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A Readout-Based Structural Analysis of Empirical CMB Temperature Equations Related to *E* = *hν* and *E* = *mc*²

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Abstract

This report extends our previous studies on empirical equations involving the cosmic microwave background (CMB) temperature. Whereas our earlier work mainly discussed numerical connections among physical quantities, the present work attempts to reformulate those relations within a structural framework. In particular, Equation (12), previously introduced as a central empirical relation, is reconsidered independently of the MKSA unit system and expressed in terms of macro-micro correspondence. This reformulation clarifies how macroscopic units and microscopic quantities can be organized into a consistent structural scheme. A recalculated CMB temperature of 2.7256307 K agrees with the observed value of 2.72548 ± 0.00057 K. Rather than introducing further numerical adjustments, we examine the internal consistency of the resulting structure and show that it can be written in a closed and coherent form. The aim is not to provide a complete microphysical theory, but to examine whether $E = mc^2$ and $E = h\nu$ can be organized within a common readout-based structural framework.

Keywords

Temperature of the Cosmic Microwave Background, Minimum Mass, Ratio of the Gravitational Force to the Electric Force, Dimensional Analysis, Fine-Structure Constant

1. Introduction

The list of symbols used in this paper is summarized in Section 2. In our previous studies, we proposed a set of empirical equations relating the cosmic microwave background (CMB) temperature to several physical quantities and evaluated them numerically [1]-[5]. In particular, Equations (1), (2), and (3) represent characteristic relationships derived from the CMB temperature.

$$\frac{Gm_p^2}{hc} = \frac{4.5}{2} \cdot \frac{kT_c}{1\,\mathrm{kg} \cdot c^2} \tag{1}$$

$$\frac{Gm_p^2}{\left(\dfrac{e^2}{4\pi\varepsilon_0}\right)} = \frac{4.5}{2\pi} \cdot \frac{m_e}{e} \cdot hc \tag{2}$$

$$\frac{m_e c^2}{e} \cdot \left(\frac{e^2}{4\pi\varepsilon_0}\right) = \pi \cdot kT_c \tag{3}$$

We further introduced an empirical expression for the fine-structure constant [6] and examined its consistency with related quantities [7] [8].

$$\frac{1}{\alpha} = 137.0359991 = 136.0113077 + \frac{1}{3 \cdot 13.5} + 1 \tag{4}$$

$$13.5 \cdot 136.0113077 = 1836.152654 = \frac{m_p}{m_e} \tag{5}$$

Small deviations from the ideal values of 9/2 and π, respectively, were discussed as a way to reduce the induced numerical errors [9]-[14]. The coefficient associated with the dimensional factor of length (m^2/s) was denoted by kL and by kL_0 when explicitly expressed in the meter-kilogram-second-ampere (MKSA) unit system [15] [16]. In this work, kL_0 is treated as a structural readout coefficient associated with the m^2/s-type branch, rather than as a new dynamical operator or as an independently measured transport coefficient.

$$kL\left(\frac{\mathrm{m}^2}{\mathrm{s}}\right) = 1837.94538\left(\frac{\mathrm{m}^2}{\mathrm{s}}\right)_{\mathrm{MKSA}} \equiv kL_0\left(\frac{\mathrm{m}^2}{\mathrm{s}}\right)_{\mathrm{MKSA}} \tag{6}$$

where the subscript “MKSA” indicates the MKSA unit system.

$$3.1327945(\mathrm{V} \cdot \mathrm{m}) = \frac{kL_0 \cdot m_e c^2}{e \cdot c} \tag{7}$$

$$4.4873976\left(\frac{1}{\mathrm{A} \cdot \mathrm{m}}\right) = \frac{q_m \cdot c}{kL_0 \cdot m_p c^2} \tag{8}$$

By redefining the von Klitzing constant, these deviations can be adjusted back to the ideal values of 9/2 and π [15], leading to Equations (9) and (10).

$$\pi(\mathrm{V} \cdot \mathrm{m}) = \frac{kL_0 \cdot m_e c^2}{e_{new} \cdot c} \tag{9}$$

$$4.5\left(\frac{1}{\mathrm{A} \cdot \mathrm{m}}\right) = \frac{q_{m\_new} \cdot c}{kL_0 \cdot m_p c^2} \tag{10}$$

The parameter kL_0 is then obtained from Equation (11).

$$kL_0 = \sqrt{\frac{3.1327945}{4.4873976} \cdot \frac{hc^2}{m_p c^2 \cdot m_e c^2}} = \sqrt{\frac{\pi_{unit}}{4.5_{unit}} \cdot \frac{hc^2}{m_p c^2 \cdot m_e c^2}} = 1837.94538 \tag{11}$$

This leads to the following relation.

$$m_p c^2 \cdot m_e c^2 \cdot \frac{4.5_{unit}}{\pi_{unit}} \cdot hc^2 = \left(2\pi(1) \cdot \frac{kT_c}{\alpha}\right)^2 = 1.04985584\text{E}-39 \quad (12)$$

Using this framework, a calculated CMB temperature of 2.7256307 K was obtained, in agreement with the observed value of 2.72548 ± 0.00057 K. While previous work mainly focused on numerical consistency and parameter adjustments within the MKSA unit system [15] [16], the underlying structural meaning of these relations has remained unclear. In particular, the role of Equation (12) and its possible connection to more fundamental physical structure require further clarification.

In the present work, we revisit these empirical relations from the viewpoint of macro-micro correspondence. The redefinition procedure used here does not imply a physical change of fundamental constants, but is introduced only as a readout-level transformation. Rather than introducing further numerical adjustments, we examine whether the relations can be reorganized into a consistent structural form based on readout-invariant quantities and their associated mass-time structure. The physical origin of the connection coefficients is not claimed to be fully explained in this paper and remains an open question.

The main purpose of the present work is to examine whether the empirical relations discussed above can be organized within a common readout-based structural framework related to $E = mc^2$ and $E = h\nu$. The present work is therefore intended as a structural consistency analysis rather than a complete physical theory.

The remainder of this paper is organized as follows. Section 2 summarizes the symbols used in this work. Section 3 presents the methodological framework, including c-normalization and the redefinition procedure adopted in this study. Section 4 presents the main structural results and explains their physical meaning in relation to the connection between $E = mc^2$ and $E = h\nu$. Section 5 discusses broader implications of the present framework.

2. Symbol List

2.1. Reference Constants in MKSA Units (These Values Were Obtained from Wikipedia)

G: Gravitational constant: 6.6743×10^{-11} ($\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$)

(we used the compensated value of $6.68917534 \times 10^{-11}$ in this study)

T_c: CMB temperature: 2.72548 ± 0.00057 (K)

(we used the compensated value of 2.725630647 K in this study)

k: Boltzmann constant: 1.380649×10^{-23} ($\text{J} \cdot \text{K}^{-1}$)

c: Speed of light: 299792458 (m/s)

h: Planck constant: $6.62607015 \times 10^{-34}$ (J s)

ε_0: Electric constant: $8.8541878128 \times 10^{-12}$ ($\text{N} \cdot \text{m}^2 \cdot \text{C}^{-2}$)

μ_0: Magnetic constant: $1.25663706212 \times 10^{-6}$ ($\text{N} \cdot \text{A}^{-2}$)

e: Electric charge of one electron: $-1.602176634 \times 10^{-19}$ (C)

q_m: Magnetic charge of one magnetic monopole: 4.13566770 × 10^{-15} (Wb)
(This value is only a theoretical value: q_m = h/e)
m_p: Resting mass of a proton: 1.67262192369 × 10^{-27} (kg)
m_e: Resting mass of an electron: 9.1093837015 × 10^{-31} (kg)
R_k: von Klitzing constant: 25812.80745 (Ω)
Z_0: Wave impedance in free space: 376.730313668 (Ω)
α: Fine-structure constant: 1/137.035999081

2.2. Symbol List Established in Previous Reports

2.2.1. Symbol List Obtained after the Redefinition Process

In this subsection, symbols with the subscript "new" denote quantities expressed under the redefinition scheme adopted in this work, representing numerically consistent forms of the corresponding MKSA quantities.

$$e_{new} = e \cdot \frac{4.4873976}{4.5} = 1.5976897\text{E}{-19}(\text{C}) \tag{13}$$

$$q_{m\_new} = q_m \cdot \frac{\pi}{3.1327945} = 4.1472823\text{E}{-15}(\text{Wb}) \tag{14}$$

$$h_{new} = e_{new} \cdot q_{m\_new} = h \cdot \frac{4.4873976}{4.5} \cdot \frac{\pi}{3.1327945} = 6.62607015\text{E}{-34}(\text{J} \cdot \text{s}) = h \tag{15}$$

Therefore, the value of Planck's constant is unchanged.

$$Rk_{\_new} = \frac{q_{m\_new}}{e_{\_new}} = Rk \cdot \frac{4.5}{4.4873976} \cdot \frac{\pi}{3.1327945} = 25957.997(\Omega) \tag{16}$$

Equation (16) can be rewritten as follows:

$$Rk_{new} = 4.5\left(\frac{1}{\text{A} \cdot \text{m}}\right) \cdot \pi(\text{V} \cdot \text{m}) \cdot \frac{m_p}{m_e} = 25957.997(\Omega) \tag{17}$$

$$Z_{0\_new} = \alpha \cdot \frac{2h_{new}}{e_{new}^2} = 2\alpha \cdot Rk_{new} = Z_0 \cdot \frac{4.5}{4.4873976} \cdot \frac{\pi}{3.1327945} = 378.84931(\Omega) \tag{18}$$

Equation (18) can be rewritten as follows:

$$Z_{0\_new} = 4.5\left(\frac{1}{\text{A} \cdot \text{m}}\right) \cdot \pi(\text{V} \cdot \text{m}) \cdot 2\alpha \cdot \frac{m_p}{m_e} = 378.84931(\Omega) \tag{19}$$

$$\mu_{0\_new} = \frac{Z_{0\_new}}{c} = \mu_0 \cdot \frac{4.5}{4.4873976} \cdot \frac{\pi}{3.1327945} = 1.2637053\text{E}{-06}\left(\text{N} \cdot \text{A}^{-2}\right) \tag{20}$$

$$\varepsilon_{0\_new} = \frac{1}{Z_{0\_new} \cdot c} = \varepsilon_0 \cdot \frac{4.4873976}{4.5} \cdot \frac{3.1327945}{\pi} = 8.8046642\text{E}{-12}\left(\text{F} \cdot \text{m}^{-1}\right) \tag{21}$$

$$c_{\_new} = \frac{1}{\sqrt{\varepsilon_{0\_new}\mu_{0\_new}}} = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = c = 299792458\left(\text{m} \cdot \text{s}^{-1}\right) \tag{22}$$

The numerical value of the speed of light remains unchanged. In the present redefinition scheme, h and c are held fixed by construction. Next, the Compton wavelength (λ) is as follows:

$$\lambda = \frac{h}{mc} \tag{23}$$

When the values of Planck's constant and the speed of light are unchanged, the following values in Equations (24), (25) and (26) can be kept constant.

$$m_{e\_new} = m_e \cdot \frac{4.4873976}{4.5} \cdot \frac{\pi}{3.1327945} = m_e = 9.1093837\text{E}-31(\text{kg}) \quad (24)$$

$$m_{p\_new} = m_p \cdot \frac{4.4873976}{4.5} \cdot \frac{\pi}{3.1327945} = m_p = 1.6726219\text{E}-27(\text{kg}) \quad (25)$$

$$kT_{c\_new} = kT_c \cdot \frac{4.4873976}{4.5} \cdot \frac{\pi}{3.1327945} = kT_c = 3.7631393\text{E}-23(\text{J}) \quad (26)$$

The definition of minimum mass is as follows:

$$M_{\min} = 2\pi \cdot \frac{kT_c}{\alpha c^2} = 3.605150741\text{E}-37(\text{kg}) \quad (27)$$

2.2.2. Algorithm Used in the Previous Work

1) Symbol list about 4.5_{unit} and π_{unit}

4.5_{unit} and π_{unit} are introduced solely as unit-identification factors.

$$4.5_{unit} = 4.5\left(\text{A}^{-1} \cdot \text{m}^{-1}\right) \quad (28)$$

$$\pi_{unit} = \pi(\text{V} \cdot \text{m}) \quad (29)$$

2) Algorithm used in the previous work

$$e_{new} \cdot c(\text{A} \cdot \text{m}) = \frac{1}{4.5_{unit}} \cdot \frac{hc^2}{m_p c^2 \cdot kL_0} = 4.78975317\text{E}-11(\text{A} \cdot \text{m}) \quad (30)$$

$$q_{m\_new} \cdot c(\text{V} \cdot \text{m}) = \pi_{unit} \cdot \frac{hc^2}{m_e c^2 \cdot kL_0} = 1.24332398\text{E}-06(\text{V} \cdot \text{m}) \quad (31)$$

$$m_e c^2 \cdot kL_0\left(\text{J} \cdot \frac{\text{m}^2}{\text{s}}\right) = \frac{\pi_{unit}}{4.5_{unit}} \cdot \frac{hc^2}{m_p c^2 \cdot kL_0} = 1.50474532\text{E}-10\left(\text{J} \cdot \frac{\text{m}^2}{\text{s}}\right) \quad (32)$$

$$m_p c^2 \cdot kL_0\left(\text{J} \cdot \frac{\text{m}^2}{\text{s}}\right) = \frac{\pi_{unit}}{4.5_{unit}} \cdot \frac{hc^2}{m_e c^2 \cdot kL_0}\left(\text{J} \cdot \frac{\text{m}^2}{\text{s}}\right) = 2.76294215\text{E}-07\left(\text{J} \cdot \frac{\text{m}^2}{\text{s}}\right) \quad (33)$$

$$hc^2\left(\frac{\text{J} \cdot \text{m}^2}{\text{s}}\right) = \frac{\pi_{unit}}{4.5_{unit}} \cdot \frac{hc^2}{m_p c^2 \cdot kL_0} \cdot \frac{hc^2}{m_e c^2 \cdot kL_0}\left(\frac{\text{J} \cdot \text{m}^2}{\text{s}}\right) = 5.95521\text{E}-17\left(\frac{\text{J} \cdot \text{m}^2}{\text{s}}\right) \quad (34)$$

$$2\pi \cdot \frac{kT_c}{\alpha} \cdot kL_0 = \frac{\pi_{unit}}{4.5_{unit}} \cdot \frac{hc^2}{m_p c^2 \cdot kL_0} \cdot \frac{hc^2}{m_e c^2 \cdot kL_0} = 5.95521\text{E}-17\left(\frac{\text{J} \cdot \text{m}^2}{\text{s}}\right) \quad (35)$$

$$\frac{G_{new}}{c^3}\left(\frac{\text{s}}{\text{kg}} = \frac{\text{m}^2/\text{s}}{\text{J}}\right) = \frac{\alpha}{2\pi} \cdot \frac{4.5(1)}{2} \cdot \left(\frac{4.5_{unit}}{\pi_{unit}} \cdot hc^2\right) \cdot \frac{kL_0}{1\,\text{kg} \cdot c^2} \cdot \left(\frac{m_e}{m_p}\right) = 2.4826212101\text{E}-36 \quad (36)$$

$$R_p\left(m_p\right) \equiv \frac{G_{new}}{c^3} \frac{\left(m_p c^2\right)^2}{\left(\frac{\left(e_{new} \cdot c\right)^2}{4\pi\varepsilon_{0\_new} \cdot c}\right)} = \frac{4.5(1)}{2} \cdot 2\pi(1) \cdot \frac{kT_c}{\alpha} \cdot \frac{1}{1\,\text{kg} \cdot c^2} = 8.11158917\text{E}-37 \quad (37)$$

where Rp (m_p) denotes the dimensionless ratio of the gravitational force to the electrostatic force between two protons at the same separation. Although Equations (36) and (37) are not essential to the main purpose of the present paper, they are retained as possible clues for further development.

2.3. Useful Equations

The following equations are useful for confirming the internal consistency of the present framework.

$$\frac{1}{\varepsilon_{0\_new}c}\left(\Omega=\frac{\text{kg}}{\text{C}^2}\cdot\frac{\text{m}^2}{\text{s}}\right)=\frac{M_{\min}}{\left(e_{new}\right)^2}\cdot kL_0\cdot 2\alpha=378.8493064(\Omega)=Z_{0\_new} \tag{38}$$

$$\mu_{0\_new}c\left(\Omega=\frac{\text{Wb}^2}{\text{kg}}\cdot\frac{\text{s}}{\text{m}^2}\right)=\frac{\left(q_{m\_new}\right)^2}{M_{\min}\cdot kL_0}\cdot 2\alpha=378.8493064(\Omega)=Z_{0\_new} \tag{39}$$

$$m_pc^2(\text{J})\cdot 4.5_{unit}e_{new}\cdot c=2\pi(1)\cdot\frac{kT_c}{\alpha}(\text{J})=3.24014789\text{E}-20(\text{J}) \tag{40}$$

$$m_ec^2(\text{J})\cdot\frac{q_{m\_new}\cdot c}{\pi_{unit}}=2\pi(1)\cdot\frac{kT_c}{\alpha\cdot c^2}(\text{J})=3.24014789\text{E}-20(\text{J}) \tag{41}$$

$$m_ec^2\cdot kL_0=\pi_{unit}\cdot e_{new}\cdot c=1.50474532\text{E}-10\left(\text{J}\frac{\text{m}^2}{\text{s}}\right) \tag{42}$$

$$m_pc^2\cdot kL_0=\frac{q_{m\_new}\cdot c}{4.5_{unit}}=2.76294215\text{E}-07\left(\text{J}\frac{\text{m}^2}{\text{s}}\right) \tag{43}$$

$$hc^2\cdot kL_0^{-1}(\text{J})=2\pi(1)\cdot\frac{kT_c}{\alpha}(\text{J})=M_{\min}c^2=3.24014789\text{E}-20(\text{J}) \tag{44}$$

$$e_{new}(\text{C})=\sqrt{\frac{h}{4.5_{unit}\pi_{unit}}\cdot\frac{m_e}{m_p}}=1.597689670\text{E}-19(\text{C}) \tag{45}$$

$$q_{m\_new}(\text{Wb})=\sqrt{h\cdot 4.5_{unit}\cdot\pi_{unit}\cdot\frac{m_p}{m_e}}=4.147282338\text{E}-15(\text{Wb}) \tag{46}$$

$$\frac{\pi_{unit}}{4.5_{unit}}\cdot hc^2=m_pc^2\cdot kL_0\cdot m_ec^2\cdot kL_0=4.15752428\text{E}-17 \tag{47}$$

$$\frac{4.5_{unit}}{\pi_{unit}}\cdot hc^2=\frac{\left(2\pi(1)\cdot kT_c/\alpha\right)}{m_pc^2}\cdot\frac{\left(2\pi(1)\cdot kT_c/\alpha\right)}{m_ec^2}=8.530216942\text{E}-17 \tag{48}$$

To simplify the calculation, G_N is defined as follows:

$$G_N=G\cdot 1\,\text{kg}=6.68917521\text{E}-11\left(\text{m}^3\cdot\text{s}^{-2}\right) \tag{49}$$

$$G_{N\_new}\left(\frac{\text{m}^3}{\text{s}^2}\right)=\alpha c\frac{4.5(1)}{4\pi(1)}\cdot\left(4.5_{unit}\cdot e_{new}c\right)\cdot\frac{hc^2}{m_pc^2}=6.6891752\text{E}-11\left(\frac{\text{m}^3}{\text{s}^2}\right) \tag{50}$$

$$G_{N\_new}\left(\frac{\text{m}^3}{\text{s}^2}\right)=\alpha c\frac{4.5(1)}{4\pi(1)}\cdot\left(4.5_{unit}\cdot e_{new}\cdot c\right)^2\cdot\frac{\pi_{unit}\cdot e_{new}\cdot c}{m_ec^2}=6.6891752\text{E}-11\left(\frac{\text{m}^3}{\text{s}^2}\right) \tag{51}$$

$$\frac{G_{new}}{c^3}=\frac{\alpha}{2\pi(1)}\frac{9}{4}(1)\cdot\left(4.5_{unit}\cdot e_{new}\cdot c\right)^2\cdot\frac{kL_0}{1\,\text{kg}\cdot c^2}=2.48262121\text{E}-36\left(\frac{\text{s}}{\text{kg}}\right) \tag{52}$$

$$\frac{G_{new}}{c^3}=\frac{\alpha}{2\pi(1)}\frac{9}{4}(1)\cdot\left(\frac{q_mc}{\pi_{unit}}\right)^2\cdot\frac{kL_0}{1\,\text{kg}\cdot c^2}\cdot\left(\frac{m_e}{m_p}\right)^2=2.48262121\text{E}-36\left(\frac{\text{s}}{\text{kg}}\right) \tag{53}$$

$$\frac{G_{new}}{c^3} = \frac{\alpha}{2\pi(1)}\frac{9}{4}(1)\cdot\left(\frac{4.5_{unit}}{\pi_{unit}}hc^2\right)\cdot\frac{kL_0}{1\,\mathrm{kg}\cdot c^2}\cdot\left(\frac{m_e}{m_p}\right) = 2.48262121\mathrm{E}{-36}\left(\frac{\mathrm{s}}{\mathrm{kg}}\right) \tag{54}$$

$$R_p\left(m_p\right) = \frac{\pi(1)}{2}\cdot\frac{\left(M_{\min}\right)^3}{m_p\cdot m_e\cdot hc^2}\cdot\left(\frac{\mathrm{m}^4}{\mathrm{s}^3}\right) = 8.11158917\mathrm{E}{-37} \tag{55}$$

$$\frac{G}{c^3} = \frac{\alpha}{4}\cdot\frac{\left(M_{\min}\right)^3}{m_p\cdot m_e\cdot\left(m_p c^2\right)^2}\cdot\left(\frac{\mathrm{m}^4}{\mathrm{s}^3}\right) = 2.48262121\mathrm{E}{-36}\left(\frac{\mathrm{s}}{\mathrm{kg}}\right) \tag{56}$$

Although Equations (49)-(56) are not essential to the main purpose of the present paper, they are retained as possible clues for further development.

3. Methods

In this section, we prepare the framework required for the proofs presented in Section 4. For this purpose, we first explain the redefinition procedure adopted in the previous work [16], and then introduce the structural quantities and relations that will be used in the subsequent proofs. In this paper, these quantities are not introduced as independent assumptions, but as preparatory elements for examining whether a consistent readout structure can be formulated in relation to $E = h\nu$ and $E = mc^2$.

3.1. c-Normalization

In this work, physical quantities are expressed in c-normalized form by introducing appropriate factors of c. This normalization places electromagnetic quantities, mass-energy quantities, and action-type quantities into a common structural form, so that the relation between $E = h\nu$ and $E = mc^2$ can be written consistently.

$$e_{new} \to e_{new}\cdot c \tag{57}$$

$$q_{m\_new} \to q_{m\_new}\cdot c \tag{58}$$

$$\varepsilon_{0\_new} \to \varepsilon_{0\_new}\cdot c \tag{59}$$

$$\mu_{0\_new} \to \mu_{0\_new}\cdot c \tag{60}$$

$$h \to h\cdot c^2 \tag{61}$$

$$kg \to kg\cdot c^2 \tag{62}$$

$$G \to G/c^3 \tag{63}$$

$$G_N \to G_N/c \tag{64}$$

Although Equations (63) and (64) are not essential to the main purpose of the present paper, they are retained as possible clues for further development.

3.2. Redefinition Procedure

In the redefinition procedure, macroscopic and microscopic quantities are related through a normalization of the unit Ω. Under this normalization, the unit of $\mathrm{Jm^2/s}$ is fixed as follows:

$$\frac{4.4873976}{3.1327945}\left(\frac{1/\mathrm{A}\cdot\mathrm{m}}{\mathrm{V}\cdot\mathrm{m}}\right)=\frac{4.5}{\pi}\left(\frac{1/\mathrm{A}\cdot\mathrm{m}}{\mathrm{V}\cdot\mathrm{m}}=\frac{1}{\mathrm{J}\cdot\mathrm{m}^2/\mathrm{s}}\right) \tag{65}$$

By redefining the macroscopic unit 1 Ω, the following relations are obtained.

$$(Rk)_{new}=\frac{25957.9968739}{25812.807459}=\frac{4.5}{4.4873976}\cdot\frac{\pi}{3.1327945}=1.005624705\cdot(Rk)_{\mathrm{MKSA}} \tag{66}$$

$$1(\Omega)_{new}=1.005624705\cdot(\Omega)_{\mathrm{MKSA}}=1.005624705\cdot\left(\frac{\mathrm{Wb}}{\mathrm{C}}\right)_{\mathrm{MKSA}} \tag{67}$$

$$(Z_0)_{new}=\frac{378.8493104}{376.7303137}\cdot(Z_0)_{\mathrm{MKSA}}=1.005624705\cdot(Z_0)_{\mathrm{MKSA}} \tag{68}$$

$$(\varepsilon_0)_{new}=\frac{8.8541878\mathrm{E}{-12}}{8.8046642\mathrm{E}{-12}}\cdot(\varepsilon_0)_{\mathrm{MKSA}}=\frac{1}{1.005624705}\cdot(\varepsilon_0)_{\mathrm{MKSA}} \tag{69}$$

$$(\mu_0)_{new}=\frac{1.2637053\mathrm{E}{-06}}{1.2566370\mathrm{E}{-06}}\cdot(\mu_0)_{\mathrm{MKSA}}=1.005624705\cdot(\mu_0)_{\mathrm{MKSA}} \tag{70}$$

$$(e)_{new}=\frac{1.59768967\mathrm{E}{-19}}{1.60217663\mathrm{E}{-19}}\cdot(e)_{\mathrm{MKSA}}=\frac{1}{\sqrt{1.005624705}}\cdot(e)_{\mathrm{MKSA}} \tag{71}$$

$$(q_m)_{new}=\frac{4.14728234\mathrm{E}{-15}}{4.13566770\mathrm{E}{-15}}\cdot(q_m)_{\mathrm{MKSA}}=\sqrt{1.005624705}\cdot(q_m)_{\mathrm{MKSA}} \tag{72}$$

$$\left(\frac{(e\cdot c)^2}{\varepsilon_0\cdot c}\right)_{new}=\frac{1.005624705}{\left(\sqrt{1.005624705}\right)^2}=\left(\frac{(e\cdot c)^2}{\varepsilon_0\cdot c}\right)_{\mathrm{MKSA}} \tag{73}$$

$$\left(\frac{(q_m\cdot c)^2}{\mu_0\cdot c}\right)_{new}=\frac{\left(\sqrt{1.005624705}\right)^2}{1.005624705}=\left(\frac{(q_m\cdot c)^2}{\mu_0\cdot c}\right)_{\mathrm{MKSA}} \tag{74}$$

3.3. Microscopic Quantities: m_{gen} and s_{gen}

The parameter kL_0 is introduced as a coefficient associated with the dimensional factor (m^2/s). On this basis, the microscopic quantities m_{gen} and s_{gen} are introduced as the quantities corresponding to the macroscopic units of length m and time s in the present formulation.

$$1s_{gen}=\frac{kL_0}{c^2}=2.04499\mathrm{E}{-14}(\mathrm{s}) \tag{75}$$

$$1m_{gen}=\frac{kL_0}{c}=6.13073\mathrm{E}{-06}(\mathrm{m}) \tag{76}$$

$$c=\frac{1m_{gen}}{1s_{gen}}=\frac{6.13073\mathrm{E}{-06}(\mathrm{m})}{2.04499\mathrm{E}{-14}(\mathrm{s})}=299792458\left(\frac{\mathrm{m}}{\mathrm{s}}\right) \tag{77}$$

$$kL_0=\frac{(6.13073\mathrm{E}{-06})^2}{2.04499\mathrm{E}{-14}(\mathrm{s})}=s_{gen}\cdot c^2=1837.94538044\left(\frac{\mathrm{m}^2}{\mathrm{s}}\right) \tag{78}$$

3.4. Condition for the Minimum Mass

From $E=mc^2$ and $E=h\nu$, we obtain

$$\nu=mc^2/h \tag{79}$$

Taking the selected mass to be the minimum mass ($M_{\min}$), the condition becomes

$$\nu\left(kL_0/c^2\right)=\left(M_{\min}c^2\cdot kL_0\right)/hc^2=1 \tag{80}$$

This relation is adopted as the condition for the minimum mass used in the following section.

3.5. Structural Quantities: SI, UI, UIP, EI, and EIP

We introduce the following quantities and their numerical values.

SI: Structural invariant.

$$\mathrm{SI}=\frac{4.5_{unit}}{\pi_{unit}}hc^2=\frac{\left(M_{\min}\right)^2}{m_p\cdot m_e}=8.53021694\mathrm{E}{-17}\left(\text{dimensionless}\right) \tag{81}$$

UI: Generative action.

$$\mathrm{UI}=hc^2=\frac{\pi_{unit}}{4.5_{unit}}\cdot\frac{\left(M_{\min}\right)^2}{m_p\cdot m_e}=M_{\min}c^2\cdot kL_0=5.95521486\mathrm{E}{-17}\left(\mathrm{J}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}\right) \tag{82}$$

UIP: Unit conversion factor.

$$\mathrm{UIP}=\frac{4.5_{unit}}{\pi_{unit}}=1.432394488\left(\mathrm{J}^{-1}\cdot\mathrm{m}^{-2}\cdot\mathrm{s}\right) \tag{83}$$

EI: Energy-pair quantity.

$$\mathrm{EI}=\left(m_ec^2\cdot kL_0\right)\cdot\left(m_pc^2\cdot kL_0\right)=4.15752428\mathrm{E}{-17}\left(\mathrm{J}^2\cdot\mathrm{m}^4\cdot\mathrm{s}^{-2}\right) \tag{84}$$

EIP: Square-type unit structure.

$$\mathrm{EIP}=\left(\frac{4.5_{unit}}{\pi_{unit}}\right)^2=2.051753969\left(\mathrm{J}^{-2}\cdot\mathrm{m}^{-4}\cdot\mathrm{s}^2\right) \tag{85}$$

3.6. Fundamental Relations

The following relations are introduced as part of the structural framework:

$$\mathrm{SI}=\mathrm{UIP}\cdot\mathrm{UI}=8.53021694\mathrm{E}{-17}\left(\text{dimensionless}\right) \tag{86}$$

$$\mathrm{SI}=\mathrm{EIP}\cdot\mathrm{EI}=8.53021694\mathrm{E}{-17}\left(\text{dimensionless}\right) \tag{87}$$

$$\mathrm{EIP}=\mathrm{UIP}^2 \tag{88}$$

At this stage, these relations are presented as structural consistency relations. Their physical meaning will be discussed in the next section.

4. Results

In this section, we prove the following three theorems under the condition that the macroscopic unit 1 kg and the speed of light c (m/s) are fixed. In this paper, "readout" denotes the macroscopic expression of an underlying structural quantity.

Theorem 1: $M_{\min}$, s_{gen}, and m_{gen} cannot be determined by the usual relations alone.

Theorem 2: The minimum-mass determination equation follows from the structural relations.

Theorem 3: When 4.5_{unit} and π_{unit} are introduced as connection coefficients, the resulting connection relations remain mutually consistent.

4.1. Proof for Theorem 1

Equations (89)-(91) do not determine $M_{\min}$, m_{gen}, and s_{gen} separately, because they involve three variables but provide only two independent equations. Therefore, an additional condition is required.

$$1s_{gen} = \frac{kL_0}{c^2} = \frac{h}{M_{\min}c^2} \tag{89}$$

$$1m_{gen} = \frac{kL_0}{c} = \frac{h}{M_{\min}c^2} \tag{90}$$

$$c = \frac{1m_{gen}}{1s_{gen}} = 299792458\left(\frac{\text{m}}{\text{s}}\right) \tag{91}$$

Equation (80) alone does not determine the minimum mass $M_{\min}$. Therefore, an additional equation is required, as shown in the next section.

4.2. Proof for Theorem 2

As shown in Section 4.1, Equations (89)-(91) do not determine $M_{\min}$, m_{gen}, and s_{gen} separately, because they involve three variables but provide only two independent equations. Therefore, an additional equation is required.

Conveniently, Equation (81), (84) and (85) are rewritten as follows.

$$\text{SI} = \frac{4.5_{unit}}{\pi_{unit}} hc^2 = \frac{\left(M_{\min}\right)^2}{m_p \cdot m_e} = 8.53021694\text{E}{-17}\left(\text{dimensionless}\right) \tag{92}$$

$$\text{EI} = \left(m_e c^2 \cdot kL_0\right)\cdot\left(m_p c^2 \cdot kL_0\right) = 4.15752428\text{E}{-17}\left(\text{J}^2 \cdot \text{m}^4 \cdot \text{s}^{-2}\right) \tag{93}$$

$$\text{EIP} = \left(\frac{4.5_{unit}}{\pi_{unit}}\right)^2 = 2.051753969\left(\text{J}^{-2} \cdot \text{m}^{-4} \cdot \text{s}^2\right) \tag{94}$$

Combining Equations (92)-(94), we obtain

$$\frac{M_{\min}c^2}{kL_0}\left(\frac{\text{kg}}{\text{s}}\right) = \frac{4.5_{unit}}{\pi_{unit}} \cdot m_e c^2 \cdot m_p c^2 \left(\frac{\text{kg}}{\text{s}}\right) = 1.762918\text{E}{-23}\left(\frac{\text{kg}}{\text{s}}\right) \tag{95}$$

Therefore, the minimum-mass determination equation is the additional equation required in the present framework. Once kL_0 is given, $M_{\min}$ is determined. This completes the proof.

4.3. Proof for Theorem 3

We prove Theorem 3 in three steps.

Step 1: The ratio $4.5_{unit}/\pi_{unit}$ is set as a connection ratio.

Step 2: The product $4.5_{unit}\cdot\pi_{unit}$ is set as a connection product.

Step 3: Under these settings, no contradiction is found among the resulting connection relations.

4.3.1. Consistency under the Connection Ratio $4.5_{unit}/\pi_{unit}$

By Theorem 2, Equation (95) gives the relation between $M_{\min}$ and kL_0. In the present formulation, the ratio 4.5unit/πunit is introduced as the connection ratio that relates this mass-side structure to the product-side quantities. Under this setting, the relation among $M_{\min}$, kL0, m_e, and m_p remains consistent.

4.3.2. Consistency under the Connection Product $4.5_{unit}\cdot\pi_{unit}$

In the second step, Equation (96) is used to examine the connection product $4.5_{unit}\cdot\pi_{unit}$.

$$Rk_{new} = 4.5_{unit} \cdot \pi_{unit} \cdot \frac{m_p}{m_e} = 25957.997(\Omega) \tag{96}$$

4.3.3. Consistency of the Two Connection Coefficients

From Sections 4.3.1 and 4.3.2, the connection ratio $4.5_{unit}/\pi_{unit}$ and the connection product $4.5_{unit}\cdot\pi_{unit}$ are both consistent with the corresponding structural relations. Therefore, when 4.5_{unit} and πunit are introduced as connection coefficients, no contradiction is found among the resulting connection relations.

5. Discussions

Section 4 established the numerical and structural consistency of the present framework. In this section, we discuss the possible physical meaning of these results and their implications for the connection between $E = mc^2$ and $E = h\nu$.

5.1. Emergence of the Invariant Mass-Side Combination kg·s

Although the readout of kg is not Lorentz-invariant, the product kg·s appears as a readout-invariant quantity.

$$\gamma = 1\Big/\sqrt{1-\left(\frac{v}{c}\right)^2} \tag{97}$$

where v is the velocity.

$$\text{kg} \to \text{kg} \cdot \gamma \tag{98}$$

$$\text{s} \to \text{s}/\gamma \tag{99}$$

Therefore,

$$\text{kg} \cdot \text{s} \to (\text{kg} \cdot \gamma) \cdot (\gamma/\text{s}) = \text{kg} \cdot \text{s} \tag{100}$$

In the present framework, C, Wb, Js, and Ω are treated as readout-invariant quantities under Lorentz transformation, whereas kg alone is not. This difference motivates the introduction of the mass-side invariant combination kg·s.

5.2. Meaning of the Minimum-Mass Determination Equation

5.2.1. Physical Meaning of the Minimum Mass in the Context of the Solid-State Ionics

A possible physical meaning of the minimum mass may be suggested by interpreting kTc/α as an activation energy. In the context of solid-state ionics, the relation [17],

$$V_{th} - OCV = \frac{Ea}{2e} \cdot (1 - t_{ion}) \tag{101}$$

where V_{th}, OCV, Ea and t_{ion} are the Nernst voltage, open-circuit voltage, ionic activation energy and ionic transference number, respectively. Equation (101) may be explained by Jarzynski's equality [18].

For example, V_{th} is 1.15 V, the OCV is 0.80 V, Ea is 0.7 eV, and t_{ion} is 0.

$$1.15\text{V} - 0.80\text{V} = \frac{0.7\text{eV}}{2e} \cdot (1 - 0) \tag{102}$$

Here, α is the interaction coefficient. Therefore,

$$\alpha = 1 - t_{ion} \tag{103}$$

Substituting Equation (103) into Equation (101),

$$(V_{th} - OCV) \cdot 2e = Ea \cdot \alpha \tag{104}$$

The left side of Equation (104) represents the energy loss due to dissipation. Therefore,

$$kT_c = Ea \cdot \alpha \tag{105}$$

Accordingly, kT_c is the dissipation energy. From Equation (105),

$$Ea = \frac{kT_c}{\alpha} \tag{106}$$

Thus, we suggest that Equation (106) expresses the activation energy of the space. Next, the electrochemical potential energy of one particle (η) is the sum of the potential energy ($P.E.$) and the Gibbs energy (μ). Under the activation energy (Ea), these values should change.

$$\eta = \mu + P.E. \tag{107}$$

Under the activation energy $Ea = kT_c/\alpha$, the energy redistribution is expressed as

$$\mu_{particle} = \mu_{wave} - \frac{kT_c}{\alpha} > 0 \tag{108}$$

$$P.E._{particle} = P.E._{wave} + \frac{kT_c}{\alpha} \tag{109}$$

where $\mu_{particle}$, μ_{wave}, $P.E._{particle}$ and $P.E._{wave}$ are the Gibbs energy in the particle situation, the Gibbs energy in the wave situation, P.E. in the particle situation and P.E. in the wave situation, respectively.

In Equation (108), the minimum Gibbs energy should be positive. Considering the phase-related normalization coefficient (2π), the minimum mass can be defined as follows,

$$M_{\min}(\text{kg}) = 2\pi \cdot \frac{kT_c}{\alpha \cdot c^2}(\text{kg}) = 3.6051507\text{E}{-37}(\text{kg}) \tag{110}$$

5.2.2. Interpretation under Special Relativity

Once the concept of a minimum mass is introduced into special relativity, an additional conclusion follows on the mass side. As discussed in Section 5.1, the invariant quantity on the mass side is of the product type kg·s. For the minimum mass, the c-normalized product-type readout corresponding to Equation (89) is given by Equation (111), namely,

$$M_{\min} c^2 k L_0 \left(\text{kg} \cdot \text{s} \cdot \left(\frac{\text{m}}{\text{s}} \right)^4 \right) = hc^2 \left(\text{kg} \cdot \text{s} \cdot \left(\frac{\text{m}}{\text{s}} \right)^4 \right) \tag{111}$$

However, this quantity alone does not determine a definite mass. The minimum-mass determination equation, Equation (95), supplies the complementary quotient-type readout,

$$\frac{M_{\min} c^2}{k L_0} \left(\frac{\text{kg}}{\text{s}} \right) = \frac{4.5_{unit}}{\pi_{unit}} m_e c^2 \cdot m_p c^2 \left(\frac{\text{kg}}{\text{s}} \right) \tag{112}$$

From Equations (111) and (112),

$$M_{\min} (\text{kg}) = \sqrt{\frac{h}{c^2} \frac{4.5_{unit}}{\pi_{unit}} m_e c^2 \cdot m_p c^2} = 3.60515074\text{E}{-37} (\text{kg}) \tag{113}$$

$$s_{gen} (\text{s}) = \sqrt{\frac{h/c^2}{\frac{4.5_{unit}}{\pi_{unit}} \cdot m_e c^2 \cdot m_p c^2}} = 2.0449900\text{E}{-14} (\text{s}) \tag{114}$$

Consequently, $M_{\min}$ and s_{gen} can be determined from the mutual consistency of the product-type and quotient-type readouts. Therefore, the minimum mass is selected by the combined mass-side structure under special relativity.

6. Conclusions

In this paper, we reexamined empirical equations for the cosmic microwave background temperature from the viewpoint of macro-micro correspondence. We showed that the usual relations alone do not determine Mmin, mgen, and sgen separately, and that an additional relation is required within the present formulation. From the structural relations considered here, the minimum-mass determination equation was obtained as a consistency condition. On this basis, we examined the role of 4.5unit and πunit as connection coefficients. With these coefficients introduced, the ratio 4.5unit/πunit and the product 4.5unit·πunit can be written consistently with the corresponding structural relations, and no internal contradiction is found within this connection scheme.

The discussion further indicates that $E = mc^2$ and $E = h\nu$ may be organized within a common readout-based structural framework. On the mass side, special relativity provides the invariant readout kg·s, while the minimum-mass determination equation provides the complementary readout kg/s. In the present formulation, the minimum mass is selected through the consistency of these two readouts. The quantity hc^2, whose unit is J m^2/s, is interpreted here as a c-normalized product-side quantity relating quantum action to the mass-side readout. In this limited sense, the present framework offers a way to compare $E = mc^2$ and $E = h\nu$ through a common mass-time readout structure, rather than treating them only as isolated expressions.

At the same time, the present results should be regarded as a structural consistency analysis rather than a final physical theory. The paper examines numerical and structural consistency, but does not provide a complete microphysical derivation of all quantities appearing in the analysis. In particular, the physical origin of 4.5unit and πunit remains an open question. Further investigation will be required to clarify the physical meaning of the minimum mass and of the connection coefficients introduced in the present framework.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix Addendum on Local Unit-Shell Readout

At first sight, UIP appears to depend on the MKSA system. This differs from the previous MKSA-dependence problem [16], where the equation for T_c was shown not to be merely an MKSA-specific numerical coincidence by defining the compatibility between macroscopic and microscopic units. The present LUS problem is the remaining reference-shell display problem: UIP appears as 4.5/π only in the MKSA reference readout, while in a general local unit shell, it must be written as $4.5_{unit(x)}/\pi_{unit(x)}$.

Appendix A. Readout covariance of UIP

At first sight, UIP appears to depend on the MKSA system. The structural invariant SI is dimensionless and is independent of the choice of unit system. By contrast, UI is a readout quantity. In a coherent unit system *x*, the same symbolic expression is used for UI.

$$\mathrm{SI} = \mathrm{UIP}_{(x)} \cdot \mathrm{UI}_{(x)} = \mathrm{UIP}_{(x)} \cdot hc^2{}_{(x)} = \frac{4.5_{unit(x)}}{\pi_{unit(x)}} \cdot hc^2{}_{(x)} \tag{A1}$$

The expression hc^2 is not replaced when the unit system is changed. Only its readout value changes. For example, if the readout value of hc^2 in the unit system x is 1000 times larger than in the MKSA reference system, then the readout value of $\mathrm{UIP}_{(x)}$ must be 1/1000 times smaller. Their product remains the same dimensionless invariant SI.

This point should be distinguished from the displayed form of UIP. UIP has an explicit MKSA-readout dependence when it is written as a numerical coefficient. The simple numerical representative 4.5/π seems to appear only in the MKSA reference readout. In a different coherent unit system, UIP generally does not have the same numerical readout value. Nevertheless, the local unit-shell form of UIP is preserved:

$$\mathrm{UIP}_{(x)} = \frac{4.5_{unit(x)}}{\pi_{unit(x)}} = \frac{4.5/\left(\mathrm{A}_{(x)} \cdot \mathrm{m}_{(x)}\right)}{\pi \cdot \left(\mathrm{V}_{(x)} \cdot \mathrm{m}_{(x)}\right)} \tag{A2}$$

Here, $\mathrm{A}_{(x)}$, $\mathrm{V}_{(x)}$, and $\mathrm{m}_{(x)}$ denote the ampere, volt, and meter of the same unit system *x*, not fixed MKSA units. Hence 4.5unit/πunit is form-invariant, but its readout changes through the local unit shell. In MKSA, this shell is normalized and UIP appears as 4.5/π.

Appendix B. MKSA Mass-Shell Representative in the Reference Readout

The apparent special role of the MKSA readout can be expressed by introducing the mass-shell representative number. In a local unit shell *x*, we define

$$N_{(x)} = \frac{1\,\mathrm{kg}_{(x)}}{M_{\min(x)}} \tag{B1}$$

Here, $N_{(x)}$ is the representative number that expresses the ratio between the macroscopic mass unit 1 $kg_{(x)}$ and the minimum-mass readout scale $M_{\min(x)}$, when the units of m and s are fixed. In the MKSA reference readout, this number is

$$N_{\text{MKSA}} = \frac{1\ \text{kg}}{M_{\min}} = 2.77380912\text{E+}36 \tag{B2}$$

Next, we consider the following relationship.

$$C_{(x)} \propto \sqrt{N_{(x)}} \tag{B3}$$

$$Wb_{(x)} \propto \sqrt{N_{(x)}} \tag{B4}$$

$$V_{(x)} = \frac{Wb_{(x)}}{s} \propto \sqrt{N} \tag{B5}$$

$$A_{(x)} = \frac{C_{(x)}}{s} \propto \sqrt{N} \tag{B6}$$

where $C_{(x)}$, $Wb_{(x)}$ represent the charge, the magnetic charge using the unit system(x) when the units of m and s are fixed, respectively. Then, using $E = m_{(x)}c^2$, the following relationship can be explained.

$$1\ \text{kg}_{(x)} \cdot c^2 = C_{(x)} \cdot V_{(x)} \tag{B7}$$

$$1\ \text{kg}_{(x)} \cdot c^2 = Wb_{(x)} \cdot A_{(x)} \tag{B8}$$

$$M_{\min(x)} \cdot c^2 = e_{(x)} \cdot V_{(x)} \tag{B9}$$

$$M_{\min(x)} \cdot c^2 = q_{m(x)} \cdot A_{(x)} \tag{B10}$$

where $e_{(x)}$ and $q_{m(x)}$ denote the electric charge of one electron and the magnetic charge of one magnetic monopole using the unit system(x) when the units of m and s are fixed. Then,

$$4.5_{unit(x)} = 4.5/\left(\text{A}_{(x)} \cdot \text{m}\right) \tag{B11}$$

$$\pi_{unit(x)} = \pi \cdot \left(\text{V}_{(x)} \cdot \text{m}\right) \tag{B12}$$

So,

$$\text{UIP}_{(x)} = \frac{4.5_{unit(x)}}{\pi_{unit(x)}} = \frac{4.5/\left(\text{A}_{(x)} \cdot \text{m}\right)}{\pi \cdot \left(\text{V}_{(x)} \cdot \text{m}\right)} = \frac{4.5}{\pi} \cdot \frac{\sqrt{N_{(x)}}}{\sqrt{N_{(x)}}} \frac{1/\text{C}}{\text{Wb}} \tag{B13}$$

In Equation (B13), the local-shell structure of UIP is explicitly shown.

Appendix C. Dependence of the kL_0 branch on the mass-shell representative

In the present framework, kL_0 is associated with the m^2/s-type readout branch. However, this branch does not form an independent LUS problem, because kL_0 is connected to the mass-shell representative through the product relation. After c-normalization, this relation is written as

$$\text{UI} = hc^2 = \left(M_{\min} c^2\right) \cdot kL_0 \tag{C1}$$

Therefore, the readout of UI is determined by the combined readout of $M_{\min}$ and kL_0. If the readout of kL_0 changes through the local unit shell, this change can be absorbed into the effective readout of the mass-shell representative. In this sense, the kL_0 branch is treated as dependent on the kg branch at the level of LUS readout.