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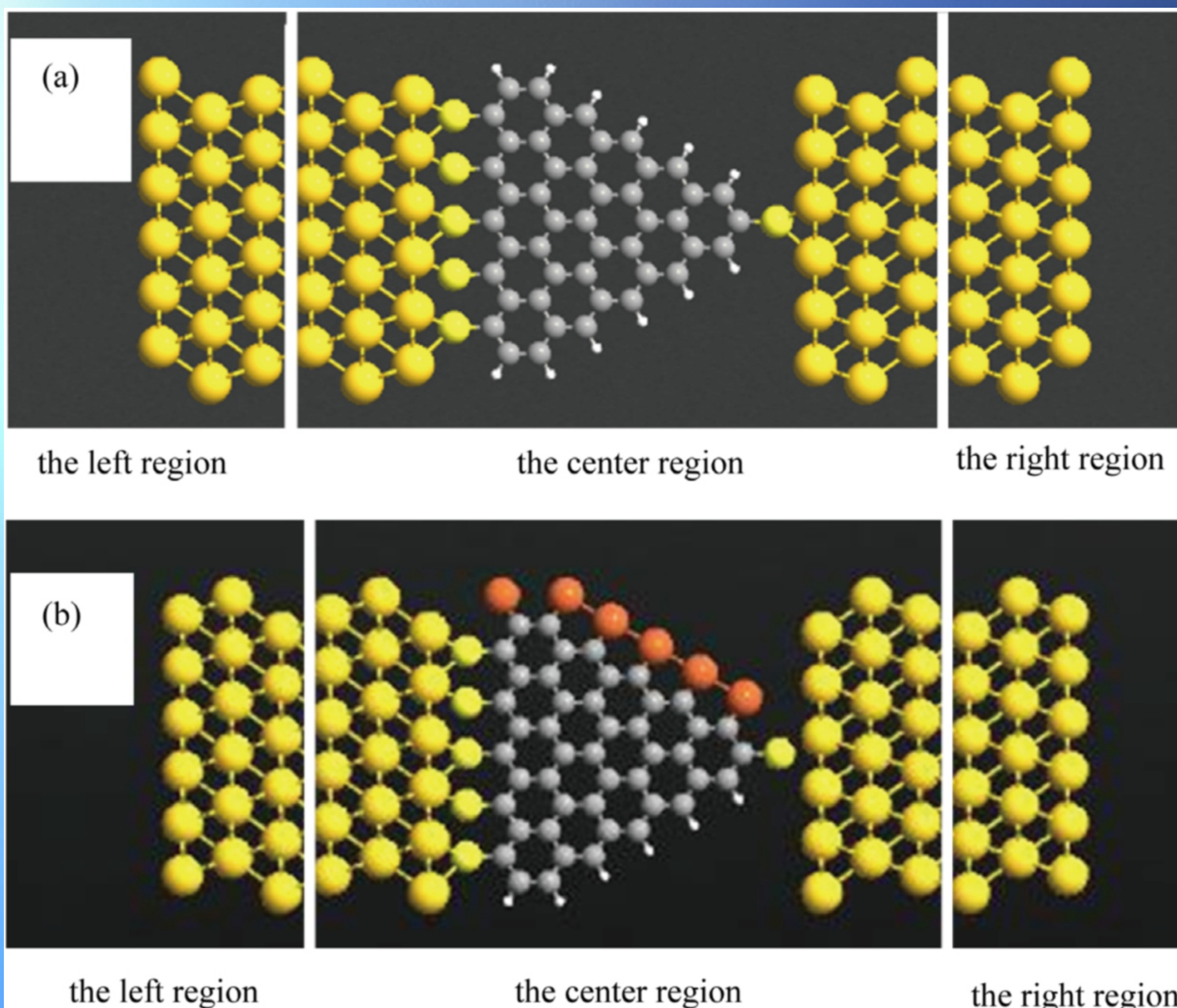

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Size of the Electron Microparticle Calculated from the Oersted Law

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Abstract

An attempt to obtain a new theoretical derivation of the size of the electron microparticle has been done. To this purpose first the Maxwell equation for the electron current has been examined for the case of the one-electron current present in the Bohr model of the hydrogen atom. It has been shown that the equation is satisfied on condition that the microstructure properties of the electron particle are taken into account. In the next step, the quanta of the magnetic field characteristic for the Bohr atom and the electron time periods specific for the electron current along the orbits were substituted in place of parameters entering the classical Oersted equation. This gives an expression for the cross-section radius of the orbits not much different than results for the radius of the electron microparticle obtained in a former electron theory.

Keywords

Cross-Section Area of the Electron Orbits and the Radius of the Electron Microparticle, Hydrogen Atom, Oersted Equation

1. Introduction

In order to obtain any classical property of an elementary microparticle, say the electron, an approach combined of both the quantum and classical physical laws seems to be necessary. In the present case—when the classical size parameter of the electron particle is aimed to be deduced—the quantum aspects can be provided by the Bohr model of the hydrogen atom. Here—for any quantum state—we have a definite orbital motion of a single electron in the electrostatic field of a positively charged proton nucleus. The motion—beyond of its orbital track—has well-defined velocity and energy parameters. However, in order to make use of the equations of classical electrodynamics, especially the Maxwell equations, knowledge of the magnetic field—together with the electric field—in the atom seems to be necessary.

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However both of the Maxwell equations—that consider the change of the magnetic induction and that concern the electric line current only (by assuming that the displacement current can be neglected)—take into account the time parameter on different footing [1]-[4]. In the first equation the time action is reduced to the use of a short interval representing the derivative of the magnetic flux with respect to time; in the second equation the time interval enters solely the current velocity, which can be assumed to be a constant term along an arbitrarily long quantity of time. This produces a stationary electric current whose charge density $e\varrho$ satisfies the equation

$$\partial \frac{e\varrho}{\partial t} = 0. \tag{1}$$

If we assume the Bohr theory as valid for the hydrogen atom, the electric current given by the one-electron orbital motion is fully stationary for any chosen quantum level n. The velocity of the current composed of a single electron particle is [5]

$$v_n = \frac{2\pi r_n}{T_n} = \frac{2\pi n^2\hbar^2}{me^2}\frac{me^4}{2\pi n^3\hbar^3} = \frac{e^2}{n\hbar} \tag{2}$$

since

$$l_n = 2\pi r_n = 2\pi \frac{n^2\hbar^2}{me^2} \tag{3}$$

is the orbit length and

$$T_n = \frac{2\pi n^3\hbar^3}{me^4} \tag{4}$$

is the time period necessary to travel the distance l_n about the atomic nucleus.

The aim of the present paper is, in the first step, to point out that the Maxwell equation concerning the electric line current

$$\boldsymbol{j} = e\boldsymbol{v}\varrho, \tag{5}$$

where $\boldsymbol{v}$ is the electron velocity and ϱ is the density of the electron particle, can be satisfied only when the microstructure properties of the electron particle are taken into account.

To this purpose we consider the quanta of the magnetic field H_n neglected in the original Bohr model [5] [6]. These quanta—introduced in Section 2—seem to be of importance (see [7]) because they lead to the quanta of the magnetic flux identical with those known experimentally since a long time in superconductors [8] [9]. Moreover, a combination of the electric and magnetic field present in the atom gives the Poynting vector which approximately provides us with a proper rate of the energy emission due to the process of the electron transition between two quantum levels [10]. In Section 3 we show that the quanta H_n fulfill the Maxwell equation for the electric current with a satisfactory accuracy.

In the next step, in Section 4, the quanta of the magnetic field H_n—which are due to the electron orbital motion in the atom—are substituted into the equation representing the Oersted law:

$$\oint H_n \mathrm{d}\boldsymbol{s} = H_n 2\pi r = \frac{4\pi}{c} I_n. \tag{6}$$

Here the path of $\mathrm{d}\boldsymbol{s}$ circumvents the circular cross-section area of the orbit, the area is assumed to have the radius r: it defines the surface of the orbital conductor at which the magnetic field is equal to H_n (see Figure 43 in Ref. [2]). Since any orbit can be occupied solely by a single electron particle, r should be independent of the quantum index n. Symbol I_n in (6) represents the current intensity which is coupled with the circulation time period T_n of the electron along the orbit n by the formula

$$I_n = \frac{e}{T_n}. \tag{7}$$

It will be found that the indices n in (6) cancel together leaving the formula for the cross-section radius r of the orbit independent of n. This r is expected to approach the radius r_e of the electron particle moving along the orbit.

2. Maxwell Equation for the Electric Current and the Magnetic Field in the Hydrogen Atom

The Maxwell equation is written briefly in the form

$$\nabla \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j} \tag{8}$$

but it seems to be more convenient to apply an integral form of (8) which is

$$\oint \boldsymbol{H}\,\mathrm{d}l = H_n l_n = \frac{4\pi}{c}\int \boldsymbol{j}\,\mathrm{d}\,\boldsymbol{f}. \tag{9}$$

The magnetic field H_n in (9) is a constant term for a given n; see below. The length l_n is given in (3). It should be noted that the integral on the left of (9) does not concern the dot product of $\boldsymbol{H}_n$ and $\boldsymbol{dl}$, but is the integral of $\boldsymbol{H}_n$ extended over the line having the length l_n [2].

The H_n can be obtained as a result of a constant electric current on the level n if we note that the current is surrounding periodically the nucleus with the frequency

$$\Omega_n = \frac{2\pi}{T_n} \tag{10}$$

where T_n is given in (4). On the other hand, the Ω_n is coupled with H_n by the formula [7] [11]

$$\Omega_n = \frac{eH_n}{mc}. \tag{11}$$

This is an effect of the Lorentz force law in which the wave-vector $\boldsymbol{k}$ of the electron particle satisfies the relation

$$\hbar\frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}t} \approx \hbar\frac{\Delta k}{T} = \frac{e}{c}\left[\boldsymbol{H}_n \times \boldsymbol{v}_n\right] = \frac{e}{c}H_n v_n. \tag{11a}$$

The last step in (11a) is due to the fact that the magnetic field is normal to the velocity vector along the orbit. For a full circulation time T_n we have $\Delta k = 2\pi k_n$. Since $\hbar k_n = mv_n$ we obtain from (11a) the relation

$$\frac{\hbar 2\pi k_n}{T_n} = mv_n\Omega_n = \frac{e}{c}H_n v_n$$

identical with (11).

A substitution of T_n from (4) into (10) gives together with (11) the equation

$$\frac{me^4}{n^3\hbar^3} = \frac{eH_n}{mc}$$

from which we obtain

$$H_n = \frac{m^2e^3c}{n^3\hbar^3}. \tag{12}$$

It is interesting to note that H_n in (12) can be obtained also from the theory of the cyclotron resonance in metals [11]. We have the relation [11]

$$\Omega_n = \frac{2\pi c}{eH_n}\frac{|E_n|}{S_n}$$

where

$$|E_n| = \frac{me^4}{2\hbar^2n^2} \tag{13}$$

is the absolute electron energy in state n [5], and

$$S_n = \pi r_n^2 \tag{14}$$

is the area occupied by the electron orbit in that state. Here the constants $|E_n|$ and S_n replace respectively the energy interval ΔE and area interval ΔS in the real space admitted in course of the change of the quantum state *n*. A substitution of $|E_n|$ from (13) and S_n from (14) valid for the hydrogen atom gives together with the formula (11) for $\Omega = \Omega_n$ the relation:

$$\frac{eH}{mc} = \Omega = \frac{2\pi c}{eH} \frac{\frac{me^4}{2\hbar^2 n^2}}{\frac{\pi\hbar^4 n^4}{m^2 e^4}} = \frac{2\pi c}{eH} \cdot \frac{me^4}{2\hbar^2 n^2} \cdot \frac{m^2 e^4}{\pi\hbar^4 n^4}.$$

This gives

$$H^2 = H_n^2 = \frac{e^6 m^4 c^2}{\hbar^6 n^6}$$

which yields the square value of H_n in (12).

3. Current Analysis Done with the Aid of a Microstructure Parameter of the Electron Particle

Usually, when the electron is considered as a charged particle having the radius r_e, the potential energy of the charge extended on a spherical surface is assumed to be approximately equal to the rest energy of the electron [2] [12]:

$$\frac{e^2}{r_e} \cong mc^2. \tag{15}$$

In effect

$$r_e \approx \frac{e^2}{mc^2}. \tag{16}$$

The current (5) is composed, first, of the volume *V* occupied by the electron particle, so

$$\rho = \frac{1}{V} = \frac{1}{\frac{4\pi}{3} r_e^3}, \tag{17}$$

next the same current should move within a tube having a cross-section area equal approximately to

$$\int \mathrm{d}\boldsymbol{f} \cong \pi r_e^2. \tag{18}$$

Since the velocity v_n for a given *n* is a constant [see (2)], we obtain for the right-hand side of (9) the formula

$$\frac{4\pi}{c} \int \boldsymbol{j} \mathrm{d}\boldsymbol{f} = \frac{4\pi}{c} e v_n \frac{1}{\frac{4\pi}{3} r_e^3} \pi r_e^2 = \frac{4\pi}{c} \frac{e^3}{n\hbar} \frac{3}{4} \frac{1}{r_e} = \frac{\pi}{c} \frac{e^3}{n\hbar} 3 \frac{mc^2}{e^2} = 3\pi \frac{emc}{n\hbar}. \tag{19}$$

The left-hand side of (9) is

$$H_n l_n = \frac{m^2 e^3 c}{n^3 \hbar^3} 2\pi \frac{n^2 \hbar^2}{me^2} = 2\pi \frac{emc}{n\hbar}. \tag{20}$$

A difference between the both sides of (9), or (19) and (20), is represented by the factor of 3/2.

4. The Quanta of the Magnetic Field and Time Periods Entering the Oersted Law Give the Radius of the Electron Microparticle

Any current is associated with the magnetic field and the lines of that field circumvent the line of the current. We assume that at the distance *r* from the center of the current cross-section area the field is H_n for any orbit *n*.

In this case the formulae (6) and (7) give the relation

$$H_n 2\pi r = \frac{4\pi}{c}\frac{e}{T_n} \tag{21}$$

from which we obtain

$$\frac{m^2 e^3 c}{n^3 \hbar^3} 2\pi r = \frac{4\pi}{c} e \frac{me^4}{2\pi n^3 \hbar^3}. \tag{22}$$

In effect the cross-section radius of the orbit which approximately can be identified with the radius of the electron microparticle becomes

$$r \approx r_e = \frac{e^2}{\pi mc^2}. \tag{23}$$

This result—evidently independent of the index n—is not much different than that given by the well-known formula (16) and the formula derived in [13]:

$$r_e = \frac{e^2}{6\pi mc^2}. \tag{24}$$

5. Summary

The Maxwell equations, when applied to electrons, usually neglect the microsize parameters of the electron particle. In Appendix we demonstrate that the Poynting vector $\boldsymbol{S}^P$ can be connected with the rest energy of the electron, therefore also with the radius r or r_e.

One of aims of the present paper was to indicate that these parameters can be essential in making the Maxwell equations satisfied for a given problem.

The Maxwell equation for the electric current has been examined for the case of the one-electron current present in the Bohr model of the hydrogen atom. It has been shown, for the magnetic field induced by the current, that the equation is satisfied on condition that the microstructure parameter of the electron radius r_e is explicitly taken into account. Here an earlier result can be pointed out that the magnetic field strength $H_n = B_n$ entering the Poynting vector constructed for the rate of the emission spectrum in the hydrogen atom cannot be reproduced from the Biot-Savart law unless the electron microstructure radius r_e is applied in the calculations (see [10] [14] [15]).

But the size of the electron microradius can be of importance for itself, especially in quantum electrodynamics, so its calculation becomes a useful task. In the next step of the paper, a substitution of H_n and T_n—characteristic respectively for the magnetic field quanta and time periods of the electron circulation in the atom—into the Oersted formula gives the expression for the cross-section radius of the electron orbit equal to

$$r = \frac{e^2}{\pi mc^2}. \tag{25}$$

The result in (25), which can be identified with the size of the radius r_e of the electron microparticle, does not differ much from the well-known formula (16) as well as the formula quoted in (24).

References

[1] Lass, H. (1950) Vector and Tensor Analysis. McGraw-Hill, New York.

[2] Matveev, A.N. (1964) Electrodynamics and the Theory of Relativity. Izd. Wyzszaja Szkola, Moscow. (In Russian)

[3] Greiner, W. (1998) Classical Electrodynamics. Springer, New York. http://dx.doi.org/10.1007/978-1-4612-0587-6

[4] Suffczynski, M. (1980) Electrodynamics. 4th Edition, PWN Warszawa. (In Polish)

[5] Sommerfeld, A. (1931) Atombau und Spektrallinien, Vol. 1. 5th Edition, Vieweg, Braunschweig.

[6] Heilbron, J.L. and Kuhn, T.S. (1969) *Historical Studies in Physical Sciences*, **1**, 211-290. http://dx.doi.org/10.2307/27757291

[7] Olszewski, S. (2016) *Reviews in Theoretical Science*, **4**, 336-352. http://dx.doi.org/10.1166/rits.2016.1066

[8] Kittel, C. (1987) Quantum Theory of Solids. 2nd Edition, Wiley, New York.

[9] Cyrot, M. and Pavuna, D. (1992) Introduction to Superconductivity and High T_c Materials. World Scientific, Singapore. http://dx.doi.org/10.1142/1039

[10] Olszewski, S. (2015) *Journal of Modern Physics*, **6**, 1277-1288. http://dx.doi.org/10.4236/jmp.2015.69133

[11] Slater, J.C. (1967) Quantum Theory of Molecules and Solids, Vol. 3. McGraw-Hill, New York.

[12] Landau, L.D. and Lifshits, E.M. (1969) Mechanics. Electrodynamics. Izd. Nauka, Moscow. (In Russian)

[13] Lindsay, R.B. and Margenau, H. (1963) Foundations of Physics. Dover, New York.

[14] Olszewski, S. (2015) *Journal of Modern Physics*, **6**, 2202-2210. http://dx.doi.org/10.4236/jmp.2015.615224

[15] Olszewski, S. *Quantum Matter*. (In Press)

[16] Olszewski, S. (2016) *Journal of Modern Physics*, **7**, 162-174. http://dx.doi.org/10.4236/jmp.2016.71018

[17] Olszewski, S. (2016) *Journal of Modern Physics*, **7**, 827-851. http://dx.doi.org/10.4236/jmp.2016.78076

[18] Olszewski, S. (2016) *Journal of Modern Physics*, **7**, 1004-1020. http://dx.doi.org/10.4236/jmp.2016.79091

Appendix: Rest Energy of the Electron Mass Connected with the Poynting Vector of the Hydrogen Atom

The value of the Poynting vector for the energy emission in the hydrogen atom can be easily calculated with the aid of H_n in (12) and the absolute value of the vector of the electric field intensity $|\boldsymbol{E}_n|$:

$$|\boldsymbol{E}_n| = \frac{e}{r_n^2} \tag{A1}$$

where r_n is the orbit radius applied in (2) and (3). For the spherical surface S having the radius r_n, so

$$S = S_n = \pi r_n^2, \tag{A2}$$

we obtain the absolute value of the Poynting vector $\boldsymbol{S}_n^P$ equal to

$$\left|\boldsymbol{S}_n^P\right| = \frac{c}{4\pi}|\boldsymbol{E}_n||\boldsymbol{H}_n|S_n = \frac{c}{4\pi}\frac{e}{r_n^2}\frac{e^3m^2c}{\hbar^3n^3}4\pi r_n^2 = c^2\frac{e^4m^2}{\hbar^3n^3} = 2\pi mc^2\frac{e^4m}{2\pi n^3\hbar^3} = \frac{2\pi mc^2}{T_n}. \tag{A3}$$

The decrement of $\left|\boldsymbol{S}_n^P\right|$ due to the change of the quantum state $n+1$ into n for large n is equal to

$$\begin{aligned}\Delta\left|\boldsymbol{S}_n^P\right| &= c^2\frac{e^4m^2}{\hbar^3}\left[\frac{1}{n^3}-\frac{1}{(n+1)^3}\right] = c^2\frac{e^4m^2}{\hbar^3}\frac{(n+1)^3-n^3}{(n+1)^3n^3}\\ &\cong 3c^2\frac{e^4m^2}{\hbar^3n^4} = \frac{mc^2}{n}6\pi\frac{e^4m}{2\pi n^3\hbar^3} = \frac{mc^2}{n}\frac{6\pi}{T_n}.\end{aligned} \tag{A4}$$

The time period T_n of the electron circulation in state n of the hydrogen atom [see (4)] is entering the denominator of the last term in (A3) and (A4). This is a characteristic substitution of the transition time Δt between two neighbouring quantum levels $n+1$ and n in the hydrogen atom obtained in a quantum aspect of the Joule-Lenz energy dissipation theory [10] [16]:

$$\Delta t \approx T_n. \tag{A5}$$

The emission rate (A4) can be compared with that given by the Joule-Lenz approach [10] [16]:

$$\frac{\Delta E}{\Delta t} = \frac{E_{n+1}-E_n}{\Delta t} = \frac{m}{2}\frac{e^4}{\hbar^2}\left[\frac{1}{n^2}-\frac{1}{(n+1)^2}\right]\frac{1}{\Delta t} \cong \frac{me^4}{\hbar^2n^3}\frac{1}{T_n}. \tag{A6}$$

We see that decrease of (A6) with increase of n is much more rapid than decrease of (A4). Moreover we have

$$\begin{aligned}\frac{e^4}{\hbar^2} &= \frac{\left(4.8\times10^{-10}\right)^4}{\left(1.06\times10^{-27}\right)^2}(\mathrm{cm/s})^2 \cong 177\times10^{14}\ \mathrm{cm^2/s^2}\\ &< 6\pi c^2 = 6\pi\left(3\times10^{10}\right)^2(\mathrm{cm/s})^2 \cong 170\times10^{20}\ \mathrm{cm^2/s^2}\end{aligned} \tag{A7}$$

which makes any (A6) much smaller than (A4). The reason of the discrepancy seems to be the choice of S equal to (A2) instead of a much smaller S equal to the toroidal surface enclosing the orbit of the electron circulation about the nucleus.

In any way the Joule-Lenz approximation for the energy emission rate in the hydrogen atom works well as it is indicated by its comparison with the quantum-mechanical theory (see [17] [18]).

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Higgs-Like Boson and Bound State of Gauge Bosons $\underline{W}^+\underline{W}^-$ II

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Abstract

The Higgs-like boson discovered at CERN in 2012 is tentatively assigned to a newly found bound state of two charged gauge bosons W^+W^- with a mass of $E_B \approx 117$ GeV, much closer to the measured 125 GeV than 110 GeV predicted in a paper with the same title earlier this year. The improvement is due to a shift from the earlier SU(2) representation assignment for the gauge bosons to the more realistic SU(3) one and that the computations are carried out with much greater accuracy.

Keywords

Bound State of W^+W^-, Higgs-Like Boson, SU(3) Group

1. Introduction

This note is a further development of the recent paper [1], in which the Higgs-like boson H(125) discovered in 2012 with mass 125.09 GeV [2] was tentatively assigned to a bound state of two charged gauge bosons W^+W^- with a mass of $E_B \approx 110$ GeV. The starting point is the action for gauge bosons ([3], 7.1.2),

$$S_{GBI} = -\frac{1}{4}\int d^4 x_I \sum_{l=1}^{8} G_{Al}^{\mu\nu}(x_I) G_{Al\mu\nu}(x_I)$$
$$G_{Al}^{\mu\nu}(x_I) = \frac{\partial W_{Al}^{\nu}(x_I)}{\partial x_{I\mu}} - \frac{\partial W_{Al}^{\mu}(x_I)}{\partial x_{I\nu}} - \varepsilon_{jkl}\, g W_{Aj}^{\mu}(x_I) W_{Ak}^{\nu}(x_I) \tag{1}$$
$$W_A^{\mu\pm}(x_I) = \left[W_{A1}^{\mu}(x_I) \pm i W_{A2}^{\mu}(x_I)\right] \big/ \sqrt{2} = \left(W_A^{0\pm}(x_I), \underline{W}_A^{\pm}(x_I)\right)$$

where $\underline{W}_A^{\pm}(x_I)$ denotes the charged gauge bosons and the superscript 0 its time component. In ([1], 1), however, the flavour index l was limited to run from 1 to 3 to reflect the assumption that these gauge bosons W^+W^-

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belong to a SU(2) representation, the lowest ranked one of a SU group. In this representation, the other gauge bosons, the massive neutral Z and the massless A are absent.

2. SU(2) vs SU(3)

Now, H(125) was generated in high energy proton-proton collisions in which all these 4 gauge bosons appear and a SU(3) representation is more appropriate. The 4 extra gauge bosons of the 8 gauge bosons in this representation degenerate to the 4 observed ones ([3], §7.2.3).

Further development is the same as that in [1] with the change of SU(2) to SU(3). This change only affects the running coupling constant ([1], 19) where the coefficient of the logarithmic term is proportional to the eigenvalue of the quadratic Casimir operator C_2 which is 2 for SU(2) and 3 for SU(3) ([4], 18.106). The renormalized coupling constant ([1], 20) now reads

$$g_R^2 = g^2\left[1 + C_2\frac{11g^2}{24\pi^2}\log\left(\frac{L_f}{L_R}\right)\right]^{-1}, \qquad L_R \to r, \qquad L_f > r \tag{2}$$

which corresponds to ([4], 18.133) for $C_2 = 3$ and is used here. The same computations that led to **Table 1** in [1] will be repeated here using (2) and with much greater accuracy.

3. Detailed Computation

In the Fortran 77 "dverk" integration subroutine, denote the integration step length by d_s. The inter gauge boson distance r that enter the computations is $r_c = k_d d_s$, where k_d is the number of steps needed to reach r_c. Only at these discrete r_c values can the solutions be printed out. This subroutine only allows $k_d \le 2^{10} = 1024$. Since the backward integrations has to start from some large r value, taken to be ≈0.5 GeV^{-1} in [1], d_s has a minimum d_{sm} = 0.5/1024 ≈ 0.000488 GeV^{-1}. Let r_{ci} be the r_c closest to r_i and $r_{ci} = k_{di}d_s$. Three step lengths, $d_s = d_{sm}$, $2d_{sm}$ and $4d_{sm}$, corresponding to k_{di}, $k_{di}/2$ and $k_{di}/4$, respectively, for a given r_{ci} will be used.

A bound state solution exists when the three conditions of ([1], 17) is exactly satisfied. This requires that $\Delta_{max} = 0$. Among the three parameters that fix ([1], 17), E_b and b_0 are continuous and can be specified to any degree of accuracy. But the third parameter r_i is according to the last paragraph limited to the discrete r_{ci} which can differ from r_i by $<d_{sm} \ne 0$. Therefore, $\Delta_{max} \ne 0$ and minima of Δ_{max} are sought. For such minima encountered here, it is sufficient to specify Δ_{max} up to 0.01%. This error margin leads to that E_B needs be accurate up to 0.001 GeV and b_0 up to 0.0001.

In [1], only $d_s = 2d_{sm} \approx 0.001$ GeV^{-1} was used. As was mentioned near ([1], 18), a criterion for the existence of a bound state solution has been taken to be $\Delta_{max} < \Delta_{err}$, an error due to the finite integration step length; $\Delta_{err} = d_s/r_{ci} = 2d_{sm}/r_{ci}$. As is seen in **Table 1** of [1] and **Table 1** below, only $r_i \approx 0.032$ - 0.033 are of interest. In [1], $\Delta_{err} = 2d_{sm}/r_i \approx 3\%$ and the criterion $\Delta_{max} < 3\%$ was used. This criterion is not absolute or derivable but is regarded as a plausible first approximation. It is satisfied by the solution ([1], 18) with $\Delta_{max} = 2.69\%$ and the 2 underlined entries in **Table 1** of [1] with $\Delta_{max} = 2.71\%$ and 2.23%.

Here, $\Delta_{err} = d_{sm}/r_i \approx 1.5\%$ and $\Delta_{err} = 4d_{sm}/r_i \approx 6\%$ are also considered. The corresponding criteria are $\Delta_{max} < \Delta_{err} \approx 1.5\%$ and $\Delta_{max} < \Delta_{err} \approx 6\%$, respectively.

Table 1. This table is the same as **Table 1** of [1] with ([1], 20) replaced by (2) here to reflect the change of the eigenvalue of the Casimir operator C_2 from 2 to 3. Only the underlined two cases satisfy the extrapolated criterion $\Delta_{max} < \Delta_{err} \approx 0.75\%$ below and can be solutions. k_{di} is the number of integration steps needed to reach r_{ci}, the printout r_c value nearest to the joint distance r_i in ([1], 17), for three different integration step lengths, $4d_{sm}$, $2d_{sm}$ and d_{sm}. * denotes that $k_{di} = 60$ was excluded due the above $k_d \le 2^{10} = 1024$ limitation in the backward integration.

L_f GeV^{-1}	0.20	0.20	0.30	0.30	0.34	0.35	0.36	0.40	0.50
k_{di}	16	33, 66	17	33, 66	17, 34, 68	17, 34, 68	17, 34, 68	17, 34, 68	15, 30*
Δ_{max} %	33.57	30.18	6.20	1.71	<u>0.50</u>	<u>0.35</u>	1.04	2.42	32.73
b_0	1.2753	1.3466	1.5277	1.4108	1.6792	1.7143	1.7486	1.8760	1.9161
E_b GeV	104.635	105.337	112.708	113.179	115.951	116.845	117.773	121.825	131.227

4. New Results

The computations in [1] are repeated using the more accurate specifications above. The changed results are Δ_{max} = 2.69% → 2.67% in ([1], 18), and 2.71% → 2.52% and 2.23% → 1.44% in ([1], Table 1). But now the criterion becomes Δ_{max} < 1.5% and is not satisfied by the solution ([1], 18) with bare g and M_W values and such a bound state no longer exists. Using SU(2) representation, the L_f = 0.30 GeV^{-1} case in ([1], Table 1) with Δ_{max} = 2.71% → 2.52% is also no longer a solution. The L_f = 0.35 GeV^{-1} case with Δ_{max} = 2.23% → 1.44% is barely < 1.5% but will not survive the extrapolated criterion Δ_{err} → 0.75% below.

Now, employ (2) with SU(3) and the more accurate E_B, b_0 and r_{ci} values mentioned above, the results are given in **Table 1**.

For L_f = 0.20 and 0.30, k_{di} = 33 and 66 refer to the same r_{ci}. Since 33/2 = 16.5 is not an integer, k_{di} = 17 and 16 refer to this r_{ci} + and − $2d_{sm}$ respectively. It is seen that Δ_{max} is lower for the smallest step length d_{sm} accompanying k_{di} = 66, as expected. The four cases with k_{di} = 17, 34 and 68 are accompanied by the step lengths d_s = $4d_{sm}$, $2d_{sm}$ and d_{sm}, respectively, correspond to the same r_{ci} and yield the same integration results. This shows that the computer accuracy is independent of these step lengths; only printouts do. Extrapolating these cases by reducing d_s one more step down to $d_{sm}/2$, $\Delta_{err} \to d_{sm}/2r_i$. The so-extrapolated criterion becomes Δ_{max} < 0.75% which is satisfied only by the two underlined cases in **Table 1**. A further reduction leads to Δ_{max} < 0.375% which is only satisfied by the L_f = 0.35 case. If the step length is reduced by half once more, Δ_{max} < 0.1875% and there is no solution for any case in **Table 1** and also in **Table 2**.

The L_f = 0.35 case with Δ_{max} = 0.35% in **Table 1** using SU(3) representation is far more close to 0 than does the 2.23% from **Table 1** of [1]. It may be regarded as a solution to the bound state and is tentatively assigned to H(125) instead. The calculated mass E_b = 116.845 GeV is much closer to 125.09 GeV [2] than does 110.02 GeV in [1]. The wave functions for these cases are close to that given by the dotted curve in Figure 1 of [1].

Equation (2) shows that L_f is an infrared cutoff and represents the size of the normalization box for a $W^{\pm}$ boson. Its mass M_W = 80.385 GeV is interpreted to have been determined when this boson is separated from other interacting particles by >L_f. L_f = 0.35 GeV^{-1} in **Table 1** appears to be compatible to some of the experimental conditions determining M_W. Also, E_b gets closer to the measured 125 GeV with increasing L_f. But why Δ_{max} is small enough for the present bound state to exist only when $L_f \approx$ 0.35 GeV^{-1} is not understood.

Table 2. Deviations Δ_{max} for different Casimir operator eigenvalues C_2 for L_f = 0.35. The underlined values satisfy the above twice extrapolated criterion Δ_{max} < 0.375%. C_2 = 3 for SU(3) is preferred by the bound state.

C_2	E_B GeV	b_0	Δ_{max} %	k_{di}
2.0	108.101	1.4113	21.30	16
2.0	109.833	1.4119	1.44	33, 66
2.0	110.083	1.4348	22.35	17
2.4	112.351	1.5818	5.57	17, 34, 68
2.8	115.372	1.6645	1.83	17, 34, 68
2.9	116.098	1.6899	0.62	17, 34, 68
2.94	116.396	1.6996	<u>0.23</u>	17, 34, 68
2.97	116.594	1.7094	<u>0.35</u>	17, 34, 68
2.99	116.771	1.7119	<u>0.26</u>	17, 34, 68
3.0	116.845	1.7143	<u>0.35</u>	17, 34, 68
3.02	116.995	1.7193	0.54	17, 34, 68
3.1	117.579	1.7407	1.43	17, 34, 68
3.2	118.345	1.7651	2.18	17, 34, 68

5. Variation of C_2

The calculations leading to **Table 1** has been repeated for $L_f = 0.35\ \text{GeV}^{-1}$ using different C_2 in (2). The results are shown in **Table 2**.

This table shows that solutions according to the twice extrapolated criterion $\Delta_{max} < 0.375\%$ exist only for $2.94 \le C_2 \le 3.0$. The bound state bosons W^+W^- prefer SU(3) and reject SU(2).

To obtain more precise prediction on the existence of bound state solutions, the Fortran 77's "dverk" subroutine may be replaced by more modern routines including finite element method.

6. Consequences

If the 2012 H(125) is indeed a bound state W^+W, it can no longer be the SM Higgs and at least the low energy end of SM is without foundation and has to be abandoned. No appreciable predictive power is lost; SM has not been able to account for basic hadron spectra and decays. SSI [3] is far more successful in this region. Mass generation of $W^\pm$ comes from pseudoscalar mesons when the relative time between the both quarks in the meson is taken into account.

Further, the presence of a Higgs condensate, disregarding the requirement that its isospin must be >0, will lead to a cosmological impasse ([5], p. 247). Such a condensate originated in the hypothesis (Nambu…) that vacuum contains a spin 0 background field that is spontaneously symmetry broken. It is reminiscent of the aether of the late 19th century that permeates the vacuum as a medium carrying light waves. Such attempts to put physics into the vacuum are against the historical examples that new physics come from some basic principles and mathematics ([3], Appendix G, Sec. 6). They do not work.

References

[1] Hoh, F.C. (2016) *Journal of Modern Physics*, **7**, 36-42. http://dx.doi.org/10.4236/jmp.2016.71004

[2] Olive, K.A., *et al.* (2015) *Chinese Physics C*, **38**, 090001. http://dx.doi.org/10.1088/1674-1137/38/9/090001

[3] Hoh, F.C. (2011) Scalar Strong Interaction Hadron Theory. Nova Science Publishers, Hauppauge.

[4] Lee, T.D. (1981) Particle Physics and an Introduction to Field Theory. Harwood Academic Publisher, New York.

[5] Wilczek, F. (2005) *Nature*, **433**, 239-247. http://dx.doi.org/10.1038/nature03281

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http://dx.doi.org/10.4236/jmp.2016.711116

The Mass and Size of Photons in the 5-Dimensional Extended Space Model

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Abstract

We propose the generalization of Einstein's special theory of relativity (STR). In our model, we use the (1 + 4)-dimensional space G, which is the extension of the (1 + 3)-dimensional Minkowski space M. As a fifth additional coordinate, the interval S is used. This value is constant under the usual Lorentz transformations in M, but it changes when the transformations in the extended space G are used. We call this model the Extended space model (ESM). From a physical point of view, our expansion means that processes in which the rest mass of the particles changes are acceptable now. In the ESM, gravity and electromagnetism are combined in one field. In the ESM, a photon can have a nonzero mass and this mass can be either positive or negative. It is also possible to establish in the frame of ESM connection between mass of a particle and its size.

Keywords

Photon, Mass, Size, 5-Dimensional Space, Extended Space Model, *Gravitation*, *Special Theory of Relativity*

1. Introduction

We consider the Extended space model (ESM), which is a generalization of Einstein's special theory of relativity (STR). The ESM is formulated in a 5-dimensional space, or more specifically in a (1 + 4)-dimensional space with the metric $(+----)$. Thus, we work in a space with coordinates (t, x, y, z, s) and metric $(+----)$. The objects under consideration are located on a cone

$$(ct)^2 - x^2 - y^2 - z^2 - s^2 = 0. \tag{1}$$

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The foundations and different properties of this theory are presented in Ref. [1]-[6]. Here we briefly recall its main statements and describe the structure of the Extended space.

In the STR, the rest mass *m* of a particle is a Lorentz scalar. For photons, $m = 0$. The main idea of the ESM is that the mass *m* is not a Lorentz scalar and can vary under external influences.

Such particle having a mass *m*, corresponds to a hyperboloid in Minkowski space, in the limiting case this hyperboloid degenerates into a cone.

$$s^2 = (ct)^2 - x^2 - y^2 - z^2. \tag{2}$$

Since the change of the mass of a particle corresponds its transition from one hyperboloid to the other, *i.e*. change of the corresponding interval, it seems natural to choose interval *s* as an additional fifth coordinate. Thus, we will work in a space with coordinates (t, x, y, z, s) and metric $(+----)$. The objects under consideration are located on a cone

$$(ct)^2 - x^2 - y^2 - z^2 - s^2 = 0. \tag{3}$$

We denote this space as $G(1,4)$. The Minkowski $M(1,3)$ space is a subspace of $G(1,4)$. An interval in the Minkowski $M(1,3)$ space plays a role of the fifth coordinate in the $G(1,4)$ space. We designate this coordinate by the letter *S*. The other coordinates are designated as T, X, Y, Z. One of the characteristic features of the ESM is that the particle's rest mass *m* is *s* variable quantity and a photon, moving in a medium with refraction index $n > 1$, and a nonzero mass is acquired. This mass can be both positive and negative.

The usual (1 + 2)-dimensional cones and hyperboloids occur as sections of the surface (3) by hyperplanes $s = s_0$. In the space $G(1,4)$ one can constructed in usual way the objects that have different tensor nature and transform appropriately under linear transformations of the $G(1,4)$ space (Ref. [7]).

The 5-*dimensional ESM has a number of advantages compared to STR*. *Firstly*, *it is a more symmetric theory*. *In this model*, *the energy*, *momentum and mass are equivalent and can be transformed into each other*. *Secondly*, *in this model there is no fundamental difference between massive and massless particles and they can be transformed into each other either*. *In addition*, *under the ESM electromagnetic and gravitational fields are combined into a single field*. *This field is investigated in papers* [2] [5]. *It is shown that with the help of rotations in G*(1,4) *space one can transform the electromagnetic field into four other fields with gravitational properties*.

In this work, we will show that it is possible to compare a nonzero mass to a system of photons, and that it is possible to establish connection between mass of a photon and its size.

The various aspects of the concept of "mass" in STR was discussed by Okun' [8]-[10].

2. The Refractive Index in the Extended Space

In Minkowski space $M(1,3)$ a 4-vector of energy and momentum

$$\tilde{p} = \left(\frac{E}{c}, p_x, p_y, p_z\right) \tag{4}$$

is associated to each particle (Ref. [11]). In the extended space of $G(1,4)$, we completes its to 5-vector

$$\overline{p} = \left(\frac{E}{c}, p_x, p_y, p_z, mc\right). \tag{5}$$

For free particles, the components of the vector (5) satisfy the equation

$$E^2 = c^2 p_x^2 + c^2 p_y^2 + c^2 p_z^2 + m^2 c^4. \tag{6}$$

It is well-known relation of relativistic mechanics, which relates the energy, momentum and mass of a particle. Its geometric meaning is that the vector (5) is isotropic, *i.e*. its length in the space $G(1,4)$ is equal to zero. However, in contrary to the usual relativistic mechanics, we now suppose that the mass *m* is also a variable, and it can vary at motion of a particle on the cones (3), (6). It should be understood so that the mass of the particle changes when it enters the region of the space that has a nonzero density of matter or field. Since in such areas the speed of light is reduced, they can be characterized by value *n*—the *refractive index* of a medium. The parameter *n* relates the speed of light in vacuum *c* with the speed *v* of light in a medium.

$$v = c/n. \tag{7}$$

We compere parameter $n \geq 1$ *to each form of medium or field. For example the refractive index of a gravitational field that is described by the Schwarzchild solution reads* [9],

$$n(r) = 1 + \frac{r_g}{r} = 1 + \frac{2\gamma M}{rc^2}. \tag{8}$$

Here $r_g < r$ *is the gravitational radius and M is a dot mass, which generates the Schwarzchild field.*

A set of variables (5) forms a 5-pulse, its components are conserved, if the space $G(1,4)$ is invariant under the corresponding direction. In particular, its fifth component p_4, having sense of mass, does not change if the particle moves in the area with constant value *n*.

The gravitational effects in ESM were discussed in Ref. [3]-[5].

3. The Vectors of the Free Particles

In the usual relativistic mechanics and field theory the mass of a particle is constant, and for particles with zero masses and nonzero rest masses different methods of description are used. The particles with nonzero rest masses are characterized by their mass *m* and speed $\vec{v}$. The particles with zero mass (photons) are characterized by frequency ω and wavelength λ. These ω and λ are connected with energy *E* and momentum $\vec{p}$ sa follows

$$E = \hbar\omega, \quad \vec{p} = \frac{2\pi\hbar}{\lambda}\vec{k}. \tag{9}$$

The 4-vector

$$\tilde{p}_m = \left(\frac{E}{c}, \vec{p}\right) = \left(\frac{mc}{\sqrt{1-\beta^2}}, \frac{m\vec{v}}{\sqrt{1-\beta^2}}\right), \quad \beta^2 = \frac{v^2}{c^2} \tag{10}$$

corresponds to a particles with nonzero rest mass.

The 4-vector

$$\tilde{p}_\omega = \left(\frac{\hbar\omega}{c}, \frac{2\pi\hbar}{\lambda}\vec{k}\right) = \left(\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}\vec{k}\right) \tag{11}$$

corresponds to a particles with zero mass.

The length of the 4*-vector* $\tilde{l} = (x_0, x_1, x_2, x_3)$ *is defined in accordance with the metric* $(+---)$ *of the Minkowski space* $M(1,3)$ *It equal to*

$$\tilde{l}^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2. \tag{12}$$

It follows from (12) *that length of a massive vector* (10) $\tilde{p}_m^2 = m^2c^2$, *therefor it is not an isotropic vector. And length of a photon vector* (11) $\tilde{p}_\omega^2 = 0$, *so it is an isotropic vector. It is the difference between massive and massless particles in the frame of STR.*

In the frame of our approach, there is no difference between massive and massless particles, and therefore one can establish a connection between two methods of description *of these two sorts of particles*. This can be done using the relation (9) and the hypothesis of de Broglie, according to which these relations hold for the massive particles. Now, substituting (9) in (5), we obtain the relation between the mass *m*, frequency ω and wavelength λ

$$\omega^2 = \left(\frac{2\pi c}{\lambda}\right)^2 + \frac{m^2c^4}{\hbar^2}. \tag{13}$$

$$\omega = \frac{mc^2}{\hbar\sqrt{1-\beta^2}}, \quad \lambda = \frac{2\pi\hbar}{mv}\sqrt{1-\beta^2}. \tag{14}$$

It follows that if $v \to 0, \lambda \to \infty$, but $\omega \to \omega_0 \neq 0$. Here ω_0 determines the energy of a particle at rest.

Now we construct 5-vectors from 4-vectors (10), (11). We suppose that a 5-vector

$$\overline{p} = (mc, 0, 0, 0, mc) \tag{15}$$

corresponds to a stationary particle of mass *m*.

The 5-vector of a particle, which moves with velocity $\vec{v}$, can be obtained by transformation to the moving coordinate system. Then the vector (15) takes the form

$$\overline{p}_m = \left(\frac{mc}{\sqrt{1-\beta^2}}, \frac{m\vec{v}}{\sqrt{1-\beta^2}}, mc \right). \tag{16}$$

Similarly the 4-vector (11) transforms into 5-vector

$$\overline{p}_\omega = \left(\frac{\hbar\omega}{c}, \frac{2\pi\hbar}{\lambda}\vec{k}, 0 \right). \tag{17}$$

At the transition to a moving coordinate system the vector (17) does not change its form, only the frequency ω changes its value.

$$\omega \to \omega' = \frac{\omega}{\sqrt{1-\beta^2}}. \tag{18}$$

Thus, in empty space in a stationary reference frame there are two fundamentally different object with zero and nonzero masses, which in the space of $G(1,4)$ correspond to the 5-vectors

$$\overline{p}_\omega = \left(\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}\vec{k}, 0 \right) \tag{19}$$

and

$$\overline{p}_m = (mc, 0, 0, 0, mc). \tag{20}$$

The vector (19) describes a photon with zero mass, the energy $\hbar\omega$, and the velocity *c*. The vector (20) describes a stationary particle of mass *m*. The photon has a momentum $p = \frac{\hbar\omega}{c}$, a massive particle has a momentum equal to zero. In the 5-dimensional space, these two vectors are isotropic, in Minkowski space only the vector (19) is isotropic.

The length of the 5-vector $\overline{l} = (x_0, x_1, x_2, x_3, x_4)$ *is defined in accordance with the metric* $(+----)$ *of the extended space* $G(1,4)$. It equal to

$$\overline{l}^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2. \tag{21}$$

It follows from the definition (21) *that* 5-*vectors* (16), (17) *are isotropic vectors*, *ie their length is equal to zero*.

$$\overline{p}_m^2 = \overline{p}_\omega^2 = 0. \tag{22}$$

If we restrict ourselves to Lorentz transformations in Minkowski space it is impossible to transform an isotropic vector into anisotropic one and vice versa. In other words in frame of the SRT photon can not acquires mass, and a massive particle can not be a photon. But in the Extended space $G(1,4)$ a photon and a massive particle can be related to each other by a simple rotation.

As it was already mentioned the parameter *n* connects the speed of light in vacuum with that in the medium: $v = c/n$. Using it, one can parametrize the fifth coordinate in the $G(1,4)$ space. The value $n = 1$ corresponds to the empty Minkowski space $M(1,3)$ in which light moves at the velocity *c*. The propagation of light in a medium with $n \neq 1$ is interpreted as an exit of a photon from the Minkowski space and its transition into another subspace of $G(1,4)$ space. This transition can be described as a rotation in the $G(1,4)$ space. All types of such rotations were studied in Ref. [1].

For hyperbolic rotation through the angle θ in the (TS) plane the photon 5-vector (19) with zero mass is transformed in the following manner (Ref. [1]):

$$\left(\frac{\hbar\omega}{c},\frac{\hbar\omega}{c}\vec{k},0\right)\Rightarrow\left(\frac{\hbar\omega}{c}\cosh\theta,\frac{\hbar\omega}{c}\vec{k},\frac{\hbar\omega}{c}\sinh\theta\right)=\left(\frac{\hbar\omega}{c}n,\frac{\hbar\omega}{c}\vec{k},\frac{\hbar\omega}{c}\sqrt{n^2-1}\right). \tag{23}$$

As a result of this transformation a particle with mass is appeared.

$$m=\frac{\hbar\omega}{c^2}\sinh\theta=\frac{\hbar\omega}{c^2}\sqrt{n^2-1}\,. \tag{24}$$

The velocity of this particle is defined by formula (7).

Under the same rotation the massive 5-vector (20) is transformed as

$$\left(mc,0,0,0,mc\right)\Rightarrow\left(mce^{\theta_\pm},0,0,0,mce^{\theta_\pm}\right); \tag{25}$$

$$e^{\theta_\pm}=n\pm\sqrt{n^2-1}.$$

Under such rotation a massive particle changes its mass

$$m\to me^{\theta},\ \ 0\le\theta<\infty \tag{26}$$

and energy but conserves its momentum.

The rotation through the angle ϕ in the (XS) plane transforms the photon vector in accordance to the law

$$\left(\frac{\hbar\omega}{c},\frac{\hbar\omega}{c},0,0,0,\right)\Rightarrow\left(\frac{\hbar\omega}{c},\frac{\hbar\omega}{c}\cos\phi,0,0,\frac{\hbar\omega}{c}\sin\phi\right)=\left(\frac{\hbar\omega}{c},\frac{\hbar\omega}{cn},0,0,\frac{\hbar\omega}{cn}\sqrt{n^2-1}\right). \tag{27}$$

Given this, the photon acquires the mass

$$m=\frac{\hbar\omega}{c^2}\sin\phi=\frac{\hbar\omega}{c^2n}, \tag{28}$$

and velocity

$$v=c\cos\phi=\frac{c}{n}. \tag{29}$$

The vector of a massive particle is transformed in accordance to the law

$$\left(mc,0,0,0,mc\right)\to\left(mc,-mc\sin\phi,0,0,mc\cos\phi\right)=\left(mc,-\frac{mc}{n}\sqrt{n^2-1},0,0,\frac{mc}{n}\right). \tag{30}$$

In this transformation the energy of a particle is conserved but its mass and momentum change

$$m\to m\cos\phi=\frac{m}{n}, \tag{31}$$

$$0\to -mc\sin\phi=-\frac{mc}{n}\sqrt{n^2-1}. \tag{32}$$

It is easy to see that vectors (23), (25) *and* (27), (30) *are isotropic*.

It is important that photon mass, which is generated by transformations (23), (27), can have either positive and negative sign. This immediately follows from the symmetry properties of $G(1,4)$ space. As to the particles that initially had positive mass, after transformations (25), (30) it remains positive.

4. The Mass of a System of Photons

It is well known that one can compare a plain wave and vector (19) to a photon only in empty space. If there is some particle or field in the space in addition to initial photon it is necessary to describe this photon by other vector. In order to find this vector let's consider the system of two photons with a same energy. We suppose that these photons are moving in the same plane but in different directions. In the empty space these photons are described by 5-vectors (19)

$$\left(\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c^2}v_x, \frac{\hbar\omega}{c^2}v_y, 0, 0\right) \tag{33}$$

and

$$\left(\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c^2}v_x, -\frac{\hbar\omega}{c^2}v_y, 0, 0\right). \tag{34}$$

Here v_x, v_y, v_z are components of photon's velocity, they satisfy the condition $v_x^2 + v_y^2 + v_z^2 = c^2$. In our case, $v_z = 0$.

The energy E of the system of two photons is $E = 2\hbar\omega$.

The momentum P of the system of two photons is $P = 2\frac{\hbar\omega}{c^2}v_x$.

These photons do not interact with each other, therefor the system of two such photons is a free system and it must be described by an isotropic 5-vector. This vector reads

$$\left(2\frac{\hbar\omega}{c}, 2\frac{\hbar\omega}{c^2}v_x, 0, 0, 2\frac{\hbar\omega}{c^2}\sqrt{c^2 - v_x^2}\right) = \left(2\frac{\hbar\omega}{c}, 2\frac{\hbar\omega}{c}\cos\alpha, 0, 0, 2\frac{\hbar\omega}{c}\sin\alpha\right). \tag{35}$$

We see that one can associate with a system of two photons a mass

$$m = 2\frac{\hbar\omega}{c^2}\sqrt{1 - \frac{v_x^2}{c^2}} = 2\frac{\hbar\omega}{c^2}\sin\alpha. \tag{36}$$

Here $\frac{v_x}{c} = \cos\alpha$ and 2α is an angle between directions of photons.

Let us compare now formulas (27) and (36). We see that in our case the angle ϕ, which determines the rotation the (XS) plane, is equal to angle α that is a half of an angle between directions of photons.

The other approach to the problem of constructing a mass of a system of photons was proposed by Rivlin [12] [13] and Fedorov [14].

5. Localization of Fields and Particles

There is a natural way in the frame of ESM to establish a connection between mass of a particle and its size. It can be done with the help of an analogy between the dispersion relation for a free particle

$$E^2 = (c\vec{p})^2 + m^2c^4 \tag{37}$$

and dispersion relation for a wave in the hollow metal waveguide

$$\omega^2 = \omega_{mn}^2 + (c\xi)^2. \tag{38}$$

Here ω_{mn} is the critical frequency of the waveguide mode, and ξ is a wave propagation constant.

The similarity of the ratios (37) and (38) drew the attention of many scientists. One can associated with the critical frequency ω_{mn} a parameter

$$M = \frac{\hbar\omega_{mn}}{c^2}. \tag{39}$$

This parameter has the *unit* of mass, and the question arises, if this quantity can be interpreted as a real mass? The mass, which acquires the electromagnetic field when it enters the waveguide. In the works of Rivlin, this problem was studied in a systematic way [12] [13]. Here we will not go into this problem, but we will mention only the fact that the mass *m* is related to the width *a* of a square waveguide by a relation

$$a = \frac{\sqrt{2}\pi\hbar}{mc}. \tag{40}$$

That is the value, which we propose to consider the characteristic linear parameter that is associated with the particle.

$$l = \frac{2\pi\hbar}{mc}. \tag{41}$$

The value (41) resembles the Compton wavelength of the electron, however, the physical meaning of it is very different. In the formula for Compton wavelength of the electron, the parameter m is the rest mass of the electron, but in Equation (41) m is the mass that a photon acquires when it is subjected by external influences.

In the empty space a free photon is described by a plane wave and has an infinite size. The mass of this photon is equal to zero, but its energy is finite. It is an idealized object. In does not exist in reality, because in reality there is no absolutely empty space. But when photon enters the space with external fields, it acquires a non-zero mass m. In accordance with the formula (40) *a finite linear parameter l can be compared to this mass m. We consider the linear parameter l as a size of a photon. Such reduction of an infinite format of a free photon to finite size of a photon in an external field us a result of action of this field.*

In ESM an external action is described by rotations in Extended space $G(1,4)$. We have discussed above the rotations from the group $L(1,4)$, and set how changing the mass of the photon at these turns. Because a linear parameter l expressed by the formula (41) using the mass of the photon, with its help it is possible to find the dependence of this parameter from the values define these rotations.

So, in the case of rotations in the plane (TS) dependence of the photon mass from the angle of rotation θ is determined by the formula (10). Substituting this expression into the formula (41), we obtain the expression for the parameter l through the angle θ.

$$l = \frac{2\pi c}{\omega \sinh \vartheta}. \tag{42}$$

In the case of rotations in the plane (XS) dependence of the photon's mass is determined by the formula (27). With its help, we obtain an expression for the parameter l through the angle ψ.

$$l = \frac{2\pi c}{\omega \sin \psi}. \tag{43}$$

The rotation angles θ, ψ *describe the value of external action. If this action tends to zero, the angles*

$$\theta, \psi \to 0, \tag{44}$$

the mass $m \to 0$ and the size of photon

$$l \to \infty. \tag{45}$$

6. Conclusion

In given work, the generalization of Einstein's Special theory of relativity is proposed. It is the (4 + 1)-dimensional Extended space model. It is shown that in the frame of this model, it is possible to compare the mass and size of the photon. *In forthcoming works, we will discuss the problem of localization of massive particles.*

References

[1] Tsipenyuk, D.Yu. and Andreev, V.A. (2000) *Bulletin of the Lebedev Physics Institute*, **6**, 23. arXiv:gr-qc/0106093.

[2] Tsipenyuk, D.Yu. and Andreev, V.A. (2002) *Bulletin of the Lebedev Physics Institute*, **6**, 1. arXiv:physics/0302006.

[3] Tsipenyuk, D.Yu. and Andreev, V.A. (2004) *Bulletin of the Lebedev Physics Institute*, **9**, 10. arXiv:physics/0506002.

[4] Tsipenyuk, D.Yu. and Andreev, V.A. (2004) *Bulletin of the Lebedev Physics Institute*, **10**, 13. arXiv:physics/0407144.

[5] Andreev, V.A. and Tsipenyuk, D.Yu. (2014) *Natural Science*, **6**, 248-253. http://dx.doi.org/10.4236/ns.2014.64028

[6] Andreev, V.A. and Tsipenyuk, D.Yu. (2015) Tunneling of the Potential Barrier and Particle's Size in the Extended Space Model. *Proceedings of International Meeting*, Bauman Moscow State Technical University, Moscow, 29 June-2 July 2015, 20-32.

[7] Kobayashi, S. and Nomizu, K. (1963) Foundation of Differential Geometry, Vol. 1. Inderscience Publishers, New York, London, Sydney.

[8] Okun', L.B. (1989) *Physics-Uspekhi*, **32**, 629. http://dx.doi.org/10.1070/PU1989v032n07ABEH002739

[9] Okun', L.B., Selivanov, K.G. and Telegdi, V.L. (1999) *Physics-Uspekhi*, **42**, 1141. http://dx.doi.org/10.1070/PU1999v042n10ABEH000597

[10] Okun', L.B. (2000) *Physics-Uspekhi*, **43**, 1366. http://dx.doi.org/10.1070/PU2000v043n12ABEH000889

[11] Landau, L.D. and Lifshitz, E.M. (1994) The Classical Theory of Fields. World Scientific, Singapore.

[12] Rivlin, L.A. (1997) *Physics-Uspekhi*, **40**, 291. http://dx.doi.org/10.1070/PU1997v040n03ABEH000215

[13] Rivlin, L.A. (1992) *Soviet Journal of Quantum Electronics*, **22**, 771.

[14] Fedorov, M.V. and Vintskevich, S.V. (2016) Invariant Mass and Propagation Speed of Light Pulses in Vacuum. arXiv:1604.00227v3.

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Revised Quantum Electrodynamics in a Condensed Form

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Abstract

This theory aims beyond the possibilities being available from the Standard Model. Examples are given by the directly obtained rest masses of the elementary particles, the deduced values of the elementary charge and of the mass of the boson detected by CERN which are close to their experimental data, and by an incorporated spin of the photon.

Keywords

Quantum Electrodynamics, Zero Point Energy, Standard Model and beyond

1. Introduction

The main features and applications of the Revised Quantum Electrodynamic theory (RQED) by the author are presented in this condensed review, without detailed deductions which have been reported elsewhere [1]-[7]. The review starts from a specification of the domain of application in Section 2, and with identifications of the fundamental concepts which form the basis of the field equations of RQED in Section 3. This is followed by the applications to elementary particles with rest mass and spin, with net electric charge, and with intrinsic electric charges in Section 4. Some associated applications to cosmology are finally given in Section 5.

2. Domain of Application

The theory of Revised Quantum Electrodynamics includes fundamental physics, with microscopic applications to elementary particles as well as to macroscopic ones on cosmological scale. It leads to new results not being available from the Standard Model and conventional approaches, and it differs essentially from the theory by Higgs. The latter is due to multistage processes by which a heavy particle is created through nonlinear broken symmetry effects, and is then in its turn decaying into all less heavy particles.

This theory thus makes an attempt in the first place to give a picture of the fundamental properties of the less

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complicated elementary particles, such as leptons and the photon. Further investigations are needed on the bound quarks and on more complex and compound particles. In addition, the theory does not deal with the classical field equations which, in their turn, apply to the formation of macroscopic charge and mass densities due to already existing particles.

3. Basis of Present Theory

The RQED theory is based on a number of fundamental concepts:

- From quantum theoretical analysis of the harmonic oscillator Planck has proved the existence of a lowest nonzero energy state, the Zero Point Energy (ZPE).
- This state generally includes photon-like electromagnetic oscillations, with and without associated electric charges.
- The ZPE fluctuations manifest themselves in the Casimir force which arises in vacuo between two metal plates separated by a small spacing. This is shown by experiments to result in a real *macroscopic* pressure. Consequently, the vacuum is not merely a state of empty space.
- The ZPE and the Casimir force thus indicate that there are sources in the *vacuum* state which have to be included into the electromagnetic field equations. This leads to an incorporated four-current, associated with a *nonzero* electric field divergence, $\operatorname{div}\boldsymbol{E}$. The result of this is a *broken* symmetry of the electric and magnetic field strengths, $\boldsymbol{E}$ and $\boldsymbol{B}$. The nonzero $\operatorname{div}\boldsymbol{E}$ thereby increases the degrees of freedom of the system, acting somewhat like a hidden variable.
- Spatial integration of $\operatorname{div}\boldsymbol{E}$ over the volumes of corresponding particle models leads both to nonzero and zero *net* electric charges, as well as to *intrinsic* charges of both polarities.
- The extended electromagnetic field equations of RQED still become both Lorentz and gauge invariant. Relativity in the form of Lorentz invariance does not only apply to plane electromagnetic waves, but also to *other* geometries such as to cylindrical waves.
- The quantized electrodynamic field equations become identical with the original equations in which the potentials and current densities are replaced by their expectation values.
- The conventional theory on the ZPE frequency spectrum is unacceptable, because it is *underdetermined*, treats all states with the *same* statistical probability of *unity*, and leads to an *infinite* total energy density. This problem is eliminated by the inclusion of a Boltzmann factor in a revised form of the spectrum deduced from conventional statistical theory.

The revised four-dimensional field equations of the *vacuum* state thus have the form [1]

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)A_\mu=\mu_0 J_\mu, \qquad \mu=1,2,3,4 \tag{1}$$

with

$$A_\mu=\left(\boldsymbol{A},\frac{i\phi}{c}\right) \tag{2}$$

standing for the four-potential and

$$J_\mu=\left(\boldsymbol{j},ic\bar{\rho}\right)=\bar{\rho}\left(\boldsymbol{C},ic\right) \tag{3}$$

for the four-current density representing a source due to the electric charge density

$$\bar{\rho}=\varepsilon_0\operatorname{div}\boldsymbol{E} \tag{4}$$

Here $\boldsymbol{C}$ is a *velocity vector* defined by

$$\boldsymbol{C}^2=c^2 \tag{5}$$

to make its modulus equal to the *scalar velocity constant c* of light. Thus $\boldsymbol{C}$ is a vector with up to 3 components. An example is given by screw-shaped cylindrical electromagnetic waves forming a photon, where $\boldsymbol{C}$ has two components, one related to the direction of propagation and one being associated with nonzero spin.

The original form of the field equations in an empty vacuum state are on the other hand given by $J_\mu=0$ in Equations (1) and (3), by $\operatorname{div}\boldsymbol{E}=0$ in Equation (4), and by $\boldsymbol{C}$ being identical with the scalar c.

4. Applications to Elementary Particles

The applications of RQED to elementary particles are based on field equations in *vacuo* with broken symmetry due to a nonzero $\operatorname{div}\boldsymbol{E}$.

4.1. Particles with Rest Mass and Spin

In all particle models of RQED the source terms due to broken symmetry give rise to *steady* states having a *nonzero* rest mass and an *associated* nonzero spin:

- This applies to particles with net charge such as the electron, muon, and tauon where the product $m_0 M_0 = (eh/4\pi)\left[1+\left(e^2/4\pi\varepsilon_0 ch\right)\right] \cong 8.455\times10^{-54}\ \mathrm{kg}\cdot\mathrm{A}\cdot\mathrm{m}^2$ of rest mass and magnetic moment is a given nonzero quantity. Such particles are prevented by the included magnetic field from "exploding" under the action of their electrostatic eigenforce.
- An example of particles with vanishing net charge is the Z boson for which a mass of 91 GeV becomes connected with a characteristic radius of about 10^{-18} m. A superposition of two such particles with opposite and cancelling magnetostatic potentials leads to a composite purely electrostatic and unstable boson, having no net charge, no magnetic field, no spin, and a minimum mass of 125 GeV within a limit of error of only a few percent [4]. This deduced particle state agrees with all the basic properties of the particle recently detected by CERN, but it has *no* relation to the theory by Higgs with its associated multistage processes of decaying particles. In addition, the latter theory does not predict the exact value of the resulting boson mass.
- A screw-shaped model for the individual photon results from RQED theory. It *includes* a spin at the expense of a very small reduction of its phase and group velocities from the velocity constant *c*, *i.e*. only in the ninth decimal of *c*. The spin is then associated with a very small but nonzero photon rest mass. Without broken symmetry and with a vanishing $\operatorname{div}\boldsymbol{E}$, such as in the Standard Model, the result would instead become a spinless photon in contradiction with experiments [1] [3], thereby propagating at the velocity *c*.
- The present photon model contributes to the understanding of the wave-particle dualism, the observed needle radiation, the photoelectric effect, and of two-slit experiments.

4.2. Particles with Net Electric Charge

Particles with net integrated electric charge, such as the electron, muon, and tauon, are given by divergent corresponding generating functions and point-charge-like geometries:

- Convergent integrated final results are still obtained, in terms of a revised renormalization procedure.
- The theory results in a deduced *fundamental* charge constant $Q_0 = (\varepsilon_0 ch)^{1/2} \cong 8.86\times10^{-19}\ \mathrm{A}\cdot\mathrm{s}$. It combines with a variational analysis on the geometrical shape of the charge-density profile to a *deduced* minimum elementary charge. The value of the latter agrees with the experimental result $e \cong 1.602\times10^{-19}\ \mathrm{A}\cdot\mathrm{s}$, within a limit of error of only a few percent [1] [3]. The form of the charge Q_0 gives an example where relativity is linked with quantum mechanics through the present theory. The form of Q_0 is directly related to the well-known fine-structure constant.

4.3. Intrinsic Electric Charges

The broken symmetry also results in *substantial* intrinsic charges of both polarities within the volume of a particle, in the cases with as well as without net integrated charges:

- Examples are given in which these intrinsic charges become an order of magnitude larger than the net integrated charge [5].
- The corresponding short-range Coulomb interaction between two such particles is then expected to become two orders of magnitude larger than the interaction due to the net charges, *i.e*. of the same magnitude as the strong force.
- This raises the question whether the interaction due to the intrinsic forces will interfere with the strong force, or become an explanation of the latter. This requires further investigations.

5. Applications to Cosmology

As based on the revised form of the ZPE frequency spectrum, the following results are obtained on cosmological scale where gravity becomes important:

- A new interpretation of dark matter and dark energy has been proposed. Dark matter is considered to originate from the ZPE density which leads to a *contracting* gravitational force, whereas dark energy originates from the ZPE density gradient leading to an *expansive* force [6] [7].
- In this way dark matter and dark energy are due to the *same* common ZPE spectrum, and this resolves the difficulty with the cosmological constant. It also resolves the related coincidence problem of equal magnitudes of dark matter and dark energy, *i.e*. near an equilibrium of the corresponding forces.
- In an application to the observable universe having a radius of about 10^{26} m, and with the present estimated proportions of dark matter, dark energy, and normal matter given by the astronomers, the present theory would result in in a deduced ZPE density of about $3\times 10^{-9}\ \mathrm{J}\cdot\mathrm{m}^{-3}$. This is in close agreement with its estimated value, and can be taken as a support of the theory.
- An experimental method in determining the ZPE density at the Earth's orbit has been proposed. It is based on measurements on the maximum Casimir force between two plates of different metals and at a vanishing air gap.

6. Summary

Conventional theory and the Standard Model including the approach by Higgs are based on a vacuum state of empty space. This leads to severely *restricted* forms of the corresponding solutions. These limitations can be removed by the present theory which opens a number of new possibilities *beyond* those so far being available. Examples are given by directly obtained rest masses of the elementary particle models, the deduced values of the elementary charge and of the mass of the boson detected by CERN being close to their experimental data, the incorporated spin of the individual photon, the large intrinsic charges of particle models and their possible relation to the strong force, and the revised ZPE frequency spectrum with its proposed relation to dark matter and dark energy. All results described in Sections 4 and 5 are thus based in the common concept of a Zero Point Energy.

References

[1] Lehnert, B. (2013) Revised Quantum Electrodynamics. In: Dvoeglazov, V.V., Ed., *Contemporary Fundamental Physics*, Nova Science Publishers, Inc., New York.

[2] Lehnert, B. (2015) *Journal of Modern Physics*, **6**, 448-452. http://dx.doi.org/10.4236/jmp.2015.64048

[3] Lehnert, B. (2015) *Journal of Modern Physics*, **6**, 1695-1700. http://dx.doi.org/10.4236/jmp.2015.611171

[4] Lehnert, B. (2015) *Journal of Modern Physics*, **6**, 2974-2079.

[5] Lehnert, B. (2013) *Progress in Physics*, **3**, 17-20.

[6] Lehnert, B. (2013) *Journal of Plasma Physics*, **79**, 327-334. http://dx.doi.org/10.1017/S0022377812001055

[7] Lehnert, B. (2016) *Journal of Modern Physics*, **7**, 1112-1119. http://dx.doi.org/10.4236/jmp.2016.710100

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As Regards the Speed in a Medium of the Electromagnetic Radiation Field

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Abstract

The velocity of the electromagnetic radiation in a perfect dielectric, containing no charges and no conduction currents, is explored and determined on making use of the Lorentz transformations. Besides the idealised blackbody radiation, whose vacuum propagation velocity is the universal constant *c*, being this value independent of the observer, there is another behaviour of electromagnetic radiation, we call *inertial radiation*, which is characterized by an electromagnetic inertial density $\Delta\left(x^{\mu}\right)$, and therefore, it happens to be described by a time-like Poynting four-vector field $\mathcal{P}^{\mu}\left(x^{\nu}\right)$ which propagates with velocity $\beta<1$. $\Delta\left(x^{\mu}\right)$ is found to be a relativistic invariant expressible in terms of the relativistic invariants of the electromagnetic field. It is shown that there is a rest frame, where the Poynting vector is equal to zero. Both phase and group velocities of the electromagnetic radiation are evaluated. The wave and eikonal equations for the dynamics of the radiation field are formulated.

Keywords

Inertial Radiation Field, Mass Field Density, Rest State, Poynting Vector, Wave and Eikonal Equations of the Radiation Field Dynamics

1. Introduction

In his famous lecture delivered for almost 93 years now to the Nordic Assembly of Naturalists at Gothenburg [1], A. Einstein expressed his lively interest concerning the problem of identifying gravitational and electromagnetic fields not as quite independent manifestations of nature, but just as two manifestations of same nature, or of a same entity. Although some previous efforts have been already conducted looking for generalized and dual

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expressions of the electromagnetic field, where the gravitational field should already appear lodging there [2]-[6], the idea of present work is to develop a contribution to see through as directly as possible, not where the concept of unification is now playing a deep role; but where the concept of inertiality appears clearly as an intrinsic hidden property of the electromagnetic field.

So, the problem is to derive a formula for velocity of the electromagnetic radiation as a certain function of the field strengths [7]. In this context, let us recall that in the case of the dynamics of massive point particles, it can conventionally be presented by two expressions, where the first one contains evolution equations for the energy and momentum, and the second part contains a connection among energy, momentum and the velocity of the particle. The relationship of the energy with the velocity is the main part of the particle dynamics which is indispensable in order to define a trajectory of motion of the point particle.

The aim of the present paper is to elaborate a method for finding *the velocity of the e-m radiation*, corresponding to the transport of the energy, momentum and angular momentum, as a function of the field strengths.

Z. Oziewicz [8] (1998) proposed that *the transportation of the energy by e-m radiation field is possible if only if the density of the momentum does not vanish with respect to all inertial observers*. He has argued that *this may happen only in the system of reference moving with light-velocity equal to c, because for other systems of reference moving with velocities less than light-velocity, one may find such a system where the Poynting vector vanishes.* Furthermore, he elaborated a method of calculation of the velocity of the system of reference where the Poynting vector vanished [9].

In this paper, we explore the dynamics of energy and momentum of the electromagnetic field by introducing the concepts of velocity of the radiation and the *field mass density*. Our method is based on the same idea of Z. Oziewicz proposed to identify the velocity of the radiation with the velocity of the system of reference, where the Poynting vector vanished. However, oppositely to our result, he concluded that there was no physical rest frame, with $v < c$, where the Poynting vector might vanish, but the frame of light.

In order to define a velocity for the electromagnetic radiation, a simple logic scheme has to be used: there exists a certain inertial reference system where the density of the momentum of the inertial *e-m* radiation field is equal to zero. The velocity of this inertial system is identified with the velocity of the radiation. So, according to this concept, in order to define the velocity of the field, it is sufficient to know the transformation laws of the field under the Lorentz-group.

The Concept of the Inertial Electromagnetic Field

In the solution of any electromagnetic problem the fundamental relations that must be satisfied are the four field equations—Maxwell equations [10]. Consider the particular case of electromagnetic phenomena in a perfect dielectric containing no charges and no conduction currents. For this case the Maxwell equations become

$$[\nabla \times \boldsymbol{B}] - \mu\epsilon \frac{\partial}{\partial t}\boldsymbol{E} = 0, \tag{1.1a}$$

$$(\nabla \cdot \boldsymbol{E}) = 0, \tag{1.1b}$$

$$[\nabla \times \boldsymbol{E}] + \frac{\partial}{\partial t}\boldsymbol{B} = 0, \tag{1.2a}$$

$$(\nabla \cdot \boldsymbol{B}) = 0. \tag{1.2b}$$

According to Maxwell theory the velocity of the electromagnetic (*e-m*) waves is defined via permittivity ϵ and permeability μ of the medium. In the same way, in the vacuum the velocity is defined by the universal constant

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}. \tag{1.3}$$

The physical sense of the speed of the electromagnetic radiation is attached to the velocity of the flux of radiation transporting energy, momentum and angular momentum. Since the speed of the light in the medium is defined by formula $v^2/c^2 = 1/\mu\epsilon$, the velocity of electromagnetic radiation depends of the index of refraction *n*, $n^2 = \epsilon\mu$, *i.e*, the velocity of the *e-m* radiation in the medium depends on the refractive index *n* of the medium.

The refractive index n is related with the *phase velocity* and the *wavelength* according to formulae

$$n = \frac{c}{v_{ph}} = \frac{\lambda_0}{\lambda}, \tag{1.4}$$

where

$$\lambda_0 = \frac{2\pi c}{\omega} \tag{1.5}$$

is the vacuum wavelength. We are addressing both cases occuring $n > 1$ and $n < 1$. The transversal fields in the medium are related as follows

$$\boldsymbol{B} = n\left[\boldsymbol{k} \times \boldsymbol{E}\right], \tag{1.6}$$

$$\left[\boldsymbol{E} \times \boldsymbol{B}\right] = n\left|\boldsymbol{E}\right|^2 \boldsymbol{k}, \tag{1.7}$$

and

$$\boldsymbol{E}^2 - \boldsymbol{B}^2 = \left(n^2 - 1\right)\boldsymbol{E}^2. \tag{1.8}$$

It is seen, that in the medium the value of the *e-m* field characteristics change in such a way that the property of transversality keeps conserved, but the relationship

$$\boldsymbol{E}^2 - \boldsymbol{B}^2 = 0, \tag{1.9}$$

which is valid for the radiation field in the vacuum, in the medium holds no more true.

This argument is taken as a pivoting idea to introducing the concept of *inertial e-m radiation field*, as a transversal *e-m* field, which is principally characterized by the main condition

$$\boldsymbol{E}^2 - \boldsymbol{B}^2 \neq 0. \tag{1.10}$$

Once the concept of inercial electromagnetic field is defined, the rest of the paper is organized as follows. First we present an alternative covariant double-scalar potential formalism for the representation of transversal electromagnetic fields (Section 2). Thereafter we derive the formula for the phase velocity of the electromagnetic radiation field (Section 3). As an application of this formula, we write the eikonal equation of the geometrical optics, and the wave equation for electromagnetic radiation field for a definite energy density, Poynting vector and mass field density (Section 4). Finally, in Section 5 the main conclusions of the work are thrown.

2. Double Potential Representation of the Transversal Electromagnetic Field

In Ref. [11] we have suggested another look on the nature of the transversal *e-m* fields. According to this viewpoint, for the transverse *e-m* fields, instead of using four potential functions, it is better to use a pair of Lorentz scalar functions. This representation automatically provides transversality of the strength vectors. Furthermore, the pair of scalar potentials are solutions of the Klein-Gordon equations, whereas the four-potentials play the role of a current density, and the Lorentz gauge condition for the four potentials takes the form of a continuity equation for the current density.

The Maxwell's equations in the vacuum are equations for the electric field strength $\vec{E}$ and the magnetic flux density $\vec{B}$. The four-potentials Φ, A_x, A_y, A_z were introduced into the Maxwell's electrodynamics in order to simplify the system of wave equations. However, thanks principally to the quantum mechanics, the essential conceptual and theoretical role of the potentials has been manifested. Indeed, in the Schrödinger, Dirac and Klein-Gordon equations the field strengths do not any longer appear, but the four potentials actually do instead. External *e-m* fields were introduced into quantum mechanics via four-potentials already at the level of Hamilton-Jacobi equations. It is to be acknowleged, that the principle of gauge invariance was born thanks to the four-potential representation of the *e-m* fields. However, the form of the potential representation has a great significance in the quantum field theory, in the theory of superconductivity, in the theory of the magnetic charge. The puzzling role playing by the vector potential representation in the quantum mechanical motion of particles was strikingly illustrated by the Aharonov-Bohm effect [12].

As we very well know, the equation

$$(\nabla \cdot \boldsymbol{B}) = 0. \tag{2.1}$$

is automatically satisfied, if the *magnetic-flux density* is represented as

$$\boldsymbol{B} = [\nabla \times \boldsymbol{A}], \tag{2.2}$$

because of the identity

$$(\nabla \cdot [\nabla \times \boldsymbol{A}]) = 0. \tag{2.3}$$

The expression for the electric field strength is in turn expressed by

$$\boldsymbol{E} = -\nabla\Phi - \frac{\partial \boldsymbol{A}}{\partial t}. \tag{2.4}$$

These formulae are such that the second group of Maxwell equations representing Faraday's law and the absence of magnetic charges are automatically satisfied. The first group of Maxwell equations are reduced to the following two equations for the potentials

$$-\Delta\boldsymbol{A} + \nabla(\nabla \cdot \boldsymbol{A}) + \frac{1}{c^2}\frac{\partial}{\partial t}\left(\nabla\Phi + \frac{\partial \boldsymbol{A}}{\partial t}\right) = 0, \tag{2.5a}$$

$$\nabla \cdot \left(\nabla\Phi + \frac{\partial}{\partial t}\boldsymbol{A}\right) = 0. \tag{2.6b}$$

Equations (2.6a, 2.6b) can be separated by choosing the so called Lorentz gauge condition

$$(\nabla \cdot \boldsymbol{A}) + \frac{1}{c^2}\frac{\partial}{\partial t}\Phi = 0. \tag{2.7}$$

Substitution of Equation (2.7) into (2.6a, 2.6b) yields wave equations for the four-potentials

$$\Delta\boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{A} = 0,\ \ \Delta\Phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi = 0. \tag{2.8}$$

Now, let us rewrite these formulae in their tensorial form. From the potentials Φ and $\boldsymbol{A}$ we pass into the four-vector representation by

$$A^\mu := (\Phi, c\boldsymbol{A}),\ \ A_\mu := (\Phi, -c\boldsymbol{A})$$

Further, let us introduce four-coordinates

$$x^\mu := (ct, \boldsymbol{r}),\ \ x_\mu := (ct, -\boldsymbol{r}).$$

In this notation, the vectors of electric field strength and magnetic flux density may be cast into the form of a screw-symmetric tensor

$$F_{\mu\nu} := (c\boldsymbol{B}, \boldsymbol{E}). \tag{2.9}$$

Then, formulae (2.4) and (2.5) are joined into one expression

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,\ \ \text{with}\ \ \partial_\mu := \frac{\partial}{\partial x^\mu}. \tag{2.10}$$

The first group of Maxwell equations takes the form

$$\partial_\mu F^{\mu\nu} = 0, \tag{2.11a}$$

whereas the second part of Maxwell equations admits the following form

$$\partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} = 0. \tag{2.11b}$$

In order to obtain the wave equation for the four-potential vector, usually, the Lorentz-gauge condition is used

$$\partial_\mu A^\mu = 0. \tag{2.12}$$

With this condition, the Maxwell equations are reduced to the wave equation for the four-potential vector

$$\partial_\mu \partial^\mu A_\nu = 0. \tag{2.13}$$

With respect to Lorentz transformations the electromagnetic fields are characterized to possess two invariants $I_1 = (\boldsymbol{E} \cdot \boldsymbol{B})$ and $I_2 = \boldsymbol{E}^2 - \boldsymbol{B}^2$. Among them, the field defined as a *pure radiation field*, or *null field*, for which the two invariants are identically zero, has a peculiar status. This field is a propagating field with the well-known properties $\boldsymbol{E} \perp \boldsymbol{B} \perp \boldsymbol{k}$, where $\boldsymbol{k}$ is the direction of propagation. Under these conditions, we refer to the *transversal character* of the e.m. radiation field [10]. However, the potential representation of this field meets difficulties concerning with its Lorentz covariant description. The radiation fields are described by two degrees of freedom, whereas the theory describes them by a four potential vector. Therefore, it is necessary for the theory to introduce subsidiary conditions, like the radiation gauge, or the Coulomb gauge. These Lorentz noncovariant conditions are used besides the Lorentz-covariant Lorentz gauge equation. These are the main difficulties which appear in the common accepted formulation. Alternatively we can handle the formalism as follows

First of all let us notice that the equation

$$(\nabla \cdot B) = 0, \tag{2.14}$$

is satisfied by defining the magnetic-flux density as

$$\boldsymbol{B} = [\nabla \phi \times \nabla \psi]. \tag{2.15}$$

For the electric field strength one obtains

$$\boldsymbol{E} = \frac{\partial \phi}{\partial t} \nabla \psi - \frac{\partial \psi}{\partial t} \nabla \phi. \tag{2.16}$$

In a tensorial form these formulas are given by

$$F_{\mu\nu} = \partial_\mu \phi \partial_\nu \psi - \partial_\nu \phi \partial_\mu \psi. \tag{2.17}$$

Obviously, the functions ϕ, ψ (*two-potentials*) are invariant with respect to Lorentz-transformations. Within these formulae, the field Lorentz-transformations directly follow from the coordinate Lorentz-transformations. The four-potential vector is expressed from two-potentials as follows

$$\boldsymbol{A} = \frac{1}{2}(\phi \nabla \psi - \psi \nabla \phi),\ \Phi = \frac{1}{2}\left(\phi \frac{\partial \psi}{\partial t} - \psi \frac{\partial \phi}{\partial t}\right), \tag{2.18}$$

or in tensorial notation

$$A_\mu = \frac{1}{2}(\phi \partial_\mu \psi - \psi \partial_\mu \phi). \tag{2.19}$$

The advantage of using this *two-potential representation* resides in the fact that we automatically introduce the two desired degrees of freedom. The first main consequence of this approach is *the property of transversality* of the electromagnetic field. In fact, in this representation, the field strength vectors satisfy the equation

$$I_1 := (\boldsymbol{B} \cdot \boldsymbol{E}) = 0. \tag{2.20}$$

Notice, however, that the dealing of the second invariant I_2 is not trivial.

It is to be noticed, that by reducing the degrees of freedom from four to two, additional algebraic relations between the fields and the four potential vector arise, they are namely

$$[\boldsymbol{A} \times \boldsymbol{E}] - \Phi \boldsymbol{B} = 0,\ (\boldsymbol{A} \cdot \boldsymbol{B}) = 0. \tag{2.21}$$

What kind of interpretation can be given to these equations? Looking for an answer, we refer the reader to Ref. [13] and references therein. Stating it briefly, the authors have found the following topological invariant

$$S = \int (\boldsymbol{A} \cdot \boldsymbol{B}) \mathrm{d}V. \tag{2.22}$$

Furthermore, it is shown that for the static magnetic field $\boldsymbol{B}$, S characterizes to what extent the magnetic lines are coupled to each other. For a single magnetic line this value estimates the screwness of the line. The relativistic generalization of this invariant was done in Ref. [14], as

$$\mathcal{J}^{\mu}=\tilde{F}^{\mu\nu}A_{\nu},\ \tilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}, \tag{2.23}$$

Or in the components

$$\vec{\mathcal{J}}=[\boldsymbol{A}\times\boldsymbol{E}]-\Phi\boldsymbol{B},\ \mathcal{J}_0=(\boldsymbol{A}\cdot\boldsymbol{B}).$$

The four-vector $\mathcal{J}^{\mu}$ satisfies the equation

$$\partial_{\mu}\mathcal{J}^{\mu}=-2(\boldsymbol{E}\cdot\boldsymbol{B}). \tag{2.24}$$

From this last equation it follows that $\mathcal{J}^{\mu}$ is conserved only for transversal fields.

Lorentz Gauge Equation as a Continuity Equation

The Lorentz gauge Equation (2.12) appears written in our two-potential representation as

$$\partial_{\mu}\left(\phi\partial^{\mu}\psi-\psi\partial^{\mu}\phi\right)=0,$$

which can be evaluated to

$$\phi\Box^2\psi-\psi\Box^2\phi=0,\ \text{ with }\ \Box^2=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2. \tag{2.25}$$

This equation separates into two Klein-Gordon type equations

$$\Box^2\psi=-\lambda^{-2}\psi,\ \Box^2\phi=-\lambda^{-2}\phi, \tag{2.26}$$

where the parameter λ is the Compton's length of the wave. Furthermore, two real Klein-Gordon equations (2.26) may be united into one Klein-Gordon type equation for the complex-valued function $\Psi=\phi+i\psi$.

We may also define in our formalism the current density as

$$j_{\mu}=\frac{1}{2}\left(\phi\partial_{\mu}\psi-\psi\partial_{\mu}\phi\right). \tag{2.27}$$

Furthermore, this current density satisfies the continuity equation

$$\partial_{\mu}j^{\mu}=0, \tag{2.28}$$

which arose above as the Lorentz-gauge condition (2.12) for the potential A^{μ}.

3. The Concept of Velocity for the Inertial Radiation Field

Consider two observers $\mathcal{A}$ and $\mathcal{B}$ with relative velocity v. Then the *e-m* fields are transformed according to the Lorentz formulae

$$\begin{aligned}\boldsymbol{E}(\mathcal{B})&=\frac{1}{v^2}\boldsymbol{v}\left(\boldsymbol{v}\cdot\boldsymbol{E}(\mathcal{A})\right)+\gamma_v\left\{\boldsymbol{E}(\mathcal{A})+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}(\mathcal{A})\right\},\\ \boldsymbol{B}(\mathcal{B})&=\frac{1}{v^2}\boldsymbol{v}\left(\boldsymbol{v}\cdot\boldsymbol{B}(\mathcal{A})\right)+\gamma_v\left\{\boldsymbol{B}(\mathcal{A})-\frac{1}{c}\boldsymbol{v}\times\boldsymbol{E}(\mathcal{A})\right\},\end{aligned} \tag{3.1}$$

where

$$\gamma_v=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{3.2}$$

Suppose that the velocity of $\mathcal{B}$-observer is perpendicular to electric and magnetic fields detected by $\mathcal{A}$-observer:

$$\boldsymbol{v}\cdot\boldsymbol{E}(\mathcal{A})=0=\boldsymbol{v}\cdot\boldsymbol{B}(\mathcal{A}). \tag{3.3}$$

Then formulae (3.1) are reduced to Heaviside formulae

$$\boldsymbol{E}(\mathcal{B})=\gamma_v\left\{\boldsymbol{E}(\mathcal{A})+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}(\mathcal{A})\right\},\quad \boldsymbol{B}(\mathcal{B})=\gamma_v\left\{\boldsymbol{B}(\mathcal{A})-\frac{1}{c}\boldsymbol{v}\times\boldsymbol{E}(\mathcal{A})\right\}. \tag{3.4}$$

These transformation formulae keep unchanged both invariants:

$$I_1=(\boldsymbol{E}\cdot\boldsymbol{B}),\ I_2=\boldsymbol{E}^2-\boldsymbol{B}^2. \tag{3.5}$$

The Poynting's vector is transformed as follows

$$\begin{aligned}\boldsymbol{E}(\mathcal{B})\times\boldsymbol{B}(\mathcal{B})&=\gamma_v^2\left\{\boldsymbol{E}\times\boldsymbol{B}-\boldsymbol{E}\times[\boldsymbol{v}\times\boldsymbol{E}]+[\boldsymbol{v}\times\boldsymbol{B}]\times\boldsymbol{B}-\left[[\boldsymbol{v}\times\boldsymbol{B}]\times[\boldsymbol{v}\times\boldsymbol{E}]\right]\right\}\\&=\left[[\boldsymbol{v}\times\boldsymbol{B}]\times[\boldsymbol{v}\times\boldsymbol{E}]\right]=\boldsymbol{v}\left(\boldsymbol{E}\cdot[\boldsymbol{v}\times\boldsymbol{B}]\right)-\boldsymbol{E}\left(\boldsymbol{v}\cdot[\boldsymbol{v}\times\boldsymbol{B}]\right)=-\boldsymbol{v}\left(\boldsymbol{v}\cdot[\boldsymbol{E}\times\boldsymbol{B}]\right).\end{aligned} \tag{3.6}$$

(Hereafter we omit the $\mathcal{A}$-observer designation.)

Now suppose that in the $\mathcal{B}$-system of references moving with velocity v with respect to the $\mathcal{A}$-system the Poynting vector is equal to zero

$$\boldsymbol{E}(\mathcal{B})\times\boldsymbol{B}(\mathcal{B})=0.$$

Then,

$$[\boldsymbol{E}\times\boldsymbol{B}]+\boldsymbol{v}\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right)+\boldsymbol{v}\left(\boldsymbol{v}\cdot[\boldsymbol{E}\times\boldsymbol{B}]\right)=0. \tag{3.7}$$

From this equation it follows that the velocity vector $\boldsymbol{v}$ is parallel (or, anti-parallel) to the Poynting vector $[\boldsymbol{E}\times\boldsymbol{B}]$. This proposition is compatible with the condition (3.3). Thus, we may take

$$\boldsymbol{v}=\lambda[\boldsymbol{E}\times\boldsymbol{B}]. \tag{3.8}$$

In order to construct an equation for the unknown constant λ, $\boldsymbol{v}$ from (3.8) is substituted into (3.7). In this way we obtain

$$[\boldsymbol{E}\times\boldsymbol{B}]+\lambda[\boldsymbol{E}\times\boldsymbol{B}]\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right)+\lambda^2[\boldsymbol{E}\times\boldsymbol{B}]\left([\boldsymbol{E}\times\boldsymbol{B}]^2\right)=0. \tag{3.9}$$

Suppose that $[\boldsymbol{E}\times\boldsymbol{B}]\neq 0$. Then Equation (3.9) is reduced into the following quadratic equation for λ:

$$1+\lambda\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right)+\lambda^2\left([\boldsymbol{E}\times\boldsymbol{B}]^2\right)=0. \tag{3.10}$$

Let us introduce the following quantities

$$P_0=\frac{1}{2}\left(\boldsymbol{E}^2+\boldsymbol{B}^2\right),\ \boldsymbol{P}=[\boldsymbol{E}\times\boldsymbol{B}] \tag{3.11}$$

and

$$\Delta^2=P_0^2-\boldsymbol{P}^2. \tag{3.12}$$

Evidently, Δ is a function of the two invariants I_1, I_2:

$$2\Delta=\sqrt{\left(\boldsymbol{E}^2-\boldsymbol{B}^2\right)^2+4(\boldsymbol{B}\cdot\boldsymbol{E})^2}=\sqrt{I_1+4I_2}. \tag{3.13}$$

Consequently, Δ is itself also an invariant. This value we interpret as the inertial field density. In this notation, the quadratic equation for λ appears as

$$\lambda^2P^2+2\lambda P_0+1=0, \tag{3.14}$$

which has two different roots

$$P^2\lambda_1=-P_0+\Delta,\ P^2\lambda_2=-P_0-\Delta.$$

By using this quantities in (3.8) we obtain two kinds of velocities

$$\boldsymbol{v}_1^2=\lambda_1^2[\boldsymbol{E}\times\boldsymbol{B}]^2=\frac{(P_0-\Delta)}{P_0+\Delta}, \tag{3.15a}$$

and

$$\boldsymbol{v}_2^2 = \lambda_2^2 \left[\boldsymbol{E}\times\boldsymbol{B}\right]^2 = \frac{\left(P_0+\Delta\right)}{P_0-\Delta}. \tag{3.15b}$$

The first velocity v_1^2 is less than the speed of light $v_1^2 \le c^2$, and other one is greater than the speed light $v_2^2 \ge c^2$. Notice also that

$$v_1 v_2 = c^2. \tag{3.16}$$

These velocities are explicitly expressed via field strengths as follows

$$\frac{v_1^2}{c^2} = \frac{\boldsymbol{E}^2+\boldsymbol{B}^2-2\Delta}{\boldsymbol{E}^2+\boldsymbol{B}^2+2\Delta}, \tag{3.17a}$$

$$\frac{v_2^2}{c^2} = \frac{\boldsymbol{E}^2+\boldsymbol{B}^2+2\Delta}{\boldsymbol{E}^2+\boldsymbol{B}^2-2\Delta}. \tag{3.17b}$$

Thus, if $\Delta \ne 0$, then there always exists a system of reference where the Poynting's vector is equal to zero. In other words, the rest state is achievable. The situation is quite analogous to the case of massive particle where the state with $P = 0$ can be achieved. Now, we are in position to specify the concept of *inertial field density*. Define this quantity as follows

$$\mathcal{M} = \Delta. \tag{3.18}$$

In this notation, the formula for the velocity is written as

$$\frac{v^2}{c^2} = \frac{P_0-\mathcal{M}}{P_0+\mathcal{M}}. \tag{3.19}$$

From this formula it follows the expression for the density of the energy

$$P_0 = \mathcal{M}\frac{c^2+v^2}{c^2-v^2}, \tag{3.20}$$

and for the Poynting's vector

$$\boldsymbol{P} = \frac{2c^2}{c^2-v^2}\mathcal{M}\boldsymbol{v}. \tag{3.21}$$

Let ψ be the rapidity, so that,

$$v = c\tanh\psi. \tag{3.22}$$

Then,

$$P_0 = \mathcal{M}\cosh\left(2\psi\right),\ P = \mathcal{M}\sinh\left(2\psi\right). \tag{3.23}$$

Comparing these formulae with the analogous formulae for the relativistic point-particle, it is seen, that in the case of *e*-*m* field, for theenergy and the momentum, the rapidity appears multiplied by the factor "2".

We have derived two kinds of velocities $v_1 < c$ and $v_2 > c$. In order to interpret these velocities we have to refer to formulae (1.6)-(1.8) for transverse fields in the medium with refractive index *n*. Within our notations of the densities of the energy and the mass we write

$$2P_0 = \boldsymbol{E}^2+\boldsymbol{B}^2 = \left(1+n^2\right)\boldsymbol{E}^2,\ 2\mathcal{M} = \left(n^2-1\right)E^2. \tag{3.24}$$

Consequently, we have

$$\frac{v_1^2}{c^2} = \frac{P_0-\mathcal{M}}{P_0+\mathcal{M}} = \frac{1}{n^2}, \tag{3.25}$$

for $n > 1$, and

$$\frac{v_2^2}{c^2}=\frac{P_0+\mathcal{M}}{P_0-\mathcal{M}}=\frac{1}{n^2}, \tag{3.26}$$

for $n<1$.

From these formulae it follows that the velocities $v_k, k=1,2$ admit an interpretation as the phase velocities related in ordinary way with the refractive index *n*.

By analogy we may also define the group velocity *V*

$$\frac{\mathrm{d}P_0}{\mathrm{d}P}=\frac{V}{c}, \tag{3.27}$$

satisfying always $V\le c$.

The group velocity is related to the phase velocity by the formula

$$V=\frac{2v}{1+\frac{v^2}{c^2}} \tag{3.28}$$

where *v* can be either v_1 or v_2.

4. Wave Mechanical Dynamics

In the previous section we have obtained formulae for phase velocities as functions of the densities of the energy, momentum and the mass. Consider classical electromagnetic waves travelling according to a scalar wave equation of the simple form

$$\nabla^2\psi-\frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\psi=0, \tag{4.1}$$

where $\psi(x,y,z,t)$ represents any component of the electric or magnetic field and $n(x,y,z)$ is the (time independent) refractive index of the medium.

By assuming

$$\psi=u(r,\omega)\exp(-i\omega t), \tag{4.2}$$

with $r=(x,y,z)$, we get from (4.1) the Helmholtz equation

$$\nabla^2u+\left(n^2k_0\right)^2u=0,\ k_0=\frac{2\pi}{\lambda_0}=\frac{\omega}{c}. \tag{4.3}$$

The solutions of the wave equation are looked of the form [15]

$$u(r,\omega)=R(r,\omega)\exp\left(i\phi(r,\omega)\right), \tag{4.4}$$

with real *R* and ϕ functions, which represent respectively, the amplitude and phase of the monochromatic waves. The wave vector is defined as

$$\vec{k}=\nabla\phi(r,\omega). \tag{4.5}$$

In the limit of geometrical optics, according to the eikonal equation

$$k^2\equiv(\nabla\phi)^2\cong(nk_0)^2, \tag{4.6}$$

The Eikonal equation in a geometrical wave theory has the form

$$(\nabla W)^2=\frac{k^2}{k_0^2}=n^2=\frac{c^2}{v^2}.$$

For the rays inside the medium with refractive index *n* defined via $P_0,\mathcal{M}$ by formulae (3.25)-(3.26) we write the eikonal equation of the form

$$(\nabla W)^2=n^2=\frac{P_0+\mathcal{M}}{P_0-\mathcal{M}},\ n^2>1,$$

and

$$\left(\nabla W\right)^2 = n^2 = \frac{P_0 - \mathcal{M}}{P_0 + \mathcal{M}},\ n^2 < 1.$$

5. Conclusions

The concept of velocity for the radiation fields is a long stated problem. This problem of the velocity of the electromagnetic waves has been always considered as a definitely solved problem: these waves have a velocity equal to $c = 1\big/\sqrt{\mu_0\epsilon_0}$ in the vacuum. From this formula, it follows that in the medium the velocity of the electromagnetic wave obeys actually the same formula

$$v/c = \frac{1}{\sqrt{\mu\epsilon}},$$

which is used also to be written as $v = \frac{c}{n}$, where n is the refractive index of the medium. To the present, this expression has been considered phenomenological in nature, it is however relativistic. Here, it is clear that we are refering to the phase velocity of the waves, which can be both smaller as well as larger than the speed of light. However, we have been able to relate this velocity to the true physical group velocity of the radiation which is always present. We have developed this study in a relativistic formalism, considering the velocity of the electromagnetic radiation field as proposed by *Zbigniew Oziewicz*. This formalism is constructed out of first principles, because it is based on the transformations of Lorentz group, in order to derive the formula for the velocity of the reference system where the Poynting vector is equal to zero. In this way we arrived to the formula connecting the phase velocity with the densities of the energy and momentum.

From the viewpoint of the theory, we have been able to describe the inertiality of the electromagnetic Field, we are able to assure that with the exception of the propagation in vacuum, where the Poynting vector is a null four-vector, in a dielectric medium, the Poynting vector migrates into a time-like four-vector, whose norm is the inertial field density, formula (3.12), that is why the rest frame of the propagation is reachable. To the present, it has been known to us only the massless quantum mechanical behavior of photons. We don't dare, for the moment, to describe them, inside matter, as mutants in a classical, massive state, but we just point out to the inertiality as an intrinsic hidden property of the electromagnetic field. How to quantify the inertial field density may surely be the subject of future works.

As we have introduced the notion of inertial field density as a measure of the *inertia* of the electromagnetic radiation, in return, we have formulated a new wave equation envolving the density of energy and the inertial field density.

References

[1] Einstein, A. (1923) Fundamental Ideas and Problems of the Theory of Relativity. Lecture Delivered to the Nordic Assembly of Naturalists at Gothenburg, 11 July 1923.

[2] Kalb, M. and Ramond, P. (1974) *Physical Review D*, **9**, 2273-2284. http://dx.doi.org/10.1103/PhysRevD.9.2273

[3] Dvoeglazov, V. (2000) *Annales de la Fondation Louis de Broglie*, **25**, 81.

[4] Malik, R.P. (2011) *Physics of Particles and Nuclei Letters*, **8**, 244-250. http://dx.doi.org/10.1134/S1547477111030149

[5] Rodríguez-Domínguez, A.R. and Martínez-González, A. (2012) *Journal of Nonlinear Mathematical Physics*, **19**, No. 4.

[6] Keller, J. (1986) *International Journal of Theoretical Physics*, **25**, 779-806. http://dx.doi.org/10.1007/BF00669917

[7] Kholmetskii, A.L. (2004) *Annales de la Fondation Louis de Broglie*, **29**, 549-557.

[8] Oziewicz, Z. (2006) Electric Field and Magnetic Field in a Moving Reference System (Abridged Abstract). In: Adhikari, M.R., Ed., *Physical Interpretations of Relativity Theory Session*, *an International Symposium on Recent Advances*, Kolkata, 47-53.

[9] Oziewicz, Z. (2008) *Review Bulletin of Calcutta Mathematical Society*, **16**, 49-66.

[10] Barut, A.O. (1980) Electrodynamics and Classical Theory of Fields and Particles. Dover Publications, Inc., New York.

[11] Yamaleev, R.M. (2005) *Annales de la Fondation Louis de Broglie*, **30**, 449-461.

[12] Ryder, L.H. (1985) Quantum Field Theory. Cambridge University Press.

[13] Keller, J. (1993, 1999) The Geometric Content of the Electron Theory, Parts I and II. *Advances in Applied Clifford Algebras*, **3**, 5-25 and **9**, 309-395.

[14] Rañada, A.F. (1992) *European Journal of Physics*, **13**, 70-76. http://dx.doi.org/10.1088/0143-0807/13/2/003

[15] Orefice, A., Giovanell, R. and Ditto, D. (2015) *Annales de la Fondation Louis de Broglie*, **40**, 95-112.

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Cosmogonic Speculations: Particle Creation from Energy Conservation in the Universe Evolution

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Abstract

Information about the Universe from Hubble's law is consistent with that coming from the evaluation of inertial forces, supporting the picture of a steady state expansion of a black hole Universe. Backed up also by the consideration of the black body self energy, the post big bang temperature decrease is consistent with particle creation if energy conservation applies at every scale. This process is shown to provide a gravitational repulsive force which can counterbalance gravitational attraction thus allowing the possibility of a steady non-inflationary expansion. That seems to provide an alternative coherent scheme for our picture of the Universe evolution, disposing of the cosmological term, of dark energy and of the bulk of dark matter.

Keywords

Cosmogony, Black Holes, Particle Creation, Self Energy, Inertial Forces

1. Introduction

The present scenario for our Universe evolution is based first of all on two sound experimental facts: Hubble's law [1] and the CMB [2] (although the latter was predicted semiquantitatively by Alpher, Herrman and Gamow [3]). In more recent times finer details have come from BOOMERAanG [4], Cobe [5], Planck [6] and the Supernovae data [7] [8] which hint at an accelerated expansion.

The prevailing fit is based on the GR equations where the theoretical interpretation of the notorious cosmological term seems to represent a formidable problem. Quoting e.g. Susskind [9] "the best efforts of the best physicists, using our best theories predict the cosmological constant… equivalent to the vacuum energy... incorrectly by 120 orders of magnitude".

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That its theoretical treatment leaves many people unsatisfied is reflected e.g. in Perlmutter's words [10]: "Dark energy has now been added to the already perplexing question of dark matter [11]. One is tempted to speculate that these ingredients are add-ons, like the Ptolemaic epicycles, to preserve an incomplete theory".

In that respect, let also quote Feynman's [12] words "Although in ordinary circumstances a physicist would rebel against a theory which so cavalierly ignores our cherished laws about the conservation of matter and energy, one must remember that *here we are not dealing with an ordinary problem but with a cosmological problem*". Although these words were referring to the steady state theory of Hoyle [13], the warning applies in general.

That a revision is needed had also already been pointed out in [14] in connection with the treatment of one of our theoretical milestones *i.e*. black body radiation. Indeed at cosmological scales the gravitational interaction of photons cannot be neglected and this results in a substantial role of the self energy, especially in the radiation dominated era.

If we enforce energy conservation at any scale, one is thus forced to admit photon (and particle) non-conservation, which explains the non-gravitational collapse of the Universe. This provides an alternative scheme where our *causal* Universe has evolved from a total zero energy quantum fluctuation at the Planck scale to the present dimensions, again with total zero energy, without any need to invoke inflation.

The crucial points will be :

1) energy vs. particle number conservation,

2) the presence of a gravitational repulsive force due to particle creation,

3) the black body handling,

4) the consideration of inertial forces.

2. Present Picture: The Unity of the Universe

a) inertial forces

We summarize the results of the calculations backing up the conjecture put forward mainly by Sciama [15] and further developed by Berry [16] concerning the constraints provided by the inertial forces on the mass of the Universe.

Consider first how the Coriolis force is generated in the rotating frame by the counter rotating Universe. Since a rotating matter distribution produces a gravitomagnetic field $\boldsymbol{h}$ (according to the parameter free effective vector formulation of gravitation [17]) proportional to the angular velocity of rotation $\boldsymbol{\omega}$

$$\boldsymbol{F}_{GM} = m\boldsymbol{v}\times\left(\frac{2GM}{c^2R}\boldsymbol{\omega}\right) = 2m\boldsymbol{v}\times\left(\frac{GM}{c^2R}\right)\boldsymbol{\omega} \tag{1}$$

the Coriolis force on a mass m

$$\boldsymbol{F}_{Cor} = 2m\boldsymbol{v}\times\boldsymbol{\omega} \tag{2}$$

is recovered

$$\boldsymbol{F}_{Cor} = \boldsymbol{F}_{GM} \tag{3}$$

if for the Universe,

$$\frac{GM_U}{c^2R_U} = 1. \tag{4}$$

The relevant point in this argument is that in the relative rotation, the magnetic field generated by distant layers of matter goes as $1/R$ *i.e*. the same behavior of radiation, rather than the usual $1/R^2$ of Newtonian forces. Therefore a relative more important role even of distant stars is a matter of fact.

As regards linear inertial forces, the same effective vector gravitation accounts for them via a different mechanism *i.e*. dipole radiation [18].

It is well known that in the non inertial frame S' of a particle m_1 attracted by another *M*, where the said particle is at rest and manifestly $\boldsymbol{a}_1' = 0$, non inertial forces have to be introduced. They come from all other particles which acquire the opposite acceleration $-\boldsymbol{a}_1$ generating an additional force given by

$$F'_{1,I} = m_1 \boldsymbol{g}_{rad} = -m_1 \frac{\mathrm{d}\boldsymbol{b}}{\mathrm{d}t} \simeq m_1 \left(\frac{G}{c^2} \int \frac{2\rho}{|r'|} \mathrm{d}V' \right) \boldsymbol{a}_1 \simeq m_1 \left(\frac{2GM_U}{c^2 R_U} \right) \boldsymbol{a}_1 \tag{5}$$

where I stands for inertial and $\boldsymbol{b}$ for the gravitomagnetic vector potential. Therefore if

$$\frac{2GM_U}{c^2 R_U} = 1 \tag{6}$$

the inertial force generated in the non inertial reference frame S' of particle 1 by the accelerated masses of the rest of the Universe exactly compensates the "true" Newtonian (N) force $F_{1,N} = m_1 \boldsymbol{a}_1 = -\frac{GMm_1\boldsymbol{r}}{r^3}$

$$F_{1,N} + F'_{1,I} = 0. \tag{7}$$

Thus the two previous conditions on the energy content of the Universe, which taken at their face value seem to be contradictory, confirm instead at a semiquantitative level the fact that we are indeed living inside a black hole. All the more so since they come from two different mechanisms.

Let us now come to an apparently disconnected issue *i.e*. the cosmological expansion.

b) the gross features of the expansion

Hubble's law [1] of the recession velocity of galaxies reads

$$v = H_0 R \tag{8}$$

where $H_0 \simeq 2.3 \times 10^{-18}\ \mathrm{sec}^{-1}$ stands for the present value of the Hubble's constant, which defines the radius of our present causal Universe

$$R_U = c/H_o \simeq 10^{26}\ \mathrm{m} \tag{9}$$

Whether our Universe would eventually contract or keep expanding should depend on a critical density ρ_c. In classical terms, following e.g. [19] [20], consider a sphere of radius R of constant mass density in flat space. The kinetic energy of a particle m at its surface is $K = 1/2\, m (\mathrm{d}R/\mathrm{d}t)^2$ and is accompanied by the attraction of the particles inside the sphere with potential energy $U = -G\frac{4\pi R^3 \rho m}{3R}$.

The steady state thus takes place when these two energies balance each other or

$$E = K + U = 0 = 1/2\, m (\mathrm{d}R/\mathrm{d}t)^2 - G\frac{4\pi R^3 \rho_c m}{3R}. \tag{10}$$

In this case

$$(1/R \times \mathrm{d}R/\mathrm{d}t)^2 = 8\pi/3 G\rho_c \tag{11}$$

and

$$\rho_c \simeq 3H_o^2/(8\pi G) \simeq 1.1 \times 10^{-26}\ \mathrm{kg/m^3} \simeq 6 m_p/\mathrm{m}^3 \tag{12}$$

m_p standing for the nucleon mass.

It is evident that these considerations represent a rough non relativistic schematization of the process. As a matter of fact in the energy density no trace of the kinetic energy of the expanding particles within the sphere appears nor their contribution to the potential energy because of the mass energy equivalence; the mass M is just an "effective" static source which is thought to represent reality and hence to determine the motion of m. Nevertheless data seem to confirm [4] to the percent the critical value for the density.

Let us then check the consistency of Equation (11) with the results from inertial forces by using Hubble's relation predictively at R_U so that the Universe density will enter the r.h.s.

$$\left. \left(\frac{1}{R} \frac{\mathrm{d}R}{\mathrm{d}t} \right)^2 \right|_{R_U} = \frac{c^2}{R_U^2} = \frac{8\pi G \rho_U}{3} = \frac{2GM_U}{R_U^3} \tag{13}$$

whence

$$\frac{2GM_U}{c^2 R_U} = 1;\ M_U c^2 - \frac{2GM_U^2}{R_U} = 0 \tag{14}$$

which implies that **the total mass is balanced by its gravitational self energy** (up to a factor of 2) **to make the total energy of the Universe zero.**

We thus see that the extrapolation of Hubble's law to the whole of our causal Universe, circumventing critical estimates of the contribution of distant regions to the critical density, yields the same black hole condition entering the evaluation of inertial forces and validates Feynman's conjecture that the critical density is just about the best density to use in cosmological problems because it allows for free particle creation [12], the mass being swallowed by its gravitational self energy. Thus Hubble's law is not only fundamental in its vector form in proving that there is no center in the Universe but its extrapolation is paramount in determining its fate.

Let us then see how mass creation embodied in the previous equation enters consistently the equation of motion for the acceleration. We differentiate with respect to t to get from
the **velocity**

$$\left(\mathrm{d}R/\mathrm{d}t\right)^2 = 8\pi/3 G\rho R^2 \tag{15}$$

the **acceleration**

$$\left(\mathrm{d}^2 R/\mathrm{d}t^2\right) = -GM/R^2 + G/R\left(\mathrm{d}M/\mathrm{d}R\right). \tag{16}$$

This last equation differs fundamentally from the standard treatment rewriting completely our picture of the Universe. Indeed whereas the first term in the r.h.s. yields the Newtonian (and GR) attractive acceleration due to the mass M enclosed within the radius R, the second term gives a **repulsive** effect due to matter creation. Therefore even without a cosmological term we have a built-in repulsive gravitational term which may make the Universe not to collapse, without dark energy *i.e.*

$$\Omega_\Lambda = 0. \tag{17}$$

The preceding **energy conserving cosmogonic equations (ECCE)** represent the minimal parameter-free self-consistent model of the gross features of the Universe in which extra parameters may be introduced only if necessary for fitting finer details of its evolution. In the r.h.s. of **Equation (16) the two terms can indeed compensate thus yielding a steady state evolution** if

$$\mathrm{d}M/M = \mathrm{d}R/R \tag{18}$$

equivalent to

$$\mathrm{d}\rho/\rho = -2\,\mathrm{d}R/R. \tag{19}$$

The scale factor which links mass and radius is manifestly G/c^2 which explains why the self energy cancellation holds true only for the Universe: in other words only for very small and dense objects (Planck) or for dilute but very big regions *i.e*. the Universe itself during its evolution. Thus we have a double constraint on the velocity and on the acceleration: if in a black hole condition the Universe expands, mass and radius are linearly linked.

Notice that although the equation of energy conservation looks like the traditional one, the consistency requirement coming from the acceleration, implies a totally different scenario. In other words whereas in a traditional picture (without the cosmological term) the fate of the Universe resembles the description of the motion of a body thrown up in the air in a gravitational field, so that according to how its velocity compares to the escape velocity determined by the density it will collapse or not, here the expansion velocity is

$$\left(\mathrm{d}R/\mathrm{d}t\right) \simeq \text{const}. \tag{20}$$

It follows that

$$R(t) \simeq t \tag{21}$$

implying at the same time for the density evolution

$$\rho \simeq 1/R^2 \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\rho R^2\right) = 0 \tag{22}$$

which is still another way to express **the linear relation between mass and radius of a black hole**, confirming the flatness of space. The previous relation can be also directly got from Equation (14) by imposing total energy conservation for the interacting Universe at all times

$$M(t)\left(c^2 - 2GM(t)/R(t)\right) = 0. \tag{23}$$

This has to be contrasted with the requirement one gets from thermodynamic considerations as well as from GR

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}\Big/\left(\rho + p/c^2\right) = -3\frac{\mathrm{d}R}{\mathrm{d}t}\Big/R \tag{24}$$

which expresses **matter conservation** (*i.e.* essentially particle number) as it can be simply obtained by imposing

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho R^3\right) = 0 \tag{25}$$

and by taking into account the energy content of pressure (which plays a role essentially only for radiation) with the well known behaviors for matter (m) and radiation (rad)

$$\rho_m \simeq 1/R^3, \quad \rho_{rad} \simeq 1/R^4. \tag{26}$$

The ratio of pressure to density $w = p/\rho$, parameter which enters the standard phenomenology for the various forms of energy has an irrelevant role with respect to the pressure by particle creation as it will be further clarified in the following.

In conclusion the standard scenario has to cope with the problem of energy conservation, the proposed one clashes with our belief of particle number conservation. However this picture does not at all correspond to Hoyle's [13]. There, particle creation is invoked to obtain a steady constant density state and to disprove the big bang. The present proposal rather somewhat resembles, apart from the fundamental difference due to particle creation, to the primeval egg by Lemaître [21].

Just to get an order of magnitude, the estimated number of particles would have resulted from

$$\frac{\Delta N}{\Delta t \Delta V} \simeq \frac{10}{1\,\text{year} \times 1\,\text{km}^3} \tag{27}$$

(which we get allowing for creation of the present number minus the negligible Planck one over the age of the Universe) *i.e.* the creation of only ten hydrogen atom per cubic kilometer per year (since it has been referred to the present dimensions the creation rate would have been higher in the past). Therefore in that respect the present proposal does not go against any experimental evidence.

3. Self Energies: The Planck Scale

The fundamental Planck quantities are

$$R_P = \left(\hbar G/c^3\right)^{1/2} \simeq 10^{-33}\ \text{cm} \tag{28}$$

$$t_P = R_P/c \simeq 10^{-44}\ \text{sec} \tag{29}$$

$$E_P = M_P c^2 = \frac{\hbar}{t_P} \simeq 10^{19}\ \text{GeV}. \tag{30}$$

As well known one way to derive them is to ask in which units our "fundamental" coupling constants $c, \hbar, G$ are equal and of the order of 1. In particular G enters through Newton's law which is therefore assumed to be valid at any scale.

In another derivation they are determined by requiring the Compton wavelength of a particle to coincide with its Schwarzschild radius. Thus the Planck energy corresponds to the energy contained in the minimum (because it cannot be compressed farther without violating the uncertainty principle) quantum Schwarzschild radius *i.e.* the smallest quantum black hole.

This gravitational quantum "fluctuation" should, according to the prevailing picture,

$$\Delta E \Delta t \geq \hbar \tag{31}$$

last for times of the order of Planck's time.

But since

$$M_P c^2 - G M_P^2 / R_P = 0 \tag{32}$$

the Planck fluctuation is energetically stable in the sense that it does not have to decay in Planck times (up to numerical factors of order 1).

The equations written for the Universe at the present time can therefore be rewritten for Planck quantities

$$\left(\mathrm{d}R/\mathrm{d}t\right)^2\Big|_P = 8\pi/3 G\rho R^2\Big|_P \tag{33}$$

etc. with the same considerations.

Thus

$$G\rho_P \simeq \frac{GM_P}{R_P^3} = \frac{c^2}{R_P^2} = \frac{1}{t_P^2} \simeq H_P^2 . \tag{34}$$

When compared to the present quantity

$$G\rho_U \simeq \frac{GM_U}{R_U^3} \simeq \frac{c^2}{R_U^2} = \frac{1}{t_U^2} \simeq H_0^2 \tag{35}$$

one has

$$\frac{H_0^2}{H_P^2} = \frac{t_P^2}{t_U^2} \simeq 10^{-122} \simeq \left(10^{-61}\right)^2 \simeq \left(\frac{R_P}{R_U}\right)^2 . \tag{36}$$

From that we can immediately obtain an expression for Hubble's parameter at a generic time t and Universe radius R in terms of the present quantities H_0, t_U, R_U. We thus see that e.g. at half the time and radius of the Universe, the corresponding Hubble's parameter would be twice H_0.

In general

$$H(t) = \frac{c}{R_{U(t)}} \simeq \sqrt{G\rho(t)} . \tag{37}$$

Thus Hubble's parameter would have **decreased** in the course of the expansion by the above $\simeq 60$ orders of magnitude due to the density dilution.

A hypothetical observer in the past, say 10 billion years ago, would have of course "measured" a younger Universe by roughly a factor ten, which would have reflected in a corresponding bigger Hubble's parameter(t). These formulas confirm the argument [22] that an observer in the past ($t < t_U$) would have seen a different history, although that has been stated for the cosmological term which we have not needed to invoke. It goes without saying that, in case, given the different role attributed to the vacuum which is the expanding Universe itself, this unnecessary extra parameter Λ would not be constant but time dependent and should be also of order c^2/R^2.

That would answer Perlmutter's ([10]) worry that *in the standard picture* it appears "a remarkable and implausible coincidence that the mass density, *just in the present epoch*, is within a factor of 2 of the vacuum energy density because the mass density is supposed to become ever more dilute in the course of the expansion, whereas the vacuum energy density Λ, a property of empty space itself, stays constant". It would dispose as well of a similar statement [23] "to make things worse, the energy density $\left(10^{-3}\ \mathrm{eV}\right)^4$ happens to be of the same order of magnitude of the matter density of the Universe ρ_m $\left(\text{indeed}\left(10^{-3}\ \mathrm{eV}\right)^4 \simeq m_p c^2/m^3\right)$.

4. Self Energies: Evolution of Radiation and Matter

Let us inquire how much this parameter free scenario can account for the present 6-parameter cosmological phenomenology in the reconstruction of the past . As regards matter, its total energy in the Universe coming from Hubble's law is

$$E_m \simeq 10^{80}\ \text{GeV} \simeq 10^{80} m_p\,. \tag{38}$$

The present energy density of radiation, coming from the CMB, is

$$\epsilon_\gamma \simeq \left(kT_\gamma\right)^4 \simeq 10^5\ \text{eV/m}^3 \tag{39}$$

which yields a total energy for the Universe at present

$$E_\gamma \simeq \epsilon_\gamma \left(R_U\right)^3 \simeq 10^{74}\ \text{GeV} \simeq 10^{74} m_p \tag{40}$$

so that

$$E_\gamma / E_m \simeq 10^{-6}\,. \tag{41}$$

Thus "matter" (by which we mean loosely speaking non radiation) dominates over radiation by such a typical order of magnitude.

The total number of photons is given by

$$N_\gamma = n_\gamma \times R^3 \simeq \left(kT_\gamma R\right)^3 \tag{42}$$

and is of the order of 10^{87}.

Self energy would correct these energies in accord with the black body condition. *The relevant point is that the self energy dependence on R is not the same as that of the energy so that this correction does not factorize. Thus self-energy can act as a gauge in the reconstruction of the history of the Universe, undisputedly proving that a creation mechanism must have been constantly at work.*

As regards nucleons traditionally from energy conservation of non interacting particles (which amounts to particle number conservation)

$$E_m \simeq \epsilon_m R^3 \simeq \left(N_m \left(mc^2 + 3/2\,kT\right)\right) \simeq n_m R^3 \left(mc^2 + 3/2\,kT\right) \tag{43}$$

one predicts

$$\epsilon_m \simeq n_m \simeq \frac{1}{R^3} \tag{44}$$

(this conclusion is somewhat altered when taking into account the temperature and the velocity dependence of baryons. It is however easy to see that the former is totally irrelevant up to a temperature of $10^{10}\ \text{K} = 1\,\text{MeV}$ and that the latter even for $v = c$ would just give a factor of 2).

On the contrary the requirement of positivity for the energy coming from the self energy implies that the present black hole condition should hold, if we enforce energy conservation, also going backward in time. Thus as a function of R

$$1 - \frac{GM_m}{c^2 R} \simeq 0 \Rightarrow \rho \simeq \frac{1}{R^2} \tag{45}$$

($\rho = \epsilon / c^2$) where the irrelevant factor of 2 has already been commented upon. This is the same behavior derived from the equations of motion in par. 2).

Again *energy conservation of gravitationally interacting particles and particle number conservation are mutually contradictory*. Indeed the requirement Equation (44) would make the self energy correction of Equation (45) to turn negative. In other words the fact that less particles were present in the past in the energy conserving case with respect to their constancy of the traditional picture means once more that particles have been created in the expansion.

As regards photons, because of the energy mass equivalence, they are also obviously source of a sizable gravitational field when sufficiently energetic and/or numerous. Their black body treatment *at ordinary conditions* is absolutely unquestionable. Indeed if we consider e.g. a black body of one cubic meter at room temperature $kT \simeq 3\times10^{-2}\ \text{eV}$, its total energy is $(kT)^4 m^3 \simeq 10^8\ \text{MeV}$. Its mass M is hence $10^{-22}\ \text{kg}$ and its gravitational self energy, assuming a uniform distribution, $GM^2/R \simeq 10^{-10} \times 10^{-44}/1 \simeq 10^{-54}$ J, completely negligible with respect to its "bare" energy $Mc^2 \simeq 10^{-5}$ J!

For the Universe at present

$$\frac{GM_\gamma}{c^2 R_U} \simeq O\left(10^{-6}\right) \tag{46}$$

which is also negligible. And this must have been so for a long time since the energy density of the Planck spectrum which comes to us from the CMB

$$u(\omega)\mathrm{d}\omega \simeq \frac{\hbar\omega^3}{\mathrm{e}^{\hbar\omega/kT}-1}\mathrm{d}\omega \tag{47}$$

is remarkably accurate.

So the spectrum has kept its black body shape (which is sort of astonishing in view of the absence of significant later scattering processes) in agreement with scaling properties of the former angular frequency ω'

$$\omega = \omega'/(1+z) \tag{48}$$

(where z is the cosmological redshift), provided also the photon temperature T_γ scales by the same amount

$$T_\gamma \simeq 1/R\,. \tag{49}$$

This is consistent with photon number $N_\gamma \simeq \left(kT_\gamma R\right)^3 \simeq n_\gamma\,(R)^3$ conservation

$$n_\gamma \simeq \frac{1}{R^3} \tag{50}$$

from which the preceding result for the energy density ϵ_γ (and for $u(\omega)$)

$$\epsilon_\gamma \simeq \frac{1}{R^4} \tag{51}$$

follows. Thus

$$\frac{\epsilon_\gamma}{\epsilon_m} \simeq \frac{1}{R^2} \tag{52}$$

and from the previous estimates, matter and photon energy density would equalize at recombination (*rec*)

$$R_{rec} \simeq 10^{23}\ \mathrm{m} \tag{53}$$

which corresponds to the standard estimate for $T_\gamma \simeq 3000$ K, although through a completely different mechanism. The total mass and radius at that time would be one thousandth of the present one. At the same time, given the constancy of the photon number in this lapse of time, it is immediate to see that gravitational self-energy completely swallows the radiation energy

$$1 - \frac{GM_\gamma}{c^2 R} \simeq 0 \tag{54}$$

at

$$R' = \frac{GE'_\gamma}{c^4} \simeq 10^{23}\,\mathrm{m}\,. \tag{55}$$

Thus at

$$R' = R_{rec} \tag{56}$$

we have, at the same time, equality of matter and photon energy densities and zero total energy for both. This is gratifying since it proves that the self energy mechanism offers an alternative doubly constrained prediction. In former times the self energy mechanism would work both for radiation and for matter thus keeping their ratio constant at the value of recombination.

The different behavior of radiation and matter is then apparent. In the first case, because of the null mass, the wavelength is closely related to the Universe dimensions. This explains why we can reach $z \simeq 1000$. As regards nucleons the dominance of the mass in the energy makes them rather insensitive to the expansion (the typical z is much smaller; in essence their number varies but not their energy.) As mentioned, at R' only $O(10^{-3})$ of the present estimated number should have been present.

To summarize, back to R_{rec} *we have conservation of* N_γ *but decrease of* N_m *till the different energy densities equalize. In still former times both energy densities increase in the same way because of the self energy constraint so that there is no radiation dominated era! But their number density varies as*

$$n_\gamma / n_m \simeq m_p c^2 / kT_\gamma \; . \tag{57}$$

Thus typically at a temperature $T \simeq 10^9$ K of primordial nucleosynthesis, about 10^4 photons per nucleon exist. Whether this poses a problem for nuclear abundances will be examined elsewhere.

Further backward in the reconstruction of the history of the "dark ages", from the self energy condition for photons

$$(kT)^4 R^3 / c^2 R \simeq 1 \tag{58}$$

one gets

$$T^2 \simeq 1/R(t) \simeq 1/t \tag{59}$$

where the second equality comes from the results of the equations of motion.

The conclusions of this paragraph are therefore a linear connection between matter creation and the dimensions of the Universe throughout, and a linear dependence between the temperature of radiation and radius R back to recombination and quadratic before.

5. Structures

a) CMB and the event horizon

As well known [24] [25] CMB temperature fluctuations $\frac{\delta T}{T} \simeq 10^{-5}$ reflect in the angular size $\theta \simeq 0.01$ (which corresponds to the multipole moment $l \simeq 200$ of the first peak of the observed angular power spectrum) of cold and hot spots of the remarkably homogeneous map of the CMB temperature. They are usually accounted for in terms of the angular diameter distance d_A and of their dimensions of wavelength $\lambda \simeq ct_{rec}$

$$\theta = \frac{\lambda}{d_A} = \frac{2}{3\sqrt{3}\left(\left(1+z_{rec}\right)^{1/2} - 1\right)} \simeq 1/\sqrt{z} \tag{60}$$

where the Universe expansion in the matter dominated era goes, according to the standard treatment, as $R \simeq t^{2/3}$ with the ensuing $z \simeq 1100$.

Acoustic oscillations are clearly due to inhomogeneities in the baryon-photon fluid before recombination resulting in "standing waves" connected to the Universe dimensions and hence for us to the event horizon. In the present scenario photons emerging from the last scattering surface at R_{rec} in order to subtend today the said angle of $\theta \simeq 0.01$ of temperature fluctuations in the CMB, must have originated at $R_{ac} \simeq 10^{21}$ m (ac = acoustic) since indeed the subtended angle would be

$$\theta \simeq \frac{R_{ac}}{R_U} \simeq \frac{10^{21}}{10^{26}} . \tag{61}$$

Correspondingly that would imply a much bigger z than at recombination.

Notice that in terms of the corresponding quantities at recombination one has

$$\frac{R_{ac}}{R_{rec}} \simeq \frac{10^{21}}{10^{23}} \simeq \frac{t_{ac}}{t_{rec}} \simeq \frac{T_{rec}^2}{T_{ac}^2} . \tag{62}$$

So the acoustic peaks are directly connected to density fluctuations in a much smaller event horizon than that at recombination with corresponding shorter wavelengths. With CMB anisotropies we can "penetrate" the last scattering barrier.

The above prediction would entail a temperature an order of magnitude bigger of the usual 3000 K (*i.e.* kT of the order of 4 eV which is reasonable given the value of $\simeq 10\,\text{eV}$ for hydrogen ionization) and thermalization would have begun much earlier and have lasted from 0.1 million yr to 10 million yr.

These spots which originated well before, when interpreted in terms of the Universe mass at recombination, seem to imply a contribution of the baryonic mass to the total energy

$$\Omega_B \simeq 10^{-2} \tag{63}$$

which is one of the parameters entering the angular distribution fits. This is simply due to a theoretical "snapshot" of the Universe disregarding its time dependence and therefore the mass creation mechanism at work. In other words we have invisible matter in a time independent description which disappears when considering its time dependence.

b) supernovae : an accelerated expansion?

In the standard approach far away objects are "measured" through the luminosity distance

$$d_L = R_U r(1+z) \simeq \frac{z}{H_0}(1+..) \tag{64}$$

which comes from an expansion of the relevant quantities from their present value and where the dots stand for terms involving derivatives of the Hubble parameter. Thus measurements of luminosity distances and redshift parameters z of a number of galaxies have led to the discovery of the accelerated expansion of the Universe. This was once more unexpected since it clashed with our traditional picture of an attractive gravity.

In the present scenario on the other hand no expansion is necessary, since *a linear dependence of the Hubble parameter on time has been predicted*.

The farther the objects, the older but also within a Universe with a smaller radius $R_U(t') = R'$. Thus the extension of Hubble's law to any epoch means that their velocity is c times the ratio of their distance to the causal Universe radius R' *at the given r* (the one in the distance ladder determined by the luminosity, with the additional assumption that standard candles be really standard in the course of time) with respect to the present R_U.

$$v = \frac{c}{R'} r = H' r . \tag{65}$$

Thus the velocity would be greater than that predicted by H_0 in a time independent description,

$$\Delta v = (H' - H_0) r = \left(\frac{c}{R'} - \frac{c}{R_U} \right) r \tag{66}$$

the distance would consequently appear to be greater for very distant objects, supernovae would look fainter and this might be ascribed, following Riess and Turner [26], to an acceleration in the past.

c) galaxy clusters

Galaxy clusters involve dimensions of Mpc *i.e.* factors 10^{-3} with respect to R_U.

Their distance from us are e.g. $O(16\,\text{Mpc})$ for the Virgo cluster and $O(100\,\text{Mpc})$ for the Coma cluster, with a mass -to-light ratio of the order of 1000.

The gravitational fields are weak; indeed from

$$\beta = v^2/c^2 \simeq GM/c^2 R = \epsilon . \tag{67}$$

$\beta \simeq 10^{-5}$ ($v \simeq 1000\,\text{km/s}$) making GR corrections inessential.

The finding that the dynamic mass entering the kinetic energy be much higher than the one determined by luminosity measurements or

$$K > |U| \quad i.e \quad E > 0 \tag{68}$$

has led to speculate on the existence of missing mass or dark matter. These speculations are based on the application of the well known virial theorem which states that

$$\langle U \rangle = -2 \langle K \rangle . \tag{69}$$

This relation for average values applies for systems remaining within a finite volume, by averaging over "sufficiently" long times. However with the given values, the orbital period of individual galaxies around the common center of mass would be of the order of some billion years. Thus the application of the virial theorem in the preceding form seems at least questionable. More than that, if energy conservation holds true and E is therefore time independent

$$E = \langle E \rangle = \langle K \rangle + \langle U \rangle = -\langle K \rangle \tag{70}$$

contradicting the the initial inequality for the energy, which is just a learned way of expressing the simple fact that a system of total positive energy cannot be bound.

Thus it seems plausible that in this domain the formation of gravitationally bound objects out of negative capacity gravitational fluctuations be overcome by the effect of the Universe expansion. As a matter of fact the virial theorem reads in reality

$$K_{tot} - 1/2 \frac{\mathrm{d}}{\mathrm{d}t} \left(\Sigma M_i \boldsymbol{r}_i \cdot \boldsymbol{v}_i \right) = -1/2 U_{tot} . \tag{71}$$

An estimate of the second term, which cannot be averaged over many cycles in this case, yields

$$1/2 \frac{r^2}{H^2} \tag{72}$$

i.e. a velocity of the indivual masses M_i

$$v = cr/R_U \simeq c \times 10^{-3} \tag{73}$$

for the present value of the H, which would be increased by an order of magnitude when correctly considering its value some ten times bigger at the formation time thus substantially decreasing the value of the kinetic energy term. This brings velocity and luminosity measurements towards agreement challenging the necessity of invoking a sizable amount of dark matter.

In conclusion *the virial theorem in an expanding Universe gets substantial contributions from the expansion questioning the bound state picture* [27] [28].

d) the rotation curves of galaxies

As repeatedly underlined the present scenario concerns the bulk features of the Universe evolution. It is however natural to wonder whether it can account also for the missing mass which seems necessary for the description of galaxies where Hubble's expansion plays practically no role.

Application of Newton's law to the rotation curves of masses m (hydrogen atoms emitting the 21 cm line) orbiting a galaxy of total bulge mass M within a radius R reproduces reasonably well experimental data in the inner part *i.e*. for $r \le R$

$$v^2/r = GM/R^3 r \tag{74}$$

whereas in the exterior predicts a decrease of v

$$v^2/r = GM/r^2 \tag{75}$$

contradicted by experimental data which show a roughly constant behavior [29].

It has been noted that the presence of extra matter (missing mass) in the halo (h) with a density a

$$\rho \simeq 1/r^2 \tag{76}$$

which implies

$$M_h(r) \simeq r \tag{77}$$

would provide the extra attraction for the rotation curves and an extra deflection for lensing ("This distribution is spacially more extended than that of the visible baryons, demonstrating clearly the existence of dark matter" [30]), although alternative explanations have been proposed as well as for the bullet cluster [31].

Indeed that would result in an extra $1/r$ term in the previous equation (accounting for a constant v in the halo) equivalent to a modification of the acceleration in Newton's law [32]. Such an additional term to Newton's force appears quite naturally for rotating objects from gravitomagnetism (of which Coriolis is a relativistic example). However, given the value of ϵ, it seems to provide too low an acceleration. The problem is under investigation.

The overall situation of the proposed history of the Universe is summarized in **Table 1** and **Figure 1**.

Table 1. Parameters of the Universe evolution. The relevant part of our gravitational history has been spaced.

Age	Total mass	Temperature (K)	$R_U\ (m)$	Process
10^{-44} s	10^{19} GeV	10^{32}	10^{-35}	Planck origin
1 s	10^{63} GeV	10^{10}	10^{9}	Neutron freezout
1 min	$10^{64\text{-}65}$ GeV	10^{9}	10^{11}	Primordial nucleosynthesis
1 yr	10^{70} GeV	10^{7}	10^{12}	Atomic processes
0.1 million yr	10^{75} GeV	30.000	10^{21}	BAO
10 million yr	10^{77} GeV	3.000	10^{23}	Decoupling
......				
1 billion yr	10^{79} GeV	20	10^{25}	Gravitation
14 billion yr	10^{80} GeV	2.7 (CMB)	10^{26}	Now

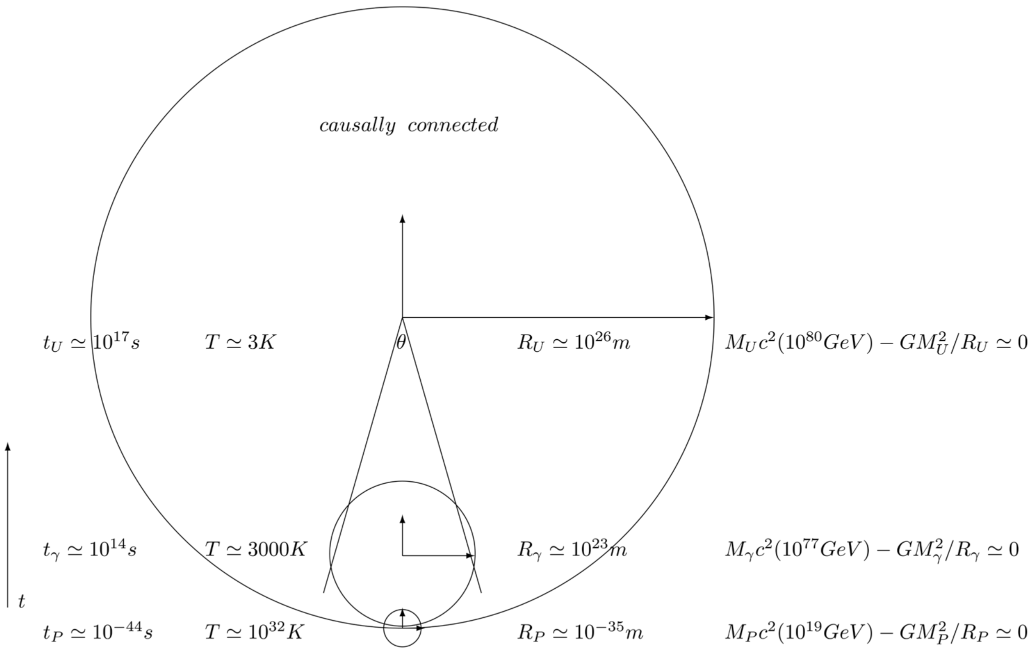

Figure 1. The Universe evolution.

The three most relevant situations *i.e.* Planck, total zero energy of radiation and present Universe are represented for a CMB observer, not in scale.

- The Planck "primeval egg" evolves in time, causally connected (c.c.), at the speed of light. No inflation has to be invoked.
- The evolution is accompanied by matter and photons creation which decreases the temperature.
- In all three cases the total energy is zero: we are inside an expanding black hole. This is backed up by the account of inertial forces.
- The evolution of the Universe is determined by the time dependent Hubble's parameter which measures its age and, within each bubble, the expansion velocity of its constituents.
- The Hubble radius and the event horizon coincide.

6. Conclusions

In the present paper a cosmogonic scenario has been proposed. At variance with respect to the standard matter conservation treatment, energy conservation, *with the essential role played by self energy*, has been enforced and seen to account for the puzzling non-gravitational collapse due to matter creation balancing gravitational attraction.

In this parameter free phenomenological treatment, the gross features of our Universe evolution from the stable quantum fluctuation at the Planck epoch have been accounted for without any need for the cosmological constant Λ, entailing a different chronology.

The stringent connection among a zero total energy Universe, (an extended use of) Hubble's law, and matter creation in the Universe expansion consistent with the equations of motion for the velocity and the acceleration, has been underlined and emphasized.

We can hence summarize our results as follows:

1) the consideration of self gravitational effects is essential,

2) they play a crucial role both at a classical and at a quantum level and both for matter and for radiation. Indeed the gravitational interaction of photons at cosmic scales necessarily alters the standard black body treatment,

3) the main features of the evolution of our Universe from the Planck to the present scale reasonably follow from energy conservation and (no need for an unbelievably) fine tuning of the cosmological parameters,

4) the evidence for the black hole nature of our Universe coming from the reproduction of inertial forces supports Hubble's formulation of its expansion. More than that *the black hole condition* $GM/c^2R \simeq 1$ *implies the constancy in time of these laws*,

5) finally the constancy of the power necessary to create matter in the course of the Universe evolution

$$W_U = E_U/t_U = M_U c^2/t_U = c^5/G = W_P = 10^{52}\ \text{W} \tag{78}$$

where the usual black hole relation has been used (seems to provide a further), piece of support for the consistency of the present approach.

In conclusion, the cosmological term, dark energy and (the bulk of) dark matter seem to be reasonably due only to our use of inadequate equations to account for the gross features of the actual evolution of the Universe.

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References

[1] Hubble, E. (1929) *Proceedings of the National Academy of Sciences of the United States of America*, **15**, 168-173. http://dx.doi.org/10.1073/pnas.15.3.168

[2] Penzias, A.A. and Wilson, R.W. (1965) *Astrophysical Journal*, **142**, 419-421. http://dx.doi.org/10.1086/148307

[3] Kragh, H. (1999) Cosmology and Controversy: The Historical Development of Two Theories of the Universe. Princeton University Press, Princeton.

[4] de Bernardis, P., *et al.* (2000) *Nature*, **404**, 955-959. arXiv:astro-ph/0004404. http://dx.doi.org/10.1038/35010035

[5] COBE Mission Design, Spacecraft and Orbit. Goddard Space Flight Center, 18 April 2008.

[6] Planck Collaboration. arXiv:1502.01582.

[7] Riess, A.G., *et al.* (1998) *The Astronomical Journal*, **116**, 1009. http://dx.doi.org/10.1086/300499

[8] Perlmutter, S., *et al.* (1999) *The Astrophysical Journal*, **517**, 565. http://dx.doi.org/10.1086/307221

[9] Susskind, L. (2006) The Cosmic Landscape. Hachette Book Group, New York.

[10] Perlmutter, S. (2003) *Physics Today*, **56**, 53. http://dx.doi.org/10.1063/1.1580050

[11] Rubin, V. (1998) Dark Matter in the Universe. Scientific American, Inc., New York, 106.

[12] Feynman, R.P. (1999) Lectures on Gravitation. Penguin Books, London.

[13] Hoyle, F. (1948) *Monthly Notices of the Royal Astronomical Society*, **108**, 372-382. http://dx.doi.org/10.1093/mnras/108.5.372

[14] Christillin, P. (2011) *The European Physical Journal Plus*, **126**, 88. http://dx.doi.org/10.1140/epjp/i2011-11088-6
[15] Sciama, D.W. (1959) The Unity of the Universe. Doubleday, Garden City, NY.
[16] Berry, M. (1976) Principles of Cosmology and Gravitation. Cambridge University Press, Cambridge.
[17] Christillin, P. and Barattini, L. (2013) ArXiv:1205.3514v3.
[18] Christillin, P. (2014) *The European Physical Journal Plus*, **129**, 175. http://dx.doi.org/10.1140/epjp/i2014-14175-2
[19] Okun, L.B. (1982) Leptons and Quarks. North Holland, Amsterdam.
[20] Maoz, D. (2006) Astrophysics in a Nutshell. Princeton University Press, Princeton.
[21] Lemaître, G. (1927) *Annales de la Société Scientifique de Bruxelles*, **A47**, 49-59.
[22] Barrow, J.D. and Shaw, D.J. (2011) *General Relativity and Gravitation*, **43**, 2555-2560. ArXiv:1105.3105.
[23] Zee, A. (2003) *Physics Letters B*, **594**, 8-12. ArXiv:hep-th/0309032v1.
[24] De Bernardis, P. and Bersanelli, M. (2013) *Il Nuovo Saggiatore*, **29**, 25.
[25] Mennella, A. and Natoli, P. (2015) *Il Nuovo Saggiatore*, **31**, 63.
[26] Riess, A.G. and Turner, M.S. (2004) *Italian edition of Scientific American*, **427**, 56.
[27] Ambartsumian, V.A. (1961) *Astronomical Journal*, **66**, 536-540. http://dx.doi.org/10.1086/108460
[28] Van den Bergh, S. (2000) ArXiv:astro-ph/0005314v1.
[29] Roos, M. (2010) ArXiv:1001.0316v1.
[30] Ellis, R.S. (2010) *Philosophical Transactions of the Royal Society A*, **368**, 967-987.
[31] Markevitch, M., *et al.* (2002) *The Astrophysical Journal*, **567**, L27-L31.
[32] Milgrom, M. (1983) *Astrophysical Journal*, **270**, 365-370. http://dx.doi.org/10.1086/161130

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Ion Acoustic Soliton and the Lambert Function

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Abstract

The Sagdeev potential method is employed to compute the width of (Ion-acoustic) soliton propagated in a cold plasma. The computation indicates that the soliton width is a continuous function (of the Mach number M), which is expressed in terms of the Lambert Function. Despite the (fairly) complex form of the function, the numerical plotting makes sense about its changes.

Keywords

Solitary and Shock Wave, Ion Acoustic Wave

1. Introduction

Over the last decades, there has been a great deal of interest and significant progress in the study of nonlinear plasma theories and many of works and researches in plasma physics devote much attention to these theories. The nonlinear theories include a large number of effects and phenomena such as the nonlinear coherent structures as shock waves, solitary waves (solitons), vortices, etc.

The collective electrical and magnetic properties of plasmas could produce interactions that take the place of collisions and permit shocks and solitons to form. A shock wave is a sudden transition (a type of propagating disturbance) in the properties of a fluid medium (liquid or gas), involving a difference in flow velocity across a narrow (ideally, abrupt) transition. In high-energy density physics, nearly any experiment involves at least one shock wave. Such shock waves may be also produced by applying pressure to a surface or by creating a collision between two materials.

Nonlinear effects in plasmas occur when a large amplitude wave is excited by an external means. Soliton (shock) waves are formed as a result of a balance between the nonlinearity and dispersion (dissipation) of the medium. For example, the dispersion in the ion acoustic wave can be counter-balanced by nonlinearity and an ion acoustic soliton can propagate. From the mathematical point of view, solitons are the stationary solutions to

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the *Korteweg-de Vries* equation:

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\beta\frac{\partial^3 V}{\partial x^3}=0, \tag{1}$$

where $V=V(x,t)$ is a function of position *x* and time *t*. Some historical events and discoveries led to the soliton theory and Equation (1) is not the only equation that has solitonic solution. The sine-Gordon and the nonlinear Schrödinger equations are among them. The solutions to the Equation (1) are traveling wave with constant profile in time and they describe the different types of models in various branches of physics and natural science (for instance, Equation (1) is modeled to describe the waves on shallow water surfaces).

As mentioned, Equation (1) may arise in the study of such diverse physical systems as fluids and plasmas. In plasma ambient, Equation (1) may be due to the balancing between nonlinearity and dispersion and for this reason its study is of special interest.

One of the possible approaches to study of the nonlinear effects is the so-called reductive perturbation technique [1]-[5]. This technique is widely employed to investigate the asymptotic behavior of nonlinear excitations and is more convenient to study of small-amplitude nonlinear perturbations, or to treat of plasma waves in a state very close to thermodynamic equilibrium.

Another successful approach to study of the electrostatic solitons and shock waves has been the Sagdeev Potential (**SP**) or Pseudo Potential (**PP**) method. The SP is one particular notion that has become immensely important in soliton and shock research [6]. The main advantage of this method over reductive perturbation technique is that, it is appropriate for arbitrary amplitude waves and one can derive all the soliton results of perturbation methods and compare it with the exact results obtained by the SP method [7].

In the present work, our aim is to compute the soliton width (**SW**) for an ion acoustic solitary wave propagated in a cold plasma. The computation is based on analogy between pseudo-particle in PP well and real particle in conservative potential well. Knowing the angular frequency of (small) oscillation of real particle about its equilibrium position and comparing this with corresponding quantity in the plasma system, one attains a formula to compute the SW. To understand briefly, the SW is defined as spatial length corresponding to a (complete) spatial oscillation of psuedo-particle in the PP well; comparing this with the definition of period of temporal oscillation for real particle in the well, the computation is straightforward. Also, for better understanding of the changes, the graph of the width function is visualized, with the help of numerical plotting (computer algebra).

The work is organized as follows:

The model and the main calculation are presented in the next section, and conclusions are given in the last section.

2. The Soliton Width as a Function of the Mach Number

We will consider a one dimensional model for propagation of ion acoustic wave in a cold plasma. The solitary wave is generated by spatial oscillation of electrostatic potential $\phi(x)$ which is the so-called psuedo-particle. The wave is travelling to the left in the (to say) *x* direction with a constant speed u_0. Therefore, in the wave-frame position, the SP is given by [8]:

$$V(\chi)=1-\mathrm{e}^{\chi}+M^2\left(1-\sqrt{1-\frac{\chi^2}{M^2}}\right), \tag{2}$$

This PP is subjected to the boundary condition $V(0)=0$, and the following dimensionless parameters are used

$$\chi=\frac{e\phi}{KT_e},\ \xi=\frac{x}{\lambda_D},\ M=\frac{u_0}{\sqrt{KT_e/m}},$$

where T_e to be electron temperature, λ_D is the ion Debye length and *M* is the Mach number.

The form of the PP would determine whether soliton like solutions may exist or not. The conditions for the existence of solitary waves are:

$$\left.\frac{\mathrm{d}^2 V}{\mathrm{d}\chi^2}\right|_{x=0}<0 \text{ and } V(\chi_m)=0,$$

where χ_m is the maximum value of χ beyond which the PP becomes imaginary[1] and it is often called amplitude of the soliton (or shock). By virtue of above conditions, it is easily to show that the values of the Mach number are confined to the interval

$$1 < M < 1.6, \tag{3}$$

It may be useful to plot the graph of the PP (2) versus its argument χ. The graph is illustrated in **Figure 1** for three values of *M*. As the figure shows, the depth of the potential well and amplitude of the soliton increase with increasing Mach number.

The PP (2) satisfies the energy condition:

$$\frac{1}{2}\left(\frac{\mathrm{d}\chi}{\mathrm{d}\xi}\right)^2 + V(\chi) = 0, \tag{4}$$

which is analogous to the principle of energy conservation

$$\frac{1}{2}m\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + U(x) = E,$$

for a real particle of mass *m* moving in a (conservative) potential $U(x)$. In view of this analogy, Equation (4) can be regarded as an energy integral of moving pseudo-particle of unite mass, pseudo-position χ, pseudo-velocity $\mathrm{d}\chi/\mathrm{d}\xi$ and pseudo-potential $V(\chi)$. Hence, by the following replacements:

$$m \to 1,\ x \to \chi,\ t \to \xi \text{ and } U \to V,$$

one can obtain similar formulas and results in the plasma system, as discussed below.

We know that for a real particle moving in the potential $U(x)$, the angular frequency of small oscillations ω, about the equilibrium position (a minimum at x_0) is given by

$$\omega^2 = \frac{1}{m}\left.\frac{\mathrm{d}^2U(x)}{\mathrm{d}x^2}\right|_{x=x_0}, \tag{5}$$

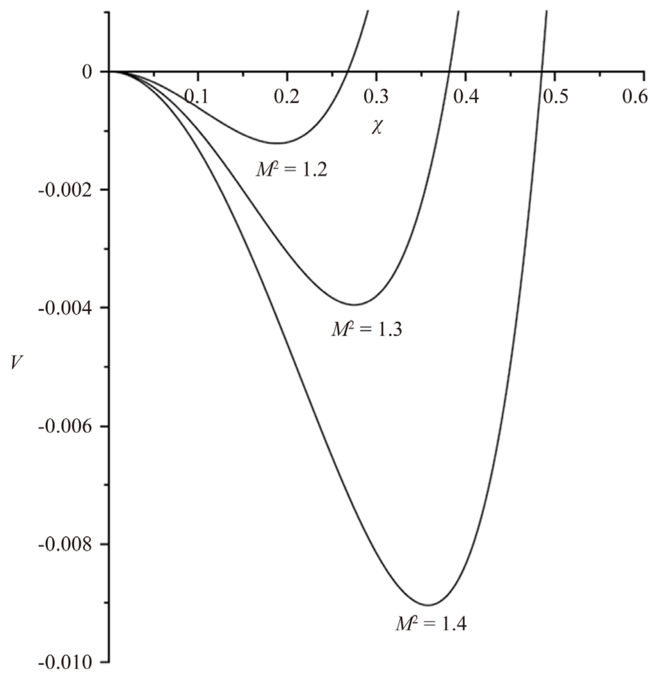

Figure 1. Psuedo-potential curves $V(\chi)$ corresponding to three values of $M^2 = 1.2, 1.3, 1.4$.

[1] χ_m is the rightmost intersection of potential curve and χ axis (**Figure 1**), and, it is an increasing function of *M*.

and as a result, the period of oscillation motion T becomes

$$T = 2\pi/\omega = 2\pi\Big/\sqrt{U''(x_0)}. \tag{6}$$

The natural question, then, is what conclusions can be drawn from this for our plasma system, more clearly, what quantities are given by similar formulas which are obtained by the above replacements in (5) and (6), that is

$$k_s^2 = \left.\frac{\mathrm{d}^2V(\chi)}{\mathrm{d}\chi^2}\right|_{\chi=\chi_0}, \tag{7}$$

and

$$\lambda_s = 2\pi/k_s = 2\pi\Big/\sqrt{V''(\chi_0)}. \tag{8}$$

where we also use $k_s = 2\pi/\lambda_s$ and the two replacement $\omega \to k_s$ in (5), $T \to \lambda_s$ in (6)[2].

It is easily to check that λ_s has the dimensions of the *Length*, and in below, we see that λ_s is SW.

We first remind the following definitions:

T: a time interval corresponding to one complete temporal oscillation of real particle (in the well).

On the other hand the width of the solitary wave is equal to the length that disturbance of the solitary wave takes place, or in SP terminology,

SW: a distance interval corresponding to one complete spatial oscillation of pseudo-particle (in the PP well), thus, it is natural to drawing the result

$$\lambda_s = \mathrm{SW} = 2\pi/k_s = 2\pi\Big/\sqrt{V''(\chi_0)}. \tag{9}$$

To use the above formula, we need to know the point χ_0, that is minimum of PP. This is given by vanishing of the first derivative of PP (2), namely

$$V'(\chi_0) = \frac{M}{\sqrt{M^2 - 2\chi_0}} - \mathrm{e}^{\chi_0} = 0. \tag{10}$$

solving this equation for χ_0, we get

$$\chi_0 = \chi_0(M) = \frac{1}{2}\left(M^2 + W\left(-M^2\mathrm{e}^{-M^2}\right)\right). \tag{11}$$

where W is the Lambert function. Substituting χ_0 into the second derivative

$$V''(\chi) = \frac{M}{\left(M^2 - 2\chi\right)^{\frac{3}{2}}} - \mathrm{e}^{x}. \tag{12}$$

and using Equation (9), we obtain the final expression for the SW as function of the Mach Number, that is

$$\lambda_s(M) = \frac{2\pi}{\sqrt{\dfrac{-M}{w\left(-M^2\mathrm{e}^{-M^2}\right)^{\frac{3}{2}}} - \mathrm{e}^{\frac{1}{2}\left(M^2 + w\left(-M^2\mathrm{e}^{-M^2}\right)\right)}}}, \tag{13}$$

Due to the presence of Lambert function, we have no clear idea of the behavior of the width function (13), then it is instructive to graph the function. Because of the transcendental form of the function, we have to use the numerical plotting and the corresponding graph is illustrated in **Figure 2**. It represents a monotonically decreasing function of M. The function takes on arbitrarily large values when $M \to 1$, and decreases rapidly over the (allowable) range $1 < M < 1.6$.

As mentioned, the soliton width is the length of a complete sweep of the pseudo-particle in the potential well and as is clear from the **Figure 1**, the potential well corresponding to the larger values of M is more narrow than

[2]The equilibrium position of PP is assumed to be at χ_0.

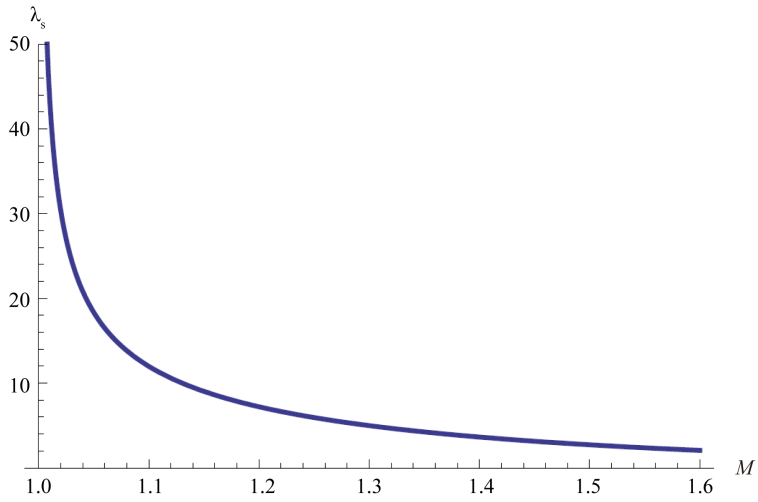

Figure 2. The graph of the soliton width λ_s versus M: rapidly monotonic decreasing function.

the smaller one, then, the solitons corresponding to larger M have shorter width.

In order to have a quantitative measure of the changes of the width function, let us compute the ratio

$$r = \frac{\Delta\lambda}{\Delta M} = \frac{\lambda_s(M_2) - \lambda_s(M_1)}{M_2 - M_1},$$

where M_1 and M_2 are near the upper and lower bounds of M respectively, that is $M_2 = 1.58$, $M_1 = 1.01$. From **Figure 2**, one finds

$$\lambda_s(M_1) \simeq 50 \text{ and } \lambda_s(M_2) \simeq 2$$

inserting these values in to the ratio, we obtain

$$r \approx -\frac{48}{0.58} \approx -82.7$$

This number is the average decreasing rate of the width per mach number, that is the change in the width divided by the change in Mach number. The negative sign indicates the decreasing nature of the function. As it is evident the amount of decreasing is relatively large.

In the above discussion, the width changes were described in terms of the Mach number itself, but, due to definition of the Mach number $\left(M = \frac{u_0}{\sqrt{KT_e/m}}\right)$, the following easy corollary can be deduced:

1) A positive (negative) change in the soliton wave velocity u_0 leads to a negative (positive) change in the SW, and

2) A positive (negative) change in the ratio (T_e/m) leads to a positive (negative) change in the SW.

In the end of this section, it is necessary to say that our ability to plot the width function λ_s (as continues function) is because of the analytic form of the SP (2). Indeed, the method presented here may be employed for any SP of analytic form [9]. When the SP has not analytic form, there is no such ability, but we may still calculate numerically λ_s at each allowable equilibrium (a minimum) position. This is based on estimation of equilibrium position and substituting into the second derivative of SP. In this case, one has a (discrete) point diagram, instead of a continues curve. One such computation has been represented in [10] for an electron-positron plasma including stationary ions in the background.

3. Conclusions and Results

The shock and soliton waves occur most likely because of the nonlinear disturbances, namely discontinuities in various variables as energy density, pressure, temperature, etc, in plasmas and other mediums. In addition to amplitude and velocity of disturbance, the determination of spatial scales within the collision less shock or soliton may be of particular interest. For example, for a shock disturbance $u(x,t)$ which obeys the Burgers equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\nu\frac{\partial^2 u}{\partial x^2}, \tag{14}$$

with the initial condition

$$u(x,0)=\begin{cases}u_+ & x>0\\ u_- & x<0\end{cases}, \tag{15}$$

it can be shown [11] that the width and speed of the shock are respectively

$$\omega=\frac{4\nu}{u_+-u_-},\quad V=\frac{1}{2}(u_+-u_-).$$

In the case of ion acoustic wave which is mediated by electric potential in the plasma, the width of the shock or soliton may be particularly important in its relation to the width of the electrostatic potential drop across the shock.

In this work, the PP approach is employed to calculate the width of (one dimensional) ion acoustic solitary wave propagating in a cold plasma. The calculation shows that the soliton width is a continuous function of the Mach number and the function is expressed in terms of the Lambert function. Because of transcendental form of the width function, we have to use the numerical method (computer algebra) to graph of the function over the (short) allowable range of the Mach number ($1<M<1.6$). The graphical representations help us to understand the behavior of the width function. The main feature of the graph is monotonic rapidly decreasing one, such that the average rate of change is about 82 unit of the length per Mach number. Contrary to the width, the amplitude of solitary wave is an increasing function of the Mach number.

References

[1] Salahuddin, M., Saleem, H. and Saddiq, M. (2005) *Physical Review E*, **66**, Article ID: 036407.

[2] Esfandyari-Kalejahi, A., Mehdipoor, M. and Akbari-Moghanjoughi, M. (2009) *Physics of Plasmas*, **16**, Article ID: 052309. http://dx.doi.org/10.1063/1.3142465

[3] Tiwari, R.S. (2008) *Physics Letters A*, **372**, 3461-3466. http://dx.doi.org/10.1016/j.physleta.2008.02.002

[4] Tiwari, R.S., Kaushik, A. and Mishra, M.K. (2007) *Physics Letters A*, **365**, 335-340. http://dx.doi.org/10.1016/j.physleta.2007.01.020

[5] Mushtaq, A. and Shah, H.A. (2005) *Physics of Plasmas*, **12**, Article ID: 072306. http://dx.doi.org/10.1063/1.1946729

[6] Sagdeev, R.Z. (1966) *Review of Plasma Physics*, **4**, 23.

[7] Roychoudhury, R. (2000) *Proceedings of Institute of Mathematics of NAS of Ukraine*, **30**, 510-515.

[8] Chen, F.F. (1984) Introduction to Plasma Physics and Controlled Fusion. 2nd Edition, Plenum, New York. http://dx.doi.org/10.1007/978-1-4757-5595-4

[9] Malakolkalami, B. and Mohammadi, T. (2014) *The Open Plasma Physics Journal*, **7**, 199.

[10] Malakolkalami, B. and Mohammadi, T. (2012) *Journal of Plasma Physics*, **78**, 05.

[11] Ablowitz, M.J. (2011) Nonlinear Dispersive Waves. Cambridge University Press, New York, 27. http://dx.doi.org/10.1017/cbo9780511998324

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Biquaternionic Form of Laws of Electro-Gravimagnetic Charges and Currents Interactions

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Abstract

One the base of differential algebra of biquaternions, the one model of electro-gravimagnetic interactions of electric and gravimagnetic charges and currents has been constructed. For this, three Newton laws analogues are used. The closed system of biquaternionic wave equations is constructed for determination of the charges-currents and electro-gravimagnetic fields and united field of interactions. The equation of charge-current transformation is like the generalization of biquaternionic presentation of Dirac equation. The properties of its solutions are described, depending on properties of external EGM field. The biquaternions of energy-pulse of EGM-field and charges-currents are considered. The energy-pulse of EGM-interactions is calculated.

Keywords

Electro-Gravimagnetic Field, Electric Charge, Gravimagnetic Charge, Current, Energy, Biquaternion, Bigradient, Dirac Equation, Newton Laws

1. Introduction

In the paper [1], we described the one biquaternionic model of electro-gravimagnetic (EGM) field, charges and currents. In this model, gravitational field (which is potential) is united with magnetic field (which is torsional) which gives possibility to enter gravimagnetic tension, gravimagnetic charge and gravimagnetic current. There we have shown that in algebra of biquaternions, the charges and currents of EGM-field are physical appearance of bigradient of EGM-intensity. Differential operator *bigradient* is the generalization of gradient operator on the space of biquaternions which characterizes a direction of more extensive change of biquaternionic functions. If bigradient of EGM-intensity is equal to zero then charges and currents are absent. Also there we constructed the

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law of inertia for a free system of mass, charges and currents, which described their motion under action only with internal electric and gravimagnetic tensions. This law is the fields analogue of the first Newton law—inertia law for a solid (considered as material point).

Here we consider the motion of electro-gravimagnetic charges and currents under action of external EGM-fields which are created by other charges and currents. The laws of their interaction are the fields analogue of the second and third Newton laws. They have been constructed in the form of biquaternionic wave equations which generalize biquaternionic form of Dirac equations. Some solutions of them are discussed. The energy-pulse of EGM-interactions is calculated.

We used also here the differential algebra of biquaternions in Hamiltonian form which was shortly described in [1] (see [2] for more detail). This form is very convenient for description of physical fields. There are voluminous literature about application of algebras of quaternions and biquaternions in fields theory, in the theory of electromagnetic fields, and quantum mechanics [3]-[17]. Differential algebra of biquaternions gives possibility to simplify mathematical record of systems of Maxwell and Dirac equations to construct their solutions and to study their properties.

The novelty of this work is the construction of laws of electro-gravymagnetic interactions on the base of biquaternions algebra. The properties of this algebra and operation of quaternionic multiplication in fields theory have visual physical interpretation and detect new objective laws which are impossible to determine without this algebra. It's natural for matter as you'll see here.

2. Biquaternions of Electro-Gravimagnetic Field, Charges and Currents

Their are the next complex characteristics of EGM-field [1]:

- complex vector of *EGM-intensity*

$$A(\tau,x) = A^E + iA^H = \sqrt{\varepsilon}E + i\sqrt{\mu}H;$$

- complex charges field:

$$\rho(\tau,x) = \rho^E/\sqrt{\varepsilon} - i\rho^H/\sqrt{\mu};$$

- complex currents field:

$$J(\tau,x) = \sqrt{\mu}\, j^E - i\sqrt{\varepsilon}\, j^H;$$

- complex scalar *field of attraction-resistance*

$$\alpha(\tau,x) = \frac{i\alpha_1}{\sqrt{\varepsilon}} + \frac{\alpha_2}{\sqrt{\mu}}.$$

Here real vectors E and H are the tensions of electric and *gravimagnetic fields*; real scalars ρ^E, ρ^H are the densities of electric and *gravimagnetic charges*; real vectors j^E, j^H are the densities of electric and gravimagnetic currents; values ε, μ are the constants of electric conductivity and magnetic permeability of the EGM-medium.

In biquaternions algebra on Minkowski space $\mathbb{M} = \{(\tau,x)\}$ the EGM-field, charges and currents may be presented by use the next biquaternions:

EGM-intensity

$$\mathrm{A}(\tau,x) = i\alpha(\tau,x) + A(\tau,x),$$

charge-current

$$\mathbf{\Theta}(\tau,x) = -i\rho(\tau,x) - J(\tau,x).$$

In the paper [1] [2] was shown that connection between EGM-intensity and charge-current has the bigradiental form:

POSTULATE 1

$$\nabla^+\mathbf{A} = (\partial_\tau + i\nabla)\circ\mathbf{A} = \mathbf{\Theta}(\tau,\mathbf{x}). \tag{1}$$

Here and further the bigradients ∇^+ and ∇^- are the next biquaternionic differential operators:

$$\nabla^{\pm}\mathbf{F} \triangleq (\partial_\tau \pm i\nabla)\circ(f+F) = (\partial_\tau f \mp i(\nabla, F)) \pm \partial_\tau F \pm i\nabla f \pm i[\nabla, F]$$
$$= (\partial_\tau f \mp i\,div F) \pm \partial_\tau F \pm i\,grad\, f \pm i\,rot F.$$

Further we name them *mutual bigradients*. They define a "directions" of more intensive changing of biquaternionic field.

3. The Power and Density of Acting Forces

Let's consider two systems of charges and currents $\mathbf{\Theta}(\tau,x), \mathbf{\Theta}'(\tau,x)$. Every of them generate own EGM-field: $\mathbf{A}(\tau,x) = ia + A$ and $\mathbf{A}'(\tau,x) = ia' + A'$, which corresponds to (1). At first let consider the case when $a' = 0$. We name a *power-force density* the next biquaternion

$$\mathbf{F} = p - iF = \mathbf{A}'\circ\mathbf{\Theta} = -A'\circ(i\rho + J) = (A', J) - i\rho A' - [A', J] \tag{2}$$

which is acting from side of A' -field on the charge and current of A-field. Really, according to definitions, the scalar part is determined as power density of acting forces:

$$p = (A', J) = c^{-1}\left((E', j^E) + (H', j^H)\right) + i\left((B', j^E) - (D', j^H)\right). \tag{3}$$

Here $B = \mu H$ is analogue of a magnetic induction (in torsional part complies with it), $D = \varepsilon E$ is a vector of an electric offset.

Selecting the real and imaginary part from the formulae (2) we get the expressions for a density of acting forces $\left(F = F^H + iF^E\right)$:

$$F^H = \rho^E E' + \rho^H H' + B' \times j^E - D' \times j^H \tag{4}$$

$$F^E = c\left(\rho^E B' - \rho^H D'\right) - c^{-1}\left(E' \times j^E + H' \times j^H\right). \tag{5}$$

Potential part of H' describes the tension of gravitational field. Torsional part of this vector describes magnetic field. The scalar part of $\mathbf{\Theta}, \mathbf{\Theta}'$ contains the densities of electric charge and gravitational mass. Its vector part contains the densities of electric and mass currents. Coming from these suggestions, in formula (4) we see the known forces, appropriately:

- *Coulomb's force* $\rho^E E'$;
- *gravimagnetic force* $\rho^H H'$ (it complies with gravitational force in a potential part H');
- *Lorentz force* $B' \times j^E$ (more exactly, it complies with it in torsional part B');
- *gravielectric force* $j^H \times D'$.

In real part of the power p (3) we see the powers of Coulomb's force, gravitational and magnetic forces. The power of gravielectric force in real part of (3) does not enter as it does not work on the mass displacement, because it is perpendicular to its velocity. It's interesting that Lorentz force also does not enter in real part of (3). It proves that this force is perpendicular to mass velocity, though directly from Maxwell equations this does not follow.

Naturally, by analogy, we assume that formula (5) describes forces, causing a change of electric current. Consequently their power stands in imaginary part p (3).

With entering the scalar field a', type of scalar and vector parts of power-force biquaternion (2) is changed, as follows

$$\mathbf{F} = \mathbf{A}'\circ\mathbf{\Theta} = \left((A', J) + a'\rho\right) - i\left(a'J + \rho A'\right) - [A', J] \tag{6}$$

You see here the additional summands which appear in presentation of the powers ($a'\rho$) and force ($-ia'J$).

Vector $a'J$ describes *absorbtion-resistance force* which acts on Θ from A'-field.

4. CC-Transformations Equation: Second Newton Law

The charge-current field $\mathbf{\Theta}$ is changed under influence of the $\mathbf{A}'$-field. As it's well known, the direction of the most intensive change the scalar field describes its gradient. By analogy we expect that change of charge-current biquaternion is more intensive toward its bigradient. Naturally to expect that this change must be toward external power-force.

POSTULATE 2. The law of a change of a charge-current field under the action of external EGM-field is

$$\kappa\nabla^{-}\mathbf{\Theta} = \mathbf{F} \triangleq \mathbf{A}' \circ \mathbf{\Theta}. \tag{7}$$

Entering the constant of interaction κ is connected with dimensionality. We name Equation (7) as *CC-transformations equation*.

If one EGM-field much stronger then second one:

$$\|\mathbf{A}'\| \gg \|\mathbf{A}\|$$

it's possible to neglect the second field change under influence of charge and current on $\mathbf{A}'$-field. In this case from Equation (7) we can find the CC-field $\mathbf{\Theta}(\tau,\mathbf{x})$, its changing under action of external $\mathbf{A}'$-field.

Revealing scalar and vector part from (7), we get the system of two differential equations:

$$i\kappa\left(\partial_\tau \rho + div\, J\right) = \left(A', J\right) + a'\rho = p, \tag{8}$$

$$i\kappa\left(\partial_\tau J - i rot\, J + \nabla\rho\right) = \left(a'J + \rho A'\right) - i\left[A', J\right] = F. \tag{9}$$

By use (3), (4) we obtain from Equation (9) the two vectorial differential equations:

$$\kappa\left(\sqrt{\varepsilon}\partial_\tau j^H + \sqrt{\mu} rot\, j^E + \mu^{-0.5} grad\, \rho^H\right) = \rho^E E' + \rho^H H' + B' \times j^E - D' \times j^H + Re\left(a'J\right), \tag{10}$$

$$\kappa\left(\sqrt{\mu}\partial_\tau j^E - \sqrt{\varepsilon} rot\, j^H + \varepsilon^{-0.5} grad\, \rho^E\right) = c\left(\rho^E B' - \rho^H D'\right) - c^{-1}\left(E' \times j^E + H' \times j^H\right) + Im\left(a'J\right). \tag{11}$$

The Equation (10) is the second Newton law analogue for CC-field. Here the value $\kappa\sqrt{\varepsilon}\, j^H$ is analogue of mass momentum. In right part you see all known forces but also two new forces: gravielectric force and absorbtion-resistance force. Last of them is proportional to currents. Their direction depends on signs of real and imaginary part $a'(\tau, x)$ which can be as positive and negative. By analogy to mechanics of media we call it such name.

Equation (10) describes the motion of gravimagnetic charges and currents under action of the external EGM-field. Consequently Equation (11) defines the motion of electric charges and currents.

The scalar Equation (8) is the law of conservation of electric and gravimagnetic charges. As you see the external EGM-field can essentially change CC-field.

5. Third Newton Law: The Laws of Charge-Currents Interactions

On the virtue of the third Newton law about acting and counteracting forces, we suppose that must be executed for electro-gravimagnetic forces the equality:

POSTULATE 3

$$\mathbf{F}' = -\mathbf{F}.$$

From here we get

fields analogue of third Newton law:

$$\mathbf{A}' \circ \mathbf{\Theta} = -\mathbf{A} \circ \mathbf{\Theta}'. \tag{12}$$

By use it we construct

the law of the charge-current interaction:

$$\kappa\nabla^{-}\mathbf{\Theta} = \mathbf{A}' \circ \mathbf{\Theta},\ \kappa\nabla^{-}\mathbf{\Theta}' = \mathbf{A} \circ \mathbf{\Theta}', \tag{13}$$

$$\mathbf{A}' \circ \mathbf{\Theta} = -\mathbf{A} \circ \mathbf{\Theta}', \tag{14}$$

$$\nabla^{+}\mathbf{A} = \mathbf{\Theta},\ \nabla^{+}\mathbf{A}' = \mathbf{\Theta}'. \tag{15}$$

Here Equation (13) correspond to the second Newton law which is written for charge-current each of interacting field. Equation (14) is the third Newton law. Together with Maxwell equations for these fields (15) they give closed system of the nonlinear differential equations for determination $\mathbf{A}, \mathbf{A}', \mathbf{\Theta}, \mathbf{\Theta}'$.

It is interesting that in scalar part of Equation (12) it requires the equality of the powers corresponding to forces, acting on charges and currents of the other field, *i.e.* it is befitted known in mechanics the identity to reciprocity of Betty, which is usually written for the forces works.

6. First Newton Law: Free EGM-Field

Let's consider A-field, which is generated $\mathbf{\Theta}$, in absence of other charges-currents. We name it *a free field*. In this case, $\mathbf{F}=0$. From (13) we get inertia law, which is analogue of *the first Newton law for charge-current*:

$$\nabla^{-}\mathbf{\Theta}=\mathbf{0}, \tag{16}$$

which is equivalent to equations:

$$\partial_{\tau}\rho+div\,J=0,\ \partial_{\tau}J-i\,rot\,J+\nabla\rho=0.$$

For initial designations we have following formulas:

$$\partial_{t}\rho^{E}+div\,j^{E}=0,\quad \partial_{t}\rho^{H}+div\,j^{H}=0; \tag{17}$$

$$\partial_{\tau}j^{E}=\sqrt{\varepsilon/\mu}\,rot\,j^{H}-c\,grad\,\rho^{E},\quad \partial_{\tau}j^{H}=-\sqrt{\mu/\varepsilon}\,rot\,j^{E}-c\,grad\,\rho^{H}. \tag{18}$$

Naturally that the well known law of charges conservation in the form (17) must be executed in absence of external EGM-field.

This law (16) was considered in [1] and solutions of this equations were defined in the next form:

$$\mathbf{\Theta}=\nabla^{-}\psi^{0}+i\sum_{j=1}^{3}\nabla^{-}\left(\psi^{j}e_{j}\right). \tag{19}$$

Scalar potentials ψ^{j} are arbitrary solutions of classic wave equation:

$$\frac{\partial^{2}\psi^{j}}{\partial\tau^{2}}-\Delta\psi^{j}=0,\quad j=0,1,2,3,4 \tag{20}$$

where Δ is Laplace operator. They may be presented so

$$\psi^{j}=\int_{R^{3}}\varphi^{j}(\xi)\exp\left(-i(\xi,x)-i\|\xi\|t\right)\mathrm{d}\xi_{1}\mathrm{d}\xi_{2}\mathrm{d}\xi_{3},\quad \forall\varphi^{j}(\xi)\in L_{1}\left(R^{3}\right). \tag{21}$$

We give here also for (16)

the solution of Cauchy problem

$$\mathbf{\Theta}(\tau,x)=-\frac{H(\tau)}{4\pi}\nabla^{+}\left\{\tau^{-1}\int_{\|y-x\|=\tau}\mathbf{\Theta}_{0}(y)\mathrm{d}S(y)\right\}, \tag{22}$$

were there are the initial conditions:

$$\mathbf{\Theta}(0,x)=\mathbf{\Theta}_{0}(x),\mathbf{A}(0,x)=\mathbf{A}_{0}(x). \tag{23}$$

From postulate 1 to follow that **A**-field is defined in the form:

$$4\pi A(\tau,x)=-\nabla^{-}\left\{\int_{r\le\tau}\frac{\mathbf{\Theta}(\tau-r,y)}{r}\mathrm{d}V(y)+\tau^{-1}\int_{r=\tau}\mathbf{A}_{0}(y)\mathrm{d}S(y)\right\}-\tau^{-1}\int_{r=\tau}\mathbf{\Theta}(0,y)\mathrm{d}S(y), \tag{24}$$

where $r=\|y-x\|,\mathrm{d}V(y)=\mathrm{d}y_{1}\mathrm{d}y_{2}\mathrm{d}y_{3}$, $\mathrm{d}S(y)$ is a differential of spheres area. About determination of these solutions see the theory of biwave equation in [2].

This formula is a generalization of the famous *Kirchhoff formula* for solution of Cauchy problem for wave equation [18].

7. The Solutions of CC-Transformation Equation by Action of Invariable External EGM-Field

Let consider Equation (7) when external EGM-field $\mathbf{A}'$ is constant, don't depend on τ and *x*. We write it in the form

$$\nabla^{-}\mathbf{\Theta}-\kappa^{-1}\mathbf{A}'\circ\mathbf{\Theta}=0. \tag{25}$$

This equation is biquaternionic generalization of Dirac equations. Its presentation by use differential equations coincides with Dirac equations when $\mathbf{A}' = im$, m is real constant. About construction full system of solutions of homogeneous and inhomogeneous Dirac equation see [4] [5].

There are simple connection between solutions of Equation (7) and Equation (22):

$$\mathbf{\Theta} = \mathbf{\Theta}_0 \exp\left(\kappa^{-1}\left(a'\tau + i\left(A', x\right)\right)\right) \tag{26}$$

where $\mathbf{\Theta}_0$ is the solution of (7):

$$\nabla^{-}\mathbf{\Theta}_0 = 0.$$

From here to follow that imaginary part of scalar field resistance-absorbtion a' creates harmonic vibration with frequency $a_1/\sqrt{\varepsilon}$, but real part $'$ increases or decreases $\|\mathbf{\Theta}_0\|$ over time depend on a sign a_2. Real part of vector A' (electric field) creates for $\mathbf{\Theta}_0$ sinusoidal deviation in R^3 along direction of E. But its imaginary part (gravymagnetic field) gives exponential growth or diminution $\|\mathbf{\Theta}\|$ along direction the vector H.

8. Energy-Impulse Biquaternions: First Thermodynamics Law

We introduce the biquaternion of energy-impulse of EGM-field (*EGM-energy-impulse*):

$$W + iP = 0.5\mathbf{A}^{*} \circ \mathbf{A}, \tag{27}$$

where $\mathbf{A}^{*}$ is conjugated $\mathbf{A}$:

$$\mathbf{A} = a + A, \qquad \mathbf{A}^{*} = \bar{a} - \bar{A} = -i\bar{\alpha} - \bar{A},$$

$\bar{a}, \bar{A}$ are complex conjugated to a, A. Here positive scalar function W is energy density of EGM field:

$$W = 0.5\left(\|\alpha\|^2 + \|A\|^2\right) = 0.5\left(\|\alpha\|^2 + \|E\|^2 + \|H\|^2\right),$$

P is real vector-function:

$$P = 0.5\left(a\bar{A} - \bar{a}A + \left[A, \bar{A}\right]\right) = \alpha_2 E/\sqrt{\mu} + \alpha_1 H/\sqrt{\varepsilon} + [E, H].$$

By $\alpha = 0$, as you see, W, P are like to an energy and Pointing vector of EM-field and satisfid to equation:

$$\partial_\tau W + div P = -\operatorname{Re}\left(J, \bar{A}\right) = c^{-1}\left(j^H H - j^E E\right). \tag{28}$$

By analogue we construct the biquaternion of energy-impulse for charge-current field (*CC-energy-impulse*):

$$0.5\mathbf{\Theta}^{*} \circ \mathbf{\Theta} = \left(\frac{\|\rho_E\|^2}{\varepsilon} + \frac{\|\rho_H\|^2}{\mu} + Q\right) + i\left(P_J - \sqrt{\frac{\mu}{\varepsilon}}\rho^E j^E - \sqrt{\frac{\varepsilon}{\mu}}\rho^H j^H\right). \tag{29}$$

It contains currents energy :

$$Q = 0.5\|J\|^2 = 0.5\left(\mu\left\|j^E\right\|^2 + \varepsilon\left\|j^H\right\|^2\right),$$

where first summand includes Joule heat $\left\|j^E\right\|^2$; second one includes kinetic energy density of mass current $\left\|j^H\right\|^2$, also it contains the energy of torsional part of currents (magnetic current). Here vector P_J is analogue of Pointing vector, but for the current:

$$P_J = 0.5 i J \times \bar{J} = c^{-1}\left[j^E, j^H\right].$$

Only if gravimagnetic and electrical currents are parallel or one from them is equal zero, then $P_J = 0$.
If to take scalar product Equation (9) with $i\bar{J}$, we get

$$\kappa\left(\partial_\tau Q - div P_J + \operatorname{Re}\left(\nabla\rho, \bar{J}\right)\right) = \operatorname{Im}\left(F, \bar{J}\right) = c^{-1}\left(\left(F^H, j^H\right) + \left(F^E, j^E\right)\right). \tag{30}$$

It is easy to see that this law is like to the first thermodynamics law. Here the sum of second and third summands in left part is designated $-U$. The function

$$U = div P_J - \sqrt{\mu/\varepsilon}\left(\nabla\rho^E, j^E\right) - \sqrt{\varepsilon/\mu}\left(\nabla\rho^H, j^H\right)$$

characterizes the own velocity of the change of current energy of $\boldsymbol{\Theta}$-field. Right part (30), which depends on power of acting external forces, can to increase or decrease this velocity.

If there are not acting external forces,

$$\partial_\tau Q = U. \tag{31}$$

It's the first thermodynamics law for a free CC-field.

9. The United Field of Interaction: Energy-Pulse of Interactions

If there are some (N) interacting CC-fields then we have for every of them

$$\kappa \nabla^- \boldsymbol{\Theta}^k = \sum_{m \neq k} \mathbf{A}^m \circ \boldsymbol{\Theta}^k, \ \nabla^+ \mathbf{A}^k = \boldsymbol{\Theta}^k, \quad k = 1, \cdots, N \tag{32}$$

$$\nabla^+ \mathbf{A}^m \circ \mathbf{A}^k = -\nabla^+ \mathbf{A}^k \circ \mathbf{A}^m, \quad k \neq m. \tag{33}$$

United CC-field, as easy to see after summing the first equation over k, is free. It satisfies to the inertia law

$$\nabla^- \boldsymbol{\Theta} = \nabla^- \sum_{m=1}^{M} \boldsymbol{\Theta}^m = \mathbf{0} \tag{34}$$

because all forces are internal, also as in Newton mechanics of interacting solids.

Let consider the laws of energy transformation at interaction of different charges-currents. Energy-pulse for united charge-current field has the form:

$$\boldsymbol{\Xi}_\Theta = 0.5 \boldsymbol{\Theta} \circ \boldsymbol{\Theta}^* = 0.5 \sum_{k=1}^{N} \boldsymbol{\Theta}^k \circ \sum_{l=1}^{N} \boldsymbol{\Theta}^{*l} = 0.5 \left(\sum_{k=1}^{N} \boldsymbol{\Theta}^k \circ \boldsymbol{\Theta}^{*k} + \sum_{k \neq l} \boldsymbol{\Theta}^k \circ \boldsymbol{\Theta}^{*l} \right) = \sum_{k=1}^{N} W_\Theta^{(k)} + i \sum_{k=1}^{N} P_\Theta^{(k)} + \delta \boldsymbol{\Xi}_\Theta.$$

Here the first summand is an amount of energy-pulse of interacting charge-current.

We introduce biquaternion of *energy-pulse interaction*. Its real part describes energy-pulse interaction for the same name charge and current, but imagine part for different name ones:

$$\delta \boldsymbol{\Xi}_\Theta = \delta W_\Theta + i \delta P_\Theta = \sum_{k \neq l} \boldsymbol{\Xi}_\Theta^{kl}, \quad \boldsymbol{\Xi}_\Theta^{kl} = 0.5 \left(\boldsymbol{\Theta}^k \circ \boldsymbol{\Theta}^{*l} + \boldsymbol{\Theta}^l \circ \boldsymbol{\Theta}^{*k} \right)$$

$$\boldsymbol{\Xi}_\Theta^{kl} = \operatorname{Re}\left(\rho^k \rho^{*l} + \left(J^k, J^{*l} \right) \right) - i \left\{ \operatorname{Re}\left(\rho^k J^{*l} + \rho^{*l} J^k \right) + \operatorname{Im}\left[J^k, J^{*l} \right] \right\},$$

or in initial designations:

$$\begin{aligned}
\boldsymbol{\Xi}_\Theta^{kl} = {} & \frac{\rho^{E(k)} \rho^{E(l)}}{\sqrt{\varepsilon_k \varepsilon_l}} + \frac{\rho^{(k)H} \rho^{H(l)}}{\sqrt{\mu_k \mu_l}} + \sqrt{\mu_k \mu_l} \left(j^{(k)E}, j^{(l)E} \right) + \sqrt{\varepsilon_k \varepsilon_l} \left(j^{(k)H}, j^{(l)H} \right) \\
& - i \left\{ \sqrt{\frac{\mu_l}{\varepsilon_k}} \rho^{(k)E} j^{(l)E} + \sqrt{\frac{\varepsilon_l}{\mu_k}} \rho^{(k)H} j^{(l)H} + \sqrt{\frac{\mu_k}{\varepsilon_l}} \rho^{(l)E} j^{(k)E} + \sqrt{\frac{\varepsilon_k}{\mu_l}} \rho^{(l)H} j^{(k)H} \right. \\
& \left. - \sqrt{\varepsilon_k \mu_l} \left[j^{(l)E}, j^{(k)H} \right] + \sqrt{\varepsilon_l \mu_k} \left[j^{(k)E}, j^{(l)H} \right] \right\}.
\end{aligned}$$

As result we get the conditions of energy transformation by charges-currents interaction:

energy separation if $\delta W_\Theta > 0$;

energy absorption if $\delta W_\Theta < 0$;

energy conservation if $\delta \boldsymbol{\Xi}_\Theta = 0$.

Vector δP_Θ shows the main direction and intensities of these energy processes.

10. Conclusions

We construct here biquaternionic forms of laws of electric and gravimagnetic charges and currents interaction by analogy to Newton laws, which gives us closed hyperbolic system of differential equations for their definition and determination of corresponding EGM-fields. For the free system of charges and currents, Equations (22) and (24) define the behavior of CC-field and EGM-field over time if their initial states are known. After calculating the bigradients from here, the all fields, charges and currents can be defined according to their definitions. It's the very suitable short form which contains the algorithm for their calculation.

At building of charge-current transformations equation, we get as known gravitational, electric and magnetic forces, so we found the new forces which are needed in experimental motivation. Considered properties (26) of solutions of CC-transformation equation by existence external EGM-field give possibility to test this model on practice.

Note also that the essential at building and studying this model of EGM-field and CC-field is the differential algebra of biquaternions [2], without which such construction of differential equations, describing interaction of charges and currents in the forms which give the fields analogies of Newton laws and are very convenient for calculations, will be practically impossible.

References

[1] Alexeyeva, L.A. (2016) *Journal of Modern Physics*, **7**, 435-444. http://www.scirp.org/journal/jmp http://dx.doi.org/10.4236/jmp.2016.75045

[2] Alexeyeva, L.A. (2012) *Int. J. Clifford Analysis, Clifford Algebras and Their Applications*, **7**, 19-39.

[3] Rastall, R. (1964) *Review of Modern Physics*, **36**, 820-832. http://dx.doi.org/10.1103/RevModPhys.36.820

[4] Edmonds Jr., J.D. (1978) *American Journal of Physics*, **46**, 430. http://dx.doi.org/10.1119/1.11316

[5] Spilker, G.L. (1983) *Report of USSR Academy of Sciences*, **272**, 1359-1363.

[6] Rotelli, P. (1989) *Modern Physics Letters A*, **4**, 933-940. http://dx.doi.org/10.1142/S0217732389001106

[7] Davies, A.J. (1990) *Physical Review D*, **41**, 2628-2630. http://dx.doi.org/10.1103/PhysRevD.41.2628

[8] Finkelstein, D., Jauch, J.M., Schiminovich, S. and Speiser, D. (1992) *Journal of Mathematical Physics*, **3**, 207-220. http://dx.doi.org/10.1063/1.1703794

[9] Adler, S.L. (1995) Quaternionic Quantum Mechanics and Quantum Fields. Oxford University Press, New York.

[10] Kravchenko, V.V. (1995) *Doklady Mathematics*, **51**, 287-289.

[11] Kassandrov, V.V. (1995) *Gravitation and Cosmology*, **1**, 216-222.

[12] De Leo, S. and Rodrigues Jr., S. (1998) *International Journal of Theoretical Physics*, **37**, 1707-1720. http://dx.doi.org/10.1023/A:1026692508708

[13] Kravchenko, V.V. (2003) Quaternionic Equation for Electromagnetic Fields in Inhomogeneous Media. In: Begehr, H., Gilbert, R. and Wah Wong, M., Eds., *Progress in Analysis*, Vol. 1, World Scientific, 361-366.

[14] Efremov, A.P. (2004) *Hypercomples Numbers in Geometry and Physics*, **1**, 111-127.

[15] Acevedo, M., Lopez-Bonilla, J. and Sanchez-Meraz, M. (2005) *Apeiron*, **12**, 371.

[16] Alexeyeva, L.A. (2013) Differential Algebra of Biquaternions. Dirac Equation and Its Generalized Solutions. Progress in Analysis. *Proceedings of the* 8*th Congress of the ISAAC*, Moscow, 22-27 August 2013, 153-161.

[17] Alexeyeva, L.A. (2013) *Mathematical Journal*, **13**, 17-35. (In Russian) http://www.math.kz/images/journal/2013-1/Alexeyeva.pdf

[18] Vladimirov, V.S. (1976) Generalized Functions in Mathematical Physics. Nauka Publisher, Moscow.

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Electronic Transport Properties of Transition-Metal Terminated Trigonal Graphene Nanoribbons

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Abstract

With the non-equilibrium Green's function method and density functional theory, we have studied the electronic properties of trigonal graphene nanoribbons, with Fe terminal and H terminal, coupled to gold electrodes. Rectification behavior can be observed when the electrode-molecule contact distance is larger than 2.2 Å. The electronic transport is greatly improved in case of Fe terminal which is analyzed in terms of transmission spectra and density of states.

Keywords

Trigonal Graphene Nanoribbons, Electronic Transport, Fe Terminal, Transmission Spectra

1. Introduction

Carbon-based nanostructures have emerged as one of the most promising candidates for nanoelectronics in recent years. Much attention has been paid to developing graphene electronics. Due to their interesting electronic properties, such as the giant carrier mobility at room temperature [1], the anomalous quantum hall effect [2], and quantum blockade [3], narrow graphene nanoribbons (GNRs) has a promising prospect of application in graphene-based devices.

With the development of experimental technology, it has become possible to tailor a GNR into well-defined shapes by cutting a grapheme along specific single crystallographic directions [4]. By tailoring GNRs into a triangular shape, Chen *et al*. have designed a rectifying and perfect spin filtering electronic device based on AGNRs [5]. Deng *et al*. investigated the effects of side groups on the electronic transport properties of the trigonal grapheme flake and found that the rectifying rations and direction could be significantly tuned by the type and the attached positions of side groups [6]. A similar triangular GNR nanostructure has also been investigated

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in which the vertex carbon of the GNR is substituted by B and N. The rectification behavior was observed because a potential barrier similar to the p-n junction was formed in the B-N region of the central molecule [7].

The presence of metal atoms adsorbed on GNRs changes its electronic and magnetic properties and thus is of great interest in physics. Wang *et al.* investigated the structure, electronic and magnetic properties of metal-terminated zigzag GNR (ZGNR) [8]. Cao *et al*. investigated the magnetic line defects located at the ribbon edge and spin-filter effect was observed [9].

In this paper, we aim to construct a two-probe nanodevice based on Fe-terminated triangular ZGNR and compare the transport properties of Fe-terminated triangular ZGNR system and H-terminated triangular ZGNR system. As it turns out, it is interesting to find that in the presence of Fe-terminal, the electronic transport is greatly enhanced compared with H-terminal.

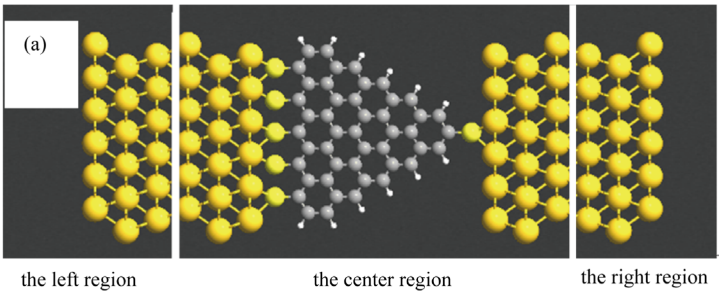
2. Model and Method

Our modeling setup is depicted in **Figure 1**. Fe-terminated and H-terminated triangular ZGNR system are denoted as Z-H (**Figure 1(a)**) and Z-Fe (**Figure 1(b)**), respectively. The vacuum layers between two adjacent images along x and y directions are set to 10 Å. The molecular device we studied is divided into three regions: left electrode, right electrode and scattering region. The scattering region involves central molecule and the first three gold layers near it.

The geometry of these two systems is first optimized with a residual force of 0.05 eV/Å. After the structure relaxation, we calculate the electronic transport properties of the system by the well test ATK package [10], which adopts a non-equilibrium Green function (NEGF) method combined with density functional theory (DFT). The single-zeta polarization (DZP) basis set for valence electrons is adapted. The Perdew-Burke-Ernzerhof (PBE) formulation of the generalized gradient approximation (GGA) exchange correlation function is used. Core electrons are modeled with Troullier-Martins nonlocal pseudopotential. An energy cut off of 200 Ry is used to define the real-space grid for numerical calculations in the electron density. The electronic Brillouin zone integration is sampled with a 1 × 1 × 100 uniform k-point mesh. The current is calculated using the widely accepted Landauer-Büttiker formula [7]:

$$I = \frac{2e}{h}\int T\left(E,V_b\right)\left[f_L\left(E-\mu_L\right)-f_R\left(E-\mu_R\right)\right]\mathrm{d}E$$

where $T(E, V_b)$ is the transmission coefficient for electrons with energy E at bias voltage V_b, $f_{L/R}$ the Fermi function, and $\mu_{L/R}$ the electrochemical potential of the left/right electrode.



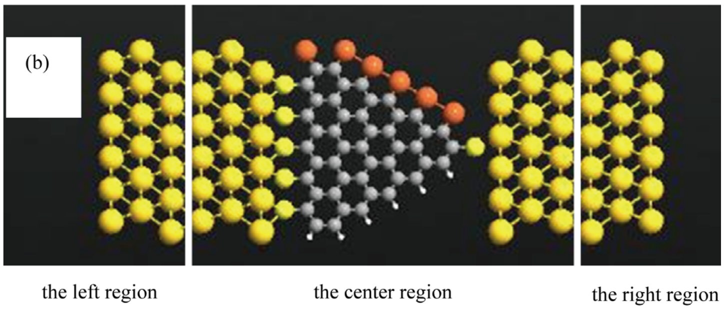


Figure 1. The geometric structures (a) Z-H and (b) Z-Fe.

3. Results and Discussion

First, we calculate the self-consistent current-voltage (I-V) characteristic for Z-H system in a bias range from −2.0 V to 2.0 V which is shown in **Figure 2(a)**. We have discussed the effect of the electrode-molecule contact distance on the electronic transport and found that when the left and right contact distance varies from 1.8 Å to 2.2 Å, no obvious rectifying behavior is observed although the central molecule itself has asymmetric structure. Four I-V curves corresponding to different contact distances are taken for example, among which three ones are obtained with electrode-molecule contact distance less than 2.3 Å and the fourth with the right contact distance equal to 2.3 Å. In the whole, the I-V curves are asymmetric due to the asymmetric molecule structure and they have similar features in the bias range from −1.0 V to 1.0 V. In such a bias range, there is no obvious current through the GNR. The current shows remarkable enhancement until the bias increases to 1.0 V (−1.0 V). The situation in negative bias is similar so that no rectification is observed. However, when the right and left contact distances are changed to 2.3 Å and 1.8 Å respectively, obvious rectifying behavior is observed. This is reasonable because the strength asymmetry of the right and left electrode-molecule coupling could lead to current rectification.

Next we turn to the Fe-terminated system. In order to illustrate the effect of Fe-terminal on the electronic transport, we plot in **Figure 2(b)** the I-V curves for Z-H and Z-Fe for comparison. The I-V curves are obtained numerically in the bias range (−2.0, 2.0) V with an interval of 0.2 V. The I-V curves of Z-Fe are also calculated with the contact distance varying from 1.8 Å to 2.2 Å and one I-V curve is chosen as an example. It is found that the electronic transport is greatly enhanced in Z-Fe compared with Z-H. The current of Z-Fe increases by one order of magnitude.

Transmission spectra analysis provides us a most intuitive picture for understanding the I-V characteristic. We have calculated transmission spectra in the bias range (−2.0, 2.0) V with an interval of 0.2 V. Without loss of generality, we plot in **Figure 3** the transmission spectra for Z-H and Z-Fe at bias 0, 1.0 V and 2.0 V as representative ones. As the current is obtained from the Landauer-Büttiker formula mentioned above, the magnitude of current in fact depends on the transmission coefficient in the integral regions, namely, the size of the integral area in the bias window. The dashed lines denote the corresponding bias windows at different biases. As can be seen from **Figure 3(a)**, **Figure 3(c)** and **Figure 3(e)**, no transmission peaks appear around the Fermi level at $V_b = 0$. In case of $V_b = 1.0$ V one peak appears and enter the bias window. Therefore, there is relatively weak enhancement in current. When the bias is further increased to 2.0 V, the peak inside the window splits into two ones and the current is larger than that at $V_b = 1.0$ V. The change of transmission spectra with the increase of bias is different for Z-Fe. From **Figure 3(b)**, **Figure 3(d)** and **Figure 3(f)** it can be found that much more peaks appear no matter at lower bias or at higher bias. As the bias increases more and more transmission peaks enter the bias window, which leads to continuous enhancement in current as shown in **Figure 2(b)**. As a result, the current of Z-Fe is much larger than that of Z-H.

To further rationalize the significant difference in the electronic transport of these two kinds of systems, we plot in **Figure 4** the density of states (DOS) at $V_b = 0$, −1.0 V, 1.0 V. It is clear that there are more electronic states around the Fermi level in Z-Fe than that in Z-H, which undoubtedly contributes to the stronger transport

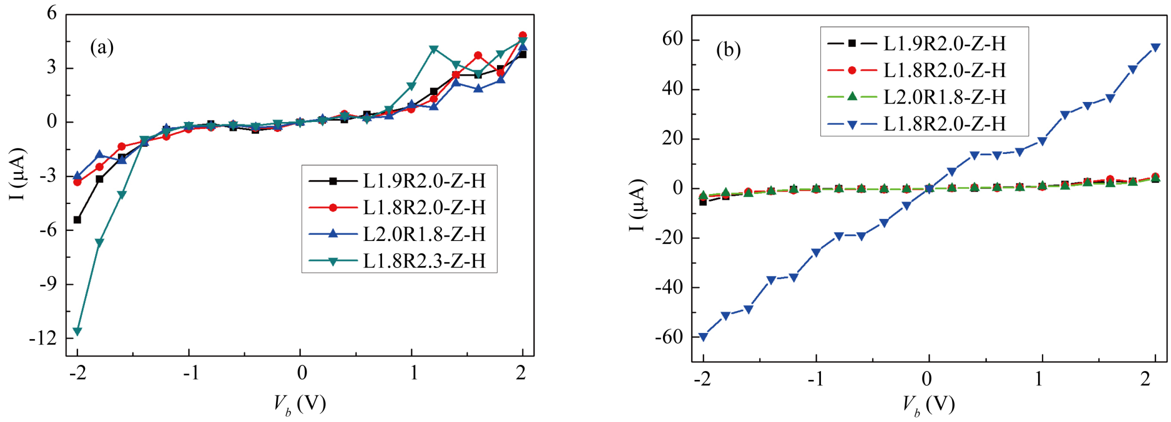


Figure 2. Calculated *I-V* curves for Z-H and Z-Fe systems.

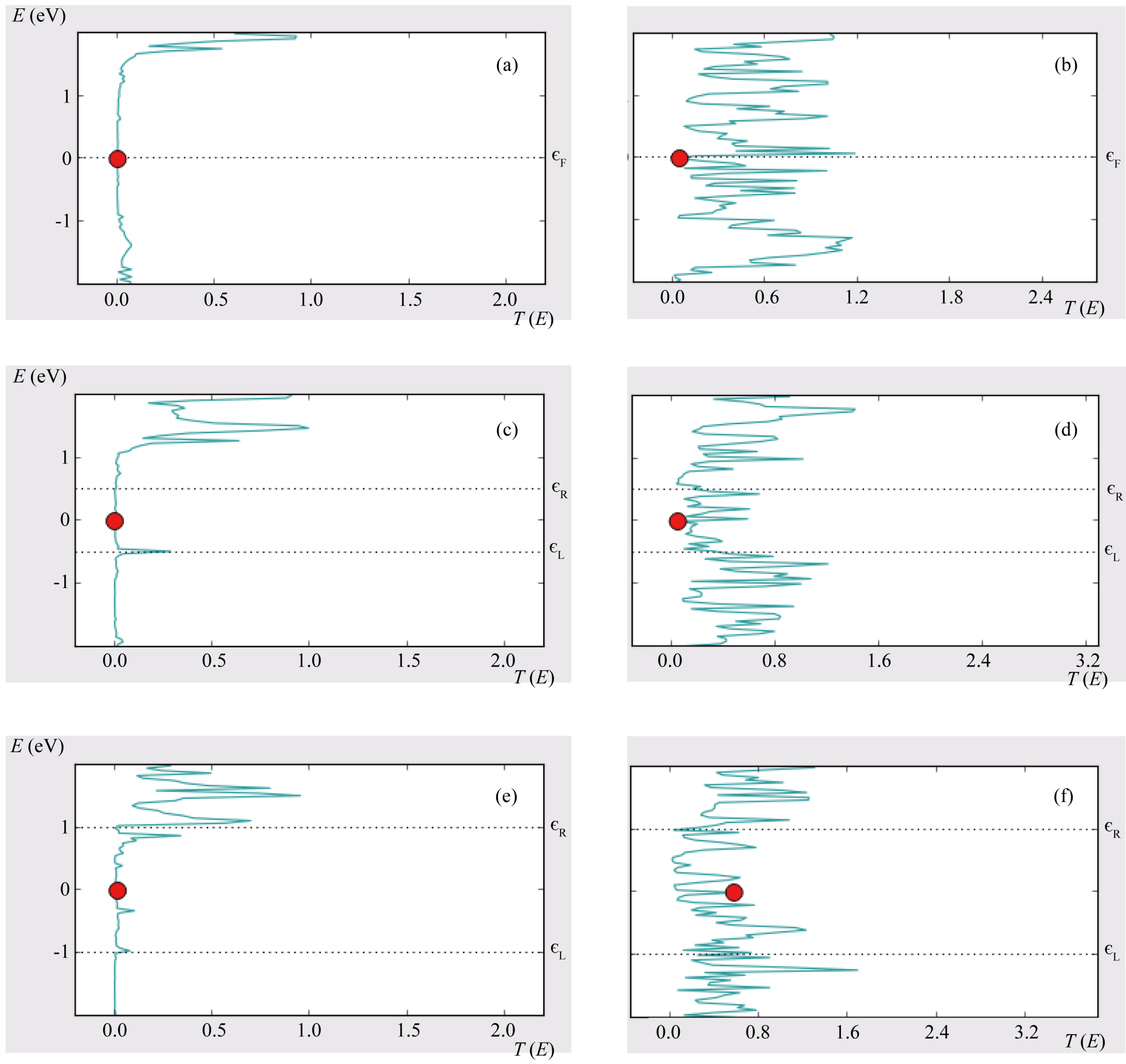


Figure 3. Transmission spectra for Z-H (the left panel) and Z-Fe (the right panel) at different bias voltages V_b = 0, 1.0 V, and 2.0 V. (a) V_b = 0; (b) V_b = 0; (c) V_b = 1.0 V; (d) V_b = 1.0 V; (e) V_b = 2.0 V; (f) V_b = 2.0 V.

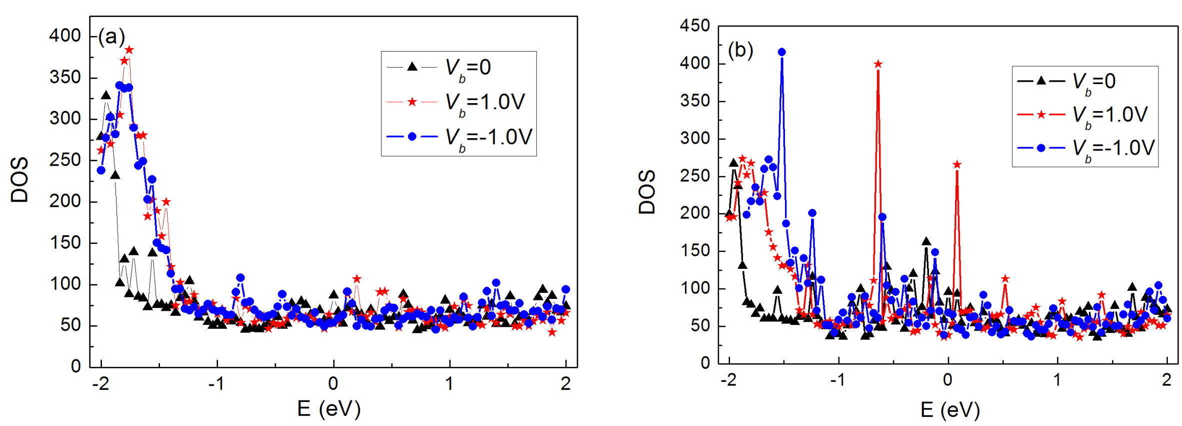


Figure 4. Density of state for (a) Z-H and (b) Z-Fe at different bias voltages 0, −1.0 V, 1.0 V.

ability of Z-Fe.

4. Summary

We have constructed two-probe nanodevices based on triangular ZGNR. Effect of electrode-molecule contact distance on the electronic transport is discussed and it is found that when the left and right contact distance varies from 1.8 Å to 2.2 Å, no obvious rectifying behavior is observed although the central molecule itself has asymmetric structure. The transport properties of Fe-terminated and H-terminated triangular ZGNRs are compared. As it turns out, it is interesting to find that in the presence of Fe-terminal, the electronic transport is greatly enhanced compared with H-terminal. The significant difference in the electronic transport of these two kinds of systems is attributed to the much more transmission peaks around the Fermi level in Fe-terminated system.

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References

[1] Morozov, S.V., Novoselov, K.S. and Katsnelson, M.I. (2008) *Physical Review Letters*, **100**, Article ID: 016602.

[2] Geim, A.K. and Novoselov, K.S. (2007) *Nature Materials*, **6**, 183-191. http://dx.doi.org/10.1038/nmat1849

[3] Zhang, Y.Y., Hu, J.P., Bernevig, B.A., Wang, X.R., *et al.* (2008) *Physical Review B*, **78**, Article ID: 155413.

[4] Campos, L.C., Manfrinato, V.R., Yamagishi, J.D.S., *et al.* (2009) *Nano Letters*, **9**, 2600-2603. http://dx.doi.org/10.1021/nl900811r

[5] He, J. and Chen, K.Q. (2012) *Journal of Applied Physics*, **112**, Article ID: 114319.

[6] Deng, X.Q., Tang, G.P. and Guo, C. (2012) *Physics Letters A*, **376**, 1839-1844. http://dx.doi.org/10.1016/j.physleta.2012.04.021

[7] Deng, X.Q., Zhang, Z.H. and Tang, G.P. (2012) *Applied Physics Letters*, **100**, Article ID: 063107.

[8] Wang, Y. and Cheng, H.P. (2011) *Physical Review B*, **83**, Article ID: 113402.

[9] Cao, C., Chen, L.N., Long, M.Q., Huang, W.R. and Xu, H. (2012) *Journal of Applied Physics*, **111**, Article ID: 113708.

[10] Brandbyge, M., Mozos, J.L., Ordejón, P., Taylor, J. and Stokbro, K. (2002) *Physical Review B*, **65**, Article ID: 165401.

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There Is a Way to Comprise Half-Integer Eigenvalues for Photon Spin

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Abstract

In this article, an attempt based on **Spin Topological Space**, **STS**, to give a reasonable detailed account of the cause of **photonic fermionization phenomena** of light photon is made.

STS is an unconventional spin space in quantum mechanics, which can be used to account for where the unconventional half-integer spin eigenvalues phenomenon of light photon comes from.

We suggest to dectect the possible existence of photonic one-third-spinization phenomenon of light photon, by using three beams of light photon in interference experiment.

Keywords

Spin Topological Space, STS, Non-Hermitian matrix, Casimir operator, photonic fermionization phenomena, half-integer spin eigenvalues, one third, one fourth spin eigenvalues of photon spin

1 Introduction

Kyle E. Ballantine, John F. Donegan and Paul R. Eastham [1] measured the total angular momentum of the beam of light with their interferometer, and observed some curious optical phenomena. They found: the eigenvalues of angular momentum of light photon obviously shifted away from the normal physical values that are ruled by the general axioms accepted in today's quantum mechanics world.

Normal angular momentum quantum numbers of the photon must be integers, in units of the Planck constant $\hbar$: eigenvalues of spin are $-1\hbar, 0\hbar, +1\hbar$ and eigenvalues of orbital are $0\hbar, 1\hbar, 2\hbar, 3\hbar, \ldots$

However, as the title of their paper, " There are many ways to spin a photon: Half-quantization of a total optical angular momentum " [1] shows: the experimental data in [1] were half-integer, $+\hbar/2$ and $-\hbar/2$, or even may be $+1.5\hbar$ and $-1.5\hbar$...! It is an important physicial experiment result, and indeed, light photon is boson, however possesses fermionic, spectrum ! curious phenomena...

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This present article, " There is a way to comprise half-integer eigenvalues for photon spin ", is in the frame of **Spin Topological Space**, **STS** [2] to consider the contributions of spin effects of light photon, and tries to clear up the cause of the photonic fermionization phenomena, which emerged from the experiment [1].

The contributions of orbital effects of light photon, which show half-integer eigenvalues, could appeal to the mechanism of Non-Hermitian orbital angular momentum $\mathbb{L}_3, \mathbb{L}^2$ [3].

Normally, in quantum mechanics, different kinds of spin particles possess different dimensional spaces, which are expressed by finite dimensional matrices, and these finite dimensional matrices are all Hermiticity.

According to **STS** , spin angular momentum $\vec{\pi}(l)$ of particles is expressed by infinite dimensional matrices in three-physical space. The first component $\pi_1(l)$ and the second component $\pi_2(l)$ are Non-Hermitian matrices; the third component $\pi_3(l)$ is Hermitian diagonal matrix. Here, mark " l " indicates the lth generation spin particles, $l = 1, 2, 3, \ldots$

2 Three groups of matrices $\vec{\pi}_{+3,-3}(2), \vec{\pi}_{+2,-1}(1), \vec{\pi}_{+3/2,-3/2}(1)$ of light photon particle $\vec{\pi}(l)$, which satisfy spin angular momentum commutation relus, play the major role **in elaborating the machanism of photonic fermionization phenomena**.

$$\vec{\pi}_{+3,-3}(2) \times \vec{\pi}_{+3,-3}(2) = i\hbar\vec{\pi}_{+3,-3}(2) \tag{1}$$

$$\vec{\pi}_{+2,-1}(1) \times \vec{\pi}_{+2,-1}(1) = i\hbar\vec{\pi}_{+2,-1}(1) \tag{2}$$

$$\vec{\pi}_{+3/2,-3/2}(1) \times \vec{\pi}_{+3/2,-3/2}(1) = i\hbar\vec{\pi}_{+3/2,-3/2}(1) \tag{3}$$

Where

$$\vec{\pi}_{+3,-3}(2) = \{ \pi_{1;+3,-3}(2), \pi_{2;+3,-3}(2), \pi_{3;+3,-3}(2) \} \tag{4}$$

$$\vec{\pi}_{+2,-1}(1) = \{ \pi_{1;+2,-1}(1), \pi_{2;+2,-1}(1), \pi_{3;+2,-1}(1) \} \tag{5}$$

$$\vec{\pi}_{+3/2,-3/2}(1) = \{ \pi_{1;+3/2,-3/2}(1), \pi_{2;+3/2,-3/2}(1), \pi_{3;+3/2,-3/2}(1) \} \tag{6}$$

Or instead of (1), (2), (3), in terms of raising matrix operator π_j^+, lowering matrix operator π_k^- and $\pi_{3;j,k}$, *i. e.* (7) below, to represent commutation rules (8), (9), (10) of light photon with three different kinds of spin state ($\hbar = 1$):

$$\{ \pi_j^+(l), \pi_k^-(l), \pi_{3;j,k}(l) \} \tag{7}$$

$$\pi_{+3}^+(2)\pi_{-3}^-(2) - \pi_{-3}^-(2)\pi_{+3}^+(2) = 2\pi_{3;+3,-3}(2) \tag{8.1}$$

$$\pi_{3;+3,-3}(2)\pi_{+3}^+(2) - \pi_{+3}^+(2)\pi_{3;+3,-3}(2) = +\pi_{+3}^+(2) \tag{8.2}$$

$$\pi_{3;+3,-3}(2)\pi_{-3}^-(2) - \pi_{-3}^-(2)\pi_{3;+3,-3}(2) = -\pi_{-3}^-(2) \tag{8.3}$$

$$\pi_{+2}^+(1)\pi_{-1}^-(1) - \pi_{-1}^-(1)\pi_{+2}^+(1) = 2\pi_{3;+2,-1}(1) \tag{9.1}$$

$$\pi_{3;+2,-1}(1)\pi_{+2}^+(1) - \pi_{+2}^+(1)\pi_{3;+2,-1}(1) = +\pi_{+2}^+(1) \tag{9.2}$$

$$\pi_{3;+2,-1}(1)\pi_{-1}^-(1) - \pi_{-1}^-(1)\pi_{3;+2,-1}(1) = -\pi_{-1}^-(1) \tag{9.3}$$

$$\pi_{+3/2}^+(1)\pi_{-3/2}^-(1) - \pi_{-3/2}^-(1)\pi_{+3/2}^+(1) = 2\pi_{3;+3/2,-3/2}(1) \tag{10.1}$$

$$\pi_{3;+3/2,-3/2}(1)\pi_{+3/2}^+(1) - \pi_{+3/2}^+(1)\pi_{3;+3/2,-3/2}(1) = +\pi_{+3/2}^+(1) \tag{10.2}$$

$$\pi_{3;+3/2,-3/2}(1)\pi_{-3/2}^-(1) - \pi_{-3/2}^-(1)\pi_{3;+3/2,-3/2}(1) = -\pi_{-3/2}^-(1) \tag{10.3}$$

Write down the explicit representations of raising matrix operators and lowering matrix operators that appear in the above three formulas (8), (9), (10):

$$\pi^{+}_{+3}(2) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\ \}_{+2} \qquad (11)$$
$$\pi^{-}_{-3}(2) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ -7,\ -8,\ ,\ \}_{-2} \qquad (12)$$

$$\pi^{+}_{+2}(1) = \mathrm{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\ \}_{+1} \qquad (13)$$
$$\pi^{-}_{-1}(1) = \mathrm{diag}\{\ ,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ ,\ \}_{-1} \qquad (14)$$

$$\pi^{+}_{+3/2}(1) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 13,\ 11,\ 9,\ 7,\ 5,\ 3,\ 1,\ -1,\ -3,\ -5,\ -7,\ ,\ \}_{+1} \qquad (15)$$
$$\pi^{-}_{-3/2}(1) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ -7,\ -5,\ -3,\ -1,\ 1,\ 3,\ 5,\ 7,\ 9,\ 11,\ -13,\ ,\ \}_{-1} \qquad (16)$$

Subscripts " +1 " and " -1 " represent the first minor top-right diagonal and the first minor down-left diagonal.
Subscripts " +2 " and " -2 " represent the second minor top-right diagonal and the second minor down-left diagonal.
Subscripts " 0 " indicates major diagonal, sometimes for convenience be omitted.

In condition for keeping photon’s Casimir operator invariant, that is, keeping

$$\pi^{2}_{+3,-3}(2) = \pi^{2}_{+2\,,\,-1}(1) = \pi^{2}_{+3/2,-3/2}(1) = 1(1+1)I_0\hbar^2 = 2I_0\hbar^2 \qquad (17)$$
$$I_0 = \mathrm{diag}\{\ ,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ ,\ \}_0 \qquad (18)$$

Further, next three groups of math series forms of the spin third component $\pi_3(l)$ of light photon are obtained as below

$$\pi_{3;\ +3,-3}(2)$$
$$= \mathrm{diag}\{,\ 3\hbar,\ 2.5\hbar,\ 2\hbar,\ \boxed{1.5\hbar,}\ 1\hbar,\ \boxed{0.5\hbar,}\ 0\hbar,\ \boxed{-0.5\hbar,}\ -1\hbar,\ \boxed{-1.5\hbar,}\ -2\hbar,\ ,\}_0 \qquad (19)$$

$$\pi_{3;\ +2\,,-1}(1)$$
$$= \mathrm{diag}\{,\ 6\hbar,\ 5\hbar,\ 4\hbar,\ 3\hbar,\ \boxed{2\hbar,}\ \boxed{1\hbar,}\ \boxed{0\hbar,}\ \boxed{-1\hbar,}\ \boxed{-2\hbar,}\ -3\hbar,\ -4\hbar,\ ,\}_0 \qquad (20)$$

$$\pi_{3;\ +3/2,-3/2}(1)$$
$$= \mathrm{diag}\{,\ 5.5\hbar,\ 4.5\hbar,\ 3.5\hbar,\ 2.5\hbar,\ \boxed{1.5\hbar,}\ \boxed{0.5\hbar,}\ \boxed{-0.5\hbar,}\ \boxed{-1.5\hbar,}\ -2.5\hbar,\ -3.5\hbar,\ -4.5\hbar,\ ,\}_0 \qquad (21)$$

(19): Alternating series form of Integer eigenvalues and Half-integer eigenvalues
(20): Integer eigenvalues series form
(21): Half-integer eigenvalues series form

(19), (20), (21) are just seperately the figures of what happening in Kyle E. Ballantine’s and his colleagues’ experiment:

Integer eigenvalues series form (20) and half-integer eigenvalues series form (21) give the accounts of " One family includes have the expected bosonic spectrum with integer eigenvalues, and other family, has a fermionic spectrum, comprising half-integer eigenvalues. " (quoted passage from the paper [1]).

By the way, (20) $\pi_{3;\ +2\,,-1}(1)$ and (21) $\pi_{3;\ +3/2,-3/2}(1)$, both of them are together involved in (19) $\pi_{3;\ +3,-3}(2)$. So it seems that there should exist the third family, alternating series form of Integer eigenvalues and Half-integer eigenvalues (19).

3 Physical behavior mechanism of photonic fermionization of light photon experiment

Now, matrices (8), (9), (10) can be used to describe the experiment rusults (17) and (19), (20), (21) of photonic fermionization phenomena of light photon, but from what kind of experimental procedure of physical behavior mechanism, these experimental results arise ?

For this reason, deeper research is given. Be concise, the sign of "(1)", is omitted in follows.

Because $\vec{\pi}_{+2,-1}$ and $\vec{\pi}_{m+2\,,\,m-1}$ are spin angular momentums in **STS**, it means：

$$\vec{\pi}_{+2,-1} \times \vec{\pi}_{+2,-1} = i\vec{\pi}_{+2,-1} \tag{2}$$

$$\vec{\pi}_{m+2\,,\,m-1} \times \vec{\pi}_{m+2\,,\,m-1} = i\vec{\pi}_{m+2\,,\,m-1} \tag{22}$$

Using the linear combinanation of (2) with (22), a new spin angular momentum $\vec{\pi}_{m/2+2\,,\ m/2-1}$ (23) is composed, and it obeys commutation rule (24)

$$\vec{\pi}_{m/2+2\,,\ m/2-1} = \tfrac{1}{2}\{\vec{\pi}_{m+2\,,\,m-1} + \vec{\pi}_{+2,-1}\}\ \ldots \tag{23}$$

$$\vec{\pi}_{m/2+2\,,\ m/2-1} \times \vec{\pi}_{m/2+2\,,\ m/2-1} = i\ \vec{\pi}_{m/2+2\,,\ m/2-1} \tag{24}$$

$\vec{\pi}_{+2,-1}$, $\vec{\pi}_{m+2\,,\,m-1}$ and $\vec{\pi}_{m/2+2\,,\ m/2-1}$ all are light photon, since their Carsimir operators equal to $2\hbar^2$, *i. e.*

$$\pi^2_{+2,-1} = \pi^2_{m+2\,,\,m-1} = \pi^2_{m/2+2\,,\ m/2-1} = 1(1+1)I_0\hbar^2 = 2I_0\hbar^2 \tag{25}$$

Write down the third component of (23), and its exlicit formulation (26. *m*) as below:

$$\pi_{3;\ m/2+2\,,\ m/2-1} = \tfrac{1}{2}\{\pi_{3;\ m+2\,,\,m-1} + \underline{\pi_{3;\,+2,-1}}\}\ ;\ m = 0, \pm1, \pm2, \pm3, \pm4, \ldots \tag{26. m}$$

$$♦\pi_{3;\,+4,\,+1} = \tfrac{1}{2}\{\pi_{3;\,+6\,,\,+3} + \underline{\pi_{3;\,+2,-1}}\}$$
$$= \text{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ \underline{3},\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\} \tag{26.4}$$

$$◇\pi_{3;\,+7/2,+1/2} = \tfrac{1}{2}\{\pi_{3;\,+5\,,\,+2} + \underline{\pi_{3;\,+2,-1}}\}$$
$$= \tfrac{1}{2}\,\text{diag}\{\ ,\ 15,\ 13,\ 11,\ 9,\ 7,\ \underline{5},\ 3,\ 1,\ -1,\ -3,\ -5,\ ,\} \tag{26.3}$$

$$♦\pi_{3;\,+3,\,0} = \tfrac{1}{2}\{\pi_{3;\,+4\,,\,+1} + \underline{\pi_{3;\,+2,-1}}\}$$
$$= \text{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ \underline{2},\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\} \tag{26.2}$$

$$◇\pi_{3;\,+5/2,-1/2} = \tfrac{1}{2}\{\pi_{3;\,+3\,,\,0} + \underline{\pi_{3;\,+2,-1}}\}$$
$$= \tfrac{1}{2}\,\text{diag}\{\ ,\ 13,\ 11,\ 9,\ 7,\ 5,\ \underline{3},\ 1,\ -1,\ -3,\ -5,\ -7,\ ,\} \tag{26.1}$$

$$♦♦\boxed{\pi_{3;\,+2,\,-1}} = \tfrac{1}{2}\{\pi_{3;\,+2,-1} + \underline{\pi_{3;\,+2,-1}}\}$$
$$= \text{diag}\{\ ,\ 6,\ 5,\ 4,\ 3,\ 2,\ \underline{1},\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\} \tag{26.0}$$

$$◇\boxed{\pi_{3;\,+3/2,-3/2}} = \tfrac{1}{2}\{\pi_{3;\,+1,\,-2} + \underline{\pi_{3;\,+2,-1}}\}$$
$$= \tfrac{1}{2}\,\text{diag}\{\ ,\ 11,\ 9,\ 7,\ 5,\ 3,\ \underline{1},\ -1,\ -3,\ -5,\ -7,\ -9,\ ,\} \tag{26.-1}$$

$$\blacklozenge\pi_{3;\,+1,-2} = \tfrac{1}{2}\{\,\pi_{3;\,0\,,-3} + \underline{\pi_{3;\,+2,-1}}\,\}$$
$$= \mathrm{diag}\{\ ,\ 5,\ 4,\ 3,\ 2,\ 1,\ \underline{0},\ -1,\ -2,\ -3,\ -4,\ -5,\ ,\} \quad (26.-2)$$
$$\lozenge\pi_{3;\,+1/2,-5/2} = \tfrac{1}{2}\{\,\pi_{3;\,-1\,,-4} + \underline{\pi_{3;\,+2,-1}}\,\}$$
$$= \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 9,\ 7,\ 5,\ 3,\ 1,\ \underline{-1},\ -3,\ -5,\ -7,\ -9,\ -11,\ ,\} \quad (26.-3)$$
$$\blacklozenge\pi_{3;\,0\,,-3} = \tfrac{1}{2}\{\,\pi_{3;\,-2\,,-5} + \underline{\pi_{3;\,+2,-1}}\,\}$$
$$= \mathrm{diag}\{\ ,4,\ 3,\ 2,\ 1,\ 0,\ \underline{-1},\ -2,\ -3,\ -4,\ -5,\ -6\ ,\} \quad (26.-4)$$

From (26), two important conclusions are given

1) There exist two diffrent families of the third component of light photon:

family **BP: B**osonization of **P**hoton, labelled by " ♦ ",

family **FP: F**ermionization of **P**hoton, labelled by " ◇ "

For light photon, the angular momentum addition of two angular momentums, one angular momentum $\mathbf{BP}_1$ with other angular momentum $\mathbf{FP}_2$, may generate two diffrent families of the third component of light photon.

$$\mathbf{BP}\blacklozenge(m) = \tfrac{1}{2}\{\,\mathbf{BP}_1(m) + \mathbf{BP}_2(0)\,\}\ ;\ m = 0, \pm2, \pm4, \pm6, \ldots \qquad (27)$$
$$\mathbf{FP}\lozenge(m) = \tfrac{1}{2}\{\,\mathbf{BP}_1(m) + \mathbf{BP}_2(0)\,\}\ ;\ m = \pm1, \pm3, \pm5, \pm7, \ldots \qquad (28)$$

BP♦ and **FP◇** alternately appear with *m*.

2) For a fixed term of the new spin angular momentum $\pi_{3;\ m/2+2\,,\ m/2-1}$, there are many options to choose from the general expression (29).

$$\mathbf{BP}\blacklozenge(m,m'),\ \mathbf{FP}\lozenge(m,m') = \tfrac{1}{2}\{\,\mathbf{BP}_1(m) + \mathbf{BP}_2(m')\,\} \qquad (29)$$
$$m,\ m' = 0, \pm1, \pm2, \pm3, \ldots$$

Family **BP♦** (27) and Family **FP◇** (28) are the simplest couple, in which one spin angular momentum $\mathbf{BP}_2(0)$, $\underline{\pi_{3;\,+2,-1}}$ is keeping invariant, as other spin angular momentum $\mathbf{BP}_1(m)$, $\vec{\pi}_{m+2\,,\,m-1}$ varies with m in expression $\pi_{3;\ m/2+2\,,\ m/2-1}$ (26. m).

(20) $\pi_{3;\ +2\,,-1}(1)$ and (21) $\pi_{3;\ +3/2,-3/2}(1)$, which are the part of expression $\pi_{3;\ m/2+2\,,\ m/2-1}$ (26. m). When m equals to 0 and -1, (20) and (21) are just (26.0) family **BP♦**(0) and (26.-1) family **FP◇**(-1).

The relationship between (20) and (21), or equivalent to that between matrices (9) and (10), could refer to the math statements (26.0) and (26.-1). They are the results of the additions of spin angular momentum photon $\pi_{3;\,+2,-1}$ with photon $\underline{\pi_{3;\,+2,-1}}$, and photo $\pi_{3;\,+1,\,-2}$ with photon $\underline{\pi_{3;\,+2,-1}}$ in Ballantine's and his colleagues' experiment.

By the way, the intervals between two adjoining **BP♦** and **FP◇** is $\frac{\hbar}{2}$

$$\Delta\pi_{3;\ m/2+2\,,\ m/2-1} = \mathbf{FP}\lozenge(m \pm 1) - \mathbf{BP}\blacklozenge(m) = \pm\tfrac{\hbar}{2} I_0 \qquad (30)$$

4 Prediction about one-third-spinization phenomenon of light photon

Proceeding in above way, parallel to the math structure of three groups of matrices (11), (12) and (13), (14), (15), (16) for photonic fermionization, we guess at the existent of so-call photonic one-third-spinization phenomenon of light photon, and four groups of matrices (31), (32) and (33), (34), (35), (36), (37), (38) are given, below labelled by " ♣ ".

$$\pi^{+}_{+4}(3) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 9,\ 8,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0,\ -1,\ ,\ \}_{+3} \tag{31}$$

$$\pi^{-}_{-5}(3) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 0,\ -1,\ -2,\ -3,\ -4,\ -5,\ -6,\ -7,\ -8,\ -9,\ -10,\ ,\ \}_{-3} \tag{32}$$

$$\pi^{+}_{+2}(1) = \mathrm{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\ \}_{+1} \tag{33}$$

$$\pi^{-}_{-1}(1) = \mathrm{diag}\{\ ,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ ,\ \}_{-1} \tag{34}$$

$$\pi^{+}_{+5/3}(1) = \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 20,\ 17,\ 14,\ 11,\ 8,\ \underline{5},\ 2,\ -1,\ -4,\ -7,\ -10,\ ,\ \}_{+1} \tag{35}$$

$$\pi^{-}_{-4/3}(1) = \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 11,\ 8,\ 5,\ 2,\ -1,\ \underline{-4},\ -7,\ -10,\ -13,\ -16,\ -19,\ ,\ \}_{-1} \tag{36}$$

$$\pi^{+}_{+4/3}(1) = \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 19,\ 16,\ 13,\ 10,\ 7,\ \underline{4},\ 1,\ -2,\ -5,\ -8,\ -11,\ ,\ \}_{+1} \tag{37}$$

$$\pi^{-}_{-5/3}(1) = \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 10,\ 7,\ 4,\ 1,\ -2,\ \underline{-5},\ -8,\ -11,\ -14,\ -17,\ -20,\ ,\ \}_{-1} \tag{38}$$

As well as photon's Casimir operator

$$\pi^{2}_{+4,-5}(3) = \pi^{2}_{+2\,,\,-1}(1) = \pi^{2}_{+5/3,-4/3}(1) = \pi^{2}_{+4/3,-5/3}(1) = 1(1 + 1)I_0\hbar^2 = 2I_0\hbar^2 \tag{39}$$

Accordingly, next four groups of math series forms of the spin third component $\pi_3(l)$ of light photon are obtained as below

$$\pi_{3;\ +4,\,-5}(3) =$$
$$\mathrm{diag}\{,\ 2,\ 5/3,\ 4/3,\ 1,\ 2/3,\ \boxed{\underline{1/3},}\ 0,\ \boxed{-1/3,}\ -2/3,\ -1,\ -4/3,\ ,\}_0 \tag{40}$$

$$\pi_{3;\ +2\,,\,-1}(1) =$$
$$\mathrm{diag}\{,\ 6,\ 5,\ 4,\ 3,\ 2,\ \boxed{\underline{1},}\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\}_0 \tag{41}$$
$$\mathrm{diag}\{,\ 18/3,\ 15/3,\ 12/3,\ 9/3,\ 6/3,\ \boxed{\underline{3/3},}\ \boxed{0/3,}\ -3/3,\ -6/3,\ -9/3,\ -12/3,\ ,\}_0$$

$$\pi_{3;\ +5/3,-4/3}(1) =$$
$$\mathrm{diag}\{,\ 17/3,\ 14/3,\ 11/3,\ 8/3,\ 5/3,\ \boxed{\underline{2/3},}\ \boxed{-1/3,}\ -4/3,\ -7/3,\ -10/3,\ -13/3,\ ,\}_0 \tag{42}$$

$$\pi_{3;\ +4/3,-5/3}(1) =$$
$$\mathrm{diag}\{,\ 16/3,\ 13/3,\ 10/3,\ 7/3,\ 4/3,\ \boxed{\underline{1/3},}\ \boxed{-2/3,}\ -5/3,\ -8/3,\ -11/3,\ -14/3,\ ,\}_0 \tag{43}$$

(41), (42), (43) combine to form (40). All of them imply that the third component eigenvalues of light photon can be integer, one-third-integer series.

Let us have some acquaintance with the relationship among (41), (42), (43), by the general formula of addition of spin angular momentum of light photon (44.*m*).

It is shown that (41), (42), (43) are (44.0), (44.-1), (44.-2), which are parts of general formular (44) below

General formula of the addition of light photon are given by (44.*m*)

$$\pi_{3;\ m/3+2\ ,\ m/3-1} \;=\; \tfrac{1}{3}\{\ \pi_{3;\ m+2\ ,\ m-1} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}\ ;\quad m = 0, \pm1, \pm2, \pm3, \ldots \qquad (44.m)$$

$$♦\pi_{3;\,+4,\,+1} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,+8,\,+5} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$=\ \mathrm{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ \underline{3},\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\ \} \qquad (44.6)$$

$$♣\pi_{3;\,+11/3,\,+2/3} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,+7,\,+4} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 23,\ 20,\ 17,\ 14,\ 11,\ \underline{8},\ 5,\ 2,\ -1,\ -4,\ -7,\ ,\ \} \qquad (44.5)$$

$$♣\pi_{3;\,+10/3,\,+1/3} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,+6,\,+3} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 22,\ 19,\ 16,\ 13,\ 10,\ \underline{7},\ 4,\ 1,\ -2,\ -5,\ -8,\ ,\ \} \qquad (44.4)$$

$$♦\pi_{3;\,+3,\,0} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,+5,\,+2} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$=\ \mathrm{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ \underline{2},\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\ \} \qquad (44.3)$$

$$♣\pi_{3;\,+8/3,-1/3} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,+4,\,+1} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 20,\ 17,\ 14,\ 11,\ 8,\ \underline{5},\ 2,\ -1,\ -4,\ -7,\ -10,\ ,\ \} \qquad (44.2)$$

$$♣\pi_{3;\,+7/3,-2/3} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,+3,\,0} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 19,\ 16,\ 13,\ 10,\ 7,\ \underline{4},\ 1,\ -2,\ -5,\ -8,\ -11,\ ,\ \} \qquad (44.1)$$

$$♦♦\underline{\pi_{3;\,+2,\,-1}} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,+2,-1} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$=\ \mathrm{diag}\{\ ,\ 6,\ 5,\ 4,\ 3,\ 2,\ \underline{1},\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\ \} \qquad (44.0)$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 18,\ 15,\ 12,\ 9,\ 6,\ \boxed{\underline{3}},\ ,\ 0\ \ -3,\ -6,\ -9,\ -12,\ ,\ \}$$

$$♣\pi_{3;\,+5/3,-4/3} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,+1,-2} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 17,\ 14,\ 11,\ 8,\ 5,\ \boxed{\underline{2}},\ -1,\ -4,\ -7,\ -10,\ -13,\ ,\ \} \qquad (44.-1)$$

$$♣\pi_{3;\,+4/3,-5/3} \;=\; \tfrac{1}{3}\{\ \pi_{3;\ 0\,,-3} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 16,\ 13,\ 10,\ 7,\ 4,\ \boxed{\underline{1}},\ -2,\ -5,\ -8,\ -11,\ -14,\ ,\ \} \qquad (44.-2)$$

$$♦\pi_{3;\,+1,-2} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,-1,-4} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$=\ \mathrm{diag}\{\ ,\ 5,\ 4,\ 3,\ 2,\ 1,\ \underline{0},\ -1,\ -2,\ -3,\ -4,\ -5,\ ,\ \} \qquad (44.-3)$$

$$♣\pi_{3;\,+2/3,-7/3} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,-2,-5} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 14,\ 11,\ 8,\ 5,\ 2,\ \underline{-1},\ -4,\ -7,\ -10,\ -13,\ -16,\ ,\ \} \qquad (44.-4)$$

$$♣\pi_{3;\,+1/3,-8/3}(1) \;=\; \tfrac{1}{3}\{\ \pi_{3;\,-3,-6} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 13,\ 10,\ 7,\ 4,\ 1,\ \underline{-2},\ -5,\ -8,\ -11,\ -14,\ -17,\ ,\ \} \qquad (44.-5)$$

$$♦\pi_{3;\ 0\,,-3} \;=\; \tfrac{1}{3}\{\ \pi_{3;\,-4,-7} \;+\; \underline{2\pi_{3;\,+2,-1}}\ \}$$
$$=\ \mathrm{diag}\{\ ,\ 4,\ 3,\ 2,\ 1,\ 0,\ \underline{-1},\ -2,\ -3,\ -4,\ -5,\ -6,\ ,\ \} \qquad (44.-6)$$

$$\mathbf{BP}♦(m) \;=\; \tfrac{1}{3}\{\ \mathbf{BP}_1(m) \;+\; 2\mathbf{BP}_2(0)\ \}\ ;\quad m \;=\; 0, \pm3, \pm6.\,, \ldots \qquad (45)$$
$$\mathbf{DP}♣(m) \;=\; \tfrac{1}{3}\{\ \mathbf{BP}_1(m) \;+\; 2\mathbf{BP}_2(0)\ \}\ ;\quad m \;=\; \pm1, \pm2, \pm4, \pm5, \ldots \qquad (46)$$

$$\Delta\pi_{3;\ m/3+2\ ,\ m/3-1} \;=\; \mathbf{DP}♣(m \pm 1) \;-\; \mathbf{BP}♦(m) \;=\; \pm\tfrac{\hbar}{3}\,I_0 \qquad (47)$$

And $\quad \pi^2_{+2,-1} \;=\; \pi^2_{m+2\ ,\ m-1} \;=\; \pi^2_{m/3+2\ ,\ m/3-1} \;=\; 1(1\;+\;1)I_0\hbar^2 \;=\; 2I_0\hbar^2 \qquad (48)$

Combine (44), (26), obtian:

$$\pi_{3;\ m/6+2,\ m/6-1} = \tfrac{1}{3}\{\ \pi_{3;\ m+2,\ m-1} + \underline{2\pi_{3;+2,-1}}\ \}\ ;\ m = 0, \pm1, \pm2, \pm3, ... \qquad (49.m)$$

$$\blacklozenge\pi_{3;+4,+1} = \tfrac{1}{3}\{\ \pi_{3;+8,+5} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \text{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ \underline{3},\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\} \qquad (49.12)$$

$$\pi_{3;+23/6,+5/6} = \tfrac{1}{3}\{\ \pi_{3;+15/2,+9/2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{6}\text{diag}\{\ ,\ 47,\ 41,\ 35,\ 29,\ 23,\ \underline{17},\ 11,\ 5,\ -1,\ -7,\ -13,\ \} \qquad (49.11)$$

$$\clubsuit\pi_{3;+11/3,+2/3} = \tfrac{1}{3}\{\ \pi_{3;+7,+4} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{3}\text{diag}\{\ ,\ 23,\ 20,\ 17,\ 14,\ 11,\ \underline{8},\ 5,\ 2,\ -1,\ -4,\ -7,\ \} \qquad (49.10)$$

$$\diamond\pi_{3;+7/2,+1/2} = \tfrac{1}{3}\{\ \pi_{3;+13/2,+7/2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{2}\text{diag}\{\ ,\ 15,\ 13,\ 11,\ 9,\ 7,\ \underline{5},\ 3,\ 1,\ -1,\ -3,\ -5,\ ,\} \qquad (49.9)$$

$$\clubsuit\pi_{3;+10/3,+1/3}(1) = \tfrac{1}{3}\{\ \pi_{3;+6,+3} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{3}\text{diag}\{\ ,\ 22,\ 19,\ 16,\ 13,\ 10,\ \underline{7},\ 4,\ 1,\ -2,\ -5,\ -8,\ \} \qquad (49.8)$$

$$\pi_{3;+19/6,+1/6} = \tfrac{1}{3}\{\ \pi_{3;+11/2,+5/2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{6}\text{diag}\{\ ,\ 43,\ 37,\ 31,\ 25,\ 19,\ \underline{13},\ 7,\ 1,\ -5,\ -11,\ -17,\ \} \qquad (49.7)$$

$$\blacklozenge\pi_{3;+3,0} = \tfrac{1}{3}\{\ \pi_{3;+5,+2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \text{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ \underline{2},\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\} \qquad (49.6)$$

$$\pi_{3;+17/6,-1/6} = \tfrac{1}{3}\{\ \pi_{3;+9/2,+3/2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{6}\text{diag}\{\ ,\ 41,\ 35,\ 29,\ 23,\ 17,\ \underline{11},\ 5,\ -1,\ -7,\ -13,\ -19,\ \} \qquad (49.5)$$

$$\clubsuit\pi_{3;+8/3,-1/3} = \tfrac{1}{3}\{\ \pi_{3;+4,+1} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{3}\text{diag}\{\ ,\ 20,\ 17,\ 14,\ 11,\ 8,\ \underline{5},\ 2,\ -1,\ -4,\ -7,\ -10,\ \} \qquad (49.4)$$

$$\diamond\pi_{3;+5/2,-1/2} = \tfrac{1}{3}\{\ \pi_{3;+7/2,+1/2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{2}\text{diag}\{\ ,\ 13,\ 11,\ 9,\ 7,\ 5,\ \underline{3},\ 1,\ -1,\ -3,\ -5,\ -7,\ ,\} \qquad (49.3)$$

$$\clubsuit\pi_{3;+7/3,-2/3} = \tfrac{1}{3}\{\ \pi_{3;+3,0} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{3}\text{diag}\{\ ,\ 19,\ 16,\ 13,\ 10,\ 7,\ \underline{4},\ 1,\ -2,\ -5,\ -8,\ -11,\ ,\} \qquad (49.2)$$

$$\pi_{3;+13/6,-5/6} = \tfrac{1}{3}\{\ \pi_{3;+5/2,-1/2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{6}\text{diag}\{\ ,\ 37,\ 31,\ 25,\ 19,\ 13,\ \underline{7},\ 1,\ -5,\ -11,\ -17,\ -23,\ ,\} \qquad (49.1)$$

$$\blacklozenge\blacklozenge\underline{\pi_{3;+2,-1}} = \tfrac{1}{3}\{\ \pi_{3;+2,-1} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \text{diag}\{\ ,\ 6,\ 5,\ 4,\ 3,\ 2,\ \underline{1},\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\} \qquad (49.0)$$

$$\pi_{3;+11/6,-7/6} = \tfrac{1}{3}\{\ \pi_{3;+3/2,-3/2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{6}\text{diag}\{\ ,\ 35,\ 29,\ 23,\ 17,\ 11,\ \underline{5},\ -1,\ -7,\ -13,\ -19,\ -25,\ ,\} \qquad (49.-1)$$

$$\clubsuit\pi_{3;+5/3,-4/3} = \tfrac{1}{3}\{\ \pi_{3;+1,-2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{3}\text{diag}\{\ ,\ 17,\ 14,\ 11,\ 8,\ 5,\ \underline{2},\ -1,\ -4,\ -7,\ -10,\ -13,\ ,\} \qquad (49.-2)$$

$$\diamond\pi_{3;+3/2,-3/2} = \tfrac{1}{3}\{\ \pi_{3;+1/2,-5/2} + \underline{2\pi_{3;+2,-1}}\ \}$$
$$= \tfrac{1}{2}\text{diag}\{\ ,\ 11,\ 9,\ 7,\ 5,\ 3,\ \underline{1},\ -1,\ -3,\ -5,\ -7,\ -9,\ ,\} \qquad (49.-3)$$

$$\clubsuit\pi_{3;\,+4/3,-5/3} \;=\; \tfrac{1}{3}\{\, \pi_{3;\;0,-3} \;+\; \underline{2\pi_{3;\,+2,-1}} \,\}$$
$$=\tfrac{1}{3}\,\mathrm{diag}\{\;,\; 16,\; 13,\; 10,\; 7,\; 4,\; \underline{1},\; -2,\; -5,\; -8,\; -11,\; -14,\; ,\} \quad (49.\text{-}4)$$

$$\pi_{3;\,+7/6,-11/6} \;=\; \tfrac{1}{3}\{\, \pi_{3;\,-1/2,\,-7/2} \;+\; 2\pi_{3;\,+2,-1} \,\}$$
$$=\tfrac{1}{6}\,\mathrm{diag}\{\;,\; 31,\; 25,\; 19,\; 13,\; 7,\; \underline{1},\; -5,\; -11,\; -17,\; -23,\; -29,\; \} \quad (49.\text{-}5)$$

$$\blacklozenge\pi_{3;\,+1,-2} \;=\; \tfrac{1}{3}\{\, \pi_{3;\,-1,-4} \;+\; \underline{2\pi_{3;\,+2,-1}} \,\}$$
$$=\;\mathrm{diag}\{\;,\; 5,\; 4,\; 3,\; 2,\; 1,\; \underline{0},\; -1,\; -2,\; -3,\; -4,\; -5,\; ,\} \quad (49.\text{-}6)$$

$$\pi_{3;\,+5/6,-1/2} \;=\; \tfrac{1}{3}\{\, \pi_{3;\,-3/2,\,-9/2} \;+\; \underline{2\pi_{3;\,+2,-1}} \,\}$$
$$=\tfrac{1}{6}\,\mathrm{diag}\{\;,\; 29,\; 23,\; 17,\; 11,\; 5,\; \underline{-1},\; -7,\; -13,\; -19,\; -25,\; -31,\; ,\} \quad (49.\text{-}7)$$

$$\clubsuit\pi_{3;\,+2/3,-7/3} \;=\; \tfrac{1}{3}\{\, \pi_{3;\,-2,-5} \;+\; \underline{2\pi_{3;\,+2,-1}} \,\}$$
$$=\tfrac{1}{3}\,\mathrm{diag}\{\;,\; 14,\; 11,\; 8,\; 5,\; 2,\; \underline{-1},\; -4,\; -7,\; -10,\; -13,\; -16,\; ,\} \quad (49.\text{-}8)$$

$$\Diamond\pi_{3;\,+1/2,-5/2} \;=\; \tfrac{1}{3}\{\, \pi_{3;\,-5/2,\,-11/2} \;+\; 2\pi_{3;\,+2,-1} \,\}$$
$$=\tfrac{1}{2}\,\mathrm{diag}\{\;,\; 9,\; 7,\; 5,\; 3,\; 1,\; \underline{-1},\; -3,\; -5,\; -7,\; -9,\; -11,\; ,\} \quad (49.\text{-}9)$$

$$\clubsuit\pi_{3;\,+1/3,-8/3}(1) \;=\; \tfrac{1}{3}\{\, \pi_{3;\,-3,-6} \;+\; \underline{2\pi_{3;\,+2,-1}} \,\}$$
$$\tfrac{1}{3}\,\mathrm{diag}\{\;,\; 13,\; 10,\; 7,\; 4,\; 1,\; \underline{-2},\; -5,\; -8,\; -11,\; -14,\; -17,\; ,\} \quad (49.\text{-}10)$$

$$\pi_{3;\,+1/6,-17/6} \;=\; \tfrac{1}{3}\{\, \pi_{3;\,-7/2,\,-13/2} \;+\; \underline{2\pi_{3;\,+2,-1}} \,\}$$
$$=\tfrac{1}{6}\,\mathrm{diag}\{\;,\; 25,\; 19,\; 13,\; 7,\; 1,\; \underline{-5},\; -11,\; -17,\; -23,\; -29,\; -35,\; ,\} \quad (49.\text{-}11)$$

$$\blacklozenge\pi_{3;\;0,-3} \;=\; \tfrac{1}{3}\{\, \pi_{3;\,-4,-7} \;+\; \underline{2\pi_{3;\,+2,-1}} \,\}$$
$$=\;\mathrm{diag}\{\;,\; 4,\; 3,\; 2,\; 1,\; 0,\; \underline{-1},\; -2,\; -3,\; -4,\; -5,\; -6,\; ,\} \quad (49.\text{-}12)$$

By the way, the intervals between above two adjoining π_3 is $\frac{\hbar}{6}$

$$\Delta\pi_{3;\;m/6+2\,,\;m/6-1} \;=\; \pi_{3;\;(m+1)/6+2\,,\;(m+1)/6-1} \;-\; \pi_{3;\;m/6+2\,,\;m/6-1} \;=\; \frac{\hbar}{6}\,I_0 \tag{50}$$

Reducing (30), (47), (50) to following limitation

$$\lim_{n\to\infty}\;\Delta\pi_{3;\;m/2n+2\,,\;m/2n-1} \;=\; \pi_{3;\;(m+1)/2n+2\,,\;(m+1)/2n-1} \;-\; \pi_{3;\;m/2n+2\,,\;m/2n-1} \;=\; \frac{\hbar}{2n}\,I_0 \tag{51}$$

And $\quad \pi^2_{+4,-5}(3) \;=\; \pi^2_{m+2\,,\,m-1}(1) \;=\; \pi^2_{m/6+2\,,\;m/6-1}(1) \;=\; 1(1\,+\,1)I_0\hbar^2 \;=\; 2I_0\hbar^2 \qquad (52)$

5 Conclusions

This paper bases on the principle of the addition of spin angular momentums in **STS** frame, trying to explain the Non-boson-spinization phenomenon of light photon that occurred in [1]. Particle’s spin angular momentums itself, which are influencing on the light photon interference, maybe, rather than the physical quantity phase of propagating light wave causing alone, from previous experiences.

By Table A. Explanation for what happening in photonic fermionization of light photon experiment [1].

By Table B. Suggestion for dectecting the possible existence of photonic one-third-spinization phenomenon of light photon, by using three beams of light photon in interference experiment.

By (51) When the numbers of beams of light photon increase, the intervals between two adjoining π_3 become narrower, and the interference patterns approach to continuous spectrum.

Table A. <u>**Interference by two beams of light photon**</u> [1]

fermionization phenomenon of light photon

boson-spinization	**fermionization**
$\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$	$\vec{\pi}_{+1,-2}$, $\vec{\pi}_{+2,-1}$
$\frac{1}{2}\{\pi_{3;+2,-1} + \underline{\pi_{3;+2,-1}}\}$ (26.0)	$\frac{1}{2}\{\pi_{3;+1,-2} + \underline{\pi_{3;+2,-1}}\}$ (26.-1)
$\pi_{3;+2,-1}(1)$ (20)	$\pi_{3;+3/2,-3/2}(1)$ (21)
..., $\boxed{2\hbar,}$ $\boxed{1\hbar,}$ $\boxed{0\hbar,}$ $\boxed{-1\hbar,}$ $\boxed{-2\hbar,}$,...	..., $\boxed{1.5\hbar,}$ $\boxed{0.5\hbar,}$ $\boxed{-0.5\hbar,}$ $\boxed{-1.5\hbar,}$,...

Table B. <u>**Interference by three beams of light photon**</u>

one-third-spinization phenomenon of light photon

boson-spinization	**one-third-spinization**	**one-third-spinization**
$\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$	$\vec{\pi}_{+1,-2}$, $\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$	$\vec{\pi}_{0,-3}$, $\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$
$\frac{1}{3}\{\pi_{3;+2,-1} + \underline{2\pi_{3;+2,-1}}\}$ (44.0)	$\frac{1}{3}\{\pi_{3;+1,-2} + \underline{2\pi_{3;+2,-1}}\}$ (44.-1)	$\frac{1}{3}\{\pi_{3;0,-3} + \underline{2\pi_{3;+2,-1}}\}$ (44.-2)
$\pi_{3;+2,-1}(1)$ (41)	$\pi_{3;+5/3,-4/3}(1)$ (42)	$\pi_{3;+4/3,-5/3}(1)$ (43)
..., $\boxed{\underline{3\hbar/3},}$ $\boxed{0\hbar/3,}$, ...	..., $\boxed{\underline{2\hbar/3},}$ $\boxed{-1\hbar/3,}$, ...	..., $\boxed{\underline{1\hbar/3},}$ $\boxed{-2\hbar/3,}$, ...

References

[1] Kyle E. Ballantine, John F. Donegan and Paul R. Eastham
Science Advances 29 Apr 2016: Vol.2, No.4, e1501748
DOI: 10.1126/sciadv.1501748

[2] ShaoXu Ren *The Origins of Spins of Elementary Particles*
ISBN 9 78-988-13649-7-5 (2014);
Journal of Modern Physics, **5**, 1848-1879
http:/dx.doi.org/10.4236/jmp.2014.517181;
Interaction of the Origins of Spin Angular Momentum
ISBN 978-988-14902-0-9 (2016 2nd edition);
Journal of Modern Physics, **7**, 737-759 http:/dx.doi.org/10.4236/jmp.2016.78070

[3] ShaoXu Ren *The Third Kind of Particles* ISBN 978-7-900500-91-5 (2011),
ISBN 978-988-15598-9-0 (2012), ISBN 988-3-659-17892-4 (2012);
Journal of Modern Physics, **5**, 800-869 http:/dx.doi.org/10.4236/jmp.2014.59090

Journal of Modern Physics, 2016, 7, 1375-1377
Published Online July 2016 in SciRes. http://www.scirp.org/journal/jmp
http://dx.doi.org/10.4236/jmp.2016.711124

Absolute Zero Occurs in Black Holes

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Abstract

The black hole is a region in space in which nothing can escape its pull. The two important parts of the anatomy of a stable black hole are the event horizon and gravitational singularity. The main discussion is regarding the temperature of a black hole. Absolute zero is a state which enthalpy and entropy is zero. The temperature of a black hole approaches the gravitational singularity in which space-time possibly ceases and entropy is zero producing absolute zero or possible sub-absolute zero.

Keywords

Black Hole, Absolute Zero, Event Horizon, Gravitational Singularity

1. Introduction

Absolute zero is the lower limit of the thermodynamic temperature scale, a state of which the enthalpy and entropy of cooled ideal gas reaches minimum value, taken as 0. The theoretical temperature is determined by extrapolating the ideal gas law; by international agreement, absolute zero is taken as −273.15 on the Celsius scale (International system of units) [1] [2]. The corresponding Kelvin temperature scale sets its zero point at absolute zero by definition.

The laws of thermodynamics dictate that absolute zero cannot be reached using any thermodynamic means. As the temperature of the substance being cooled approaches the temperature of the cooling agent, the variable approaches infinity.

Absolute zero and even sub-absolute zero gas occurs in our natural universe. Absolute zero exists in black holes.

A black hole is a region of space in which the gravitational field is so powerful that nothing, including electromagnetic radiation such as visible light, can escape its pull—a kind of bottomless pit in space-time. At its center, there lies an infinitely small, infinitely dense singularity, a place where the normal laws of physics break down [3]. The theory of general relativity predicted that a sufficiently compact mass can deform space-time to

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form a black hole [4] [5]. The boundary at the region from which no escape is possible is called the event horizon. Although crossing the event horizon has enormous effect on the fate of the object crossing it, it appears to have no locally detectable features. In many ways, a black hole acts like an ideal black body, as it reflects no light [6] [7]. Moreover, quantum field theory in curved space time predicts that event horizons emit Hawking radiation with the same spectrum as a black body of a temperature inversely proportional to its mass.

At the center of a black hole, as described by general relatively, there lies a gravitational singularity. Gravitational singularity is a region where space-time curvature becomes infinite [8]. For a non-rotating black hole, this region takes the shape of a single point and for a rotating black hole, it is a smear that lies in the plane of rotation [9]. In both cases, the singular region has zero volume. It can also be shown that the singular region contains all the mass of the black hole solution [10]. The singular region can thus be thought of having infinite density. The point at which there is gravitational singularity is where entropy ceases.

The discussion is dealing in black holes with stable systems. Stable systems are inherently self-corrections. If something disturbs them from their equilibrium state, they naturally correct themselves and return to the one equilibrium. Unstable black holes are not self-correcting under perturbations. They are the opposite. Once disturbed, their natural dynamics magnifies the disturbances [11].

A black hole behaves as though its horizon has a temperature and that temperature is inversely proportional to the hole's mass; $T \approx (6 \times 10^{-8})\, M$.

Thus as the energy (mass) increases, the temperature decreases. Here misexpressed in units of solar masses (2×10^{33} grams). The temperature is in degrees Kelvin, that is, degrees Celsius above absolute zero.

The formula for the temperature of a black hole is:

$$T = \left(\frac{hc^3}{8GM\ \text{kg}}\right) \approx \left(6.169 \times 10^{-8} \frac{M_\odot}{M}\right) \text{K}$$

where h is the Dirac constant, c is the speed of light, k_8 is the Boltzmann constant, G is the gravitational constant, M is the mass of the black hole and $M_\odot$ is the mass of the sun.

This has important consequences; the bigger the black hole the colder it is, and thus the slower it loses mass.

If this is the case, the temperature approaches absolute zero as a mass from the event horizon gets closer to the gravitational singularity. The possibility of sub-absolute zero occurs inside the black hole at the site of zero entropy, the gravitational singularity.

Ultracold quantum gas made up of potassium atoms recently produced sub-absolute zero. The sub-absolute zero gas mimics "dark energy"; the mysterious force pushes the universe to expand in an ever faster rate against the inward pull of gravity [12].

Absolute zero occurs past the event horizon and toward the gravitational singularity of the black hole.

There have been multiple theories in relation toward the understanding of temperature and black holes. The only way to satisfy the second law of thermodynamics is to admit that black holes have entropy. Further formulae have been used in the thermodynamic relationship between energy, temperature, and entropy. The Bekenstein-Hawking have attempted to explain such relationship. Further conjuncture using string theory and loop quantum gravity further offer explanations of entropy and proportionality of the area of the horizon.

Just as there are many astrophysicists, theoretical physicists, experimental physicists, etc., there are many explanations about the possibilities of properties of a black hole. Temperature of a black hole is included in one discussion of the properties.

2. Discussion

When discussing about temperature of a black hole, a clarification of the definition of temperature itself is very important.

Temperature is a measurement of the amount of random kinetic energy in a macroscopic system. This definition is generally incorrect. A more acceptable definition is, for two macroscopic systems that can exchange energy, and energy will be transferred from the higher temperature object to the lower temperature object.

In order to determine the temperature of a system, determination which way energy will be transferred to or from another system is required. The two systems will divide up the total energy as to maximize their combined entropy so when the two systems are allowed to exchange energy, there is some transfer of energy that will result in the total entropy being maximized.

Mathematically, the important quantity is the rate of exchange in entropy with respect to energy for both systems. Temperature is the inverse of that quantity.

In solids, liquids, and gases, temperature does provide a measure of the energy in the system. But for a system of magnets in a magnetic field, when energy increases, the temperature increases to infinity and then heads back to zero.

The basis of current thought is black holes have zero entropy. That would oppose the second law of thermodynamics that entropy cannot spontaneously decrease. I believe that any mass inside a black hole, entropy decrease from the event horizon as it approaches the gravitational singularity. That is why radiation may be emitted at or near the event horizon (Hawking radiation) [13]. But when mass enters the gravitational singularity (center of black hole), entropy is zero as space-time is at a standstill in a stable black hole.

Therefore, as entropy is zero, the temperature becomes absolute zero (or possibly sub-absolute zero).

There is a possibility that while matter at the gravitational singularity (center of black hole) emits no radiation, thereby no energy; the singularity produces absolute zero (or possibly sub-absolute zero). Sub-absolute zero gas mimics "dark energy", the mysterious force that pushes the universe to expand at an every faster rate against the inward pull of gravity. There is a possibility that the dark energy of the universe is nothing but the product of the gravitational singularity producing sub-absolute zero emission of dark energy.

3. Summary

As a matter gets closer to the gravitational singularity, there will be a point in the black hole in which atoms stop moving because of cessation of space-time. The point is absolute or possibly sub-absolute zero. Therefore, absolutely zero (and possibly sub-absolute zero) does occur in nature.

The point at which atoms are not moving near or at the gravitational singularity. This means that at the very least, absolute zero exists in nature.

References

[1] "Unit of Thermodynamic Temperature (Kelvin)". SI Brochure, 8th Edition, 13 March 2010 [1967], Bureau International des Poids et Mesures, Section 2.1.1.5. **Note:** The Triple Point of Water Is 0.01˚C, Not 0˚C; Thus 0 K Is −273.15˚C, Not 0273.16˚C.

[2] Arora, C.P. (2001) Thermodynamics. Tata McGraw-Hill, Noida, 43.

[3] Wald (1984) General Relativity. University of Chicago Press, 200-300.

[4] Wald, R.M. (1997) Gravitational Collapse and Cosmic Censorship. arXiv:gr-qc/9710068.

[5] Overbye, D. (2015) Black Hole Hunters. NSAS.

[6] Schultz, B.F. (2003) Gravity from the Ground up. Cambridge University Press, Cambridge, 110. http://dx.doi.org/10.1017/CBO9780511807800

[7] Davies, P.C.W. (1978) *Reports on Progress in Physics*, **41**, 1313-1355. http://dx.doi.org/10.1088/0034-4885/41/8/004

[8] Carroll (2004) Spacetime and Black Holes. 205.

[9] Carroll (2004) Spacetime and Black Holes. 264-265.

[10] Carroll (2004) Spacetime and Black Holes. 252.

[11] Braun, S., *et al.* (2013) *Science*, **339**, 52-55. http://dx.doi.org/10.1126/science.1227831

[12] Wald, R.M. (2001) *Living Reviews in Relativity*, **4**, 6. arXiv:gr-qc/9912119. http://dx.doi.org/10.12942/lrr-2001-6

[13] Hawking, S.W. (1975) *Communications in Mathematical Physics*, **43**, 199-220. http://dx.doi.org/10.1007/BF02345020

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A Bridge Connecting Classical Physics and Modern Physics

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Abstract

In classical physics, time and space are absolute and independent, so time and space can be treated separately. However, in modern physics, time and space are relative and dependent: time and space must be treated together. In 4-d s-t frames, we treat time and space independently, then add a constraint to link them together. In teaching, there is a big gap between classical and modern physics. We hope that we are able to find a frame connecting them to make learning simpler. 3-d s-t frame is the best candidate to serve this purpose: time and space are able to be treated dependently by defining the unit of time as T and the unit of space as λ in this frame. Furthermore, the ratio, λ/T, is the velocity, c, of the medium. This paper shows the equivalence between a 4-d s-t frame and a 3-d s-t frame by properly converting coordinates of two frames.

Keywords

4-d s-t Frames, 3-d s-t Frame, Proper Time, Proper Length, Time Dilation, Length Contraction

1. Introduction

In order to help students to be prepared for modern physics, many professors ask students to forget what they learn in classical physics. If a student ask too many questions based on what they learn in classical physics, it will be difficult for any professor to have progress in teaching modern physics. Even though, Einstein already demonstrated that time and space are related from the thought experiment by emitting photons from the middle of a car of the moving train, we still treat them as independent in a 4-d-s-t frames. In 4-d s-t frame, the representation of 3 dimensions of (x, y, and z) and dimension of time (t) are all perpendicular each other. The only way to depict this is to have three separate graphs of x vs. t, y vs. t, and z vs. t with time which can be negative on those graphs. Later, Minkoski added the same constraint for a pair of inertial frames to make time and space related in the following equation.

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$$x^2 + y^2 + z^2 - (ct)^2 = x'^2 + y'^2 + z'^2 - (ct')^2 = \text{const.} \tag{1}$$

We believe that time and spaces are dependent and should be shown in a frame which is applied not only to modern physics but also to classical physics. A 3-d s-t frame serves that purpose. We can draw the time dilation and length contraction between two inertial 3-d s-t frames. We can show that a 4-d s-t frame is the approximation of a 3-d s-t frame, when the velocity of a moving frame is much less than the velocity of the medium. We also show that a 3-d s-t frame is equivalent to a 4-d s-t frame after properly converting coordinates between two frames.

2. Time Dilation Shown in 3-d s-t Frames

There is a pair of inertial frames [1]: The 3-d s-t frame O' travels at the velocity u with respect to the 3-d s-t frame O. We can assume that the frame O is constructed in the platform and the frame O' is constructed in the train traveling along the railroad. The radius of the sphere at t is ct for the frame O and the radius of the sphere at t' is ct' for the frame O′ where c is the velocity of the medium. The motion of the train is a vertical line described from the frame O′ and is a slanted line described from the frame O in **Figure 1(a)**.

Because of $(ct)^2 - (ut)^2 = (ct')^2$, then we are able to derive

$$t = t' \Big/ \left(\sqrt{1-(u/c)^2} \right) \tag{2}$$

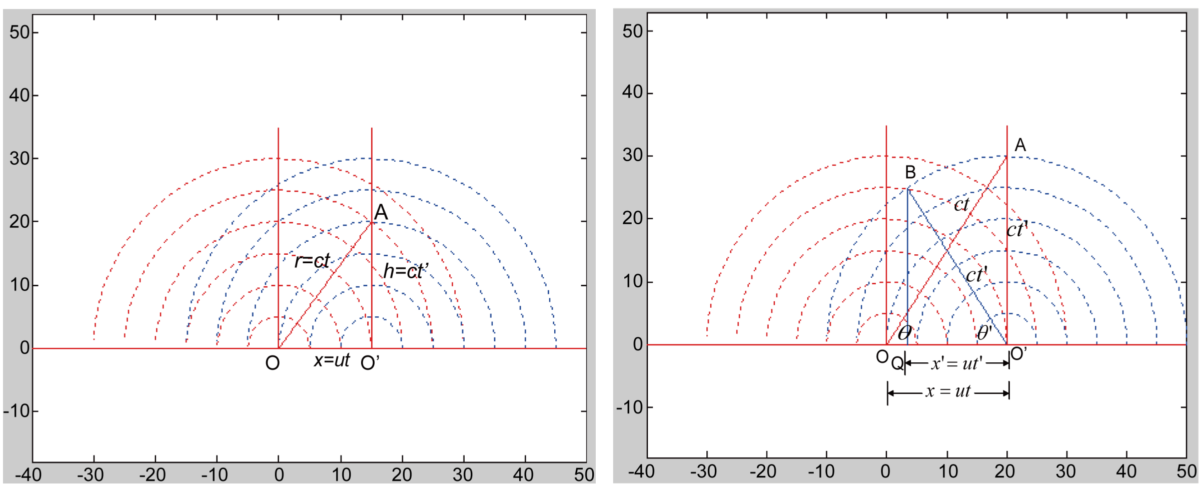
which is called time dilation [2]. The time, t', recorded from the event of the train moving along the railroad in the frame O′ is called proper(original) time, because the train comes into contact with the track in front at the same location (head of the train) viewed from the frame O′. The time recorded from the event of the train moving along the railroad in the frame O is called special time, because the train comes into contact with the tract in front at different locations (different parts of the tract), as viewed from the frame O.

The special time for measuring an event happened at the different locations in the 3-d s-t frame O is always longer than the proper time for measuring an event that happens at the same location in the 3-d s-t frame O′. In modern physics, it is called time dilation. We are able use the same graph to derive length contraction formula.

Because of $\cos\theta = \dfrac{ut}{ct} = \dfrac{u}{c}$ and $\cos\theta' = \dfrac{ut'}{ct'} = \dfrac{u}{c}$, then

$$\theta = \theta' . \tag{3}$$

Because of $\cot\theta = \cot\theta'$, then $\dfrac{ut}{ct'} = \dfrac{ut'}{QB}$ where



(a) (b)

Figure 1. (a) Red circles represent the polar coordinates on the platform. We are able to construct a new space-time frame by presenting time with polar coordinates on the platform to describe the motion of the train using the line OA. **(**b) Blue circles represent the polar coordinate on the train. We are able to construct a new space-time frame by presenting time with polar coordinate on the train to describe the motion of the platform using the line O′B.

$$\mathrm{QB}=\sqrt{\mathrm{O'B}^2-\mathrm{O'Q}^2}=\sqrt{(ct')^2-(ut')^2}\,. \tag{4}$$

The motion of the frame O′ is described as the slanted line OA from the frame O and the motion of the frame O is described as the slanted line OB from the frame O′ in **Figure 1(b)**.

Because of $x=ut$ and $x'=ut'$, then the above equation can be expressed as

$$\frac{x}{ct'}=\frac{x'}{ct'\sqrt{1-(u/c)^2}} \tag{5}$$

then it can be expressed as

$$x'=x\sqrt{1-(u/c)^2} \tag{6}$$

which is called length contraction [2]. If we want to measure the length of any object, we can lay the object along the railroad. Now, let us assume that there is a rod, which can be extended to the length of $x=ut$, is laid alongside the station platform. There is an observer O on the platform and another observer O′ on the train and both measure the rod's length using a sensor attached to the front of the train, *i.e*. the origin O′ of the moving frame S′. The length of the rod as measured by an observer on the stationary frame S with respect to the rod, is defined as proper (original) length, *x*, while the length of the rod as measured by an observer on the moving frame S′ with respect to the rod, is defined as special length, x'. This equation shows that the special length, x', is always less than the proper length, *x*. This result states that the length of a rod as measured by an observer in a moving frame is shorter than the one measured by an observer on a stationary frame.

3. Approximately Absolute Time Can Be Shown in 3-d s-t Frames

For a pair of inertial 3-d s-time frames, if the velocity of moving frame is much less than the velocity of medium, then time is approximately same for both frames shown in the following graph **Figure 2(a)**. If $u\ll c$, then $(u/c)\approx 0$ and $t\approx t'$ from the formula of time dilation

$$t=\frac{t'}{\sqrt{1-(u/c)^2}}\,. \tag{7}$$

From the figure of **Figure 2(a)**, if $u\ll c$, then $r\approx h$ where *r* is the hypotenuse and *h* is the height of the right triangle. Dived *c* on both sides, then $(r/c)\approx(h/c)$ where $t=r/c$, $t'=h/c$ and

$$t\approx t'\,. \tag{8}$$

From 3-d s-t frames, we can understand why time can be treated as absolute value regardless frame i in classic physics, where $u << c$, regardless of the frame of reference.

Figures 2(a)-2(d) show the motion of moving train at different percentage velocity of the medium with wave property.

4. Converting between Coordinates of a 4-d s-t Frame and Coordinates of a 3-d s-t Frame

The coordinates of a particle in the 4-d s-t frame are (x,y,z,t). The coordinates of a particle in the 3-d s-t frame are $(\hat{x},\hat{y},\hat{z},\hat{t})$. Converting between a 4-d s-t frame and a 3-d s-t frame is following: $t=\hat{t}T$; $x=\hat{x}\lambda$; $y=\hat{y}\lambda$; $z=\hat{z}\lambda$; $c=\dfrac{\lambda}{T}$, the velocity of a medium with property of wave (light, sound,…), where *T* is period of the wave, λ is the wavelength of the wave (see **Figure 3**).

If the unit of time is *sec*, then the unit of radius for polar circle is chosen to be the period of the wave-medium, then the unit of x-axis should be chosen as the wavelength of the wave. We construct this new space-time frame to make space and time dependent by using the period and the wavelength of the wave-medium. Furthermore, the ratio of wavelength to period is equal to the velocity of the wave-medium, $c=\dfrac{\lambda}{T}$.

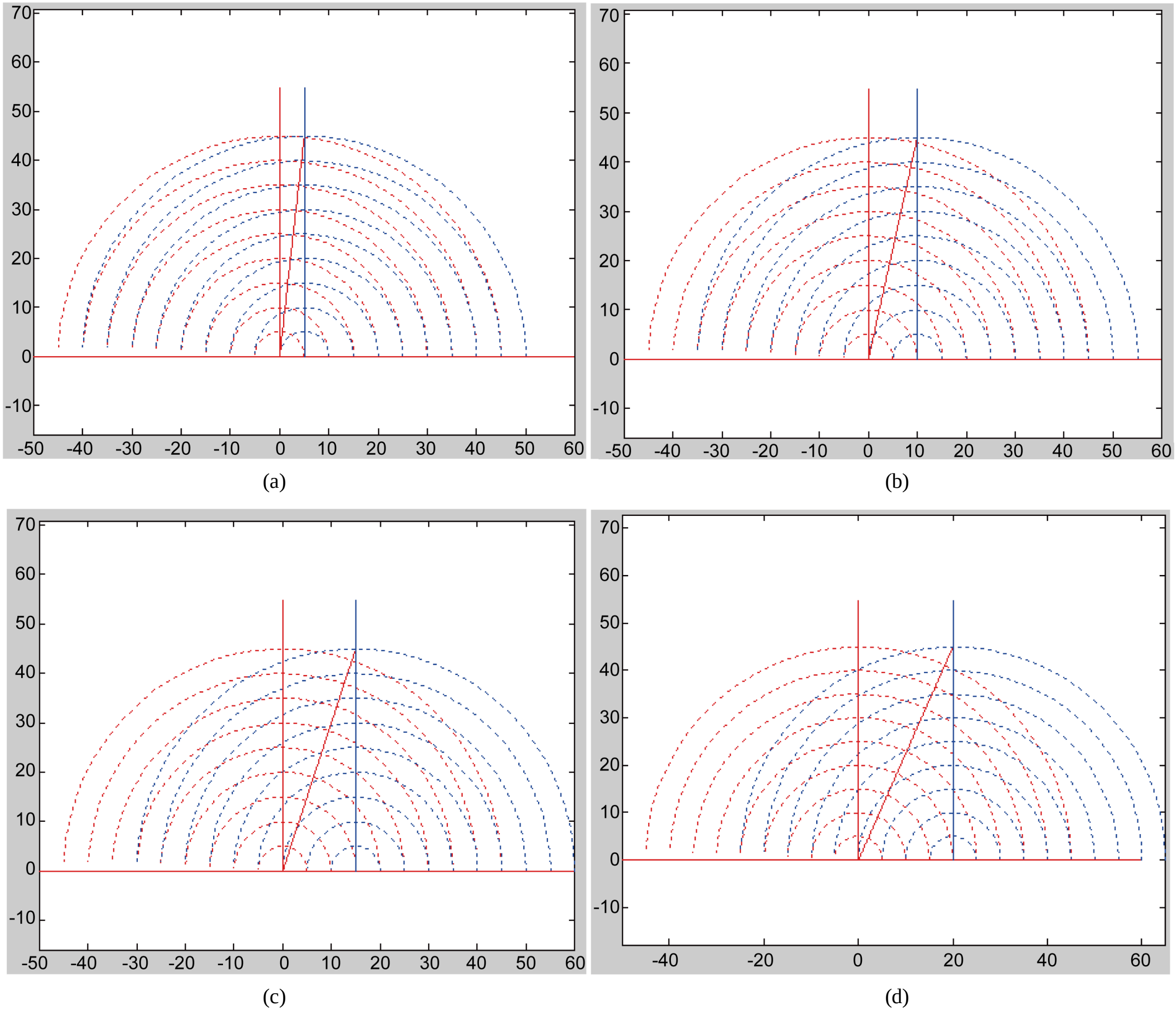


Figure 2. (a) $u = (1/9)c$; (b) $u = (2/9)c$; (c) $u = (3/9)c$; (d) $u = (4/9)c$.

In order to describe the motion of an object in 3-dimensional space along the locations of x-axis, y-axis, and z-axis, we can construct a new space-time frame. Spheres with different radius representing different outgoing time, polar coordinates will be formed from circles of intersections between spheres and x-y plane, y-z plane, and z-x plane [3]. We are able to use the red polar coordinates of x-y plane, the blue polar coordinates of y-z plane, and the gray polar coordinates of z-x plane to describe the locations of a moving object moving along x-axis, y-axis, and z-axis in **Figure 4**. This kind of new coordinate frame, which embeds the time axis into space axes is called 3-d s-t frame. This new coordinate frame eliminates one dimension from the 4-d s-t frame and solves the puzzle of visualizing a time axis which perpendicular to 3 already perpendicular spacial dimensions.

5. Equivalence between a 4-d s-t Frame and a 3-d s-t Frame

In the following section, we try to find relations between kinetic terms in a 4-d s-t frame and kinetic terms in a 3-d s-t frame through: $t = \hat{t}T$; $x = \hat{x}\lambda$; $y = \hat{y}\lambda$; $z = \hat{z}\lambda$; $c = \frac{\lambda}{T}$ where T is the period of wave and λ is the wavelength of wave. We also try to find the forms of dynamic formula in a 3-d s-t frame through the same conversion.

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}\left(\hat{x}\lambda\right)}{\mathrm{d}\left(\hat{t}T\right)} = \frac{\lambda}{T}\frac{\mathrm{d}\hat{x}}{\mathrm{d}\hat{t}} = c\frac{\mathrm{d}\hat{x}}{\mathrm{d}\hat{t}} \tag{9}$$

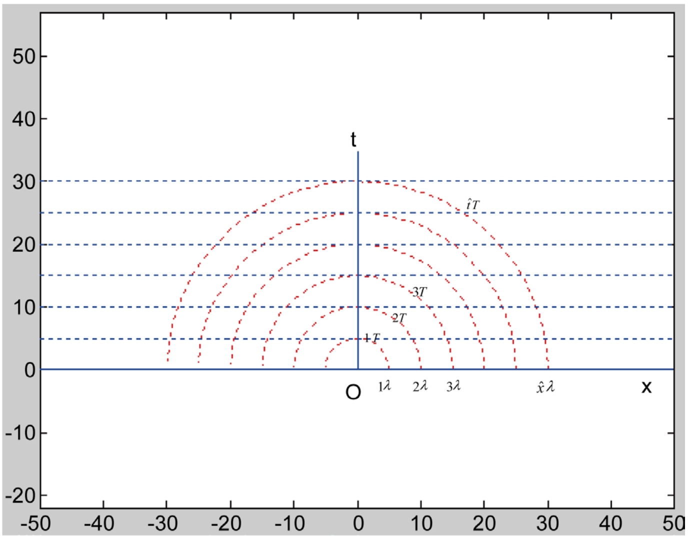


Figure 3. Constructing a new space-time frame by presenting time with unit of T and length with unit of λ using polar coordinates.

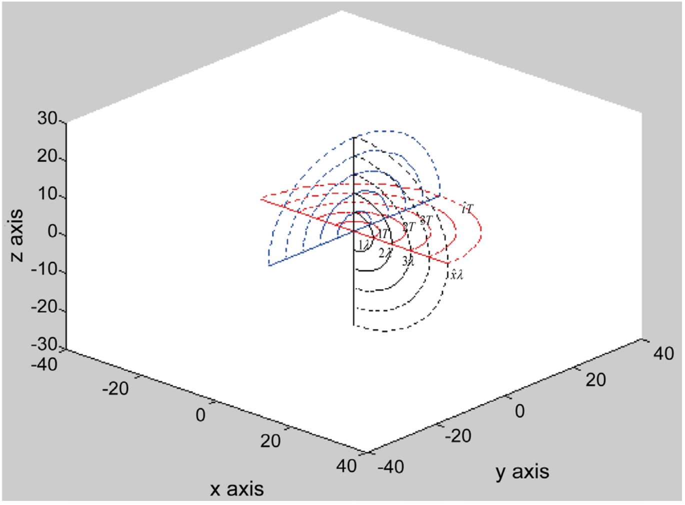


Figure 4. Three-dimensional space-time frame formed by three polar planes on three different planes.

$$\rightarrow v_x = \frac{\lambda}{T}\hat{v}_x = c\hat{v}_x \tag{10}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}(\hat{y}\lambda)}{\mathrm{d}(\hat{t}T)} = \frac{\lambda}{T}\frac{\mathrm{d}\hat{y}}{\mathrm{d}\hat{t}} = c\frac{\mathrm{d}\hat{y}}{\mathrm{d}\hat{t}} \tag{11}$$

$$\rightarrow v_y = \frac{\lambda}{T}\hat{v}_y = c\hat{v}_y \tag{12}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\mathrm{d}(\hat{z}\lambda)}{\mathrm{d}(\hat{t}T)} = \frac{\lambda}{T}\frac{\mathrm{d}\hat{z}}{\mathrm{d}\hat{t}} = c\frac{\mathrm{d}\hat{z}}{\mathrm{d}\hat{t}} \tag{13}$$

$$\rightarrow v_z = \frac{\lambda}{T}\hat{v}_z = c\hat{v}_z \tag{14}$$

$$\frac{\mathrm{d}v_x}{\mathrm{d}t}=\frac{\mathrm{d}\left(c\hat{v}_x\right)}{\mathrm{d}\left(\hat{t}T\right)}=\frac{c}{T}\frac{\mathrm{d}\hat{v}_x}{\mathrm{d}\hat{t}}=\frac{\lambda}{T^2}\frac{\mathrm{d}\hat{v}_x}{\mathrm{d}\hat{t}} \tag{15}$$

$$\rightarrow a_x=\frac{c}{T}\hat{a}_x=\frac{\lambda}{T^2}\hat{a}_x \tag{16}$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t}=\frac{\mathrm{d}\left(c\hat{v}_y\right)}{\mathrm{d}\left(\hat{t}T\right)}=\frac{c}{T}\frac{\mathrm{d}\hat{v}_y}{\mathrm{d}\hat{t}}=\frac{\lambda}{T^2}\frac{\mathrm{d}\hat{v}_y}{\mathrm{d}\hat{t}} \tag{17}$$

$$\rightarrow a_y=\frac{c}{T}\hat{a}_y=\frac{\lambda}{T^2}\hat{a}_y \tag{18}$$

$$\frac{\mathrm{d}v_z}{\mathrm{d}t}=\frac{\mathrm{d}\left(c\hat{v}_z\right)}{\mathrm{d}\left(\hat{t}T\right)}=\frac{c}{T}\frac{\mathrm{d}\hat{v}_z}{\mathrm{d}\hat{t}}=\frac{\lambda}{T^2}\frac{\mathrm{d}\hat{v}_z}{\mathrm{d}\hat{t}} \tag{19}$$

$$\rightarrow a_z=\frac{c}{T}\hat{a}_z=\frac{\lambda}{T^2}\hat{a}_z \tag{20}$$

Distance of a particle with a constant velocity

$$x=x_0+v_xt \tag{21}$$

$$\rightarrow \lambda\hat{x}=\lambda\hat{x}_0+\frac{\lambda}{T}\hat{v}_x\left(\hat{t}T\right) \tag{22}$$

$$\rightarrow \hat{x}=\hat{x}_0+\hat{v}_x\hat{t} \tag{23}$$

$$y=y_0+v_yt \tag{24}$$

$$\rightarrow \lambda\hat{y}=\lambda\hat{y}_0+\frac{\lambda}{T}\hat{v}_y\left(\hat{t}T\right) \tag{25}$$

$$\rightarrow \hat{y}=\hat{y}_0+\hat{v}_y\hat{t} \tag{26}$$

$$z=z_0+v_zt \tag{27}$$

$$\rightarrow \lambda\hat{z}=\lambda\hat{z}_0+\frac{\lambda}{T}\hat{v}_z\left(\hat{t}T\right) \tag{28}$$

$$\rightarrow \hat{z}=\hat{z}_0+\hat{v}_z\hat{t} \tag{29}$$

Distance of a particle with a constant acceleration

$$x=x_0+v_{0x}t+\frac{1}{2}a_xt^2 \tag{30}$$

$$\rightarrow \lambda\hat{x}=\lambda\hat{x}_0+\frac{\lambda}{T}\hat{v}_{0x}\left(\hat{t}T\right)+\frac{1}{2}\left(\frac{\lambda}{T^2}\right)\left(\hat{a}_x\hat{t}^2T^2\right) \tag{31}$$

$$\rightarrow \hat{x}=\hat{x}_0+\hat{v}_{0x}\hat{t}+\frac{1}{2}\hat{a}_x\hat{t}^2 \tag{32}$$

$$y=y_0+v_{0y}t+\frac{1}{2}a_yt^2 \tag{33}$$

$$\rightarrow \lambda\hat{y}=\lambda\hat{y}_0+\frac{\lambda}{T}\hat{v}_{0y}\left(\hat{t}T\right)+\frac{1}{2}\left(\frac{\lambda}{T^2}\right)\left(a_y\hat{t}^2T^2\right) \tag{34}$$

$$\rightarrow \hat{y}=\hat{y}_0+\hat{v}_{0y}\hat{t}+\frac{1}{2}\hat{a}_y\hat{t}^2 \tag{35}$$

$$z=z_0+v_{0z}t+\frac{1}{2}a_zt^2 \tag{36}$$

$$\rightarrow \lambda\hat{z} = \lambda\hat{v}_{0z} + \frac{\lambda}{T}\hat{v}_{0z}\left(\hat{t}T\right) + \frac{1}{2}\left(\frac{\lambda}{T^2}\right)\left(\hat{a}_z\hat{t}^2T^2\right) \tag{37}$$

$$\rightarrow \hat{z} = \hat{z}_0 + \hat{v}_{0z}\hat{t} + \frac{1}{2}\hat{a}_z\hat{t}^2. \tag{38}$$

It shows that all kinetic formulas in a 3-d s-t frame are same as in classical physics.

Newton's Second Law:

$$F = ma \tag{39}$$

$$\rightarrow F = m\left(\frac{\lambda}{T^2}\hat{a}\right) = \frac{\lambda}{T^2}m\hat{a} = \frac{\lambda}{T^2}\hat{F} \tag{40}$$

$$\rightarrow F = \frac{\lambda}{T^2}\hat{F} \tag{41}$$

and

$$\rightarrow \hat{F} = m\hat{a}. \tag{42}$$

Momentum:

$$p = mv \tag{43}$$

$$\rightarrow p = m\left(\frac{\lambda}{T}\hat{v}\right) = \frac{\lambda}{T}\left(m\hat{v}\right) = \frac{\lambda}{T}\hat{p} \tag{44}$$

$$\rightarrow p = \frac{\lambda}{T}\hat{p} \tag{45}$$

and

$$\rightarrow \hat{p} = m\hat{v}. \tag{46}$$

Kinetic Energy:

$$E_k = \frac{1}{2}mv^2 \tag{47}$$

$$\rightarrow E_k = \frac{1}{2}m\left(\frac{\lambda}{T}\hat{v}\right)^2 = \frac{\lambda^2}{T^2}\left(\frac{1}{2}m\hat{v}^2\right) = \frac{\lambda^2}{T^2}\hat{E}_k \tag{48}$$

$$\rightarrow E_k = \frac{\lambda^2}{T^2}\hat{E}_k \tag{49}$$

and

$$\rightarrow \hat{E}_k = \frac{1}{2}m\hat{v}^2. \tag{50}$$

The other form of Newton's Second Law:

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} \tag{51}$$

$$\rightarrow \frac{\lambda}{T^2}\hat{F} = \frac{\mathrm{d}\left(\frac{\lambda}{T}\hat{p}\right)}{\mathrm{d}\left(\hat{t}T\right)} = \frac{\lambda}{T^2}\frac{\mathrm{d}\hat{p}}{\mathrm{d}\hat{t}} \tag{52}$$

$$\rightarrow \hat{F} = \frac{\mathrm{d}\hat{p}}{\mathrm{d}\hat{t}}. \tag{53}$$

It shows that the all forms of dynamic formula in a 3-d s-t frame are same forms of dynamic formula in classical physics.

a) In Modern Physics:

For time dilation

$$t = \frac{t_0'}{\sqrt{1-(u/c)^2}} \tag{54}$$

$$\to \hat{t}T = \frac{\hat{t}_0' T}{\sqrt{1-(u/c)^2}} \tag{55}$$

$$\to \hat{t} = \frac{\hat{t}_0'}{\sqrt{1-(u/c)^2}}. \tag{56}$$

For length contraction

$$x' = x_0\sqrt{1-(v/c)^2} \tag{57}$$

$$\to \hat{x}'\lambda = \hat{x}_0\lambda\sqrt{1-(v/c)^2} \tag{58}$$

$$\to \hat{x}' = \hat{x}_0\sqrt{1-(v/c)^2}. \tag{59}$$

It shows that the all forms of kinetic formula in a 3-d s-t frame are same forms of kinetic formula in modern physics.

6. Discussion

This paper expounds that a 4-d s-t frame is the approximation of a 3-d s-t frame, when the velocity of a moving frame is much less than the velocity of the medium. While, showing the equivalence between a 4-d s-t frame and a 3-d s-t frame by proper converting coordinates of two frames. In the "Time Dilation and Length Contraction Shown in Three-Dimensional Space-Time Frames", demonstrates that we are able to visualize time dilations and length contractions through graphs on **Figure 1(a)** and **Figure 1(b)** when 3-d s-t frames are used to describe motions of particles moving at high velocity which is percentage of the velocity of light.

We are able to describe the motion of a particle at very small velocity, relative to the velocity of light in a 3-d s-t frame in classical physics. The following graph (see **Figure 5**) illustrates how to utilize this new 3-d s-t Frame to describe the motion of particles moving along the *x*-axis with different velocities [2]. If the velocity of sound, which is the medium of the system, is V_m = 343 m/sec, then the radius of the circle representing 1sec would be

$$1R = (V_m)(1\,\text{sec}) = 343\text{ m} \tag{60}$$

the radius of the circle representing 2 secs would be

$$2R = (V_m)(2\,\text{sec}) = 686\text{ m} \tag{61}$$

etc. In **Figure 5**, we use different lines OA, OB, OC, OD, OE, FG to describe the different motions of particles which velocities are much less than V_m = 343 m/sec, the velocity of sound which is the medium of the system.

We also able use the velocity of sound

$$V_m = \lambda/T = 17.15\text{ m}/0.05\,\text{sec} = 343\text{ m/sec} \tag{62}$$

to construct a polar coordinate by letting $\lambda = 17.15\text{ m}$ and $T = 0.05\,\text{sec}$ to describe motions of particles of the system.

A single particle moves in a helix in space. A particle moves in circles on the x-y plane while also moving with constant speed along the z-axis. The description of the motion is shown in **Figure 6**. The red sine-wave represents the motion of the particle on x-axis, the blue cosine-wave represents the motion of the particle on y-axis, and the black arrow represents the motion of the particle on z-axis. More complicated motions of particles can similarly be described in a 3-d s-t frame as needed [3].

P.S.: For all figures except **Figure 5** in the paper, there are numbers along x, y, z axes. The purpose of these numbers aids drawing from the MATLAB program of the computer.

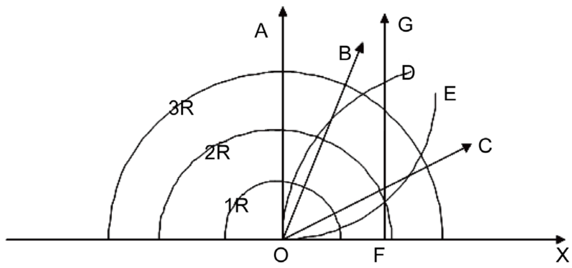


Figure 5. OA represents a particle that remains stationary at O, OB represents a particle moving with a relatively slow constant velocity, OC represents a particle moving with a relatively fast constant velocity, OD represents a particle moving with a constant acceleration, OE represents a particle moving with a constant deceleration, and FG represents a particle that remains stationary at F.

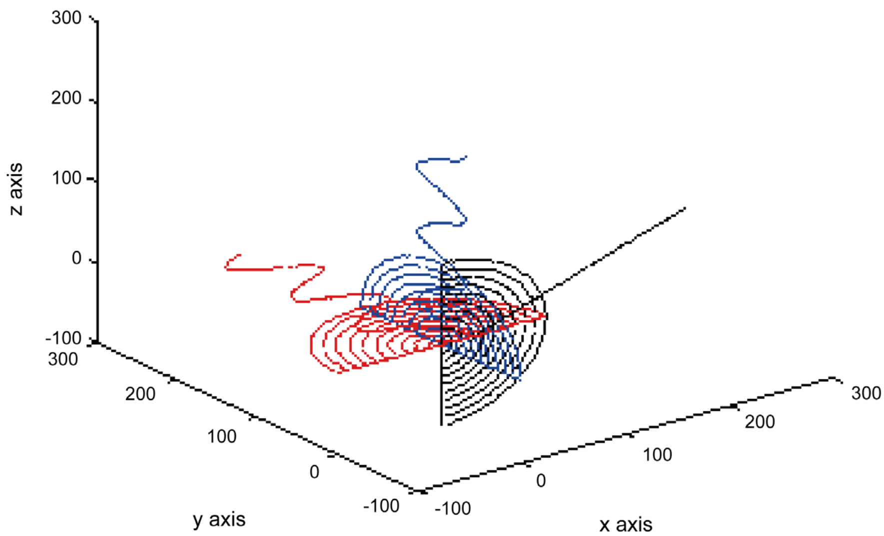


Figure 6. A single particle moving in a helix in space: A particle moves in circles on the x-y plane while also moving with constant speed along the z-axis.

It might be worth to discuss rewriting textbooks of classical physics and modern physics described in 3-d s-t frames for teaching physics [4].

We believe that if students study physics using 3-d s-t frames, they will benefit by this approach [5].

7. Conclusion

A 4-d s-t frame is independent and is only tied together using an equation constraint. A 3-d s-t frame is dependent and the constraint is embedded into the system [6]. A 3-d s-t frame can represent both time dilation and length contraction in graphs. In 3-d s-t frames, one important distinction between space and time is that space is bidirectional and time is unidirectional, *i.e.* an object can proceed forward and backward in space but can only proceed forward in time. In the proposed 3-d s-t frame, the representation of time as concentric spheres of different radii centered about the origin of the spatial axes inherently restricts time to a single direction, *i.e.* negative time (circles with negative radii) has no meaning. Essentially, a 4-d s-t frame and a 3-d s-t frame can be considered as equal by proper converting coordinates of two frames, because their forms of laws of physics are equivalent. Thus, a 3-d s-t frame can serve as a bridge connecting classical physics and modern physics.

References

[1] Chen, T. and Chen, Z. (2008) *Concepts of Physics*, **V**, No. 3. http://merlin.fic.uni.lodz.pl/concepts/www/V_3/523.pdf

[2] Chen, T. and Chen, Z. (2009) *Concepts of Physics*, **VI**, No. 2. http://merlin.phys.uni.lodz.pl/concepts/www/VI_2/221.pdf

[3] Chen, T. and Chen, Z. (2005) Motions of Particles Described in a Three-Dimensional Space-Time Frame. *Proceeding of the* 10*th Asian Technology Conference in Mathematics*, Korea National University of Education, South Korea, 12-16 December 2008. http://epatcm.any2any.us/EP/EP2005/2005P163/fullpaper.pdf

[4] Chen, T. and Chen, Z. (2011) *The Journal of Frontiers in Science*. http://www.sapub.org/journal/archive.aspx?journalid=1015&issueid=337

[5] Chen, T. and Chen, Z. (2016) *Astrophysics and Cosmology Journal of Modern Physics*, **7**, 611-626. http://www.scirp.org/journal/jmp http://dx.doi.org/10.4236/jmp.2016.77062

[6] Chen, T. and Chen, Z. (2016) *Journal of Modern Physics*, **7**, 1192-1199. http://www.scirp.org/journal/jmp http://dx.doi.org/10.4236/jmp.2016.710108