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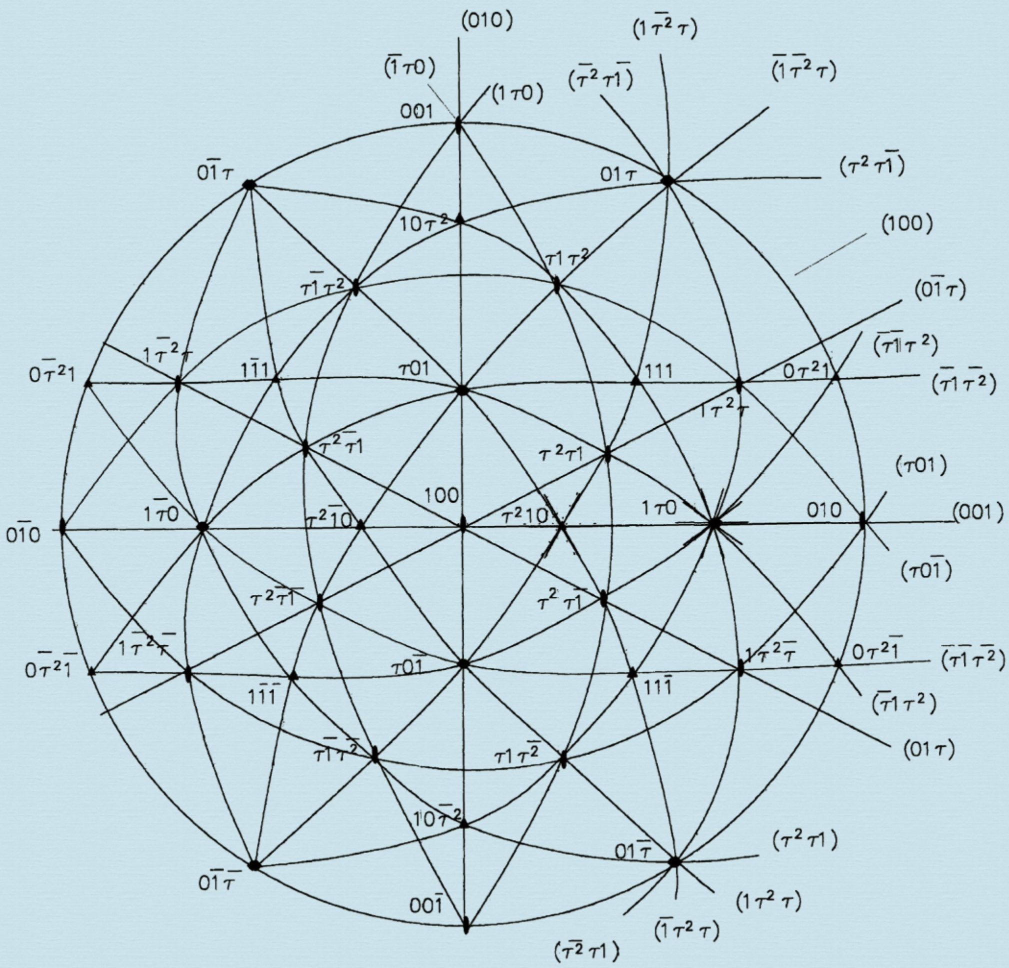

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Table of Contents

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Various Empirical Equations to Unify between the Gravitational Force and the Electromagnetic Force

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Abstract

Previously, we proposed several empirical equations to describe the relationship between an electromagnetic force and the temperature of the cosmic microwave background (CMB).We attempted to justify why our empirical equations cannot be coincidental from the mathematical connections between our three equations. However, there are small errors in our empirical equations, which may lead to "indeed or not" arguments. After evaluating our equations, we discovered a method to improve the accuracy of the numerical calculations. For the value of the CMB, we used 2.72642 K instead of 2.72548 K. Regarding the factor of 9/2, we used 4.48870 instead of 4.5. Regarding the factor of π, we used 3.13189 instead of 3.14159. Then, the error becomes less than 10^{-5}. This means that our equations cannot be coincidental. Furthermore, we attempt to provide hints on how to construct the background theory.

Keywords

Temperature of the Cosmic Microwave Background, Wagner's Equation, Jarzynski's Equality

1. Introduction

Our main research explains the current independent constant voltage loss (0.35 V) in Sm doped Ceria (SDC) electrolytes in solid oxide fuel cells (SOFC). This voltage loss cannot be explained by Wagner's equation. During the oxygen ion hopping conduction with enough electron's atmosphere, this voltage loss may be explained by Jarzynski's equality. The discovered equivalent circuit is different from the usual RC circuit, and seems to be useful to explain the gravitational force. Then, we discovered Equations (1)-(3) [1] [2] and [3].

Previously, we reported the empirical equation for a gravitational force in terms of the temperature of the cosmic microwave background (CMB) [1] [2]:

$$\frac{Gm_p}{\frac{\lambda_p}{2}} \times 1\,\text{kg} = \frac{9}{2} kT_c \tag{1}$$

where G, m_p, λ_p, k, and T_c are the gravitational constant, the rest mass of a proton, the Compton wavelength, the Boltzmann constant and the temperature of the CMB, respectively. One kilogram is the standard unit of mass, as previously explained [1]. The error was 0.217%.

Below, we present the empirical equation for the ratio between a gravitational force and an electric force [3]:

$$\frac{Gm_p^2}{\frac{e^2}{4\pi\varepsilon_0}} = 4.5 \times \frac{m_e}{e} \times \hbar c \times \left(1 \frac{\text{C}}{\text{J}\cdot\text{m}} \times \frac{1}{1\,\text{kg}}\right) \tag{2}$$

where e, ε_0, m_e, $\hbar$ and c are the electric charge of one electron, the electric constant, the rest mass of an electron, the reduced Planck constant and the speed of light, respectively. As previously noted, 1 C/J/m is the standard electrostatic quantity [3]. The error was only 0.0578%.

Below, we present that Coulomb's law with distance can be expressed in terms of the CMB [3].

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{1}{r^2} \frac{e\pi}{m_e c^2} kT_c \times \left(1 \frac{\text{J}\cdot\text{m}}{\text{C}}\right) \tag{3}$$

where r is the distance between two electrons. The error was 0.274%.

We tried to explain Equation (1), theoretically [1] [2]. However, after the discoveries of empirical Equations (2) and (3), we noticed that there were confusions in our definitions between the Gibbs volume entropy and PV (P is pressure and V is volume). To elucidate the equivalent circuit, there need the more difficult thermodynamic concept. So, we abandoned the theoretical explanation for Equations (2) and (3) [3].

Our conclusion in the previous report was that the mathematical connections among Equations (1)-(3) provide evidence that they are not coincidental [3]. There remain following two main problems in our previous works.

<u>Problem1</u>: We could not explain three equations theoretically.

It means that we cannot provide the suitable equivalent circuit in our equations. The equivalent circuit may be different from the usual LRC circuits. There may be the strong combinations with the difficult advanced thermodynamics, which include the inevitable dissipation.

<u>Problem 2</u>: There are small errors in our empirical equations

Equations (1)-(3) are not complex and connected each other. But there remains the possibility that the small errors may lead to "indeed or not" arguments.

In this report, we searched for a compensation method to solve the above Problem 2 and to provide hints to solve Problem 1. The rest of the paper is organized as follows. In Section 2, we present the symbol list, the calculation results for frequently used values and important equations. In Section 3, we present new expressions for Equations (1)-(3). In Section 4, we present the newly discovered empirical equations. In Section 5, we present our compensation methods and the verification of newly discovered empirical equations. In Section 6, we attempt to provide hints on how to construct a background theory.

2. Symbol List and Frequently Used Values

2.1 Symbol List

These values were obtained from Wikipedia.

G: gravitational constant: 6.6743×10^{-11} ($m^3 \cdot kg^{-1} \cdot s^{-2}$)

T_c: temperature of the cosmic microwave background: 2.72548 (K)

k: Boltzmann constant: 1.380649×10^{-23} ($J \cdot K^{-1}$)

c: speed of light: 299,792,458 (m/s)

h: Planck constant: $6.62607015 \times 10^{-34}$ (Js)

$\hbar$: Dirac constant (reduced Planck constant): $1.054571817 \times 10^{-34}$ (Js)

ε_0: electric constant: $8.8541878128 \times 10^{-12}$ ($N \cdot m^2 \cdot C^{-2}$)

μ_0: magnetic constant: $1.25663706212 \times 10^{-6}$ ($N \cdot A^{-2}$)

e: electric charge of one electron: $-1.602176634 \times 10^{-19}$ (C)

q_m: magnetic charge of one magnetic monopole: $4.13566770 \times 10^{-15}$ (Wb) (this value is only a theoretical value, $q_m = h/e$)

m_p: rest mass of a proton: $1.672621923 \times 10^{-27}$(kg)

m_e: rest mass of an electron: $9.1093837 \times 10^{-31}$ (kg)

λ_p: Compton wavelength for a proton: 1.32141×10^{-15} (m)

λ_e: Compton wavelength for an electron: $2.4263102367 \times 10^{-12}$ (m)

r_e: classic electron radius: $2.8179403227 \times 10^{-15}$ (m)

a_0: Bohr radius: $0.529177210 \times 10^{-10}$ (m)

R_∞: Rydberg constant: 10,973,731.568 (m^{-1})

α: fine-structure constant: 1/137.0359991

R_k: von Klitzing constant: 25,812.80745 (Ω)

Z_0: wave impedance in free space: 376.730313668 (Ω)

2.2. Calculation Results for the Frequently Used Values

The calculation results for several frequently used values are presented in this section. The number of significant figures used is 5.

$$\frac{e^2}{4\pi\varepsilon_0} = \frac{\left(1.6022\times10^{-19}\right)^2}{4\pi\times8.8542\times10^{-12}} = 2.3071\times10^{-28}\left(\mathrm{J\cdot m}\right) \tag{4}$$

$$\frac{e}{4\pi\varepsilon_0} = \frac{1.6022\times10^{-19}}{4\pi\times8.8542\times10^{-12}} = 1.4400\times10^{-9}\left(\mathrm{J\cdot m/C}\right) \tag{5}$$

$$\frac{e^2}{4\pi\varepsilon_0}\frac{e}{4\pi\varepsilon_0} = 2.3071\times10^{-28}\times1.4400\times10^{-9} = 3.3221\times10^{-37}\left(\mathrm{J^2\cdot m^2\cdot C^{-1}}\right) \tag{6}$$

$$Gm_p^2 = 6.6743\times10^{-11}\times\left(1.6726\times10^{-27}\right)^2 = 1.8672\times10^{-64}\left(\text{J}\cdot\text{m}\right) \tag{7}$$

$$Gm_e^2 = 6.6743\times10^{-11}\times\left(9.1093\times10^{-31}\right)^2 = 5.5384\times10^{-71}\left(\text{J}\cdot\text{m}\right) \tag{8}$$

When T_c is 2.7264 K,

$$kT_c = 1.3807\times10^{-23}\times2.7264 = 3.7642\times10^{-23}\left(\text{J}\right) \tag{9}$$

2.3. Important Equations

The important equations are presented in this section.

$$\frac{e^2}{4\pi\varepsilon_0\hbar c} = \alpha = \frac{1}{137.036} \tag{10}$$

$$\frac{q_m^2}{\mu_0\pi\hbar c} = \frac{1}{\alpha} = 137.036 \tag{11}$$

$$\hbar c = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^{\frac{1}{2}}\left(\frac{q_m^2}{\mu_0\pi}\right)^{\frac{1}{2}} \tag{12}$$

3. New Expressions for Equations (1)-(3)

The mathematical connections among Equations (1)-(3) have already been proven [3]. The purpose of this section is to explain newly discovered equations in Section 4.

3.1. New Expression for Equation (1)

For convenience, Equation (1) is rewritten as Equation (13).

$$\frac{Gm_p}{\frac{\lambda_p}{2}} = \frac{9}{2}kT_c \tag{13}$$

Here,

$$\lambda_p = \frac{h}{m_p c} \tag{14}$$

From Equations (13) and (14),

$$\frac{Gm_p m_p c^2}{hc} = \frac{9}{4}kT_c \tag{15}$$

From Equations (12) and (15),

$$Gm_p^2 = \frac{4.5\pi}{c^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^{\frac{1}{2}}\left(\frac{q_m^2}{\mu_0\pi}\right)^{\frac{1}{2}}kT_c \tag{16}$$

3.2. New Expression for Equation (2)

For convenience, Equation (2) is rewritten as Equation (17).

$$\frac{Gm_p^2}{\frac{e^2}{4\pi\varepsilon_0}} = 4.5\times\frac{m_e}{e}\times\hbar c \tag{17}$$

From Equations (12) and (17),

$$Gm_p^2 = 4.5\times\frac{m_e}{e}\times\left(\frac{e^2}{4\pi\varepsilon_0}\right)^{\frac{3}{2}}\left(\frac{q_m^2}{\mu_0\pi}\right)^{\frac{1}{2}} \tag{18}$$

3.3. New Expression for Equation (3)

From Equation (3),

$$\frac{m_e c^2}{e\pi}\times\frac{e^2}{4\pi\varepsilon_0} = kT_c \tag{19}$$

4. Our Newly Discovered Empirical Equations

4.1. Empirical Equation between the Rest Mass Values of an Electron and a Proton

The empirical equation between the rest mass values of a proton and an electron is presented in Equation (20). Using Equation (20), we discovered three empirical equations with the rest mass of a proton. And we discovered three empirical equations with the rest mass of an electron.

$$\frac{m_p}{m_e} = \frac{1}{4.5\times\pi}\frac{q_m}{e} \tag{20}$$

$$\frac{m_p}{m_e} = \frac{1.6726\times10^{-27}}{9.1094\times10^{-31}} = 1836.2 \tag{21}$$

$$\frac{1}{4.5\times\pi}\frac{q_m}{e} = \frac{1}{4.5\times3.1416}\times\frac{4.1357\times10^{-15}}{1.6022\times10^{-19}} = 1825.9 \tag{22}$$

$$\text{Error} = \frac{1836.2}{1825.9} - 1 = 0.00563 \tag{23}$$

4.2. Three Empirical Equations with the Rest Mass of a Proton

From Equation (16),

$$Gm_p^2 = \frac{4.5}{2}\frac{eq_m}{c}kT_c \tag{24}$$

From Equations (18) and (20),

$$Gm_p^2 = 4.5\times4.5\times\pi\frac{m_p}{q_m}\times\left(\frac{e^2}{4\pi\varepsilon_0}\right)^{\frac{3}{2}}\left(\frac{q_m^2}{\mu_0\pi}\right)^{\frac{1}{2}} \tag{25}$$

Therefore,

$$Gm_p = 4.5\times4.5\times\frac{ec}{2}\times\left(\frac{e^2}{4\pi\varepsilon_0}\right) \tag{26}$$

From Equations (19) and (20),

$$4.5\times\frac{m_pc^2}{q_m}\times\left(\frac{e^2}{4\pi\varepsilon_0}\right)=kT_c \tag{27}$$

4.3. Three Empirical Equations with the Rest Mass of an Electron

From Equations (20) and (24),

$$G\left(\frac{q_m}{e}\times\frac{m_e}{4.5\pi}\right)^2=\frac{4.5}{2}\frac{eq_m}{c}kT_c \tag{28}$$

Therefore,

$$Gm_e^2=\frac{4.5^3}{2}\frac{e^3}{q_mc}\pi^2kT_c \tag{29}$$

From Equations (20) and (26),

$$G\frac{q_m}{e}\times\frac{m_e}{4.5\pi}=4.5\times4.5\times\frac{ec}{2}\times\left(\frac{e^2}{4\pi\varepsilon_0}\right) \tag{30}$$

Hence,

$$Gm_e=4.5^3\times\pi\times\frac{e^2c}{2q_m}\times\left(\frac{e^2}{4\pi\varepsilon_0}\right) \tag{31}$$

For convenience, Equation (19) is rewritten as Equation (32).

$$\frac{m_ec^2}{e\pi}\times\frac{e^2}{4\pi\varepsilon_0}=kT_c \tag{32}$$

5. Our Compensation Methods

In section 3, we propose seven empirical equations. They are Equations ((20), (24), (26), (27), (29), (31) and (32)). Regarding the factor of 9/2, we used 4.48870 instead of 4.5 which was determined from Equation (26). Regarding the factor of π, we used 3.13189 instead of 3.14159, which was determined from Equations ((20) and (31)). Regarding the value of the CMB, we used 2.72642 K instead of 2.72548 K, which was determined from Equations ((24), (27), (29) and (32)).

5.1. Verification of Equation (20)

From Equation (20) with the compensation method,

$$\frac{m_p}{m_e}=\frac{1}{4.4887\times3.1319}\times\frac{4.1357\times10^{-15}}{1.6022\times10^{-19}}=1836.2 \tag{33}$$

Equation (33) is equal to Equation (21). Therefore, the compensation method is perfect.

5.2. Verification of Equation (24)

For convenience, Equation (9) is rewritten as Equation (34).

$$kT_c=1.3807\times10^{-23}\times2.7264=3.7642\times10^{-23}\,(\mathrm{J}) \tag{34}$$

From Equations (24) and (34) with the compensation method,

$$\frac{4.4887}{2}\frac{1.6022\times10^{-19}\times4.1357\times10^{-15}}{2.9979\times10^{8}}3.7642\times10^{-23}=1.8672\times10^{-64} \quad (35)$$

Equation (35) is equal to Equation (7). Therefore, the compensation method is perfect.

5.3. Verification of Equation (26)

$$Gm_p=6.6743\times10^{-11}\times1.6726\times10^{-27}=1.1164\times10^{-37} \quad (36)$$

For convenience, Equation (4) is rewritten as Equation (37).

$$\frac{e^2}{4\pi\varepsilon_0}=\frac{\left(1.6022\times10^{-19}\right)^2}{4\pi\times8.8542\times10^{-12}}=2.3071\times10^{-28}\left(\mathrm{J\cdot m}\right) \quad (37)$$

From Equations (26) and (37) with the compensation method,

$$4.4887^2\times\frac{1.6022\times10^{-19}\times2.9979\times10^{8}}{2}\times2.3071\times10^{-28}=1.1164\times10^{-37} \quad (38)$$

Equation (36) is equal to Equation (38). Therefore, the compensation method is perfect.

5.4. Verification of Equation (27)

From Equation (27) with the compensation method,

$$4.4887\times\frac{1.6726\times10^{-27}\times\left(2.9979\times10^{8}\right)^2}{4.1357\times10^{-15}}\times2.3071\times10^{-28}=3.7642\times10^{-23} \quad (39)$$

Equation (39) is equal to Equation (9. Therefore, the compensation method is perfect.

5.5. Verification of Equation (29)

From Equation (29) with the compensation method,

$$\frac{4.4887^3}{2}\frac{\left(1.6022\times10^{-19}\right)^3}{4.1357\times10^{-15}\times2.9979\times10^{8}}\times3.1319^2\times3.7642\times10^{-23}$$
$$=5.5384\times10^{-71} \quad (40)$$

Equation (40) is equal to Equation (8). Therefore, the compensation method is perfect.

5.6. Verification of Equation (31)

$$Gm_e=6.6743\times10^{-11}\times9.1094\times10^{-31}=6.0799\times10^{-41} \quad (41)$$

From Equation (31) with the compensation method,

$$4.4887^3\times3.1319\times\frac{\left(1.6022\times10^{-19}\right)^2\times2.9979\times10^{8}}{2\times4.1357\times10^{-15}}\times2.3071\times10^{-28}$$
$$=6.0799\times10^{-41} \quad (42)$$

Equation (41) is equal to Equation (42). Therefore, the compensation method is perfect.

5.7. Verification of Equation (32)

From Equation (32) with the compensation method,

$$\frac{9.1094\times10^{-31}\times\left(2.9979\times10^{8}\right)^{2}}{1.6022\times10^{-19}\times3.1319}\times2.3071\times10^{-28}=3.7642\times10^{-23} \tag{43}$$

Equation (43) is equal to Equation (9). Therefore, the compensation method is perfect.

6. Discussion

With only three compensation values, we explained 7 equations perfectly. Therefore, we can justify why the discovered compensation values should not coincide. We attempt to provide hints to construct a background theory.

6.1. Consideration of Our Compensation Method

Regarding the value of the CMB, we used 2.72642 K instead of 2.72548 K. This does not indicate experimental measurement error. Our compensation methods use 4.48870 instead of the factor 4.5. Furthermore, regarding the factor of π, we used 3.13189 instead of 3.14159. This small deviation may be due to unknown theoretical reasons.

For convenience, Equation (20) is rewritten as Equation (44).

$$\frac{m_p}{m_e}=\frac{1}{4.5\times\pi}\frac{q_m}{e} \tag{44}$$

Here,

$$4.5\times\pi=14.13716694 \tag{45}$$

The accurate equation should be

$$\frac{q_m}{e}\times\frac{m_e}{m_p}=14.05809432 \tag{46}$$

Therefore, the two compensation values cannot be independent. Here,

$$\frac{q_m}{e}=R_k \tag{47}$$

$$R_k\alpha=\frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}=\frac{Z_0}{2} \tag{48}$$

Therefore, we believe that the two compensation values should be related to 137.036. However, we cannot discover an accurate relationship. Perhaps there needs a computer program that can perform vector analysis. With our methods, vector analysis cannot be performed.

6.2. Consideration of the Relationship with the Jarzynski Equality

A nonequilibrium work relation is described by the Jarzynski equality [4].

$$\left\langle e^{-\beta W} \right\rangle = e^{-\beta \Delta F_T} \tag{49}$$

where β is $1/kT$ and W and ΔF_T are the work and the difference in the Helmholtz free energy, respectively. The angular brackets $\langle \text{-} \rangle$ indicate an ensemble of realizations of the process. For adiabatic processes, ΔF_T is zero. The following equation should be added [4]:

$$\left\langle e^{-\beta X} \right\rangle = 1 \tag{50}$$

where X is the energy in the same adiabatic process and ensemble of realizations. According to Jarzynski, quantum theory can be explained with Equation (50). Hence, X is investigated in detail without background theory. In Equation (50), the temperature (T) may be equal to the CMB. Previously, we attempted to explain Equation (1) [1] [2]. However, after the discovery of many empirical equations, we strongly feel that X may be separated into electromagnetic potential energy and kinetic energy, including the rest mass energy.

6.3. Relationship between G and kT_c

In our four empirical equations, we used the gravitational constant (G). We noticed that G is not independent of the CMB. From Equation (26),

$$m_p = 4.5 \times 4.5 \times \frac{ec}{2G} \times \left(\frac{e^2}{4\pi\varepsilon_0} \right) \tag{51}$$

From Equations (24) and (51),

$$G\left[4.5 \times 4.5 \times \frac{ec}{2G} \times \left(\frac{e^2}{4\pi\varepsilon_0} \right) \right]^2 = \frac{4.5}{c} \frac{eq_m}{2} kT_c \tag{52}$$

Hence,

$$\frac{4.5^3}{2q_m} ec^3 \left(\frac{e^2}{4\pi\varepsilon_0} \right)^2 = GkT_c \tag{53}$$

From Equation (54) with the compensation method,

$$\begin{aligned} &\frac{4.4887^3}{2 \times 4.1357 \times 10^{-15}} \times 1.6022 \times 10^{-19} \times \left(2.9979 \times 10^8\right)^3 \times \left(2.3071 \times 10^{-28}\right)^2 \\ &= 2.5124 \times 10^{-33} \end{aligned} \tag{54}$$

$$GkT_c = 6.6743 \times 10^{-11} \times 3.7642 \times 10^{-23} = 2.5124 \times 10^{-33} \tag{55}$$

Equation (54) is equal to Equation (55). Therefore, the compensation method is perfect. Equation (53) is a little similar with Hawking radiation. Therefore, the relationship between Hawking radiation and Equation (53) should be examined.

6.4. Consideration for the Four Special Lengths

About for four special lengths (Classical electron radius, Compton wavelength for an electron, Bohr radius and Rydberg constant), the following Equation (56) is well known.

$$r_e 2\pi = \alpha \lambda_e = \alpha^2 a_0 2\pi = \alpha^3 \frac{1}{2R_\infty} \quad (56)$$

When π is changed from 3.14159 to 3.13189 in Equation (56), the relationship between four lengths seemed to be changed. But there are no problems. We used Equation (4). For convenience, Equation (4) is rewritten below.

$$\frac{e^2}{4\pi\varepsilon_0} = \frac{\left(1.6022\times10^{-19}\right)^2}{4\pi\times8.8542\times10^{-12}} = 2.3071\times10^{-28}\left(\mathrm{J\cdot m}\right) \quad (57)$$

Then, in Equation (57), we used 3.14159 as π.

$$r_e = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{m_e c^2} \quad (58)$$

So, the value of r_e cannot be changed. Consequently, Equation (56) can be unchanged.

6.5. Consideration for the Degree of Freedom outside the Lorenz Invariant Scheme

The factor of 4.5 and π were considered to be related with the degree of freedom inside a proton and an electron [3]. So, our compensations in this report are related with the degree of freedom outside the Lorenz invariant scheme. The degree of freedom inside an electron seems to be 2π. We thought that 2π is related with a spin of electron. Angrick et al. confirmed that the spin of electron cannot be ignored thermodynamically [5]. Furthermore, Aquino *et al.* discovered the new method for vector analysis [6]. We hope that the degree of freedom outside the Lorenz invariant scheme will be clarified experimentally in detail.

7. Conclusions

Previously, we discovered an empirical equation (Equation (1)) relating a gravitational force and the CMB [1]. Next, we discovered the empirical equation (Equation (2)) for the ratio between a gravitational force and an electric force. Coulomb's law with distance can be expressed in terms of the CMB (Equation (3)). Our conclusion was that the mathematical connections among Equations (1)-(3) provide evidence that they are not coincidental [3]. However, there are small errors in our empirical equations, which may lead to "indeed or not" arguments.

Therefore, we attempted to reduce the errors. For this purpose, we searched for an empirical equation between the remaining mass values of an electron and a proton (Equation (20)). Next, we discovered three empirical equations (Equations ((24), (26) and (27)) with the rest mass of a proton. We then discovered three empirical equations (Equations ((29), (31) and (32)) with the rest mass of an electron.

Then, we discovered the compensation methods. For the value of the CMB, we used 2.72642 K instead of 2.72548 K. However, we felt that this was not the result of experimental measurement errors. This small deviation may be attributed to unknown theoretical reasons. Regarding the factor of 9/2, we used 4.48870 instead of 4.5. Regarding the factor of π, we used 3.13189 instead of 3.14159. Then, the errors in the seven equations were less than 10^{-5}. Therefore, we can justify why the discovered compensation values should not coincide.

Furthermore, we attempted to provide hints on how to construct a background theory. X in the Jarzynski equality is investigated in quantum physics. X may be separated into electromagnetic potential energy and kinetic energy, including the rest mass energy. We noted that G is not independent of the CMB. Equation (53) is similar with Hawking radiation. Therefore, the relationship between Hawking radiation and Equation (54) should be examined. The relationship between four special lengths can be unchanged. The degree of freedom outside the Lorenz invariant scheme is important to explain our compensations.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Miyashita, T. (2020) *Journal of Modern Physics*, **11**, 1180-1192. https://doi.org/10.4236/jmp.2020.118074

[2] Miyashita, T. (2020) *Journal of Modern Physics*, **11**, 1159-1160. https://doi.org/10.4236/jmp.2020.1110096

[3] Miyashita, T. (2021) *Journal of Modern Physics*, **12**; Will Be Published.

[4] Jarzynski, C. (2017) *Physical Review X*, **7**, Article ID: 011008. https://doi.org/10.1103/PhysRevX.7.011008

[5] Angrick. C., Braun, J., Ebert, H. and Donath, M. (2021) *Journal of Physics: Condensed Matter*, **33**, Article ID: 115501. https://doi.org/10.1088/1361-648X/abd338

[6] Aquino, F.W. and Wong, B.M. (2018) *Journal of Physical Chemistry Letters*, **9**, 6456-6462. https://doi.org/10.1021/acs.jpclett.8b02786

Appendix

The possible problem was discovered in Equation (26) during typesetting for publication. Equation (26) may be the following Equation (59).

$$Gmp = 4.5 \times 4.5 \times \frac{ec}{2} \times \frac{\pi}{3.14159} \times \left(\frac{e^2}{4\pi\varepsilon_0} \right) \tag{59}$$

Then, the calculated kTc cannot be determined.

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Particle-Wave Duality Resulting from the Granulation of Fields in a Hypercubic Lattice

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Abstract

The possibility of granulated discrete fields is considered in which there are at least three distinct base granules. Because of the limited size of the granules, the motion of an endlessly extended particle field must to be split into an inner and an outer part. The inner part moves gradually in a point particle-like fashion, the outer is moving step-wise in a wave-like manner. This dual behaviour is reminiscent of the particle-wave duality. Field granulation can be caused by deviations of the structure of the lattice at the boundaries of the granule, causing some axes of the granule to be tilted. The granules exhibit relativistic effects, inter alia, caused by the universality of the coordination number of the lattice.

Keywords

Discrete Space, Granular Fields, Hypercubic Lattice, Motion in a Lattice, Particle-Wave Duality, Relativistic Effects

1. Introduction

A duality in the movement of particles in a discrete space can only be established if the possibility of motion in a lattice can be described. The problem of motion in a discrete space has a long history. It is first formulated by the Arrow Paradox of the ancient Greek philosopher Zeno of Elea. According to the interpretation of Grünbaum [1], the Arrow paradox states that it would be impossible to perceive any difference between a moving and a motionless object at the smallest moment of a discrete space-time. More recently, B. Russell [2] also highlighted the problems with the movement of point-shaped particles in a lattice. When the object is a field, the entities in space determining it are coupled to the vertices of the lattice, then when the field moves, the vertices must also move. Because of the time immobility of the vertices of a lattice, Čapek Milič and others have seen this as an impossibility [3].

Here too, the immobility of the vertices is a basality, but then in a four-dimensional lattice. It is shown that three-dimensional areas within a four-dimensional discrete space can be in motion relative to the surrounding lattice when the areas have boundaries that provide for the transition of the moving inner region and the still outer space. When motion in a discrete space is restricted to bounded areas, it is obvious to investigate if fields in a discrete space can also be composed of bounded areas. The second purpose of the article is therefore to get an idea of the properties of the most common fields consisting of granules, *i.e.* bounded three-dimensional areas in a four-dimensional discrete space. For this purpose, the so-called discrete field needs only be composed of three different types of basic granules, including a moving granule, done in Chapter 2.

Chapter 3 deals with the properties the divers basic granules must have in order the granulated fields correspond as closely as possible to reality. Considering fields as an ensemble of granularities of limited dimensions opens up new explanation opportunities, especially when describing the motion of an endlessly expanded particle field. In that case, two modes of motion are needed consistent with the de Broglie's hypothesis when the granular fields are part of a multidirectional hypercubic lattice, as shown in Chapter 4.

The purpose of how granule movement in a lattice can take place, is explained in Chapter 5, showing that the granularities may originate from aberrations of the structure of a hypercubic lattice in the boundaries. To show how the movement of a granule looks like, a Minkowski diagram-like visualisation has been applied to a lattice. It is also shown that in a lattice, under certain conditions, relativistic effects are present within the moving granule.

2. The Granulation of Fields in a Lattice-Like Discrete Space

In a hypercubic lattice, all vertices in time and space are connected by eight edges (coordination number). If this four-dimensional lattice is taken as a frame of reference, where the fields are somehow connected to the vertices, all the phenomena from the past and the future are also linked. Consequently, there are two points of view to look at phenomena in the form of fields, both of which are used in this article:

- One is the four-dimensional perspective within which a phenomenon is the time development of an unchanging four-dimensional presence extending from the past into the future. Also called eternalism or block-universe [4].
- The other is the spatial perspective in which a phenomenon is a concatenation of a whether or not in time changing three-dimensional presence.

2.1. The Discrete Field

In a continuous space, a field is defined as a quantity present at any location in space. In a lattice as discrete space, the location consists of vertices where each vertex has a limited number of relations with other vertices. A field in a lattice needs to be expressed as a set of quantities related to the vertices. The field thus discretized limits the possibilities in the description of a field.

In a hypercubic lattice all vertices have the same coordination number. Assuming this also holds when fields are present, it means that there is a large number of vertices with edges having field characteristics in addition to the function of position determination in space. So there are field vertices of which at least two of the eight edges are needed to determine the position within the space and the rest can be field edges. Then, to allow for a diversity of field strengths, massive concentrations of field vertices are needed. Because large field strength must occur everywhere in space, the space would only consist of field vertices. If so, the vertices lose their function as the smallest entities of a fourth-dimensional space-time. This can be solved by a much larger coordination number or by positioning the field vertices in the boundaries of granules.

2.2. The Granular Character of the Discrete Field

Another possibility is that interaction stems from the coincidence over time of three-dimensional regions with field action. Chapter 5 outlines how such an area could look like. The discrete field is then a coherent set of three-dimensional areas of various sizes L, expressed in the unit distance Δx of a lattice.

From a three-dimensional perspective, the three-dimensional area is a flat spatial plane that extends over time, called the active plane. A variety of field strengths is possible when the discrete field consists of a series of active planes at regular distance L. The spatial density of the field is then $1/L$. The probability of two active planes of different fields with the same action coincide is $1/L^2$.

With the active planes of size L^2 at mutual spatial distances L, the discrete field can be split into granules of size L^3 having the action at the boundaries of the granule. This allows the discrete field to be divided into separate non-overlapping granules of size L^3.

2.3. The Field Granule from Discretization of the Units

The likelihood of a granulated field also arises from the transformation of the usual units into the two units Δx and S of a hypercubic lattice with multiple directions [5], where Δx is the smallest distance of a lattice and S stands for the number of lattices in which a phenomenon is present. The conversion results in an expression of the field strength in one lattice or subspace:

$$E_{sub} = 1\big/L_d^2 \qquad 1/\Delta t,$$

in which L_d is a quantity analogue to the discrete distance between two particles.

The inverse proportionality with an area indicates the existence of a field granule, or field granule of spatial size L_d^3. The conversion also results in a smallest field granule for the electron (electron radius) has the enormous local magnitude of about $L_{d0} = 3.38 \times 10^{19} \Delta x$, expressed in smallest lattice distance.

3. The Discrete Field in One Subspace Described with Field Granules

Suppose that the granules of the discrete field have the shape of a four-dimensional cube, with the dimension L^3 in space and an extension L in time, where the cause of the field effects is present in the spatial boundaries. Chapter 5 examines the possibility that the field effect is caused by aberrations of the structure of the four-dimensional hypercubic lattice.

It turns out that these active boundaries are repeated after the time T_{field} and are interconnected by the corner vertices of the cube. The corner vertices form the basis of interconnectedness of granules allowing the various types of granules to form extensive fields.

3.1. The Three Different Base Field Granules in Each Subspace

Three different types of field granules are required to describe the most common phenomena.

Still field granule;

The still field granule is immobile relative to the local discrete space. The field granule is regularly repeated every T_{field}. There can be different types of still field granules, each representing another type of action like electric or magnetic. It is assumed that the cause of the action is present in one of the side planes of the cubic field granule, making the presence frequency of an active plane $1/T_{\text{field}}$. An example of a such plane with local deviation of the axis declination is described in Section 5.2. The inner space of the field granule is void, that is, a regular lattice.

Moving granule;

The moving granule, called moving space, is a cube with four moving side planes that ensures the regular interior space to move with relativistic effects. Section 5.5 describes the moving space in a lattice. This possibility is visually illustrated in Figure 1. The moving plane is a three-dimensional bounded area in R^4, whose entire inner space is moving with speed v and the boundaries are such it forms the transition between the still outer space and the moving inner space. A more precise description of such a boundary is shown in Section 5.3 in the case of a lattice. The relativistic effects are showed in Section 5.4.

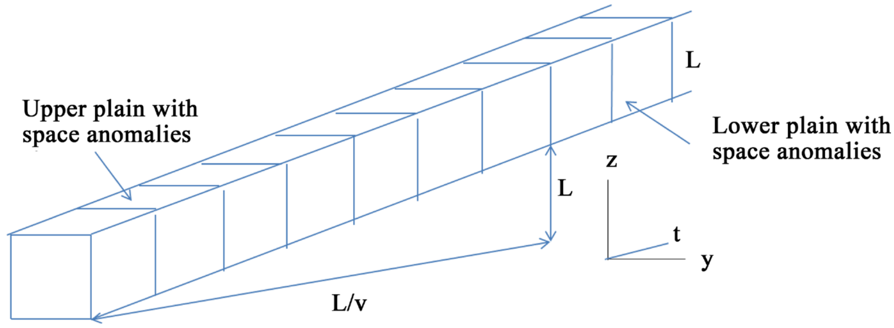

Figure 1. The moving granule. In the moving space, the entire space between top and bottom planes moves integrally with respect to the space around the granule caused by the anomalies in the space structure in the upper and bottom planes. The anomalies caused that every $t = 1/L$ the space to be changed from a still to a moving space at the top and vice versa on the bottom plain.

Jumping field granule moving at maximum speed;

A repeating space jump in time is another mode of movement of a field granule in a lattice. The field granule is present at one location during the time T before jumping to a successive location L $(=T)$ in the direction of motion. In R^4, the sequence of jumping field granules is located along the space-time diagonal. Seen from R^3, the presence of the field granule moves with discrete speed $v_d = L/T = 1$. In Figure 2, the four-dimensional situation is roughly outlined, with the jumping field granule depicted as a three-dimensional cube.

To be in accordance with reality, it is supposed that the side faces of the cube to consist of two orthogonal planes with a E and B action.

The granular field is an ordered combination of various types of field granules forming constructions regularly distributed in space and time and interconnected via the corners. Next, it is investigated what the properties of granular fields will be if they are described as a combination of said granules.

3.2. The Movement of a Homogeneous Electric and Magnetic Granular Field

Take the constructive combination of still and jumping field granules. The still E (or B) field is a time chain of still field granules. Let the moving field be a time chain of still E (or B) and jumping E, B field granules of the same size L, interconnected at the corner vertices, replacing one of a still field granules in the time

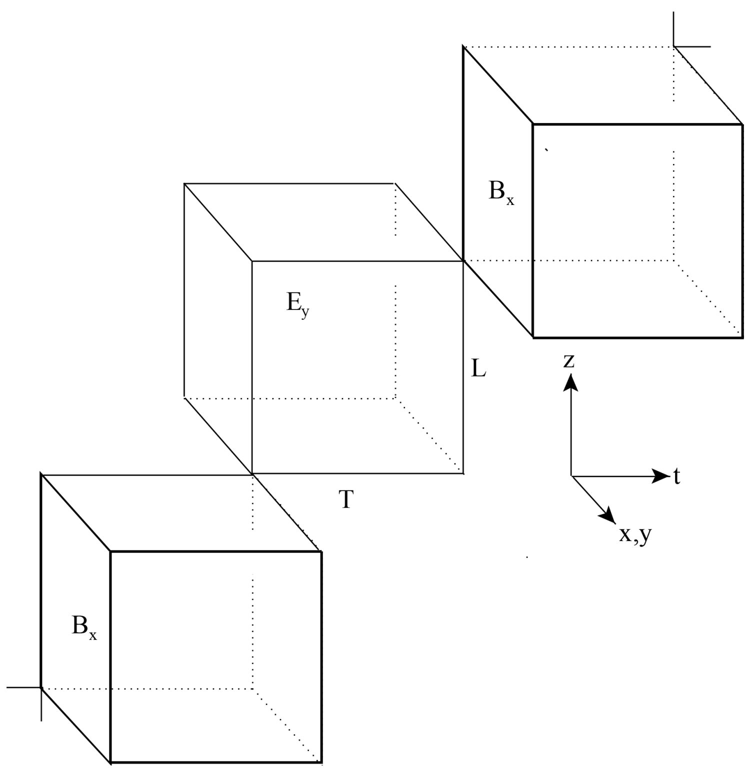

Figure 2. A jumping field granule. The jump-like motion of field granules is depicted in R^4 as a series of cubes. The cubes are interconnected sequentially in the corner vertices, such that the cubes are present along the diagonal of space-time.

chain by a jumping field granule at each $T_{\text{jump}} = L/v$. So, the time chain of field granules makes each T_{jump} a jump Lin the direction of movement. Because the jumping field granule consists of both E and B fields as presumed, the E field is present at all times. The jumping field granule is the cause of the presence of the B field during the short period $T(=L)$ every T_{jump}.

It is obvious that the field strength is proportional to the time the field is present to perform its operation, *i.e.* has the probability of coinciding with another E or B field granule. Then the fraction B/E is equal to the ratio of the time periods in which the fields are present, so that $B/E = T/T_{\text{jump}} = v$:

$$B = vE, \tag{1}$$

or when B is the main field: $E = vB$.

Equation (1) is in accordance with the empirical expression of a moving electric or magnetic field, showing that the discrete field can be expressed in field granules.

The disadvantage of het above model is that it misses the reason for the repetition time T_{jump}, making the model incomplete.

3.3. The Motion of a Granular Particle Field

Due to the spatial in homogeneity of the particle field, the motion of a granular particle field must be split into two of the above types of motion acting simultaneously. Described in granules, an endlessly extending particle field will consist of a series of concentric field granules of increasing size. A moving granulated particle field necessarily consists of the equal speed of all individual granules of different sizes. This makes that none of the three basic granules alone can describe the motion of the discrete particle field, the solution is a combined presence of a moving space (granule) and jumping granules. To this end, the motion of the particle field is split into an inner and outer sphere, the inner part is being moved by one moving granule with rib L_{moving}. The outer part by a series of combined still and jumping granules of suitable size moving in the manner of a homogeneous field as described in the previous paragraph.

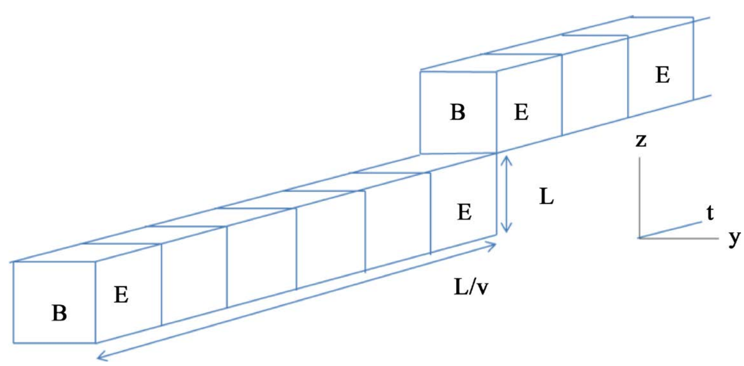

Figure 3. A moving homogeneous field. A moving homogeneous field is a combination of a still and a jumping field, depicted in R^4 as a series of cubes. During the time $t = L/v$ the field stands still, then the jumping granule causes a jump over the distance L. The cubes are interconnected forming a constructive unity.

The smallest part of the particle is present as a locally still field within the moving space and will, as part of the moving space, move gradually with discrete velocity $v_d = 1/L_{\text{moving}}$ relative to the surrounding space. Because the inner part is part of the moving space, it also has time dilation and length contraction.

Using the visualisations of **Figure 1** and **Figure 3**, the moving granule of **Figure 1** is implanted into the granule of the moving field of **Figure 3**. Many concentric granules a la **Figure 3** of increasing size form the outer shell of the moving particle field.

3.4. The Momentum of the Inner Field

The concentric field granules of the particle field have increasing discrete sizes L_{di} and the associated discrete repeat distances in time T_{di}. Let R_{particle} be the averaged effective spatial size of the many granules and T_{particle} the mean repeat distance of the granules, determined by:

$$1/R_{\text{particle}} = \sum 1/L_{di} \qquad 1/\Delta x, \tag{2}$$

both provided with the corresponding discrete units. Remarkably, these two quantities have a significant difference in size, as can be seen from ref. [6]:

$$T_{\text{particle}} = (2\pi/\alpha) R_{\text{particle}} . \tag{3}$$

Wherein α is the fine structure constant. This means that in one subspace, the cube-shaped granules are only sporadically present over time. In the time in between, the field granules a represent in other subspaces. Call the particle mass in one subspace the average density of the still field granules at a given location, represented by:

$$m_{\text{subspace}} = 1/T_{\text{particle}} \qquad 1/\Delta t. \tag{4}$$

This is conform the particle mass in one subspace of reference [6].

Take a moving particle field. All particle granules with size $L_{di} \le L_{\text{moving}}$ are present within the moving space. It is reasonable that $1/T_{\text{particle}}$ is mainly determined by the field granules smaller than L_{moving}. Relative to the surroundings of the moving space, the density of the particle granules at successive positions in the surrounding space is given by multiplying $1/T_{\text{particle}}$ by the discrete velocity. This will be called the momentum per subspace:

$$p_{\text{subspace}} = v_d / T_{\text{particle}} \qquad 1/\Delta t. \tag{5}$$

In a lattice is $\Delta x = \Delta t$ making v_d dimensionless.

3.5. The Momentum of the Outer Particle Field

Each T (=L_{moving}), the moving space makes a unit-step in the direction of motion so that after $t = T^2$ the total size L_{moving} of the cube has been covered. The outer part of the moving particle field must move in conjunction with the inner part, *i.e.* making a minimum jump L_{moving} every $t = T^2$ or a multiple thereof. This is possible if the outer particle field consists of a series of concentric shells of increasing discrete size L_{di}, with $L_{d1} = L_{\text{moving}}$. To be in motion, each shell consists of several still field granules and one jumping field granule, which makes every discrete $t_{di} = L_{di}/v_d$ a jump L_{di} in the direction of motion.

Implicitly, every jump L_{di} fits exactly into L_{di+1}, making L_{di} a plural of L_{d1}. This means that, seen in R^3, the spatial distribution of the outer particle field is spaced by a multiple of L_{moving}. This means that during T^2 there is a still outer field, after which the still field is present at the next spatial location. Therefore there is a greater chance of interaction at mutual distances L_{moving}, which can be interpreted as the wavelength λ_d. By doing that, the relation between the discrete wavelength λ_d of a moving particle in a discrete space and the discrete velocity becomes $v_d = 1/\lambda_d$. Thus the number of outer field granules entering a new area, or the momentumof the particle field, is determined by

$$p_{\text{subspace}} = 1/L_{\text{moving}} = 1/\lambda_d \qquad 1/\Delta x. \tag{6}$$

This relation applies next to relation (5).

In one subspace, the wave length λ_d is expressed in units of one lattice. In the next chapter, relations (5) and (6) will be expanded to a space with multiple lattices. The interpretation $\lambda_d = L_{\text{moving}}$ links the wavelength to the particle mass via Equation (9).

3.6. Variety of Speeds of the Particle Field

The jumping outer part of the moving particle resembles the jerking motion described in [7] [8] or [9]. The description of a moving particle given here is not only more extensive than in these references, it also enables a wide variety of velocities. To clarify this, take the situation where the moving particle consists of a series of concentric moving granules. Then, going from the outside in, there is an increase of velocity and time dilation caused by the successive moving spaces. According to the relativistic relations, the effect of one of the moving granules sized L_i on velocity and time dilation is determined by $T_{i+1}^2 = T_i^2\left(1-v_i^2\right)$ with $v_i = 1/L_i$. With k moving spaces, the cumulative effect is $T_{k+1}^2 = T^2\Pi\left(1-v_i^2\right) \mid i = 1,\cdots,k$. This makes that the velocity v_{tot} of the inner moving granule is determined by: $T^2\left(1-v_{tot}^2\right) = T^2\Pi\left(1-v_i^2\right) \mid i = 1,\cdots,k$, making

$$v_{tot}^2 = 1 - \Pi\left(1 - 1/L_i^2\right) \approx \sum v_i^2 - \cdots$$

With this expression any velocity can be obtained by a suitable combination of L_i.

4. Granulated Fields in a Multidirectional Discrete Space

In a hypercubic lattice, the movement of the particle field can only take place along one of the three spatial axis, which is very unrealistic. In a more realistic multidirectional discrete space, the entire space is split up into $n_{\text{directions}}$ subspaces each with their own set of spatial directions [5]. The number of spatial directions $n_{\text{directions}}$ comprises at least 10^{22} subspaces. The set of subspaces $n_{\text{directions}}$ is a small subset of all subspaces $n_{\text{subspaces}}$; $n_{\text{subspaces}} \gg n_{\text{directions}}$, allowing the particle field n_{particle} to be in a multiple of $n_{\text{directions}}$.

4.1. Movement of a Particle Field in Multiple Subspaces

When applying the mode of motion of a moving spherical symmetrical particle field from the previous chapter in a multidirectional space, the field is present in at least $n_{\text{directions}}$ subspaces. Each subspace, if necessary, provided with a subspace with an axis in the direction of motion. The necessity for the outermost regions of the moving particle field to have a subspace with an axis in the direction of movement ensures that the number of these subspaces is also $n_{\text{directions}}$. Consequently, the moving particle field is present in $n_{\text{directions}}$ subspaces with an axis in the direction of motion next to the $n_{\text{directions}}$ subspaces with random axes. The different subspaces are interconnected via the space-point. The space-point of a multidirectional lattice is a time series of vertices connected in time, the vertices being part of subspaces with different spatial directions ref. [5].

4.2. The Momentum of a Particle Field in a Multidirectional Lattice

In section 3.4 it has been determined that per subspace the densityof a field granule of the particle is $m_{\text{subspace}} = 1/T_{\text{particle}}$. Multiplication of this particle mass per subspace by the number of subspaces n_{particle} gives the total densityof the particle field expressed in the two units of a multidirectional discrete space:

$$m_{\text{discrete}} = n_{\text{particle}} / T_{\text{particle}} \qquad S/\Delta t. \tag{7}$$

In reference [6] it has been shown that only the discrete units Δx and S are needed to express the physical quantities in a multidirectional lattice. Also in reference [6], the same mass expression (7) has been found.

With (5), the density of the particle granules in consecutive positions with respect to the particle's surrounding space is

$$p_{\text{discrete}} = v_d n_{\text{particle}} / T_{\text{particle}} \qquad S/\Delta t.$$

By inserting the mass expression (7) herein, the momentum $p_{d\,\text{particle}}$ expressed in discrete units is

$$p_{d\ \text{particle}} = m_{\text{discrete}} v_d \qquad S/\Delta t, \tag{8}$$

also in accordance with reference [6].

4.3. The Movement of the Outer Field Granules

As explained in section 4.1, the outermost field of the moving particle is presented as a series of jump field grains in $n_{\text{directions}}$ subspaces that all having an axis in the direction of motion. Equation (6) describes the momentum in one subspace, *i.e.* the frequency with which the outer field granules in one subspace enter new areas of space. For $n_{\text{directions}}$ subspaces, the frequency is augmented by that number by which the momentum in the multi directional lattice becomes

$$p_{d\ \text{particle}} = n_{\text{directions}} / \lambda_d \qquad S/\Delta x, \tag{9}$$

λ_d is the discrete wavelength in the subspaces involved. Equation (9) links the wavelength to the particle mass.

In ref. [6] it is shown that the de Broglie relation in a multidirectional lattice is

$$p_d = h_d / \lambda_d \qquad S/\Delta x, \tag{10}$$

where in h_d is the discrete Planck's constant,being the Planck's constant expressed in the discrete units. With (9), the discrete Planck's constant becomes:

$$h_d = n_{\text{directions}} \qquad S. \tag{11}$$

Relation (11) has also been found in reference [6] by using the same reasoning as in section 3.6 to obtain the wavelength of the granulated photon field.

4.4. Relativistic Mass Increase of Moving Granulated Fields

In case the number of edges per vertex is the same for every vertex of a hypercubic lattice, it is shown in section 5.4 that length contraction and time dilation (14) is present within the moving space, which is also true for the largest granule L_{moving} of the moving space. Because according to Equation (6) $\lambda_d = L_{\text{moving}}$, the wave length is subject to length contraction

$$\lambda'_d = \lambda_{d0}\sqrt{1-v_d^2} \qquad \Delta x. \tag{12}$$

Using the equation (10), the relativistic momentum is

$$p'_d = h_d / \lambda'_d$$

With $p = m \cdot v$ and considering that the v is the velocity in the surrounding space, makes the particle mass relativistic

$$m'_d = m_{d0} \Big/ \sqrt{1-v_d^2} \qquad S/\Delta x. \tag{13}$$

From the length contraction of the wave length follows the increase of the particle mass.Based on this, the following observations can be made.

According to Section 3.4, each granule of the outer paticle field L_{di} is a plural of L_{moving}. When the size of L_{moving} changes, the larger granules follow immediately. Apparently the successive particle granules are spatially interconnected.

Secondly, the densification by length contraction increases the presence of the outer field-units and therewith the likelihood of an encounter with another field (interaction). This is in line with the interpretation of the momentum of a granular field as the density of the particle granules at successive positions in space.

4.5. The Dual Description of the Particle Momentum in a Multidirectional Lattice

In a multidirectional discrete space, the endlessly extended particle field is present as a series of concentric granules of different sizes in each of the $n_{\text{directions}}$ ($>10^{22}$) subspaces. To describe motion of an endlessly extended particle field with granules of limited sizes, it will consist of inner and outer granules. The inner field granules show a gradual displacement, represented mathematically by equation (8). The outer particle field shows step wise displacement represented by (9).

Because the largest granule within the moving space and the smallest granule of the outer part are somehow coupled, Equations (8) and (9) act simultaneously. This means that that in the case of granulated fields, the momentum of a particle is mathematically described in two ways, resembling the particle-wave duality.

5. Field Granules Arising from Anomalies in the Structure of the Lattice

This chapter examines whether the granulation of fields can be the result of deviations in the structure of a lattice. A lattice consists of vertices and edges with a certain coordination number (8 for a hypercubic lattice). The structure of such a lattice is the organisation of the vertices and edges. The space structure of a hypercubic lattice with four orthogonal axes in every vertex is very simple and regular in every vertex, it is called the regular lattice.

A multitude of local structural deviations from the regular lattice are possible. Some of these will be given here whose pattern in deviations is repeated regularly along the time axis. The examples, necessarily at the level of vertices and edges, are presented visually for quick understanding.

5.1. An Example of a Simple Anomaly in the Structure of the Lattice

To become familiar with structural deviations, an example of a very simple anomaly in the structure of a regular lattice is given in Figure 4(a).

Generally, a structural deviation in one vertex necessarily implies that the two adjacent vertices also have a structural deviation. A series of adjacent deviating

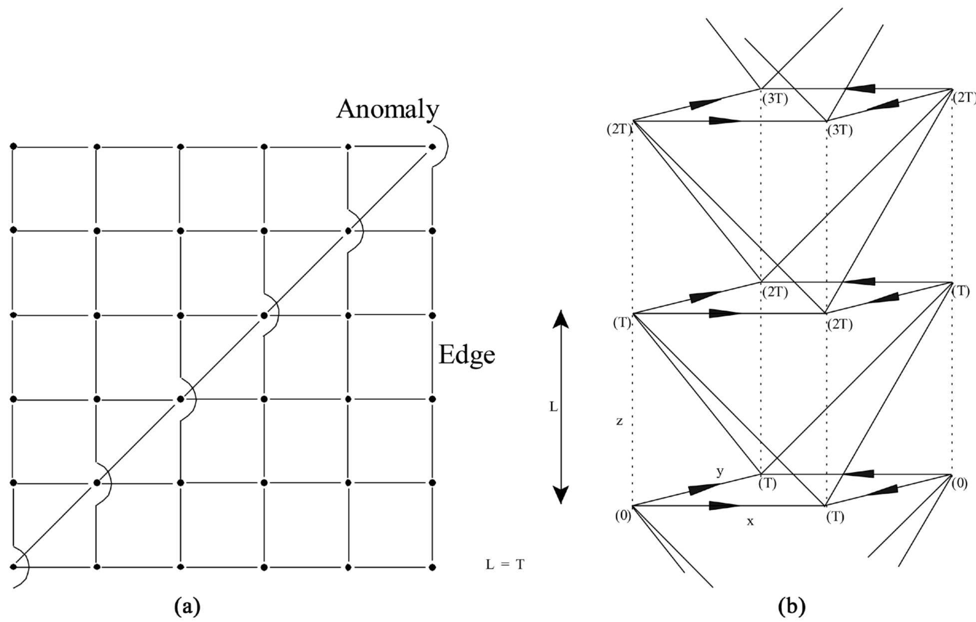

Figure 4. Constructions of anomalies within one hypercubic lattice. The left (a) shows a simple anomaly in a two-dimensional lattice forming a straight diagonal anomaly-line. (b) shows a construction of anomaly-lines in a four-dimensional lattice. The anomaly-lines diagonally in space and time are indicated by an arrow. The figure shows that a cube-like pattern of anomaly-lines of rib L is present at regular intervals in time and space: ($t = 0$, $z = 0$), ($t = T$, $z = L$), etc. Spatially seen, the pattern moves in the z-direction with speed 1.

vertices constitutes a so-called anomaly-line. Note that an anomaly-line as in Figure 4(a) locally runs diagonally between two axes, and secondly that it has no beginning or end.

In R^4, constructions of anomaly-lines can exist when several lines are combined to endlessly extending repetitive structures. Figure 4(b) illustrates such a construction. Herein four anomaly-lines merge at regular distances in space-time into one vertex. The sketched image is diagonal in space and time and is present in R^4as a static column-like construction in four dimensions.

In the construction a recurring pattern can be recognized consisting of a cube-shaped assembly of eight anomaly-lines. Each cube consists of four pure spatial anomaly-lines and four lines diagonally in space-time. By focussing on the progression in the time of the cube-shaped pattern, the pattern makes each T a step L along the z-axis. The series of positions of the corner vertices of the cube can be expressed by the set $\{z,t\} = \{(iZ,iT) \mid i:\text{integer}, Z = T = L\}$.

Seen in R^3 the cube moves with velocity $v_d = iZ/iT = 1$.

The assembly of anomaly lines can also be arranged so that the formed construction consists of a series of cube-like patterns extending endlessly in time. Seen from R^3, these cubes stand still at one location.

5.2. Stand-Still Three-Dimensional Regions with Tilted Axis Caused by Anomalies

With the possibility of the last section, no physics can be present. Namely, at two intersecting moving columns there is no conceivable transition in two fully different columns.

To provide for possible interactions, consider structural deviations whose anomalies are present on the boundaries of a three-dimensional region of R^4. These anomalies affect the entire inner space-time region by tilting some axes of the region. These so-called tilted axes are not parallel to similar axes of the lattice surrounding the region. Because the organization of the internal axes is locally the same as in a hypercubic lattice, the space is regular within the said region.

Due to the tilting of the axes within an entire area, there is a chance that regions overlap cannot always occur, *i.e.* it has an interaction as result. These three-dimensional regions in R^4 form an active plane within R^3.

An active plane forms an endless corridor in time with, at the top and bottom of the corridor, the anomalies that cause the axes to tilt. Figure 5 shows an example of an active plane in the form of a time slice of the three-dimensional corridor.

The cohesion of slices along the y-axis is enhanced when a z-line is missing in the corridor at every T^2, caused by two open edges, *i.e.* an edge with a missing vertex, at the top and bottom. A missing z-axis means that there is a gap in time in the active plane. Then the organization of the anomalies along the y-axis in the top and bottom of the corridor can be such (for example in the manner of **Appendix A**) that every T (=L) the active plane has an open edge at the corner vertices.

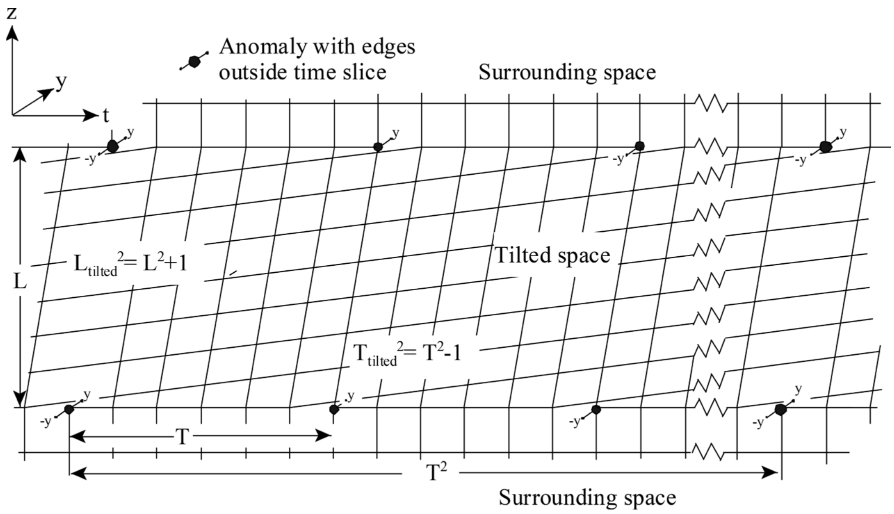

Figure 5. Time slice with tilted axes of a corridor of a still active plane. The time cross-section of the corridor of height L forms a slice along the z-and t-axis of thickness Δy (=1). The lines depicted are the connecting edges, the vertices are located at the intersections of the lines. At regular intervals T, the anomalies at the top and bottom lack a connecting edge causing the z- and t-axes to be relatively tilted. The missing edge on each T, drawn as an open, unconnected edge in the y-direction, is thought to be connected to a vertex of the adjacent slice.

Based on Figure 5, two distinct active planes can be distinguished, one with tilted z- and t-axes and the other with tilted y-axis and t-axis. This distinction makes that, apart from opposite spatial tilt direction, the active plane can have six different spatial orientations in R^3.This is reminiscent of the six different E, B operating axes.

Active planes of opposite tilt cannot merge, which means that the merging of active planes is not possible in all cases. The said unconnected open edge at the corners each T allows to arrange several active planes into a cube, with which various field granulesof Section 3.1 can be formed.

5.3. Anomalies Causing the Movement of a Three-Dimensional Regions

By taking a small variation in the structure of Figure 5, an entire spatial plane is in motion. Figure 6 shows the structure of such plane, in which the time axis makes a spatial step Δz in the direction of motion each T. Herein, the anomalies at top and bottom are such that all time lines of the corridor and boundaries are relatively inclined in the direction of motion, where the tilted time axes having no beginning or end.

The time slice of the moving plane consists of the repeated occurrence of T segments in which one of the segments makes a jump in time due to a missing z-axis. The moving plane is formed by a series of L (=T) adjacent time slices in such a way that segments with missing z-axis are never present at the same time.

Seen in R^4, the corridor with the moving plane looks static. In R^3, the plane is in consecutive positions along the z-axis over time, which means it is in motion.

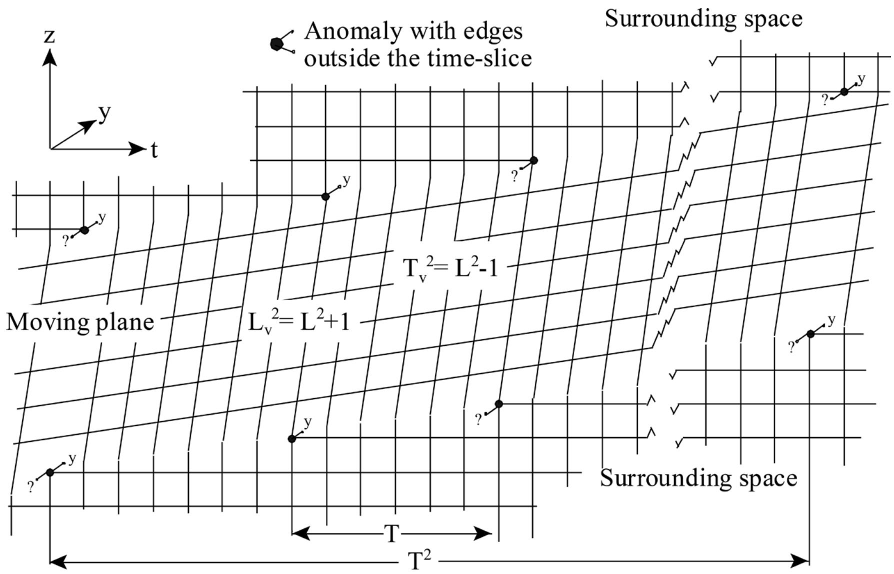

Figure 6. A time slice of a moving plane. Depicted the edges of a time slice of a moving plane with the vertices at the intersections. In contrast to the tilted plane, the inner vertices are shifted by the unit-distance on each T (=L). This organization results in two or four unconnected edges each T, which are connected with adjacent slices. In a multidirectional lattice, the tilted time lines should be read as the (moving) space-points.

As can be seen in Figure 6, all axes with in the corridor are tilted in the same way so that the entire space of the corridor has a locally regular space structure that moves at local speed $v_d = 1/T$.

In the case of a moving space-point the set of subspaces must be such that one of them has an axis in the direction of motion. This subspace sets the space-point in motion.

5.4. Length Contraction and Time Dilation within a Moving Plane

An obvious definition for the distance between two vertices on an axis is the number of intermediate edges. Using this definition, the time distance within the moving plane is shorter than the comparable time distance in the surrounding space because of the missing z-axis in the moving plane. The time distance T^2 in surrounding space has a corresponding time interval T_v^2 within the moving plane, which, due to the time gap, is one unit-distance less: $T_v^2 = T^2 - 1$. Seen from the surrounding space, combined with $v = 1/T$, the time distance T_v within the moving plane becomes: $T_v = T\sqrt{1-v^2}$.

As Figure 6 shows, the time axis within the moving plane is literally stretched relative to the time axis of the surroundings. Take the time distance Δt within the moving plane. In surrounding space, this time interval lasts longer, *i.e.* it comprises a larger number of $\Delta t'$ vertices met: $\Delta t'/\Delta t = T/T_v$. Using the above expression for T_v, the relation between the time intervals $\Delta t'$ and Δt becomes

$$\Delta t' = \Delta t/\sqrt{1-v^2},$$

being the time dilation of events.

In the corridor the smaller number of vertices overtime is compensated by an increase in vertices in the direction of movement. To clarify this: The corridor extending indefinitely in time has, in addition to the boundary in upper and lower surfaces, also two side surfaces as boundaries. At the location of the side surfaces, the inner vertices are connected to the vertices outside of the corridor.

Take the side surface $L.\ T$ expressed in distances of the surrounding space. The same side surface, expressed in distances of the corridor, comprises of $L_v T_v$ vertices, where L_v is the mean number of vertices forming the height. At the surfaces, each vertex of the corridor is connected with a vertex of surrounding space, making $L_v T_v = LT$. As a result, the average number of vertices within the moving plane in the direction of the velocity is

$$\Delta z' = \Delta z\sqrt{1-v^2}\ .$$

Take a number of vertices Δz within the moving plane in the direction of motion. Seen from surrounding space, this distance comprises less intermediate edges $\Delta z'$, making

$$\Delta z' = \Delta z\sqrt{1-v^2}\ , \tag{14}$$

being the expression of the length contraction.

In a lattices-like discrete space, length contraction arises from the same coordination number of all vertices instead of the principle of relativity. In addition, it is based on a high regularity in the presence of anomalies in a hypercubic lattice as discrete space.

5.5. The Moving Space

A moving field granule consists of a cube with four moving side planes. All these planes are moving with the same speed v in the same spatial direction. This caused all the vertices an edges of the cube's interior also moves with speed v due to the interconnections with the side planes. Like the side planes, the entire space within the moving space will also have time dilation and length contraction as well as a regular space structure.

The consequence of the regular space structure is that each field granule positioned within the moving space has the same internal structure as placed in the non-moving situation, which is the subject of sections 3.3 and 4.2.

6. Concluding Remarks

This article is about the possibility that fields are made up of granules, being three-dimensional cubic-like regions in R^4 of finite sizes. There are some indications, to see Chapter 2, that fields in a discrete space-time have such a property.

Consequence of granulation is that the motion of the particle field needs to be split into an inner and an outer part. The inner part moves gradually with a moving space as point-like particles (8), the outer part coupled thereto consists of a series of jumping granules (9) that exhibit wave characteristics.

The dual description of the particle motion results in an expression for the discrete Planck's constant (11) in a multidirectional discrete space, the same as obtained in ref. [6].

As shown in Chapter 5, deviations in the regular structure of a hypercubic lattice offer a promising possibility of being the origin of granulations. The attractive thing about space deviations is that it opens up the possibility that all phenomena are the result of structural deviations, *i.e.* expressions of space itself.

Results

A discrete space in the form of a hypercubic lattice leads to field granulation.

In the case of granulated fields, the particle motion needs to be split into parts, reminiscent of the particle-wave duality. The particle wave-duality is thus an indication of the granularity of fields besides the existence of the electron radius.

Motion in a static four-dimensional lattice takes place as a space-time corridor in R^4 with tilted time axes, to be visualized in a Minkowski diagram-like fashion.

In a hypercubic lattice, the moving corridor is the cause of both movement and relativistic features. The latter arises from regularities in the presence of the anomalies in space structure and from the same coordination number of all vertices.

If the granulation of fields is based on structural anomalies, the multidirectional hypercubic lattice is an ultimate substantival space [10].

With the finding here that motion is relative to space itself, the luminiferous aether discussion of the early 20e century can be extended with the possibility of a multidirectional hypercubic lattice as the fixed frame of reference.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Grünbaum, A. (1967) Modern Science and Zeno's Paradox. Wesleyan University Press, Middletown, Connecticut.

[2] Russell, B. (1926) Our Knowledge of the External World. Allen & Unwin, London, 179.

[3] Čapek, M. (1961) The Philosophical Impact of Contemporary Physics. Princeton University Press, Princeton, New Jersey, 68-69.

[4] Crisp, T.M. (2007) Presentism, Eternalism and Relativity Physics. In: Craig, W.L. and Smith, Q., Eds., *Einstein, Relativity and Absolute Simultaneity*, Routledge, London, 262-278.

[5] de Groot, C.T. (2017) *Journal of Modern Physics*, **8**, 1175-1189. https://doi.org/10.4236/jmp.2017.88078

[6] de Groot, C.T. (2020) *Journal of Modern Physics*, **11**, 432-447. https://doi.org/10.4236/jmp.2020.113027

[7] van Bendegem, J.P. (1995) *Logique & Analyse*, **38**, 127-150.

[8] Grünbaum, A. (1973) Philosophical Problems of Space and Time. 2nd Edition, Rei-

del, Dordrecht, 632. https://doi.org/10.1007/978-94-010-2622-2

[9] Forrest, P. (1995) *Synthese*, **103**, 327-354. https://doi.org/10.1007/BF01089732

[10] Dasgupta, S. (2015) *Philosophy Compass*, **10**, 601-624. https://doi.org/10.1111/phc3.12219

Appendix A. The Interconnection of Time Slices

In the description of an active plane of **Figure 5** and **Figure 6**, there are one or two open edges each T, both at bottom and at the top of the time slices. The vertex with one open edge is connected to a vertex of one of the two adjacent slices, the vertex with two open edges is connected to both adjacent slices. These open edges form the mutual connection of a bundle of time slices. A bundle of *L* time slices forms a spatial plane, whereby the double open edges uniting the plane and the single open edge causing the plane to have an open edge every *T*.

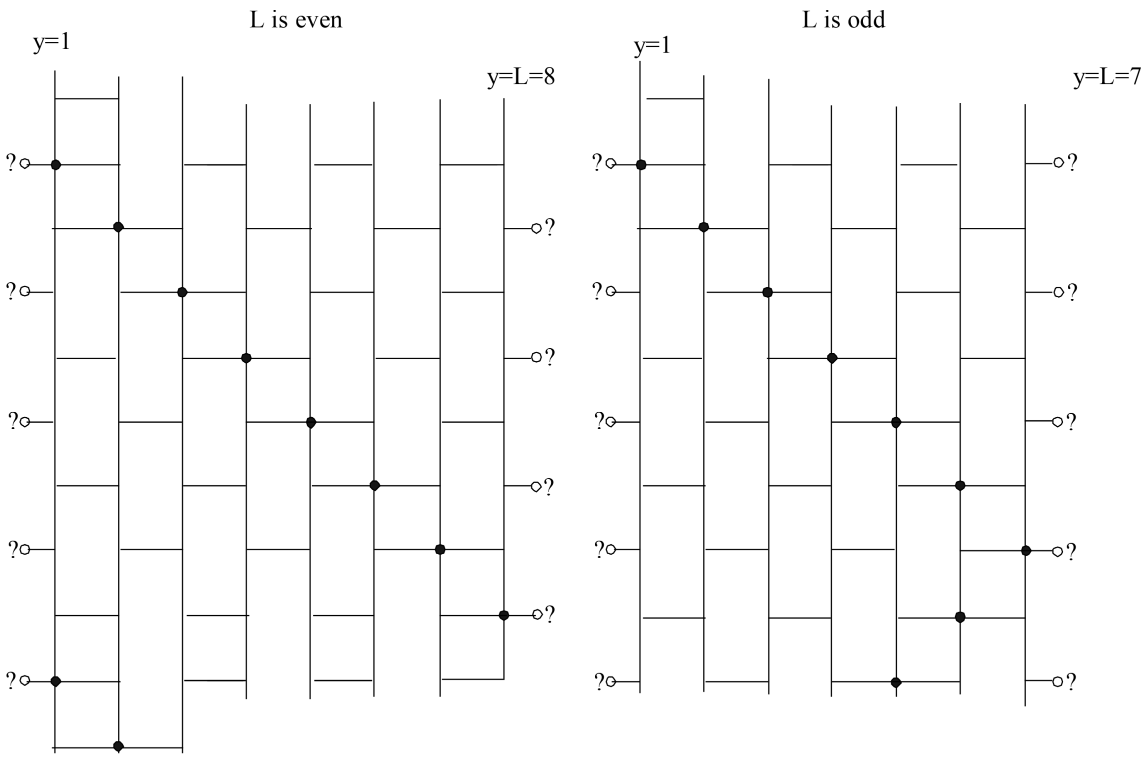

Figure A1. The edges connecting the time slices. Depicted in the *y*,*t*-plane is the possible organization of the coupling of the time slices via one or two open edges. Due to two open edges there is a regular pattern over the time distance T^2 in the *y*,*t*-plane. There are two possible organizations, in the left figure the plane has one open edge on each *T* at alternating left and right corner vertex, on the right plane there is one open edge on each corner every 2*T*.

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A Present Day Perspective on Einstein-Podolsky-Rosen and Its Consequences

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Abstract

Ever since it first appeared in 1935, the famous paper by Einstein, Podolsky and Rosen, questioning the completeness of quantum mechanics as a theory, has courted controversy. The initial arguments with Bohr have never been forgotten or gone away; the ideas of Bell have remained; many experiments have been performed purporting to support the stance of Bohr. More recently, however, an experiment performed by a group in Basel has questioned this accepted position and, theoretically, this new perspective has received support from at least two sources. It is the work behind these two sources, especially the second, together with the experimental work at Basel, which form the basis for this examination of the present position as far as this extremely important position for physical science is concerned. Needless to say, considering the views expressed in these two approaches, it is also necessary and appropriate to consider some possible consequences if this new view becomes accepted. Due to the fact that the recent support for the Einstein, Podolsky, Rosen argument makes use of results in iso-mathematics, iso-mechanics and iso-chemistry, these possible consequences include the exact representation of nuclear data, the achievement of an attractive force between identical valence electrons with the ensuing exact representation of molecular data, the prediction of new clean energies and the prediction of the possible recycling of nuclear waste via stimulated decay—none of which is allowable utilising traditional quantum mechanics. Hence here, as well as discussing the resolution of the long standing issue provoked by the well-known Einstein, Podolsky, Rosen article, some of these consequences will be discussed with a view to provoking more general, open-minded discussion within the scientific community.

Keywords

Einstein-Podolsky-Rosen Issue, Mayants' Approach, Basel Experiment,

Hadronic Mechanics

1. Introduction

The question of uncertainty affects many areas, including my own particular field of interest—thermodynamics, although, in that case, the effect may be felt indirect. For a moment consider the situation in thermodynamics. In traditional classical thermodynamics there are no uncertainties; all the variables, for example the internal energy and total number of particles, possess definite values. However, when systems composed of a large number of particles are to be considered, the methods of statistical mechanics have to be employed due to our present state of knowledge. As a consequence, when incorporated into thermodynamics, the realm known as statistical thermodynamics is entered. This is, in some crucial ways, totally different from classical thermodynamics because the introduction of statistical techniques has introduced uncertainty into the picture. No longer are there definite values for the internal energy or total number of particles; rather average values are considered. These average values, as with the average values of other thermodynamic variables, can fluctuate in this new regime. Hence, a degree of uncertainty is introduced which leads to the derivation of thermodynamic uncertainty relations. It is important to note, though, that these relations have been introduced via the recourse to statistical methods to describe details of the system under consideration. They have been introduced because, in a system composed of a large number of particles, it is not possible to write down all the equations of motion of the individual particles, let alone solve the resulting set of simultaneous equations. The uncertainty, therefore, has been introduced as a result of our inability to solve the exact problem; there is no inherent uncertainty in the original system, just as there is no inherent uncertainty in traditional classical thermodynamics. This reasoning follows for all statistical thermodynamic theories and indicates a very real difference between classical and statistical thermodynamics.

Indeed, the same reasoning may be seen to apply to many, if not all, problems considered utilising probability theory. For example, in introducing probability, it is popular to consider the tossing of a coin. If the coin is simply tossed, the outcome when it lands—head or tails—is totally uncertain. However, this is not so if someone is in possession of all the initial conditions pertaining to the toss. If the initial speed is known, the height to which the coin rises may be found, as may the time taken to reach that height. Similarly, the time taken to fall back to a given level may be found. If the rate of rotation is also known, that, together with the total time of flight, should enable the state of the coin on reaching the desired final level to be ascertained. Hence, the uncertainty associated with this problem really arises through a lack of knowledge of the initial conditions in the problem; it is not an inherent property of the actual system.

It may be seen, therefore, that neither statistical thermodynamics nor probability may be termed complete theories in the sense that neither provides exact solutions to problems. In both, uncertainty is introduced as a result of the inability to write down and solve a set of exact equations and/or a lack of knowledge of initial conditions.

The above may be viewed as simple, even naïve, reflections but it must be remembered always that quantum theory is entirely probabilistic and it might be wondered if thoughts similar to those above about the nature of probability and statistical methods influenced Einstein's thinking since, in so many cases, people have recourse to probabilistic and/or statistical methods to compensate either for a lack of knowledge in a problem or for a lack of ability to solve the exact problem with which they are faced. Whatever his reason, it seems Einstein remained distrustful of the completeness of quantum mechanics as a physical theory and, in 1935, his reservations became fully apparent in his article written with the collaboration of Boris Podolsky and Nathan Rosen.

2. Einstein, Podolsky and Rosen.

It seems that, from the moment it first appeared in 1935, the now famous paper by Einstein, Podolsky and Rosen [1], entitled "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?", has courted controversy and that still appears to be the case today. This is not surprising since, as its title clearly states, it predicted that "quantum mechanics is not a complete theory" and this because, in what followed, it was claimed that determinism could be recovered, at least under certain conditions. If contemporary accounts [2] are to be believed, the immediate response from several eminent scientists was almost one of fury. Apparently Pauli was one who was particularly annoyed, partly because the article was published in an American journal and he was concerned in case it turned the American public against the quantum theory. However, it is claimed that, in Copenhagen, Bohr was in a state of shock as well as anger and reacted by abandoning all projects on which he was working to address the problems initiated by the Einstein, Podolsky, Rosen paper. Bohr eventually devoted three months of intense work to construct a rebuttal which he published in the same journal that Einstein, Podolsky and Rosen had used – Physical Review. At the time it seemed this might be the end of the affair with scientists either convinced by Bohr's reply or feeling the whole issue to be philosophical rather than physical; it seemed experimental results were not in question and so the whole issue could be quietly forgotten.

However, precisely what did the Einstein, Podolsky, Rosen article say about physics and physical systems? It claimed that a property of a physical system is an element of physical reality if it can be predicted accurately without disturbing the system. The example cited consisted of two particles that are linked together and it was used to show that the position and momentum of a given particle may be found by taking suitable measurements of the second particle without the first being disturbed. Thus, it was claimed that both attributes of the first particle are elements of physical reality. However, quantum mechanics does not allow this and so the contention was that it could not be a complete theory. In fact, it was declared in the article that

> "if without in any way disturbing a system we can predict with certainty (*i.e.* with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. It seems to us that this criterion, while far from exhausting all possible ways of recognising a physical reality, at least provides us with one such way, whenever the conditions set down in it occur."

This quote has been used on numerous occasions since it first appeared but is, nevertheless, well worth repeating here in order to provide a more complete, if brief, background to the whole set of questions surrounding this important paper by one of the last century's most influential scientists. It is also worth reiterating what followed in the original article.

Actually the authors considered two particles, 1 and 2, with respective momentum and position coordinates (p_1, q_1) and (p_2, q_2) in a state with definite total momentum $P = p_1 + p_2$ and definite relative distance $Q = q_1 + q_2$. Since P and Q commute, this is possible. The particles are allowed to interact before observations are made on particle 1 long after such interaction has ceased. If p_1 is measured, it follows that p_2 is known without particle 2 having been disturbed. An immediate consequence of this is that, in the language of the original article, p_2 is an element of reality. If q_1 is then measured, q_2 is known without particle 2 being disturbed in any way and so q_2 is also an element of reality. Hence, the conclusion of the thought experiment is that both p_2 and q_2 are elements of reality. However, according to quantum mechanics both cannot be elements of reality simultaneously. The conclusion which followed, therefore, was that quantum mechanics was not complete.

Following the above quote, the authors spent time describing entangled states and their argument fundamentally reduced to a description of quantum entanglement for position and momentum before concluding by saying:

> "Thus, by measuring either A or B, we are in a position to predict with certainty, and without in any way disturbing the second system, either the value of the quantity P or the value of the quantity Q. In accordance with our criterion for reality, in the first case we must consider the quantity P as being an element of reality, in the second case the quantity Q is an element of reality. But as we have seen, both wave functions belong to the same reality. Previously we proved that either (1) the quantum-mechanical description of reality given by the wave-function is not complete, or (2) when the operators corresponding to the two physical quantities do not commute the two quantities cannot have simultaneous reality…. We are thus forced to conclude that the quantum-mechanical description of physical reality given

by wave functions is not complete."

Here, in the quote, a slightly different notation was used from that above but this does not detract from the argument presented but the earlier notation enables in some ways a clearer linking up with what follows shortly when the later work of David Bohm is considered.

In all this, Einstein, Podolsky and Rosen made an assumption of locality, something which appears entirely reasonable; what happens in one place doesn't immediately affect what happens in another. However, the conclusion of the Einstein, Podolsky, Rosen argument definitely appears to contradict conventional quantum mechanics which asserts that the pair of particles involved in the thought experiment is described by a wave-function that only yields probabilistic predictions and simply cannot yield exact values of either the position or momentum of either particle. It also appears to contradict another quantum mechanical belief that the position and momentum of a particle may not have definite values ascribed to them simultaneously as that would violate the Heisenberg uncertainty principle. In his rebuttal, Bohr claimed he felt the trend of the Einstein, Podolsky, Rosen argument didn't appear to meet the actual situation faced in atomic physics adequately. He argued that the paradox didn't present a practical challenge to the application of quantum mechanics to real physical problems and, at the time, most physicists seemed to accept this. It might be noted, though, that this 1935 paper contains neither a paradox nor any logical flaw; rather it concludes that objective reality is incompatible with quantum mechanics being complete. It is perhaps worth remembering that Bohr harboured a deep concern over the use of language in the interpretation of quantum mechanics. For example, he advocated *the use of the word* "*phenomenon*" *to refer exclusively to observations obtained under specified circumstances*, *including an account of the whole experiment.* [3] Einstein, however, believed that a deeper theoretical framework which allowed the description of phenomena independently of these conditions should be sought and this is what he meant by the term objective reality.

Of course, it must be remembered always that the Einstein, Podolsky, Rosen argument is based on a thought experiment and thought experiments are just that thought experiments. As such they are very difficult to interpret due to the assumptions made not always being totally clear, possibly not even to the originators themselves. In fact, in a purely thought experiment, it is easy to imagine a situation where a fundamental assumption is made with no-one realising that has occurred. Remember that all indulge in thought experiments some even when they are asleep but their true validity, or otherwise, only becomes apparent when contemplation has ceased and the actual thoughts committed to paper and resulting concrete scrutiny. Supposedly, the essence of a good practical experiment is that it should be readily repeatable. It is relatively easy to see how this could be true, but could equally well be untrue, of any thought experiment. Hence, important results derived via thought experiments should always be treated with extreme care. In fact, the above mentioned objection of Bohr to the Einstein, Podolsky, Rosen problem might even seem to indicate he felt this thought experiment had little to do with genuine physical problems. Nevertheless, thought experiments provide an extremely useful tool for practical science and as far as the thought experiment leading to the Einstein, Podolsky, Rosen argument is concerned, it is one which has been viewed and examined now over a large number of years and, seemingly, has always led to a genuine problem in physics.

However, while many scientists might have seen Bohr's rebuttal as signalling the close of the argument, some twenty years later another eminent physicist entered the arena by making a genuine breakthrough in the understanding of the issues posed by the Einstein, Podolsky, Rosen article. This was David Bohm who, in 1951 [4], changed the entire setting of the original challenge to quantum theory in a way which made those issues clearer and easier to understand. In Bohm's thought experiment, although two particles were still involved, instead of considering position and momentum, he simplified matters by considering only one variable of physical interest and that was spin, although some might feel this case more complicated in that it involves spin with its less well-known features rather than the seemingly better understood ideas of position and momentum As with the original Einstein, Podolsky, Rosen thought experiment, the two particles were supposed localised at a distance from each other so that spin measurements are totally separated from one another in both space and time, meaning that one cannot influence the other. However, the two particles are entangled so that once the spin on one is found to be "up", for example, the other one must be "down", and this would be the case for all directions. Quantum mechanics, though, claims this not to be the case; the spin in different directions does not have simultaneous reality. Hence, another thought experiment giving results which raise serious questions about quantum theory. It is not without interest to recall the article by Bohm and Aharonov in 1957 [5]. In this article, after a brief review of the original Einstein, Podolsky, Rosen work, it is shown that it involves a kind of correlation of the properties of distant non-interacting systems, which is quite different from previously known kinds of correlation. They then continue by hypothesising a situation which would still be consistent with all experimental data available at the time but would avoid the paradox. Crucially they then draw attention to an experiment already performed, the results of which may be interpreted as providing the "first clear empirical proof that the aspects of the quantum theory discussed by Einstein, Podolsky and Rosen do represent real properties of matter" [5]. The experiment to which reference was made is that by Wu and Shaknov [6] and which they noted exhibited the spin correlations of Bohm's version of the Einstein, Podolsky, Rosen paradox. They pointed out that the paper argued against the idea that particles are not entangled or that quantum entanglement of particles might dissipate with distance. This view has not been refuted as yet the entanglement of particles is real and doesn't dissipate with distance.

The next big step in the ongoing discussion of the important issues raised by Einstein, Podolsky and Rosen was provided by the theoretical work of J. S. Bell in the 1960's. However, one big impetus for that work was provided by comments attributed to Bohm and Aharonov in their 1957 article [5]. In that paper they claimed that a delayed choice would be necessary if experimenters were to determine whether or not the so-called Einstein-Podolsky-Rosen particles behaved in the way Einstein and his two colleagues found unacceptable; or, in other words, experimenters would have to choose which spin direction to measure *only after* the said particles were in flight. This, they pointed out, would ensure that neither one particle, nor the experimental apparatus used, could send a signal to the other particle.

It was this requirement which was later brought to the forefront of everyone's attention by J. S. Bell in his ground breaking contributions to the debate. As has been pointed out previously [2], Bell wrote two vitally important papers in this particular area of physics. Somewhat ironically, the first of these papers appeared after the second but both appeared in the mid-1960's, some thirty years after the original Einstein, Podolsky, Rosen article was published. In the first of these [7], entitled "On the problem of hidden variables in quantum theory", he tackled the problem of an error in earlier work by von Neumann. Basically, von Neumann had assumed that the expected value of the sum of a number of observable quantities equalled the sum of the expected values of the separate observable quantities. Bell was concerned by this seemingly reasonable assumption in that he knew it was physically indefensible if the observable quantities are replaced by operators which do not commute. In his eventual article, Bell proved that von Neumann's assumption was not appropriate and so all results deriving from it were, at least, questionable. This, of course, meant that the argument concerning the existence of hidden variables in quantum theory was resurrected. It seems almost natural now to feel that, although Bell already knew of the 1935 Einstein, Podolsky, Rosen article, he should turn his attention more critically to its contents [8]. Bell already knew of the various arguments used against this paper but he—possibly alone—recognised something missed by everyone else—Einstein and his two colleagues were correct or, at the very least, had come up with an important truth. Again as has been pointed out previously [2], this wasn't a claim that quantum mechanics was incomplete but that quantum mechanics and Einstein's insistence on locality and realism were incompatible with one another. Bell's result was expressed in mathematical terms which contained inequalities. He suggested that if these inequalities could be seen to be violated as a result of experiments then that would provide evidence in support of quantum mechanics but, if the inequalities were preserved, then that would provide evidence in favour of the thesis put forward by Einstein and his co-workers. This powerful paper quite naturally provoked several highly distinguished experimental physicists to design experiments to test the inequalities. The results of all the experiments conducted over a period of many years favoured the validity of quantum theory over Einstein's belief in realism and locality.

Bell had set out to use mathematics to show that Einstein's ideas concerning causality and locality are incompatible with the statistical predictions of quantum mechanics. He also indicated that one of the big problems was created by the locality requirement because that requirement means that the results of measurement on one system may not be affected by operations on another system with which it has interacted in the past. Bell's result showed that either the hidden variable ideas were correct or quantum mechanics was, but not both! Of course, it then followed that if quantum mechanics was correct then non-locality must be an actual feature of the world—even if that be only on a microscopic scale. However, it might be worth noting the actual words Bell used in his introduction to this paper;

> "The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causalityand locality. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics."

Considering some of the introductory comments, it is possibly not without interest to note that Bell refers specifically here to the *statistical predictions of quantum mechanics*. This very mention of the word "statistical" might to some be an admission of some lack of completeness in the theory under discussion; that is, the incompleteness of quantum mechanics. Also, the stressing of the word "statistical" here leads conveniently to the consideration of another approach to the whole problem of the completeness or not of quantum theory.

3. Work of Kurt Gödel and Lazar Mayants

In any discussion of completeness, possibly the first work to spring to mind is that of Kurt Gödel who, in 1931 [9], published results which have, ever since, been regarded as of great importance in both mathematical logic and the philosophy of mathematics. The results are summed up in two incompleteness theorems and are concerned with the limits of provability in formal axiomatic theories. The first theorem proves that, in any consistent formal system within which a certain amount of arithmetic may be performed, there will be statements of the language of that system which may neither be proved nor disproved within the system. The second incompleteness theorem then proceeds to show that such a formal system cannot prove its own consistency. On the face of it, there is nothing here to link directly with the issues raised by Einstein, Podolsky and Rosen but the talk of completeness raises questions which might be in need of consideration. Some have wondered, since physics is so dependent on mathematics, if a lack of completeness in mathematics—as seemingly shown by

Gödel indicates lack of completeness in physics. As far as quantum theory is concerned, it might be wondered quite reasonably if quantum theory is the system and the manipulations carried out constitute the mathematics, in which case Gödel's theorems might be felt to apply, but this is mere speculation. Nevertheless, it seems this is an approach which might reasonably be born in mind by any investigating the cited work of Einstein, Podolsky and Rosen.

In two books published in the 1980's and 1990's [10] [11], Mayants introduced a novel new way of examining several outstanding issues of the then current physical thinking. It is interesting to note that, in the Foreward to the first of these books, Professor Henry Margenau wrote

> "In this book he presents a unique, extremely detailed, and embracive version of a subject that has suffered for a long time from numerous internal imperfections. His approach is new and original, the material covered features not only the foundations of the science of probability but also most of its applications, including statistical and quantum mechanics. The key methodological principle underlying the book is of extraordinary significance and deserves special attention."

Considering the origin, this testimony in itself surely seems to suggest taking the contents very seriously and listening carefully to any conclusions deduced using this "new and original" approach. It is also of interest to note a quote by the author in his Preface from the 1959 Russian edition of Dirac's book, *The Principles of Quantum Mechanics*. Dirac says

> "Our notions about the principles of the physical world are changing in the course of time by consecutive stages. We are now at a certain stage but there is no reason to think that this stage is the last one. We can anticipate further changes in the future, which will perhaps be as drastic as in the past."

With these thoughts of Dirac following from the strong recommendation of Margenau in mind, it seems obvious to continue further to explore and consider the basic ideas of Mayants and how they may be utilised to shed further light on the problems existing in physics after the publication of the Einstein, Podolsky, Rosen article as well as those which have emerged since as a result of perusals of that paper.

The first of Mayants' books listed above lays out his approach in great detail and should really be read from cover to cover to gain a full appreciation of his approach and what he is really attempting to do. However, probably two basic concepts are at the heart of all he does; these are the concepts of concrete and abstract objects. In fact, he takes the first of these as the actual starting point for his entire presentation and it is regarded as a primary concept which may not be reduced to any simpler concepts. He perceives concrete objects in the usual, one might say trivial, sense as being quite simply definite, actually existing objects. It is noted also from the very start that such objects are those which are the subject of any experimental study. While the concept of a concrete object is relatively straightforward and agrees with our immediate linguistic interpretation of such a named object, that of an abstract object is less so and, as Mayants himself does in his book, is probably best introduced and understood via examples.

For example, to adapt a situation used by Mayants himself, consider an actual room, which would obviously be a concrete object, and suppose it to contain a number of people, which would again be concrete entities, who differ in their sex and age. Hence, this total number of people may be divided into four groups if by age is meant that an individual is either old or young. Then, to quote Mayants, "a young woman in this room" is simply the image of all the concrete persons belonging to the group of young women and since discussing it may only be achieved by ignoring any differences between the concrete members of this group, it may be termed "an abstract object". Again, the notion of "a man in this group" results from ignoring features, including age, of the concrete members of the group and so must be an abstract object corresponding to the concrete persons. Finally, the idea of "a person in this group" results from discounting all the differences between the concrete persons assembled in the room and is, therefore, an abstract object corresponding to all these persons.

Mayants then continues to discuss several other actual examples to further illustrate his thesis before utilising ideas from set theory to come up with a suitable mathematical definition of this notion of an abstract object. Here though the above example will be deemed sufficient as a good basis for the idea which will be used later. Possibly the crucial point about concrete and abstract objects is that, in any experiment, the scientist involved is a concrete scientist who conducts a concrete experiment on concrete objects with the aid of concrete devices and obtains concrete results.

Having noted this seemingly trivial point about concrete and abstract objects, it may be realised that general laws which hold for each concrete object may be felt of as laws valid for the corresponding abstract object also. This notion lies at the basis of the various scientific theoretical disciplines which, in truth, deal with abstract objects. Of course, the correctness, or otherwise, of any conclusions drawn from developments in any such theory must necessarily be checked by experiments involving the corresponding concrete objects. Implicitly, this possibly obvious fact is taken for granted or, probably in most cases, goes unnoticed and is tacitly ignored. In fact, the necessity for recognising the existence of and difference between concrete and abstract objects is rarely noted at all. This final point follows because, in most cases, confusion of a concrete object with an abstract one rarely causes any confusion or misunderstanding. However, it is Mayants' contention that, in some situations, this confusion is totally inadmissible since it can, and does, lead to paradoxes and erroneous conclusions.

The conclusion to follow from the above is that it must be very clearly understood whether concrete or abstract objects are meant in any situation under investigation. If this is not done, a degree of confusion could easily result. As an example, consider (as Mayants did) a very simple question such as "How long is an elephant's trunk?" As it stands this question is indeed totally meaningless. This is due to the fact that "an elephant" is an abstract object which has a trunk since *any* concrete elephant has a trunk but the trunk of the abstract elephant has no definite length since concrete elephants have trunks of different lengths. The question posed only possesses real meaning when it refers to a specific concrete elephant. As with most of the examples cited by Mayants, this one is very simple but well serves to illustrate this seemingly trivial point concerning concrete and abstract objects and their crucial differences as well as their important links.

Before looking at any paradoxes, after this early stressing of the notions of concrete and abstract objects and their respective properties, Mayants proceeds first to give a detailed discussion of what he terms "probabilistics" or, in other words, he talks about basic ideas in both probability and statistics. In this section, though, he also discusses the Lagrange and Hamilton equations so familiar from classical mechanics. Considering what is to follow in a later section of this article as well as the fact that he has dealt at length with the seemingly trivial point about concrete and abstract objects, it is possibly surprising that he restricts his discussion of these two sets of equations to the case of conservative fields and, therefore, to the situation where a potential energy enters the discussion. However, from the point of view of Mayants' text, this proves a totally acceptable course of action to pursue, although it does quite obviously introduce a restriction on the possible use of at least some of the results which follow. Following on from this section, he proceeds to give a careful outline of the fundamentals of probabilistic physics. This includes a resumé of results in classical statistical mechanics and quantum mechanics. The book concludes with a final section devoted to methodological problems and it is here that he applies his mode of thinking, including the crucial notions of concrete and abstract objects, to some of what are regarded as paradoxes in physics – including the Einstein, Podolsky, Rosen problem which, as has been seen already, should not really be seen as being a paradox. Although this book contains so much more of interest and should really be read in its entirety, the basic notions of concrete and abstract objects, which are at the heart of his discussion of the Einstein, Podolsky, Rosen problem and, indeed, of issues raised by Bell's work, have been highlighted sufficiently clearly for progress to be made in addressing these latter two pieces of work from Mayants' point of view.

Put in a nutshell, Mayants feels there is nothing wrong with the reasoning in the Einstein, Podolsky, Rosen paper but the conclusion seems to contradict conventional quantum mechanics which asserts the pair of particles involved is described by a wave-function which gives only probabilistic predictions and simply cannot yield exact values for either position or momentum. Again, it seems to contradict the quantum mechanical understanding that the position and momentum of a particle may have precise values simultaneously. The important word here in Mayants reasoning is "seems". His assertion is that there is simply no contradiction – only a confusion between concrete and abstract particles. As he points out, the Einstein, Podolsky, Rosen thought experiment is, like all experiments, just that – an experiment. As such, it must involve a procedure which is carried out using concrete particles. However, quantum mechanics involved probabilities which, in turn, refer to abstract particles. To Mayants, this alone removes any apparent contradiction between the conclusions of the Einstein, Podolsky, Rosen thought experiment and conventional quantum mechanics. To him it reduces to a case of like not being compared with like. This, it must be admitted, is an argument that may well satisfy some but, equally, will certainly not convince many others. Nevertheless, it is a line of reasoning of which all interested in this ongoing controversy should be aware.

Having "disposed" of the immediate so-called problem posed by the Einstein, Podolsky, Rosen thought experiment, Mayants turned his attention to what he seems to have regarded as the most paradoxical inference it revealed. As far as conventional quantum theory is concerned, the actual state of either particle in the pair depends on what physical quantity relating to its partner has been measured and this is not dependent on the distance separating the two particles. If this is so, it should mean that a particle should realise immediately if a measurement has been made on the second particle, regardless of the distance separating them. The particle should realise the result of such a measurement as soon as it is made. However, there may be no interaction between the particles and it is accepted that no signal may pass between them with a speed in excess of that of light. This has provoked thoughts of "action at a distance". Of course, it must always be wondered precisely what is meant here by the "speed of light" since that speed is well-known not to be a constant but something which depends on the refractive index of the medium through which the light is passing.

Mayants continues by noting that quantum theory has nothing to do with measurement in that it is concerned with abstract systems which, in a sense, do not exist but measurement, being an experimental procedure, may only be made on concrete systems. As for the "action at a distance" puzzle, that is really no puzzle at all. After all, a measurement of any quantity on a concrete particle simply reveals the actual value that quantity has. But, since in the Einstein, Podolsky, Rosen case the sum of the moment a of the two particles is fixed, once that of one particle is measured, that of the other follows immediately from the law of conservation of momentum – the distance between the two particles is simply not a factor in the discussion.

In the final analysis though, the Einstein, Podolsky, Rosen thought experiment is just that and, as such, might not be realisable in practice or might require further technical developments before it can be fully realisable practically. Hence, it is conceivable that, at least at some points in time, no inferences deemed controversial are, in fact, directly verifiable. As a result, other experiments, which are realisable in principle, have been proposed. Some of these are a result of

Bohm's modification of the Einstein, Podolsky, Rosen situations and/or are related to Bell's work. These have been regarded as versions of the Einstein, Podolsky, Rosen experiment but, in reality, they are not for the following reasons.

Mayants' view is that there are really two types of science: that which requires no probabilistic input and probability related science. In the first of these, any theoretical laws applying to an abstract object are valid for any corresponding concrete object at the same time and so the theory may be verified for of these concrete objects. Classical mechanics provides an excellent example of such a case where the mechanical behaviour of each mechanical system may be predicted with certainty.

On the other hand, theoretical laws applying to abstract objects in a probability related science are of a probabilistic nature and may not be verified on a separate concrete object; their verification relies on performing an appropriate statistical experiment on a large number of concrete objects and collecting statistical data. It is seen that definite values of properties of a particular concrete object may not be predicted in principle in probability related sciences. This is true for any probability related area of science including quantum physics.

Mayants' next points out that the Einstein, Podolsky, Rosen thought experiment involves concrete mechanical systems but the concrete systems considered in most of the experiments conducted to test this thought experiment belong to probability related sciences. In his view, the apparent paradoxes in experiments related to Bell's work are really linked with the incorrect identification as special versions of the original thought experiment to be tested. He asserts that a qualitative understanding of the issues involved may be gained simply by considering the strict distinction between concrete and abstract objects.

The next focus of Mayants' analysis is the situation discussed by Bohm. Although many felt Bohm's suggestion helped clarify the issues involved, Mayants indicates the situation considered by Bohm more complicated than that of Einstein, Podolsky and Rosen. This is because the original proposal dealt with well-known quantities, momentum and position, whereas the Bohm suggestion involved spin with its less well-known features. Again though, Mayants' focus remains with the distinction between concrete and abstract objects. As he stresses, whenever measurement is discussed, it is concrete objects which are involved but only abstract objects are involved whenever quantum mechanical matters are under discussion. Hence, in the case of Bohm's suggested experiment, the spin of one concrete particle along some direction, if the spin of its counterpart along the same direction is known, is quite simply determined by the law of conservation of spin. However, this raises the question of whether, or not, it is possible to verify this directly by experiment. The crucial point here is that both measurements must be made on one and the same pair of concrete particles. In any experimental situation it would be vital to avoid confusing particles belonging to different pairs. This would imply that both measurements would need to be made in one random test or, in other words, practically simultaneously. It might be remembered here that a concrete particle has only one spin component which is related to the axis chosen at the time.

The real experiments performed as a result of Bohm's suggestion and Bell's work yielded results which confirmed the belief that two spatially very remote particles belonging to one pair and whose initial total spin was zero have spins of opposite values, regardless of the direction along which they have been measured. Again the question of how one particle could know the result for the other raised its head. Yet again though the answer lies with the fact that a simple conservation law was involved – the law of spin conservation. Later similar real experiments were performed by Aspect [12] involving polarization of photons instead of spin. These led to similar results but again Mayants points out that, in this case, it is just the law of conservation of polarization which is involved.

By introducing a clear distinction between concrete and abstract objects and by drawing on the validity of the various conservation laws, Mayants appears to have cleared up the problems associated for so many years with the Einstein, Podolsky, Rosen thought experiment. But have they? It is undoubtedly the case that many exist who haven't heard of the work and ideas of Lazar Mayants but it is also undoubtedly true that, to many, the issues raised by the famous Einstein, Podolsky, Rosen paper still exist. It is also surprising to some that the 1998 paper by Santilli [13], which offered a different clarification of the whole saga, remains relatively unknown. However, more of that paper and the following work when more recent experimental work has been discussed.

One final word in relation to the ideas of Mayants seems appropriate as, as will be seen later, it is also highly relevant to the work and ideas of Santilli. In his earlier book [10] Mayants, as has been noted already, quoted a passage from Dirac's book on quantum mechanics. He did, though, use a second quote which is, in fact, the final paragraph of the 1958 English edition of Dirac's monograph, again entitled *The Principles of Quantum Mechanics*, and it is this which seems apt to repeat at this juncture. Dirac said:

> It would seem that we have followed as far as possible the path of logical development of the ideas of quantum mechanics as they are at present understood. The difficulties, being of a profound character, can be removed only by some drastic change in the foundations of the theory, probably a change as drastic as the passage from Bohr's orbit theory to the present quantum mechanics.

This powerful statement from such an eminent theoretical physicist surely deserves careful contemplation.

4. Modern Experimental Developments

As will be discussed more later, a conference took place via Zoom in September 2020 at which many issues relating to the problems raised by the Einstein, Podolsky, Rosen article were discussed. All the talks may be heard at either http://www.world-lecture-series.org/level-xii-epr-teleconference-2020 or https://www.youtube.com/channel/UCwHACC6p2QTrmdbSIz-mPuw/videos.

One talk was given by Gerald Eigen and it concerned an experiment concerned with measurement of the polarization correlation of the two-photon system produced in proton-electron annihilation. The talk may be heard in its entirety on either of the above sites but should be appearing in print in the conference proceedings before too long. This experiment shows, seemingly quite definitely, that the validity of Bell's inequality is confirmed and, therefore, so is the validity of quantum mechanics. However, as the author himself agrees at the conference, this experiment actually confirms the validity of quantum mechanics for point particles under electromagnetic interactions. This restriction must be noted since it implies that this experiment deals with the verification if the basic axioms of the Copenhagen interpretation of quantum mechanics hold – namely it deals with point-like particles under potential interactions.

Recently, the matter has resurfaced with the announcement of experimental results supporting the Einstein, Podolsky, Rosen assertions at Basel [14]. This has provoked further contemplation of this whole issue of completeness and just what it really means. The Basel team noted that the phenomenon dated back to a thought experiment of 1935 and that it allowed measurement results to be predicted precisely but, of course, as mentioned earlier, it must be remembered always that thought experiments are just that thought experiments and such are very difficult to interpret due to the assumptions made not always being totally clear, possibly not even to the originators themselves. However, again as mentioned earlier, via a thought experiment, Einstein, Podolsky and Rosen showed that precise predictions are possible theoretically in certain circumstances. Briefly, such a notion may be explained as follows: two quantum mechanical degrees of freedom are said to be entangled, or non-separable, if the quantum state of one may not be described independently of the other. Hence, if measurements are performed on both, entanglement means that correlations between the outcomes are an inevitable result. Einstein, Podolsky and Rosen pointed out that, for sufficiently strong correlations, local measurements in one region could apparently change the quantum state in a spatially separated region. This was dubbed by Schrodinger [15] as "steering". Put another way, steering results in measurements on one system being used to predict the results of corresponding measurements on the second system with arbitrary precision in principle. Again, to make the position absolutely clear, it was also the case that the two systems could be separated spatially. The resulting paradox is that an observer may use measurements on the first system to make precise statements about the second system, in fact more precise than an observer who has direct access to that second system but not the first.

The Basel team used lasers to cool atoms to a small fraction of a degree above the absolute zero of temperature. At such low temperatures, the atoms are thought to behave completely according to the rules of quantum mechanics and form a Bose-Einstein condensate. In this ultra-cold cloud, the atoms collide with one another constantly, causing their spins to become entangled. The researchers involved then took measurements of the spin in spatially separated regions of the condensate. By using high-resolution imaging, they were able to measure the spin correlations between the separate regions directly and simultaneously localise the atoms in precisely defined positions. Hence, in this experiment, the researchers seem to have succeeded in using measurements in a given region to predict precisely the results for another region.

However, while this work appeared covered openly in the popular press and so became known to a wide variety of people, it is of further interest to note that the authors themselves raised more awareness of other similar experimental work in this field. They noted that, as they put it, complimentary to our work, spatially distributed multipartite entanglement has been observed [16], as has entanglement of spatially separated modes [17].

Hence, it is seen that work is afoot in several establishments which is aimed at investigating the interesting phenomenon of "entanglement". In the past, experiments have concentrated on using light or individual atoms to study the predictions of Einstein, Podolsky and Rosen but the Basel group has successfully used many particle systems consisting of reasonably large numbers of particles to observe the behaviour predicted by Einstein, Podolsky and Rosen in 1935. In fact, as commented on in *Science*, volume 360, in the three articles quoted here, so-called distributed entanglement was achieved in a very challenging setting involving a large number of cold particles. A quick perusal of the articles shows that, in all three, the entanglement was created initially in an atomic cloud which was allowed to expand. Local measurements in different parts of the cloud which were separated spatially showed that the entanglement survived the expansion.

At the time of writing, it is not known if any serious objections to this work have arisen. If such have emerged, the arguments surrounding the issues raised in the Einstein, Podolsky, Rosen article will, no doubt, rumble on. If none has, or does, emerge then it is conceivable that a new era for physics might be opening up since it is surely the case that applications will follow which we all hope will be of benefit to mankind rather than the opposite. The Basel group itself is seemingly already speculating on possible applications of this purely academic research with which they've been involved. One speculation is that the method they've used might be used for entanglement-enhanced imaging of electromagnetic field distributions and also in quantum information tasks, but that is pure speculation at this stage.

One crucial final point needs to be clarified at this juncture and that is concerned with the apparent divergence of result from these two experiments. However, the two do not, in a very real sense, contradict one another. The Basel experiment deals with what might be termed extended particles while that of Eigen is concerned with point particles that, as such, may only sense potential interactions. Hence, the Basel experiment is not in opposition with this work by

Eigen since it confirms the need for a completion of quantum mechanics for extended particles

The issues raised in this section have been concerned solely with experimentation linked closely with the problems and questions still surrounding that shortish paper of 1935. There was no mention of any further theoretical thoughts on the subject. However, as was mentioned earlier, it is possibly surprising that little has been heard of the 1998 article by Santilli [13] and it is the lack of knowledge of this work which poses a significant question for the scientific community, although, when you read even just the abstract for that paper, maybe some answers become apparent. With talk of such concepts as nonlinear, nonlocal, non-canonical, axiom-preserving isotopies and spin-isospin symmetry and iso-spaces, some will be put off by the implied effort to understand properly what follows in the body of the paper, while others will dismiss the work out-of-hand because it depends crucially on concepts unfamiliar to them. This may be an improper attitude towards proposed new science but many will have forged impressive curricula vitae based on what they regard as well-established concepts and procedures and will be reluctant to jeopardise their personal positions. However, this attitude is completely understandable and these comments should, in no way, be seen as criticism of anyone or any point of view related to this general matter. It does, though, imply a hugely important question for the scientific community when do we agree to examine with a truly open mind, radical new proposals for help in solving age-old problems? It seems there was no difficulty in examining and accepting a wide range of results from Riemannian geometry, as well as the uncertainties introduced by quantum mechanics, into physics and chemistry some one hundred years ago, so why not afford the same respect to hadronic mechanics or are the fundamental results of quantum mechanics to remain sacrosanct even when they don't answer all the important questions facing the scientific community?

These are vitally important questions in general but are particularly apposite when considering the so-called Einstein, Podolsky, Rosen paradox and work related to it. Basically, the Einstein, Podolsky, Rosen claims that quantum mechanics is an incomplete theory because its description of physical reality does not include all elements of reality, while every element of physical reality should be precisely represented in a complete theory. Santilli's new approach has important consequences as far as this argument is concerned. Traditionally, commuting quantities are believed to be independent but, in the so-called iso-topic completion of quantum mechanics, iso-commuting quantities can be mutually interacting, although it should be understood that such interactions are structurally different from those of action-at-a-distance/potential type. Fundamentally, quantum mechanics may be considered an incomplete theory in that it does not contain the element of reality given by the nonlocal structure of interactions expected from the mutual wave overlapping. Hadronic mechanics overcomes this problem.

It is important to realise though that, as Santilli himself points out, hadronic mechanics is not intended to represent all elements of reality; it is not meant to be a final theory. Physics is, after all, a discipline which will never admit final theories. Hadronic mechanics simply provides one type of completion of quantum mechanics that of axiom preserving type. It might also be noted at this point that Santilli has also shown via his new mathematics that von Neumann's theorem on hidden variables is quite simply inapplicable under isotopies – note, not violated, but inapplicable! He has also established that the oft-quoted Bell's inequality is not valid universally but holds for the conventional form of quantum mechanics specifically.

5. Santilli's Hadronic Mechanics

In the preceding few paragraphs, mention has been made of the work and philosophy of Ruggero Santilli. Some unfamiliar terms have been used, some implications have been made. It is now appropriate to examine in detail both Santilli's work as well as the philosophy behind it and background to it. Hopefully, the scientific community will be in a position to both examine and judge openly and unambiguously.

To fully understand and appreciate Santilli's standpoint on the whole issue of completeness of quantum mechanics, it is probably necessary to go back to his starting point. This was when, as a postgraduate student at the University of Torino, he had the opportunity to study some of the original papers written by Lagrange. He immediately found that Lagrange's original notion for his analytical mechanics required the realisation that not all forces are derivable from a potential. He also studied and accepted the Einstein-Podolsky-Rosen argument due to the inability by quantum mechanics to represent energy-producing processes in physics and chemistry because of their irreversibility over time, while quantum mechanical axioms are reversible over time due to the invariance of the Lie brackets under anti-Hermiticity, $[A,B] = -[A,B]^{\dagger}$. During his Ph. D. studies, Santilli read Bohr's rejection of the Einstein-Podolsky-Rosen argument but could not accept it due to Bohr's silence on a number of elements of reality, already known in 1935, which support a suitable "completion" of quantum mechanics. These included the inability of quantum mechanics to incorporate the external terms added by Lagrange to his celebrated analytic equations to represent non-potential forces which are the origin of irreversible processes. Therefore, Santilli decided to dedicate part of his research life to the verification of the Einstein-Podolsky-Rosen argument,.

Hence, the most popular, favoured form of the famous Lagrange equations of analytical mechanics in use today is not the most general because it ignores all non-potential based forces. Incidentally, the derivations of this popular form of the equations also depend on the mass involved being a constant which must make applications in special relativity problematical. At a slightly later time, Santilli also took the opportunity to study some of the original writings of Hamilton on his celebrated ideas concerning analytical mechanics. He found that Hamilton had essentially the same view as Lagrange and his original formulations* of his mechanical methods also incorporated terms to allow for non-potential based forces. This realisation of unmentioned problems of generality in conventional mechanics, this to include quantum mechanics led Santilli to propose what was possibly his first important contribution in this field; this was the formulation and proof of the following [17]:-

> A macroscopic system with forces that are non-conservative and/or irreversible over time cannot be consistently decomposed into a finite number of elementary particles all with solely conservative forces derivable from a potential and, vice versa, a finite number of elementary particles all in conservative conditions cannot consistently yield, under the correspondence principle or other means, a macroscopic system with non-conservative and/or irreversible forces.

The importance of this result lies first in the fact that it acknowledges the potential existence and importance of non-conservative forces as well as indicating immediately that quantum mechanics, for example, cannot be deemed a universal theory valid for all possible conditions in nature or, in other words, quantum mechanics cannot be regarded as a totally complete theory. This statement does, of course, raise once again the actual meaning of the term "completeness" as applied in quantum mechanics and it might be felt it implies something even more general and far reaching than that implied in the celebrated Einstein, Podolsky, Rosen paper.

The validity of this theorem has serious implications for physical science. Possibly the most important is related to the fact that twentieth century science is based on Lie algebras with antisymmetric brackets $[A,B]=-[B,A]$ that appear in the time evolution of the physical quantity $Q(r, p)$; that is, in

$$\mathrm{d}Q/\mathrm{d}t=[Q,H]$$

where H is the Hamiltonian and the brackets are the usual Poisson brackets. Very early in his studies, Santilli noted that, when the external terms mentioned earlier are included, the correct equation is

$$\frac{\mathrm{d}Q}{\mathrm{d}t}=[Q,H]+\frac{\partial Q}{\partial p}F=(Q,H)$$

where F represents the external terms, those not derivable from a potential and (Q, H) a notation introduced by Santilli as an extension to the usual Poisson brackets. This seemingly small change does, though, create difficulties since this inclusion of the so-called external forces results in a violation of some of the conditions which are usually felt to characterise an algebra, specifically the inclusion of the external terms results in a violation for the new bracket (Q, H) to characterize an algebra as understood in mathematics (due to the violation of the right associative and scalar axioms).

The philosophy adopted by Santilli for the resolution of the impasse was to characterize the time evolution of irreversible systems via new brackets (Q, H) which:

1) verify all axioms to characterize an algebra as understood in mathematics;

2) the said algebra has to be a covering of Lie algebras as a necessary condition to allow a completion of quantum mechanics in the Einstein-Podolsky-Rosen sense;

3) the said brackets must violate the symmetry of the Lie brackets under anti-Hermiticity

$[A,B]=-[A,B]^{\dagger}$ as a necessary condition to represent irreversible systems.

Following an extensive search in European mathematical libraries, Santilli finally located the definition by the American mathematician A. A. Albert [18] of brackets (Q, H) as being Lie-admissible when the attached totally antisymmetric brackets $\{Q,H\}^{*}=(Q,H)-(H,Q)$ verify all the axioms of Lie algebras; and Jordan-admissible when the totally symmetric brackets

$\{Q,H\}^{*}=(Q,H)+(H,Q)$ verify all the axioms of Jordan algebras.

Santilli published his Ph. D. thesis in 1967 in two papers, the first [19] on the algebraic resolution of the indicated impasse with the introduction of the Lie-admissible and Jordan-admissible parametric brackets

$$(A,B)=aAB-bBA=n(AB-BA)+m(AB+BA),$$

where $a = n + m$ and $b = -m + n$ are non-null positive scalars. As is well known, from the late 1980s on there was the appearance of a very large number of papers on the simpler q-deformations of Lie algebras with brackets $(A,B)=AB-qBA$, none of which quoted their origination by Santilli [19] some twenty years earlier.

In the second paper of his Ph.D. thesis [20] Santilli introduced the parametric Lie-admissible completion of Hamilton's equations

$$\frac{\mathrm{d}r}{\mathrm{d}t}=a\frac{\partial H}{\partial p},\ \frac{\mathrm{d}p}{\partial t}=-b\frac{\mathrm{d}H}{\partial r}$$

and their operator counterpart

$$i\frac{\mathrm{d}Q}{\mathrm{d}t}=(Q,H)=aQH-bHQ$$

which provide an approximate representation of the external terms and related irreversible systems. Rather importantly from an historical as well as scientific viewpoint, Santilli showed in this second paper [20] that the totally symmetric component of the Lie-admissible brackets represents Lagrange's and Hamilton's external terms, thereby realizing Jordan's dream that his algebras would one day see physical application.

Following a decade of teaching positions in the U.S.A., Santilli resumed full time work on Lie-admissible formulations at Harvard University in 1977 and succeeded in achieving the following identical Lie-admissible reformulation of the brackets of the classical time evolution of irreversible systems [21] [22].

$$\frac{\mathrm{d}Q}{\mathrm{d}t}=\left[Q,H\right]+\frac{\partial Q}{\partial p}F=\frac{\partial Q}{\partial r}\frac{\partial H}{\partial p}-\frac{\partial H}{\partial r}S\frac{\partial Q}{\partial p}=\left(Q,H\right)$$

which represent all infinitely possible, non-singular external terms F and related irreversible systems for $S=1-\partial Q/\partial p\,F/\partial H/\partial p$. In the same monographs [21] [22], Santilli introduced the operator image of the above classical Lie-admissible time evolution in the infinitesimal form

$$i\frac{\mathrm{d}Q}{\mathrm{d}t}=\left(A,H\right)=QRH-HSQ=AQ\left\langle H-H\right\rangle Q$$

which represents all infinitely possible non-singular external terms F for $R = 1$, $S = 1 - F/H$, and related finite Lie-admissible group form

$$Q\left(t\right)=e^{\left(HRti\right)}Q\left(0\right)e^{\left(IitSH\right)}=U\left(t\right)Q\left(0\right)U\left(t\right)^{\dagger}$$

showing the clear non-unitary character of the theory $UU^{\dagger}\neq 1$. In the same works [21] [22] Santilli introduced the name of hadronic mechanics to denote a Lie-admissible completion of quantum mechanics for the representation of irreversible processes, while uniquely and unambiguously admitting quantum mechanics for $R = S = 1$ (see the subsequent works [23] [24] [25] recent work [26]and review [27]).

Following the achievement of maturity in the construction [23] [24] and verification [25] of hadronic mechanics, Santilli extended the results to the formulation of hadronic chemistry [28] (see also reviews [29] [30] for the study of the following limitations of quantum chemistry:

1) the lack of an exact representation of molecular data when derived from first principles, with deviations of the theory from experimental data on binding energies of the order of 2% which is a multiple of the thermal energy released in chemical reactions such as that in the formation of the water molecule;

2) the inability to permit accurate thermochemical calculations in energy producing chemical reactions, such as those for the combustion of fossil fuels, due to the reversible structure of quantum chemistry compared to the irreversibility of the indicated chemical reactions;

3) the evident prediction by the Schrödinger equation of quantum chemistry that the identical electrons of valence bonds should repel each other due to their same charge with a repulsive force not overcome by available quantum models of valence bonds, and consequential absence of a quantitative model of molecular structure compatible with experimental evidence;

4) the incorrect prediction that all molecules are paramagnetic from the lack of an attractive force between valence electron pairs;

5) it should be noted that the irreversible Lie-admissible branch of hadronic mechanics and chemistry stimulated considerable interest for the development of at compatible, irreversible statistical mechanics, evidently as an intermediate step toward the achievement of a direct link between mechanics and thermodynamics. Among numerous contributions are J. Fronteau *et al.* [31] Dunning-Davies [32], A. Bhalekar [33], and others. An important collection of historical

as well as more recent works on irreversible non-potential statistical mechanics has been provided by A. Schober [34].

Recall that both Lie algebras and twentieth century applied mathematics have an associative modular structure in the sense that the action to the right on a Hilbert state $H\psi = E\psi$ is equivalent to the action to the left $\psi H = \psi E'$, $E' = \text{E}$, thus admitting one single universal enveloping associative algebra. Lie-admissible mathematics, also called geno-mathematics, was conceived by Santilli [35] [36] [37] for the intent of reducing irreversibility to the following primitive bimodular axioms:

1) A modular action to the right $H > \psi = HS\psi = E\psi$ representing motion forward in time and modular action to the left $\psi < H = \psi RH = yE'$, representing motion backward in time, irreversibility being then assured whenever $R \neq S$ (or $E' \neq E$);

2) A forward geno-unit $I^{>} = 1/S$ and a backward geno-unit $I^{<} = 1/R$ with corresponding forward and backward geno-numbers [38] geno-spaces, geno-geometries and geno-differential calculus [39];

3) Bimodular Lie-admissible lifting of the various branches of Lie's theory, including enveloping associative algebras, Lie algebras and Lie transformation groups [23] [24]. To understand the significance of Santilli's Lie-admissible mathematics in physics, recall that non-unitary time evolutions are known to violate causality when formulated in the conventional Hilbert space over a conventional numeric field. A first significance of the Lie-admissible mathematics is to show that causality is fully verified by Lie-admissible time evolutions because they verify no unitarity conditions provided they are formulated on geno-spaces over geno-fields, called geno-unitarity.

Lie-admissible mathematics is nowadays classified into single-valued, multi-valued and hyper-valued [40] [41] [42] in the sense that multiplications and other operations produce one single result, or produce an ordered number of results, or verify Vougiouklis hyperstructural laws, respectively. Single-valued Lie-admissible mathematics is recommended for the representation of simple irreversible processes in physics [25] and chemistry [28]; multi-valued Lie-admissible mathematics is recommended for the representation of complex biological processes [40], such as the representation of the growth in time of seashells [43]; while hyper-valued Lie-admissible mathematics has allowed the initiation of a quantitative representation of living organisms [44] as a collection of an extremely large number of extended constituents in continuous entanglement verifying the Einstein-Podolsky-Rosen argument and, therefore, in continuous communications, resulting in an extremely large number of complex interconnections beyond human comprehension that can be solely representable via hyperstructures.

The above methods have been the basis for Santilli's studies of the Einstein, Podolsky, Rosen argument reviewed later and which have seen him offer possible solutions to some of the questions raised concerning the often unstated restrictions imposed nowadays by widely accepted theories of which quantum mechanics is one. Santilli has often pointed out the many positive contributions quantum mechanics has made to scientific knowledge while, at the same time, being concerned by doubts expressed by leading scientific figures of earlier years. A huge impression was left by Fermi [45] who stated that "*there are doubts as to whether the usual concepts of geometry hold for such small regions of space* (*those of nuclear forces*)". This is, by itself, an extremely powerful statement by one of the leading scientific figures of his age but is it well-known, do people pay it due attention? The answer to both those questions is probably "No". Santilli also alludes to a statement included in Blatt and Weisskopf's book *Theoretical Nuclear Physics* [46] in which they speculate on page 31 on the possibility "that the intrinsic magnetism of a nucleon is different when it is in close proximity to another nucleon". In fact, this statement acted as a major spur to Santilli who claims to have produced a complete theory of total nuclear magnetic moments via his so-called hadronic generalisation of quantum mechanics. Whether or not he has achieved this is for the scientific community as a whole to decide but, until his work is read with open minds and properly digested, no final verdict can be sensibly announced. This indicates, once again, the urgent need for a totally open-minded examination of Santilli's work Another major influence was Dirac's concerns as expressed in an earlier quote when discussing Mayants' views and contributions to the debate. He was further influenced by the views of various philosophers of science, especially Karl Popper, who ended up being a strong supporter of Santilli's proposal to construct a covering, or extension, of conventional quantum mechanics.

All these, and more, acted as stimuli for Santilli. However, when considering any problem, Santilli appears driven by an unshakeable belief in the idea that science, in general, doesn't admit complete and final theories, and could not progress without the introduction of some new mathematics. One immediate example illustrating this is provided by Newtonian mechanics, which had been so successful for so long, finding itself regarded as a special limiting case of relativistic mechanics towards the beginning of the last century. Also, Einstein's general theory of relativity brought to the fore in the world of physics new mathematical methods. This new mathematics involved tensors and was reliant on earlier purely mathematical work by such as Riemann, Ricci and Bianchi. Hence, the huge change in physics at the beginning of the twentieth century was accompanied by new mathematics being introduced and used in physics and a well-established theory clearly being seen to be approximate and not final. Accordingly, Santilli turned his attention to producing new mathematics in order to deal with these new problems. To do this, he turned to the work of Marius Sophus Lie for some of his inspiration. After much intellectual effort, Santilli proposed so-called hadronic mechanics which is basically an image of quantum mechanics formulated via several completely new forms of mathematics, termed by him iso-, geno-, and hyper-mathematics, with so-called isoduals for antimatter. The corresponding iso-, geno-, and hyper-mechanics are then found to represent single-valued reversible, single-valued irreversible, and multi-valued irreversible systems respectively. Fundamentally, hadronic mechanics preserves all the usual laws and principles of orthodox quantum mechanics but represents what might be termed a completion of that subject, as seemingly required by the well-known argument of Einstein, Podolsky and Rosen [1]. It is strongly suspected by many that Santilli's hadronic mechanics genuinely achieves this objective. In fact, in the introduction to one of his books [47], Popper went so far as to make the following assertion:

> "I should like to say that he (Santilli)—one who belongs to a new generation seems to me to move on a different path. Far be it from me to belittle the giants who founded quantum mechanics under the leadership of Planck, Einstein, Bohr, Born, Heisenberg, de Broglie, Schrödinger, and Dirac. Santilli too makes it very clear how greatly he appreciates the work of these men. But in his approach he distinguishes the region of the arena of incontrovertible applicability of quantum mechanics (he calls it atomic mechanics) from nuclear mechanics and hadronics, and his most fascinating arguments in support of the view that quantum mechanics should not, without new tests, be regarded as valid in nuclear and hadronic mechanics, seem to me to augur a return to sanity: to that realism and objectivism for which Einstein stood, and which had been abandoned by those two very great physicists, Heisenberg and Bohr".

However, the whole truth will be known only after the wider scientific community has examined the veritable mountain of material with an open mind. Incidentally, the names for these three new branches of mathematics/mechanics were constructed for the following reasons: firstly the "iso" prefix, being short for isotopic which comes from the Greek and is meant to indicate the property of axiom-preserving for the new theory; secondly, the "geno" prefix comes from genotopic which again follows from its Greek meaning which suggests an axiom-inducing property of that new theory; and finally, the term hyperstructural basically arose from ideas of multivalued functions. Further, iso-mechanics is fundamentally a non-unitary theory but is reversible; geno-mechanics preserves this property of non-unitarity but introduces ideas of irreversibility; hyper-mathematics goes even further and, while preserving non-unitarity and irreversibility, introduces multi-valuedness which increases the number of degrees of freedom open to the investigator and thus permits the study of far more complicated structures than was allowed previously. However, as far as the present discussion is concerned, only Santilli's aforementioned iso-mathematics is strictly relevant. To cover all aspects of even iso-mathematics and the related iso-mechanics in one article would be an impossible task. However, below an attempt will be made to give a flavour of the entire theory with special emphasis on those topics directly relevant to the specific issue under discussion here – the

Einstein, Podolsky, Rosen proposals from their 1935 article.

5.1. Santilli's Iso-Mathematics

From the very beginning Santilli recognised the wonderful success and consistency of traditional quantum mechanics but also noted clear limitations in its range of applicability such as the fact that not all of nature can be reduced to isolated points because of interactions due to wave-overlapping (at mutual distances smaller than the coherent wavelength of each pair) under which quantum mechanics is exact no longer. Therefore, he began a search for generalised methods which gave the traditional theory as a particular case when the mutual distances are such as to render all non-quantum mechanical effects ignorable. The so-called isotopies, which are maps (or liftings) of any given linear, local, unitary structure into the most general possible nonlinear, nonlocal, non-unitary forms but which reduce to the original form in special circumstances, were the result. The isotopies are axiom preserving because of the latter property. The basis for these generalised methods lies in a generalisation of the notion of number and, ultimately, this relies on a generalisation of the whole idea of the unit which, until these suggestions of Santilli, has always been +1. More generally, everything revolves around the generalisation of the usual basic $n \times n$ dimensional unit given by $I = \mathrm{diag}(1,1,1,\cdots)$ into an $n \times n$ matrix $\hat{I}$ which is well-behaved, non-singular and Hermitian but whose elements have an arbitrary dependence, which is both nonlinear and nonlocal in general, on all required quantities and their derivatives of arbitrary order, such as coordinates $r = (x, y, z, \cdots)$ and wave-functions $\psi(t, r)$ as well as time, local temperature, local density, etc. Hence, the starting point is the mapping

$$\boldsymbol{I} \to \hat{\boldsymbol{I}}$$

which is termed *isotopic lifting* or *lifting* for short. More detail for all of what follows may be found in the original papers [48] by Santilli as well as in monographs [23] [24] he wrote subsequently.

In order for his new theory to preserve the basic abstract axioms of conventional methods, Santilli introduced the additional lifting of the conventional associative product of two quantities A and B as follows:

$$A \times B = AB \to A \hat{\times} B = A \times \hat{T} \times B$$

where

$$\hat{I} = \hat{T}^{-1}.$$

Then, it is seen that $\hat{I}$ is the correct right and left unit of the new theory:

$$\hat{I} \hat{\times} A = \hat{T}^{-1} \times \hat{T} \times A = A = A \hat{\times} \hat{I} = A \times \hat{T} \times \hat{T}^{-1},$$

where $\hat{I}$ is called the iso-unit and $\hat{T}$ the isotopic element.

In those same original articles cited above, it was also shown that the above two maps lift an associative algebra ξ with the usual unit 1, elements, A, B, … and a conventional associative product $A \times B$ in an axiom preserving manner into a new algebra $\hat{\xi}$ with a new unit $\hat{I} = \hat{T}^{-1}$, and new product $A \hat{\times} B = A \times \hat{T} \times B$. Also, as with the original, the new product is associative:

$$\xi : A \times (B \times C) = (A \times B) \times C \to \hat{\xi} : A \hat{\times} (B \hat{\times} C) = (A \hat{\times} B) \hat{\times} C.$$

It was shown also [21] that the original and new algebras are locally isomorphic under the condition of positive definiteness $\hat{I} > 0$, but are anti-automorphic if $\hat{I} < 0$. Since the associative law continues to hold at the isotopic level, $\hat{\xi}$ is termed an iso-associative algebra.

When examined in detail, it is seen that, in conventional dynamics, the anti-symmetric part ξ^- attached to the associative algebra ξ with the familiar product $[A, B] = A \times B - B \times A$, which is characteristic of a Lie algebra and results in the time evolution $i\,\mathrm{d}A/\mathrm{d}t = [A, H]$, where H is the Hamiltonian, with an exponentiated form $A(t) = \exp(iHt) \times A(0) \times \exp(-iHt)$, constituting a one parameter Lie group.

The isotopies briefly discussed already allow a generalisation of this. The anti-symmetric algebra $\hat{\xi}^-$ attached to the iso-associative algebra $\hat{\xi}$ leads to $[A \hat{,} H] = A \hat{\times} B - B \hat{\times} A$ which was proved to preserve the Lie axioms at the isotopic level. The fundamental dynamical equations of the isotopic theory are then given by

$$i\,\mathrm{d}A/\mathrm{d}t = [A \hat{,} H] = A \hat{\times} H - H \hat{\times} A = A \times \hat{T} \times H - H \times \hat{T} \times A$$

with an exponentiated form

$$A(t) = \exp\left(iH \times \hat{T} \times t\right) \times A(0) \times \exp\left(-it \times \hat{T} \times H\right),$$

where $\hat{T}$ retains its earlier meaning.

The above constitutes an extremely brief outline of the structural elements of the isotopies of Lie's theory and also serves as a brief introduction to the methods introduced by Santilli in a veritable library of articles and books in an attempt to combat at least some of the inadequacies he observed—as had others before and after him in modern science. The latter equations above are highly nonlinear, nonlocal and non-Hamiltonian but, in isotopic spaces have been shown to retain the conditions of linearity, locality and canonicity. All this is based purely on a generalisation of the basic unit of conventional methods. However, for consistency, it proves necessary for *all* conventional mathematical methods to be lifted (to use Santilli's own terminology). This would include lifting numbers, fields, angles, differential calculus, trigonometric and hyperbolic functions, special functions and transforms, vector, metric and Hilbert spaces, algebras, geometries, mechanics, etc, This has obviously been an enormous task which has been accomplished by Santilli over a lifetime of endeavour and obviously only a brief resumé can be included here. Nevertheless, an attempt will be made to cover all necessary aspects of this mammoth work by liberal use of references for any interested in the minute detail of required manipulations so that more will be able to follow the reasoning behind his resolution of problems posed by the Einstein, Podolsky, Rosen article.

The above discussion hopefully gives a brief outline of the sort of terminology and manipulations that are required in the new mathematics underlying Santilli's approach to so many issues facing modern day physics; an approach based totally on the belief that further progress will only be made if new mathematics is introduced. This conviction was based, no doubt, on the way in which people like Newton and Einstein approached the resolution of the theoretical problems of interest to them. While in the above an introduction to the iso-product and to a generalisation of Lie algebra has been discussed, several other definitions and applications are necessary for a truly full understanding of this new mathematical topic of iso-mathematics but, rather than include all the mathematical details here, brief introductions accompanied by original references will be included. This approach is for several reasons, one of which is to not include too much mathematical detail which could easily interrupt the fundamental narrative and secondly too much mathematical detail could detract from the main issue which is the physics involved. The other issues of relevance concerning iso-mathematics are

1) The application to the solution of the Lorentz problem; that is, the invariance of locally varying light speeds $C = c/n$. It must always be remembered that Einstein's assumption was simply that the speed of light in a vacuum is constant, not that the speed of light is constant since, as is well-known, the speed of light does vary in different media. This problem is studied in detail in chapter 8 of [23].

2) As mentioned earlier, it also proves necessary to extend the idea of numbers and, to this end, Santilli introduced the notion of iso-numbers and the details are worked out in chapter 2 of [23].

3) In chapter 3 of [23], the idea of iso-spaces is introduced and discussed in detail while so-called iso-functions are examined in chapter 6 of [39].

4) Iso-differential calculus was first introduced in [39] but it might be helpful to some to consider [49] also.

5) The simple construction of iso-mathematics via non-unitary transforms, together with the important topic of the invariance of isotopic elements and iso-units under iso-unitary transforms are dealt with in detail in [50].

With all this, and more, in place, attention turned to developing what is now referred to as iso-mechanics and, following that the generalisation or, as Santilli terms it, covering of quantum mechanics termed hadronic mechanics. The full details of the construction of this may be found in [23] and [24]. Suffice it to say that hadronic mechanics is based on some fundamental structures:

a) An enveloping algebra $\hat{\xi}$ with generic elements $\hat{A}, \hat{B}, \cdots$ (which are the same polynomials in r and p of the usual quantum algebra only written in iso-space) called the iso-associative envelope and characterised by the iso-associative product:

$$\hat{A} \hat{\times} \hat{B} = \hat{A} \times \hat{T} \times \hat{B} \quad \text{with isounit} \quad \hat{I} = \hat{T}^{-1};$$

b) The iso-fields $\hat{C}(\hat{c}, +, \hat{x})$ of iso-complex numbers $\hat{c} = c \times \hat{I}$, or its iso-real particularisation $\hat{R}(\hat{n}, +, \hat{x})$; and

c) The iso-Hilbert space $\mathfrak{H}$ with isostates $\hat{\psi}, \hat{\phi}, \cdots$ and iso-inner product over $\hat{C}$

$$\hat{\psi} \uparrow \hat{\phi} = \langle \hat{\psi} | \times \hat{T} \times | \hat{\phi} \rangle = \hat{I} \times \int \mathrm{d}\hat{x}^3 \hat{\psi}^\uparrow \times \hat{T} \times \hat{\phi} \in \hat{C}(\hat{c}, +, \hat{x})$$

The isotopies of both the Schrödinger and Heisenberg representations are then identified. Eventually after some further manipulations it is found that, as a general rule, hadronic mechanics preserves all the axioms of quantum mechanics and actually quantum and hadronic mechanics coincide for all isotopies for which $\hat{I} > 0$ at the abstract level. Hence, the axiomatic consistencies of hadronic mechanics are guaranteed and it is evident that hadronic mechanics merely provides a more general nonlinear, nonlocal and non-potential realisation of the same axioms as traditional quantum mechanics. It is, in short, a more general theory of which the traditional one might be viewed as a special case dealing with a restricted number of situations. Hadronic mechanics was conceived and constructed precisely to allow detailed investigations of physical situations which lie outside the scope of traditional quantum mechanics. Having been constructed though, it also allowed further perusal and a deeper understanding of issues raised by the aforementioned article by Einstein, Podolsky and Rosen [1], as well as possible resolutions of other problems which had been outstanding in science for many years. Further, having been deeply interested in environmental issues since his early years in science, it allowed Santilli to put forward other ideas aimed at at least alleviating some of these problems if not actually solving them completely.

5.2. Resolution of Einstein, Podolsky, Rosen Issues

The above discussion of the new mathematics devised by Santilli is indeed a very brief introduction as it is meant to be. As stated earlier, anyone truly interested in an in depth examination of these totally new ideas should consult the enormous library of books and articles devoted to it, probably starting with [23] and [24] cited here. It was in a publication of 1998 [13] that Santilli made his first contribution to the Einstein, Podolsky, Rosen debate. In this article, he concerned himself with examining isorepresentations of Lie-Isotopic *SU*(2) algebra. However, he ended the paper with a discussion of applications of the preceding theory to nuclear physics and, possibly more importantly in the present context, to issues concerning local realism. He proved under the conditions imposed by his earlier mathematics that Bell's inequality and the von Neumann theorem are *inapplicable* under isotopies and this allowed a limited completion of quantum mechanics along the lines of that envisaged by Einstein, Podolsky and Rosen. It is possibly worth noting at this point that this approach from as long ago as 1998 immediately brings to mind the prediction of Dirac in the final paragraph of his celebrated book on quantum mechanics and quoted earlier when discussing Mayants' approach and ideas. However, for completeness, it is quoted in full here again:

> It would seem that we have followed as far as possible the path of logical development of the ideas of quantum mechanics as they are at present understood. The difficulties, being of a profound character, can be removed only by some drastic change in the foundations of the theory, probably a change as drastic as the passage from Bohr's orbit theory to the present quantum mechanics.

There can be no doubt that Santilli did, and does, provide a truly "drastic change" in the theory due to his new mathematics. Is he correct? Only time will give the real answer to that query but his above mentioned article is only the beginning, not the end, of his involvement with this issue which has plagued theoretical physics for more than eighty years.

One other issue that has always concerned Santilli is that most contemporary theory is, and was, concerned with point particles or, sometimes, particles exhibiting spherical symmetry. He wanted to extend ideas to encompass extended particles and, in an article of 2019 [51], he continued his study of the ideas initiated by Einstein, Podolsky and Rosen according to which quantum mechanics could be (to use their words) "completed" into a much broader theory which recovered classical determinism. What he achieved in this article was to show, by using the previously achieved isotopic lifting of applied mathematics into iso-mathematics and quantum mechanics into the isotopic branch of hadronic mechanics, that extended particles seem to approach classical determinism in the interior of hadrons, nuclei and stars and also appear to recover it in the limiting conditions of gravitational collapse. As in so much of his work, Santilli stresses again in this paper that he is concerned with extended particles immersed in hyperdense media with ensuing linear *and* non-linear, local *and* non-local, Hamiltonian *and* non-Hamiltonian interactions. Yet again it might be noticed the reference to non-Hamiltonian interactions; something missing or, at least, hidden from so many modern discussions of mechanics but which caught his attention as a research student in Italy and has remained with him throughout his professional life. The main result, though, of this article was to show that the standard deviations of coordinates and momenta for particles in hyperdense media are characterised by the isotopic element which, being very small always, $\hat{T} \ll 1$, reduces the uncertainties in a manner inversely proportional to a non-linear increase of the density, pressure, temperature and other characteristics of the medium, while allowing $\hat{T} = 0$ under exteme limiting conditions with the resulting recovery of full determinism as predicted by Einstein, Podolsky and Rosen.

As is seen, Santilli appears to have returned to the problems raised by Einstein, Podolsky and Rosen after a lapse of some twenty years and one must wonder if the experimental results published by the Basel group had any effect. Around the time of this 2019 article, Santilli began organising an international conference, to be held in Florida in September 2020, devoted to the issues raised by the Einstein, Podolsky, Rosen article. In the event, the meeting had to take place via Zoom and, as was mentioned earlier, for anyone interested in what transpired, all the talks have been recorded and are available from the World Lecture Series website as well as from a YouTube link with the full proceedings are to appear in written form soon.

This conference coincided with the appearance of a further series of three papers published in 2020 and which mark the end of at least this stage of the development of Santilli's ideas on the subject. However, as he himself stresses continually, there can never be a true end to any scientific investigations, only something that seems to be the end at a precise moment in time a possible end dictated by the extent of appropriate knowledge at the time. The three papers in question [52] [53] [54] bear a joint title *Studies on A. Einstein, B. Podolsky and N. Rosen argument that "quantum mechanics is not a complete theory"* but each bearing its own explanatory sub-title. In the first of these [52], subtitled *Basic Methods*, Santilli begins by restating the fundamental notions put forward in his 1998 [13] paper; that is, he showed that the objections to the Einstein, Podolsky, Rosen position were indeed valid for point-like particles in a vacuum (so-called exterior dynamical systems) but, for extended particles in hyperdense physical media (so-called interior dynamical systems) those same objections are inapplicable note not violated but simply inapplicable since the latter systems seem to admit a classical counterpart when examined within the isotopic branch of hadronic mechanics. Again, as noted earlier, within a more recent article [51], he showed that quantum uncertainties associated with extended particles seem to tend to zero in the interior of hadrons, nuclei and stars and to be identically so in the limit of gravitational collapse. In this first paper of the mentioned trilogy, he went on to review, upgrade and specialise the basic mathematical, physical and chemical methods required in a further detailed analysis of the Einstein, Podolsky, Rosen issue. The details of the work included in this piece are largely mathematical but it is claimed that what was achieved was;

1) A review and upgrade of the so-called Lie-admissible and Lie-isotopic "completions" of twentieth century applied mathematics for the representation of time irreversible and reversible interior systems;

2) A review and upgrade of the "completions" of quantum mechanics and chemistry into the Lie-admissible and Lie-isotopic branches of hadronic mechanics and chemistry;

3) A review and upgrade of the main aspects of the studies to this point; that is, the "completions" of the Newton differential calculus into forms applicable to irreversible and reversible interior systems of extended particles.

Hence, once again, the paper is largely of a mathematical character and it is worth remembering that, in his terminology, "completion" is referring to an extension of existing theory to make the new, more general theory directly applicable to a range of physical topics not covered by presently accepted methods. Also, in this paper he introduces the reader to another class of generalised methods called *genotopies*, which are a natural generalisation of the isotopies when the iso-unit is no longer Hermitian. Again, the mathematical details are included in the text of the article or via copious references therein and it seems appropriate to exclude this mathematical detail here but for the interested reader to follow it up at leisure in detail. The important point in this review is simply to make people fully aware of a mammoth piece of work devoted to extending the scope of theoretical physics and to draw attention to some of the results achieved, specifically, but not exclusively, those connected to issues relating to the Einstein, Podolsky, Rosen paper [1].

In the second paper of this trilogy [53], subtitled *Apparent Confirmation of the EPR Argument*, he set out to study in detail the iso-symmetries for interior dynamical systems and also confirmed the results of the apparent proof, appearing in his 1998 article [13], that these interior dynamical systems do, in fact, admit classical counterparts. It was also confirmed that the previously published [51] apparent proof that Einstein's determinism is approached progressively for extended particles in the interiors of hadrons, nuclei and stars and is totally verified in the limit of gravitational collapse. All this, as stated here, was simply confirmation of results obtained previously but here looked at afresh and, therefore, worthy of separate consideration since more detail is included to help the reader properly understand and appreciate precisely what has been achieved. However, the paper contained new results as well. For the first time, it is thought, it was shown that the recovering of Einstein's determinism in interior dynamical systems actually implies the apparent removal of quantum mechanical divergencies. This latter point was found due to the rapid convergence of the iso-series of hadronic mechanics, the removal of the singularity in Dirac's delta distribution and other factors which the interested reader can learn about in detail in the original reference. Again, though, it might be noted that everything depends on the validity of Santilli's hadronic mechanics and so, the reference above is to iso-series in hadronic mechanics and not simply series. This seemingly small point is important because it illustrates, yet again, the requirement that *all* physical and mathematical quantities involved in all discussions must be those associated with the new language of hadronic mechanics. This may prove an insurmountable obstruction to some but, if the words of Dirac, which appear as the last paragraph of his extremely well-known book on quantum mechanics and which were quoted earlier in full, are to have any true bearing on this vitally important issue for physical science, then getting to grips with this proposed new mathematics is a task large though it may be which should be attempted.

The concluding article of the trilogy [54], subtitled *Illustrative Examples and Applications*, begins by apparently showing for the first time that the final statement of the Einstein, Podolsky, Rosen article [1] that the wave function of traditional quantum mechanics does not provide a complete description of physical reality, is indeed valid. In point of fact, the claim here is that the study has produced an axiom preserving "completion" of the quantum mechanical wave function due to deep wave-overlapping when represented via iso-mathematics. It is shown also that this new approach permits what would otherwise be an impossible representation of the attractive force between identical electron pairs in valence coupling, as well as the representation of the characteristics of various systems both physical and chemical which occur in nature. This "completion" of the usual wave function of quantum mechanics into what might sensibly be termed the hadronic iso-wave function of this new theory also allows the exact representation of all the characteristics of the neutron in its synthesis from the hydrogen atom here a return to the ideas of Rutherford which were deemed incorrect under traditional quantum mechanics. Also, it allows a complete description of the properties of the deuteron again something not achievable with traditional quantum mechanics. A little more will be said about some of these extra achievements in the next section. Although these points may not be felt pertinent as far as a discussion of the Einstein, Podolsky, Rosen issues are concerned, it seems they do fit in in order to give a rounded idea of what is claimed by this totally new approach in theoretical physics and chemistry. Also, all the points to be discussed are, it is felt, of direct relevance to the wider issue of nuclear and particle physics in general.

The above is but a very short introduction to a large body of results produced over a lifetime by someone fascinated by the prospect of extending the range of theoretical physics' theory to cover extended particles but also because of a lack of belief in quantum mechanics as a final theory due to its linearity being at odds with an expectation that the complexity of the Universe cannot always be represented by any linear theory. It is not without interest to note that another scientist who studied non-linear generalisations of quantum mechanics was Heisenberg. In fact, Santilli revealed at the above-mentioned conference that he met with Heisenberg in the early 1970's to enquire about the latter's non-linear theory. The outcome was to discover that, while Santilli was not expecting any non-linear interactions to be derivable from a potential, Heisenberg was seemingly remaining with a Hamiltonian approach dependent on potentials. Nevertheless, the interchange evidently did spur Santilli to proceed to conceive the representation of all non-linear interactions with the isotopic element, in which case the resulting iso-Schrödinger equation does verify the superposition principle and the characterisation of the constituents of a bound state with non-linear internal interactions not derivable from a potential is allowed. Finally, since a quotation from the writings of Dirac has been used more than once already, it is also of interest to note that he, Dirac, was making himself acquainted with some of Santilli's ideas but this possibly interesting and important development was interrupted tragically by Dirac's death.

6. Some Details of Some Relevant Applications of the Above

Although not specifically related to matters concerning the Einstein, Podolsky, Rosen issues, it is interesting to note to how many different outstanding problems, including those alluded to above, Santilli has turned his attention with this new approach and, seemingly, with so much success. As mentioned earlier, one of his earliest worries concerned the range of applicability of quantum mechanics. Having noted the comments and concerns of some truly notable scientists of the early part of the last century, he devised so-called Hadronic Mechanics and succeeded in explaining a wide variety of otherwise unexplainable phenomena. These are catalogued in detail in his book *Foundations of Hadronic Chemistry* [28], as well as some being listed here in the abstract, but it is worth noting, and speculating on, some of them here since they, if not all, could well be of interest to anyone concerned with problems in quantum mechanics but, more specifically, in nuclear physics. As noted on page 35 of his book, explaining that the experimental data on the Bose-Einstein correlation in proton anti-proton annihilation at both high and low energy provided experimental verification of hadronic mechanics in particle physics. Such experimental data may be represented by traditional quantum mechanics only after the introduction of arbitrary parameters which seem to have no physical origin. However, hadronic mechanics is easily able to explain things because it proves capable of dealing with the off-diagonal terms appearing in expectation values. This latter property is not allowed in orthodox quantum mechanics because, for a quantity to be observable, its expectation value must be diagonal in form. This, of course, introduces mathematical terms into the discussion which, ideally, should be avoided but, suffice it to say, that the phenomenon may not be explained by orthodox quantum mechanics because it is too restricted as a theory. Another experimental verification, in the sense of the previous example, has been provided by the ability of the new theory to explain data concerning the anomalous behaviour of the mean-life of the kaon with energy. This has been examined successfully over various energy ranges and is important because, as with the example of the Bose-Einstein correlation, it establishes the existence of effects in the interior of kaons which are nonlinear, non-local and, most importantly, non-potential (that is, non-conservative).

As Santilli has stated quite categorically on several occasions but, possibly most clearly at the beginning of section 3 of his article in the *Journal of New Energy* [55], he has always thought of physical particles as being particles which may be defined rigorously in our spacetime. He points out that hadronic mechanics was conceived and developed in order to identify the constituents of all unstable hadrons with genuine physical particles. Has he succeeded? Time will tell, but the positive evidence is there for all to see and is mounting. As has been seen already, any discussion of this topic inevitably seems to introduce mathematical ideas and notation at some point. Again as stated already, this is unfortunate but doesn't detract from an appreciation of the picture emerging and might serve as a spur for professionals to investigate the detail further in order to reach a truly informed opinion of the work.

From the point of view of physics, it seems that Santilli obtained inspiration from early ideas of Rutherford. It was in 1920 [56] that Rutherford postulated the existence of a new particle, which was, in essence a "compressed hydrogen atom"; that is, it was composed of an electron compressed entirely within the proton. This he called a neutron. Presumably Rutherford thought that, when a hydrogen atom is compressed, for example, in the core of a star, the high pressures involved could result in it being reduced in size to that of a proton, with an electrically neutral particle emerging finally. Twelve years later, Chadwick [57] established the existence of the neutron experimentally. However, Rutherford's original conception of this particle was dismissed by many of the founders of quantum mechanics for a variety of seemingly good reasons at the time: the model would require a positive binding energy; both constituents possess spin ½ and so, the resulting particle would not be permitted to have spin ½ by normal quantum mechanics; orthodox quantum mechanics would also not allow the correct magnetic moment to follow in this model. Hence, the rejection of Rutherford's model of a neutron and this heralded a change in the direction of physics' research. Up to that time, physics had been based on the notion that the constituents of so-called bound states have to be capable of being isolated and identified in laboratories. The rejection of Rutherford's conception appears to have altered this view. This then was the spur for Santilli and, having devised the new mathematics referred to earlier, he first succeeded in producing a consistent model of the meson, π^0, as a bound state of an electron and a positron. This model is not possible in conventional quantum mechanics for a number of reasons, one of which concerns binding energy. Quantum bound states possess negative binding energies and this implies a total mass less than the sum of the constituent masses. For a π^0 meson, this would imply a rest energy appreciably less than its actual rest energy of 135 Mev. This problem, as are all others, is resolved by hadronic mechanics or, at least, that is the claim with all the evidence clearly available for examination by those with a mind so to do. The model Santilli proposes does, in fact, explain all the characteristics of the said particle – zero spin, electrically neutral, null magnetic moment, a rest energy of 135 Mev, a mean-life of approximately 10^{-16} sec., a charge radius of about 1 m (that is, 10^{-15} m), decay according to

$$\pi^0 \to e^+ + e^-$$

and this model of the smallest of hadrons has now been extended successfully to all mesons. Further, although the theory does not view quarks as actual physical particles, but rather as mathematical objects with a composite structure, this new model for hadrons does prove compatible with the current quark theories, always assuming that quarks have a composite structure. For those interested, further details of this model may be found in a variety of publications but especially in volume **4** of the *Journal of New Energy* [55], as mentioned earlier. In fact this reference is a veritable goldmine of information on this general topic of hadronic mechanics and its consequences both for physics itself and probably for mankind as a whole through its consideration of the possibilities offered by the theory for alternative new clean energies.

However, what could conceivably turn out to be Santilli's most important achievement was his success in using the new hadronic mechanics to resurrect the Rutherford model for the structure of the neutron successfully. This model recognises a neutron as being composed of a bound state of a proton and an electron at a distance of 1 fm; that is, at a distance of 10^{-15} m. As mentioned earlier, such a model is prohibited by conventional quantum mechanics, so, if Santilli's ideas are valid, what are the consequences for physics? The answer is, quite simply, enormous! The abandonment of the original approach to the structure of physical particles would have a profound and far-reaching effect on research in the area of particle physics obviously. However, it is the possible ecological implications which are staggering and of so much direct relevance to absolutely everyone. The orthodox approach has conceivably prevented the study of the neutron as a major source of clean energy and actually seems to have obstructed the study of new forms of clean nuclear energy. These are now being studied via hadronic mechanics, as is the associated problem of the safe disposal of the nuclear waste presently causing so much trouble.

The main characteristics of the neutron, such as its having a rest energy of 939.6 Mev, a mean-life of 916 secs., spin 1/2, and a charge radius of 0.8×10^{-13} cm., were all explained in a model of the neutron devised by Santilli using hadronic mechanics in 1990 [58]. This was a non-relativistic treatment, but a relativistic treatment soon followed and appeared in 1993 [59]. The crucial point about this is that the model was precisely that proposed by Rutherford so many years earlier. Using hadronic mechanics, Santilli was able to derive all the properties of the neutron when it was viewed as being composed of an electron totally compressed inside a proton. This model, remember, had been abandoned because this structure was inexplicable using orthodox quantum mechanics. However, the fact that the Rutherford model may be explained using this new technique cannot, in itself, be regarded as justification for the new hadronic approach. The real justification is provided by the fact that there appears to be experimental verification of the structure in that experimental verification of the synthesis of neutrons from protons and electrons seems to have been achieved in the 1980's by a group in Brazil under C. Borghi, although the results were published only in 1993 [60]. Although this is exciting, it is by no means conclusive evidence and that is precisely why caution is exercised when reporting and discussing this development. However, the possible ramifications are so important that it is vital for this experiment to be repeated independently several times so that a genuine conclusion may be reached which may be accepted by all in the scientific community.

The ramifications alluded to concern the possibility of utilising these new theoretical ideas to produce new clean energies for mankind. This again is a topic to which Santilli has devoted much time and energy over the years. Basically, many of these new energies are characterised by processes in the interior of hadrons, rather than in nuclei or atoms. It might be noted that energy is required if unstable hadrons are to be synthesised from physical particles; in the case of the neutron, 0.80 Mev is required to synthesise it from protons and electrons. However, as Santilli points out [55], "once created, unstable hadrons become a large reservoir of energy, which is released in their decay". Some of these proposed new energies, therefore, are produced by using mechanisms capable of stimulating the decay of unstable hadrons, or by simply using the energy produced in their natural decay. In this article, he goes on to describe the way in which energy could conceivably be produced via stimulated neutron decay. He also draws attention to the quantity of energy involved, pointing out that the electron emitted in neutron decay would possess energy roughly 100,000 times more than that of electrons hitting a computer screen. Again, it is noted that this mechanism is possible only if the neutron is composed of the physical particles, the proton and the electron. The main ideas behind the proposal are that the neutron does actually decay spontaneously. Also, its mean-life is not fixed but depends on local conditions; for example, if it's a constituent of some unstable nuclei, the mean-life is a few seconds; in a vacuum, it's more of the order of fifteen minutes; in other unstable nuclei, it's even longer; and in natural, light, stable nuclei, it's infinite. However, the neutron itself is naturally unstable and so it is felt it should be possible to stimulate its decay and hence control its mean-life. The actual proposal suggests testing this possibility through the use of photons with the resonating frequency of 1.204 Mev, plus the additional threshold energy required to satisfy conservation requirements of

$$\gamma + n \rightarrow p^{+} + e^{-} + \upsilon .$$

Here the figure of 1.204 Mev for the resonating frequency is another consequence of the hadronic model of the neutron adopted. It has been found, by studying nuclei, that most nuclei do not permit reactions such as that represented by the above equation due to violation of conservation laws. However, some do and it is these which offer the possibility of a new form of usable energy, termed by Santilli *hadronic energy*. In his book, Santilli chooses, as a representative example, molybdenum ($_{42}Mo^{100}$) but also draws attention to the fact that other natural, light elements, such zinc ($_{30}Zn^{70}$), possess the required prerequisites. Most of this is still in need of experimental verification. It seems that, if successful, these tests would offer a prize too valuable to be ignored. It is to be hoped, therefore, that the necessary experiments will be performed in the very near future, so that existing doubts may be cleared up, one way or the other, finally.

A further important reason for having the predictions of hadronic mechanics fully and openly tested is provided by the rapid accumulation of highly radioactive nuclear waste around the world. This is proving a major problem for many countries. The U.S.A. has been seen to have a major problem of disposal and also to have an additional problem posed by those opposed to the current method for attempting to achieve that disposal. Britain, on the other hand, while facing problems concerning disposal of its own nuclear waste, faces additional protests from those opposed to its business of helping in the disposal of nuclear waste from other countries. In both instances, and in others, people are extremely worried by the perceived threat posed by the actual disposal method as well as that posed by the transportation of that waste across country. All of these worries have been exacerbated by the rapidly growing terrorism threat facing so much of the world. There can be no doubt that a great many people, some with scientific knowledge, some without such knowledge, harbour genuine worries. There can be no doubt also that those worries, and indeed fears, are not unjustified. The above discussion surrounding the composition of the neutron obviously offers the possibility of a resolution of the difficulties and concerns. These essentially reborn ideas concerning the structure of the neutron, if valid, offer the possibility of recycling nuclear waste by way of stimulating its decay in such a way as to reduce the extremely long lifetimes to hours or, at worst, days. It is envisaged that this could be achieved by the use of relatively light equipment and that the nuclear power plants could achieve this within their own boundaries, thus eliminating all transportation of these highly dangerous materials. If the idea works, although jobs in the industry presently formed around the disposal of nuclear waste would vanish, many new jobs in a much safer nuclear waste disposal industry would appear. A new industry might conceivably be expected to grow for the development, production and sale of the new equipment, since it would be a vital requirement for nuclear power plants throughout the world.

The basic idea revolves around the fact that the nuclei concerned are large and naturally unstable. One idea is to expose the highly radioactive nuclear waste to an intense, coherent flow of photons with the required resonating frequency. It is felt that this may be achieved via a synchrotron of about three metres diameter; a size which could be accommodated in nuclear power plants. A typical example is provided by uranium (${}_{92}U^{238}$) which has a life-time of the order of 10^9years. A double stimulated transmutation of this element could change it into plutonium (${}_{94}Pu^{238}$). Again, this is an unstable quantity and has harmful emissions as well, but its life-time is a mere 86 days and it could well be retained under suitable shields for that period of time. It may be superfluous to draw extra attention to this point, but it is worth noting the different life-times involved here 86 days as against 10^9 years! The phenomenal advantage of this stimulated transmutation is immediately evident. Will it work? The theory certainly suggests that it should, but only experimentation will give the actual answer to that question. Possibly the bigger, more relevant, question to ask at this time is whether or not the scientific community and national governments are prepared to finance the experiments necessary to test this thesis?

At this moment in time, it is worth realising that the cost of carrying out the proposed experiments would probably be of the order of a several hundred thousand pounds, maybe even a million pounds. This sounds a lot of money, and indeed it is. However, an experiment to detect neutralinos those particles predicted by theory as candidates for so-called "dark matter" which seems so important to preserve the currently accepted standard model in cosmology has been running for many years with little, or no, success so far. Nevertheless, it has been announced that those running this experiment are installing yet another new detector at the cost of one and a half million pounds! It has also been announced that, in America, a new extremely powerful super-computer has been used to create a three-dimensional model of two colliding black holes. Since this is purely a computer experiment, it must be noted from the very outset that any results obtained will be totally dependent on the original input model and information. Both these factors will be completely dependent on present day knowledge and, possibly more importantly, theories. Hence, both will be influenced heavily by "conventional wisdom". Nevertheless, the results from this computer experiment are being heralded as very exciting and it is proposed to use this information to restart another sequence of very expensive experiments to seek evidence of such collisions, including yet another search for gravitational waves. This latter search is again, incidentally, another extremely expensive series of experiments which has continued for a great many years with, as yet, little generally accepted success. This second proposed venture will undoubtedly eat up further millions of pounds of scientific research money. Fundamentally, no-one interested in science should be opposed to either of these two possible areas of research. Both will add, either positively or negatively, to human knowledge and, as such, are important. However, even if successful, neither will produce any immediate major benefit for mankind. If several hundred thousand pounds were to be spent checking out Santilli's theories, the worst that could happen would be negative results; in which case several hundred thousand pounds would have been wasted, but yet again, knowledge would have been gained. Negative knowledge may be, but knowledge nevertheless. If successful though, mankind's energy worries would recede into the background, at least for the immediate future, and nuclear power would become a so much safer option. Also, with the problem of the disposal of nuclear waste dealt with, the genuine worries of so many, when issues surrounding nuclear power are raised, would be assuaged.

However, the scientific establishment tends to regard orthodox quantum mechanics as a sacrosanct part of "conventional wisdom", so it must be thought doubtful that it will sanction work which directly challenges that "foundation stone of modern science". The positions of national governments are far more difficult to assess. They will consult scientific advisers who will be members of the scientific establishment, so the line of their advice is probably predictable. They will be under pressure from a wide variety of areas of "big business" but, no doubt, the most vociferous will be those wreaking profits from the present highly questionable methods of nuclear waste disposal. The individual members of those governments will also, though, be under pressure from members of their electorates. If news of this possibility of there being a truly safe, in-house method of disposing of nuclear waste did become fully public, then it is probably this final factor that would weigh most strongly with national governments since, at the end of the day when all the political manœuvering and gesturing has been discarded, it is the thought of votes at the next election which would end up being of paramount importance. Can the possibility of the existence of such a prize really be ignored any longer?

The success in describing the above mentioned model for the neutron using this new hadronic mechanics also opened the way to view afresh models for other systems, in particular the deuteron. Here an unresolved problem had lain around for years; that was the inability of conventional quantum mechanics to explain the value of one for the spin of the deuteron. The deuteron was felt to be composed of two particles, each having spin a half and the basic axioms of quantum mechanics would imply, therefore, a spin value of zero for the ground state of such a system. The new hadronic mechanics clears up this problem also. Following on from the reduction of the neutron to a hadronic bound state of a proton and an electron, the deuteron is viewed as a three-body situation comprising two protons and one electron – or, more accurately in Santilli's language, two iso-protons and one iso-electron. This model is able to represent accurately all the characteristics of the deuteron, including its spin. This success led Santilli to extend the notion to all nuclei. The result was to produce a new hadronic structure model of nuclei in terms of combinations of iso-protons and iso-electrons, which reduces to the usual model involving protons and electrons as a first approximation. This all seems at first sight to be merely another huge amount of almost unintelligible theory which will have little or no effect as far as the ordinary person is concerned. Amazingly, that is not the case. If this theory does turn out to be correct, the implications for society are immense because it could result in a number of new forms of clean energy for mankind's use; forms which are not possible with the old proton – neutron model. It does appear, therefore, that this is an area worthy of further open-minded investigation simply because the possible prize at the end is so attractive and, indeed, necessary considering the massive environmental problems and energy demands facing our world at the moment.

7. Some Speculative Thoughts

To begin with one or two quotes from the original paper [1] by Einstein, Podolsky and Rosen which might usefully be brought to the fore:

Firstly, it was stated that

> "...every element of the physical reality must have a counterpart in the physical theory."

Secondly, it might be remembered that the authors claimed that

> "The elements of the physical reality cannot be determined by a priori philosophical considerations, but must be found by an appeal to results of experiments and measurements"

Which would seem to be an almost self-evident statement if the usual norms of scientific practice are to be observed.

They also claimed from their thought experiment that

"by measuring either A or B we are in a position to predict with certainty, and without in any way disturbing the second system, either the value of the quantity P (that is p_k) or the value of the quantity Q (that is q_r)."

Finally, it was stated that

"Previously we proved that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. Starting then with the assumption that the wave function does give a complete description of the physical reality, we arrived at the conclusion that two physical quantities, with non-commuting operators, can have simultaneous reality. Thus the negation of (1) leads to the negation of the only other alternative (2). We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete."

Two primary elements of the Einstein, Podolsky, Rosen argument may now be noted separately:

(i) It is possible to define both position and momentum of two previously interacting quantum particles/systems.

(ii) Measurement may not (non-locally) disturb system two if system one is measured, unless a hidden variable not yet defined within the context of *wave function* is identified.

Point two is clearly implied from the last sentence in the paper:

"We believe, however, that such a theory is possible."

and the aforementioned sentence:

"...every element of the physical reality must have a counterpart in the physical theory."

It is important to note at this juncture, the concerns of Heisenberg regarding such fanciful methods of deduction and exploration as thought experiment and human imagining alone, which appear to closely parallel Einstein's views of the same, as already noted above.

From page 15 of the book on quantum mechanics by Heisenberg [61], concerning the reality of uncertainty, he states that

"In this connection one should particularly remember that the human language permits the construction of sentences which do not involve any consequence and which therefore have no content at all––in spite of the fact that these sentences produce some kind of picture in our imagination; e.g. the statement that besides our world there exists another world with which any connection is impossible in principle, does not lead to any experimental

> consequence, but does produce a kind of picture in the mind. Obviously such a statement can neither be proved nor disproved. One should be especially careful in using the words "reality", "actually", etc., since these words very often lead to statements of the type just mentioned."

This point should be noted as it is of importance for what follows.

If the notion of the Einstein, Podolsky, Rosen argument is sound, it might be expected that the scheme would be used in some sort of demonstrable way. If the idea is good and leads to accurate measurement, some practical usage might have been made of it after all these years. Entangled science aside, is the basic notion in point one above actually demonstrable?

For a moment consider the usual interpretations of the Einstein, Podolsky, Rosen ideas, and imagine two quantum particles which have interacted, and are now moving directly away from each other at a 180 degree relation. This is the interpretation most used, that akin to the thinking of Kumar [62] [63] which defines the Einstein, Podolsky, Rosen idea as "two particles, A and B, [which] interact briefly and then move off in opposite directions."

It might be wondered if this scheme is actually able to measure anything, and is it used? It seems just possible that the answer to this query might be in the affirmative. Positron Emission Tomography scanning (the PET scan) appears to use this idea to measure biological processes and define the locations thereof. A PET scanner is essentially a gamma ray detector. In PET scans, Blood Oxygenation Level-Dependent relations indicative of tissue oxygen metabolism are detected through positron/electron annihilations created by way of an injected radioactive oxygen tracer such as ^{15}O, which has a half-life of 123 seconds. As the unstable nucleus of a ^{15}O atom decays having been absorbed by dynamic oxygen using tissues such as neurons, it emits a positron. The positron annihilates when brought in contact with an electron, emitting two (gamma) annihilation photons which travel in exactly opposite directions, a 180 degree relation of two quantum particles moving at a constant mutual speed, allowing accurate measurement of the location of the source interaction in space, and also, inference could easily be drawn from one particle measurement to the values of the other.

It may be concluded that the basic notion is in fact quite functional as a system of measurement when used in a general way. It is clear also that scientific observers could easily infer the position and momentum of one particle from measurement of the other, which travels in an exact mirror opposite direction, both at a known speed.

It does seem, therefore, that the Einstein, Podolsky, Rosen scheme does allow actual measurement, as it should in reality, and is not just a fanciful idea one may draw up to form a picture in one's head, and so, answers in this one aspect at least, both Heisenberg's and Einstein's standards of a workable theory as represented in good science.

Now consider the nonlocal aspects of the Einstein, Podolsky, Rosen theory and assess the outcome of experiments. Local realism insists that measurement of one separated system part could not ever superluminally affect the other separated parts of the system (presumably unless some missing, hidden variable is in play). It might be recalled that, in the Copenhagen interpretation of Quantum Mechanics, the wave function is an entirely probabilistic entity! However, it is found that nonlocal measurement effects moving well in excess of light speed are evidenced and those results then repeated in experiments involving entanglement.

In an article by Yin, *et al.* [64], it may be read:

> "In the well-known EPR paper, Einstein *et al.* called the nonlocal correlation in quantum entanglement as 'spooky action at a distance'. If the spooky action does exist, what is its speed? All previous experiments along this direction have locality and freedom-of-choice loopholes. Here, we strictly closed the loopholes by observing a 12-hour continuous violation of Bell inequality and concluded that the lower bound speed of 'spooky action' was four orders of magnitude of the speed of light if the Earth's speed in any inertial reference frame was less than 10^{-3} times that of the speed of light."

How reliable are the results reported in this online article is a question that might come to the minds of many immediately. The answer to that is not known at the present time but, if acceptable, experiments must be repeatable and must be repeated by different personnel. Hence, only time will give the answer to the question. However, in the interim, this all leads to other questions concerning details of the quantum mechanical scenario.

Here, the new theories appear to come good and the possibility arises that the matter may be resolved in favour of a hidden variable: the scalar wave within the aether [63].

8. Consequences and Further Speculation

Obviously, if correct, the views of both Mayants and Santilli as laid out above imply serious consequences for physical science as a whole but possibly for traditional quantum mechanics in particular. Mayants lays great stress on the difference between what he terms "concrete" and "abstract" objects, while Santilli points out that traditional quantum mechanics really applies only to point particles and possibly spherically symmetric particles but not to generally extended particles. Using different terminology, both investigators draw attention to the basic limitations imposed in quantum mechanics by the initial assumptions made in constructing the theory. Both in their different ways set out to look beyond those assumptions and see what conclusions might be drawn.

The biggest question raised must surely be concerning the position of the whole idea of *uncertainty*, especially if Santilli's conclusions are totally valid because they indicate a return to the notion of Einstein determinism or, in other words, indicate a return to the original idea of particles being genuine physical objects with genuine physical dimensions and, hence, not hampered by ideas of uncertainty concerning position, momentum, or any other physical property. At this point, the quote from Heisenberg mentioned earlier should be revisited because, in the light of what has just been said, that quote must make anyone wonder if the actual notion of uncertainty is one of those things about which Heisenberg was advising caution. In fact, it appears this well-established property of traditional quantum physics might even be termed a particulate anthropomorphism in that it seems almost as if a human quality, in this case uncertainty, has been assigned to a physical particle. It is perfectly reasonable if such an idea is put forward but it must be asked if it is a human idea or a physical one; it is certainly something which may be pictured but does it really possess physical content?

However, such thoughts immediately raise the question of what could actually be causing the observed measurement results of some well authenticated quantum experiments. If not uncertainty, what is the physical cause of the measurement problem and seeming duality between particle and wave? Of course, if Mayants' reasoning is followed through to its logical conclusion, duality is always the mark of confused thinking as, it might be felt, are most, if not all, paradoxes. Hence, it must be wondered what could be causing the plainly available uncertain effects witnessed in some experiments? Following this line of thought through indicates that there must be some genuine physical reason and it cannot be simply the result of some confused human imagining! This seems to be leading away from direct thoughts about the Einstein, Podolsky, Rosen article but, in truth, is staying very close to it since, if the implications of that article are to have been proved valid, then the consequences are of vital importance to the continued progress of theoretical physical science and, in this, the position of the notion of uncertainty is central. Another consequence must be the existence of so-called hidden variables. Here remember that, in Santilli's approach, he did not claim to *disprove* any existing results but rather to show that those results were simply *not applicable* in the extended situations he was considering. Hence, his work has to be considered as an extension to existing theory and not an alternative. One suggestion for coping with this notion of hidden variables in the present context is to resurrect the old idea of an aether. In many ways this notion has not really left science but has often been replaced by the vacuum to which so many different properties have been attributed [65]. Further, more recently, an apparently new solution to the usual Maxwell electromagnetic equations has emerged. An examination of the expressions for the electric and magnetic fields in terms of the usual scalar and vector potentials indicates that a scalar (longitudinal) wave solution exists if the magnetic field is zero but does so whether the electric field is zero or non-zero [66]. These two factors raise the thought that there is a hidden variable which is the aether and longitudinal pressure waves forming up to explain these otherwise puzzling occurrences concerning what appear to be examples of uncertainties.

If the existence of such an aether is considered for a moment, it might be recalled that boundary layer theory is well established following the initial work by Prandtl [67], the details of which may be found in most books on fluid mechanics such as that by Cole [68]. This theory may then be applied to the aether; that is, the boundary between the aether itself and any body passing through it or over which it passes. Imagine, for a moment, an aetherial boundary layer around a concrete physical particle. It might be argued that the original uncertainty equation describes only effects. The boundary layer as a particle-surrounding scalar wave accounts for the causal mechanism of uncertainty effects, the measurement uncertainty is then caused by an actual wave surrounding the actual particle. In recent times physics has excluded the aether and, hence, the wave around each quantum particle. The uncertainty of the momentum and *x* component of velocity in the Heisenberg equations themselves might conceivably be caused by this wave obscuring the precise detail of those aspects of the particle. The overall change as diffusion then refers to the heat within the scalar wave and hence its initial (quantum) size, delta in the Heisenberg equations then referring to the amount of change in temperature above absolute zero, in a causal analysis and proper treatment [62]. That wave is the source of "diffusion" effects. Note how, in the paper *Entropy in a column of gas under gravity* [69], heat first added to the system creates gravitational potential (in part) and not only increase in temperature. That gravitational potential is, by these present theories, the creation of the scalar waves which create a gravitational field.

If this is so, and this, at the present moment, highly speculative theory correct, a violation of measurement "uncertainty" should be observed in experiments if the scalar waves around the particles are deprived of heat. Indeed, this is exactly what is seen in experiments. The back action limit, the quantum limit on measurement precision bounded by uncertainty, is violated, and now, just as might be expected, absolute zero may be approached *arbitrarily close* to deprive the actual source of the heat needed to create these uncertainty effects. As Clark and colleagues have pointed out recently [70]:

> "Here we propose and experimentally demonstrate that squeezed light can be used to cool the motion of a macroscopic mechanical object below the quantum backaction limit. We first cool a microwave cavity optomechanical system using a coherent state of light to within 15 per cent of this limit. We then cool the system to more than two decibels below the quantum backaction limit using a squeezed microwave field generated by a Josephson parametric amplifier."

Uncertainty is experimentally demonstrable as a function of heat instantiated within the boundary scalar wave surrounding the particle. It appears likely that, as heat is further reduced as absolute zero is approached more closely, the cause of quantum uncertainty and fluctuation which is the omnidirectional motion of aether particles within the particle boundary scalar wave is then reduced, perhaps by way of energy reduction of the aether particle itself and/or alignment of said omnidirectional particle motions, leading to the absence of any wave-forming particulate energy value at absolute zero temperature.

From the point of view considered here it would seem that quantum fluctuation effects and related uncertainty are caused by omnidirectional aether particle motion. Uncertainty itself within quantum particulate measurement dynamics is then actually caused by the boundary wave, surrounding a quantum particle as a function of heat. It might seem also that uncertainty effects emerge as a function of quantum scale, as the aether particle size is more closely approached.

Again, new experiments are seen where, as might be expected, heat is reduced to permit the proliferation of related condensate and Einstein, Podolsky, Rosen effects to emerge. Note, for example the paper by Fadel, *et al.* [14] referred to in some detail earlier in which it is stated that

> "While spin-squeezed and other nonclassical states of atomic ensembles were used to enhance measurement precision in quantum metrology, the notion of entanglement in these systems remained controversial because the correlations between the indistinguishable atoms were witnessed by collective measurements only. Here we use high resolution imaging to directly measure the spin correlations between spatially separated parts of a spin-squeezed Bose-Einstein condensate. We observe entanglement that is strong enough for Einstein-Podolsky-Rosen steering: we can predict measurement outcomes for non-commuting observables in one spatial region based on a corresponding measurement in another region with an inferred uncertainty product below the Heisenberg relation."

Some Final Thoughts on the Aether

Before concluding, it might be appropriate to reflect on the demise of the aether theories over the last hundred years and more. In the intervening time, several people have doggedly pursued investigations into theories involving the aether concept, often at personal cost. Among those was Kenneth Thornhill and it might benefit many to read his work which is readily available on the internet [71]. In the cited article, he starts by showing that Planck's energy distribution for a black body radiation field may be derived for a gas-like aether with Maxwellian statistics. The gas consists of an infinite variety of particles whose masses are integral multiples of the mass of the unit particle. Also the frequency of electromagnetic waves correlates with the energy per unit mass of the particles, not with their energy, thus differing from Planck's quantum hypothesis. Identifying the special wave-speed, usually called the speed of light, with the wave-speed in the 2.7 K background radiation field, leads to a mass of 0.5×10^{-39} kg for the unit aether particle. Interestingly, in this article he also shows that the speed of light should vary with the square root of the background temperature. It is not without interest to note that this suggestion by Thornhill would obviate any need for introducing theories of inflation to protect the Big Bang notion. More may be found on the whole question of the constancy, or otherwise, of the speed of light in the article by Farrell and Dunning-Davies [72].

Also, before ending this section, attention should be drawn to a companion paper by Thornhill [73] in which he discusses in detail the fact that, in a gas-like aether, the duality between the oscillating electric and magnetic fields, which are transverse to the direction of propagation of electromagnetic waves, becomes a triality with the longitudinal oscillations of the motion of the aether if electric field, magnetic field and motion are coexistent and mutually perpendicular. He points out that it must be shown that, if electromagnetic waves also comprise longitudinal condensational oscillations of a gas-like aether, analogous to sound waves in a material gas, then all three aspects of such waves must propagate together along identical wave fronts. This he shows to be the case. Further he finds that the equations governing the motion and the electric and magnetic field strengths in such an aether, together with their common characteristic hyperconoid, are all invariant under Galilean transformation.

Talk of reintroducing the notion of an aether will be regarded as highly controversial but, if the present indications concerning the validity of the Einstein, Podolsky, Rosen arguments are correct, serious consequences follow for theoretical physical science and, given the discussions relating to physical properties of the vacuum, it seems that thoughts concerning an aether must re-enter the arena of scientific thought.

9. Conclusions

For eighty-five years now the arguments of the 1935 article of Einstein, Podolsky and Rosen have continued to cast a cloud over the entire area of quantum mechanics. However, that area of physics has always caused concern in the thoughts and minds of many concerning its real meaning and understanding; after all, accepting the idea of uncertainty as described in quantum mechanics does not come easily to most. Added to this vague doubt probably existing in the minds of many, the Einstein, Podolsky, Rosen article can have done little to dampen these doubts. A further problem with the whole idea must have been provided by the adoption of the so-called Copenhagen interpretation whereby the wavefunction was connected irrevocably with probability because, as soon as probability became a central tenet of quantum mechanics then, at least from a purely linguistic point of view, the theory became incomplete. Linguistically, completeness implies providing a totally exact answer to questions. Hence, in the case of quantum mechanics, that would have meant a theory with the ability to provide exact answers to questions of position, momentum, etc. Once it was clearly acknowledged that this was not the case, the theory became, almost by definition, incomplete. Of course, this argument is based on purely linguistic considerations and it is certainly not inconceivable that another more specific idea of what is meant by "completeness" in a purely physical scientific context is in the minds of some and leads them to claim completeness for the theory.

Here attention has been restricted deliberately to the examinations of the issue by Mayants and Santilli, both of which claim the theory incomplete. This has been a deliberate choice since the motivation for writing was provided by the results of the Basel experimental work but this, in turn, led to a perceived need to bring to light theories which, until now, although in the public domain, have been shunned from deep, open-minded assessment by many. The view here is not that these two approaches are necessarily correct but that they are deserving of completely open-minded public examination. Also, as far as the second discussed theory that of Santilli is concerned, further consequences, not strictly within the remit of this general discussion, have been included. These consequences are exemplified by those listed in the abstract, some of which have been examined in more detail in section 6. This has been to show how the proposed extension to physical science theory, if correct, could have seriously important consequences in a number of areas for each and every one of us.

Further it is suggested here that the notion of "uncertainty" within physical systems is only an anthropomorphic effects descriptor, not a causal description of physics. Apparent fluctuation effects in quantum systems and uncertain measurement effects are in fact caused by a real object and are not the result of probabilistic considerations; it is suggested that one possible explanation could be provided by the existence of an aether and the scalar waves within it. Quantum mechanics as interpreted by the Copenhagen interpretation is, in point of fact, incomplete as stated already. If accepted, this suggestion would imply that the wave function would need to be augmented in its interpretation to include representation of aetherial and scalar wave dynamics. If this were so, it appears that the adjusted theory would conceivably satisfy Einstein's highest standards as a physical theory.

Finally, consider once again the final paragraph of the 1958 English edition of Dirac's well-known monograph, entitled *The Principles of Quantum Mechanics:*

> It would seem that we have followed as far as possible the path of logical development of the ideas of quantum mechanics as they are at present understood. The difficulties, being of a profound character, can be removed only by some drastic change in the foundations of the theory, probably a change as drastic as the passage from Bohr's orbit theory to the present quantum mechanics.

From this, it would seem that Dirac is suggesting, even supporting, the introduction of fundamentally new theory to help in the resolution of difficulties inherent in the subject of quantum mechanics and this in 1958. Possibly the time has come to look afresh at new, possibly revolutionary, ideas and here it would seem appropriate to examine in a totally open-minded fashion the work of such as Mayants and, more importantly, Santilli since it seems distinctly possible that his ideas really do offer a way forward to the above-mentioned suggestion of Dirac. To Dirac it appeared that traditional quantum mechanics had taken theory as far as it could and a radical change to the foundations was necessary if further progress was to be made. It is now time to decide after fair, open-minded, public assessment if Santilli's new theories provide at least a first step in achieving that goal.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Einstein, A., Podolsky, B. and Rosen, N. (1935) *Physical Review*, **44**, 777. https://doi.org/10.1103/PhysRev.47.777

[2] Aczel, A.D. (2002) Entanglement. John Wiley & Sons Ltd., Chichester.

[3] Bohr, N. (1948) *Dialectica*, **2**, 312-319. https://doi.org/10.1111/j.1746-8361.1948.tb00703.x

[4] Bohm, D. (1951) Quantum Theory. Prentice Hall, Englewood Cliffs.

[5] Bohm, D. and Aharonov, Y. (1957) *Physical Review*, **108**, 1070. https://doi.org/10.1103/PhysRev.108.1070

[6] Wu, C.S. and Shaknov, I. (1950) *Physical Review*, **77**, 136. https://doi.org/10.1103/PhysRev.77.136

[7] Bell, J.S. (1966) *Reviews of Modern Physics*, **38**, 447. https://doi.org/10.1103/RevModPhys.38.447

[8] Bell, J.S. (1964) *Physics*, **1**, 195. https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195

[9] Gödel, K. (1931) *Monatsheft für Mathematik und Physik*, **38**, 173-198. https://doi.org/10.1007/BF01700692

[10] Mayants, L. (1984) The Enigma of Probability and Physics. D. Reidel Pub. Co., Dordrecht. https://doi.org/10.1007/978-94-009-6294-1

[11] Mayants, L. (1994) Beyond the Quantum Paradox. Taylor & Francis, London.

[12] Aspect, A. in Lingren, I., Rosen, A. and Svanberg, S. (eds.) (1983) Atomic Physics. Vol. 8, Plenum Press, New York.

[13] Santilli, R.M. (1998) *Acta Applicandae Mathematicae*, **50**, 177-190. https://doi.org/10.1023/A:1005831620903

[14] Fadel, M., *et al.* (2018) *Science*, **360**, 409-413. https://doi.org/10.1126/science.aao1850

[15] Schrodinger, E. and Born, M. (1935) *Proceedings of the Cambridge Philosophical Society*, **31**, 555-563. https://doi.org/10.1017/S0305004100013554

[16] Kunkel, P., *et al.* (2018) *Science*, **360**, 413-416. https://doi.org/10.1126/science.aao2254

[17] Lange, K., *et al.* (2018) *Science*, **360**, 416-418. https://doi.org/10.1126/science.aao2035

[18] Albert, A.A. (1948) *Transactions of the American Mathematical Society*, **64**, 552-593. https://doi.org/10.1090/S0002-9947-1948-0027750-7

[19] Santilli, R.M. (1967) *Nuovo Cimento*, **41**, 570-576. https://doi.org/10.1007/BF02902200

[20] Santilli, R.M. (1969) *Meccanica*, **1**, 3-21. https://doi.org/10.1007/BF02129014

[21] Santilli, R.M. (1978) Foundation of Theoretical Mechanics. Vol. I, Springer-Verlag, Heidelberg.

[22] Santilli, R.M. (1978) Foundation of Theoretical Mechanics. Vol. II, Springer-Verlag, Heidelberg.

[23] Santilli, R.M. (1995) Elements of Hadronic Mechanics. Vol. I, Ukraine Academy of Sciences, Kiev.

[24] Santilli, R.M. (1995) Elements of Hadronic Mechanics. Vol. II, Ukraine Academy of Sciences, Kiev.

[25] Santilli, R.M. (1995) Elements of Hadronic Mechanics. Vol. III, Ukraine Academy of Sciences, Kiev.

[26] Santilli, R.M. (2006) *Nuovo Cimento B*, **121**, 443-486.

[27] Gandzha, I. and Kadisvily, J. (2010) New Sciences for a New Era. Sankata Printing Press, Kathmandu.

[28] Santilli, R.M. (2001) Foundations of Hadronic Chemistry, with Applications to New Clean Energies and Fuels. Kluwer Academic Publishers, Dordrecht. https://doi.org/10.1007/978-1-4020-4004-7_7

[29] Tange, V.M. (2015) Foundation of Chemistry. https://doi.org/10.1007/s10698-015-9218-z

[30] Trell, E. (2003) *International Journal of Hydrogen Energy*, **28**, 251-253. https://doi.org/10.1016/S0360-3199(02)00031-9

[31] Fronteau, J., Tellez-Arenas, A. and Santilli, R.M. (1979) *Hadronic Journal*, **3**, 130.

[32] Dunning-Davies, J. (2006) *Progress in Physics*, **4**, 24.

[33] Bhalekar, A. (2013) *AIP Conference Proceedings*, **1558**, 702. https://doi.org/10.1063/1.4825588

[34] Schober, A. (1984) Irreversibility and Nonpotentiality in Statistical Mechanics. Hadronic Press, Palm Harbor.

[35] Santilli, R.M. (1979) *Hadronic Journal*, **3**, 440.

[36] Santilli, R.M. (1982) Lie-Admissible Approach to the Hadronic Structure. Vol. I, Hadronic Press, Palm Harbor.

[37] Santilli, R.M. (1982) Lie-Admissible Approach to the Hadronic Structure. Vol. II, Hadronic Press, Palm Harbor.

[38] Santilli, R.M. (1993) *Algebras*, *Groups and Geometries*, **10**, 273.

[39] Santilli, R.M. (1996) *Rendiconti Circolo Matematico Palermo*, No. 42, 7-82.

[40] Santilli, R.M. (1997) Isotopic, Genotypic and Hyperstructural Methods in Theoretical Biology. Ukraine Academy of Sciences.

[41] Bhalekar, A.A. and Vougiouklis, T. (2015) *AIP Conference Proceedings*, **1648**, Article ID: 510001. https://doi.org/10.1063/1.4912706

[42] Vougiouklis, T. (2011) *Algebras*, *Groups and Geometries*, **28**, 83-104.

[43] Illert, C.R. and Santilli, R.M. (1995) Foundations of Theoretical Conchology. Hadronic Press, Palm Harbor.

[44] Santilli, R.M. and Vougiouklis, T. (2020) *Algebras*, *Groups and Geometries*, **37**. (In Press)

[45] Fermi, E. (1950) Nuclear Physics. University of Chicago Press, Chicago.

[46] Blatt, J.M. and Wiesskopf, V.F. (1963) Theoretical Nuclear Physics. John Wiley, New York.

[47] Popper, K. (1982) Quantum Theory and the Schism in Physics. Hutchinson, London.

[48] Santilli, R.M. (1978) *Hadronic Journal*, **1**, 223, 574, 1279.

[49] Georgiev, S. (2014) Foundation of the Iso-Differential Calculus. Vols. I to VI, Nova Science Publishers, Inc., New York.

[50] Santilli, R.M. (1997) *Foundations of Physics*, **27**, 1169-1177. https://doi.org/10.1007/BF02551439

[51] Santilli, R.M. (2019) *Ratio Mathematica*, **37**, 5.

[52] Santilli, R.M. (2020) *Ratio Mathematica*, **38**, 5.

[53] Santilli, R.M. (2020) *Ratio Mathematica*, **37**, 71.

[54] Santilli, R.M. (2020) *Ratio Mathematica*, **37**, 139.

[55] Santilli, R.M. (1999) *Journal of New Energy*, **4**, 106.

[56] Rutherford, E. (1920) *Proceedings of the Royal Society A*, **97**, 374. https://doi.org/10.1098/rspa.1920.0040

[57] Chadwick, J. (1932) *Proceedings of the Royal Society A*, **136**, 692. https://doi.org/10.1098/rspa.1932.0112

[58] Santilli, R.M. (1990) *Hadronic Journal*, **13**, 513-515. https://doi.org/10.1002/clc.4960130716

[59] Santilli, R.M. (1993) JJINR Comm., E4-93-352.

[60] Borghi, C., *et al.* (1993) *J. Nucl. Phys. (Russian)*, **56**, 147.

[61] Heisenberg, W. (1930) The Physical Principles of the Quantum Theory. Dover, New York.

[62] Kumar, M. (2011) Quantum: Einstein, Bohr and the Great Debate about the Nature of Reality. W. W. Norton & Company, New York.

[63] Norman, R. and Dunning-Davies, J. (2018) *Hadronic Journal*, **41**, 1.

[64] Yin, *et al.* (2013) Bounding the Speed of Spooky Action at a Distance.

[65] Rafelski, J. and Müller, B. (1985) The Structured Vacuum. Verlag Harri Deutsch, Frankfurt.

[66] Freedom of Information Act Electronic Reading Room. https://www.cia.gov/library/readingroom/docs/CIA-RDP96-00792R000500240001-6.pdf

[67] Prandtl, L. (1904) Proc. 3rd Internat. Math. Congr.

[68] Cole, G.H.A. (1962) Fluid Dynamics. Methuen, London.

[69] Landsberg, P.T., Dunning-Davies, J. and Pollard, D. (1994) *American Journal of Physics*, **63**, 712. https://doi.org/10.1119/1.17502

[70] Clark, *et al.* (2017) *Nature*, **541**, 191-195. https://www.nature.com/articles/nature20604

[71] Thornhill, C.K. (1983) *Spec. Sc. Tech.*, **8**, 263.

[72] Farrell, D.J. and Dunning-Davies, J. (2007) New Research on Astrophysics, Neutron Stars and Galaxy Clusters. Nova Science Publishers, New York.

[73] Thornhill, C.K. (1983) *Spec. Sc. Tech.*, **8**, 273.

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Does Our Universe Conform with the Existence of a Universal Maximum Energy-Density ρ_{max}^{uni}?

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Abstract

Recent astronomical observations of high redshift quasars, dark matter-dominated galaxies, mergers of neutron stars, glitch phenomena in pulsars, cosmic microwave background and experimental data from hadronic colliders do not rule out, but they even support the hypothesis that the energy-density in our universe most likely is upper-limited by ρ_{max}^{uni}, which is predicted to lie between 2 to 3 the nuclear density ρ_0. Quantum fluids in the cores of massive NSs with $\rho \approx \rho_{max}^{uni}$ reach the maximum compressibility state, where they become insensitive to further compression by the embedding spacetime and undergo a phase transition into the purely incompressible gluon-quark superfluid state. A direct correspondence between the positive energy stored in the embedding spacetime and the degree of compressibility and superfluidity of the trapped matter is proposed. In this paper relevant observational signatures that support the maximum density hypothesis are reviewed, a possible origin of ρ_{max}^{uni} is proposed and finally the consequences of this scenario on the spacetime's topology of the universe as well as on the mechanisms underlying the growth rate and power of the high redshift QSOs are discussed.

Keywords

General Relativity: Neutron Stars, Incompressible Superfluids, Quantum Chromodynamics, Cosmology: Big Bang Physics, Dark Matter and Dark Energy, Quasars, First Generation of Stars

1. Introduction

Very recently we have witnessed a historical breakthrough in observational astronomy: the beginning of the Multi-Messenger era. The accurate and successful calibration of instrumentations operating remotely at different frequency regimes has successfully enabled us to detect astronomical events with unprecedented accuracy. Thanks to modern communication technologies that enable pairing remote telescopes and form global networks of virtual telescopes that provided the highest resolution imaging currently possible at any wavelength in astronomy [1] [2]. Based thereon, observation of highly energetic astrophysical events, such as mergers of neutron stars and black holes, GRBs, Kilonova resulting from NS-mergers, imaging the event horizons of black holes [3] [4] as well as quasars at extremely high redshift [5] delivered a lot of data that enabled us to gain deeper insight of the underlying physics. Other breakthroughs have been recorded on the micro-scales, such as the Higgs bosons at the Large Hadronic Collider (LHC), and the fluidity character of quark-gluon plasmas at the RHIC (see [6] [7]).

On the other hand, while modern observations are continuously shedding light on the huge diversity of the state of matter both on the micro and macro-scales, they also raise new questions related to the validity of solutions of problems that have been thought to have settled.

Based on the recent theoretical and observational studies, it is argued in this paper that:

- If ρ_{max}^{uni} indeed exist, then the remnant of the merger of the two neutron stars in GW170817 should be a massive neutron star (NS) whose core is made of an incompressible quark-gluon superfluid that obeys the laws of quantum field theories (see **Figure 1** and the reference [8]). However, the ambient

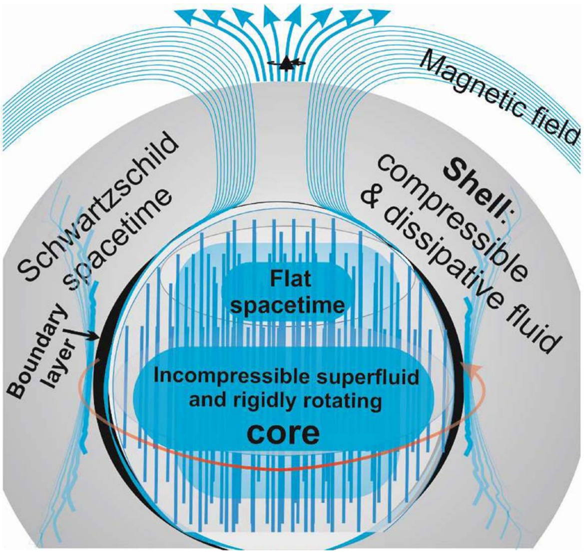

Figure 1. A schematic description of the internal structure of a neutron star and the topology of the embedding spacetime. There are three different regions that are relevant for the present discussion: the central core, which is made of incompressible superfluid and embedded in a Minkowski-type spacetime. The surrounding shell, inside which the matter is compressible, dissipative and embedded in a Schwarzschild-type spacetime. The geometrically thin boundary layer between the core and the surrounding shell, where matter and spacetime repeatedly undergo phase transitions.

media remain compressible and dissipative, though these properties are doomed to disappear at the end of the luminous lifetime of NSs, rendering them invisible.

These cores appear to be generic for almost all massive NSs, whose masses and dimensions are set to grow as NSs age. Within the context of general relativity, a physically consistent and causality-preserving treatment of the two totally distinct fluids: incompressible-superfluid and compressible-dissipative normal fluid, is possible, if the embedding spacetime is of a bimetric type: a Minkowski flat at the background of the core surrounded by a Schwarzschild curved one.

- The above argument is extended to show that the history and large-scale structures of our universe may indeed tolerate the existence of ρ_{max}^{uni}. Also, I address and discuss several relevant observational signatures both on the micro and macroscopic scales that support the maximum energy density hypothesis.
- If such a density upper-limit does exist, then what could be the underlying physical mechanisms that may lead to the existence of ρ_{max}^{uni}?

2. Observational Signatures

2.1. The Origin of Power of High Redshift QSOs and Their Growth Problem

Early large-scale structures are considered to have formed during the first two billion years after the big bang. Indeed, the three UV-emission lines from the luminous star-forming galaxy GN-z11 which has been recently analyzed using Hubble Space Telescope (HST) imaging data, most likely correspond to emission at $z = 10.957 \pm 0.001$, *i.e.* to roughly 420 Myr after the big bang [9] [10]. Taking into account that GN-z11 is roughly 100 less massive than our Galaxy, then GN-z11 is expected to host a SMBH at its center. Similarly, recent observations of the quasar ULAS J1342+0928 reveal that its redshift is $z = 7.54$ (see [5] and the references therein), which implies that the object must have been there already when the universe was 700 Myr old. Such rapid growths cannot be explained, unless the seeds are massive BHs of at least $10^6 M_\odot$ [11] [12]. Therefore primordial black holes that were formed either within the first 10^{-23} s or during the first 10^{-5} s after the big bang should be ruled out, due to lacking observational evidence for their existences [13].

Irrespective of the underlying growths mechanisms, their mass increase should follow the equation of mass conservation:

$$\frac{\mathrm{d}\mathcal{M}}{\mathrm{d}t} = \sum \int_S \bar{\boldsymbol{f}} \cdot \mathrm{d}\boldsymbol{s} = a_0 \left(\frac{\beta_v}{\beta_c^4} \right) \rho \mathcal{M}^2 + \dot{\mathcal{M}}_{ext}, \tag{1}$$

where $a_0 = 16\pi G^2/c^3 \approx 10^{-6}$ and β_v, β_c, ρ correspond to the relative fluid velocity, v/c, sound speed, V_s/c and the density of the inflowing matter through the boundary in units of 10^{-10} g/cc, respectively. Here $\mathcal{M}/t$ are in units of $5\times10^7 M_\odot$ per year. $\dot{\mathcal{M}}_{ext}$ denotes other growth mechanisms, such as direct collapse of primordial gas clouds, runaway collisions of dense star in clusters, hyper-Eddington accretion as well as episodic galaxy mergers (see [12] and the references therein). Assuming $\mathrm{d}\left(\beta_v \rho / \beta_c^4\right) / \mathrm{d}\mathcal{M} \ll 1$, then the time-dependent mass should evolve as:

$$\frac{\mathcal{M}(\tau)}{\mathcal{M}_0} = \frac{1}{1 - a_0 \left(\frac{\beta_v}{\beta_c^4}\right) \rho \tau \mathcal{M}_0}, \tag{2}$$

where τ corresponds to the elapsed time for a SMBH of initial mass $\mathcal{M}_0$ to reach $\mathcal{M}(\tau)$. For $\mathcal{M}(\tau)/\mathcal{M}_0 = 100$ and $\mathcal{M}_0 = 1$, we obtain roughly:

$$\tau_{100} \approx \frac{10^6}{\rho} \times \left(\frac{\beta_c^4}{\beta_v}\right) = \begin{cases} > 10^9 & R_A = R_{LSO}\left(\beta_v = \beta_c \approx 1\right) \\ \mathcal{O}\left(10^9\right) & \text{Super-Eddington Accr.} \\ & \text{collapse of clouds, collision and merger.} \end{cases} \tag{3}$$

R_{LSO} here is the last stable orbit, where $\beta_v = \beta_c \approx 1$. The Eddington accretion is used to estimate the density ρ. In the case of super-Eddington accretion caused by direct collapse of clouds, star collision and mergers, the matter is expected to be increasingly hotter as the center of mass is approached, hence more difficult to compress, thereby giving rise to slow and episodic accretion.

The conclusion here is that current scenarios are incapable of providing viable solution to the rapid growth of SMBHs within the first 600 Myr after the big bang. This problem will soon become more prominent, when advanced observations shed the light on the existence of QSOs at $z \geq 10$, which would logically imply that their existence of QSOs at $z \gg 10$ cannot be ruled out.

Moreover, high redshift QSOs may have been there already before the big bang, and that their nature must be more complicated than being objects with mathematical singularities at their centers.

2.2. Compact Objects and the Mass-Gap

Astronomical observations reveal a gap in the mass spectrum of compact objects: neither black holes nor NSs have ever been observed in the mass range $\left[2.5 M_\odot \leq \mathcal{M}_{NS} \leq 5 M_\odot\right]$. Even the nature of the remnant of the well-studied NS-merger event, GW170817, whose mass is predicted to be around 2.79, is still not conclusively determined on whether it is a massive NS or a stellar-mass BH (see [8] [14] and the references therein).

Recently it was suggested that when massive NSs cool down during their cosmic evolution, their incompressible superfluid cores should grow both in mass and volume to finally fade away as dark energy objects (DEOs; see the references [15] [16]). Based on observational data of glitching pulsars, and specifically of the Crab and Vela pulsars, this scenario appears to successfully explain various aspects of the glitch phenomena, such as the growth of mass and volume of their cores, as well as of the mechanisms underlying the under and overshootings observed to accompany their prompt spin-up (see [17] [18] and the references therein). Recalling that the Crab and Vela are considered to the most well-studied pulsars in our galaxy, the remarkable agreement of the scenario with observations was possible only, because of the assumption that their cores are made of incompressible quark-gluon superfluids.

Based thereon, isolated massive NSs either in the local or early universe should metamorphose into DEOs, whose embedding spacetimes is of Minkowski type.

2.3. Dark matter in the Early Universe

Dark matter (DM) in cosmology is an essential pillar for understanding the entire cosmic web: starting from structure formation, clusters of galaxies and galaxy formation down to stellar-mass objects. Except for gravitational interaction with normal matter, the physics of dark matter continues to be a conundrum in understanding our universe.

For the present discussion, we select three properties that support the constant density hypothesis:

When a massive ultra-compact object[1] (UCO) run out of all their secondary energiesc[2], the embedding spacetime should have compressed the matter inside their interiors up to the maximum limit, at which matter undergoes a phase transition into incompressible gluon-quark superfluid, which corresponds to the lowest possible quantum energy state. Objects of this type are both electromagnetically and gravitationally passive and are classified here as DEOs.

Indeed, the ultra diffuse galaxy, Dragonfly 44, is considered to be a dark matter dominated galaxy [19] [20]. Its dwarf-like shape and the extremely low surface brightness indicate that it might be the relic of an old massive galaxy, in which the stars have run out of their secondary energies during the course of their cosmic evolution. Also, it was argued that the lack of X-ray emission is an indication that most of the compact objects in the galaxy are relatively old and therefore turned dark by now. Assuming star formation rates and supernovae statistics to apply for this galaxy during its active phase in the past, then roughly 1% of its population must be in the form of NSs and 0.1% stellar black holes. While BHs need to swallow matter in order to be detected, NSs may still eject highly relativistic particles from their polar caps and therefore at least 0.001% of the NS population should be detectable. However, such signatures appear to be missing in the D44 galaxy.

In the context of our scenario, the old and massive NSs in D44 must have metamorphosed into DEOs that subsequently migrated either outwards into the halo or inwards where they conglomerate to form a cluster of massive DEOs. The time duration required for these objects to randomly migrate from the central region of a galaxy to the surrounding halo or vise versa may is of order $\tau_{diff} \sim L^2/\nu_{eff}$, where L is the average radius of the galaxy and ν_{eff} is a measure

[1]A family of objects that include pulsars, neutron stars, and magnetars.

[2]All other types of energies except the rest energy.

of their gravitational scattering, $\sigma_{DNS\text{-}Star}$, times the duration of their momentum exchange with other objects $\tau_{DNS\text{-}Star}$. Assuming the masses of both types of objects to be of order the solar mass, then $\sigma_{DNS\text{-}Star} \approx 10^{-4} R_{\odot}^{2}$. On the other hand, $\tau_{DNS\text{-}Star}$ can be set to be of order $\sqrt{\sigma_{DNS\text{-}Star}}/V_{rel}$, where V_{rel} is the relative velocity of the two arbitrary encountering objects (see [21] for further details(. Setting a maximum relative velocity of 100 km/s, we obtain τ_{diff} of order 10^3 times the age of the universe.

Consequently, massive NSs that run out of their secondary energies and became DEOs either prior to or post the big bang may be considered as DM-candidates. Their poor collisions and mergers may be attributed to the topology of the bimetric spacetimes embedding DEOs. This picture is in line with recent observational and theoretical studies indicating that DM may have been present already in the pre-big bang era and/or during cosmic inflation [22] [23]. However, unlike our present macroscopic approach, the underlying assumption of the theoretical study in [24] is that DM should be made of microscopic particles of unknown origin that emerged out of scalar fields.

2.4. Merger of NSs and Glitching Pulsars

Observations reveal that NSs occupy a relatively narrow mass range that lies between $1.2M_{\odot}$ and $2.2M_{\odot}$, though the error bars in many cases are relatively large. Despite the use of modern instrumentations and global coordination of multi-messenger observations, their radii, which should decrease with increasing mass, are still rather uncertain,.

Both theory and numerical calculations indicate that the central density of neutron stars must be larger than the nuclear ρ_0. However probing the state of ultracold supranuclear dense matter under terrestrial conditions is unfeasible and the governing physics remains rather uncertain. Therefore, when modeling the internal structure of NSs, the regularity condition usually imposed at the centers of UCO is actually identical to the incompressibility condition, which is a limiting case of:

$$\frac{\Delta\varepsilon}{\Delta r} \xrightarrow[\Delta r \to 0]{} \nabla\varepsilon = 0. \tag{4}$$

Microscopically Δr is limited from below by the average separation between two arbitrary baryons Δ_{bb}. Using $\Delta r = \Delta_{bb}$ and $\Delta\varepsilon\Delta t \approx \hbar$, we obtain that $\Delta r \gg 10^{10} \times \Delta_{bb}$. Among others, this implies that incompressibility is a macroscopic property, but that quantum fluids must fulfill on the microscopic scale at $r = 0$. In this case, the pressure here ceases to behave as a local thermodynamical quantity and it becomes solely a Lagrangian multiplier: a mathematical term that dictates the global dynamics of the quantum fluid, which in turn depends strongly on the topology of the embedding spacetime. This pseudo-pressure should be carefully constructed, so that its transition into the ultrabaric regime remains forbidden, $\mathrm{d}p/\mathrm{d}\varepsilon \geq 1$, and therefore causality condition is grossly violated (see Figure 7 in the reference [15]).

Indeed, we consider the following observational signatures to support our scenario:

- Isolated NSs with $\mathcal{M}_{NS} \geq 2.5M_{\odot}$ are practically missing and theoretically difficult to model, irrespective of EOSs used.
- Based on the multi-messenger observations of the NS-merger event GW-170817, the remnant should have a mass of about $2.69M_{\odot}$, though its nature is still an open question. In particular, observations did not conclusively rule out that the remnant might be a massive NS. In fact recent numerical calculations of merging two incompressible superfluid cores trapped in a potential that mimics an embedding curved spacetime show that such cores are capable of merging without decaying (see **Figure 2** and the reference [25]). Combining this numerical observation with the possibility that the merger of the overlying compressible and dissipative ambient media of both objects may switch off the dynamo action of the remnant, thereby diminishing most magnetic activity not a short period.
- The discrete events of glitches observed in pulsars are considered to be the responses of their cores to the long-term cooling of the objects. Moreover, the majority of glitching pulsars appear to have the following common features:
    - Glitches are associated with the evolution of pulsars and young NSs, they occur instantly but the corresponding relaxation time increases as they age. Recently, accurate observations revealed certain association of under and overshootings with the prompt spin-up events of glitching pulsars [17] [18].

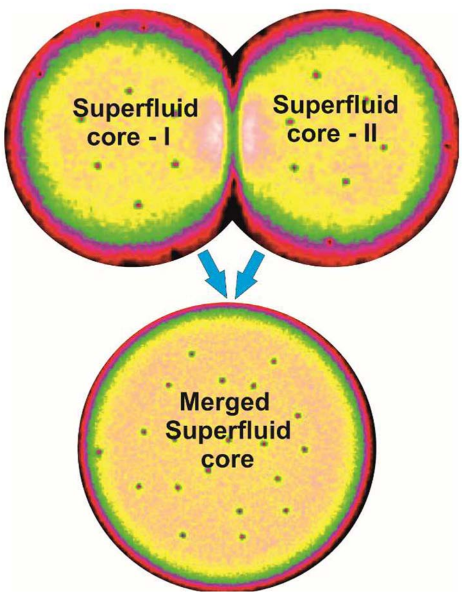

Figure 2. By means of numerical solving the modified time-dependent Gross-Pitaevskii equation in 2D, the merger of two superfluid cores trapped in a symmetric external potential have been calculated. The energy loss due to merger of the two cores was found to decrease with decreasing the numerical diffusion, *i.e.* with increasing the spatial and temporal accuracies.

- In the $\dot{P}$-P diagram, almost all observed glitching pulsars appear to be younger than 10^7 yr and their magnetic fields are stronger than 10^{11} Gauss.
- The reoccurrence of glitches decreases with pulsar's age [26] [27].
- UCOs, with magnetic fields larger than the quantum critical value $B > B_c = m^2c^3/e\hbar \approx 4.4\times10^{13}\,\mathrm{G}$, are found to occupy a narrow range in B-age diagram, where the majority of magnetars are found to be younger than 10^5 yr (see **Figure 3**; and the references [28] [29]).

As it was shown in [8] [30], these features may easily be explained by invoking the bimetric spacetime scenario. Accordingly, cores of massive NSs are made of incompressible superfluids embedded in Minkowski spacetimes, whereas the surrounding compressible and dissipative media are embedded in Schwarzschild-type spacetimes. This scenario is consistent with general relativity as the mass-energy inside the cores dictates the topology of the embedding spacetime and vise versa.

2.5. The Missing First Generation of Massive Neutron Stars

Within the framework of the big bang scenario, the first generation of stars must

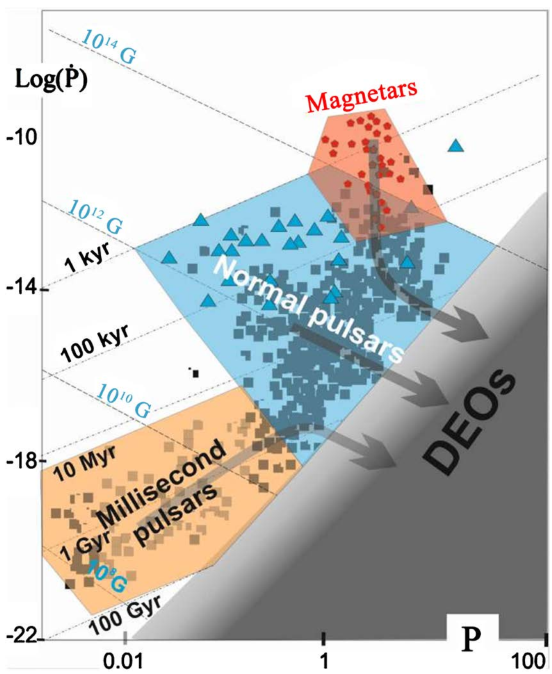

Figure 3. The derivative of the period time, $\dot{P}$, versus period P of the population of UCOs and their tendency to migrate (marked with arrows) toward the lower-right region, where they end up as DEOs. There are three distinct classes that can be identified: millisecond pulsars (gray squares/orange region), glitching and normal pulsars (triangles & squares/blue region) and magnetars (pentagons/red region). All types of UCOs should migrate into the lower right region, where they end up as DEOs. While migration of strongly magnetized isolated and massive UCOs should proceed relatively fast, those with intermediate masses in binaries and/or in accretion mode generally need much longer time to migrate.

have formed approximately 500 Myr after the big bang [31]. The progenitor clouds were made mainly of atomic hydrogen with an average temperature of about 1000 K. This relatively high temperature enables cloud to collapse under their own self-gravity, if the mass content is larger than the critical Jeans mass amounting to 10^2 up to 10^4 solar masses. Hence the resulting stars must have been extraordinary luminous and therefore short-living, and expected to subsequently collapse into BHs or massive NSs. While BHs are generally accepted to be the remnants of collapsed population III stars, NSs however have been neither investigated nor observed. Moreover, given the relatively large dimensions and long time scales characterizing the progenitor clouds in combination with rotation, it is not at clear, why the centrally collapsing portion of the cloud with its relatively short dynamical time scale should proceed self-similar and fail to blow away the outer slowly contracting shells or forbids fragmentation of the collapsing cloud. We note that the effect of numerical diffusion in the simulations of the collapsing clouds is to enhance the coupling of the different parts of the cloud and facilitate their direct collapse. In real life, however, the cores of collapsing clouds will be much hotter and the advected magnetic fields via ambipolar diffusion will soon be strong enough to fragment the cloud. Such sophisticated and robust numerical solvers are still to be developed in order to account for important effects, such as radiation transfer, ambipolar diffusion, multi-component fluids, large scale magnetic fields and so on.

Nevertheless, to form a BH, the spacetime embedding the progenitor should have compressed the central core and increased its central density up to the critical density ρ_{cr}, beyond which normal repulsive forces fail to oppose further compression. In this case, the event horizon r_H should surpass the actual radius of the central core, and the enclosed average density must surpass the critical value:

$$\rho_{cr} \approx 1.6\times 10^{27}\frac{1}{r_H^2}. \tag{5}$$

For r_H of order 10^6 cm, one obtains $\rho_{cr} \approx 6\,\rho_0$, where ρ_0 is the nuclear density.

Obviously, the reaction time scale of the core at the verge of collapse is of order micro-seconds, which is comparable to the light crossing time. Hence the core is well-equipped to oppose all types of external forces including compressional forces. In this case, the accumulated matter on the surface most likely would bounce rather than triggering a collapse of the object into a BH. Indeed, most sophisticated numerical modeling of core-collapse supernovea give rise to bouncing rather than to direct collapse into a stellar BH [32], provided the domain of calculations does not contain an apparent horizon, which is manually created by changing the EOSs long before ρ_{max}^{uni} has bee reached and/or surpassing the maximum compressibility limit [33]. Moreover, most of today's numerical solvers are still neither capable of modeling incompressible supranuclear dense matter inside the cores of massive objects that are on the verge of collapsing into BHs nor treating the topology of the embedding spacetime in a self-consistent manner and therefore are irrelevant for the present discussion.

On the other hand, the multi-messenger observations of NS merger event, GW170817, suggest that violent merger of NSs may not necessarily lead to BH formation, but that forming a dynamically stable massive NS with $\mathcal{M}_{NS} > 2.79 M_{\odot}$ should not be ruled out either [8]. The luminous lifetime of such a massive NS would be extremely short and would soon turn invisible DEO.

Consequently, as the first generation of stars must have been massive, their collapse should have generated a numerous number of massive NSs. Their luminous lifetime would be relatively short and therefore they should have entered their final evolutionary phase as DEOs long ago. On the other hand, supernovae statistics predict that at least 1% of star populations in star-forming clouds should be NSs. Although this rate is expected to be even higher in the early universe, yet isolated NSs older than one Gyr have not been observed. The topology of the spacetime embedding DEOs is of a bimetric type, which in turn prohibits their merger and makes them to excellent BHs and DM-candidates.

2.6. Hadronic Collisions: LHC and RHIC

Among many recent explorations related to particle collisions in labs, there are several discoveries that should be relevant for the present discussion:

- Fluidity of quark-gluon plasmas

Quark-gluon-plasmas emerging from smashed nuclei both at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) were verified to behave nearly as perfect liquids [6] [34]. This was a turning point in physics, as the entropy of such colliding particles under normal laboratory conditions is extremely high and the outcome is expected to be a dilute gas: an assembly of loosely connected particles that running out of the center of collisions almost in spherical symmetric patterns.

Recalling that QGPs are imperfect due to unitarity, the cross-sections of colliding particles ought to be bounded from below. Indeed, based on infinitely coupled supersymmetric Yang-Mills gouge theory (SYM) and using anti-de Sitter space/conformal field theory (ADS/CFD) duality conjecture, the lower-limit was verified to be:

$$\left(\frac{\eta}{s}\right)_{SYM} = \frac{1}{4\pi}, \tag{6}$$

which is roughly equal to the lower-limit obtained using the uncertainty principle [35].

While the physics governing particle collisions under terrestrial conditions are totally different from those at the center of massive NSs, the emergence of quarks and plasma out of nucleons as liquids can be considered as a strong evidence for the here-proposed hypothesis. Indeed, let the velocity of particles immediately after collisions be of order the sound speed: $V \approx V_s = c/\sqrt{3}$. This yields a dynamical time scale $\tau_d = \lambda/V_s = \alpha_\lambda r_b/c$. On the other hand, the viscous time scales as:

$$\tau_{vis} \approx \frac{\lambda_\lambda^2}{v_{vis}} = \frac{\alpha_\lambda^2 r_b^2}{\alpha_{vis} V_s \langle \ell_{vis} \rangle} = \frac{\alpha_\lambda^2}{\alpha_{vis}\alpha_\ell}\tau_d, \tag{7}$$

where α_{vis} denotes the effective viscous velocity in units of the sound speed and α_ℓ the corresponding viscous length scale in units of r_b. Given that elliptic flows start shaping already during the initial expansion, then $\frac{\alpha_\lambda^2}{\alpha_{vis}\alpha_\ell}$ should be of order unity, which yields the Reynolds number:

$$Re = \frac{\tau_{diff}}{\tau_{dyn}} \approx 1. \tag{8}$$

Going back in time, this result would imply that the confined QGPs, inside the sample of nucleons under lab conditions, are dissipative liquids with non-vanishing viscosity over entropy, *i.e.* $0 < \eta/s < 1$.

Therefore the energy state of the QGPs here should decay into a lower energy state to finally hardonize into stable baryons.

However, in low Reynolds number flows, the effect of viscosity is to smooth out irregularities and prohibits the formation of distinct flow patterns, such as the micro-elliptical flows of deconfined QGPs, whose patterns considerably deviate from a pure spherical expansion. This implies that the dynamical time scale, on which particle collisions at the LHC/RHIC occur, are still too long compared to the time scales on which gluon effects are operating during the hardonization of the QGP.

In the case of NSs, as they cool down on the cosmic time, their heat contents decrease and the supranuclear dense matter inside their cores approaches the limit of maximum compressibility. Here both the temperature and entropy should go to the zero limit. From statistical thermodynamical point of view, the number of possible microstates of constituents of incompressible superfluid reduces to just one single quantum energy state, where all constituents are forced to occupy. This corresponds to the zero-degree of freedom and therefore to vanishing entropy and viscosity:

$$\lim_{T\to 0}\eta = \lim_{T\to 0} s = \frac{k_B}{4\pi}\sum_{n=1}^{\infty}\ln(\Omega_i) = \frac{k_B}{4\pi}\ln(1) = 0 \tag{9}$$

This implies that $\frac{\eta}{s}$, η should be more sensitive to temperature's variation than entropy, *i.e.*,

$$\lim_{T\to 0}\left(\frac{\eta}{s}\right) = \lim_{T\to 0}\frac{\left(\frac{d\eta}{dT}\right)}{\left(\frac{ds}{dT}\right)} = \lim_{T\to 0} T^\gamma = 0, \tag{10}$$

where γ is an arbitrary positive constant. This convergence behaviour is in line with energy conservation, as viscous energy extraction from an incompressible fluid is upper-limited by the availability of secondary energies: $dQ^{vis} < TdS$.

- The transition from a maximally compressible into a purely incompressible fluid-phase should be associated with instantaneous input of free energy. This is a consequence of global energy conservation: The gravitational field, or equivalently, the energy stored in the curved spacetime embedding the ambient compressible media may be calculated using the surface integral (see [36] for further details):

$$\Delta E_{tot}^{g} = \frac{1}{16\pi}\int_{S^n}^{S^{n+1}} \mathrm{d}^2 S^j \left(\frac{\partial}{\partial x^k} g_{jk} - \frac{\partial}{\partial x^j} g_{kk} \right), \tag{11}$$

where the domain of integration, $\mathcal{D}^{n+1}$, is bounded by the surfaces S^n and S^{n+1}, and g_{jk} is the spacetime metric. As UCOs cool down, the topology of the manifold $\mathcal{D}^{n+1}$ changes from a curved into a flat one, and therefore ΔE_{tot}^{g} becomes free. As the matter has reached the maximum compressibility limit at zero-entropy, ΔE_{tot}^{g} cannot be absorbed locally, and therefore it is conjectured that this energy goes into enhancing the surface tension, *i.e.* the "bag energy" confining the enclosed continuum of gluon-quark medium.

For $\rho = \rho_{max}^{uni}$, the energy per particle may be estimated as follows: since the separation between baryons vanishes, *i.e.* $\Delta_{bb} = 0$ (see **Figure 4**). In this state the distance between quarks initially belonging to different baryons reduces to $\Delta_{qq} = \ell_{qq}$, which is assumed to be the smallest possible distance between quarks at $T = 0$. By reducing Δ_{qq} to ℓ_{qq}, an energy of order $\Delta\varepsilon^{+} \approx c\hbar/(2\ell_{qq}) \approx 1/3\,\mathrm{GeV}$ becomes available, but which cannot be absorbed locally[3]. $\Delta\varepsilon^{+}$ may be viewed as an excited energy state of the carriers of the rest nuclear forces between baryons, which is subsequently converted into tensorial surface tensions that keep the enclosed quarks and gluons inside super-baryons stably confined. When integrating $\Delta\varepsilon^{+}$ over all baryons that reached $\rho = \rho_{max}^{uni}$, one should obtain energy that is equal to the rest energy of the individual and infinitely-separated baryons. For $\rho = \rho_{max}^{uni}$, this extra-energy is necessary for facilitating the phase transition of the fermionic QGPs inside individual baryons into the Bosonic superfluid phase, where the mass-energy of the constituents is uniformly distributed with restored Chiral symmetry.

The process here is reversible, as once the symmetry is broken, the super-baryon starts decaying, thereby liberating $\Delta\varepsilon^{+} = \Delta E_{tot}^{g}$ in a run-away hadronization procedure.

A straightforward consequence of this scenario would be that the first generation of massive NSs should have metamorphosed into invisible dark energy objects (DAOs), to end up doubling their initial mass. At a certain point in their cosmic history, the cores, would have to decay, thereby hardonizing their entire contents and giving rise to a monstrous explosion through which each DEO would liberate roughly 10^{57} GeV. Here the entire content of baryons is jettisoned into space almost with the speed of light. Recalling that the lifetime of neurons

[3] $\Delta\varepsilon^{+}$ is still lower than the energy required to raise the quarks and gluons inside baryons at $\rho = \rho_{max}^{uni}$ and vanishing entropy to the next excited energy state. This energy, which originates from vacuum energy between baryons, is termed here as "dark energy".

in free space is extremely short compared to protons, then the present scenario may nicely explain the dominant abundant of hydrogen in our universe.

- As DEOs are made of SuSu matter, which is characterized by a single characteristic speed: the speed of light and as they are embedded in purely flat spacetime, then the decaying front should propagate almost with the speed of light and would turn the object apart in less than 10^{-5} s. The straightforward consequence here is that the ultra-relativistic outward-propagating baryons would leave the center sufficiently fast, before the embedding spacetime starts changing its curvature significantly. At such high speeds, the baryons are too energetic and elements heavier than hydrogen are unlikely to form, which in turn, may explain the dominant abundance of hydrogen atoms in our universe.

3. A Possible Origin of ρ_{cr}^{uni}

Given the fundamental constants, such as the Planck constant $\hbar$, the speed of light c and the gravitational constant G, one may construct the following scales:

$$\begin{aligned} \ell_p &= \sqrt{\frac{\hbar G}{c^3}} = \mathcal{O}\left(10^{-33}\right)\text{cm}, \\ m_p &= \sqrt{\frac{\hbar c}{G}} = \mathcal{O}\left(10^{-5}\right)\text{g}. \end{aligned} \tag{12}$$

The inclusion of G in these two expressions is purely ad hoc and supported by neither astronomical observations nor experimental data. However, it suggests a range of scales, where the effects of both GR and quantum fields may overlap theoretically.

On the other hand, the spacetime embedding an incompressible supranuclear dense superfluid, such as inside the cores of massive NSs, is conformally flat [30], gravity-effects diminish, and therefore the characteristic length and time scales governing such fluids are independent of G.

Moreover, the incompressibility condition, *i.e.* $\mathrm{d}\varepsilon/\mathrm{dVol} = 0$, implies that the separation between the massive constituents making the mass-energy of the fluid should saturate around a minimum length scale ℓ_{min}^{uni}. Recalling that the effective masses of baryons are endowed by quarks and gluons, ℓ_{min}^{uni} is expected to mainly be related to quarks. Indeed, the separation between nucleons in atomic nuclei, Δ_{bb}, is generally larger than the radius of a single nucleon, which can be reduced through compression exerted either by the curvature of the embedding spacetime or by momentum-exchange during collisions with other nuclei.

Based on experimental and theoretical studies as well as observations of NSs, we argue that ℓ_{min}^{uni} should lie in the interval:

$$\left[\frac{1}{4} \le \frac{\ell_{min}^{uni}}{r_b} \le 1\right], \tag{13}$$

where r_b is the radius of the a baryon at zero-temperature.

Under normal terrestrial conditions, the nuclear density is roughly

$n_0 \approx 0.16/\text{cm}^3$, where the radius of the proton lies within the range: $\left[0.84 \le r_p \le 0.87\right]$ fm [37].

$\ell_{min}^{uni} < 1/4$ should be excluded as the resonance energy[4] would largely exceed the energy required for deconfining quarks inside individual baryons. Also, it would correspond to matter density $\rho \gg 10 \times \rho_0$; much larger than the critical density beyond which intermediate massive NSs collapse into BHs (see Equation (5)).

On the other hand, using $\ell_{min}^{uni} \approx r_b$, the resulting resonance energy amounts to $\Delta\varepsilon \approx 0.23\,\text{GeV}$. Summing over all possible numbers of quark-bonds, would yield roughly the rest energy of an individual baryon.

Based thereon, the volume of a massive NS at the end of its luminous lifetime would be the sum of the volumes of the enclosed baryons at zero-temperature, *i.e.*

$$\mathcal{V}_{NS}^{t=\infty} = \sum_{n=1}^{N} \mathcal{V}_n^b = N \times \mathcal{V}_0^b, \tag{14}$$

where $N, \mathcal{V}_0^b$ denote the number of merged baryons and the volume of a single baryon at zero-temperature, respectively.

For an intermediate massive NS with $N = 10^{57}$ baryons and $\ell_{qq} = \Delta_{qq} = r_b = 0.85\,\text{fm}$, we obtain a radius $R_{NS}^{t=\infty} = N^{1/3} r_b = 1.0168 \times 10^6\,\text{cm}$. While $R_{NS}^{t=\infty}$ here is larger than the corresponding Schwarzschild radius, it is still smaller than the last stable orbit, though these comparisons are irrelevant, as the spacetime inside $R_{NS}^{t=\infty}$ is Minkowski flat and not curved Schwarzschild.

Recalling that the action of compression of baryons by the embedding curved spacetime under zero entropy conditions, is to mainly transform the positive energy ΔE_{tot}^g (see Equation (11)) into storable energy $\mathrm{d}W$. From global energy conservation, one finds:

$$\Delta E_{tot}^g = \mathrm{d}W = -P\mathrm{d}\mathcal{V} = -P\left(\mathrm{d}\sum_{i=1}^{N} \Delta V_i\right), \tag{15}$$

where $\mathcal{V}$ consists of the sum of compressible space between baryons, ΔV_{bb}, and those of the incompressible gluon-quark plasma inside baryons at zero-temperature, $\mathcal{V}_0^b$. For a given number N of baryons, we obtain $\mathcal{V} = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \Delta V_i + N\mathcal{V}_0^b$, where $\Delta V_i(r)$ is a radius-dependent quantity which is expected to increase as the surface of the object is approached.

Once the neighboring baryons are compressed together and came into direct contact with each other (see **Figure 4**), ΔV_i reaches the lower limit:

$$\Delta V_i \xrightarrow[\Delta_{qq} \to \ell_{qq}]{} \mathcal{V}_0^b, \tag{16}$$

which corresponds to the maximum compressibility state of matter. Under these critical conditions, the available free energy due to spacetime compression cannot be stored locally. One reasonable possibility would be that the quantum fluid undergoes a phase transition into a pure incompressible state, and the free energy

[4] *i.e.* maximum energy resulting from the uncertainty in the length scale: $\Delta\varepsilon_{UP} \le c\hbar/\ell$.

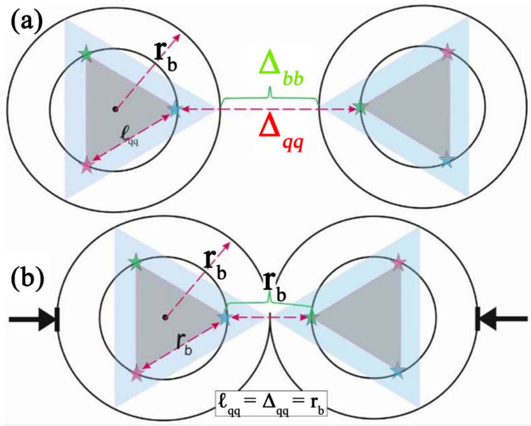

Figure 4. A schematic description of arbitrary two baryons in the maximally compressible state on the verge of merging to form incompressible gluon-quark superfluid. Here the optimal separation between three quark flavour inside a baryon at ρ_{max}^{uni} and $T = 0$ is $\ell_{qq} = \Delta_{qq} = r_b = 0.85\,\text{fm}$.

goes into enhancing the pairing processes of quarks and energize the confining force of the quarks inside the super-baryon.

As a consequence, the enhanced energetics of the super-baryon implies that the embedding curved spacetime must flatten. The limiting case here would be a vanishing gravitational energy, ΔE_{tot}^{g}. This energy state corresponds to the end cosmic phase of NSs, where they become invisible DEOs embedded in Minkowski spacetimes. In this case, the total mass-energy of a single DEO consists of:

$$\begin{aligned} E_{tot}(t=\infty) &= E_{rest} + \Delta E_{tot}^{g}\Big|_{t=0} \\ &= N \times 0.931\,\text{GeV} + E_g\Big|_{t=0} \\ &= 2 \times N \times 0.931\,\text{GeV}, \end{aligned} \tag{17}$$

where $\Delta E_{tot}^{g}\Big|_{t=0}$ is the initial gravitational energy stored in the curved spacetime embedding a newly born pulsar (see **Figure 5**).

In the absence of destructive external forces, the gravitational and thermodynamical properties of DEOs suggest that these should be long-living objects with a lifetime most likely much longer than the current age of the universe. Once a fully evolved DEO starts decaying, its hadronization process goes instantly, thereby liberating its total $\Delta E_{tot}^{g}\Big|_{t=0}$. Recalling that the DEOs are governed by one single speed, the speed of light, its hardonization process would be associated with a giant explosion through which the entire content of the SB is jettisoned into almost a flat spacetime with ultra-relativistic speeds, *i.e.* sufficiently fast, before the embedding spacetime starts re-curving.

Due to the relatively short lifetime of neutrons in free space, the hardonization process of the entire DEO would give rise to creating hydrogen and light elements.

Finally, there are another two issues that support the present scenario: The hyperon puzzle in NS and the predicted radii of the pre-merged NSs in GW170817.

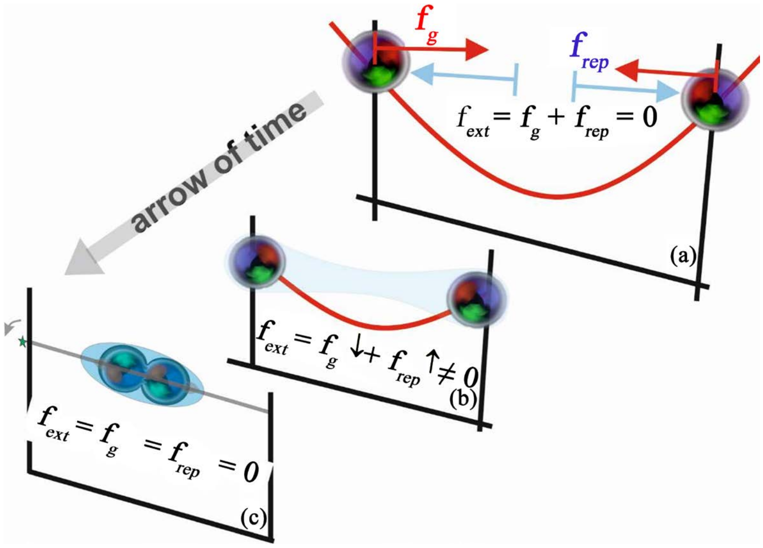

Figure 5. The forces between two arbitrary baryons in hydrostatic equilibrium inside massive NSs. The repulsive force f_{rep}, is set to oppose f_g, the compressional force, exerted by the embedding curved spacetime. The attractive force f_{ext}, ist vanishingly small initially (a). As NSs spin and cool down, the separating distance between baryons decreases and the strength of attractive forces, denoted by f_{ext}, becomes increasingly significant (b). The energy associated with f_{ext} is extracted from the positive energy of the embedding curved spacetime and therefore the spacetime should flatten. In the limit, when $\Delta_{bb} = \ell_{qq}$, baryons merge together, where the repulsive and attractive forces vanish, whilst the embedding spacetime becomes completely flat. The energy extracted from the spacetime goes into tensorial surface tensions that confine the enclosed quarks, ensure uniform mass-energy distribution, and facilitate a phase transition into a Bosonic superfluid with restored Chiral symmetry.

In the former case, it was shown that the inclusion of hyperons production in the EOSs reduces the mass of the NSs significantly and makes them incompatible with observations, whereas hybrid stars with cores made of deconfined quark would reverse this reduction tendency [38]. Also, noting that the onset of hyperon production occurs for $\rho \geq 2\rho_0$, the formation of incompressible gluon-quark superfluid in their cores is a viable scenario. In the latter case, however recent studies of the internal structure of the NSs in GW170817 using sophisticated EOSs with tidal deformability, predict their radii to be roughly 11 km. Our scenario predicts that isolated NSs would end up their luminous lifetime as DEOs having radii of order $R_{NS}^{t=\infty} \approx 10\ \text{km}$, which is a reasonable prediction, when noting that the volume of isolated NSs should shrink as they evolve on the cosmic time.

Asymptotic freedom of SuSu-matter

Although the density and entropy regimes of quantum fluids inside the cores of ultra-compact objects are totally different from those of particle collisions under terrestrial conditions, the QCD properties in the former case are expected to be even simpler than in the latter case. Indeed, the spatial variation of local thermodynamical quantities both in the maximally compressible and purely incompressibility fluid phases would vanish. The incompressible superfluid core is characterized by one single length scale: the radius of the core, R_{core}, and one single time scale $\tau_{core} = R_{core}/c$. As all constituents occupy the same quantum state, the momentum transfer Q^2 between them saturates around the maximum possible value, where the divergence of the tensorial momentum, $\boldsymbol{Q}^{\mu\nu}$ are forbidden, *i.e.*

$$\boldsymbol{Q}^{\mu\nu}_{;\nu} = 0$$

In terms of the renormalization group equation for the running coupling constant α_s [39]:

$$Q^2 \frac{\partial \alpha_s\left(Q^2\right)}{\partial Q^2} = \beta\left(\alpha_s\left(Q^2\right)\right), \tag{18}$$

the conditions to be imposed on Q^2 at both boundaries are identical and therefore $\alpha_s\left(Q^2\right)$ must remain constant. In fact, the Chiral symmetry is fully restored $\left(\langle \bar{q}q \rangle = 0\right)$ in this regime, thereby rendering the cores long-term dynamical stability.

3.1. The Bimetric Universe and CMB Radiation

While the existence of a universal maximum energy density promotes the hypothesis that our universe is infinitely large and old, it suggests that it went neither through an inflationary phase nor creating BHs. The model still conforms with the smooth and uniform temperature of the cosmic microwave background radiation (CMB). Here the instant decay of a finite and perfectly symmetric cluster of DEOs, whose contents are made of quark-gluon superfluids with restored Chiral symmetry may have the potential of creating the CMB we observe today. In this case, our location in the embedding spacetime must have been pretty close to the center of a spherically symmetric gigantic explosion, which is termed here as a local big bang (LBB). The progenitor of the LBB is a giant cluster of DEOs embedded in flat spacetimes. Have the cluster started decaying, its whole content undergoes a rapid hadronization process, and the energy stored in the system undergo a reversible process: part of this energy goes back into curving the embedding spacetime, while the rest goes into forming a soup of highly energetic particles and radiation.

Here a race starts between the outward-oriented ultra-relativistically propagation of fluid flows and topology fitting of the embedding spacetime. Hence two fronts were created: The hardonization front that must go through the entire cluster, and the topological front, "f_1" at the interface between the flat the curved spacetimes (see **Figure 6**). Although the material front propagates at ultra-relativistic speed, the topological front "f_1" propagates with the speed of light. This generates a time delay that increases with the cosmic age. Note that the hardonization front is associated with the creation of interacting particles, generating entropy, and turning the matter into compressible dissipative media. In the course of this process, the embedding spacetime ought to undergo a

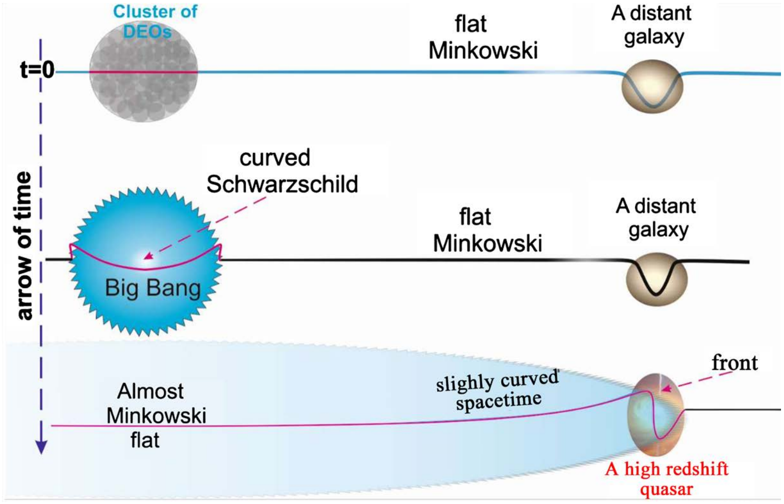

Figure 6. The structure of the bimetric spacetime of our expanding universe. Accordingly, the progenitor of the big bang is a cluster of DEOs that conglomerated over dozens of Gyr at the background of an infinitely flat spacetime. Immediately after the instantaneous decay of the cluster into normal dissipative matter, the topology of the local flat spacetime changed into a curved Schwarzschild, thereby initiating a spherically symmetric expansion front, that separated the two topologies and propagated with the speed of light. Had the expansion front marched through distant, quiet and old galaxies that are embedded in locally curved spacetimes, but superimposed on an infinitely flat spacetime, then the motions of their entire contents of objects would be considerably perturbed and turn turbulent, thereby setting these galaxies into active mode, that identified today as powerful high redshift QSOs.

topological change from Minkowski flat into a curved Schwarzschild. However, as the amount of energy that goes into curving the embedding spacetime is finite, the curvature should decrease as the expansion goes on, thereby asymptotically converging into a perfectly flat spacetime (see **Figure 6**).

Mathematically, let the rest mass-energy of the cluster of DEOs on the verge of an LBB explosion be $\mathcal{E}_{cl}$. As the cluster is embedded in flat spacetime, the gravitational energy vanishes, *i.e.*. $\mathcal{E}_g = 0$. Assuming mass-energy conservation, then the total energy shortly before and after LBB should remain constant, *i.e.*:

$$\left\{\left[\mathcal{E}_{cl}^{b} + \mathcal{E}_{cl}^{vac}\right] + \mathcal{E}_g\right\}_{t\le 0} = \left\{\mathcal{E}_{cl}^{b} + \left[\mathcal{E}_{cl}^{vac} + \mathcal{E}_g\right]\right\}_{t>0}, \tag{19}$$

where $\mathcal{E}_{cl}^{b}$ is the rest energy of the total baryons at the nuclear density, ρ_0, and zero-entropy. $\mathcal{E}_{cl}^{vac}$ is the work needed for compressing the constituents in the system from ρ_0, up to the maximum compressibility limit, where $\rho = \rho_{max}^{uni}$. $\mathcal{E}_g$ is the gravitational energy, which is calculated from the integral:

$$\begin{aligned}\mathcal{E}_g &= \frac{1}{16\pi}\int_{S^n}^{S^{n+1}} \mathrm{d}^2 S^j \left(\frac{\partial}{\partial x^k} g_{jk} - \frac{\partial}{\partial x^j} g_{kk}\right) \\ &= \begin{cases} 0 & \text{flat space time} \\ \mathcal{E}_{cl}^{b} & \text{Schwarzschild space time} \end{cases}\end{aligned} \tag{20}$$

Based thereon, the total mass involved in the big bang may be predicated as follows: for an average density of $\rho_{now} \approx 10^{-29}$ g/cc, and a radius of

$R^{uni} > c \times \tau_{age}^{uni} \approx 10^{28}\,\text{cm}$, we obtain a total mass of $\mathcal{E}_{cl}^{b} \approx 5 \times 10^{22}$. Inside the DEO cluster the density is roughly uniform and has the value ρ_{max}^{uni}. This yields a radius of approximately 2 AU for the DEO cluster. Note that a SMBH with $R_S = 2\,\text{AU}$ yields an enclosed mass of roughly 10^8 which falls in the lower mass-regime of currently observed quasars. Alternatively, a SMBH of 10^{22} would yields a Schwarzschild radius of order 10^{27} cm, which should be ruled out as well.

Indeed, the cluster of DEOs presented here has a uniform density and is embedded in a Minkowski spacetime, so that the necessity for an exponential growth to smooth out density irregularities becomes it unnecessary.

3.2. Summary & Discussion

In this paper, we have discussed the possibility that our universe may permit the existence of a universal maximum energy density ρ_{max}^{uni}, which characterizes the state of incompressible superfluids at the center of massive NSs. Based on theoretical and observational studies of pulsars and neutron stars, ρ_{max}^{uni} is predicted to lie between $2 \times \rho_0$ and $3 \times \rho_0$. Under these conditions and at zero-temperature, the irreducible distance between quarks is predicated to be of order: $\ell_{qq} \approx 0.85\,\text{fm}$, *i.e.* to the experimentally revealed value of the radius of a stable baryon.

The underlying assumption here is that UCOs should be embedded in bimetric spacetimes: their incompressible quark-gluon superfluid cores are embedded in Minkowski-type spacetimes, whereas the ambient compressible and dissipative media are embedded in curved spacetimes.

Based on this bimetric spacetime scenario and on the universal maximum density hypothesis, solutions to several debates both in physics and astrophysics could now be provided:

- Incompressible fluids are treatable using the field equations of General Relativity and may be modeled in a self-consistent manner, without violating the causality condition or creating physically unrealistic ultrabaric regimes (see Figure 7 in [15]).
- The quantum mechanisms underlying the glitch phenomena in pulsars, the increasing intervals between successive glitches as the objects age, the origin of under and over-shootings of the rotational frequencies observed to associate the glitch events of the Vela pulsar can be well-explained.
- Formation of stable massive NSs without collapsing into stellar BHs, such as the remnant of GW170817, may be considered as a reliable evolutionary track. In this case, the remnant of the first generation of massive NSs should have fully metamorphosed into invisible DEOs by now.
- The cosmological origin of high redshift QSOs and dark matter in the early universe conform with our scenario that the universe should be infinitely large and old.
- The dominance and origin of the chemical abundance of the light elements in the universe is a natural consequence of the collective decay of the DEO-cluster

that led to LBB explosion through which hardonizion run away.

- The scenario of a bimetric universe provides a simple and reasonable explanation to the flatness problem of the universe, the origin of dark matter and dark energy.
- The present scenario is capable of providing a new robust mechanism for the origin and power of high redshift QSOs. Indeed, the topological front that separating the Schwarzschild from the Minkowski spacetimes carries sufficient energy to significantly deform the locally curved spacetime embedding distant galaxies, thereby turning the well-ordered motions of their contents of objects into turbulent motion. This excited energy state would considerably increase the rate of destruction and collision of stars and would turn the galaxy into an active mode for a considerable cosmic time.

The duration of this activity starts from the moment at which the topological front collides with locally curved spacetime embedding the respective galaxy and would continue until the remaining stellar components have settled back into a significantly lower energy state [40].

One of the far-reaching consequences here is that the big bang should be a recurrent phenomenon in our infinitely large and old universe. Indeed, the existence of a maximum universal density ρ_{cr}^{Uni} would render the inflationary phase unnecessary and would naturally explain why our universe escaped its collapse into a giant BH.

The scenario here predicts that massive stars should have sufficient time to collapse into massive UCOs, cool down and to subsequently turn invisible at the end of their luminous lifetimes. Their further collapse into BHs is prohibited by the existence of ρ_{cr}^{Uni}, at which the topology of the embedding spacetime changes into a Minkowski-type spacetime (see Hujeirat 2021 in preparation).

Nevertheless, our scenario addresses several new problems that need to be answered, such as:

1) What is the physical nature of the well-observed stellar and massive black holes?

2) What are the physical mechanisms that drive DEOs to conglomerate into clusters, how DEOs interact with each other or with other gravitationally bound astrophysical objects?

3) What are the predicated lifetimes of DEOs, and what are the mechanisms underlying their decay?

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Abbott, *et al.* (2017) *The Astrophysical Journal Letters*, **848**, L12.

[2] Falcke, H., Laing, R., Testi, L. and Zensus, A. (2012) ESO Headquarters Astronomi-

cal News.

[3] Nicholl, M., Margalit, B., Schmidt, P., *et al.* (2021) Tight Multi-Messenger Constraints on the Neutron Star Equation of State from GW170817 and a Forward Model for Kilonova Light Curve Synthesis.

[4] Psaltis, D., *et al.* (2020) *Physical Review Letters*, **125**, Article ID: 141104.

[5] Yang, J., Wang, F., Fan, X., *et al.* (2020) Pōniuā'ena: A Luminous z = 7.5 Quasar Hosting a 1.5 Billion Solar Mass Black Hole.

[6] Pasechnik, R. and Sumbera, M. (2017) *Universe*, **3**, 7. https://doi.org/10.3390/universe3010007

[7] Shuryak, E.V. (2004) What RHIC Experiments and Theory Tell Us about Properties of Quark-Gluon Plasma?

[8] Hujeirat, A.A. and Samtaney, R. (2020) *Journal of Modern Physics*, **11**, 1779-1784. https://doi.org/10.4236/jmp.2020.1111110

[9] Fan, X., Wang, F., Yang, J., *et al.* (2019) *The Astrophysical Journal Letters*, **870**, L11. https://doi.org/10.3847/2041-8213/aaeffe

[10] Oesch, P.A., *et al.* (2016) *The Astrophysical Journal*, **819**, 129.

[11] Shimasaku, K. and Izumi, T. (2019) *The Astrophysical Journal Letters*, **872**, L29. https://doi.org/10.3847/2041-8213/ab053f

[12] Smith, A., Bromm, V. and Abraham Loeb, A. (2017) The First Supermassive Black Holes.

[13] Carr, B.J., Kohri, K., Sendouda, Y. and Yokoyama, J. (2010) New Cosmological Constraints on Primordial Black Holes.

[14] Abbott, *et al.* (2019) *Physical Review X*, **9**, Article ID: 011001.

[15] Hujeirat, A.A. (2018) *Journal of Modern Physics*, **9**, 532-553. https://doi.org/10.4236/jmp.2018.94037

[16] Hujeirat, A.A. and Samtaney, R. (2020) *Journal of Modern Physics*, **11**, 1779-1784. https://doi.org/10.4236/jmp.2020.1111110

[17] Hujeirat, A.A. and Samtaney, R. (2020) *Journal of Modern Physics*, **11**, 1785-1798. https://doi.org/10.4236/jmp.2020.1111111

[18] Ashton, G., Lasky, P.D., Graber, V. and Palfreyman, J. (2019) Rotational Evolution of the Vela Pulsar during the 2016 Glitch.

[19] Bogdan, A. (2020) The Archetypal Ultra-Diffuse Galaxy, Dragonfly 44, Is Not a Dark Milky Way.

[20] Argueelles, C.R., Dieaz, M.I., Krut, A., *et al.* (2021) *Monthly Notices of the Royal Astronomical Society*, **502**, 4227-4246. https://doi.org/10.1093/mnras/staa3986

[21] Gustafsson, B., Ross, P. and Church, M.B.D. (2016) *AandA*, **593**, A85. https://doi.org/10.1051/0004-6361/201423916

[22] Barkana, R. (2018) *Nature*, **555**, 71-74. https://doi.org/10.1038/nature25791

[23] Arbey, A., Ellis, J., Mahmoudi, F. and Robbins, G. (2018) *Journal of High Energy Physics*, **2018**, Article No. 132. https://doi.org/10.1007/JHEP10(2018)132

[24] Tenkanen, T. (2019) *Physical Review Letters*, **123**, Article ID: 061302. https://doi.org/10.1103/PhysRevLett.123.061302

[25] Sivakumer, A., Muruganandam, P. and Hujeirat, A.A. (2021) On the Merger of Superfluid Cores of Neutron Stars the Nature of the Remnant of GW170817. (In Preparation)

[26] Roy, J., Yashwant Gupta, Y. and Lewandowski, W. (2012) *Monthly Notices of the*

Royal Astronomical Society, **424**, 2213-2221.
https://doi.org/10.1111/j.1365-2966.2012.21380.x

[27] Espinoza, C.M., Lyne, A.G., Stappers, B.W. and Kramer, C. (2011) *Monthly Notices of the Royal Astronomical Society*, **414**, 1679-1704.
https://doi.org/10.1111/j.1365-2966.2011.18503.x

[28] Ashton, G., Prix, R. and Jones, D.I. (2017) *Physical Review D*, **96**, Article ID: 063004.
https://doi.org/10.1103/PhysRevD.96.063004

[29] Ng, C., Champion, D.J., Bailes, M., *et al.* (2015) The High Time Resolution Universe Pulsar Survey XII: Galactic Plane Acceleration Search and the Discovery of 60 Pulsars.

[30] Hujeirat, A.A. (2020) *Journal of Modern Physics*, **11**, 1779-1784.
https://doi.org/10.4236/jmp.2020.1111110

[31] Bromm, V. (2013) Formation of the First Stars.

[32] Fischer, T., Wu, M.-R., Benjamin Wehmeyer, B., *et al.* (2021) Core-Collapse Supernova Explosions Driven by the Hadron-Quark Phase Transition as a Rare r-Process Site.

[33] Chon, S., Hosokawa, T., Kazuyuki Omukai, K., *et al.* (2021) *Monthly Notices of the Royal Astronomical Society*, **502**, 700-713. https://doi.org/10.1093/mnras/stab061

[34] McInnes, B. (2017) *Nuclear Physics B*, **921**, 39-58.
https://doi.org/10.1016/j.nuclphysb.2017.05.004

[35] Mueller, B. (2007) From Quark-Gluon Plasma to the Perfect Liquid.

[36] Witten, E. (1981) *Communications in Mathematical Physics*, **80**, 381-402.
https://doi.org/10.1007/BF01208277

[37] Karr, J.-P. and Marchand, D. (2019) *Nature*, **575**, 61.
https://doi.org/10.1038/d41586-019-03364-z

[38] Bombaci, I. (2016) The Hyperon Puzzle in Neutron Stars.

[39] Gracey, J.A. (2020) *Physical Review D*, **101**, Article ID: 125022.
https://doi.org/10.1103/PhysRevD.101.125022

[40] Hujeirat, A.A. (2021) The Bimetric Spacetime Topology of the Universe and the Origin of Power of High Redshift Quasars. (In Preparation)

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A "Potency-Act" Interpretation of Quantum Physics

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Abstract

Some considerations are presented on the so called "ontological interpretations" of quantum physics, starting from a remark by Werner Heisenberg on the relation between the probabilistic character of quantum states and the Aristotelian notion of "potency". We show how an interesting revival of the original idea by Heisenberg can be found in the recent scientific and epistemological literature, in order to solve some paradoxical aspects emerging within some of the usual interpretations of quantum physics. Moreover a way seems to be open in order to rediscover the role of Aristotelian-Thomistic notion of "analogy" of "causal agents" operating even in the physical world. The "Potency-Act" interpretation of quantum physics appears aside the role of the Aristotelian notion of "Form" when it is compared with the recent notion of "information" in the context of the physics of "complex systems" and the biology of "living systems".

Keywords

Quantum Mechanics, Quantum Field Theory, Ontological Interpretation, Potency-Act

1. Introduction

In the present paper we propose some informal comments on the "ontological" interpretations of quantum physics starting from Heisenberg's suggestion of comparing qualitatively some quantum results with the Aristotelian-Thomistic "Potency-Act" theory.

After presenting a sketch of the most referenced "ontological" interpretations of quantum mechanics, and more generally of quantum physics (here both referenced shortly as QM), in Section 2, we propose to reconsider, as especially "up to date", Heisenberg's suggestion of retaining QM as a sort of contemporary rediscovery of Aristotelian "Potency-Act" theory, within the modern context of a mathematical formulation of scientific theories.

In Section 3 we consider and comment an essential quotation from Heisenberg concerning his own idea of establishing a comparison between QM states ψ and their ontological interpretation in terms of Aristotelian "Potency-Act" doctrine.

Section 4 is devoted to some recent contributions by authors following the "Potency-Act" interpretation of QM in the light of the further developments of quantum physics.

In Section 5 we compare the "probabilistic" interpretation of the wave function ψ with the Aristotelian-Thomistic theory of causality, which includes, beside "deterministic" causality, also "probabilistic" causes and even "free" ones, like *e.g.,* human free will. The last matter, which here is not treated, involves also cognitive sciences and anthropology.

Section 6 deals with the example of a simple physical system like a "beam splitter" interferometer in which a single particle (*e.g.,* a photon or an electron) is observed to behave like a coherent wave train, according to QM prediction.

Section 7 applies "Potency-Act" ontological interpretation of QM to vacuum polarization and vacuum fluctuations.

Some concluding remarks are presented in Section 8.

2. Conventional Interpretations of QM

In his well known book *The Road to Reality. A Complete Guide to the Laws of the Universe* [1] at p. 786, Roger Penrose lists six known "ontological" interpretations of QM, *i.e.*:

1) "Copenhagen",
2) Many worlds,
3) Environmental decoherence,
4) Consistent histories,
5) Pilot wave,
6) New theory with objective *R*.

Moreover, in the last years, a new approach to QM has been opened by quantum information leading to a formalism, even if not to a proper new interpretation, in terms of *qubits* (see *e.g.* [2] [3]).

First we will shortly summarize all of them and later we will go back to an original suggestion by Werner Heisenberg.

1) *The "Copenhagen" interpretation*

The so called "*Copenhagen*" *interpretation,* (for a review see, *e.g.* [4]) initially proposed by Niels Bohr, offers a non-realistic viewpoint according to which QM does not provide a genuine and predictive description of what really is happening within the world, but it allows to make computations about the measurements an observer is able to do thanks to a suitable detecting apparatus. The irreducible wave and particle frames are considered as "complementary". Wave function amplitudes $|\psi|$ are related to the local probability density of some particle presence when an observation is done and are responsible of light and matter interference phenomena. Particles are observed as *quanta* of momentum $p = \hbar k$ non-exactly localizable as points in space and instant events in time, because of Heisenberg uncertainty principle, stating the relations between momentum/ position, and energy/time:

$$\Delta p \Delta x \geq \frac{1}{2}\hbar, \quad \Delta E \Delta t \geq \frac{1}{2}\hbar. \tag{1}$$

Then the QM theory appears simply as a theoretical instrument for provisional computation of probabilities of observation and not as a true description of the real world.

2) *The "many worlds" or "multiverse" interpretation*

The "many worlds" or "multiverse" interpretation (for a review see *e.g.* [5]) is intriguing but it appears somehow exotic and non-genuinely scientific because of its non-falsifiability (according to K.R. Popper's scientificity criterion, *see e.g.* [6], part I, chap 2, section 6). Since it assumes that all the possible states $|\psi_n\rangle$ involved into the state function of a system:

$$|\psi\rangle = \sum_n c_n |\psi_n\rangle, \tag{2}$$

are all really existent in some world, So an infinite number of "parallel worlds" are intended to be "actually" existent, each within its "real" universe. All the parallel universes are non-connected components of a whole, so that only one of them may be observed by us. The wave function involves a superposition of many "real" worlds together.

While the "*Copenhagen*" *interpretation* is "instrumentalisitc" (in the sense of Popper [6] and T.S. Kuhn [7]) (or even idealistic or "conventionalistic" in the sense of P. Feyerabend, see, e.g. [8]), the "many worlds" interpretation is excessively "realistic". Moreover, the latter interpretation, even if it is fascinating, manifestly violates the "Occam's razor" simplicity principle [9]. which is normally assumed as scientifically basic. Why to suppose the true "actual" existence of infinite worlds when only one of them is observable by us?

3) *The "Environmental decoherence" interpretation*

The "*Environmental decoherence*" *interpretation* includes the interaction with the environment, *i.e.*, what is considered as "external" to the system on which an observer (human or instrumental) is performing measurements.

So the wave function governing the "whole" is required to include, beside the wave function $|\psi^S\rangle$ related to the system on which one is performing measurements and the wave function $|\psi^A\rangle$ describing the instrumental apparatus needed to exploit measurements, also an almost inaccessible wave function for the "external" environment $|\psi^{env}\rangle$, which must be considered no longer as "external".

$$|\psi\rangle = |\psi^S\rangle \otimes |\psi^A\rangle \otimes |\psi^{env}\rangle. \tag{3}$$

Interactions of the system with the environment and the apparatus appear both as the "causal agent" determining the collapse from coherence of the wave function. Such interaction, which may be even "non-local", is recognized as the physical basis of entanglement [10].

The causal contribution of environment to decoherence needs to be added to the interaction between the measurement apparatus and the system itself. Such approach is "holistic" in the sense that a quantum system cannot be treated as an isolated one, being always influenced by the environment.

4) *Consistent histories*

The "*Consistent histories*" *interpretation* [11] considers QM as an intrinsically probabilistic theory, in which probability does not depend on observational and measurement operations, but is involved in the wave equation itself as a "true law of nature".

Ontologically it appears that "probabilistic causation" is a kind of causality relation in the physical microscopic world, aside "deterministic causation" in the physical macroscopic world. Introduction of "probability" within the causality is no longer interpreted as a lack of causation, or as a consequence of our ignorance about the physical world, but as a new way of actuating causation in QM, aside the "deterministic" causality in classical mechanics.

5) *Pilot wave*

The "*pilot wave*" *interpretation* proposed by Louis De Broglie and later modified by David Bohm attempts a classical-like approach to quantum mechanics suggesting that the particle trajectories are driven by a pilot wave, being oscillations of some matter field ψ just as a photon is a quantum of oscillating electromagnetic fields E, B. In Bohm's variant of the theory, a sort of "quantum potential" is guessed and evaluated, the effect of which is to drive particles along definite trajectories leading to interference and wave effects. But the last approach, in its original form, is not compatible with Einstein's relativity.

> "The most thoroughly worked-out theory of this type is the pilot wave theory developed by de Broglie and presented by him at the Fifth Solvay Conference held in Brussels in 1927, revived by David Bohm in 1952, and currently an active area of research by a small group of physicists and philosophers.
> According to this theory, there are particles with definite trajectories, that are guided by the quantum wave function." [12]

6) *New theory with objective R*

The "*objective reduction*" *interpretation* is looking for an evolutionary continuous process performed by an evolution operator U, avoiding jumps about the quantum states of a system.

> "Often referred as 'objective-*R*' theory. '*R*' stands for the quantum state reduction when a measurement is taking place. Supporters of this interpretation believe that today's quantum mechanics is not here to stay and will be

replaced by a theory at which *R* is a real process. Physicists and mathematicians who work in the field, try to balance the inconsistence of *R* process (discontinuous process, 'jump' from a state to another) with the unitary and simple (at least in Schrödinger's equation) time evolution of the state vector (often denoted as *U*-process)." [13]

3. Heisenberg's Interpretation of QM and the Aristotelian "Potency-Act" Theory

Surprisingly, but not too much, Werner Heisenberg, in his relevant book *Physics and Philosophy,* suggested the idea that a meaningful "ontological" interpretation of quantum theory could be related, in some sense, to the Aristotelian concepts of "potency" (*Latin*, "*potentia*") and therefore "act" (*Latin*, "*actus*"), even if the latter is not explicitly referred-to by him. Here are his own words.

> "A first and very interesting step toward a real understanding of quantum theory was taken by Bohr, Kramers and Slater in 1924. These authors tried to solve the apparent contradiction between the wave picture and the particle picture by the concept of the probability wave. […]
> The probability wave of Bohr, Kramers, Slater, however, meant more than that; it meant a tendency for something. It was a quantitative version of the old *concept of 'potentia' in Aristotelian philosophy.* It introduced something standing in the middle between the idea of an event and the *actual* event, a strange kind of physical reality just in the middle between possibility and reality. Later when the mathematical framework of quantum theory was fixed, Born took up this idea of the probability wave and gave a clear definition of the mathematical quantity in the formalism, which was to be interpreted as the probability wave. It was not a three-dimensional wave like elastic or radio waves, but a wave in the many-dimensional configuration space, and therefore a rather abstract mathematical quantity." [14] (pgs 11-12 [emphasis mine]).

An historical sketch of Heisenberg's "Aristotelian" interpretation of quantum states can be found in [15], where the author explains how (Section 1):

> "the relationship between this use and Aristotle's notion was not made by Heisenberg in full detail, beyond noting their common character: that of signifying the system's objective capacity to be found later to possess a property in actuality. For such actualization, Heisenberg required measurement to have taken place, an interaction with external systems that disrupts the otherwise independent, natural evolution of the quantum system. The notion of state actualization was later taken up by others, including Shimony, in the search for a law-like measurement process. Yet, the relation of quantum potentiality to Aristotle's original notion has been viewed as mainly terminological, even by those who used it thus."

The Aristotelian-Thomistic approach suggests that physical entities may behave according two possible ways of existence:

1) *Actual existence*

2) *Potential existence*

The *actual existence* of a particle, or a quantum, which is non-necessarily localizable as a point, but may possess even an extended structure (string, brane, cloud, etc.), is the only one way of existence we may observe and measure.

The *potential existence* is the way of existence which is "hidden" (*i.e.,* non-observable) into a wave carrying "information" (the Schrödinger's wave function ψ) about the probability of presence of a particle, which can be "actualized" by a suitable "causal agent". The latter agent may act either by a direct interaction (measurement, environment, interaction with other particles or fields) with the system governed by ψ, according to the "conservation laws", or acting disregarding the conservation laws in a non-observable way thanks to the uncertainty Heisenberg's relation.

Among all the "potential states" of matter, the "higher is the probability" for a particle or a system to come to actual existence, the "nearest to act" such potency will be considered (*Latin*, "*potentia proxima*"). It must be emphasized that a "potential existence" is different from being nothing and being actually, as a sort of intermediate state. And different levels of potency are allowed to matter. So the quantum *vacuum* may be considered as an entity which exhibits a "proximate potency" (measured by a probability) to jump to act thanks to Heisenberg's uncertainty, when a suitable "causal agent" is acting on it. "Information" (candidate to be interpreted as the Aristotelian "formal cause") hidden into the wave function requires an agent ("efficient cause") to become operating on potency organizing it as something actually existent.

So the wave function ψ can be interpreted as a mathematical representation of the "nearest potency" states of the quantum system governed by it. Each eigenstate of the Hamiltonian, or the angular momentum, or the spin or other operator describing the system represents an "actual state" of the system as may be measured by an observer thanks to his apparatus, or into which the system collapses interacting with the environment or with any other "causal agent". Collapsing of the wave function represents the passage from "potency" to "act".

i) The "Potency-Act" interpretation of QM is ontologically "realistic" at least as it is realistic the "consistent histories" interpretation. Since it is assumed that probability is intrinsic to the ontology of the "causal agents" acting on a quantum physical system, independently of the measurement and observation instruments. Probability is assumed to be intrinsic to the nature of a microscopic causation process and does not arise as a statistical effect of measurement procedures.

ii) The "many worlds" or "parallel universes" of the Everett's interpretation of QM are admissible in the frame of "Potency-Act" interpretation as worlds existing, but "in potency" and not actually, while only one of them is assumed to be "actually" existent and therefore "observable", as what we usually call "the real world".

iii) As a remarkable epistemological consequence it follows that the "Occam's razor" principle which requires to minimize the number of entities and assumptions within a scientific theory, is fully satisfied. No superfluous actual worlds are required: "Entities must not be multiplied without necessity" (*Latin*, "*entia non sunt multiplicanda sine necessitate*").

4. The "Potency-Act" Interpretation after Heisenberg

Significantly it must be emphasized that recently, about a century after Heisenberg, a "Potency-Act" Aristotelian-Thomistic interpretation of QM has come to the attention of several researchers and philosophers of QM ontology. Among them we may mention, *e.g.* Ignacio Silva [16], Robert R. Bishop and Joseph E. Brenner [17], Alfred Driessen [18], Boris Bozňjak [19], and others.

According to Silva:

> "With this interpretation Heisenberg abandons the Parmenidean/Cartesian materialistic ontology to embrace a new ontology for quantum phenomena: the ontology of act and potency. He moves from the rigid mechanism of Cartesian ontology to a richer Aristotelian ontology. According to Heisenberg, developments in quantum mechanics have given rise to a more subtle concept of reality than that based in classical physics. The univocal notion of being assumed in classical physics should be left aside for an analogical notion of being. This thought opens a path for a new development of the concepts of philosophy of nature of act and potency." [16] (pg 641).

A "classical" suggestion of what happens in a binary quantum system (like the electron *spin*), thanks to which one can guess what it happens, is provided by a coin tossed on air and later fallen down to ground. When the coin is flying on air its state is undetermined, being "potentially" either "head" or "tail", in a sort of mixture of both states.

Only after its fall onto the ground it reaches an "actual", definite "eigenstate", which may be, with the same probability $\frac{1}{2}$, either "head" or "tail".

The electron *spin,* as any other binary quantum system, behaves just as a "superposition" (linear combination) of two eigenstates $\left|\uparrow\right\rangle$ (*up*) and $\left|\downarrow\right\rangle$ (*down*):

$$\left|\psi\right\rangle = \alpha_{\uparrow}\left|\uparrow\right\rangle + \alpha_{\downarrow}\left|\downarrow\right\rangle. \tag{4}$$

We can interpret such a superposition as a state "in potency" of the electron, which collapses into an "actual" eigenstate ("up" or "down") after an actuating causal process (interaction with other systems, measurement apparatus, environment). So a binary system like the "Schrödinger's cat" is not actually dead and alive at the same time, but is potentially dead or alive before being observed, since it is "actually alive" and "potentially dead" until the poison ampule is broken and vice-versa after the latter event. According to "Potency-Act" interpretation of QM no paradoxical situation happens.

Only the universe within which the cat is in its "actual eigenstate" is the "actual universe", while the so called "parallel universe" is "in potency".

Therefore in a "Potency-Act" interpretation of QM we may also assume that "waves", as mathematically described by the wave function ψ, involve the states of the system "in potency", while the "particles" (*i.e.,* the extended structures like *quanta,* strings, branes, clouds, etc.) involve the states of the system "in act". The mathematical formalism of operators, implying the uncertainty principle for non-commuting physical quantities:

$$\Delta A \Delta B \geq \frac{1}{2}\left|\left\langle \left[A, B\right]\right\rangle\right|, \tag{5}$$

$\left[A, B\right] = AB - BA$ being the commutator, when referred to the conjugate operators of position x and momentum p of a "particle":

$$\Delta x \Delta p \geq \frac{1}{2}\hbar, \tag{6}$$

being $\left[x, p\right] = i\hbar$, simply declares that the point like representation of a structured system is unphysical, being too simplistic.

5. Different Types of Causal Agents

The "realistic" character of probability which is presupposed in the "Potency-Act" interpretation of QM, has its remote ontological foundation in the Aristotelian-Thomistic theory of causality.

In fact, according to the latter physical and metaphysical view of reality, three kinds of behavior of "causal agents" may be observed in nature.

1) The "deterministic" causal agents (*Latin*, *ad unum*);

2) The "probabilistic" causal agents (*Latin*, *ut in pluribus* or highly probable; *ut in paucioribus* or less probable);

3) The "free" causal agents (*Latin, ad utrumlibet* or ambivalent).

The first ones behave like causes in "classical physics" (like forces and fields), according to which the evolution phase trajectory of a system is "bi-univocally" determined assigning the initial (or the boundary) conditions beside the physical laws governing the examined phenomenon.

The second ones behave like causes in "quantum physics" (like quantum potentials and quantum fields), since the effect of their action is non-deterministic and non-univocal, so that the phase trajectories of a system are undetermined and only probability clouds can be estimated. Heisenberg himself says that:

> "in modern physics the concept of possibility, that played such a decisive role in Aristotle's philosophy, has moved again into a central place." [20]

The third ones behave like "human freedom", being unpredictable in each individual choice which may be indifferently oriented, if it is left alone.

Manifestly according to the "Aristotelian-Thomistic" philosophy of nature, on which the "Potency-Act" interpretation of QM is based, the probability is intrinsic to the system and is not induced by external or subjective factors. Moreover:

> "the transition from the 'possible' to the 'actual' takes place as soon as the interaction between the object and the measuring device, and thereby with the rest of the world, has come into play; it is not connected with the act of registration of the result in the mind of the observer." [14] (pgs 54-55).

According to Heisenberg the "causal agent" of the transition from a "potential" to an "actual" state of a system has an objective character and is neither a sort of subjective mental projection of mind, in the sense of idealistic philosophy nor a spontaneous event happening in absence of an "adequate causation".

6. The Example of the "Beam Splitter" Interferometer

Let us now apply the "Potency-Act" interpretation of QM to the interference phenomenon happening in a wave "splitter apparatus" like the one shown in **Figure 1**.

A photon or an electron described by the wave function ψ is incoming into the apparatus (see "IN" in **Figure 1**, where it splits into two half particles (waves of half amplitude). The transmitted half wave proceeds horizontally with unaltered phase. So the reflected wave which is running down maintains its original phase being submitted to an "internal" reflection. In S1, S2 each one of the coherent half wave meets, at the same distance, an ordinary glass, where is submitted to "external" reflection, which reverses its phase (phase shift of π). Eventually the half waves reach "OUT" where they split a second time into half a half wave, the red wave reflecting "internally", the blue wave reflecting "externally". On "OUT" two observers (detectors) are placed on the horizontal and on the vertical directions.

The horizontal observer detects two superposed waves in phase concordance, while the vertical observer detects nothing since the two superposed waves emerging from the apparatus are in phase opposition and destroy themselves.

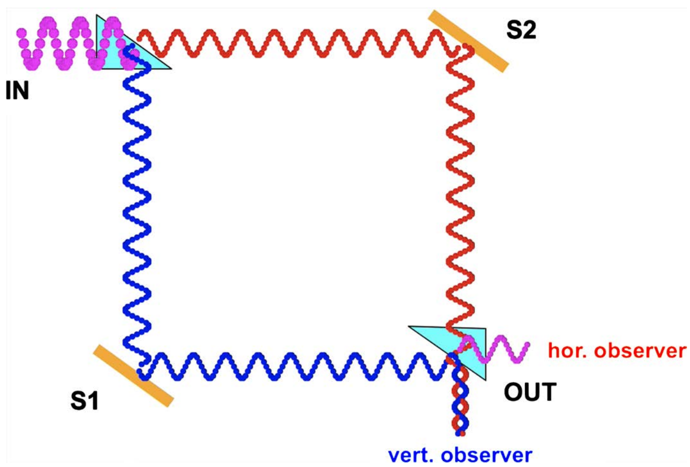

Figure 1. Interference in a photon/electron wave splitter apparatus (click for animation, http://www.albertostrumia.it/sites/default/files/Animations/QMechanics/SplitterOnde.m4v).

According to the "many worlds" interpretation the photon/electron wave observed by the horizontal detector lives in our world, while the other wave which is not observable by the vertical detector lives in a "parallel universe". A somehow strange situation!

While following a "Potency-Act" interpretation we can say the horizontal observer sees the particle which is "actually" existent, while the vertical observer cannot see what has come back from "act" to "potency", because of the action of a suitable "causal agent" (the different numbers of external and internal reflections along the two ways implying destructive interference). The actual world required is the only one which is observable, while the so called "parallel world" (non-observable) needs no longer to be supposed as actually existing, but only "potentially" existing as hidden into the capabilities of matter.

7. The Quantum Fluctuation of Vacuum

Another relevant question involves the "quantum fluctuations of vacuum". It is known that particle-antiparticle pairs (*e.g.,* electron-positron) may emerge from "quantum vacuum" under the "external causal action" of an electromagnetic field.

Therefore people is investigating if the universe may have arisen because of an "internal causal action" arising spontaneously as a quantum fluctuation. Is such an internal action coming from some "informational causal agent" acting from "outside" the material universe, being in itself "immaterial" as any information?

Moreover, does such information play a "teleonomical" role driving the evolution of universe from a vacuum fluctiation towards "attractors" like elementary particles, nuclei, atoms, molecules, stars, galaxies, planetary systems, living beings?

As Boris Kožnjak observed:

> "Final causes in fact provide a better ontology for quantum mechanics than spontaneous causation. The idea of 'spontaneity' is unanalyzable and therefore of little use in quantum mechanics.
> In addition, it is ontologically sterile in the context of quantum measurement, as shown by a historical and conceptual review of the role of efficient causation in experimental physics." [19]

8. Conclusions

We have proposed an introductory sketch on the recent revival of interest in Aristotelian-Thomistic "Potency-Act" theory of matter states (and more generally "entities") in order to provide an intriguing ontological interpretation of quantum physics, as originally was suggested by Werner Heisenberg at the very beginning of quantum mechanics. New perspectives appear especially promising even in order to rediscover the Thomistic doctrine of "analogy of being" (*Latin*, "*analogia entis*") [21].

The latter way of investigation, about quantum physics, appears to be complementary to a parallel rediscovery of the "Form-Matter" Aristotelian-Thomistic doctrine, in the context of information theory and its relations to complexity, self organization and the biology of living systems [22]. Even if it would be, at present, at least arbitrary to identify the Aristotelian "form" with our "information" and the Aristotelian "potency" with "quantum probability" as mathematically described by the wave function ψ, the early suggestion by Heisenberg seems today more and more relevant.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Penrose, R. (2004) The Road to Reality. A Complete Guid to the Laws of the Universe. Jonathan Cape, London.

[2] Deutsch, D. (2004) It from Qubit. In: Barrow, J., *et al.*, Eds., *Science & Ultimate Reality*, Cambridge University Press, Cambridge, 90-102. https://doi.org/10.1017/CBO9780511814990.008

[3] Dür, W. and Heusler, S. (2014) *The Physics Teacher*, **52**, 489. https://arxiv.org/abs/1312.1463 https://doi.org/10.1119/1.4897588

[4] Faye, J. (2019) Copenhagen Interpretation of Quantum Mechanics. https://plato.stanford.edu/archives/win2019/entries/qm-copenhagen

[5] Vaidman, L. (2018) Many-Worlds Interpretation of Quantum Mechanics. https://plato.stanford.edu/archives/fall2018/entries/qm-manyworlds

[6] Popper, K.R. (1992) The Logic of Scientific Discovery. Routledge, London and New York.

[7] Kuhn, T.S. (1996) The Structure of Scientific Revolutions. The University of Chicago Press, Chicago and London.

[8] Feyerabend, P. (1993) Against Method. Verso, London.

[9] Baker, A. (2016) Simplicity. Stanford Encyclopedia of Philosophy. https://plato.stanford.edu/archives/win2016/entries/simplicity

[10] Zeh, H.D. (1995) Decoherence: Basic Concepts and Their Interpretation. https://arXiv:quant-ph/9506020

[11] Griffiths, R.B. (2019) The Consistent Histories Approach to Quantum Mechanics. Stanford Encyclopedia of Philosophy. https://plato.stanford.edu/entries/qm-consistent-histories

[12] Myrvold, W. (2018) Philosophical Issues in Quantum Theory. Stanford Encyclopedia of Philosophy. https://plato.stanford.edu/archives/fall2018/entries/qt-issues

[13] Lazarou, D.K. (2018) Interpretation of Quantum Theory. An Overview. Stanford

Encyclopedia of Philosophy. https://arxiv.org/abs/0712.3466

[14] Heisenberg, W. (1962) Physics and Philosophy. Harper (Reprint Penguin Classics, 2000), New York.

[15] Jaeger, G. (2017) *Philosophical Transactions of the Royal Society A*, **375**, Article ID: 20160390. https://doi.org/10.1098/rsta.2016.0390

[16] Silva, I. (2013) Werner Heisenberg and Thomas Aquinas on Natural Indeterminism. The Dominican Council, John Wiley & Sons Ltd., Oxford and Malden. https://doi.org/10.1111/nbfr.12036

[17] Bishop, R.R. and Brenner, J.E. (2017) Potentiality, Actuality and Non-Separability in Quantum and Classical Physics: Res Potentiae in the Macroscopic World. Stanford Encyclopedia of Philosophy. https://arxiv.org/abs/1801.01471

[18] Driessen, A. (2020) *Acta Philosophica*, **29**, 395-413. http://philsci-archive.pitt.edu/16265/1/Aristotle_and_Quantum_Mechanics.pdf

[19] Kožnjak, B. (2020) *Journal for General Philosophy of Science*, **51**, 459-480. https://doi.org/10.1007/s10838-020-09500-y

[20] Heisenberg, W. (1960) *Wort und Wirklichkeit*, **1**, 32-62.

[21] Strumia, A. (2007) *Acta Biomedica*, **78**, 32-38. https://www.mattioli1885journals.com/index.php/actabiomedica/article/view/4608

[22] Strumia, A. (2020) From Fractals and Cellular Automata to Biology. Information as Order Hidden within Chance. World Scientific, Singapore. https://www.worldscientific.com/worldscibooks/10.1142/11743 https://doi.org/10.1142/11743

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An Explanation for the Violation of Lepton Universality in Beauty-Quark Decays: The Binary Isotope Mixture of Beauty-Quarks

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Abstract

This paper purposes an explanation for the recent evidence for the violation of lepton universality in beauty-quark decays at CERN's Large Hadron Collider. A beauty meson (B^+) transforms into a strange meson (K^+) with the emission of either electron-positron (e^+e^-) or muon-antimuon ($\mu^+\mu^-$). The ratio (R_K) of branching fractions for $B^+ \to K^+\mu^+\mu^-$ and $B^+\to K^+e^+e^-$ decays is measured to be $R_K = 0.846$ instead of 1 in the violation of lepton universality in the Standard Model. This paper proposes that the violation is derived from the binary isotope mixture of two beauty-quarks, b_7 (4979 MeV mass) and b_8 (143,258 MeV mass) whose masses are calculated from the periodic table of elementary particles. b_7 is the observable B, while b_8 is the hidden B to preserve the generation number symmetry between the three lepton family generations and the three quark family generations in the Standard Model. The preservation of the generation number symmetry forbids b_8 to decay into $K^+\mu^+\mu^-$. In the transition state involving the virtual particles (γ, W± and Z°) before the decay, b_7 and b_8 emerge to form the binary isotope mixture from B. The rates of emergence as the rates of diffuse in Graham's law of diffusion are proportional to inverse square root of mass. The rate ratio between b_8/b_7 is $(4979/143{,}258)^{1/2} = 0.1864$. Since b_7 decays into K^+, e^+e^-, and $\mu^+\mu^-$, while b_8 decays into K^+, e^+e^-, and forbidden $\mu^+\mu^-$, the calculated ratio (R_K) of branching fractions for $B^+\to K^+\mu^+\mu^-$ and $B^+\to K^+e^+e^-$is $0.5/(0.1864 \times 0.5+ 0.5) = 0.843$ in excellent agreement with the observed 0.846. The agreement between the calculated R_K and the observed R_K confirms the validity of the periodic table of elementary particles which provides the answers for the dominance of matter over antimatter, dark-matter, and the mass hierarchy of elementary particles.

Keywords

Beauty-Quark Decays, Violation of Lepton Universality, Periodic Table of

Elementary Particles, Binary Isotope Mixture of Beauty-Quarks, Ratio of Branching Fractions, Dark Matter, Dominance of Matter over Antimatter

1. Introduction

For about sixty years, the Standard Model (SM) of particle physics has provided the model for various properties and interactions of fundamental particles, and has been confirmed by numerous experiments. In lepton universality of the SM, the different charged leptons, the electron, muon and tau, have identical electroweak interaction strengths. Lepton universality has been confirmed in a wide range of particle decays. A recent measurement of beauty-quark decays based on proton-proton collision data collected with the LHCb detector at CERN's Large Hadron Collider shows the evidence for the breaking of lepton universality in beauty-quark decays [1]. A beauty meson (B^+) transforms into a strange meson (K^+) with the emission of leptons ($\ell^+\ell^-$) which is either electron-positron (e^+e^-) or muon-antimuon ($\mu^+\mu^-$). Lepton universality of the SM predicts that the ratio (R_K) of branching fractions for $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ decays is 1. The observed R_K is 0.846 instead of 1 in the violation of lepton universality in the SM. The measurement has a significance of 3.1 standard deviations.

If confirmed by future measurements, this violation of lepton universality would imply new physics beyond the Standard Model. Since the SM is unable to explain cosmological observations of the dominance of matter over antimatter, dark-matter, or the mass hierarchy of elementary particles, the new physics derived from the violation of lepton universality can explain the SM's shortcomings. In the previous papers [2]-[8], the SM's shortcoming to account for the mass hierarchy of elementary particles is solved by the periodic table of elementary particles where the masses of all leptons, quarks, gauge bosons, the Higgs boson, and hadrons can be calculated precisely, and the calculated masses are in excellent agreement with the observed masses. As shown in this paper, the new physics from the violation of lepton universality actually confirms the validity of the periodic table of elementary particles.

This paper proposes that the violation is derived from the binary isotope mixture of two beauty-quarks, b_7 (4979 MeV mass) and b_8 (143,258 MeV mass) whose masses are calculated from the periodic table of elementary particles [7]. b_7 is the observable B, while b_8 is the hidden B to preserve the generation number symmetry between the three lepton family generations and the three quark family generations in the SM. The preservation of the generation number symmetry forbids b_8 to decay into $K^+\mu^+\mu^-$. In the transition state involving the virtual particles (γ, W± and Z°) before the decay, b_7 and b_8 emerge to form the binary isotope mixture from B. The rates of emergence as the rates of diffuse in Graham's law of diffusion are proportional to inverse square root of mass. The rate ratio between b_8/b_7 is $(4979/143{,}258)^{1/2} = 0.1864$. Since b_7 decays into K^+, e^+e^-, and $\mu^+\mu^-$, while b_8 decays into K^+, e^+e^-, and forbidden $\mu^+\mu^-$, the calculated ratio (R_K) of branching fractions for $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ is $0.5/(0.1864 \times 0.5 + 0.5) = 0.843$ in excellent agreement with the observed 0.846. The agreement between the calculated R_K and the observed R_K confirms the validity of the periodic table of elementary particles.

Section 2 describes the periodic table of elementary particles and the calculation of the masses of leptons and quarks including the masses of b_7 and b_8. Section 3 describes the beauty-quark decay and the calculation of R_K for the branching fractions of $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$. Section 4 explains dark matter and the dominance of matter over antimatter.

2. The Periodic Table of Elementary Particles

The periodic table of elementary particles for baryonic matter and dark matter [2]-[8] is based on the seven principal mass dimensions (d's) for stable baryonic matter leptons (electron and neutrinos), gauge bosons (all forces), gravity, and dark matter (five sterile dark matter neutrinos) and the seven auxiliary mass dimensions (a's) for unstable leptons (muon and tau) and quarks (d, u, s, c, b, and t) as in Figure 1 and Table 1.

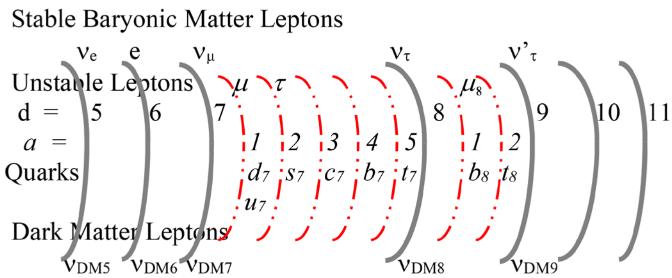

Figure 1. Leptons and quarks in the seven principal mass dimensional orbitals (solid lines) denoted by the principal mass dimensional orbital number d and the seven auxiliary dimensional mass orbitals (dash-dotted lines) denoted by the auxiliary mass dimensional orbital number *a*.

Table 1. The periodic table of elementary particles for baryonic matter and dark matter.

d	a = 0 Stable Baryonic Matter Leptons	a = 0 Dark Matter Leptons	1 Unstable Leptons	2	1 Quarks	2	3	4	5	a = 0 Bosons
5	ν_e	ν_{DM5}								B_5 = A electromagnetism
6	e	ν_{DM6}								B_6 = g* strong (basic gluon for quarks)
7	ν_μ	ν_{DM7}	μ_7	τ_7	d_7/u_7	s_7	c_7	b_7	t_7	$B_7 = Z^0_L$ left-handed BM weak
8	ν_τ	ν_{DM8}	μ_8 (absent)		b_8 (absent)	t_8				$B_8 = Z^0_R$ right-handed DM weak
9	ν'_τ (high-mass ν_τ)	ν_{DM9}								B_9 = dark matter repulsive force
10										B_{10} = particle-antiparticle asymmetry
11	Gravitino									B_{11} = gravity

d = principal mass dimension number, a = auxiliary mass dimension number, DM = dark matter, BM = baryonic matter.

The periodic table of elementary particles provides the answers for the dominance of matter over antimatter [8], dark-matter [5], and the mass hierarchy of elementary particles [2]-[8]. The masses of all leptons, quarks, gauge bosons, the Higgs boson, gravity, and dark matter can be calculated by the periodic table of elementary particles [2]-[8]. Since this paper deals with mostly beauty quark, only the masses of leptons and quarks are calculated.

The mass of mass dimensional fermion and the mass of mass dimensional boson are related to each other with three simple formulas as the follows.

$$M_{\mathrm{d,B}} = M_{\mathrm{d,F}} / \alpha_{\mathrm{d}} \tag{1}$$

$$M_{\mathrm{d+1,F}} = M_{\mathrm{d,B}} / \alpha_{\mathrm{d+1}} \tag{2}$$

$$M_{\mathrm{d+1,B}} = M_{\mathrm{d,B}} / \alpha_{\mathrm{d+1}}^2 , \tag{3}$$

where d is the mass dimension number, F is fermion, and B is boson. Each dimension has its own α_{d}, and all α_{d}'s except α_7 (α_w) of the seventh dimension (weak interaction) are equal to α, the fine structure constant of electromagnetism.

The lepton mass formula and the quark mass formula are derived from the incorporation of basic gluon ($g^* = B_6 = M_{F6}/\alpha = M_e/\alpha = 70$ MeV from Equation (1)) from **Table 1** to electron. The incorporation of basic gluon as flux quanta follows the composite fermion theory for the FQHE (fractional quantum Hall effect) [9] [10]. In the composite fermion model for FQHE, the formation of composite fermion is through the attachment of an even number of magnetic flux quanta to electron, while the formation of composite boson is through the attachment of an odd number of magnetic flux quanta to electron. In the same way, the formation of composite fermion is through the attachment of an even number of basic gluons to electron, while the formation of composite boson is through the attachment of an odd number of basic gluons to electron. The formation of composite boson is equal to the formation of composite di-leptons, so the formation of composite lepton is through the attachment of one half of an odd number of basic gluons to electron. As a result, the muon (μ) mass formula is as follows.

$$\begin{aligned} M_{\mu_7} &= Me + 3M_{g^*}/2 \\ &= Me + 3M_e/2\alpha \\ &= 105.5488\ \mathrm{MeV} \end{aligned} \tag{4}$$

which is in excellent agreement with the observed 105.6584 MeV [11] for the mass of muon. The masses of leptons follow the Barut lepton mass formula [12] as follows.

$$M_{\mathrm{lepton}} = M_e + \frac{3M_e}{2\alpha} \sum_{a=0}^{n} a^4 , \tag{5}$$

where $a = 0$, 1, and 2 are for e, μ_7, and τ_7, respectively. The calculated mass of τ_7 is 1786.2 MeV in good agreement with the observed mass as 1776.82 MeV. According to Barut, the second term, $\sum_{a=0}^{n} a^4$ of the mass formula is for the

Bohr-Sommerfeld quantization for a charge-dipole interaction in a circular orbit. The more precise calculated mass of τ for the tau lepton mass formula is as follows.

$$\begin{aligned} M_\tau &= Me + \left(\frac{3M_e}{2\alpha} - M_e\right)\sum 2^4 \\ &= Me + \left(17\frac{3M_e}{2\alpha} - 17M_e\right), \\ &= 1777.47\ \text{MeV} \end{aligned} \quad (6)$$

which is in excellent agreement with observed 1776.82 MeV, and means that during this dipole-interaction in a circular orbit for τ, an electron with total mass of $17M_e$ is lost. $17M_e$ is shown as the observed 17 MeV for $34M_e$ in the light boson (17 eē) [13] [14].

Quark has fractional charge (±1/3 or ±2/3), 3-color gluons (red, green, and blue) for 3g*, and both the principal mass dimensions and axillary mass dimensions, so similar to Equation (4), d and u in the principal mass dimension involves e/3 or 2e/3 and 3g* as follows.

$$\begin{aligned} &\text{principal mass dimensional oribital at d} = 6 \\ M_{\text{principal } q} &= \frac{1 \text{ or } 2Me}{3} + \frac{3\left(3M_{\text{g*}}\right)}{2} \\ &= \frac{1 \text{ or } 2Me}{3} + \frac{3\left(3M_{\text{B6}}\right)}{2} \\ &= \frac{1 \text{ or } 2Me}{3} + \frac{9Me}{2\alpha} \end{aligned} \quad (7)$$

For quarks in the auxiliary mass dimensions, 3-color basic gluons (3g*) become 3-color auxiliary basic gluons (3g*_{a7}) at d = 7. From Equations (1) and (3), $\alpha_w = \alpha_7 = \alpha$ of week interaction = $(M_{B6}/M_{B7})^{1/2} = (M_{F6}/\alpha/M_{B7})^{1/2} = (M_e/\alpha/M_Z)^{1/2} = 0.02771$. Based on Equation (2), auxiliary basic gluon is derived from muon as follows.

$$M_{\text{g*}_{a7}} = M_{\mu 7}\alpha_w \quad (8)$$

Similar to Equation (4), the masses of quarks in the auxiliary mass dimension are as follows.

$$\begin{aligned} &\text{auxiliary mass dimensional orbital at d} = 7 \\ M_{\text{auxiliary } q7} &= \frac{3\left(3M_{\text{g*}_{a7}}\right)}{2}\sum_{a=1}^{n} a^4 = \frac{9M_{\mu 7}\alpha_w}{2}\sum_{a=1}^{n} a^4 \end{aligned} \quad (9)$$

The quark mass formula at d = 7 is the combination of Equations (7) and (9) as follows.

$$M_{q7} = \frac{1 \text{ or } 2Me}{3} + \frac{9Me}{2\alpha} + \frac{9M_{\mu 7}\alpha_w}{2}\sum_{a=1}^{n} a^4 \quad (10)$$

where a = 1, 2, 3, 4, and 5 for u_7/d_7, s_7, c_7, b_7, and t_7, respectively.

The quark mass at a = 5 for the auxiliary mass dimension at d = 7 is the maximum mass below the mass of B_7, so the next auxiliary mass dimension has to start from B_7. There are b and t at d = 8, so it is necessary to have μ_8 for the masses of b and t. Like μ_7 in Equation (4), the mass of μ_8 is as follows.

$$\begin{aligned} M_{\mu_8^0} &= 2Me + 3M_{g^*{}_7}/2 \\ &= 2Me + 3M_{B7}/2 \\ &= 2Me + 3M_{Z^0}/2 \\ &= 136.78\ \text{GeV} \end{aligned}, \tag{11}$$

Since at d = 7, there are 3-color basic gluons, at d = 8, 3-*color* basic gluons are not needed, and only one basic gluon ($g^*{}_7$) at d = 7 is used. Similar to Equations (7) and (9), the quark mass formulas for the principal and auxiliary mass dimensions are as follows.

$$\begin{aligned} &\text{principal mass dimensional orbital at d} = 7 \\ &M_{\text{principal quark}} = 3M_{g^*{}_7}/2 = 3M_{B_7}/2 = 3M_Z/2 \end{aligned} \tag{12}$$

$$\begin{aligned} &\text{auxiliary mass dimensional orbital at d} = 8 \\ &M_{\text{auxiliary quark}} = \frac{3\left(M_{g^*{}_{a8}}\right)}{2}\sum_{a'=1}^{n'} a'^4 = \frac{3\mu_8^0\alpha}{2}\sum_{a'=1}^{n'} a'^4 \end{aligned} \tag{13}$$

The quark mass formula at d = 8 is the combination of Equations (10) and (11) as follows.

$$M_{q8} = \frac{3M_Z}{2} + \frac{3M_{\mu_8^0}\alpha}{2}\sum_{a'=1}^{n'} a'^4 \tag{14}$$

where a' = 1 and 2 for b'_8 and t'_8, respectively.

Combining Equations (10) and (14), the quark mass formula is as follows.

$$M_{\text{quark}} = \frac{1\text{ or }2Me}{3} + \frac{9Me}{2\alpha} + \frac{9M_{\mu_7}\alpha_w}{2}\sum_{a=1}^{n} a^4 + \frac{3M_Z}{2} + \frac{3M_{\mu_8^0}\alpha}{2}\sum_{a'=1}^{n'} a'^4 \tag{15}$$

where a =1, 2, 3, 4, and 5 for d/u. s, c, b, and t, respectively, and a' = 1 and 2 for b and t respectively. The calculated masses ford, u, s, c, b_7, and t are 328.4 MeV, 328.6 MeV, 539.3 MeV, 1606.6 MeV, 4979.3 MeV, and 175.4 GeV, respectively. The calculated mass of $b_8\left(=b_7+b'_8\right)$ is 143,258 MeV. In the SM, there are three generations of leptons. Extra-muon μ_8 is outside of the three generations of leptons in the SM, so μ_8 is absent as shown in **Table 1**. As shown in **Table 1**, to be symmetrical to the absent μ_8, b_8 quark is also absent. In other words, b_8 quark is hidden. The calculated mass of top quark is 175.4 GeV in good agreement with the observed 172.4 GeV [11]. The periodic table of elementary particles calculates accurately the particle masses of all leptons, quarks, gauge bosons, hadrons, the Higgs boson, and the cosmic rays by using only five known constants: the number (seven) of the extra spatial dimensions in the observed four-dimensional space time from the eleven-dimensional membrane, the mass of electron, the masses of Z and W bosons, and the fine structure constant [5] [6] [7].

3. Beauty-Quark Decays

As described in the previous section, the mass of b_7 is 4974.6 MeV, while the mass of b_8 is 143,258 MeV. In the SM, there are three generations of leptons. Extra-muon μ_8 is outside of the three generations of leptons in the SM, so μ_8 is absent as shown in **Table 1**. To be symmetrical to the absent μ_8, b_8 quark is also absent. In other words, b_8 quark is hidden. As a result, the observable b_7 and the hidden b_8 form the binary isotope mixture.

A beauty meson (B^+) transforms into a strange meson (K^+) with the emission of leptons ($\ell^+\ell^-$) which is either electron-positron (e^+e^-) or muon-antimuon ($\mu^+\mu^-$). The decay process includes a transition state where the decay process is mediated by virtual particles that can have a physical mass larger than the mass difference between the initial- and final-state particles. In the SM, these virtual particles include the electroweak-force carriers, the γ, W± and Z° bosons, and the top quark as **Figure 2**.

In the transition state before the decay, from B^+, the binary isotope mixture of beauty quarks emerges along with the virtual particles (γ, W± and Z°) that can have a physical mass larger than the mass difference between the initial- and final-state particles. In the transition state, the binary isotope mixture is virtual. The rates of emergence as the rates of diffuse in Graham's law of diffusion are proportional to inverse square root of mass.

$$R_b = \left(M_{b_7}/M_{b_8}\right)^{1/2} = \left(4979/143258\right)^{1/2} = 0.1864 \tag{16}$$

The preservation of the generation number symmetry to have the absent μ_8 forbids b_8 to decay into $K^+\mu^+\mu^-$, while b_7 can decay into $K^+\mu^+\mu^-$ and $K^+e^+e^-$. As a result, b_7 decays into K^+, e^+e^-, and $\mu^+\mu^-$, while b_8 decays into K^+, e^+e^-, and forbidden $\mu^+\mu^-$. (The forbidden $\mu^+\mu^-$ simply turns into γ.) The calculated ratio (R_K) of branching fractions for $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ is as follows.

$$R_K = 0.5/\left(0.5R_b + 0.5\right) = 0.5/\left(0.5\times 0.1864 + 0.5\right) = 0.843 \tag{17}$$

The calculated ratio (R_K) of branching fractions for $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ is 0.843 in excellent agreement with the observed 0.846. The agreement between the calculated R_K and the observed R_K confirms the validity of the periodic table of elementary particles.

On molecular level, the emergence of the binary isotope mixture of b_7 and the hidden b_8 from B^+ is similar to the diffusion of the binary isotope mixture of

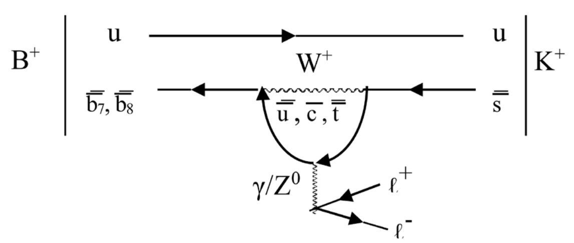

Figure 2. In the transition state before the decay, the binary isotope mixture of the two beauty quarks as the observable $\bar{b}_7$ (4979 MeV mass) and the hidden $\bar{b}_8$ (143,258 MeV mass) emergences from B^+. $\bar{b}_7$ decays into K^+, e^+e^-, and $\mu^+\mu^-$, while b_8 decays into K^+ and only e^+e^-. The virtual particles in the transition state include the electroweak-force carriers, the γ, W± and Z° bosons, and the top quark.

gaseous molecules. One of the well-known examples of the binary isotope mixture is the binary isotope mixture of $^3He/^4He$ from the degassing of the Earth's mantle through magmatism that results in the irreversible loss of helium to space, and high $^3He/^4He$ ratios observed in oceanic basalts have been considered the main evidence for a "primordial" undegassed deep mantle reservoir [15]. The initial ratio of the degassing rates from Graham's law between $^3He/^4He$ is as follows.

$$R_{\text{He}} = \left(M_{^4\text{He}} / M_{^3\text{He}}\right)^{1/2} = \left(4/3\right)^{1/2} = 1.15 \tag{18}$$

As a result, the initial rate ratio of degassing is 1.15 which is confirmed by the observation. Another example of the binary isotope mixture is the neon isotope mixture of$^{20}Ne/^{22}Ne$ [16]. The isotopic diffusivity ratio for neon ($^{20}Ne/^{22}Ne$) in silicate glasses appears to equal the inverse square-root of the isotopic masses as 1.05.

4. Dark Matter and the Dominance of Matter over Antimatter

It is speculated that since the SM is unable to explain cosmological observations of the dominance of matter over antimatter, the apparent dark-matter content of the Universe, or explain the patterns seen in the interaction strengths of the particles, the new physics derived from the violation of lepton universality can explain the SM's shortcomings [1]. In the Section 2, the periodic table of elementary particles explains the patterns seen in the interaction strengths of the particles for leptons and quarks. In this Section, dark matter and the dominance of matter over antimatter are explained by the periodic table of elementary particles.

The origins of dark matter and baryonic matter were explained in the references [5] [8] [17]. There are five types of dark matter and one type of baryonic matter with the mass ratio of 5:1 [8] [17]. In the periodic table of elementary particles, the five types of dark matter are the five sterile massive right-handed neutrinos, ν_{DM5}, ν_{DM6}, ν_{DM7}, ν_{DM8}, and ν_{DM9} [5]. At 72.8% dark energy, the calculated values for baryonic matter and dark matter (with the 1:5 ratio) are 4.53% (=(100 − 72.8)/6) and 22.65% (=4.53 × 5), respectively, in excellent agreement with observed 4.56% and 22.7%, respectively [18].

The dominance of matter over antimatter can be explained by B_{10} in the table for the principal mass dimensional bosons derived from the periodic table of elementary particles as in **Table 2**.

The lowest energy gauge boson (B_5) at d = 5 is the Coulomb field for electromagnetism. The second gauge lowest boson (B_6) at d = 6 is basic gluon (g* = 70 MeV ≈ one half of pion) is the strong force as the nuclear force in the pion theory [19] where pions mediate the strong interaction at long enough distances (longer than the nucleon radius) or low enough energies. B_6 is denoted as basic gluon, g*. At short enough distances (shorter than the nucleon radius) or high enough energies, gluons emerge to confine fractional charge quarks. Fractional charge quarks are confined by gluons in QCD (quantum chromodynamics). No

Table 2. The masses of the principal mass dimensional bosons (gauge bosons) $M_{d,B} = M_{d,F}/\alpha_d$ and $M_{d+1,B} = M_{d,B}/\alpha_{d+1}^2$ from Equations (1) and (3).

B_d	M_d	GeV (calculated)	Gauge boson	Interaction
B_5	$M_e\alpha$	3.7×10^{-6}	A = photon	Electromagnetic
B_6	M_e/α	7×10^{-2} (70.02 MeV)	g* = basic gluon	Strong
B_7	$M_Z = M_{B6}/\alpha_w^2$	91.1876 (given)	Z_L	weak (left) for baryonic matter
B_8	$M_7/\alpha^2 = M_Z/\alpha^2$	1.71×10^6	Z_R	weak (right) for dark matter
B_9	$M_8/\alpha^2 = M_Z/\alpha^4$	3.22×10^{10}		dark matter repulsive force
B_{10}	$M_9/\alpha^2 = M_Z/\alpha^6$	6.04×10^{14}		particle-antiparticle asymmetry
B_{11}	$M_{10}/\alpha^2 = M_Z/\alpha^8$	1.13×10^{19}	G	gravity

isolated fractional charge quark is allowed, and only collective integer charge quark composites are allowed. In general, collective fractional charges are confined by the short-distance confinement force field where the sum of the collective fractional charges is integer [20]. As a result, fractional charges are confined and collective. The confinement force field includes gluons for collective fractional charge quarks in hadrons and the magnetic flux quanta for collective fractional charge quasiparticles in the fractional quantum Hall effect (FQHE) [21] [22] [23].

The third lowest boson (B_7) at d = 7 is Z_L for the left-handed weak interaction among leptons and quarks. Massive weak bosons produce short-distance interaction. B_8 at d = 8 is Z_R for the right-handed weak interaction among dark matter neutrinos as dark matter neutrino oscillation. The symmetry between Z_R and Z_L provides the neutrino oscillation for both baryonic matter neutrinos [24] and dark matter neutrinos.

B_9 as the gauge boson represents dark matter repulsive force. The condensed baryonic gas at the critical surface density (derived from the acceleration constant a_0 in MOND [25] [26]) induces the creation tensor for dark matter repulsive force to transform dark matter in the region into repulsive dark matter repulsing one another, corresponding to the Farnes' repulsive dark matter [8] [27]. Before the emergence of dark matter repulsive force, dark matter in the CMB was not repulsive.

B_{10} at d = 10 is for the gauge boson for particle-antiparticle asymmetry to provide the slight excess of particle in particle-antiparticle at the Big Bang, while B_8 has particle-antiparticle symmetry. (B_9 emerged long after the Big Bang.) As a result, the excess of particle is α^4 (2.8×10^{-9}) per particle-antiparticle (photon) for the ratio between B_8 and B_{10}. Since baryonic matter is 1/6 of dark matter and baryonic matter [5] [8] [17], the baryonic matter excess is 4.7×10^{-10} which is in a good agreement with 6×10^{-10} for the ratio of the numbers between baryonic matter and photons in the Big Bang nucleosynthesis [28].

B_{11} is for gravity. F_{11} (8.275×10^{16} GeV) relates to spin 3/2 gravitino, while B_{11}

(1.134×10^{19} GeV) relates to spin 2 graviton. In supersymmetry, gravitino and graviton mediate the supersymmetry between fermion and boson in space dimension and gravitation. There are 11 space dimensions in the 11 space time dimensional membrane. As a result, the supersymmetry involves 11 $F_{11} + B_{11}$, which is equal to 1.225×10^{19} GeV in excellent agreement with the Planck mass (1.221×10^{19} GeV) derived from observed gravity as $(\hbar c/G)^{1/2}$ where c is the speed of light, G is the gravitational constant, and ħ is the reduced Planck constant.

5. Summary

In summary, this paper purposes an explanation for the recent evidence for the violation of lepton universality in beauty-quark decays at CERN's Large Hadron Collider. A beauty meson (B^+) transforms into a strange meson (K^+) with the emission of either electron-positron (e^+e^-) or muon-antimuon ($\mu^+\mu^-$). The ratio (R_K) of branching fractions for $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ decays is measured to be $R_K = 0.846$ instead of 1 in the violation of lepton universality in the Standard Model. This paper proposes that the violation is derived from the binary isotope mixture of two beauty-quarks, b_7 (4979 MeV mass) and b_8 (143,258 MeV mass) whose masses are calculated from the periodic table of elementary particles. b_7 is the observable B, while b_8 is the hidden B to preserve the generation number symmetry between the three lepton family generations and the three quark family generations in the Standard Model. The preservation of the generation number symmetry forbids b_8 to decay into $K^+\mu^+\mu^-$. In the transition state involving the virtual particles (γ, W± and Z°) before the decay, b_7 and b_8 emerge to form the binary isotope mixture from B. The rates of emergence as the rates of diffuse in Graham's law of diffusion are proportional to inverse square root of mass. The rate ratio between b_8/b_7 is $(4979/143,258)^{1/2} = 0.1864$. Since b_7 decays into K^+, e^+e^-, and $\mu^+\mu^-$, while b_8 decays into K^+, e^+e^-, and forbidden $\mu^+\mu^-$, the calculated ratio (R_K) of branching fractions for $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$ is $0.5/(0.1864 \times 0.5 + 0.5) = 0.843$ in excellent agreement with the observed 0.846. The agreement between the calculated R_K and the observed R_K confirms the validity of the periodic table of elementary particles which provides the answers for the dominance of matter over antimatter, dark-matter, and the mass hierarchy of elementary particles.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] LHCb Collaboration (2021) Test of Lepton Universality in Beauty-Quark Decays. arXiv:2103.11769 [hep-ex]. https://arxiv.org/abs/2103.11769

[2] Chung, D. (1997) *Speculations in Science and Technology*, **20**, 259-268. https://doi.org/10.1023/A:1026433207862

[3] Chung, D. and Hefferlin, R. (2013) *Journal of Modern Physics*, **4**, 21-26. https://doi.org/10.4236/jmp.2013.44A004

[4] Chung, D. (2016) *Journal of Modern Physics*, **7**, 1591-1606. https://doi.org/10.4236/jmp.2016.712144

[5] Chung, D. (2018) *Journal of Modern Physics*, **9**, 2308-2319. https://doi.org/10.4236/jmp.2018.913146

[6] Chung, D. (2018) *Journal of Modern Physics*, **9**, 2638-2656. https://doi.org/10.4236/jmp.2018.914164

[7] Chung, D. (2019) *Journal of Modern Physics*, **10**, 1310-1341. https://doi.org/10.4236/jmp.2019.1011087

[8] Chung, D. (2020) *Journal of Modern Physics*, **11**, 1091-1122. https://doi.org/10.4236/jmp.2020.117069

[9] Kamilla, R., Wu, X. and Jain, J. (1996) *Physics Review Letters*, **76**, 1332. https://doi.org/10.1103/PhysRevLett.76.1332

[10] Jain, J. (2007) Composite Fermions. Cambridge University Press, New York.

[11] Particle Data Group (2018) *Physical Review D*, **98**, Article ID: 030001. https://doi.org/10.1103/PhysRevD.98.030001

[12] Barut, A. (1979) *Physical Review Letters*, **42**, 1251. https://doi.org/10.1103/PhysRevLett.42.1251

[13] Krasznahorkay, A., *et al.* (2016) *Physical Review Letters*, **116**, Article ID: 042501. https://doi.org/10.1103/PhysRevLett.116.042501

[14] Feng, J., *et al.* (2016) *Physical Review Letters*, **117**, Article ID: 071803. https://doi.org/10.1103/PhysRevLett.117.071803

[15] Class, C. and Goldstein, S. (2005) *Nature*, **436**, 1107-1112. https://doi.org/10.1038/nature03930

[16] Rama, S. and Hart, S. (1965) *Science*, **147**, 737-738. https://doi.org/10.1126/science.147.3659.737

[17] Chung, D. and Krasnoholovets, V. (2013) *Journal of Modern Physics*, **4**, 77-84. https://doi.org/10.4236/jmp.2013.47A1009

[18] Jarosik, N., *et al.* (2010) Seven-Year Wilson Microwave Anisotropy Probe (WMAP) Observations: Sky Maps, Systematic Errors, and Basic Results. http://lambda.gsfc.nasa.gov/product/map/dr4/pub_papers/sevenyear/basic_results/wmap_7yr_basic_results.pdf

[19] Fujita, J. and Miyazawa, H. (1957) *Progress of Theoretical Physics*, **17**, 360. https://doi.org/10.1143/PTP.17.360

[20] Chung, D. (2016) *Journal of Modern Physics*, **7**, 1150-1159. https://doi.org/10.4236/jmp.2016.710104

[21] Tsui, D., Stormer, H. and Gossard, A. (1982) *Physical Review Letters*, **48**, 1559-1562. https://doi.org/10.1103/PhysRevLett.48.1559

[22] Stormer, H. (1999) *Reviews of Modern Physics*, **71**, 875-889. https://doi.org/10.1103/RevModPhys.71.875

[23] Laughlin, R. (1983) *Physical Review Letters*, **50**, 1395-1398. https://doi.org/10.1103/PhysRevLett.50.1395

[24] Loureiro, A., *et al.* (2019) *Physical Review Letters*, **123**, Article ID: 081301. https://doi.org/10.1103/PhysRevLett.123.081301

[25] Milgrom, M. (1989) *Astronomy and Astrophysics*, **211**, 37-40.

[26] Milgrom, M. (2009) *Monthly Notices of the Royal Astronomical Society*, **398**, 1023-1026. https://doi.org/10.1111/j.1365-2966.2009.15255.x

[27] Farnes, J. (2018) *Astronomy & Astrophysics*, **620**, A92. https://doi.org/10.1051/0004-6361/201832898

[28] Steigman, G. (2007) *Annual Review of Nuclear and Particle Science*, **57**, 463-491. https://doi.org/10.1146/annurev.nucl.56.080805.140437

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Standard Model Masses Explained

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Abstract

The Standard Model of particle physics requires nine lepton and quark masses as inputs, but does not incorporate neutrino masses required by neutrino oscillation observations. This analysis addresses these problems, explaining Standard Model particle masses by describing fundamental particles as solutions of Einstein's equations, with radii 1/4 their Compton wavelength and half of any charge on rotating particles located on the surface at each end of the axis of rotation. The analysis relates quark and lepton masses to electron charge and mass, and identifies neutrino masses consistent with neutrino oscillation observations.

Keywords

Standard Model Masses, Spherical Fundamental Particles, Einstein's Equations

1. Introduction

The particle physics Standard Model involves three trios of charged fermions, three neutrinos, three spin one bosons with average charge zero, and a scalar Higgs boson. The nine charged fermion masses must be provided as inputs to the Standard Model. Neutrinos are massless in the Standard Model, in conflict with observations of neutrino oscillations. This analysis resolves these problems by describing particles as solutions of Einstein's equations, with radii 1/4 their Compton wavelength l and half of any charge on rotating particles located on the surface at each end of the axis of rotation, to explain Standard Model masses consistent with PDG 2020 [1] and neutrino mass error bars. Particle internal mass and pressure distribution is described by cubic equations for l in each charge state, allowing at most three particles in each charge state. This analysis replaces the Standard Model assumption of massless neutrinos that conflicts with neutrino oscillation observations, and avoids point particle infinite density. Charge neutrality of the universe is important to the analysis.

2. Cubic Equations for Wavelengths from Mass and Pressure Distribution

Internal particle mass and pressure distribution, involving volume $\sim l^3$, surface area $\sim l^2$, and diameter $l/2$, results in cubic equations in l and allows at most three particles in each charge state. Describing mass and pressure distribution with surface and linear elements requires minimum surface shell thicknesses and axial core radii near the Planck length $l_P = \sqrt{\frac{\hbar G}{c^3}}$. Total particle mass is the sum of mass equivalents of pressure, $m/2$, in the volume, mass equivalent of surface pressure $\frac{\pi}{4} S l^2$, and core mass Ll. So $\frac{4}{3}\pi\rho\left(\frac{l}{4}\right)^3 = \frac{4}{3}\pi\frac{\rho}{2}\left(\frac{l}{4}\right)^3 + 4\pi S\left(\frac{l}{4}\right)^2 + 2L\left(\frac{l}{2}\right)$. This cubic, written as $Al^3 - Bl^2 - Cl = 0$ with $A = \frac{\pi}{96}\rho$, $B = \frac{\pi S}{4}$, and $C = 2L$, has discriminant $\Delta = B^2C^2 - 4AC^3$.

3. Charged Fermions

The Standard Model involves three constituent quarks in protons. With electron charge $-e$ and two trios of quarks, hydrogen atom charge neutrality requires two charge $\frac{2e}{3}$ up quark and one charge $\frac{-e}{3}$ down quark constituent in each proton. Using the fine structure constant $\frac{e^2}{\hbar c} = \frac{1}{137}$, electrostatic potential energy from repulsion between equal surface charges at opposite ends of the rotation axis is $\left(\frac{qe}{6}\right)^2 \Big/ \left(\frac{l}{2}\right) = q^2 mc^2 \frac{e^2}{18\hbar c} = q^2 \frac{mc^2}{2466}$. With electron mass m_e, electrostatic potential energy of charged ground state fermions is identical if up quark mass = $4m_e$ and down quark mass = $9m_e$, well within PDG 2020 error bars.

Discriminant Δ is positive for fermions regardless of the sign of B, so the cubic has three real roots corresponding to three fermion Compton wavelengths in each charge state [2]. Wavelengths in each charge state are projections on the l axis of vertices of Nickalls triangles, centered on average wavelength $l_{avg}(q)$ and circumscribed by circles with radius $R(q)$ intersecting the l axis at $l_{avg}(q) + R(q)$. Largest wavelength $l_1(q) = l_{avg}(q) + R(q)\cos[\theta(q)]$, smallest wavelength $l_3(q) = l_{avg}(q) + R(q)\cos[\theta(q) + 2\pi/3]$, and intermediate wavelength $l_2(q) = l_{avg}(q) + R(q)\cos[\theta(q) + 4\pi/3]$. Nickalls triangle offset distance, in **Figure 1**, is $d(q) = \sqrt{R^2(q) - D^2(q)}$, where $D(q) = R(q) - l_1(q) - l_{avg}(q)$. With up quark mass = $4m_e$ and down quark mass = $9m_e$, hydrogen atom charge neutrality relates lepton wavelengths to quark wavelength in charge states $\frac{2e}{3}$ and $\frac{-e}{3}$ by $d(-e) = d\left(\frac{-e}{3}\right) - 2d\left(\frac{2e}{3}\right)$. Strange quarks, with lowest mass and longest wavelength of 2nd and 3rd generation fermions, have greatest "leverage" on Nickalls triangle orientation, and PDG 2020 masses satisfy the $d(q)$ relation if strange quark mass is increased 0.5% to 93.45 MeV/c².

Fermions described [3] as Godel solutions to Einstein's equations have matter energy density $\rho c^2 = \frac{48\hbar c}{\pi l^4}$, vacuum energy density $-\frac{1}{2}\rho c^2 = -\frac{24\hbar c}{\pi l^4}$, and fermion internal gravitational constants $G_{Fi} = \frac{3c^3 l_i^4(q)}{\hbar\left[0.2 l_i(q) - l_{avg}(q)\right]^2}$. Fermion equatorial tangential velocity > *c*, allowing closed time-like curves, is pathological in Godel cosmologies but not a problem in fermions unchanged from creation to annihilation.

4. Neutral Particles

Considering our universe as a closed vacuum-dominated Friedmann solution of Einstein's equations so large it is almost flat, with Hubble constant $H_0 = 67.8\ \text{km}\cdot\text{sec}^{-1}\cdot\text{Mpc}^{-1}$ and $\Omega_\Lambda = 0.692$, cosmic vacuum energy density is $\rho_v = 5.98\times10^{-30}\ \text{g/cm}^3$ [4]. Electron neutrinos are neutrino ground state if electron neutrino vacuum energy density is the negative of cosmic vacuum energy density, with electron neutrino wavelength $l_1 = \sqrt[4]{\frac{24\hbar}{\pi\rho c}}$ and mass $m_1 = 1.36\times10^{-3}\ \text{eV}$. Neutrino oscillation data [5] $m_2^2 - m_1^2 = 7.37\times10^{-5}(\text{eV})^2$ and $m_3^2 - \frac{1}{2}\left(m_1^2 + m_2^2\right) = 2.50\times10^{-3}(\text{eV})^2$ estimates $m_2 = 8.58\times10^{-3}\ \text{eV}$, $m_3 = 5.08\times10^{-2}$ and neutrino mass sum 0.61 eV, about half Vagnozzi's [6] 0.12 eV upper bound on neutrino mass sum.

Electroweak symmetry breaking requires three vector bosons with zero average charge: $W_\pm$ (mass $m_w = 80379\ \text{MeV/c}^2$ and $l_w = 2.45\times10^{-3}\ \text{F}$) and Z (mass $m_Z = 91187.6\ \text{MeV/c}^2$ and $l_z = 2.16\times10^{-3}\ \text{F}$). Two equal $W_\pm$ wavelengths occur if discriminant $\Delta = 0$, requiring $B^2 = 4AC$, $\rho = \frac{3}{4}\pi\frac{S^2}{L}$, and vector boson internal gravitational constants $G_B = \frac{25c^3 l^2}{3\hbar}$. Scalar Higgs bosons (mass $m_H = 125100\ \text{MeV/c}^2$ and $l_H = 1.58\times10^{-3}\ \text{F}$) have no rotation axis, and no *l* term in mass equivalent distribution. So only one charge zero scalar particle is allowed, $Al - B = 0$, $S = \frac{2\hbar c}{\pi l_H^3}$, and Higgs internal gravitational constant $G_H = \frac{c^3 l_H^2}{12\hbar}$.

These results account for the masses of all Standard Model particles with zero average charge.

5. Relation to Point Particle Idealization

Classical physics idealizes the force of gravity between extended bodies as the force between masses concentrated at the center of mass of each body. The mathematical construct corresponding to the center of mass of extended Standard Model particles provides a link to the idealization of point particles in quantum mechanics. At the instant of particle-antiparticle creation, the centers of mass of the respective Einstein solutions coincide and the spin axes are aligned. Correspondingly, at the instant of particle-antiparticle annihilation, the centers of mass of the respective Einstein solutions coincide and the spin axes are aligned.

6. Conclusion

Describing Standard Model particles as solutions of Einstein's equations, with radii 1/4 their Compton wavelength and charge on rotating particles located on the surface at each end of the axis of rotation, explains Standard Model masses consistent with PDG 2020 and neutrino mass error bars. Study of interacting Einstein solutions with radii 1/4 Compton wavelength of their mass seems justified.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Zyla, P.A., *et al.* (2020) (Particle Data Group 2020) *Progress of Theoretical and Experimental Physics*, **2020**, 083C01.

[2] Nickalls, R.W.D. (1993) *The Mathematical Gazette*, **77**, 354. https://doi.org/10.2307/3619777

[3] Mongan, T.R. (2020) *Journal of Modern Physics*, **11**, 1993-1998. https://doi.org/10.4236/jmp.2020.1112126

[4] Mongan, T.R. (2012) Holography, Large Scale Structure, Supermassive Black Holes and Minimum Stellar Mass. arXiv:1301.0304

[5] Capozzi, F. (2016) *Nuclear Physics B*, **908,** 218. [arXiv:1601.07777].

[6] Vagnozzi, S. (2019) Cosmological Searches for the Neutrino Mass Scale and Mass Ordering. PhD Thesis, Stockholm University, Stockholm. [arXiv:1907.08010].

[7] Mongan, T. (2001) *General Relativity and Gravitation*, **33**, 1415. https://doi.org/10.1023/A:1012065826750

[8] Bousso, R. (2002) *Reviews of Modern Physics*, **74**, 825. [arXiv:hep-th/0203101]. https://doi.org/10.1103/RevModPhys.74.825

[9] Mongan, T.R. (2021) *Journal of Modern Physics*, **12**, 295-299. https://doi.org/10.4236/jmp.2021.123022

Appendix: Cosmological Context

Our universe can be described as a closed vacuum-dominated Friedmann solution of Einstein's equations so large it is almost flat. If there are compact extra dimensions, the universe could originate by quantum fluctuation from nothing [7], with dark energy identified as energy associated with compact dimensions. The holographic principle [8] indicates only $N = \frac{3}{8G\rho_v}\left(\frac{c}{l_P}\right)^2 = 4.69\times10^{122}$ bits of information will ever be available to describe the observable universe. With no source or sink for information outside a closed universe, the number of bits of information in the universe remains constant. Two states $\pm\frac{e}{6}$ of each bit define electric charge e, and origin of the universe by quantum fluctuation from nothing requires zero net charge in the universe. Energy difference between bit states requires a small antimatter/matter ratio, consistent with observations [9].

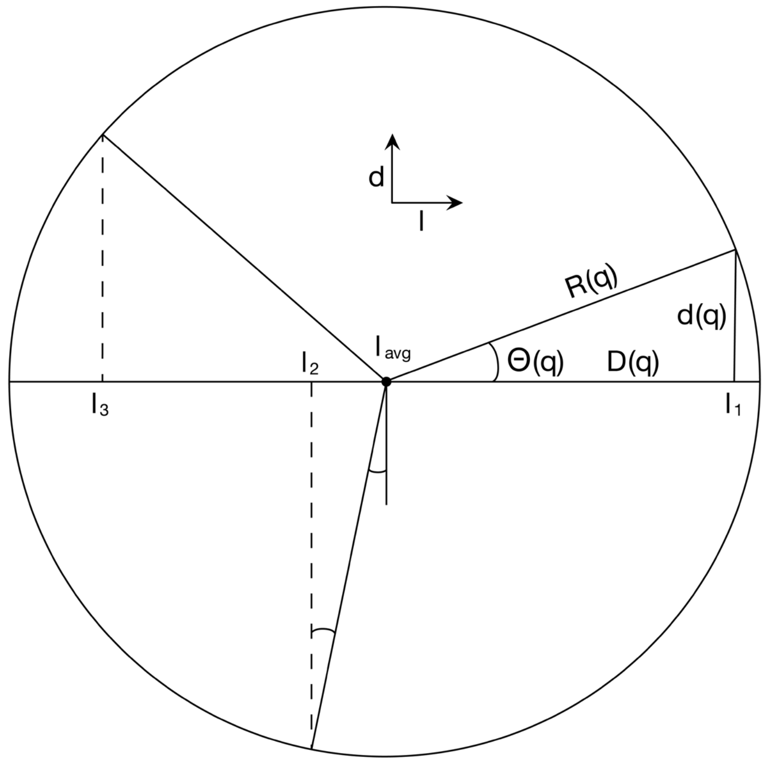

Figure 1. Particle wavelengths related to Nickalls triangle vertices on circumscribing circle.

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Mass, Fine Structure Constant, and the Classification of Elementary Particles by Masses

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Abstract

The equations of motion of physical bodies are given, the characteristic parameters of which become the basis for determining a fundamental property of all matter—"mass". The equations of motion are characterized by two constants, the derivative of one of which is the fine structure constant. Using these constants, energy scales are compiled, which are the basis for classifying particles by mass.

Keywords

Mass, Energy, Structure, Quantum Numbers, Elementary Particles

1. Introduction

In modern theoretical views, the origin of the mass and the fine structure constant (the symbol α) one relates with the phenomena of interaction [1]-[6], thus the search for a connection between the mass of elementary particles (EP) and α seems to be quite natural. However, the establishment of this connection is complicated by the fact that the contents of α and mass are not sufficiently revealed. Regarding the content of α, Feynman remarks very nicely: "one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man" [7]. In practice, the conclusion given in [8] [9] that the concept of mass is shrouded in serious uncertainties and is one of the most important unsolved problems of modern physics remains relevant.

In the proposed work, based on the results of the approach [10] [11] [12], which is named as "Structural theory of the physical world" or simply the Structural theory (ST), a new interpretation of the genesis of α is put forward, a connection is established between the mass of the EP and α, a variant of the classification of EP by masses is proposed.

2. The Equations of Motion and Mass of Physical Bodies

In the ST, a hypothesis is accepted on the existence of some particles of the conventionally smallest hierarchical level (ε-particles), the elementary act of interaction between which (the ε-act) takes place with a strictly defined duration at a strictly defined distance, while due to the ε-act, the particles displace at the same distance. Hence, the number of successively realized ε-acts can be used as a single tool for determining both length and time. Taking the interval of length and time associated with the ε-act (ε-interval) equal to ξ_d cm and ξ_τ sec, respectively, let us give a dimensional definition of the smallest path ξ_d and time ξ_τ intervals, as a result of this they are called the coefficients of the dimensions of length and time.

Δ-elements of three types are modeled from ε-particles: Δ_i, Δ_j, and Δ_k which perform oscillatory motion in mutually perpendicular directions with amplitude H_Δ.

A pair of identical Δ-elements (type $2\Delta_i$, $2\Delta_j$, $2\Delta_k$) is characterized by α_0-multiple repetitions of the amplitude

$$H_\Delta^2 = H_c, \tag{1}$$

that is, the total number of ε-acts

$$H_{00} = \alpha_0 H_c, \tag{2}$$

where

$$\alpha_0 = \sum_{n=1}^{7} n^2 = \sum_{n=1}^{7}\sum_{l=0}^{n-1}(2l+1) = 140 \tag{3}$$

constant determined by the number of sequentially realized states Δ-pairs, n and l are the analogs of the principal and azimuthal quantum numbers for the considered hierarchical level.

Of the six Δ-elements, the γ-particles are modeled, which are the basis of the known EPs (electrons, muons, mesons, proton, neutron, etc.), as well as γ_{0i} particles of the general Δ-composition $2\Delta_i$, $2\Delta_j$, $2\Delta_k$, the presence of which explains the common quantitative laws their movements in all physical bodies.

The resulting trajectory of motion of γ_{0i}-particles, determined by excluding time from the periodic parametric equations describing the behavior of each Δ-pair separately, is formed as a torus, the volume of which is called the trajectorial one and is determined by the integrals

$$\int_L S\mathrm{d}l = \int_{S_j} curl S \mathrm{d}S_j \tag{4}$$

where $S = S_i + S_k$, the axial vectors S_i, S_k and S_j are defined using the vector products $H_j \times H_k, H_i \times H_j$ and $H_k \times H_i$, and $\mathrm{d}l = \mathrm{d}l_i + \mathrm{d}l_k$ is the sum of elementary paths due to $2i$- and $2k$-pairs.

The above equation differs from the Stokes equation in that it describes the motion of one particle and with its help, the volume is computed: by the circulation of an axial vector S along a closed path (the left side of the equation) or a mixed product of three vectors, one of which is $curlS$ (the right side of the equation). The numerical values of the circulation path L, axial vectors S_i, S_k and S_j, determined using curvilinear integrals [12], are respectively equal to:

$$L = \alpha_0 2\pi r_c, \quad S = \alpha_0 \pi r_c^2, \quad S_j = \alpha_0 4\pi r_c^2 \tag{5}$$

where the moduli of oscillations of pairs are taken to be equal to each other: $|H_i| = |H_j| = |H_k| = H_c$, besides, it is indicated that

$$r_c = H_c \xi_d / 2 \tag{6}$$

Let a circle be inscribed in a square with the side $H_c \xi_d$. Multiplying the length $4H_c \xi_d$ and area of this square $H_c^2 \xi_d$ by π/4, we obtain the circumference inscribed in the square and the area of the circle. Hence, we can conclude that with the help of the coefficient π/4 we pass from the spent ε-acts to the measured quantities related to the formation of rounded trajectories of motion. The formulas of the series (5) characterize a planar figure, at the same time, the trajectory of motion of γ_{0i}-particles formed by adding three mutually perpendicular oscillations is a three-dimensional figure, that is, a greater number of ε-acts are spent on the formation of a real trajectory of motion. Taking this circumstance into account, it is assumed that in the transition from three-dimensional to flat figures, the amplitude H_c of the oscillation decreases by the factor χ_c, thereby introducing a new constant H_0, we transform Equation (2) to the form

$$H_0 = \alpha_0 H_c / \chi_c = \alpha_c H_c \tag{7}$$

where is denoted

$$\alpha_c = \alpha_0 / \chi_c \tag{8}$$

Taking into account the designations (5) and (7), we represent the formulas of series (5) in the form

$$L = \alpha_c 2\pi R, \quad S = \alpha_c \pi r_c^2, \quad S_j = \alpha_c 4\pi r_c^2 \tag{9}$$

The trajectorial volume of γ_{0i}-particles obtained from integrals (4) taking into account formulas (9) and interaction conditions are determined by the equations

$$\alpha_c 2\pi r_c \left[\alpha_c^2 \pi r_c^2\right] = \alpha_c \pi \frac{r_c}{2}\left[\alpha_c^2 \pi 4 r_c^2\right] = 2\pi^2 \alpha_c^3 r_c^3 \tag{10}$$

$$\alpha_c 2\pi R\left[\pi H_i^2\right] = \alpha_c \pi \frac{r_c}{2}\left[\alpha_c^2 \pi 4 r_c^2\right] \tag{11}$$

$$\lambda\left[H_i^2\right] = H_i\left[\pi \alpha_c^2 H_c^2\right] = H_i\left[\pi H_0^2\right] \tag{12}$$

$$\left(\alpha_c 2\pi R_c + \lambda\right)\left[H_i^2\right] = \left(H_0 + H_i\right)\left[\pi H_0^2\right] \tag{13}$$

where oriented surfaces (axial vectors) are taken in square brackets.

Equation (10) describes the motion of γ_{0i}-particles during intrinsic interaction involving only Δ-eigenpairs with equal moduli of oscillation amplitudes: $|H_{ic}| = |H_{jc}| = |H_{kc}| = H_c$, where the additional subscript "*c*" marks the characteristics of eigenpairs. With its interaction, the trajectory of motion of γ_{0i}-particles is a torus with equal radii $r_c = H_c \xi_d / 2$. With the aid of Equation (11), the motion of γ_{0i}-particles involved in the interaction with their own partner is described. As an example of this type of interaction, one can consider the electric and gravitational interactions [10] [11]. In this case, the interaction mechanism is reduced to the following: the proper particles are making the shuttle motion relative to their bases and form complexes with the bases of their partners, exchanging Δ-pairs. Particles with the participation of which the electrostatic interaction is realized at a distance are called γ_E-particles, the index "*E*" at γ indicates the nature of the interaction. With the help of the index "*i*" at the amplitude H_i of the oscillation in Equations (10)-(13), it is indicated the participation in the interactions of its own or third-party partners. When $H_i < H_c$ the trajectory of motion is in the form of a torus with the large radius *R* and the small radius $r = H_i \xi_d / 2$.

Equation (11) is often convenient to represent in the form

$$\alpha_c 2\pi R\left[H_i^2\right] = \alpha_c H_c \left[\alpha_c^2 \pi H_c^2\right] \tag{14}$$

Interactions with third-party partners are implemented mainly by external influences: mechanical, radiation, heat transfer, etc. The results of interaction are reduced to the transfer or exchange of doublets from γ_{0i}-particles (β_ε-pairs) of third-party origin. In a free state, β_ε-pairs are photons characterized by the Δ compositions of the type $2j2\bar{i}2k/2\bar{j}2i2k$ [10] [11], where the dashes above the Δ-element indicate that the Δ-pair is moving in the opposite direction.

The presence in the numerator and denominator of Δ-pairs with the same directions and phases of motion determines the direction of the photon motion, while λ in Equation (12) it is not the wavelength at all, this is the smallest transverse path, with the passage of which the manifestation of β_ε-pairs (or photon) as an integral formation. In various media γ_{0i}-particles from outside β_ε-pairs form complexes with the particles of the medium, the motion of which is already described by the Equation (13).

In [10] [11] it was shown that proceeding from the equations of motion (4), (10-14) it is possible to derive all the equations of classical mechanics, electrodynamics, and thermodynamics having an empirical origin.

From the quantitative laws of motion (10-14) it follows that the smallest intervals of the longitudinal and transverse paths associated with the manifestation of the integrity of the γ_{0i}-particles are equal to

$$l_i = H_i \xi_d, l_0 = H_0 \xi_d, l_{0i} = \left(H_0 + H_i\right)\xi_d \tag{15}$$

$$\lambda = \frac{\pi H_0^2 \xi_d}{H_i}, \lambda_0 = \frac{\pi H_0^2 \xi_d}{H_o} = \pi H_0 \xi_d, \lambda_{0i} = \frac{\pi\left(H_0 + H_i\right)H_0^2}{H_i^2} \tag{16}$$

The corresponding time intervals for longitudinal and transverse motion are the same and determined by the formulas

$$\tau_i = H_i \xi_\tau, \tau_0 = H_0 \xi_\tau, \tau_{0i} = \left(H_0 + H_i\right) \xi_\tau \tag{17}$$

where taking into account the fact that during longitudinal displacement a certain number of ε-intervals is spent on weaving a perpendicular surface, a new time coefficient is introduced

$$\xi_\tau = \pi H_0^2 \xi_t \tag{18}$$

Based on Equations (15) and (16), the speed of longitudinal motion is determined by the formula

$$v = \frac{H_i}{H_0 + H_i} c \tag{19}$$

where is denoted

$$c = \frac{\xi_d}{\xi_\tau} \tag{20}$$

Using formulas (16), (19), and (20), we compose the following identities $\lambda c \equiv \lambda c, \lambda_0 c \equiv \lambda_0 c, \lambda v \equiv \lambda v$ and represent their right-hand sides as a product of dimensional and dimensionless components

$$\begin{gathered} \lambda c = \frac{\pi H_0^2 \xi_d^2}{H_i \xi_\tau}, \lambda_0 c = \pi H_0 \frac{\xi_d^2}{\xi_\tau}, \\ \lambda v = \frac{\pi H_0^2 H_i}{H_i \left(H_0 + H_i\right)} \frac{\xi_d^2}{\xi_\tau} = \frac{\pi H_0^2}{\left(H_0 + H_i\right)} \frac{\xi_d^2}{\xi_\tau} \end{gathered} \tag{21}$$

Dividing and multiplying the right-hand sides of the given equalities by a strictly constant value with the dimension of mass ξ_m, we obtain:

$$\lambda c = h m_i^{-1}, \lambda_0 c = h m_0^{-1}, \lambda v = h m^{-1} \tag{22}$$

or

$$m_i c \lambda = h, m_0 c \lambda = h, m v \lambda = h \tag{23}$$

where the following is denoted:

$$m_i = \frac{\xi_m H_i}{\pi H_0^2}, m_0 = \frac{\xi_m H_0}{\pi H_0^2}, m = \frac{\xi_m \left(H_0 + H_i\right)}{\pi H_0^2} \tag{24}$$

$$h = \frac{\xi_m \xi_d^2}{\xi_\tau} \tag{25}$$

From formula (25) it follows that the constant h is a combination of the dimension coefficients; it is shown in the ST that h is numerically equal to Planck's constant.

The symbol m_i denotes the mass of interaction with third-party partners, the mass of "rest" m_0 is the result of its interaction, m is the total mass, which can be represented as the sum

$$m = m_0 + m_i \tag{26}$$

By analogy with ξ_d and ξ_τ, the constant ξ_m is called the coefficient of a dimension of the mass. Many other dimensions can be combined with ξ_d, ξ_τ

and ξ_m.

Integrating from ξ_d, ξ_τ and ξ_m the energy dimension coefficient

$$\xi_\varepsilon = \frac{\xi_m \xi_6^2}{\xi_\tau}, \tag{27}$$

and multiplying by dimensionless components of a certain mass (24), and taking into account (26), we obtain

$$m_i c^2 = \frac{\xi_\varepsilon H_i}{\pi H_0^2}, m_0 c^2 = \frac{\xi_{\varepsilon i}}{\pi H_0}, mc^2 = \frac{\xi_\varepsilon \left(H_0 + H_i\right)}{\pi H_0^2}, \tag{28}$$

whence it follows that the content of the unity of mass and energy is in the unity of their dimensionless components.

By slightly transforming Equations (16), (22), and (28), the energy of γ_{0i} -particles can be represented as the reciprocal of the transverse path:

$$m_i c^2 = \frac{ch}{\lambda}, m_0 c^2 = \frac{ch}{\alpha_c 2\pi r_c}, mv^2 = \frac{ch}{\alpha_c 2\pi r + \lambda}, \tag{29}$$

where taking into account formulas (20), (25), and (27), we denote

$$\xi_\varepsilon \xi_d = ch \tag{30}$$

Based on Equations (21), (24), (28), and (29), we can conclude that the fundamental property of matter, mass, is determined by the ratio of the perpendicular surfaces of transverse motion to the value of the corresponding trajectory volumes—the reciprocal of the transverse path.

We emphasize that the above definitions refer to inertial mass.

3. Numerical Values of Dimension Coefficients, α_c and H_c

It was shown in [12] and [13] that the Newton gravity constant G is also a combination of the dimension coefficients:

$$G = \frac{\xi_d^3}{2\pi \xi_m \xi_\tau^2}, \tag{31}$$

whence, taking into account formulas (20) and (25), we obtain the connection between ξ_d, ξ_τ, ξ_m and world constants $c, h, \hbar = h/2\pi$ and G:

$$\begin{aligned} \xi_d &= \left(\frac{2\pi Gh}{c^3}\right)^{1/2} = 1.015\times 10^{-34}\,\text{m}, \\ \xi_\tau &\left(\frac{2\pi Gh}{c^5}\right)^{1/2} = 3.38\times 10^{-43}\,\text{s}, \\ \xi_m &= \left(\frac{ch}{G}\right)^{1/2} = 2.176\times 10^{-8}\,\text{kg} \end{aligned} \tag{32}$$

Because an electron in the state of proper interaction consists of two γ_{0i} -particles [10] [11], taking into account the second equality (24), its mass m_e is determined by the formula

$$m_e = \frac{2\xi_m}{\pi H_0} = \frac{\xi_m}{H_{e0}}, \tag{33}$$

respectively, we represent the second formula of series (29) in the form

$$m_e c^2 = \frac{eh}{\alpha_c 2\pi r_e} \tag{34}$$

where the following notation is used

$$H_{e0} = H_0/2, r_e = r_c/2 \tag{35}$$

Using formulas (32) and (33), we determine the numerical values of the constants H_{e0} and H_0:

$$H_{e0} = 7.6034\times10^{21}, H_0 = 1.5207\times10^{22} \tag{36}$$

where m_e is taken as $m_e = 9.11\times10^{-31}\,\text{kg}$ [14].

Multiplying and dividing the right side of Equation (34) by the square of a constant value with the dimension of the charge ξ_q, that is, by the factor of the dimension of the charge, we obtain

$$m_e c^2 = \frac{c\hbar\xi_q^2}{\alpha_c \xi_q^2 r_e} = \frac{\xi_q^2}{r_e}, \tag{37}$$

where the numerical value ξ_q^2 is selected in such a way that the following condition is met

$$\frac{c\hbar}{\alpha_c \xi_q^2} = 1 \tag{38}$$

Comparing formula (37) with the well-known formula for the proper interaction of an electron $m_e c^2 = e^2/r_e$, we can conclude that the coefficient of dimensionality of the charge ξ_q is equal to the value of the electric charge, that is,

$$\xi_q = e \tag{39}$$

Hence, taking into account condition (38), it follows that

$$\alpha_c \xi_q^2 = c\hbar = \alpha_c e^2 \tag{40}$$

or

$$\alpha_c = \alpha^{-1} \tag{41}$$

that is, the constant α_c determined using formulas (3) and (8) is numerically equal to the reciprocal of the fine structure constant α.

Based on (7), (36), and (41) we determine the numerical value of the constant $H_c = 1.11\times10^{20}$, whence, taking into account formulas (6), (32), and (35), we obtain the value of the classical radius of the electron,

$$r_e = \frac{H_c \xi_d}{4} = 2.82\times10^{-15}\,\text{m} \tag{42}$$

which is an additional confirmation of the results (39), and (41).

Transforming the equation of motion (14) to the form

$$\frac{H_i^2}{H_o \pi H_0^2} = \frac{\xi_d}{\alpha_c 2\pi r} \tag{43}$$

taking into account (19), (28), (39) and (40), we obtain the equation for the electrical interaction of unit charges

$$\varepsilon_q = mv^2 = \frac{\xi_\varepsilon \xi_d}{\alpha_c 2\pi r} = \frac{ch}{\alpha_c 2\pi r} = \frac{e^2}{r}, \tag{44}$$

whence formula (40) and equality (41) follow again.

In [11] it is shown that the dimensional value of the charge q is determined by the product of the number N_q of γ_E-particles and the coefficient of dimension ξ_q, that is, $q = N_q \cdot \xi_q$; thus, taking into account formula (44), the Coulomb equation for charges q_1 and q_2 is represented by the formula

$$\varepsilon_q = \frac{N_{q1} N_{q2} \xi_q^2 \xi_d}{r} = \frac{N_{q1} N_{q2} e^2}{r} = \frac{q_1 q_2}{r} \tag{45}$$

Four important conclusions follow from the derivation of the Coulomb equation:

- the magnitude of the elementary charge e is equal to the dimension coefficient of the charge ξ_q
- the square of the elementary charge e^2, introduced based on the Coulomb equation to give the property of matter the "charge" of dimensional content is the essence of a combination of constants $\alpha_c, 2\pi$ and coefficients of dimensions ξ_e, ξ_τ and ξ_m:

$$e^2 = \frac{\xi_m \xi_d^3}{\alpha_c 2\pi \xi_\tau^2} \tag{46}$$

- the constant α_c^{-1} is numerically equal to the fine structure constant $\alpha_c = \alpha^{-1}$
- the radius of the closed transverse path is the distance of interaction of the number charge and structure formation.

From series (23) in integral form follows the basic equation of mechanics

$$mv = m_i c, \tag{47}$$

according to which the momentum P is uniquely determined by the interaction mass in the corresponding direction. Both momentum P and paths l_i (15) and λ (16) are criteria for identifying the particle under consideration and are related to the realization of the same variable:

$$l_i = H_i \xi_d, \lambda = \pi H_0^2 \xi_d / H_i, P = \xi_p H_i / \pi H_0^2, \tag{48}$$

where $\xi_p = \xi_m c$ is the coefficient of a dimension of momentum. From (48) it follows that λ is inversely proportional to H_i, while the momentum is directly proportional; thus the fulfillment of the condition $mv\lambda = m_i c\lambda = h$ becomes a condition for the manifestation of the integrity of the particle. Accordingly, when passing a transverse path $n\lambda$, where $n = 1, 2, 3, \cdots$, the integer number of particles appears n time, that is $mvn\lambda = nh$. If the path is equal to $n\lambda$: to the value of a closed transverse path, for example, a circle with a length, then

$$2\pi r = n\lambda \quad \text{or} \quad mvr = n\hbar \tag{49}$$

Whence, taking into account Equations (11) and (7), it follows

$$n \cdot H_i = H_c \quad \text{или} \quad n\alpha_c H_i = H_0 \tag{50}$$

Replacing the variable H_i by a variable *n* using formula (50), we represent Equations (11), (12), (49), and (35) in the form of the Bohr relations characterizing the stationary orbits of an electron in a hydrogen atom.

$$\lambda = 2\pi n\alpha_c^2 r_e, 2\pi r = 2\pi n^2 \alpha_c{}^2 r_e \tag{51}$$

$$v = \frac{c}{n\alpha_c + 1} \approx \frac{c}{n\alpha_c}, m_e vr = n\hbar, \varepsilon = \frac{m_e c^2}{2n\alpha_c\left(n\alpha_c + 1\right)} \approx \frac{m_e c^2}{2n^2\alpha_c^2} \tag{52}$$

From the above results, it follows that the orbits of the electron are trajectories of transverse motion, while the principal quantum number indicates the multiplicity of manifestation of the integrity of the electron in stationary orbits. Taking into account the first formula (51), the quantity λ can be represented as the sum of transverse paths λ_i and λ_{ki} caused by the proper 2*i*- and 2*k*-pairs.

$$\lambda = \lambda_i + \lambda_{ki} = \pi\left(n_i + n_{ki}\right)\alpha_c^2 H_{ec}\xi_d = \pi n\alpha_c^2 H_{ec}\xi_d \tag{53}$$

where is denoted

$$n_i + n_{ki} = n \tag{54}$$

n_i and n_{ki} indicate the multiplicity of manifestation of the corresponding transverse paths and are called the Eigenquantum numbers. From Equation (54) and the equality of the moduli of the amplitudes $|H_{ic}| = |H_{kc}|$ follows the always fulfilled condition

$$n_i = n_{ki} \tag{55}$$

At $n_i + n_{ki} = 1$, condition (55) implies

$$n_i = n_{ki} = 0.5 \tag{56}$$

In the case of excited H atoms, the γ_{0i}-particles of photons absorbed by the atom form complexes with the electron under consideration, followed by exchange 2*k*- or 2*j*-pairs, and under new Δ-compositions, new trajectories of motion are formed. By analogy with its interaction, introducing the transverse paths $\lambda_{kf} = \pi n_{kf}\alpha_c^2 H_{ce}\xi_d$ and $\lambda_j = \pi n_j\alpha_c^2 H_{ce}\xi_d$ caused by 2*k$_f$*- and 2*j*-pairs, the final electron orbit is represented by the equation

$$n\left(\lambda_i + \lambda_{ki} + \lambda_{kf} + \lambda_j\right) = n\pi\left(n_i + n_{ki} + n_{kf} + n_j\right)\alpha_c^2 H_{ec}\xi_d = \pi n^2\alpha_c^2 H_{ec}\xi_d, \tag{57}$$

where proper and third-party Δ_k-pairs are marked with additional indices "*i*" and "*f*", respectively, and the following is denoted

$$n = n_i + n_{ki} + n_{kf} + n_j \tag{58}$$

Equation (57) implies that for $n_{kf} = n_j = 0$, trajectories are formed in the form of circles, at $n_j = 0$, we have elliptical orbits, the condition $n_j \neq 0$ indicates the orientation of the orbits relative to the plane formed by 2*i*- and 2*k*-pairs.

From equation (58) and condition (56), it follows that for a given *n*, the sum $n_{kf} + n_j$, like the azimuthal quantum number *l*, takes $n-1$ values, that is,

$$m_l = n_{kf} + n_j \tag{59}$$

From this, it follows that the content that l is related to interactions with the participation of third-party partners, while n_{kf}, as well as n_j, can take from zero to l-values, that is, the sum $n_{kf} + n_j$ can be realized by $2l+1$ variants. It is easy to verify that we obtain the indicated number of variants by determining the magnetic quantum number by the difference

$$m_e = n_{kf} - n_j \tag{60}$$

that is, given l the participation of third-party partners, it is possible to form a subgroup with $2l+1$ different states.

Just as n_i and n_{ki} are called eigen quantum numbers, n_{kf} and n_j are called the third-party quantum numbers, because their origin is related to the participation of third-party partners in interactions.

Taking into account condition (55), the proposed set of quantum numbers n_i, n_{ki}, n_{kf} and n_j are not only an alternative to quantum numbers n, l and m_l, but also reveals an additional aspect of their content—the Δ-content is determined, therefore, the trajectory of motion of the composite γ_{0i}-particles—the nature and number of interacting partners is indicated because each Δ-content is formed with the participation of a specific partner by interaction.

The total number of possible options for interaction, respectively, and the possible number of partners is determined by the sum $\sum_n \sum_{l=0}^{n-1} (2l+1)$. For example, this sum characterizes the state of an electron in many-electron atoms and nucleons in nuclei (shell theory of the nucleus [15]). In [10], it is shown that systems of eight units (initially given by seven partners) predetermine the patterns of formation of complete families, ranging from Δ-elements from ε-particles to subsequent generations of a higher hierarchical level, including for many-electron atoms and nuclei. The sum (3) becomes a completeness criterion for a particular hierarchical generation in terms of the number of possible formations.

4. The Classification of Elementary Particles by Masses

From the content of the constant α_0 it follows that with the participation of the correction (8), it can be used as a basis for determining the mass of the elementary particles and their classification by masses. To this end, introducing the interaction

$$H_i = n^p \alpha_c^q H_0, \tag{61}$$

it is possible to compose a set of energy scales with a characteristic classification parameter $q = 0, \pm 1, \pm 2$, each of which, in turn, using the principal and azimuthal quantum numbers, is divided into groups and corresponding subgroups. The parameter p in (61) indicates an increase ($p = +1$) or a decrease ($p = -1$) in the interaction potential relative to the initial potential of a particular energy scale. Thus, in addition to the generally accepted set of quantum numbers, the number q is additionally introduced, which becomes a characteristic parameter for the classification of composite elements, elementary particles, over the energy.

Introducing (61) into Equation (14), we obtain the dependence of the interaction distance on the parameter q, n and p:

$$R = \frac{H_e \xi_d}{2n^{2p}\alpha_c^{2q}} \tag{62}$$

where the numerical value ξ_d is given in the series (32).

Thus, the intrinsic spectrum of interaction distances corresponds to a specific energy scale. The energy scales with $q = -2$ and $q = -1$ (the interaction distance $1.25 \times 10^{-5} \div 6.65 \times 10^{-10}$ m) are characterized by electromagnetic interactions in atomic-molecular systems and systems of the nucleus-electron type. In particular, for the energy scale $q = -1$ at $p = -1$, taking into account formula (50), we obtain the Bohr Equations (51) and (52) characterizing the stationary orbits of the hydrogen atom.

It should be noted that elementary particles are not directly formed as a result of electrostatic and magnetic interactions; only under certain conditions, after certain transformations, the photons without the rest mass are born

Based on definitions (24), (26), and (61), the mass of one γ_{0i}-particle is given by the equation

$$m = \left(1 + n^p \alpha_c^q\right) m_0 \tag{63}$$

at small interaction distances, when $q \geq 1$ and $n^p \alpha_c^q \gg 1$, the mass is determined mainly by the third-party interactions.

$$m = n^p \alpha_c^q m_0 \tag{64}$$

Accordingly, the masses of elementary particles, taking into account the number of -particles N_γ in their composition, are described by the equations

$$m = \left(1 + N_\gamma n^p \alpha_c^q\right) m_0 \quad \text{or} \quad m = N_\gamma n^p \alpha_c^q m_0 \tag{65}$$

As already noted, in the framework of energy scales with negative q values, the photons without a "rest" mass are mainly described. Now let us present a Table, where the masses of the individual γ_{0i}-particles are given in MeV units, starting with $q = 0$ (**Table 1**).

Table 1. The values q, n and the mass of the γ_{0i}-particles.

q	p	n						
		1	2	3	4	5	6	7
0	+1	0.2555	0.511	0.7665	1.022	1.2775	1.533	1.7885
	−1		0.12775	0.08516	0.064	0.0511	0.04258	0.0365
1	+1	35.0127	70.0254	105.0381	140.051	175.064	210.076	245.0888
	−1		17.506	11.671	8.753	7.003	5.835	5.008
2	+1	4798	9596	14,396	19,192	23,990	28,788	33,586
	−1		2399	1599	11,995	959.6	799.6	685

$q = 0$. From the γ_{0i} particles of this scale, one can model the lightest lepton (electron) with mass $m_e = 2m_0 \left(N_\gamma = 2, n = 1\right)$, estimate the masses of the first generation quarks u and d: ≈2 MeV and ≈4 MeV, accordingly, to operate with the difference between the masses of the neutron (n^0) and proton (p^+) - 1.23 MeV, estimate the mass defect (*i.e.*, the binding energy) between nucleons, starting from the deuterium nucleus (2.23 MeV) and up to the heavy nuclei (≈7 - 9 MeV per nucleon)

$q = 1$. γ_{0i} -particles of this scale are most promising for determining the mass of muonic leptons (104 - 106 MeV) and τ-lepton (1777 MeV), strange s-quark (104 MeV) and charmed c-quark (1270 MeV), π-mesons (137 - 140 MeV), p^+ (938.28) and n^0 (939.57 MeV), the constant of the energy scale of quantum chromodynamics Λ_{QCD} (217^{+25}_{-23} MeV).

$q = 2$. Particles of this energy scale can be used to simulate the b quark (4.2 GeV) and the top t quark (173 GeV), but basically, this group covers the energy parameters of the weak interaction [16], the Higgs scalar field constant (<H> = 247 GeV), the masses of Higgs bosons (24.3 - 125 GeV), gauge bosons $W^{\pm}$(80 GeV), vector boson Z (91 GeV), heavy bosons W_r and Z_{CM} (405 and 505 GeV) [17], etc. In the Standard Model of elementary particle physics (SM), it is just with processes in the energy region at $q = 2$ according to the Higgs scheme that the mechanism of production of massive elementary particles: $W^{\pm}$ and Z-bosons is explained.

It follows from Equation (29) that it is impossible to obtain particles with the mass of $\xi_m \approx 1.22 \times 10^{19}\,\text{GeV}$ or $\xi_m/\alpha \approx 6.5 \times 10^{14}\,\text{GeV}$ (respectively, the energy scales of Superstring theory [18] and the Grand Unification [19] because for this purpose it is necessary to approach an unrealizable condition $H_{iS}^2 = H_0^3$. The values of the given order of magnitudes can only be the auxiliary parameters when carrying out the computing operations.

In general, by appropriate selection N_γ, q and *p*, it is possible to compute the mass of almost any elementary particle. However, to avoid arbitrariness and errors, first of all, it is necessary to take into account the γ_{0i} = composition of the considered elementary particles, information about which can be obtained from the experimentally observed results of theirs decay. Besides, based on the proposed model, it will be very useful to determine besides the mass the other characteristics of given elementary particles: spin, magnetic moment, etc. It is not excluded that particles with very close masses can be formed by different combinations of N_γ, q and *p*, respectively, and the properties of these particles, especially their decay products, will be very different. As an example, consider the production of $\pi^{\pm}$-mesons from accelerated electron-positron pairs according to the scheme

$$e^- + e^+ \to \pi^- + \pi^+, \tag{66}$$

and their subsequent decay through the channels

$$\pi^{\pm} \to \mu^+ + \nu_\mu; \mu^+ \to e^+ + \nu_e + \tilde{\nu}_\mu \tag{67}$$

$$\pi^- \to \mu^- + \tilde{\nu}_\mu; \mu^- \to e^- + \tilde{\nu}_e + \nu_\mu \tag{68}$$

From the particle balance it follows that as a result of the above transformations, six additional neutrino particles were formed: electron neutrinos ν_e and antineutrinos $\tilde{\nu}_e$, and two muonic neutrinos ν_μ and antineutrinos $\tilde{\nu}_\mu$. When explaining the indicated and any other transformations with the participation of elementary particles in ST, one does not operate with the ideas of physical vacuum and secondary quantization. By analogy with chemistry, any transformations with the participation of elementary particles are considered as processes of connection, decay, and exchange of constituent elements. The very process of acceleration of electrons and positrons is reduced to the formation of complexes with the participation of particles of the accelerating medium and accelerated charges, thus the already mentioned complexes participate in collisions. In particular, decays of π^+ and π^- mesons through channels (67) and (68) are represented by the following schemes:

$$e^+\beta_{\varepsilon1}\beta_{\varepsilon2}\gamma_{e\nu} \to e^+\beta_{\varepsilon1}\tilde{\gamma}_{\mu\nu}\gamma_{e\nu} + \nu_\mu \tag{69}$$

$$e^+\beta_{\varepsilon1}\tilde{\gamma}_{\mu\gamma}\gamma_{e\gamma} \to e^+\beta_{\varepsilon1} + \tilde{\nu}_\mu + \nu_e \tag{70}$$

$$e^-\beta_{\varepsilon1}\beta_{\varepsilon2}\tilde{\gamma}_{e\nu} \to e^-\beta_{\varepsilon1}\gamma_{\mu\nu}\tilde{\gamma}_{e\nu} + \tilde{\nu}_\mu \tag{71}$$

$$e^-\beta_{\varepsilon1}\beta_{\varepsilon2}\tilde{\gamma}_{e\nu} \to e^-\beta_{\varepsilon1}\gamma_{\mu\nu}\tilde{\gamma}_{e\nu} + \tilde{\nu}_\mu \tag{72}$$

where $e^+\beta_{\varepsilon1}\beta_{\varepsilon2}\gamma_{e\nu} - \pi^+$-muons, $e^-\beta_{\varepsilon1}\beta_{\varepsilon2}\tilde{\gamma}_{e\nu} - \pi^-$ = meson, $e^+\beta_{\varepsilon1}\tilde{\gamma}_{\mu\nu}\gamma_{e\nu}$ and $e^-\beta_{\varepsilon1}\gamma_{\mu\nu}\tilde{\gamma}_{e\nu}$ respectively μ^+ and μ^- muons, $\beta_{\varepsilon1}$ and $\beta_{\varepsilon2}$ doublets of γ_{0i}-particles with different potentials $\gamma_{e\nu}$, $\tilde{\gamma}_{e\nu}$, $\gamma_{\mu\nu}$ and $\tilde{\gamma}_{\mu\nu}$ inclusions of six Δ-elements, transforming during decay into an electron and muon neutrinos and antineutrinos, respectively.

As a rule, the potential of $\beta_{\varepsilon1}$-pairs determines the kinetic energy of the resulting electrons and positrons. The neutrino particles themselves are characterized by Δ-compositions of the type $\left[2j(ik)\right]_I\left(\bar{i}\,\bar{k}\right)_{II}$ where the indices "I" and "II" indicate the temporal state of the compound Δ-pairs [10] [11].

According to schemes (69) and (71), π^+ and π^- mesons consist of six γ_{0i}-particles, and their mass is represented by the sum

$$m_\pi \approx (4\times17.5 + 2\times35)\,\text{MeV} = 140\ \text{MeV} \tag{73}$$

that is from four γ_{0i}-particles with the mass 17.5 MeV ($N_\gamma = 4$, n = 2, p = −1, q = 1) and two μ^+-particles with a mass of 35.5 MeV ($N_\gamma = 2$, n = 1, p = −1, q = 1). According to schemes (70) and (72), μ^+- and μ^--mesons consist of five γ_{0i}-particles with a total mass

$$m_{\mu^\pm} \approx (4\times17.5 + 35.5)\,\text{MeV} = 105.5\ \text{MeV} \tag{74}$$

That is, the mass of μ-mesons is composed of four γ_{0i}-particles with mass 17.5 MeV ($N_\gamma = 4$, n = 2, p = −1, q = 1) and from one particle of mass 35.5 MeV ($N_\gamma = 1$, n = 2, p = −1, q = 1)

In [20], the following formula was proposed

$$m_\mu = \frac{3m_e}{2\alpha} \tag{75}$$

where the relationship between the masses of electron (m_e), muon m_μ, and α is written. Proceeding from the fact that an electron in the state of its interaction consists of two γ_{0i} -particles, the given ratio could be explained by the sum of the masses of six γ_{0i} -particles with masses of 17.5 MeV. Nevertheless, based on the $\mu^{\pm}$ -meson decay schemes (70) and (72), the most probable variant of the definition m_μ is the sum (74).

Based on the composition of the mesons decay products π^0, $\pi^0 \to 2f_\gamma$ where f_γ are gamma quanta, we can conclude that π^0 -meson consists of four γ_{0i} -particles with a mass of 35.5 MeV each, that is, the composition of π^0 a meson is very different from $\pi^{\pm}$ mesons.

We do not say at all that with the help of the given Table it is possible to unambiguously determine the mass of any elementary particle, however, concerning a specific elementary particle, the selection of combinations from various options can become an additional factor in compiling its correct model. So, it is very tempting to represent the proton mass m_p as the sum

$$m_p \approx 4(210 + 17.5 + 7)\,\text{MeV} = 938\,\text{MeV} \tag{76}$$

where groups with $N_\gamma = 4$, $q = 1$,and $n = 6$. $p = +1$; $n = 2$, $p = -1$; $n = 5$, $p = -1$ are taken into account.

An interesting option is also

$$m_p \approx 4(217 + 17.5)\,\text{MeV} \approx 938\,\text{MeV} \tag{77}$$

where 217 MeV is the energy scale of quantum chromodynamics.

Due to the stability of the proton, taking into account the third formula of a series (24), its mass can be uniquely determined by the relation

$$m_p = \frac{\xi_m \left(H_i + H_p\right)}{\pi H_0^2} \tag{78}$$

where H_p is the total potential of interactions involving composite γ_{0i} particles of a proton.

Thus, the calculation of the proton mass is reduced to the calculation of the constant H_p.

We especially note that the mass of 17.5 MeV, which appears in formulas (73), (74), (76), and (77), is quite close to the key quantity considered in [21] [22].

From the foregoing, it follows that the proposed method for determining the mass based on the mechanism of motion of physical bodies seems to be a promising basis for computing the masses of elementary particles, regardless of the interactions in which they participate, whether they belong to fermions or bosons, leptons or baryons.

5. The Characteristic Constants of Interactions and Equations of Motion

Any motions, the parameters of which determine the mass of physical bodies, are caused by interactions. So, for the electromagnetic interaction, one uses α and e^2, related by an equation $\alpha = e^2/c\hbar$; practically similar formulas are used to describe the strong and weak interaction. The gravitational coupling constant α_G, widely discussed [23], is represented by the formula

$$\alpha_G = \frac{Gm^2}{c\hbar} \tag{79}$$

where m is the mass of gravitationally interacting particles.

In particular, α_G for the electron (α_{Ge}) and proton (α_{Gp}) are determined by the formulas

$$\alpha_{Ge} = \frac{Gm_e^2}{c\hbar} \text{ and } \alpha_{Gp} = \frac{Gm_p^2}{c\hbar} \tag{80}$$

Whence, taking into account formulas (20), (24), (25), (31), and (33) it follows

$$\alpha_{Ge} = \frac{4}{\pi^2 H_0^2}, \alpha_{Gp} = \frac{H_p^2}{\pi^2 H_0^4} \tag{81}$$

where $H_0 = \alpha_c H_c$ are products of constants α_c and H_c (7), $H_i = H_p \gg H_0$ is taken into account, H_p is the proton interaction potential. The presence of relation between H_c and α_c directly follows from the basic equations of motion (10)-(14). Namely, by α_c-multiple changes in the constant H_c, the energy scales are selected and they can be computed using α_c and H_c.

Similarly, based on formulas (20), (25), and (40), it can be shown that the square of the charge is the product of dimensional and dimensionless constants:

$$e^2 = \frac{c\hbar}{\alpha_c} = \frac{\xi_m \xi_d^3}{2\pi \alpha_c \xi_\tau^2} \tag{82}$$

That is, when choosing the dimension of the property of matter (based on the Coulomb equation), constants $\alpha_c = \alpha_0 / \chi_c$ (8) and 2π were included in the value of e^2.

Thus, when describing various interactions and related phenomena, we operate with primary constants of three types: structural content H_c and α_0; geometric origin π and χ_c; the coefficients of dimensions of length ξ_d, time ξ_τ (or ξ_t), and mass ξ_m. Dimension coefficients are introduced only to give a dimensional content to physical quantities, combinations of them are universal constants of physics (20), (25), and (31). The origin of the geometric constants of the type π and χ_c is related to the transition from actually formed trajectories to simplified linear trajectories with measured parameters. So, when interacting with partners, the trajectory of motion is formed as a spatial figure from mutually perpendicular χ_d-intervals. The transition to the measured parameters occurs in two ways:

- by curving the outer outlines, which turns the initial figure also into a spatial figure in the form of a torus;
- representation of the trajectory in a linear format, while the large radius of the torus is considered as the distance of interaction, while the small radius remains unobservable even within the framework of quantum mechanics. Hence it follows that the formation of a real trajectory requires more ε-in-

tervals than is assumed in calculations with measured parameters. A decrease in the number of ε-acts due to these operations is taken into account by constant π and χ_c.

Constants of structural origin H_c and α_0 are characteristic criteria for the quantitative description of the phenomena of the physical world at both the subatomic and atomic-molecular levels. Starting from a certain hierarchical level, the constant α_0 becomes the initial value to determine length, time, matter-energy, and related parameters.

The constant, the value of which is calculated by a certain set of quantum numbers, is associated with the regularities of structure formation at various hierarchical levels. As a result of the transition to measured values, the constant $\alpha_c = \alpha_0 / \chi_c$ is transformed into a constant, the reciprocal of which is known as the fine structure constant, The presence of a relation between H_c and α_c directly follows from the basic Equations (10)-(14). It is precisely by multiple changes α_c in the constant H_c that energy scales are selected that characterize interactions of various natures and are the quantitative basis for the classification of elementary particles by mass.

6. Conclusions

The equations of motion of physical bodies are characterized by longitudinal and transverse components, the characteristic parameters of which become the basis for determining the property of matter "mass". The main states of the equations of motion H_c and α_c are related, respectively, to the amplitude of oscillation of the initial particles of modeling the physical world and the patterns of structure formation at various hierarchical levels.

The origin of the inverse value of the fine structure constant is represented by the ratio of α_0 and another constant related to the transition from a real trajectory of motion to a simplified trajectory with measurable parameters.

Using these constants, one can develop energy scales, which are a quantitative basis for describing the interactions of various classes and classifications of elementary particles by mass.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Jammer, M. (1961) Concept of Mass in Classical and Modern Physics. Harvard University Press, Cambridge.

[2] Wilczek, P. (2006) *Modern Physics Letters*, **21**, 701-712. https://doi.org/10.1142/S0217732306020135

[3] Gordon, K. (2013) The Mysteries of Mass. *Scientific American*, 32-39.

[4] Braibant, S., Giacomelli, G. and Spurio, M. (2009) Particles and Fundamental Interactions. An Introduction to Particle Physics. Springer Verlag, Heidelberg.

[5] Bazarov, A., *et al*. (2018) *Physical Review D*, **98**, Article ID: 054517. https://doi.org/10.1103/PhysRevD.98.054517

[6] Langacker, P. (2017) The Standard Model and Beyond. Second Edition, CRC Press, Boca Raton, 636 p. https://doi.org/10.1201/b22175

[7] Feynman, R.P. (1985) QED: The Strange Theory of Light and Matter. Princeton University Press, Princeton, 129.

[8] Kragh, H. (2003) *Archive for History of Exact Sciences*, **57**, 395-431. https://doi.org/10.1007/s00407-002-0065-7

[9] Bedford, D. and Krumm, P. (2004) *American Journal of Physics*, **72**, 969-970. https://doi.org/10.1119/1.1646135

[10] Kirakosyan, Kh. (2011) Structural Physics. Structural Content of the Basic Concepts and Physical Law. "Flinta", Moscow, 420 p. (In Russian)

[11] Kirakosyan, Kh. (2018) *Theoretical Physics*, **3**, 1-27. https://doi.org/10.22606/tp.2018.31001

[12] Kirakosyan, Kh. (2019) *New Horizons of Mathematical Physics*, **3**, 83-103. https://doi.org/10.22606/nhmp.2019.33001

[13] Kirakosyan, Kh. (2018) *Theoretical Physics*, **3**, 33-36. https://doi.org/10.22606/tp.2018.32002

[14] CODATA Internationally Recommended 2014 Values of the Fundamental Physical Constants. https://physics.nist.gov/cuu/Constants

[15] Caurier, E., *et al*. (2005) *Reviews of Modern Physics*, **77**, 427-488. https://doi.org/10.1103/RevModPhys.77.427

[16] Higgs, P.W. (1964) *Physical Review Letters*, **13**, 508. https://doi.org/10.1103/PhysRevLett.13.508

[17] PDG (2008) *Physics Letters B*, **667**, 1.

[18] Becker, K., Becker, M. and Schwarz, J. (2007) String Theory and M. Theory. A Modern Introduction. Cambridge Univ. Press, Cambridge.

[19] Ross, G. (1984) Grand Unified Theories. Westview Press, Boulder.

[20] Nambu, Y. (1952) *Progress of Theoretical Physics*, **7**, 595-596. https://doi.org/10.1143/PTP.7.5.595

[21] Krasznahorkay, A.J., *et al*. (2016) *Physical Review Letters*, **116**, Article ID: 042501. https://doi.org/10.1103/PhysRevLett.116.042501

[22] Krasznahorkay, A.J., *et al*. (2019) New Evidence Supporting the Existence of the Hypothetic X17 Particle.

[23] Barrow, J.D. (2002) The Constants of Nature from Alpha to Omega—The Numbers That Encode the Deepest Secrets of the Universe. Vintage.

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Time Quanta Present in Simple Quantum Systems

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Abstract

Differences of the time periods in two independent quantum systems are examined on a semiclassical level. The systems are the electron in the hydrogen atom and a free-electron particle moving in a one-dimensional potential box, respectively. It is demonstrated that in both systems the relativistic correction to the time interval can be expressed as a multiple of the same quantum of time. The size of the quantum is proportional to the ratio of the Planck's constant and the rest energy of the electron particle.

Keywords

Time Periods of the Electron Oscillations in the Bohr Hydrogen Atom and in a One-Dimensional Potential Box, Relativistic Correction to the Difference of the Time Periods, Time Quanta

1. Introduction

The problem of the time calculation seldom discussed in the quantum theory can remain actual even for very simple quantum systems. These systems are often of the periodic character, so we need the time periods of the electron motion as parameters. This kind of parameter is usually approached in a semiclassical way. Perhaps the best known example concerns the time periods of the electron orbital motion in the hydrogen atom [1] [2].

Other well-known examples of a semiclassical motion are the free particle in a potential box and the harmonic oscillator. In the last case a difficulty concerning time is provided by an unstable size of the velocity connected with a moving particle.

In order to make the problem of calculations of the time periods to be very simple, only the electron in the hydrogen atom and the particle moving in a one-dimenssional potential box are considered exactly in the paper. The time quanta in the harmonic oscillator are calculated only in an approximate way.

The main aim of the paper was to point out that a relativistic change of a difference between two oscillation periods of time in a quantum system can be obtained with the aid of the time quantum which has its size independent of the examined system.

2. Electron Circulating in the Hydrogen Atom and Its Time Periods

A successful Bohr theory of the atom (see e.g. [1]) assumed planar circular orbits about the nucleus. The radius of the nth orbit is defined by formula [2]

$$r_n = \frac{n^2\hbar^2}{me^2} \tag{1}$$

and the quantum numbers of the orbits are labelled by

$$n = 1, 2, 3, \cdots, \infty. \tag{2}$$

m is the electron mass, - e - electron charge. The electron velocity on the nth orbit is [2]

$$v_n = \frac{e^2}{n\hbar}, \tag{3}$$

so the time period T_n connected with the electron circulating along the same orbit is deduced from the formula

$$v_n = \frac{2\pi r_n}{T_n} \tag{4}$$

giving

$$T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi n^2\hbar^2}{me^2}\frac{n\hbar}{e^2} = \frac{2\pi n^3\hbar^3}{me^4}. \tag{5}$$

The task is to calculate the difference

$$T_{n_2} - T_{n_1} \tag{6}$$

transformed into a relativistic interval of time (see Sec. 3) for an arbitrary pair of the quantum states, say n_2 and n_1. Let us assume that

$$n_2 > n_1, \tag{6a}$$

so the difference in (6) is a positive number.

3. Relativistic Difference of the Time Periods Given in (6)

In order to improve (6) we apply the relativistic theory in the sense that the components entering (6) should be multiplied respectively by the factors (see e.g. [3] [4])

$$\sqrt{1 - \frac{v_{n_2}^2}{c^2}} \approx 1 - \frac{1}{2}\frac{v_{n_2}^2}{c^2}, \tag{7}$$

$$\sqrt{1-\frac{v_{n_1}^2}{c^2}} \approx 1-\frac{1}{2}\frac{v_{n_1}^2}{c^2}. \tag{8}$$

In effect we get a proper relativistic change of the time periods connected with the difference (6). Two terms in each of the power expansions (7) and (8) seem to be sufficient because

$$v_{n_2} \ll c \tag{9}$$

and

$$v_{n_1} \ll c. \tag{10}$$

We obtain:

$$T_{n_2}^{\mathrm{rel}} - T_{n_1}^{\mathrm{rel}} = \sqrt{1-\frac{v_{n_2}^2}{c^2}}T_{n_2} - \sqrt{1-\frac{v_{n_1}^2}{c^2}}T_{n_1} \cong T_{n_2} - T_{n_1} - \frac{1}{2c^2}\left(v_{n_2}^2 T_{n_2} - v_{n_1}^2 T_{n_1}\right). \tag{11}$$

Therefore

$$\begin{aligned} T_{n_2}^{\mathrm{rel}} - T_{n_1}^{\mathrm{rel}} &\cong T_{n_2} - T_{n_1} - \frac{1}{2c^2}\left(\frac{e^4}{n_2^2\hbar^2}\frac{2\pi n_2^3\hbar^3}{me^4} - \frac{e^4}{n_1^2\hbar^2}\frac{2\pi n_1^3\hbar^3}{me^4}\right) \\ &= \frac{2\pi\hbar^3}{me^4}\left(n_2^3 - n_1^3\right) - \frac{2\pi\hbar}{2mc^2}\left(n_2 - n_1\right) \\ &= T_{n_2} - T_{n_1} - \frac{h}{2mc^2}\left(n_2 - n_1\right). \end{aligned} \tag{12}$$

At the end of (12) appeared a supplement having a dimension of time and being proportional to the difference

$$n_2 - n_1. \tag{13}$$

Also we have that

$$T_{n_2} - T_{n_1} \sim n_2 - n_1 \tag{14}$$

because

$$\left(n_2 - n_1\right)\left(n_2^2 + n_2 n_1 + n_1^2\right) = n_2^3 - n_1 n_2^2 + n_2^2 n_1 - n_2 n_1^2 + n_2 n_1^2 - n_1^3 = n_2^3 - n_1^3. \tag{15}$$

These results make (6) proportional to both (13) and (15).

4. Electron Oscillation in a One-Dimensional Potential Box and Its Properties

For a constant potential within the potential box of length L we have the electron energy states [5]

$$E_n = \frac{n^2 h^2}{8mL^2} \tag{16}$$

where n is the integer quantum number, m the electron mass, and h is the Planck constant.

Formula (16) is equivalent to the kinetic energy expression

$$E_n = \frac{m}{2}v_n^2, \tag{17}$$

therefore by putting (16) equal to (17) we obtain the electron velocity in the potential box:

$$v_n = \frac{nh}{2mL}. \tag{18}$$

For a given n this velocity can be considered as having a constant value for the whole length L.

Due to the boundary conditions at the box ends, the electron behaves like a free particle oscillating along the length

$$L + L = 2L \tag{19}$$

for each oscillation time period T_n. A non-relativistic approach to T_n gives from (18):

$$T_n = \frac{2L}{v_n} = \frac{4mL^2}{nh}. \tag{20}$$

Our aim is to examine the difference

$$T_{n_2} - T_{n_1}. \tag{21}$$

The assumption of $n_2 > n_1$ implies a negative time difference in (21); see (20).

5. A Relativistic Difference of the Time Periods Entering (21)

A constant velocity (18) implies an approximately free-electron motion along the distance L entering (19). A relativistic modification of the time difference (21) becomes similar to that obtained in the case of the difference (6):

$$T_{n_2}^{\mathrm{rel}} - T_{n_1}^{\mathrm{rel}} = \sqrt{1 - \frac{v_{n_2}^2}{c^2}}\, T_{n_2} - \sqrt{1 - \frac{v_{n_1}^2}{c^2}}\, T_{n_1} \cong T_{n_2} - T_{n_1} - \frac{1}{2c^2}\left(v_{n_2}^2 T_{n_2} - v_{n_1}^2 T_{n_1}\right), \tag{22}$$

where the term in brackets in (22) is a positive number; see (24). We have

$$T_{n_2} - T_{n_1} = \frac{4mL^2}{h}\left(\frac{1}{n_2} - \frac{1}{n_1}\right) = \frac{4mL^2}{h}\frac{n_1 - n_2}{n_1 n_2} < 0 \tag{23}$$

because of the assumption done below (21). But in the next step:

$$\begin{aligned} \frac{1}{2c^2}\left(v_{n_2}^2 T_{n_2} - v_{n_1}^2 T_{n_1}\right) &= \frac{1}{2c^2}\left[\left(\frac{n_2 h}{2mL}\right)^2 \frac{4mL^2}{n_2 h} - \left(\frac{n_1 h}{2mL}\right)^2 \frac{4mL^2}{n_1 h}\right] \\ &= \frac{1}{2c^2}\left(\frac{n_2 h}{m} - \frac{n_1 h}{m}\right) = \frac{h}{2mc^2}\left(n_2 - n_1\right) > 0. \end{aligned} \tag{24}$$

A sum of results obtained in (23) and (24) (taken with a minus sign) gives

$$T_{n_2}^{\mathrm{rel}} - T_{n_1}^{\mathrm{rel}} = T_{n_2} - T_{n_1} - \frac{h}{2mc^2}\left(n_2 - n_1\right) \tag{25}$$

which is formally equal to the result calculated in (12). Evidently see (23) the difference

$$T_{n_2} - T_{n_1}$$

is proportional to $n_1 - n_2$. The component (24)—because of its minus sign in (22) makes the whole expression (25) to get the same property of proportionality.

A supplement obtained at the end of (25) is identical to that entering (12) calculated for the hydrogen atom. This enables us to consider the term having

$$\frac{h}{2mc^2} \tag{26}$$

as a quantum of time correcting the difference of the time periods

$$T_{n_2} - T_{n_1} \tag{27}$$

obtained either for the hydrogen atom, or in the potential box.

6. Approximate Approach to the Time Quanta in Case of the Harmonic Oscillator

In this case we assume—for the sake of simplicity—that the time periods T belonging to different quantum states are equal giving the circular frequency

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \tag{28}$$

where k is the oscillation constant and m is the mass of the oscillator [6]. All energy states are given by the formula

$$E_n = E_m^{(\mathrm{kin})} + E_n^{(\mathrm{pot})} = \frac{ka_n^2}{2} \tag{29}$$

where a_n is the oscillator amplitude characteristic for state n.

The states are equally separated by the terms $\hbar\omega$:

$$E_n = n\hbar\omega = \frac{ka_n^2}{2}. \tag{30}$$

The velocity v_n of the oscillating particle between the limits

$$-a_n < x < a_n \tag{31}$$

has its maximum at a central particle position $x = 0$.

In calculating the relativistic correction to the time period T we take into account—for the sake of simplicity—the average particle velocity

$$\bar{v}_n = \frac{4a_n}{T}. \tag{32}$$

In effect the relativistic correction to T becomes:

$$T_n^{\mathrm{rel}} \cong \sqrt{1 - \frac{\bar{v}_n^2}{c^2}}T \cong \sqrt{1 - \left(\frac{4a_n}{T}\right)^2 \frac{1}{c^2}}T \cong \left[1 - \frac{1}{2}\left(\frac{4a_n}{T}\right)^2 \frac{1}{c^2}\right]T. \tag{33}$$

But because of (30) we have

$$a_n^2 = \frac{2E_n}{k} \tag{34}$$

which gives with the aid of the formula (28) for ω

$$T_n^{\text{rel}} - T = -\frac{1}{2}\frac{(4a_n)^2}{Tc^2} = -\frac{16E_n}{kTc^2} = -\frac{16n\hbar\omega}{kTc^2} = -\frac{8}{\pi}\frac{n\hbar\omega^2}{kc^2} = -\frac{8}{\pi}\frac{n\hbar}{mc^2} = -\frac{8}{\pi^2}\frac{nh}{2mc^2}. \quad (35)$$

7. Summary

We examined the differences of the time periods possessed by the electron either in the case of its circulation in the hydrogen atom, or when the electron is enclosed as a particle moving in a one-dimensional potential box. The approach to the time properties of each of these physical systems is modified by the relativistic theory.

It is found that contribution given by the relativistic term of time for both systems is the same. This enables us to consider the relativistic corrections introduced to the differences of the time periods as representing the quanta of time: the size of quanta is proportional to the difference of the quantum numbers which define separately any of the time periods present in each system. Therefore, in order to obtain the correction, the quantum numbers multiply solely one term independent of the physics of the examined system. This term is a constant ratio

$$\frac{h}{2mc^2} \cong 0.4\times 10^{-20}\ \text{sec} \quad (36)$$

built up of the fundamental constants of nature. It can be noted that the denominator entering the fraction in (36) is equal to the Dirac's energy expression separating theseas of electron particles and antiparticles; see e.g. [7].

An approximate treatment concerning (36) was applied in calculating the time quanta of the harmonic oscillator. In this case the non-relativistic oscillation periods of time are assumed to be equal for all quantum levels and the relativistic correction concerns—in average—solely an individual oscillation period of time. In effect of the used approximations the result in (36) should be multiplied by a constant factor

$$\frac{8}{\pi^2} \quad (37)$$

which is not much different than unity.

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The paper is elaborated in course of my prayers to the Almighty God for a young Victoria—suddenly deceased.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Bohr, N. (1992) The Theory of Spectra and the Atomic Constitution. Cambridge

University Press, Cambridge, UK.

[2] Sommerfeld, A. (1931) Atombau und Spektrallinien. Vol. 1, 5th Edition, Vieweg, Braunschweig.

[3] Jauncey, G.E.M. (1948) Modern Physics. 3rd Edition, Van Nostrand, New York.

[4] Landau, L.D. and Lifshits, E.M. (1969) Mechanics. Electrodynamics. Izd. Nauka, Moscow (in Russian).

[5] Eyring, H., Walter, J. and Kimball, G.E. (1957) Quantum Chemistry. Wiley, New York.

[6] Sommerfeld, A. (1949) Mechanik. Akademische Verlagsgesellschaft, Leipzig.

[7] Slater, J.C. (1967) Quantum Theory of Molecules and Solids. Vol. 3, McGraw Hill, New York.

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Quasicrystals' Resonant Response with Translational Symmetry

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Abstract

Diffraction in quasicrystals is in logarithmic order and icosahedral point group symmetry. Neither of these features are allowed in Bragg diffraction, so a special theory is required. The present work displays exact agreement between the analytic metric with a numeric description of diffraction in quasicrystals that is based on quasi-structure factors. So far, we treated the hierarchic structure as ideal; now, we detail the theory by including two significant features: firstly, the steady state wave function of the incident radiation demonstrates how harmonics, in metrical space and time, enable coherent interaction between the periodic wave packet and hierarchic quasicrystal; secondly, mapping of the hierarchic structure for any influence of defects will allow estimation of possible error margins in the analysis. The hierarchic structure has the required logarithmic periodicity: superclusters, containing about 10^3 atoms, convincingly map phase contrast images; while higher orders leave space for subsidiary speculation. The diffraction is completely explained for the first time.

Keywords

Quasicrystal, Icosahedra, Hierarchic, Integral, Periodic, Resonant Response, Harmonic, Irrational, Geometric Series, Metric

1. Introducing Harmonics in Hierarchy

"Metallic phase with long range order and no translational symmetry" [1], how come? The diffraction pattern is in geometric series, is irrational, aperiodic and anharmonic. This cannot happen by Bragg's law; however, a metric results from a separable irrational part of the indices [2] [3]. This metric commensurates the periodic incident radiation with the irrational and geometric indexation that is due to the hierarchic geometry. We have previously shown how the diffraction occurs with a lattice parameter a that is measured, analyzed, verified, complete and fundamentally classical. This novel diffraction contrasts with Bragg's law in crystals, where the order is integral, periodic, and harmonic.

In Quasicrystals, the irrational diffraction is simulated by two independent routes: one numeric by Quasi-Structure Factors (QSFs) [4]; the second derived analytically from the irrational indexation. The two independent results match exactly. Illustrations of harmonization in quasi-Bloch waves, as below, explain the metrication and coherent scattering in geometric-series, *i.e.* scattering from the irrational quasi-lattice. Knowing the ideal model and understanding the diffraction, we are now able to turn to atomic mapping, and to defects.

As a consequence of thermodynamics, all crystals are defective. The hierarchic icosahedra (HI) constitute a special case for two reasons: firstly, as Pauling noted [5], icosahedra are known in crystalline systems with large unit cells, but they include central holes. These are problematic when the cells are of regular size. Nevertheless, they are now systematically analyzed and we can fairly understand the conformability of common defects to the ideal hierarchic structure. Secondly, quasicrystals (QCs) are often rapidly quenched, so that plural defects are easily frozen into the structure. We need to investigate how they are accommodated without causing the diffraction to become diffuse owing to disorder.

The digitization and harmonization are found present in the scattered radiation during its diffractive interaction with the geometric quasi-lattice. Quasi-Bloch waves, in the resonant response of the probe, commensurate and harmonize in scattering. Their behavior resembles quantum transitions of the harmonic wave functions used in time-independent atomic physics. Both processes are consequences of wave-particle duality that is the crowning result of 19th century physics. The diffraction is mediated by the wave-packet, that will be briefly reviewed to illustrate harmonization.

Notice that our main literature review is given in ref. [1]. The present work is unique in its derivation of complete diffraction, by conventional description, in 4-dimensions, and with a systematic and extensible model that is perfectly hierarchic; perfectly icosahedral; and in perfect geometric series[1], like the diffraction.

2. Scattering Radiation

Digitization and harmony are discovered in the scattered wave, so we need to review generally the nature of radiant scatterers in the broader scope of physics. The Michelson-Morley experiment falsified the ether hypothesis. An attempt was made to salvage it with the Lorentz transformation, but this was not as successful as Einstein's foundational relativity. That "physical laws are invariant in all inertial reference systems", has been verified in many ways. A consequence is the Pythagorean style equation for rest mass: $m_0^2c^4 = E^2 - p^2c^2$. After quantization by Planck's law for energy E, and the de Broglie hypothesis for momentum p, and with simplification of units $\hbar = c = 1$ for the reduced Planck constant

[1]Further descriptions can be found in recent listings in http://www.xraylithography.com.

$\hbar$ and the speed of light c, the rest mass reduces to:

$$m_0^2 = \omega^2 - k^2 = (\omega + k)(\omega - k). \tag{1}$$

The brackets symbolize in turn *particulate* conservation laws and response that is *wave-like*, resonant and harmonic. The former is real; the latter imaginary. The particle-wave duality is thus formulated in respective real and imaginary parts of the *normal wave packet* [6]:

$$\varphi = A \cdot \exp\left(\frac{X^2}{2\sigma^2} + X\right)$$

with imaginary:

$$X = i\left(\bar{\omega}t - \bar{k}x\right) \tag{2}$$

where σ depends on initial conditions, and describes the coherence of the packet in space and time[2]; and A^2 is a normalizing constant[3]. The angular frequencies ω and wave vectors k are in fact distributed, but they are represented in equation 1 by mean values. The intensity $\phi^* \phi$ is a probability density function for a particle, or for a photon having zero mass, $m_0 = 0$. Notice that the response is elastic because its absolute, measurable value is unity: $(e^X)^* e^X = 1$, everywhere and at all time.

Incidentally, a profound solution for Equation (1) has negative mass. This implies, by the first order derivation $E = m'c^2$ with $m' = m_0\left(1 - v_g^2/c^2\right)^{-1/2}$, that $\omega < 0$ and $k < 0$. These implications are supported by the facts that $(\omega < 0 \cup k > 0) \supset$ singularities in ω and k when $|m_0| = |k|$. The singularities are not observed (*cf.* [7]); whereas, by contrast, the phase velocity $v_p = \omega/k > 0$ and group velocity $v_g = k/m' = \mathrm{d}\omega/\mathrm{d}k > 0$ are always positive [6] (cf. the Switching Principle [8] [9]).

Meanwhile, Equation (2) represents the steady state for the incident radiation and, after a transition involving a change in vector k, it represents, likewise, the steady state of the diffracted wave. When the incident wave strikes the QC, it interacts with the QC field to form quasi-Bloch waves. You can think of these as lattice images observed in thin foils in the two-beam condition. The waves, as they proceed through the QC, oscillate (by the pendellösung effect) between the two beams (in crystals: [10]; *cf.* in QCs: [11] [2]). In wedge specimens, this oscillation produces images of 'thickness fringes'. The process requires and ensures harmonic interaction, in both space and time, in the propagation direction as in the transverse. An example will be given in the next section, though the 'quasi-lattice image' will not be a true lattice image[4] because of the metric.

Notice that Equations (2) linearize the second order Equation (1) of special relativity, and so they perform a similar function for the free particle as Dirac's

[2]Typically, the coherence has transverse components, σ_y, σ_z as well, but these are only implied here for simplicity.

[3]Typically, $A^* A = m_0^2 / \int \exp\left(X^2/\sigma^2\right) \mathrm{d}\tau$.

[4]Because of the metrical displacement (except at geometric intercepts) of the quasi-Bloch wave from the quasi-lattice. This will be illustrated below.

equation does for the bound electronic states in an atom. Moreover, the equations 2 separate the propagation direction from the transverse direction, and this has many consequences including: solutions for negative mass [12][5], phase velocity [6], uncertainty, Newton's second law, electron spin, magnetic radius and fine structure constant [13][6], reduction of the wave packet [14] etc. The equations apply in harmonious diffraction by quasicrystals and crystals, as they do in the Schrödinger equation that operates on steady-state, harmonic bases. The diffraction orders and quantum numbers respectively describe interaction requirements that are quantized by *necessary constructive interference over space and time.* Equation (2) describes the fundamental radiant packet that gives required information about the structure of quasicrystals. The formalism enables our understanding of the fundamental interaction required in the coherent diffraction.

3. Properties of Hierarchic Icosahedra

The quasicrystal responds to incident radiation with sharp, icosahedral diffraction that is in irrational and geometric order. The diffraction must be long range because sharp. The principal axes are indexed in 3-dimensions (**Figure 1**) and so is the diffraction pattern [4]. However, phase-contrast imaging contradicts the classic order because it is anharmonic owing to multiple, aperiodic, interplanar spacings. It emerges that the quasicrystal is hierarchically ordered and uniquely icosahedral, with dense unit cells that are bound within icosahedral clusters. The hierarchy is infinitely extensible. There is translational symmetry: it is in the radiant quasi-Bloch-waves that are excited by the quasi-Bragg condition so as to commensurate with the aperiodic ordering. In doing so, the waves harmonize and digitize the hierarchic quasi-lattice. These functions are numerically and analytically derived, so that the quasi-lattice is measured and consistently verified. Harmony is essential to the digitized interaction as it is in quantum basis state solutions to Schrödinger's equation.

It will become clear how the structure is hierarchic and therefore geometric. The scattering radiation is periodic (Equation (2)), but is excited by the geometric quasi-lattice, to form hierarchically commensurate quasi-Bloch-waves. Corresponding periodic excitations are routinely analyzed in crystals [10], but are now adapted for QCs [15] [11]. In **Figure 2(b)** a crystalline Bloch wave (blue) is compared with a corresponding quasi-Bloch-wave (red wave) that metrically stretches the blue wave by the inverse coherence factor $1/c_s$. This is calculated both numerically and analytically [2]. Whereas the crystalline Bloch wave is not commensurate with the geometric, irrational and logarithmically-periodic quasi-lattice; the quasi-Bloch-wave is commensurate with the geometric and hierarchic

[5]To avoid unphysical singularities when $k = -m_0$ in the antiparticle, the switching principle is switched back [6].

[6]Dirac used a rank 8 matrix [Pais A, in Paul Dirac, the man and his work, ed. Goddard, P., *Cambridge* (1998) ISBN 0521583829] to calculate the fine structure constant; the approximate answer by dispersion dynamics is the same but given in rank 2 [13].

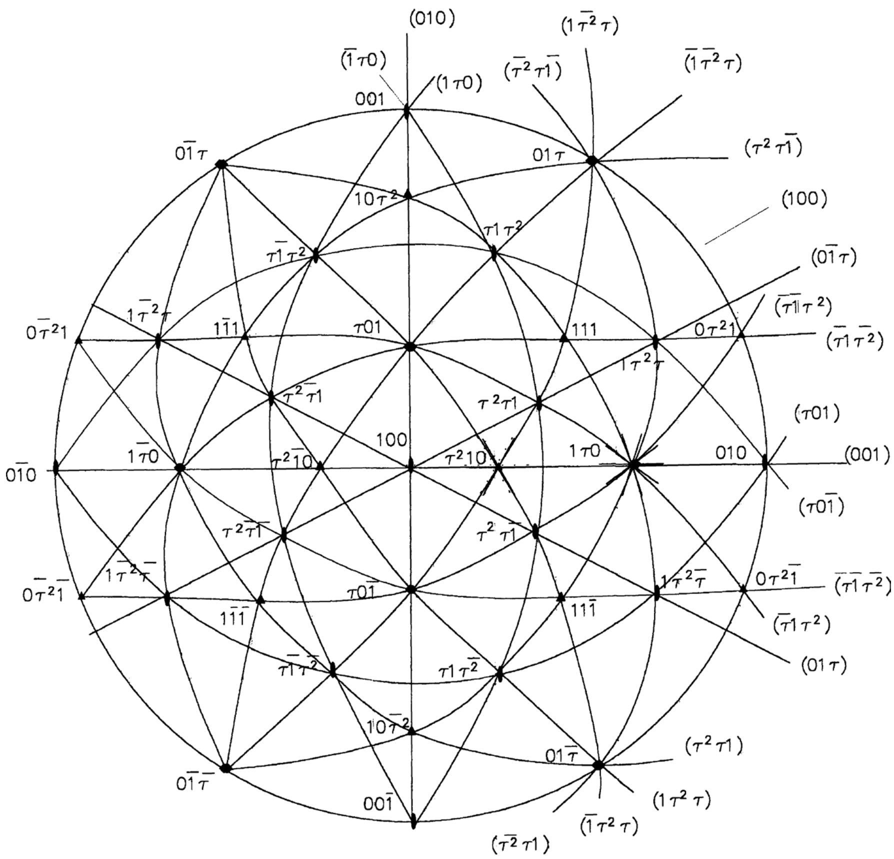


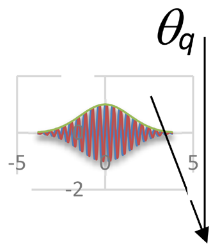
Figure 1. Stereogram of the principal axes for the icosahedral structure. Normal to them are the principal diffraction planes. They are all 3-dimensional with indices τ^m where the golden section $\tau = \left(1+\sqrt{5}\right)/2$. After William of Ockham, "Dimensions should not be multiplied without necessity." All of the beam intensities in the original data [1] have been indexed and calculated [16].

structure, and is invariant about all translations $a\tau^m$. This translational symmetry opposes Shechtman's claim [1]. The harmonization is obtained through the Fibonacci series digitization of periodic cycles between geometric intercepts (Equation (3) below). The harmonization is both long-range and local around each intercept.

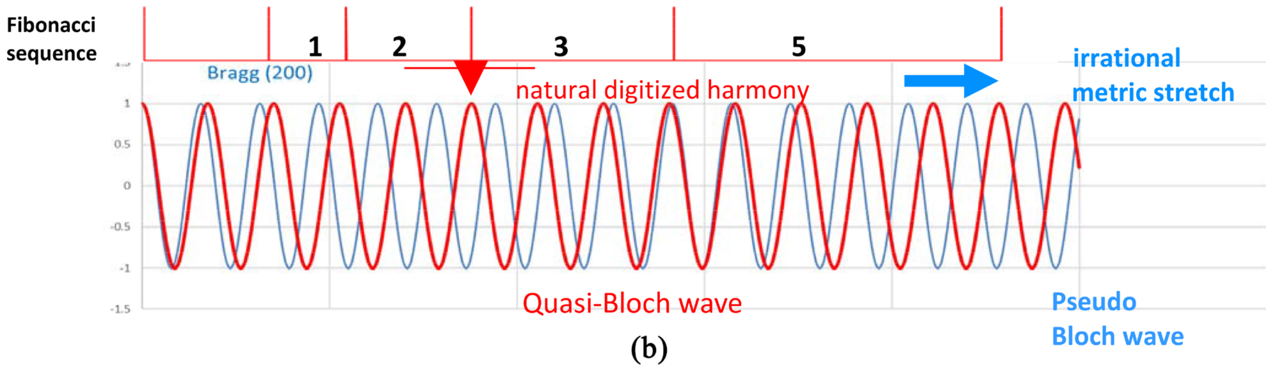
So long as we understand how the diffraction occurs, there is no need to decide whether 'hierarchic symmetry' should be called 'translational'. Principal features of the complex structure and diffraction of hierarchic icosahedra justify the following summary:

Summary:

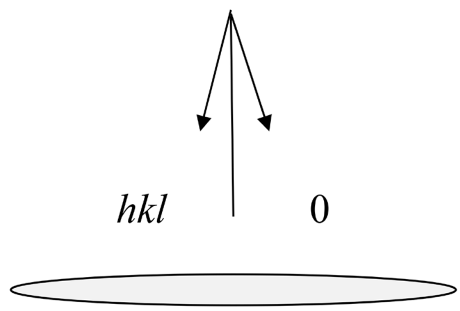
- The point group symmetry is icosahedral, in the diffraction pattern, consistent with the structure [1].
- The pattern is sharp [1].
- The orders are in geometric series [4] [16] [17].
- The series are irrational. They do not therefore obey Bragg's law for crystal diffraction [4] [16] [17].



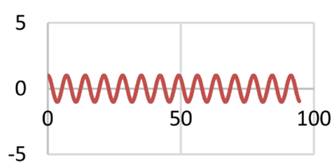
(a)



(b)



(c)



(d)

Figure 2. (a) Incident, time-dependent, beam probe (Equation (2)), rank $\Re^{4,}$ inclined at quasi-Bragg angle from normal: $\theta_q = \sin^{-1}\left(\lambda\sqrt{h^2+k^2+l^2}\right)/(2ac_s)$. (b) Crystalline Bloch waves (blue) are commensurate with their corresponding periodic crystal lattice at the Bragg condition. When this wave is stretched horizontally by the inverse coherence factor $1/c_s$, the quasi-Bloch-wave (QBW in red) commensurates with the irrational, geometric and hierarchic, quasi-lattice. Its geometric order is represented by the intercepts on the horizontal line above it [17] [16]. The digitized number of periodic cycles between successive intercepts is in Fibonacci sequence (denominator in Equation (3)), and the diffraction is logarithmically periodic. The natural and irrational parts of the indices are separable: the irrational part is expressed by the metric stretch; the natural part scatters with sharp, coherent diffraction. (c) Diffracted beams emitted beneath foil, including indices. (d) In a transmission electron microscope, beams can be magnetically refocused to produce a quasi-lattice image of the *probe* at the base of the specimen foil. The lattice image is the interference due to the superposition.

- The stereogram and planar indexations are 3-dimensional (**Figure 1** [18]).
  "Dimensions should not be multiplied without necessity".
  But, inclusion of time for harmony *is* necessary: the rank is $\Re^4$.
- All structure factors under Bragg's law are zero. The Quasi-Structure-Factor contains a numerical metric c_s that is peculiar to the icosahedral hierarchy, and that is calculated iteratively over successive orders of cluster and super cluster[7]: the formulaic function of the metric is a virtual breathing strain [4] [16] [17], cohering with hierarchy.
- Quasi-structure-factors map intensities in experimental diffraction patterns [4] [16] [17].
- Indices $\tau^m = F_m(1,\tau) = \partial_{(m,1)} + F_{m+1}(0,1) + F_m(0,1)\tau$ where the Fibonacci sequences F_m (0, 1) are of natural numbers on bases 0, 1 [2]. The metric is analyzed by separating the irrational parts of the indices [2] [3].
- The metric function is given by the irrational residue in the formula [2]:

$$\frac{1}{c_s} = 1 + \frac{\tau^m - F_{m+4}/2}{F_{m+1}} = \frac{1}{0.894} \tag{3}$$

- Diffraction, on the irrational indices, metricates, digitizes and harmonizes the geometric series (**Figure 2**) [2].
- These features are necessities for radiation scattering by coherent diffraction [2].
- The quasi-lattice-parameter a is measured, analyzed, harmonized, verified and complete [2J
- The dense unit cell is consistent with the fact that diatomic QCs occur only with the atomic diameter ratio: $d_{\text{solute}}/d_{\text{solvent}} = \sqrt{1+\tau^2} - 1$ [4] [16] [17]
- Phase contrast optimum defocus images in thin specimens [19] display clear hierarchies (containing ~10^3 atoms), with dimensions that match corresponding hierarchic structures that are simulated. The hierarchic structure is infinitely extensible [4] [16] [17] (present work).

4. Mapping Simulations of Phase Contrast Images

Figure 3 illustrates the extremely dense unit cell. At its center is the small *Mn* atom with its comparatively large atomic scattering factor, surrounded by 12 tightly packed *Al* atoms. The ratio of atomic diameters is $d_{\text{Mn}}/d_{\text{Al}} = \sqrt{1+t^2} - 1$. Notice the 5-fold axis that is normal to two circles of 5 *Al* atoms. This structure will prove significant in images and maps: when two planes are imaged together, they adopt the appearance of 10-fold rotational symmetry; but when imaged alone in sufficiently thin specimens, the plane appears 5-fold.

[7]The QSF F^p for a supercluster order p, with Miller indices h, k, l, is equal to the QSF order p-1 multiplied by the function for the phase due to the stretching factor τ^{2p}, where $\boldsymbol{h}$ is the corresponding normal vector for the (hkl) plane; c_s the metric; and $\boldsymbol{r}$ the vector representing the position of either an atomic site in the unit cell, or the center in a subcluster [2]:

$$F_{hkl}^{p} = \sum_{i=1}^{\text{all atoms}} \cos\left(2\pi \cdot c_s \left(\overline{h_{hkl}} \cdot \tau^{2p} \overline{r_{cc}}\right)\right) \cdot F_{hkl}^{p-1}$$

The unit cell is edge sharing, not face sharing as in crystals. The stoichiometry is therefore Al_6Mn.

In **Figure 4** the unit cells are hierarchically arranged within clusters. Notice that the unit edge width of the icosahedral unit cell stretches to τ^2 in the icosahedral cluster. The hierarchy extends infinitely to superconductors order p with stretching factors τ^{2p} compared to the cluster.

The atoms illustrated in **Figure 3** and **Figure 4** were localized on Cartesian axes as was first done to calculate structure factors [4] [16]. The 5-fold [1 τ0] axis was identified and the position (x_0,y_0,z_0) of each atom in a quasicrystal of selected size (superclusters orders 1 and 2 in figure 5a). These atoms were all projected onto the (1 τ0) plane at points (x_2,y_2,z_2) using rotation formulae the about z axis:

$$y_2 = (y_0 - x_0 t)/(1+t^2),$$
$$x_2 = (x_0 t^2 - y_0 t)/(1+t^2), \text{and } z_2 = z_0 \quad (4)$$

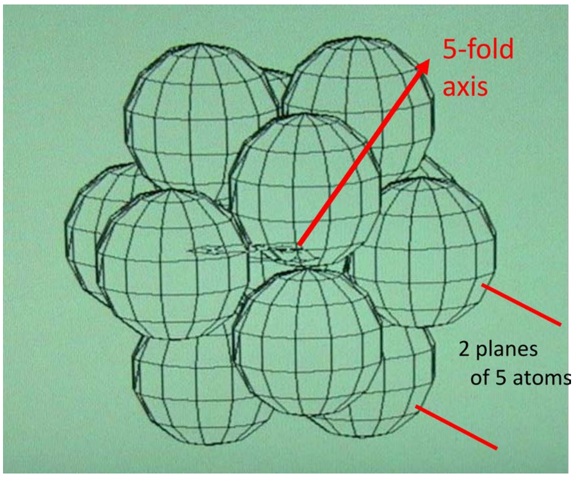


Figure 3. Extremely dense icosahedral unit cell. At its center is the small *Mn* atom with its comparatively large atomic scattering factor, surrounded by twelve tightly packed *Al* atoms.

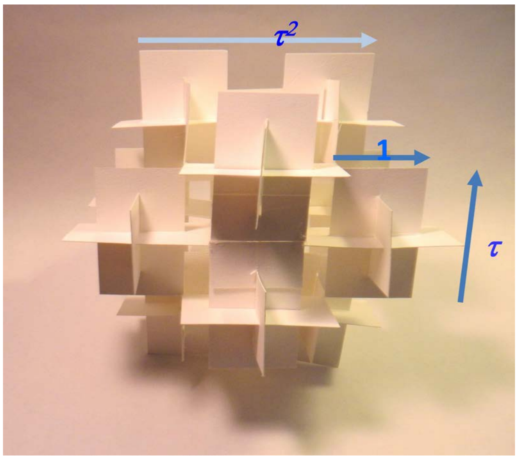
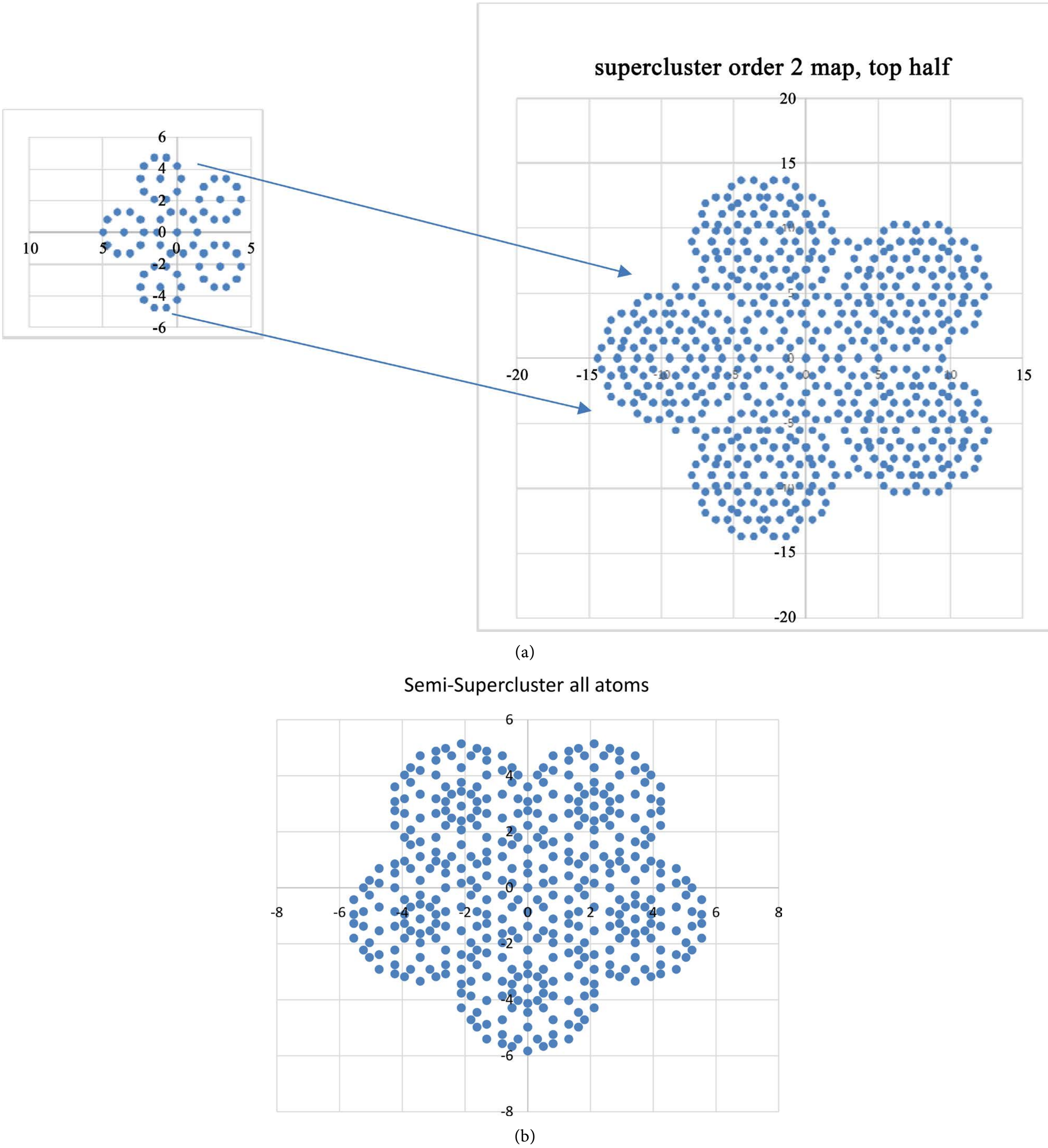


Figure 4. Hierarchy of icosahedral unit cells in the icosahedral cluster. The stretching factor is τ^2. The structure is infinitely extensible. The triad of golden rectangles can represent any member of the hierarchic series.



(a)

(b)

Figure 5. (a) At left is a map of *Mn* atoms in a semi-supercluster (cluster in **Figure 4** scaled up by τ^2), simulating phase-contrast images from foils ~2 nm thick owing to the large atomic scattering factor of *Mn*. Two planes of unit cells (**Figure 3** & **Figure 4**) map as a circle of 10 unit cells within a single circle of 5 clusters. These structures are experimentally evident in **Figure 6** [19]. At right, the semi-supercluster order 2 (scaled up by τ^4) would be ~5 nm thick. The full supercluster map would appear to have 10-fold symmetry, though individual second order superclusters are not distinguishable in imaging, because they overlap neighboring superclusters. (b) Map of all atoms, both *Mn* and *Al*, in the same semi-supercluster as shown on the left inset in figure 5a. The difference represents the *Al* atoms. Notice that when the two outermost cluster layers (representing *Al* and *Mn*) merge—owing to limited phase-contrast microscope resolution—the clusters touch tangentially as in the experimental **Figure 6** below. Figures 5a and 5b together simulate figure 6 with convincing accuracy. Mapping scales are given in units of the measured quasi-lattice-parameter a [2].

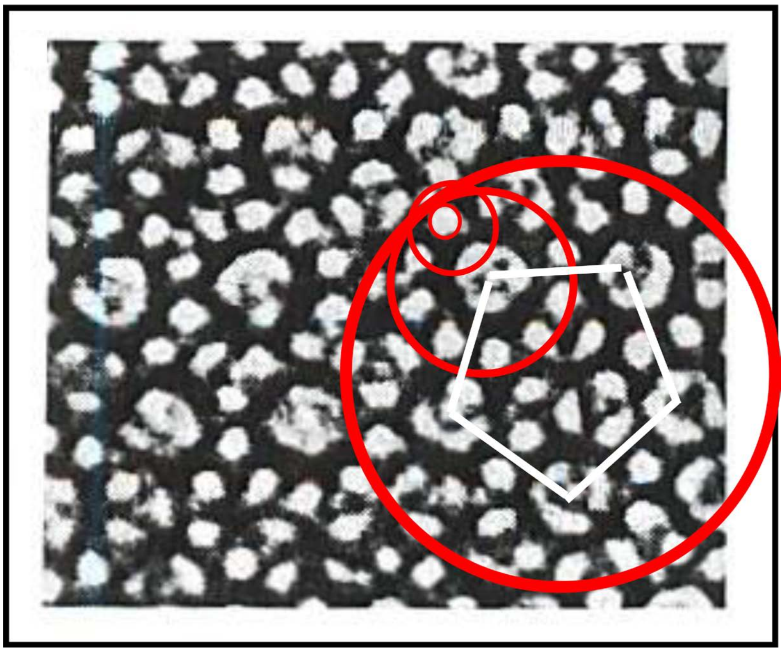


Figure 6. Hierarchic structure (red circles) on phase-contrast, optimum-defocus, transmission electron microscope image of Al_6M_n [19] [16] (p. 66), observed parallel to the 5-fold axis. The image shows hierarchic arrangement of M_n atoms (smallest circle) and (with increasing diameter) a unit cell (13 atoms), cluster (~12^2 atoms) and supercluster (~12^3 atoms). The centers of the clusters are located at the apices of the white pentagon. The structure is mapped in Figure 5b.

Their hypotenuse on (x_2,y_2) gives a radius that was plotted against z_0 in figures 5. Moreover, after calculating the heights of the projections, atoms within a given foil thickness were selected for display.

The simulated map of *Mn* atoms in one half of a supercluster, that is contained in a thin film about 2 nm thick, is shown in **Figure 5a** at left. The mapping calculation employs the same structure as is used for quasi structure factors [2] [4], except that Al atoms are ignored. The image shows the two 'circles' of M_n (**Figure 4**) that appear in 10-fold circles in the cluster; but only one 5-fold circle of clusters in the supercluster. By contrast, in the hierarchic semi-supercluster order 2, with a thickness of ~5 nm, the clusters appear to be 10-fold owing to the twin 'circles'. These features explain many characteristic structures in the phase contrast images [19].

Moreover, in transmission electron microscopy, optimum-phase-contrast sacrifices realism in image location because images change with defocus. Since the atomic scattering power of *Mn* is comparatively large, $f_{Mn}^2 \approx 4 \times f_{Al}^2$ [10], the skeleton *Mn* projections in the inset in **Figure 5a** should move toward 5b in a through focal series. Then, with typical loss of resolution, the skeleton will transform to the observed image in **Figure 6**. Here clusters touch tangentially as in a defocused projection of **Figure 5b**.

The mapping simulations match—both as patterns and by lateral measurements—the electron microscope image in **Figure 6**. They confirm the hierarchic concept[8], so that we can now proceed to discuss the most common defect structures.

[8]The maps dispel the common misconception that the structure is decagonal.

5. Elementary Defects

We begin by dividing the defects into 1) vacancies; or into 2) perfect interstitials without deformity; or 3) other defects, typically deformed. The second of these is significant because of the high density of the unit cell, that makes it the likely cause both for the driving force for the ideal icosahedral hierarchy, and for filling of intrinsic defects. There is evidence for this dominance of the unit cell in dynamic studies of QC growth [20] [21]. There, systematic realignment of atoms can be observed in unit cells and clusters that coalesce into superclusters and, presumably, into higher orders. It is obvious in the rearrangements, that the fundamental icosahedral orientation is maintained during the rearrangements. This must be the case where each cell shares two or more edges with its neighbors. The rigid orientation would occur to a greater or lesser extent if there were, during solidification, multiple nucleation sites, possibly of different sizes—atom, cell, cluster, etc. The coalescence is evident in growth and is surely significant in rapid cooling.

Because the structure is edge sharing, it gives the impression of being porous. Compared with typical crystals, the QC is slightly soft and viscous [22]. *The unit cell is extremely dense*, but higher orders contain "holes" (see Pauling's observation in [5]) especially at cluster and supercluster centers, but also on each side of shared edges. In higher orders, these spaces require filling. Guiding the study must be a concern for consistency in the icosahedral point group symmetry that reciprocates in the diffraction pattern. We will first consider vacancies and interstitials in the smallest structure, the unit cell (**Figure 3**), and proceed to clusters, and to superclusters in ascending hierarchic order (**Figure 4**). To ease comprehension, represent 3-dimensional icosahedra by corresponding 2-dimensional cross-sections: for the unit cell, that is the golden rectangle $\tau \times 1$ in **Figure 7**.

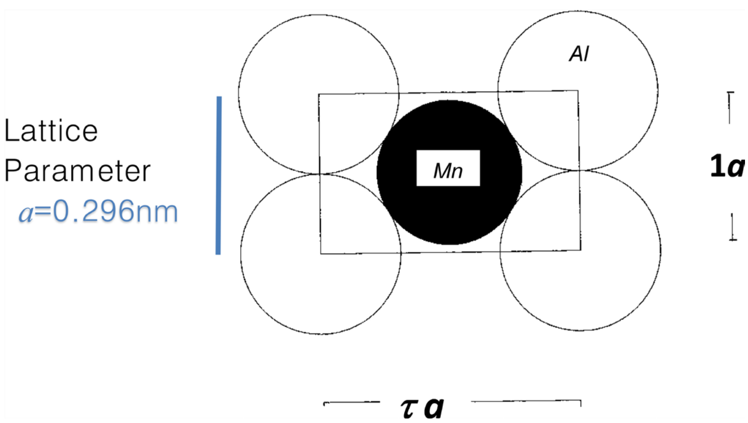


Figure 7. The cross-section above, that is taken from the unit cell shown in **Figure 3** is identical to the golden rectangles in **Figure 4**. The quasi-lattice-parameter a is measured as the edge width [2J. It is equal to the diameter of closely packed *Al*, $a = d_{Al}$. Notice that the diameter of the solute atom in this dense diatomic structure is given by $d_{\mathrm{Mn}} = \left(\sqrt{1+t^2}-1\right)a$. The ratio $d_{\mathrm{solvent}}/d_{\mathrm{solute}}$ is universal in diatomic quasicrystals. Each unit cell has 15 identical sections at various orientations. The sections that will appear again in **Figure 8** are scaled versions of the section here.

This figure illustrates the dominating cross-section of the dense atomic model in **Figure 3**, including the unit cell and quasi-lattice parameter. The unit cell contains 15 identical cross-sections at different orientations. This orientational consistency can be thought of as roughly approximating cylindrical symmetry, with Euclidean axes. Because of edge sharing, various subsidiary spaces fill gaps on either side of the edges that join the dense unit cells. We simplify our understanding of the subsidiary cells by thinking of the 3-dimensional HI by its dominant 2-dimensional cross-section.

We can then consider in turn vacancies, perfect interstitials, and strained interstitials. These features can be made consistent with the sharp diffraction pattern by preserving icosahedral symmetry in all defects, or most of them. So in **Figure 8**,

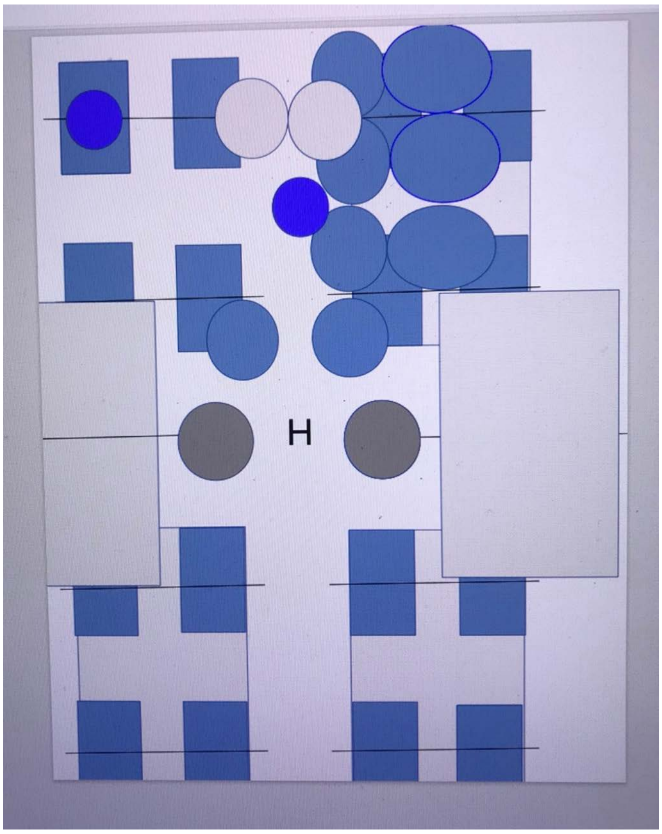


Figure 8. Hierarchic cross-section of a supercluster containing at its corners 4 cross-sections of clusters, each containing 4 rectangular cross-sections of *blue unit cells* (**Figure 7**). Blue circles represent *Al* atoms at corners of selected unit cells. The most elementary defects are considered: 1. *Vacancies* occur at *sites* that are closer spaced than the atomic diameters. The two 'mobile sites' (blue ellipses) may then be shared by one atom like an extended edge site, *e.g.* separated by $1/\tau$ at the top right corner cluster. 2. *Perfect interstitials*: *e.g.* the dark blue unit *Mn* atom (center top) between four clusters, occupying space—left open by the edge sharing clusters—without distortion. 3. *Off-plane* regular (hierarchic) structures (off white and dark grey) 4. *Other distorted interstitials* such as octagonal structures, not represented in the diagram. All of the critical dimensions in the figure belong to the geometric series τ^m, *e.g.* 1 for the diameter of the *Al* atom; $\tau \times 1$ for the rectangular unit cell (as in **Figure 7**, with units of *a*); $\tau^3 \times \tau^2$ for the cluster; $\tau^5 \times \tau^4$ for the supercluster; $1 \times \tau^{-1}$ for the cluster hole; $\tau^2 \times \tau$ for the supercluster hole H; *etc.*

icosahedra are represented by their 2-dimensional cross-sections: the unit cell, by the blue golden rectangle $\tau \times 1$; the cluster, by four unit cells $\tau^3 \times \tau^2$; the super-cluster, by four clusters $\tau^5 \times \tau^4$, *etc.*

In isolation, or in the melt before solidification, the *unit cell* is dense. However, when it is integrated into an hierarchic cluster, some sites are vacated because of insufficient space. We suppose that mobile *Al* atoms share neighboring sites that are closer together (separated by $1/\tau$) than the unit diameter of the atom. These mobile atoms are represented by elliptical wave functions on the *Al* atoms at corners of adjacent golden rectangles in the (top right) cluster in **Figure 8**. By contrast, typical the *Al* atoms are circular with unit (icosahedral) diameter. The vacancy is common at the cluster level in the hierarchy. For example, the central "hole" is icosahedral and may be represented, in cross-section, by the golden rectangle $1 \times 1/\tau$. It is so small that there is room for only three Al atoms on twelve ideal corner sites. These have been simulated [[4] p.54] in agreement with phase-contrast images.

The largest individual volume, at any tier of hierarchy, is the central "hole". In the supercluster it is marked with the letter H in **Figure 8**. This hole has the cross-section $\tau^2 \times \tau$. Its center is surrounded, in front and behind, by off-white cluster cross-sections, and closer in by *Al* atoms in dark grey. The "hole" is larger than the unit cell so there is more than one way in which the space can be filled. It could also be filled by an octahedral structure which may not show in diffraction because the octahedron is a subgroup of the icosahedral group.

An estimate can easily be calculated for the contribution to scattering caused by interstitial filling in the "hole". The supercluster has cross-section $\tau^5 \times \tau^4$; so that the volume of the "hole" is $\tau^{-6} \sim 5.6\%$ less voluminous. The same defect ratio holds for in-fill ratios at higher orders. This is why they can be neglected in first calculations but might be revealed in further detail by concerted research. Meanwhile, we have, for the most part, been able to leave the defect holes empty in QSF calculations, since the effects of their tentative, possible inclusions were small. In this paper, primacy is given to logarithmic periodicity, hierarchic structure and resonant response; details of the structural "jig-saw puzzle"—what we here call defects—are relegated, because their effects are comparatively small and insignificant.

6. Conclusions

Whatever may be the structural details of quasicrystals, whether systematic or accidental; the ideal hierarchic model has provided complete understanding of diffraction in geometric series with irrational indices. Quasicrystals have demonstrated that quantum physics and Bragg diffraction are not beautiful mathematics but empirical physics: the proof of the numeric metric by analytic separation of the irrational residue is a benefit of observation over speculative expectation. The coherent scattering, that is here described, depends on constructive interference—in time and space—of the incident radiation. We have shown how logarithmically-periodic solids digitize and harmonize incident periodic waves. The radiation responds to the hierarchic QC fields by forming quasi-Bloch waves on a special *metric*. This formation commensurates the periodic, incident radiation with the varied and irrationally-spaced, hierarchic quasicrystal. Quasi-structure factors show that the resulting translational periodicity about $a\tau^m$, scatters the radiation coherently into geometric reciprocal space. There *is* long range order and this is evident in the diffraction. However, the corresponding *translational symmetry occurs by resonant response of the scattering radiation.* Without harmonics, the scattering would be random owing to destructive interference within the wave packet. Spectacularly, this doesn't happen.

Moreover, in this work, we have united time-dependent wave optics in 4-dimensions (Section 2) with more typical 3-dimensional optics that is adapted from Bragg optics. The time dependence is necessary to show how the quantum effects—evident in both atomic states and in diffraction—occur by harmonies, in the physical domain, and are commonly glossed in purely mathematical descriptions. The harmonics are furthermore associated with QC defects that are represented, for easy comprehension, by 2-dimensional cross-sections. These are scaled for the various hierarchies. Based, as they are, on the exact description of the diffraction, the present physical description of harmonic quanta, has wider application in general physics.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Shechtman, D., Blech, I., Gratias, D. and Cahn, J.W. (1984) *Physical Review Letters*, **53**, 1951-1953. https://doi.org/10.1103/PhysRevLett.53.1951

[2] Bourdillon, A.J. (2020) *Journal of Modern Physics*, **11**, 581-592. https://doi.org/10.4236/jmp.2020.114038

[3] Bourdillon, A.J. (2020) Quasicrystal Bloch Wave Symmetry about at^m. https://www.youtube.com/watch?v=gZClVNKhhVw

[4] Bourdillon, A.J. (2011) Logarithmically Periodic Solids. Nova Science, Hauppauge.

[5] Pauling, L. (1985) *Nature*, **317**, 512-514. https://doi.org/10.1038/317512a0

[6] Bourdillon, A.J. (2017) Dispersion Dynamics in the Hall Effect and Pair Bonding in HiT_c. Nova Science, Hauppauge.

[7] Dirac, P.A.M. (1958) The Principles of Quantum Mechanics. Oxford University Press, Oxford. https://doi.org/10.1063/1.3062610

[8] Feynman, R.P. (1949) *Physical Review*, **76**, 749-769. https://doi.org/10.1103/PhysRev.76.769

[9] Stueckelberg, E.C.G. (1941) *Helvetica Physics Acta*, **14**, 588-594.

[10] Hirsch, P., Howie, A., Nicholson, R.B., Pashley, D.W. and Whelan, M.J. (1977) Electron Microscopy of Thin Crystals. Krieger, New York.

[11] Bourdillon, A.J. (2010) Quasicrystals' 2D Tiles in 3D Superclusters. UHRL, San Jose,

p. 37 *seq*.

[12] Bourdillon, A.J. (2018) *Journal of Modern Physics*, **9**, 1304-1316. https://doi.org/10.4236/jmp.2018.96079

[13] Bourdillon, A.J. (2018) *Journal of Modern Physics*, **9**, 2295-2307. https://doi.org/10.4236/jmp.2018.913145

[14] Bourdillon, A.J. (2020) *Journal of Modern Physics*, **11**, 1926-1937. https://doi.org/10.4236/jmp.2020.1112121

[15] Bourdillon, A.J. (2019) *Journal of Modern Physics*, **10**, 624-634. https://doi.org/10.4236/jmp.2019.106044

[16] Bourdillon, A.J. (2009) Quasicrystals and Quasi Drivers.

[17] Bourdillon, A.J. (2009) *Solid State Communications*, **149**, 1221-1225. https://doi.org/10.1016/j.ssc.2009.04.032

[18] Bourdillon, A.J. (2013) *Micron*, **51**, 21-25. https://doi.org/10.1016/j.micron.2013.06.004

[19] Bursill, L.A. and Peng, J.L. (1985) *Nature*, **316**, 50-51. https://doi.org/10.1038/316050a0

[20] Nagao, K., Inuzuka, T., Nishimoto, K. and Edagawa, K. (2015) *Physical Review Letters*, **115**, Article ID: 075501. https://doi.org/10.1103/PhysRevLett.115.075501

[21] Thi, N.H., Gastaldi, J., Schenk, T., Reinhart, G., Mangelinck-Noel, N., Cristiglio, V., Billia, B., Grushko, B., Hartwig, J., Klein, H. and Baruchel, J. (2006) *Physical Review E*, **74**, Article ID: 021605. https://doi.org/10.1103/PhysRevE.74.031605

[22] Puckerman, B.E. (2011) Quasicrystals: Types, Systems, and Techniques. Nova Science, Hauppauge.

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Cosmological Duality in Four Time and Four Space Dimensions

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Abstract

We describe a duality transformation in a cosmological model of four time and four space dimensions ((4 + 4)-dimensions). In particular, we show that *via* the Fourier transform, at the level of the zero-point energy of quantum mechanics and the de Sitter space, a Gaussian distribution in four dimensions leads to a dual Gaussian distribution also in four dimensions, with duality transformation $\sigma \to \frac{1}{\sigma}$, in the standard deviation σ. Moreover, we show that as a consequence of such a duality in σ a duality of the cosmological constant Λ can be obtained. Finally, we comment on the possibility that both the oriented matroid theory as well as the surreal number theory are related to the formalism presented in this work.

Keywords

Cosmology, (4 + 4)-Dimensional Gravitational Theory, Classical and Quantum Gravity

1. Introduction

It is known that the (5 + 5)-dimensional space-time (five time and five space dimensions) is a common signature to both type IIA strings and type IIB strings [1]. In fact, versions of *M*-theory [2] lead to type IIA and to type IIB string in space-time of signatures (5 + 5). It turns out that by duality transformations string theories of signatures (5 + 5) can be related to other string signatures such as (1 + 9) [3].

Just as the (1 + 3)-dimensional signature can be considered as a reduced world of the de Sitter (1 + 4)-dimensional or anti-de Sitter (2 + 3)-dimensional signatures *via* the cosmological constant $\Lambda > 0$ and $\Lambda < 0$ (see Refs [4] [5] and references therein), respectively, here, we shall assume that up to two cosmological constants, the (4 + 4)-world emerges from (5 + 5)-dimensional world. Specifically, we show that *via* a Fourier transform a complete duality symmetry of the standard deviation

$$\sigma^2 \leftrightarrow \frac{1}{\sigma^2}$$

of a Gaussian of distribution of 4-space coordinates associated with the de Sitter space (anti-de Sitter) and the vacuum zero-point energy yields to a Gaussian of 4-time coordinates of the same vacuum scenario. This is in fact one of our main contribution and its importance emerges when we notice that only in 4-dimensions such a totally duality symmetry is achieved. In the process, we discover that in 4-dimensions the cosmological constant $\Lambda > 0$ associated with the de Sitter space ($\Lambda < 0$, anti de Sitter space) is dual to the cosmological constant $\Lambda_P = \frac{3}{l_P}$, where $l_P = \left(\frac{\hbar G}{c^3}\right)^{1/2}$ is the Planck length, with G the gravitational Newton constant, $\hbar$ the Planck constant and c the light velocity.

There are at least two frameworks where a (4 + 4)-world has emerged as interesting physical scenario. First, it has been proved [6] that the Dirac equation in (4 + 4)-dimensions admit a Majorana-Weyl physical spinor state with only 8 real components which can be identified with the 4-complex components of the usual electron components. Second, in Ref. [7] it has been shown that a general Kruskal-Szekeres transform, in black-hole physics, implies 8 hidden regions instead of just 4-regions as it is usually believed. It turns out that this 8-regions admit better interpretation in a world of (4 + 4)-dimensions.

Technically, this work is organized as follows. In Section 2, we consider the geodesic of a point particle in the de Sitter (anti-de Sitter) space and show that the classical harmonic oscillator equation in 4-dimensions emerges. In Section 3, we quantize the system obtained in the previous section and focus in the case of zero-point energy showing that this case admits a Gaussian distribution solution. While in Section 4, by the use of the Fourier transform we investigate a duality scenario. Thus combining the zero-point energy and de Sitter vacuum space we discover a duality of the cosmological constant. Finally in Section 5, we express a number of final remarks. In particular, from our analysis of the Gaussian distribution from 4-space dimensions to 4-time dimensions we conclude that the space-time involved may be considered corresponds to a (4 + 4)-dimensional spacetime. Moreover, we briefly comment about the possibility that, for further work, the mathematical structures of matroid theory and surreal number theory may be interesting routes for a connection with our approach.

2. Geodesic Equation of the de Sitter (Anti-de Sitter) Space

Let us start recalling some geometrical aspects of the de Sitter space (or anti de Sitter space). In particular, it is well known that the Christoffel symbols in the de

Sitter space satisfy the equation

$$\Gamma^{\mu}_{\alpha\beta} = \frac{x^{\mu}}{l_0^2} g_{\alpha\beta}. \tag{1}$$

Here, the metric $g_{\alpha\beta}$ is given by

$$g_{\alpha\beta} = \delta_{\alpha\beta} + \frac{x_{\alpha} x_{\beta}}{l_0^2 - x^{\tau} x^{\sigma} \delta_{\tau\sigma}}, \tag{2}$$

and l_0^2 is determined by

$$l_0^2 = x^{\tau} x^{\sigma} \delta_{\tau\sigma} + \left(x^5\right)^2, \tag{3}$$

where $\delta_{\tau\sigma}$ is the Kronecker delta (see Ref. [8] and references therein). The indices $\alpha, \beta, \cdots$ etc run from 1 to 4. Thus, using (1) the geodesic equation

$$\ddot{x}^{\mu} + \Gamma^{\mu}_{\alpha\beta} \dot{x}^{\alpha} \dot{x}^{\beta} = 0, \tag{4}$$

yields

$$\ddot{x}^{\mu} + \frac{x^{\mu}}{l_0^2} g_{\alpha\beta} \dot{x}^{\alpha} \dot{x}^{\beta} = 0. \tag{5}$$

In general, for arbitrary variable $A(\tau)$ one defines $\dot{A} \equiv \frac{\mathrm{d}A}{\mathrm{d}\tau}$ and $\ddot{A} \equiv \frac{\mathrm{d}^2 A}{\mathrm{d}\tau^2}$. Further, the line element is given by $c^2 \mathrm{d}\tau^2 = g_{\alpha\beta} \mathrm{d}x^{\alpha} \mathrm{d}x^{\beta}$. Thus, one sees that $g_{\alpha\beta} \dot{x}^{\alpha} \dot{x}^{\beta} = c^2$ and therefore (5) becomes

$$\ddot{x}^{\mu} + \frac{c^2 x^{\mu}}{l_0^2} = 0. \tag{6}$$

One recognizes in (6) the classical harmonic oscillator equation in 4-dimensions, with $\kappa = \frac{c^2}{l_0^2}$ as the spring constant.

3. Quantization; Zero-Point "Energy"

Let us now quantize the system described by the expression (6) from which one finds the constant of motion

$$\varepsilon = \frac{\mathcal{P}^{\tau} \mathcal{P}^{\sigma} \delta_{\tau\sigma}}{2} + \frac{x^{\tau} x^{\sigma} \delta_{\tau\sigma}}{2 l_0^2}, \tag{7}$$

with $\mathcal{P}^{\tau} = \frac{\dot{x}^{\tau}}{c}$. Therefore, at the quantum level one promotes the quantities ε and $\mathcal{P}^{\tau}$ by the operators $\hat{\varepsilon} = -i l_P \frac{\partial}{c \partial \tau}$ and $\hat{\mathcal{P}}_{\mu} = -i l_P \frac{\partial}{\partial x^{\mu}}$, respectively. Here, we assume that l_P is the Planck length, namely $l_P = \left(\frac{\hbar G}{c^3}\right)^{1/2}$. Note that the reason to write the quantum operators $\hat{\varepsilon}$ and $\hat{\mathcal{P}}_{\mu}$ in terms of l_P rather than $\hbar$ can be traced back to the geodesic Equation (4) which is independent of the test particle mass and therefore the quantities ε and $\mathcal{P}^{\sigma}$ that play the analogue role of energy and momentum, are dimensionless. Thus, the formula (7) leads to the Schrödinger equation

$$\hat{\varepsilon}\psi = \frac{\hat{\mathcal{P}}_\tau \hat{\mathcal{P}}_\sigma \delta^{\tau\sigma}}{2}\psi + \frac{x^\tau x^\sigma \delta_{\tau\sigma}}{2l_0^2}\psi. \tag{8}$$

One may assume the usual proposal for the physical state $\psi = \mathrm{e}^{i\frac{c\varepsilon\tau}{l_P}}\varphi\left(x^\mu\right)$. This leads to

$$\varepsilon\varphi = \frac{\hat{\mathcal{P}}_\tau \hat{\mathcal{P}}_\sigma \delta^{\tau\sigma}}{2}\varphi + \frac{x^\tau x^\sigma \delta_{\tau\sigma}}{2l_0^2}\varphi. \tag{9}$$

Of course, in analogy to the usual harmonic oscillator equation, (9) must imply that ε is quantized. Since one is mainly interested in the lowest energy state, here we shall be concerned only with the zero-point "energy". For this purpose for each coordinate in $x^\mu = \left(x, y, z, \xi\right)$ one must have the formula

$$\varepsilon_{(x,y,z,\xi)} = \frac{l_P \omega}{2}, \tag{10}$$

with $\omega = \frac{1}{l_0}$ and

$$\varphi = \varphi_x\left(x\right)\varphi_y\left(y\right)\varphi_z\left(z\right)\varphi_\xi\left(\xi\right). \tag{11}$$

Thus, in the zero-point "energy", for each coordinate in $\left(x, y, z, \xi\right)$ one must have the equation

$$\frac{l_P\omega}{2} = -\frac{l_P^2}{2}\frac{1}{\varphi_x}\frac{\mathrm{d}^2\varphi_x}{\mathrm{d}x^2} + \frac{\omega^2 x^2}{2}, \tag{12}$$

where φ_x denotes the function φ evaluated for each coordinate in $\left(x, y, z, \xi\right)$. Solving (12) for φ_x one discovers that

$$\varphi_x \sim \mathrm{e}^{-\frac{\sigma^2 x^2}{2}}, \tag{13}$$

where

$$\sigma = \left(\frac{\omega}{l_P}\right)^{1/2} = \left(\frac{1}{l_P l_0}\right)^{1/2} \tag{14}$$

is the inverse of the standard deviation. Following similar steps, for the other coordinates y, z, ξ one finally discovers that

$$\varphi = \frac{\sigma^q}{\left(2\pi\right)^{q/2}}\mathrm{e}^{-\frac{\sigma^2\left(x^2+y^2+z^2+\xi^2\right)}{2}} \tag{15}$$

or

$$\varphi = \frac{\sigma^q}{\left(2\pi\right)^{q/2}}\mathrm{e}^{-\frac{\sigma^2\left(x^\mu x^\nu \delta_{\mu\nu}\right)}{2}}. \tag{16}$$

Here, q is a positive integer number to be determined below. Of course, (15) (or (16)) corresponds to Gaussian distribution in 4-dimensions.

4. Duality and the Fourier Transform

It turns out that the Fourier transform of (15) or (16) is given by

$$^{*}\varphi=\frac{(2\pi)^{q/2}}{\sigma^{q}}\mathrm{e}^{-\frac{t^{\tau}t^{\sigma}\delta_{\tau\sigma}}{2\sigma^{2}}}. \tag{17}$$

In fact, let us show in some detail that (15) (or (16)) implies (17). For this purpose let $^{*}\mathcal{F}(t)$ be the Fourier transform that sends us from the one dimensional *x*-space to the *t*-space

$$^{*}\mathcal{F}(t)=\int_{-\infty}^{\infty}\mathrm{e}^{itx}\mathcal{F}(x)\mathrm{d}x. \tag{18}$$

Here the function $\mathcal{F}(x)$ is given by $\mathcal{F}(x)=\frac{\sigma}{\sqrt{2\pi}}\mathrm{e}^{-\frac{\sigma^{2}x^{2}}{2}}$, where typically the factor $\frac{\sigma}{\sqrt{2\pi}}$ is introduced for normalizing the function $\mathcal{F}(x)$. The Fourier transform $^{*}\mathcal{F}(t)$ is obtained as follows; by definition one has

$$^{*}\mathcal{F}(t)=\frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{itx}\mathrm{e}^{-\frac{\sigma^{2}x^{2}}{2}}\mathrm{d}x, \tag{19}$$

then, completing the squares one finds

$$^{*}\mathcal{F}(t)=\frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{-\frac{1}{2}\left(\sigma x-\frac{it}{\sigma}\right)^{2}}\mathrm{e}^{-\frac{t^{2}}{2\sigma^{2}}}\mathrm{d}x. \tag{20}$$

Now, introducing the variable $u=\sigma x-\frac{it}{\sigma}$ so that the differential remains as $\mathrm{d}u=\sigma\mathrm{d}x$ one finds that the Fourier transform (20) becomes

$$^{*}\mathcal{F}(t)=\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{t^{2}}{2\sigma^{2}}}\int_{-\infty}^{\infty}\mathrm{e}^{-\frac{u^{2}}{2}}\mathrm{d}u. \tag{21}$$

Making the integral, one ends up with

$$^{*}\mathcal{F}(t)=\mathrm{e}^{-\frac{t^{2}}{2\sigma^{2}}}. \tag{22}$$

Thus, the Fourier transform of a Gaussian distribution in the *x*-space, with standard deviation $\frac{1}{\sigma}$ is another Gaussian distribution, but now in the *t*-space and with standard deviation σ. This is of course well known fact which is expressed in simply words as; the Fourier transform of Gaussian is another Gaussian. However, in the general case of higher dimensions we shall put spacial attention to the duality relation $\sigma\leftrightarrow\frac{1}{\sigma}$ of the deviation standard.

So, let us generalize the above procedure to any dimension *d*. One has

$$\begin{aligned}^{*}\mathcal{F}(t_{1},\cdots,t_{d})=&\frac{\sigma^{q}}{(2\pi)^{q/2}}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\mathrm{e}^{-\frac{1}{2}\left(\sigma x^{1}-\frac{it^{1}}{\sigma}\right)^{2}}\cdots\mathrm{e}^{-\frac{1}{2}\left(\sigma x^{d}-\frac{it^{d}}{\sigma}\right)^{2}}\\&\times\mathrm{e}^{-\frac{(t^{1})^{2}}{2\sigma^{2}}\cdots-\frac{(t^{d})^{2}}{2\sigma^{2}}}\mathrm{d}x^{1}\cdots\mathrm{d}x^{d}.\end{aligned} \tag{23}$$

Since the term $\mathrm{e}^{-\frac{(t^1)^2}{2\sigma^2}\cdots-\frac{(t^d)^2}{2\sigma^2}}$ is independent of the variables $x^1,\cdots,x^d$ one can write (23) as

$$
\begin{aligned}
&{}^{*}\mathcal{F}(t_1,\cdots,t_d)\\
&=\frac{\sigma^q}{(2\pi)^{q/2}}\mathrm{e}^{-\frac{(t^1)^2}{2\sigma^2}\cdots-\frac{(t^d)^2}{2\sigma^2}}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\mathrm{e}^{-\frac{1}{2}\left(\sigma x^1-\frac{it^1}{\sigma}\right)^2}\cdots\mathrm{e}^{-\frac{1}{2}\left(\sigma x^d-\frac{it^d}{\sigma}\right)^2}\mathrm{d}x^1\cdots\mathrm{d}x^d.
\end{aligned}
\tag{24}
$$

One can again make the change of variable $u_i=\left(\sigma x^i-\frac{it^i}{\sigma}\right)$, so that $\mathrm{d}u_i=\sigma\mathrm{d}x^i$ and hence (24) becomes

$$
{}^{*}\mathcal{F}(t_1,\cdots,t_d)=\frac{\sigma^q}{(2\pi)^{q/2}\sigma^d}\mathrm{e}^{-\frac{(t^1)^2}{2\sigma^2}\cdots-\frac{(t^d)^2}{2\sigma^2}}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\mathrm{e}^{-\frac{1}{2}u_1^2}\cdots\mathrm{e}^{-\frac{1}{2}u_d^2}\mathrm{d}u_1\cdots\mathrm{d}u_d. \tag{25}
$$

Performing the integrals in (25) yield

$$
{}^{*}\mathcal{F}(t_1,\cdots,t_d)=\frac{\sigma^q(2\pi)^{d/2}}{(2\pi)^{q/2}\sigma^d}\mathrm{e}^{-\frac{(t^1)^2}{2\sigma^2}\cdots-\frac{(t^d)^2}{2\sigma^2}}. \tag{26}
$$

Thus, if $d=2q$ then one finds

$$
{}^{*}\mathcal{F}(t_1,\cdots,t_d)=\frac{(2\pi)^{q/2}}{\sigma^q}\mathrm{e}^{-\frac{(t^1)^2}{2\sigma^2}\cdots-\frac{(t^d)^2}{2\sigma^2}}. \tag{27}
$$

In particular, by setting $q=2$ one obtains $d=4$ and thus (16) leads to the Gaussian distribution

$$
\mathcal{G}=\frac{\sigma^2}{2\pi}\mathrm{e}^{-\frac{\sigma^2}{2}\left[(x^1)^2+(x^2)^2+(x^3)^2+(x^4)^2\right]} \tag{28}
$$

and therefore according to (27) the Fourier transform of $\mathcal{G}$ is given by

$$
\mathcal{G}^*=\frac{2\pi}{\sigma^2}\mathrm{e}^{-\frac{1}{2\sigma^2}\left[(t^1)^2+(t^2)^2+(t^3)^2+(t^4)^2\right]}, \tag{29}
$$

in agreement with (17).

Note the surprising duality relation between $\mathcal{G}$ and $\mathcal{G}^*$, namely

$$
\sigma^2\leftrightarrow\frac{1}{\sigma^2}, \tag{30}
$$

which only is fully achieved in $d=4$. In this sense one can say that the 4-dimensions associated with $\mathcal{G}^*$ and and 4-dimensions associated with $\mathcal{G}$ are dual to each other. Observe that $\mathcal{G}^{**}\sim\mathcal{G}$ which is the usual requirement for a duality symmetry. From (14) one knows that $\sigma^2=\frac{1}{l_P l_0}$ so that (30) can be written as

$$
l_P l_0\leftrightarrow\frac{1}{l_P l_0}. \tag{31}
$$

Thinking about the magnetic monopole g and the electric charge e duality, namely

$$e \leftrightarrow \frac{n\hbar}{e}, \tag{32}$$

with $ge = n\hbar$, one is tempted to assume that (31) implies a duality of the form

$$l_P l_0 = n\alpha, \tag{33}$$

where α is a constant to be determined. Considering that at the present, the Planck length is of the order $l_P \sim 10^{-33}\,\text{cm}$ and assuming the de Sitter length is of the order of the radius of the Universe $l_0 \sim 10^{+28}\,\text{cm}$, for $n = 1$, one obtains that α is of the order of 10^{-5} cm^2 which is too large to be identified with any fundamental atomic radius. However, an interesting and attractive possibility is to assume the condition $\alpha \sim 1\,\text{cm}^2$, which implies that $l_0 \sim 10^{+33}\,\text{cm}$.

It is remarkable that there exist a relation between the de Sitter length l_0 and the cosmological constant Λ, namely (see [8] and references therein)

$$\frac{2\Lambda}{(d-1)(d-2)} = \frac{1}{l_0^2}. \tag{34}$$

In $d = 4$ one gets

$$\frac{\Lambda}{3} = \frac{1}{l_0^2}. \tag{35}$$

Thus, the duality relation (31) can be written as

$$\left(\frac{\Lambda_P\Lambda}{9}\right)^{1/2} \leftrightarrow \frac{1}{\left(\frac{\Lambda_P\Lambda}{9}\right)^{1/2}}, \tag{36}$$

where one has defined

$$\frac{\Lambda_P}{3} = \frac{1}{l_p^2}. \tag{37}$$

Since one is assuming the formula (33) one sees that $\Lambda_P\Lambda = \beta^2$, with $\beta^2 = \frac{9}{n^2\alpha^2}$ and hence one has discovered a cosmological constant duality

$$\Lambda \leftrightarrow \frac{1}{\Lambda_P}. \tag{38}$$

So, considering the case $n = 1$ and $\alpha = 1$, since l_p is of the order of 10^{-33} cm, one finds that Λ_P is of the order of 10^{66} cm^{-2} which is, of course, very large, but according to (38) the cosmological constant Λ will be very small of the order 10^{-66} cm^{-2}. It is worth mentioning that the type of duality (38) has been previously described in the context of $\mathcal{S}$-duality for linearized gravity [9].

5. Final Remarks

In the above discussion, we have focused in the de Sitter space with $\Lambda > 0$, however similar conclusions must be achieved in the case of anti de Sitter space with $\Lambda < 0$ (see Refs [4] [5] and references therein). Further, in the usual quantum mechanics, the Fourier transform relate in one to one correspondence the configuration space of *q*-coordinate with the conjugate momentum *p*-coordinates. If there are *n* such *q*-coordinates there must *n* *p*-coordinates and this mean that the total phase space must be (*n* + *n*)-dimensional. Following this route of though one can say that the Fourier transform between the 4 x^{μ} -coordinates and the 4 t^{α} -coordinates in our approach must lead inevitably to a (4 + 4)-dimensional space-time. In fact, combining (23), (28) and (29) one may write

$$f^{*} = \hat{f}\left(\mathrm{e}^{-\frac{\sigma^{2}x^{\mu}x^{\nu}\delta_{\mu\nu}}{2}+\frac{t^{\mu}t^{\nu}\delta_{\mu\nu}}{2\sigma^{2}}}\right), \tag{39}$$

with $\mu,\nu = 1,2,3,4$. Here,

$$\hat{f}\left(\mathrm{e}^{-\frac{\sigma^{2}x^{\mu}x^{\nu}\delta_{\mu\nu}}{2}+\frac{t^{\mu}t^{\nu}\delta_{\mu\nu}}{2\sigma^{2}}}\right) = \frac{\sigma^{2}}{2\pi}\mathrm{e}^{\frac{t^{\mu}t^{\nu}\delta_{\mu\nu}}{2\sigma^{2}}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{e}^{-\frac{1}{2}ix^{\mu}t^{\nu}\delta_{\mu\nu}}\mathrm{e}^{-\frac{\sigma^{2}x^{\mu}x^{\nu}\delta_{\mu\nu}}{2}}\mathrm{d}x^{1}\mathrm{d}x^{2}\mathrm{d}x^{3}\mathrm{d}x^{4} \tag{40}$$

and $f^{*} = \frac{2\pi}{\sigma^{2}}$. So, from (39) one may conclude that the space-time involved may be considered to correspond to a (4 + 4)-dimensional. Therefore, a surprising scenario emerges from our formalism: The point is that our result refers to both vacuum de Sitter space associated with the "macroscopic" x^{μ}-world of 4-dimensional space and the zero-point "energy" linked to the lowest possible energy at that quantum "microscopic" t^{α}-world of 4-dimensions, and *vice versa*. The key result is that these two worlds are related by duality symmetry; a standard deviation σ of a x^{μ}-world is dual to the $\frac{1}{\sigma}$ in the t^{α}-world. This means that a x^{μ}-world with thin and tall (small σ) Gaussian distribution is dual to wide and low (large σ) Gaussian distribution in the other t^{α}-world and *vice versa*. Moreover, this is verified by observing that the large cosmological constant Λ_{P} in the quantum world is dual to the small cosmological constant Λ in the classical cosmological world. This seems to explain, in a cosmological context, the smallness of the cosmological constant.

Thinking in the possible geometrization of the duality symmetry in (4 + 4)-dimensions space, one is tempted to propose the modified line element

$$\mathrm{d}s^{2} = -\frac{c^{2}\mathrm{d}t^{\mu}\mathrm{d}t^{\nu}\delta_{\mu\nu}}{\sigma^{2}l_{P}^{4}} + \sigma^{2}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu}\delta_{\mu\nu}. \tag{41}$$

Here, we have introduced proper units in order to have $\mathrm{d}s^{2}$ dimensionless, the *t*-coordinates in seconds and *x*-coordinates in centimeters. So, (41) considers explicitly the standard deviation σ in a dual form. It remains to explore the full consequences of (41).

The idea that duality emerges as a key concept in vacuum cosmology in 4-dimensions has been explored before in [10]. Here, we have shown that duality also seems to play a fundamental role in vacuum cosmology in (4 + 4)-dimensions, at both macroscopic and microscopic level. So, one wonders whether duality can be the central mathematical notion in quantum gravity. This route of though shall bring us sooner or later to look for a mathematical structure as a frame for the duality concept. In turns out that such a formalism already exist, namely oriented matroid theory (see Refs. [11]-[17] and references therein). This mathematical framework is a generalization of both graphs and matrices and establishes that every oriented matroid has a dual. This means that although every graph corresponds to a matroid, there are matroids which are not graphic. So, this clarify why the complete graphs K_5 and $K_{3,3}$ do not have a dual; they have an associated matroid $M(K_5)$ and $M(K_{3,3})$ but the corresponding dual matroids $M^*(K_5)$ and $M^*(K_{3,3})$ are not graphic. Thus, oriented matroid theory has entered its key concept: duality. So, one may be interesting for further work to establish the relation between the duality described in this work and oriented matroid theory.

Finally, let us just mention another source of interest for further work. We refer to the surreal number theory. This is a mathematical structure that was discovered by Conway [18] (see also Ref. [19]) and Gonshor [20]. Among its interesting properties of such numbers is that natural, integers and real numbers are contained in the surreal number framework. In Refs. [21] and [22] it was established the main reasons why we believe that such a numbers may be physically interesting. In particular one may be interested in considering surreal number theory in the context of quantum gravity in (4 + 4)-dimensions. Let us clarify this comment. Classical general relativity in (4 + 4)-dimensions can be formulated in R^8, that is in a continuum 8-dimensional space-time background. So quantizing gravity may be though as the framework, which must imply at mechanism for starting with a continuum differential geometry in 8-dimensions and then discretize such a geometry. But what if we go backwards? That is, we start with a discrete mathematical structure of surreal numbers $\mathcal{S}$ and we end up with a continuum 8-dimensional manifold based on R. Consider the set [18]

$$s = \{S_L \mid S_R\} \tag{42}$$

and call S_L and S_R the left and right sets of s, respectively. Surreal numbers are defined in terms of two axioms:

Axiom 1. Every surreal number corresponds to two sets S_L and S_R of previously created numbers, such that no member of the left set $S_L \in S_L$ is greater or equal to any member s_R of the right set S_R.

Let us denote by the symbol $\not\geq$ the notion of no greater or equal to. So the axiom establishes that if s is a surreal number then for each $s_L \in S_L$ and $s_R \in S_R$ one has $s_L \not\geq s_R$. This is denoted by $S_L \not\geq S_R$.

Axiom 2. One number $s = \{S_L \mid S_R\}$ is less than or equal to another number $s' = \{S'_L \mid S'_R\}$ if and only if the two conditions $S_L \not\geq s'$ and $s \not\geq S'_R$ are satisfied.

This can be simplified by saying that $s \leq s'$ if and only if $S_L \ngeq s'$ and $s \ngeq S'_R$.

Observe that Conway definition relies in an inductive method; before a surreal number *x* is introduced one needs to know the two sets S_L and S_R of surreal numbers. Following Conway algorithm one finds numbers, of the type

$$s = \frac{m}{2^n}, \tag{43}$$

where *m* is an integer and *n* is a natural number, $n > 0$. Of course, the numbers (43) are dyadic rationals which are dense in the real *R*.

A different but equivalent definition of surreal numbers is due to Gonshor [20]:

Definition 1. A surreal number is a function *f* from initial segment of the ordinals into the set $\{+,-\}$.

For instance, if f is the function so that $f(1) = +$, $f(2) = +$, $f(3) = -$, $f(4) = +$ then f is the surreal number $(++-+)$.

It can be shown that the positive sectors of surreal numbers are given by [23]

$$\mathcal{J}_{(+)}(l_1, l_2) = \begin{cases} (\mathrm{I})\, l_1, & \text{if } l_2 - l_1 = 0, \\ (\mathrm{II})\, l_1 - \frac{1}{2}, & \text{if } l_2 - l_1 = 1 \\ (III)\, l_1 - \frac{1}{2} \pm \sum_{k=1}^{l_2-(l_1+1)} \frac{1}{2^{k+1}}, & \text{if } l_2 - l_1 > 1 \end{cases} \tag{44}$$

Here, the numbers l_1 and l_2 take values in the set $\{1,2,3,\ldots\}$. The negative sector is obtained as $\mathcal{J}_{(-)}(l_1,l_2) = -\mathcal{J}_{(+)}(l_1,l_2)$. This implies that from (I) and (II) one gets $\mathcal{J}_{(+)}(1,1) = 1$, $\mathcal{J}_{(+)}(1,2) = \frac{1}{2}$ and from (III) one obtains $\mathcal{J}_{(+)}(1,3) = \left\{\frac{1}{4}, \frac{3}{4}\right\}$, $\mathcal{J}_{(+)}(1,4) = \left\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right\}$ and so on. Since

$\mathcal{J}_{(-)}(l_1,l_2) = -\mathcal{J}_{(+)}(l_1,l_2)$ one also has $\mathcal{J}_{(-)}(1,1) = -1$, $\mathcal{J}_{(=)}(1,2) = -\frac{1}{2}$ and $\mathcal{J}_{(=)}(1,3) = \left\{-\frac{1}{4}, -\frac{3}{4}\right\}$, $\mathcal{J}_{(+)}(1,4) = \left\{-\frac{1}{8}, -\frac{3}{8}, -\frac{5}{8}, -\frac{7}{8}\right\}$ and so on.

How can be related the present formalism to surreal numbers? The first thing that we would like to highlight is that just as in (4 + 4)-world the duality symmetry is a key concept, the starting point in both the Conway and the Gonshor formulation is duality scenario. In the case of Conway formulation we see from (42) that a surreal numbers is defined in terms of dual sets S_L and S_R, left and right sets of *s*, respectively. While in the Gonshor definition of surreal number is a function *f* from initial segment of the ordinals into the dual set $\{+,-\}$. Since these two formulations are equivalents one conclude the key concept in both is duality. The second observation is that the zero-point "energy" in (10), namely $\varepsilon = \frac{l_P \omega}{2}$ remind us the spin structure of $\frac{1}{2}$-spin in the sense that

$s=\frac{\hbar}{2}$. However, theoretically there are many type of fermions, including $s=\frac{3\hbar}{2}$ (gravitino) and $s=\frac{5\hbar}{2}$. And these spin structures start looking as dyadic rational (43) arising in the surreal number algorithm. In fact, this kind of observation motivate to propose the dyadic rational $\frac{m}{2^n}$-spin as an alternative tool for quantum gravity and dark matter [21] [22] and [24]. Thus, it is tempted to propose for each coordinate in $(x,y,z,\xi)=(x^1,x^2,x^3,x^4)$ the equation

$$\frac{\mathcal{J}_{(+)}(l_1,l_2)l_P\omega}{2}=-\frac{l_P^2}{2}\frac{1}{\varphi_x}\frac{\mathrm{d}^2\varphi_x}{\mathrm{d}x^2}+\frac{\mathcal{J}_{(+)}^2(l_1,l_2)\omega^2x^2}{2}, \tag{45}$$

which generalize the zero-point "energy" for the harmonic oscillator (12). This equation can be rewritten as

$$\frac{l_P\tilde{\omega}}{2}=-\frac{l_P^2}{2}\frac{1}{\varphi_x}\frac{\mathrm{d}^2\varphi_x}{\mathrm{d}x^2}+\frac{\tilde{\omega}^2x^2}{2}, \tag{46}$$

with $\tilde{\omega}=\mathcal{J}_{(+)}(l_1,l_2)\omega=\frac{\mathcal{J}_{(+)}(l_1,l_2)}{l_0}$. Hence the general solution of (45) must be similar to (28), namely

$$\mathcal{G}=\frac{\tilde{\sigma}^2}{2\pi}\mathrm{e}^{-\frac{\tilde{\sigma}^2\mathcal{J}_{(+)}(l_1,l_2)}{2}\left[(x^1)^2+(x^2)^2+(x^3)^2+(x^4)^2\right]}, \tag{47}$$

with $\tilde{\sigma}^2=\frac{\tilde{\omega}}{l_P}$. This means that the Fourier transform of $\mathcal{G}$ must be given by

$$\mathcal{G}^*=\frac{2\pi}{\tilde{\sigma}^2}\mathrm{e}^{-\frac{1}{2\tilde{\sigma}^2}\left[(t^1)^2+(t^2)^2+(t^3)^2+(t^4)^2\right]}. \tag{48}$$

Observe that in this case the zero-point "energy" will be

$$\tilde{\varepsilon}(l_1,l_2)=l_P\tilde{\omega}=l_P\omega\mathcal{J}_{(+)}(l_1,l_2), \tag{49}$$

with $\mathcal{J}_{(+)}(l_1,l_2)$ given by (44). One observes that considering the point (II) in (44) the expression (49) exactly reproduces the usual eigenvalues of the energy of the harmonic oscillator, with $n=l_1-1$. We think that this rough derivation of (49) may motivate the subject for further work.

Let us conclude this work by summarizing our main results. First, by taking a duality transformation as a key symmetry we explore some aspects a cosmological model of four time and four space dimensions ((4 + 4)-dimensions). In particular, we show that, at the level of zero-point energy in a de Sitter space, a Fourier transform of a Gaussian distribution in four dimensions leads to a dual Gaussian distribution also in four dimensions, with duality transformation $\sigma\to\frac{1}{\sigma}$, in the standard deviation σ. Moreover, we show that as a consequence of such a duality in σ a duality of the cosmological constant Λ, associated with the Sitter space, can be obtained. Finally, since duality symmetry is a fundamental concept in both the oriented matroid theory as well as the surreal number theory we roughly explain how these mathematical theories can be connected to the present work.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Green, M.B., Schwarz, J.H. and Witten, E. (1987) Superstring Theory I and II. Cambridge University Press, Cambridge.

[2] Duff, M.J. (1996) *International Journal of Modern Physics A*, **11**, 5623-5641. https://doi.org/10.1142/S0217751X96002583

[3] Hull, C.M. (1998) *JHEP*, **11**, 17. https://doi.org/10.1088/1126-6708/1998/11/017

[4] Yan, M.-L. (2015) De Sitter Invariant Special Relativity. University of Science and Technology of China, Hefei.

[5] Licata, I., Chiatti, L. and Benedetto, E. (2017) De Sitter Projective Relativity. Springer Briefs in Physics, Springer, Berlin. https://doi.org/10.1007/978-3-319-52271-5

[6] Nieto, J.A. and Espinoza, M. (2017) *International Journal of Geometric Methods in Modern Physics*, **14**, Article ID: 1750014. https://doi.org/10.1142/S0219887817500141

[7] Nieto, J.A. and Madriz, E. (2019) *Physica Scripta*, **94**, Article ID: 115303. https://doi.org/10.1088/1402-4896/ab2d96

[8] Avila, G., Castillo, S.J. and Nieto, J.A. (2016) *Journal of Interdisciplinary Mathematics*, **19**, 955-975.

[9] Nieto, J.A. (1999) *Physics Letters A*, **262**, 274-281. https://doi.org/10.1016/S0375-9601(99)00702-1

[10] Tkach, V.I., Socorro, J., Rosales, J.J. and Nieto, J.A. (1999) *Physical Review D*, **60**, Article ID: 067503. https://doi.org/10.1103/PhysRevD.60.067503

[11] Nieto, J.A. (2004) *Advances in Theoretical and Mathematical Physics*, **8**, 177-188. https://doi.org/10.4310/ATMP.2004.v8.n1.a4

[12] Nieto, J.A. (2006) *Advances in Theoretical and Mathematical Physics*, **10**, 747-757. https://doi.org/10.4310/ATMP.2006.v10.n5.a5

[13] Nieto, J.A. (2004) *Journal of Mathematical Physics*, **45**, 285. https://doi.org/10.1063/1.1625416

[14] Nieto, J.A. (2014) *Nuclear Physics B*, **883**, 350-372. https://doi.org/10.1016/j.nuclphysb.2014.04.001

[15] Nieto, J.A. and Marn, M.C. (2000) *Journal of Mathematical Physics*, **41**, 7997. https://doi.org/10.1063/1.1319518

[16] Nieto, J.A. (2013) *Physics Letters B*, **718**, 1543-1547.

https://doi.org/10.1016/j.physletb.2012.12.034

[17] Nieto, J.A. (2010) *Physics Letters B*, **692**, 43-46. https://doi.org/10.1016/j.physletb.2010.07.010

[18] Conway, J.H. (1976) On Number and Games. London Mathematical Society Monographs. Academic Press, Cambridge.

[19] Knuth, D.E. (1974) Surreal Numbers: How Two Ex-Students Turned on to Pure Mathematics and Found Total Happiness: A Mathematical Novelette. Addison-Wesley Publishing Co., Boston.

[20] Gonshor, H. (1986) An Introduction to the Theory of Surreal Numbers. London Mathematical Society Lectures Notes Series, Vol. 110, Cambridge Univ. Press, Cambridge. https://doi.org/10.1017/CBO9780511629143

[21] Nieto, J.A. (2016) *Journal of Modern Physics*, **7**, 2164-2176. https://doi.org/10.4236/jmp.2016.715188

[22] Nieto, J.A. (2018) *Frontiers in Physics*, **6**, Article No. 106. https://doi.org/10.3389/fphy.2018.00106

[23] Avalos-Ramos, C., Felix-Algandar, J.A. and Nieto, J.A. (2020) *IOSR Journal of Mathematics*, **16**, 35.

[24] Aviles-Niebla, C., Nieto-Marn, P.A. and Nieto, J.A. (2020) *International Journal of Geometric Methods in Modern Physics*, **17**, Article ID: 2050180. https://doi.org/10.1142/S0219887820501807

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A New Approach: About the Appearance of "Dark Matter" Effects in the Process of Expansion of the Universe*

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Abstract

The paper considers cosmological objects belonging to fundamentally different classes that do not intersect with each other. Firstly, these are objects that make up a pure Hubble stream. Secondly, these are objects that have constant proper distances. These include planets, stars, and galaxies in gravitationally coupled systems. They all do not participate in the Hubble stream as independent objects. It is shown that the commoving reference system and proper reference system standardly used in cosmology change places with each other when switching from considering Hubble objects to "planets". The features of the evolution (more precisely, devolution, degradation, reverse development) of the latter were analyzed and it was found that the cosmological acceleration of all "planets", in contrast to Hubble objects, coincides in order of magnitude with the fundamental value of H^2R. As applied to the Pioneers anomaly, this approach allowed us to obtain the calculated value of cosmological acceleration, which coincides in order of magnitude with the observed value. It seems that this approach is applicable also to other local gravitationally coupled systems and allows us to explain the characteristic flattened shape of the orbital curves of stars and galaxies by the fact that the influence of the fundamental cosmological acceleration H^2R is added to the system's own gravitational field.

Keywords

Dark Matter, Expansion of the Universe, Anomalous Pioneer Effect

1. Introduction

The expansion of the universe is described by general relativity. General relativi-

* *In dedicaton to Olga Kreydik.*

ty in combination with the cosmological principle unambiguously leads to Friedmann's cosmological models, *i.e.* to space-time with the Friedman-Robertson-Walker (FRU) metric:

$$Ds^2 = -c^2dt^2 + R^2\left(t\right)\left\{d\chi^2 + Sk^2\left(\chi\right)\left[d\theta^2 + \sin^2\theta d\Phi^2\right]\right\}$$

Here χ is the commoving coordinate, $R(t)$ (sometimes R) is a scale factor with the dimension of distance, increasing with time in the expanding universe, and $Sk(\chi) = (sin\chi, \chi$ or $sh\chi)$, where χ depends on the used model of the universe (closed, flat, open). Along with the factor $R(t)$ and the commoving coordinate χ, the scale factor R_0, which does not depend on t, and the dimensionless scale factor $a = R(t_1)/R(t_2)$ are often used.

The proper distance D from the origin to the location of interested star (galaxy) is the product of the scale factor $R(t)$ (its value is usually taken equal to the size of the universe at the present time) by the commoving coordinate:

$$D = R\left(t\right)\chi \quad (1)$$

Obviously, $\chi = D/R(t)$ coincides with $a = R(t_1)/R(t_2)$ for $D = R(t_1)$ and $R(t) = R(t_2)$. The overall speed of a star, including escape velocity and peculiar velocity, is:

$$D' = R\left(t\right)'\chi + R\left(t\right)\chi' = v_{reg} + v_{pec} \quad (2)$$

Since in what follows we will be interested in the forces and accelerations arising in the process of the expansion of the universe, we will also write down the second derivative of the distance (further, instead of $R(t)$, we will use R):

$$D'' = \left(R''\chi + R'\chi'\right) + \left(R'\chi' + R\chi''\right) = \left(v_{reg}\right)' + \left(v_{pec}\right)' \quad (3)$$

The commoving frame of reference (hereinafter referred to as FR) must exactly accompany the selected objects (in cosmology participants in the Hubble flow), must be glued to them so that, as a result, for any proper motions, these objects remain motionless in such a FR. Therefore, in the commoving, or Lagrangian [1] FR $t-\chi$ (often more convenient $t - R_0\chi$), the world lines of galaxies, strictly obeying the Hubble law, are vertical straight lines $\chi = \chi_i$. The deviation of any world line from the vertical, indicates that the corresponding object has dropped out of the purely Hubble flow, *i.e.* about the presence of his peculiar speed.

A different picture is observed in proper, or Euler's [1] FR $t - D$, which is much more familiar to us. Here, the same Hubble objects with $\chi = \chi_i$ represent a bundle of world lines $D = R(t)\chi_i$, diverging from the origin. Here there is a dependence on time, and it is clear that the universe is expanding. The connection between the two cases is given by the law of motion (1), which expresses the proper, or Euler, coordinates D through the commoving, or Lagrangian, χ_i and time.

In a contracting universe, expression (1) for the proper distance will change to

$$D = \left(R_0 - R\right)\chi$$

leading to the appropriate transformation of the expressions for speed and acceleration.

2. Analysis of Cosmological Accelerations

And now let us represent in the same FR $t-D$ and FR $t-\chi$ the world lines of cosmological objects that do not take sovereign participation in the global expansion of the universe. These include objects that are part of local gravitationally coupled systems, such as the planets of the Solar System or galaxies that rotate in circular orbits in a cluster of galaxies and, therefore, have fixed proper distances D_i from the center of rotation. We again see the world lines in the form of vertical lines. But now these are world lines $D = D_i$ of "non-Hobbles" (planet-like objects) in FR $t-D$, but not world lines $\chi = \chi_i$ of Hubble objects in FR $t - \chi$, which are usually dealt with in cosmology. The verticality of the world lines is indisputable evidence that they are depicted in the commoving FR. Thus, in the case of vertical world lines, we are dealing with two fundamentally different classes of cosmological objects. First, these are objects with world lines $\chi = \chi_i$, which are full-fledged participants in the Hubble flow, forming the Hubble flow; and, secondly, these are objects with $D = D_i$ (it will be more convenient to use D_i instead of R_0 to denote time-independent distances in FR $t-D$ that do not independently participate in this flow. These classes are associated with different commoving FR. And not just different, but in a sense diametrically opposite, mutually complementary.

We emphasize again: for any objects with $D = D_i$, the commoving FR is by no means the FR $t-\chi$, as for Hubble objects, but the FR $t-D$ instead. Naturally, when moving from considering Hubble objects to analyzing the motion of local objects, *i.e.* when passing from FR $t-\chi$ to FR $t-D$ the Lagrangian and Euler descriptions trade places, and therefore now the law of motion is not $D = R(t)\chi_i$, as in (1) for Hubble objects, but vice versa:

$$\chi = D_i / R(t) \tag{4}$$

As it follows from (4), the proper distance χ for planet-like objects necessarily decreases with time t (as does the proper distance D for the above-mentioned narrowing universe).

The general expressions for the velocity χ' and acceleration χ'' of cosmological ojects in FR $t-\chi$ are easy to find by differentiation (4) or directly from (1)-(3):

$$\chi' = D'/R - (R'/R)\chi , \tag{5}$$

$$\chi'' = D''/R - (R''/R)\chi - 2(R'/R)\left[D'/R - (R'/R)\chi\right] \tag{6}$$

For planet-like objects (*i.e.* with $D' = 0$, $D'' = 0$) from (5)-(6) it is easy to obtain expressions for the velocity χ' and acceleration χ'' in FR $t-\chi$:

$$\chi' = -(R'/R)\chi = -H\chi \tag{5a}$$

and

$$\chi'' = -(R''/R)\chi + 2(R'/R)^2 \chi = H^2\chi \tag{6a}$$

The formula for the velocity χ' is interesting in that it is an analogue of the Hubble law, but in the space χ, not D, and with negative H. The latter means that the object under consideration does not expand, but, on the contrary, narrows in FR $t-\chi$. In order to describe the motion of such an object along the χ axis, it is convenient to use the distance $f = 1 - \chi$, measured along the same χ axis, but in the direction of motion, *i.e.* from $\chi = 1$ to $\chi = 0$. Then

$$\chi' = -Hf = -H(1-\chi) \tag{7}$$

The formula (6a) expresses the acceleration χ'' through the acceleration R'' and the speed R' of the expansion of the universe. Let's pay attention to the fact that the two terms of the final acceleration χ'' have not only opposite signs, but also completely different status. The real existence of the expansion of the universe with the speed R' does not raise the slightest doubt. And the sign of this term is correct, it is it that provides the necessary decrease in time in the modulus of negative velocity (5a). As for the first term in (6a): its existence, of course, cannot be considered firmly established, even despite the results of observations of Sn1a, since other explanations of these results are also possible.

For planets, in accordance to expression (3), it is true:

$$D'' = (R''\chi + R'\chi') + (R'\chi' + R\chi'') = (v_{reg})' + (v_{pec})' = 0$$

Therefore, according to (6), (7) for $R'' = 0$, we have:

$$\chi'' = 2(R'/R)^2 f = 2H^2(1-\chi) \tag{6b}$$

and, at $R'' = RH^2\chi$

$$\chi'' = (R'/R)^2 f = H^2(1-\chi) \tag{6c}$$

Thus, we finally come to the following conclusion: the existence of the acceleration R'' of the expansion of the universe is not a prerequisite for the occurrence of accelerations χ'' acting on objects moving in the FR $t-\chi$, and, moreover, does not have any fundamental effect (sign change) to the final result.

Now let us compare the cosmological accelerations of Hubble and planetary objects. As follows from expression (3), for Hubble objects

$$D'' = \chi''D = H^2 R\chi_i \tag{8}$$

But the authors of [2] assume that for all cosmological objects with an increase in χ, the acceleration increases, reaching a maximum of H^2R at $\chi = 1$. As a result, they obtained the theoretical value of the cosmological acceleration for the Pioneers, which is very far from the observed value.

But what is the difference in the behavior of Hubble and planetary objects? Of course, we are able to compare the behavior of objects only in the same FRs, specifically, their proper FRs, *i.e.* in FR $t-D$ for Hubble and in FR $t-\chi$ for planets. In this case, the main difference immediately becomes obvious from which all the others should follow. This, of course, is the difference in the signs of the velocities: unconditionally positive for the Hubble velocity D' (our Universe is expanding). And, as it turns out, negative value for the proper speed of the planets. What follows from this? That the proper distance D of any Hubble object can only increase over time, *i.e.* change from smallest to largest. That is why the value of the cosmological acceleration D'' in formula (7) for Hubble objects grows. In contrast, the proper distance χ can change over time only from larger to smaller. Thus, the projection χ of the representing point of any object with $D = D_i$ in the FR $t - \chi$ necessarily moves along the χ axis, and in a completely definite way in the direction from the point $\chi = 1$ (larger) to the point $\chi = 0$ (smaller). Therefore, formula (7) is inapplicable to the motion of all planet-like objects, such as stars in galaxies and galaxies in clusters. The fact is that the law of motion (1) and, consequently, the formula (7) obtained on its basis are valid only for the expanding universe, *i.e.* for a set of cosmological objects with their own distances increasing over time. Planetary objects do not belong to such. On the contrary, as follows from the law of motion (4), their own distances χ decrease with time. For them, the point with $\chi = 1$ is always the starting point of movement. In the light of the above, we have the only opportunity to measure the distance along the χ axis: in the direction from the point $\chi = 1$ to the point $\chi = 0$. Therefore, instead of (8), we must write:

$$D_i'' = H^2 Rf = H^2 R(1-\chi) = H^2 R(1 - D_i/R) \tag{9}$$

Here $f = 1 - \chi$ is the distance measured along the χ axis in the direction from $\chi = 1$ to $\chi = 0$.

Let us estimate what happens to the cosmological acceleration of planet-like objects in the transition from (8) to (9), when the factor χ in (8) is replaced by the factor $f = (1 - \chi)$ in (9). Since the factor χ is usually extremely small, D_i'' in this case increases by many orders of magnitude. Thus, in [2], using formula (8), the cosmological acceleration of the Pioneers was calculated (at that moment, the Pioneers' proper distance was 40 AU), equal to 2×10^{-23} ms^{-2}, which is 13 orders of magnitude different from the observed value 2×10^{-10} ms^{-2}. The use of formula (9) instead of (8) gives a calculated value that coincides with the observed one. And for the largest known gravitationally bound object (a cluster of galaxies with a diameter of 30 million light years) we have $f = 1 - \chi = 1 - 10^{-3}$. Thus, the value $f = (1 - \chi)$ that determines the value of D_i'' in (9) is practically equal to 1 for all existing (or at least known to us) gravitationally coupled systems in the visible universe, from planets ($\chi = 10^{-15}$) and to the largest galaxy clusters ($\chi = 10^{-3}$). Therefore, practically for all such objects, $D_i'' = H^2 R$.

Well, it seems that we got a consistent picture, in which the final acceleration χ'' of the planet (necessarily positive in the expanding universe for any values of R'') causes a decrease in the modulus of its negative velocity from (5a) over time.

This acceleration affects the components of all local gravitationally coupled systems. Such as Pioneers, planets, stars in galaxies and galaxies in clusters of galaxies. It turns out that this acceleration is practically independent of χ and is equal to the fundamental value $RH^2 = C^2/R$.

3. Conclusions

It seems that the results obtained are quite enough to explain both the observed anomalous Pioneer effect and the flattened shape of the rotation curves of galaxies. Moreover, a persistent feeling arises that within the framework of the outlined approach, over time, it will be possible to explain most of the effects and phenomena currently associated with the hypothesis of dark matter.

Indeed, what is actually happening? As a result of the local process of stopping the expansion of space, in addition to the “seed” gravitational field, a field of cosmological inertial fields arises. The two fields are summed up, forming a more intense final field in a vast area of space, which, due to the principle of equivalence, is perceived as purely gravitational. Thus, around each center of gravitational attraction (galaxies, clusters of galaxies), in addition to its own gravitational field, a much more extended and much more intense final field is formed at large distances. It is very likely that it is the totality of these fields that is perceived as the gravitational field of the totality of visible matter and dark matter.

The existence of the cosmological inertial field in the expanding universe and its effect on gravitationally bound systems can explain not only the shape of the observed rotation curves, but also numerous other phenomena today explained by the existence of dark matter [3]; such as the results of gravitational microlensing, the formation of galaxies in an obscenely short period of time, or even the well-known picture of the collision of two clusters of galaxies MACSJ0025, in which these clusters, together with "dark matter" (and in our case, with cosmological inertial fields) freely fly through each other, while clouds of intergalactic gas, together with their cosmological inertial fields, are decelerated by collisions. The fact is that particles of hot intergalactic gas moving chaotically, especially in the process of cloud collisions, can have both positive and negative acceleration. Therefore, the total acceleration of all particles, *i.e.* the field of inertia of an intergalactic gas cloud can turn out to be arbitrarily small, in contrast to the fields of inertia of galaxies in clusters that have flown through each other. Thus, inertial fields can have additional degrees of freedom and thus provide a significant variety in the behavior of various galaxies and their clusters.

It seems that it is the cosmological inertial field that we are discussing (can we call it the inertial field of the universe, the Mach field?). That may turn out to be the physical embodiment of “dark matter”.

And in conclusion, let us note that we obtained all the above results exclusively within the framework of the general theory of relativity, without invoking any new entities or hypotheses, such as hitherto unknown particles that make up dark matter, or a modification of Newton’s laws. It is the application of general relativity to two classes of cosmological objects (Hubble and planet-like) that made it possible to calculate the cosmological accelerations of planet-like objects in this work.

P.S.: In this work, we were interested in cosmological accelerations only for vanishingly small values of χ, which is quite sufficient when considering local systems. But what happens across the entire χ range? It turns out in [4], which while at small χ the cosmological attraction force considered in this article prevails, at large χ the repulsive force becomes the main one. So, for a particle with a peculiar velocity, which has the same modulus as in (5a), but opposite sign (direction) $\chi'' = -4H^2\chi$.

So, what happens? Until now, by default, it is assumed that the same cosmological acceleration $\chi'' = H^2\chi$ acts on each particle, regardless of the direction (sign) of its peculiar velocity. Summing up the result for all particles, we got the final positive acceleration, which could explain the observed cosmological acceleration of the global expansion of the universe. But now the picture is getting more complicated. A braking force acts on the para particles (the v_{reg} and v_{pec} directions are antiparallel), which slows down their dispersal, *i.e.* slowing down the expansion of the universe. And on the ortho particles (the directions v_{reg} and v_{pec} are parallel) twice as intense accelerating field, "accelerating" the expansion of the universe. But besides, as was shown in [4], different factors are involved in the expressions for the ortho (χ) and pair ($-\chi$) accelerations. Ortho-force is proportional to the factor $f = (1 - \chi)$, and para-force is proportional to the factor χ. Therefore, at small χ we have a net effect that slows down the expansion of the universe, and at large χ an accelerating one. Of course, the above reasoning is very arbitrary. The point is that the decelerating and accelerating forces are applied to different objects that are in no way connected with each other, except for the general location χ. But in this case, the center of gravity of the cloud (set) of such particles will shift in a completely definite (by the value of χ) direction. It is believed [5] that in the real universe, the acceleration of expansion "turned on" after several billion years after the Big Bang.

Summing up the orto/para effects, we obtain for two particles, the final result of the type $\chi_1'' + \chi_2'' = 2H^2 f - 4H^2\chi$, *i.e.* the sign (direction) of the final acceleration now really depends on the value of χ. So, at $\chi = 0.1$ we get $f = 0.9$ and the final "braking acceleration" $\chi_1'' + \chi_2'' = 1.4H^2\chi$. And at $\chi = 0.9$ we have $\chi_1'' + \chi_2'' = -3.4H^2\chi$, *i.e.* real acceleration of expansion. The braking and accelerating forces balance each other at $\chi = 0.33$, *i.e.* at $f = 0.67$. This corresponds to $13.8 \times 0.67 = 9.2$ billion since the Big Bang, which is in good agreement with the data from [5].

It is also worth noting that cosmological acceleration in the universe acts as a typical positive feedback loop. In the sense, the system pushes each particle in the direction of its motion (of course, in FR $t - \chi$).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Wallander, S.V. (1978) Lectures on Hydromechanics. Leningrad State University Publishing House, Leningrad.

[2] Carrera, M. and Giulini, D. (2010) *Reviews of Modern Physics*, **82**, 169. https://www.ugr.es/~zink/cosmo/GiuliniRMP.pdf

https://doi.org/10.1103/RevModPhys.82.169

[3] Zasov, A.V., Saburova, A.S., Khoperskov, A.V. and Khoperskov, S.A. (2017) *UFN*, **187**, 3-44. https://doi.org/10.3367/UFNr.2016.03.037751

[4] Sitnikov, L.S. (2020) On Cosmological Inertial Fields in the Expanding Universe. https://drive.google.com/open?id=1MstLNxIuqcP34TMLiWnqQKgjz0kShUUC

[5] Ask Ethan # 52: How Long Has It Been Since the Expansion of the Universe Began to Accelerate? https://habr.com/ru/post/395005/

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Main Problems of the Theoretical Physics and Artifacts of Local Physics

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Abstract

Shortcomings of the Boltzmann physical kinetics are considered. Boltzmann equation is only plausible equation. The cosequences originated from this fact are considered in the different fields of theoretical physics from the point of view of nonlocal physics. Namely: main principles of nonlocal physics; generalized hydrodynamic equations; magnetic field evolution in the superconductor of the second type; Hubble expansion; special theory of relativity; the problem of the interaction of matter (M) with physical vacuum (PV) is considered including the PV—M energy exchange. Application nonlocal physics to the problem of the dark matter existence—dark matter does not exist, analytical investigation.

Keywords

Nonlocal Physics, Interaction of Matter With Physical Vacuum, Antimatter Evolution After Big Bang, Dark Matter Problem, Transport Processes in Physical Vacuum

1. Introduction

By the end of the twentieth century it was found that 96% of matter and energy in the Universe is of unknown origin. The terminology “dark matter” and “dark” energy was introduced in the scientific language. But even this would not be such a depressing fact, if it were not for the belief that this dark matter and dark energy are not diagnosed, and only the indirect effects on space objects can judge the existence of dark matter and dark energy. We even had to face the opinion of religious-minded people who claimed that four percent is all that the Creator left for the study to man. In practice, science has faced the most serious challenge since the fundamental monograph of I. Newton “Mathematical principles of natural philosophy” was published in 1687. On the Internet you can find a reaction to the current situation, bordering on despair. I initially assumed that the nonlocal physics developed by me would lead to very significant changes in the description of natural phenomena. That is to say, 4% of new information compared to 96% of the classical theory, but not Vice versa! What effect as a result! It is impossible to believe that more than three hundred years after Newton, world science came to such a disappointing result. Then the question arises—where is the error? The answer to this question is given by nonlocal physics. In other words, the origin of all difficulties lies in the total oversimplification, which follows from the fundamental shortcomings of the local statistical theory of transport processes.

In fact, the crisis has evolved over the years. The first heavy blow in this way the unresolved physical problems, was the suicide of the great physicist L. Boltzmann; according to the belief of many (including Acad. M. A. Leontovich) connected with unsolved problems and criticism of Boltzmann kinetic theory, the first of the following list of unsolved problems of fundamental physics (local physical kinetics of dissipative processes):

1) Kinetic theory of entropy and the problem of "primary" fluctuation.

2) A strict theory of turbulence.

3) Quantum non-relativistic and relativistic hydrodynamics, the theory of charge separation in atomic structures and nuclei. High temperature superconductivity.

4) The theory of ball lightning.

5) The theory of dark matter.

6) The theory of dark energy. Hubble expansion of the Universe.

7) The destiny of antimatter after the Big Bang.

8) Unified theory of dissipative structures, from atomic structure to cosmology.

These problems can not be solved within the framework of local physics, but find their natural solution as a special case of solving the problem 8—the creation of a unified theory of dissipative structures—from atomic structures to cosmology, [1]-[8].

I repeat—the origin of difficulties in theoretical physics consists in the total Oversimplification following from the principles of local physics and reflects the general shortcomings of the local kinetic transport theory based on the Boltzmann kinetic theory.

2. Boltzmann Equation is Only a Plausible Equation

The Boltzmann equation [1] works from the molecular to cosmological level, but has an amazing origin and equally obvious drawbacks. Regarding the origin, the equation is based on Newtonian mechanics, which contains in the equation the second derivative by time. But the Boltzmann kinetic equation has only the first temporal derivative. This fact leads to the irreversibility of the processes; hence the irreversible nature of the evolution of H-functions and the inevitable question: where does the initial fluctuation appear from, if Boltzmann kinetic theory does not contain fluctuations in principle? You do not even need (initially) to write equations. Indeed, Boltzmann physical kinetics is based on a reduced description of dissipative processes and the principle of local thermodynamic equilibrium (LTE).

It is assumed that the distribution function (DF) is not changed within a physically infinitely small volume (say, a $PhSV_1$) that contains, however, enough particles for the introduction of macroscopic parameters (such as temperature and concentration), which are constant within PhSV. But PhSV is an *open* thermodynamic system that responds to the environment only after its interaction with foreign particles, penetrated from a neighboring PhSV; in other words, after a time τ of order of the average time *between* collisions. Then in the simplest case of the gas objects nonlocal parameter τ can be considered as a corresponding relaxation time.

Transport processes in open dissipative systems are considered in physical kinetics. Therefore, the kinetic description is inevitably related to the system diagnostics. Such an element of diagnostics in the case of theoretical description in physical kinetics is the concept of the physically infinitely small volume (PhSV). The correlation between theoretical description and system diagnostics is well-known in physics. Suffice it to recall the part played by test charge in electrostatics or by test circuit in the physics of magnetic phenomena.

The traditional definition of PhSV contains the statement to the effect that the PhSV contains a sufficient number of particles for introducing a statistical description; however, at the same time, the PhSV is much smaller than the volume V of the physical system under consideration; in a first approximation, this leads to local approach in investigating the transport processes.

It is assumed in classical hydrodynamics that local thermodynamic equilibrium is first established within the PhSV, and only after that the transition occurs to global thermodynamic equilibrium if it is at all possible for the system under study. Let us consider the hydrodynamic description in more detail from this point of view. Assume that we have two neighboring physically infinitely small volumes $PhSV_1$ and $PhSV_2$ in a non-equilibrium system. The one-particle distribution function (DF) $f_{sm,1}(\mathbf{r}_1,\mathbf{v},t)$ corresponds to the volume $PhSV_1$, and the function $f_{sm,2}(\mathbf{r}_2,\mathbf{v},t)$—to the volume $PhSV_2$. It is assumed in a first approximation that $f_{sm,1}(\mathbf{r}_1,\mathbf{v},t)$ does not vary within $PhSV_1$, same as $f_{sm,2}(\mathbf{r}_2,\mathbf{v},t)$ does not vary within the neighboring volume $PhSV_2$. It is this assumption of locality that is implicitly contained in the Boltzmann equation (BE) [2]-[8]. However, the assumption is too crude. Indeed, a particle on the boundary between two volumes, which experienced the last collision in $PhSV_1$ and moves toward $PhSV_2$, introduces information about the $f_{sm,1}(\mathbf{r}_1,\mathbf{v},t)$ into the neighboring volume $PhSV_2$. Similarly, a particle on the boundary between two volumes, which experienced the last collision in $PhSV_2$ and moves toward $PhSV_1$, introduces information about the DF $f_{sm,2}(\mathbf{r}_2,\mathbf{v},t)$ into the neighboring volume $PhSV_1$. The relaxation over translational degrees of freedom of particles of like masses occurs during several collisions. As a result, "Knudsen layers" are formed on the boundary between neighboring physically infinitely small volumes, the characteristic dimension of which is of the order of path length. Therefore, a correction must be introduced into the DF in the PhSV, which is proportional to the mean time between collisions and to the substantive derivative of the DF.

Let a particle of finite radius be characterized as before by the position $\mathbf{r}$ at the instant of time t of its center of mass moving at velocity $\mathbf{v}$. Then, the situation is possible where, at some instant of time t, the particle is located on the interface between two volumes. In so doing, the lead effect is possible (say, for $PhSV_2$), when the center of mass of particle moving to the neighboring volume $PhSV_2$ is still in $PhSV_1$. However, the delay effect takes place as well, when the center of mass of particle moving to the neighboring volume (say, $PhSV_2$) is already located in $PhSV_2$ but a part of the particle still belongs to $PhSV_1$.

Moreover, even the point-like particles (*starting after the last collision near the boundary between two mentioned volumes*) *can change the distribution functions in the neighboring volume. The adjusting of the particles dynamic characteristics for translational degrees of freedom takes several collisions. As result, we have in the definite sense* "*the Knudsen layer*" *between these volumes. This fact unavoidably leads to fluctuations in mass and hence in other hydrodynamic quantities. Existence of such* "*Knudsen layers*" *is not connected with the choice of space nets and fully defined by the reduced description for ensemble of particles of finite diameters in the conceptual frame of open physically small volumes, therefore with the chosen method of measurement.*

This entire complex of effects defines non-local effects in space and time. The corresponding situation is typical for the theoretical physics—we could remind about the role of probe charge in electrostatics or probe circuit in the physics of magnetic effects.

The physically infinitely small volume (PhSV) is an *open* thermodynamic system *for any division of macroscopic system by a set of PhSVs.*

Let us give some explanations on the qualitative level of investigation. Suppose that the distribution function (DF) f corresponds to $PhSV_1$ and DF $f-\Delta f$ is connected with $PhSV_2$ for Boltzmann particles. In the boundary area in the first approximation, fluctuations will be proportional to the mean free path (or, equivalently, to the mean time between the particle collisions). Then for PhSV the correction for DF should be introduced as

$$f^a = f - \tau\, Df/Dt \tag{2.1}$$

in the left hand side of classical KE describing the translation of DF in phase space. As the result

$$Df^a/Dt = J^B, \tag{2.2}$$

where J^B is the Boltzmann local collision integral. Important to notice that it is only qualitative explanation of the generalized BE derivation obtained earlier (see for example [2]-[8]) by different strict methods from the BBGKY chain of kinetic equations. The structure of the KE_f is generally as follows

$$\frac{Df}{Dt} = J^B + J^{nl}, \tag{2.3}$$

where J^{nl} is the non-local integral term incorporating in particular the time delay effect. The generalized Boltzmann physical kinetics, in essence, involves a local approximation

$$J^{nl} = \frac{D}{Dt}\left(\tau \frac{Df}{Dt}\right) \tag{2.4}$$

for the second collision integral, here τ being proportional to the mean time *between* the particle collisions. All of the known methods of deriving kinetic equation relative to one-particle DF lead to approximation (2.4), including the method of many scales, the method of correlation functions, and the iteration method. We can draw here an analogy with the Bhatnagar-Gross-Krook (BGK) approximation for J^B,

$$J^B = \frac{f_0 - f}{\tau}, \tag{2.5}$$

which popularity as a means to represent the Boltzmann collision integral is due to the huge simplifications it offers. In other words – the local Boltzmann collision integral admits approximation via the BGK algebraic expression, but more complicated non-local integral can be expressed as differential form (2.4). The ratio of the second to the first term on the right-hand side of Equation (2.3) is given to an order of magnitude as $J^{nl}/J^B \approx O\left(\mathrm{Kn}^2\right)$ and at large Knudsen numbers (defining as ratio of mean free path of particles to the character hydrodynamic length) these terms become of the same order of magnitude. It would seem that at small Knudsen numbers answering to hydrodynamic description the contribution from the second term on the right-hand side of Equation (2.3) is negligible.

This is not the case, however. When one goes over to the hydrodynamic approximation (by multiplying the kinetic equation by collision invariants and then integrating over velocities), the Boltzmann integral part vanishes, and the second term on the right-hand side of Equation (2.3) gives a single-order contribution in the generalized Navier—Stokes description. Mathematically, we cannot neglect a term with a small parameter in front of the higher derivative. Physically, the appearing additional terms are due to viscosity and they correspond to the small-scale Kolmogorov turbulence [2]-[8].

The integral term J^{nl} turns out to be important both at small and large Knudsen numbers in the theory of transport processes.

Thus, $\tau\, Df/Dt$ is the distribution function fluctuation, and writing Equation (2.2) without taking into account Equation (2.1) makes the BE non-closed. From viewpoint of the fluctuation theory, Boltzmann employed the simplest possible closure procedure $f^a = f$ in (2.2).

Then, the additional GBE terms (as compared to the BE) are significant for any Kn, and the order of magnitude of the difference between the BE and GBE solutions is impossible to tell beforehand. For GBE the generalized H-theorem is proven [2]-[8].

Boltzmann equation (BE) fully ignores non-local effects and contains only the local collision integral J^B. The foregoing nonlocal effects are insignificant only in equilibrium systems, where the kinetic approach changes to methods of statistical mechanics.

Results [2]-[8]:

1) Kinetic theory must be non-local in principle.

2) The effect is of the order of the Knudsen number; since nonlocal effects are proportional to the Knudsen number; then we have an opportunity of the description of nonlocal effects in the framework of the two scale approximation.

3) The effect is due to reduced description and not associated with a specific division of a physical system by a net of PhSV.

4) Accurate derivation of the kinetic equation (KE) relative to one-particle DF leads to corrections of the order of the Knudsen number even before the decoupling of the Bogolyubov hierarchy.

5) This means that in the Boltzmann equation the terms of the order of the Knudsen number are lost; these terms of the order of the Knudsen number, important at large and at small Knudsen numbers.

6) The Boltzmann equation does not belong even to the class of minimal models as being the only plausible equation.

7) The Boltzmann equation in this sense is the wrong equation.

It is clear that this is a revolution in the theory of dissipative processes, in particular in hydrodynamics. In the hydrodynamic Navier–Stokes equation, which is a direct consequence of the Boltzmann equation (BE), the terms of the order proportional to a viscosity are partly lost. It leads to the problem of turbulence and the problems of existence and uniqueness of solutions of the Navier–Stokes equations.

The rigorous approach to derivation of kinetic equation relative to one-particle DF f $\left(KE_{f_1}\right)$ is based on employing the hierarchy of Bogolyubov equations [2]-[8]. Generalized Boltzmann physical kinetics brings the strict approximation of non-local effects in space and time and after transfer to the local approximation leads to parameter τ. The appearance of the nonlocal τ parameter is consistent with the Heisenberg uncertainty relation. In the general case, the parameter τ is the non-locality parameter; in quantum hydrodynamics, its magnitude is correlated with the "time-energy" uncertainty relation [4]. But in principle generalized kinetic nonlocal equation (and therefore generalized hydrodynamic equations (GHE)) needn't in using of the "time-energy" uncertainty relation for estimation of the value of the non-locality parameter τ. Moreover the "time-energy" uncertainty relation does not produce the exact relations and from position of non-local physics is only the simplest estimation of the non-local effects.

Really, let us consider two neighboring physically infinitely small volumes $PhSV_1$ and $PhSV_2$ in a non-equilibrium system. Obviously the time τ should tends to diminish with increasing of the velocities u of particles invading in the nearest neighboring physically infinitely small volume ($PhSV_1$ or $PhSV_2$):

$$\tau = H_\tau / u^n \ . \tag{2.6}$$

But the value τ cannot depend on the velocity direction and naturally to tie τ with the particle kinetic energy, then

$$\tau = H_\tau / mu^2 \ , \tag{2.7}$$

where H_τ is a coefficient of proportionality, which reflects the state of physical system. In the simplest case H_τ is equal to Plank constant $\hbar$ and relation (2.7) became compatible with the Heisenberg relation. The non-locality parameter τ plays the same role as the transport coefficients in local hydrodynamics. The different models can be introduced for the τ definition, but the corresponding results not much different like in local kinetic theory for different models of the particles interaction.

It is known that Ehrenfest adiabatic theorem is one of the most important and widely studied theorems in Schrödinger quantum mechanics. It states that if we have a slowly changing Hamiltonian that depends on time, and the system is prepared in one of the instantaneous eigenstates of the Hamiltonian then the state of the system at any time is given by an the instantaneous eigenfunction of the Hamiltonian up to multiplicative phase factors. The adiabatic theory can be naturally incorporated in generalized quantum hydrodynamics based on local approximations of non-local terms. The adiabatic theorem and consequences of this theory deliver the general quantization conditions for non-local quantum hydrodynamics, [4].

3. Shortcomings of the Schrödinger Quantum Mechanics

We now turn to the logic of the development of the non-local theory:

A) In 1926 Madelung published a brilliant article [9] in which he transformed the quantum postulate (Schrödinger equation) in hydrodynamics. In other words, the evolution of a *single* bound electron was possible to interpret as some effective *flow*.

B) In 1964 John Stewart Bell [10] found that local statistical theory of dissipative processes is incorrect in principle.

C) In 2007 I found that the Schrödinger equation and hydrodynamic Madelung’s form are a deep particular case of nonlocal kinetic equations (see for example [11] [12]) as a result of the transition to the local limit of non-local equations.

This means that generalized physical kinetics (as created earlier by me) has been extended to quantum physics in the form of quantum hydrodynamics. I would even say such emotional words—the biggest secret of the Schrödinger equation (SE) is a strange thing—why, in fact, it generally works? Honestly, it starts to work when we go beyond the postulate that it is. Here just note again that:

1) SE is not able to give a self-consistent description of the nucleus - electron shell.

2) SE does not lead to an independent analogue of the hydrodynamic energy equation.

3) SE is not a dissipative equation and therefore cannot be applied to the description of dissipative processes in nanotechnology.

Generalized hydrodynamic equations (GHE) should contain Schrödinger equation (SE) as a deep special case. This affirmation was proved in articles [11] [12]. In other words, we formulated in explicit form all assumptions (all steps) that should be implemented to obtain SE from GHE.

So, we can state that SE is a deep special case of generalized hydrodynamic equations. This means that a new quantum mechanics of dissipative processes has been created.

The Boltzmann equation essentially "does not work" at the distances of the order of the radius of interaction of particles and, therefore, can not be effectively used in the theoretical study of nanotechnology even in the framework of "plausible" models.

It is established that the theory of transport processes (including quantum mechanics) can be presented within the framework of the universal theory (unified theory of dissipative systems) based on nonlocal physical description. It is shown, in particular, that the equations of nonlocal physics lead to the appearance of solitons, which supports the Schrödinger opinion, who interpreted quantum mechanics from the point of view of the existence of waves of matter. The Schrödinger equation is not dissipative. Therefore, the generalized quantum hydrodynamics is a tool for solving problems in the theory of dissipative nano-systems.

4. Modification of Maxwell Equations

Notice that the application of the above principles also leads to the modification of the system of Maxwell equations. While the traditional formulation of this system does not involve the continuity equation, its derivation explicitly employs the equation

$$\frac{\partial \rho^a}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{j}^a = 0, \tag{4.1}$$

where ρ^a is the charge per unit volume, and $\mathbf{j}^a$ a the current density, both calculated without accounting for the fluctuations. As a result, the system of Maxwell equations written in the standard notation, namely

$$\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{B} = 0, \quad \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{D} = \rho^a, \quad \frac{\partial}{\partial \mathbf{r}} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \frac{\partial}{\partial \mathbf{r}} \times \mathbf{H} = \mathbf{j}^a + \frac{\partial \mathbf{D}}{\partial t} \tag{4.2}$$

contains

$$\rho^a = \rho - \rho^{fl}, \quad \mathbf{j}^a = \mathbf{j} - \mathbf{j}^{fl}. \tag{4.3}$$

The ρ^{fl} and $\mathbf{j}^{fl}$ fluctuations calculated using the GBE are given, for example, in Ref. [7], see also [4]. In rarefied media both effects lead to Johnson's flicker noise observed in 1925 for the first time by J.B. Johnson by the measurement of current fluctuations of thermo-electron emission.

As an example of applying of the generalized Maxwell equations we consider the distribution of the magnetic field in a superconductor Type II from position of nonlocal physics. Usually superconductors are categorized as Type-I or Type-II. Type-I superconductors support only Meissner and normal states, while Type-II superconductors form magnetic vortices in sufficiently strong applied magnetic fields. When the Type I superconductor is placed in the magnetic field, it suddenly or easily looses its superconductivity at critical magnetic field H_c. After H_c, the Type I superconductor will become conductor. Example of Type I superconductors: Aluminum ($H_c = 0.0105$ Tesla), Zinc ($H_c = 0.0054$ Tesla).

Type II superconductors are those superconductors which loose their superconductivity gradually but not easily or abruptly when placed in the external magnetic field. Type II superconductors start to loose their superconductivity at lower critical magnetic field H_{c1} and completely loose their superconductivity at upper critical magnetic field H_{c2}. The state between the lower critical magnetic field H_{c1} and upper critical magnetic field H_{c2} is known as vortex state or intermediate state. After magnetic field H_{c2}, the Type II superconductor will become conductor. Except for the elements vanadium, technetium and niobium, the Type II category of superconductors is comprised of metallic compounds and alloys. This new category of superconductors was identified by L.V. Shubnikov [13] at the Kharkov Institute of Science and Technology in the Ukraine in 1936 when he found two distinct critical magnetic fields (known now as H_{c1} and H_{c2}) in PbTl2. Since a Type II will allow some penetration by an external magnetic field into its surface, this creates some rather novel mesoscopic phenomena like superconducting filaments, stripes or flux-lattice vortices.

The superconducting current flows in a layer with a thickness equal to the penetration depth, which makes it pointless to make large-diameter wires from a superconductor. Actually used for passing through a current-carrying superconducting cable consists of many thin superconducting filaments in a matrix. This design makes it possible to use the superconductor cross-section as efficiently as possible, provides sufficiently high mechanical properties, and provides shunting of superconductor sections in case of loss superconducting properties (for example, when the critical field is accidentally exceeded). The typical superconducting "hair" of such a cable has a diameter of several microns.

The Abrikosov vortex is a vortex of superconducting current (supercurrent) that circulates around a normal (non-superconducting) core (the vortex filament), inducing a magnetic field with a magnetic flux equivalent to the magnetic flux quantum. It was discovered by the physicist A. A. Abrikosov in 1957. In his work "On the magnetic properties of superconductors of the second group" [14], it was theoretically shown that the penetration of a magnetic field into a Type II superconductor occurs in the form of quantized vortex filaments. Each such thread (vortex) has a normal (non-superconducting) core with a radius of the order of the superconductor coherence length.

From position of the BCS theory around this normal cylinder, in a region with a radius of the order of the depth of penetration of the magnetic field, a vortical undamped current of Cooper pairs (supercurrent) flows, oriented so that the magnetic field (created by it) is directed along the normal core; that is, coincides with the direction of the external magnetic field. In this case, each vortex carries one quantum of the magnetic flow and

$$B(r) \sim \sqrt{\frac{\lambda}{r}} \mathrm{e}^{-r/\lambda}, \tag{4.4}$$

where λ is the London depth. But the Abrikosov solution [14] is, strictly speaking, incorrect. In the ideal case, which is considered in his work, there may be a state different from the predicted one. In order to honestly describe the Abrikosov state, we need to take into account fluctuations and consider a superconductor with finite dimensions. This is quite a difficult task which needs applying of the nonlocal physics methods. Abrikosov considered the ideal case and neglected fluctuations. The solution turned out to be correct in the sense that a vortex state is actually observed in real superconductors (see also [15]-[21]).

Let us consider the magnetic field distribution in the "hair' from the position of non-local physics. The filament is considered as the long cylinder which radius is r_0. Let us use equation (see Equation (10.22) in [22]) written in the form

$$\left[1+2\frac{\tau}{\tau_r}\right]\frac{\partial^2}{\partial t^2}\frac{\partial \mathbf{B}}{\partial t}+\frac{1}{\tau_r}\frac{\partial}{\partial t}\frac{\partial \mathbf{B}}{\partial t}=\frac{\tau}{\tau_r}v_\phi^2\frac{\partial}{\partial t}\Delta\mathbf{B}-v_\phi^2\frac{\partial}{\partial t}\left\{\frac{\partial}{\partial \mathbf{r}}\times\left[\frac{\partial}{\partial \mathbf{r}}\times\mathbf{B}\right]\right\}, \tag{4.5}$$

where τ_r is relaxation time, v_ϕ is phase velocity. Let us transform (4.5)

$$\left[1+2\frac{\tau}{\tau_r}\right]\frac{\partial^2}{\partial t^2}\frac{\partial \mathbf{B}}{\partial t}+\frac{1}{\tau_r}\frac{\partial}{\partial t}\frac{\partial \mathbf{B}}{\partial t}=\frac{\tau}{\tau_r}v_\phi^2\frac{\partial}{\partial t}\Delta\mathbf{B}+v_\phi^2\frac{\partial}{\partial t}\Delta\mathbf{B} \tag{4.6}$$

or

$$\left[1+2\frac{\tau}{\tau_r}\right]\frac{\partial^2}{\partial t^2}\frac{\partial \mathbf{B}}{\partial t}+\frac{1}{\tau_r}\frac{\partial}{\partial t}\frac{\partial \mathbf{B}}{\partial t}=\left[1+\frac{\tau}{\tau_r}\right]v_\phi^2\frac{\partial}{\partial t}\Delta\mathbf{B} \tag{4.7}$$

Let us introduce the scales

$$t \leftrightarrow \tau_r,\quad r \leftrightarrow r_0 = v_\phi \tau_r,\quad B \leftrightarrow B_0 \tag{4.8}$$

Then

$$\left[1+2\frac{\tau}{\tau_r}\right]\frac{1}{\tau_r^3}\frac{\partial^2}{\partial \tilde{t}^2}\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}}+\frac{1}{\tau_r^3}\frac{\partial}{\partial \tilde{t}}\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}}=\left[1+\frac{\tau}{\tau_r}\right]v_\phi^2\frac{\partial}{\partial t}\Delta\tilde{\mathbf{B}} \tag{4.9}$$

or

$$\left[1+2\frac{\tau}{\tau_r}\right]\frac{\partial^2}{\partial \tilde{t}^2}\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}}+\frac{\partial}{\partial \tilde{t}}\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}}=\left[1+\frac{\tau}{\tau_r}\right]v_\phi^2\tau_r^3\frac{\partial}{\partial t}\Delta\tilde{\mathbf{B}} \tag{4.10}$$

or

$$\left[1+2\frac{\tau}{\tau_r}\right]\frac{\partial^2}{\partial\tilde{t}^2}\frac{\partial\tilde{\mathbf{B}}}{\partial\tilde{t}}+\frac{\partial}{\partial\tilde{t}}\frac{\partial\tilde{\mathbf{B}}}{\partial\tilde{t}}=\left[1+\frac{\tau}{\tau_r}\right]\tau_r\frac{\partial}{\partial t}\tilde{\Delta}\tilde{\mathbf{B}} \tag{4.11}$$

or

$$\left[1+2\frac{\tau}{\tau_r}\right]\frac{\partial^2}{\partial\tilde{t}^2}\frac{\partial\tilde{\mathbf{B}}}{\partial\tilde{t}}+\frac{\partial}{\partial\tilde{t}}\frac{\partial\tilde{\mathbf{B}}}{\partial\tilde{t}}=\left[1+\frac{\tau}{\tau_r}\right]\frac{\partial}{\partial\tilde{t}}\tilde{\Delta}\tilde{\mathbf{B}} \tag{4.12}$$

We use spherical coordinate system. It is naturally supposing that in superconducting regime $\mathbf{B}=\mathbf{B}(r)$. Then after integration (4.12) on time we obtain

$$\left[1+2\frac{\tau}{\tau_r}\right]\frac{\partial^2\tilde{B}}{\partial\tilde{t}^2}+\frac{\partial\tilde{B}}{\partial\tilde{t}}=\left[1+\frac{\tau}{\tau_r}\right]\tilde{\Delta}\tilde{B}+\tilde{f}(\tilde{r}), \tag{4.13}$$

where $\tilde{f}(\tilde{r})$ is a function of $\tilde{r}$. The specific type of this function is determined by the task conditions. If $\tau\ll\tau_r$ we have

$$\frac{\partial^2\tilde{B}}{\partial\tilde{t}^2}+\frac{\partial\tilde{B}}{\partial\tilde{t}}=\tilde{\Delta}\tilde{B}+\tilde{f}(\tilde{r}). \tag{4.14}$$

For the stationary case

$$\tilde{\Delta}\tilde{B}+\tilde{f}(\tilde{r})=0 \tag{4.15}$$

or in the mentioned spherical coordinate system we reach in the case of the radial symmetry

$$\frac{\partial^2\tilde{B}}{\partial\tilde{r}^2}+\frac{2}{\tilde{r}}\frac{\partial\tilde{B}}{\partial\tilde{r}}+\tilde{f}(\tilde{r})=0. \tag{4.16}$$

Let us choose the arbitrary function $\tilde{f}(\tilde{r})$ in the form (4.17). Obviously this form corresponds to the magnetic field distribution around a rectilinear infinite conductor with current $(C=const)$

$$\tilde{f}(\tilde{r})=C\left[\tilde{B}(\tilde{r})-\frac{1}{\tilde{r}}\right] \tag{4.17}$$

we have

$$\frac{\partial^2\tilde{B}}{\partial\tilde{r}^2}+\frac{2}{\tilde{r}}\frac{\partial\tilde{B}}{\partial\tilde{r}}+C\left(\tilde{B}-\frac{1}{\tilde{r}}\right)=0 \tag{4.18}$$

or

$$\tilde{r}\frac{\partial^2\tilde{B}}{\partial\tilde{r}^2}+2\frac{\partial\tilde{B}}{\partial\tilde{r}}+C\tilde{r}\tilde{B}-C=0. \tag{4.19}$$

Equation (4.19) has analytical solution. For example for the case $C = 1$ we have equation

$$\tilde{r}\frac{\partial^2\tilde{B}}{\partial\tilde{r}^2}+2\frac{\partial\tilde{B}}{\partial\tilde{r}}+\tilde{r}\tilde{B}-1=0 \tag{4.20}$$

and its solution in the complex form

$$\tilde{B}(\tilde{r})=\frac{C_1\mathrm{e}^{-i\tilde{r}}}{\tilde{r}}-\frac{iC_2\mathrm{e}^{i\tilde{r}}}{2\tilde{r}}+\frac{1}{\tilde{r}}. \tag{4.21}$$

The real part of the solution (4.21) is written as

$$\tilde{B}_{real}(\tilde{r}) = \frac{1}{\tilde{r}}\left(C_1 \cos\tilde{r} + \frac{C_2}{2}\sin\tilde{r} + 1\right). \tag{4.22}$$

This statement can be verified by directly substituting the solution (4.21) and (4.22) into the equation (4.20). Extremes can be found from

$$\tilde{B}_{real}(\tilde{r}) = \frac{1}{\tilde{r}}\left(C_1 \cos\tilde{r} + \frac{C_2}{2}\sin\tilde{r} + 1\right). \tag{4.23}$$

Really from the equation

$$\frac{\partial\tilde{B}}{\partial\tilde{r}} = -\frac{1}{\tilde{r}^2}\left(C_1 \cos\tilde{r} + \frac{C_2}{2}\sin\tilde{r} + 1\right) + \frac{1}{\tilde{r}}\left(-C_1 \sin\tilde{r} + \frac{C_2}{2}\cos\tilde{r}\right) \tag{4.24}$$

we find

$$\frac{1}{\tilde{r}}\left(C_1 \cos\tilde{r} + \frac{C_2}{2}\sin\tilde{r} + 1\right) + \left(C_1 \sin\tilde{r} - \frac{C_2}{2}\cos\tilde{r}\right) = 0. \tag{4.25}$$

If $\tilde{r} \to \infty$ we have for the long-range order (for the case $C = 1$)

$$\tan\tilde{r} = \frac{C_2}{2C_1}, \tag{4.26}$$

or

$$\tilde{r} = \arctan\left[\frac{C_2}{2C_1}\right] + \pi n, \quad n = 0, 1, 2, \cdots \tag{4.27}$$

It is known that short-range order is the order in the mutual arrangement of atoms or molecules in a substance, which (in contrast to the long-range order) is repeated only at distances commensurate with the distances between atoms, that is, the short—range order is the presence of a pattern in the arrangement of neighboring atoms or molecules. As you see **Figures 1-7** demonstrate the

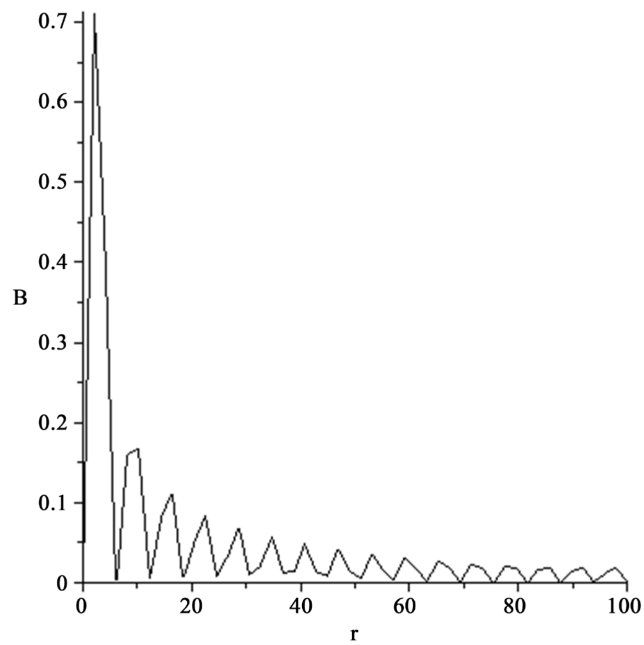

Figure 1. The dependence $\tilde{B}(\tilde{r})$, $C = 1$.

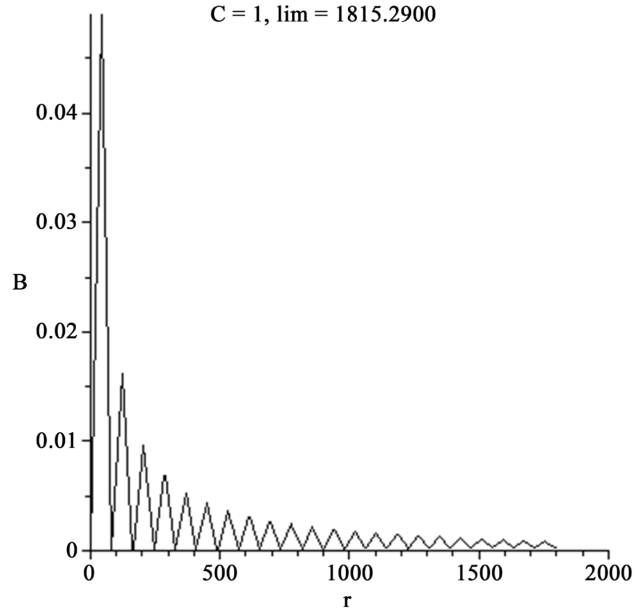

Figure 2. The dependence $\tilde{B}(\tilde{r})$, $C = 1$; lim = 1815.2900.

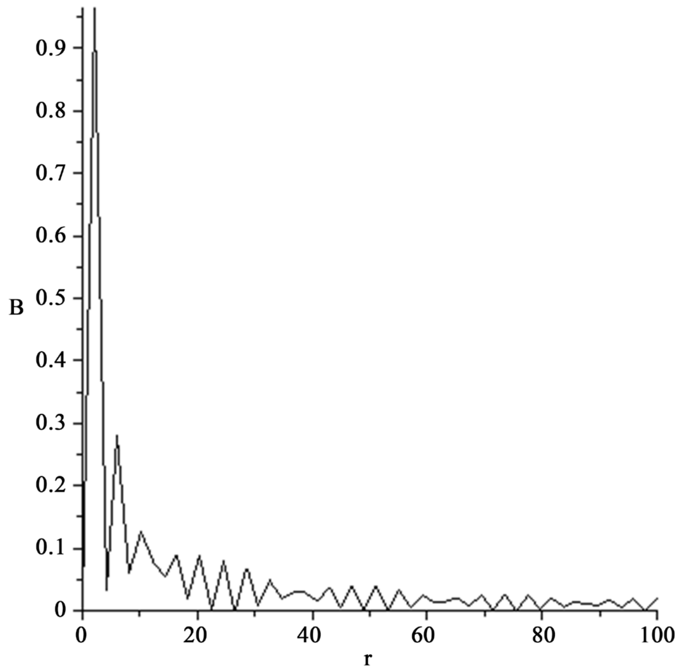

Figure 3. The dependence $\tilde{B}(\tilde{r})$, $C = 2$.

appearance of short- and long-range order.

Obviously we can use the simple Maple program

dsolve[interactive]({r*diff(B(r),r$2)+2*diff(B(r),r)+C*r*B(r)-C = 0, B(0) = 0, D(B)(0) = 0.5*C});

We find results reflected on **Figure 1** (C = 1) - **Figure 7** (C = 100).

Figure 2 reveals *the finite domain* of the $\tilde{B}(\tilde{r})$ evolution. It should be noticed that the Maple plotter hides the details of the picture at large values $\tilde{r}$.

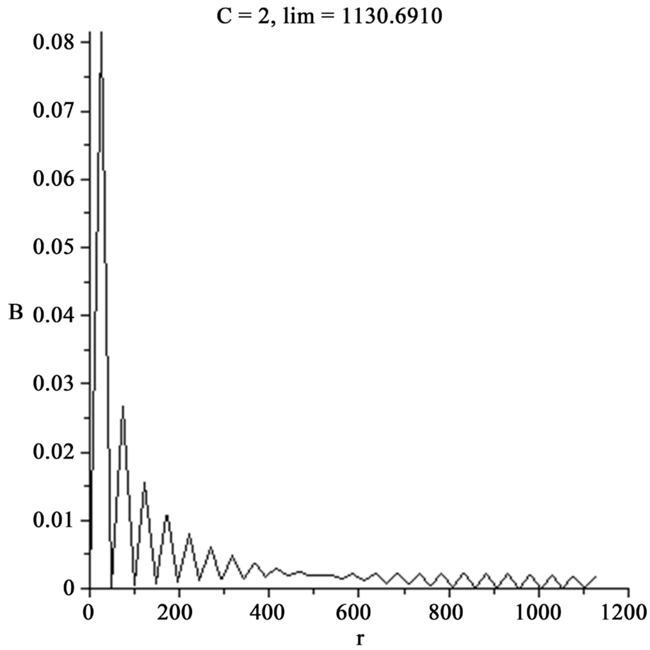

Figure 4. The dependence $\tilde{B}(\tilde{r})$, $C = 2$; lim = 1130.6910.

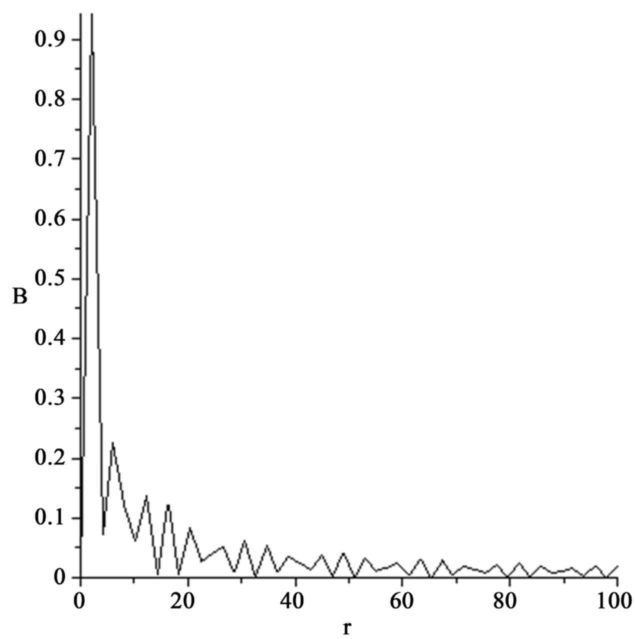

Figure 5. The dependence $\tilde{B}(\tilde{r})$, $C = 3$.

From Figures 1-7 follow:

1) The magnetic field distribution in a superconductor Type II has the character of damping waves.

2) There are modes when vibrational beats appear in a system of damped waves, Figures 3-5.

3) The solutions exist in the *limit* domain (in contrast to Abrikosov's theory).

4) As we see we reveal the quantization picture of the magnetic field.

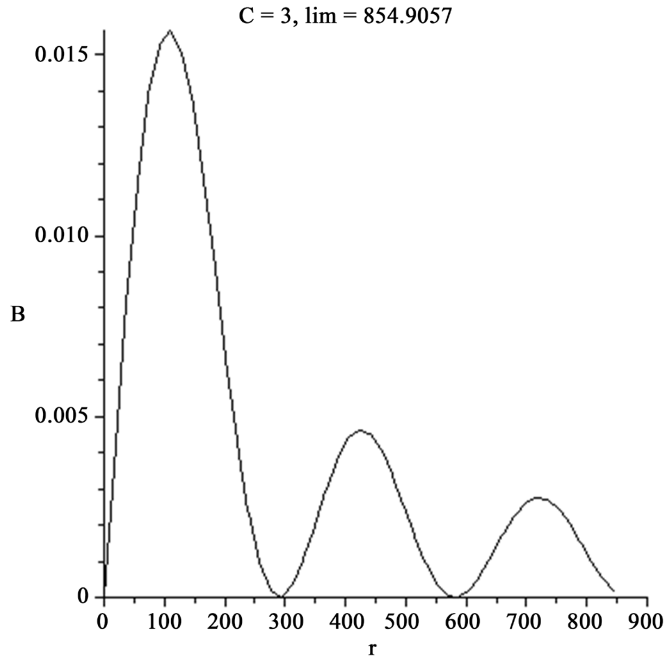

Figure 6. The dependence $\tilde{B}(\tilde{r})$, $C = 3$; lim = 854.9057.

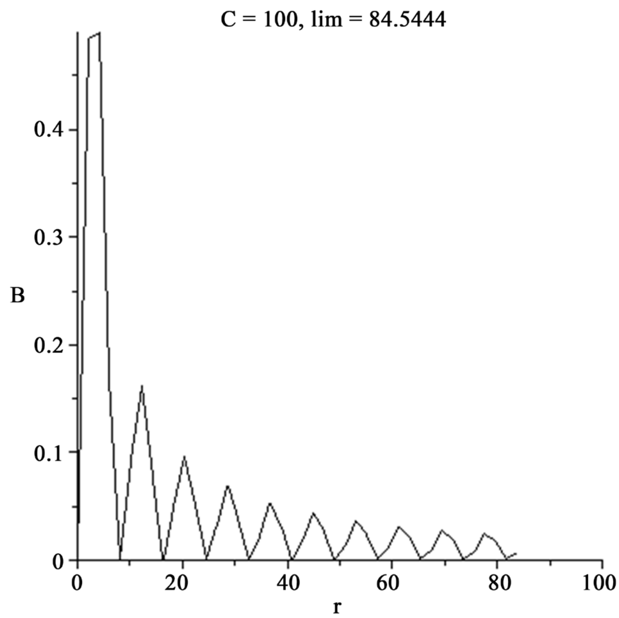

Figure 7. The dependence $\tilde{B}(\tilde{r})$, $C = 100$; lim = 84.5444.

5. Generalized Hydrodynamic Equations

The generalized hydrodynamic equations (GHE) can be obtained from the nonlocal kinetic equation in the frame of the Enskog procedure, [2] [3] [4]. Generally speaking to GHE should be added the system of generalized Maxwell equations (for example in the form of the generalized Poisson equation for electric potential) and gravitational equations (for example in the form of the generalized Poisson equation for gravitational potential). For example

$$\frac{\partial}{\partial \mathbf{r}}\cdot \mathbf{F}^{(1)} = -4\pi\gamma_N\left[\rho-\tau\left(\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot\rho\mathbf{v}_0\right)\right]. \tag{5.1}$$

(Continuity equation for species α)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho_\alpha-\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0)\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)\right.\right.\\&\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0\mathbf{v}_0)+\overleftrightarrow{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial \mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}=R_\alpha.\end{aligned} \tag{5.2}$$

(Continuity equation for mixture)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho-\sum_\alpha\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0)\right]\right\}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho\mathbf{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)\right.\right.\\&\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0\mathbf{v}_0)+\overleftrightarrow{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial \mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}=0.\end{aligned} \tag{5.3}$$

(Momentum equation for species α)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial \mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.\right.\\&\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}-\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0)\right)\right]\\&-\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial \mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.\right.\\&\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}\times\mathbf{B}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho_\alpha\mathbf{v}_0\mathbf{v}_0+p_\alpha\overleftrightarrow{\mathrm{I}}-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right.\right.\right.\\&\left.+p_\alpha\overleftrightarrow{\mathrm{I}}\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha(\mathbf{v}_0\mathbf{v}_0)\mathbf{v}_0+2\overleftrightarrow{\mathrm{I}}\left(\frac{\partial}{\partial \mathbf{r}}\cdot(p_\alpha\mathbf{v}_0)\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\overleftrightarrow{\mathrm{I}}p_\alpha\mathbf{v}_0\right)\\&\left.\left.-\mathbf{F}_\alpha^{(1)}\rho_\alpha\mathbf{v}_0-\rho_\alpha\mathbf{v}_0\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0[\mathbf{v}_0\times\mathbf{B}]\right]\right\}\\&=\int m_\alpha\mathbf{v}_\alpha J_\alpha^{st,el}\mathrm{d}\mathbf{v}_\alpha+\int m_\alpha\mathbf{v}_\alpha J_\alpha^{st,inel}\mathrm{d}\mathbf{v}_\alpha.\end{aligned} \tag{5.4}$$

(Momentum equation for mixture)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho\mathbf{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial \mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.\right.\\&\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}-\sum_\alpha\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\rho_\alpha\mathbf{v}_0)\right)\right]\\&-\sum_\alpha\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial \mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}\right.\right.\\&\left.\left.-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}\times\mathbf{B}+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\rho\mathbf{v}_0\mathbf{v}_0+p\overleftrightarrow{\mathrm{I}}-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right.\right.\right.\\&\left.+p_\alpha\overleftrightarrow{\mathrm{I}}\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha(\mathbf{v}_0\mathbf{v}_0)\mathbf{v}_0+2\overleftrightarrow{\mathrm{I}}\left(\frac{\partial}{\partial \mathbf{r}}\cdot(p_\alpha\mathbf{v}_0)\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\overleftrightarrow{\mathrm{I}}p_\alpha\mathbf{v}_0\right)\\&\left.\left.-\mathbf{F}_\alpha^{(1)}\rho_\alpha\mathbf{v}_0-\rho_\alpha\mathbf{v}_0\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0[\mathbf{v}_0\times\mathbf{B}]\right]\right\}=0.\end{aligned} \tag{5.5}$$

(Energy equation for α species)

$$\frac{\partial}{\partial t}\left\{\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\right]\right\}$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\right.\right.\right.$$
$$\left.+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}p_\alpha v_0^2\overset{\leftrightarrow}{I}\right.$$
$$\left.+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\overset{\leftrightarrow}{I}+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\overset{\leftrightarrow}{I}\right)-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\overset{\leftrightarrow}{I}$$
$$-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}-\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]$$
$$\left.\left.-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\right]\right\}-\left\{\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0-\tau_\alpha\left[\mathbf{F}_\alpha^{(1)}\cdot\left(\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)\right.\right.\right.$$
$$\left.\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha\overset{\leftrightarrow}{I}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right)\right]\right\} \tag{5.6}$$
$$=\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,el}\mathrm{d}\mathbf{v}_\alpha+\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,inel}\mathrm{d}\mathbf{v}_\alpha.$$

(Energy equation for mixture)

$$\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\frac{3}{2}p+\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\right]\right\}$$
$$+\frac{\partial}{\partial \mathbf{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\mathbf{v}_0+\frac{5}{2}p\mathbf{v}_0+\mathbf{v}_0\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\right.\right.\right.$$
$$\left.+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}p_\alpha v_0^2\overset{\leftrightarrow}{I}\right.$$
$$\left.+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\overset{\leftrightarrow}{I}+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\overset{\leftrightarrow}{I}\right)-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\overset{\leftrightarrow}{I}$$
$$-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}-\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]$$
$$\left.\left.-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\right]\right\}-\left\{\mathbf{v}_0\cdot\sum_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}-\sum_\alpha\tau_\alpha\left[\mathbf{F}_\alpha^{(1)}\cdot\left(\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)\right.\right.\right. \tag{5.7}$$
$$\left.\left.\left.+\frac{\partial}{\partial \mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial \mathbf{r}}\cdot p_\alpha\overset{\leftrightarrow}{I}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right)\right]\right\}=0.$$

Here $\mathbf{F}_\alpha^{(1)}$ are the forces of the non-magnetic origin, $\mathbf{B}$—magnetic induction, $\overset{\leftrightarrow}{I}$—unit tensor, q_α—charge of the α-component particle, p_α—static pressure for α-component, ε_α—internal energy for the particles of α-component, $\mathbf{v}_0$ —hydrodynamic velocity for mixture, τ_α —non-local parameter.

GHE are extremely important for astrophysics special cases when density $\rho \to 0$ (the initial stage of evolution of the Universe, the Big Bang; transport processes in physical vacuum) and when density $\rho \to \infty$ (evolution of the black hole). Both limiting cases have no physical or mathematical meaning in "classical" hydrodynamics. Thus, we have a unified statistical theory of dissipative structures, which has a hydrodynamic shape defined by the genesis of GHE. Then we obliged to deliver come comments concerning application of special (SRT) and general (GRT) relativistic theory in theoretical astrophysics.

6. Nonlocal Physics and Special and General Theory of Relativity

Let us investigate the possible connection between nonlocal physics and special theory of relativity (SRT). The underlying SRT experiment belongs to Albert A. Michelson and Edward W. Morley. The Michelson–Morley experiment was performed and published in November, 1887 (which has tremendous variants in the following years) [23]. They compared the speed of light in perpendicular directions, in an attempt to detect the relative motion of matter through some stationary medium called aether.

The result was negative, in that the expected difference between the speed of light in the direction of movement through the presumed aether, and the speed at right angles, was found not to exist. Michelson–Morley type experiments have been repeated many times with steadily increasing sensitivity. The result is considered as the evidence against the aether theory, and initiated a line of research that eventually led to special relativity, which rules out a stationary aether. Figure 8 shows the Michelson, Morley device. Calculate the time for which the light will pass the distance to the mirror in the horizontal direction. In one second light travels c meters, and the aether wind blows it in v meters back. Therefore, the actual speed will be equal $(c-v)$. It means that light will reach the mirror over $t = \frac{L_\Rightarrow}{c-v}$.

Obviously the way back will take time $t = \frac{L_\Rightarrow}{c+v}$. Total time spent:

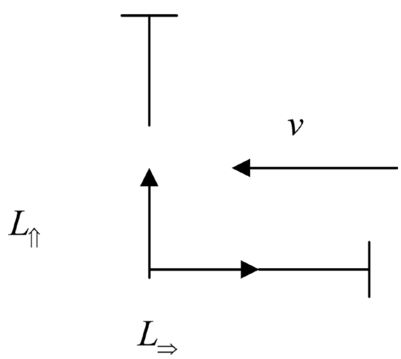

Figure 8. The principal scheme of Michelson - Morley experiment.

$$t_1 = \frac{L_{\Rightarrow}}{c+v} + \frac{L_{\Rightarrow}}{c-v} = \frac{2L_{\Rightarrow}c}{c^2-v^2}. \tag{6.1}$$

Calculate the elapsed time for moving in the vertical direction. In one second the aether wind will shift the light on the v meters to the left.

In other words the stationary observer should see the real velocity of the vertical movement as $u^2+v^2=c^2$, or $u = c\cos\phi = \sqrt{c^2-v^2}$. Then the real rate of convergence of the light with mirror (reflected on **Figure 8** as the upper mirror) is $u=\sqrt{c^2-v^2}$. Moving in the opposite direction leads to the symmetrical situation. Thus,

$$t_2 = \frac{2L_{\Uparrow}}{\sqrt{c^2-v^2}}. \tag{6.2}$$

The time difference is

$$\begin{aligned} t_1 - t_2 &= \frac{2L_{\Rightarrow}c}{c^2-v^2} - \frac{2L_{\Uparrow}}{\sqrt{c^2-v^2}} = \frac{2c}{c^2-v^2}\left[L_{\Rightarrow} - L_{\Uparrow}\frac{\sqrt{c^2-v^2}}{c}\right] \\ &= \frac{2c}{c^2-v^2}\left[L_{\Rightarrow} - L_{\Uparrow}\sqrt{1-\frac{v^2}{c^2}}\right]. \end{aligned} \tag{6.3}$$

Some conclusions:

1) The mentioned above configuration known as Michelson-Morley experiment (1887) would have detected the earth's motion through the supposed luminiferous aether that most physicists at the time believed was the medium in which light waves propagated. From the first glance the null result of that experiment leads to the condition (see (6.3))

$$t_1 - t_2 = 0. \tag{6.4}$$

2) But relation (6.4) obtained for the *closed* thermodynamic system without taking into account the direct influence of physical vacuum PV (or in old terminology) luminiferous aether.

3) In principal we consider the *open* thermodynamic system interacting with physical vacuum (PV). But the relation (6.1)-(6.3) obtained for classic dynamic system without taking into account the direct influence of PV on the mentioned system.

4) Nevertheless we intend to construct simplified theory *excluding* the direct PV influence. This fact leads to kinematic relation

$$L_{\Rightarrow} = L_{\Uparrow}\sqrt{1-\frac{v^2}{c^2}}. \tag{6.5}$$

Let us consider this situation in detail from the position of the nonlocal physics. The Michelson installation can be considered as a device delivering particles moving with the velocities v in the open probe contour filled by PV. The corresponding nonlocal parameter is (see also (2.7))

$$\tau_D = \frac{H_D}{v^2}. \tag{6.6}$$

For photons we have (υ_f is frequency)

$$\tau_{ph} h \upsilon_f = h, \tag{6.7}$$

or

$$\tau_{ph} = \frac{1}{\upsilon_f}. \tag{6.8}$$

Now we can rewrite the relation (6.3) in the form

$$t_1 - t_2 = \frac{2c}{c^2 - v^2}\left[L_{\Rightarrow} - L_{\Uparrow}\sqrt{1 - \frac{v^2}{c^2}}\right] = \frac{2c\tau_D}{\tau_D c^2 - H_D}\left[L_{\Rightarrow} - L_{\Uparrow}\sqrt{1 - \frac{H_D}{\tau_D c^2}}\right]. \tag{6.9}$$

Michelson's experiment assumes that the length of the instrument's shoulders does not change when rotated to a right angle.

In this case from the position of nonlocal physics

$$t_1 - t_2 = 0 \tag{6.10}$$

only if

$$H_D \to 0 \tag{6.11}$$

or if the PV viscosity tends to zero. As we see this affirmation contradicts the Heisenberg's uncertainty principle.

Moreover, it could be said that the uncertainty principle reflects the existence of Physical Vacuum.

From the physical point of view it means that for the realizations (6.10) we should exclude the influence of PV in the explicit form. Then (if $H_D \to 0$)

$$t_1 - t_2 \to \frac{2}{c}\left[L_{\Rightarrow} - L_{\Uparrow}\right] = 0. \tag{6.12}$$

H.A. Lorentz was aware in the aether existence. In his lectures on theoretical physics delivered at the University of Leiden he wrote [24], (Chapter 1, Paragraph 8: "Reduction in the direction of movement") he wrote:

"The negative result of Michelson's experiment can be explained by assuming that the length of the instrument's shoulders changes when rotated to a right angle....This size dependence on the orientation relative to the Earth's motion is not as surprising as it might seem at first. Indeed, the size is determined by molecular forces, and since the latter are transmitted over the aether, on the contrary it would be strange if the state of motion of the latter does not affect the size of bodies. The nature of molecular forces unknown to us; however, if we assume that they are transmitted through the aether in the same way as the electrical forces, then we can build a theory of this reduction, which gives the value of the latter, exactly corresponding to the value necessary to explain the zero effect of the Michelson's experiment. The size of this reduction will be 6.5 cm for the diameter of the Earth and 1/200 microns for the rod length of 1 m."

No difficulties to reproduce the Lorentz' estimation; for example for the rod reduction $L_{rr} = \sqrt{1 - \frac{9 \times 10^2}{9 \times 10^{10}}} = L\left(1 - \frac{1}{2} \times 10^{-8}\right)$, m we have ($v_{earth} = 30$ km/s,

$c = 3\times10^5 \text{ km/s}$.

The origin of contradiction consists in situation when the open thermodynamic system is considered as a closed one.

In other words the mirrors should not be considered as fixed objects.

Then special theory of relativity is *kinematic* theory which allows avoiding PV effects in the *explicit* form from consideration.

This affirmation is valid for the situation when the ratio of the object velocity to the light velocity in an inertial coordinate system is not small.

Criticism of general relativity (GR) is well known topic in theoretical physics (see for example [25]). No reason to discuss here the shortcomings of GRT. Nevertheless we should notice:

1) GRT does not contain the conservation laws, in other words GRT can't serve as a basement for investigation of transport processes in physical systems.

2) GRT is a phenomenological generalization of the Poisson equation with the aim to organize the transfer to the Newtonian gravitation. Not surprisingly, this equation has wave solutions. It was meant to be, not the other way around.

3) Poisson equation has no interpretation even in Madelung-Schrödinger quantum mechanics (QM). Then we have no chance to obtain the unified GRT-QM theory.

4) GRT postulates that the rate of transmission of gravitational interactions is equal to the speed of light. Laplace and Poincaré [26] [27] believed that the rate of transmission of gravitational interactions is several orders of magnitude higher than the speed of light; extremely important problem since the time of Laplace and Poincaré.

5) Formally speaking the Newtonian gravity propagates with the infinite speed. This conclusion is connected only with the description in the frame of local physics. Usual affirmation - general relativity (GR) reduces to Newtonian gravity in the weak-field, low-velocity limit. In literature you can find criticism of this affirmation because the conservation of angular momentum is implicit in the assumptions on which GR rests. Finite propagation speeds and conservation of angular momentum are incompatible in GR. Therefore, phenomenological GR was forced to claim that gravity is not a force that propagates in any classical sense.

6) In many cases, the result of the calculations is simply adjusted to the desired one using a small number of experimental data (including the situation with the Λ-term and Soldner problem discussed in [6]).

7) GTR has no reasonable asymptotic behavior at a density tending to infinity (a black hole) and at a density tending to zero (a physical vacuum). The appearance of singularity can not be considered as a reasonable solution.

Then black hole formation is not a robust prediction of the general theory of relativity. The idea of a black hole was known for a hundred years before Einstein.

The following effects of the principal significance exist as a direct consequence of nonlocal physics (see monographs [6] [28]):

1) The birth of the Universe is convoying of appearance of the repulsion forces. In the existing terminology we discover the "negative pressure" and "dark energy" in all cases. This fundamental result does not depend on the mechanism of external perturbations. In other words, the anti-gravity in the physical vacuum exists, if there is dissipation of energy or in the absence of dissipation at all.

2) Physical Vacuum (PV) is not a speculative object; it is a reality as "matter" and "fields". In other words, the physical vacuum is "the third" physical reality along with matter and fields.

3) The fundamental statistical theory on the dissipative non-equilibrium theory should be reconsidered.

4) The theoretical basement of the advanced technologies of the 21st century is nonlocal physics.

7. Hubble Effect and Nonlocal Physics

A special place in astrophysics is the effect of Hubble—expanding groups of galaxies, accompanied by a proportional increase in the rate of expansion groups based on the distance from the main center of gravity. The proportionality factor is the Hubble constant (Hu, for Universe Hu = $2.3 \times 10^{-18}\ \mathrm{s}^{-1}$), which as it turns out, is not a constant value, $v = \mathrm{Hu}\ r$. We introduce the Hubble parameter notation using the first two letters of the surname (as in similarity theory) in order to avoid confusion with the Boltzmann *H*-function.

The main origin of the Hubble effect (*including the matter expansion with acceleration*) *is self—catching of expanding matter by the self-consistent gravitational field in conditions of the weak influence of the central massive bodies*, [4] [6].

It would seem that we are dealing with the well studied problem in classical gas dynamics of point explosion. Not at all, the classical dynamics of a point explosion has no a Hubble mode.

Let us consider now the results of so called inflation theory and Hubble observations from the position of nonlocal physics. It was shown by B.V. Alexeev that the following inequality takes place (so called the generalized Boltzmann—Alexeev theorem) [3] [4] [7] [8]. One obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(H - \tau\frac{\mathrm{d}H}{\mathrm{d}t}\right) \le 0\,. \tag{7.1}$$

We introduce the H^a - function in accordance with the definition

$$H^a = H - \tau\frac{\mathrm{d}H}{\mathrm{d}t}\,. \tag{7.2}$$

Then the inequality is valid that makes up the conclusion of the generalized H-theorem,

$$\frac{\mathrm{d}H^a}{\mathrm{d}t} \le 0\,. \tag{7.3}$$

If we suppose that τ is constant not depending on time, inequality (7.3) can be considered as combination of two principles—Boltzmann's principle

$$\frac{\mathrm{d}H}{\mathrm{d}t} \le 0 \tag{7.4}$$

and Prigogine's principle [3] [4] [29] [30]

$$\frac{\mathrm{d}^2 H}{\mathrm{d}t^2} \ge 0 . \tag{7.5}$$

The reference level for the energy function can be chosen arbitrarily, in this case we use

$$H - \tau \frac{\mathrm{d}H}{\mathrm{d}t} = 0 . \tag{7.6}$$

or

$$\frac{\mathrm{d} \ln H}{\mathrm{d}t} = \frac{1}{\tau} \tag{7.7}$$

or if nonlocal parameter $\tau = const$ we have

$$H = H_0 \mathrm{e}^{\frac{t}{\tau}} . \tag{7.8}$$

But the Boltzmann H – function is proportional to the velocity v_{0r}, then

$$v_{0r}^2 = \hat{H}_0 \mathrm{e}^{\frac{t}{\tau}} \tag{7.9}$$

or

$$v_{0r} = \sqrt{\hat{H}_0} \mathrm{e}^{\frac{t}{2\tau}} \tag{7.10}$$

and

$$2 v_{0r} \frac{\partial v_{0r}}{\partial t} = \hat{H}_0 \mathrm{e}^{\frac{t}{\tau}} \frac{1}{\tau} , \tag{7.11}$$

or

$$2 v_{0r} \frac{\partial v_{0r}}{\partial t} = v_{0r}^2 \frac{1}{\tau} , \tag{7.12}$$

or

$$\mathrm{d}v_{0r} = \frac{1}{2\tau} v_{0r} \mathrm{d}t \tag{7.13}$$

or

$$\mathrm{d}v_{0r} = \frac{1}{2\tau} \mathrm{d}r \tag{7.14}$$

or for the $\tau = const$

$$v_{0r} = \mathrm{Hu}\ r , \tag{7.15}$$

where the Hubble coefficient Hu is introduced

$$\mathrm{Hu} = \frac{1}{2\tau} . \tag{7.16}$$

Let us comment now so called the inflationary theory of the Universe expansion in the early stages of the evolution of the Universe. It is assumed in the mentioned theory that in the period from 10^{-42} sec to 10^{-36} sec the Universe was in the inflationary stage of its development. Its main feature is the maximum strong negative pressure of the substance, leading to an exponential increase in the kinetic energy of the Universe and its volume by many orders of magnitude [31]. The inflationary model involves replacing the power law of expansion $\sim \sqrt{t}$ with an exponential law $\sim \exp(\text{Hu}t)$ where Hu as before is the Hubble constant of the inflationary stage, in a general way, depending on time. The value of the Hubble constant at the inflation stage is taken as 10^{-42} sec^{-1} - 10^{-36} sec^{-1}, that is, it gigantic exceeds its current value. Such a law of expansion can be provided by the states of physical fields ("inflation field") corresponding to the equation of state, that is, negative pressure; this stage is called inflationary (from lat. *inflatio*—inflation).

The model of cosmic inflation has opponents, including Roger Penrose [32]. The arguments of the opponents are that the solutions offered by the inflationary model are only "sweeping the litter under the carpet". The Rodger Penrose consideration contains the inflation theory criticism from the position of local physics.

We should remark:

1) The basic relation of the inflation theory concerning the exponential growth of the early Universe and the Hubble relation are *the simplest consequence of the nonlocal physics*.

2) The inflation theory has no chance to solve so called "initial value problems" concerning the Planck time and the problems of the relic gravitational.

3) The negative pressure does not exist.

4) The general relativistic theory is not applicable to the early Universe when the density $\rho \to 0$ and Big Bang corresponds to the explosion of the Physical Vacuum (PV).

5) The early evolution of the PV can not be considered in the frame of local physics in principal.

6) The proportionality coefficient in the Hubble dependence is commonly referred to as the Hubble (Hu) constant. The value of the Hubble constant continues to be refined, and now the most accurate value is close to 70 km/s per 1 megaparsec (1 MPC is about 3 million light years). For example, if a galaxy is located at a distance of 100 MPC from us, we can expect that it is moving away from our Galaxy at a speed of about 70 × 100 = 7000 km/s. The Hu value is determined from observations of galaxies whose distances are measured without red shift (primarily from the brightest stars or Cepheids). The independent estimates of Hu give this parameter a value of 66 - 78 km/s per mega parsec. This means that galaxies located at a distance of 100 mega parsecs are moving away from us at a speed of 6600 - 7800 km/s. At present (2019), the values obtained by calculating the distances to galaxies from the luminosity of the Cepheids ob-

served in them on the Hubble space telescope give an estimate of 74.03 ± 1.42 (km/s)/mega parsec, and the values obtained by measuring the parameters of relic radiation at the Planck space Observatory showed a value of 67.4 ± 0.5 (km/s)/mega parsec as of 2018.

7) The problem of estimating Hu is complicated by the fact that, in addition to the cosmological speeds due to the expansion of the Universe, galaxies still have their own (circular) speeds, which can be several hundred km/s (for members of massive clusters of galaxies—more than 1000 km/s). This leads to the fact that Hubble's law is poorly fulfilled or not fulfilled at all for objects located at a distance closer than 10 - 15 million LY, that is, just for those galaxies whose distances are most reliably determined without redshift. As we see the cause of the discrepancy is much more complicated, because local physics is not applicable to this situation in principal. For this reason, if galaxies are relatively close to us—say, a few mega parsecs away—the speed of random motion makes the usual Hubble's law inapplicable; such galaxies can either move away or approach us. Hubble's law is fairly accurate only for distant galaxies, and then within certain limits—at very large distances of billions of light years (thousands of mega parsecs), the Hubble constant differs from the one accepted for closer galaxies.

8) However, Hubble's law is currently used as the simplest and most reliable way to determine the distances to galaxies and their clusters. This is the most important parameter describing the current rate of expansion of the Universe.

9) Extremely important to underline that the Hubble problem has two aspects matter evolution in space and the PV evolution in space.

8. The Problem of Dark Matter: Dark Matter does not Exist. Analytical Investigation

The problem of dark matter was considered by me before in the frame of the nonlocal description (see for example monographs [4] [6]). But here I intend to show that fundamental results of the observation astronomy can be obtained in the analytical form.

About forty years after Zwicky's initial observations Vera Rubin [33] [34] [35] [36], astronomer at the Department of Terrestrial Magnetism at the Carnegie Institution of Washington presented findings based on a new sensitive spectrograph that could measure the velocity curve of edge-on spiral galaxies to a greater degree of accuracy than had ever before been achieved. Together with Kent Ford, Rubin announced at a 1975 meeting of the American Astronomical Society the astonishing discovery that most stars in spiral galaxies orbit at roughly the same speed reflected schematically on **Figure 9**.

For example, the rotation curve of the type B corresponds to the galaxy NGC3198. The following extensive radio observations determined the detailed rotation curve of spiral disk galaxies to be flat (as the curve **B**), much beyond as seen in the optical band. Obviously the trivial balance between the gravitational and centrifugal forces leads to relation between orbital speed V and galactocentric distance r as $V^2 = \gamma_N M/r$ beyond the physical extent of the galaxy of mass M (the curve A). The obvious contradiction with the velocity curve B having a 'flat' appearance out to a large radius, was explained by introduction of a new physical essence—dark matter because for spherically symmetric case the hypothetical density distribution $\rho(r) \sim 1/r^2$ leads to $V = const$. The result of this activity is known undetectable dark matter which does not emit radiation, inferred solely from its gravitational effects. But it means that upwards of 50% of the mass of galaxies was contained in the dark galactic halo.

Strict consideration leads to the system of the generalized hydrodynamic equations (GHE) (5.1)-(5.7) [2]-[7]. Regime B (**Figure 9**) cannot be obtained in the frame of local statistical physics in principal and authors of many papers introduce different approximations for additional "dark matter density" (as usual in Poisson equation) trying to find coincidence with data of observations. From the wrong position of local theories Poisson Equation (5.1) contains "dark matter density", continuity Equations (5.2) and (5.3) contain the "flux of dark matter density", motion Equations (5.4) and (5.5) include "dark energy", the energy Equations (5.6) and (5.7) has "the flux of dark energy" and so on to the "senior dark velocity moments". This entire situation is similar to the turbulent theories based on local statistical physics and empirical corrections for velocity moments.

The character features reflected on **Figure 9** can be explained in the frame of Newtonian gravitation law and the non-local kinetic description [2]-[7]. Let us discuss mathematical and physical models involved into the mentioned consideration:

1) The typical galaxy contains the tremendous quantity of stars. As a result the hydrodynamic application for the investigation of the universe objects is typical in astrophysics. For example our home galaxy Milky Way contains over 200 billion stars including our Sun. The Milky Way has a diameter of 100,000 light years and belongs to the type of a barred spiral galaxy. The Milky Way has three main parts: a disk, in which the Solar System resides, a bulge at the core, and an all encompassing halo. This situation allows to apply the methods of statistical physics for the system description.

2) Peculiar features of the halo movement can be explained without new concepts like "dark matter". It is shown (see for example [6]) that the transformation

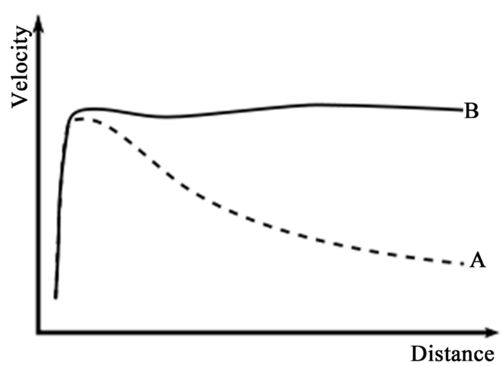

Figure 9. Rotation curve of a typical spiral galaxy: predicted (**A**) and observed (**B**).

of the Kepler's regime into the flat rotation curves depends on the similarity parameter

$$\tilde{\gamma}_N = \gamma_N \rho_0 x_0^2 / u_0^2 , \tag{8.1}$$

where γ_N is gravitational constant, x_0 and u_0 are the character length and the soliton velocity correspondingly. Parameter $\tilde{G} = \tilde{\gamma}_N$ plays the role of similarity criteria in traditional hydrodynamics. Important conclusions:

3) The obtained results demonstrate evolution the rotation curves from the Kepler regime (in the case of small $\tilde{G}$, like curve **A** on **Figure 9**) to observed orbital curves with the shelf (large $\tilde{G}$, like curve **B** on **Figure 9**) for typical spiral galaxies. Then parameter $\tilde{G}$ defines the transfer from the Kepler's to Rubin scenario.

4) The stars with planets (like Sun) correspond to gravitational soliton with small $\tilde{G}$ and therefore originate the Kepler rotation regime.

5) Important to underline that the shown transformation of the Kepler's regime into the flat rotation curves for different solitons explains the "mysterious" fact of the dark matter absence in the Sun vicinity.

I don't intend to repeat the corresponding calculations (see [6]). I intend to explain the obtained results in the frame of *analytical* investigation. We use the spherical coordinate system with the independent variables r, θ, φ (radial distance *r*, polar angle θ and azimuth angle φ):

Let us write down GHE (5.1)-(5.7) in spherical coordinate system for the one species system. We use continuity equation and motion equation written together for the physical system with radial symmetry

$$\begin{aligned} &\frac{\partial}{\partial t}\left\{\rho - \tau\left[\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial\left(r^2 \rho v_{0r}\right)}{\partial r}\right]\right\} \\ &+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left[\rho v_{0r} - \tau\left(\frac{1}{r^2}\frac{\partial\left(r^2 \rho v_{0r}^2\right)}{\partial r} - \rho g_r\right)\right]\right\} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2 \frac{\partial p}{\partial r}\right) = 0 \end{aligned} \tag{8.2}$$

$$\begin{aligned} &\frac{\partial}{\partial t}\left\{\rho v_{0\varphi} - \tau\left[\frac{\partial}{\partial t}\left(\rho v_{0\varphi}\right) + \frac{1}{r^2}\frac{\partial\left(r^2 \rho v_{0r} v_{0\varphi}\right)}{\partial r}\right]\right\} \\ &+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left[\rho v_{0r} v_{0\varphi} - \tau\left(\frac{1}{r^2}\frac{\partial\left(r^2 \rho v_{0r}^2 v_{0\varphi}\right)}{\partial r} - g_r \rho v_{0\varphi}\right)\right]\right\} \\ &-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2 \frac{\partial\left(p v_{0\varphi}\right)}{\partial r}\right) = 0, \end{aligned} \tag{8.3}$$

where $v_{0\varphi}$ is an orbital velocity. Two differential equations (8.2) and (8.3) are compatible even in the non-stationary case if

$$v_{0\varphi} = const \tag{8.4}$$

for the arbitrary τ.

Let us prolong the investigation of stationary case comparing the nonlocal continuity Equation (8.2) written in the form after integration

$$\rho v_{0r} v_{0\varphi} - \tau \left(\frac{1}{r^2} v_{0\varphi} \frac{\partial \left(r^2 \rho v_{0r}^2 \right)}{\partial r} - g_r \rho v_{0\varphi} + v_{0\varphi} \frac{\partial p}{\partial r} \right) = 0 \tag{8.5}$$

and the motion equation onto $\mathbf{e}_\varphi$ -direction (8.3) we find

$$\left(\rho v_{0r}^2 + p \right) \frac{\partial v_{0\varphi}}{\partial r} = 0 \tag{8.6}$$

or for satisfying (8.6) we should admit that $\frac{\partial v_{0\varphi}}{\partial r} = 0$ or

$$v_{0\varphi} = const\,. \tag{8.7}$$

for all *r*. Extremely important that the relation (8.7) includes the Newton gravitation regime. Really, from the mathematical point of view the Newton's law has an asymptotic character if $r \to 0$ or $r \to \infty$

$$F \sim \frac{1}{r^2} \tag{8.8}$$

or

$$v_{0\varphi,N} \sim \frac{1}{r^{1/2}}\,. \tag{8.9}$$

After substitution (8.9) into (8.6) we have

$$\left(\rho v_{0r}^2 + p \right) \frac{1}{r^{3/2}} \to 0 \tag{8.10}$$

if $r \to \infty$; total energy also tends to zero if $r \to \infty$, $\left(\rho v_{0r}^2 + p \right) \to 0$.

Let us investigate the Poisson Equation (5.1), which can be written in dimensionless form as ($\tilde{G}$ is the dimensionless Newton gravitational constant $\tilde{\gamma}_N$)

$$\frac{\partial}{\partial \tilde{\mathbf{r}}} \cdot \tilde{\mathbf{g}} = -4\pi \tilde{G} \left[\tilde{\rho} - \tilde{\tau} \left(\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial \tilde{\mathbf{r}}} \cdot \tilde{\rho} \tilde{\mathbf{v}}_0 \right) \right]. \tag{8.11}$$

We suppose for the simplicity the *stationary* case with the *radial* symmetry. Then

$$\frac{\partial}{\partial \tilde{r}} \left(\tilde{r}^2 \tilde{g}_r \right) = -\tilde{G} \left[\tilde{\rho} \tilde{r}^2 - \tilde{\tau} \frac{\partial \left(\tilde{r}^2 \tilde{\rho} \tilde{v}_{0r} \right)}{\partial \tilde{r}} \right] \tag{8.12}$$

or after integration

$$\tilde{r}^2 \tilde{g}_r = -\tilde{G} \int_0^{\tilde{r}} \left[\tilde{\rho} \tilde{r}^2 \right] \mathrm{d}\tilde{r} + \tilde{G} \int_0^{\tilde{r}} \left[\tilde{\tau} \frac{\partial \left(\tilde{r}^2 \tilde{\rho} \tilde{v}_{0r} \right)}{\partial \tilde{r}} \right] \mathrm{d}\tilde{r}\,. \tag{8.13}$$

Let us suppose that nonlocal dimensionless parameter $\tilde{\tau}$ does not depend on $\tilde{r}$. In this case we find

$$\tilde{r}^2 \tilde{g}_r = -\tilde{G} \int_0^{\tilde{r}} \left[\tilde{\rho} \tilde{r}^2 \right] \mathrm{d}\tilde{r} + \tilde{G} \tilde{\tau} \tilde{r}^2 \tilde{\rho} \tilde{v}_{0r} \tag{8.14}$$

or

$$\tilde{r}^2 \tilde{g}_r = -\tilde{G}\tilde{M} + \tilde{G}\tilde{\tau}\tilde{r}^2 \tilde{\rho}\tilde{v}_{0r}, \quad (8.15)$$

where $\tilde{M}$ is the dimensionless mass of the central body. As a result we obtain non-local gravitation law

$$\tilde{g}_r = -\tilde{G}\frac{\tilde{M}}{\tilde{r}^2} + \tilde{\tau}\tilde{G}\tilde{\rho}\tilde{v}_{0r} \quad (8.16)$$

and only in the local case we reach the classical Newton law

$$\tilde{g}_r = -\tilde{G}\frac{\tilde{M}}{\tilde{r}^2}. \quad (8.17)$$

In the nonlocal case gravitational acceleration

$$\tilde{g}_r = -\tilde{G}\left[\frac{\tilde{M}}{\tilde{r}^2} - \tilde{\tau}\tilde{\rho}\tilde{v}_{0r}\right] \quad (8.18)$$

turns into zero in a point $\tilde{r}_{cr}$ if

$$\frac{\tilde{M}}{\tilde{r}_{cr}^2} = \tilde{\tau}\tilde{\rho}\tilde{v}_{0r}, \quad (8.19)$$

where

$$\tilde{r}_{cr} = \sqrt{\frac{\tilde{M}}{\tilde{\tau}\tilde{\rho}\tilde{v}_{0r}}}. \quad (8.20)$$

if $\tilde{v}_{0r,cr} > 0$. Then the well known relation should be modified

$$m\tilde{g}_r = -\tilde{G}m\left[\frac{\tilde{M}}{\tilde{r}^2} - \tilde{\tau}\tilde{\rho}\tilde{v}_{0r}\right] \quad (8.21)$$

or the orbital velocity is equal

$$V_{orb} = \sqrt{\tilde{G}\left[\frac{\tilde{M}}{\tilde{r}} - \tilde{\tau}\tilde{\rho}\tilde{v}_{0r}\tilde{r}\right]}. \quad (8.22)$$

The energy flux density can be negative $\tilde{\rho}\tilde{v}_{0r} < 0$ that is, can be $V_{orb} > V_{orb,Newton}$. This equality may mean the introduction of additional mass which in local physics as an "additional dark matter" is considered.

Then

$$V_{orb} = \sqrt{\tilde{G}\frac{\tilde{M}}{\tilde{r}}\left[1 - \frac{\tilde{\tau}\tilde{\rho}\tilde{v}_{0r}\tilde{r}^2}{\tilde{M}}\right]}. \quad (8.23)$$

For the bounded system $\tilde{\rho}\tilde{v}_{0r} < 0$; we have formally the additional (if you want "dark" mass)

$$M_{dark}(\tilde{r}) = \tilde{\tau}\tilde{\rho}|\tilde{v}_{0r}|\tilde{r}^2. \quad (8.24)$$

It means also that in the well known Soldner formulae the additional mass $M_{dark}(\tilde{r})$ should be inserted which leads to increasing effective mass *M*. For example (see [6])

$$\delta_N = \frac{2\gamma_N}{R_{Sun}c^2}\left(M_{Sun} + M_{dark}(\tilde{r})\right). \quad (8.25)$$

From the nonlocal Newton-Kepler relation (8.25) follows that anti-lensing effect can take place. If the parameter $\tilde{\tau}$ is not dependent on $\tilde{r}$ the effective gravitational acceleration

$$\tilde{g}_r = -\tilde{G}\left[\frac{\tilde{M}}{\tilde{r}^2} - \tilde{\tau}\tilde{\rho}\tilde{v}_{0r}\right] \tag{8.26}$$

can be positive, negative, or zero and only in the local case we reach the classical Newton law.

Let us compare (8.26) with the correction (8.27) following from the general relativity (GR)

$$g_r = -G\left(\frac{M}{r^2} - \frac{4}{c^2}\frac{GM^2}{r^3} + \dots\right). \tag{8.27}$$

As we see the corrections (8.26) and (8.27) have the same structure. Comparing formulas (8.26) and (8.27) allow defining the nonlocal parameter from the GR point of view.

Conclusions:

1) The kind of the halo rotation (8.7) corresponds to the Vera Rubin effect. In other words the orbital speed is constant for different radius.

2) This kind of the halo rotation corresponds to the Vera Rubin effect even in the presence of the mass transfer in the r direction.

3) This analytical result does not depend on the τ choice.

4) The theory can be applied to the investigation of the ring rotation around the cosmic objects like Saturn. For example it means that *the rotation inside of each Saturn rings corresponds to the Vera Rubin model*. This effect can be verified by the direct observation or with the help of robotic spacecrafts.

5) The Newton law of gravitation for the statistical physical system should be modified.

Dark matter is artifact of local physics. Dark matter does not exist.

9. Transport Processes in Physical Vacuum. Preliminary Remarks

GHE have extremely important for astrophysics special cases when density $\rho \to 0$ (the initial stage of evolution of the Universe, the Big Bang) and when density $\rho \to \infty$ (evolution of the black hole). Both limiting cases have no physical or mathematical meaning in "classical" hydrodynamics. Thus, we have a unified statistical theory of dissipative structures, which has a hydrodynamic shape. We will, as already mentioned, refer to the corresponding system of equations as the fundamental equations of the unified theory (UT)

If the matter is absent, non-local evolution equations have nevertheless non- trivial solutions corresponding evolution of PV which description in time and 3D space on the level of quantum hydrodynamics demands only quantum pressure p, the self-consistent force $\mathbf{R}$ (acting on unit of the space volume) and velocity $\mathbf{v}_0$. The system of non local equations is written for the case when the usual matter is absent ($\rho = 0$); also radiation, gravitation (as well as other mass forces) and electromagnetic fields are absent. No reason to speak about special or general relativity in this situation, because these theories don't work in the described conditions.

We intend to consider from the position of the nonlocal physics the interaction of Physical Vacuum with matter.

We intend to consider the problem of the fundamental significance—is it possible to transfer the PV energy to matter? All results are obtained from the first principles of physics.

Another great problem of theoretical physics consists in asymmetry of matter and antimatter in the visible Universe.

What is the root of the problems from the theoretical point of view? The origin of difficulties consists in total Oversimplification following from principles of local physics and reflects the general shortcomings of the local kinetic transport theory based on the Boltzmann kinetic theory. Let us realize this program of investigation.

The process by which the inequality between matter and antimatter particles developed is called barogenesis. In modern physics, antimatter is defined as matter which is composed of the antiparticles of the corresponding particles of "ordinary" matter. Minuscule numbers of antiparticles can be found in natural processes like cosmic ray collisions and some types of radioactive decay. Only a tiny fraction of these have successfully been bound together in experiments to form anti-atoms. We consider this problem from the position of nonlocal physics.

With this aim we write down the GHE system of Equations (5.1)-(5.7) for the two component mixture of charged particles (without taking into account the component's internal energy) in the dimensionless form, where dimensionless symbols are marked by tildes. Summary of wave matter equations with the PV influence (subscripts "i" and "e" correspond to the positively and negatively charged components, subscript "υ" corresponds to PV).

We use the system of dimensional transport equations defining the interaction Matter—Physical Vacuum:

- Poisson equation

$$\frac{\partial^2 \psi}{\partial x^2} = -4\pi e\left\{\left[n_i - \tau_i\left(\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u)\right)\right] - \left[n_e - \tau_e\left(\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u)\right)\right]\right\}, \quad (9.1)$$

- continuity equation for positive ion component

$$\frac{\partial}{\partial t}\left\{\rho_i - \tau_i\left[\frac{\partial \rho_i}{\partial t} + \frac{\partial}{\partial x}(\rho_i u)\right]\right\} + \frac{\partial}{\partial x}\left\{\rho_i u - \tau_i\left[\frac{\partial}{\partial t}(\rho_i u) + \frac{\partial}{\partial x}(\rho_i u^2) + \frac{\partial p_i}{\partial x} - \rho_i F_{i,eff}\right]\right\} = 0, \quad (9.2)$$

- continuity equation for electron component

$$\frac{\partial}{\partial t}\left\{\rho_e - \tau_e\left[\frac{\partial \rho_e}{\partial t} + \frac{\partial}{\partial x}(\rho_e u)\right]\right\} + \frac{\partial}{\partial x}\left\{\rho_e u - \tau_e\left[\frac{\partial}{\partial t}(\rho_e u) + \frac{\partial}{\partial x}(\rho_e u^2) + \frac{\partial p_e}{\partial x} - \rho_e F_{e,eff}\right]\right\} = 0, \quad (9.3)$$

- momentum equation

$$\frac{\partial}{\partial t}\left\{\rho u-\tau_i\left[\frac{\partial}{\partial t}(\rho_i u)+\frac{\partial}{\partial x}\left(p_i+\rho_i u^2\right)-\rho_i F_{i,eff}\right]-\tau_e\left[\frac{\partial}{\partial t}(\rho_e u)+\frac{\partial}{\partial x}\left(p_e+\rho_e u^2\right)-\rho_e F_{e,eff}\right]\right\}$$
$$-\rho_i F_{i,eff}-\rho_e F_{e,eff}+F_{i,eff}\tau_i\left(\frac{\partial \rho_i}{\partial t}+\frac{\partial}{\partial x}(\rho_i u)\right)+F_{e,eff}\tau_e\left(\frac{\partial \rho_e}{\partial t}+\frac{\partial}{\partial x}(\rho_e u)\right)$$
$$+\frac{\partial}{\partial x}\left\{\rho u^2+p-\tau_i\left[\frac{\partial}{\partial t}\left(\rho_i u^2+p_i\right)+\frac{\partial}{\partial x}\left(\rho_i u^3+3p_i u\right)-2\rho_i u F_{i,eff}\right]\right. \tag{9.4}$$
$$\left.-\tau_e\left[\frac{\partial}{\partial t}\left(\rho_e u^2+p_e\right)+\frac{\partial}{\partial x}\left(\rho_e u^3+3p_e u\right)-2\rho_e u F_{e,eff}\right]\right\}=0;$$

- energy equation for positive ion component

$$\frac{\partial}{\partial t}\left\{\rho_i u^2+3p_i-\tau_i\left[\frac{\partial}{\partial t}\left(\rho_i u^2+3p_i\right)+\frac{\partial}{\partial x}\left(\rho_i u^3+5p_i u\right)-2\rho_i F_{i,eff}u\right]\right\}$$
$$+\frac{\partial}{\partial x}\left\{\rho_i u^3+5p_i u-\tau_i\left[\frac{\partial}{\partial t}\left(\rho_i u^3+5p_i u\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial x}\left(\rho_i u^4+8p_i u^2+5\frac{p_i^2}{\rho_i}\right)-F_{i,eff}\left(3\rho_i u^2+5p_i\right)\right]\right\} \tag{9.5}$$
$$-2u\rho_i F_{i,eff}+2\tau_i F_{i,eff}\left[\frac{\partial}{\partial t}(\rho_i u)+\frac{\partial}{\partial x}\left(\rho_i u^2+p_i\right)-\rho_i F_{i,eff}\right]=-\frac{p_i-p_e}{\tau_{ei}},$$

- energy equation for electron component

$$\frac{\partial}{\partial t}\left\{\rho_e u^2+3p_e-\tau_e\left[\frac{\partial}{\partial t}\left(\rho_e u^2+3p_e\right)+\frac{\partial}{\partial x}\left(\rho_e u^3+5p_e u\right)-2\rho_e F_{e,eff}u\right]\right\}$$
$$+\frac{\partial}{\partial x}\left\{\rho_e u^3+5p_e u-\tau_e\left[\frac{\partial}{\partial t}\left(\rho_e u^3+5p_e u\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial x}\left(\rho_e u^4+8p_e u^2+5\frac{p_e^2}{\rho_e}\right)-F_{e,eff}\left(3\rho_e u^2+5p_e\right)\right]\right\} \tag{9.6}$$
$$-2u\rho_e F_{e,eff}+2\tau_e F_{e,eff}\left[\frac{\partial}{\partial t}(\rho_e u)+\frac{\partial}{\partial x}\left(\rho_e u^2+p_e\right)-\rho_e F_{e,eff}\right]=-\frac{p_e-p_i}{\tau_{ei}},$$

where u is translational velocity of the quantum object, ψ —scalar potential, n_i and n_e are the number density of the charged species, $F_{i,eff}$ and $F_{e,eff}$ are the forces acting on the unit mass of ion and electron with taking into account the PV influence.

For the τ_i and τ_e approximations, the relations are used in the forms

$$\tau_i=\frac{\hbar}{m_i u^2},\quad \tau_e=\frac{\hbar}{m_e u^2}. \tag{9.7}$$

We begin with introduction the scales for velocity

$$[u]=u_0 \tag{9.8}$$

and for coordinate x $\frac{\hbar}{m_e u_0}=x_0$, for $\psi\to\psi_0$, for $n\to n_0$.

Generalized Poisson Equation (9.1) now is written as

$$\frac{\partial^2\psi}{\partial x^2}=-4\pi e\left\{\left[n_i-\tau_i\left(\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u)\right)\right]-\left[n_e-\tau_e\left(\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}(n_e u)\right)\right]\right\}, \tag{9.9}$$

or

$$\psi_0 \frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} = -4\pi e \rho_0 x_0^2 \left\{ \left[\frac{1}{m_i} \tilde{\rho}_i - \tau_i \frac{1}{m_i} \left(\frac{\partial \tilde{\rho}_i}{\partial t} + \frac{u_0}{x_0} \frac{\partial}{\partial \tilde{x}} (\tilde{\rho}_i \tilde{u}) \right) \right] - \left[\frac{1}{m_e} \rho_e - \tau_e \frac{1}{m_e} \left(\frac{\partial \tilde{\rho}_e}{\partial t} + \frac{u_0}{x_0} \frac{\partial}{\partial \tilde{x}} (\tilde{\rho}_e \tilde{u}) \right) \right] \right\}. \tag{9.10}$$

If the potential scale ψ_0 and the density scale ρ_0 are chosen as

$$\psi_0 = \frac{m_e}{e} u_0^2, \tag{9.11}$$

$$\rho_0 = \frac{m_e^4}{4\pi \hbar^2 e^2} u_0^4. \tag{9.12}$$

we find

$$\frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} = -4\pi x_0^2 \frac{m_e^3}{4\pi \hbar^2} u_0^2 \left\{ \left[\frac{1}{m_i} \tilde{\rho}_i - \tau_i \frac{1}{m_i} \left(\frac{\partial \tilde{\rho}_i}{\partial t} + \frac{u_0}{x_0} \frac{\partial}{\partial \tilde{x}} (\tilde{\rho}_i \tilde{u}) \right) \right] - \left[\frac{1}{m_e} \tilde{\rho}_e - \tau_e \frac{1}{m_e} \left(\frac{\partial \tilde{\rho}_e}{\partial t} + \frac{u_0}{x_0} \frac{\partial}{\partial \tilde{x}} (\tilde{\rho}_e \tilde{u}) \right) \right] \right\}. \tag{9.13}$$

Let us introduce the wave variable for the plane flow

$$\xi = x - Ct, \tag{9.14}$$

using the scales

$$\frac{\hbar}{m_e u_0} = x_0, \tag{9.15}$$

$$t_0 = \frac{x_0}{C} \tag{9.16}$$

we reach the dimensionless form of the independent variable (9.14)

$$\tilde{\xi} = \tilde{x} - \tilde{t}. \tag{9.17}$$

Using (9.7) and (9.15) one obtains

$$\frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} = -m_e \left\{ \left[\frac{1}{m_i} \tilde{\rho}_i - \frac{\hbar}{u^2 m_i^2} \left(-\frac{C}{x_0} \frac{\partial \tilde{\rho}_i}{\partial \tilde{\xi}} + \frac{u_0}{x_0} \frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho}_i \tilde{u}) \right) \right] \left[-\frac{1}{m_e} \tilde{\rho}_e - \frac{\hbar}{u^2 m_e^2} \left(-\frac{C}{x_0} \frac{\partial \tilde{\rho}_e}{\partial \tilde{\xi}} + \frac{u_0}{x_0} \frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho}_e \tilde{u}) \right) \right] \right\} \tag{9.18}$$

Let be

$$C = u_0. \tag{9.19}$$

We have finally from (9.15)-(9.18)

$$\frac{\partial^2 \tilde{\psi}}{\partial \tilde{\xi}^2} = -\left\{ \frac{m_e}{m_i} \left[\tilde{\rho}_i - \frac{1}{\tilde{u}^2} \frac{m_e}{m_i} \left(-\frac{\partial \tilde{\rho}_i}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho}_i \tilde{u}) \right) \right] - \left[\tilde{\rho}_e - \frac{1}{\tilde{u}^2} \left(-\frac{\partial \tilde{\rho}_e}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}} (\tilde{\rho}_e \tilde{u}) \right) \right] \right\}. \tag{9.20}$$

We transform now the continuity equations for i and e components. One obtains for the positive charged i species

$$\frac{\partial}{\partial t}\left\{\rho_i-\tau_i\left[\frac{\partial\rho_i}{\partial t}+\frac{\partial}{\partial x}(\rho_i u)\right]\right\}+\frac{\partial}{\partial x}\left\{\rho_i u-\tau_i\left[\frac{\partial}{\partial t}(\rho_i u)+\frac{\partial}{\partial x}(\rho_i u^2)+\frac{\partial p_i}{\partial x}-\rho_i F_{i,eff}\right]\right\}=0. \quad (9.21)$$

In the moving coordinate system all dependent hydrodynamic values are functions of (ξ,t). We investigate the possibility of the quantum object formation of the soliton type. For this solution there is no explicit dependence on time for coordinate system moving with the phase velocity u_0. Then using the scale relation

$$\frac{1}{t_0}=\frac{u_0}{x_0} \quad (9.22)$$

and (9.7) we have

$$-\frac{\partial\tilde{\rho}_i}{\partial\tilde{\xi}}+\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u})+\frac{u_0}{x_0}\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{\hbar}{m_i u^2}\left[-\frac{\partial\tilde{\rho}_i}{\partial\tilde{\xi}}+\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u})\right]\right\}-\frac{u_0}{x_0}\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{\hbar}{m_i u^2}\left[-\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u})+\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u}^2)+\frac{p_0}{\rho_0 u_0^2}\frac{\partial\tilde{p}_i}{\partial\tilde{\xi}}-\tilde{\rho}_i F_{i,eff}\frac{x_0}{u_0^2}\right]\right\}=0 \quad (9.23)$$

Let us introduce the scale

$$F_0\to\frac{u_0^2}{x_0}, \quad (9.24)$$

the scale

$$p\to p_0=\rho_0 u_0^2. \quad (9.25)$$

and transform (9.23)

$$\frac{\partial\tilde{\rho}_i}{\partial\tilde{\xi}}-\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u})-\frac{m_e}{m_i}\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\left[-\frac{\partial\tilde{\rho}_i}{\partial\tilde{\xi}}+\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u})\right]\right\}+\frac{m_e}{m_i}\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\left[-\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u})+\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u}^2)+\frac{\partial\tilde{p}_i}{\partial\tilde{\xi}}-\tilde{\rho}_i\tilde{F}_{i,eff}\right]\right\}=0. \quad (9.26)$$

The forces of the electric origin have the form

$$\rho_i F_i=-\frac{u_0^2}{x_0}\rho_0\frac{m_e}{m_i}\frac{\partial\tilde{\psi}}{\partial\tilde{\xi}}\tilde{\rho}_i, \quad (9.27)$$

$$\rho_e F_e=\frac{u_0^2}{x_0}\rho_0\frac{\partial\tilde{\psi}}{\partial\tilde{\xi}}\tilde{\rho}_e, \quad (9.28)$$

but we should add the volume forces of the PV origin. We find

$$\frac{\partial\tilde{\rho}_i}{\partial\tilde{\xi}}-\frac{\partial}{\partial\tilde{\xi}}(\tilde{\rho}_i\tilde{u})+\frac{m_e}{m_i}\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\left[\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_i+\tilde{\rho}_i+\tilde{\rho}_i\tilde{u}^2-2\tilde{\rho}_i\tilde{u}\right)-\tilde{\rho}_i\tilde{F}_i-\frac{1}{1+\frac{\tilde{\rho}_e}{\tilde{\rho}_i}\frac{m_i}{m_e}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right]\right\}=0, \quad (9.29)$$

where

$$\tilde{F}_i = -\frac{m_e}{m_i}\frac{\partial \tilde{\psi}}{\partial \tilde{\xi}}. \tag{9.30}$$

Let us realize transformation of the continuity equation for *e*-species to the dimensionless form

$$\begin{aligned}&\frac{\partial \rho_e}{\partial t}+\frac{\partial}{\partial x}(\rho_e u)-\frac{\partial}{\partial t}\left\{\tau_e\left[\frac{\partial \rho_e}{\partial t}+\frac{\partial}{\partial x}(\rho_e u)\right]\right\}\\&-\frac{\partial}{\partial x}\left\{\tau_e\left[\frac{\partial}{\partial t}(\rho_e u)+\frac{\partial}{\partial x}(\rho_e u^2)+\frac{\partial p_e}{\partial x}-\rho_e F_{e,eff}\right]\right\}=0,\end{aligned} \tag{9.31}$$

Using the relations $\frac{1}{t_0}=\frac{u_0}{x_0}$ and $\tau_e=\frac{\hbar}{m_e u^2}$ one obtains

$$\begin{aligned}&-\frac{\partial \tilde{\rho}_e}{\partial \tilde{\xi}}+\frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}_e\tilde{u})+\frac{u_0}{x_0}\frac{\partial}{\partial \tilde{\xi}}\left\{\frac{\hbar}{m_e u^2}\left[-\frac{\partial \tilde{\rho}_e}{\partial \tilde{\xi}}+\frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}_e\tilde{u})\right]\right\}\\&-\frac{u_0}{x_0}\frac{\partial}{\partial \tilde{\xi}}\left\{\frac{\hbar}{m_e u^2}\left[-\frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}_e\tilde{u})+\frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}_e\tilde{u}^2)+\frac{p_0}{\rho_0 u_0^2}\frac{\partial \tilde{p}_e}{\partial \tilde{\xi}}-\tilde{\rho}_e F_{e,eff}\frac{x_0}{u_0^2}\right]\right\}=0\end{aligned} \tag{9.32}$$

Let us introduce the scale $F_0 \to \frac{u_0^2}{x_0}$ and the scale $p \to p_0 = \rho_0 u_0^2$; I remind that $\upsilon_{qu}=\frac{h}{m_e}$ is introduced by me quantum kinematic viscosity and $x_0=\frac{\hbar}{m_e u_0}=\frac{\upsilon_{qu}}{2\pi u_0}$. We have

$$\begin{aligned}&\frac{\partial \tilde{\rho}_e}{\partial \tilde{\xi}}-\frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}_e\tilde{u})-\frac{\partial}{\partial \tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\left[-\frac{\partial \tilde{\rho}_e}{\partial \tilde{\xi}}+\frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}_e\tilde{u})\right]\right\}\\&+\frac{\partial}{\partial \tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\left[\frac{\partial}{\partial \tilde{\xi}}\left(\tilde{p}_e+\tilde{\rho}_e+\tilde{\rho}_e\tilde{u}^2-2\tilde{\rho}_e\tilde{u}\right)-\tilde{\rho}_e\tilde{F}_e-\frac{1}{1+\frac{\tilde{\rho}_i}{\tilde{\rho}_e}\frac{m_e}{m_i}}\frac{\partial \tilde{p}_\upsilon}{\partial \tilde{\xi}}\right]\right\}=0,\end{aligned} \tag{9.33}$$

where

$$\tilde{\rho}_e\tilde{F}_e=\frac{\partial \tilde{\psi}}{\partial \tilde{\xi}}\tilde{\rho}_e. \tag{9.34}$$

We transform now the motion equation;

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho u-\tau_i\left[\frac{\partial}{\partial t}(\rho_i u)+\frac{\partial}{\partial x}\left(p_i+\rho_i u^2\right)-\rho_i F_{i,eff}\right]\right.\\&\left.-\tau_e\left[\frac{\partial}{\partial t}(\rho_e u)+\frac{\partial}{\partial x}\left(p_e+\rho_e u^2\right)-\rho_e F_{e,eff}\right]\right\}\\&-\rho_i F_{i,eff}-\rho_e F_{e,eff}+F_{i,eff}\tau_i\left(\frac{\partial \rho_i}{\partial t}+\frac{\partial}{\partial x}(\rho_i u)\right)+F_{e,eff}\tau_e\left(\frac{\partial \rho_e}{\partial t}+\frac{\partial}{\partial x}(\rho_e u)\right)\\&+\frac{\partial}{\partial x}\left\{\rho u^2+p-\tau_i\left[\frac{\partial}{\partial t}\left(\rho_i u^2+p_i\right)+\frac{\partial}{\partial x}\left(\rho_i u^3+3p_i u\right)-2\rho_i u F_{i,eff}\right]\right.\\&\left.-\tau_e\left[\frac{\partial}{\partial t}\left(\rho_e u^2+p_e\right)+\frac{\partial}{\partial x}\left(\rho_e u^3+3p_e u\right)-2\rho_e u F_{e,eff}\right]\right\}=0;\end{aligned} \tag{9.35}$$

We find

$$
\begin{aligned}
&-\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}\tilde{u}-\tau_i\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_i+\tilde{\rho}_i\tilde{u}^2\right)-\tilde{\rho}_iF_{i,eff}\frac{x_0}{u_0^2}\right]\right.\\
&\left.-\tau_e\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_e+\tilde{\rho}_e\tilde{u}^2\right)-\tilde{\rho}_eF_{e,eff}\frac{x_0}{u_0^2}\right]\right\}\\
&-\frac{x_0}{u_0^2}\left(\tilde{\rho}_iF_{i,eff}+\tilde{\rho}_eF_{e,eff}\right)+F_{i,eff}\tau_i\frac{1}{u_0}\left(-\frac{\partial\tilde{\rho}_i}{\partial\tilde{\xi}}+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}\right)\right)\\
&+F_{e,eff}\tau_e\frac{1}{u_0}\left(-\frac{\partial\tilde{\rho}_e}{\partial\tilde{\xi}}+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}\right)\right)\\
&+\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}\tilde{u}^2+\tilde{p}-\tau_i\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_i+\tilde{\rho}_i\tilde{u}^2\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^3+3\tilde{p}_i\tilde{u}\right)-2\tilde{u}\tilde{\rho}_iF_{i,eff}\frac{x_0}{u_0^2}\right]\right.\\
&\left.-\tau_e\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_e+\tilde{\rho}_e\tilde{u}^2\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^3+3\tilde{p}_e\tilde{u}\right)-2\tilde{u}\tilde{\rho}_eF_{e,eff}\frac{x_0}{u_0^2}\right]\right\}=0;
\end{aligned}
\tag{9.36}
$$

Using the relations $\tau_i=\frac{\hbar}{m_iu^2}$, $\tau_i=\frac{\hbar}{m_iu^2}$, $\frac{\hbar}{m_eu_0}=x_0$ and $F_0=\frac{u_0^2}{x_0}$ we have

$$
\begin{aligned}
&-\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}\tilde{u}-\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_i+\tilde{\rho}_i\tilde{u}^2\right)-\tilde{\rho}_i\tilde{F}_{i,eff}\right]\right.\\
&\left.-\frac{1}{\tilde{u}^2}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_e+\tilde{\rho}_e\tilde{u}^2\right)-\tilde{\rho}_e\tilde{F}_{e,eff}\right]\right\}\\
&-\left(\tilde{\rho}_i\tilde{F}_{i,eff}+\tilde{\rho}_e\tilde{F}_{e,eff}\right)+\tilde{F}_{i,eff}\frac{u_0^2}{x_0}\frac{\hbar}{m_iu^2}\frac{1}{u_0}\left(-\frac{\partial\tilde{\rho}_i}{\partial\tilde{\xi}}+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}\right)\right)\\
&+\tilde{F}_{e,eff}\frac{u_0^2}{x_0}\frac{\hbar}{m_eu^2}\frac{1}{u_0}\left(-\frac{\partial\tilde{\rho}_e}{\partial\tilde{\xi}}+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}\right)\right)\\
&+\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}\tilde{u}^2+\tilde{p}-\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_i+\tilde{\rho}_i\tilde{u}^2\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^3+3\tilde{p}_i\tilde{u}\right)-2\tilde{u}\tilde{\rho}_i\tilde{F}_{i,eff}\right]\right.\\
&\left.-\frac{1}{\tilde{u}^2}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{p}_e+\tilde{\rho}_e\tilde{u}^2\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^3+3\tilde{p}_e\tilde{u}\right)-2\tilde{u}\tilde{\rho}_e\tilde{F}_{e,eff}\right]\right\}=0;
\end{aligned}
\tag{9.37}
$$

or

$$
\begin{aligned}
&\frac{\partial}{\partial\tilde{\xi}}\left\{\left(\tilde{\rho}_i+\tilde{\rho}_e\right)\tilde{u}^2+\left(\tilde{p}_i+\tilde{p}_e\right)-\left(\tilde{\rho}_i+\tilde{\rho}_e\right)\tilde{u}\right\}\\
&+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[\frac{\partial}{\partial\tilde{\xi}}\left(2\tilde{p}_i+2\tilde{\rho}_i\tilde{u}^2-\tilde{\rho}_i\tilde{u}-\tilde{\rho}_i\tilde{u}^3-3\tilde{p}_i\tilde{u}\right)-\tilde{\rho}_i\tilde{F}_i-\frac{1}{1+\frac{\tilde{\rho}_e}{\tilde{\rho}_i}\frac{m_i}{m_e}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right]\right.\\
&\left.+\frac{1}{\tilde{u}^2}\left[\frac{\partial}{\partial\tilde{\xi}}\left(2\tilde{p}_e+2\tilde{\rho}_e\tilde{u}^2-\tilde{\rho}_e\tilde{u}-\tilde{\rho}_e\tilde{u}^3-3\tilde{p}_e\tilde{u}\right)-\tilde{\rho}_e\tilde{F}_e-\frac{\tilde{\rho}_e}{\tilde{\rho}_e+\tilde{\rho}_i\frac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right]\right\}\\
&-\left(\tilde{\rho}_i\tilde{F}_i+\tilde{\rho}_e\tilde{F}_e\right)-\frac{\tilde{\rho}_i}{\tilde{\rho}_i+\tilde{\rho}_e\frac{m_i}{m_e}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}-\frac{\tilde{\rho}_e}{\tilde{\rho}_e+\tilde{\rho}_i\frac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\tilde{F}_i+\frac{1}{\rho_i+\tilde{\rho}_e\dfrac{m_i}{m_e}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\frac{\partial}{\partial\tilde{\xi}}\left[\tilde{\rho}_i\left(\tilde{u}-1\right)\right]\\
&+\left(\tilde{F}_e+\frac{1}{\tilde{\rho}_e+\tilde{\rho}_i\dfrac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)\frac{1}{\tilde{u}^2}\frac{\partial}{\partial\tilde{\xi}}\left[\tilde{\rho}_e\left(\tilde{u}-1\right)\right]\\
&+2\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}}\left[\tilde{\rho}_e\tilde{F}_e+\frac{\tilde{\rho}_e}{\tilde{\rho}_e+\tilde{\rho}_i\dfrac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right]\right\}+2\frac{m_e}{m_i}\frac{\partial}{\partial\tilde{\xi}}\left(\frac{1}{\tilde{u}}\left[\tilde{\rho}_e\tilde{F}_e+\frac{\tilde{\rho}_e}{\tilde{\rho}_e+\tilde{\rho}_i\dfrac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right]\right)=0;
\end{aligned}
\tag{9.38}
$$

where

$$
\tilde{F}_e=\frac{\partial\tilde{\psi}}{\partial\tilde{\xi}},\quad \tilde{F}_i=-\frac{m_e}{m_i}\frac{\partial\tilde{\psi}}{\partial\tilde{\xi}}.
\tag{9.39}
$$

Let us derivate the dimensionless energy equation for positive ion component.

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left\{\rho_i u^2+3p_i-\tau_i\left[\frac{\partial}{\partial t}\left(\rho_i u^2+3p_i\right)+\frac{\partial}{\partial x}\left(\rho_i u^3+5p_i u\right)-2\rho_i F_{i,eff}u\right]\right\}\\
&+\frac{\partial}{\partial x}\left\{\rho_i u^3+5p_i u-\tau_i\left[\frac{\partial}{\partial t}\left(\rho_i u^3+5p_i u\right)\right.\right.\\
&\left.\left.+\frac{\partial}{\partial x}\left(\rho_i u^4+8p_i u^2+5\frac{p_i^2}{\rho_i}\right)-F_{i,eff}\left(3\rho_i u^2+5p_i\right)\right]\right\}\\
&-2u\rho_i F_{i,eff}+2\tau_i F_{i,eff}\left[\frac{\partial}{\partial t}\left(\rho_i u\right)+\frac{\partial}{\partial x}\left(\rho_i u^2+p_i\right)-\rho_i F_{i,eff}\right]=-\frac{p_i-p_e}{\tau_{ei}},
\end{aligned}
\tag{9.40}
$$

or

$$
\begin{aligned}
&-\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_i\tilde{u}^2+3\tilde{p}_i-\tau_i\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^2+3\tilde{p}_i\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}\right)-2\tilde{\rho}_i\tilde{F}_{i,eff}\tilde{u}\right]\right\}\\
&+\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}-\tau_i\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}\right)\right.\right.\\
&\left.\left.+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^4+8\tilde{p}_i\tilde{u}^2+5\frac{\tilde{p}_i^2}{\tilde{\rho}_i}\right)-\tilde{F}_{i,eff}\left(3\tilde{\rho}_i\tilde{u}^2+5\tilde{p}_i\right)\right]\right\}\\
&-2\tilde{u}\tilde{\rho}_i\tilde{F}_{i,eff}+2\tau_i\tilde{F}_{i,eff}\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^2+\tilde{p}_i\right)-\tilde{\rho}_i\tilde{F}_{i,eff}\right]=-t_0\frac{\tilde{p}_i-\tilde{p}_e}{\tau_{ei}}.
\end{aligned}
\tag{9.41}
$$

Using the relations $\tau_i=\dfrac{\hbar}{m_i u^2}$, $\dfrac{\hbar}{m_e u_0}=x_0$ we find

$$
\begin{aligned}
&-\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_i\tilde{u}^2+3\tilde{p}_i-\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^2+3\tilde{p}_i\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}\right)-2\tilde{\rho}_i\tilde{F}_{i,eff}\tilde{u}\right]\right\}\\
&+\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}-\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}\right)\right.\right.
\end{aligned}
$$

$$+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^4+8\tilde{p}_i\tilde{u}^2+5\frac{\tilde{p}_i^2}{\tilde{\rho}_i}\right)-\tilde{F}_{i,eff}\left(3\tilde{\rho}_i\tilde{u}^2+5\tilde{p}_i\right)\Bigg]\Bigg\}$$
$$-2\tilde{u}\tilde{\rho}_i\tilde{F}_{i,eff}+2\tilde{F}_{i,eff}\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^2+\tilde{p}_i\right)-\tilde{\rho}_i\tilde{F}_{i,eff}\right]=-t_0\frac{\tilde{p}_i-\tilde{p}_e}{\tau_{ei}}, \tag{9.42}$$

After introduction the τ_{ei} nonlocal parameter $u^2\left(m_e+m_i\right)\tau_{ei}=\hbar$ and $\frac{\hbar}{m_e u_0}=x_0$, we have

$$\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}-\tilde{\rho}_i\tilde{u}^2-3\tilde{p}_i\right\}+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^2+3\tilde{p}_i\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}\right)-2\tilde{\rho}_i\tilde{F}_{i,eff}\tilde{u}\right]\right\}+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}\right)\right.\right.$$
$$\left.\left.-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^4+8\tilde{p}_i\tilde{u}^2+5\frac{\tilde{p}_i^2}{\tilde{\rho}_i}\right)+\tilde{F}_{i,eff}\left(3\tilde{\rho}_i\tilde{u}^2+5\tilde{p}_i\right)\right]\right\} \tag{9.43}$$
$$-2\tilde{u}\tilde{\rho}_i\tilde{F}_{i,eff}+2\tilde{F}_{i,eff}\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^2+\tilde{p}_i\right)-\tilde{\rho}_i\tilde{F}_{i,eff}\right]$$
$$=-\left(\tilde{p}_i-\tilde{p}_e\right)\tilde{u}^2\left(1+\frac{m_i}{m_e}\right),$$

and finally

$$\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_i\tilde{u}^3+5\tilde{p}_i\tilde{u}-\tilde{\rho}_i\tilde{u}^2-3\tilde{p}_i\right\}$$
$$+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left[\frac{\partial}{\partial\tilde{\xi}}\left(2\tilde{\rho}_i\tilde{u}^3+10\tilde{p}_i\tilde{u}-\tilde{\rho}_i\tilde{u}^4-\tilde{\rho}_i\tilde{u}^2-3\tilde{p}_i-8\tilde{p}_i\tilde{u}^2-5\frac{\tilde{p}_i^2}{\tilde{\rho}_i}\right)\right]\right\}$$
$$+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}\left(\tilde{F}_i+\frac{1}{\rho_i+\tilde{\rho}_e\frac{m_i}{m_e}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)\left[3\tilde{\rho}_i\tilde{u}^2+5\tilde{p}_i-2\tilde{\rho}_i\tilde{u}\right]\right\}$$
$$-2\tilde{u}\tilde{\rho}_i\left(\tilde{F}_i+\frac{1}{\rho_i+\tilde{\rho}_e\frac{m_i}{m_e}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)+2\left(\tilde{F}_i+\frac{1}{\rho_i+\tilde{\rho}_e\frac{m_i}{m_e}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)\frac{1}{\tilde{u}^2}\frac{m_e}{m_i}$$
$$\times\left[\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_i\tilde{u}^2+\tilde{p}_i-\tilde{\rho}_i\tilde{u}\right)-\tilde{\rho}_i\left(\tilde{F}_i+\frac{1}{\rho_i+\tilde{\rho}_e\frac{m_i}{m_e}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)\right]$$
$$=-\left(\tilde{p}_i-\tilde{p}_e\right)\tilde{u}^2\left(1+\frac{m_i}{m_e}\right), \tag{9.44}$$

Let's transform of the energy equation for electron component to the dimensionless form:

$$\frac{\partial}{\partial t}\left\{\rho_e u^2+3p_e-\tau_e\left[\frac{\partial}{\partial t}\left(\rho_e u^2+3p_e\right)+\frac{\partial}{\partial x}\left(\rho_e u^3+5p_e u\right)-2\rho_e F_{e,eff}u\right]\right\}$$
$$+\frac{\partial}{\partial x}\left\{\rho_e u^3+5p_e u-\tau_e\left[\frac{\partial}{\partial t}\left(\rho_e u^3+5p_e u\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial x}\left(\rho_e u^4+8p_e u^2+5\frac{p_e^2}{\rho_e}\right)-F_{e,eff}\left(3\rho_e u^2+5p_e\right)\right]\right\}$$
$$-2u\rho_e F_{e,eff}+2\tau_e F_{e,eff}\left[\frac{\partial}{\partial t}\left(\rho_e u\right)+\frac{\partial}{\partial x}\left(\rho_e u^2+p_e\right)-\rho_e F_{e,eff}\right]=-\frac{p_e-p_i}{\tau_{ei}}, \tag{9.45}$$

or

$$-\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_e\tilde{u}^2+3\tilde{p}_e-\tau_e\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^2+3\tilde{p}_e\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^3+5\tilde{p}_e\tilde{u}\right)\right.\right.$$
$$\left.\left.-2\tilde{\rho}_e F_{e,eff}\tilde{u}\frac{x_0}{u_0^2}\right]\right\}+\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_e\tilde{u}^3+5\tilde{p}_e\tilde{u}-\tau_e\frac{u_0}{x_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^3+5\tilde{p}_e\tilde{u}\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^4+8\tilde{p}_e\tilde{u}^2+5\frac{\tilde{p}_e^2}{\tilde{\rho}_e}\right)-\frac{x_0}{u_0^2}F_{i,eff}\left(3\tilde{\rho}_i\tilde{u}^2+5\tilde{p}_i\right)\right]\right\}$$
$$-2\tilde{u}\tilde{\rho}_e F_{e,eff}\frac{x_0}{u_0^2}+2\tau_e F_{e,eff}\frac{1}{u_0}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^2+\tilde{p}_e\right)-\tilde{\rho}_e F_{e,eff}\frac{x_0}{u_0^2}\right]$$
$$=-t_0\frac{\tilde{p}_e-\tilde{p}_i}{\tau_{ei}}, \tag{9.46}$$

or after using $\tau_e=\frac{\hbar}{m_e u^2}$, $\frac{\hbar}{m_e u_0}=x_0$ we reach the equation

$$-\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_e\tilde{u}^2+3\tilde{p}_e-\frac{1}{\tilde{u}^2}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^2+3\tilde{p}_e\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^3+5\tilde{p}_e\tilde{u}\right)-2\tilde{\rho}_e\tilde{F}_{e,eff}\tilde{u}\right]\right\}$$
$$+\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_e\tilde{u}^3+5\tilde{p}_e\tilde{u}-\frac{1}{\tilde{u}^2}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^3+5\tilde{p}_e\tilde{u}\right)\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^4+8\tilde{p}_e\tilde{u}^2+5\frac{\tilde{p}_e^2}{\tilde{\rho}_e}\right)-\tilde{F}_{i,eff}\left(3\tilde{\rho}_i\tilde{u}^2+5\tilde{p}_i\right)\right]\right\}$$
$$-2\tilde{u}\tilde{\rho}_e\tilde{F}_{e,eff}+2\tilde{F}_{e,eff}\frac{1}{\tilde{u}^2}\left[-\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}\right)+\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^2+\tilde{p}_e\right)-\tilde{\rho}_e\tilde{F}_{e,eff}\right]$$
$$=-t_0\frac{\tilde{p}_e-\tilde{p}_i}{\tau_{ei}}, \tag{9.47}$$

As before we use $u^2\left(m_e+m_i\right)\tau_{ei}=\hbar$ and $\frac{\hbar}{m_e u_0}=x_0$, then

$$\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_e\tilde{u}^3+5\tilde{p}_e\tilde{u}-\tilde{\rho}_e\tilde{u}^2-3\tilde{p}_e\right\}$$
$$+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\left[\frac{\partial}{\partial\tilde{\xi}}\left(2\tilde{\rho}_e\tilde{u}^3+10\tilde{p}_e\tilde{u}-\tilde{\rho}_e\tilde{u}^4-\tilde{\rho}_e\tilde{u}^2-3\tilde{p}_e-8\tilde{p}_e\tilde{u}^2-5\frac{\tilde{p}_e^2}{\tilde{\rho}_e}\right)\right]\right\}$$
$$+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\tilde{F}_e\left[3\tilde{\rho}_e\tilde{u}^2+5\tilde{p}_e-2\tilde{\rho}_e\tilde{u}\right]\right\}-2\tilde{u}\tilde{\rho}_e\tilde{F}_{e,eff}$$

$$+2\tilde{F}_{e,eff}\frac{1}{\tilde{u}^2}\left[\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^2+\tilde{p}_e-\tilde{\rho}_e\tilde{u}\right)-\tilde{\rho}_e\tilde{F}_{e,eff}\right]=-\left(\tilde{p}_e-\tilde{p}_i\right)\tilde{u}^2\left(1+\frac{m_i}{m_e}\right). \quad (9.48)$$

Writing in the explicit form we use for example

$$\tilde{F}_{e,eff}=\tilde{F}_e+\frac{1}{\tilde{\rho}_e+\tilde{\rho}_i\frac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}, \quad (9.49)$$

then

$$\begin{aligned}&\frac{\partial}{\partial\tilde{\xi}}\left\{\tilde{\rho}_e\tilde{u}^3+5\tilde{p}_e\tilde{u}-\tilde{\rho}_e\tilde{u}^2-3\tilde{p}_e\right\}\\&+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\left[\frac{\partial}{\partial\tilde{\xi}}\left(2\tilde{\rho}_e\tilde{u}^3+10\tilde{p}_e\tilde{u}-\tilde{\rho}_e\tilde{u}^4-\tilde{\rho}_e\tilde{u}^2-3\tilde{p}_e-8\tilde{p}_e\tilde{u}^2-5\frac{\tilde{p}_e^2}{\tilde{\rho}_e}\right)\right]\right\}\\&+\frac{\partial}{\partial\tilde{\xi}}\left\{\frac{1}{\tilde{u}^2}\left(\tilde{F}_e+\frac{1}{\tilde{\rho}_e+\tilde{\rho}_i\frac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)\left[3\tilde{\rho}_e\tilde{u}^2+5\tilde{p}_e-2\tilde{\rho}_e\tilde{u}\right]\right\}\\&-2\tilde{u}\tilde{\rho}_e\left(\tilde{F}_e+\frac{1}{\tilde{\rho}_e+\tilde{\rho}_i\frac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)+2\left(\tilde{F}_e+\frac{1}{\tilde{\rho}_e+\tilde{\rho}_i\frac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)\frac{1}{\tilde{u}^2}\\&\times\left[\frac{\partial}{\partial\tilde{\xi}}\left(\tilde{\rho}_e\tilde{u}^2+\tilde{p}_e-\tilde{\rho}_e\tilde{u}\right)-\tilde{\rho}_e\left(\tilde{F}_e+\frac{1}{\tilde{\rho}_e+\tilde{\rho}_i\frac{m_e}{m_i}}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}\right)\right]\\&=-\left(\tilde{p}_e-\tilde{p}_i\right)\tilde{u}^2\left(1+\frac{m_i}{m_e}\right).\end{aligned} \quad (9.50)$$

If $\frac{\partial p_\upsilon}{\partial x}-R_{\upsilon,x}=0$, then motion equation for the PV evolution is written as follows

$$\frac{\partial p_\upsilon}{\partial t}+v_{\upsilon,x}\frac{\partial p_\upsilon}{\partial x}+4p_\upsilon\frac{\partial v_{\upsilon,x}}{\partial x}=0 \quad (9.51)$$

or to the equation which has the wave solutions

$$\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}-\tilde{v}_{\upsilon,x}\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}-4\tilde{p}_\upsilon\frac{\partial\tilde{v}_{\upsilon,x}}{\partial\tilde{\xi}}=0. \quad (9.52)$$

Let us demonstrate derivation of the wave PV energy equation for the case $\tau_\upsilon\neq 0$. From Equation (5.7) follows in the PV limit ($\rho\to 0$)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{3p_\upsilon-\tau_\upsilon\left[3\frac{\partial p_\upsilon}{\partial t}+5\frac{\partial}{\partial x}\left(v_\upsilon p_\upsilon\right)-2R_r v_\upsilon\right]\right\}\\&+\frac{\partial}{\partial x}\left\{5pv_\upsilon-\tau_\upsilon\left[5\frac{\partial}{\partial t}\left(pv_\upsilon\right)+7\frac{\partial}{\partial x}\left(pv_\upsilon^2\right)-2R_r v_\upsilon^2\right]\right\}\\&-\frac{\partial}{\partial x}\left(\tau_\upsilon\frac{\partial}{\partial x}\left(p_\upsilon v_\upsilon^2\right)\right)-5\frac{\partial}{\partial x}\left(\tau_s\frac{\partial}{\partial x}\frac{p^2}{\rho}\right)=0,\end{aligned} \quad (9.53)$$

or

$$\frac{\partial}{\partial t}\left\{3p_\upsilon-\tau_\upsilon\left[3\frac{\partial p_\upsilon}{\partial t}+5p_\upsilon\frac{\partial v_\upsilon}{\partial x}+3v_\upsilon\frac{\partial p_\upsilon}{\partial x}\right]\right\}$$
$$+5p_\upsilon\frac{\partial v_\upsilon}{\partial x}+3v_\upsilon\frac{\partial p_\upsilon}{\partial x}-\frac{\partial}{\partial x}\left\{\tau_\upsilon\left[5\frac{\partial}{\partial t}\left(p_\upsilon v_\upsilon\right)+5v_\upsilon^2\frac{\partial p_\upsilon}{\partial x}+7p\frac{\partial v_\upsilon^2}{\partial x}\right]\right\} \quad (9.54)$$
$$-\frac{\partial}{\partial x}\left(\tau_\upsilon\frac{\partial}{\partial x}\left(p_\upsilon v_\upsilon^2\right)\right)-5\frac{\partial}{\partial x}\left(\tau_s\frac{\partial}{\partial x}\frac{p^2}{\rho}\right)=0,$$

Write down the energy PV Equation (9.54) for the case $\tau_\upsilon = const$ in the dimensionless form, we find

$$3\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}-5\tilde{p}_\upsilon\frac{\partial\tilde{v}_\upsilon}{\partial\tilde{\xi}}-3\tilde{v}_\upsilon\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}+5\frac{m_e}{m_e+m_i}\frac{\partial}{\partial\tilde{\xi}}\left(\frac{1}{\tilde{u}_s^2}\frac{\partial}{\partial\tilde{\xi}}\frac{(\tilde{p}_i+\tilde{p}_e)^2}{\tilde{\rho}_i+\tilde{\rho}_e}\right)$$
$$+\tilde{\tau}_\upsilon\frac{\partial}{\partial\tilde{\xi}}\left[3\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}-8\tilde{v}_\upsilon\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}-10\tilde{p}_\upsilon\frac{\partial\tilde{v}_\upsilon}{\partial\tilde{\xi}}+6\tilde{v}_\upsilon^2\frac{\partial\tilde{p}_\upsilon}{\partial\tilde{\xi}}+16\tilde{p}_\upsilon\tilde{v}_\upsilon\frac{\partial\tilde{v}_\upsilon}{\partial\tilde{\xi}}\right]=0. \quad (9.55)$$

The following figures reflect some results of calculations realized according to the system of of eight ordinary non-linear Equations (9.20), (9.29), (9.33), (9.38), (9.44),(9.50), (9.52) and (9.55) with the help of Maple.

Some comments to the system of Equations (9.20), (9.29), (9.33), (9.38), (9.44), (9.50), (9.52) and (9.55):

1) The problem belongs to the class of Cauchy problems.

2) In comparison with the Schrödinger theory connected with behavior of the wave function, no special conditions are applied for dependent variables including the domain of the solution existing. This domain is defined automatically in the process of the numerical solution of the concrete variant of calculations.

3) From the introduced scales

$$u_0,\quad x_0=\frac{\hbar}{m_e}\frac{1}{u_0},\quad \psi_0=\frac{m_e}{e}u_0^2,\quad \rho_0=\frac{m_e^4}{4\pi\hbar^2e^2}u_0^4,\quad p_0=\rho_0u_0^2=\frac{m_e^4}{4\pi\hbar^2e^2}u_0^6$$

only two parameters are independent – the phase velocity u_0 of the quantum object, and external parameter *H*, which is proportional to Plank constant $\hbar$ and in general case should be inserted in the scale relation as $x_0=\frac{H}{m_eu_0}=\frac{n\hbar}{m_eu_0}$. It leads to exchange in all scales $\hbar\leftrightarrow H$.

The following notations on figures are used: r-density $\tilde{\rho}_i$, s-density $\tilde{\rho}_e$, u-velocity $\tilde{u}$, h-PV velocity, p-pressure $\tilde{p}_i$, q-pressure $\tilde{p}_e$, w-PV pressure $\tilde{p}_\upsilon$ and v-self consistent potential $\tilde{\psi}$. Other notations: $m_e\to\mathrm{L}$, $m_i\to\mathrm{T}$, $\frac{\partial}{\partial\tilde{\xi}}\to\mathrm{D}$, independent variable *t* responds to $\tilde{\xi}$. Explanations placed under all following figures, Maple program contains Maple's notations for example the expression $D(u)(0)=0$ means in the usual notations $\frac{\partial\tilde{u}}{\partial\tilde{\xi}}(0)=0$.

Let's compare the two configurations—taking into account the influence of the physical vacuum and disregarding this influence. Cauchy conditions placed under all following figures.

Figure 10 reflects results without the PV influence.

Figure 11 reflects results with taking into account the PV influence (but the PV nonlocal parameter $\tilde{\tau}_{\upsilon} = 0$).

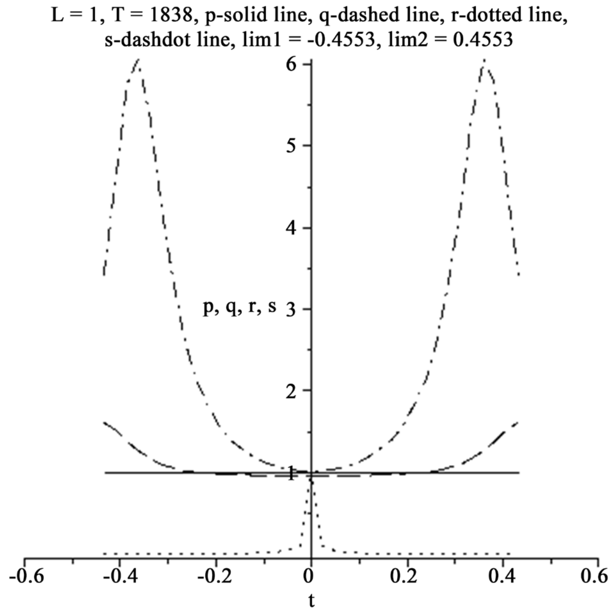

Figure 10. p-pressure $\tilde{p}_i$ (solid line), q-pressure $\tilde{p}_e$ (dashed line), r-density $\tilde{\rho}_i$ (dotted line), s-density $\tilde{\rho}_e$ (dashdot line), Cauchy conditions: A) v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 1, q(0) = 0.95, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0.

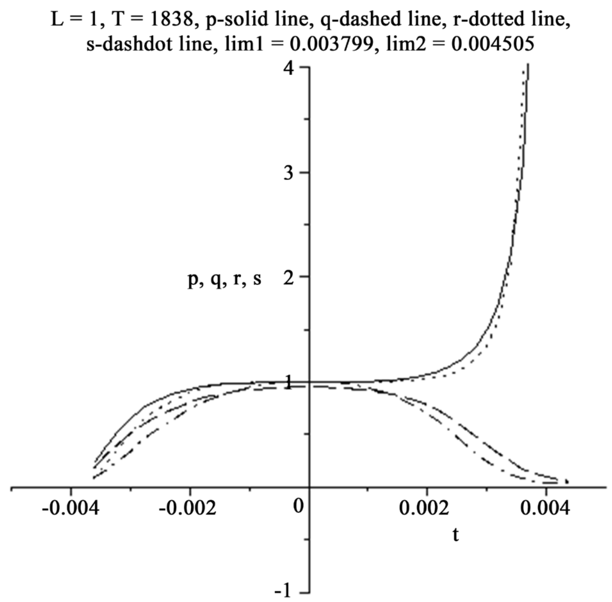

Figure 11. p-pressure $\tilde{p}_i$ (solid line), q-pressure $\tilde{p}_e$ (dashed line), r-density $\tilde{\rho}_i$ (dotted line), s-density $\tilde{\rho}_e$ (dashdot line), Cauchy conditions: v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 1, q(0) = 0.95, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0, w(0) = 1, D(w)(0) = 0, h(0) = 0, D(h)(0) = 1.

For the Cauchy conditions A) the PV influence leads to the destruction of the atom structure (like hydrogen) and appearance the object with the positive charged shell and the character size, which is smaller in hundreds times than previous object (like neutron).

Figure 12 reflects results without the PV influence, but for other Cauchy conditions type B: B) v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 0.95, q(0) = 1, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0.

For the Cauchy conditions B) the PV influence leads to the destruction of the structure with the positive shell (like neutron) and to appearance of the object with the negative charged shell and the character size, which is smaller in hundreds times than classical hydrogen atom.

Figure 13 reflects the results with the PV influence ($\tilde{\tau}_\upsilon = 0$ or D = 0).

In other words the anti-hydrogen has the very small cross section (in comparison with the hydrogen atoms), leaves the birth area and is now on the periphery of the Universe (see also [4] [6]).

But it is not the full scenario of events. We prolong our investigation. Let be now $\tilde{\tau}_\upsilon \neq 0$. In this case we should use the PV energy equation in the form:

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{3p_\upsilon-\tau_\upsilon\left[3\frac{\partial p_\upsilon}{\partial t}+5p_\upsilon\frac{\partial v_\upsilon}{\partial x}+3v_\upsilon\frac{\partial p_\upsilon}{\partial x}\right]\right\}\\&+5p_\upsilon\frac{\partial v_\upsilon}{\partial x}+3v_\upsilon\frac{\partial p_\upsilon}{\partial x}-\frac{\partial}{\partial x}\left\{\tau_\upsilon\left[5\frac{\partial}{\partial t}\left(p_\upsilon v_\upsilon\right)+5v_\upsilon^2\frac{\partial p_\upsilon}{\partial x}+7p\frac{\partial v_\upsilon^2}{\partial x}\right]\right\}\\&-\frac{\partial}{\partial x}\left(\tau_\upsilon\frac{\partial}{\partial x}\left(p_\upsilon v_\upsilon^2\right)\right)-5\frac{\partial}{\partial x}\left(\tau_s\frac{\partial}{\partial x}\frac{p^2}{\rho}\right)=0,\end{aligned} \tag{9.56}$$

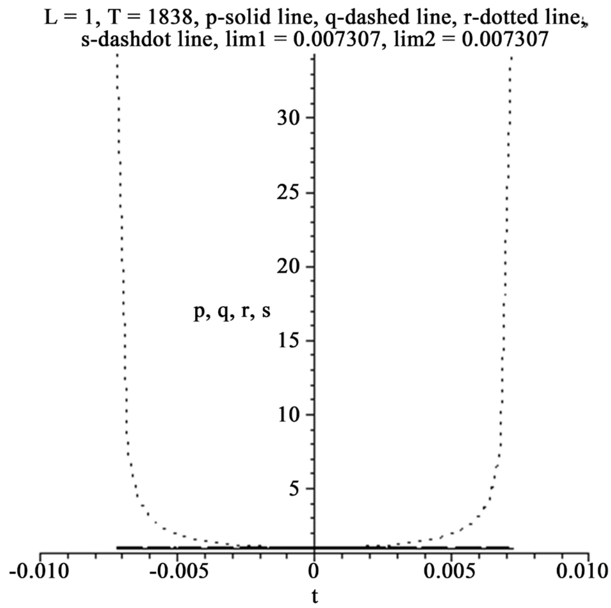

Figure 12. p-pressure $\tilde{p}_i$ (solid line), q-pressure $\tilde{p}_e$ (dashed line), r-density $\tilde{\rho}_i$ (dotted line), s-density $\tilde{\rho}_e$ (dashdot line), Cauchy conditions: B) v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 0.95, q(0) = 1, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0.

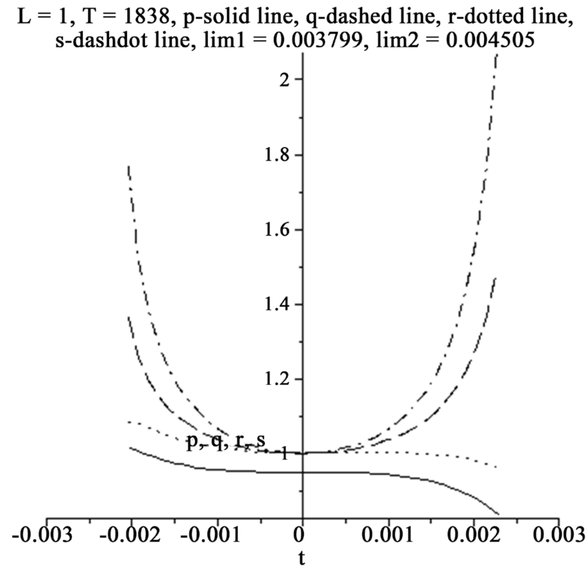

Figure 13. p-pressure $\tilde{p}_i$ (solid line), q-pressure $\tilde{p}_e$ (dashed line), r-density $\tilde{\rho}_i$ (dotted line), s-density $\tilde{\rho}_e$ (dashdot line), Cauchy conditions: v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 0.95, q(0) = 1, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0, w(0) = 1, D(w)(0) = 0, h(0) = 0, D(h)(0) = 1.

As we see from Equation (9.56) the nonlocal PV parameter is in reality a function of coordinates and time. Interesting to notice, that PV loses the interaction with Matter if $\tilde{\tau} \to \infty$. If $0 < \tilde{\tau}_\upsilon < \infty$ we should use the additional dimensionless PV energy equation in the form (9.55) supposing $\tilde{\tau}_\upsilon = const$.

Let be $\tilde{\tau}_\upsilon = 1$ (D = 1), we find

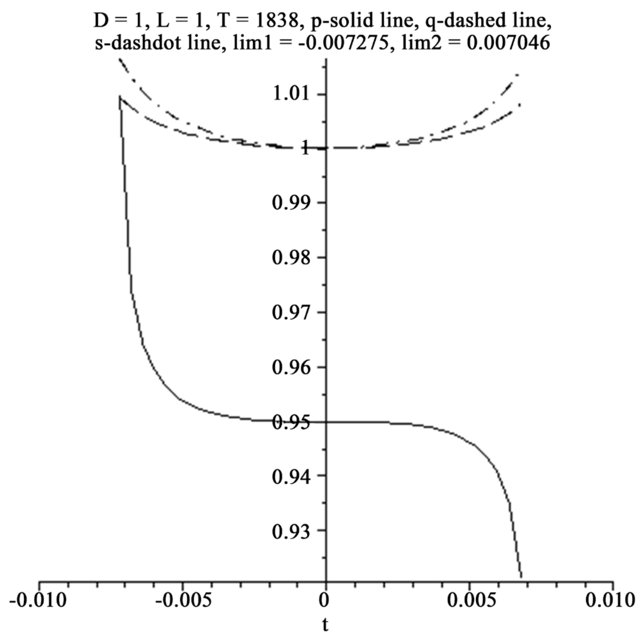

Figure 14. p-pressure $\tilde{p}_i$ (solid line), q-pressure $\tilde{p}_e$ (dashed line), s-density $\tilde{\rho}_e$ (dashdot line), Cauchy conditions: v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 0.95, q(0) = 1, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0.

Let us compare now the results of two calculations realized in the same conditions but for the different nonlocal PV parameters (D = 0 and D = 1).

For **Figure 15** and **Figure 16**: p-pressure $\tilde{p}_i$ (solid line), q-pressure $\tilde{p}_e$

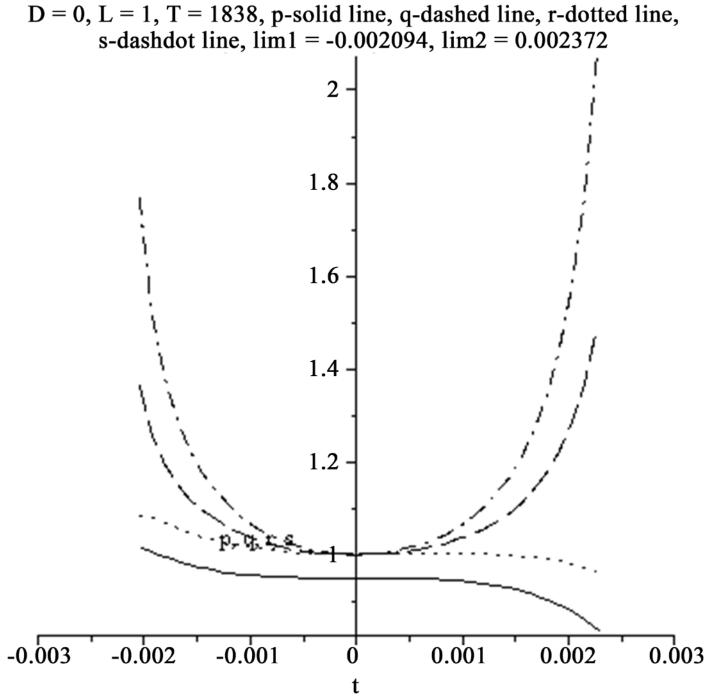

Figure 15. p-pressure $\tilde{p}_i$ (solid line), q-pressure $\tilde{p}_e$ (dashed line), r-density (dotted line), s-density $\tilde{p}_e$ (dashdot line). Cauchy conditions: v(0)=1, r(0)=1, s(0)=1, u(0) = 1, p(0) = 0.95, q(0) = 1, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0, w(0) = 1, D(w)(0) = 0, h(0) = 0, D(h)(0) = 1.

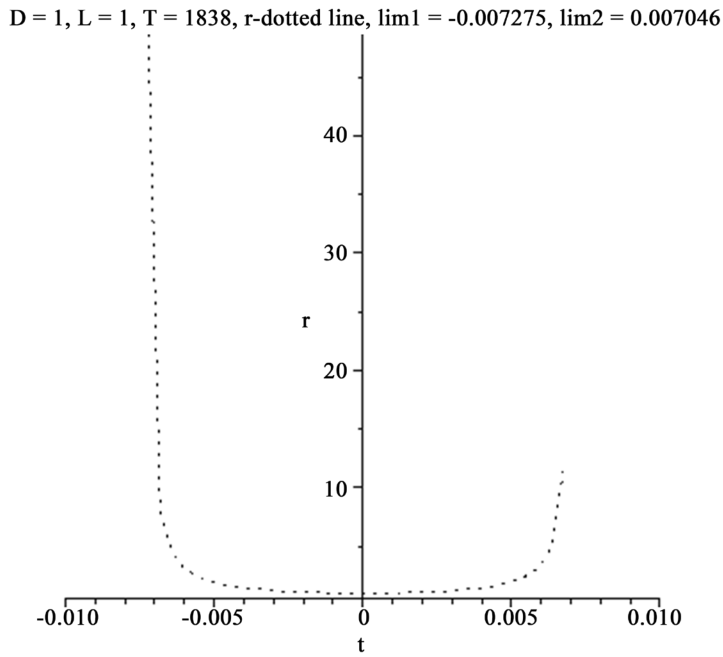

Figure 16. r-density $\tilde{p}_i$ (dotted line)

(dashed line), r-density $\tilde{\rho}_i$ (dotted line), s-density $\tilde{\rho}_e$ (dashdot line), Cauchy conditions: v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 0.95, q(0) = 1, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0, w(0) = 1, D(w)(0) = 0, h(0) = 0, D(h)(0) = 1.

Take a look at Figure 15 (with $\tilde{\tau}_\upsilon = 0$, or D = 0, left here) and Figure 16 (with $\tilde{\tau}_\upsilon = 1$ or D = 1), right here). These configurations have the extremely important feature (for the chosen Cauchy conditions, compare with Figure 10). In the first case (D = 0) this object has the negative charged shell (like hydrogen), but in the second case (D = 1) we reveal the object with the positive charged shell with the larger cross section.

This situation can be explained be the following way. At an early stage in the development of the universe the value of the PV parameter is closer to zero. As a result the antimatter (as I wrote before) having the small cross section, leaves the central part of domain. As the universe ages, the non-locality parameter $\tilde{\tau}_\upsilon$ increases, which leads to the situation responding configuration 16, right.

Figure 17 and Figure 18 reflect some other calculations.

Figures 11-18 display quantum objects placed in bounded region of 1D space, all parts of this objects are moving with practically the same velocity. Important to underline that no special boundary conditions were used for all cases. Then this soliton is product of the self-organization of ionized matter.

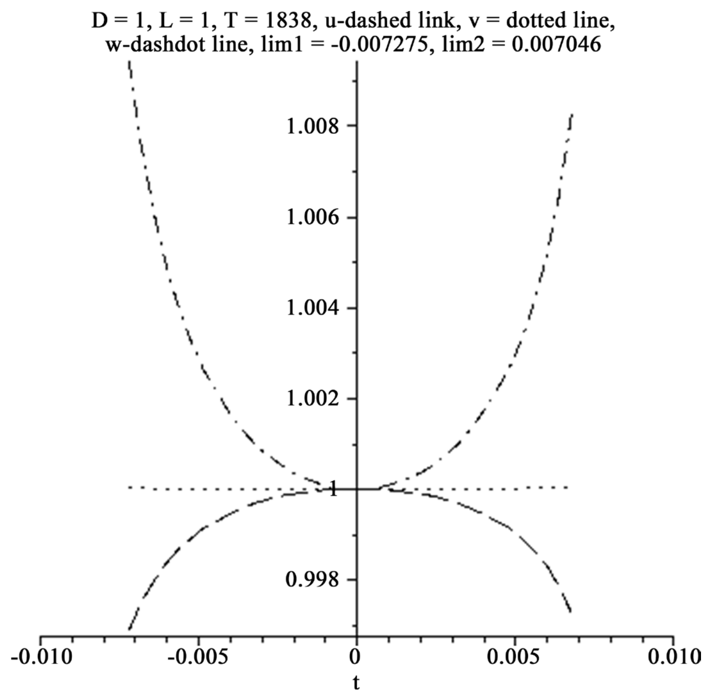

Figure 17. u-velocity $\tilde{v}$ (dashed line), v-electric potential $\tilde{\psi}$ (dotted line), w-PV pressure $\tilde{p}_\upsilon$ (dashdot line). v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 0.95, q(0) = 1, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0, w(0) = 1, D(w)(0) = 0, h(0) = 0, D(h)(0) = 1.

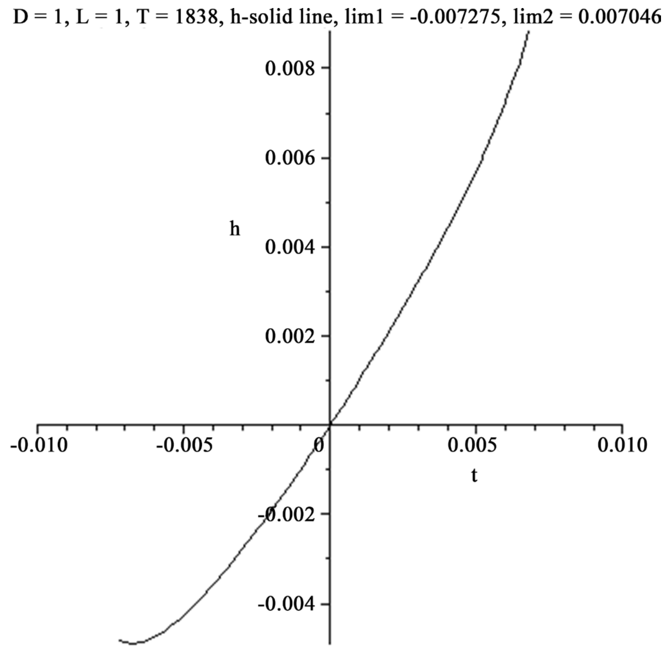

Figure 18. h-velocity $\tilde{v}_\upsilon$ (solid line), v(0) = 1, r(0) = 1, s(0) = 1, u(0) = 1, p(0) = 0.95, q(0) = 1, D(v)(0) = 0, D(r)(0) = 0, D(s)(0) = 0, D(u)(0) = 0, D(p)(0) = 0, D(q)(0) = 0, w(0) = 1, D(w)(0) = 0, h(0) = 0, D(h)(0) = 1.

10. About the Energy Echange between Ordinary Matter and Physical Vacuum

Let us consider the spherical 1D stationary flow and its interaction with the surrounding physical vacuum. It means that we should introduce into consideration two kinds of nonlocal paramaters for matter τ_s and for physical vacuum (PV) τ_υ. The generalized nonlocal hydrodynamic equations can be written as follows (see also the system of Equations (5.1) - (5.7)):

continuity equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\rho v_{0r}-\tau_s\left[\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^2\right)}{\partial r}-R_r\right]\right\}\right\}-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau_s r^2\frac{\partial p}{\partial r}\right)=0. \quad (10.1)$$

This equation can be immediately integrated

$$\rho v_{0r}-\tau_s\left[\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^2\right)}{\partial r}+\frac{\partial p}{\partial r}-R_r\right]=0, \quad (10.2)$$

where R_r is the external force acting in the radial direction on the volume unit.

Motion equation for matter for the stationary case is written as

$$\begin{aligned}&\frac{\partial p}{\partial r}-R_r+\tau_s F_r\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}\right)}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\rho v_{0r}^2-\tau_s\left[\frac{1}{r^2}\frac{\partial\left(r^2\rho v_{0r}^3\right)}{\partial r}-2R_r v_{0r}\right]\right\}\right\}\\&-2\frac{\partial}{\partial r}\left(\frac{\tau_s}{r^2}\frac{\partial\left(r^2 p v_{0r}\right)}{\partial r}\right)-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau_s r^2\frac{\partial\left(p v_{0r}\right)}{\partial r}\right)=0,\end{aligned} \quad (10.3)$$

where F_r is the external force acting in the radial direction on the mass unit,

$$R_r = \rho F_r \,. \tag{10.4}$$

Equation (10.3) can be written in the form ($\tau_s = const$)

$$\begin{aligned} & r^2\left(\frac{\partial p}{\partial r}-R_r\right)+\tau_s F_r\frac{\partial\left(r^2\rho v_{0r}\right)}{\partial r}+\frac{\partial}{\partial r}\left(r^2\rho v_{0r}^2\right)-\tau_s\frac{\partial^2\left(r^2\rho v_{0r}^3\right)}{\partial r^2} \\ & +2\tau_s\frac{\partial}{\partial r}\left(r^2 R_r v_{0r}\right)-2\tau_s r^2\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial\left(r^2 p v_{0r}\right)}{\partial r}\right)-\tau_s\frac{\partial}{\partial r}\left(r^2\frac{\partial\left(p v_{0r}\right)}{\partial r}\right)=0. \end{aligned} \tag{10.5}$$

Energy equation for the matter for the stationary case has the form

$$\begin{aligned} & \frac{\partial}{\partial r}\left\{r^2 v_{0r}\left(\rho v_{0r}^2+5p\right)\right\}-\tau_s\frac{\partial^2}{\partial r^2}\left(r^2\left(\rho v_{0r}^2+7p\right)v_{0r}^2\right) \\ & +\tau_s\frac{\partial}{\partial r}\left(3r^2 R_r v_{0r}^2+5r^2 pF\right)-2R_r v_{0r} r^2+2\tau_s F_r\frac{\partial}{\partial r}\left(r^2\rho v_{0r}^2\right) \\ & +2\tau_s F_r r^2\left(\frac{\partial p}{\partial r}-R_r\right)-\tau_s\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(p v_{0r}^2\right)\right)-5\tau_s\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(\frac{p^2}{\rho}\right)\right)=0, \end{aligned} \tag{10.6}$$

We should add to the previous Equations (10.2), (10.3) and (10.6) the self-consistent equations describing the PV motion. The nonlocal parameter τ_υ for PV should be introduced. Then for the matter and PV description we find

$$\rho_s v_s-\tau_s\left[\frac{1}{r^2}\frac{\partial\left(r^2\rho_s v_s^2\right)}{\partial r}+\frac{\partial p_s}{\partial r}-\frac{\partial p_\upsilon}{\partial r}\right]=0\,, \tag{10.7}$$

$$\begin{aligned} & r^2\frac{\partial}{\partial r}\left(p_s-p_\upsilon\right)+\tau_s\frac{\partial p_\upsilon}{\partial r}\frac{1}{\rho_s}\frac{\partial\left(r^2\rho_s v_s\right)}{\partial r}+\frac{\partial}{\partial r}\left\{r^2\rho_s v_s^2\right\} \\ & -\tau_s\frac{\partial}{\partial r}\left[\frac{\partial\left(r^2\rho_s v_s^3\right)}{\partial r}\right]+2\tau_s\frac{\partial}{\partial r}\left\{r^2\frac{\partial p_\upsilon}{\partial r}v_s\right\} \\ & -2\tau_s r^2\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial\left(r^2 p_s v_s\right)}{\partial r}\right)-\tau_s\frac{\partial}{\partial r}\left(r^2\frac{\partial\left(p_s v_s\right)}{\partial r}\right)=0. \end{aligned} \tag{10.8}$$

$$\begin{aligned} & \frac{\partial}{\partial r}\left\{r^2 v_s\left(\rho v_{0r}^2+5p_s\right)\right\}-\tau_s\frac{\partial}{\partial r}\left\{\frac{\partial}{\partial r}\left(r^2\left(\rho_s v_s^2+7p_s\right)v_s^2\right)\right\} \\ & +\tau_s\frac{\partial}{\partial r}\left\{3r^2\frac{\partial p_\upsilon}{\partial r}v_s^2+5r^2\frac{p_s}{\rho_s}\frac{\partial p_\upsilon}{\partial r}\right\}-2r^2\frac{\partial p_\upsilon}{\partial r}v_s \\ & +2\tau_s\frac{1}{\rho_s}\frac{\partial p_\upsilon}{\partial r}\frac{\partial}{\partial r}\left(r^2\rho_s v_s^2\right)+2\tau_s r^2\frac{1}{\rho_s}\frac{\partial p_\upsilon}{\partial r}\left(\frac{\partial p_s}{\partial r}-\frac{\partial p_\upsilon}{\partial r}\right) \\ & -\tau_s\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(p_s v_s^2\right)\right)-5\tau_s\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(\frac{p_s{}^2}{\rho_s}\right)\right)=0. \end{aligned} \tag{10.9}$$

For the PV description we use

$$R_\upsilon=\frac{\partial p_\upsilon}{\partial r}\,, \tag{10.10}$$

$$\frac{\partial}{\partial r}\left[3p_\upsilon\frac{\partial v_\upsilon}{\partial r}+v_\upsilon\frac{\partial p_\upsilon}{\partial r}+\frac{4}{r}p_\upsilon v_\upsilon\right]-\frac{2}{r}\left[v_\upsilon\frac{\partial p_\upsilon}{\partial r}-p_\upsilon\frac{\partial v_\upsilon}{\partial r}\right]=0\,, \tag{10.11}$$

$$5p_\upsilon \frac{\partial v_\upsilon}{\partial r} + 3v_\upsilon \frac{\partial p_\upsilon}{\partial r} + \frac{10}{r} p_\upsilon v_\upsilon - \tau_\upsilon \frac{\partial}{\partial r}\left[16 p_\upsilon v_\upsilon \frac{\partial v_\upsilon}{\partial r} + 5v_\upsilon^2 \frac{\partial p_\upsilon}{\partial r}\right] - \frac{14}{r^2}\tau_\upsilon p_\upsilon v_\upsilon^2$$
$$-20\frac{1}{r}\tau_\upsilon v_\upsilon^2 \frac{\partial p_\upsilon}{\partial r} - \frac{60}{r}\tau_\upsilon p_\upsilon v_\upsilon \frac{\partial v_\upsilon}{\partial r} + 4\tau_\upsilon v_\upsilon \frac{\partial p_\upsilon}{\partial r}\frac{\partial v_\upsilon}{\partial r} - \frac{5}{r^2}\frac{\partial}{\partial r}\left[\tau_s r^2 \frac{\partial}{\partial r}\frac{p_s^2}{\rho_s}\right] = 0 \quad (10.12)$$

We use the system of equation written in the dimensionless form using the scales

$$[r_0],\ [u_0],\ [p_0],\ [F] = \frac{p_0}{r_0},\ [t_0] = \frac{r_0}{u_0},\ r_0 = u_0 t_0 .$$

In particular

$$\tilde{R}_\upsilon = \frac{\partial \tilde{p}_\upsilon}{\partial \tilde{r}}, \quad (10.13)$$

$$\frac{\partial}{\partial \tilde{r}}\left[3\tilde{p}_\upsilon \frac{\partial \tilde{v}_\upsilon}{\partial \tilde{r}} + \tilde{v}_\upsilon \frac{\partial \tilde{p}_\upsilon}{\partial \tilde{r}} + \frac{4}{\tilde{r}}\tilde{p}_\upsilon \tilde{v}_\upsilon\right] - \frac{2}{\tilde{r}}\left[\tilde{v}_\upsilon \frac{\partial \tilde{p}_\upsilon}{\partial \tilde{r}} - \tilde{p}_\upsilon \frac{\partial \tilde{v}_\upsilon}{\partial \tilde{r}}\right] = 0, \quad (10.14)$$

In this case system of Equations (10.7)-(10.12) written in the dimensionless form contains the dimensionless parameter, $\mathrm{T} \leftrightarrow \tilde{\tau}$ and need ten Cauchy conditions. These conditions we write down for the external surface of the spherical object. Then we investigate the evolution of the surface perturbation on the following scenario of the PV + Matter behavior.

We take into account the possible variations of the PV nonlocal parameter τ_υ. In this case we should introduce two nonlocal parameters for Matter τ_s and PV τ_υ. I underline again that nonlocal parameters in nonlocal physics play the same role as kinetic coefficients in usual local Boltzmann kinetic theory.

The Maple program was used in calculations ($\tau_\upsilon \neq 0$) including the energy income. In this case we need the values without taking into account the influence of PV. The Maple notations were used:

$v_s \to$ u —matter velocity with PV, v—matter velocity without PV;

$p_s \to$ p —matter pressure with PV, w—matter pressure without PV;

$\rho_s \to$ r —matter density with PV, m—matter density without PV.

For the surrounding PV motion we use:

$v_\upsilon \to$ s —PV velocity, $p_\upsilon \to$ q —PV pressure, $\tilde{r} \to$ t, nonlocal parameter for Matter $\tilde{\tau}_s \to$ T, nonlocal parameter for PV $\tilde{\tau}_\upsilon \to$ A. Let us introduce the energy income from PV to Matter defined as

$$a(t) = \left(w(t) + m(t) * v(t)^2\right) - \left(p(t) + r(t) * u(t)^2\right). \quad (10.15)$$

In the calculations we should use parameters T, A and of course Cauchy conditions which could be found in the program text.

w(1) = 1, v(1) = 1, m(1) = 1, D(w)(1) = 1, D(v)(1) = 1, D(m)(1) = 1, a(1) = 0, p(1) = 1, D(p)(1) = 1, q(1) = 1, D(q)(1) = 1, r(1) = 1, D(r)(1) = 0, u(1) = 1, D(u)(1) = 0, s(1) = 1, D(s)(1) = 1.

The **Figures 19-34** reflects the corresponding calculations including the rate of income $\frac{\partial \tilde{a}}{\partial \tilde{r}}$. Figures contain the boundary of the solution existing.

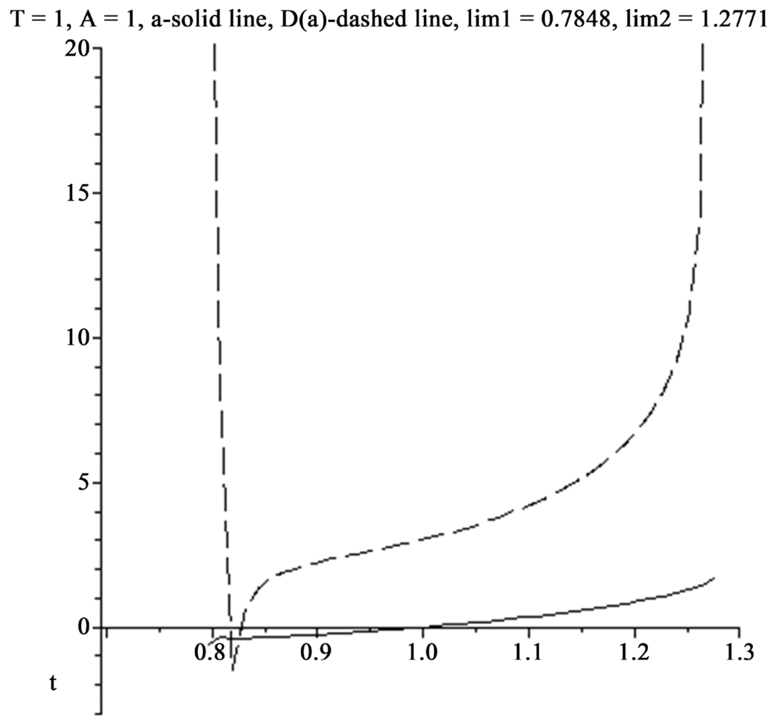

Figure 19. Energy income $\tilde{a}(\tilde{r})$, the rate of the energy income $\frac{\partial \tilde{a}}{\partial \tilde{r}}$, $\mathrm{T} = \mathrm{A} = 1$.

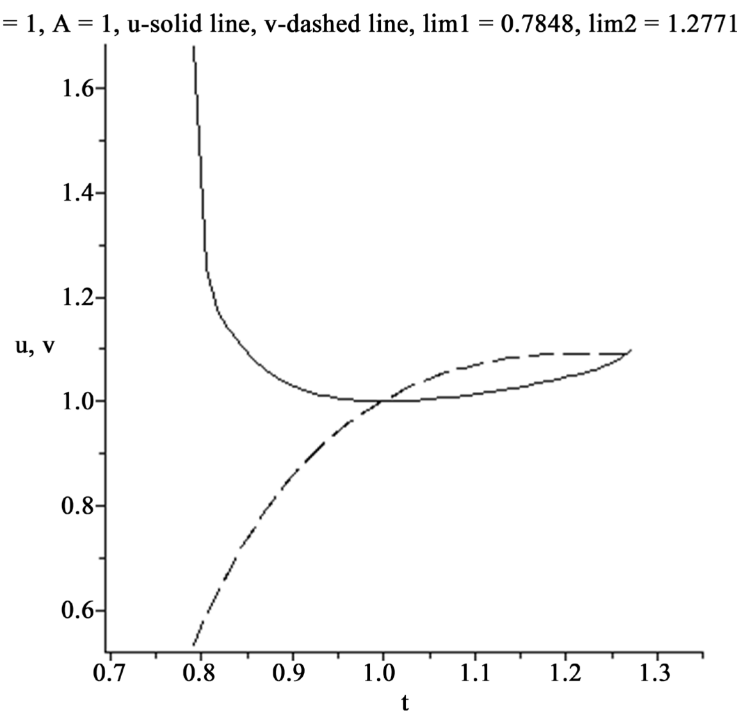

Figure 20. Evolution of matter velocity $\tilde{v}_s(\tilde{r})$ (u(t) - with taking into account PV, v(t)) - without PV influence), $\tilde{\tau} = 1$, $\mathrm{A} = 1$.

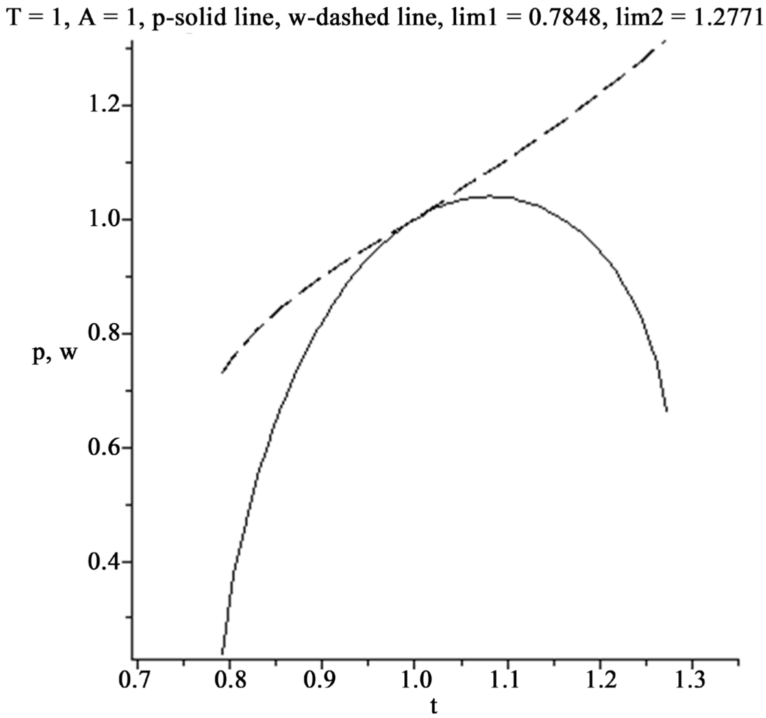

Figure 21. Evolution of matter pressure $\tilde{p}_s(\tilde{r})$ (p(t) with PV influence, w(t) without PV influence), $\tilde{\tau}=1$, $A=1$.

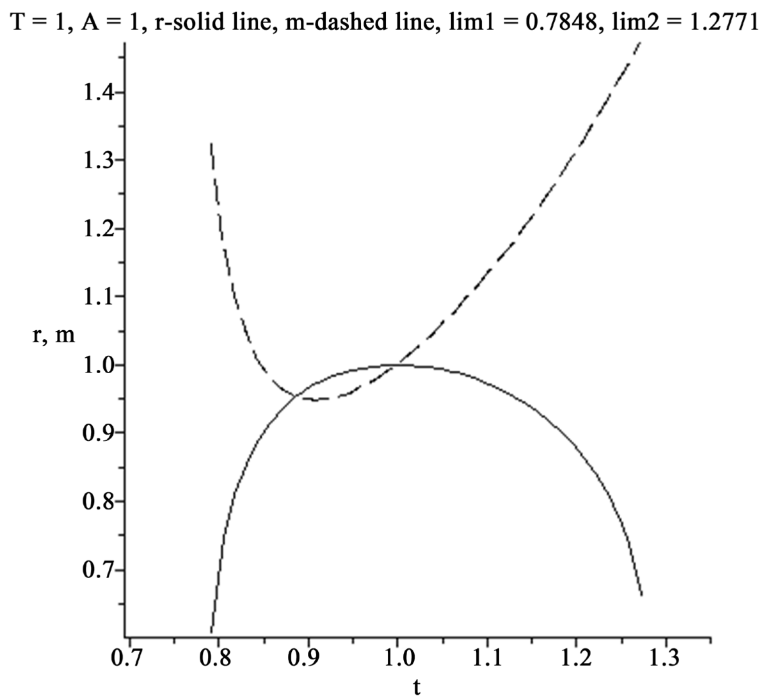

Figure 22. Evolution of matter density $\tilde{\rho}_s(\tilde{r})$ (r(t) - with PV influence, m(t)) without PV influence), $\tilde{\tau}=1$, $A=1$.

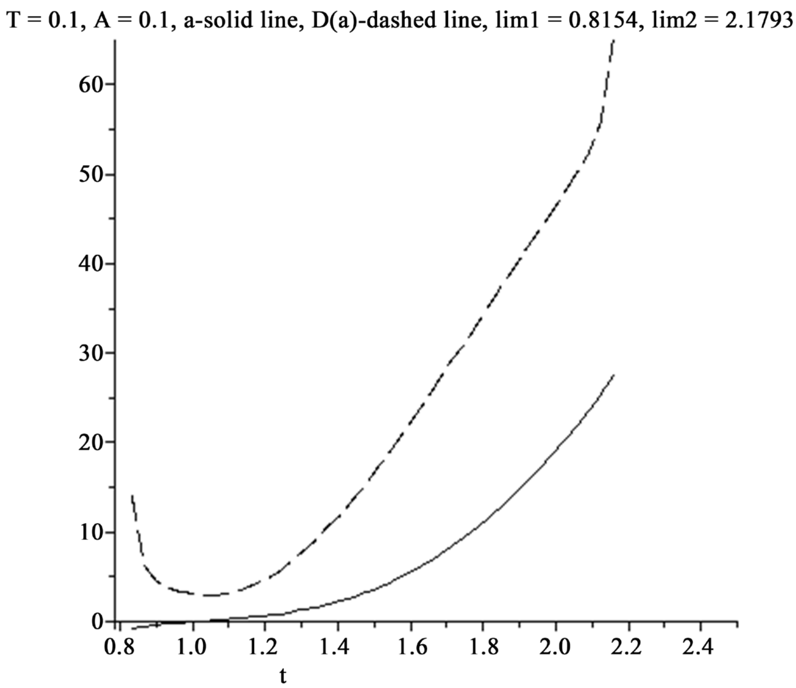

Figure 23. Energy income $\tilde{a}(\tilde{r})$, the rate of the energy income $\frac{\partial \tilde{a}}{\partial \tilde{r}}$, $\mathrm{T} = \mathrm{A} = 0.1$.

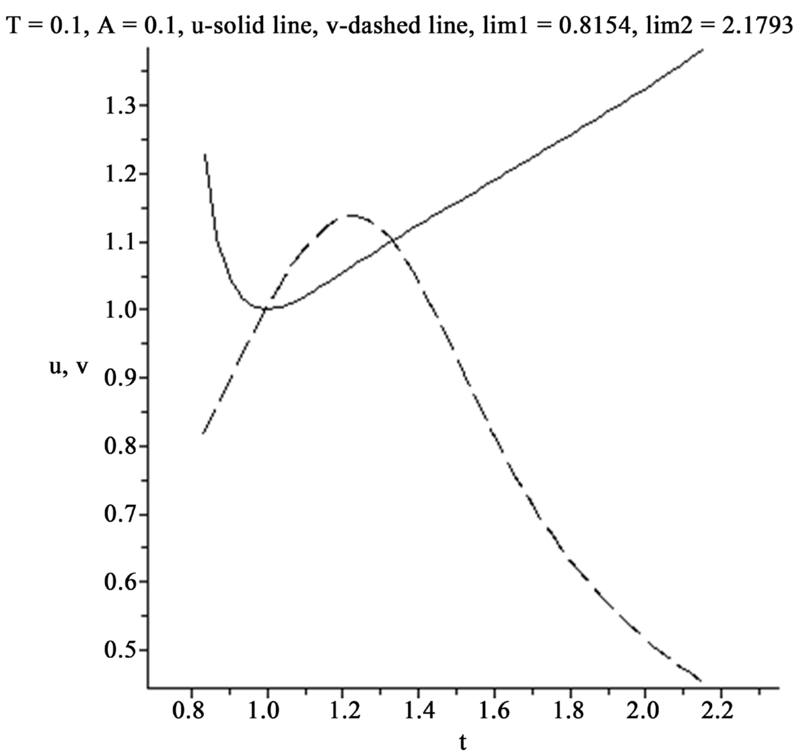

Figure 24. Evolution of matter velocity $\tilde{v}_s(\tilde{r})$ (u(t) with PV influence, v(t) without PV influence), $\tilde{\tau} = 0.1$, $\mathrm{A} = 0.1$.

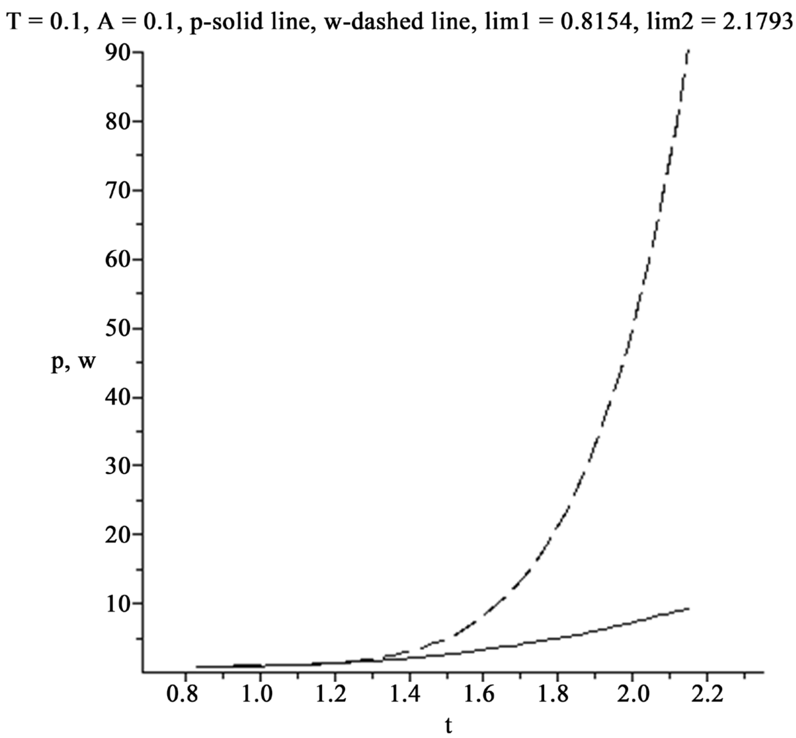

Figure 25. Evolution of matter pressure $\tilde{p}_s(\tilde{r})$ (p(t) - with PV influence, w(t) - without PV influence, $\tilde{\tau} = 0.1$, $A = 0.1$.

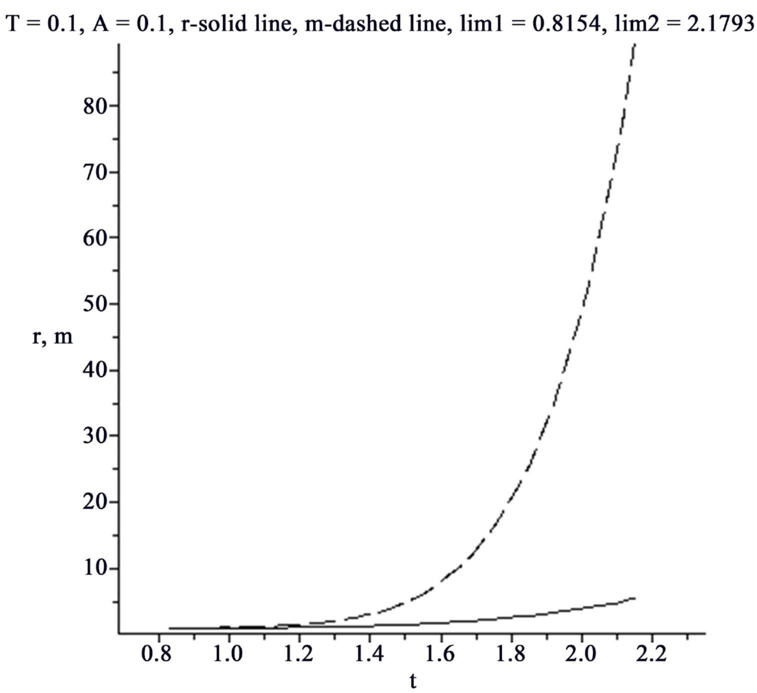

Figure 26. Evolution of matter density $\tilde{\rho}_s(\tilde{r})$ (r(t) with PV influence, m(t)) without PV influence), $\tilde{\tau} = 0.1$, $A = 0.1$.

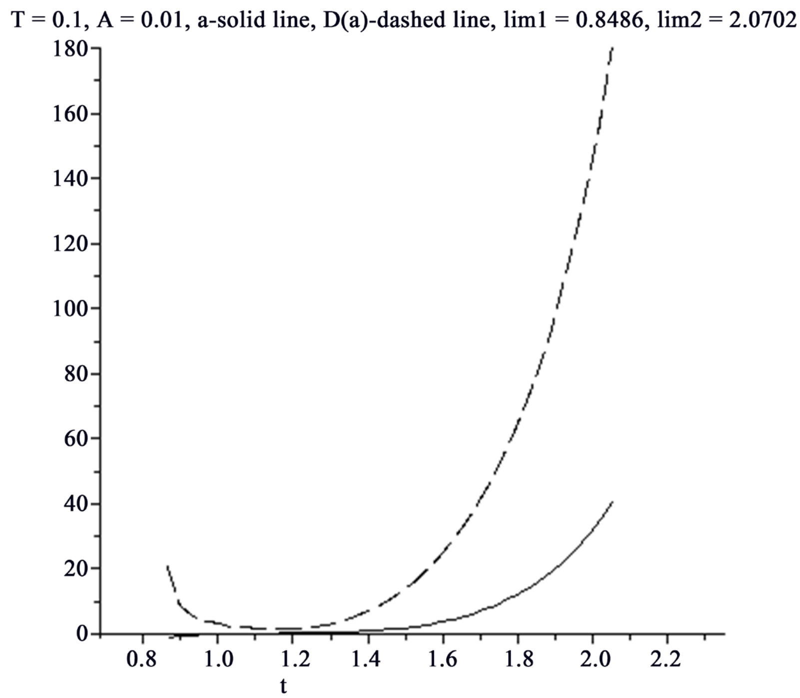

Figure 27. Energy income $\tilde{a}(\tilde{r})$, the rate of the energy income $\frac{\partial \tilde{a}}{\partial \tilde{r}}$, $\mathrm{T} = 0.1$, $\mathrm{A} = 0.01$.

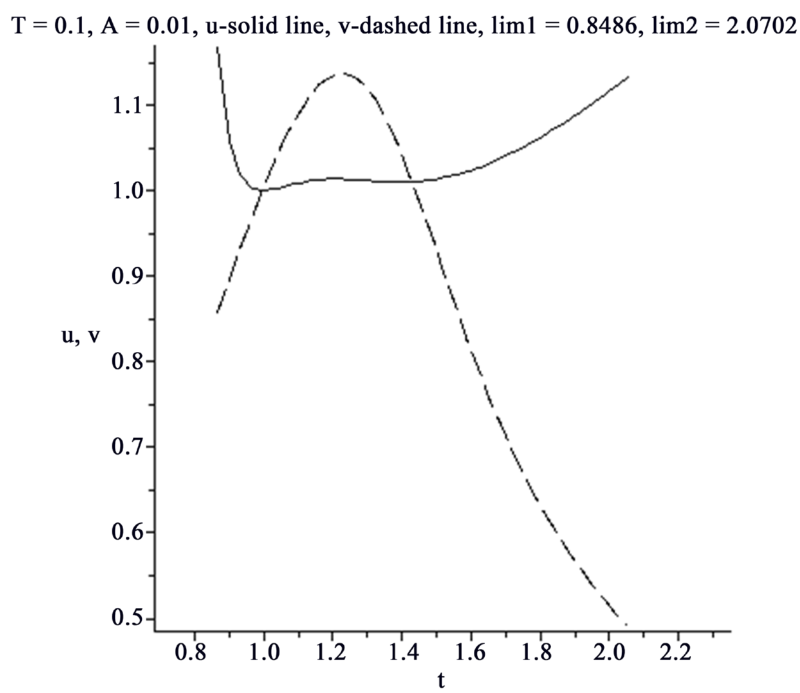

Figure 28. Evolution of matter velocity $\tilde{v}_s(\tilde{r})$ (u(t) - with PV influence, v(t) - without PV influence), $\tilde{\tau} = 0.1$, $\mathrm{A} = 0.01$.

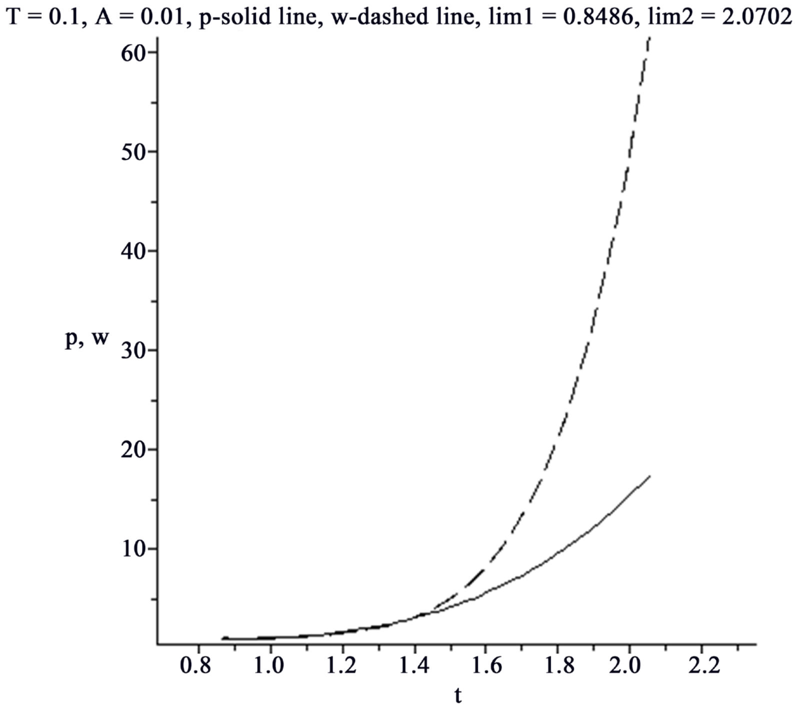

Figure 29. Evolution of matter pressure $\tilde{p}_s(\tilde{r})$ (p(t)) - with PV influence, w(t) - without PV influence), $\tilde{\tau} = 0.1$, $A = 0.01$.

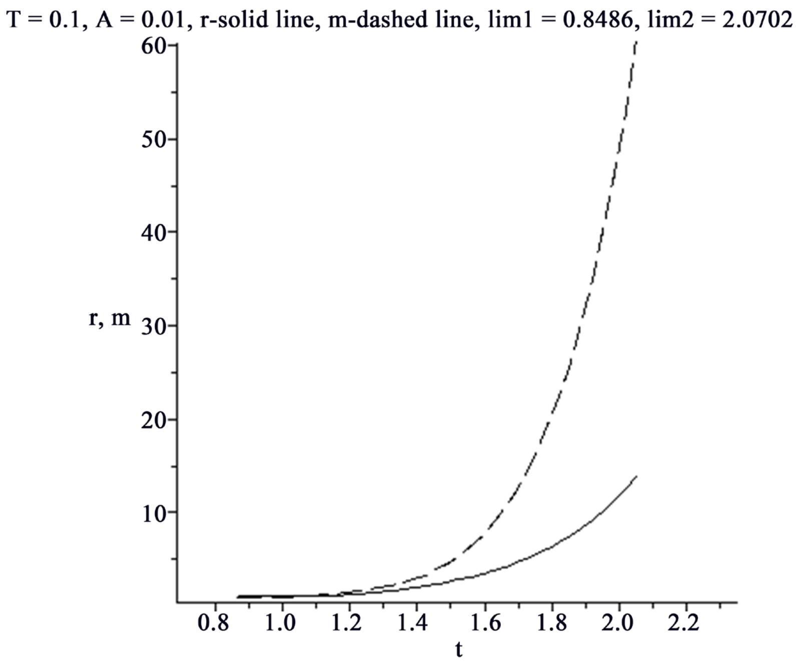

Figure 30. Evolution of matter density $\tilde{\rho}_s(\tilde{r})$ (r(t) - with PV influence, m(t) - without PV influence), $\tilde{\tau} = 0.1$, $A = 0.1$.

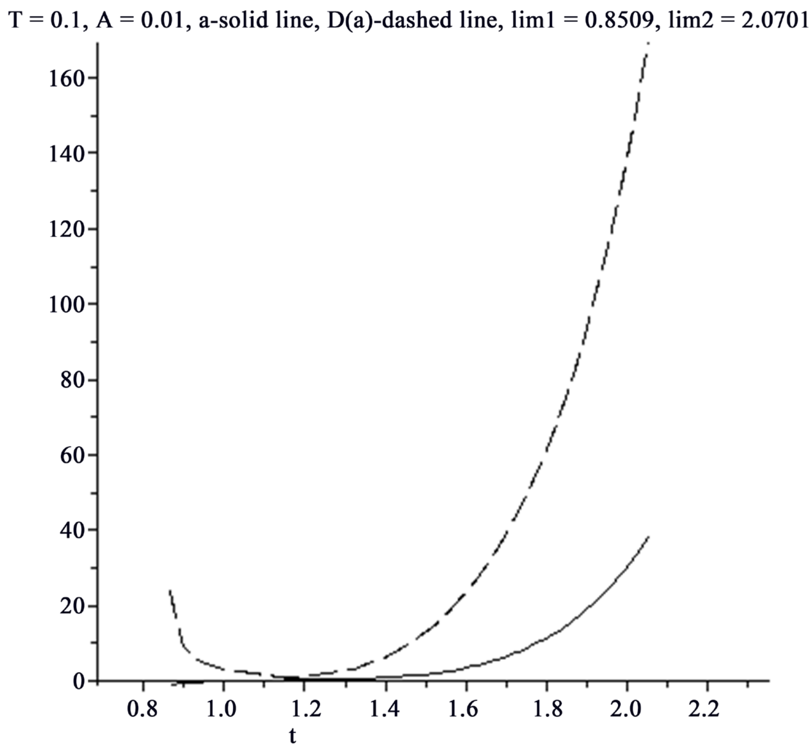

Figure 31. Energy income $\tilde{a}(\tilde{r})$, the rate of the energy income $\frac{\partial \tilde{a}}{\partial \tilde{r}}$, $\mathrm{T}=0.1$, $\mathrm{A}=0.001$.

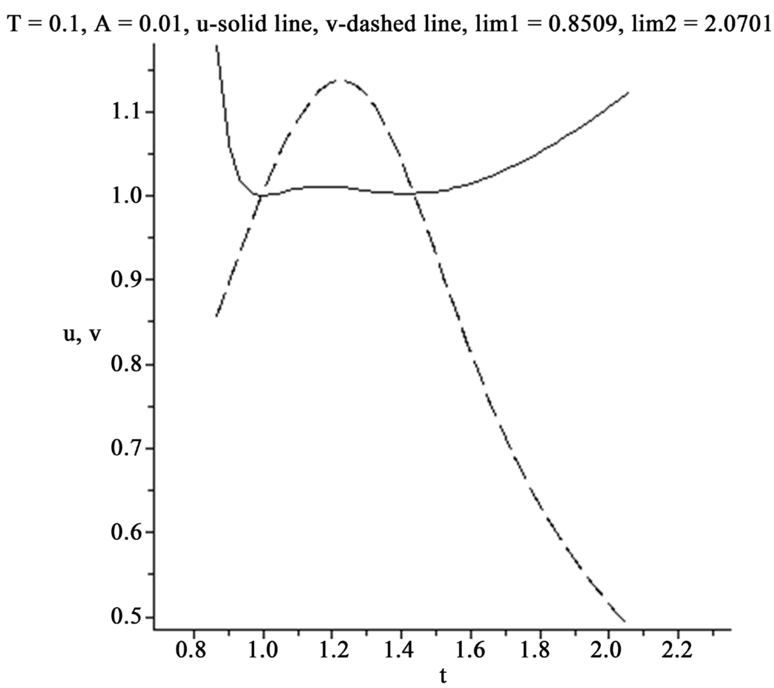

Figure 32. Evolution of matter velocity $\tilde{v}_s(\tilde{r})$ (u(t) with PV influence, v(t) without PV influence), $\tilde{\tau}=0.1$, $\mathrm{A}=0.001$.

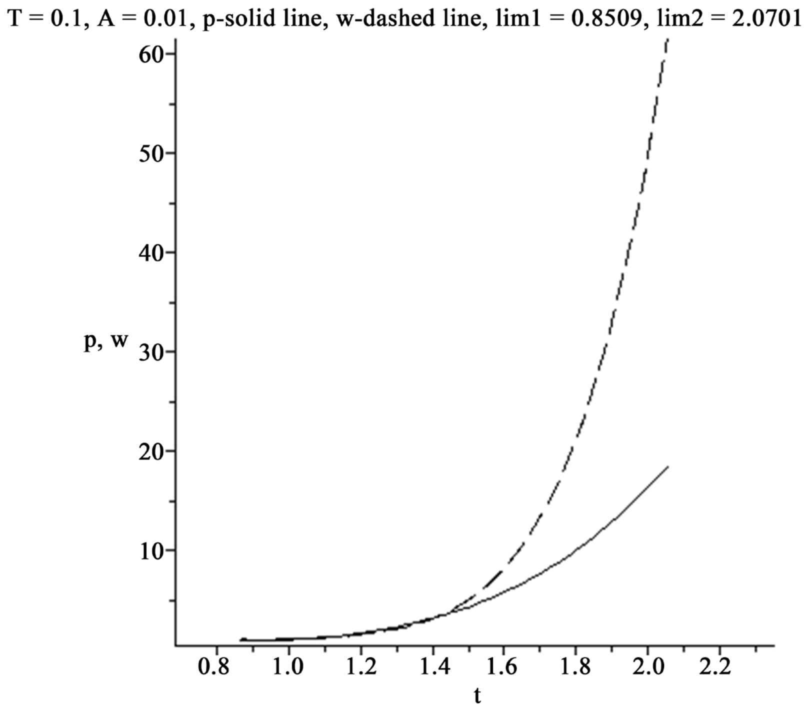

Figure 33. Evolution of matter pressure $\tilde{p}_s(\tilde{r})$ (p(t) with PV influence, w(t) without PV influence), $\tilde{\tau} = 0.1$, $A = 0.001$.

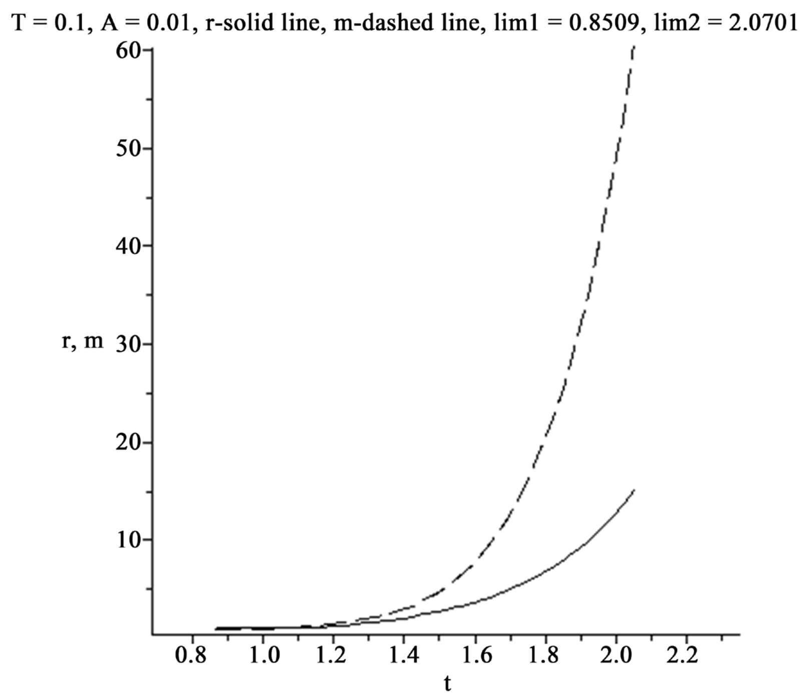

Figure 34. Evolution of matter density $\tilde{\rho}_s(\tilde{r})$ (r(t) with PV influence, m(t) without PV influence, $\tilde{\tau} = 0.1$, $A = 0.001$.

Conclusion to Item 10:

1) In many cases mentioned calculations lead to so to speak "volume quantization"—the solutions exist only in the finite domain of space, (see for example Figures 19-34).

2) The left boundary of the solution existing can be smaller than the sphere boundary $\tilde{r} = 1$.

3) The linear size of these domains demonstrate the weak dependence on nonlocal parameters $\tilde{\tau}_s$ and $\tilde{\tau}_\upsilon$. From the other side the parameters of the energy income show the strong dependence on nonlocal parameters. The nonlocal theory can show the explosion of object which size is significantly less than the size of the visible Universe.

4) Extremely important that the direct calculations demonstrate *the possibility of the energy income from PV to Matter*.

11. Final Remarks: The Destiny of Anti-Matter after Big Bang (BB) and Conclusion

The problem antimatter evolution is considered from positions of the Newtonian theory of gravitation and non-local kinetic theory. It is found that disappearing of antimatter after Big Bang can be explained as a result of antimatter interaction with physical vacuum. This interaction leads to appearance of the antimatter particles which cross-sections are significantly less than the cross-sections of the ordinary matter. As a result the main part of antimatter is concentrated now on the out part of the visible Universe.

Application nonlocal physics to the problem of the dark matter existence leads to affirmation – dark matter does not exist.

Physical Vacuum (PV) is not a speculative object; it is a reality as "matter" and "fields". In other words, the physical vacuum is "the third" physical reality along with matter and fields. At an early stage in the development of the universe the value of the PV parameter is closer to zero. As a result the antimatter particles having the small cross sections, leaves the central part of the BB domain. Anti-matter particles after the Big Bang are placed now mainly on the periphery of the Universe. We observe now the effects of the matter antimatter annihilations on the periphery of our Universe.

The birth of the universe is convoying of appearance of the repulsion forces. In the existing terminology we discover the "negative pressure" and "dark energy" in all cases. This fundamental result does not depend on the mechanism of external perturbations. In other words, the anti-gravity in the physical vacuum exists, if there is dissipation of energy or in the absence of dissipation at all.

We find the solutions of the transport equations defining the evolution the physical vacuum (PV).

It means:

1) If the matter is absent, non-local evolution equations have nevertheless non-trivial solutions corresponding evolution of PV which description in time

and 3D space on the level of quantum hydrodynamics demands only quantum pressure p, the self-consistent force $\mathbf{F}$ (acting on unit of the space volume) and velocity $\mathbf{v}_0$. The system of non local equations is written for the case when the usual matter is absent ($\rho = 0$), also radiation, gravitation (as well as other mass forces) and electromagnetic fields are absent.

2) In all other cases we consider from the position of the nonlocal physics the interaction of Physical Vacuum with the external electromagnetic and gravitational fields taking into account the possible technical applications like EM-engine.

3) The general relativity (GR) and the special relativity (SR) are not applicable for the description of processes in Physical Vacuum.

4) An image of the entire universe consists mostly of colossal collections of galaxies interspersed with vast empty spaces, known as voids. The appearance of void is usually associated with Big Bang and appearance of the Cold Spot. Correlation between CMB fluctuations, voids and clusters (Kovács *et al.* 2017) was found including Cold Spot related to Eridanus Supervoid (Szapudi *et al*, 2015). But there many observed voids. Cosmic voids are vast spaces which contain very few or no galaxies. Voids typically have a diameter of 10 to more than 100 megaparsecs. The existence of limited spherical formations in the Universe filled with interacting matter and physical vacuum is the justification for the existence of voids and parallel universes.

5) It means that we should wait for the tremendous discoveries in the nonlocal theory of voids. In particular, some voids may have been formed by their own local PV explosions (like Big Bang). But the result of such local PV explosions could lead to the appearance of so-called "parallel universes". These post-explosion universes may have had a different set of fundamental physical constants and therefore a different flow of time. Then we should search the voids with abnormal transport processes (from the point of view of the Earth science).

6) Direct calculations demonstrate *the possibility of the energy income from PV to Matter*.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Boltzmann, L. (1872) *Sitz. Ber. Kaiserl. Akad. Wiss*, **66**, No. 2, s.275.

[2] Alekseev, B.V. (1982) Matematicheskaya Kinetika Reagiruyushchikh Gazov (Ma-

thematical Theory of Reacting Gases). Nauka, Moscow.

[3] Alexeev, B.V. (2004) Generalized Boltzmann Physical Kinetics. Elsevier, Amsterdam, 368 p. https://doi.org/10.1016/B978-044451582-7/50027-8

[4] Alexeev, B.V. (2015) Unified Non-Local Theory of Transport Processes. Elsevier, Amsterdam, 644 p.

[5] Alexeev, B.V. (2016) Unified Non-Local Relativistic Theory of Transport Processes. Elsevier, Amsterdam, 455 p.

[6] Alexeev, B.V. (2017) Nonlocal Astrophysics. Dark Matter, Dark Energy, Physical Vacuum. Elsevier, Amsterdam, 454 p.

[7] Alexeev, B.V. (1994) *Philosophical Transactions of the Royal Society of London*, **349**, 417-443. https://doi.org/10.1098/rsta.1994.0140

[8] Alexeev, B.V. (1995) *Physica A*, **216**, 459-468. https://doi.org/10.1016/0378-4371(95)00044-8

[9] Madelung, E. (1927) *Zeitschrift für Physic*, **40**, 322-326. https://doi.org/10.1007/BF01400372

[10] Bell, J.S. (1964) *Physics*, **1**, 195. https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195

[11] Alexeev, B.V. (2008) *Journal of Nanoelectronics and Optoelectronics*, **3**, 143-158. https://doi.org/10.1166/jno.2008.207

[12] Alexeev, B.V. (2008) *Journal of Nanoelectronics and Optoelectronics*, **3**, 316-328. https://doi.org/10.1166/jno.2008.311

[13] Reinders, L.J. (2018) The Life, Science and Times of Lev Vasilevich Shubnikov. Springer International Publishing, Cham. https://doi.org/10.1007/978-3-319-72098-2

[14] Abrikosov, A.A. (1957) *Journal of Experimental and Theoretical Physics*, **5**, 1174.

[15] Nikulov, A.V. (1999) The Vortex Lattice Melting Theory as Example of Science Fiction. In: Ausloos, M. and Kruchinin, S., Eds., *NATO Science Series*: *Symmetry and Pairing in Superconductors*, Kluwer Academic Publishers, Dordrecht, 131. http://arXiv.org/abs/cond-mat/9811051 https://doi.org/10.1007/978-94-011-4834-4_12

[16] Nikulov, A.V. (1999) What Is the Vortex Lattice Melting, Reality or Fiction? In: Kossowski, R., *et al*., Eds., *NATO Science Series*: *Physics and Materials Science of Vortex States*, *Flux Pinning and Dynamics*, Kluwer Academic Publishers, Dordrecht, 609. http://arXiv.org/abs/physics/0202021 https://doi.org/10.1007/978-94-011-4558-9_31

[17] Kleiner, W.H., Roth, L.M. and Autler, S.H. (1964) *Physical Review A*, **133**, 1226. https://doi.org/10.1103/PhysRev.133.A1226

[18] Maki, K and Takayama H. (1971) *Progress of Theoretical Physics*, **46**, 1651-1665. https://doi.org/10.1143/PTP.46.1651

[19] Nikulov, A.V., Remisov, D.Yu. and Oboznov V.A. (1995) *Physical Review Letters*, **75**, 2586. https://doi.org/10.1103/PhysRevLett.75.2586

[20] Nikulov, A.V., Dubonos, S.V. and Koval, Y.I. (1997) *Journal of Low Temperature Physics*, **109**, 643-652. https://doi.org/10.1007/BF02435484

[21] Nikulov, A.V. (1995) *Physical Review B*, **52**, 10429-10432. https://doi.org/10.1103/PhysRevB.52.10429

[22] Alexeev, B.V. (2021) *Journal of Modern Physics*, **12**, 552-593. https://www.scirp.org/journal/jmp https://doi.org/10.4236/jmp.2021.125037

[23] Michelson, A.A. and Morley, E.W. (1887) *American Journal of Science*, **34**, 333-345. https://doi.org/10.2475/ajs.s3-34.203.333

[24] Lorentz, H.A. (1901) Aether Theories and Aether Models (1901-1902). Lectures on Theoretical Physics Delivered at the University of Leyden, Volume 1. Edited by H. Bremecamp.

[25] Logunov, A.A., Loskutov, Yu.M. and Mestvirishvili, M.A. (1987) *TMF*, **73**, 163-186. https://doi.org/10.1007/BF01017582

[26] Laplace, P.S. (1966) Mechanique Celeste. English Transl. Reprinted by Chelsea Publ., New York.

[27] Poincaré, J.H. (1983) About Science. Nauka, Moscow.

[28] Alexeev, B.V. (2018) Extremal States in Nonlocal Physics. Lambert Academic Publishing, Riga.

[29] Prigogine, I. (1962) Introduction to Thermodynamics of Irreversible Processes. 2nd Edition, Wiley Interscience, London.

[30] Nikolis, G. and Prigogine, I. (1977) Self-Organization in Nonequilibrium Systems. Wiley-Interscience Publication, Hoboken.

[31] Rubakov, V.A. (2019) *Science and Life*, **11**, 46-50.

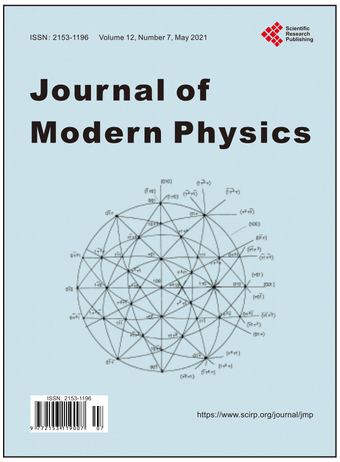
[32] Penrose, R. (2011) The Road to Reality: A Complete Guide to the Laws of the Universe. Knopf (US), New York, Bodley Head, London.

[33] Rubin, V. and Ford, W.K. (1970) *Astrophysical Journal*, **159**, 379. https://doi.org/10.1086/150317

[34] Rubin, V., Thonnard, N. and Ford, W.K. (1980) *Astrophysical Journal*, **238**, 471. https://doi.org/10.1086/158003

[35] Zwicky, F. (1933) *Helvetica Physica Acta*, **6**, 110-127.

[36] Zwicky, F. (1937) *Astrophysical Journal*, **86**, 217. https://doi.org/10.1086/143864



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