

Distinguishing between FLRW and Taub-NUT Cosmologies

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Abstract

The interpretation of cosmological redshift as evidence for the expansion of space is a central component of the standard Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological framework. In this paper, we compare FLRW cosmology with a stationary Taub-NUT gravitational-redshift cosmology using redshift-based observables. We discuss three possible discriminants: the Hubble diagram, redshift drift, and astrometric radial velocities. The supernova Hubble diagram is fit with both a Taub-NUT redshift-distance relation and a reference flat FLRW model. The fits show that present supernova redshift-distance data alone do not uniquely distinguish the two descriptions. However, future observations at high redshift ($z > 2$) may provide an observational means of distinguishing between the two cosmological theories. In addition, redshift drift provides a direct future observational test: expanding FLRW models predict a systematic, nonzero drift, whereas a stationary Taub-NUT model predicts zero cosmological drift for sources at fixed coordinate distance. Astrometric radial velocities are included for conceptual completeness, but quantitative estimates show that they are not practical at cosmological distances.

Keywords

Cosmological Models, Redshift Drift, Hubble Diagram, Taub-NUT Spacetime, Stationary Cosmology

1. Introduction

The standard theory of cosmology is based on the FLRW metric, in which cosmological redshifts are interpreted as arising from the evolution of the cosmic scale factor. This interpretation is supported by a large body of observations, including the supernova Hubble diagram [1] [2], the cosmic microwave background [3], baryon acoustic oscillations [4], and the growth of large-scale structure. The pur-

pose of the present paper is to ask whether redshift-based observables can distinguish an expanding FLRW interpretation from a specific stationary gravitational-redshift interpretation based on the Taub-NUT solution.

The historical relation between redshift and distance is usually associated with [5], although the development of expanding-universe cosmology involved the earlier theoretical work of [6] and [7]. Hubble himself discussed alternative interpretations of redshift, including gravitational effects and scattering processes, and [8] emphasized that the interpretation of redshift as velocity was an assumption. In modern cosmology, the expansion interpretation is no longer merely historical; it is embedded in a quantitatively successful framework. Nevertheless, direct tests of the time evolution of cosmological redshift remain important because they probe the kinematic structure of spacetime without relying on standard candles or standard rulers.

In this paper, “stationary cosmology” means cosmological models based on a time-independent metric, so that the metric coefficients satisfy

$$\frac{\partial g_{\mu\nu}}{\partial t} = 0. \quad (1)$$

In the specific Taub-NUT interpretation considered here [9] [10], the observed redshift is attributed to a gravitational redshift rather than to the expansion of space. The source distance entering the model is a radial coordinate of the stationary Taub-NUT geometry. The central question is therefore: can the redshift-distance relation and redshift-drift measurements distinguish between the expanding FLRW framework and the stationary Taub-NUT framework?

1.1. Motivation and Scope

The Hubble diagram is an important empirical relation [1] [2] [5], but it is not by itself a direct measurement of the expansion of space. The interpretation that cosmological redshift is caused by the expansion of space is therefore an assumption of the FLRW framework. Although supported indirectly by a broad range of observations, a direct observational verification has not yet been achieved.

Occam's Razor

Scientific theories should be based on the smallest possible number of assumptions. One way to reduce assumptions is to determine through observation whether an assumption corresponds to reality. Another is to show that an assumption is unnecessary. For example, in [9] [10], we eliminated the assumption of the cosmological principle in our theory of cosmology. Here we demonstrate how, through astronomical observations, the Hubble diagram and redshift drift, we can determine if the assumption of the cosmological redshift is actually due to the expansion of space, which would make it a fact instead of an assumption, or if on the contrary it is not due to the expansion of space, in which case this interpretation would need to be eliminated from cosmological theory.

It is clear that a useful cosmological test should reduce the dependence on in-

interpretive assumptions wherever possible. Here, we suggest that the redshift distance relationship and redshift drift could provide a test of the expansion of space. Redshift drift is more direct because it measures whether the redshift of a distant source changes with observer time [11]-[14]. In an expanding FLRW model, comoving sources generally exhibit a nonzero cosmological redshift drift. In a stationary metric, a source at fixed coordinate distance has no cosmological drift. This contrast motivates the present comparison.

Other cosmological probes, including the cosmic microwave background, baryon acoustic oscillations, weak lensing, number counts, and structure formation, are essential to modern precision cosmology. They are outside the scope of this paper, which is limited to redshift-based discriminants.

1.2. Methods

We consider three possible methods for distinguishing an expanding FLRW description from a stationary Taub-NUT description:

- 1) The supernova Hubble diagram;
- 2) The redshift drift;
- 3) Astrometric radial velocities.

The first two are practical or potentially practical observational tests. Astrometric radial velocities are included mainly for conceptual completeness because, as shown quantitatively in Appendix A.1., the perspective-acceleration signal is far too small to be useful at cosmological distances.

2. Taub-NUT Cosmology

The Taub-NUT cosmology considered here is a stationary model based on the Taub-NUT solution of Einstein's field equations and on the cosmological interpretation developed in [9] [10]. In this model, the cosmological redshift is interpreted as a gravitational redshift. The redshift is written in terms of the metric coefficient as

$$(1+z)^2 = g_{00}(r). \quad (2)$$

The Taub-NUT metric used here is

$$g_{00}(r) = \frac{2\alpha r + n^2 - r^2}{r^2 + n^2}, \quad (3)$$

which gives

$$z(r) = \left(\frac{2\alpha r + n^2 - r^2}{r^2 + n^2} \right)^{1/2} - 1. \quad (4)$$

Here, r is the radial coordinate of the Taub-NUT geometry and is expressed in Gpc in the empirical comparison below. The parameter α is the length-scale parameter associated with the mass term in the metric, and n is the NUT parameter. In the present phenomenological fit, both are expressed in the same distance units as r . The physically allowed domain is the range for which

$$2\alpha r + n^2 - r^2 > 0, \quad (5)$$

so that the square root in Equation (4) is real. The comparison below uses the positive-redshift branch relevant to the observed supernova sample.

2.1. Distance Measure Used in the Comparison

The observational supernova distance is inferred from the distance modulus,

$$d_L (\text{Gpc}) = \frac{10^{(\mu-25)/5}}{1000}, \quad (6)$$

where μ is the published corrected distance modulus. **Figure 1** compares z_{HD} with this inferred luminosity distance. For the Taub-NUT overlay, the practical comparison identifies the radial coordinate r with the empirical distance scale d_L in Gpc. This identification is a phenomenological comparison, not a full derivation of the luminosity-distance relation in the Taub-NUT spacetime. A fully self-consistent treatment of photon propagation, flux dilution, and luminosity distance in this stationary metric remains an important extension of the present work.

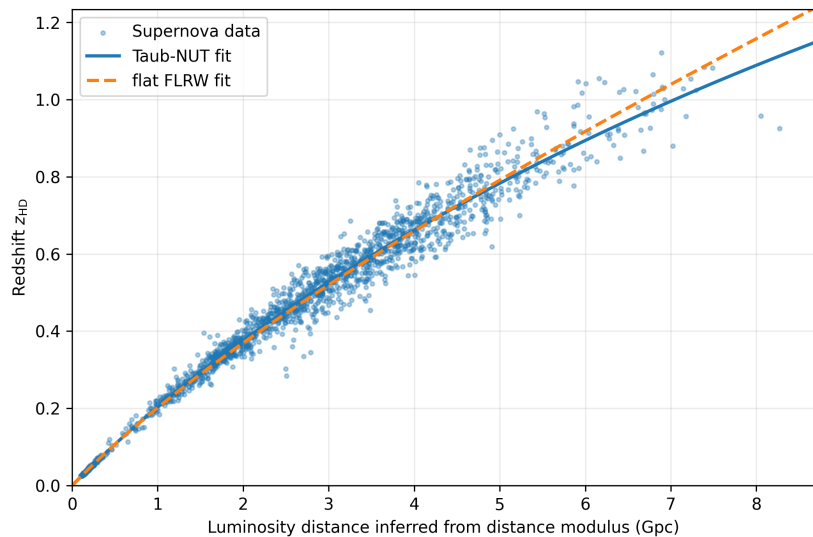


Figure 1. Supernova redshift z_{HD} versus luminosity distance inferred from the distance modulus. The solid curve is the best-fit Taub-NUT relation with $\alpha = 318.72 \pm 53.40$ Gpc and $n = 37.61 \pm 3.23$ Gpc. The dashed curve is a reference flat FLRW model with $\Omega_m = 0.30$, $\Omega_\Lambda = 0.70$, and best-fit $H_0 = 69.30 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$. Individual error bars are omitted for clarity.

2.2. Reproducible Supernova Fit

In this analysis, the fit uses the Pantheon + SH0ES supernova compilation with columns z_{HD} , z_{HDERR} , μ , and σ_μ . Only objects with positive z_{HD} , positive μ , and finite positive σ_μ were retained. This gives $N = 1768$ supernovae in the range

$$0.02509 \leq z_{\text{HD}} \leq 1.12132. \quad (7)$$

The distance d_L was computed from Equation (6). The Taub-NUT parameters α and n were obtained by a nonlinear least-squares fit of Equation (4) to $z_{\text{HD}}(d_L)$. For the reference FLRW comparison, a spatially flat Λ CDM model with $\Omega_m = 0.30$ and $\Omega_\Lambda = 0.70$ was used, with H_0 fit to the same data.

For the goodness-of-fit calculation, the effective uncertainty in redshift was computed by combining the tabulated z_{HD} uncertainty with the distance-modulus uncertainty propagated into redshift through the local slope of the fitted curve,

$$\sigma_{z,\text{eff}}^2 = \sigma_{z_{\text{HD}}}^2 + \left(\frac{dz}{dd_L} \frac{\ln 10}{5} d_L \sigma_\mu \right)^2. \quad (8)$$

This treatment uses diagonal uncertainties only and is therefore an approximate model-comparison statistic. A full covariance-matrix analysis would be a natural next refinement.

Table 1. Approximate model comparison using the same supernova sample.

Model	Parameters	χ^2	χ_ν^2	RMSE in z	AIC	BIC
Taub-NUT	α, n	1793.69	1.0157	0.03682	1797.69	1808.64
Flat FLRW	H_0	1795.06	1.0159	0.03773	1797.06	1802.54

Table 1 shows that the present redshift-distance data do not provide a decisive separation between the two descriptions in this approximate comparison. The Taub-NUT fit gives a slightly smaller χ^2 and RMSE, while the FLRW model gives a smaller BIC because it uses one fitted parameter rather than two. Thus, the Hubble diagram by itself is not sufficient to establish the Taub-NUT interpretation; rather, it shows that a stationary gravitational-redshift curve can mimic much of the observed redshift-distance relation over the present supernova range.

2.3. High-Redshift Hubble Diagram

High-redshift supernovae observed with JWST may eventually extend the Type Ia supernova Hubble diagram into the approximate range $2 \lesssim z \lesssim 4$. This is potentially important because extrapolations of the Taub-NUT and FLRW curves can separate at high redshift. This possible separation is illustrated in **Figure 2**. However, the current observational situation must be stated cautiously: Type Ia supernova constraints at $z > 2$ remain limited, and independent luminosity-distance measurements in this range are still developing [15]-[17].

3. Redshift Drift as a Discriminator

The observed period of a periodic cosmological source is related to its emitted period by [18]-[20]

$$T_{\text{obs}} = (1+z)T_{\text{emit}}. \quad (9)$$

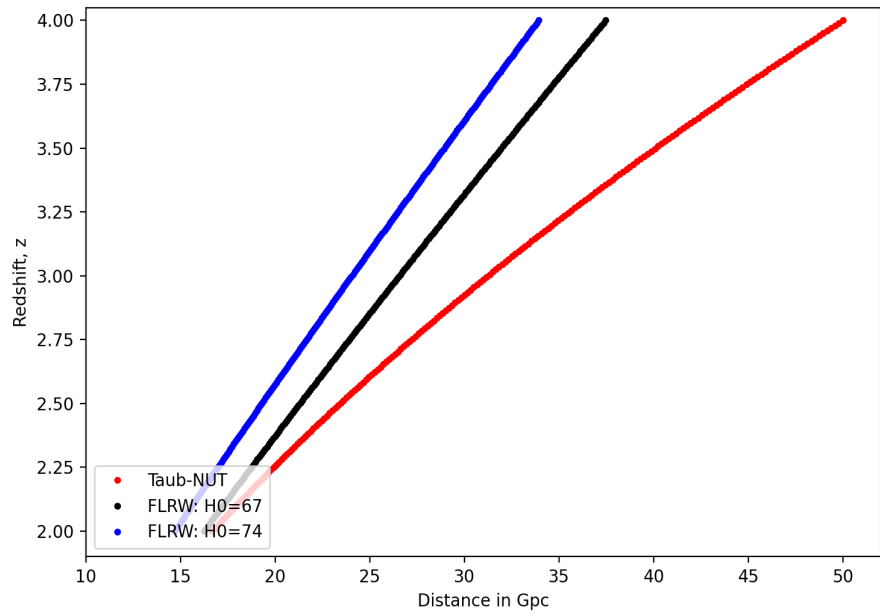


Figure 2. Projected comparison in the high-redshift range $2 \leq z \leq 4$, shown in the same orientation as the originally submitted manuscript. The red curve is the Taub-NUT prediction. The black and blue curves are flat Λ CDM luminosity-distance examples with $\Omega_m = 0.30$, $\Omega_\Lambda = 0.70$, and $H_0 = 67$ or $74 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. This figure illustrates a possible future discriminant; it does not imply that present high-redshift supernova data already establish the distinction.

In an expanding FLRW cosmology, the redshift evolves with observer time. In a stationary metric, by contrast, the metric coefficients are time independent. For the Taub-NUT relation

$$z(r) = \sqrt{g_{00}(r)} - 1, \quad (10)$$

the observer-time derivative is

$$\dot{z} \equiv \frac{dz}{dt_0} = \frac{g'_{00}(r)}{2[1+z]} \dot{r}, \quad (11)$$

where t_0 is observer time, $g'_{00} = dg_{00}/dr$, and $\dot{r} = dr/dt_0$. Therefore, for a source at fixed coordinate distance,

$$\dot{r} = 0, \quad (12)$$

and hence

$$\dot{z} = 0. \quad (13)$$

The corresponding FLRW prediction is the Sandage-Loeb relation [11] [12]

$$\dot{z} = (1+z)H_0 - H(z), \quad (14)$$

where $H(z)$ is the Hubble parameter at emission. The finite redshift change over an observing interval Δt_0 is

$$\Delta z \simeq \dot{z} \Delta t_0, \quad (15)$$

and the associated spectroscopic velocity drift is conventionally written

$$\Delta v = c \frac{\Delta z}{1+z}. \quad (16)$$

Thus, the two frameworks make sharply different predictions

$$\text{stationary cosmology at fixed } r : \Delta v = 0, \quad (17)$$

whereas

$$\text{FLRW cosmology : } \Delta v \neq 0 \quad (18)$$

in general.

This statement applies to the cosmological component of the drift. In practice, any observed $\dot{z}$ must be corrected for peculiar accelerations, source motions, observer acceleration, line-of-sight gravitational effects, source evolution, and instrumental systematics. A robust test, therefore, requires a population of sources and a redshift-dependent drift pattern. The expected FLRW signal is extremely small, of order $1 - 10 \text{ cm}\cdot\text{s}^{-1}$ over decade-long baselines for suitable high-redshift sources. Nevertheless, future high-resolution spectrographs such as ESPRESSO and ANDES are designed to approach the long-term stability required for this test [21]-[23].

4. Relation to Recent Tests of Gravity

The present comparison is related to a broader effort to test gravity and cosmology through observational signatures beyond the weak-field solar-system regime. Recent work on modified-gravity signatures, infrared corrections, strong-field accretion, and black-hole dynamics illustrates the general importance of connecting alternative gravitational models with measurable observables [24]-[27]. In the present paper, however, the focus remains narrower: whether redshift-based cosmological observables can distinguish the FLRW interpretation from a stationary Taub-NUT gravitational-redshift interpretation.

5. Conclusion

We have compared FLRW cosmology with a stationary Taub-NUT gravitational-redshift cosmology using redshift-based observables. The supernova analysis shows that the present redshift-distance data do not by themselves decisively distinguish the two descriptions. The approximate fit gives comparable goodness-of-fit values for a two-parameter Taub-NUT curve and a one-parameter reference flat FLRW curve. Therefore, the manuscript does not claim that Taub-NUT cosmology is proven.

The more decisive test is redshift drift. FLRW cosmology predicts a systematic redshift drift for comoving sources because the scale factor evolves with time. A stationary Taub-NUT cosmology predicts zero cosmological redshift drift for sources at fixed coordinate distance. Thus, a statistically significant detection of the FLRW redshift-drift pattern would strongly favor an expanding spacetime interpretation, whereas a robust null result after controlling for local and source-specific effects would motivate renewed consideration of stationary alternatives.

Astrometric radial velocities provide a useful conceptual comparison because they would measure radial motion without using spectroscopic redshift. However, the perspective-acceleration signal at cosmological distances is too small to be practical. Consequently, redshift drift and future high-redshift Hubble diagrams remain the most promising redshift-based discriminants between expanding and stationary cosmological scenarios.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

A.1. Astrometric Radial Velocity and the Expansion of Space

Astrometric radial velocities can in principle be obtained from perspective acceleration rather than spectroscopic redshift [28]. If a source has transverse velocity v_t , radial velocity v_r , and distance d , its proper motion is

$$\mu = \frac{v_t}{d}. \quad (19)$$

If the distance varies as $d(t) = d_0 + v_r t$, then

$$\dot{\mu} = -\mu \frac{v_r}{d}, \quad (20)$$

or equivalently

$$v_r = -\left(\frac{\dot{\mu}}{\mu}\right)d. \quad (21)$$

Combining these relations gives

$$\dot{\mu} = -\frac{v_t v_r}{d^2}. \quad (22)$$

The inverse-square distance dependence makes the signal impractically small at cosmological distances.

For a representative source at $d = 100$ Mpc with $v_t = 300 \text{ km} \cdot \text{s}^{-1}$ and $v_r \approx H_0 d \approx 7000 \text{ km} \cdot \text{s}^{-1}$, using typical peculiar-velocity scales from the literature [29]-[32], the perspective acceleration is approximately

$$|\dot{\mu}| \approx 1.4 \times 10^{-9} \mu\text{as} \cdot \text{yr}^{-2}. \quad (23)$$

Over a 20 yr observing baseline, the angular curvature from this acceleration is only

$$\Delta\theta_{\dot{\mu}} = \frac{1}{2} \dot{\mu} T^2 \approx 2.8 \times 10^{-7} \mu\text{as}. \quad (24)$$

The corresponding observational requirements are summarized in **Table A1**.

Table A1. Number of repeated observations required to detect proper motion and perspective acceleration for a source at $d = 100$ Mpc over a 20 yr baseline.

Facility	Proper Motion Observations	Perspective-Acceleration Observations
Gaia	~ 16	$\sim 6 \times 10^{18}$
HST Multi-Epoch	~ 600	$\sim 2 \times 10^{20}$
Current VLBI	$\sim 1 - 6$	$\sim 2 \times 10^{17} - 2 \times 10^{18}$
GaiaNIR Concept	~ 1	$\sim 6 \times 10^{16}$
SKA-VLBI Optimistic	~ 1	$\sim 9 \times 10^{15}$
Speculative Interferometer	~ 1	$\sim 2 \times 10^{13}$

Assumptions: $d = 100$ Mpc, $v_t = 300 \text{ km} \cdot \text{s}^{-1}$, $v_r = 7000 \text{ km} \cdot \text{s}^{-1}$, and a 20 yr baseline. The estimates include statistical averaging only and neglect systematic errors, source-structure effects, calibration floors, spacecraft stability, and photon-counting limitations.

As shown in **Table A1**, the required number of observations becomes enormous for perspective-acceleration measurements at cosmological distances. If one assumes, unrealistically, one statistically independent observation every second, the required observing time is

$$T_{\text{req}} \approx \left(\frac{2\sigma_{\theta}}{\dot{\mu}\sqrt{R}} \right)^{2/5}, \quad (25)$$

where $R = 3.16 \times 10^7 \text{ yr}^{-1}$. The resulting times remain decades to centuries even under highly optimistic assumptions. Thus astrometric radial velocities cannot be used as a practical cosmological expansion test at 50 - 100 Mpc.

A.2. Stationary and Non-Expanding Cosmological Alternatives

Many stationary or non-expanding cosmological models have been proposed in the literature, including general-relativistic stationary models [33]-[37], gravitational-redshift interpretations [38]-[40], and tired-light mechanisms [41]-[44]. The purpose of mentioning these models is not to place them on the same empirical footing as standard cosmology. Rather, their existence motivates the value of direct observational tests, especially redshift drift, that can distinguish expanding from stationary interpretations.