \quad \sigma_{\mu\nu} = 0, \quad \theta = 3H_{K}, \quad H_{K} \equiv \frac{\dot{K}}{K}. \tag{133}$$

Equation (132) therefore reduces for every propagation direction to

$$\frac{\dot{p}}{p} = -H_{K}, \tag{134}$$

or

$$\dot{p} = -H_{K}p. \tag{135}$$

The momentum change is isotropic and linear in p. It is generated by the same material deformation rate that appears in the bulk metric. Integrating Equation (134) between emission and observation gives

$$\ln\left(\frac{p_{0}}{p_{e}}\right) = -\int_{t_e}^{t_0} H_{K}\left(t\right)\mathrm{d}t = -\ln\left(\frac{K_{0}}{K_{e}}\right), \tag{136}$$

and hence

$$p_{0}K_{0} = p_{e}K_{e}. \tag{137}$$

Since photon frequency is proportional to the locally measured momentum,

$$1 + z \equiv \frac{\nu_{e}}{\nu_{0}} = \frac{p_{e}}{p_{0}} = \frac{K_{0}}{K_{e}}. \tag{138}$$

The operational redshift relation introduced in the earlier work is therefore reproduced by null-geodesic propagation in the finite-material bulk selected by the closure conditions of Section 2. For the homogeneous isotropic background distribution,

$$f = f(t, p), \tag{139}$$

the total derivative along a phase-space characteristic is

$$\frac{\mathrm{d}f}{\mathrm{d}\lambda} = \frac{\mathrm{d}t}{\mathrm{d}\lambda}\left[\frac{\partial f}{\partial t} + \frac{\mathrm{d}p}{\mathrm{d}t}\frac{\partial f}{\partial p}\right]. \tag{140}$$

Using Equations (131) and (135),

$$\frac{\mathrm{d}f}{\mathrm{d}\lambda} = p\left[\frac{\partial f}{\partial t} - H_K p \frac{\partial f}{\partial p}\right]. \tag{141}$$

Since $p > 0$ on the future-directed photon mass shell, the collisionless Liouville equation is equivalent to

$$\frac{\partial f}{\partial t} - H_K p \frac{\partial f}{\partial p} = 0. \tag{142}$$

This is exactly the previously established $K(t)$ kinetic transport equation.

Theorem 2 (One-rate closure). *In the geodesic, shear-free bulk derived in Section* 2, *the* $K(t)$ *kinetic redshift operator is the reduced phase-space form of null-geodesic Liouville transport generated by the material deformation rate*

$$H_K = \frac{\theta}{3}. \tag{143}$$

It is not an additional photon interaction or a second redshift mechanism.

Proof. The null-geodesic energy equation gives

$$\frac{\dot{p}}{p} = -\frac{\theta}{3} \tag{144}$$

when $a_\mu = 0 = \sigma_{\mu\nu}$. The finite-material closure gives

$$\frac{\theta}{3} = \frac{\dot{K}}{K} = H_K. \tag{145}$$

Therefore

$$\dot{p} = -H_K p. \tag{146}$$

Substitution into the collisionless Liouville equation yields Equation (142). The geometric momentum law and the kinetic transport operator are thus the same phase-space evolution written in two forms. □

Adding a second term proportional to $-H_K p$ would alter the characteristic equation to

$$\frac{\dot{p}}{p} = -2H_K \tag{147}$$

and would produce

$$1 + z = \left(\frac{K_0}{K_e}\right)^2, \tag{148}$$

contradicting Equation (138). More generally, any additional frequency-loss term would represent either a non-geodesic force or a nonzero collision operator and would require a separate covariant source in the kinetic equation. Neither is present in the construction considered here. The invariant particle-number current and photon stress tensor are

$$N^{\mu}_{(\gamma)} = \int_{\mathcal{P}_x} p^{\mu} f \, \mathrm{d}\Pi, \tag{149}$$

$$T^{\mu\nu}_{(\gamma)} = \int_{\mathcal{P}_x} p^{\mu} p^{\nu} f \, \mathrm{d}\Pi, \tag{150}$$

where in a local orthonormal frame comoving with the material and using c = 1, the invariant measure on the future null mass shell $\mathcal{P}_x$ is defined by

$$\mathrm{d}\Pi = \frac{g_{\gamma}}{(2\pi\hbar)^3} \frac{d^3 p}{p^{\hat{0}}}$$

with $g_{\gamma} = 2$ for photons.

Collisionless Liouville transport gives

$$\nabla_{\mu} N^{\mu}_{(\gamma)} = 0, \quad \nabla_{\mu} T^{\mu\nu}_{(\gamma)} = 0 \tag{151}$$

for the separately conserved photon sector assumed in Equation (104). The number-current component of hypothesis (H2) in Section 2 is therefore not an additional independent postulate. It is the number-conservation statement of collisionless Liouville transport for a sector with no intersector coupling, and Equation (151) exhibits it explicitly. In the comoving isotropic bulk,

$$N^{\mu}_{(\gamma)} = n_{\gamma} u^{\mu}, \tag{152}$$

so the number-current equation reduces to

$$\dot{n}_{\gamma} + \theta n_{\gamma} = 0 \tag{153}$$

and therefore

$$\dot{n}_{\gamma} + 3 H_K n_{\gamma} = 0. \tag{154}$$

This is the covariant conservation law used in Section 2 to establish the KT-GR closure. The full kinetic derivation now shows that the same result follows from the invariant photon distribution without introducing a sink, source, or separate reservoir.

5. Recovery of the $K(t)$ Photon Results

The transport equation obtained in Section 4 may now be solved without further geometric assumptions. The result recovers the photon-sector relations established in the earlier kinetic treatment and identifies them as exact consequences of collisionless Liouville propagation in the derived finite-material bulk.

The characteristic equations of Equation (142) are

$$\frac{\mathrm{d}t}{\mathrm{d}s} = 1, \quad \frac{\mathrm{d}p}{\mathrm{d}s} = -H_K p, \quad \frac{\mathrm{d}f}{\mathrm{d}s} = 0, \tag{155}$$

where s parameterizes characteristics of the reduced phase-space equation and is not the affine parameter λ of Section 4; the two are related by $\mathrm{d}s = p\,\mathrm{d}\lambda$. Using

$$H_K = \frac{\dot{K}}{K}, \tag{156}$$

the momentum characteristic satisfies

$$\frac{\mathrm{d}\ln p}{\mathrm{d}t} = -\frac{\mathrm{d}\ln K}{\mathrm{d}t}, \tag{157}$$

$$\ln\left[p(t)K(t)\right] = \ln\left(p_i K_i\right), \tag{158}$$

$$p(t)K(t) = p_i K_i, \tag{159}$$

where

$$K_i \equiv K(t_i). \tag{160}$$

Therefore

$$p_i = \frac{K(t)}{K_i} p. \tag{161}$$

Since f is constant along each characteristic,

$$f(t,p) = f_i\left(\frac{K(t)}{K_i} p\right). \tag{162}$$

Equation (162) is valid for an arbitrary homogeneous and isotropic initial photon distribution. It states that collisionless evolution translates the distribution uniformly in logarithmic momentum without changing its occupation number along the phase-space flow. The characteristic solution immediately determines the evolution of a thermal photon distribution. Let the initial distribution be Bose-Einstein,

$$f_i(p) = \frac{1}{\exp\left[\left(pc-\mu_i\right)/\left(k_B T_i\right)\right]-1}. \tag{163}$$

Substitution into Equation (162) gives

$$f(t,p) = \frac{1}{\exp\left[\left(\frac{K(t)}{K_i} pc - \mu_i\right)\Big/\left(k_B T_i\right)\right]-1}. \tag{164}$$

Writing the evolved distribution in the standard form

$$f(t,p) = \frac{1}{\exp\left[\left(pc-\mu(t)\right)/\left(k_B T(t)\right)\right]-1} \tag{165}$$

requires

$$T(t) = T_i \frac{K_i}{K(t)}, \quad \mu(t) = \mu_i \frac{K_i}{K(t)}. \tag{166}$$

It follows that

$$\frac{\mu(t)}{k_B T(t)} = \frac{\mu_i}{k_B T_i} \tag{167}$$

is invariant under the transport.

An initially Planckian distribution has

$$\mu_i = 0. \tag{168}$$

Equation (166) then gives

$$\mu(t) = 0, \quad T(t) = T_i \frac{K_i}{K(t)}. \tag{169}$$

Thus a Planck spectrum remains exactly Planckian under collisionless, frequency-independent $K(t)$ transport. No thermalization process is required to restore the spectral form after propagation. This statement applies after the initial thermal spectrum has been established and while the collisionless assumptions of Section 4 remain valid.

Applying Equation (169) between an emission epoch t_e and the present epoch t_0 gives

$$\frac{T_e}{T_0} = \frac{K_0}{K_e}. \tag{170}$$

Using Equation (138),

$$1 + z = \frac{K_0}{K_e}, \tag{171}$$

one obtains

$$T(z) = T_0(1+z). \tag{172}$$

This relation agrees with current background temperature measurements [17]-[19]. The usual background temperature-redshift relation is therefore reproduced without an additional photon interaction. The characteristic solution also fixes the photon number and energy densities. In a local orthonormal frame comoving with the material, and for photon degeneracy $g_\gamma = 2$,

$$n_\gamma(t) = \frac{g_\gamma}{(2\pi\hbar)^3} \int f(t,p)\,\mathrm{d}^3 p, \tag{173}$$

$$\rho_\gamma(t) = \frac{g_\gamma}{(2\pi\hbar)^3} \int pc\, f(t,p)\,\mathrm{d}^3 p. \tag{174}$$

Define the initial-momentum variable

$$q \equiv \frac{K(t)}{K_i} p. \tag{175}$$

Then

$$\mathrm{d}^3 p = \left(\frac{K_i}{K(t)}\right)^3 \mathrm{d}^3 q, \quad p = \frac{K_i}{K(t)} q. \tag{176}$$

Using Equation (162) in Equation (173) gives

$$\begin{aligned} n_\gamma(t) &= \left(\frac{K_i}{K(t)}\right)^3 \frac{g_\gamma}{(2\pi\hbar)^3} \int f_i(q)\,\mathrm{d}^3 q \\ &= n_{\gamma i}\left(\frac{K_i}{K(t)}\right)^3. \end{aligned} \tag{177}$$

Likewise,

$$\begin{aligned}\rho_\gamma(t) &= \left(\frac{K_i}{K(t)}\right)^4 \frac{g_\gamma}{(2\pi\hbar)^3}\int qc\, f_i(q)\,\mathrm{d}^3q \\ &= \rho_{\gamma i}\left(\frac{K_i}{K(t)}\right)^4.\end{aligned} \tag{178}$$

Therefore

$$n_\gamma \propto K^{-3}, \quad \rho_\gamma \propto K^{-4}. \tag{179}$$

These are phase-space results. They are not inserted as independent dilution or cooling laws. Differentiating Equations (177) and (178) gives

$$\dot{n}_\gamma = -3H_K n_\gamma, \quad \dot{\rho}_\gamma = -4H_K\rho_\gamma. \tag{180}$$

For an isotropic photon gas,

$$P_\gamma = \frac{\rho_\gamma}{3}. \tag{181}$$

The energy-density equation may therefore be written as

$$\dot{\rho}_\gamma + 3H_K\left(\rho_\gamma + P_\gamma\right) = 0. \tag{182}$$

This is precisely the photon-sector continuity equation obtained from $\nabla_\mu T^{\mu\nu}_{(\gamma)} = 0$. The reduction of the distribution function, the moment integrals, and covariant stress-energy conservation thus give the same evolution. The scaling of the mean photon energy follows directly:

$$\left\langle E_\gamma \right\rangle \equiv \frac{\rho_\gamma}{n_\gamma} \propto K^{-1}. \tag{183}$$

This agrees with the individual characteristic law $pK = \text{constant}$. The number density contributes three powers of K^{-1} from the local momentum-space Jacobian, while the photon energy contributes the fourth. For the Bose-Einstein family, the invariant entropy current is

$$S^\mu_{(\gamma)} = -k_B\int_{\mathcal{P}_x} p^\mu\left[f\ln f - (1+f)\ln(1+f)\right]\mathrm{d}\Pi. \tag{184}$$

Collisionless Liouville transport preserves the distribution along the phase-space flow and gives

$$\nabla_\mu S^\mu_{(\gamma)} = 0. \tag{185}$$

In the homogeneous isotropic bulk,

$$S^\mu_{(\gamma)} = s_\gamma u^\mu, \tag{186}$$

so

$$\dot{s}_\gamma + 3H_K s_\gamma = 0, \tag{187}$$

and therefore

$$s_\gamma \propto K^{-3}. \tag{188}$$

Since the proper volume of a fixed comoving bulk region scales as K^3, its total collisionless photon entropy is constant. The transport preserves the fine-grained kinetic entropy and requires neither entropy production nor an entropy reservoir.

The results of this section may be collected as

$$pK = \text{constant}, \quad T \propto K^{-1}, \quad \mu \propto K^{-1},$$
$$\frac{\mu}{k_B T} = \text{constant}, \quad n_\gamma \propto K^{-3}, \quad \rho_\gamma \propto K^{-4}. \tag{189}$$

For an initially Planckian spectrum,

$$\mu = 0 \tag{190}$$

at every later collisionless epoch. All of these relations arise from one Liouville flow and one deformation rate. None requires a second redshift operator, post-decoupling photon production, or transfer of photon energy into an unmodeled reservoir.

6. Conservation Closure

The results of Sections 3-5 now permit the matter, radiation, and substrate ledgers to be closed within one covariant bulk description. The purpose of this section is not to introduce new evolution laws, but to show that the number, energy, and deformation scalings already derived are mutually consistent and do not require an unmodeled reservoir. For a conserved comoving matter species with number current

$$N_m^\mu = n_m u^\mu, \tag{191}$$

separate conservation gives

$$\nabla_\mu N_m^\mu = 0. \tag{192}$$

Using

$$\theta = 3H_K, \tag{193}$$

Equation (192) becomes

$$\dot{n}_m + 3H_K n_m = 0, \tag{194}$$

and therefore

$$n_m \propto K^{-3}. \tag{195}$$

For nonrelativistic matter with conserved rest mass per particle,

$$\rho_m = m n_m, \tag{196}$$

so

$$\rho_m \propto K^{-3}, \quad P_m \simeq 0. \tag{197}$$

This is the usual comoving-number result expressed in terms of the physical material deformation ratio. The photon sector satisfies

$$\dot{n}_\gamma + 3H_K n_\gamma = 0, \tag{198}$$

and hence

$$n_\gamma \propto K^{-3}. \tag{199}$$

For the baryonic component of the conserved nonrelativistic matter sector, write

$$n_b \equiv n_m. \tag{200}$$

It follows immediately that the baryon-to-photon ratio

$$\eta \equiv \frac{n_b}{n_\gamma} \tag{201}$$

is constant throughout the collisionless post-decoupling regime:

$$\dot{\eta} = 0. \tag{202}$$

Thus the transport does not require post-decoupling photon production to maintain the photon abundance relative to conserved matter. For any fixed comoving bulk region $\mathcal{D}$, the proper volume is

$$V_{\mathcal{D}}(t) = K^3(t) V_{\mathcal{D},0}, \tag{203}$$

where

$$V_{\mathcal{D},0} = \int_{\mathcal{D}} \sqrt{\gamma}\, \mathrm{d}^3 x. \tag{204}$$

The total photon number is therefore

$$N_\gamma = n_\gamma V_{\mathcal{D}}, \tag{205}$$

and

$$\dot{N}_\gamma = 0. \tag{206}$$

The same result follows directly from integrating the conserved number current over the comoving spatial region. The photon energy contained in the same comoving region is

$$E_\gamma = \rho_\gamma V_{\mathcal{D}}. \tag{207}$$

Since

$$\rho_\gamma \propto K^{-4}, \quad V_{\mathcal{D}} \propto K^3, \tag{208}$$

one obtains

$$E_\gamma \propto K^{-1}. \tag{209}$$

The decrease in photon energy does not appear as a local sink term. Instead, the photon continuity equation,

$$\dot{\rho}_\gamma + 3H_K(\rho_\gamma + P_\gamma) = 0, \tag{210}$$

may be multiplied by $V_{\mathcal{D}}$ to give

$$\frac{\mathrm{d}E_\gamma}{\mathrm{d}t} = -P_\gamma \frac{\mathrm{d}V_{\mathcal{D}}}{\mathrm{d}t}. \tag{211}$$

For

$$P_\gamma = \frac{\rho_\gamma}{3}, \quad \frac{\dot{V}_{\mathcal{D}}}{V_{\mathcal{D}}} = 3H_K, \tag{212}$$

Equation (211) is exactly equivalent to Equation (209). The change in photon energy is therefore the standard covariant pressure-work term associated with the deformation of the material volume. The substrate obeys its own pressure-work identity,

$$\dot{\rho}_s + 3H_K\left(\rho_s + P_s\right) = 0. \tag{213}$$

Defining

$$E_s = \rho_s V_{\mathcal{D}}, \tag{214}$$

one obtains

$$\frac{\mathrm{d}E_s}{\mathrm{d}t} = -P_s \frac{\mathrm{d}V_{\mathcal{D}}}{\mathrm{d}t}. \tag{215}$$

Likewise, separately conserved matter satisfies

$$\frac{\mathrm{d}E_m}{\mathrm{d}t} = -P_m \frac{\mathrm{d}V_{\mathcal{D}}}{\mathrm{d}t}. \tag{216}$$

Adding the three sectors gives

$$\frac{\mathrm{d}E_{\mathrm{tot}}}{\mathrm{d}t} = -P_{\mathrm{tot}} \frac{\mathrm{d}V_{\mathcal{D}}}{\mathrm{d}t}, \tag{217}$$

where

$$E_{\mathrm{tot}} = E_m + E_\gamma + E_s. \tag{218}$$

Equation (217) is the integrated form of local covariant stress-energy conservation in the homogeneous bulk. It is not a statement of a globally conserved scalar energy for the entire spacetime. This distinction is essential. General relativity guarantees

$$\nabla_\mu T^{\mu\nu}_{\mathrm{tot}} = 0, \tag{219}$$

which is a local conservation law. It does not, in a time-dependent curved geometry, generally supply a unique global energy obtained by summing the contents of an arbitrary cosmological spatial slice [20]-[23]. The correct ledger is the local pressure-work identity together with separate number conservation for uncoupled sectors. Within that ledger, no missing photon energy must be assigned to a hidden bath, external absorber, or independent redshift reservoir. The absence of post-decoupling photon production can be stated as an exact source theorem. Let the photon kinetic equation be written in the general form

$$\mathcal{L}\left[f\right] = C\left[f\right], \tag{220}$$

where $C\left[f\right]$ is the collision operator. The corresponding number-current divergence is

$$\nabla_\mu N^\mu_{(\gamma)} = \int_{\mathcal{P}_x} C\left[f\right]\mathrm{d}\Pi. \tag{221}$$

For collisionless propagation,

$$C\left[f\right] = 0, \tag{222}$$

so

$$\nabla_\mu N^\mu_{(\gamma)} = 0. \tag{223}$$

In the homogeneous bulk this is equivalent to

$$\dot{n}_\gamma + 3H_K n_\gamma = 0. \tag{224}$$

Theorem 3 (Post-decoupling photon-source closure). *Assume that the photon sector has no direct intersector coupling and is described by classical collisionless transport on the null-geodesic flow of the derived bulk. Then*

$$\nabla_\mu N^\mu_{(\gamma)} = 0. \tag{225}$$

For every comoving region contained within the homogeneous bulk,

$$N_\gamma = \text{constant}. \tag{226}$$

Within this kinetic description, post-decoupling photon creation or destruction requires a nonzero zeroth moment of the collision operator or an explicit interaction source, neither of which is present in the construction.

Proof. Collisionless evolution gives

$$C[f] = 0. \tag{227}$$

The zeroth momentum moment of the Boltzmann equation therefore yields

$$\nabla_\mu N^\mu_{(\gamma)} = 0. \tag{228}$$

In the homogeneous isotropic bulk,

$$N^\mu_{(\gamma)} = n_\gamma u^\mu, \tag{229}$$

so there is no net photon-number flux through the boundary of a comoving bulk region. Integrating the conserved current over that region gives

$$\dot{N}_\gamma = 0. \tag{230}$$

□

Observationally, spectral-distortion limits constrain any post-thermalization photon injection [24] [25]. The energy moment is equally explicit. Taking the first momentum moment of Equation (220) gives

$$\nabla_\mu T^{\mu\nu}_{(\gamma)} = \int_{\mathcal{P}_x} p^\nu C[f] \,\mathrm{d}\Pi. \tag{231}$$

For $C[f] = 0$,

$$\nabla_\mu T^{\mu\nu}_{(\gamma)} = 0. \tag{232}$$

The photon energy change is therefore fully accounted for by Equation (211). A physical transfer of photon energy to another sector would require

$$\nabla_\mu T^{\mu\nu}_{(\gamma)} = Q^\nu_\gamma \neq 0, \tag{233}$$

with a compensating source in the remaining sectors,

$$\sum_A Q^\nu_A = 0. \tag{234}$$

No such exchange vector occurs in the post-decoupling model considered here.

The complete post-decoupling ledger is therefore

$$\begin{gathered} J \propto K^3, \quad n_b \propto K^{-3}, \quad n_\gamma \propto K^{-3}, \quad \rho_\gamma \propto K^{-4}, \\ \eta = \text{constant}, \quad N_\gamma = \text{constant}. \end{gathered} \tag{235}$$

The same deformation rate governs material volume, photon momentum, number density, energy density, and pressure work. The construction contains one redshift process, one collisionless photon population, and one covariant conservation ledger.

7. Scope

The construction established in Sections 2-6 is a local homogeneous-bulk theory derived under hypotheses (H1)-(H5). Within this regime, the Einstein and kinetic equations force geodesic, shear-free deformation, factorization of the spatial metric, constant sectional curvature in the observable bulk, and exact agreement between the general-relativistic Liouville equation and the independently established $K(t)$ transport equation. The closure applies during epochs in which the photon background is governed by collisionless Liouville transport. Within that regime, no additional photon force, frequency-loss term, or independent redshift interaction is present. The observed momentum evolution is generated entirely by the null-geodesic Liouville flow associated with the same deformation rate H_K. The observable interior metric is time dependent, but the expansion of space itself is not taken as primitive. The scalar $K(t)$ instead represents the physical isotropic deformation ratio of a finite material substrate, with Robertson-Walker geometry derived only after the closure conditions are applied.

The results established here close the local kinematic, kinetic, thermodynamic, and conservation sectors of the homogeneous framework. The constitutive and global completion of the finite material system remain separate questions and do not enter the local redshift, transport, thermodynamic, or conservation results established here. At the homogeneous level, the derived field equations may share the mathematical form of Robertson-Walker cosmology, while the present construction differs in its primitive ontology, finite-domain structure, and constitutive content [26]-[29]. The homogeneous background equations alone do not determine the additional physical structure that distinguishes these formulations.

The present paper therefore establishes, conditional on hypotheses (H1)-(H5), the local general-relativistic and kinetic closure of the $K(t)$ framework: the redshift kernel, material deformation, photon transport, Planck-spectrum preservation, number scaling, energy scaling, and covariant conservation ledger are generated by one deformation rate and one collisionless Liouville flow.

8. Conclusions

This paper completes the collisionless local general-relativistic closure of the $K(t)$ program and establishes the homogeneous-bulk foundation of Flexible Static Cosmology (FLSC). Beginning from a covariant finite-material construction and the independently established $K(t)$ transport law, the analysis derives rather than assumes the Robertson-Walker form of the observable bulk. A single deformation rate governs material evolution, null-geodesic photon momentum, and covariant volume evolution. The Einstein equations, material current, Liouville equation, photon-number evolution, and pressure-work ledger close consistently under that same rate. The construction recovers the operational redshift relation, exact Planck-spectrum preservation, the observed temperature relation, photon-number and energy-density scalings, and a constant baryon-to-photon ratio from one collisionless phase-space flow.

These results establish that the corresponding homogeneous cosmological observables do not uniquely require an expanding-space ontology. FLSC therefore provides a static cosmology within ordinary general relativity, with cosmological redshift and background photon transport governed by a single deformation rate.

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