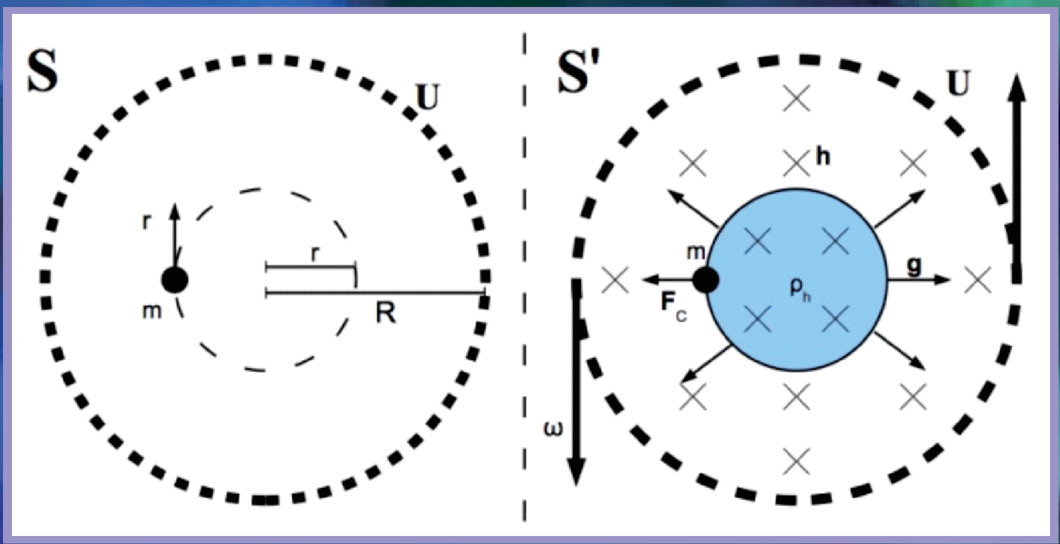
ISSN: 2153-1196 Vol. 3, Special Issue, September 2012

Scientific Research

Special Issue on Gravitation, Astrophysics and Cosmology

Journal of Modern Physics



ISSN: 2153-1196

www.scirp.org/journal/jmp

Journal Editorial Board

ISSN: 2153-1196 (Print)   ISSN: 2153-120X (Online)

http://www.scirp.org/journal/jmp

Journal of Modern Physics, 2012, 3, 1097-1355
Published Online September 2012 in SciRes (http://www.SciRP.org/journal/jmp/)

Scientific Research

TABLE OF CONTENTS

Volume 3 Special Issue September 2012

***Journal of Modern Physics*, 2012, 3, 1097-1355**
Published Online September 2012 in SciRes (http://www.SciRP.org/journal/jmp/)

The figure on the front cover is from the article published in Journal of Modern Physics, 2012, Vol. 3, Special Issue, pp. 1298-1300 by Lorenzo Barattini and Paolo Christillin

Journal of Modern Physics (JMP)

Journal Information

SUBSCRIPTIONS

The *Journal of Modern Physics* (Online at Scientific Research Publishing, www.SciRP.org) is published monthly by Scientific Research Publishing, Inc., USA.

Subscription rates:

Print: $59 per issue.
To subscribe, please contact Journals Subscriptions Department, E-mail: sub@scirp.org

SERVICES

Advertisements
Advertisement Sales Department, E-mail: service@scirp.org

Reprints (minimum quantity 100 copies)
Reprints Co-ordinator, Scientific Research Publishing, Inc., USA.
E-mail: sub@scirp.org

COPYRIGHT

PRODUCTION INFORMATION

For manuscripts that have been accepted for publication, please contact:
E-mail: jmp@scirp.org

Journal of Modern Physics, **2012, 3, 1097-1102**
doi:10.4236/jmp.2012.329144 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Astrophysical Constraints on the Scale of Left-Right Symmetry in Inverse Seesaw Models

Debasish Borah
Department of Physics, Tezpur University, Tezpur, India
Email: dborah@tezu.ernet.in

Received July 1, 2012; revised August 9, 2012; accepted August 17, 2012

ABSTRACT

We revisit the recently studied supersymmetric gauged inverse seesaw model [1] to incorporate astrophysical constraints on lightest supersymmetric particle (LSP) lifetime such that LSP constitutes the dark matter of the Universe. The authors in [1] considered light sneutrino LSP that can play the role of inelastic dark matter (iDM) such that desired iDM mass splitting and tiny Majorana masses of neutrinos can have a common origin. Here we consider a generalized version of this model without any additional discrete symmetry. We point out that due to spontaneous R-parity $\left(R_p = (-1)^{3(B-L)+2S}\right)$ breaking in such generic supersymmetric gauged inverse seesaw models. LSP can not be perfectly stable but decays to standard model particles after non-renormalizable operators allowed by the gauge symmetry are introduced. We show that strong astrophysical constraints on LSP lifetime makes sneutrino dark matter more natural than standard neutralino dark matter. We also show that long-livedness of sneutrino dark matter constrains the left right symmetry breaking scale $M_R < 10^4$ GeV .

Keywords: PACS Numbers; 12.10.-g; 12.60.Jv; 11.27.+d

1. Introduction

Left-Right Symmetric Models (LRSM) [2-6] provide a framework within which spontaneous parity breaking as well as tiny neutrino masses [7-10] can be successfully implemented without reference to very high scale physics such as grand unification. Incorporating Supersymmetry (SUSY) into it comes with other advantages like providing a solution to the gauge hierarchy problem, and providing a Cold Dark Matter candidate which is the lightest supersymmetric particle (LSP). In Minimal Supersymmetric Standard Model (MSSM), the stability of LSP is guaranteed by R-parity, defined as $R_p = (-1)^{3(B-L)+2S}$ where S is the spin of the particle. This is a discrete symmetry put by hand in MSSM to keep the baryon number (B) and lepton number (L) violating terms away from the superpotential. In generic implementations of Left-Right symmetry, R-parity is a part of the gauge symmetry and hence not ad-hoc like in the MSSM. In one class of models [11-14], spontaneous parity breaking is achieved without breaking R-parity. This was not possible in minimal supersymmetric left right (SUSYLR) models where the only way to break parity is to consider spontaneous R-parity violation [15]. In minimal SUSYLR model parity, $SU(2)_R$ gauge symmetry as well as R-parity break simultaneously by the vacuum expectation value of right handed sneutrino.

Here we study a different SUSYLR model which belong to a more general class of models where both R-parity and D-parity break spontaneously [16] by the vacuum expectation value (vev) of a Higgs field carying $U(1)_{B-L}$ gauge charge ± 1 and hence odd under R-parity. Spontaneous R-parity breaking models have received lots of attention recently due to their rich phenomenology [17-19]. In such generic spontaneous R-parity breaking models, the scalar superpartner of right-handed neutrino acquire a non-zero vev which breaks $U(1)_{B-L}$ symmetry spontaneously. Such a scenario gives rise to tree level mixing between neutralinos and light neutrinos and hence the neutralino dark matter candidate is lost in such a model unless one talks about long lived gravitino dark matter. However the model we study in this letter, although breaks R-parity spontaneously, does not give rise to tree level mixing terms between LSP and standard model fermions. Thus we can have a dark matter candidate in such a model without introducing the least understood gravity sector into account. Recently right handed sneutrino dark matter in such a model was discussed in [1]. However, the authors in [1] (also in [20]) considered an additional discrete Z_2 symmetry so as to guarantee a perfectly stable LSP. Here we consider a generalized version of this model without any additional symmetries apart from the gauge symmetry. We point out that LSP dark matter, although stable at the renormalizable level, decays after higher dimensional gauge invariant terms are introduced. The strength of such operators will be tightly constrained from the fact that LSP lifetime should be longer than the age of the Universe and large enough so as to agree with astrophysical observations of nearby galaxies and clusters [21]. Astrophysical constraints on such operators within the framework of MSSM was studied in [22]. Here we follow a similar analysis in our model and show that astrophysical constraints not only put an upper bound on the left-right symmetry breaking scale but also make the sneutrino dark matter more natural than standard neutralino dark matter. It is worth mentioning that constraints on the left-right symmetry breaking scale in such a model were derived recently in [23] from the requirement of successful gauge coupling unification and disappearance of transitory domain walls formed as a result of spontaneous discrete symmetry breaking.

This letter is organized as follows. In Section 2 we briefly review the model. In Section 3 we discuss the higher dimensional operators in the model and astrophysical constraints. We summarise the constraints from gauge coupling unification and domain wall disappearance from our earlier work [23] in Section 4 and finally conclude in Section 5.

2. The Model

Spontaneous R-parity breaking can be achieved even without giving vev to the sneutrino fields. If the $U(1)_{B-L}$ symmetry is broken by a Higgs field which has odd $B-L$ charge then R-parity is spontaneously broken. We call this model as Minimal Higgs Doublet (MHD) Model. The minimal such model [16,24] has the following particle content

$$L(2,1,-1), L_c(1,2,1), S(1,1,0), Q\left(2,1,\frac{1}{3}\right), Q_c\left(1,2,-\frac{1}{3}\right)$$

$$H=\begin{pmatrix} H_L^+ \\ H_L^0/\sqrt{2} \end{pmatrix} \sim (2,1,1), H_c=\begin{pmatrix} H_R^+ \\ H_R^0/\sqrt{2} \end{pmatrix} \sim (1,2,-1),$$

$$\bar{H}=\begin{pmatrix} H_L^0/\sqrt{2} \\ h_L^- \end{pmatrix} \sim (2,1,-1), \bar{H}_c=\begin{pmatrix} h_R^0/\sqrt{2} \\ H_R^- \end{pmatrix} \sim (1,2,1),$$

$$\Phi_1(2,2,0), \Phi_2(2,2,0)$$

where the numbers in brackets correspond to the quantum numbers corresponding to

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}.$$

The symmetry breaking pattern is

$$\begin{aligned} &SU(2)_L \times SU(2)_R \\ &\times U(1)_{B-L} \xrightarrow{\langle H, H_c\rangle} SU(2)_L \\ &\times U(1)_Y \xrightarrow{\langle \Phi \rangle} U(1)_{em} \end{aligned} \quad (1)$$

Neutrino masses arise naturally in this model by so called inverse seesaw mechanism by virtue of the presence of singlet superfields $S(1,1,0)$ (one per generation). The renormalizable superpotential relevant for the spontaneous parity violation and neutrino mass is given as follows

$$\begin{aligned} W_{ren} &= h_l^{(i)} L^T \tau_2 \Phi_i \tau_2 L_c + h_q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q_c \\ &\quad + \iota f L^T \tau_2 S H + \iota f^* L_c^T \tau_2 S H_c + M_s S S \\ &\quad + \mu_{ij} Tr \tau_2 \Phi_i^T \tau_2 \Phi_j + f_{h\varphi}\left(H^T \Phi_i H_c + \bar{H}^T \Phi_i \bar{H}_c\right) \\ &\quad + m_h H^T \tau_2 \bar{H} + m_h H_c^T \tau_2 \bar{H}_c \end{aligned} \quad (2)$$

We denote the vev of the neutral components of $\Phi_1, \Phi_2, H_L, \bar{H}_L, H_R, \bar{H}_R$ as

$$\begin{aligned} &\left\langle (\Phi_1)_{11} \right\rangle = v_1, \left\langle (\Phi_2)_{22} \right\rangle = v_2, \left\langle H_L, \bar{H}_L \right\rangle \\ &= v_L, \left\langle H_R, \bar{H}_R \right\rangle = v_R \end{aligned}$$

The neutrino mass matrix in the basis (ν, ν_c, S) is given by

$$M_{\nu\rho} = \begin{pmatrix} 0 & M_D & F v_L \\ M_D^T & 0 & F' v_R \\ F^T v_L & F'^T v_R & M_s \end{pmatrix} \quad (3)$$

where

$$M_D = \left(\phi_{12}^0 h_1 + \phi_{22}^0 h_2\right), F = f/\sqrt{2}, F' = f^*/\sqrt{2}.$$

After orthogonalization we get the following expression for ν mass

$$M_\nu = -M_D M_R^{-1} M_D^T - \left(M_D + M_D^T\right) v_L / v_R \quad (4)$$

where

$$M_R = \left(F' v_R\right) M_s^{-1} \left(F'^T v_R\right) \quad (5)$$

It should be noted from the neutrino mass matrix that these mass terms allow the mixing of an R-parity odd singlet fermion S with an R-parity even neutrino. Note that the superpotential preserves R parity. The mild R parity violation occurring in the neutrino mass matrix should be understood as an accidental consequence of $B-L$ gauge symmetry breakdown.

Neutrino mass can arise from type III seesaw mechanism [25] if we introduce fermionic triplets instead of singlets. However when we have a TeV scale intermediate $U(1)_{B-L}$ symmetry, the fermion triplets will spoil the gauge coupling unification [16,26] and hence fermion singlets will serve a better purpose in this case.

3. Non-Renormalizable Operators and Astrophysical Constraints

The authors of [22] considered explicit R-parity violating terms in the MSSM superpotential of the form $\lambda\left(LLE^c\right)+\varepsilon\left(LH\right)$ which lead to the decay of LSP dark matter candidate in the model. Similar analysis within the framework of grand unified theories can be found in [27,28]. We take the conservative lower bound $\left(\tau_{LSP}>10^{27}\ \text{s}\right)$ on LSP lifetime coming from the recent Fermi telescope observation of nearby galaxy and clusters [21].

In the model we are studying, the effective terms in the superpotential leading to LSP decay can arise after introduing dimension four and dimension five operators as follows:

$$W_{non-ren} \supset \frac{f_1}{\Lambda}\left(L^T\tau_2 H + L_c^T\tau_2 H_c\right)\left(H^T\tau_2\bar{H} + H_c^T\tau_2\bar{H}_c\right) + \frac{f_2}{\Lambda^2}\left(L^T\tau_2\Phi_i\tau_2 L_c\right)\left(L^T\tau_2 H + L_c^T\tau_2 H_c\right)$$

The first term give rise to terms like $\varepsilon\left(LH\right)$ in the low energy effective theory after gauge symmetry is spontaneously broken. The strength of such a term is dictated by $\varepsilon_{non-ren}\sim\left\langle H_c\right\rangle^2/\Lambda$. Here $\left\langle H_c\right\rangle$ is the left-right symmetry breaking scale which has a lower bound of $M_{W_R}\geq 2.5\ \text{TeV}$ [29]. And, the cut-off scale Λ is the generic grand unified theory (GUT) scale $\Lambda=\Lambda_{GUT}\sim 2\times10^{16}\ \text{GeV}$. Using these values and assuming generic order one dimensionless coefficients f_1, we have

$$\varepsilon_{non-ren} > 10^{-10}\ \text{GeV} \tag{6}$$

As shown in [22], the decay width of neutralino corresponding to a term $\varepsilon\left(LH\right)$ in the superpotential is given by

$$\Gamma_\chi \propto \varepsilon^2\frac{G_F^2 m_\chi^3}{768\pi^3} \tag{7}$$

with constant of proportionality of order unity. Now, for generic neutralino dark matter with mass of the order of $100\ \text{GeV}$, the astrophysical constraint on LSP lifetime $\left(\tau_{LSP}>10^{27}\ \text{s}\right)$ gives rise to

$$\varepsilon_{astro} < 10^{-22}\ \text{GeV} \tag{8}$$

Clearly the astrophysical bound (8) does not agree with the strength of $\varepsilon_{non-ren}$ arising from generic non-renormalizable operators in the theory. If we fine tune f_1 to be as small as electron Yukawa coupling 10^{-5}, then ε can be as small as 10^{-15}. But this lies around seven orders of magnitude above the upper bound set by astrophysical constraints (8). Thus, standard neutraino dark matter is very unlikely in these models unless we have unnatural fine tuning of the dimensionless coefficients in the non-renormalizable operators. It should be noticed that a term like $\varepsilon\left(LH\right)$ arise at tree level in generic spontaneous R-parity violating models with non-zero right handed sneutrino vev [17-19].

The second term in the non-renormalizable superpotential gives rise to an effective term of the form $\lambda_{non-ren}\left(LL_cL_c\right)$ which opens the decay channel of sneutrino into two standard model fermions. The strength of such a term is given by $\lambda_{non-ren}\sim\left\langle\phi\right\rangle\left\langle H_c\right\rangle/\Lambda^2$ where $\left\langle\phi\right\rangle\sim10^2\ \text{GeV}$ and $\left\langle H_c\right\rangle>2.5\ \text{TeV}$. For $\Lambda=\Lambda_{GUT}$ such a term is of strength

$$\lambda_{non-ren} > 10^{-27} \tag{9}$$

The decay width of a sneutrino to standard model fermion-antifermion pairs is given by

$$\Gamma_{\tilde{\nu}} = \frac{\lambda^2 m_{\tilde{\nu}}}{8\pi}\left(1-\frac{4m_f^2}{m_{\tilde{\nu}}^2}\right)^{3/2} \tag{10}$$

Now, for sneutrino LSP mass of the order of $100\ \text{GeV}$, the astrophysical constraint on LSP lifetime $\left(\tau_{LSP}>10^{27}\ \text{s}\right)$ gives rise to

$$\lambda_{astro} < 10^{-26} \tag{11}$$

which agrees with the generic $\lambda_{non-ren}$ arising from the non-renormalizable operators in the theory (9). Thus sneutrino LSP in such a model can be a viable dark matter candidate provided it satisfies other relevant constraints of relic density, direct detection etc. Recently it was shown that such a right handed sneutrino dark matter (within the framework of a similar left right model) can satisfy relic density as well as direct detection constraints [1].

For right handed sneutrino dark matter to obey the relevant astrophysical constraints (11), the left right symmetry breaking scale should however have an upper bound. Requiring $\left\langle\phi\right\rangle\left\langle H_c\right\rangle/\Lambda_{GUT}^2<10^{-26}$ gives rise to a bound on the left-right symmetry breaking scale

$$\left\langle H_c\right\rangle < 10^4\ \text{GeV} \tag{12}$$

for generic GUT scale and order one dimensionless couplings. However, as studied in [16,23] and summarised in the next section, successful gauge coupling unification in such a minimal model puts a lower bound on left-right symmetry breaking scale $\sim10^{12}\ \text{GeV}$.

4. Constraints on M_R from Unification and Domain Wall Disappearance

Similar to generic SUSYLR models, here also the intermediate symmetry breaking scales are constrained by demanding successful gauge coupling unification at a very high scale $M_G\left(>2\times10^{16}\text{GeV}\right)$. The couplings of

$U(1)_{B-L}$ and $SU(2)_{L,R}$ meet much before the allowed Unification scale if the intermediate symmetry breaking scale M_R is lower than a certain value. For the minimal SUSYLR model with Higgs doublets, this lower bound on M_R is found to be of the order of 10^{12} GeV. We also consider two additional heavy colored superfields so that the $SU(3)_c$ coupling meet the other two couplings at one point. They are denoted as $\chi\left(3,1,1,-\frac{2}{3}\right),\bar{\chi}\left(\bar{3},1,1,\frac{2}{3}\right)$ and can be accommodated within $SO(10)$ GUT theory in the representations $120,\overline{126}$. Here we assume that the structure of the GUT theory is such that these fields survive the symmetry breaking and can be as light as the $SU(2)_R$ breaking scale. The resulting gauge coupling unification as shown in the **Figure 1**.

As discussed in details in [23], the succesful disappearance of domain walls in this model do not put any strict constraints on the left-right symmetry breaking scale and can be anywhere between a TeV scale and the Planck scale. Thus unification and domain wall disappearance constraints are compatible with each other. The discrepancy between the astrophysical limit $M_R < 10^4$ GeV and the limit from successful unification $M_R \geq 10^{12}$ GeV can be removed by including a parity odd singlet to our model. As studied in [16,26,30], such a framework allows even a TeV scale M_R from the requirement of successful gauge coupling unification as can be seen from **Figure 2**. It should be noted that the authors of [1] indeed considered such a model with parity odd singlet which allows a TeV scale M_R. Such TeV scale M_R is not just a requirement from astrophysical constraints as we have found above, but these TeV scale gauge bosons also contribute to the dark matter annihilations [1] in the early universe producing the correct relic density at present.

5. Results and Conclusions

We have discussed the issue of stability of LSP dark matter in a specific version of SUSYLR model with inverse seesaw mechanism of neutrino mass where both D-parity and R-parity are spontaneously broken. We point out that, although LSP is a stable particle in the renormalizable version of the model, it can decay into standard model fermions after the non-renormalizable terms are introduced. The requirement that LSP dark matter should be long lived so as to satisfy strict astrophysical and cosmological bounds constrains the strength of these higher dimensional operators suppressed by

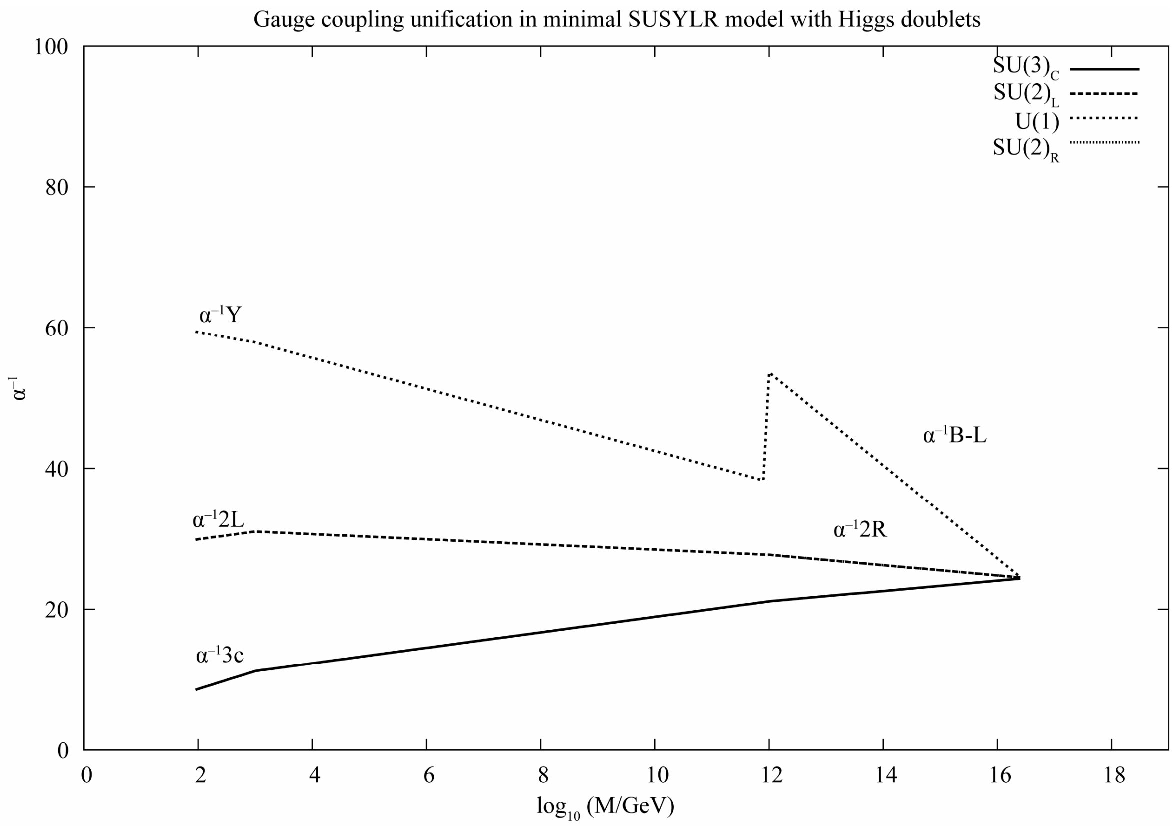


Figure 1. Gauge coupling unification in minimal SUSYLR model with higgs doublets, $M_{susy} = 1$ TeV, $M_R = 10^{12}$ GeV, $M_{GUT} = 10^{16.4}$ GeV. The figure is redrawn from [16,23].

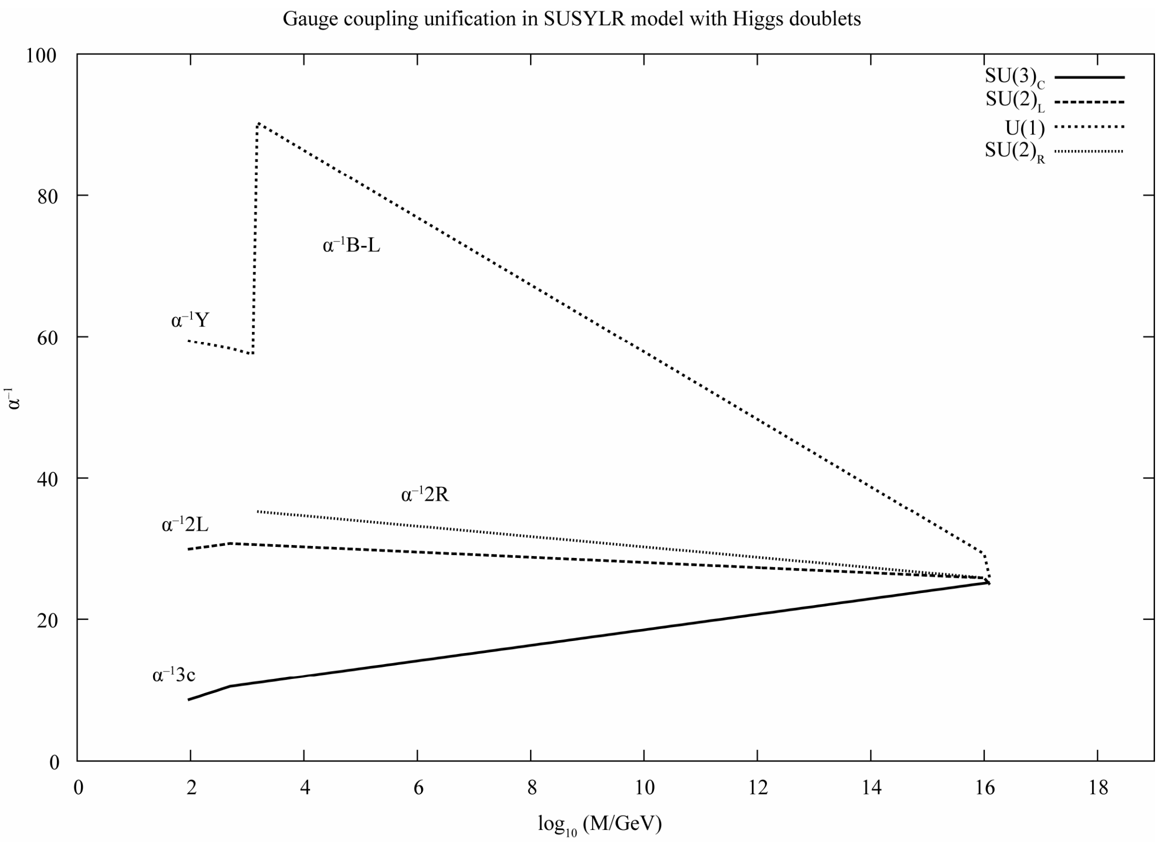


Figure 2. TeV scale M_R is possible with the introduction of parity odd singlets into the MHD model. The figure is redrawn from [16,26] with $M_{susy} = 500\ \text{GeV}$ and $M_{GUT} = 10^{16}\ \text{GeV}$.

GUT scale. We point out that standard neutralino dark matter (decaying through dimension four operators in the superpotential) scenario is disfavored in this model unless one considers unnatural fine-tuning of the dimensionless coefficients in the higher dimensional operators. However, right handed sneutrino dark matter (decaying through dimension five operators in the superpotential) satisfy the astrophysical bounds more naturally and can be a viable dark matter candidate provided it satisfies other relevant constraints like relic density, direct detection etc.

Interestingly, the dimension five operators leading to sneutrino decay involve the left right symmetry breaking scale. The requirement that the strength of such an operator should be small enough to satisfy astrophysical bounds constrains the left right symmetry breaking scale. For generic GUT scale and order one dimensionless couplings, we find this bound to be $M_R < 10^4\ \text{GeV}$. However, as studied in [16,23], successful gauge coupling unification puts a lower bound $M_R \geq 10^{12}\ \text{GeV}$. The mismatch between these two bounds can be fixed by introducing a parity odd singlet [16,26,30] which allow M_R as low as a TeV from the requirement of successful gauge coupling unification. Such TeV scale gauge bosons, apart from satisfying the astrophysical constraints also opens up new dark matter annihilation channels [1] producing the correct relic density in the present Universe.

6. Acknowledgements

I would like to thank Prof Urjit A. Yajnik, IIT Bombay for useful comments and discussions.

REFERENCES

[1] H. An, P. B. Dev, Y. Cai and R. N. Mohapatra, "Sneutrino Dark Matter in Gauged Inverse Seesaw Models for Neutrinos," *Physical Review Letters*, Vol. 108, 2012, Article ID: 081806. doi:10.1103/PhysRevLett.108.081806

[2] J. C. Pati and A. Salam, "Lepton Number as the Fourth Color," *Physical Review*, Vol. D10, 1974, pp. 275-289.

[3] R. N. Mohapatra and J. C. Pati, "A Natural Left-Right Symmetry," *Physical Review*, Vol. D11, 1975, pp. 2558-2569.

[4] G. Senjanovic and R. N. Mohapatra, "Exact Left-Right Symmetry and Spontaneous Violation of Parity," *Physical Review*, Vol. D12, 1975, pp. 1502-1513.

[5] R. N. Mohapatra and R. E. Marshak, "Local B-L Symme-

try of Electroweak Interactions, Majorana Neutrinos and Neutron Oscillations," *Physical Review Letters*, Vol. 44, 1980, pp. 1316-1319. doi:10.1103/PhysRevLett.44.1316

[6] N. G. Deshpande, J. F. Gunion, B. Kayser and F. I. Olness, "Left-Right Symmetric Electroweak Models with Triplet Higgs," *Physical Review*, Vol. D44, 1991, pp. 837-858.

[7] S. Fukuda, *et al*., (Super-Kamiokande Collaboration), "Constraints on Neutrino Oscillations Using 1258 Days of Super-Kamiokande Solar Neutrino Data," *Physical Review Letters*, Vol. 86, 2001, pp. 5656-5660. doi:10.1103/PhysRevLett.86.5656

[8] Q. R. Ahmad, *et al*., (SNO Collaboration), "Direct Evidence for Neutrino Flavor Transformation from Neutral Current Interactions in the Sudbury Neutrino Observatory," *Physical Review Letters*, Vol. 89a, 2002, Article ID: 011301. doi:10.1103/PhysRevLett.89.011301

[9] Q. R. Ahmad, *et al*., (SNO Collaboration), "Measurement of Day and Night Neutrino Energy Spectra at SNO and Constraints on Neutrino Mixing Parameters," *Physical Review Letters*, Vol. 89b, 2002, Article ID: 011302. doi:10.1103/PhysRevLett.89.011302

[10] J. N. Bahcall and C. Pena-Garay, "Solar Models and Solar Neutrino Oscillations," *New Journal of Physics*, Vol. 6, 2005, pp. 63-79. doi:10.1088/1367-2630/6/1/063

[11] C. S. Aulakh, A. Melfo and G. Senjanovic, "Minimal Supersymmetric Left-Right Model," *Physical Review*, Vol. D57, 1998, pp. 4174-4178.

[12] C. S. Aulakh, K. Benakli and G. Senjanovic, "Reconciling Supersymmetry and Left-Right Symmetry," *Physical Review Letters*, Vol. 79, 1997, pp. 2188-2191. doi:10.1103/PhysRevLett.79.2188

[13] K. S. Babu and R. N. Mohapatra, "Minimal Supersymmetric Left-Right Model," *Physics Letters*, Vol. B668, 2008, pp. 404-409.

[14] S. Patra, A. Sarkar, U. Sarkar and U. Yajnik, "Spontaneous Parity Violation in a Supersymmetric Left-Right Symmetric Model," *Physics Letters*, Vol. B679, 2009, pp. 386-389.

[15] R. Kuchimanchi and R. N. Mohapatra, "No Parity Violation without R-Parity Violation," *Physical Review*, Vol. D48, 1993, pp. 4352-4360.

[16] D. Borah, "Supersymmetric Left-Right Models with B-L Odd Higgs Doublets," *International Journal of Modern Physics*, Vol. A26, 2011, pp. 1305-1326.

[17] P. F. Perez and S. Spinner, "Spontaneous R-Parity Breaking and Left-Right Symmetry," *Physics Letters*, Vol. B673, 2009, pp. 251-254.

[18] V. Barger, P. F. Perez and S. Spinner, "Minimal Gauged U(1)(B-L) Model with Spontaneous R-Parity Violation," *Physical Review Letters*, Vol. 102, 2009, Article ID: 181802. doi:10.1103/PhysRevLett.102.181802

[19] P. F. Perez and S. Spinner, "The Minimal Theory for R-Parity Violation at the LHC," *Journal of High Energy Physics*, Vol. 2012, No. 4, 2012, pp. 118-155. doi:10.1007/JHEP04(2012)118

[20] P. S. B. Dev and R. N. Mohapatra, "Electroweak Symmetry Breaking and Proton Decay in SO(10) SUSY-GUT with TeV W(R)," *Physical Review*, Vol. D82, 2010, Article ID: 035014.

[21] L. Dugger, T. E. Jeltema and S. Profumo, "Constraints on Decaying Dark Matter from Fermi Observations of Nearby Galaxies and Clusters," *Journal of Cosmology and Astroparticle Physics*, Vol. 1012, 2010, pp. 15-37.

[22] V. Berezinsky, A. S. Joshipura and J. W. F. Valle, "Gravitational Violation of R-Parity and Its Cosmological Signatures," *Physical Review*, Vol. D57, 1998, pp. 147- 151.

[23] D. Borah and S. Mishra, "Spontaneous R-Parity Breaking, Left-Right Symmetry and Consistent Cosmology with Transitory Domain Walls," *Physical Review*, Vol. D84, 2011, Article ID: 055008.

[24] D. Borah and S. Patra, "Type III Seesaw and Dark Matter in a Supersymmetric Left-Right Model," arXiv:0910.0146.

[25] R. Foot, H. Lew, X. G. He and G. C. Joshi, "Seesaw Neutrino Masses Induced by a Triplet of Leptons," *Zeitschrift fur Physik*, Vol. C44, 1989, p. 441.

[26] D. Borah, S. Patra and U. Sarkar, "TeV scale Left Right Symmetry with Spontaneous D-Parity Breaking," *Physical Review*, Vol. D83, 2011, Article ID: 035007.

[27] A. Arvanitaki, *et al*., "Astrophysical Probes of Unification," *Physical Review*, Vol. D79, 2009, Article ID: 105022.

[28] A. Arvanitaki, *et al*., "Decaying Dark Matter as a Probe of Unification and TeV Spectroscopy," *Physical Review*, Vol. D80, 2009, Article ID: 055011.

[29] A. Maiezza, M. Nemevsek, F. Nesti and G. Senjanovic, "Left-Right Symmetry at LHC," *Physical Review*, Vol. D82, 2010, Article ID: 055022.

[30] P. S. B. Dev and R. N. Mohapatra, "TeV Scale Inverse Seesaw in SO(10) and Leptonic Non-Unitarity Effects," *Physical Review*, Vol. D81, 2010, Article ID: 013001.

Journal of Modern Physics, **2012, 3, 1103-1122**
doi:10.4236/jmp.2012.329145 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

To the Theory of Galaxies Rotation and the Hubble Expansion in the Frame of Non-Local Physics

Boris V. Alexeev
Moscow Lomonosov State University of Fine Chemical Technologies, Prospekt Vernadskogo, Moscow, Russia
Email: Boris.Vlad.Alexeev@gmail.com

Received May 4, 2012; revised June 9, 2012; accepted June 17, 2012

ABSTRACT

The unified generalized non-local theory is applied for mathematical modeling of cosmic objects. For the case of galaxies the theory leads to the flat rotation curves known from observations. The transformation of Kepler's regime into the flat rotation curves for different solitons is shown. The Hubble expansion with acceleration is explained as result of mathematical modeling based on the principles of non-local physics. Peculiar features of the rotational speeds of galaxies and effects of the Hubble expansion need not in the introduction of new essence like dark matter and dark energy. The origin of difficulties consists in the total Oversimplification following from the principles of local physics.

Keywords: Dark Matter; Dark Energy; Galaxy: Halo; Galaxy: Kinematics and Dynamics; Hubble Expansion; Hydrodynamics

1. Introduction

More than ten years ago, the accelerated cosmological expansion was discovered in direct astronomical observations at distances of a few billion light years, almost at the edge of the observable Universe. This acceleration should be explained because mutual attraction of cosmic bodies is only capable of decelerating their scattering. It means that we reach the revolutionary situation not only in physics but also in the natural philosophy on the whole. Practically we are in front of the new challenge since Newton's *Mathematical Principles of Natural Philosophy* was published. As result, new idea was introduced in physics about existing of a force with the opposite sign, which is called universal antigravitation. Its physical source is called as dark energy that manifests itself only because of postulated property of providing antigravitation.

It was postulated that the source of antigravitation is "dark matter" which inferred to exist from gravitational effects on visible matter. However, from the other side dark matter is undetectable by emitted or scattered electromagnetic radiation. It means that new essences—dark matter, dark energy—were introduced in physics only with the aim to account for discrepancies between measurements of the mass of galaxies, clusters of galaxies and the entire universe made through dynamical and general relativistic means, measurements based on the mass of the visible "luminous" matter. It could be reasonable if we are speaking about small corrections to the system of knowledge achieved by mankind to the time we are living. But mentioned above discrepancies lead to affirmation, that dark matter constitutes 80% of the matter in the Universe, while ordinary matter makes up only 20%.

Dark matter was postulated by Swiss astrophysicist Fritz Zwicky of the California Institute of Technology in 1933. He applied the virial theorem to the Coma cluster of galaxies and obtained evidence of unseen mass. Zwicky estimated the cluster's total mass based on the motions of galaxies near its edge and compared that estimate to one based on the number of galaxies and total brightness of the cluster. He found that there was about 400 times more estimated mass than was visually observable. The gravity of the visible galaxies in the cluster would be far too small for such fast orbits, so something extra was required. This is known as the "missing mass problem". Based on these conclusions, Zwicky inferred that there must be some non-visible form of matter, which would provide enough of the mass, and gravity to hold the cluster together.

The work by Vera Rubin (see for example [1,2]) revealed distant galaxies rotating so fast that they should fly apart. Outer stars rotated at essentially the same rate as inner ones (~254 km/s). This is in marked contrast to the solar system where planets orbit the sun with velocities that decrease as their distance from the centre increases. By the early 1970s, flat rotation curves were routinely detected. It was not until the late 1970s, however, that the community was convinced of the need for dark matter halos around spiral galaxies. The mathematical modeling (based on Newtonian mechanics and local physics) of the rotation curves of spiral galaxies was realized for the various visible components of a galaxy (the bulge, thin disk, and thick disk). These models were unable to predict the flatness of the observed rotation curve beyond the stellar disk. The inescapable conclusion, assuming that Newton's law of gravity (and the local physics description) holds on cosmological scales, that the visible galaxy was embedded in a much larger dark matter (DM) halo, which contributes roughly 50% - 90% of the total mass of a galaxy. As result another models of gravitation were involved in consideration—from "improved" Newtonian laws (such as modified Newtonian dynamics and tensor-vector-scalar gravity [3]) to the Einstein's theory based on the cosmological constant [4]. Einstein introduced this term as a mechanism to obtain a stable solution of the gravitational field equation that would lead to a static Universe, effectively using dark energy to balance gravity.

Computer simulations with taking into account the hypothetical DM in the local hydrodynamic description include usual moment equations plus Poisson equation with different approximations for the density of DM (ρ_{DM}) containing several free parameters. Computer simulations of cold dark matter (CDM) predict that CDM particles ought to coalesce to peak densities in galactic cores. However, the observational evidence of star dynamics at inner galactic radii of many galaxies, including our own Milky Way, indicates that these galactic cores are entirely devoid of CDM. No valid mechanism has been demonstrated to account for how galactic cores are swept clean of CDM. This is known as the "cuspy halo problem". As result, the restricted area of CDM influence introduced in the theory. As we see the concept of DM leads to many additional problems.

I do not intend to review the different speculations based on the principles of local physics. I see another problem. It is the problem of Oversimplification—but not "trivial" simplification of the important problem. The situation is more serious—total Oversimplification based on principles of local physics, and obvious crisis, we see in astrophysics, simply reflects the general shortcomings of the local kinetic transport theory. It is important to underline that we should have expected this crisis of local statistical physics after the discovery of Bell's fundamental inequalities [5]. The antigravitation problem in application to the theory of galaxies rotation and the Hubble expansion is solved further in the frame of non-local statistical physics and the Newtonian law of gravitation.

I deliver here some main ideas and deductions of the generalized Boltzmann physical kinetics and non-local physics. For simplicity, the fundamental methodic aspects are considered from the qualitative standpoint of view avoiding excessively cumbersome formulas. A rigorous description can be found, for example, in the monograph [6].

In 1872 L. Boltzmann [7,8] published his kinetic equation for the one-particle distribution function (DF) $f(\mathbf{r},\mathbf{v},t)$. He expressed the equation in the form

$$Df/Dt = J^B(f), \qquad (1)$$

where J^B is the local collision integral, and $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + \mathbf{F}\cdot\frac{\partial}{\partial \mathbf{v}}$ is the substantial (particle) derivative, $\mathbf{v}$ and $\mathbf{r}$ being the velocity and radius vector of the particle, respectively. Boltzmann Equation (1) governs the transport processes in a one-component gas, which is sufficiently rarefied that only binary collisions between particles are of importance and valid only for two character scales, connected with the hydrodynamic time-scale and the time-scale between particle collisions. While we are not concerned here with the explicit form of the collision integral, note that it should satisfy conservation laws of point-like particles in binary collisions. Integrals of the distribution function (*i.e.* its moments) determine the macroscopic hydrodynamic characteristics of the system, in particular the number density of particles n and the temperature T. The Boltzmann equation (BE) is not of course as simple as its symbolic form above might suggest, and it is in only a few special cases that it is amenable to a solution. One example is that of a maxwellian distribution in a locally, thermodynamically equilibrium gas in the event when no external forces are present. In this case the equality $J^B = 0$ and $f = f_0$ is met, giving the maxwellian distribution function f_0. A weak point of the classical Boltzmann kinetic theory is the way it treats the dynamic properties of interacting particles. On the one hand, as the so-called "physical" derivation of the BE suggests, Boltzmann particles are treated as material points; on the other hand, the collision integral in the BE brings into existence the cross sections for collisions between particles. A rigorous approach to the derivation of the kinetic equation for f (noted in following as KE_f) is based on the hierarchy of the Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) [6,9-13] equations.

A KE_f obtained by the multi-scale method turns into the BE if one ignores the change of the distribution function (DF) over a time of the order of the collision time (or, equivalently, over a length of the order of the particle interaction radius). It is important to note [6,14] that accounting for the third of the scales mentioned above leads (*prior* to introducing any approximation destined to break the Bogolyubov chain) to additional terms, generally of the same order of magnitude, appear in the BE. If the correlation functions is used to derive KE_f from the BBGKY equations, then the passage to the BE means the neglect of non-local effects.

Given the above difficulties of the Boltzmann kinetic theory, the following clearly inter related questions arise. First, what is a physically infinitesimal volume and how does its introduction (and, as the consequence, the unavoidable smoothing out of the DF) affect the kinetic equation? This question can be formulated in (from the first glance) the paradox form—what is the size of the point in the physical system? Second, how does a systematic account for the proper diameter of the particle in the derivation of the KE_f affect the Boltzmann equation? In the theory developed here, I shall refer to the corresponding KE_f as Generalized Boltzmann Equation (GBE). The derivation of the GBE and the applications of GBE are presented, in particular, in [6]. Accordingly, our purpose is first to explain the essence of the physical generalization of the BE.

Let a particle of finite radius be characterized, as before, by the position vector $\mathbf{r}$ and velocity $\mathbf{v}$ of its center of mass at some instant of time t. Let us introduce physically small volume (**PhSV**) as element of measurement of macroscopic characteristics of physical system for a point containing in this **PhSV**. We should hope that **PhSV** contains sufficient particles N_{ph} for statistical description of the system. In other words, a net of physically small volumes covers the whole investigated physical system.

Every **PhSV** contains entire quantity of point-like Boltzmann particles, and *the same* DF f *is prescribed for whole* **PhSV** *in Boltzmann physical kinetics.* Therefore, Boltzmann particles are fully "packed" in the reference volume. Let us consider two adjoining physically small volumes $\mathbf{PhSV_1}$ and $\mathbf{PhSV_2}$. We have in principle another situation for the particles of finite size moving in physical small volumes, which are *open* thermodynamic systems.

Then, the situation is possible where, at some instant of time *t*, the particle is located on the interface between two volumes. In so doing, the lead effect is possible (say, for $\mathbf{PhSV_2}$), when the center of mass of particle moving to the neighboring volume $\mathbf{PhSV_2}$ is still in $\mathbf{PhSV_1}$. However, the delay effect takes place as well, when the center of mass of particle moving to the neighboring volume (say, $\mathbf{PhSV_2}$) is already located in $\mathbf{PhSV_2}$ but a part of the particle still belongs to $\mathbf{PhSV_1}$.

Moreover, *even the point-like particles* (*starting after the last collision near the boundary between two mentioned volumes*) *can change the distribution functions in the neighboring volume. The adjusting of the particles dynamic characteristics for translational degrees of freedom takes several collisions. As result*, *we have in the definite sense* "*the Knudsen layer*" *between these volumes. This fact unavoidably leads to fluctuations in mass and hence in other hydrodynamic quantities. Existence of such* "*Knudsen layers*" *is not connected with the choice of space nets and fully defined by the reduced description for ensemble of particles of finite diameters in the conceptual frame of open physically small volumes*, *therefore—with the chosen method of measurement.*

This entire complex of effects defines non-local effects in space and time. The corresponding situation is typical for the theoretical physics—we could remind about the role of probe charge in electrostatics or probe circuit in the physics of magnetic effects.

Suppose that DF f corresponds to $\mathbf{PhSV_1}$ and DF $f-\Delta f$ is connected with $\mathbf{PhSV_2}$ for Boltzmann particles. In the boundary area in the first approximation, fluctuations will be proportional to the mean free path (or, equivalently, to the mean time between the collisions). Then for **PhSV** the correction for DF should be introduced as

$$f^a = f - \tau\, Df/Dt \tag{2}$$

in the left hand side of classical BE describing the translation of DF in phase space. As the result

$$Df^a/Dt = J^B, \tag{3}$$

where J^B is the Boltzmann local collision integral.

Important to notice that it is only qualitative explanation of GBE derivation obtained earlier (*see for example* [6]) *by different strict methods from the BBGKY—chain of kinetic equations.* The structure of the KE_f is generally as follows

$$\frac{Df}{Dt} = J^B + J^{nonlocal}, \tag{4}$$

where $J^{nonlocal}$ is the non-local integral term incorporating the non-local time and space effects. The generalized Boltzmann physical kinetics, in essence, involves a local approximation

$$J^{nonlocal} = \frac{D}{Dt}\left(\tau \frac{Df}{Dt}\right) \tag{5}$$

for the second collision integral, here in the simplest case τ being the mean time *between* the particle collisions. We can draw here an analogy with the Bhatnagar-Gross-Krook (BGK) approximation for J^B,

$$J^B = \frac{f_0 - f}{\tau}, \tag{6}$$

which popularity as a means to represent the Boltzmann collision integral is due to the huge simplifications it offers. In other words—the local Boltzmann collision integral admits approximation via the BGK algebraic expression, but more complicated non-local integral can be expressed as differential form (5). The ratio of the second to the first term on the right-hand side of Equation (4) is given to an order of magnitude as $J^{nonlocal}/J^B \approx O\left(\mathrm{Kn}^2\right)$ and at large Knudsen numbers (Kn defining as ratio of mean free path of particles to the character hydrodynamic length) these terms become of the same order of magnitude. It would seem that at small Knudsen numbers answering to hydrodynamic description the contribution from the second term on the right-hand side of Equation (4) is negligible.

This is not the case, *however.* When one goes over to the hydrodynamic approximation (by multiplying the kinetic equation by collision invariants and then integrating over velocities), the Boltzmann integral part vanishes, and the second term on the right-hand side of Equation (4) gives a single-order contribution in the generalized Navier-Stokes description. Mathematically, we cannot neglect a term with a small parameter in front of the higher derivative. Physically, the appearing additional terms are due to viscosity and they correspond to the small-scale Kolmogorov turbulence [6,15]. The integral term $J^{nonlocal}$ turns out to be important both at small and large Knudsen numbers in the theory of transport processes. Thus, $\tau Df/Dt$ is the distribution function fluctuation, and writing Equation (3) without taking into account Equation (2) makes the BE non-closed. From viewpoint of the fluctuation theory, Boltzmann employed the simplest possible closure procedure $f^a = f$.

Then, the additional GBE terms (as compared to the BE) are significant for any Kn, and the order of magnitude of the difference between the BE and GBE solutions is impossible to tell beforehand. For GBE the generalized H-theorem is proven [6,16].

It means that the local Boltzmann equation does not belong even to the class of minimal physical models and corresponds so to speak to "the likelihood models". This remark refers also to all consequences of the Boltzmann kinetic theory including "classical" hydrodynamics.

Obviously the generalized hydrodynamic equations (GHE) will explicitly involve fluctuations proportional to τ. In the hydrodynamic approximation, the mean time τ between the collisions is related to the dynamic viscosity μ by

$$\tau p = \Pi \mu, \tag{7}$$

[17,18]. For example, the continuity equation changes its form and will contain terms proportional to viscosity. On the other hand, if the reference volume extends over the whole cavity with the hard walls, then the classical conservation laws should be obeyed, and this is exactly what the monograph [6] proves. Now several remarks of principal significance:

1) All fluctuations are found from the strict kinetic considerations and tabulated [6]. The appearing additional terms in GHE are due to viscosity and they correspond to the small-scale Kolmogorov turbulence. The neglect of formally small terms is equivalent, in particular, to dropping the (small-scale) Kolmogorov turbulence from consideration and is the origin of all principal difficulties in usual turbulent theory. Fluctuations on the wall are equal to zero, from the physical point of view this fact corresponds to the laminar sub-layer. Mathematically it leads to additional boundary conditions for GHE. Major difficulties arose when the question of existence and uniqueness of solutions of the Navier-Stokes equations was addressed.

O. A. Ladyzhenskaya has shown for three-dimensional flows that under smooth initial conditions a unique solution is only possible over a finite time interval. Ladyzhenskaya even introduced a "correction" into the Navier-Stokes equations in order that its unique solvability could be proved (see discussion in [19]). GHE do not lead to these difficulties.

2) It would appear that in continuum mechanics the idea of discreteness can be abandoned altogether and the medium under study be considered as a continuum in the literal sense of the word. Such an approach is of course possible and indeed leads to the Euler equations in hydrodynamics. However, when the viscosity and thermal conductivity effects are to be included, a totally different situation arises. As is well known, the dynamical viscosity is proportional to the mean time τ between the particle collisions, and a continuum medium in the Euler model with $\tau = 0$ implies that neither viscosity nor thermal conductivity is possible.

3) The non-local kinetic effects listed above will always be relevant to a kinetic theory using one particle description—including, in particular, applications to liquids or plasmas, where self-consistent forces with appropriately cut-off radius of their action are introduced to expand the capability of GBE [20-25]. Fluctuation effects occur in any open thermodynamic system bounded by a control surface transparent to particles. GBE (3) leads to generalized hydrodynamic equations [6] as the local approximation of non local effects, for example, to the continuity equation

$$\frac{\partial \rho^a}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot \left(\rho \mathbf{v}_0\right)^a = 0, \tag{8}$$

where ρ^a, $\mathbf{v}_0^a$, $\left(\rho \mathbf{v}_0\right)^a$ are calculated in view of non-locality effect in terms of gas density ρ, hydrodynamic velocity of flow $\mathbf{v}_0$, and density of momentum flux $\rho \mathbf{v}_0$; for locally Maxwellian distribution, ρ^a, $\left(\rho \mathbf{v}_0\right)^a$ are defined by the relations

$$\begin{aligned} &\left(\rho - \rho^a\right)\big/\tau = \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot \left(\rho \mathbf{v}_0\right), \\ &\left(\rho \mathbf{v}_0 - \left(\rho \mathbf{v}_0\right)^a\right)\big/\tau = \frac{\partial}{\partial t}\left(\rho \mathbf{v}_0\right) \\ &+ \frac{\partial}{\partial \mathbf{r}} \cdot \rho \mathbf{v}_0 \mathbf{v}_0 + \overleftrightarrow{\mathrm{I}} \cdot \frac{\partial p}{\partial \mathbf{r}} - \rho \mathbf{a} \end{aligned} \tag{9}$$

where $\vec{I}$ is a unit tensor, and $\mathbf{a}$ is the acceleration due to the effect of mass forces.

In the general case, the parameter τ is the non-locality parameter; in quantum hydrodynamics, the "time-energy" uncertainty relation defines its magnitude. The violation of Bell's inequalities [5] is found for local statistical theories, and the transition to non-local description is inevitable. The following conclusion of principal significance can be done from the generalized quantum consideration [22,23]:

1) Madelung's quantum hydrodynamics is equivalent to the Schrödinger equation (SE) and leads to description of the quantum particle evolution in the form of Euler equation and continuity equation.

2) SE is consequence of the Liouville equation as result of the local approximation of non-local equations.

3) Generalized Boltzmann physical kinetics defines the strict approximation of non-local effects in space and time and after transmission to the local approximation leads to parameter τ, which on the quantum level corresponds to the uncertainty principle "time-energy".

4) GHE lead to SE as a deep particular case of the generalized Boltzmann physical kinetics and therefore of non-local hydrodynamics.

In principal GHE needn't in using of the "time-energy" uncertainty relation for estimation of the value of the non-locality parameter τ. Moreover, the "time-energy" uncertainty relation does not lead to the exact relations and from position of non-local physics is only the simplest estimation of the non-local effects.

Really, let us consider two neighboring physically infinitely small volumes $\mathbf{PhSV_1}$ and $\mathbf{PhSV_2}$ in a non-equilibrium system. Obviously the time τ should tend to diminish with increasing of the velocities u of particles invading in the nearest neighboring physically infinitely small volume ($\mathbf{PhSV_1}$ or $\mathbf{PhSV_2}$):

$$\tau = H/u^n . \tag{10}$$

However, the value τ cannot depend on the velocity direction and naturally to tie τ with the particle kinetic energy, then

$$\tau = H/mu^2 , \tag{11}$$

where H is a coefficient of proportionality, which reflects the state of physical system. In the simplest case H is equal to Plank constant $\hbar$ and relation (11) becomes compatible with the Heisenberg relation.

Finally, we can state that introduction of open control volume by the reduced description for ensemble of particles of finite diameters leads to fluctuations (proportional to Knudsen number) of velocity moments in the volume. This fact defines the significant reconstruction of the theory of transport processes. Obviously the mentioned non-local effects can be discussed from viewpoint of breaking of the Bell's inequalities [5] because in the non-local theory the measurement (realized in $\mathbf{PhSV_1}$) has influence on the measurement realized in the adjoining space-time point in $\mathbf{PhSV_2}$ and verse versa.

In the following sections I intend to apply the unified generalized non-local theory for mathematical modeling of cosmic objects. For the case of galaxies the theory leads to the flat rotation curves known from observations. The transformation of Kepler's regime into the flat rotation curves for different solitons is shown. The Hubble expansion with acceleration is explained as result of mathematical modeling based on the principals of non-local physics. Therefore the answers for the following questions are formulated:

1) Why the concept of the dark matter is not significant in the Solar system?

2) Why the galaxy rotation curves have the character flat form?

3) Is it possible to obtain the continuous transition from the Kepler regime to the flat halo curves?

4) Why after Big Bang explosion (or after the explosion in the Hubble boxes) the Hubble expansion exists with acceleration? ([26-28], Nobel Prize for the observers S. Perlmutter, A. G. Riess, B. Schmidt of the year 2011).

In other words—is it possible using only Newtonian gravitation law and non-local statistical description to forecast the flat gravitational curve of a typical spiral galaxy (Section 2) and the Hubble expansion (including the Hubble expansion with acceleration, PRS-regime), (Section 3)? The last question has the positive answer.

2. Disk Galaxy Rotation and the Problem of Dark Matter

About forty years after Zwicky's initial observations Vera Rubin, astronomer at the Department of Terrestrial Magnetism at the Carnegie Institution of Washington presented findings based on a new sensitive spectrograph that could measure the velocity curve of edge-on spiral galaxies to a greater degree of accuracy than had ever before been achieved. Together with Kent Ford, Rubin announced at a 1975 meeting of the American Astronomical Society the astonishing discovery that most stars in spiral galaxies orbit at roughly the same speed reflected schematically on **Figure 1**.

For example, the rotation curve of the type **B** corresponds to the galaxy NGC3198. The following extensive radio observations determined the detailed rotation curve of spiral disk galaxies to be flat (as the curve **B**), much beyond as seen in the optical band. Obviously the trivial balance between the gravitational and centrifugal forces leads to relation between orbital speed V and galactocentric distance r as $V^2 = \gamma_N M/r$ beyond the physical extent of the galaxy of mass M (the curve **A**). The

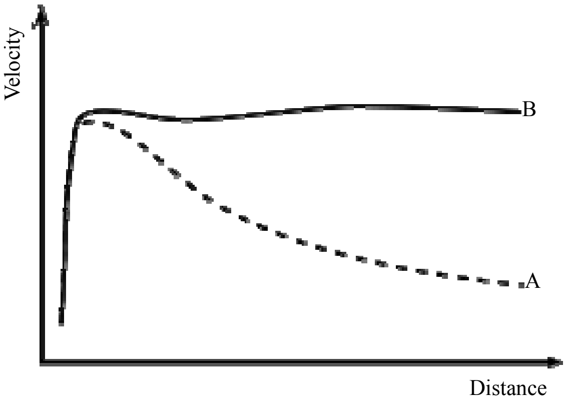


Figure 1. Rotation curve of a typical spiral galaxy: predicted (A) and observed (B).

obvious contradiction with the velocity curve **B** having a "flat" appearance out to a large radius, was explained by introduction of a new physical essence—dark matter because for spherically symmetric case the hypothetical density distribution $\rho(r)\sim 1/r^2$ leads to $V = const$. The result of this activity is known—undetectable dark matter which does not emit radiation, inferred solely from its gravitational effects. But it means that upwards of 50% of the mass of galaxies was contained in the dark galactic halo.

Strict consideration leads to the following system of the generalized hydrodynamic equations (GHE) [6,22-25, 29-31] written in the generalized Euler form:

(Continuity equation for species α)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho_\alpha-\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\right)\right]\right\}\\ &+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right)+\vec{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}=R_\alpha.\end{aligned}\tag{12}$$

(Continuity equation for mixture)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho-\sum_\alpha\tau_\alpha\left[\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\right)\right]\right\}\\ &+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho\mathbf{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0\right)+\vec{\mathrm{I}}\cdot\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}=0.\end{aligned}\tag{13}$$

(Momentum equation for species α)

$$\begin{aligned}&\frac{\partial}{\partial t}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}\\ &-\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}\left(\rho_\alpha\mathbf{v}_0\right)\right)\right]\\ &-\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}\times\mathbf{B}\\ &+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho_\alpha\mathbf{v}_0\mathbf{v}_0+p_\alpha\vec{\mathrm{I}}-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0+p_\alpha\vec{\mathrm{I}}\right)\right.\right.\\ &+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\left(\mathbf{v}_0\mathbf{v}_0\right)\mathbf{v}_0+2\vec{\mathrm{I}}\left(\frac{\partial}{\partial\mathbf{r}}\cdot\left(p_\alpha\mathbf{v}_0\right)\right)\\ &\left.\left.+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\vec{\mathrm{I}}p_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\rho_\alpha\mathbf{v}_0-\rho_\alpha\mathbf{v}_0\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\left[\mathbf{v}_0\times\mathbf{B}\right]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\left[\mathbf{v}_0\times\mathbf{B}\right]\right]\right\}\\ &=\int m_\alpha\mathbf{v}_\alpha J_\alpha^{st,el}\mathrm{d}\mathbf{v}_\alpha+\int m_\alpha\mathbf{v}_\alpha J_\alpha^{st,inel}\mathrm{d}\mathbf{v}_\alpha.\end{aligned}\tag{14}$$

(Momentum equation for mixture)

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\rho\mathbf{v}_0-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}-\sum_\alpha\mathbf{F}_\alpha^{(1)}\left[\rho_\alpha-\tau_\alpha\left(\frac{\partial\rho_\alpha}{\partial t}+\frac{\partial}{\partial\mathbf{r}}(\rho_\alpha\mathbf{v}_0)\right)\right]\\
&-\sum_\alpha\frac{q_\alpha}{m_\alpha}\left\{\rho_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial p_\alpha}{\partial\mathbf{r}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0\times\mathbf{B}\right]\right\}\times\mathbf{B}\\
&+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\rho\mathbf{v}_0\mathbf{v}_0+p\overleftrightarrow{\mathrm{I}}-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\rho_\alpha\mathbf{v}_0\mathbf{v}_0+p_\alpha\overleftrightarrow{\mathrm{I}}\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha(\mathbf{v}_0\mathbf{v}_0)\mathbf{v}_0+2\overleftrightarrow{\mathrm{I}}\left(\frac{\partial}{\partial\mathbf{r}}\cdot(p_\alpha\mathbf{v}_0)\right)\right.\right.\\
&\left.\left.+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\overleftrightarrow{\mathrm{I}}p_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\rho_\alpha\mathbf{v}_0-\rho_\alpha\mathbf{v}_0\mathbf{F}_\alpha^{(1)}-\frac{q_\alpha}{m_\alpha}\rho_\alpha[\mathbf{v}_0\times\mathbf{B}]\mathbf{v}_0-\frac{q_\alpha}{m_\alpha}\rho_\alpha\mathbf{v}_0[\mathbf{v}_0\times\mathbf{B}]\right]\right\}=0
\end{aligned}\tag{15}$$

(Energy equation for α species)

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\right]\right\}\\
&+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0-\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)\right.\right.\\
&+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}p_\alpha v_0^2\overleftrightarrow{\mathrm{I}}+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\overleftrightarrow{\mathrm{I}}+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\overleftrightarrow{\mathrm{I}}\right)\\
&-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\overleftrightarrow{\mathrm{I}}-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}-\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]\\
&\left.\left.-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\right]\right\}\\
&-\left\{\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0-\tau_\alpha\left[\mathbf{F}_\alpha^{(1)}\cdot\left(\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial\mathbf{r}}\cdot p_\alpha\overleftrightarrow{\mathrm{I}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right)\right]\right\}\\
&=\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,el}\mathrm{d}\mathbf{v}_\alpha+\int\left(\frac{m_\alpha v_\alpha^2}{2}+\varepsilon_\alpha\right)J_\alpha^{st,inel}\mathrm{d}\mathbf{v}_\alpha.
\end{aligned}\tag{16}$$

(Energy equation for mixture)

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\frac{\rho v_0^2}{2}+\frac{3}{2}p+\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{\rho_\alpha v_0^2}{2}+\frac{3}{2}p_\alpha+\varepsilon_\alpha n_\alpha\right)\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)-\mathbf{F}_\alpha^{(1)}\cdot\rho_\alpha\mathbf{v}_0\right]\right\}\\
&\qquad +\\
&+\frac{\partial}{\partial\mathbf{r}}\cdot\left\{\frac{1}{2}\rho v_0^2\mathbf{v}_0+\frac{5}{2}p\mathbf{v}_0+\mathbf{v}_0\sum_\alpha\varepsilon_\alpha n_\alpha-\sum_\alpha\tau_\alpha\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0+\frac{5}{2}p_\alpha\mathbf{v}_0+\varepsilon_\alpha n_\alpha\mathbf{v}_0\right)\right.\right.\\
&+\frac{\partial}{\partial\mathbf{r}}\cdot\left(\frac{1}{2}\rho_\alpha v_0^2\mathbf{v}_0\mathbf{v}_0+\frac{7}{2}p_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{1}{2}p_\alpha v_0^2\overleftrightarrow{\mathrm{I}}+\frac{5}{2}\frac{p_\alpha^2}{\rho_\alpha}\overleftrightarrow{\mathrm{I}}+\varepsilon_\alpha n_\alpha\mathbf{v}_0\mathbf{v}_0+\varepsilon_\alpha\frac{p_\alpha}{m_\alpha}\overleftrightarrow{\mathrm{I}}\right)\\
&-\rho_\alpha\mathbf{F}_\alpha^{(1)}\cdot\mathbf{v}_0\mathbf{v}_0-p_\alpha\mathbf{F}_\alpha^{(1)}\cdot\overleftrightarrow{\mathrm{I}}-\frac{1}{2}\rho_\alpha v_0^2\mathbf{F}_\alpha^{(1)}-\frac{3}{2}\mathbf{F}_\alpha^{(1)}p_\alpha-\frac{\rho_\alpha v_0^2}{2}\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\frac{5}{2}p_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]\\
&\left.\left.-\varepsilon_\alpha n_\alpha\frac{q_\alpha}{m_\alpha}[\mathbf{v}_0\times\mathbf{B}]-\varepsilon_\alpha n_\alpha\mathbf{F}_\alpha^{(1)}\right]\right\}\\
&-\left\{\mathbf{v}_0\cdot\sum_\alpha\rho_\alpha\mathbf{F}_\alpha^{(1)}-\sum_\alpha\tau_\alpha\left[\mathbf{F}_\alpha^{(1)}\cdot\left(\frac{\partial}{\partial t}(\rho_\alpha\mathbf{v}_0)+\frac{\partial}{\partial\mathbf{r}}\cdot\rho_\alpha\mathbf{v}_0\mathbf{v}_0+\frac{\partial}{\partial\mathbf{r}}\cdot p_\alpha\overleftrightarrow{\mathrm{I}}-\rho_\alpha\mathbf{F}_\alpha^{(1)}-q_\alpha n_\alpha[\mathbf{v}_0\times\mathbf{B}]\right)\right]\right\}=0.
\end{aligned}\tag{17}$$

Here $\mathbf{F}_{\alpha}^{(1)}$ are the forces of the non-magnetic origin, $\mathbf{B}$ —magnetic induction, $\overset{\leftrightarrow}{I}$ —unit tensor, q_{α} —charge of the α-component particle, p_{α} —static pressure for α-component, ε_{α} —internal energy for the particles of α-component, $\mathbf{v}_0$ — hydrodynamic velocity for mixture, τ_{α} —non-local parameter.

GHE can be applied to the physical systems from the Universe to atomic scales. All additional explanations will be done by delivering the results of modeling of corresponding physical systems with the special consideration of non-local parameters τ_{α}. Generally speaking to GHE should be added the system of generalized Maxwell equations (for example in the form of the generalized Poisson equation for electric potential) and gravitational equations (for example in the form of the generalized Poisson equation for gravitational potential).

In the following I intend to show that the character features reflected on **Figure 1** can be explained in the frame of Newtonian gravitation law and the non-local kinetic description created by me. With this aim let us consider the formation of the soliton's type of solution of the generalized hydrodynamic equations for gravitational media like galaxy in the self consistent gravitational field. Our aim consists in calculation of the self-consistent hydrodynamic moments of possible formation like gravitational soliton.

Let us investigate of the gravitational soliton formation in the frame of the non-stationary 1D Cartesian formulation. Then the system of GHE consist from the generalized Poisson equation reflecting the effects of the density and the density flux perturbations, continuity equation, motion and energy equations. The GHE derivation can be found in [6,15,29]. This system of four equations for non-stationary 1D case is written as the deep particular case of Equations (12)-(17) in the form:

(Poisson equation)

$$\frac{\partial^2 \Psi}{\partial x^2} = 4\pi\gamma_N \left[\rho - \tau \left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) \right) \right], \tag{18}$$

(Continuity equation)

$$\frac{\partial}{\partial t}\left\{ \rho - \tau \left[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) \right] \right\} + \frac{\partial}{\partial x}\left\{ \rho u - \tau \left[\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial p}{\partial x} + \rho \frac{\partial \Psi}{\partial x} \right] \right\} = 0, \tag{19}$$

(Motion equation)

$$\begin{aligned} &\frac{\partial}{\partial t}\left\{ \rho u - \tau \left[\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial p}{\partial x} + \rho \frac{\partial \Psi}{\partial x} \right] \right\} + \frac{\partial \Psi}{\partial x}\left[\rho - \tau \left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) \right) \right] \\ &+ \frac{\partial}{\partial x}\left\{ \rho u^2 + p - \tau \left[\frac{\partial}{\partial t}(\rho u^2 + p) + \frac{\partial}{\partial x}(\rho u^3 + 3pu) + 2\rho u \frac{\partial \Psi}{\partial x} \right] \right\} = 0, \end{aligned} \tag{20}$$

(Energy equation)

$$\begin{aligned} &\frac{\partial}{\partial t}\left\{ \rho u^2 + 3p - \tau \left[\frac{\partial}{\partial t}(\rho u^2 + 3p) + \frac{\partial}{\partial x}(\rho u^3 + 5pu) + 2\rho u \frac{\partial \Psi}{\partial x} \right] \right\} \\ &+ \frac{\partial}{\partial x}\left\{ \rho u^3 + 5pu - \tau \left[\frac{\partial}{\partial t}(\rho u^3 + 5pu) + \frac{\partial}{\partial x}\left(\rho u^4 + 8pu^2 + 5\frac{p^2}{\rho} \right) \right.\right. \\ &\left.\left. + \frac{\partial \Psi}{\partial x}(3\rho u^2 + 5p) \right] \right\} + 2\frac{\partial \Psi}{\partial x}\left\{ \rho u - \tau \left[\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + p) + \rho \frac{\partial \Psi}{\partial x} \right] \right\} = 0, \end{aligned} \tag{21}$$

where u is translational velocity of the one species object, Ψ —self consistent gravitational potential ($\mathbf{g} = -\partial\Psi/\partial\mathbf{r}$ is acceleration in gravitational field), ρ is density and p is pressure, τ is non-locality parameter, γ_N is Newtonian gravitation constant.

Let us introduce the coordinate system moving along the positive direction of x-axis in ID space with velocity $C = u_0$ equal to phase velocity of considering object

$$\xi = x - Ct\,. \tag{22}$$

Taking into account the De Broglie relation we should wait that the group velocity u_g is equal $2u_0$. In moving coordinate system all dependent hydrodynamic values are function of (ξ, t). We investigate the possibility of the object formation of the soliton type. For this solution there is no explicit dependence on time for coordinate system moving with the phase velocity u_0. Write down the system of Equations (18)-(21) in the dimensionless form, where dimensionless symbols are marked by tildes.

For the scales $\rho_0, u_0, x_0 = u_0 t_0, \Psi_0 = u_0^2, \gamma_{N0} = u_0^2 / (\rho_0 x_0^2)$, $p_0 = \rho_0 u_0^2$ and conditions $\tilde{C} = C/u_0 = 1$, the equations take the form:

(Generalized Poisson equation)

$$\frac{\partial^2 \tilde{\Psi}}{\partial \tilde{\xi}^2} = 4\pi \tilde{\gamma}_N \left[\tilde{\rho} - \tilde{\tau}\left(-\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}\tilde{u}) \right) \right], \quad (23)$$

(Continuity equation)

$$\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} - \frac{\partial \tilde{\rho}\tilde{u}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}}\left\{ \tilde{\tau}\left[\frac{\partial}{\partial \tilde{\xi}}\left[\tilde{p} + \tilde{\rho}\tilde{u}^2 + \tilde{\rho} - 2\tilde{\rho}\tilde{u} \right] + \tilde{\rho}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} = 0, \quad (24)$$

(Motion equation)

$$\begin{aligned} &\frac{\partial}{\partial \tilde{\xi}}\left(\tilde{\rho}\tilde{u}^2 + \tilde{p} - \tilde{\rho}\tilde{u} \right) + \frac{\partial}{\partial \tilde{\xi}}\left\{ \tilde{\tau}\left[\frac{\partial}{\partial \tilde{\xi}}\left(2\tilde{\rho}\tilde{u}^2 - \tilde{\rho}\tilde{u} + 2\tilde{p} - \tilde{\rho}\tilde{u}^3 - 3\tilde{p}\tilde{u} \right) + \tilde{\rho}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] \right\} \\ &+ \frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}}\left\{ \tilde{\rho} - \tilde{\tau}\left[-\frac{\partial \tilde{\rho}}{\partial \tilde{\xi}} + \frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}\tilde{u}) \right] \right\} - 2\frac{\partial}{\partial \tilde{\xi}}\left\{ \tilde{\tau}\tilde{\rho}\tilde{u}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right\} = 0, \end{aligned} \quad (25)$$

(Energy equation)

$$\begin{aligned} &\frac{\partial}{\partial \tilde{\xi}}\left(\tilde{\rho}\tilde{u}^2 + 3\tilde{p} - \tilde{\rho}\tilde{u}^3 - 5\tilde{p}\tilde{u} \right) - \frac{\partial}{\partial \tilde{\xi}}\left\{ \tilde{\tau}\frac{\partial}{\partial \tilde{\xi}}\left(2\tilde{\rho}\tilde{u}^3 + 10\tilde{p}\tilde{u} - \tilde{\rho}\tilde{u}^2 - 3\tilde{p} - \tilde{\rho}\tilde{u}^4 - 8\tilde{p}\tilde{u}^2 - 5\frac{\tilde{p}^2}{\tilde{\rho}} \right) \right\} \\ &+ \frac{\partial}{\partial \tilde{\xi}}\left\{ \tilde{\tau}\left(3\tilde{\rho}\tilde{u}^2 + 5\tilde{p} \right)\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right\} - 2\tilde{\rho}\tilde{u}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} - 2\frac{\partial}{\partial \tilde{\xi}}\left\{ \tilde{\tau}\tilde{\rho}\tilde{u}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right\} \\ &+ 2\tilde{\tau}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}}\left[-\frac{\partial}{\partial \tilde{\xi}}(\tilde{\rho}\tilde{u}) + \frac{\partial}{\partial \tilde{\xi}}\left(\tilde{\rho}\tilde{u}^2 + \tilde{p} \right) + \tilde{\rho}\frac{\partial \tilde{\Psi}}{\partial \tilde{\xi}} \right] = 0, \end{aligned} \quad (26)$$

Some comments to the system of four ordinary non-linear Equations (23)-(26):

1) Every equation from the system is of the second order and needs two conditions. The problem belongs to the class of Cauchy problems.

2) In comparison for example, with the Schrödinger theory connected with behavior of the wave function, no special conditions are applied for dependent variables including the domain of the solution existing. This domain is defined automatically in the process of the numerical solution of the concrete variant of calculations.

3) From the introduced scales $\rho_0, u_0, x_0 = u_0 t_0, \Psi_0 = u_0^2$, $\gamma_{N0} = u_0^2 / (\rho_0 x_0^2)$, $p_0 = \rho_0 u_0^2$, only three parameters are independent, namely, ρ_0, u_0, x_0.

4) Approximation for the dimensionless non-local parameter $\tilde{\tau}$ should be introduced (see (11)). In the definite sense it is not the problem of the hydrodynamic level of the physical system description (like the calculation of the kinetic coefficients in the classical hydrodynamics). Interesting to notice that quantum GHE were applied with success for calculation of atom structure [22-25], which is considered as two species charged e, i mixture. The corresponding approximations for non-local parameters τ_i, τ_e and τ_{ei} are proposed in [22,23]. In the theory of the atom structure [23] after taking into account the Balmer's relation, (11) transforms into

$$\tau_e = n\hbar / (m_e u^2), \quad (27)$$

where $n = 1, 2, \cdots$ is principal quantum number. As result the length scale relation was written as $x_0 = H/(m_e u_0) = n\hbar/(m_e u_0)$. But the value $\nu^{qu} = \hbar/m_e$ has the dimension $[cm^2/s]$ and can be titled as *quantum viscosity*, $\nu^{qu} = 1.1577\,\mathrm{cm}^2/\mathrm{s}$. Then

$$\tau_e = n\nu^{qu}/u^2 . \quad (28)$$

Introduce now the quantum Reynolds number

$$\mathrm{Re}^{qu} = u_0 x_0 / \nu^{qu} . \quad (29)$$

As result from (27)-(29) follows the condition of quantization for Re^{qu}. Namely

$$\mathrm{Re}^{qu} = n, n = 1, 2, \cdots \quad (30)$$

5) Taking into account the previous considerations I introduce the following approximation for the dimensionless non-local parameter

$$\tilde{\tau} = 1/\tilde{u}^2 , \quad (31)$$

$$\tau = u_0 x_0 / u^2 = \nu_0^k / u^2 , \quad (32)$$

where the scale for the kinematical viscosity is introduced $\nu_0^k = u_0 x_0$. Then we have the physically transparent result—non-local parameter is proportional to the kinematical viscosity and in inverse proportion to the square of velocity.

The system of generalized hydrodynamic Equations (23)-(26) (solved with the help of Maple) have the great possibilities of mathematical modeling as result of changing of eight Cauchy conditions describing the character features of initial perturbations which lead to the soliton formation. The following Maple notations on figures are used: *r*—density $\tilde{\rho}$, *u*—velocity $\tilde{u}$, *p*—pressure $\tilde{p}$ and *v*—self consistent potential $\tilde{\Psi}$.

Explanations placed under all following figures, Maple program contains Maple's notations—for example the expression $D(u)(0)=0$ means in the usual notations $\left(\partial\tilde{u}/\partial\tilde{\xi}\right)(0)=0$, independent variable t responds to $\tilde{\xi}$.

We begin with investigation of the problem of principle significance—is it possible after a perturbation (defined by Cauchy conditions) to obtain the gravitational object of the soliton's kind as result of the self-organization of the matter? With this aim let us consider the initial perturbations (SYSTEM I): $\mathbf{u(0)=1}$, $\mathbf{p(0)=1}$, $\mathbf{r(0)=1}$, $\mathbf{D(u)(0)=0}$, $\mathbf{D(p)(0)=0}$, $\mathbf{D(r)(0)=0}$, $\mathbf{D(v)(0)=0}$, $\mathbf{v(0)=1}$.

The **Figures 2-4** reflect the result of solution of Equations (23)-(26) with the choice of scales leading to $\tilde{\gamma}_N = 1$. **Figures 2-5** correspond to the approximation of the non-local parameter $\tilde{\tau}$ in the form (31). **Figure 2** displays the gravitational object placed in bounded region of 1D Cartesian space, all parts of this object are moving with the same velocity. Important to underline that no special boundary conditions were used for this and all following cases. Then this soliton is product of the self-organization of gravitational matter. **Figures 3** and **4** contain the answer for formulated above question

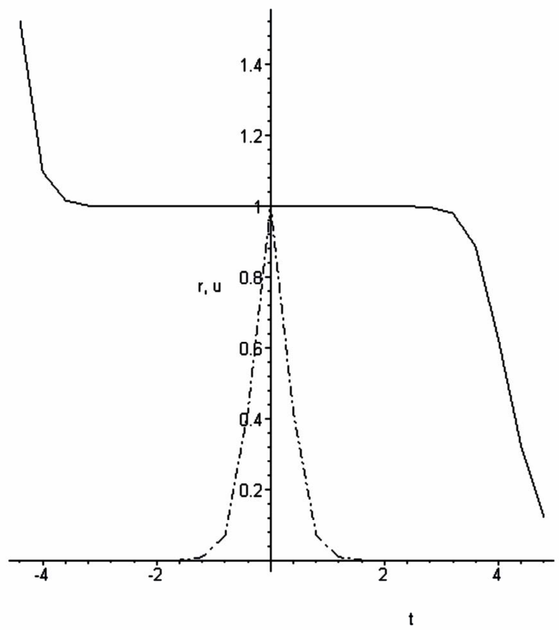


Figure 2. *r* density $\tilde{\rho}$ (dash dotted line), u velocity $\tilde{u}$ in gravitational soliton.

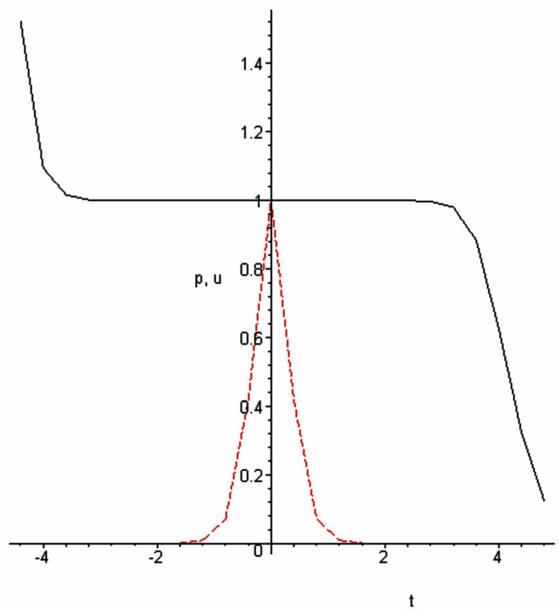


Figure. 3. p pressure $\tilde{p}$ (dashed line), u velocity $\tilde{u}$ in gravitational soliton.

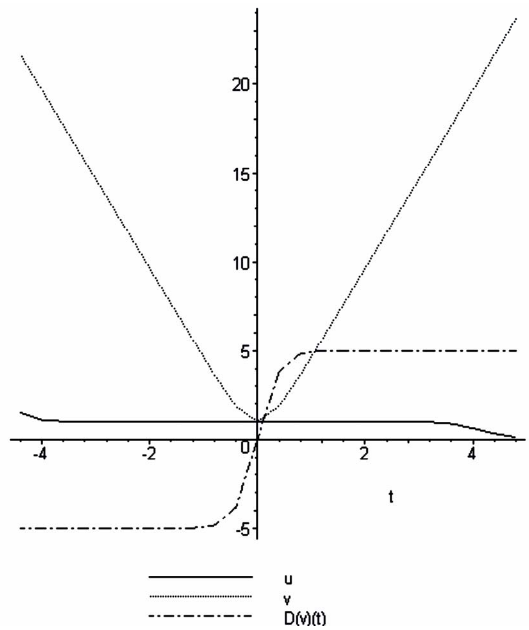


Figure 4. *u* velocity $\tilde{u}$, *v* self consistent potential $\tilde{\Psi}$, $D(\mathrm{v})(t) = \partial\tilde{\Psi}/\partial\tilde{\xi}$ in soliton.

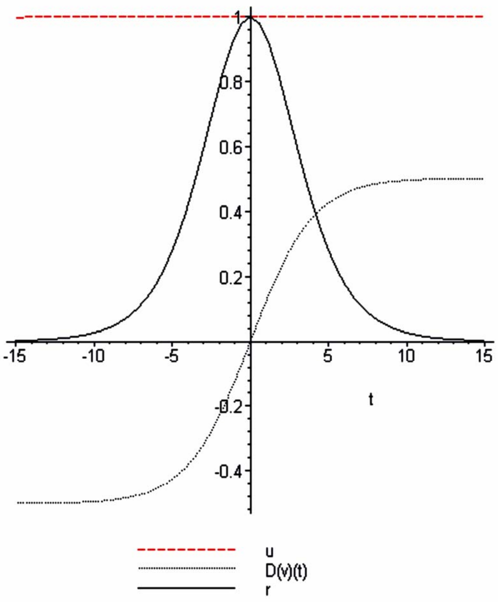


Figure 5. ***u*** **velocity** $\tilde{u}$ **,** ***r*** **density** $\tilde{\rho}$**,** $D(\mathrm{v})(t) = \partial\tilde{\Psi}/\partial\tilde{\xi}$ **in soliton, (** $\tilde{\gamma}_N = 0.01$ **).**

about stability of the object. The derivative (see **Figure 4**) $\frac{\partial\tilde{\Psi}}{\partial\tilde{\xi}} = \frac{\partial\Psi}{\partial\xi}\frac{x_0}{u_0^2} = -g(\xi)/(u_0^2/x_0) = -\tilde{g}(\xi)$ is proportional to the self-consistent gravitational force acting on the soliton and in its vicinity. Therefore the stability of the object is result of the self-consistent influence of the gravitational potential and pressure.

Extremely important that the self-consistent gravitational force has the character of the flat area which exists in the vicinity of the object. This solution exists only in the restricted area of space; the corresponding character length is defined automatically as result of the numerical solution of the problem. The non-local parameter $\tilde{\tau}$, in the definite sense plays the role analogous to kinetic coefficients in the usual Boltzmann kinetic theory. The influence on the results of calculations is not too significant. The same situation exists in the generalized hydrodynamics. Really, let us use the another approximation for $\tilde{\tau}$ in the simplest possible form, namely

$$\tilde{\tau} = 1. \tag{33}$$

The following **Figures 6-10** reflect the results of solution of Equations (23)-(26) with the choice of scales leading to $\tilde{\gamma}_N = 1$, but with the approximation of the non-local parameter $\tilde{\tau}$ in the form (33).

Spiral galaxies have rather complicated geometrical forms and 3D calculations can be used. But reasonable to suppose that influence of halo on galaxy kernel is not too significant and to use for calculations the spherical coordinate system. The 1D calculations in the Cartesian coordinate system correspond to calculations in the spherecal coordinate system by the large radii of curvature, but have also the independent significance in another character scales. Namely for explanations of the meteorological front motion (without taking into account the Earth rotation). In this theory cyclone or anticyclone corresponds to moving solitons. In the Earth scale the

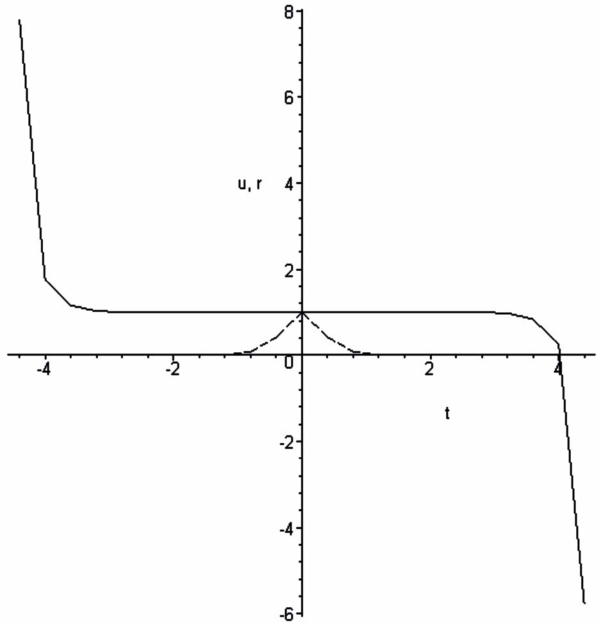


Figure 6. ***r*** **density** $\tilde{\rho}$ **(dashed line),** ***u*** **velocity** $\tilde{u}$ **in gravitational soliton.**

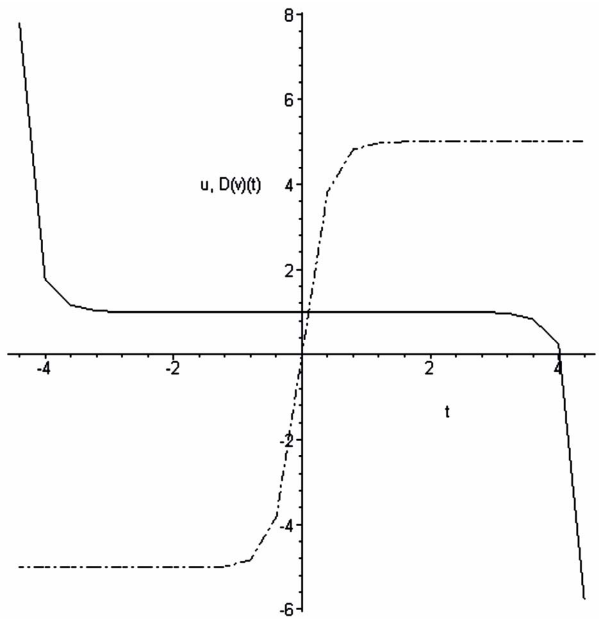


Figure 7. ***u*** **velocity** $\tilde{u}$ **,** $D(\mathrm{v})(t) = \partial\tilde{\Psi}/\partial\tilde{\xi}$ **(dash-dotted line).**

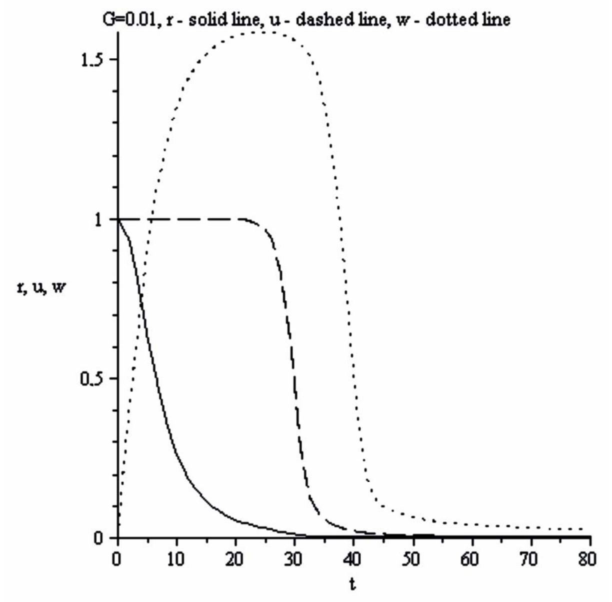


Figure 8. ***r*** **density** $\tilde{\rho}$**,** ***u*** **velocity** $\tilde{u}$**,** ***w*** **orbital velocity** $\tilde{w}$**.** $G = 0.01$**.**

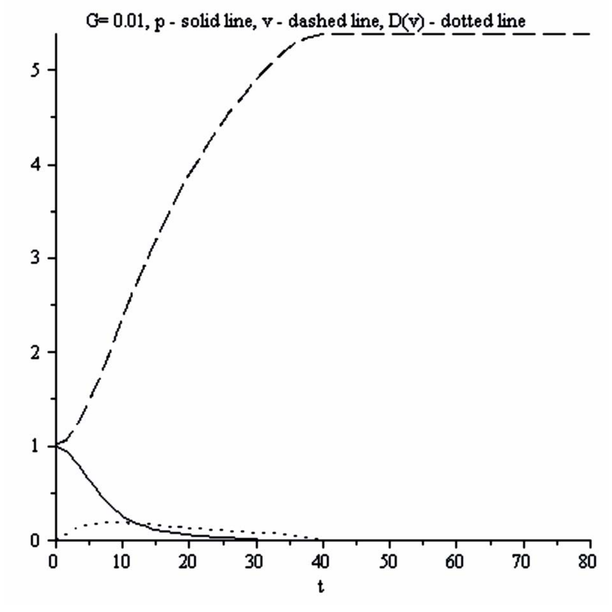


Figure 9. ***p*** **pressure** $\tilde{p}$**,** ***v*** **self consistent potential in gravitational soliton,** $\tilde{\Psi}$**,** $D(\mathrm{v})(t) = \partial\tilde{\Psi}/\partial\tilde{\xi}$ **in gravitational soliton.**

scales can be used: $\rho_{air} = 1.29 \cdot 10^{-3}\, g/\mathrm{cm}^3$, $u_0 = 1\, m/s$, $x_0 = 10\, km$ and $\tilde{\gamma}_N \sim 0.01$. **Figure 5** reflects the results of the corresponding calculation and in particular reflects correctly the wind orientation in front and behind of the soliton.

The full system of 3D non-local hydrodynamic equations in moving (along x axis) Cartesian coordinate system and the corresponding expression for derivatives in

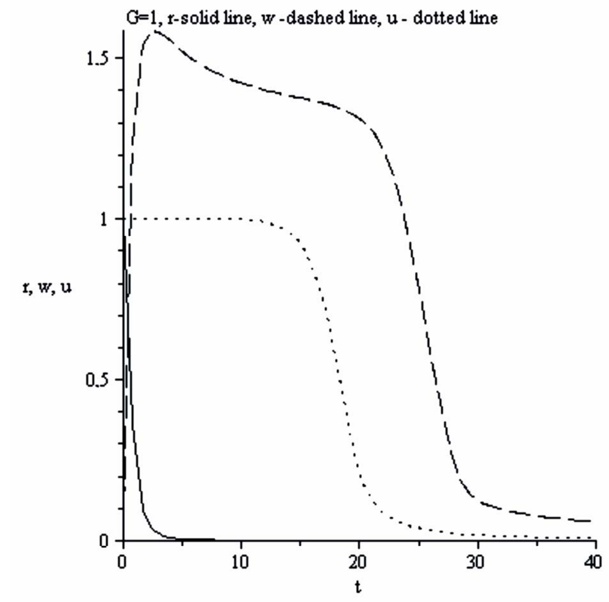


Figure 10. ***r*** **density** $\tilde{\rho}$**,** ***u*** **velocity** $\tilde{u}$ **in gravitational soliton,** ***w*** **orbital velocity** $\tilde{w}$**.** $G = 1$**.**

the spherical coordinate system can be found in [29,30]. The following figures reflect the result of soliton calculations for the case of spherical symmetry for galaxy kernel. The velocity $\tilde{u}$ corresponds to the direction of the soliton movement for spherical coordinate system on following figures. Self-consistent gravitational force F acting on the unit of mass permits to define the orbital velocity w of objects in halo, $w = \sqrt{Fr}$, or

$$\tilde{w} = \sqrt{\tilde{r}\frac{\partial\tilde{\Psi}}{\partial\tilde{r}}}, \tag{34}$$

where r is the distance from the center of galaxy. All calculations are realized for the conditions (SYSTEM I) but for different parameter

$$G = \tilde{\gamma}_N = \gamma_N/\gamma_{N0} = \gamma_N \rho_0\, x_0^2/u_0^2. \tag{35}$$

Parameter G plays the role of similarity criteria in traditional hydrodynamics. Important conclusions:

1) The following **Figures 8-15** demonstrate evolution of the rotation curves from the Kepler regime (**Figures 8** and **9**; small G, like curve **A** on **Figure 1**) to observed (**Figures 14** and **15**; large G, like curve **B** on **Figure 1**) for typical spiral galaxies.

2) The stars with planets (like Sun) correspond to the gravitational soliton with small *G* and therefore originate the Kepler rotation regime.

3) Regime **B** cannot be obtained in the frame of local statistical physics in principal and authors of many papers introduce different approximations for additional "dark matter density" (as usual in Poisson equation) trying to find coincidence with data of observations.

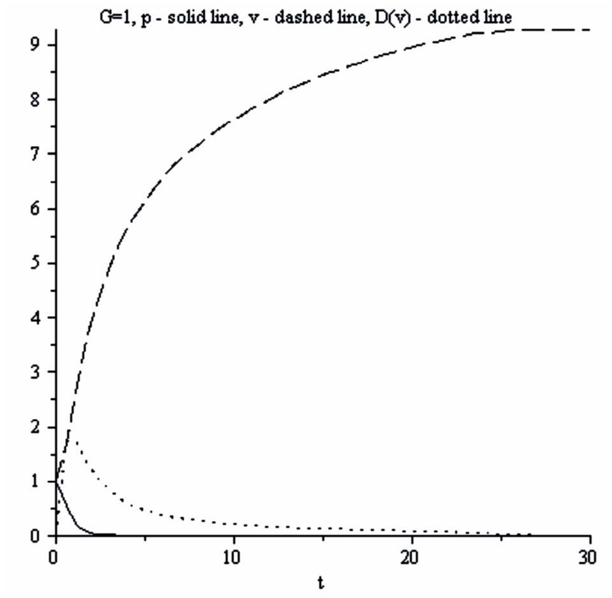


Figure 11. ***p*** **pressure** $\tilde{p}$**,** ***v*** **self consistent potential** $\tilde{\Psi}$**,** $D(\mathrm{v})(t) = \partial\tilde{\Psi}/\partial\tilde{\xi}$ **in gravitational soliton.**

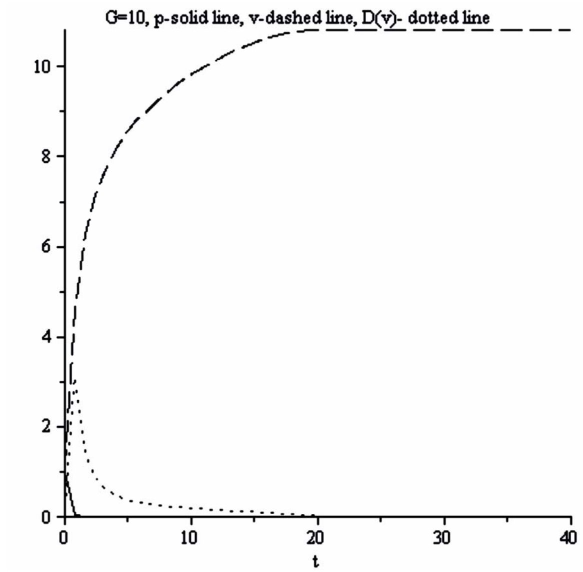


Figure 13. ***p*** **pressure** $\tilde{p}$**,** ***v*** **self consistent potential** $\tilde{\Psi}$**,** $D(\mathrm{v})(t) = \partial\tilde{\Psi}/\partial\tilde{\xi}$ **.**

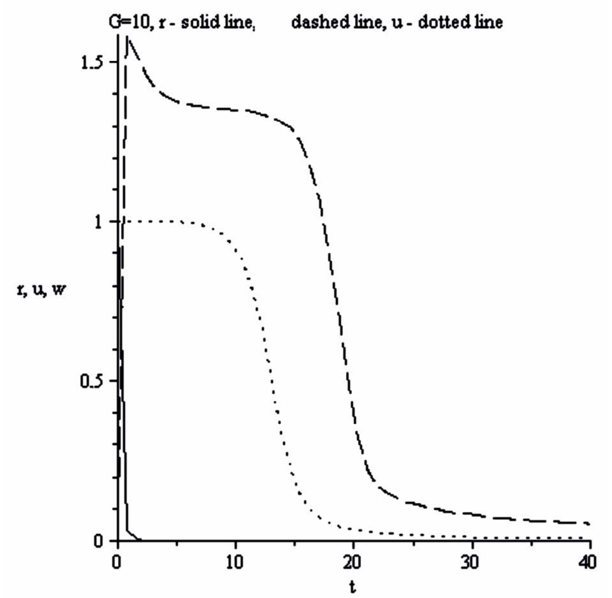


Figure 12. ***r*** **density** $\tilde{\rho}$**,** ***u*** **velocity** $\tilde{u}$ **in gravitational soliton,** ***w*** **orbital velocity** $\tilde{w}$ **.** $G = 10$ **.**

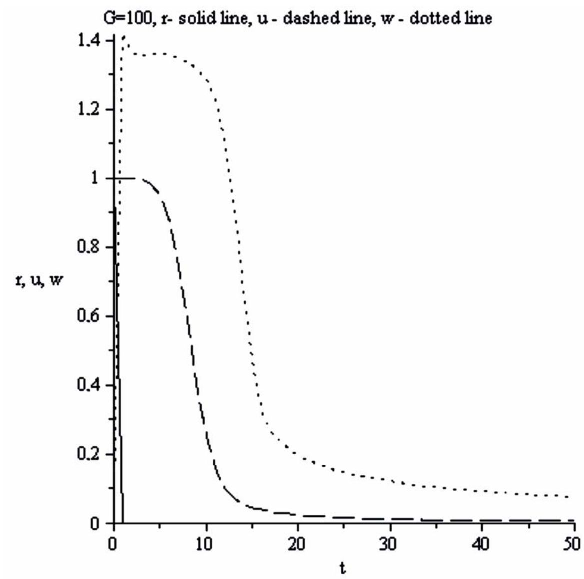


Figure 14. ***r*** **density** $\tilde{\rho}$**, u velocity** $\tilde{u}$ **in gravitational soliton,** ***w*** **orbital velocity** $\tilde{w}$ **.** $G = 100$ **.**

4) From the wrong position of local theories Poisson Equation (18) contains "dark matter density", continuity Equation (19) contains the "flux of dark matter density", motion Equation (20) includes "dark energy", energy Equation (21) has "the flux of dark energy" and so on to the "senior dark velocity moments". This entire situation is similar to the turbulent theories based on local statistical physics and empirical corrections for velocity moments.

As we see peculiar features of the halo movement can be explained without new concepts like "dark matter". Important to underline that the shown transformation of the Kepler's regime into the flat rotation curves for different solitons explains the "mysterious" fact of the dark matter absence in the Sun vicinity.

3. Hubble Expansion and the Problem of Dark Energy

In simplest interpretation of the *local theories* the dark

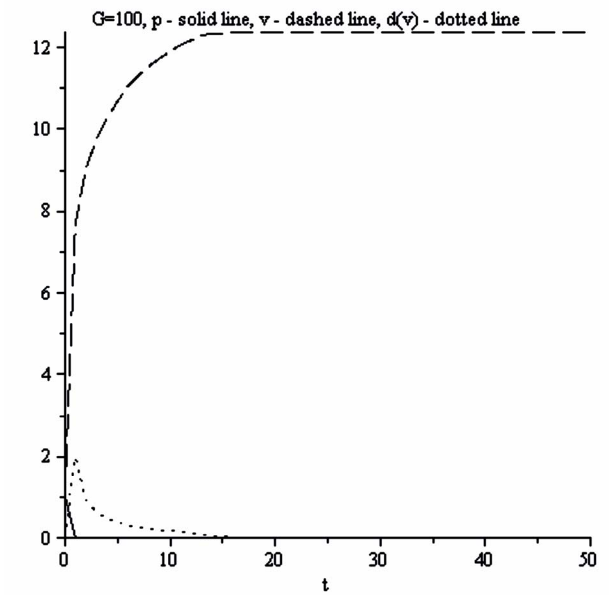


Figure. 15. ***p*** **pressure** $\tilde{p}$**,** ***v*** **self consistent potential** $\tilde{\Psi}$**,** $\boldsymbol{D}(\mathrm{v})(\boldsymbol{t}) = \partial\tilde{\Psi}/\partial\tilde{\xi}$ **.**

energy is related usually to the Einstein cosmological constant. In review [4] the modified Newton force is written as

$$F(r) = -\frac{\gamma_N M}{r^2} + \frac{8\pi\gamma_N}{3}\rho_{\mathrm{v}} r, \tag{36}$$

where ρ_{v} is the Einstein-Gliner vacuum density [33]. In the limit of large distances the influence of central mass M becomes negligibly small and the field of forces is determined only by the second term in the right side of (36). It follows from relation (36) that there is a distance r_{v} at which the sum of the gravitation and antigravitation forces is equal to zero. In other words r_{v} is "the zero-gravitational radius". For so called Local Group of galaxies estimation of r_{v} is about 1Mpc.

From the non-local statistical theory the physical picture follows which leading to the Hubble flow without new essence like dark energy and without modification of Newton force like (36).

Namely:

The main origin of Hubble effect (*including the matter expansion with acceleration*) *is self—catching of expanding matter by the self—consistent gravitational field in conditions of weak influence of the central massive bodies.*

The formulated result is obtained in the frame of the linear theory [25,31]. Is it possible to obtain the corresponding result on the level of the general non-linear description? Such an investigation was successfully realized and leads to a direct mathematical model supporting the well known observations of S. Perlmutter, A. Riess (USA) and B. Schmidt (Australia). These researchers studied Type 1a supernovae and determined that more distant galactic objects seem to move faster. Their observations suggest that not only is the Universe expanding, its expansion is relentlessly speeding up.

Effects of gravitational self-catching should be typical for Universe. The existence of "Hubble boxes" is discussed in review [4] as typical blocks of the nearby Universe. Gravitational self-catching takes place for Big Bang having given birth to the global expansion of Universe, but also for Little Bang in so called Local Group (using the Hubble's terminology) of galaxies. Then the evolution of the Local Group (the typical Hubble box) is really fruitful field for testing of different theoretical constructions (see **Figure 16**). The data were obtained by Karachentsev and his collaborators in 2002-2007 in observation with the Hubble Space Telescope [4,32]. Each point corresponds to a galaxy with measured values of distance and line-of-site velocity in the reference frame related to the center of the Local Group. The diagram shows two distinct structures, the Local Group and the local flow of galaxies. The galaxies of the Local Group occupy a volume with the radius up to ~1.1 - 1.2 Mpc, but there are no galaxies in the volume whose radius is less than 0.25 Mpc. These galaxies move both away from the center (positive velocities) and toward the center (negative velocities). These galaxies form a gravitationally bound quasi-stationary system. Their average radial velocity is equal to zero. The galaxies of the local flow are located outside the group and all of them are moving from the center (positive velocities) beginning their motion near $R \approx 1\,\mathrm{Mpc}$ with the velocity $v \sim 50\,\mathrm{km/s}$. By the way the measured by Karachentsev the average Hubble parameter for the Local Group is 72 ± 6 $\mathrm{km\cdot s^{-1}Mpc^{-1}}$.

Let us choose these values as scales:

$$x_0 = 1\,\mathrm{Mpc}, u_0 = 50\,\mathrm{km/s}. \tag{37}$$

Recession velocities increase as the distance increases in accordance with the Hubble law. The straight line correspond the dependence from observations

$$v = H(r)r \tag{38}$$

for the region outside of the Local Group. In the non-dimensional form

$$\tilde{v} = \tilde{H}(\tilde{r})\tilde{r} \tag{39}$$

where

$$\tilde{H} = \frac{x_0}{u_0} H(\tilde{r}). \tag{40}$$

For the following calculations we should choose the corresponding scales (especially for estimating G) for modeling of the Local Group evolution

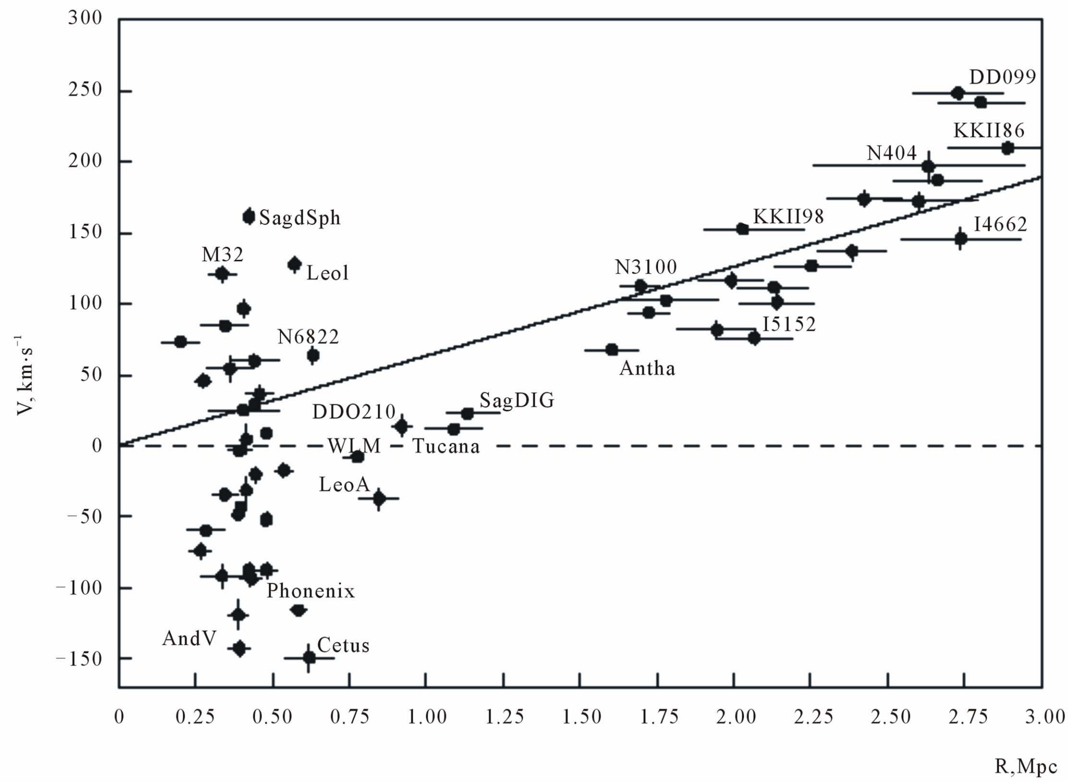


Figure 16. Velocity-distance diagram for galaxies at distances of up to 3 Mpc for local group of galaxies.

$$G \equiv \tilde{\gamma}_N = \gamma_N / \gamma_{N0} = \gamma_N \rho_0 x_0^2 / u_0^2 . \quad (41)$$

For the density scale estimation the average density of the local flow could be used. But the corresponding data are not accessible and I use the average density of the Local Group which can be taken from references [32,34] with $\rho_0 = 4.85 \times 10^{-29}\ \mathrm{g/cm^3}$. Then from (41) we have $G \cong 1$.

Let us go now to the mathematical modeling. The non-local system of hydrodynamic equations describing the explosion with the spherical symmetry is written as (see [30], Appendix 2)

$$g_r = -\frac{\partial \psi}{\partial r}, \quad (42)$$

(Poisson equation)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) = 4\pi\gamma_N \left[\rho - \tau\left(\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial\left(r^2 \rho v_r\right)}{\partial r}\right)\right], \quad (43)$$

(Continuity equation)

$$\frac{\partial}{\partial t}\left\{\rho - \tau\left[\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial\left(r^2\rho v_r\right)}{\partial r}\right]\right\} + \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\rho v_r - \tau\left[\frac{\partial}{\partial t}\left(\rho v_r\right) + \frac{1}{r^2}\frac{\partial\left(r^2 \rho v_r^2\right)}{\partial r} - \rho g_r\right]\right\}\right\} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2 \frac{\partial p}{\partial r}\right) = 0, \quad (44)$$

(Motion equation)

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\{\rho v_r - \tau\left[\frac{\partial}{\partial t}\left(\rho v_r\right) + \frac{1}{r^2}\frac{\partial\left(r^2\rho v_r^2\right)}{\partial r} + \frac{\partial p}{\partial r} - \rho g_r\right]\right\} - g_r\left[\rho - \tau\left(\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial\left(r^2 \rho v_r\right)}{\partial r}\right)\right] \\
&+ \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left(\rho v_r^2 - \tau\left[\frac{\partial}{\partial t}\left(\rho v_r^2\right) + \frac{1}{r^2}\frac{\partial\left(r^2\rho v_r^3\right)}{\partial r} - 2 g_r \rho v_r\right]\right)\right\} + \frac{\partial p}{\partial r} - \frac{\partial}{\partial r}\left(\tau \frac{\partial p}{\partial t}\right) - 2\frac{\partial}{\partial r}\left(\frac{\tau}{r^2}\frac{\partial\left(r^2 p v_r\right)}{\partial r}\right), \quad (45) \\
&- \frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2 \frac{\partial\left(p v_r\right)}{\partial r}\right) = 0
\end{aligned}$$

(Energy equation)

$$\frac{\partial}{\partial t}\left\{\frac{1}{2}\rho v_r^2+\frac{3}{2}p-\tau\left[\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v_r^2+\frac{3}{2}p\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\left(\frac{1}{2}\rho v_r^2+\frac{5}{2}p\right)\right)-\rho g_r v_r\right]\right\}$$
$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left\{\left(\frac{1}{2}\rho v_r^2+\frac{5}{2}p\right)v_r-\tau\left[\frac{\partial}{\partial t}\left(\left(\frac{1}{2}\rho v_r^2+\frac{5}{2}p\right)v_r\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{1}{2}\rho v_r^2+\frac{7}{2}p\right)v_r^2\right)\right.\right.\right.$$
$$\left.\left.\left.-\rho g_r v_r^2-\left(\frac{1}{2}\rho v_r^2+\frac{3}{2}p\right)g_r\right]\right\}\right\}-\left\{\rho g_r v_r-\tau\left[g_r\left(\frac{\partial}{\partial t}(\rho v_r)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho v_r^2\right)+\frac{\partial p}{\partial r}-\rho g_r\right)\right]\right\}$$
$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(\tau r^2\frac{\partial}{\partial r}\left(\frac{1}{2}pv_r^2+\frac{5}{2}\frac{p^2}{\rho}\right)\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau p g_r\right)=0. \tag{46}$$

The system of Equations (43)-(46) belongs to the class of the 1D non-stationary equations and can be solved by known numerical methods. But for the aims of the transparent vast mathematical modeling of self-catching of the expanding matter by the self-consistent gravitational field I introduce the following assumption. Let us allot the quasi-stationary Hubble regime when only the implicit dependence on time for the unknown values exists. It means that for the intermediate (Hubble) regime the substitution

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial r}\frac{\partial r}{\partial t}=v_r\frac{\partial}{\partial r} \tag{47}$$

can be introduced. As result we have the following sys− tem of the 1D dimensionless equations:

$$\frac{1}{\tilde r^2}\frac{\partial}{\partial \tilde r}\left(\tilde r^2\frac{\partial\tilde\psi}{\partial\tilde r}\right)=4\pi G\left[\tilde\rho-\tilde\tau\left(\tilde v_r\frac{\partial\tilde\rho}{\partial\tilde r}+\frac{1}{\tilde r^2}\frac{\partial\left(\tilde r^2\tilde\rho\tilde v_r\right)}{\partial\tilde r}\right)\right], \tag{48}$$

$$\tilde v_r\frac{\partial}{\partial\tilde r}\left\{\tilde\rho-\tilde\tau\left[\tilde v_r\frac{\partial\tilde\rho}{\partial\tilde r}+\frac{1}{\tilde r^2}\frac{\partial\left(\tilde r^2\tilde\rho\tilde v_r\right)}{\partial\tilde r}\right]\right\}$$
$$+\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left\{\tilde r^2\left\{\tilde\rho\tilde v_r-\tilde\tau\left[\tilde v_r\frac{\partial}{\partial\tilde r}(\tilde\rho\tilde v_r)+\frac{1}{\tilde r^2}\frac{\partial\left(\tilde r^2\tilde\rho\tilde v_r^2\right)}{\partial\tilde r}+\tilde\rho\frac{\partial\tilde\psi}{\partial\tilde r}\right]\right\}\right\}-\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left(\tilde\tau\tilde r^2\frac{\partial\tilde p}{\partial\tilde r}\right)=0, \tag{49}$$

$$\tilde v_r\frac{\partial}{\partial\tilde r}\left\{\tilde\rho\tilde v_r-\tilde\tau\left[\tilde v_r\frac{\partial}{\partial\tilde r}(\tilde\rho\tilde v_r)+\frac{1}{\tilde r^2}\frac{\partial\left(\tilde r^2\tilde\rho\tilde v_r^2\right)}{\partial\tilde r}+\frac{\partial\tilde p}{\partial\tilde r}+\tilde\rho\frac{\partial\tilde\psi}{\partial\tilde r}\right]\right\}+\frac{\partial\tilde\psi}{\partial\tilde r}\left[\tilde\rho-\tilde\tau\left(\tilde v_r\frac{\partial\tilde\rho}{\partial\tilde r}+\frac{1}{\tilde r^2}\frac{\partial\left(\tilde r^2\tilde\rho\tilde v_r\right)}{\partial\tilde r}\right)\right]$$
$$+\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left\{\tilde r^2\left\{\tilde\rho\tilde v_r^2-\tilde\tau\left[\tilde v_r\frac{\partial}{\partial r}\left(\tilde\rho\tilde v_r^2\right)+\frac{1}{\tilde r^2}\frac{\partial\left(\tilde r^2\tilde\rho\tilde v_r^3\right)}{\partial\tilde r}+2\frac{\partial\tilde\psi}{\partial\tilde r}\tilde\rho\tilde v\right]_r\right\}\right\}$$
$$+\frac{\partial\tilde p}{\partial\tilde r}-\frac{\partial}{\partial\tilde r}\left(\tilde v_r\tilde\tau\frac{\partial\tilde p}{\partial\tilde r}\right)-2\frac{\partial}{\partial\tilde r}\left(\frac{\tilde\tau}{\tilde r^2}\frac{\partial\left(\tilde r^2\tilde p\tilde v_r\right)}{\partial\tilde r}\right)-\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left(\tilde\tau\tilde r^2\frac{\partial\left(\tilde p\tilde v_r\right)}{\partial\tilde r}\right)=0, \tag{50}$$

$$\tilde v_r\frac{\partial}{\partial\tilde r}\left\{\tilde\rho\tilde v_r^2+3\tilde p-\tilde\tau\left[\tilde v_r\frac{\partial}{\partial\tilde r}\left(\tilde\rho\tilde v_r^2+3\tilde p\right)+\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left(\tilde r^2\tilde v_r\left(\tilde\rho\tilde v_r^2+5\tilde p\right)\right)+2\tilde\rho\frac{\partial\tilde\psi}{\partial\tilde r}\tilde v_r\right]\right\}$$
$$+\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left\{\tilde r^2\left\{\left(\tilde\rho\tilde v_r^2+5\tilde p\right)\tilde v_r-\tilde\tau\left[\tilde v_r\frac{\partial}{\partial\tilde r}\left(\left(\tilde\rho\tilde v_r^2+5\tilde p\right)\tilde v_r\right)+\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left(\tilde r^2\left(\tilde\rho\tilde v_r^2+7\tilde p\right)\tilde v_r^2\right)+2\tilde\rho\frac{\partial\psi}{\partial\tilde r}\tilde v_r^2+\left(\tilde\rho\tilde v_r^2+3\tilde p\right)\frac{\partial\tilde\psi}{\partial\tilde r}\right]\right\}\right\}$$
$$+2\left\{\tilde\rho\frac{\partial\tilde\psi}{\partial\tilde r}\tilde v_r-\tilde\tau\frac{\partial\tilde\psi}{\partial\tilde r}\left[\tilde v_r\frac{\partial}{\partial\tilde r}(\rho\tilde v_r)+\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left(\tilde r^2\tilde\rho\tilde v_r^2\right)+\frac{\partial\tilde p}{\partial\tilde r}+\tilde\rho\frac{\partial\tilde\psi}{\partial\tilde r}\right]\right\}$$
$$-\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left(\tilde\tau\tilde r^2\frac{\partial}{\partial\tilde r}\left(\tilde p\tilde v_r^2+5\frac{\tilde p^2}{\tilde\rho}\right)\right)-2\frac{1}{\tilde r^2}\frac{\partial}{\partial\tilde r}\left(\tilde r^2\tilde\tau\tilde p\frac{\partial\tilde\psi}{\partial\tilde r}\right)=0 \tag{51}$$

The system of generalized hydrodynamic equations (48)-(51) have the great possibilities of mathematical modeling as result of changing of eight Cauchy conditions describing the character features of the local flow evolution. The following Maple notations on figures are used: *r*—density $\tilde{\rho}$, *u*—velocity $\tilde{v}_r$, *p*—pressure $\tilde{p}$ and *v*—self consistent potential $\tilde{\psi}$, *h*—$\tilde{H}$ and independent variable *t* is $\tilde{r}$. Explanations placed under all following figures, Maple program contains Maple's notations—for example the expression $D(u)(0)=0$ means in the usual notations $(\partial\tilde{u}/\partial\tilde{r})(0)=0$.

As mentioned before, the non-local parameter $\tilde{\tau}$, in the definite sense plays the role analogous to kinetic coefficients in the usual Boltzmann kinetic theory. The influence on the results of calculations is not too significant, (see (31, 33)). The same situation exists in the generalized hydrodynamics. As before I introduce the following approximation for the dimensionless non-local parameter (see (31), here $\tilde{u}=\tilde{v}_r$) $\tilde{\tau}=1/\tilde{u}^2$. Let us define also the dimensionless acceleration-deceleration function for the quasi-stationary regime

$$Q=\frac{\partial\tilde{v}_r}{\partial\tilde{r}}=\frac{\partial\tilde{u}}{\partial\tilde{r}}, \qquad (52)$$

as an analogue of the dimensionless deceleration function q which was used in [28].

One obtains for the approximation (31) and SYSTEM 2:

$\mathbf{v(1)=1,\ u(1)=1,\ r(1)=1,\ p(1)=1,\ D(v)(1)=0,\ D(u)(1)=0,\ D(r)(1)=0,\ D(p)(1)=0}$.

Figures 17 and **18** correspond to *G* = 1. From these cal- culations follow:

1) As it was waiting the quasi-stationary regime exists only in the restricted (on the left and on the right sides) area. Out of these boundaries the explicit time dependent regime should be considered. But it is not the Hubble regime.

2) In the Hubble regime one obtains the negative area (low part of the dash-dotted curve of **Figure 17**). It corresponds to the self-consistent force acting along the expansion of the local flow.

3) The dependence of $\tilde{H}(\tilde{r})$ is not linear (see **Figure 18**), more over the curvature contains maximum. The area of acceleration placed between two areas of the deceleration.

Let us show now the result of calculations for another $\tilde{\tau}$ approximation in the simplest possible form, namely (see also (33)) $\tilde{\tau}=1$. One obtains for this $\tilde{\tau}$ - approximation and SYSTEM 2 for *G* = 1, see **Figures 19-22**.

We can add to the previous conclusions:

4) Approximation $\tilde{\tau}=const$ conserves all principal characters of the previous dependences, but the area of the Hubble regime becomes larger.

5) Approximation $\tilde{\tau}=const$ allows realizing the

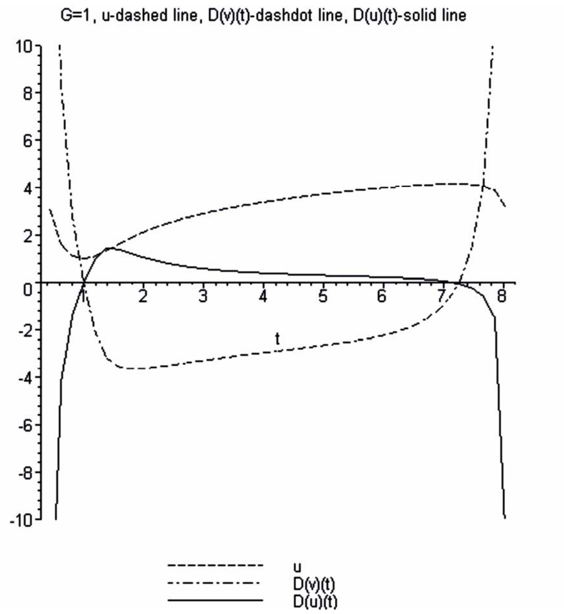


Figure 17. Dependence of the acceleration-deceleration function Q (in Maple notation $D(u)(t)=\partial\tilde{u}/\partial\tilde{r}$), derivation of the self-consistent potential $D(v)(t)=\partial\tilde{\Psi}/\partial\tilde{\xi}$ and velocity $u=\tilde{u}$ on the radial distance $\tilde{r}$.

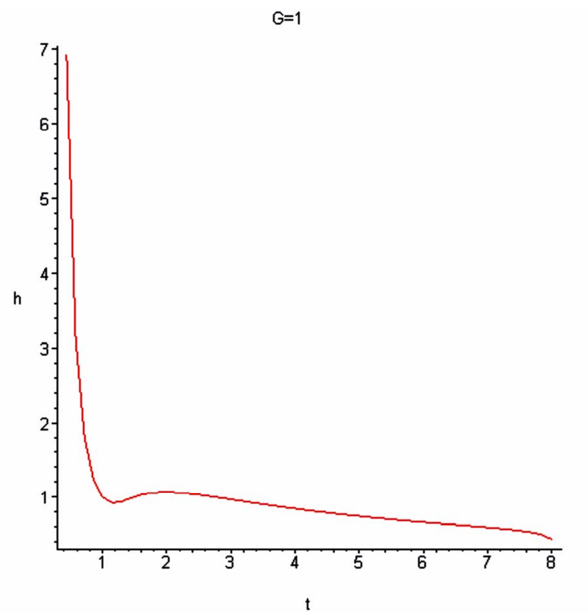


Figure 18. Dependence of the dimensionless Hubble parameter on the radial distance.

numerical transition to the "classical" gas dynamics of explosions. By the $\tilde{\tau}\to 0$ there are no Hubble regimes in principal.

6) Diminishing of G leads to diminishing of the area of the Hubble regime with the positive acceleration of the matter catched by the self-consistent gravitational field.

7) Dependence of $\tilde{H}(\tilde{r})$ does not contain the maxi-

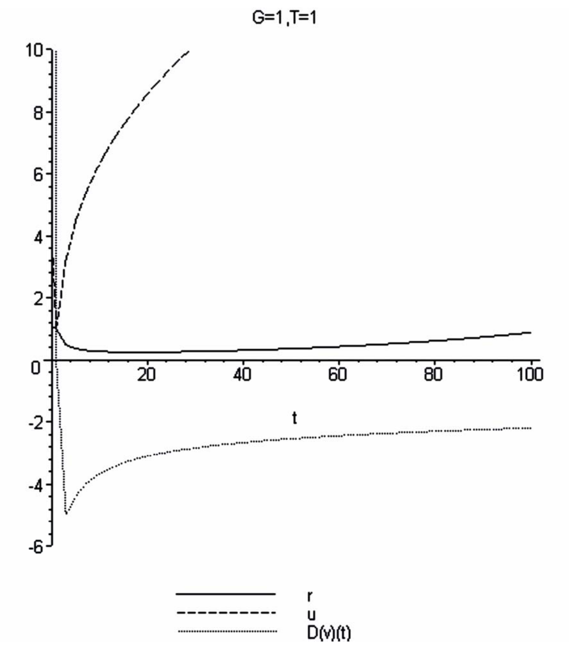


Figure 19. ***r*** **density** $\tilde{\rho}$**,** ***u*** $\tilde{u}$**,** $D(\mathrm{v})(t)=\partial\tilde{\Psi}/\partial\tilde{\xi}$ **.**

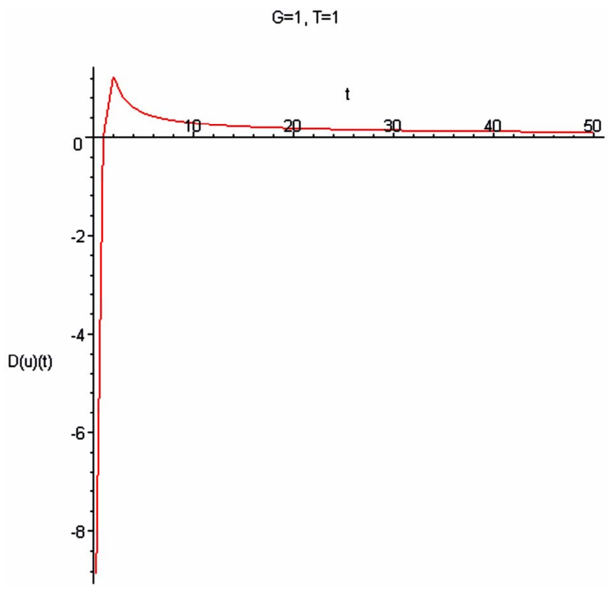


Figure 20. Dependence of the acceleration-deceleration $D(u)(t)=\partial\tilde{u}/\partial\tilde{r}$ **on** $\tilde{r}$ **.**

mum on the curve for the small value of parameter G (A-regime). It is reasonable to find from the observation the Hubble boxes where A-regime is realizing. Consideration of the Local Group evolution of galaxies (see **Figure 16**) leaves the impression that this burst responds to the PRS-regime.

As we see the Hubble expansion with acceleration is explained as result of mathematical modeling based on the principles of non-local physics. Peculiar features of

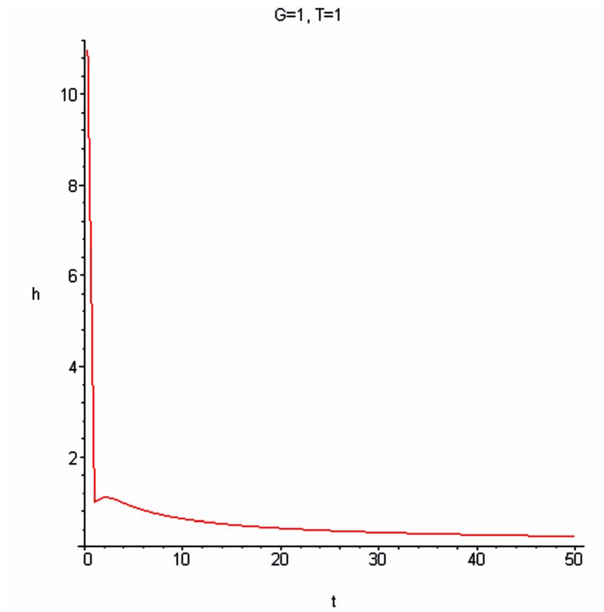


Figure 21. Dependence of the dimensionless Hubble parameter on the radial distance for *G* = 1.

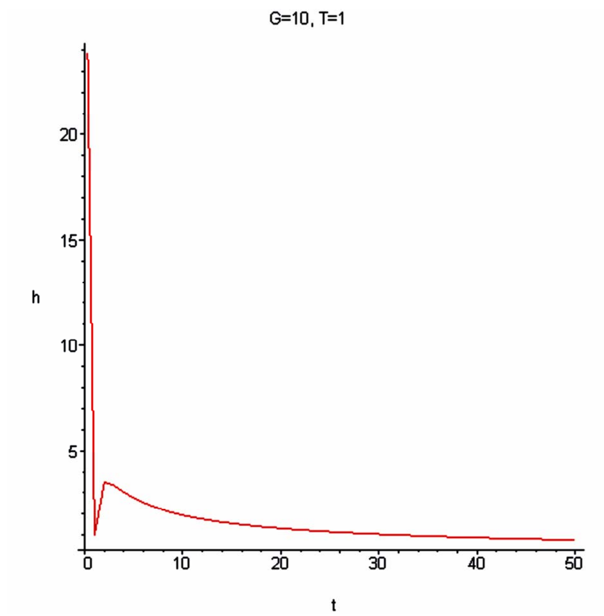


Figure 22. Dependence of the dimensionless hubble parameter on the radial distance for *G* = 10.

the rotational speeds of galaxies and the Hubble expansion with acceleration need not in the introduction of new essence like dark matter and dark energy.

4. Conclusion

The unified generalized non-local theory is applied for mathematical modeling of cosmic objects with success. For the case of galaxies the theory leads to the flat rotation curves known from observations. The transformation of Kepler's regime into the flat rotation curves for different solitons is shown. The origin of Hubble effect (including the matter expansion with acceleration) is self-catching of the expanding matter by the self-consistent gravitational field in conditions of weak influence of the central massive bodies. The Hubble expansion with acceleration is obtained as result of mathematical modeling based on the principles of non-local physics. Peculiar features of the rotational speeds of galaxies and effects of the Hubble expansion need not in the introduction of new essence like dark matter and dark energy. The origin of difficulties consists in the total Oversimplification following from the principles of local physics.

REFERENCES

[1] V. Rubin and W. K. Ford Jr., "Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions," *Astrophysical Journal*, Vol. 159, 1970, p. 379. doi:10.1086/150317

[2] V. Rubin, N. Thonnard and W. K. Ford Jr., "Rotational Properties of 21 S_c Galaxies with a Large Range of Luminosities and Radii from NGC 4605 (R = 4 kpc) to UGC 2885 (R = 122 kpc)," *Astrophysical Journal*, Vol. 238, 1980, pp. 471-487. doi:10.1086/158003

[3] M. Milgrom, "The MOND Paradigm," ArXiv Preprint, 2007. http://arxiv.org/abs/0801.3133v2

[4] A. D. Chernin, "Dark Energy and Universal Antigravitation," *Physics-Uspekhi*, Vol. 51, No. 3, 2008, pp. 267-300. doi:10.1070/PU2008v051n03ABEH006320

[5] J. S. Bell, "On the Einstein Podolsky Rosen Paradox," *Physics*, No. 1, 1964, pp. 195-200.

[6] B. V. Alexeev, "Generalized Boltzmann Physical Kinetics," Elsevier, Amsterdam, 2004.

[7] L. Boltzmann. "Weitere Studien über das Warmegleichgewicht unter Gasmoleculen," *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften*, Vol. 66, 1872, p. 275.

[8] L. Boltzmann, "Vorlesungen über Gastheorie," Verlag von Johann Barth, Zweiter Unveränderten Abdruck. 2 Teile, Leipzig, 1912.

[9] N. N. Bogolyubov, "Problemy Dinamicheskoi Teorii v Statisticheskoi Fizike, (Dynamic Theory Problems in Statistical Physics)," Moscow Leningrad Gostekhizdat, 1946.

[10] M. Born and H. S. Green. "A General Kinetic Theory of Liquids," *Proceedings of the Royal Society*, Vol. 188, No. 1012, 1946, pp. 10-18. doi:10.1098/rspa.1946.0093

[11] H. S. Green, "The Molecular Theory of Fluids," North-Holland, Amsterdam, 1952.

[12] J. G. Kirkwood, "The Statistical Mechanical Theory of Transport Processes 2: Transport in Gases," *Journal of Chemical Physics*, Vol. 15, No. 1, 1947, pp. 72-76. doi:10.1063/1.1746292

[13] J. Yvon, "La Theorie Statistique des Fluides et l'Equation d'etat," Hermann, Paris, 1935.

[14] B. V. Alekseev, "Matematicheskaya Kinetika Reagiruyushchikh Gazov," (Mathematical Theory of Reacting Gases), Nauka, Moscow, 1982.

[15] B. V. Alexeev, "The Generalized Boltzmann Equation, Generalized Hydrodynamic Equations and their Applications," *Philosophical Transactions of the Royal Society of London*, Vol. 349, 1994, pp. 417-443. doi:10.1098/rsta.1994.0140

[16] B. V. Alexeev, "The Generalized Boltzmann Equation," *Physica A*, Vol. 216, No. 4, 1995, pp. 459-468. doi:10.1016/0378-4371(95)00044-8

[17] S. Chapman and T. G. Cowling, "The Mathematical Theory of Non-Uniform Gases," University Press, Cambridge, 1952.

[18] I. O. Hirschfelder, Ch. F. Curtiss and R. B. Bird, "Molecular Theory of Gases and Liquids," John Wiley and sons, Inc., New York, Chapman and Hall, London, 1954.

[19] Yu. L. Klimontovich, "About Necessity and Possibility of Unified Description of Hydrodynamic Processes," *Theoretical and Mathematical Physics*, Vol. 92, No. 2, 1992, p. 312.

[20] B. V. Alekseev, "Physical Basements of the Generalized Boltzmann Kinetic Theory of Gases," *Physics-Uspekhi*, Vol. 43, No. 6, 2000, pp. 601-629. doi:10.1070/PU2000v043n06ABEH000694

[21] B. V. Alekseev, "Physical Fundamentals of the Generalized Boltzmann Kinetic Theory of Ionized Gases," *Physics-Uspekhi*, Vol. 46, No. 2, 2003, pp. 139-167. doi:10.1070/PU2003v046n02ABEH001221

[22] B. V. Alexeev, "Generalized Quantum Hydrodynamics and Principles of Non-Local Physics," *Journal of Nanoelectronics and Optoelectronics*, Vol. 3, No. 2, 2008, pp. 143-158. doi:10.1166/jno.2008.207

[23] B. V. Alexeev, "Application of Generalized Quantum Hydrodynamics in the Theory of Quantum Soliton Evolution," *Journal of Nanoelectronics and Optoelectronics*, Vol. 3, No. 3, 2008, pp. 316-328. doi:10.1166/jno.2008.311

[24] B. V. Alexeev, "Generalized Theory of Landau Damping," *Journal of Nanoelectronics and Optoelectronics*, Vol. 4, No. 1, 2009, pp. 186-199. doi:10.1166/jno.2009.1021

[25] B. V. Alexeev, "Generalized Theory of Landau Damping in Collisional Media," *Journal of Nanoelectronics and Optoelectronics*, Vol. 4, No. 3, 2009, pp. 379-393. doi:10.1166/jno.2009.1054

[26] S. Perlmutter, *et al*., "Measurements of Ω and Λ from 42 High-Redshift Supernovae," *Astrophysical Journal*, Vol. 517, No. 2, 1999, pp. 565-586. doi:10.1086/307221

[27] A. G. Riess, *et al*., "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," *Astronomical Journal*, Vol. 116, No. 3, 1998, pp. 1009-1038. doi:10.1086/300499

[28] A. G. Riess, *et al*., "Type IA Supernova Discoveries at $z > 1$ from the Hubble Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution," *Astrophysical Journal*, Vol. 607, 2004, pp. 665-687. doi:10.1086/383612

[29] B. V. Alexeev, "Non-local Physics. Non-Relativistic Theory," Lambert Academic Press (in Russian), 2011.

[30] B. V. Alexeev and I. V. Ovchinnikova, "Non-Local Physics. Relativistic Theory," Lambert Academic Press, 2011, (in Russian).

[31] B. V. Alexeev, "Application of Non-Local Physics in the Theory of Hubble Expansion," Nova Science Publishers, Inc., 2011.

[32] I. D. Karachentsev and O. G. Kashibadze, "Masses of the Local Group and of the M81 Group Estimated from Distortions in the Local Velocity Field," *Astrophysics*, Vol. 49, No. 1, 2006, pp. 3-18.

[33] E. B. Gliner, "The Vacuum-Like State of a Medium and Friedman Cosmology," *Soviet Physics—Doklady*, Vol. 15, No. 6, 1970, pp. 559-561.

[34] L. S. Sparke and J. S. Gallagher III, "Galaxies in the Universe: Introduction," Cambridge University Press, Cambridge, 2000.

Journal of Modern Physics, **2012, 3, 1123-1127**
doi:10.4236/jmp.2012.329146 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Can Effects of Dark Matter Be Explained by the Turbulent Flow of Spacetime?

F. Elliott Koch, Angus H. Wright
School of Physics, University of New South Wales, Sydney, Australia
Email: f.koch@unsw.edu.au, mhchan@phy.cuhk.edu.hk

Received July 9, 2012; revised August 11, 2012; accepted August 18, 2012

ABSTRACT

For the past forty years the search for dark matter has been one of the primary foci of astrophysics, although there has yet to be any direct evidence for its existence [1]. Indirect evidence for the existence of dark matter is largely rooted in the rotational speeds of stars within their host galaxies, where, instead of having a $\sim r^{-1/2}$ radial dependence, stars appear to have orbital speeds independent of their distance from the galactic center, which led to proposed existence of dark matter [1,2]. We propose an alternate explanation for the observed stellar motions within galaxies, combining the standard treatment of a fluid-like spacetime with the possibility of a "bulk flow" of mass through the Universe. The differential "flow" of spacetime could generate vorticies capable of providing the "perceived" rotational speeds in excess of those predicted by Newtonian mechanics. Although a more detailed analysis of our theory is forthcoming, we find a crude "order of magnitude" calculation can explain this phenomena. We also find that this can be used to explain the graviational lensing observed around globular clusters like "Bullet Cluster".

Keywords: Dark Matter; Galaxy; Relativity; Gravitation

1. Introduction

In the pursuit of determining a model that accurately predicts the past, present, and future of the evolution of the Universe, physicists have generated a range of possible candidates. Currently, the most generally accepted being the ΛCDM model, which contains, among others, parameters dealing with the existence of a cosmological constant (Λ) and cold dark matter (CDM). Furthermore, the existence of the dark matter component of this cosmology, and others, is not that which is generally considered contentious. Dark matter has rather become somewhat of a staple in the diet of cosmologies. However, there are observational reasons to give pause to the assumed existence of Universal cold dark matter, which then should lead us to question whether or not there are other alternative models.

Models of dark matter succeed in accounting for the galactic rotation curves observed throughout the Universe, by increasing the mass of the galaxy beyond the observed. There are, however, simple problems with the dark matter halo model that have yet to be fully explained (e.g. [3-7]). One of these problems is the disparity between the observed (stellar) mass function, usually defined in terms of the Schechter function, and the theoretically expected cosmological halo mass function [8]. One of the defining problems of galactic formation and evolution is determining the origins of this disparity. This is an example of how our understanding of dark matter (or lack there of) is still grounds for much debate. However, if it may be possible to utilise a different model for the origin of galactic formation then it is possible that some of these questions may be answered.

Recently, there has been some observational evidence for the "bulk flow" of matter through the Universe [9-13]. If these measurements prove to be true, then the nature of this flow is of interest beyond that of the distribution of matter in the Universe. Specifically relevant to this discussion is the interpretation that the "flow" observed is caused not by an *en masse* transit of matter through the Universe, but rather by the motion of spacetime itself. Whilst this concept is indeed foreign it can be considered somewhat preferential to the former case, from an isotropic viewpoint, as the motions of objects in the Universe need not be preferentially oriented in this regime. More importantly, variation in the "bulk flow" of spacetime fluid through the Universe could produce eddies in spacetime and provide the additional unexplained velocity to rotational speeds of stars beyond the central bulge of galaxies.

As the intention of this paper is to merely propose an alternative theoretical explanation for observations consistent with the existence of dark matter the structure is as follows: Section 2 describes the treatment of spacetime as a fluid. Sections 3 and 4 discuss classical fluid dynamics and relativistic fluid dynamics and how votices in the spacetime "fluid" can produce observations consisten with dark matter. Finally we present concluding remarks and propose future work in Section 5.

2. Spacetime as a Fluid and the Differential Rotation of Spacetime

In General Relativity it is common for theories to compare the nature of the spacetime coordinate system to an ideal fluid, often called the "cosmic fluid". This treatment is integral for the formulation of many concepts in GR, including the formation and propagation of gravitational waves. Additionally, Kerr-Newman geometry for a rotating black hole, which predicts an effective "differencial rotation" of spacetime. The principle effect regarding black hole studies is the implication that, within $r_0 \equiv M + \sqrt{M^2 - a^2\cos^2\theta}$, it is impossible to remain stationary with respect to distant "stationary" observer beyond r_0.

The Kerr-Newman geometery has been used to successfully describe the light curves of rapidly rotating neutron stars [14]. Doppler boosting and time-delay-induced pertubations from frame-dragging cause "soft lags" in pulse profiles of neutron stars which have been measured in X-ray spectra.

As a starting point for our proposition we use this geometry, simply as an example of how the differential rotation of spacetime is possible. We leave all other black-hole allegories or implications of a large, dense, rotating mass located at the "centre of the Universe" behind.

3. Vortices in Classical Fluid Dynamics

If spacetime is able to experience a form of differential rotation, then it is of interest to examine how this would impact the fluid treatment we noted earlier.

Classical fluid dynamics states that eddies/vortices with angular velocity ω^{vort} occur for any differential flow as:

$$\omega^{vort} = \nabla \times \mathbf{v} \tag{1}$$

and

$$\frac{\partial \omega^{vort}}{\partial t} = \nabla \times \left(\omega^{vort} \times \mathbf{v} \right).$$

Due to the chaotic nature of this relationship, when applied to astronomical distances and timescales even small perturbations can ultimately produce large scale phenomena.

Essentially, if we treat spacetime as a classical fluid with turbulence caused by a differential flow as observed by Osborne *et al.* [12] (and others), spatial variations in the flow would generate eddies that could not only host galaxies, but adequately explain their rotational dynamics as well.

Treating spacetime as a classical fluid, and assuming the velocity of any "bulk flow" is $\mathbf{v} = v_x(x,y)\hat{x} + v_y(x,y)\hat{y}$, we find $\omega_z^{vort} = \left(\delta v_x^2 + \delta v_y^2\right)^{1/2}$ where $\delta v_i = \partial v_i / \partial x_{j\neq i}$. van Albada *et al.* [15] utilised light curves to show that rotational velocities of stars deviated from Newtonian mechanics by ~ 50 km/s at a radial distance of ~ 20 kpc from the galactic centre. We find that, to account for this discrepancy $\Delta v \equiv \left(\delta v_x^2 + \delta v_y^2\right)^{1/2}$, $\omega^{vort} = \Delta v \sim 3\,\mathrm{kms^{-1}kpc^{-1}}$ for observed velocities of the outer most stars of a galaxy which correspond to the largest values of Δv. So, given some bulk flow through in the local Universe, there only need be a systematic variation of $\sim 3\,\mathrm{kms^{-1}kpc^{-1}}$ within the flow to produce an eddy large enough to provide the unaccounted velocity to the outermost stars of a galaxy. Furthermore, as the velocity of the eddy is dependent on radius from the centre of the eddy (*i.e.* the galactic centre), the effect will diminish as the radius from the galactic centre decreases. Accounting for the increase in observable galactic mass with decreasing radius, it can be supposed that as one approaches the galactic centre the motion of stars becomes primarily governed by gravitation. Formulated simply: as $r \to r_c$, $v_* \to v_N$, where r_c is the radius of the galactic "bulge" and v_N is the orbital speed of stars predicted by Newtonian mechanics.

In this way, we propose that the observed motion of galactic stars over the entire disk may be explained by the presence of eddies in spacetime caused by appreciably small variations in "bulk flow".

Thus far we have treated spacetime as a classical fluid, but as we are dealing with distortions of spacetime relativistic effects must be addressed. Greenberg found that, independent of any relativistic geometry used, the angular velocity 2-form of an eddy/vortex is

$$\omega_{\alpha\beta} = \frac{1}{2}\left(u_{\alpha;\beta} - u_{\beta;\alpha}\right) \tag{2}$$

where u_β is the velocity 4-vector of the "fluid" [16]. The most noticeable difference between Equations (1) and (2) is the coupled nature of spacetime. Therefore any changes in the "flow" over time could also produce vortices in spacetime, much like those produced by variations in differential flow in classical dynamics. Coupling temporal and spatial variations could further enhance the turbulent nature of spacetime, and thus the production of eddies where galaxies could grow.

4. Relativis Fluid Dynamics

Examining the relativistic case, we can choose a reference frame such that $u_t \neq 0$, and the spatial components u_i are all zero. The reasoning behind this is that, for an external observer, the motion of a "fluid" through a stationary reference frame is observationally indifferent to a static "fluid" in a co-moving reference frame. Using this restriction, Equation (2) becomes

$$\omega_{ti} = -\omega_{it} = \frac{1}{2} u_{t;i} \tag{3}$$

Since this essentially a 2-form for a vortex in spacetime, the magnitude of the resulting rotation would be

$$\omega^2 = \omega_{\alpha\beta}\omega^{\alpha\beta} = \omega_{\alpha\beta} g^{\alpha\gamma} g^{\beta\delta} \omega_{\gamma\delta}, \tag{4}$$

which is the magnitude of the rotation for the vortex, squared. Since it contains products of $\omega_{\mu\nu}\omega_{\nu\mu}$, then, in principle, ω^2 can be non-zero. Additionally, the covariant derivative embedded in $\omega_{\mu\nu}$ provides us with information on variations within $g_{\mu\nu}$ (spacetime) that yield $\omega^2 \neq 0$, possibly explaining observed phenomena.

We reserve the comprehensive mathematical analysis for a forthcoming paper and merely present a simplified "glimpse" of the interpretation of Equation (4). If a vortex in spacetime is to be used as a possible explanation for the observed rotational velocities of stars, and on average, these velocities are independent of angular position within the galaxy as well as vertical position within the disc, we have assumed that $u_{t;\phi} = u_{t;z} = 0$, hence

$$\begin{aligned} \omega^2 &= 2\omega_{rt}^2 \left[g^{tt} g^{rr} - \left(g^{tr} \right)^2 \right] \\ &= \frac{1}{4} \left(\partial_r u_t - \Gamma^t_{rt} u_t \right)^2 \left[g^{tt} g^{rr} - \left(g^{tr} \right)^2 \right], \end{aligned} \tag{5}$$

because $g^{tr} = g^{rt}$ and $\omega_{tr} = -\omega_{rt}$. This can be used as a restriction for properties of $g_{\mu\nu}$ and expand upon existing geometries such as the Kerr-Newman and/or Friedmann-Robertson-Walker metrics, which will be presented in subsequent publications. Here we merely present evidence that small perturbations in $g_{\mu\nu}$ and u_t may be used to explain the rotational speeds of stars in other galaxies.

Assuming the metric for the local spacetime is essentially flat, $g_{\mu\nu} = \delta_{\mu\nu} + \delta g_{\mu\nu}$, as well as the variations in u_t $\delta u_t \equiv \partial_r u_t$ and the variations in g as $\delta g u_t \equiv \Gamma^t_{rt} u_t$, we can approximate the magnitudes of these "variations" that could explain observed phenomena. Since we are assuming cross terms are small and $g^{tt} \sim g^{rr} \sim 1$, $\omega^2 \sim \left(\delta u_t - \delta g u_t \right)^2$. In the extreme case where the rotational speed of stars for a galaxy, as predicted by theory is zero, $v_* = \omega r_?$ or $\omega = v_* / r_? \sim 10^{-4}\,\mathrm{kms^{-1}\,Lyr^{-1}}$ and that $\delta u_t = 0$ implying that the stellar motion is purely from variations in the geometry. Coupling this with the afore mentioned indifference of motion through spacetime and spacetime moving with the "fluid" we can approximate the magnitude of δg. Based on observations of Osborne *et al.* [12], we assume that $u_t \sim 100\,\mathrm{kms^{-1}}$. Therefore, we determine that $\delta g \sim 10^{-6}\,\mathrm{Lyr^{-1}}$ could produce effects consistent with stellar rotational velocities observed. Additionally, Osborne *et al.* [12] also found that the "flow" changed by $\sim 50\,\mathrm{kms^{-1}}$ between the redshifts of 0.4 and 0.8. Again, if this is largely due to variations in the geometry of spacetime, this would imply $\delta g \sim 10^{-8}\,\mathrm{kms^{-1}}$. Though this is smaller than the previous approximation, that approximation did not account for the motion of stars from Newtonian mechanics.

5. Summary

If the observed effects attributed to dark matter are indeed caused by the turbulent flow of spacetime, then we can simply hypothesise that any galaxy in a cluster formed in this way should all rotate in the same direction. Albeit only significant to 1.6σ, evidence for this effect has been detected by Longo *et al.* [17]. There is no reason to expect that this observation would also be caused in the dark matter Regime for reasons other than chance. As this is a simple method to determine if turbulent spacetime flows may be the cause of galactic rotational curves, Doppler measurements of stellar velocities in galaxies are extremely important. Furthermore, if rotational curves for distant galaxies can be found, isotropy measurements could serve as an additional constraint for the validity of this theory.

Finally, we find that chaotic flows could exist on both "large" and "small" scales. Large scale turbulence is dominated by the "flow" velocity and the uniformity thereof. In this context, "large" scale turbulence would be on a galactic scale, with "large" eddies being comparable to the size of a galaxy, which could be used to explain why galaxies are not "sheared" apart. Small scale turbulence is dominated by the viscosity of the flow, which would most likely be caused by the gravitational attraction of masses present in the region of the eddy, as mentioned previously in relation to the galactic rotation curves. The scale of a "small eddy" would be comparable to that of stellar clusters. Since these clusters are still affected by distortions in spacetime, this could explain the observed gravitational lensing caused by some stellar clusters that could not be explained by modified Newtonian dynamics (MOND) [18].

This concept can, therefore, effectively explain the major observations that lead to the introduction of dark matter, and removes the need for the existence of a massive dark matter halo about galaxies. If this theory is correct, galaxies co-located within a differentially moving frame, should all rotate in the same direction (*i.e.* same chirality). Additionally, when observed from an external reference frame, there should be variations in the local spacetime of a galaxy $\delta g u_t \sim 10^{-4}\,\mathrm{kms}^{-1}\,\mathrm{Lyr}^{-1}$ where u_t is the observed "flow" of the surrounding galaxies.

Finally can now draw some different, interesting, conclusions from the disparity between the observed stellar mass function and the cosmological halo mass function. Using the model we propose here, the origins of galactic evolution lie not in vast halos of dark matter, but rather in the turbulence of spacetime. The turbulence itself traces back its origins to spatial and temporal variations in the motion of spacetime. If we are able to conceptualise this method of seeding galaxies, then we can also recognise that the formation of the vortices in which galaxies originate is highly dependent on the mechanics of the local spacetime. This, when interpreted simply, means that the expected distribution of galactic mass (*i.e*. the observed stellar mass function) should not be conformal to a simplistic power law (*i.e*. the cosmological halo mass function), but rather should be more complex in nature. The true distribution of vortex sizes in a field of uniform variation should be expected to be inately coupled with galactic mass. This may well explain the shape of the Schechter function, and why galactic mass function does not follow a simple power law, without the need for complex models to explain the reduction of star formation in the low and high mass dark matter halo regimes.

As stated above, many of these conclusion are too complicated to present in any detail within this paper, of which will be presented in following papers. The purpose of this paper is to merely propose an alternate explanation for the observed effects of dark matter and a possible explanation for why dark matter is not consistent with some other theories and observation.

6. Acknowledgements

The authors would like to thank Graeme Salter, Jonathan Horner, Annant Tanna and Bradley Hansen for insightful discussions regarding the formulation of this theory. FEK would also like to thank UNSW and the ARC for financial support used during the development of this theory under grants PS27228-FRGP, PS27261-ECR and DP-120105045.

REFERENCES

[1] T. A. Porter, R. P. Johnson and P. W. Graham, "Dark Matter Searches with Astroparticle Data," *Annual Review of Astronomy and Astrophysics*, Vol. 49, 2011, pp. 155-194. doi:10.1146/annurev-astro-081710-102528

[2] P. J. E. Peebles, "Principles of Physical Cosmology," 1993.

[3] A. Klypin, A. V. Kravtsov, O. Valenzuela and F. Prada, "Where Are the Missing Galactic Satellites?" *Astrophysical Journal*, Vol. 522, No. 1, 1999, pp. 82-92. doi:10.1086/307643

[4] S. Garbari, J. I. Read and G. Lake, "Limits on the Local Dark Matter Density," *Monthly Notices of the Royal Astronomical Society*, Vol. 416, No. 3, 2011, pp. 2318-2340. doi:10.1111/j.1365-2966.2011.19206.x

[5] I. D. Karachentsev, "Missing Dark Matter in the Local Universe," *Astrophysical Bulletin*, Vol. 67, No. 2, 2012, pp. 123-134. doi:10.1134/S1990341312020010

[6] V. Poitras, "Constraints on _(t)-Cosmology with Power Law Interacting Dark Sectors," *Journal of Cosmology and Astroparticle Physics*, Vol. 6, 2012, p. 39. doi:10.1088/1475-7516/2012/06/039

[7] D. Sluse, V. Chantry, P. Magain, F. Courbin and G. Meylan, "COSMOGRAIL: The Cosmological Monitoring of Gravitational Lenses. X. Modeling Based on High-Precision Astrometry of a Sample of 25 Lensed Quasars: Consequences for Ellipticity, Shear, and Astrometric Anomalies," *A & A*, Vol. 538, 2012, p. A99.

[8] B. P. Moster, R. S. Somerville, C. Maulbetsch, F. C. van den Bosch, A. V. Macci`o, T. Naab and L. Oser, "Constraints on the Relationship between Stellar Mass and Halo Mass at Low and High Redshift," *Astrophysical Journal*, Vol. 710, No. 2, 2010, pp. 903-923. doi:10.1088/0004-637X/710/2/903

[9] B. A. Benson, S. E. Church, P. A. R. Ade, J. J. Bock, K. M. Ganga, J. R. Hinderks, P. D. Mauskopf, B. Philhour, M. C. Runyan and K. L. Thompson, "Peculiar Velocity Limits from Measurements of the Spectrum of the Sunyaev Zeldovich Effect in Six Clusters of Galaxies," *Astrophysical Journal*, Vol. 592, 2003, pp. 674-691.

[10] S. Bhattacharya and A. Kosowsky, "Dark Energy Constraints from Galaxy Cluster Peculiar Velocities," *Physical Review D*, Vol. 77, 2008, Article ID: 083004. doi:10.1103/PhysRevD.77.083004

[11] H. A. Feldman, R. Watkins and M. J. Hudson, "Cosmic Flows on 100 h-1 Mpc Scales: Standardized Minimum Variance Bulk Flow, Shear and Octupole Moments," *Monthly Notices of the Royal Astronomical Society*, Vol. 407, No. 4, 2010, pp. 2328-2338. doi:10.1111/j.1365-2966.2010.17052.x

[12] S. J. Osborne, D. S. Y. Mak, S. E. Church and E. Pierpaoli, "Measuring the Galaxy Cluster Bulk Flow from WMAP Data," *Astrophysical Journal*, Vol. 737, No. 2, 2011, p. 98. doi:10.1088/0004-637X/737/2/98

[13] S. J. Turnbull, M. J. Hudson, H. A. Feldman, M. Hicken, R. P. Kirshner and R. Watkins, "Cosmic Flows in the nearby Universe from Type Ia Supernovae," *Monthly Notices of the Royal Astronomical Society*, Vol. 420, No. 1, 2012, pp. 447-454. doi:10.1111/j.1365-2966.2011.20050.x

[14] T. M. Braje, R. W. Romani and K. P. Rauch, "Light Curves of Rapidly Rotating Neutron Stars," *Astrophysical Journal*, Vol. 531, No. 1, 2000, pp. 447-452. doi:10.1086/308448

[15] T. S. van Albada and R. Sancisi, "Dark Matter in Spiral Galaxies," *Royal Society of London Philosophical Transactions Series A*, Vol. 320, No. 1556, 1986, pp. 447-464. doi:10.1098/rsta.1986.0128

[16] C. W. Misner, K. S. Thorne and J. A. Wheeler, "Gravitation," W.H. Freeman and Co., San Francisco, 1973.

[17] M. J. Longo, "Detection of a Dipole in the Handedness of Spiral Galaxies with Redshifts z ~ 0.04," *Physics Letters B*, Vol. 699, No. 4, 2011, pp. 224-229. doi:10.1016/j.physletb.2011.04.008

[18] R. Ibata, A. Sollima, C. Nipoti, M. Bellazzini, S. C. Chapman and E. Dalessandro, "The Globular Cluster NGC 2419: A Crucible for Theories of Gravity," *Astrophysical Journal*, Vol. 738, No. 2, 2011, p. 186. doi:10.1088/0004-637X/738/2/186

Journal of Modern Physics, **2012, 3, 1128-1130**
doi:10.4236/jmp.2012.329147 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

How Dark Energy Affects the MOND Theory in Clusters

Man Ho Chan
Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Hong Kong, China
Email: mhchan@phy.cuhk.edu.hk

Received June 25, 2012; revised July 23, 2012; accepted August 1, 2012

ABSTRACT

Modified Newtonian Dynamics (MOND) is one of the successful theories to explain the dark matter problem in galaxies. However, the data from clusters and the cosmic microwave background (CMB) indicate some dark matter should exist in larger scales. In addition, recent dynamical studies of clusters show that the effect of dark energy should not be ignored in cluster scale. In this article, I will demonstrate how dark energy affects the cluster mass calculation by using MOND. Also, I will show that the calculated cluster mass is consistent with the total matter to baryonic matter ratio obtained by the CMB data.

Keywords: Gravitation-Galaxies; Clusters; General-Dark Matter

1. Introduction

The dark matter problem is one of the key issues in modern astrophysics. The existence of cold dark matter (CDM) particles is the generally accepted model to tackle the darkmatter problem. However, no such particles have been detected directly. In addition, the CDM model also encounter many well-known unresolved issues such as the cusp problem [1,2], the missing satellite problem [3] and more recently the observation of the tidal dwarf galaxies [4]. Another alternative theory uses the Modified Newtonian Dynamics (MOND) as the weak acceleration-limit of Einstein's general relativity to explain the dark matter problem [5-7]. It is consistent with a wide range of observational data including the rotation curves of galaxies and the Tully-Fisher relation [6]. However, the recent data from gravitational lensing and hot gas in clusters challenge the original idea of MOND without any dark matter (classical MOND) [7-9]. Sanders (1999) studied 93 X-ray emitting clusters and pointed out that the cluster dark matter problem cannot be solved by MOND alone. Some 2 eV active neutrinos are needed to account for the missing mass in clusters [10]. Later, studies of gravitational lensing and hot gas in clusters show that the existence of 2 eV neutrinos is still not enough to explain the missing mass in clusters. Therefore, some more massive dark matter particles (e.g. sterile neutrinos) is required to account for the missing mass [9,11,12]. It can be shown that the equilibrium configuration of these sterile neutrinos is consistent with the missing mass in clusters [13]. On the other hand, the data from the Cosmic Microwave Background (CMB) indicate a large amount of dark matter is needed to explain the CMB spectral shape [14]. Angus (2009) shows that the existence of ~11 eV neutrinos is consistent with the CMB data and the analytic results of the Miniboone experiment [15]. Therefore, the mainstream of the discussion in MOND recently is not opposing the existence of dark matter, but the existence of CDM [16].

It has recently been recognized that dark energy exists in our universe. Angus (2009) shows that if MOND theory is needed to satisfy the fitting in CMB spectrum, a large amount of dark energy is required. Therefore, both CDM and MOND theories should consider the effect of dark energy. The local dynamic effects of dark energy were first reported by Chernin, Teerikorpi and Baryshev (2003) [17]. Later, Bisnovatyi-Kogan and Chernin (2012) show that the dark energy may affect the clusters at a few Mpc scale by Newtonian dynamics [18]. In the MOND regime, the calculated cluster mass is smaller than the one calculated by the Newtonian dynamics [6]. Therefore, the effect of the dark energy in clusters under the MOND theory will be larger. In this article, I will demonstrate how dark energy affects the cluster mass calculation by using MOND. Also, I will show that the calculated cluster mass is consistent with the total matter to baryonic matter ratio obtained by the CMB data.

2. MOND with Dark Energy in Clusters

The effective gravitational acceleration in MOND is given by [5,6]

$$g = \sqrt{g_n a_0} \tag{1}$$

when $g \ll a_0$, where g_n is the Newtonian gravity and $a_0 = 1.2 \times 10^{-8}\ \text{cm}\cdot\text{s}^{-2}$. If we assume that the hot gas with uniform temperature T in cluster is a pressure supported system, we have [6]

$$\left(\frac{kT}{m_g}\right)\frac{\mathrm{d}\rho}{\mathrm{d}r} = -\rho g' \quad (2)$$

where m_g is the mass of a gas particle, ρ is the density profile of the hot gas and g' is the total gravity of the system. The global dark energy density is $\rho_\lambda = 7 \times 10^{-30}$ g·cm^{-3} [18]. This dark energy density contributes to the antigravity in the system. Since there are no MOND effects before recombination, no MOND effects should influence the CMB [7]. Therefore, the amount of dark energy should be the same for Newtonian and MOND limit. The major difference is the effect of the dark energy in MOND limit may be smaller than that in the Newtonian limit. Therefore, the anti-gravity in the MOND regime is

$$g_\lambda = -\sqrt{\frac{8\pi G a_0 r \rho_\lambda}{3}} \quad (3)$$

Since the total gravity can be written as $g' = g + g_\lambda$, by using Equations (1)-(3), we have

$$\left(\frac{kT}{m_g}\right)\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} = -\sqrt{GMa_0} + \sqrt{\frac{8\pi G a_0 r^3 \rho_\lambda}{3}} \quad (4)$$

where M is the total mass of the cluster. By using the gas model in clusters for large r, $\mathrm{d}\ln\rho/\mathrm{d}\ln r \approx -3\beta$ [19]. Therefore, from the above equation, we get

$$M = \frac{1}{Ga_0}\left(\frac{3\beta kT}{m_g} + \sqrt{\frac{8\pi G a_0 r^3 \rho_\lambda}{3}}\right)^2 = M_m\left(1 + \frac{C_2}{C_1}\right)^2 \quad (5)$$

where $M_m \approx 6 \times 10^{12}\ M_\odot(T/1\ \mathrm{keV})^2$ is the total cluster mass without dark energy in classical MOND [10], $C_1 = 3\beta kT/mg$ and $C_2 = \sqrt{8\pi G a_0 \rho_\lambda r^3/3}$. For a typical cluster, $\beta = 0.66$, $T = 5 \times 10^7$ K and $r = 1.5$ Mpc, $C_2/C_1 \approx 0.17$. Therefore, the total cluster mass is $1.17^2 \approx 1.4$ times larger than the one calculated by the classical MOND. For larger clusters, the effect of dark energy will be much more significant. Therefore, the cluster mass probed from the hot gas by MOND is underestimated if we do not consider the dark energy. It means that more dark matter should exist in clusters.

In fact, observational data shows that the mass of hot gas can be fitted empirically by [20]

$$M_g \approx 1.7 \times 10^{12} M_\odot \left(\frac{T}{1\ \mathrm{keV}}\right)^2 \quad (6)$$

where M_g is the total mass of hot gas in a cluster. Therefore, the predicted cluster mass by the classical MOND is 3.5 times larger than the observed baryonic mass ($M_m/M_g \approx 3.5$) [10]. Sanders (2007); Angus, Famaey and Buote (2008) propose that the existence of active or sterile neutrinos in clusters may account for the missing mass. Angus (2009) obtains a good fit to the CMB spectrum by assuming all non-baryonic matter is composed by the acitve and sterile neutrinos. The fitted cosmological density parameters of baryons and matter are $\Omega_b h^2 = 0.0024$ and $\Omega_m h^2 = 0.117$ respectively [7]. Therefore, we have $\Omega_m/\Omega_b \approx 5$. This ratio is indeed larger than the ratio predicted by the classical MOND. Nevertheless, if we include the effect of dark energy by using the Equation (5) for a typical cluster, the ratio becomes $M_m/M_g = 1.17^2\ M_m/M_g \approx 4.8$, which is very closed to the ratio obtained by the cosmological density parameters. Therefore, the calculated ratio of total matter to baryonic matter in clusters by using MOND matches the result of the CMB if we include the effect of dark energy.

3. Discussion and Conclusions

In this article, we consider the effect of dark energy in clusters. In the MOND regime, the contribution of the anti-gravity effect by dark energy density is significant to the total cluster mass calculation. The total cluster mass for a typical cluster can be 40% larger than the one calculated by the classical MOND. It represents a larger amount of dark matter should exist in clusters. Therefore, the existence of 2 eV active neutrinos in clusters is not enough to account for the missing mass. Since more massive active neutrinos (>2 eV) may violate the experimental bounds [10], the existence of sterile neutrinos are required for the explanation in MOND. On the other hand, the calculated total mass to baryonic mass ratio is consistent with the cosmological data from the CMB spectrum if we include the effect of dark energy.

Since the CDM scenario encounters many fundamental problems including the cusp and the missing satellite problem, the MOND together with the existence of sterile neutrino hot dark matter (HDM) is the only theory which can retain in the recent challenges. Since neutrinos contain mass, there should exist right-handed neutrinos which may indeed be the massive sterile neutrinos [21]. The existence of eV order sterile neutrinos can explain the missing mass in the clusters and our universe [7]. Also it can explain the recent analysis of the Miniboone experiment and get a good fit in the CMB spectrum [7, 15]. The free stremaing scale of the eVorder sterile neutrinos is $\lambda \sim$Mpc [10], which can form structure in clusters and contribute to the total mass in clusters. Since the free streaming scale is larger than the size of a typical galaxy, no hot dark matter can form detectable structure within galaxies. As a result, the classical MOND alone is able to explain the rotation curves of galaxies without the help of HDM.

To conclude, the existence of dark energy can affect the calculated cluster mass by MOND significantly. Also, the MOND + HDM scenario may be one of the best theories to explain the dark matter problem in the future.

REFERENCES

[1] G. Gentile, P. Salucci, U. Klein, D. Vergani and P. Kalberla, "The Cored Distribution of Dark Matter in Spiral Galaxies," *Monthly Notices of the Royal Astronomical Society*, Vol. 351, No. 3, 2004, pp. 903-922. doi:10.1111/j.1365-2966.2004.07836.x

[2] G. Gilmore, M. Wilkinson, J. Kleyna, A. Koch, W. Evans, R. F. G. Wyse and E. K. Grebel, "Observed Properties of Dark Matter: Dynamical Studies of dSph Galaxies," *Nuclear Physics B*: *Proceedings Supplements*, Vol. 173, 2007, pp. 15-18. doi:10.1016/j.nuclphysbps.2007.08.143

[3] B. Moore, S. Ghigna, F. Governato, G. Lake, T. Quinn, J. Stadel and P. Tozzi, "Dark Matter Substructure within Galactic Halos," *The Astrophysical Journal Letters*, Vol. 524, No. 1, 1999, p. L19. doi:10.1086/312287

[4] M. Milgrom, "MOND and the Mass Discrepancies in Tidal Dwarf Galaxies," *The Astrophysical Journal Letters*, Vol. 667, No. 1, 2007, p. L45. doi:10.1086/522049

[5] M. Milgrom, "A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis," *Astrophysical Journal*, Vol. 270, 1983, pp. 365-370. doi:10.1086/161130

[6] R. H. Sanders and S. S. McGaugh, "Modified Newtonian Dynamics as an Alternative to Dark Matter," *Annual Review of Astronomy and Astrophysics*, Vol. 40, 2002, pp. 263-317. doi:10.1146/annurev.astro.40.060401.093923

[7] G. W. Angus, "Is an 11 eV Sterile Neutrino Consistent with Clusters, the Cosmic Microwave Background and Modified Newtonian Dynamics?" *Monthly Notices of the Royal Astronomical Society*, Vol. 394, No. 1, 2009, pp. 527-532. doi:10.1111/j.1365-2966.2008.14341.x

[8] R. H. Sanders, "The Virial Discrepancy in Clusters of Galaxies in the Context of Modified Newtonian Dynamics," *The Astrophysical Journal Letters*, Vol. 512, No. 1, 1999, p. L23. doi:10.1086/311865

[9] P. Natarajan and H. Zhao, "MOND Plus Classical Neutrinos Are Not Enough for Cluster Lensing," *Monthly Notices of the Royal Astronomical Society*, Vol. 389, No. 1, 2008, pp. 250-256. doi:10.1111/j.1365-2966.2008.13552.x

[10] R. H. Sanders, "Neutrinos as Cluster Dark Matter," *Monthly Notices of the Royal Astronomical Society*, Vol. 380, No. 1, 2007, pp. 331-338. doi:10.1111/j.1365-2966.2007.12073.x

[11] R. Takahashi and T. Chiba, "Weak Lensing of Galaxy Clusters in Modified Newtonian Dynamics," *The Astrophysical Journal*, Vol. 671, No. 1, 2007, p. 45. doi:10.1086/522564

[12] G. W. Angus, B. Famaey and D. A. Buote, "X-Ray Group and Cluster Mass Profiles in MOND: Unexplained Mass on the Group Scale," *Monthly Notices of the Royal Astronomical Society*, Vol. 387, No. 4, 2008, pp. 1470-1480. doi:10.1111/j.1365-2966.2008.13353.x

[13] G. W. Angus, B. Famaey and A. Diaferio, "Equilibrium Configurations of 11 eV Sterile Neutrinos in MONDian Galaxy Clusters," *Monthly Notices of the Royal Astronomical Society*, Vol. 402, No. 1, 2010, pp. 395-408. doi:10.1111/j.1365-2966.2009.15895.x

[14] D. N. Spergel, *et al*., "Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology," *The Astrophysical Journal Supplement Series*, Vol. 170, No. 2, 2007, p. 377. doi:10.1086/513700

[15] M. Maltoni and T. Schwetz, "Sterile Neutrino Oscillations after First MiniBooNE Results," *Physical Review D*, Vol. 76, 2007, Article ID: 093005.

[16] M. Milgrom, "MD or DM? Modified Dynamics at Low Accelerations vs Dark Matter," *Proceedings of Science* (*Quarks*, *Strings and the Cosmos—Hector Rubinstein Memorial Symposium*), Stockholm, 9-11 August 2010.

[17] A. Chernin, P. Teerikorpi and Y. Baryshev, "Why Is the Hubble Flow So Quiet?" *Advances in Space Research*, Vol. 31, No. 2, 2003, pp. 459-467. doi:10.1016/S0273-1177(02)00731-7

[18] G. S. Bisnovatyi-Kogan and A. D. Chernin, "Dark Energy and Key Physical Parameters of Clusters of Galaxies," *Astrophysics and Space Science*, Vol. 338, 2012, p. 337.

[19] I. King, "Density Data and Emission Measure for a Model of the Coma Cluster," *The Astrophysical Journal*, Vol. 174, 1972, p. L123. doi:10.1086/180963

[20] J. J. Mohr, B. Mathiesen and A. E. Evrard, "Properties of the Intracluster Medium in an Ensemble of Nearby Galaxy Clusters," *The Astrophysical Journal*, Vol. 517, No. 2, 1999, p. 627. doi:10.1086/307227

[21] A. A. Aguilar-Arevalo, *et al*., "Search for Electron Neutrino Appearance at the $\Delta m^2 \sim 1\ \mathrm{eV}^2$ Scale," *Physical Review Letters*, Vol. 98, 2007, Article ID: 231801.

Journal of Modern Physics, **2012, 3, 1131-1141**
doi:10.4236/jmp.2012.329148 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

On Supersymmetry and the Origin of Dark Matter

Shawqi Al Dallal[1], Walid J. Azzam[2*]
[1]College of Graduate Studies and Research, Ahlia University, Manama, Bahrain
[2]Department of Physics, College of Science, University of Bahrain, Sakhir, Bahrain
Email: *wjazzam@gmail.com

Received June 22, 2012; revised July 20, 2012; accepted July 29, 2012

ABSTRACT

Dark matter was first suspected in clusters of galaxies when these galaxies were found to move with too high a speed to be retained in the cluster by their gravitational influence on each other. Some current theories favor cold dark matter models where particles are created with low velocity dispersions and thus would become trapped in baryonic gravitational potentials. According to the standard Big-Bang model, dark matter is of nonbaryonic origin, otherwise the observed abundance of helium in the Universe would be violated. In this work, recent theoretical and observational developments are used to form a consistent picture of the events in the early Universe that gave rise to dark matter. According to the model that will be presented in this paper, supersymmetry plays a major role. In addition, the possibility that dark matter evolves in a spacetime manifold different from that of the observed Universe is discussed.

Keywords: Dark Matter; Observations; Candidates; Supersymmetry

1. Introduction

During the last two decades, dark matter (DM) has occupied a pioneering position in research concerning cosmology and theoretical physics. There is compelling evidence that about 90% of the mass of the Universe is invisible, that is, it neither emits nor absorbs electromagnetic radiation. Data implying nonluminous matter surfaced in the 1930s. The first glimpse came when Oort analyzed the Doppler shift in the spectra of stars in the galactic disk and concluded that the mass of visible stars cannot explain the amount of gravitating matter implied by the measured velocities [1]. This finding was then confirmed by Zwicky who, by observing the Coma Cluster of galaxies, concluded that the velocity dispersions in rich clusters of galaxies require a huge amount of mass to keep them bound than could be accounted for by the luminous galaxies themselves [2]. The strongest evidence for dark matter comes from studies of the mass of individual galaxies. Mass estimates of individual galaxies can be obtained from the velocity dispersions or rotation curves of stars and gas making up the galaxy itself, or from the positions and velocities of globular clusters and satellite galaxies. Although these rotation curves trace the disk, several lines of argument strongly suggest that much of the nonluminous mass is in the spherical component that makes up the galactic halo [3-7].

Several models have been proposed to explain the origin of dark matter. Weakly interacting massive particles (WIMPs) and massive compact halo objects (MACHOs) are the prime candidates [8,9]. The proposal of WIMPs as a potential candidate is motivated by the fact that primordial nucleosynthesis provides only 0.2 of the cosmological density parameter ($\Omega_0 = \rho/\rho_C = 0.2$), whereas inflationary theory and observational evidence suggest $\Omega_0 = 1$ [10-12]. The MACHOs' approach is based on finding a more natural explanation for DM using microlensing techniques. This approach, as we are going to see, may prove to be an invaluable tool for investigating the possibility of DM clumps in the galactic halo. Other potential DM candidates, such as primordial black holes and holeums, will also be discussed. Modified Newtonian dynamics (MOND) was a new approach that was proposed as a modification of Newton's law of gravity to explain the galactic rotation curves [13], and it will be briefly described in this paper.

Recently, new strategies have been developed to directly observe DM by mapping its distribution in the Universe through its gravitational interaction with ordinary matter. Moreover, models and observational data give strong hints about the origin of DM [14,15].

The primary motivation for this paper is twofold: 1) to highlight the difficulties associated with the existing models that attempt to explain the nature and origin of DM. 2) to discuss some possible theoretical frames, supported by recent theoretical and observational data, in order to have a deeper understanding of the origin of DM and the associated problems.

*Corresponding author.

2. Observational Evidence for Dark Matter

Observational evidence for dark matter has a long history starting with the work of Jan Oort and Fritz Zwicky in the early 1930s. In 1932, while studying the spectra of nearby stars in the Milky Way Galaxy, Oort noticed that the Doppler shifts of these stars indicated velocities that were too high to be accounted for by ordinary visible matter. In fact, the stars were moving so fast that they should have escaped from the Galaxy. Since the stars were clearly bound to the galaxy, Oort suggested that there must be additional, nonvisible matter in our Galaxy. Shortly after that, in 1933, Fritz Zwicky reached a similar conclusion while studying the velocities of galaxies in the Coma Cluster [16]. By measuring the velocities of galaxies near the edge of the cluster and making use of the virial theorem which states that

$$-2\langle \mathrm{K}\rangle = \langle \mathrm{U}\rangle \tag{1}$$

where $\langle \mathrm{K}\rangle$ and $\langle \mathrm{U}\rangle$ are the system's mean kinetic and potential energies, respectively, and where it is understood that the system has reached equilibrium or steady-state, Zwicky was able to estimate the cluster's total mass, which turned out to be approximately 400 times that of the luminous mass. Thus, Zwicky concluded that there must be a huge amount of nonvisible matter holding the cluster together [16].

Much later, in the 1960s and 1970s, work by Vera Rubin and her collaborators led to new observational evidence for dark matter. Rubin investigated the rotation curves for edge-on spiral galaxies, and surprisingly found flat rotation curves that extended all the way to the edges of the galaxies [17]. To appreciate why these observations were completely unexpected, consider a star of mass m orbiting at a distance r from the center of a spiral galaxy with a velocity v. From Newtonian mechanics

$$mv^2/r = GM_r m/r^2 \tag{2}$$

where M_r is the galaxy's mass contained within the star's orbit, and G is the gravitational constant. Solving for M_r and then differentiating with respect to r, we obtain

$$\mathrm{d}M_r/\mathrm{d}r = v^2/G \tag{3}$$

But for a spherically symmetric system, the mass continuity equation gives

$$\mathrm{d}M_r/\mathrm{d}r = 4\pi r^2 \rho(r) \tag{4}$$

where $\rho(r)$ is the mass density as a function of orbital distance. By equating Equations (3) and (4) we obtain

$$\rho(r) = v^2/4\pi G r^2 \tag{5}$$

which indicates that the density varies as r^{-2}, whereas the observed number density of visible stars seems to fall off much more sharply, specifically as $r^{-3.5}$. To reconcile this discrepancy, one needs to invoke the existence of nonluminous or dark matter.

Building on the work of Rubin, other investigators studied the velocity dispersions of elliptical galaxies. The velocity dispersion, σ, refers to the range of velocities about some mean value. The results obtained by different groups were in line with those of Rubin, and again pointed to the existence of large amounts of dark matter [18].

Another source of observational evidence for the existence of dark matter comes from gravitational lensing, which does not rely on orbital dynamics but rather uses the effects of general relativity to predict the mass. The results obtained for the mass-to-light ratio are in agreement with those obtained from dynamical studies [19].

Further evidence for the existence of dark matter comes from Big Bang nucleosynthesis, structure formation studies, and investigations of the anisotropies in the Cosmic Microwave Background, especially those by COBE, Boomerang, and WMAP [20-23]. The WMAP power spectrum results for the Cosmic Microwave Background provided compelling evidence for the existence of dark matter. The first peak in the power spectrum is related to baryonic matter, whereas the third peak, which was resolved by WMAP, is directly related to the density of dark matter [24,25].

The above observations succeeded in ruling out certain models for structure formation like cosmic strings, and lent support to other theories like cosmic inflation. WMAP also succeeded in establishing the ΛCDM model which is currently considered the Standard Model of Cosmology. In this model, the universe is flat and is dominated by dark energy but with appreciable contributions from dark matter [24]. In this so-called concordance model, the total density parameter, Ω_0, has three contributions:

$$\Omega_0 = \Omega_b + \Omega_{DM} + \Omega_\Lambda \tag{6}$$

where the observed value for: the baryon density Ω_b = 0.04; for the dark matter density Ω_{DM} = 0.23; and for the dark energy density Ω_Λ = 0.73. Of course, for a flat universe, these contributions add up to give a total Ω_0 = 1.

3. WIMPs as DM Candidates

The interest in WIMPs as dark matter candidates arises from a combination of particle physics, astrophysics, and cosmological arguments. The main astrophysical motivation for WIMPs is the success of cold dark matter theory to explain the origin of galaxies and large scale structure of the Universe [26]. WIMPs, if they exist, are all stable particles. All current theories assume that galaxies and the structure of the Universe arise from the gravitational growth of density fluctuations. As mentioned earlier, these assumptions were verified by the Cosmic Background Explorer satellite (COBE), and more recently by the Wilkinson Microwave Anisotropy Probe (WMAP). The cosmological motivation for WIMPs is that their mass could be adjusted to give an Ω_{DM} that could even reach 1, something that has been referred to as the WIMP miracle. It requires a WIMP having a mass of the order of 10 GeV and an asymmetry equal to the baryon asymmetry [27]. WIMPs interact only through the gravitational and weak forces, and therefore they are considered as the primary candidates for dark matter.

3.1. Axions

Many hypothetical particles have been proposed as dark matter candidates. Among these is a hypothetical elementary particle called the axion that was first postulated in 1977 to resolve the strong CP (charge conjugation and parity) problem in quantum chromodynamics. Axions were considered as a potential candidate for cold dark matter. They have no electric charge, a very small mass, and an interaction cross section for both the strong and weak nuclear forces. Therefore, they interact only weakly with ordinary matter [28]. The axion mass is given by (see for example [29])

$$ma = \left\{\left[m_u m_d\right]^{1/2} \Big/ \left(m_u + m_d\right)\right\}\left(m_\pi f_\pi\right)\left(1/f_a\right) \tag{7}$$

where $m_u \approx 4$ MeV and $m_d = 8$ MeV are the up and down quark masses, respectively, and m_π = 135 MeV is the pion mass, f_a is the axion decay constant, and $f_\pi \approx 93$ MeV is the pion decay constant. Astrophysical constraints require that $f_a \geq 10^9$ GeV, implying an axion mass $m_a \leq 10$ MeV [30]. Axion theories predict that the universe would be filled with Bose-Einstein condensates of primordial axions, and thus plausibly explain the dark matter problem [31]. In 2005, it was thought that the PVLAS dark matter detector had received a signal due to axions. However, it was shown later that the PVLAS result was incorrect [32]. In 2009 some authors casted doubt on the existence of axions, arguing that cosmological observations imply that axions create a greater fine tuning problem than the one they are hypothesized to solve [33].

3.2. Cosmions

Another particle that was of interest three decades ago is the cosmion. It was proposed to solve the solar neutrino problem [34]. This particle acts as an efficient transporter of heat in the Sun's core and thereby reduces the emission rate of ^{8}B neutrinos. In order for the cosmion solution to work, its mass must be in the range of $4 \leq m_d \leq 10$ GeV [27]. It is clear that the relatively recent discovery of neutrino oscillations in the Sudbury Neutrino Observatory provides a natural way to explain the solar neutrino problem [35,36].

3.3. Supersymmetric Particles

Supersymmetric models provide a whole set of possible particles as DM candidates. Some supersymmetric particles were introduced by particle theories to solve problems entirely unrelated to the cosmology of dark matter. Among these is the lightest supersymmetric particle (LSP) which is a stable particle in models with R-parity conservation. If LSP exists, it may account for the observed missing mass of the Universe. In order to fit observations, LSP must be neutral, non-colored [37], interacts only through weak and gravitational interactions, and must have a mass of 100 GeV to 1 TeV. With these constraints, theoretical studies limit the LSP to either the gravitino, the sneutrino, or the neutralino, a mixture of neutral Majorana fermions, namely, the photino, the higgsino, and the zino [38,39]. In what follows, each of these particles will be discussed separately as a potential DM candidate.

3.3.1. Gravitino

The gravitino is the supersymmetric partner of the graviton. It has a spin of 3/2, and is not a WIMP. If it exists, it is the fermion mediating supergravity interactions. According to the Standard Model, the mass of the graviton must not exceed 1 TeV/c^2 [40]. The gravitino has a mass

$$m_{gravitino} = F \Big/ \left[3^{1/2} M^*_{pl}\right] \tag{8}$$

where F is the supersymmetry-breaking scale squared, and $M^*_{pl} = \left[8\pi G\right]^{-1/2} \approx 2.4 \times 10^{18}$ GeV. The gauge hierarchy problem requires that $F \approx (10^{11}\ \text{GeV})^2$, and therefore all the superpartners including the gravitino have a weak-scale mass [29]. Two possible options emerge from the stability status of the gravitino. In the first option, the gravitino is a stable dark matter candidate that obeys the *R*-parity conservation. In such a case, gravitinos would have been created in the very early universe. It turns out that the calculated density of stable gravitinos is much higher than the observed dark matter density [41]. The second option is that the gravitino is unstable. In this case, it will decay only through gravitational interaction with a lifetime of the order of $M^2_{pl} \big/ m^3$, without contributing to the observed dark matter density. In the above relation, $M_{pl} = hc/G = 1.2 \times 10^{19}$ GeV is the Planck mass, and m is the mass of the gravitino. Assuming m is of the order of TeV, would imply a lifetime of the order of 10^5 seconds, which goes well beyond the era of nucleosynthesis. The decay products of the gravitino may destroy almost all nuclei created in this era, which is inconsistent with observations [42]. Other possible solutions to the cosmological gravitino problem include the split supersymmetry model where the gravitino mass far exceeds the TeV scale, or models in which the *R*-parity is violated, which would preclude the synthesis of primordial nuclei [43].

3.3.2. Sneutrino

Another dark matter particle that is of interest is the sneutrino. According to the Minimal Supersymmetric Standard Model (MSSM), the sneurtino is ruled out as a DM particle, because it exhibits large scattering and annihilation cross sections. Its abundance is limited and it shows null results in direct detection experiments for all masses near $m_{weak} \approx$ 10 GeV - 1 TeV [44,45]. The sneutrino interacts via Z boson exchange and would have been detected by now had it existed. However, extended modules involving right-handed (RH) sneutrinos reopen the possibility of the sneutrino as a DM particle [46,47].

3.3.3. Neutralino

The most interesting supersymmetric DM particle is the neutralino. It is a hypothetical WIMP dark matter particle that is predicted by supersymmetry [48,49]. The superpartners of the Z-boson (zino), the photon (photino), and the neutral Higgs (higgsino) have the same quantum number, and therefore they can mix to form four eigenstates of the mass operator called the neutralino. The properties of each neutralino are determined by the details of the mixing, and they would have weak scale masses in the range 100 GeV to 1 TeV. The neutralino is stable in models where *R*-parity is conserved, and the lightest of the four neutralinos is the LSP. The lightest neutralino is considered as a prime candidate for cold dark matter in the Universe. Neutralino DM particles can be detected by observing gamma rays and neutrinos resulting from their annihilation, preferably in regions of high DM density such as the galactic centers. So far, no experimental evidence of neutralino annihilation has been found.

3.4. Sterile Neutrino

In contrast to fermion masses described in quantum field theories that have terms that couple left-and right-handed fields together, no right-handed neutrino field is predicted by the Standard Model (SM). So all observed neutrinos exhibit left-handed helicities, where spins are antiparallel to momenta. Furthermore, all antineutrinos have left-handed helicities. Therefore, all neutrinos and antineutrinos are massless. Adding a right-handed neutrino may give them mass through the same mechanism that generates mass for quarks and charged leptons. This is achieved by adding a Majarona mass term to the Lagrangian, and thus extending the SM model to include more than two sterile neutrinos. When electroweak symmetry is broken, mass eigenstates will consist primarily of a combination of left-handed neutrinos called *active neutrinos*, whereas those dominated by right-handed neutrinos are called *sterile neutrinos* (ν_s). Sterile neutrinos do not interact via any fundamental interaction of the SM except for gravity. In general, they are not considered DM candidates. However, there exists a range for the Yukawa coupling in the SM where sterile neutrinos may be dark matter candidates. The mixing angle in this case is defined by

$$\nu_s = \nu_r \cos(\theta) + \nu_l \sin(\theta) \qquad (9)$$

where ν_r and ν_l are a linear combination of right-handed and left-handed gauge eigenstates, repectively. All the mechanisms of production of sterile neutrinos require very small masses and mixing angles to be viable candidates for DM [29]. Sterile neutrinos can be produced by oscillations at temperatures $T \approx$ 100 MeV [50]. Being neutral particles, sterile neutrinos do not interact electromagnetically, weakly, or strongly with known particles, and therefore they are very difficult to detect. Because of their mass, however, they interact gravitationally, and they are heavy enough to explain cold dark matter.

3.5. SuperWIMPs

SuperWIMPs are superweak interacting massive particles that have the required relic density, but their interaction is much weaker than the weak interaction. In spite of their superweak interaction, superWIMPs scenarios correctly predict signals emanating from cosmic rays. In the early Universe, one scenario assumes that WIMPs freeze out but later decay to produce superWIMPs that form the dark matter that exists today. Because superWIMPs are very weakly interacting, they will not affect the WIMPs' freeze out in the early Universe. This causes the WIMPs to decouple with a relic density $\Omega_{\mathrm{WIMP}} = \Omega_{\mathrm{DM}}$. SuperWIMPs inherit their relic density from WIMPs and therefore produce the required DM density. If superWIMPs interact only gravitationally, the natural time scale for WIMPs to decay to superWIMPs is $(1/Gm^3_{\mathrm{weak}}) \approx 10^3$ to 10^7 seconds [29]. Superwimps may also be produced after reheating, in the era where the inflation potential is transferred to SM particles. If the temperature is high enough, significant amounts of superWIMPs are generated [51-53]. The superWIMP relic number density is linearly proportional to the reheating temperature T_R, with the constant of proportionality equal to the gravitino production cross section [29]. For a gravitino mass $m_{\mathrm{gravitino}} \leq$ 100 GeV, the constraint on Ω_{DM} implies $T_R \leq 10^{10}$ GeV [54]. Thus, the gravitino is a typical superWIMP particle [55,56].

Another example of a superWIMP is the axino [57,58]. The axino is the supersymmetric partner of the axion. If both axions and axinos contribute to DM, then this would constitute an interesting multicomponent DM scenario [59]. SuperWimp candidates in the form of KK graviton and axion states also exist in the Universal Extra Dimentions (UED) models [55]. The KK graviton is the lightest KK state for all R^{-1} 800 GeV, where R is the compactification radius [60]. The lightest stabilized KK states by

KK-parity conservation have very similar properties to their supersymmetric counterparts [29]. To sum up, there are many superWIMP candidates that inherit their relic density from WIMPs, and are thus produced with the required relic density.

4. Kaluza-Klein Dark Matter

Extra spatial dimensions provide an alternative to weak-scale physics. The possibility of the existence of extra spatial dimensions dates back to the work of Kaluza and Klein in the 1920s. In the Universal Extra Dimensions (UED) model, all particles are restricted to move in a flat and compact extra dimension of size 10^{-18} m or smaller. In minimal UED, there is one extra dimension of size R, compactified on a circle. In this model, every SM particle has an infinite number of partner particles with mass nR^{-1} at every Kaluza-Klein (KK) level n. These particles have the same spin, in contrast to superpatners. The KK parity in UED models is preserved, implying that the lightest KK particle (LKP) is stable and a possible dark matter candidate [61,62]. The required LKP mass is 600 GeV $\leq m_{B'} \leq$ 1.4 TeV, where $m_{B'}$ is the LKP mass, the level 1 partner of the hypercharge gauge boson, sometimes called the KK photon. The detection of KK dark matter particles can be achieved by elastic scattering via coupling with nuclei through the exchange of Higgs bosons and KK quarks. Indirect detection of KK dark matter has several attractive features. Firstly, almost 60% of KK dark matter annihilates into charged lepton pairs (20% for each generation), 33% of the annihilation produces pairs of up quarks, and 3.6% produces neutrino pairs. The remaining 3.4% generate down quarks and Higgs bosons. Secondly, the low velocity cross section is the maximum possible for a thermal relic. Finally, KK dark matter spin-dependent elastic scattering cross sections for protons can be quite large, making the capture of such particles in the Sun an efficient process, which leads to the production of large neutrino fluxes.

5. Primordial Black Holes and Holeums

Theoretical investigations have shown that black holes may have been formed in the early universe due to initial inhomogeneities [63]. Hawking [64,65] argues that primordial black holes (PBH) were formed in a wide spectrum of masses ranging from 2.17 × 10^{-8} kg, corresponding to the Planck mass, up to 10^{17} solar masses. Formation of PBH is triggered when the gravitational attraction in certain overdense regions in the early Universe overcomes the pressure forces and the velocity expansion. This condition is realized when the potential energy of self-gravitation exceeds the kinetic energy of expansion. Hawking has shown that quantum effects cause black holes to create and emit particles as if they were blackbodies of temperature

$$T = hc^3 / 16\pi k_B G M_{BH} \tag{10}$$

where h and k_B are the Planck and Boltzmann constants, respectively. As Equation (10) shows, the Hawking temperature, T, is inversely proportional to the black hole mass, M_{BH}, and thus, as the black hole radiates, its temperature increases. Dimensional arguments indicate that the lifetime will be less than the age of the Universe only if $M \leq 10^{15}$ g [66]. Non-rotating PBH with initial mass 5 × 10^{15} g would have just evaporated within the present age of the Universe, whereas a black hole created maximally rotating would have just evaporated if its initial mass was 7 × 10^{15} g. If the Big Bang spews PBHs with enough mass, they will be manifested as dark matter. Many models have been proposed to describe the mechanism of evaporation. The Hagedron model assumes that the PBH mass would be converted in an extremely short time to hadronic matter at T_{PBH} ~ 140 - 160 MeV [67]. In the quark-gluon deconfinement phase transition model, the emitted free quarks and gluons would hadronize at some distance from the PBH horizon at a temperature T ~ 100 - 300 MeV [68]. Diffused gamma rays in the galactic halo may serve as an indicator of the PBH density in the Galactic halo, and thus of dark matter [68,69]. Cline *et al*. [68] assumed a clumping factor of 5 × 10^{15} g corresponding to a density of PBH of 10^{10} pc^{-3} in the Galactic halo. In this case, they estimated the number of PBHs to be ~10^{22}. For a decay rate of 3 × 10^4 s^{-1}, they found a photon flux of ~10^{38} erg/s released into the halo. Cline [70] further assumed a Page-Hawking bound of 2 × 10^4 pc^{-3} and obtained a diffuse gamma-ray flux of ~0.12 photons $m^{-2} \cdot s^{-1} \cdot sr^{-1}$. This result is consistent with the value obtained by [71,72]. An important remark concerning the above estimation of the Galactic flux is that the evaporation is restricted to PBHs with masses corresponding to the present epoch.

Detection of PBHs was the subject of intensive research since their existence was postulated in the early 1970s. The Hawking evaporation is the key process that allows a potential detection of PBHs. The evaporation is accomplished by a burst of emitted particles and gamma rays. Thus, the population of the galactic halo with PBHs can be inferred only by a careful analysis of possible signals emanating from their evaporation products. No such signal has been reported so far.

Chavda & Chavda [73] have shown that PBH in the early universe did not decay until gravity decoupled from other interactions. In this case, they demonstrated that micro PBH, having masses between 8 × 10^{18} GeV and 10^{19} GeV, formed gravitational bound states called holeums when the temperature of the Universe was between 10^{30} K and 10^{29} K. Being coupled, these PBH will not evaporate by the Hawking mechanism, unless they are ionized. The condition leading to the formation of stable bound states of PBHs are met when extremely high number density, vastly stronger gravity, and enormously large rates of interaction dominate the fireball. The frequency $\nu_{nn'}$ of the gravitational radiation emitted by a holeum when it makes a transition from a higher state n' to a lower state n is given by

$$\nu_{nn'} = \nu_0 \left[m/m_p \right]^5 \left\{ 1/n^2 - 1/n'^2 \right\} \tag{11}$$

where, $\nu_0 = m_p c^2/4h$, and m_p is the Planck mass. The energy spectrum given by the above equation is identical to that of the hydrogen atom. In other words, the holeum is a gravitational analog of the hydrogen atom. Chavda & Chavda [73] consider holeums as an essential component of DM that populates the Galactic halo. Holeum theory, in spite of its richness, is still in the infant stage, and a lot of future theoretical and observational work has to be accomplished before testing its adequacy as a DM candidate.

6. Massive Astrophysical Compact Halo Objects

Massive astrophysical compact halo objects (MACHOs) are any kind of astronomical object in the Galactic halo that may account for dark matter. Generally, these bodies emit no light, or in certain cases they may emit very faint radiation in the far infrared region of the spectrum. Therefore, they are very difficult to detect using conventional methods. MACHOs include objects such as black holes, neutron stars, brown dwarfs, or freely floating planets. Detection of MACHOs becomes possible when they pass in front of a star through microlensing [74]. The MACHO gravity amplifies light by gravitational micro-lensing, causing the star to appear brighter. The increase and subsequent decrease of light intensity caused by microlensing has a symmetric form, with no change in wavelength. Two important quantities characterize microlensing. The first is the *Einstein angle*, also called the *Einstein radius*, which is given by

$$\theta_E = \left\{ \left[4GM \left(d_S - d_L \right) \right] / \left[c^2 d_S d_L \right] \right\}^{1/2} \tag{12}$$

where G once again is the gravitational constant, M is the lens mass, d_L is the distance of the lens, and d_S is the distance of the source. A typical value for the Einstein radius of a bulge microlensing event is 1 milliarcsecond, which is a very small quantity. The second important quantity of a microlenseing event is the amplification factor A, which is given by

$$A(u) = \left[u^2 + 2 \right] / \left[u \left(u^2 + 4 \right) \right] \tag{13}$$

where u is a unitless number defined as the angular separation between the source and the lens. An important property of $A(u)$ is that it is always greater than 1, and therefore microlensing can only increase the brightness of the source. As u approaches infinity, $A(u)$ approaches 1, that is, at large separations microlensing becomes negligible. Finally, for perfect alignment (u = 0), $A(u)$ becomes infinite.

Certain theories postulate the existence of PBHs or holeums surrounding our galaxy. The black holes can be detected by observing possible bright gas, or an accretion disk formed by the pulling of nearby gas. Alternatively, they can be identified by a burst of gamma rays and particles resulting from their evaporation. However, there is no evidence so far of a microlensing event by a PBH. Neutron stars and old white dwarfs may radiate away enough energy to become cold and therefore undetectable. Nevertheless, the Universe is not old enough for these objects to reach this stage of evolution. Brown dwarfs are "aborted stars" and emit very faint infrared radiation, basically from their gravitational contraction.

Gravitational microlensing has inspired many groups to look for MACHOs in the Galactic halo. One group (MACHO group) claimed that it observed microlensing events accounting for up to 20% of dark matter in the Galaxy with an optical depth toward the Large Magellanic Cloud (LMC) of 1.2×10^{-7} and toward the Galactic bulge of 2.43×10^{-6} [8]. The EROS2 collaboration operates with higher sensitivity by a factor of 2, but has not confirmed the results of the MACHO group. The NICMOS instrument aboard the Hubble Space Telescope showed that less than one percent of the Galactic halo mass is composed of red dwarfs [75,76]. Microlensing was also used to discover exoplanets [77-80]. Microlensing has been a powerful tool for discovering planet size bodies. However, the bulk of the discovered events falls way short of accounting for the Galactic DM.

7. Modified Newtonian Dynamics

Modified Newtonian Dynamics (MOND) is a theory that was put forward by Milgrom [81,82] to modify Newton's law of gravity in order to explain the galactic rotation problem without evoking the need for dark matter. MOND assumes that acceleration is not linearly proportional to the gravitational force at small values. Stars in their journey around the galaxy are assumed to be governed solely by gravitational forces, and therefore, objects in the outer edges of the galactic disk are supposed to have much lower orbital velocities than those close to the center. However, observations reveal that stars at all distances from the center exhibit almost the same speed. Therefore, the rotation curve flattens and extends to much higher distances than the furthest observed visible matter at the edge of the Galaxy. This behavior is usually attributed to the existence of dark matter in the Galactic halo. The same phenomenon is observed in all galaxies.

MOND assumes that acceleration due to gravity does not simply depend upon the mass *m*, but rather on a quantity of the form $m/\mu(a/a_o)$, where μ is some function approaching unity for a large argument, and approaching (a/a_o) for a small argument, where *a* is the acceleration due to gravity, and a_o is a natural constant equal to 10^{-10} m/s^2. In our everyday world $(a/a_o) = 1$, and therefore, the change in Newton's law of gravity is negligible. Applying the above concepts to a star orbiting the Galactic center, one can easily obtain an expression for the orbital velocity

$$v = \left[GMa_o\right]^{1/4} \tag{14}$$

The above equation predicts that the velocity of a star in a circular orbit far away from the Galactic center is constant, and is independent of its distance from the center. If *a* and *M* are known, the constant a_o can be calculated. For our Galaxy [81] found 1.2×10^{-10} m/s^2. This is an extremely small quantity, and [81] interpreted this constant as the acceleration that will take an object from rest to the speed of light in the lifetime of the Universe. Many interpretations and inconsistencies have been advanced to discuss the validity of MOND. One interpretation is that the behavior of dark matter in the Galaxy dictates the results of MOND, and in this case DM is tightly correlated with visible matter according to a fixed relation. Compatibility issues between MOND and the observed world have been proposed. It has been argued that acceleration is not the only parameter to be considered. To verify MOND, one may consider large systems, such as galaxies or galaxy clusters, that possess the required dynamics to permit comparison with observation. In this case, MOND agrees with observation within the uncertainties of the data. To test the validity of MOND, experiments should be conducted only outside the Solar System. One proposed experiment involves flying the future LISA pathfinder spacecraft through the Earth-Sun saddle point. MOND was also successful in predicting rotation curves for the majority of low surface brightness galaxies (LSB) [83]. Smolin *et al*. [84] were unsuccessful in establishing a theoretical basis for MOND from quantum gravity. Recently, a study of a gravity-induced redshift of galactic clusters strongly supported general relativity, but was inconsistent with MOND [85]. In 2006, criticism of MOND based on the Bullet Cluster system was advanced. This is a system of two colliding clusters, and whenever a phenomenon associated with either MOND theory or DM is present, they appear to emanate from a physical location that has the same center of gravity. However, the effect produced by DM in this colliding system appears to emanate from different points in space and not just from the center of mass of the visible part in the system. This is easy to discern due to the higher energy collisions of the gas in the vicinity of the colliding galactic clusters [13]. This observation cannot be explained by a purely baryonic model. To sum up, MOND was not able to address all issues raised by observations. Tensor-Vector-Scalar (TeVeS) gravity theory is a relativistic theory proposed as an equivalent to MOND [86]. This theory was able to explain structure formation without cold dark matter, but required ~2 eV massive neutrinos. However, other authors claim that TeVeS cannot explain Cosmic Microwave Background anisotropies and structure formation at the same time. Another theory known as nonsymmetric gravitation theory was proposed to explain the rotation curves of galaxies [87]. However it was unable to address other issues associated with dark matter. Furthermore, conformal gravity theory claims to offer an alternative explanation to DM [88].

8. Supersymmetry, Superstrings, and Dark Matter

Supersymmetry is one of the great achievements of particle physics. It is regarded as a necessary feature of quantum theories of gravity. It is derived from the idea that there should be a fundamental symmetry in nature between fermions and bosons.

In supersymmetry, there is one superpartner particle state for every ordinary state. In a previous section we discussed the possibility that LSP constitutes potential candidates for DM. It is believed that these particles have not yet been observed because supersymmetry is a broken symmetry, and consequently the superpartners are heavier than the known elementary particles. Several arguments have been presented to estimate the range of a typical superpartner mass. It is argued that a range of the order of 100 GeV to 1000 GeV is consistent with electroweak symmetry breaking and with the unification of the electroweak and strong nuclear forces.

The prospect of detecting superpartners relies to a great extent on accelerators capable of achieving such high energies. The Large Hadron Collider (LHC) is designed to reach energies exceeding the above limit. Therefore, if supersymmetry is correct, physicists have good reasons to believe that LHC can find the new spectrum of predicted particles. Nevertheless, not finding superpartners in future experiments does not rule out their existence, since there is no compelling evidence that they necessarily evolve in our spacetime dimensions.

The last few decades have witnessed the emergence of superstring theory as the leading candidate for a unified description of fundamental particles and forces in nature including gravity. In this theory, particles arise as excitations of strings and interactions are simply given by the geometric splitting and joining of these strings. There are five kinds of superstring theories, but recent developments have shown that what was thought to be a set of completely different theories is in fact a different way of looking at the same thing. The unified string theory is called the M theory. Among the superstring theories is a symmetry group known as $E_8 \times E_8$ Heterotic string theory that was historically thought to be the most promising theory describing the physics beyond the Standard Model. It was discovered in 1987 by Gross, Harvey, Martinec, and Rohm [89]. For a long time it was thought to be the only string theory relevant to our Universe. The symmetry group $E_8 \times E_8$ essentially describes two universes living alongside each other. Each of the E_8 symmetries can be naturally broken and reduced to the kind of symmetries used in particle physics to describe the Universe. So, only one E_8 component is needed to describe our Universe, leaving a complete duplicate set of possibilities. The symmetry between the two halves of the group was broken at the Planck era when gravity split away from the other forces in nature. Some theorists interpret the $E_8 \times E_8$ group in terms of two interpenetrating universes but influencing each other only through gravity. A speculative idea is that the other world is a shadow universe and is identified with dark matter. An interesting perspective would be to investigate the possibility that the E_8 component represents a "supersymmetric universe", a world populated by supersymmetric particles, and remains bound to our Universe through gravitational interaction. Although this idea is speculative at this stage, it provides a frame that accommodates most of the observed properties of dark matter and is supported at the same time by recent theoretical works as explained below. Such developments may preclude our particle accelerators from discovering supersymmetric particles by conventional methods, and the signatures of missing energies may provide an indirect test of their existence.

The strength of the $E_8 \times E_8$ symmetry group comes from the fact that it gives a natural explanation for the origin of DM, while bypassing some unnecessary details that are typically required by any theory concerned with the origin and evolution of the Universe. Among these details are the scenario of inflation and the discrepancy between $\Omega_0 = 1$, as imposed by inflation, and $\Omega_0 = 0.2$, as obtained from primordial nucleosynthesis. However, in the recent past [14] analyzed old published data of 160 distant galaxies and reported a systematic rotation of the plane of polarization over cosmological distances. The discovery could mean that light travels at two slightly different speeds depending on the direction of movement, or it could mean that the Big Bang spewed two universes, each with an opposite twist.

Another important development was provided by [15]. He introduced a model in which two weakly coupled systems maintain opposite running thermodynamic arrows of time, and concluded that there exists a real possibility that at some distance from us there are regions that exhibit such peculiar directions for the arrow of time. He argued that the extended absorber theory indicates that we would see them (the other universe) at an era later than our own due to the light travel time to them. Moreover, [15] discussed the way these regions have arisen, and considered the possibility that our Universe will have a Big Crunch in the (our) future. Furthermore, [15] showed that these regions cannot communicate electromagnetically, and he identified the properties of their content with that attributed to dark matter. The analysis of the observational data presented by [14] suggests the existence of another universe. On the other hand, Schulman's [15] theoretical work indicates the existence of a DM universe evolving in different spacetime dimensions. Accepting these results at face value, we conclude that they are in agreement with the general features of the $E_8 \times E_8$ superstring theory.

Recently, an international team of astrophysicists presented a map of the distribution of dark matter in the Universe [90]. The map was constructed using gravitational lensing data. This result constituted direct evidence for the existence of dark matter. It was shown that DM forms along filaments that span hundreds of millions of light years. These filaments cross each other forming nodes of higher density DM. The most important aspect of these results is that DM tends to clump and form large scale structures similar to those observed for the distribution of visible matter in the Universe. This view supports the $E_8 \times E_8$ Heterotic string theory in providing insight into the spacetime manifold of dark matter.

9. Conclusions

Dark matter has been one of the most challenging topics in cosmology for the past 80 years, both for observers and theoreticians. In this paper, we have highlighted the difficulties associated with the current detection strategy, which is primarily based on incomplete theoretical models. A wide spectrum of particles has been proposed as DM candidates. In spite of the ever growing sophistication of the detection techniques, none of the proposed DM particles has been discovered so far. The Large Hadron Collider (LHC) will certainly be the right machine to determine the road map for identifying the valid theoretical models in particle physics, which in turn will have a great impact on future search strategies. The recent discovery of the Higgs boson has provided new momentum for the Standard Model as a cornerstone in our understanding of the Universe. Although energies of supersymmetric particles fall in the detection range of LHC, a failure to discover any of them does not necessarily disprove their existence, but will rather raise questions about their nature and the strategy to be adopted to discover them, or alternatively may motivate scientists to embark on new physics.

MACHO searches, on the other hand, have not been conclusive in accounting for the missing mass. Furthermore, accepting PBH and holeums as potential candidates for DM is premature at this stage, since no signal from their potential evaporation has been received despite intensive searches. However, their existence may in the future be indirectly inferred through other strategic searches, such as the intensity of the Galactic gamma ray background, which is an important component of their evaporation products.

It was shown that the $E_8 \times E_8$ Heterotic string theory offers a general framework to understand the nature of DM. It seems to be consistent with certain experimental data [14] and theoretical models [15] that suggested the possibility that DM may evolve in another spacetime manifold. This result conforms to Kaluza-Klein theory, where extra dimensions are needed to accommodate the KK particles. Furthermore, it was pointed out that dark matter surveys, using microlensing techniques, indicate the existence of a large scale structure of dark matter, similar to the distribution of ordinary matter populating the visible Universe.

The essence of the dark matter problem is that it is interdisciplinary in nature, and thus requires a search strategy that utilizes more than one approach. Dark matter will certainly remain a challenge for theoreticians and observers for some time to come.

REFERENCES

[1] K. Freeman and G. McNamara, "In Search of Dark Matter," Springer, Berlin, 2006.

[2] F. Zwicky, "The Redshift of Extragalactic Nebulae," *Helvetica Physica Acta*, Vol. 6, 1933, pp. 110-127.

[3] J. P. Ostriker, *et al*., "The Size and Mass of Galaxies, and the Mass of the Universe," *Astrophysical Journal*, Vol. 193, 1974, pp. L1-L4. doi:10.1086/181617

[4] S. M. Faber and J. S. Gallagher, "Masses and Mass-to-Light Ratios of Galaxies," *Annual Review of Astronomy and Astrophysics*, Vol. 17, No. 1, 1979, pp. 135-187.

[5] M. Davis, *et al*., "On the Virgo Supercluster and the Mean Mass Density of the Universe," *Astrophysical Journal*, Vol. 238, 1980, pp. L113-L116. doi:10.1086/183269

[6] V. Trimble, "Existence and Nature of Dark Matter in the Universe," *Annual Review of Astronomy and Astrophysics*, Vol. 25, No. 1, 1987, pp. 425-472.

[7] N. A. Bahcall, *et al*., "Where is Dark Matter?" *Astrophysical Journal*, Vol. 447, 1995, pp. L81-L85.

[8] C. Alcock, *et al*., "The MACHO Project: Microlensing Results from 5.7 Years of LMC Observations," *Astrophysical Journal*, Vol. 542, No. 1, 2000, pp. 281-307. doi:10.1086/309512

[9] P. Tisserand, *et al*., "Limits on the MACHO Content of the Galactic Halo from the EROS-2 Survey of the Magellanic Clouds," *Astronomy & Astrophysics*, Vol. 469, No. 2, 2007, pp. 387-404.

[10] S. Sarkar, "Supersymmetric Inflation and Large-Scale Structure," 1996.

[11] D. N. Schramm and M. S. Turner, "Big-Bang Nucleosynthesis Enters the Precision Era," *Reviews of Modern Physics*, Vol. 70, No. 1, 1998, pp. 303-318.

[12] K. Jedamzik and M. Pospelov, "Big Bang Nucleosynthesis and Particle Dark Matter," *New Journal of Physics*, Vol. 11, 2009, Article ID: 105028.

[13] D. Clowe, *et al*., "A Direct Empirical Proof of the Existence of Dark Matter," *Astrophysical Journal*, Vol. 648, No. 2, 2006, pp. L109-L113.doi:10.1086/508162

[14] B. Nodland and J. P. Ralston, "Indication of Anistropy in Electromagnetic Propagation over Cosmological Distances," *Physical Review Letters*, Vol. 78, No. 16, 1997, pp. 3043-3046.

[15] L. S. Schulman, "Opposite Thermodynamic Arrows of Time," *Physical Review Letters*, Vol. 83, No. 26, 1999, pp. 5419-5422.

[16] F. Zwicky, "On the Masses of Nebulae and of Clusters of Nebulae," *Astrophysical Journal*, Vol. 86, 1937, pp. 217-246. doi:10.1086/143864

[17] V. Rubin, N. Thonnard and W. K. Ford Jr., "Rotation Properties of 21 Sc Galaxies with a Large Range of Luminosities and Radii from NGC 4605 (R = 4 kpc) to UGC 2885 (R = 122 kpc)," *Astrophysical Journal*, Vol. 238, 1980, pp. 471-487. doi:10.1086/158003

[18] S. M. Faber and R. E. Jackson, "Velocity Dispersions and Mass-to-Light Ratios for Elliptical Galaxies," *Astrophysical Journal*, Vol. 204, 1976, pp. 668-683. doi:10.1086/154215

[19] X.-P. Wu, T. Chiueh, L.-Z. Fang and Y.-J. Xue, "A Comparison of Different Cluster Mass Estimates: Consistency or Discrepancy?" *Monthly Notices of the Royal Astronomical Society*, Vol. 301, No. 3, 1998, pp. 861-871. doi:10.1046/j.1365-8711.1998.02055.x

[20] N. W. Boggess, *et al*., "The COBE Mission: Its Design and Performance Two Years after the Launch," *Astrophysical Journal*, Vol. 397, No. 2, 1992, pp. 420-429. doi:10.1086/171797

[21] A. Melchiorri, *et al*., "A Measurement of Ω from the North American Test Flight of Boomerang," *The Astrophysical Journal Letters*, Vol. 536, No. 2, 2000, pp. L63-L66. doi:10.1086/312744

[22] E. M. Leitch, *et al*., "Degree Angular Scale Interferometer 3 Year Cosmic Microwave Background Polarization Results," *Astrophysical Journal*, Vol. 624, No. 2, 2005, pp. 10-20. doi:10.1086/428825

[23] A. C. S. Readhead, *et al*., "Polarization Observations with the Cosmic Background Imager," *Science*, Vol. 306, No. 5697, 2004, pp. 836-844.

[24] G. Hinshaw, *et al*., "Five-Year Wilkinson Microwave Anisotropy Probe Observations: Data Processing, Sky Maps, and Basic Results," *Astrophysical Journal Supplement Series*, Vol. 180, No. 2, 2009, pp. 225-245. doi:10.1088/0067-0049/180/2/225

[25] E. Komatsu, *et al*., "Five-Year Wilkinson Microwave Anisotropy Probe Observations: Cosmological Interpretation," *Astrophysical Journal Supplement Series*, Vol. 180, No. 2, 2009, pp. 330-376.

doi:10.1088/0067-0049/180/2/330

[26] V. Springle, *et al*., "Simulations of the Formation, Evolution and Clustering of Galaxies and Quasars," *Nature*, Vol. 435, 2005, pp. 629-636.

[27] J. R. Primack and D. Seckel, "Detection of Cosmic Dark Matter," *Annual Review of Astronomy and Astrophysics*, Vol. 38, 1988, pp. 751-807.

[28] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Pseudoparticles," *Physical Review Letters*, Vol. 38, No. 25, 1977, pp. 1440-1443.

[29] J. L. Feng, "Dark Matter Candidates from Particle Physics and Methods of Detection," *Annual Review of Astronomy and Astrophysics*, Vol. 48, 2010, pp. 495-545.

[30] G. G. Raffelt, "Astrophysical Axion Bounds," *Lecture Notes in Physics*, Vol. 741, 2008, pp. 51-71.

[31] P. Sikivie and Q. Yang, "Bose-Einstein Condensation of Dark Matter Axions," *Physical Review Letters*, Vol. 103, No. 11, 2009, Article ID: 111301.

[32] C. Robilliard, *et al*., "No Light Shining through a Wall: Results from a Photore Generation Experiment," *Physical Review Letters*, Vol. 99, No. 19, 2007, Article ID: 190403.

[33] K. J. Mack and P. J. Steinhardt, "Cosmological Problems with Multiple Axion-Like Fields," *The Journal of Cosmology and Astroparticle Physics*, Vol. 5, 2011, p. 1.

[34] A. Renzini, "Effects of Cosmions in the Sun and in Globular Cluster Stars," *Astronomy & Astrophysics*, Vol. 171, No. 1-2, 1987, pp. 121-122.

[35] Y. Fukuda, *et al*., "Evidence for Oscillation of Atmospheric Neutrinos," *Physical Review Letters*, Vol. 81, No. 8, 1998, pp. 1562-1567.

[36] Q. R. Ahmad, *et al*., "Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory," *Physical Review Letters*, Vol. 89, 2002, Article ID: 011301.

[37] P. C. McGuire and P. Steinhardt, "Cracking Open the Window for Strongly Interacting Massive Particles as the Halo Dark Matter," 2001, arXiV:astro-ph/0105567v1.

[38] H. Pagels and J. P. Primack, "Supersymmetry, Cosmology, and New Physics at TeraElectronvolt Energies," *Physical Review Letters*, Vol. 48, No. 4, 1982, pp. 223-226.

[39] G. Blumenthal, *et al*., "Galaxy Formation by Dissipationless Particles Heavier than the Neutrino," *Nature*, Vol. 299, 1982, pp. 37-38.

[40] T. Moroi, *et al*., "Cosmological Constraints on the Light Stable Gravitino," *Physics Letters B*, Vol. 303, 1993, pp. 289-294.

[41] N. Okada and O. Seto, "A Brane World Cosmological Solution to the Gravitino Problem," *Physical Review D*, Vol. 71, 2005, Article ID: 023517.

[42] A. de Gouvea, *et al*., "Cosmology of Supersymmetric Models with Low-Energy Gauge Mediation," *Physical Review D*, Vol. 56, 1997, pp. 1281-1299.

[43] F. Takayama and M. Yamaguchi, "Gravitino Dark Matter without R-Parity," *Physics Letters B*, Vol. 485, No. 4, 2000, pp. 388-392. doi:10.1016/S0370-2693(00)00726-7

[44] T. Falk, *et al*., "Heavy Sneutrinos as Dark Matter," *Physics Letters B*, Vol. 339, No. 3, 1994, pp. 248-251. doi:10.1016/0370-2693

[45] C. Arina and N. Fornengo, "Sneutrino Cold Dark Matter, a New Analysis: Relic Abundance and Detection Rates," *Journal of High Energy Physics*, Vol. 11, 2007, p. 29.

[46] D. G. Cerdeño, *et al*., "Very Light Right-Handed Sneutrino Dark Matter in the NMSSM," *Journal of Astronomy and Astroparticle Physics*, Vol. 11, 2011, p. 27.

[47] B. Dumont, *et al*., "Mixed Sneutrino Dark Matter in Light of the 2011 XENON and LHC Results," 2012. arXiv:1206.1521.

[48] H. Goldberg, "Constraint on the Photino Mass from Cosmology," *Physical Review Letters*, Vol. 50, No. 19, 1983, pp. 1419-1422.

[49] J. Ellis, *et al*., "Towards a Supersymmetric Cosmology," *Physics Letters B*, Vol. 147, No. 1-3, 1984, pp. 27-33. doi:10.1016/0370-2693

[50] S. Dodelson and L. M. Widrow, "Sterile Neutrinos as Dark Matter," *Physical Review Letters*, Vol. 72, 1994, pp. 17-20. doi:10.1103/PhysRevLett.72.17

[51] L. M. Krauss, "New Constraints on 'INO' Masses from Cosmology (I). Supersymmetric INOS," *Nuclear Physics B*, Vol. 227, No. 3, 1983, pp. 556-569. doi:10.1016/0550-3213

[52] D. V. Nanopoulos, *et al*., "After Primordial Inflation," *Physics Letters B*, Vol. 127, No. 1-2, 1983, pp. 30-34. doi:10.1016/0370-2693

[53] R. Juszkiewicz, *et al*., "Constraints on Cosmologically Regenerated Gravitinos," *Physics Letters B*, Vol. 158, No. 6, 1985, pp. 463-467. doi:10.1016/0370-2693

[54] M. Bolz, *et al*., "Thermal Production of Gravitinos," *Nuclear Physics B*, Vol. 606, No. 1-2, 2001, pp. 518-544. doi:10.1016/S0550-3213

[55] J. L. Feng, *et al*., "Graviton Cosmology in Universal Extra Dimensions," *Physical Review D*, Vol. 68, No. 8, 2003, Article ID: 085018.

[56] J. R. Ellis, *et al*., "Prospects for Sparticle Discovery in Variants of the MSSM," *Physics Letters B*, Vol. 603, No. 1, 2004, pp. 51-62.

[57] K. Rajagopal, *et al*., "Cosmological Implications of Axinos," *Nuclear Physics B*, Vol. 358, No. 2, 1991, pp. 447-470. doi:10.1016/0550-3213

[58] L. Covi, *et al*., "Axinos as Dark Matter," *Journal of High Energy Physics*, Vol. 5, No. 8, 2001, p. 33.

[59] H. Baer and A. D. Box, "Fine-Tuning Favors Mixed Axion/Axino Cold Dark Matter over Neutralinos in the Minimal Supergravity Model," *European Physical Journal C*, Vol. 68, No. 3, 2010, pp. 523-537.

[60] J. Cembranos, *et al*., "Resolving Cosmic Gamma Ray Anomalies with Dark Matter Decaying Now," *Physical Review Letters*, Vol. 99, No. 9, 2007, Article ID: 191301.

[61] H. C. Cheng, *et al*., "Kaluza-Klein Dark Matter," *Physical Review Letters*, Vol. 89, 2002, Article ID: 211301.

[62] G. Servant and T. M. T. Tait, "Is the Lightest Kaluza-Klein Particle a Viable Dark Matter Candidate?" *Nuclear*

Physics B, Vol. 650, No. 1-2, 2003, pp. 391-419. doi:10.1016/S0550-3213(02)01012-X

[63] S. W. Hawking, "Gravitationally Collapsed Objects of Very Low Mass," *Monthly Notices of the Royal Astronomical Society*, Vol. 152, 1971, p. 75.

[64] S. W. Hawking, "Particle Creation by Black Holes," *Communications in Mathematical Physics*, Vol. 43, No. 3, 1971, pp. 199-220.

[65] S. W. Hawking, "Black Hole Explosions?" *Nature*, Vol. 248, No. 5443, 1974, pp. 30-31.

[66] D. N. Page, "Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole," *Physical Review D*, Vol. 13, 1976, pp. 198-206.

[67] R. Hadgedron, "Statistical Thermodynamics of Strong Interactions at High Energies," *Nuovo Cimento*, Vol. 3, 1965, pp. 147-186.

[68] D. B. Cline and D. A. Sanders, "Further Evidence for Some Gamma-Ray Bursts Consistent with Primordial Black Hole Evaporation," *Astrophysical Journal*, Vol. 486, No. 1, 1997, pp. 169-178. doi:10.1086/304480

[69] S. Al Dallal, "Primordial Black Holes and Holeums as Progenitors of Galactic Diffuse Gamma-Ray Background," *Advances in Space Research*, Vol. 46, No. 4, 2010, pp. 468-471. doi:10.1016/j.asr.2010.05.005

[70] D. B. Cline, "A Gamma-Ray Halo 'Glow' from Primordial Black Hole Evaporation," *Astrophysical Journal*, Vol. 501, No. 1, 1998, pp. L1-L3. doi:10.1086/311433

[71] J. L. Osborne, *et al*., "The Diffuse Flux of Energetic Extragalactic Gamma Rays," *Journal of Physics G*, Vol. 20, No. 7, 1994, pp. 1089-1101.

[72] D. D. Dixon, *et al*., "Evidence for a Galactic Gamma-Ray Halo," *New Astronomy*, Vol. 3, No. 7, 1998, pp. 539-561. doi:10.1016/S1384-1076

[73] L. K. Chavda and A. L. Chavda, "Dark Matter and Stable Bound States of Primordial Black Holes," *Classical and Quantum Gravity*, Vol. 19, No. 11, 2002, pp. 2927-2938. doi:10.1088/0264-9381/19/11/311

[74] B. Paczynski, "Gravitational Microlensing by the Galactic Halo," *Astrophysical Journal*, Vol. 304, 1986, pp. 1-5. doi:10.1086/164140

[75] D. S. Graff and K. Frees, "Analysis of a Hubble Space Telescope Search for Red Dwarfs: Limits on Baryonic Matter in the Galactic Halo," *The Astrophysical Journal Letters*, Vol. 456, No. 1, 1996, p. L49. doi:10.1086/309850

[76] J. R. Najita, *et al*., "From Stars to Superplanets: The Low-Mass Initial Mass Function in the Young Cluster IC 348," *Astrophysical Journal*, Vol. 541, No. 2, 2000, pp. 977-1003. doi:10.1086/309477

[77] I. A. Bond, *et al*., "OGLE 2003-BLG-235/MOA 2003-BLG-53: A Planetary Microlensing Event," *The Astrophysical Journal Letters*, Vol. 606, No. 2, 2004, pp. L155-L158. doi:10.1086/420928

[78] A. Udalski, *et al*., "A Jovian-Mass Planet in Microlensing Event OGLE-2005-BLG-071," *The Astrophysical Journal Letters*, Vol. 628, No. 2, 2005, pp. L109-L112. doi:10.1086/432795

[79] A. Gould, *et al*., "Microlens OGLE-2005-BLG-169 Implies That Cool Neptune-like Planets Are Common," *The Astrophysical Journal Letters*, Vol. 644, No. 1, 2006, pp. L37-L40. doi:10.1086/505421

[80] B. S. Gaudi, *et al*., "Discovery of a Jupiter/Saturn Analog with Gravitational Microlensing," *Science*, Vol. 319, No. 5865, 2008, pp. 927-930.

[81] M. Milgrom, "A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis," *Astrophysical Journal*, Vol. 270, 1983, pp. 365-370. doi:10.1086/161130

[82] M. Milgrom, "A Modification of the Newtonian Dynamics-Implications for Galaxies," *Astrophysical Journal*, Vol. 270, 1983, pp. 371-389. doi:10.1086/161131

[83] R. H. Sanders, "Modified Newtonian Dynamics and Its Implications," *Proceedings of the Space Telescope Science Institute Symposium*, Baltimore, 2-5 May 2001, p. 62.

[84] L. Smolin, "The Trouble with Physics: The Rise of String Theory, the Fall of a Science, and What Comes Next," Houghton Mifflin Co., New York, 2006.

[85] R. Wojtak, *et al*., "Gravitational Redshift of Galaxies in Clusters as Predicted by General Relativity," *Nature*, Vol. 477, No. 7366, 2011, pp. 567-569.

[86] J. D. Bekenstein, "The Modified Newtonian Dynamics-MOND-and Its Implications for New Physics," *Contemporary Physics*, Vol. 47, No. 6, 2006, p. 387. doi:10.1080/00107510701244055

[87] J. Moffat, "Gravitational Theory, Galaxy Rotation Curves and Cosmology without Dark Matter," *Journal of Cosmology and Astroparticle Physics*, Vol. 5, 2005, p. 3.

[88] P. D. Mannheim, "Alternatives to Dark Matter and Dark Energy," *Progress in Particle and Nuclear Physics*, Vol. 56, 2006, pp. 340-445.

[89] D. J. Gross, *et al*., "Heterotic String Theory: (II). The Interacting Heterotic String," *Nuclear Physics B*, Vol. 267, No. 1, 1986, pp. 75-124. doi:10.1016/0550-3213(86)90146-X

[90] L. van Wawebeck, *et al*., "Detection of Correlated Galaxy Ellipticities on CFHT Data: First Evidence for Gravitational Lensing by Large Scale Structures," *Astronomy & Astrophysics*, Vol. 358, No. 1, 2000, pp. 30-44.

Journal of Modern Physics, **2012, 3, 1142-1151**
doi:10.4236/jmp.2012.329149 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Phase-Space Areas of the Body Motion in the Solar System Deduced from the Bohr-Sommerfeld Atomic Theory and Approximate Invariance of Their Ratios for the Pairs of Planets and Satellites

Stanislaw Olszewski, Tadeusz Kwiatkowski
Institute of Physical Chemistry, Polish Academy of Science, Warsaw, Poland
Email: solszewski@ichf.edu.pl

Received July 5, 2012; revised August 4, 2012; accepted August 11, 2012

ABSTRACT

Energy-time and momentum-position phase spaces defined by the electron orbits in the hydrogen-like atom exhibit special properties of equivalence. It is demonstrated that equivalence of the same kind can be obtained for the phase-space areas defined by the orbit pairs of planets, or satellites, which compose the solar system. In the choice of the examined areas it is useful to be guided by the Bohr-Sommerfeld atomic theory.

Keywords: Ratios of the Phase-Space Areas and Their Invariance; Planets and Satellites; Bohr-Sommerfeld Atomic Theory

1. Introduction

In physics, but not only in this domain, we look very often for parameters which allow us to identify the examined objects. For example a typical parameter of this kind is the mass which helps us to identify a given particle, or a system of particles collected together, say, in an atom. Also large systems of particles, especially those composed of mainly the same atoms or molecules, have some properties which are characteristic for the whole ensemble. A typical parameter is here a definite temperature associated—at special conditions—with the change of the system phase, for example the change is from a vapour to a liquid. Sometimes a combination of several parameters is required to be characteristic for a system.

In mechanics of a circular motion of a body along a specific closed orbit there are important phase integrals

$$S = \oint p \mathrm{d} q \tag{1}$$

met in the Sommerfeld quantum conditions; see e.g. [1]. Here p is the body momentum and q is the body position on the orbit. The integral (1) is the phase-space area of the variables (p, q) circumvented by the body in course of its motion along the orbit. The body energy E—which is conserved on the orbit—can be represented as a function of S and the frequency ν of the body motion, equal to the reciprocal value of the circulation period T, becomes

$$\nu = \frac{1}{T} = \frac{\partial E}{\partial S}. \tag{2}$$

In some special cases, however, for example for a linear oscillator, there is a linear dependence between E and S, so $\frac{\partial E}{\partial S}$ is a constant independent of E or the amplitude of oscillation. In this case we have

$$E = S\nu. \tag{3}$$

A characteristic property of (3) is that the ratio

$$S = \frac{E}{\nu} \tag{4}$$

called the adiabatic invariant, remains unchanged for slow changes of the oscillator Hamiltonian [2].

But in case of the body orbital motion in the solar system the simplifications of (3) and (4) do not hold. For from the reversed Formula (2) we obtain (see e.g. [3]):

$$\frac{1}{\nu} = T = \frac{2\pi}{\left(GM_S\right)^{1/2}} a^{3/2} = \frac{\partial S}{\partial E}, \tag{5}$$

where G is the gravitational constant, a—the major semiaxis of the Kepler orbit and M_S is the central mass which for the planetary orbital motion is the mass of the Sun. Since

$$E = -\frac{GM_S M_p}{2a}, \tag{6}$$

where M_p is the planetary mass, we obtain

$$\begin{aligned}\partial S &= \frac{2\pi a^{3/2}}{\left(GM_S\right)^{1/2}}\partial E = \frac{2\pi a^{3/2}}{\left(GM_S\right)^{1/2}}\left(\frac{\partial E}{\partial a}\right)\partial a \\ &= \frac{2\pi a^{3/2}}{\left(GM_S\right)^{1/2}}\frac{GM_S M_p}{2a^2}\partial a \\ &= \pi\left(GM_S\right)^{1/2}\frac{M_p}{a^{1/2}}\partial a.\end{aligned} \tag{7}$$

The integration performed for (7) gives, with the accuracy to a constant term,

$$S = 2\pi\left(GM_S\right)^{1/2} M_p a^{1/2}, \tag{8}$$

whereas the product of $|E|$ and T becomes

$$\begin{aligned}|E|T &= \frac{GM_S M_p}{2a}\frac{2\pi}{\left(GM_S\right)^{1/2}}a^{3/2} \\ &= \pi\left(GM_S\right)^{1/2} M_p a^{1/2} = \frac{1}{2}S.\end{aligned} \tag{9}$$

But a similar relation can be obtained for the product of the planetary momentum P and average distant R of the moving body from the Sun, if we note that for the Kepler motion of planets the eccentricity parameter e is regularly small, so

$$R \approx a, \tag{10}$$

$$\begin{aligned}P &\approx \frac{2\pi R}{T}M_p \approx \frac{2\pi a}{2\pi a^{3/2}}\left(GM_S\right)^{1/2} M_p \\ &= \frac{\left(GM_S\right)^{1/2} M_p}{a^{1/2}}.\end{aligned} \tag{11}$$

In this case

$$\pi RP \approx \pi\left(GM_S\right)^{1/2} a^{1/2} = \frac{1}{2}S, \tag{12}$$

therefore

$$|E|T \approx \pi RP \approx \frac{1}{2}S. \tag{13}$$

Evidently, the products entering (13) together with S in (8) represent the phase-space areas of (p,q).

However, any of the factors E, T, R, and P entering (13) depends on a in a different way, and a separate dependence on a applies to S. A question which may arise here is as follows: if we have a pair of the planetary objects in which one of the objects has its

$$E_1, T_1, R_1, P_1 \tag{14}$$

at some $a = a_1$, and M_{p1}, and the other object in this pair has its

$$E_2, T_2, R_2, P_2 \tag{15}$$

at some $a = a_2$, and M_{p2}, is it possible to combine the phase-space areas obtained in terms of parameters entering (14) and (15) in such a way that the resulted combination is approximately independent of a_1, a_2, M_{p1}, and M_{p2}? In effect, such a combination may become approximately constant for the whole of the planetary system.

We find an affirmative answer to that question, also for the satellitary systems, and details of the corresponding formulae are given below. It should be noted, however, that a search for the required formalism is much facilitated when similarities which exist in description of the Kepler's planetary problem and the electron motion in the framework of the Bohr-Sommerfeld atomic theory are taken into account.

2. Parameters Useful in Calculating the Phase-Space Areas of the Planetary Motion

Similarities between the planetary motion performed in the gravitational field of the Sun and the electron motion in the electrostatic field of the atomic nucleus are well known. However, any quantitative reference between parameters characteristic for both kinds of the examined motions is rather difficult to assess. This difficulty seems, in general, to be quite obvious if we note a macroscopic nature of the planetary objects and microscopic properties of an atom. Nevertheless, a geometric and mechanical similarity in the behaviour of the solar system to that of an atomic system considered in the framework of the old quantum theory is evident. This similarity may suggest a quantitative search for comparison between the macroworld of planets and the microworld connected with the electron motion in an atom. A search of this kind can materialize in definite calculations when the properties of the phase space associated with an electron circulating in the atom are compared with the phase space of the planetary, or satellitary, motion in the solar system.

In practice, the position-momentum and energy-time phase spaces can be defined both for the orbiting planets and the circulating electron. Here, for the sake of simplicity, the elliptical character of orbits can be approximately neglected equally in the macroscopic and the microscopic case. Let us begin with the action function S_n for a planet on the orbit n which satisfies the relation

$$S_n = 2\pi P_n R_n = 2\left|E_n\right| T_n, \tag{16}$$

whereas a similar action function for an electron moving on a circular trajectory corresponding to the atomic state n is

$$S_n = 2\pi p_n^e r_n^e = 2\left|E_n^e\right| T_n^e. \tag{17}$$

Symbols P_n, R_n, E_n and T_n in (16) denote respectively the momentum, orbital radius

$$R_n = \frac{1}{2} a_n \left(\sqrt{1 - e_n^2} + 1 \right), \tag{18}$$

energy and time period on the planetary orbit n having the major semiaxis a_n and the eccentricity e_n [4-6], whereas p_n^e, r_n^e, E_n^e and T_n^e in (17) are similar parameters for an electron in its quantum state, or on an orbit, labelled also by n. In our calculations we are guided by the atomic properties which are given by the formulae [7]:

$$p_n^e = m_e v_n^e = \frac{2\pi m_e Z e^2}{nh}, \tag{19}$$

$$r_n^e = \frac{n^2 h^2}{4\pi^2 m_e Z e^2}, \tag{20}$$

$$E_n^e = -\frac{2\pi^2 m_e Z^2 e^4}{n^2 h^2}, \tag{21}$$

$$T_n^e = \frac{n^3 h^3}{4\pi^2 m_e Z^2 e^4}. \tag{22}$$

Here Ze is the nucleus charge, e the electron charge, m_e the electron mass, and h is the Planck constant.

3. Change of the Phase-Space Area Due to the Orbit Change

When the orbit n is changed to another orbit, say n', the parts of the difference in the action function equal to

$$\Delta S_{n'n} = S_{n'} - S_n = \Delta S_{n'n}^{(d)} + \Delta S_{n'n}^{(nd)} \tag{23}$$

can be examined both in the planetary and the electron case. These parts can be classified as diagonal (d) changes represented by the formulae

$$\frac{1}{2} \Delta S_{n'n}^{(d)} (p, r) = \pi (R_{n'} - R_n)(P_{n'} - P_n), \tag{24}$$

$$\frac{1}{2} \Delta S_{n'n}^{(d)} (E, T) = (|E_{n'}| - |E_n|)(T_{n'} - T_n) \tag{25}$$

concerning respectively the momentum-position and energy-time phase-space areas entering S_n and $S_{n'}$ in (16). Similar non-diagonal (nd) changes entering $\Delta S_{n'n}$ are

$$\frac{1}{2} \Delta S_{n'n}^{(nd)} (p, r) = \pi \left[(R_{n'} - R_n) P_n + (P_{n'} - P_n) R_n \right], \tag{26}$$

$$\frac{1}{2} \Delta S_{n'n}^{(nd)} (E, T) = (T_{n'} - T_n)|E_n| + (|E_{n'}| - |E_n|) T_n. \tag{27}$$

In the next step, it can be noted that relations concerning components of $\Delta S_{n'n}$ for planets defined in (24)-(27) can be calculated also for the changes $\Delta S_{n'n}$ for electrons, on condition P_n, R_n, E_n and T_n entering the mentioned formulae are replaced by p_n^e, r_n^e, E_n^e and T_n^e. The electron parameters give [6]:

$$(1/2) \Delta S_{n'n}^{(d)} (p^e, r^e) \approx -\frac{(\Delta n)^2}{n} h, \tag{28}$$

$$(1/2) \Delta S_{n'n}^{(d)} (E^e, T^e) \approx -\frac{3(\Delta n)^2}{n} h, \tag{29}$$

for small $\Delta n = n' - n$, so the ratio

$$\frac{\Delta S_{n'n}^{(d)} (p^e, r^e)}{\Delta S_{n'n}^{(d)} (E^e, T^e)} = 1/3 \tag{30}$$

is practically a constant number independent of the orbit index n. In a further step

$$\frac{1}{2} \Delta S_{n'n}^{(nd)} (p^e, r^e) \approx \frac{h}{2} \left[\Delta n + 2 \frac{(\Delta n)^2}{n} \right], \tag{31}$$

$$\frac{1}{2} \Delta S_{n'n}^{(nd)} (E^e, T^e) \approx \frac{h}{2} \left[\Delta n + 6 \frac{(\Delta n)^2}{n} \right], \tag{32}$$

so the ratio of (31) and (32) is approximately also a constant:

$$\frac{\Delta S_{n'n}^{(nd)} (p^e, r^e)}{\Delta S_{n'n}^{(nd)} (E^e, T^e)} \approx 1, \tag{33}$$

which holds especially on condition when $\Delta n \ll n$.

A check of the properties of (30) and (33) for the solar system has been done on the numerical way; see [6]. An almost constant behaviour of (30) is widely confirmed, although this ratio becomes close rather to unity, especially for planets, than to 1/3 obtained in (30). On the other hand, the ratio (33) fluctuates more than (30) for different planetary, or satellitary, pairs chosen to occupy n and n'. This behaviour can be attributed to unequal second terms entering the square brackets in (31) and (32). In the most part of the examined planetary cases the ratio between (31) and (32) is about 2 instead of the result approximately equal to 1 obtained in (33).

4. Phase-Space Areas Made Free from the Orbit Index and Their Comparison

A more convenient way of comparison between the phase-space areas for an electron moving in the atom and similar areas obtained for circulating planets is when these areas are made, at least approximately, free from the orbital index. This situation, in fact, can be obtained beginning with the areas for the moving electron. For example, such an area in the momentum-position space about state n can be defined as

$$\pi \left(r_{n+1}^e - r_n^e \right) \left(p_{n+1}^e + p_n^e \right) \frac{1}{2} \approx n \frac{2}{n} \frac{h}{2} = h. \tag{34}$$

The result in (34) holds on condition n is assumed to be a large number. The area in (34) is a belt of thickness

$$r_{n+1}^{e} - r_{n}^{e} = \left[(n+1)^2 - n^2\right]\frac{h^2}{4\pi^2 m_e Ze^2} \approx \frac{2nh^2}{4\pi^2 m_e Ze^2} \tag{35}$$

plotted in the coordinate space, and the length of the belt is proportional to

$$\pi\left(p_{n+1}^{e} + p_{n}^{e}\right)\frac{1}{2} = \frac{2\pi^2 m_e Ze^2}{2h}\left(\frac{1}{n+1} + \frac{1}{n}\right) \approx \frac{2\pi^2 m_e Ze^2}{nh} \tag{36}$$

in the momentum space. A similar area can be defined when the roles of p_e and r_e are reversed. In this case we obtain the phase-space area equal to

$$\pi\left|p_{n+1}^{e} - p_{n}^{e}\right|\left(r_{n+1}^{e} + r_{n}^{e}\right)\frac{1}{2} \approx \frac{1}{2}h. \tag{37}$$

Here the thickness of the belt is

$$\left|p_{n+1}^{e} - p_{n}^{e}\right| = \frac{2\pi^2 m_e Ze^2}{h}\left|\frac{1}{n+1} - \frac{1}{n}\right| \approx \frac{2\pi m_e Ze^2}{hn^2} \tag{38}$$

and the belt length is proportional to

$$\pi\left(r_{n+1}^{e} + r_{n}^{e}\right)\frac{1}{2} \approx \frac{h^2}{4\pi m_e Ze^2}n^2. \tag{39}$$

The results obtained in (34) and (37) are: 1) close to h; 2) they differ only by a factor of 2.

A similar evaluation can be done for the phase space areas composed of the electron energies and the time periods of the rotational motion about the nucleus; see (21) and (22). We obtain for one electron state the areas

$$\left(T_{n+1}^{e} - T_{n}^{e}\right)\frac{\left|E_{n+1}^{e}\right| + \left|E_{n}^{e}\right|}{2} = 3n^2 h\frac{1}{2n^2} = \frac{3}{2}h, \tag{40}$$

$$\left\|E_{n+1}^{e}\right| - \left|E_{n}^{e}\right\|\frac{T_{n+1}^{e} + T_{n}^{e}}{2} = \frac{2}{n^3}\frac{n^3}{2}h = h, \tag{41}$$

because for a large n we find:

$$T_{n+1}^{e} - T_{n}^{e} \approx 3n^2\frac{h^3}{4\pi^2 m_e Z^2 e^4}, \tag{42}$$

$$\frac{1}{2}\left(T_{n+1}^{e} + T_{n}^{e}\right) \approx \frac{n^3 h^3}{4\pi^2 m_e Z^2 e^4}, \tag{43}$$

$$\left\|E_{n+1}^{e}\right| - \left|E_{n}^{e}\right\| \approx \frac{2}{n^3}\frac{2\pi^2 m_e Z^2 e^4}{h^2}, \tag{44}$$

$$\frac{1}{2}\left(\left|E_{n+1}^{e}\right| + \left|E_{n}^{e}\right|\right) \approx \frac{1}{n^2}\frac{2\pi^2 m_e Z^2 e^4}{h^2}. \tag{45}$$

The areas calculated in (40) and (41) are: 1) close to h; 2) differ only by a factor of $3/2$. Evidently, the area (40) is a kind of the belt having its length extended in the energy space and the belt thickness is in the period-of-time space, whereas (41) represents the belt of reversed shape properties.

Because of a similar character of the electron motion in the Bohr-Sommerfeld atom and the planetary motion in the solar system, we expect similar phase-space properties for both kinds of the motion. An essential difference is here that the electron has the same mass on any orbit n, but this does not apply to the planetary orbits. Therefore a more realistic comparison should concern the case when momenta expressions entering the examined areas are replaced by velocities, and—consequently—the energies of the moving bodies are divided by the orbiting mass. For electrons we obtain on any orbit n

$$v_{n}^{e} = \frac{p_{n}^{e}}{m_e} \tag{46}$$

for the velocity, and

$$\frac{1}{2}\left|\overline{V}_{n}^{epot}\right| = \frac{\left|E_{n}^{e}\right|}{m_e} \tag{47}$$

for the reduced energy. Because of the virial theorem, the expression (47) is equal to a half of the absolute average value of the potential acting on a moving electron. In effect of the substitutions (46) and (47) we obtain the following belt areas for the electron case:

$$A = \pi\left(r_{n+1}^{e} - r_{n}^{e}\right)\left(v_{n+1}^{e} + v_{n}^{e}\right)\frac{1}{2} = \frac{h}{m_e}, \tag{19a}$$

$$B = \pi\left(v_{n+1}^{e} - v_{n}^{e}\right)\left(r_{n+1}^{e} + r_{n}^{e}\right)\frac{1}{2} = \frac{1}{2}\frac{h}{m_e}, \tag{22a}$$

$$C = \frac{1}{2}\left(T_{n+1}^{e} - T_{n}^{e}\right)\frac{1}{2}\left(\left|\overline{V}_{n+1}^{epot}\right| + \left|\overline{V}_{n}^{epot}\right|\right) = \frac{3}{2}\frac{h}{m_e}, \tag{25a}$$

$$D = \frac{1}{2}\left\|\overline{V}_{n+1}^{epot}\right| - \left|\overline{V}_{n}^{epot}\right\|\frac{1}{2}\left(T_{n+1}^{e} + T_{n}^{e}\right) = \frac{h}{m_e}. \tag{26a}$$

Section 5 examines similar expressions calculated for the planetary and satellitary orbits.

5. Belt Areas in the Phase-Space Calculated for Planets and Satellites

In the first step let us examine the equivalence of the belt areas in the position-momentum space and areas in the energy-time period space without the mass reduction for the body momentum and energy performed below [see (56) and (57)]. In this case we have

$$A(R,P) = \pi\left(R_{n+1} - R_n\right)\left(P_{n+1} + P_n\right)\frac{1}{2} \tag{48}$$

similar to (19a),

$$B(R,P) = \pi(P_{n+1} - P_n)(R_{n+1} + R_n)\frac{1}{2} \tag{49}$$

similar to (22a),

$$C(T,E) = (T_{n+1} - T_n)(|E_{n+1}| + |E_n|)\frac{1}{2} \tag{50}$$

similar to (25a), and

$$D(T,E) = \left\| E_{n+1} \right| - \left| E_n \right\| (T_{n+1} + T_n)\frac{1}{2}, \tag{51}$$

similar to (26a) calculated for an electron in the Bohr-Sommerfeld atom. The factor of 1/2 entering the formulae (48)-(51) provides us with the arithmetical mean of the corresponding parameters defining the belt lengths. The belt areas (48)-(51) are calculated in **Tables 1** and **2** together with the ratios $B(R,P)/A(R,P)$ and $D(T,E)/C(T,E)$ which roughly satisfy the relations:

$$\frac{B(R,P)}{A(R,P)} \approx \frac{D(T,E)}{C(T,E)} \approx 2 \tag{52}$$

The input data in **Tables 1** and **2** are taken from [5,6] and **Table 3**. For the same planetary and satellitary pairs another ratio equal to

Table 1. Belt areas between the neighbouring planet orbits calculated (in Js) for the momentum-position and energy-time phase spaces and their ratios; for *A*(*R*,*P*) and *B*(*R*,*P*) see Equations. (48) and (49); for *C*(*T*,*E*) and *D*(*T*,*E*) see, respectively, (50) and (51). The entering data are based on [5,6].

Planets Pair	$A(R,P)$	$B(R,P)$	$\frac{B}{A}$	$C(T,E)$	$D(T,E)$	$\frac{D}{C}$
Venus-Mercury	0.149×10^{41}	0.403×10^{41}	2.70	0.199×10^{41}	0.351×10^{41}	1.77
Earth-Venus	0.227×10^{41}	0.300×10^{40}	0.13	0.342×10^{41}	0.841×10^{40}	0.25
Mars-Earth	0.237×10^{41}	0.962×10^{41}	4.07	0.394×10^{41}	1.120×10^{41}	2.84
Jupiter-Mars	0.215×10^{44}	0.391×10^{44}	1.82	0.255×10^{44}	0.351×10^{44}	1.37
Saturn-Jupiter	0.308×10^{44}	0.669×10^{44}	2.17	0.523×10^{44}	0.883×10^{44}	1.69
Uranus-Saturn	0.138×10^{44}	0.330×10^{44}	2.40	0.245×10^{44}	0.438×10^{44}	1.79
Neptune-Uranus	0.293×10^{43}	0.396×10^{42}	0.13	0.449×10^{43}	0.196×10^{43}	0.44
Pluto-Neptune	0.116×10^{43}	0.900×10^{43}	7.77	0.198×10^{43}	0.982×10^{43}	4.97

Table 2. Belt areas between the neighbouring satellite orbits calculated (in Js) for the momentum-position and energy-time phase spaces and their ratios; for the symbols meaning see (48)-(51). The entering data are based on [5,6] and Table 3.

Satellites Pair	$A(R,P)$	$B(R,P)$	$\frac{B}{A}$	$C(T,E)$	$D(T,E)$	$\frac{D}{C}$
Deimos-Phobos	0.516×10^{27}	0.920×10^{27}	1.78	0.968×10^{27}	1.374×10^{27}	1.42
Europa-Io	0.867×10^{36}	1.507×10^{36}	1.74	0.138×10^{37}	0.202×10^{37}	1.46
Ganymede-Europa	0.144×10^{37}	0.260×10^{37}	1.81	0.209×10^{37}	0.195×10^{37}	0.94
Callisto-Ganymede	0.319×10^{37}	0.341×10^{37}	1.07	0.513×10^{37}	0.534×10^{37}	1.04
Enceladus-Mimas	0.130×10^{33}	0.188×10^{33}	1.45	0.195×10^{33}	0.126×10^{33}	0.64
Tethys-Enceladus	0.852×10^{33}	6.400×10^{33}	7.51	0.122×10^{34}	0.600×10^{34}	4.93
Dione-Tethys	0.246×10^{34}	0.205×10^{34}	0.83	0.372×10^{34}	0.086×10^{34}	0.23
Rhea-Dione	0.746×10^{34}	1.505×10^{34}	2.02	0.110×10^{35}	0.115×10^{35}	1.05
Titan-Rhea	0.844×10^{36}	2.009×10^{36}	2.38	0.108×10^{37}	0.177×10^{37}	1.64
Iapetus-Titan	0.279×10^{37}	0.561×10^{37}	2.01	0.577×10^{37}	0.859×10^{37}	1.49
Ariel-Miranda	0.764×10^{33}	0.348×10^{34}	4.55	0.107×10^{34}	0.318×10^{34}	2.98
Umbriel-Ariel	0.158×10^{34}	0.108×10^{34}	0.69	0.241×10^{34}	0.192×10^{34}	0.80
Titania-Umbriel	0.498×10^{34}	0.746×10^{34}	1.50	0.727×10^{34}	0.515×10^{34}	0.71
Oberon-Titania	0.506×10^{34}	0.559×10^{34}	1.10	0.773×10^{34}	0.825×10^{34}	1.07

Table 3. Corrected data for one of the satellites of Saturn and two satelites of Uranus; see [5] and [6]. The period T_n is given in sidereal days (1 sidereal day = 86400 s); R_n is the mean distance of the satellite from planet center (in 10^3 m); M_{sn} is the satellite mass (in kg); E_n is the satellite energy (in J); P_n is the satellite momentum (in kg·ms^{-1}).

n	Satelite of Saturn	R_n	T_n	M_{sn}	$\lvert E_n \rvert$	P_n
7	Iapetus	3561000	79.33	1.88×10^{21}	0.1001×10^{29}	0.6138×10^{25}
n	Satelite of Uranus	R_n	T_n	M_{sn}	$\lvert E_n \rvert$	P_n
4	Titania	436300	18.704	3.48×10^{21}	0.2311×10^{29}	0.1269×10^{26}
5	Oberon	583400	13.463	2.92×10^{21}	0.1450×10^{29}	0.0920×10^{26}

$$\eta = \frac{C(T,E)+D(T,E)}{A(R,P)+B(R,P)} \approx 1.5 \tag{53}$$

is calculated in **Table 4**.

In the next step, we examine the relations between the phase-space areas when the mass reduction of the body momentum and energy mentioned at the end of Section 4 is done.

The planetary mass $M_{pn} = M_n$, or the satellite mass $M_{sn} = M_n$ enter both sides of Equation (16). The reduction of this mass gives instead of (16):

$$\frac{\pi P_n R_n}{M_n} = \frac{|E_n| T_n}{M_n} \tag{54}$$

or

$$\pi v_n R_n = \frac{1}{2}\left|\bar{V}_n^{pot}\right| T_n \tag{55}$$

where

$$v_n = \frac{2\pi R_n}{T_n} = \frac{P_n}{M_n} \tag{56}$$

is the velocity of a circulating celestial body and

$$\frac{1}{2}\left|\bar{V}_n^{pot}\right| = \frac{|E_n|}{M_n} \tag{57}$$

is the absolute value of the average potential of that body. This is so because $\frac{1}{2}\left|\bar{V}_n^{pot}\right|$ in (57) multiplied by M_n and differentiated with respect to the distance R_n equals to a half of the gravitational force acting between the motion center and the orbiting body.

A substitution of (56) and (57) instead of P_n and $|E_n|$ entering (48)-(53) [see a similar operation done for the atomic case in (46), (47) and the formulae (19a), (22a), (25a) and (26a)] gives:

$$A(R,v) = \pi\left(R_{n+1}-R_n\right)\left(\frac{P_{n+1}}{M_{n+1}}+\frac{P_n}{M_n}\right)\frac{1}{2}, \tag{58}$$

$$B(R,v) = \pi\left|\frac{P_{n+1}}{M_{n+1}}-\frac{P_n}{M_n}\right|\left(R_{n+1}+R_n\right)\frac{1}{2}, \tag{59}$$

$$C\left(T,V^{pot}\right) = \left(T_{n+1}-T_n\right)\left(\frac{|E_{n+1}|}{M_{n+1}}+\frac{|E_n|}{M_n}\right)\frac{1}{2}, \tag{60}$$

$$D\left(T,V^{pot}\right) = \left|\frac{|E_{n+1}|}{M_{n+1}}-\frac{|E_n|}{M_n}\right|\left(T_{n+1}+T_n\right)\frac{1}{2}. \tag{61}$$

Expressions

$$v_n = \frac{P_n}{M_n},\ v_{n+1} = \frac{P_{n+1}}{M_{n+1}} \tag{62}$$

are the velocities of the bodies on the orbits n and $n+1$, whereas

$$\frac{1}{2}\bar{V}_n^{pot} = \frac{E_n}{M_n},\ \frac{1}{2}\bar{V}_{n+1}^{pot} = \frac{E_{n+1}}{M_{n+1}} \tag{63}$$

represent the average body potentials on these orbits.

The ratios obtained for the hydrogen-like atom on the basis of (19a), (22a), (25a) and (26a) are:

$$\begin{gathered} A/B = 2;\ A/C = 2/3;\ A/D = 1;\ B/C = 1/3; \\ B/D = 1/2;\ C/D = 3/2. \end{gathered} \tag{64}$$

These ratios can be compared with similar ratios obtained for planets and satellites. We find (see **Tables 5** and **6**) that $A(R,v)/B(R,v)$ is very close to 2 and the ratio $C\left(T,V^{pot}\right)/D\left(T,V^{pot}\right)$ is very close to 3/2. The remaining ratios of (64), viz. $A(R,v)/C\left(T,V^{pot}\right)$, $A(R,v)/D\left(T,V^{pot}\right)$, $B(R,v)/C\left(T,V^{pot}\right)$ and $B(R,v)/D\left(T,V^{pot}\right)$ are presented in **Tables 7** and **8**. They are plainly close to the corresponding ratios (64). This agreement is evident especially in the case of planets; see **Table 7**.

In the case of satellites the strongest deviations of the observed data from the ratios (64) are obtained for the pairs of Deimos-Phobos and Iapetus-Titan (see **Table 8**).

Another ratio, similar to that calculated in (53) for the areas (48)-(51), can be calculated for the areas (58)-(61). This is

$$\chi = \frac{C\left(T,V^{pot}\right)+D\left(T,V^{pot}\right)}{A(R,v)+B(R,v)}. \tag{65}$$

Its values for different planetary and satellitary pairs

Table 4. Ratios between the averaged belt areas calculated for planets and satellites. Column first: the ratios of energy-time and momentum-position phase spaces calculated according to the Formula (53). Column second: the ratios of potential-time and velocity-position phase spaces calculated according to (65). The ratios η and χ approach a similar ratio calculated for the atomic electron orbits in (66).

Planets Pair	η	χ	Satellites Pair	η	χ
	Equation (53)	Equation (65)		Equation (53)	Equation (65)
Venus-Mercury	1.00	1.82	Deimos-Phobos	1.63	1.90
Earth-Venus	1.66	1.69	Europa-Io	1.43	1.72
Mars-Earth	1.26	1.72	Ganymede-Europa	1.00	1.73
Jupiter-Mars	1.00	2.10	Callisto-Ganymede	1.59	1.76
Saturn-Jupiter	1.44	1.77	Enceladus-Callisto	1.01	1.67
Uranus-Saturn	1.46	1.81	Tethys-Enceladus	1.00	1.68
Neptune-Uranus	1.94	1.73	Dione-Tethys	1.01	1.68
Pluto-Neptune	1.09	1.68	Rhea-Dione	1.00	1.70
			Titan-Rhea	1.00	1.87
			Iapetus-Titan	1.71	1.99
			Ariel-Miranda	1.00	1.70
			Umbriel-Ariel	1.63	1.71
			Titania-Umbriel	1.00	1.74
			Oberon-Titania	1.50	1.69

Table 5. Belt areas between the neighbouring planet orbits calculated (in $Jkg^{-1}s$) for the velocity-position and potential-time phase spaces; for $A(R,v)$ and $B(R,v)$ see Equations (58) and (59); for $C(T,V^{pot})$ and $D(T,V^{pot})$ see, respectively, (60) and (61). The input data are equal to those applied in calculating Table 1. The ratios A/B and C/D have their counterparts in the data of the Bohr-Sommerfeld theory given in (64). These are A/B = 2 and C/D = 3/2.

Planets Pair	$A(R,v)$	$B(R,v)$	$\frac{A}{B}$	$C(T,V^{pot})$	$D(T,V^{pot})$	$\frac{C}{D}$
Venus-Mercury	0.659×10^{16}	0.321×10^{16}	2.05	1.045×10^{16}	0.736×10^{16}	1.42
Earth-Venus	0.421×10^{16}	0.212×10^{16}	1.99	0.641×10^{16}	0.429×10^{16}	1.49
Mars-Earth	0.659×10^{16}	0.338×10^{16}	1.95	1.021×10^{16}	0.692×10^{16}	1.48
Jupiter-Mars	0.321×10^{17}	0.174×10^{17}	1.85	0.593×10^{17}	0.447×10^{17}	1.33
Saturn-Jupiter	0.231×10^{17}	0.118×10^{17}	1.95	0.366×10^{17}	0.252×10^{17}	1.45
Uranus-Saturn	0.373×10^{17}	0.192×10^{17}	1.95	0.600×10^{17}	0.420×10^{17}	1.43
Neptune-Uranus	0.313×10^{17}	0.158×10^{17}	1.98	0.483×10^{17}	0.330×10^{17}	1.46
Pluto-Neptune	0.210×10^{17}	0.122×10^{17}	1.72	0.342×10^{17}	0.218×10^{17}	1.57

are listed in **Table 4**. On the basis of calculations done in the Bohr-Sommerfeld theory of the hydrogen-like atom the values of η given by (53) and those of χ given by (65) should approach the same value

$$\eta^{at} = \chi^{at} = \frac{C+D}{A+B} = \frac{1+3/2}{1+1/2} = 5/3; \qquad (66)$$

see (19a), (22a), (25a) and (26a) at the end of Section 4.

6. Discussion

The aim of the present paper was to detect, and check, some regularities which can be obtained for the fragments of the phase-space areas defined by parameters characteristic for the motion of celestial bodies in the solar system. In this search we were guided by similar phase-space properties derived for the electron motion in

Table 6. Belt areas between the neighbouring satellite orbits calculated (in Jkg^{-1}s) for the velocity-position and potential-time phase spaces; for $A(R,v)$ and $B(R,v)$ see Equations (58) and (59); for $C(T,V^{pot})$ and $D(T,V^{pot})$ see, respectively, (60) and (61). The input data are equal to those applied in calculating Table 2. The ratios A/B and C/D have their counterparts in the data of the Bohr-Sommerfeld theory given in (64). These are A/B = 2 and C/D = 3/2.

Satellites Pair	$A(R,v)$	$B(R,v)$	$\frac{A}{B}$	$C(T,V^{pot})$	$D(T,V^{pot})$	$\frac{C}{D}$
Deimos-Phobos	0.776×10^{11}	0.404×10^{11}	1.92	0.130×10^{12}	0.094×10^{12}	1.39
Europa-Io	0.122×10^{14}	0.062×10^{14}	0.91	0.188×10^{14}	0.128×10^{14}	1.47
Ganymede-Europa	0.154×10^{14}	0.078×10^{14}	1.97	0.239×10^{14}	0.163×10^{14}	1.47
Callisto-Ganymede	0.243×10^{14}	0.124×10^{14}	1.95	0.382×10^{14}	0.263×10^{14}	1.46
Enceladus-Mimas	0.221×10^{13}	0.115×10^{13}	1.92	0.336×10^{13}	0.223×10^{13}	1.51
Tethys-Enceladus	0.215×10^{13}	0.106×10^{13}	2.02	0.322×10^{13}	0.218×10^{13}	1.49
Dione-Tethys	0.275×10^{13}	0.142×10^{13}	1.94	0.421×10^{13}	0.280×10^{13}	1.50
Rhea-Dione	0.436×10^{13}	0.218×10^{13}	2.00	0.664×10^{13}	0.449×10^{13}	1.48
Titan-Rhea	0.153×10^{14}	0.080×10^{14}	1.92	0.255×10^{14}	0.181×10^{14}	1.41
Iapetus-Titan	0.325×10^{14}	0.173×10^{14}	1.87	0.571×10^{14}	0.420×10^{14}	1.36
Ariel-Miranda	0.117×10^{13}	0.059×10^{13}	1.97	0.179×10^{13}	0.121×10^{13}	1.48
Umbriel-Ariel	0.120×10^{13}	0.060×10^{13}	1.99	0.183×10^{13}	0.124×10^{13}	1.48
Titania-Umbriel	0.222×10^{13}	0.112×10^{13}	1.98	0.345×10^{13}	0.236×10^{13}	1.46
Oberon-Titania	0.157×10^{13}	0.079×10^{13}	1.98	0.239×10^{13}	0.160×10^{13}	1.49

Table 7. The ratios $A(R,v)/C(T,V^{pot})$, $A(R,v)/D(T,V^{pot})$, $B(R,v)/C(T,V^{pot})$ and $B(R,v)/D(T,V^{pot})$ calculated for the belt areas of the velocity-position and potential-time spaces for planets. These ratios, as well as those calculated in Table 5, approach the ratios of the phase-space areas obtained from the electron orbits in the Bohr-Sommerfeld atom; see (64).

Planets Pair	$\frac{A(R,v)}{C(T,V^{pot})}$	$\frac{A(R,v)}{D(T,V^{pot})}$	$\frac{B(R,v)}{C(T,V^{pot})}$	$\frac{B(R,v)}{D(T,V^{pot})}$
Venus-Mercury	0.63	0.90	0.31	0.44
Earth-Venus	0.66	0.98	0.33	0.49
Mars-Earth	0.65	0.95	0.33	0.49
Jupiter-Mars	0.54	0.72	0.29	0.40
Saturn-Jupiter	0.63	0.92	0.32	0.47
Uranus-Saturn	0.62	0.89	0.32	0.46
Neptune-Uranus	0.65	0.95	0.33	0.48
Pluto-Neptune	0.61	0.96	0.36	0.56

the Bohr-Sommerfeld atom. The calculations applied the data calculated from [5] and those listed in **Tables 1** and **2** in [6]. However, several printing errors were discovered in **Table 2** of the last reference for the data of Iapetus, the satellite of Saturn, and Titania and Oberon, the satellites of Uranus. The corrected data are given in **Table 3** of the present paper.

For planets the belt areas defined in the momentum-position and energy-time phase spaces exhibit their ratios $B(R,P)/A(R,P)$ and $D(T,E)/C(T,E)$ [see (48)-(51)] considerably different from unity only for Earth-Venus, Mars-Earth, Neptune-Uranus, and Pluto-Neptune pairs; see **Table 1**. Similar belts for the phase-space areas defined for the neighbouring orbits of the satellites (**Table 2**) show the ratios $B(R,P)/A(R,P)$ and $D(T,E)/C(T,E)$ still closer to unity than in the planet

Table 8. The ratios $A(R,v)/C(T,V^{pot})$, $A(R,v)/D(T,V^{pot})$, $B(R,v)/C(T,V^{pot})$ and $B(R,v)/D(T,V^{pot})$ calculated for the belt areas of the velocity-position and potential-time spaces for satellites. These ratios, as well as those calculated in Table 6, approach the ratios of the phase-space areas obtained from the electron orbits in the Bohr-Sommerfeld atom; see (64).

Satellites Pair	$\frac{A(R,v)}{C(T,V^{pot})}$	$\frac{A(R,v)}{D(T,V^{pot})}$	$\frac{B(R,v)}{C(T,V^{pot})}$	$\frac{B(R,v)}{D(T,V^{pot})}$
Deimos-Phobos	0.60	0.83	0.31	0.43
Europa-Io	0.65	0.95	0.33	0.48
Ganymede-Europa	0.64	0.94	0.33	0.48
Callisto-Ganymede	0.64	0.92	0.32	0.47
Enceladus-Mimas	0.66	0.99	0.34	0.52
Tethys-Enceladus	0.67	0.99	0.33	0.49
Dione-Tethys	0.65	0.98	0.34	0.51
Rhea-Dione	0.66	0.97	0.33	0.49
Titan-Rhea	0.60	0.85	0.31	0.44
Iapetus-Titan	0.57	0.77	0.30	0.41
Ariel-Miranda	0.65	0.97	0.33	0.49
Umbriel-Ariel	0.67	0.99	0.34	0.50
Titania-Umbriel	0.64	0.94	0.33	0.48
Oberon-Titania	0.66	0.98	0.33	0.49

case. The only strong deviations from unity are exhibited for the satellite pairs of Thetys-Enceladus, Dione-Thetys and Ariel-Miranda.

Some properties concerning the belt areas in the phase-space and their ratios can be deduced in an analytic way. For example, an approximate equivalence between $A(R,v)$ and $C(T,V^{pot})$ [see (58) and (60)] can be attained on condition $A(R,v)$ is multiplied by the factor of 3/2. For, from (58) and (62) we obtain:

$$\begin{aligned}&\frac{3}{2}\pi\left(R_{n+1}-R_n\right)\left(v_{n+1}+v_n\right)\frac{1}{2}\\&=\frac{3}{2}\pi\Delta R\frac{2\pi R}{T}=3\pi^2\frac{R}{T}\Delta R\end{aligned}\tag{67}$$

where

$$\Delta R=R_{n+1}-R_n,\tag{68}$$

$$v=\frac{2\pi R}{T}\approx\left(v_{n+1}+v_n\right)\frac{1}{2}.\tag{69}$$

Similarly, from (60) and (63):

$$\begin{aligned}&\left(T_{n+1}-T_n\right)\left(\left|\bar{V}_{n+1}^{pot}\right|+\left|\bar{V}_n^{pot}\right|\right)\frac{1}{4}\\&=\Delta T\frac{\varkappa M_s}{2}\left(\frac{1}{2R_{n+1}}+\frac{1}{2R_n}\right)\approx\Delta T\varkappa M_s\frac{1}{2R},\end{aligned}\tag{70}$$

where $\varkappa$ is the gravitational constant. Here

$$\Delta T=T_{n+1}-T_n,\tag{71}$$

$$\frac{1}{R}\cong\frac{1}{2}\left(\frac{1}{R_{n+1}}+\frac{1}{R_n}\right).\tag{72}$$

From the Kepler's law (see e.g. [4])

$$T_n^2\cong\frac{4\pi^2}{\varkappa M_s}R_n^3\tag{73}$$

We obtain by differentiation the relation:

$$2T\Delta T\cong\frac{4\pi^2}{\varkappa M_s}3R^2\Delta R\tag{74}$$

valid approximately for any n. A substitution of (74) into (70) gives

$$\begin{aligned}\varkappa M_s\frac{1}{2R}\Delta T&=\varkappa M_s\frac{1}{2R}\frac{4\pi^2}{\varkappa M_s}\frac{3R^2}{2T}\Delta R\\&=3\pi^2\frac{R}{T}\Delta R,\end{aligned}\tag{75}$$

which is identical to (67). Hence, the ratio between $A(R,v)$ and $C(T,V^{pot})$ [see (58) and (60)] should be approximately equal to

$$\frac{A(R,v)}{C(T,V^{pot})}\approx\frac{2}{3},\tag{76}$$

which is the result obtained also for A/C in the atom on the basis of (19a) and (25a). In calculations we took into account the fact that the average $\bar{V}^{pot}$ entering (63) are not much different from the actual V^{pot} because of a quasi-circular character of the orbit possessed by a moving body.

Other ratios than 2/3 given in (76) can be deduced between the areas (58), (59) and (60), (61). These ratios are identical to the ratios between the belt areas (34), (37), (40) and (41) [or (19a), (22a), (25a) and (26a)] obtained for the Bohr-Sommerfeld hydrogen atom; see (64). In **Tables 7** and **8** we calculate these ratios for the planetary and satellitary systems.

A characteristic point is that the ratios calculated in **Tables 7** and **8** are only feebly dependent on the choice of the planetary or satellitary pair. In this sense these ratios have an almost constant character similar to that postulated at the end of Section 1.

In particular, the average deviations of the ratios in **Tables 7** and **8** from the ratios calculated in (64) are respectively:

$$\left|\frac{A/C-(A/C)^{\text{Table}}}{A/C}\right| = 0.06; \quad \cdots = 0.04; \tag{77}$$

$$\left|\frac{A/D-(A/D)^{\text{Table}}}{A/D}\right| = 0.09; \quad \cdots = 0.07; \tag{78}$$

$$\left|\frac{B/C-(B/C)^{\text{Table}}}{B/C}\right| = 0.03; \quad \cdots = 0.02; \tag{79}$$

$$\left|\frac{B/D-(B/D)^{\text{Table}}}{B/D}\right| = 0.05; \quad \cdots = 0.05; \tag{80}$$

The first numbers calculated in (77)-(80) concern the data taken from **Table 7**, the second numbers concern the data taken from **Table 8**.

REFERENCES

[1] J. C. Slater, "Quantum Theory of Atomic Structure," McGraw-Hill, New York, 1960.

[2] L. D. Landau and E. M. Lifshitz, "Mechanics," Pergamon Press, Oxford, 1960.

[3] A. Sommerfeld, "Mechanik (in German)," Akademische Verlagsgesellschaft, Leipzig, 1943.

[4] V. M. Blanco and S. W. McCuskey, "Basic Physics of the Solar System," Addison-Wesley, Reading, 1961.

[5] K. L. Lang, "Astrophysical Data: Planets and Stars," Springer, New York, 1991.

[6] S. Olszewski, "Equivalence of Energy-Time and Momentum-Position Phase Spaces of the Solar System," *Earth, Moon and Planets*, Vol. 74, No. 2, 1996, pp. 151-173. doi:10.1007/BF00056411

[7] A. Sommerfeld, "Atombau und Spektrallinien," Vol. 1, 5th Edition, Vieweg, Braunschweig, 1931.

Journal of Modern Physics, **2012, 3, 1152-1171**
doi:10.4236/jmp.2012.329150 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Astrophysical and Cosmological Probes of Dark Matter

Matts Roos
Department of Physics, University of Helsinki, Helsinki, Finland
Email: matts.roos@helsinki.fi

Received June 17, 2012; revised July 16, 2012; accepted July 26, 2012

ABSTRACT

Dark matter has been introduced to explain substantial mass deficits noted at different astronomical scales, in galaxies, groups of galaxies, clusters, superclusters and even across the full horizon. Dark matter does not interact with baryonic matter except gravitationally, and therefore its effects are sensed only on the largest scales. Although it is still unknown whether dark matter consists of particles or of a field or has some other nature, it has a rich phenomenology. This review summarizes all the astrophysical and cosmological probes that have produced overwhelming evidence for its existence. The breadth of the subject does not permit details on the observational methods (the reference list then helps), thus the review is intended to be useful mainly to cosmologists searching to model dark matter.

Keywords: pacs{95.35.+d, 98.65.-r, 98.62.-g, 98.80.-k, 98.90.+s}

1. Introduction

Apparently the matter content of the Universe is dominated by an unknown form of dark matter (DM) without interactions with ordinary baryonic matter, perhaps not even with itself. It only interacts via the gravitational field, manifesting its effects on astrophysical and cosmological scales. The purpose of this review is to summarize the phenomenology of all such effects, that can serve as probes of dark matter. Regardless of the ultimate, correct explanation of its particle nature or field nature, theory needs to address all these effects.

This review does not cover the historical development, except by glimpses, because the rapid development of observational means tends to render all discoveries older than a decade unimportant.

Beginning from the first controversial conclusions from the motion of stars near the Galactic disk on missing matter in the Galactic disk (Section 2), and that of Fritz Zwicky in 1933 [1] of missing matter in the Coma cluster (Section 3), we describe the kinematics of virially bound systems (Section 3) and rotating spiral galaxies (Section 4). An increasingly important method to determine the weights of galaxies, clusters and gravitational fields at large, independently of electromagnetic radiation, is lensing, strong as well as weak (Section 5). Next follows a discussion of dark matter in elliptical galaxies (Section 6) and mass-to-light ratios which probe dark matter in all systems, notably in dwarf spheroidals (Section 7). Different ways to measure missing mass in groups and clusters derive from the comparison of visible light and X-rays (Section 8). Mass autocorrelation functions relate galaxy masses to dark halo masses (Section 9).

In radiation the most important tools are the temperature and polarization anisotropies in the Cosmic Microwave Background (CMB) (Section 10), which give information on the mean density of both dark and baryonic matter as well as on the geometry of the Universe. The large scale structures of matter exhibit similar fluctuations evident in the Baryonic Acoustic Oscillations (BAO) (Section 11). The amplitude of the temperature variations in the CMB prove, that galaxies could not have formed in a purely baryonic Universe (Section 12). Simulations of large scale structures also show that DM must be present (Section 13). The best quantitative estimates of the density of DM come from overall parametric fits to cosmological models, notably the Cold Dark Matter model "ΛCDM" with a cosmological constant Λ, of CMB data, BAO data, and redshifts of supernovae of type Ia (SNe Ia) (Section 14). A particularly impressive testimony comes from merging clusters (Section 15). We conclude this review with a brief summary (Section 16).

2. Stars near the Galactic Disk

In 1922 the Dutch astronomer Jacobus Kapteyn [2] studied the vertical motions of all known stars near the Galactic plane and used these data to calculate the acceleration of matter. This amounts to treating the stars as members of a "star atmosphere", a statistical ensemble in which the density of stars and their velocity dispersion defines a "temperature" from which one obtains the gravitational potential. This is analogous to how one obtains the gravitational potential of the Earth from a study of the atmosphere. Kapteyn found that the spatial density is sufficient to explain the vertical motions.

Later in the same year the British astronomer James Jeans [3] reanalyzed Kapteyn's data and found a mass deficit: to each bright star two dark stars had to be present. The result contradicted grossly the expectations: if the potential provided by the known stars was not sufficient to keep the stars bound to the Galactic disk, the Galaxy should rapidly be losing stars. Since the Galaxy appeared to be stable there had to be some missing matter near the Galactic plane.

In 1932 the Dutch astronomer Jan Hendrik Oort [4] reanalyzed the vertical motions and came to the same conclusion as Jeans. There was indeed a mass deficit which Oort proposed to indicate the presence of some dark matter in our Galaxy. The possibility that this missing matter would be nonbaryonic could not even be thought of at that time. Note that the first neutral baryon, the neutron, was discovered by James Chadwick [5] only in the same year, in 1932.

However, it is nowadays considered, that this does not prove the existence of DM in the disk. The potential in which the stars are moving is not only due to the disk, but rather to the totality of matter in the Galaxy which is dominated by the Galactic halo. The advent of much more precise data in 1998 led Holmberg & Flynn [6] to conclude that no DM was present in the disk.

Oort determined the mass of the Galaxy to be 10^{11} M_{sun}, and thought that the nonluminous component was mainly gas. Still in 1969 he thought that intergalactic gas made up a large fraction of the mass of the universe [7]. The general recognition of the missing matter as a possibly new type of non-baryonic DM dates to the early eighties.

3. Virially Bound Systems

The planets move around the Sun along their orbits with orbital velocities balanced by the total gravity of the Solar system. Similarly, stars move in galaxies in orbits with orbital velocities v determined by the gravitational field of the galaxy, or they move with velocity dispersion σ. Galaxies in turn move with velocity dispersion σ under the influence of the gravitational field of their environment, which may be a galaxy group, a cluster or a supercluster. In the simplest dynamical framework one treats massive systems (galaxies, groups and clusters) as statistically steady, spherical, self-gravitating systems of N objects with average mass m and average velocity v or velocity dispersion σ. The total kinetic energy E of such a system is then (we now use σ rather than v)

$$E = (1/2) Nm\sigma^2 . \tag{1}$$

If the average separation is r, the potential energy of $N(N-1)/2$ pairings is

$$U = -(1/2) N(N-1) Gm^2/r . \tag{2}$$

The *virial theorem* states that for such a system

$$E = -U/2 . \tag{3}$$

The total dynamic mass M_{dyn} can then be estimated from σ and r

$$M_{dyn} = Nm = 2r\sigma^2/G . \tag{4}$$

This can also be written

$$\sigma^2 \propto (M_{dyn}/L) IR , \tag{5}$$

where I is a surface luminosity, R is a scale, and M_{dyn}/L is the *mass-to-light ratio*. Choosing the scale to be the half light radius R_e, this implies a relationship between the observed central velocity dispersion σ_0, I_e and R_e called the *Fundamental Plane*. of the form

$$R_e \propto (\sigma_0)^a (I_e)^b . \tag{6}$$

The virial theorem predicts the values $a = 2$, $b = 1$ for the coefficients. This relationship is found in ellipticals [8,9] and in some other types of stellar populations, but with somewhat different coefficients.

3.1. Halo Density Profiles

The shapes of DM halos in galaxies and clusters need to be simulated or fitted by empirical formulae. Mostly the shape is taken to be spherically symmetric so that the total gravitating mass profile $M(r)$ depends on three parameters: the mass proportion in stars, the halo mass and the length scale. A frequently used radial density profile parametrization is

$$\rho_{DM}(r) = \rho_0 \Big/ \left[(r/r_s)^\alpha (1 + r/r_s)^{3-\alpha} \right], \tag{7}$$

where ρ_0 is a normalization constant and $0 \le \alpha \le 3/2$. Standard choices are $\alpha = 1$ for the Navarro-Frenk-White profile (NFW) [10], and $\alpha = 3/2$ for the profile of Moore *et al.* [11], both *cusped* at $r = 0$.

Another parametrization is the Einasto profile ([12] and earlier references therein)

$$\rho_{DM}(r) = \rho_e \exp\left\{ -d_n \left[(r/r_e)^{1/n} - 1 \right] \right\}, \tag{8}$$

where the term d_n is a function of n such that ρ_e is the density at r_e, which defines a volume containing half of the total mass. At $r = 0$ the density is then finite and *cored*.

The Burkert profile [13] has a constant density core

$$\rho_{DM}(r) = \rho_0 \Big/ \left[(1 + r/r_s)\left(1 + (r/r_s)^2\right) \right], \tag{9}$$

which fitted dwarf galaxy halos well in 1995, but no longer does so, see Section 7.

Some clusters are not well fitted by any spherical approximation. The halo may exhibit a strong ellipticity or triaxiality in which case none of the above profiles is good.

The dependence of the physical size of clusters on the mass, characterized by the mass concentration index $c \equiv r_{vir}/r_s$, has been studied in ΛCDM simulations [14]. At intermediate radii c is a crucial quantity in determining the density shape.

3.2. The Coma Cluster

Historically, the first observation of dark matter in an object at a cosmological distance was made by Fritz Zwicky in 1933 [1]. While measuring radial velocity dispersions of member galaxies in the Coma cluster (that contains some 1000 galaxies), and the cluster radius from the volume they occupy, Zwicky was the first to use the virial theorem to infer the existence of unseen matter. He found to his surprise that the dispersions were almost a factor of ten larger than expected from the summed mass of all visually observed galaxies in the Coma. He concluded that in order to hold galaxies together the cluster must contain huge amounts of some non-luminous matter. From the dispersions he concluded that the average mass of galaxies within the cluster was about 160 times greater than expected from their luminosity (a value revised today), and he proposed that most of the missing matter was dark.

Zwicky's suggestion was not taken seriously at first by the astronomical community which Zwicky felt as hostile and prejudicial. Clearly, there was no candidate for the dark matter because gas radiating X-rays and dust radiating in the infrared could not yet be observed, and non-baryonic matter was unthinkable. Only some forty years later when studies of motions of stars within galaxies also implied the presence of a large halo of unseen matter extending beyond the visible stars, dark matter became a serious possibility.

Since that time, modern observations have revised our understanding of the composition of clusters. Luminous stars represent a very small fraction of a cluster mass; in addition there is a baryonic, hot *intracluster medium* (ICM) visible in the X-ray spectrum. Rich clusters typically have more mass in hot gas than in stars; in the largest virial systems like the Coma the composition is about 85% DM, 14% ICM, and only 1% stars [15].

In modern applications of the virial theorem one also needs to model and parametrize the radial distributions of the ICM and the dark matter densities. In the outskirts of galaxy clusters the virial radius roughly separates bound galaxies from galaxies which may either be infalling or unbound. The virial radius r_{vir} is conventionally defined as the radius within which the mean density is 200 times the background density.

Matter accretion is in general quite well described within the approximation of the *Spherical Collapse Model*. According to this model, the velocity of the infall motion and the matter overdensity are related. Mass profile estimation is thus possible once the infall pattern of galaxies is known [16].

In **Figure 1** the Coma profile is fitted [15] with Equation (7) with $\alpha = 0$ which describes a centrally finite profile which is almost flat. The separation of different components in the core is not well done with Equation (7) because the Coma has a binary center like many other clusters [17].

3.3. The AC 114 Cluster

Dark matter is usually dissected from baryons in lensing analyses by first fitting the lensing features to obtain a map of the total matter distribution and then subtracting the gas mass fraction as inferred from X-ray observations [19,20]. The total mass map can then be obtained with parametric models in which the contribution from cluster-sized DM halos is considered together with the main galactic DM halos [21]. Mass in stars and in stellar remnants is estimated converting galaxy luminosity to mass assuming suitable stellar mass to light ratios.

One may go one step further by exploiting a parametric model which has three kinds of components: cluster-sized DM halos, galaxy-sized (dark plus stellar) matter halos, and a cluster-sized gas distribution [17,18]. As an example we show the results of such an analysis of the dynamically active cluster AC 114 in **Figure 2**.

In systems of merging clusters DM may become spatially segregated from baryonic matter and thus observable. We shall meet several such cases in Section 15.

3.4. The Local Group

The Local Group is a very small virial system, dominated by two large galaxies, the M31 or Andromeda galaxy, and the Milky Way. The M31 exhibits blueshift, falling in towards us. Evidently our Galaxy and M31 form a bound system together with all or most of the minor galaxies in the Local Group. The Local Group extends to about 3 Mpc and the velocity dispersions of its members is about 200 $\mathrm{km \cdot s^{-1}}$.

In this group the two large galaxies dominate the dynamics, so that it is not meaningful to define a statistically average pairwise separation between galaxies, nor an average mass nor an average orbital velocity. The total kinetic energy E is still given by the sum of all the group members, and the potential energy U by the sum of all the galaxy pairs, but here the pair formed by the M31 and the Milky Way dominates, and the pairings of the smaller members with each other are negligible.

An interesting recent claim is, that the mass estimate

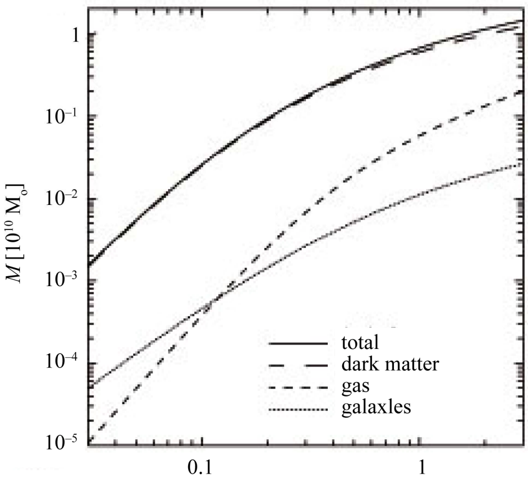


Figure 1. Density profile of matter components enclosed within a given radius *r* in the Coma cluster, versus r/r_{vir}. From E. L. Lokas and G. A. Mamon [15].

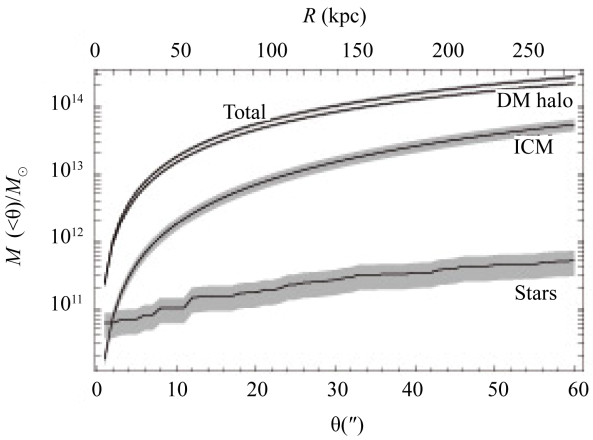


Figure 2. Density profile of matter components in the cluster AC 114, enclosed within a given projected radius. From M. Sereno *et al.* [18].

of the Local Group is also affected by the accelerated expansion, the "dark energy". A. D. Chernin *et al*. [22] have shown that the potential energy U is reduced in the force field of dark energy, so that the virial theorem for N masses m_i with baryocentric radius vectors r_i takes the form

$$E = -(1/2)U + U_2 , \tag{10}$$

where U is defined as in Equation (3), and

$$U_2 = -(4\pi\rho_v/3)\sum m_i r_i^2 \tag{11}$$

is a correction which reduces the potential energy due to the background dark energy density ρ_v. In the Local Group this correction to the mass appears to be quite substantial, of the order of 30% - 50%.

The dynamical mass of the local group is $3.2 - 3.7\times10^{12}$ solar masses whereas the total visible mass of the Galaxy + M31 is only 2×10^{11} solar masses. Thus there is a large amount of dark matter missing.

3.5. The local Universe

In a large volume beyond the local group, Tully in 1984 [23] measured the velocities of 2367 galaxies with radial velocities below 3000 km·s^{-1}. He found that the mass density parameter (which is normalized to the critical mass) in this "Local Universe" was $\Omega_m = 0.08$, in clear conflict with the global value, $\Omega_{m,global} = 0.27\pm0.02$ (as we shall see in Section 14).

More recently Karachentsev [24] has extended this analysis out to a volume of a diameter of 96 Mpc, containing 11,000 galaxies appearing single, in pairs, in triplets and in groups. Most of them belong to the Local Supercluster and constitute $<15\%$ of the mass of Virgo. The radial velocities are $v < 3500$ km·s^{-1}. These galaxies can be treated as a virial system with average density $\Omega_{m,local} = 0.08\pm0.02$, again surprisingly small compared to the global density. Karachentsev quotes three proposed explanations for this mass deficit.

- Dark matter in the systems of galaxies extends far beyond their virial radius, so that the total mass of a group or cluster is 3 - 4 times larger than the virial estimate. However, this contradicts other existing data.
- The diameter of the considered region of the Local universe, 90 Mpc, does not correspond to the true scale of the "homogeneity cell"; our Galaxy may be located in- side a giant void sized about 100 - 500 Mpc, where the mean density of matter is 3 to 4 times lower than the global value. However, the location of our Galaxy is characterized by an excess, rather than by a deficiency of local density at all scales up to 45 Mpc.
- Most of the dark matter in the Universe, or about two thirds of it, is not associated with groups and clusters of galaxies, but distributed in the space between them in the form of massive dark clumps or as a smooth "ocean". It is as yet difficult to evaluate this proposal.

Clearly the physics in the Local Universe does not prove the existence of dark matter, rather it brings in new problems.

4. Rotation Curves of Spiral Galaxies

Spiral galaxies are stable gravitationally bound systems in which visible matter is composed of stars and interstellar gas. Most of the observable matter is in a relatively thin disc, where stars and gas rotate around the galactic center on nearly circular orbits. The galaxy kinematics is measured by the Doppler shift of well-known emission lines of particular tracers of the gravitational potential: HI, CO and H_α.

If the circular velocity at radius r is v in a rotating galaxy with mass $M(r)$ inside r, the condition for stability is that the centrifugal acceleration v/r should equal the gravitational pull $GM(r)/r^2$, and the radial dependence of v would then be expected to follow Kepler's law

$$v^2 = GM(r)/r \tag{12}$$

The surprising result for spiral galaxy rotation curves is, that the velocity does not follow Kepler's inverse-root law, but stays rather constant after attaining a maximum. The most obvious solution to this is that the galaxies are embedded in extensive, diffuse halos of dark matter If the mass $M(r)$ enclosed inside the radius r, is proportional to r it follows that $v(r) \approx$ *constant*.

The rotation curve of most galaxies can be fitted by the superposition of contributions from the stellar and gaseous disks, sometimes a bulge, and the dark halo, modeled by a quasi-isothermal sphere. The inner part is difficult to model because the density of stars is high, rendering observations of individual star velocities difficult. Thus the fits are not unique, the relative contributions of disk and dark matter halo is model-dependent, and it is sometimes not even sure whether galactic disks do contain dark matter. Typically, dark matter constitutes about half of the total mass.

In **Figure 3** we show the rotation curves fitted for eleven well-measured galaxies [25] of increasing halo mass. One notes, that the central dark halo component is indeed much smaller than the luminous disk component.

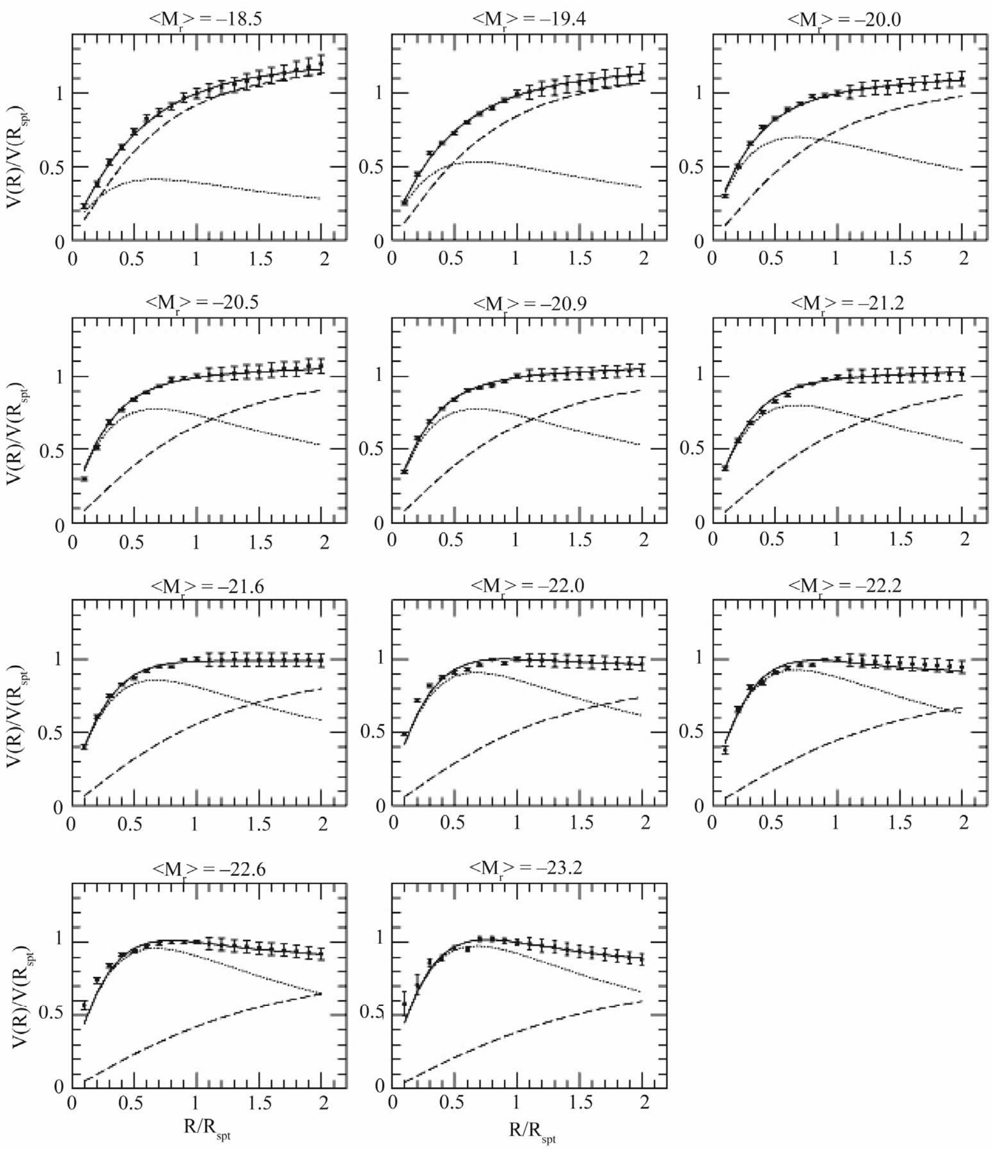


Figure 3. Best disk—halo fits to the universal rotation curve (dotted line is disk, dashed line is halo). Each object is identified by the halo virial mass, increasing downwards. From P. Salucci *et al.* [25].

At large radii, however, the need for a DM halo is obvious. On galactic scales, the contribution of DM generally dominates the total mass. Note the contribution of the baryonic component, negligible for light masses but increasingly important in the larger structures.

The mass discrepancy emerges also as a disagreement between light and mass distributions: light does not trace mass, the ratio

$$\left(\mathrm{d}M/\mathrm{d}r\right)/\left(\mathrm{d}L/\mathrm{d}r\right) \tag{13}$$

is not constant, but increases with radius [26].

Gentile *et al*. [27] have shown that cusped profiles are in clear conflict with data on spiral galaxies. Central densities are rather flat, scaling approximately as $\rho_0 \propto r_{luminous}^{-2/3}$. The best-fit disk + NFW halo mass model fits the rotation curves poorly, it implies an implausibly low stellar mass-to-light ratio and an unphysically high halo mass. Clearly the actual profiles are of very uncertain origin.

One notes in **Figure 3** that the shape of the rotation curve depends on the halo virial mass so that the distribution of gravitating matter, unlike luminous matter, is luminosity dependent. The old idea that the rotation curve stays constant after attaining a maximum is thus a simplification of the real situation. The rotation velocity can be expressed by a *Universal Rotation Curve* [25]. All spiral galaxies lie on a curve in the 4-dimensional space of luminosity, core radius, halo central density and fraction of DM, see **Figure 4**.

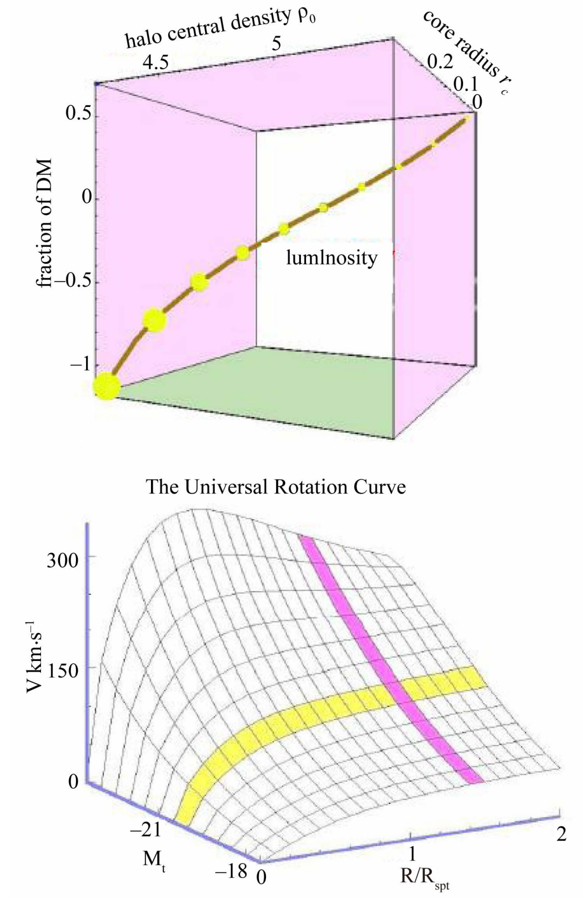


Figure 4. Upper: The 4-dimensional space of luminosity, core radius, halo central density and fraction of DM. Under: The smooth surface of spiral galaxy rotation curves in the space of normalized radius $R/R_{optical}$, magnitude M and rotation velocity V in km·s^{-1}. P. Salucci priv. comm. and ref. [25].

Our Galaxy is complicated because of what appears to be a noticeable density dip at 9 kpc and a smaller dip at 3 kpc, as is seen in **Figure 5** [28]. To fit the measured rotation curve one needs at least three contributing components: a central bulge, the star disk + gas, and a DM halo [28-30]. For small radii there is a choice of empirical rotation curves, and no DM component appears to be needed until radii beyond 15 kpc.

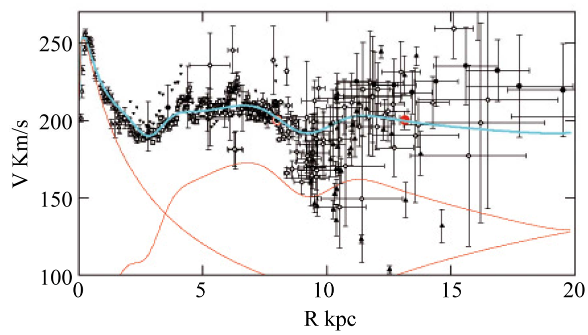


Figure 5. Decomposition of the rotation curve of the Milky Way into the components bulge, stellar disk + interstellar gas, DM halo (the red curves from left to right). From Y. Sofue *et al.* [28].

5. Strong and Weak Lensing

A consequence of the *Strong Equivalence Principle* (SEP) is that a photon in a gravitational field moves as if it possessed mass, and light rays therefore bend around gravitating masses. Thus celestial bodies can serve as *gravitational lenses* probing the gravitational field, whether baryonic or dark without distinction.

Since photons are neither emitted nor absorbed in the process of gravitational light deflection, the surface brightness of lensed sources remains unchanged. Changing the size of the cross-section of a light bundle only changes the flux observed from a source and magnifies it at fixed surface-brightness level. If the mass of the lensing object is very small, one will merely observe a magnification of the brightness of the lensed object an effect called *microlensing*. Microlensing of distant quasars by compact lensing objects (stars, planets) has also been observed and used for estimating the mass distribution of the lens-quasar systems.

In *Strong Lensing* the photons move along geodesics in a strong gravitational potential which distorts space as well as time, causing larger deflection angles and requiring the full theory of General Relativity. The images in the observer plane can then become quite complicated because there may be more than one null geodesic connecting source and observer; it may not even be possible to find a unique mapping onto the source plane *cf* **Figure 6**. Strong lensing is a tool for testing the distribution of mass in the lens rather than purely a tool for testing General Relativity. An illustration is seen in **Figure 7** where the lens is an elliptical galaxy [32].

At cosmological distances one may observe lensing by composed objects such as galaxy groups which are ensembles of "point-like", individual galaxies. Lensing effects are very model-dependent, so to learn the true magnification effect one needs very detailed information on the structure of the lens.

Weak Lensing refers to deflection through a small angle when the light ray can be treated as a straight line (**Figure 6**), and the deflection as if it occurred discontinuously at the point of closest approach (the thin-lens approximation in optics). One then only invokes SEP to account for the distortion of clock rates.

The large-scale distribution of matter in the Universe is inhomogeneous in every direction, so one can expect that everything we observe is displaced and distorted by weak lensing. Since the tidal gravitational field and the deflection angles depend neither on the nature of the matter nor on its physical state, light deflection probes the total projected mass distribution. Lensing in infrared light offers an additional advantage of being able to sense distant background galaxies, since their number density is higher than in the optical range.

Background galaxies would be ideal tracers of distortions if they were intrinsically circular, because lensing transforms circular sources into ellipses. Any measured ellipticity would then directly reflect the action of the gravitational tidal field of the interposed lensing matter, and the statistical properties of the distortions would reflect the properties of the matter distribution. But many galaxies are actually intrinsically elliptical, and the ellipses are randomly oriented. This introduces noise into the inference of the tidal field from observed ellipticities. A useful feature in the sky is a fine-grained pattern of faint and distant blue galaxies appearing as a "wall paper". This makes statistical weak-lensing studies possible, because it allows the detection of the coherent distortions imprinted by gravitational lensing on the images of the galaxy population.

Thus weak lensing has become an important technique to map non-luminous matter. A reconstruction of one of the largest and most detailed weak lensing surveys undertaken with the Hubble Space Telescope is shown in **Figure 8** [33]. This map covers a large enough area to see extended filamentary structures.

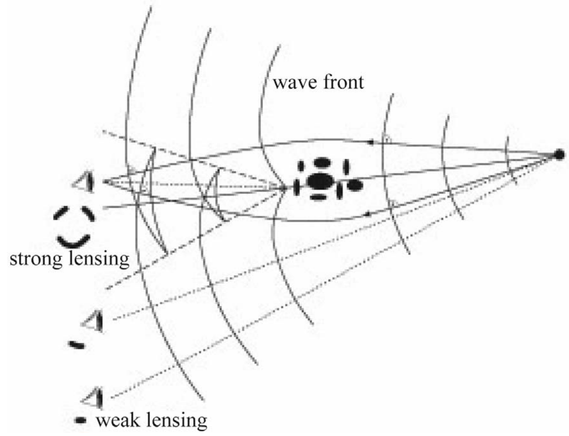


Figure 6. Wave fronts and light rays in the presence of a cluster perturbation. From N. Straumann [31].

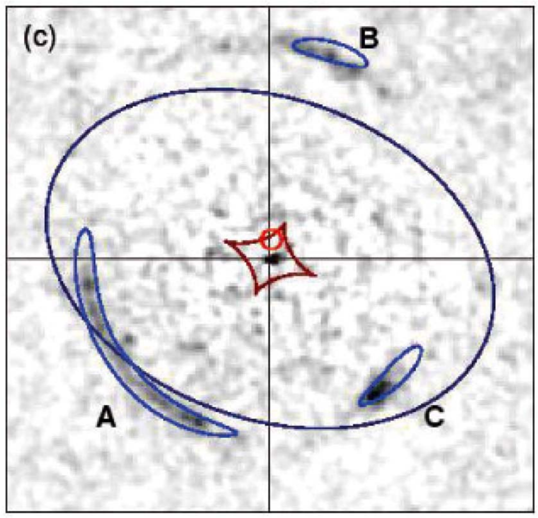


Figure 7. This image resulted from color-subtraction of a lensing singular isothermal elliptical galaxy. The strongly lensed object forms two prominent arcs A, B and a less extended third image C. From R. J. Smith *et al.* [32].

A very large review on lensing by R. Massey *et al*. [34] can be recommended. We show several examples of lensing by clusters in Section 15.

6. Elliptical Galaxies

Elliptical galaxies are quite compact objects which mostly do not rotate so their mass cannot be derived from rotation curves. The total dynamical mass is then the virial mass as derived from the velocity dispersions of stars and the anisotropies of their orbits. However, to disentangle the total mass profile into its dark and its stellar components is not straightforward, because the dynamical mass decomposition of dispersions is not unique. The luminous matter in the form of visible stars is a crucial quantity, indispensable to infer the dark component. When available one also makes use of strong and

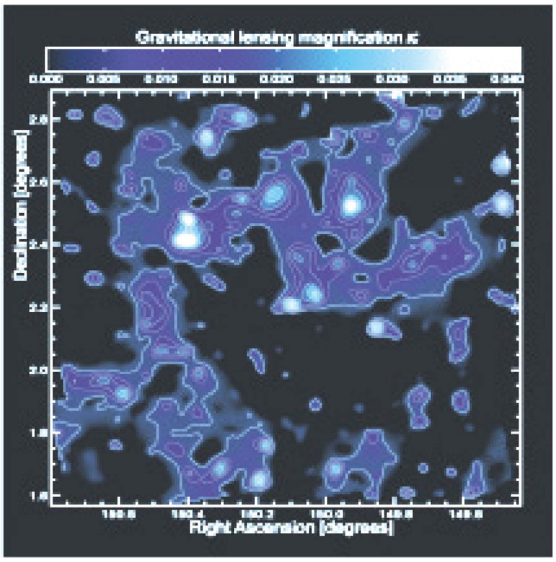


Figure 8. Map of the dark matter distribution in the 2-square degree COSMOS field: the linear blue scale on top shows the gravitational lensing magnification *κ*, which is proportional to the projected mass along the line of sight. From R. Massey *et al.* [33].

weak lensing data, and of the X-ray properties of the emitting hot gas. The gravity is then balanced by pressure gradients as given by Jeans' Equation.

Inside the half light radius R_e the contribution of the dark matter halo to the central velocity dispersion is often very small, $< 100 \ \text{km}\cdot\text{s}^{-1}$, so that the dark matter profile is intrinsically unresolvable. The outer mass profile is compatible with NFW, Equation (7), and with Burkert, Equation (9), as well. Important information on the mass distribution can be obtained from the Fundamental Plane, Equation (6). which yields the coefficients $a = 1.8$, $b = 0.8$. Note that this is in some tension with the Virial Theorem, perhaps due to variations in the central dispersions, σ_0, of the stellar populations.

O. Tiret *et al*. [36] concluded from a study of 23 giant elliptical galaxies with central velocity dispersions ≥ 330 km·s^{-1}, that the mass within 5 - 10 kpc is dominated by the stars, not by DM. On the average the dark matter component contributes less than 5% to the total velocity dispersions.

The ELIXR survey is a volume-limited (≤110 Mpc) study by P. J. Humphrey *et al*. [35], of optically selected, isolated, L* elliptical galaxies in particular the NGC 1521, for which X-ray data from *Chandra* and *XMM* exist. The isolation condition selects the appropriate galaxy halo and reduces the influence of a possible group-scale or cluster-scale halo.

Most of the baryons are in a morphologically relaxed hot gas halo detectable out to ≈ 200 kpc, that is well described by hydrostatic models. The baryons and the dark matter conspire to produce a total mass density profile that can be well-approximated by a power law, $\rho_{tot} \propto r^{-\alpha}$ over a wide range (as has been noted before, see references in [35,36]).

The fitting method involves solving the equation of hydrostatic equilibrium to compute temperature and density profile models, given parametrized mass and entropy profiles. The models are then projected onto the sky and fitted to the projected temperature and density profiles. A fit ignoring DM was poor, but inclusion of DM improved the fit highly significantly: DM was required at 8.2σ. We show this fit in **Figure 9**. In several studies [36,37], for most of the radii the dark matter contribution is very small although statistically significant.

7. Mass to Luminosity Ratios and Dwarf Spheroidals

The mass-to-light ratio of an astronomical object is defined as $\Upsilon \equiv M/L$. Stellar populations exhibit values $\Upsilon = 1-10$ in solar units, in the solar neighborhood $\Upsilon = 2.5-7$, in the Galactic disk $\Upsilon = 1.0-1.7$ from C. Flynn *et al*. [38].

Dwarf spheroidal galaxies (dSph) are the smallest stellar systems containing dark matter and exhibit very high *M*/*L* ratios, $\Upsilon = 10-100$. In Andromeda IX $\Upsilon =$ 93 + 120/–50, in Draco $\Upsilon = 330 \pm 125$. The dwarf spheroidals have radii of ≈ 100 pc and central velocity dispersions $\approx 10 \ \text{km}\cdot\text{s}^{-1}$ which is larger than expected for self-gravitating, equilibrium stellar populations. The generally accepted picture has been, that dwarf galaxies have slowly rising rotation curves and are dominated by dark matter at all radii.

However, R. A. Swaters *et al*. [39] have reported ob-

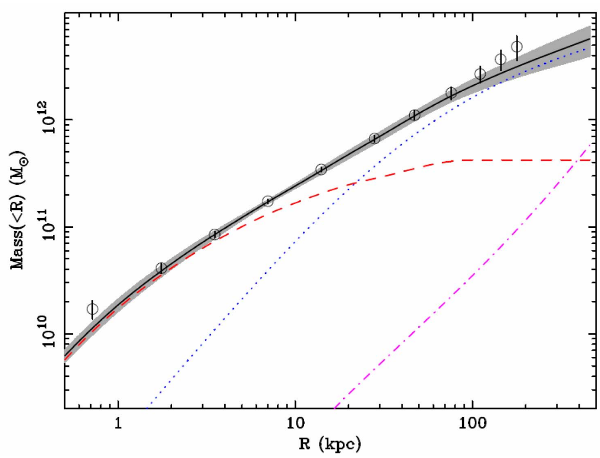


Figure 9. Radial mass profile of the elliptical galaxy NGC 1521 from a model calculation (not fitted to the measured points shown). The solid black line indicates the total enclosed mass (1*σ* errors in grey), the dashed red line is the stellar mass, the dotted blue line is the dark matter, and the dash-dot magenta line is the gas mass contribution. From P. J. Humphrey *et al*. [35].

servations of H I rotation curves for a sample of 73 dwarf galaxies, among which eight galaxies have sufficiently extended rotation curves to permit reliable determination of the core radius and the central density. They found that dark matter only becomes important at radii larger than three or four disk scale lengths. Their conclusion is, that the stellar disk can explain the mass distribution over the optical parts of the galaxy, and dark matter only becomes relevant at large radii. However, the required stellar mass-to-light ratios are high, up to 15 in the R-band.

Comparing the properties of dwarf galaxies in both the core and outskirts of the Perseus Cluster, Penny and Conselice [40] found a clear correlation between mass-to-light ratio and the luminosity of the dwarfs, such that the faintest dwarfs require the largest fractions of dark matter to remain bound. This is to be expected, as the fainter a galaxy is, the less luminous mass it will contain, therefore the higher its dark matter content must be to prevent its disruption. Dwarfs are more easily influenced by their environment than more massive galaxies.

The distance to the Perseus Cluster prevents an easy determination of Υ, so S. J. Penny and C. J. Conselice [40] instead determined the dark matter content of the dwarfs by calculating the minimum mass needed in order to prevent tidal disruption by the cluster potential, using their sizes, the projected distance from the cluster center to each dwarf and the mass of the cluster interior. Three of 15 dwarfs turned out to have mass-to-light ratios smaller than 3, indicating that they do not require dark matter.

Ultra-compact dwarf galaxies (UCDs) are stellar systems with masses of around 10^7 - 10^8 M_{sun} and half mass radii of 10 - 100 pc. A remarkable properties of UCDs is that their dynamical mass-to-light ratios are on average about twice as large as those of globular clusters of comparable metallicity, and also tend to be larger than what one would expect based on simple stellar evolution models. UCDs appear to contain very little or no dark matter.

H. Baumgardt and S. Mieske [41] have presented collisional N-body simulations which study the coevolution of a system composed of stars and dark matter. They find that DM gets removed from the central regions of such systems due to dynamical friction and mass segregation of stars. The friction timescale is significantly shorter than a Hubble time for typical globular clusters, while most UCDs have friction times much longer than a Hubble time. Therefore, a significant dark matter fraction remains within the half-mass radius of present-day UCDs, making dark matter a viable explanation for their elevated mass-to-light ratios.

A different type of systems are the ultra-faint dwarf galaxies (UFDs). When interpreted as steady state objects in virial equilibrium by V. Belokurov [42], would be the most DM dominated objects known in the Universe. Their half-light radii range from 70 pc to 320 pc.

A special case is the ultra-faint dwarf disk galaxy Segue 1 studied by M. Xiang-Gruess *et al.* [43] which has a baryon mass of only about 1000 solar masses. One interpretation is that this is a thin non-rotating stellar disk not accompanied by a gas disk, embedded in an axisymmetric DM halo and with a ratio $f \equiv M_{halo}/M_b \approx 200$. But if the disk rotates, f could be as high as 2000. If Segue 1 also has a magnetized gas disk, the dark matter halo has to confine the effective pressure in the stellar disk and the magnetic Lorentz force in the gas disk as well as possible rotation. Then f could be very large [43].

8. Small Galaxy Groups Emitting X-Rays

There are examples of groups formed by a small number of galaxies which are enveloped in a large cloud of hot gas (ICM), visible by its X-ray emission. One may assume that the electron density distribution associated with the X-ray brightness is in hydrostatic equilibrium, and one can extract the ICM radial density profiles by fits.

The amount of matter in the form of hot gas can be deduced from the intensity of this radiation. Adding the gas mass to the observed luminous matter, the total amount of baryonic matter, M_b, can be estimated, see M. Markevitch *et al*. [44] and C. De Boni and G. Bertin [45]. In clusters studied, the gas fraction increases with the distance from the center; the dark matter appears more concentrated than the visible matter.

The temperature of the gas depends on the strength of the gravitational field, from which the total amount of gravitating matter, M_{grav}, in the system can be deduced. In many such small galaxy groups one finds M_{grav}/M_b ≥ 3, testifying to a dark halo present. An accurate estimate of M_{grav} requires that also dark energy is taken into account, because it reduces the strength of the gravitational potential. There are sometimes doubts whether all galaxies appearing near these groups are physical members. If not, they will artificially increase the velocity scatter and thus lead to larger virial masses.

On the scale of large clusters of galaxies like the Coma, it is generally observed that DM represents about 85% of the total mass and that the visible matter is mostly in the form of a hot ICM.

9. Mass Autocorrelation Functions

If galaxy formation is a local process, then on large scales galaxies must trace mass. This requires the study of how galaxies populate DM halos. In simulations one attempts to track galaxy and DM halo evolution across cosmic time in a physically consistent way, providing positions, velocities, star formation histories and other physical properties for the galaxy populations of interest.

Guo *et al*. [46] use abundance matching arguments to derive an accurate relation between galaxy stellar mass and DM halo mass. They combine a stellar mass function based on spectroscopic observations with a precise halo/subhalo mass function obtained from simulations. Assuming this stellar mass-halo mass relation to be unique and monotonic, they compare it with direct observational estimates of the mean mass of halos surrounding galaxies of given stellar mass inferred from gravitational lensing and satellite galaxy dynamics data, and use it to populate halos in simulations. The stellar mass-halo mass relation is shown in **Figure 10**.

The implied spatial clustering of stellar mass turns out to be in remarkably good agreement with a direct and precise measurement. By comparing the galaxy autocorrelation function with the total mass autocorrelation function, as averaged over the Local Supercluster (LSC) volume, one concludes that a large amount of matter in the LSC is dark.

A similar study is that of Boyarsky *et al*. [47] who find a universal relation between DM column density and DM halo mass, satisfied by matter distributions at all observable scales in halo sizes from 10^8 to 10^{16} M_{sun}, as shown in **Figure 11**. Such a universal property is difficult to explain without dark matter.

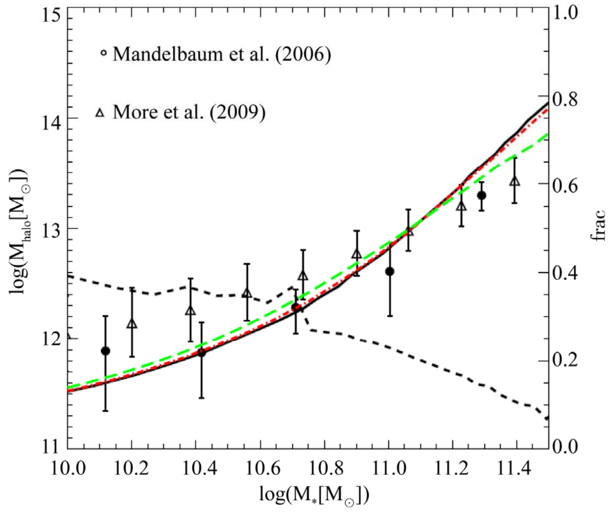


Figure 10. Dark matter halo mass M_{halo} as a function of stellar mass M_*. The thick black curve is the prediction from abundance matching assuming no dispersion in the relation between the two masses. Red and green dashed curves assume some dispersion in logM_*. The dashed black curve is the satellite fraction as a function of stellar mass, as labeled on the axis at the right-hand side of the plot. From Qi Guo *et al*. [46].

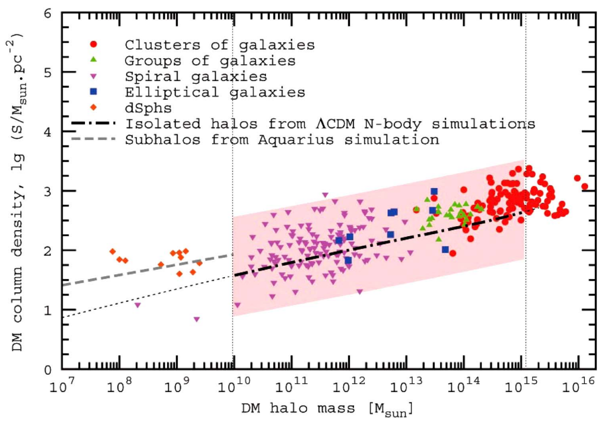


Figure 11. Dark matter column density vs. dark matter halo mass in solar units. From A. Boyarsky *et al.* [47].

10. Cosmic Microwave Background (CMB)

The tight coupling between radiation and matter density before decoupling caused the primordial adiabatic perturbations to oscillate in phase. Beginning from the time of last scattering, the receding horizon has been revealing these frozen density perturbations, setting up a pattern of standing acoustic waves in the baryon—photon fluid. After decoupling, this pattern is visible today as temperature anisotropies with a certain regularity across the sky.

The primordial photons are polarized by the anisotropic Thomson scattering process, but as long as the photons continue to meet free electrons their polarization is washed out, and no net polarization is produced. At a photon's last scattering however, the induced polarization remains and the subsequently free-streaming photon possesses a quadrupole moment.

Temperature and polarization fluctuations are analyzed in terms of multipole components or powers. The resulting distribution of powers versus multipole ℓ, or multipole moment $k = 2\pi/\ell$, is the *power spectrum* which exhibits conspicuous *Doppler peaks*. In **Figure 12** we display the radiation temperature (TT) and temperature—E-polarization correlation (TE) power spectra from the 7-year data of WMAP as functions of multipole moments [49]. The spectra can then be compared to theory, and theoretical parameters determined. Many experiments have determined the power spectra, so a wealth of data exists.

Baryonic matter feels attractive self-gravity and is pressure-supported, whereas dark matter only feels attractive self-gravity, but is pressureless. Thus the Doppler peaks in the CMBR power spectrum testify about baryonic and dark matter, whereas the troughs testify about rarefaction caused by the baryonic pressure. The position of the first peak determines $\Omega_m h^2$. Combining the TT data with determinations of the Hubble constant h, the WMAP team can determine the total mass density parameter $\Omega_m = \Omega_b + \Omega_{dm}$. The ratio of amplitudes of the

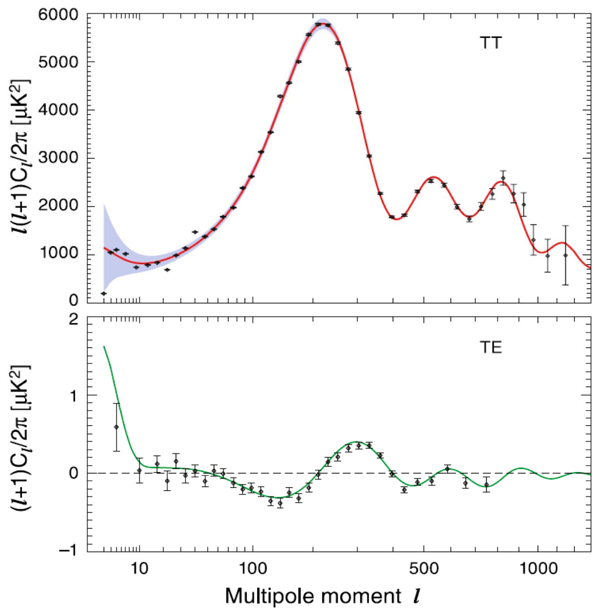


Figure 12. The CMB radiation temperature (TT) and temperature-polarization (TE) power spectra from the seven-year WMAP 94 GHz maps show anisotropies which can be analyzed by power spectra as functions of multipole moments. The solid line shows the best-fit prediction for the flat ΛCDM model. From D. Larson *et al.* [49].

second-to-first Doppler peaks determines the baryonic density parameter to be $\Omega_b = 0.0449 \pm 0.0028$ and the dark matter component to be $\Omega_{dm} = 0.222 \pm 0.026$ [49], thus $\Omega_m = 0.267$.

Power spectra at higher multipole moments have been measured with the Atacama Cosmology Telescope (ACT) [50] at 148 GHz and 218 GHz, as well as the cross-frequency spectrum between these two channels. and found to be in agreement with the 7-year WMAP 94 GHz maps in the common range $400 \le \ell \le 1000$. The ACT has also been able to measure the lensing of the CMB signal at a significance of 2.8σ, which slightly smooths out the acoustic peaks, **Figure 13**.

In a fit of the flat ΛCDM model to the data the dark matter density parameter comes out slightly higher than WMAP and the baryonic density slightly lower so the total density parameter for WMAP and ACT added is $\Omega_m = 0.276 \pm 0.016$ [51].

Information on the TE correlations comes from several measurements, among them WMAP [49], and on the E-mode polarization power spectrum alone (EE) from the QUAD collaboration [52], **Figure 14**.

The results show two surprises: Firstly, since $\Omega_m \ll 1$, a large component $\Omega_\Lambda \approx 0.74$ is missing, of unknown nature, and termed *dark energy*. The second surprise is that ordinary baryonic matter is only a small fraction of the total matter budget. The remainder is then dark matter, of unknown composition. Of the 4.5% of baryons in

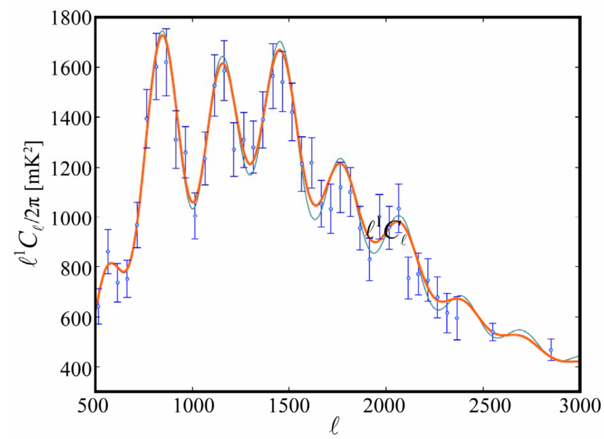


Figure 13. The ACT 148 GHz power spectrum multiplied by ℓ^4 is shown for lensed (orange curve) and unlensed models (green curve). From S. Das *et al.* [50].

the Universe only about 1% is stars.

11. Baryonic Acoustic Oscillations (BAO)

A cornerstone of cosmology is the Copernican principle, that matter in the Universe is distributed homogeneously, if only on the largest scales of superclusters separated by voids. On smaller scales we observe inhomogeneities in the forms of galaxies, galaxy groups, and clusters. The common approach to this situation is to turn to non-relativistic hydrodynamics and treat matter in the Universe as an adiabatic, viscous, non-static fluid, in which random fluctuations around the mean density appear, manifested by compressions in some regions and rarefactions in other. The origin of these density fluctuations was the tight coupling established before decoupling between radiation and charged matter density, causing them to oscillate in phase. An ordinary fluid is dominated by the material pressure, but in the fluid of our Universe three effects are competing: gravitational attraction, density dilution due to the Hubble flow, and radiation pressure felt by charged particles only.

The inflationary fluctuations crossed the post-inflationary Hubble radius, to come back into vision with a wavelength corresponding to the size of the Hubble radius at that moment. At time t_{eq} the overdensities began to amplify and grow into larger inhomogeneities. In overdense regions where the gravitational forces dominate, matter contracts locally and attracts surrounding matter, becoming increasingly unstable until it eventually collapses into a gravitationally bound object. In regions where the pressure forces dominate, the fluctuations move with constant amplitude as sound waves in the fluid, transporting energy from one region of space to another.

Inflationary models predict that the primordial mass density fluctuations should be adiabatic, Gaussian, and exhibit the same scale invariance as the CMB fluctu-

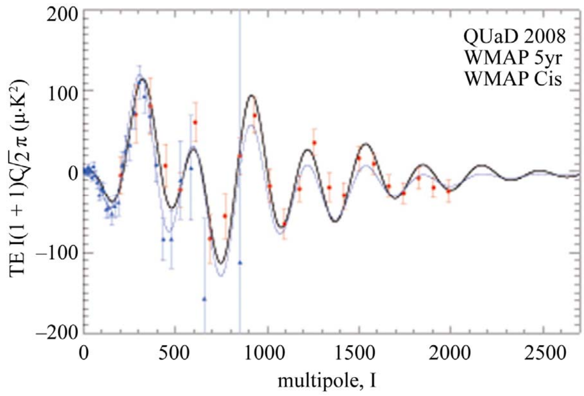


Figure 14. The E-mode polarization power spectrum (EE) from the CMB observations of the QUaD collaboration *et al.* [52].

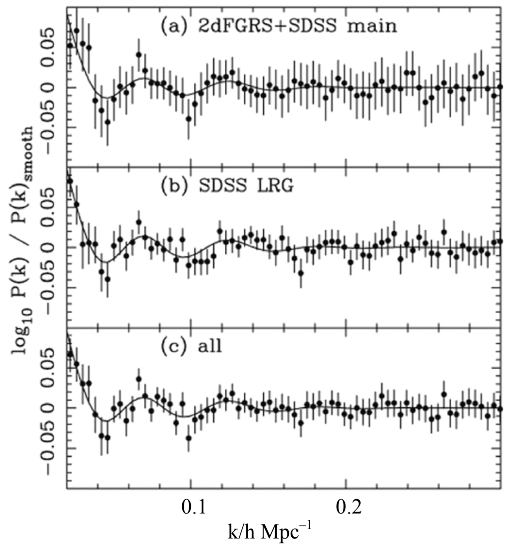


Figure 15. BAO in power spectra calculated from (a) the combined SDSS and 2dFGRS main galaxies; (b) the SDSS DR5 LRG sample; and (c) the combination of these two samples (solid symbols with 1σ errors). The data are correlated and the errors are calculated from the diagonal terms in the covariance matrix. A Standard ΛCDM distance—redshift relation was assumed to calculate the power spectra with $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$. From W. J. Percival *et al.* [53].

ations. The baryonic acoustic oscillations can be treated similarly to CMB, they are specified by the dimensionless *mass autocorrelation function* which is the Fourier transform of the power spectrum of a spherical harmonic expansion. The power spectrum is shown in **Figure 15** [53].

As the Universe approached decoupling, the photon mean free path increased and radiation could diffuse from overdense regions into underdense ones, thereby smoothing out any inhomogeneities in the plasma. The situation changed dramatically at recombination, at time 380,000 yr after Big Bang, when all the free electrons suddenly disappeared, captured into atomic Bohr orbits, and the radiation pressure almost vanished. Now the baryon acoustic waves and the CMB continued to oscillate independently, but adiabatically, and the density perturbations which had entered the Hubble radius since then could grow with full vigor into baryonic structures.

The scale of BAO depends on Ω_m and on the Hubble constant, h, so one needs information on h to break the degeneracy. The result is then $\Omega_m \approx 0.26$. In the ratio Ω_b/Ω_m the h-dependence cancels out, so one can also quantify the amount of DM on very large scales by $\Omega_b/\Omega_m = 0.18 \pm 0.04$.

12. Galaxy Formation in Purely Baryonic Matter?

We have seen in Section 10 that the baryonic density parameter, Ω_b, is very small. The critical density Ω_{crit} is determined by the expansion speed of the Universe, and the mean baryonic density of the Universe (stars, interstellar and intergalactic gas) is only $\Omega_b = 0.045$ [49].

The question arises whether the galaxies could have formed from primordial density fluctuations in a purely baryonic medium. We have also noted, that the fluctuations in CMB and BAO maintain adiabaticity. The amplitude of the primordial baryon density fluctuations would have needed to be very large in order to form the observed number of galaxies. But then the amplitude of the CMB fluctuations would also have been very large, leading to intolerably large CMB anisotropies today. Thus galaxy formation in purely baryonic matter is ruled out by this argument alone.

Thus one concludes, that the galaxies could only have been formed in the presence of gravitating dark matter which started to fluctuate early, unhindered by radiation pressure. This conclusion is further strengthened in the next Section.

13. Large Scale Structures Simulated

In the ΛCDM paradigm, the nonlinear growth of DM structure is a well-posed problem where both the initial conditions and the evolution equations are known (at least when the effects of the baryons can be neglected).

The Aquarius Project [54] is a Virgo Consortium program to carry out high-resolution DM simulations of Milky-Way—sized halos in the ΛCDM cosmology. This project seeks clues to the formation of galaxies and to the nature of the dark matter by designing strategies for exploring the formation of our Galaxy and its luminous and dark satellites.

The galaxy population on scales from 50 kpc to the size of the observable Universe has been predicted by hierarchical ΛCDM scenarios, and compared directly with a wide array of observations. So far, the ΛCDM paradigm has passed these tests successfully, particularly those that consider the large-scale matter distribution and has led to the discovery of a universal internal structure for DM halos. As was noted in Section 12, the observed structure of galaxies, clusters and superclusters, as illustrated by **Figure 16**, could not have formed in a baryonic medium devoid of dark matter.

Given this success, it is important to test ΛCDM predictions also on smaller scales, not least because these are sensitive to the nature of the dark matter. Indeed, a number of serious challenges to the paradigm have emerged on the scale of individual galaxies and their central structure. In particular, the abundance of small DM subhalos predicted within CDM halos is much larger than the number of known satellite galaxies surrounding the Milky Way (M. Boylan-Konchin *et al*. [48] and references therein).

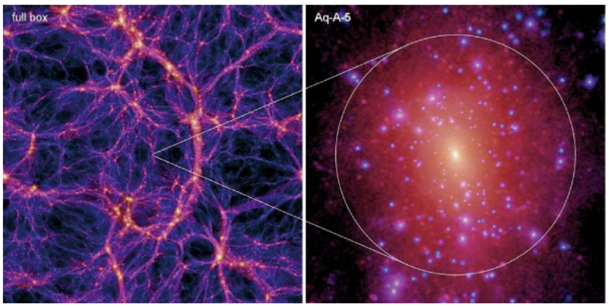


Figure 16. The left panel shows the projected dark matter density at z = 0 in a slice of thickness 13.7 Mpc through the full box (137 Mpc on a side) of the 900^3 parent simulation. The right panel show this halo resimulated at a different numerical resolution. The image brightness is proportional to the logarithm of the squared DM density projected along the line-of-sight. The circles mark r_{50}, the radius within which the mean density is 50 times the background density. From V. Springel *et al*. [54].

14. Dark Matter from Overall Fits

In Section 10 we have seen that the WMAP 7-year CMB data together with the Hubble constant value testify about the existence of DM [49,51]. In Section 11 we addressed the BAO data [53] with the same conclusion. In overall fits one combines these with supernova data (SN Ia) which offer a constraint nearly orthogonal to that of CMB in the $\Omega_\Lambda - \Omega_m$ -plane. The Union compilation of 307 selected SN Ia includes the recent large samples of SNe Ia from the Supernova Legacy Survey, the ESSENCE Survey, the older data sets, as well as the recently extended data set of distant supernovae observed with HST. M. Kowalski *et al*. [55] present the latest results from this compilation and discuss the cosmological constraints and its combination with CMB and BAO measurements. The CMB constraint is close to the line $\Omega_\Lambda + \Omega_m = 1$, whereas the supernova constraint is close to the line $\Omega_\Lambda - 1.6\times\Omega_m = 0.2$. The BAO data constrain Ω_m, but hardly at all Ω_Λ. This is shown in **Figure 17**.

Defining the vacuum energy density parameter by $\Omega_k = 1 - \Omega_\Lambda - \Omega_m$, a flat Universe corresponds to $\Omega_k = 0$. For a non-flat ΛCDM Universe with a cosmological constant responsible for dark energy, a simultaneous fit to the data sets gives

$$\begin{aligned} \Omega_m &= 0.285 + 0.020/{-0.019} \pm 0.011, \\ \Omega_k &= -0.009 + 0.009 + 0.002/{-0.010} - 0.003, \end{aligned} \quad (14)$$

where the first error is statistical and the second error systematic. Clearly one notes that the Universe is consistent with being flat. Subtracting $\Omega_b = 0.045$ from $\Omega_m = 0.285$ one obtains the density parameter for DM, $\Omega_{dm} \approx 0.24$. Assuming flatness, M. Kowalski *et al*. [55] find $\Omega_m = 0.274 \pm 0.016 \pm 0.013$. This compares well with the combined 7-year WMAP data and the ACT data, $\Omega_m = 0.276 \pm 0.016$ [51]. If one fits different models having more free parameters, one gets slightly different results, but all within these 1σ errors.

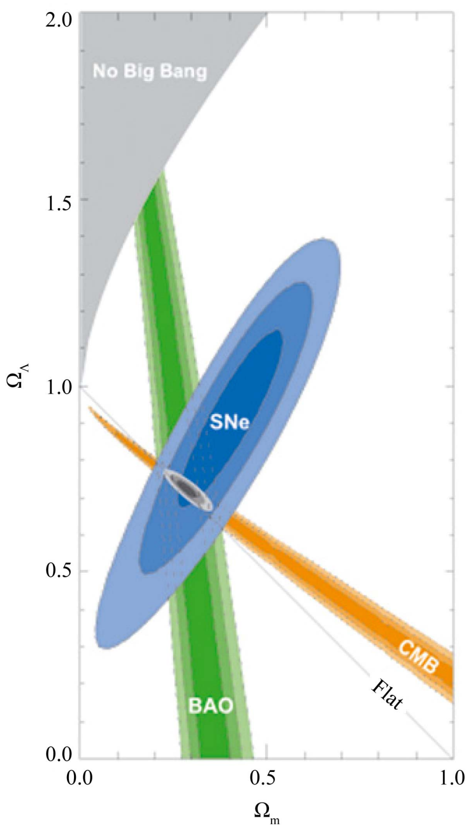


Figure 17. 68.3%, 95.4% and 99.7% confidence level contours on Ω_Λ and Ω_m obtained from CMB, BAO and the Union SN set, as well as their combination (assuming w = –1). Note the straight line corresponding to a flat Universe with $\Omega_\Lambda + \Omega_m = 1$. From M. Kowalski *et al*. [55].

15. Merging Galaxy Clusters

In isolated galaxies and galaxy clusters all matter components contributing to the common gravitational potential are more or less centrally-symmetrically coincident. This makes the dissection of DM from the baryonic components difficult and dependent on parametrization, as we have discussed in Section 3. In merging galaxy clusters however, the separate distributions of galaxies, intracluster gas and DM may become spatially segregated permitting separate observations. The visually observable galaxies behave as collisionless particles, the baryonic intracluster plasma is fluid-like, experiences ram pressure and emits X-rays, but non-interacting DM does not feel that pressure, it only makes itself felt by its contribution to the common gravitational potential.

Major cluster mergers are the most energetic events in the Universe since the Big Bang. Shock fronts in the intracluster gas are the key observational tools in the study of these systems. When a subcluster traverses a larger cluster it cannot be treated as a solid body with constant mass moving at constant velocity. During its passage through the gravitational potential of the main cluster it is shrinking over time, stripped of gas envelope and decelerating. Depending on the ratio of the cluster masses, the gas forms a bow shock in front of the main cluster, and this can even be reversed at the time when the potentials coincide.

We shall now meet several examples of galaxy cluster mergers where the presence of DM could be inferred from the separation of the gravitational potential from the position of the radiating plasma.

15.1. The Bullet Cluster 1E0657-558

The exceptionally hot and X-ray luminous galaxy cluster 1E0657-558, the *Bullet cluster* at redshift $z = 0.296$, was discovered by Tucker *et al*. in 1995 [56] in *Chandra* X-ray data. Its structure as a merger of a 2.3×10^{14} M_{sun} subcluster with a main 2.8×10^{14} M_{sun} cluster was demonstrated by Markevitch et al. [57,58] and Clowe *et al*. [59,60]. This was presented as the first clear example of a bow shock in a heated intracluster plasma.

With the advent of high-resolution lensing Bradač *et al*. [61,62] developed a technique combining multiple strongly-lensed *Hubble Space Telescope* multi-color images of identified galaxies, with weakly lensed and elliptically distorted background sources. The reconstructed gravitational potential does not trace the X-ray plasma distribution which is the dominant baryonic mass component, but rather approximately traces the distribution of bright cluster member galaxies, *cf* **Figure 18**.

The center of the total mass is offset from the center of the baryonic mass peaks, proving that the majority of the matter in the system is unseen. In front of the bullet cluster which has traversed the larger one about 100 Myr ago with a relative velocity of 4500 km·s^{-1}, a bow shock is evident in the X-rays. The main cluster peak and the distinct subcluster mass concentration are both clearly offset from the location of the X-ray gas [62].

A recent analysis of this system [72] confirms the results of references [59,60,62], and in addition finds that dark matter forms three distinct clumps.

15.2. The Galaxy Cluster Pair MACS J0025.4-1222

Another merging system with similar characteristics but with lower spatial resolution has been reported by Bradač *et al*. [19], the post-merging galaxy cluster pair MACS J0025.4-1222, also called the *Baby Bullet*. It has an apparently simple geometry, consisting of two large subclusters of similar richness, about 2.5×10^{14} M_{sun}, both at redshift $z = 0.586$, colliding in approximately the plane of the sky. Multiple images due to strong lensing of four distinct components could be identified. The combined strong and weak lensing analysis follows the method in ref. [62].

The two distinct mass peaks are clearly offset by 4σ from the main baryonic component, which is the radiating hot gas observed by *Chandra*. The relative merging velocity is estimated to be 2000 km·s^{-1}. In **Figure 19** we show linearly spaced surface mass density contours and X-ray brightness contours. The majority of the mass is spatially coincident with the identified galaxies which implies, that the cluster must be dominated by a relatively collisionless form of dark matter.

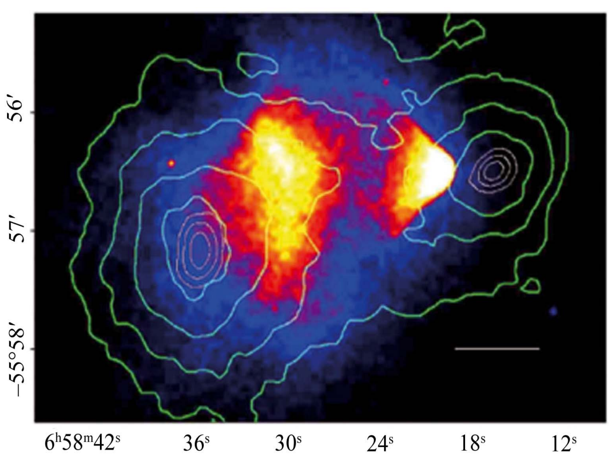


Figure 18. The merging cluster 1E0657-558. On the right is the smaller *Bullet cluster* which has traversed the larger cluster. The colors indicate the X-ray temperature of the plasma: blue is coolest and white is hottest. The green contours are the weak lensing reconstruction of the gravitational potential of the cluster. From D. Clowe *et al.* [59].

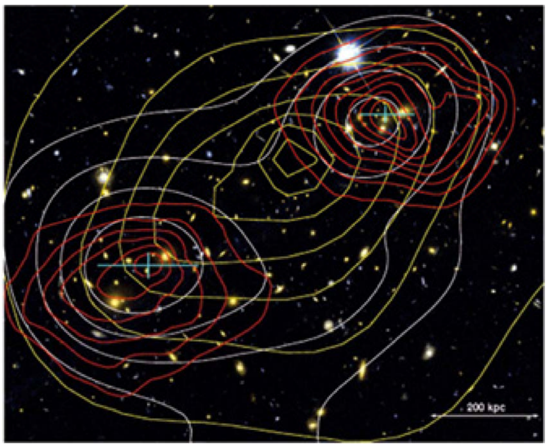


Figure 19. The color composite of the cluster MACS J0025.4-1222. Overlaid in red contours is the surface mass density (linearly spaced) from the combined weak and strong lensing mass reconstruction. The X-ray brightness contours (also linearly spaced) are overlaid in yellow and the I-band light is overlaid in white. The measured peak positions and error bars for the total mass of the two cluster components are shown as cyan crosses. From M. Bradač *et al.* [19].

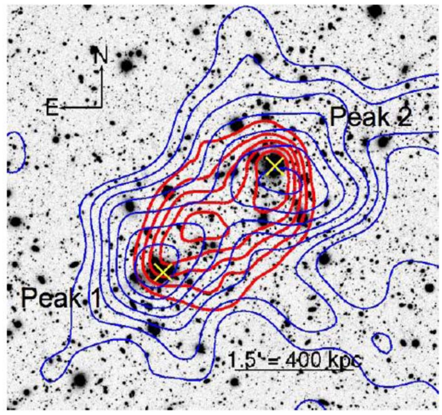


Figure 20. The A1758N merger from B. Ragozzine *et al.* [63]. The blue contours represent the weak lensing mass reconstruction made from a background galaxy density of 24.0 galaxies/arcmin^2. The outer blue contour begins at surface mass density $\kappa = 0.07$ and each contour increases in steps of 0.045 up to $\kappa = 0.34$. The red contours follow the X-ray gas mass obtained in the Chandra exposure. The NW cluster's BCG aligns with the X-ray gas and the weak lensing peak. The SE cluster's BCG and weak lensing peak are well separated from the X-ray gas, which has a bright peak near the midpoint of the two weak lensing peaks.

15.3. The Merging System A1758

A much more complicated merging system is A1758 at redshift $z = 0.279$, analyzed by the same team as above, B. Ragozzine *et al*. [63], and consisting of four clusters undergoing two separate mergers. The weak lensing mass peaks of the two northern clusters A1758N are separated at the 2.5σ level, whereas the two southern clusters are not well separated and have a disturbed X-ray morphology. There is no evidence for a merger between A1758N and A1758S in the X-ray signature and they have a projected separation of 2.0 Mpc. Note however the SZ results from the Arcminute Microkelvin Imager (AMI) in Cambridge (UK) on this system [64,65], which sees a hint of a signal between the A1758N and A1758S.

A1758N introduces a new geometry that is different from the previously discussed mergers: one weak lensing peak overlaps an X-ray peak, while the other weak lensing peak is clearly separated from the X-ray component, *cf* **Figure 20**.

Since no strong lensing has yet been confirmed, conclusions about cluster masses and DM would have to wait for better lensing data.

15.4. The Merging Cluster Abell 2146

Chandra observations of the cluster Abell 2146 at a redshift of $z = 0.234$ have revealed two shock fronts, H. R. Russell *et al*. [64]. The X-ray morphology suggests a recent merger where a subcluster containing a dense core has passed through the center of a second cluster, the remnant of which appears as the concentration of gas to the NW. The strongly peaked core has just emerged from the primary core, and is trailing material that has been ram pressure stripped in the gravitational potential. This material appears as a warmer stream of gas behind the subcluster core, and trails back to the hottest region of the disrupted main cluster.

Four steep surface brightness edges can be defined: two in the SE sector in front of the subcluster core and another two in the NW sector, *cf*. **Figure 21**. The interpretation is [64] that an upstream shock is generated as the gravitational potential minimum fluctuates rapidly during the core passage, reaching an extreme minimum when the two cluster cores coalesce. This causes a significant amount of the outer cluster gas to flow inwards. When the subcluster core exits the main core the gravitational potential rapidly returns to its premerger level and expels much of the newly arrived gas which in turn collides with the residual infall, forming an inward traveling shock front. Behind the subcluster, the ambient cluster gas that was pushed aside during its passage will fall back and produce tail shocks.

Since no weak lensing analysis is available as yet, nothing can be said about the possible role of collisionless dark matter.

15.5. The Merging Cluster Abell 2744

Newly acquired data with the *Advanced Camera for Surveys* on the *Hubble Space Telescope*, *HST*, shows that the cluster Abell 2744 is a complicated merger between three or four separate bodies, as analyzed by J. Merten *et*

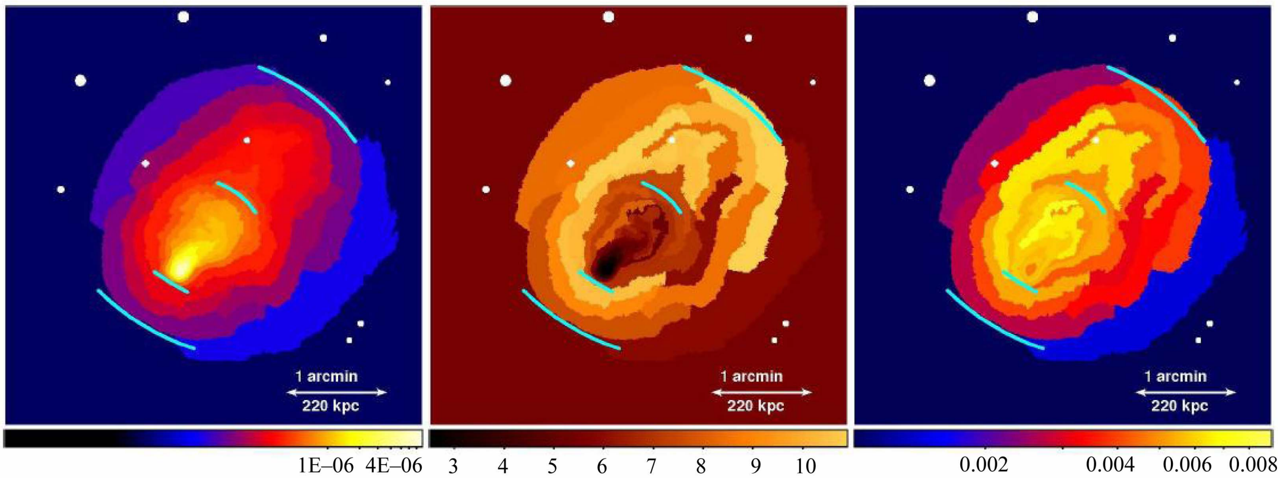


Figure 21. Left: Projected emission measure per unit area map for the merging cluster Abell 2146. Center: Projected temperature map. Right: Projected "pressure" map produced by multiplying the emission measure and temperature maps. From H. R. Russell *et al.* [64].

al. [65]. The position and mass distribution of the Southern core have been tightly constrained by the strong lensing of 11 background galaxies producing 31 multiple images. The N and NW clumps lack such images from strong lensing, indicating that they are less massive. There is also weak lensing information from *HST*, *VLT*, and *Subaru* available.

The joint gravitational lensing analysis combines all the strongly lensed multiply-imaged systems and their redshifts with weak lensing shear catalogues from all three telescopes to reconstruct the cluster's lensing potential, shown in **Figure 22**. The Core, NW and W clumps are clear detections in the surface-mass density distribution with $11\sigma, 4.9\sigma$ and 3.8σ significance over background, respectively. Somewhat fainter with 2.3σ significance is the N structure, but it clearly coincides with a prominent X-ray substructure found by M. S. Owers *et al*. [66].

To determine the geometric configuration of the collision, the location of shock fronts and velocities, densities and temperatures in the intracluster medium, all existing X-ray data from *Chandra* [66] were included and reanalyzed. Overlaying the lensing mass reconstruction and the luminosity contours of the emission in **Figure 23** shows an extremely complex picture of separations between the dark matter and baryonic components.

In the core region which is the most massive structure within the merging system, there is no large separation between the distributions of total mass and baryons. The separation of the peaks in the lensing and X-ray maps is similar to that in the *Bullet cluster* [58] and *Baby Bullet* [19]. The Northern mass substructure is ≈ 2.6 times lighter than the Core, and the X-ray emission lags behind the dark matter to the South.

The substructure in the Northwest is the second most

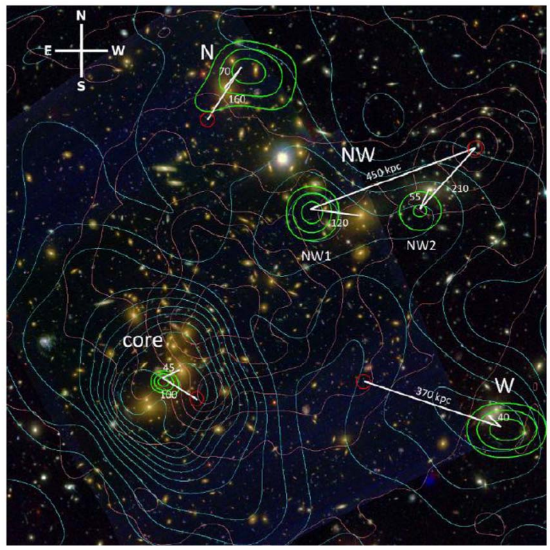


Figure 22. The surface-mass density contours of the merging cluster Abell 2744 are shown in cyan and the X-ray luminosity contours in magenta. The peak positions of the Southern core, the N, NW, and W clumps are indicated by the green likelihood contours. The small red circles show the positions of the local overdensities in the gas distribution, associated with each DM clump. The white rulers show the separation between DM peaks and the bright cluster galaxies and local gas peaks. From J. Merten *et al.* [65].

massive and there might also be a second peak in the more Western area of the NW mass clump. However, it is difficult to ascertain whether this is a single, separate DM structure and to derive decisive separation between DM, X-ray luminous gas and bright cluster member galaxies. The X-ray peak to the Northwest of the NW2 mass peak appears to be an X-ray feature with no associated matter or galaxies, a "ghost" clump.

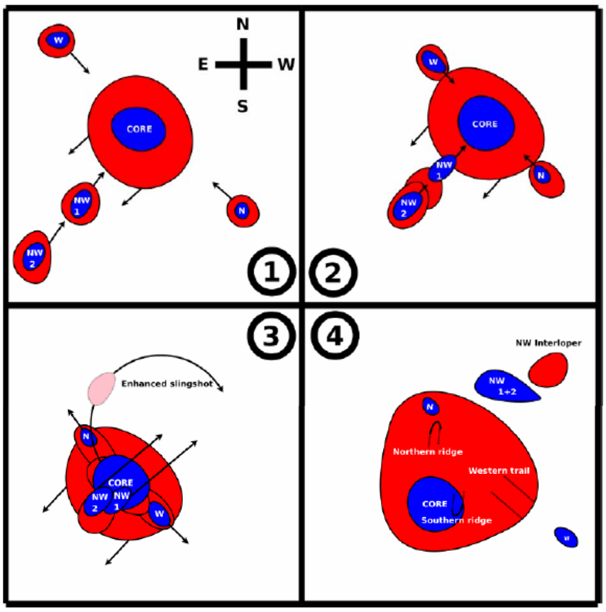


Figure 23. The proposed merger scenario of the cluster Abell 2744 in time-ordered sequence. The NE-SW sub-clusters merge first (1) with the core, followed very soon (2) by the second merger, in the SE-NW direction. The gas slingshots (3) away to its present position at the extreme NW. In (4) we see the present setup. From J. Merten *et al.* [65].

One possible interpretation [65] of the complex merging scenario that has taken place in Abell 2744 is a near simultaneous double merger $0.12-0.15$ Gyr ago. first in the NE-SW direction, *cf.* **Figure 23**. The Western clump probably passed closest through the main cluster, as it had its ICM ram-pressure stripped completely. The second merger, in the SE-NW direction, could even have consisted of two small clumps falling into the core, attracted by the core and the Northern and Western clumps. After a first core passage, gas initially trails its associated DM but, while the dark matter slows down,the gas slingshots past it due to a combination of low rampressure stripping and adiabatic expansion and cooling [66], ending up as the "ghost" clump. This scenario still requires further observations as well as verification via numerical simulations.

15.6. "El Gordo", the Fat Cluster ACT-CL J0102-4915

The Atacama Cosmology Telescope has presented properties for an exceptionally massive merging cluster, the ACT-CL J0102-4915 nicknamed *El Gordo* at redshift $z = 0.87$. It was discovered by Marriage *et al.* [67] selected by its bright Sunyaev-Zel'dovich (SZ) effect, confirmed optically and through its *Chandra* X-ray data [68]. It is the most significant SZ cluster detection to date by nearly a factor of two, with an SZ decrement comparable to the *Bullet* cluster 1E0657-558 [62].

As can be seen from **Figure 24**, the galaxy distribution is double peaked, whereas the peak in the X-ray emission lies between the density peaks. The X-ray peak forms a relatively cool bullet of low entropy gas like in the 1E0657-558. The steep fall-off in the X-ray surface brightness towards the SE, as well as the "wake" in the main cluster gas toward the NW, indicate that the bullet is apparently moving toward the SE. The SZ and X-ray peaks are offset similar to that reported for the bullet-like cluster Abell 2146 [64].

In the absence of a weak lensing mass reconstruction, the galaxy distribution can only be used as a proxy for the total mass distribution. Thus to conclude that an offset between baryonic and DM has been demonstrated is yet premature.

15.7. The Cluster Merger DLSCL J0916.2+2951

A newly discovered [71] major cluster merger at z = 0.53 is DLSCL J0916.2+2951, in which the collisional cluster gas has become clearly dissociated from the collisionless galaxies and dark matter. The cluster was identified using optical and weak-lensing observations as part of the Deep Lens Survey. Follow-up observations with Keck, Subaru, Hubble Space Telescope, and Chandra show that the cluster is a dissociative merger which constrain the DM self-interaction cross-section to σ/m (DM) ≤ 7 cm^2/g. The system is observed at least 0.7 ± 0.2 Gyr since first pass-through, thus providing a picture of cluster mergers 2 - 5 times further progressed than similar systems observed to date.

16. Comments and Conclusions

What we have termed "dark matter" is generic for observed gravitational effects on all scales: galaxies, small and large galaxy groups, clusters and superclusters, CMB anisotropies over the full horizon, baryonic oscillations over large scales, and cosmic shear in the large-scale matter distribution. The correct explanation or nature of dark matter is not known, whether it implies unconventional particles or modifications to gravitational theory. but gravitational effects have convincingly proved its existence in some form.

The few per cent of the mass of the Universe found as baryonic matter in stars and dust clouds is well accounted for by nucleosynthesis. If there exist particles which were very slow at time t_{eq} when galaxy formation started, they could be candidates for cold dark matter. They must have become non-relativistic much earlier than the leptons, and then decoupled from the hot plasma.

Whenever laboratory searches discover a new particle, it must pass several tests in order to be considered a

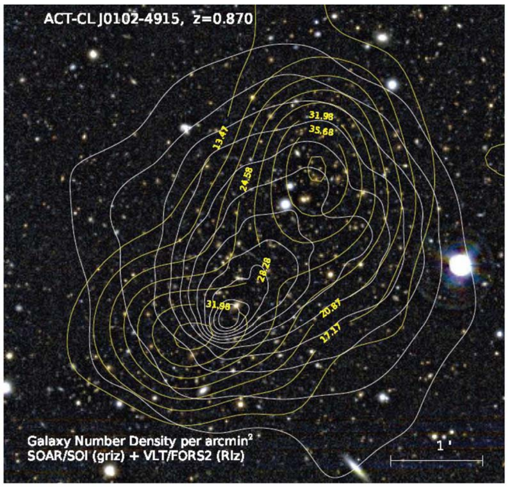


Figure 24. Isopleth contours in yellow of the number density of galaxies in the cluster ATC-CL J0102-4915. The white contours show the X-ray emission. From F. Menenteau *et al.* [68].

viable DM candidate: it must be neutral, compatible with constraints on self-interactions (essentially collisionless), consistent with Big Bang nucleosynthesis, and match the appropriate relic density. It must be consistent with direct DM searches and gamma-ray constraints, it must leave stellar evolution unchanged, and be compatible with other astrophysical bounds.

The total dynamical mass of an astronomical system is derivable from the velocity dispersions or the rotation velocities of its components via the use of the Virial Theorem or Kepler's law, respectively. A most important probe is strong gravitational lensing which measures the total mass, but also weak lensing, the oscillations in the Cosmic Microwave Background and in the ambient baryonic medium. Probes separating dark matter from total matter require in addition observations of visible light, infrared radiation, X-rays, the Sunyaev-Zel'dovich effect, and supernovae. Depending on the system under study there are many ways to combine these tools using empirical halo models, simulating stellar population models and galaxy formation models, comparing mass-to-light ratios and mass autocorrelation functions. The most remarkable systems are merging galaxy clusters which, by their motion, separate non-collisional dark matter from optically visible galaxies and hot, radiating gas.

Regardless of the nature of dark matter, all theories attempting to explain it share the burden to explain the gravitational effects described in here. Thus there remains much to be done.

17. Acknowledgements

I am grateful to Sylvain Fouquet, Carmen Rodriguez-Gonzalvez, and Will Dawson for clarifying comments and addenda.

REFERENCES

[1] F. Zwicky, "Die Rotverschiebung Von Extragalaktischen Nebeln," *Helvetica Physica Acta*, Vol. 6, No. 2, 1933, p. 110.

[2] J. C. Kapteyn, "First Attempt at a Theory of the Arrangement and Motion of the Sidereal System," *The Astrophysical Journal*, Vol. 55, No. 1, 1922, p. 302. doi:10.1086/142670

[3] J. H. Jeans, "The Motion of Stars in a Kapteyn Universe," *Monthly Notices of the Royal Astronomical Society*, Vol. 82, No. 1, 1922, p. 122.

[4] J. H. Oort, "The Force Exerted by the Stellar System in the Direction Perpendicular to the Galactic Plane and Some Related Problems," *Bulletin of the Astronomical Institutes of the Netherlands*, Vol. 6, No. 1, 1932, p. 249.

[5] J. Chadwick, "Possible Existence of a Neutron," *Nature*, Vol. 129, No. 3252, 1932, pp. 312-313.

[6] J. Holmberg and C. Flynn, "The Local Density of Matter Mapped by Hipparchos," *Monthly Notices of the Royal Astronomical Society*, Vol. 313, No. 2, 2000, pp. 209-216. doi:10.1046/j.1365-8711.2000.02905.x

[7] J. H. Oort, "Infall of Gas from Intergalactic Space," *Nature*, Vol. 224, No. 5225, 1969, pp. 1158-1163. doi:10.1038/2241158a0

[8] I. Jørgensen, M. Franx and P. Kjærgaard, "The Fundamental Plane for cluster E and S0 Galaxies," *Monthly Notices of the Royal Astronomical Society*, Vol. 280, No. 1, 1996, p. 167.

[9] J. B. Hyde and M. Bernardi, "The Luminosity and Stellar Mass Fundamental Plane of Early-Type Galaxies," *Monthly Notices of the Royal Astronomical Society*, Vol. 396, No. 2, 2009, pp. 1171-1185. doi:10.1111/j.1365-2966.2009.14783.x

[10] J. F. Navarro, C. S. Frenk and S. D. M. White, "A Universal Density Profile from Hierarchical Clustering," *Astrophysical Journal*, Vol. 490, 1997, p. 493.

[11] B. Moore, *et al*., "Resolving the Structure of Cold Dark Matter Halos," *Astrophysical Journal Letters*, Vol. 499, No. 1, 1998, p. L5. doi:10.1086/311333

[12] J. Einasto, *et al*., "Dynamic Evidence on Massive Coronass of Galaxies," *Nature*, Vol. 250, No. 26, 1974, p. 309.

[13] A. Burkert, "The Structure of Dark Matter Halos in Dwarf Galaxies," *Astrophysical Journal Letters*, Vol. 447, No. 1, 1995, p. L25. doi:10.1086/309560

[14] A. Klypin, S. Trujillo-Gomez and J. Primack, "Dark Matters Halos in the Standard Cosmological Model: Results from the Bolshoi Simulation," *Astrophysical Journal*, Vol. 740, No. 2, 2011, p. 102. doi:10.1088/0004-637X/740/2/102

[15] E. L. Lokas and G. Mamon, "Dark Matter Distribution in the Coma Cluster from Galaxy Kinematics: Breaking the Mass-Anisotropy Degeneracy," *Monthly Notices of the Royal Astronomical Society*, Vol. 343, No. 2, 2003, pp. 401-412. doi:10.1046/j.1365-8711.2003.06684.x

[16] G. Cupani, M. Mezzetti and F. Mardirossian, "Cluster Mass Estimation through Fair Galaxies," *Monthly Notices of the Royal Astronomical Society*, Vol. 403, No. 2, 2010, pp. 838-847. doi:10.1111/j.1365-2966.2009.16157.x

[17] M. J. Valtonen and G. G. Byrd, "A Binary Model for the Come Cluster of Galaxies," *Astrophysical Journal*, Vol. 230, 1979, pp. 655-666. doi:10.1086/157124

[18] M. Sereno, M. Lubini and Ph. Jetzer, "A Multi-Wavelength Strong Lensing Analysis of Baryons and Dark Matter in the Dynamically Active Cluster AC 114," 2009, Preprint arXiv:0904.0018 [astro-ph.CO].

[19] M. Bradač *et al*., "Revealing the Properties of Dark Matter in the Merging Cluster MACSJ0025.4-1222," *Astrophysical Journal*, Vol. 687, No. 2, 2008, pp. 959-967.

[20] S. W. Allen, *et al*., "Improved Constraints on Dark Energy from Chandra X-Ray Observations of the Largest Relaxed Galaxy Clusters," *Monthly Notices of the Royal Astronomical Society*, Vol. 383, No. 3, 2008, pp. 879-896. doi:10.1111/j.1365-2966.2007.12610.x

[21] M. Limousin, *et al*., "Strong Lensing in Abell 1703: Constraints on the Slope of the Inner Dark Matter Distribution," *Astronomy and Astrophysics*, Vol. 489, No. 1, 2008, pp. 23-35. doi:10.1051/0004-6361:200809646

[22] A. D. Chernin, *et al*., "Dark Energy and the Mass of the Local Group," 2009, Preprint arXiv:0902.3871 [astro-ph.CO].

[23] R. B. Tully, "Nearby Groups of Galaxies. II. An All-Sky Survey within 3000 Kilometers per Second," *Astrophysical Journal*, Vol. 321, 1984, pp. 280-304. doi:10.1086/165629

[24] I. D. Karachentsev, "Missing Dark Matter in the Local Universe," 2012. Preprint arXiv:1204.3377 [astro-ph. CO].

[25] P. Salucci, *et al*., "The Universal Rotation Curve of Spiral Galaxies. II. The Dark Matter Distribution out to the Virial Radius," *Monthly Notices of the Royal Astronomical Society*, Vol. 378, No. 1, 2007, pp. 41-47.

[26] A. Bosma and P. C. van der Kruit, "The Local Mass-to-Light Ratio in Spiral Galaxies," *Astronomy and Astrophysics*, Vol. 79, No. 3, 1979, pp. 281-286.

[27] G. Gentile, *et al*., "The Cored Distribution of Dark Matter in Spiral Galaxies," *Monthly Notices of the Royal Astronomical Society*, Vol. 351, No. 2, 2004, pp. 903-922. doi:10.1111/j.1365-2966.2004.07836.x

[28] Y. Sofue, M. Honma and T. Omodaka, "Unified Rotation Curve of the Galaxy Decomposition into de Vaucouleurs Bulge, Disk, Dark Halo, and the 9-kpc Rotation Dip," *Publications of the Astronomical Society of Japan*, Vol. 61, No. 1, 2009, p. 227.

[29] R. Catena and P. Ullio, "A Novel Determination of the Local Dark Matter Density," 2009, Preprint arXiv:0907.0018[astro-ph.CO].

[30] M. Weber and W. de Boer, "Determination of the Local Dark Matter Density in Our Galaxy," *Astronomy and Astrophysics*, Vol. 509, No. A25, 2010, 10 pages. doi:10.1051/0004-6361/200913381

[31] N. Straumann, "Matter in the Universe," *Space Science Reviews*, Vol. 100, No. 1-4. 2002, pp. 29-38. doi:10.1023/A:1015801707999

[32] R. J. Smith, *et al*., "Discovery of Strong Lensing by an Elliptical Galaxy at z = 0.0345," *Astrophysical Journal Letters*, Vol. 625, No. 2, 2005, p. L103. doi:10.1086/431240

[33] R. Massey, *et al*., "Dark Matter Maps Reveal Cosmic Scaffolding," *Nature*, Vol. 445, No. 7125, 2007, pp. 286-290.

[34] R. Massey, T. Kitching and J. Richard, "The Dark Matter of Gravitational Lensing," *Reports on Progress in Physics*, Vol. 73, No. 8, 2010, Article ID: 086901. doi:10.1088/0034-4885/73/8/086901

[35] P. J. Humphrey, *et al*., "The ELIXR Galaxy Survey. II: Baryons and Dark Matter in an Isolated Elliptical Galaxy," 2012, Preprint arXiv:1204.3095 [astro-ph.CO].

[36] O. Tiret, *et al*., "The Inner Structure of Very Massive Elliptical Galaxies: Implications for the Inside-Out Formation Mechanism of z 2 Galaxies," 2010, Preprint arXiv:1204.3095 [astro-ph.CO].

[37] R. Salinas, *et al*., "Kinematic Properties of the Field Elliptical NGC 7507," *Astronomy and Astrophysics*, Vol. 538, No. A87, 2012, 17 pages. doi:10.1051/0004-6361/201116517

[38] C. Flynn, *et al*., "On the Mass-to-Light Ratio of the Local Galactic Disk and the Optical Luminosity of the Galaxy," *Monthly Notices of the Royal Astronomical Society*, Vol. 372, No. 3, 2006, pp. 1149-1160. doi:10.1111/j.1365-2966.2006.10911.x

[39] R. A. Swaters, *et al*., "Are Dwarf Galaxies Dominated by Dark Matter?" 2011, Preprint arXiv:1101.3120 [astro-ph.CO].

[40] S. J. Penny and C. J. Conselice, "Dwarf Galaxies in Clusters as Probes of Galaxy Formation and Dark Matter," 2010, Preprint arXiv:1009.2922 [astro-ph.CO].

[41] H. Baumgardt and S. Mieske, "High Mass-to-Light Ratios of UCDs: Evidence for Dark Matter?" 2008, Preprint arXiv:0809.2783 [astro-ph.CO].

[42] V. Belokurov, *et al*., "Cats and Dogs, Hair and a Hero: A Quintet of New Milky Way Companions," *Astrophysical Journal*, Vol. 654, No. 2, 2007, p. 897. doi:10.1086/509718

[43] M. Xiang-Gruess, Y.-Q. Lou and W. J. Duschl, "Dark Matter Dominated Dwarf Disc Galaxy Segue 1," 2009, Preprint arXiv:1009.2922 [astro-ph.CO].

[44] M. Markevitch, *et al*., "Mass Profiles of the Typical Relaxed Galaxy Clusters A2199 and A496," *Astrophysical Journal*, Vol. 527, No. 2, 1999, p. 545. doi:10.1086/308124

[45] C. De Boni and G. Bertin, "The Relative Concentration of Visible and Dark Matter in Clusters of Galaxies," *Nuovo Cimento B*, Vol. 123, No. 1, 2008, p. 31.

[46] Q. Guo, *et al*., "How Do Galaxies Populate Dark Matter Halos?" *Monthly Notices of the Royal Astronomical Society*, Vol. 404, No. 3, 2010, pp. 1111-1120.

[47] A. Boyarsky, *et al*., "New Evidence for Dark Matter," 2009, Preprint arXiv:0911.1774 [astro-ph.CO].

[48] M. Boylan-Konchin, J. S. Bullock and M. Kaplinghat,

"The Milky Way's Bright Satellites as an Apparent Failure of ΛCDM," 2012, Preprint arXiv:1111.2048 [astro-ph.CO].

[49] D. Larson, *et al.*, "Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and WMAP-Derived Parameters," *The Astrophysical Journal Supplement Series*, Vol. 192, No. 2, 2011, p. 16. doi:10.1088/0067-0049/192/2/16

[50] S. Das, *et al.*, "The Atacama Cosmology Telescope: A Measurement of the Cosmic Microwave Background Power Spectrum at 148 and 218 GHz from the 2008 Southern Survey," 2010, Preprint arXiv:1009.0847 [astro-ph.CO].

[51] J. Dunkley, *et al.*, "The Atacama Cosmology Telescope: Cosmological Parameters from the 2008 Power Spectra," 2010, Preprint arXiv:1009.0866 [astro-ph.CO].

[52] P. G. Castro, *et al.*, "Cosmological Parameters from the QUAD CMB Polarization Experiment," *Astrophysical Journal*, Vol. 701, No. 2, 2009, p. 857. doi:10.1088/0004-637X/701/2/857

[53] W. J. Percival, *et al.*, "Measuring the Baryon Acoustic Oscillation Scale Using the SDSS and 2dFGRS," *Monthly Notices of the Royal Astronomical Society*, Vol. 381, 2007, p. 1053.

[54] V. Springel, *et al.*, "The Aquarius Project: The Subhalos of Galactic Halos," *Monthly Notices of the Royal Astronomical Society*, Vol. 391, No. 4, 2008, pp. 1685-1711. doi:10.1111/j.1365-2966.2008.14066.x

[55] M. Kowalski, *et al.*, "Improved Cosmological Constraints from New, Old and Combined Supernova Datasets," *Astrophysical Journal*, Vol. 686, No. 2, 2008, p. 749. doi:10.1086/589937

[56] W. H. Tucker, H. Tananbaum and R. A. Remillard, "1E0657-56: A Contender for the Hottest Known Cluster of Galaxies," *Astrophysical Journal*, Vol. 496, No. 1, 1998, p. L5. doi:10.1086/311234

[57] M. Markevitch, *et al.*, "A Textbook Example of a Bow Shock in the Merging Galaxy Cluster 1E 0657-56," *Astrophysical Journal*, Vol. 567, No. 1, 2002, p. L27. doi:10.1086/339619

[58] M. Markevitch, *et al.*, "Direct Constraints on the Dark Matter Self-Interaction Cross Section from the Merging Galaxy Cluster 1E 0657-56," *Astrophysical Journal*, Vol. 606, No. 2, 2004, p. 819. doi:10.1086/383178

[59] D. Clowe, *et al.*, "A Direct Empirical Proof of the Existence of Dark Matter," *Astrophysical Journal*, Vol. 648, No. 2, 2006, p. L109. doi:10.1086/508162

[60] D. Clowe, A. Gonzalez and M. Markevitch, "Weak-Lensing Mass Reconstruction of the Interacting Cluster 1E 0657558: Direct Evidence for the Existence of Dark Matter," *Astrophysical Journal*, Vol. 604, No. 2, 2004, p. 596. doi:10.1086/381970

[61] M. Bradač, *et al.*, "Strong and Weak Lensing United. I: The Combined Strong and Weak Lensing Cluster Mass Reconstruction Method," *Astronomy and Astrophysics*, Vol. 437, No. 1, 2005, p. 39.

[62] M. Bradač, *et al.*, "Strong and Weak Lensing United. III. Measuring the Mass Distribution of the Merging Galaxy 1ES 0657-558," *Astrophysical Journal*, Vol. 652, No. 2, 2006, p. 937. doi:10.1086/508601

[63] B. Ragozzine, *et al.*, "Weak Lensing Results of the Merging Cluster A1758," 2011, Preprint arXiv:1111.4983 [astro-ph.CO].

[64] C. Rodriguez-Gonzalvez, *et al.*, "Sunyaev-Zel'dovich Observation of the Bullet-Like Cluster Abell 2146 with the Arcminute Microkelvin Imager," 2010, Preprint arXiv: 1011.0325 [astro-ph.CO].

[65] AMI Consortium, "Detailed Sunyaev-Zel'dovich Study with AMI of 19 LoCuSS Galaxy Clusters: Masses and Temperatures out to the Virial Radius," *Monthly Notices of the Royal Astronomical Society*, Vol. 425, 2012, p. 162.

[66] H. R. Russell, *et al.*, "Chandra Observation of Two Shock Fronts in the Merging Galaxy Cluster Abell 2146," *Monthly Notices of the Royal Astronomical Society*, Vol. 406, 2010, p. 1721.

[67] J. Merten, *et al.*, "Creation of Cosmic Structure in the Complex Galaxy Cluster Merger Abell 2744," 2011, Preprint arXiv:1109.0953 [astro-ph.CO].

[68] M. S. Owers, *et al.*, "The Dissection of Abell 2744: A Rich Cluster Growing through Major and Minor Mergers," *Astrophysical Journal*, Vol. 728, No. 1, 2011, p. 27. doi:10.1088/0004-637X/728/1/27

[69] T. A. Marriage, *et al.*, "The Atacama Cosmology Telescope: Sunyaev Zel'dovich Selected Galaxy Clusters at 148 GHz in the 2008 Survey," *Astrophysical Journal*, Vol. 737, No. 2, 2011, p. 61. doi:10.1088/0004-637X/737/2/61

[70] F. Menenteau, *et al.*, "The Atacama Cosmology Telescope: ACT-CL J0102-4915 El Gordo, a Massive Merging Cluster at Redshift 0.87," 2012, Preprint arXiv:1109. 0953 [astro-ph.CO].

[71] W. E. Dawson, *et al.*, "Discovery of a Dissociative Galaxy Cluster Merger with Large Physical Separation," *Astrophysical Journal*, Vol. 747, No. 2, 2012, p. L42. doi:10.1088/2041-8205/747/2/L42

[72] D. Paraficz, *et al.*, "The Bullet Cluster Revisited: New Results from New Constraints and Improved Strong Lensing Modeling Technique," 2012, Preprint arXiv:1209. 0384 [astro-ph.CO].

Journal of Modern Physics, 2012, 3, 1172-1177
doi:10.4236/jmp.2012.329151 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Prediction of Cosmological Constant Λ in Veneziano Ghost Theory of QCD*

Lijuan Zhou[1], Weixing Ma[2], Leonard S. Kisslinger[3]
[1]Department of Information and Computing Science, Guangxi University of Technology, Liuzhou, China
[2]Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, China
[3]Department of Physics, Carnegie-Mellon University, Pittsburgh, USA
Email: Leonard Kisslinger kissling@andrew.cmu.edu

Received June 15, 2012; revised July 14, 2012; accepted July 24, 2012

ABSTRACT

Based on the Veneziano ghost theory of QCD, we estimate the cosmological constant Λ, which is related to the vacuum energy density, ρ_Λ, by $\Lambda = 8\pi G\rho_\Lambda$. In the recent Veneziano ghost theory ρ_Λ is given by the absolute value of the product of the local quark condensate and quark current mass: $\rho_\Lambda = \frac{2N_f H}{m_{\eta'}} c\left|m_q < 0\right| : \bar{q}q :| 0 >|$. By solving Dyson-Schwinger Equations for a dressed quark propagator, we found the local quark condensate $\langle 0|: \bar{q}q :|0\rangle \simeq -(235\ \text{MeV})^3$, the generally accepted value. The quark current mass is $m_q \simeq$ 4.0 Mev. This gives the same result for ρ_Λ as found by previous authors, which is somewhat larger than the observed value. However, when we make use of the nonlocal quark condensate, $\langle 0|: \bar{q}(x)q(0) :|0\rangle = g(x)\langle 0|: \bar{q}q :|0\rangle$, with *g*(*x*) estimated from our previous work, we find Λ is in a good agreement with the observations.

Keywords: Cosmological Constant Λ; Veneziano Ghost Theory of QCD; Local Quark Vacuum Condensate; Nonlocal Quark Condensate; Quantum Chromodynamics-QCD

1. The Cosmological Constant Λ and the QCD Veneziano Ghost Theory

The starting point of most cosmological study is Albert Einstein's Equations, which is a set of ten equations in Einstein's theory of general relativity. The original Einstein field equations can be written as the form [1]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} \tag{1}$$

in units of $\hbar = c = 1$, where G is the gravitational constant ($G = 6.7087(10)\times 10^{-39}\,\text{GeV}^{-2}$, sometime called Newton's constant), $R_{\mu\nu}\,(\mu,\nu = 0,\cdots,3)$ is the Ricci tensor, R is the trace of Ricci tensor (it is like the radius of curvature of space-time), $g_{\mu\nu}(x)$ represents the metric tensor, which is a function of position *x* in spacetime. $T^{\mu\nu}$ is the energy-momentum tensor, which describes the distribution of matter and energy. Equation (1) describes a non-static universe. However, Einstein believed, at that time, that our universe should be static. In order to get a static universe, in 1917 Einstein introduced a new term, $\Lambda g_{\mu\nu}$, in Equation (1) to balance the attractive force of gravity, giving his modified equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu}. \tag{2}$$

The Λ in Equation (2) is the so-called cosmological constant, which is a dimensional parameter with units of $(length)^{-2}$. Indeed, Equation (2) allows a static universe [2], called Einstein's universe, which is one of the solution [3] of Friedmann's simplified form of Einstein's equation with a Λ term. However, almost one hundred years ago the observations of redshifts of galaxies led to Hubbles Law [4] and the interpretation that the universe is expanding. This led Einstein to declare his static cosmological model, and especially the introduction of the Λ term to his original field equation theory, his "biggest blunder".

Note that the term $\Lambda g_{\mu\nu}$ in Equation (2) corresponds

*This work was supported in part by National Natural Science Foundation of China (10647002), Guangxi Science Foundation for Young Researchers under contract No. 0991009, and Guangxi Education Department with grant No.200807MS112, Department of Science and Technology of Guangxi under funds No. 2011GXNSFA018140, Department of Guangxi Education for the Excellent Scholars of Higher Education, 2011-54, Doctoral Science Foundation of Guangxi University of Technology, 11Z16, and in part by the Pittsburgh Foundation.

to adding a vacuum term to $T_{\mu\nu}$,

$$T_{\mu\nu}(vac) = \rho_\Lambda g_{\mu\nu}. \quad (3)$$

Therefore, the cosmological constant Λ is related to the vacuum energy density, ρ_Λ by [3]

$$\Lambda = 8\pi G \rho_\Lambda. \quad (4)$$

The vacuum energy density, called dark energy density, and a model with Λ representing dark energy were reintroduced about three decades ago. See Ref. [5] for a review of the physics and cosmology of Λ, with references to the many models that have been published. To explain our uniform and flat universe via inflation a cosmological constant was added to the Friedmann equation [6]. From studies of radiation from the early universe, the Cosmic Microwave Background Radiation (CMBR), by a number of projects, including WMAP [7], the inflation scenerio was verified, and it was shown that about 73% of the total energy in the universe is dark energy. As clearly shown by Friedmann's equation with a cosmological constant, dark energy corresponds to negative pressure, or anti-gravity. This was confirmed by studies of distant type 1a supernovae [8,9], which showed an acceleration of the expansion of the universe, and was consistent with dark energy being 73% of the energy in the universe. Also, dark energy causes distant galaxies to accelerate away from us, in contrast to the tendency of ordinary forms of energy to slow down the recession of distant objects. See Ref. [5] for other of the many references to CMBR, supernovae, galaxy and other studies of dark energy.

The existence of a non-zero vacuum energy would, in principle, have an effect on gravitational physics on all scales. The value of Λ in our present universe is not well known, and it is an empirical issue which will ultimately be settled by observation. A precise determination of this number (Λ) or ρ_Λ will be one of the primary goals of observational cosmology in the near future. Recently the possiblity of determining the cosmological constant by observations has been discussed [10].

A major outstanding problem is that most quantum field theories predict a huge cosmological constant Λ from the energy of the quantum vacuum. This conclusion also follows from dimensional analysis and effective field theory down to the Planck scale, by which we would expect a cosmological constant of the order of M_{pl}^4 (M_{pl} is the Planck mass with $M_{pl} = G^{-1/2} = 1.22\times10^{19}\,\mathrm{GeV}$. The Planck energy is thought to be the energy where conventional physical theories break down and a new theory of quantum gravity is required). We know that the measured value is on the order of $10^{-35}\,\mathrm{s}^{-2}$, or $10^{-47}\,\mathrm{GeV}^4$, or $10^{-29}\,\mathrm{g/cm}^3$, or about 10^{-120} in reduced planck units (M_{pl}). That is, there is a large difference between the magnitude of the vacuum energy expected from zero -point fluctuations and scalar potential, $\rho_\Lambda^{theory} = 2\times10^{110}\,\mathrm{erg/cm}^3$, and the observed value, $\rho_\Lambda^{observe} = 2\times10^{-10}\,\mathrm{erg/cm}^3$, a discrepancy of a factor of 10^{120}. This is the largest discrepancy—the worst theoretical prediction in the history of physics. At the same time, some supersymmetric theories require a cosmological constant that is exactly zero. Therefore, we face a big difficulty in understanding the observational $\rho_\Lambda^{observe}$. This problem has been referred to as the long-standing cosmological constant problem.

Vacuum energy is predicted to be created in cosmological phase transitions. In the standard model of particle physics with the temperature (T) of the universe as a function of time (t), there are two important phase transitions. At $t \simeq 10^{-11}$ seconds, with $T \simeq$ 140 GeV the universe undergoes the electroweak phase transition (EWPT), with the vacuum expectation value of the Higgs field, $\langle 0|:\Phi^{Higgs}:|0\rangle$, going from zero to a finite value corresponding to a Higgs mass $\simeq$ 140 GeV. At $t \simeq 10^{-5}$ seconds, with T $\simeq$ 150 MeV, the universe undergoes the QCD phase transition (QCDPT), when a universe consisting of a dense quark-gluon plasma becomes our current universe with hadrons. The latent heat for this phase transition is the quark condensate, $\langle 0|:\bar{q}q:|0\rangle$, also a vacuum energy, which is an essential part of the present work.

First we review the work of F. R. Urban, A. R. Zhitnitsky [11,12], which is based on the QCD Veneziano ghost theory [13-16] In this model the cosmological vacuum energy density ρ_Λ can be expressed in terms of QCD parameters for $N_f = 2$ light flavors as follows [10,11]

$$\rho_\Lambda = c\frac{2HN_f}{m_{\eta'}}\left|m_q\langle 0|:\bar{q}(0)q(0):|0\rangle\right|, \quad (5)$$

where m_q is the current quark mass and $c = c_{QCD}\times c_{grav.}$. The first factor c_{QCD} is a dimensionless coefficient with value of $c_{QCD} \simeq 1$ [10,11], which is entirely of QCD origin and is related to the definition of QCD on a specific finite compact manifold such as a torus, $\rho_\Lambda \simeq c_{QCD}\frac{2N_f\left|m_q\langle\bar{q}q\rangle\right|}{Lm_{\eta'}}$ with L being the size of the manifold and $m_{\eta'}$ the mass of η' meson. A precise computation of c_{QCD} has been calculated in a conventional lattice QCD approach by studying corrections of order $1/s$ to the vacuum energy [10,11]. Note that c_{QCD} depends on the manifold where the theory is defined. The second factor c_{grav} has a purely gravitational origin and is defined as the relation between the size L of the manifold we live in, and the Hubble constant H, $L = (c_{grav.}H_0)^{-1}$. One can define this size of the manifold as $L \simeq 17H_0^{-1}$ where $H_0 = 2.1\times10^{-42}\times h$ GeV and $h = 0.71$ (H_0, Hubble constant today). Therefore, one can explicitly obtain an estimate for the linear length L of the torus, and then obtain the value of $c_{grav.}$ with $c_{grav.} = 0.0588$.

In Section 2 we briefly review our previous calculation of the quark condensate [17] using Dyson-Schwinger equations (DSEs) [18,19], and discuss the quark current mass m_q, which are needed to calculate ρ_Λ, as shown in Equation (5). Since our values for the local quark condensate $\langle 0|:\bar{q}(0)q(0):|0\rangle$ and the current quark mass are approximately the same as in Ref. [10,11] we find the same value for ρ_Λ as in that work, with a factor 6 discrepancy when compared to the observed vacuum energy density. In Section 3 we use a nonlocal quark condensate, based on earlier research, and find good agreement between $\rho_\Lambda^{nonlocal\ theory}$ and $\rho_\Lambda^{observed}$. Finally, we give our Summary and concluding remarks in Section 4.

2. Local Quark Condensate, Current Quark Mass, ρ_Λ

In this section we review our previous work on the quark condensate, the current quark mass, and the resulting value for the cosmological constant/vacuum energy density.

2.1. The Local Quark Condensate

The quark propagator is defined by

$$S_q^{ab}(x) = \left\langle 0 \left| T\left[q^a(x)\bar{q}^b(0)\right] \right| 0 \right\rangle, \quad (6)$$

where $q^a(x)$ ($q^b(x)$) is a quark field with color *a* (*b*), and *T* is the time-ordering operator. The nonperturbative part of the quark propagator is given by

$$S_q^{NP}(x) = -\frac{1}{12}\left[\langle 0|:\bar{q}(x)q(0):|0\rangle + x_\mu \langle 0|:\bar{q}(x)\gamma^\mu q(0):|0\rangle\right]. \quad (7)$$

For short distances, the Taylor expansion of the scalar part, $\langle 0|:\bar{q}(x)q(0):|0\rangle$, of $S_q^{NP}(x)$ can be written as (see, e.g., Refs.[17,20])

$$\langle 0|:\bar{q}(x)q(0):|0\rangle = \langle 0|:\bar{q}(0)q(0):|0\rangle - \frac{x^2}{4}\langle 0|:\bar{q}(0)\left[ig_s\sigma G(0)\right]q(0):|0\rangle + \cdots. \quad (8)$$

In Equation (8) the vacuum expectation values in the expansion are the local quark condensate, the quark-gluon mixed condensate, and so forth.

The Dyson-Schwinger Equations [18,19] were used to derive the local quark condensate in Ref. [17]. See this reference for details and a discussion of approximations. Note that as shown in Equation (8), the quark-gluon mixed condensate provides the small-*x* dependence of the nonlocal $\langle 0|:\bar{q}(x)q(0):|0\rangle$ quark condensate. However, for the present work this small-*x* expansion is not useful, and we shall use a known expression for the nonlocality, described below. Therefore we only give the results for the local quark condensate. Also note that the vacuum condensates can act as a medium [21,22], which influences the properties of particles propagating through it.

Using the solutions of DSEs with three different sets of the quark-quark interaction parameters (see Ref.[17]) leads to our theoretical predictions for the local quark vacuum condensate listed in **Table 1**.

Set 1 results are consistent with many other calculations, such as QCD sum rules [23-25], Lattice QCD [26-28] and Instanton model predictions [29-31]. These numerical results will be used to calculate Λ/ρ_Λ in the Subsection 2.3 below.

2.2. The Current Mass of Light Quarks

As we have seen from Equation (5) to predict Λ we need to know the basic quark current mass m_q. Since one cannot produce a beam of quarks, it is difficult to determine the quark masses. Using various models the effective quark masses have been estimated, but we need the current quark masses of the light *u* and *d* quark. Estimates of these masses and references can be foud in the Particle Data Physics booklet [32]. They are

$$\begin{aligned} 1.7 < m_u < 3.3\ \text{MeV} \\ 4.1 < m_d < 5.8\ \text{MeVe}. \end{aligned} \quad (9)$$

From this we estimate that the current quark mass is

$$m_q \simeq 4.0\ \text{MeV} \quad (10)$$

2.3. Cosmological Constant Λ with $\langle 0|:\bar{q}(0)q(0):|0\rangle$ and m_q

From Equation (2), $\Lambda = 8\pi G\rho_\Lambda$, the vacuum energy density, while ρ_Λ, is given in Equation (5) as

$$\rho_\Lambda = c\frac{2HN_f}{m_{\eta'}} m_q \left|\langle 0|:\bar{q}(0)q(0):|0\rangle\right| \quad (11)$$

Since our values for m_q and $\langle 0|:\bar{q}(0)q(0):|0\rangle$ are the standard ones, we find the same value for ρ_Λ as in

Table 1. Predictions of local quark condensate in QCD vacuum, $\langle 0|:\bar{q}q:|0\rangle_\mu^f$ with *f* standing for quark flavor and μ denotes renormalization point, $\mu^2 = 10\ \text{GeV}^2$.

Set no. of quark interactions	$\langle 0\|:\bar{q}q:\|0\rangle_{u,d}$ for *u* and *d* quarks
Set 1	$-0.0130(\text{GeV})^3 \sim -(235\ \text{MeV})^3$
Set 2	$-0.0078(\text{GeV})^3 \sim -(198\ \text{MeV})^3$
Set 3	$-0.0027(\text{GeV})^3 \sim -(139\ \text{MeV})^3$

Ref.[11]

$$\rho_\Lambda^{theory} \simeq \left(3.6\times10^{-3}\ \text{eV}\right)^4, \quad (12)$$

while the value observed [33] is

$$\rho_\Lambda^{observed} \simeq \left(2.3\times10^{-3}\ \text{eV}\right)^4. \quad (13)$$

Although the theoretical and observed values are similar, they still differ by

$$\rho_\Lambda^{theory} / \rho_\Lambda^{observed} \simeq 6.0$$

3. Cosmological Constant Λ with Nonlocal Quark Condensate

As mentioned above, the expression

$$\langle 0|:\bar{q}(x)q(0):|0\rangle = \langle 0|:\bar{q}(0)q(0):|0\rangle - \frac{x^2}{4}\langle 0|:\bar{q}(0)\left[ig_s\sigma G(0)\right]q(0):|0\rangle + \cdots$$

does not work except for very small x. Therefore we shall use the nonlocal quark condensate derived from the quark distribution function (see Refs.[34,35]). Using the form in Ref.[35],

$$\langle 0|:\bar{q}(x)q(0):|0\rangle = g\left(x^2\right)\langle 0|:\bar{q}(0)q(0):|0\rangle, \quad (14)$$

with

$$g(x) = \frac{1}{\left(1+\lambda^2x^2/8\right)^2}. \quad (15)$$

The value of λ^2 estimated in Ref.[36] is $\lambda^2 \simeq 0.8\,\text{GeV}^2$. Using $1/\Lambda_{QCD}$ as the length scale, or $x^2 = \left(1/0.2\ \text{GeV}\right)^2$, one obtains

$$g\left(1/\Lambda_{QCD}\right) = \frac{1}{2.25^2} = \frac{1}{6.25}. \quad (16)$$

From this we obtain

$$\langle 0|:\bar{q}(x)q(0):|0\rangle = \frac{1}{6.25}\langle 0|:\bar{q}(0)q(0):|0\rangle, \quad (17)$$

and

$$\begin{aligned}\rho_\Lambda^{nonlocal\ theory} &\simeq \frac{1}{6}\left(3.6\times10^{-3}\ \text{eV}\right)^4 \\ &= \left(2.3\times10^{-3}\ \text{eV}\right)^4 \simeq \rho_\Lambda^{observed}\end{aligned} \quad (18)$$

Therefore, using the modification of the quark condensate via the nonlocal condensate, one obtains excellent agreement between the theoretical and observed cosmological constants.

4. Summary and Concluding Remarks

The cosmological constant Λ is an important physical quantity, which was introduced by A. Einstein who modified the field equations of his general theory of relativity to obtain a stationary universe. The constant has recently been used to explain the observed accelerated expansion of the universe, but its observational value is about 120 orders of magnitude smaller than the one theoretically computed in the framework of the currently accepted quantum field theories. Namely, quantum field theory predicted that vacuum energy density, ρ_Λ, is of the order of M_{pl}^4, with $M_{pl} = 1.22\times10^{19}\,\text{GeV}$, which is about 120 orders of magnitude larger than the observed value of $\rho_\Lambda^{observed} = \left(2.3\times10^{-3}\ \text{eV}\right)^4$. This difference is the so called cosmological constant problem, the worst problem of fine-tuning in physics.

Based on the Veneziano ghost theory of QCD, using a local quark condensate, we obtained the same result for ρ_Λ as in Refs[11,12], about a factor of 6 larger than $\rho_\Lambda^{observed}$. However, $\langle 0|:\bar{q}(0)q(0):|0\rangle$ is just an approximation to $\langle 0|:\bar{q}(x)q(0):|0\rangle$. Using the nonlocal quark condensate $\langle 0|:\bar{q}(x)q(0):|0\rangle = g(x)\langle 0|:\bar{q}(0)q(0):|0\rangle$ we find that the theoretical and observed values of ρ_Λ are approximately equal.

The cosmological constant Λ is a potentially important contributor to the dynamical history of the universe. Unlike ordinary matter, which can clump together or disperse as it evolves, the vacuum energy is a property of spacetime itself, and is expectd to be the same everywhere. If the cosmological costant is the valid model of dark energy, a sufficiently large cosmological constant will cause galaxies and supernovae to accelerate away from us, as has been observed, in contrast to the tendency of ordinary forms of energy to slow down the recession of distant objects. The value of Λ in our present universe is not well known. A precise determination of this constant will be one of the primary goals of both theoretical cosmology and observational cosmology in the near future.

One might doubt the correctness of the Veneziano QCD ghost theory that we used in this work, since it is an analogue of two-dimensional theory based on the Schwinger model [18,19], replacing the vector gauge field by two scalar fields. These scalar fields have positive and negative norms and cancel with each other, leaving no trace in the physical subspace. They have small contribution to the vacuum energy in the curved space. It is known that the QCD ghost must be an intrinsically vector field in order for the $U(1)$ problem to be consistently resolved within the framework of QCD. It seems to be necessary to examine if the Veneziano mechanism works in terms of the vector ghost fields instead of the scalar fields used here. However, Ohta and others in Refs. [36-38] have discussed the same problem in more realistic four dimensional models, and show that the QCD ghost produces vacuum energy density ρ_Λ proportional to the Hubble parameter which has approximately the right magnitude $\sim\left(3\times10^{-3}\,\mathrm{eV}\right)^4$.

There is now considerable evidence that the universe began as fireball in the cosmological vacuum, the so-called "Big Bang", with extremely high temperature and high energy density. One knows that the quark condensate is vastly changed by the QCD phase transition, and this implies that there is a tempreature (T) dependence of $\left\langle 0\middle|:\bar{q}(x)q(0):\middle|0\right\rangle$ and Λ. Λ is probably dependent on temperature T and momentum p of virtual particles which produce vacuum condensates, as mentioned above. We can predict the Λ dependence on temperature T and momentum p by solving the temperature dependent Dyson-Schwinger Equations. In this case, Λ is a function of T and p. Such a new study could show the behavior of the Λ during the evolution of the universe. This work is under its way and should be complete soon.

REFERENCES

[1] A. Einstein, "The Foundation of the General Theory of Relativity," 1916. http:// www.alberteinstein.info

[2] A. Einstein, "Die Feldgleichungen der Gravitation, Sitzungsberichte der Preussischen Akademie der Models," 1915, p. 387.

[3] J. B. Hartle, "Gravity: An Introduction to Einstein's General Relativity," Addison Wesley, Boston, 2008, p. 482.

[4] E. Hubble, "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae," *Proceedings of the National Academy of Sciences of USA*, Vol. 15, No. 3, 1929, 168-173. doi:10.1073/pnas.15.3.168

[5] S. M. Carroll, "The Cosmological Constant," *Living Reviews in Relativity*, Vol. 4, No. 1, 2000. arXiv: astro-ph/0004075

[6] A. H. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," *Physical Review D*, Vol. 23, 1981, pp. 347-356.

[7] D. N. Spergel, L. Verde, H. V. Peiris, *et al*., "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Obsevations: Determinaion of Cosmological Parameters," *Astrophysical Journal Supplement*, Vol. 148, No. 1, 2003, p. 175. arXiv:astro-ph/0302209

[8] A. G. Riess, J. Astron, *et al*., "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," *Astronomical Journal*, Vol. 116, No. 3, 1998, p. 1009. arXiv:astro-ph / 9805201

[9] S. Perlmutter, *et al*., "Measurements of Omega and Lambda from 42 High-Redshift Supernovae," *Astrophysical Journal*, Vol. 517, No. 2, 1999, p. 565. arXiv:astro-ph/9812133

[10] D. J. Shaw and J. D Barrow, "A Testable Solution of the Cosmological Constant and Coincidence Problems," *Physical Review D*, Vol. 83, 2011, Article ID: 043518.

[11] F. R. Urban and A. R. Zhitnitsky, "The Cosmological Constant from the Veneziano Ghost which Solves the U(1) Problem in QCD," *Physical Letters B*, Vol. 688, 2010, p. 9.

[12] F. R. Urban and A. R. Zhitnitsky, "Cosmological Constant, Violation of Cosmological Isotropy and CMB," *Physical Review D*, Vol. 80, 2009, Article ID: 063001.

[13] G. Veneziano, "U(1) without Instantons," *Nuclear Physics B*, Vol. 159, No. 1-2, 1979, pp. 213-224. doi:10.1016/0550-3213(79)90332-8

[14] E. Witten, "Current Algebra Theorems for the U(1) 'Goldstone Boson'," *Nuclear Physics B*, Vol. 156, No. 2, 1979, pp. 269-283. doi:10.1016/0550-3213(79)90031-2

[15] J. D. Barrow and D. J. Shaw, "Quantum Field Effects Near Singularities," *Physical Review Letters*, Vol. 1, 2011, p. 10130. arXiv:1201.1138v1

[16] H. Hata, T. Kugo and N. Ohta, "Skew-Symmetric Tensor Gauge Field Theory Dynamically Realized in the QCD U(1) Channel," *Nuclear Physics B*, Vol. 178, No. 3, 1981, pp. 527-544. doi:10.1016/0550-3213(81)90170-X

[17] L.-J. Zhou, L. S. Kisslinger and W.-X. Ma, "Nonzero Mean Squared Momentum of Quarks in the Non-Perturbative QCD Vacuum," *Physical Review D*, Vol. 82, 2010, Article ID: 034037.

[18] F. J. Dyson, "The S-Matrix in Quantum Electrodynamics ," *Physical Review*, Vol. 75, 1949, p. 1736.

[19] J. S. Schwinger, "On the Green's Functions of Quantized Fields. I," *Proceedings of the National Academy of Sciences of USA*, Vol. 37, No. 7, 1951, pp. 452-455. doi:10.1073/pnas.37.7.452

[20] A. G. Williams, G. Krein and C. D. Roberts, "Modelling the Quark Propagator," *Annals of Physics*, Vol. 210, No. 2, 1991, pp. 464-485. doi:10.1016/0003-4916(91)90051-9

[21] L.-J. Zhou and W.-X. Ma, "Structure of Nonlocal Vacuum Condensate of Quarks," *Chinese Physics Letters*, Vol. 20, No. 12, 2003, p. 2137. doi:10.1088/0256-307X/20/12/014

[22] L.-J. Zhou and W.-X. Ma, "Quark Virtuality and QCD Vacuum Condensates," *Chinese Physics Letters*, Vol. 21, No. 8, 2004, p. 1471. doi:10.1088/0256-307X/21/8/016

[23] W.-X. Ma, L.-J. Zhou, Gu Yun-ting, *et al*., "Quark Mass Dependence of Nucleon Magnetic Moment and Charge Radii," *Communications in Theoretical Physics*, Vol. 44, 2005, p. 333.

[24] M. V. Polyakov and C. Weiss, "Mixed Quark-Gluon Condensate from Instantons," *Physics Letters B*, Vol. 387, No. 4, 1996, pp. 841-847. doi:10.1016/0370-2693(96)01098-2

[25] H. G. Dosch and S. Narison, "Direct Extraction of the Chiral Quark Condensate and Bounds on the Light Quark Masses," *Physics Letters B*, Vol. 417, 1998, pp. 173-176. doi:10.1016/S0370-2693(97)01370-1

[26] M. Kremer and G. Schierholz, "Calculation of the Condensate $g_s\left\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\right\rangle$ on the Lattice," *Physics Letters B*, Vol. 194, No. 2, 1987, pp. 283-286. doi:10.1016/0370-2693(87)90543-0

[27] D. B. Leinweber, "QCD Sum Rules For Skeptics," Report No. DOE/Er/40427-17-N 95, 1995.

[28] L. Giusti, "The QCD Chiral Condensate from the Lattice," *Nuclear Physics B*, Vol. 538, No. 1-2, 1999, pp. 249-277. doi:10.1016/S0550-3213(98)00659-2

[29] P. Hernandez, "Scalar Condensate and Light Quark Masses from Overlap Fermions," *Nuclear Physics B*, Vol. 106-107, 2002, pp. 766-771. doi:10.1016/S0920-5632(01)01838-2

[30] M. Jamin, J. A. Oller and A. Pich, "Light Quark Masses from Scalar Sum Rules," *The European Physical Journal C*: *Particles and Fields*, Vol. 24, No. 2, 2002, pp. 237-243. doi:10.1007/s100520200930

[31] T. Doi, N. Ishii, M. Oka and H. Suganuma, "Quark-Gluon Mixed Condensate $g\left\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\right\rangle$ from Lattice QCD," *Nuclear Physics A*, Vol. 721, 2003, pp. 934-937. doi:10.1016/S0375-9474(03)01246-6

[32] K. Nakamura *et al*., "Review of Particle Physics," *Journal of Physics G*, Vol. 37, 2010, Article ID: 075021.

[33] M. Tegmark, M. A. Strauss, M. R. Blanton, *et al*., "Cosmological Parameters from SDSS and WMAP," *Physical Review D*, Vol. 69, 2004, Article ID: 103501.

[34] H. Jung and L. S. Kisslinger, "Non-Local Quark Condensate for the Nucleon Sea-Quark Distribution," *Nuclear Physics A*, Vol. 586, No. 4, 1995, pp. 682-692. doi:10.1016/0375-9474(95)00004-K

[35] M. B. Johnson and L. S. Kisslinger, "Hadronic Coupling via QCD Sum Rules Using Three-Point Functions: Vacuum Susceptibilities," *Physical Review D*, Vol. 57, No. 2, 1998, p. 2847.

[36] N. Ohta, "Dark energy and QCD Ghost," *Physical Letters B*, Vol. 695, No. 1-4, 2011, pp. 41-44. doi:10.1016/j.physletb.2010.11.044

[37] J. B. Kogut and L. Susskind, "Hamiltonian Formulation of Wilson's Lattice Gauge Theory," *Physical Review D*, Vol. 11, 1975, pp. 359-408.

[38] P. Nath and R. L. Arnowitt, "U(1) Problem: Current Algebra and the θ Vacuum," *Physical Review D*, Vol. 23, No. 2, 1981, pp. 473-476.

Journal of Modern Physics, 2012, 3, 1178-1184
doi:10.4236/jmp.2012.329152 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Relativistic Cosmology and the Pioneers Anomaly

Marcelo Samuel Berman, Fernando de Mello Gomide
Instituto Albert Einstein/Latinamerica, Av. Sete de Setembro 4500 # 101 80250-210, Curitiba, Brazil
Email: msberman@institutoalberteinstein.org, marsambe@yahoo.com, lfgomide@hotmail.com

Received June 16, 2012; revised July 15, 2012; accepted July 23, 2012

ABSTRACT

NASA spacecrafts has suffered from three anomalies. The Pioneers spacecrafts were decelerated, and their spin when not disturbed, was declining. On the other hand, fly-bys for gravity assists, appeared with extra speeds, relative to infinity. The Pioneers and fly-by anomalies are given now exact general relativistic full general solutions, in a rotating expanding Universe. We cite new evidence on the rotation of the Universe. Our solution seems to be the only one that solves the three anomalies.

Keywords: Relativistic Cosmology; Pioneers Anomaly

1. Introduction

Detailed description of the subjects treated in this paper may be found in the two books recently published by Berman in 2012 [1,2]). Additional paper references are Berman in 2007 [3]; in 2011 [4,5]; in 2012 [6]) and with co-authors Costa, (Berman and Costa in 2012 [7]) and with Gomide {Berman and Gomide in 2012 [8] and [9]-(version of the year 2010)} and book form as a Chapter in an edited book [10], by Berman and Gomide.

Anderson *et al*., in 2008 [11], and Lämmerzahl *et al*., in 2006 [12], have alerted the scientific community about the fly-by anomaly: during Earth gravity assists, spacecraft has suffered from an extra-energization, characterized by a positive extra speed, such that, measured "at infinity", the hyperbolic orbiting object presented an empirically calculated $\Delta V/V$ around 10^{-6}. A formula was supplied,

$$\frac{\Delta V}{V} = \frac{2\omega R}{c} \tag{1.1}$$

where ω, R and c stand for the angular speed and radius of the central mass, and the speed of light in vacuo. T. L. Wilson, from NASA, (Houston) and H.-J. Blome (Aachen), delivered a lecture in Montreal, on July 17, 2008, and called the attention to the fact that the most trusted cause for both this anomaly, and the Pioneers, would be "rotational dynamics" (Wilson and Blome, in 2008 [13]). One of us, had, by that time, published results on the Pioneers Anomaly, through the rotation of the Universe (Berman, in 2007 [3]). Now, we shall address the three anomalies.

The Pioneers Anomaly is the deceleration of about $-9\times10^{-8}\ \text{cm}\cdot\text{s}^{-2}$ suffered by NASA space-probes travelling towards outer space. It has no acceptable explanation within local Physics, but if we resort to Cosmology, it could be explained by the rotation of the Universe. Be cautious, because there is no center or axis of rotation. We are speaking either of a Machian or a General Relativistic cosmological vorticity. It could apply to each observed point in the Universe, observed by any observer. Another explanation, would be that our Universe obeys a variable speed of light Relativistic Cosmology, without vorticities. However, we shall see later that both models are equivalent. Thermal emission cannot be invoked, for it should also decelerate elliptical orbiters, but the deceleration only affects hyperbolic motion. It does not explain fly-bys, either. A secondary Pioneers anomaly refers to spinning down of the spacecraft, when they were not disturbed. Again, thermal emission cannot explain it.

In previous papers (Berman and Gomide in 2010, updated for this Journal in 2012 [9]; in 2012 [10]), by considering an exact but particular solution of Einsteins field equations for an expanding and rotating metric, found, by estimating the deceleration parameter of the present Universe, as $q \approx -1/2$, that the Universe appeared to possess a field of decelerations coinciding approximately with the Pioneers anomalous value (Anderson *et al*. in 2002 [14]). We now shall consider the condition for an exact match with the Pioneers deceleration,with a large class of solutions in General Relativity. Sections 5 and 6, treat the second Pioneers anomaly, and the fly-by. In Section 7, an alternative cosmological model will be presented, following an idea by Godlowski *et al*. in 2004 [15], which allows us to work with a non-modified RWs metric.

The key result for all these subjects, is that hyperbolic motion, extends towards infinity, and, thus, qualify for cosmological alternatives, and boundary conditions. The fly-bys, and the Pioneers, are in hyperbolic trajectories, when the anomalies appear, so that Cosmology needs to be invoked.

Ni in 2008 [37] and 2009 [38], has reported observations on a possible rotation of the polarization of the cosmic background radiation, around 0.1 radians. As such radiation was originated at the inception of the Universe, we tried to estimate a possible angular speed or vorticity, by dividing 0.1 radians by the age of the Universe , obtaining about 10^{-19} rad·s^{-1}. Compatible results were obtained by Chechin in 2010 [33], Su and Chu in 2009 [41], Godlowski in 2011 [35] and Chechin in 2012 [34].

The numerical result is very close to the theoretical estimate, by Berman in 2007 [3],

$$\omega = c/R = 3\times10^{-18}\ \text{rad}\cdot\text{s}^{-1} \tag{1.2}$$

where c, R represent the speed of light in vacuum, and the radius of the causally related Universe.

When one introduces a metric temporal coefficient g_{00} which is not constant, the new metric includes rotational effects. The metric has a rotation of the tri-space (identical with RWs tri-space) around the orthogonal time axis. This will be our framework, except for Section 7.

2. On the Four Kinds of Rotation in Relativistic Cosmology

The purpose of this Section is basically to focus on new rotational formulations in Relativistic Cosmology, the first, due to Berman [1-10]; based on a seminal me- tric that was proposed by Gomide and Uehara [36] when the purpose of those two authors was something else, unrelated to rotation, and the second, was an idea by Godlovski *et al.* [15], developed in several articles by Berman (see for instance papers [2,4] and books [1,2].

Consider the metric line-element:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \tag{2.1}$$

If the observer is at rest,

$$\mathrm{d}x^i = 0\ \left(i = 1,2,3\right)$$

while,

$$\mathrm{d}x^0 = \mathrm{d}t \tag{2.2}$$

This last equality defines a proper time; we called cosmic time, in Cosmology.

From the geodesics' equations, we shall have:

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}s^2} + \Gamma^i_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s} = \Gamma^i_{00} \tag{2.3}$$

We then find:

$$g^{ij}\frac{\partial g_{i0}}{\partial t} = 0 \tag{2.4}$$

This defines a Gaussian coordinate system, which in general implies that:

$$\frac{\partial g_{i0}}{\partial t} = 0 \tag{2.5}$$

We must now reset our clocks, so that, the above condition is universal (valid for all the particles in the Universe), and then our metric will assume the form:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - g_{ij}\left(x,t\right)\mathrm{d}x^i\mathrm{d}x^j \tag{2.6}$$

If we further impose that, in the origin of time, we have:

$$g_{i0}\left(t=0\right) = 0 \tag{2.7}$$

then by (2.5), we shall have:

$$g_{i0}\left(t\right) = 0 \tag{2.8}$$

The above defines a Gaussian normal coordinate system.

For a commoving observer, in a freely falling perfect fluid, the quadrivelocity u^μ will obey:

$$u^i = 0 \tag{2.9}$$

while, if we normalize the quadrivelocity, we find, from the condition:

$$g_{\mu\nu}u^\mu u^\nu = 1 \tag{2.10}$$

that,

$$g_{00}u^0 = 1 \tag{2.11}$$

Though later we shall discuss the case $g_{00} = g_{00}\left(t\right) \neq 1$, it is usually imposed:

$$g_{00} = u^0 = 1 \tag{2.12}$$

When dealing with Robertson-Walker's metric, this is the usual procedure. By this means, we have a tri-space, orthogonal to the time axis.

Gaussian coordinate systems, in fact, imply that, with $g_{0i} = 0$, there are no rotations in the metric, and in each point we may define a locally inertial reference system.

Gaussian normal coordinates were called "synchronous"; in an arbitrary spacetime, when we pick a spacelike hypersurface S_0, and we eject geodesic lines orthogonal to it, with constant coordinates x^1, x^2 and x^3, while $x^0 \equiv t + t_0$, where $t_0 = 0$ on S_0, then t is the proper time, whose origin is $t = 0$ on S_0 (see MTW in 1973 [16]).

In the above treatment, cosmic time is "absolute", so that the measure of the age of the Universe, according to this "time", is not subject to a relative nature.

Now, we might ask whether the tri-space, orthogonal to the time axis, could rotate relative to this axis. Berman in 2008 [17,18], has exactly defined this original idea, by identifying this rotation, which is different from all others, as will shall show bellow, with a time-varying metric coefficient $g_{00}(t)$. In the next Section, we relate the angular speed of the tri-space, relative to the time axis, with $g_{00}(t)$ by means of,

$$\omega = \frac{1}{2}\frac{\dot{g}_{00}}{g_{00}} \tag{2.13}$$

In the above, we still may have a perfect fluid model. Book treatments can be found in Berman [1,2].

Other type of rotation, is Raychaudhuri's vorticity [19-21], which is attached to non-perfect fluids (see, for instance, Berman in 2007 [19]). A third type of rotation, is what we usually call rotation of the metric, and is defined by non-diagonal terms, in the metric. For instance, Kerr's metric [22-24].

A fourth kind of rotation, is also attached to a perfect fluid model, like Berman's one: it is the Godlowski *et al*. in 2004 [15] idea, which is developed in Section 7 below. See also Berman [1,2,4-7].

3. Field Equations for Gomide-Uehara-R.W.-Metric

Consider first a temporal metric coefficient which depends only on t. The line element becomes:

$$\mathrm{d}s^2 = -\frac{R^2(t)}{\left(1+kr^2/4\right)^2}\left[\mathrm{d}\sigma^2\right] + g_{00}(t)\mathrm{d}t^2 \tag{3.1}$$

The field equations, in General Relativity Theory (GRT) become:

$$3\dot{R}^2 = \kappa\left(\rho + \frac{\Lambda}{\kappa}\right)g_{00}R^2 - 3kg_{00} \tag{3.2}$$

and,

$$6\ddot{R} = -g_{00}\kappa\left(\rho + 3p - 2\frac{\Lambda}{\kappa}\right)R - 3g_{00}\dot{R}\dot{g}^{00} \tag{3.3}$$

Local inertial processes are observed through proper time, so that the four-force is given by:

$$F^\alpha = \frac{\mathrm{d}}{\mathrm{d}\tau}\left(mu^\alpha\right) = mg^{00}\ddot{x}^\alpha - \frac{1}{2}m\dot{x}^\alpha\left[\frac{\dot{g}_{00}}{g_{00}^2}\right] \tag{3.4}$$

Of course, when $g_{00} = 1$, the above equations reproduce conventional Robertson-Walker's field equations.

In order to understand Equation (3.4), it is convenient to relate the rest-mass m, to an inertial mass M_i, with:

$$M_i = \frac{m}{g_{00}} \tag{3.5}$$

It can be seen that M_i represents the inertia of a particle, when observed along cosmic time, *i.e*., coordinate time. In this case, we observe that we have two acceleration terms, which we call,

$$a_1^\alpha = \ddot{x}^\alpha \tag{3.6}$$

and,

$$a_2^\alpha = -\frac{1}{2g_{00}}\left(\dot{x}^\alpha \dot{g}_{00}\right) \tag{3.7}$$

The first acceleration is linear; the second, resembles rotational motion, and depends on g_{00} and its time-derivative.

If we consider a_2^α a centripetal acceleration, we conclude that the angular speed ω is given by,

$$\omega = \frac{1}{2}\left(\frac{\dot{g}_{00}}{g_{00}}\right) \tag{3.8}$$

The case where g_{00} depends also on r, θ and ϕ was considered also by Berman in 2008 [18] and does not differ qualitatively from the present analysis, so that, we refer the reader to that paper.

4. The Exact Solution to the Pioneers Anomaly

Consider the possible solution for the rotating case. We equate (1.2) and (3.8). We try a power-law solution for R, and find,

$$g_{00} = Ae^{t^{1-1/m}} \quad (A = \text{constant}).$$

The scale-factor assumes a power-law, as in constant deceleration parameter models (Berman in 1983 [25]; and Gomide in 1988 [26])

$$R = (mDt)^{1/m} \tag{4.1}$$

where, m, D = constants, and,

$$m = q + 1 > 0 \tag{4.2}$$

where q is the deceleration parameter. We may choose q as needed to fit the observational data.

We find,

$$H = (mt)^{-1}$$

If we now solve for energy-density of matter, and cosmic pressure, for a perfect fluid, the best way to present the calculation, and the most simple, is showing the matter energy-density ρ and the σ-or gravitational density parameter, to be defined below. We find

$$\rho = \frac{3t^{-2}}{m^2 A\kappa}e^{-t^{1-1/m}} + 3k(mDt)^{-2/m} - \frac{\Lambda}{\kappa}$$

$$\sigma = \left(\rho + 3p - 2\frac{\Lambda}{\kappa}\right) = \left[\frac{6(1-m)t^{-2} + 3t^{-1/m}}{m^2 A\kappa}\right]e^{-t^{1-1/m}}$$

For the present Universe, the infinite time limit makes the above densities become zero.

It is possible to define,

$\rho_{grav} = -\dfrac{3H^2}{\kappa g_{00}}$ (negative energy-density of the gravitational field)

Now, let us obtain the gravitational energy of the field,

$$E_{grav} = \rho_{grav} V = -\left(4/3\right)\pi R^3 \left(\frac{3H^2}{\kappa g_{00}}\right) = -\frac{c^4 R^{4-3m}}{2Gm^2 A\left(mD\right)^{1/m-3}} \quad (4.3)$$

5. The Second Pioneers Anomaly

The universal angular acceleration, is given by

$$\alpha_u = \dot{\omega} = -cH/R = -c^2/R^2 \quad (5.1)$$

The spins of the Pioneers were telemetered, and as a surprise, it shows that the on-board measurements yield a decreasing angular speed, when the space-probes were not disturbed. Turyshev and Toth in 2010 [27], published the graphs (Figures 2.16 and 2.17 in their paper), from which it is clear that there is an angular deceleration of about 0.1 RPM per three years, or,

$$\alpha \approx -1.2\times10^{-10}\ \text{rad/s}^2 \quad (5.2)$$

As the diameter of the space-probes is about 10 meters, the linear acceleration is practically the Pioneers anomalous deceleration value ,in this case, -6×10^{-8} cm·s^{-2}. The present solution of the second anomaly, confirms our first anomaly explanation.

I have elsewhere pointed out that we are in face of an angular acceleration frame-dragging field, for it is our result (5.1) above, for the Universe, that causes the result (5.2), through the general formula,

$$\alpha = -\frac{cH}{l} \quad (5.3)$$

where l is the linear magnitude of the localized body suffering the angular acceleration frame-dragging. For subatomic matter, this angular acceleration can become important.

6. The Solution of the Fly-By Anomaly

Consider a two-body problem, relative to an inertial system. The additional speed, measured at infinity, relative to the total speed, measured at infinity, is proportional to twice the tangential speed of the earth, $w_e R_e$, divided by the total speed $V \to wR \approx c$ taken care of the Universe angular speed. In fact, we write

$$\frac{\Delta V}{V} = \frac{V + \omega_e R_e - \left(V - \omega_e R_e\right)}{c} = \frac{2\omega_e R_e}{c} \approx 3\times10^{-6} \quad (6.1)$$

The trick, is that infinity, in a rotating Universe, like ours, has a precise meaning, through the angular speed Formula (1.2).

7. The Godlowski *et al*. Rotation

We, now, shall follow an idea by Godlowski *et al*. in 2004 [15], and supply another General Relativistic model, of an expanding and rotating Universe. Their idea, is that the homogeneous and isotropic models, may still rotate relative to the local gyroscope, by means of a simple replacement, in the Friedman-RWs equations, of the kinetic term, by the addition of a rotational kinetic one.

Einsteins field equations, for a perfect fluid with perfect gas equation of state, and RWs metric, are two ones. The first, is an energy-density equation, the second is a definition of cosmic pressure, which can be substituted by energy momentum conservation. But, upon writing the $\dot{R}^2$ term, we shall add an extra rotational term, namely $\left(\omega R\right)^2$, in order to account for rotation. If we keep (3.1), the field equations become, for a flat Universe

$$6H^2 = \kappa\rho + \Lambda \quad (7.1)$$

with

$$p = \beta\rho \quad (7.2)$$

and

$$\dot{\rho} = -3\sqrt{2}H\rho\left(1+\beta\right) \quad (7.3)$$

The usual solution, with Bermans constant deceleration parameter models, render (Berman in 1983 [25]; and Gomide in 1988 [26]),

$$R = \left(mDt\right)^{1/m} \quad (7.4)$$

$$H = \left(mt\right)^{-1} \quad (7.5)$$

$$\ddot{R} = -qH^2R = -\left(m-1\right)H^2R \quad (7.6)$$

Notice that we may have a negative deceleration parameter, implying that the Universe accelerates, probably due to a positive cosmological "constant", but, nevertheless, it is subjected to a negative rotational deceleration, a kind of centripetal one, that acts on each observed point of the Universe, relative to each observer, given by relation (1.2), so that,

$$\ddot{R} = -qH^2R = qa_{cp} \quad (7.7)$$

We now supply the necessary relations among the constants, so that the above equations be observed, namely,

$$m = \frac{3}{2}\sqrt{2}\left(1+\beta\right) = \pm\frac{\sqrt{6}}{\sqrt{\kappa\rho_0 + \Lambda_0}}$$

$$\rho = \rho_0 t^{-2} \quad (7.8)$$

$$\Lambda = \Lambda_0 t^{-2} \tag{7.9}$$

8. Final Comments

If we calculate the centripetal acceleration corresponding to the above angular speed (1.2), we find, for the present Universe, with $R \approx 10^{28}$ cm and $c \simeq 3.10^{10}$ cm/s,

$$a_{cp} = -\omega^2 R \cong -9\times10^{-8}\ \text{cm/s}^2 \tag{8.1}$$

Our model of Section 4 has been automatically calculated alike with (1.2) and (8.1). This value matches the observed experimentally deceleration of the NASA Pioneers' space-probes. Equation (3.3) shows that one can have a positive cosmological lambda term accelerating the Universe, *i.e.*, $\ddot{R} \geq 0$ along with a centripetal deceleration that is felt by any observer, relative any observed point, given by (8.1). Berman and Gomide, in 2010, update for this Journal in 2012 [9], had obtained a Machian General Relativistic solution, though particular. We call it Machian, because it parallels the semirelativistic Machian solution by Berman in 2007 [3].

A cosmologist has made very important criticisms on our work. First, he says why do not the planets in the solar system show the calculated deceleration on the Pioneers? The reason is that elliptical orbits are closed, and localized. You do not feel the expansion of the universe in the sizes of the orbits either. In General Relativity books, authors make this explicit. You do not include Hubbles expansion in Schwarzschilds metric. But, those space probes that undergo hyperbolic motion, which orbits extend towards infinity, they acquire cosmological characteristics, like, the given P.A. deceleration. Second objection, there are important papers (Rievers and Lämmerzahl in 2011 [28]; Francisco *et al.* in 2011 [29]; Cuesta in 2011 [30]) which resolve the P.A. with non-gravitational Physics. Our answer, that is OK, we have now alternative explanations. However, in the Introduction of this paper, we have responded why thermal emission is no good an explanation because it does not explain the other two anomalies neither why the elliptical orbiters did not suffer the same deceleration; as to Cuesta in 2011 [30] he also has no explanation for the other two anomalies. This does not preclude ours. Third, cosmological reasons were discarded, including rotation of the Universe. The problem is that those discarded cosmologies, did not employ the correct metric. For instance, they discarded rotation by examining Godel model, which is non expanding, and with a strange metric. The two kinds of rotating and expanding metrics we employ now, were not discarded or discussed by the authors cited by this cosmologist. Then, the final question, is how come that a well respected author, dismissed planetary Coriolis forces induced by rotation of distant masses, by means of the constraints in the solar system. The answer is the same above, and also that one needs to consider Machs Principle on one side, and the theoretical meaning of vorticities, because one is not speaking in a center or an axis of rotation or so. When we say, in Cosmology, that the Universe rotates, we mean that there is a field of vorticities, just that. The whole idea is that Cosmology does not enter the Solar System except for non-closed orbits that extend to outer space. For the Gomide Uehara RWs metric, it is the tri-space that rotates relative to the orthogonal time-axis.

Another cosmologist pointed out a different "problem". He was discussing the prior paper, to the present one (Berman and Gomide in 2010, updated in 2012 for this Journal [9]). He objects, that the angular speed formula of ours, is coordinate dependent. Now, when you choose a specific metric, you do it thinking about the kind of problem you have to tackle. After you choose the convenient metric, you forget tensor calculus, and you work with coordinate-dependent relations. They work only for the given metric, of course. It must be stressed once more, what has been discussed in several prior papers by these authors, or by Berman alone, that the point most important that is taken into consideration remains the zero-total energy of the Universe, whose pioneer pseudo-tensor calculation has been made in an unpublished Master of Science Thesis by Berman, in 1981 [31], which was advised by the second author of this paper and became the seminal zero-energy calculation on the Universe's energy.

The solutions of Section 4, and Section 7, are in fact a large class of solutions, for they embrace any possible deceleration parameter value, or, any power-law scale-factor. Our solution with the rotation of the Universe, is the only unified explanation that applies to the three NASA anomalies.

As stated in the Abstract of this paper, detailed description of the subjects treated here, may be found in the two books recently published by Berman in 2012 [1] and [2]. Additional paper references are Berman in 2007 [3]; in 2011 [4,5] and in 2012 [2]) and with co-authors Costa, (Berman and Costa, in 2010 updated for this Journal in 2012 [7]) and with Gomide (Berman and Gomide, in 2010, updated in 2012 [9]; in 2011, updated in 2012 for this Journal, in the present paper [8,9]).

9. Acknowledgements

One of the authors (MSB) thanks Marcelo Fermann Guimarães, Nelson Suga, Mauro Tonasse, Antonio F. da F. Teixeira, and for the important incentive offered by Miss Solange Lima Kaczyk, now, a brand new advocate, continued during the last five years of his research in Cosmology.

REFERENCES

[1] M. S. Berman, "General Relativity and the Pioneers Anomaly," Nova Science Publishers, New York, 2012.

[2] M. S. Berman, "Realization of Einstein's Machian Program," Nova Science Publishers, New York, 2012.

[3] M. S. Berman, "The Pioneer Anomaly and a Machian Universe," *Astrophysics and Space Science*, Vol. 312, 2007, p. 275. http://arxiv.org/abs/physics/0606117.

[4] M. S. Berman, "The Two Pioneers Anomalies and Universal Rotation," *Astrophysics and Space Science*, Vol. 336, No. 2, 2011, pp. 337-339. doi:10.1007/s10509-011-0825-4-

[5] M. S. Berman, "General Relativity with Variable Speed of Light and Pioneers Anomaly," *Astrophysics and Space Science*, Vol. 336, No. 2, 2011, pp. 327-329.

[6] M. S. Berman, "Realization of Einsteins Machian Program: The Pioneers and Fly-By Anomalies," *Astrophysics and Space Science*, Vol. 337, No. 1, 2012, pp. 477-481.

[7] M. S Berman and N. C. A. da Costa, "On the Stability of our Universe," 2010. http://arxiv.org/abs/1012.4160.

[8] M. S. Berman and F. M. Gomide, "Relativistic Cosmology and the Pioneers Anomaly," *Journal of Modern Physics*, *Special Issue on Gravitation*, *Astrophysics and Cosmology*, 2011.

[9] M. S. Berman and F. M. Gomide, "General Relativistic Treatment of the Pioneers Anomaly," *Journal of Modern Physics*, *Special Issue on Gravitation*, *Astrophysics and Cosmology*, 2012.

[10] M. S. Berman and F. de M. Gomide, "On the Rotation of the Zero-Energy Expanding Universe," In: J. R. O'Connell and A. L. Hale, Eds., *The Big-Bang-Theory*, *Assumptions and Problems*, Nova Science Publishers, New York, 2012.

[11] J. D. Anderson, *et al*., PRL100, 2008, 091102.

[12] C. Lämmerzahl, *et al*., In: Dittus, Lammerzahl and Turyshev, Eds., *Lasers*, *Clocks*, *and Dragg-Free Control*, Springer, Heidelberg, 2008.

[13] T. L. Wilson and H.-J. Blome, Adv. Space Research, Vol. 44, 2009, pp. 1345-1353 Los Alamos Archives, arXiv: 0908.4067.

[14] J. D. Anderson, *et al*., "Study of the Anomalous Acceleration of Pioneer 10 and 11," *Physical Review D*, Vol. 65, 2002, Article ID: 082004.

[15] Godlowski, W.et al. (2004) - Los Alamos Archives, arxiv:astro-ph/0404329 .

[16] C. W. Misner, K. S. Thorne and J. A. Wheeler, "Gravitation," Freeman, San Franscisco, 1973.

[17] M. S. Berman, "A General Relativistic Rotating Evolutionary Universe," *Astrophysics and Space Science*, Vol. 314, No. 4, 2008, pp. 319-321. doi:10.1007/s10509-008-9772-0

[18] M. S. Berman, "A General Relativistic Rotating Evolutionary Universe, Part II," *Astrophysics and Space Science*, Vol. 315, No. 1-4, 2008, pp. 367-369. doi:10.1007/s10509-008-9830-7

[19] M. S. Berman, "Introduction to General Relativistic and Scalar-Tensor Cosmologies," Nova Science Publisher, New York, 2007.

[20] A. K. Raychaudhuri, "Theoretical Cosmology," Clarendon Press, Oxford, 1975.

[21] A. K. Raychaudhuri and S. Banerji and A. Banerjee, "General Relativity, Astrophysics and Cosmology," Springer, New York, 1992. doi:10.1007/978-1-4612-2754-0

[22] R. P. Kerr, *Physical Review Letters*, Vol. 11, 1963, p. 237.

[23] R. H. Boyer and R. W. Lindquist, "Maximal Analytic Extension of the Kerr Metric," *Journal of Mathematical Physics*, Vol. 8, No. 2, 1967, p. 265. doi:10.1063/1.1705193

[24] M. S. Berman, "A Primer in Black Holes, Mach's Principle and Gravitational Energy," Nova Science Publishers, New York, 2012.

[25] M. S. Berman, "Special Law of Variation for Hubbles Parameter," *IL Nuovo Cimento B*, Vol. 74, No. 2, 1983, pp. 182-186. doi:10.1007/BF02721676

[26] M. S. Berman and F. M. Gomide, "Cosmological Models with Constant Deceleration Parameter," GRG, Vol. 20, 1988, pp. 191-198.

[27] S. G. Turyshev and V. T. Toth, "The Pioneer Anomaly," 2010. Los Alamos Archives, arXiv: 1001.3686.

[28] B. Rievers and C. Lammerzahl, 2011. Los Alamos Archives, arXiv: 1104.3985.

[29] F. Francisco, *et al*., 2011. Los Alamos Archives.

[30] H. J. M. Cuesta, 2011. Los Alamos Archives, arXiv: 1105.2832 .

[31] M. S. Berman, M. S. Thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos, 1981. http://www.sophia.bibl.ita.br/biblioteca/index.html

[32] R. J. Adler, M. Bazin and M. Schiffer, "Introduction to General Relativity," 2nd Edition, McGraw-Hill, New York, 1975.

[33] L. M. Chechin, "The Cosmic Vacuum and the Rotation of Galaxies," *Astronomy Reports*, Vol. 54, No. 8, 2010, pp. 719-723. doi:10.1134/S1063772910080044

[34] L. M. Chechin, "Cosmic Structure Formation after the Big Bang," In: J. R. O'Connell and A. L. Hale, Eds., *The Big Bang*: *Theory*, *Assumptions and Problems*, Nova Science Publishers, New York, 2012, pp.103-172 .

[35] W. Godlowski, "Global and Local Effects of Rotation: Observational Aspects," *International Journal of Modern Physics D*, Vol. 20, 2011, p. 1643.

[36] F. M. Gomide and M. Uehara, *Astronomy and Astrophysics*, Vol. 95, 1981, p. 362.

[37] W.-T. Ni, Progress Theor. Phys. Suppl. Vol. 172, 2008, pp. 49-60.

[38] W.-T. Ni, "Cosmic Polarization Rotation, Cosmological Models, and the Detectability of Primordial Gravitational Waves," *International Journal of Modern Physics A*, Vol. 24, No. 18, 2009, pp. 3493-3500. doi:10.1142/S0217751X09047107

[39] de V. Sabbata and M. Gasperini, *Lettere al Nuovo Cimento*, Vol. 25, 1979, p. 489.

[40] L. L. So and T. Vargas, 2006.

[41] S. C. Su and M. C. Chu, "Is the Universe Rotating?" *The Astrophysical Journal*, Vol. 703, No. 1 2009, p. 354 doi:10.1088/0004-637X/703/1/354

[42] S. Weinberg, "Cosmology," OUP, Oxford, 2008.

Journal of Modern Physics, **2012, 3, 1185-1189**
doi:10.4236/jmp.2012.329153 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Do We Need Dark Energy to Explain the Cosmological Acceleration?

Felix M. Lev
Artwork Conversion Software Inc., Manhattan Beach, USA
Email: felixlev314@gmail.com

Received June 28, 2012; revised July 30, 2012; accepted August 7, 2012

ABSTRACT

The phenomenon of the cosmological acceleration discovered in 1998 is usually explained as a manifestation of a hypothetical field called dark energy which is believed to contain more than 70% of the energy of the Universe. This explanation is based on the assumption that empty space-time background should be flat and hence a nonzero curvature of the background is a manifestation of a hidden matter. We argue that quantum theory should proceed not from space-time background but from a symmetry algebra. Then the cosmological acceleration can be easily and naturally explained from first principles of quantum theory without involving empty space-time background, dark energy and other artificial notions. We do not assume that the reader is an expert in the given field and the content of the paper can be understood by a wide audience of physicists.

Keywords: Quantum Theory; De Sitter Invariance; Cosmological Constant

1. Introduction

The discovery of the cosmological acceleration (see e.g. Refs. [1,2]) has ignited a vast discussion on how this phenomenon should be interpreted. The results of the observations are usually represented in terms of the parameter which is called the cosmological constant (CC) and denoted by Λ. The meaning of this quantity will be discussed below. According to Refs. [3,4], the observational data on the value of Λ define it with the accuracy better than 5%. Therefore the possibilities that $\Lambda = 0$ or $\Lambda < 0$ are practically excluded.

The fact that $\Lambda > 0$ is usually explained as a manifestation of a hypothetical field called dark energy. The explanation has its roots in the well known debate between Einstein and de Sitter and one of the problems in the debate was whether the curved empty space-time background has a physical meaning or not. This problem is discussed in a vast literature and it encounters serious difficulties known as the CC problem or dark energy problem. The arguments leading to dark energy are discussed in detail in Section 2. On the other hand, in Section 3 we argue that quantum theory should start not from the choice of the space-time background but from the choice of a symmetry algebra. Then the cosmological acceleration can be easily and naturally explained from first principles of quantum theory without involving empty space-time background, dark energy and other artificial notions.

We do not assume that the reader is familiar with the Einstein equations and de Sitter symmetry. We tried to make the presentation of the material as simple as possible and we believe that the content of the paper can be understood by a wide audience of physicists.

2. Arguments Leading to Dark Energy

The majority of works dealing with the CC problem proceed from the assumption that the gravitational constant G is the fundamental physical quantity, the goal of the theory is to express Λ in terms of G and to explain why Λ is so small. For this reason we first discuss whether indeed G can be treated as a fundamental constant and whether the theory should explain the value of Λ .

The quantity G defines the gravitational force in the Newton law of gravity. Numerous experimental data show that this law works with a very high accuracy. However, this only means that G is a good *phenomenological* parameter. At the level of the Newton law one cannot prove that G is the exact constant which does not change with time, does not depend on masses, distances etc.

General Relativity (GR) is a classical (*i.e.* non-quantum) theory based on the minimum action principle. We will not assume that the reader is familiar with the Einstein equations. The only features of these equations which are important for our discussion are the following. The left-hand-side of these equations contain quantities describing the properties of space-time—the Ricci tensor $R_{\mu\nu}$, the metric tensor $g_{\mu\nu}$ and the tensor of the scalar curvature R_c $(\mu,\nu=0,1,2,3)$, while the right-hand-side contains stress energy tensor of matter $T_{\mu\nu}$. The Einstein equations derived from the minimum action principle read

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R_c=\left(8\pi G/c^4\right)T_{\mu\nu} \tag{1}$$

Therefore *G* is the coefficient of proportionality between the left-hand-side and rihgt-hand-side. General Relativity cannot calculate *G* or give a *theoretical* explanation why this value should be as it is. A problem arises whether the quantity *G* should be treated as a fundamental or phenomenological constant. For example, the quantity *c* is usually treated as fundamental and then the problem of calculating *c* does not arise. One can say that the value of *c* is as it is simply because we wish to measure time in seconds and distances in meters. One might think that the quantity *G* can be treated analogously and its value is as it is simply because we wish to measure masses in kilograms and distances in meters (in the spirit of Planck units). However, treating *G* as a fundamental constant can be justified only if there are strong reasons to believe that the Lagrangian of GR is the only possible Lagrangian. Let us consider whether this is the case.

The Lagrangian of GR should be invariant under general coordinate transformations and the simplest way to satisfy this requirement is a choice when it is proportional to R_c. In this case the Newton gravitational law is recovered in the nonrelativistic approximation and the theory is successful in explaining several well-known phenomena. However, the argument that this choice is simple and agrees with the data, cannot be treated as a fundamental requirement. Another reason for choosing the linear case is that here equations of motions are of the second order while in quadratic, cubic cases etc. they will be of higher orders. However, this reason also cannot be treated as fundamental. It has been argued in the literature that GR is a low energy approximation of a theory where equations of motion contain higher order derivatives. In particular, a rather popular approach is when the Lagrangian contains a function $f(R_c)$ which should be defined from additional considerations. In that case the constant *G* in the Lagrangian is not the same as the standard gravitational constant. It is believed that the nature of gravity will be understood in the future quantum theory of gravity but efforts to construct this theory has not been successful yet. Hence there are no solid reasons to treat *G* as a fundamental constant.

Special Relativity works with Minkowski space, which is also called the space of events. It is very important to note that Minkowski space has a physical meaning only as *a space of events for real bodies*. In particular, the notion of empty space has no physical meaning since it contradicts the physical principle that a definition of a physical quantity is a description of how this quantity should be measured. In particular, one can discuss how coordinates of *real bodies* can be measured but there is no way to measure coordinates of the empty space which exists only in our imagination.

Physicists consider others spaces of events, for example de Sitter (dS) space. It is a set of points characterized by five coordinates (t,x,y,z,u) which satisfy the restriction $x^2+y^2+z^2+u^2-t^2=R^2$ where *R* is some parameter, we work in units where $c=1$ and hence time t has the same dimension as the spacial coordinates (x,y,z,u). The dS space is invariant under the action of the dS group, which contains only conventional and hyperbolic rotations. Therefore the action of the dS group on dS space does not depend on *R* at all and, instead of the quantities (t,x,y,z,u) satisfying the above restriction, one can characterize points on the dS space by dimensionless quantities

$$\left(\xi_0=t/R,\xi_1=x/R,\xi_2=y/R,\xi_3=z/R,\xi_4=u/R\right)$$

satisfying the restriction $\xi_1^2+\xi_2^2+\xi_3^2+\xi_4^2-\xi_0^2=1$.

An analogy of this situation follows. Suppose that a one-dimensional man lives on a circumference in the xy plane with the center in the origin and radius *R*. The man does not know that in the two-dimensional world the circumference is described by the coordinates (x,y) satisfying $x^2+y^2=R^2$ since he has no information about x, y and *R*. However he can measure distances and describe the geometry of his one-dimensional world in terms of a dimensionless parameter $\varphi\in[0,2\pi]$.

Consider a vicinity of the North pole of dS space assuming that the pole has the coordinates $(0,0,0,0,R)$. If we consider only such points of dS space that *u* is close to *R* and all the values of (t,x,y,z) are much less than *R* then in this vicinity, geometry is very close to that of Minkowski space. The dimension of the quantities (t,x,y,z) in this vicinity depends on the dimension in which *R* is measured. The curvature of dS space in terms of (t,x,y,z) is $\Lambda=3/R^2$. Then the experimental results [1-3] say that *R* is of order 10^{26} m. This discussion shows that in dS theory Λ is not present at all; it appears only when one wishes to parametrize dS space by dimensionful coordinates. Hence the question of why Λ is as it is, is not fundamental since the answer is: because we want to measure distances in meters. In particular, there is no guaranty that Λ will not change with time.

When the Lagrangian is linear in R_c, the most general Einstein equations are not (1) but

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R_c+\Lambda g_{\mu\nu}=\left(8\pi G/c^4\right)T_{\mu\nu} \tag{2}$$

As follows from this expression, in GR the curvature and the metric depend on the presence of matter. In the formal limit, when matter disappears, solutions of Equation (2) are Minkowski space when $\Lambda = 0$, dS space when $\Lambda > 0$ and anti-de Sitter (AdS) space when $\Lambda < 0$. In this connection the following extremely important question arises. As discussed above, these spaces have a physical meaning only as *spaces of events for real bodies*. At the same time, in GR those spaces arise as solutions of the Einstein equations when matter is absent. In other words, those spaces arise only as empty spaces. Of course, in mathematics one can consider different spaces without thinking about the physical meaning of empty space. But in physics the notion of empty space has no meaning. We believe that these remarks show that the formal limit of GR when matter disappears is unphysical.

In textbooks on gravity written before 1998 (when the cosmological acceleration was discovered) it is often claimed that Λ is not needed since its presence contradicts the philosophy of GR: matter creates curvature of space-time, so in the absence of matter space-time should be flat (*i.e.* Minkowski) while empty dS space is not flat. As noted above, such a philosophy has no physical meaning since the notion of empty space is unphysical. That's why the discovery of the fact that $\Lambda \neq 0$ has ignited many discussions. The most popular approach is as follows. One can move the term with Λ in Equation (2) from the left-hand side to the right-hand one:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R_c = \left(8\pi G/c^4\right) T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{3}$$

Then the term with Λ is treated as the stress-energy tensor of a hidden matter which is called dark energy: $\left(8\pi G/c^4\right) T_{\mu\nu}^{DE} = -\Lambda g_{\mu\nu}$. With the observed value of Λ this dark energy contains approximately 75% of the energy of the Universe. In this approach *G* is treated as a fundamental constant and one might try to express Λ in terms of *G*. The existing quantum theory of gravity cannot perform this calculation unambiguously since the theory contains strong divergences. With a reasonable cutoff parameter, the result for Λ is such that in units where $\hbar = c = 1$, $G\Lambda$ is of order unity. This result is expected from dimensionful considerations since in these units, the dimension of *G* is $length^2$ while the dimension of Λ is $1/length^2$. However, this value of Λ is greater than the observed one by 122 orders of magnitude. This problem is called the CC problem or dark energy problem.

Several authors criticized this approach from the following considerations. GR without the contribution of Λ has been confirmed with a good accuracy in experiments in the Solar System. If Λ is as small as it has been observed then it can have a significant effect only at cosmological distances while for experiments in the Solar System the role of such a small value is negligible. The authors of Ref. [5] titled "Why All These Prejudices Against a Constant?", note that even in a special case $f\left(R_c\right) = R_c$, the most general form of the Einstein equations is as in Equation (2) and so it is not clear why we should think that only a special case (1) is allowed. If we accept the theory containing a phenomenological constant *G* which is taken from the outside then why can't we accept a theory containing two independent phenomenological constants?

It is also well known since the 1930s that on quantum level space-time coordinates are not measurable (see e.g. Ref. [6]). Hence on quantum level space-time cannot be described by differential geometry. There exist many papers the authors of which propose their solutions of the CC problem. In the next section we give simple arguments showing that the CC problem does not exist and the cosmological acceleration can be easily and naturally explained from first principles of quantum theory.

3. Quantum Approach to Cosmological Acceleration

The usual approach to dS symmetry on quantum level is as follows. Since classical dS space is invariant under the action of the dS group, in dS quantum theory operators of dS angular momenta

$$M^{ab}\left(a,b = 0,1,2,3,4; M^{ab} = -M^{ba}\right)$$

should satisfy the commutation relations of the dS algebra

$$\begin{aligned}&\left[M^{ab}, M^{cd}\right] \\ &= -i\left(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}\right)\end{aligned} \tag{4}$$

where η^{ab} is the diagonal metric tensor such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = -\eta^{44} = 1$. This approach is in the spirit of the well-known Klein's Erlangen program in mathematics.

However, as we argue in Refs. [7,8], quantum theory should not be based on classical space-time background and the approach should be the opposite. Each system is described by a set of independent operators. By definition, the rules how these operators commute with each other define the symmetry algebra. In particular, by definition, dS symmetry on quantum level means that the operators commute according to Equation (4). In semiclassical approximation, quantum theory can recover results obtained by classical one with dS space (see below). In that case dS space is meaningful only as a space of events for real particles but not as an empty space-time background.

The anti-de Sitter (AdS) symmetry on quantum level can be defined analogously but the value of η^{44} in Equation (4) is 1 instead of –1. Poincare symmetry is a special case of dS or AdS symmetry obtained as follows. If R is a parameter with the dimension *length* and the energy-momentum operator P^{μ} is *defiend* as $P^{\mu} = M^{4\mu}/R$ then in the formal limit $R \to \infty$ one gets commutation relations of the Poincare algebra from Equation (4). It is clear that on quantum level dS and AdS theories can be constructed without parameters having the dimension of length. Such parameters may be used if one wishes to interpret the results in classical approximation or in Poincare limit but they are not fundamental. In particular, neither Λ nor G can be fundamental in agreement with the discussion in the preceding section.

The next step in our construction is the definition of elementary particle. Although theory of elementary particles exists for a rather long period of time, there is no commonly accepted definition of elementary particle in this theory. In Refs. [7,8] we argue that, in the spirit of Wigner's approach to Poincare symmetry, a general definition, not depending on the choice of the classical background and on whether we consider a local or nonlocal theory, is that a particle is elementary if the set of its wave functions is the space of an irreducible representation (IR) of the symmetry algebra in the given theory. The meaning of IR is that the linear space of all possible wave functions cannot be decomposed into a sum of spaces where the operators act independently. Hence the term "irreducible" can be treated as a mathematical synonym of "elementary". There exists a wide literature describing how IRs of the dS, AdS and Poincare algebra can be constructed. Such a construction can be used not only for describing elementary particles but even for describing the motion of a macroscopic body as a whole. For example, when we consider a system of two macroscopic bodies such that the distance between them is much greater than their sizes, it suffices to describe each body as a whole by using the IR with the corresponding mass.

The following important remarks are in order. In Quantum Field Theory, Lagrangians and Minkowski space play only an auxiliary role for constructing the operators $\left(P^{\mu}, M^{\mu\upsilon}\right)$ for systems of interacting fields. Hence if we consider only systems of free particles, neither Lagrangian nor Minkowski space is needed. Analogous remarks are valid in dS theory. In particular, for describing systems of free particles, neither Lagrangian nor dS space is needed.

The above notions are sufficient for describing systems of free particles in Poincare, dS and AdS quantum theories. In particular, in semiclassical approximation one can calculate the relative acceleration of two free particles in such theories. One might think that since the particles are free, their relative acceleration will be zero. This is true in Poincare invariant theory but in the dS and AdS cases the relative acceleration is not zero. The calculation of the relative acceleration involves the following steps.

At the starting point we have no space-time and no dimensionful parameters. The only information we have is how wave functions describing particles under consideration are constructed and how the operators M^{ab} act on such wave functions. This is the maximum possible information in quantum theory.

The next step is that we introduce a parameter R with the dimension *length* and instead of the dS operators $M^{4\mu}$ work with the energy operator $E = M^{40}/R$ and the momentum operator P such that $P^{k} = M^{4k}/R$ $(k = 1,2,3)$. Then we *define* classical time t as a parameter describing the evolution according to the Schroedinger equation and *define* the position operator $\mathbf{r}_j$ of particle j $(j = 1,2)$ such that it acts on wave functions $\psi(\mathbf{p}_j)$ of particle j in momentum representation as $i\hbar\partial/\partial\mathbf{p}_j$ (as in standard quantum mechanics).

A standard quantum-mechanical calculation, which is described in detail in Refs. [7,9-11] (where we discussed different properties of dS quantum theory), shows that in the dS case the classical relative acceleration $\mathbf{a}$ of two *free* particles is $\mathbf{a} = \Lambda c^2\mathbf{r}/3$ where $\mathbf{r}$ is the classical vector of the relative distance between the particles and $\Lambda = 3/R^2$. From the formal point of view, the result is the same as in GR on dS space. However, our result has been obtained by using only standard quantum-mechanical notions while dS space, its metric, connection etc. have not been involved at all. The derivation clearly demonstrates that R is not a fundamental quantity but simply a parameter defining the scale of classical space-time coordinates $(\mathbf{r}, t)$ (in agreement with the remarks in the preceding section).

4. Conclusion

In Section 2 we argue that neither G nor Λ can be fundamental and the notion of empty space-time background is not physical. Hence the discussion on whether the empty space-time background can be curved or not does not have a physical meaning. In Section 3 we argue that quantum theory should start not from the choice of the empty space-time background but from the choice of a symmetry algebra. In view of this approach, space-time coordinates have a physical meaning only on classical level when they are applied for describing real bodies but not for describing the empty space-time background. In this approach the data of Refs. [1-3] that $\Lambda > 0$, should be interpreted not such that the space-time background is dS space but that the dS algebra is more pertinent than the Poincare or AdS ones. As shown in our Ref. [8] and references therein, this opens a radically new approach to gravity where the quantity G is not taken from the outside but (in principle) can be calculated. The above discussion shows that the phenomenon of cosmological acceleration can be easily and naturally explained from first principles of quantum theory without involving space-time background, dark energy and other artificial notions.

5. Acknowledgements

The author is grateful to Volodya Netchitailo for reading the manuscript and important remarks and to the anonymous referee for important suggestions on improving the manuscript.

REFERENCES

[1] S. Perlmutter, G. Aldering, G. Goldhaber, *et al.*, "Measurement of Omega and Lambda from H42 High-Redshift Supernovae," *Astrophysical Journal*, Vol. 517, No. 2, 1999, pp. 565-586. doi:10.1086/307221

[2] A. Melchiorri, P. A. R. Ade, P. de Bernardis, *et al.*, "A Measurement of Omega from the North American Rest Flight of Boomerang," *Astrophysical Journal*, Vol. 536, No. 2, 2000, pp. L63-L66. doi:10.1086/312744

[3] D. N. Spergel, R. Bean, O. Dore, *et al.*, "Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology," *Astrophysical Journal*, Vol. 170, No. 2, 2007, pp. 377-408. doi:10.1086/513700

[4] K. Nakamura and Particle Data Group, "Review of Particle Physics," *Journal of Physics*, Vol. G37, No. 7A, 2010, Article ID: 075021.

[5] E. Bianchi and C. Rovelli, "Why All These Prejudices against a Constant?" 2010, arXiv:1002.3966v3

[6] T. D. Newton and E. P. Wigner, "Localized States for Elementary Systems," *Review of Modern Physics*, Vol. 21, No. 3, 1949, pp. 400-405. doi:10.1103/RevModPhys.21.400

[7] F. Lev, "Positive Cosmological Constant and Quantum Theory," *Symmetry*, *Special Issue*: *Quantum Symmetry*, Vol. 2, No. 4, 2010, pp. 1945-1980.

[8] F. Lev, "De Sitter Symmetry and Quantum Theory," *Physical Review*, Vol. D85, No. 3, 2012, Article ID: 065003.

[9] F. Lev, "Finiteness of Physics and Its Possible Consequences," *Journal of Mathematical Physics*, Vol. 34, No. 2, 1993, pp. 490-527. doi:10.1063/1.530257

[10] F. Lev, "The Problem of Interactions in de Sitter Invariant Theories," *Journal of Physics*, Vol. A32, No. 7, 1999, pp. 1225-1240.

[11] F. Lev, "Could Only Fermions Be Elementary?" *Journal of Physics*, Vol. A37, No. 9, 2004, pp. 3287-3304.

Journal of Modern Physics, 2012, 3, 1190-1198
doi:10.4236/jmp.2012.329154 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

A Guess Model of Black Holes and the Evolution of Universe

Zhenhua Mei[1], Shuyu Mei[2]
[1]College of Chemistry and Molecular Engineering, Qingdao University of Science and Technology, Qingdao, China
[2]Medical College, Qingdao University, Qingdao, China
Email: mzh62@qust.edu.cn

Received June 10, 2012; revised July 16, 2012; accepted July 23, 2012

ABSTRACT

Based on the gravitational theory, fundamental data, and comprehensible suppositions, an evolution model of the universe was proposed. The universe exists in explosion and constringency mobile equilibrium state. The critical sizes of celestial bodies were calculated in their evolution process.

Keywords: Black Hole; Evolution; Universe; Gravitation; Entropy

1. Introduction

As the theoretical ratiocination (Einstein's theory of relativity) and the cumulating of the more and more obtained data or phenomena from astronomic observation of the celestial bodies, people began to have a general impression of the universe, such as the clangorous words expressed: big bang, expanding universe, and black hole *et al*. (Ginsburg 1985). From a point of view that the microcosm decides the macrocosm, more information of the universe can be predicted from present knowledge of the elementary particles.

In this paper, some parameters of evolution of the universe were calculated. The main thoughts were the Newton's universal gravitation and the structures of matter.

2. Preparation of Research

2.1. Fundamental Data and Suppositions

The gravitational constant, G, is 6.670×10^{-11} N m^2·kg^{-2}. The radius of hydrogen atom, a_0, is 5.29×10^{-11} m. The mass density of atomic nucleus, ρ_n, was determined (Erdei 1976) to be 2×10^{17} kg·m^{-3}. The mass of neutron, m_n, is 1.67495×10^{-27} kg, radius r_n, 1.2598×10^{-15} m. The combining energy of nucleus is 8 MeV. The gravitational acceleration on earth surface is 9.81 m·s^{-2}. The sun has the mass of 1.989×10^{30} kg, density of 1.409×10^3 kg·m^{-3}, radius of 6.960×10^8 m.

Suppositions: the combining energy of nucleus (strong interaction in nucleus) came from gravitation, and the nucleus consisted of gravitons; the combinative form between nucleus was its overlaps one another.

2.2. Formula Derivation A—Gravitational Acceleration of a Particle from Solid Sphere

According to Newton's law, the gravitational force between two distant particles has the form:

$F = GmM/r^2$, and $F = ma$. Hence, the gravitational acceleration, $a = GM/r^2$.

However, to a neighboring large body (has a mass of M, radius of R), the radius of the body cannot be neglected; its accurate result can be obtained by processing a mathematical integral.

As **Figure 1** expressed, a solid sphere has a radius of R, a homogeneous density of ρ, the particle has a mass, m, and has a distance, nR, from the center, the gravitational acceleration, a, of the particle was derived as follows:

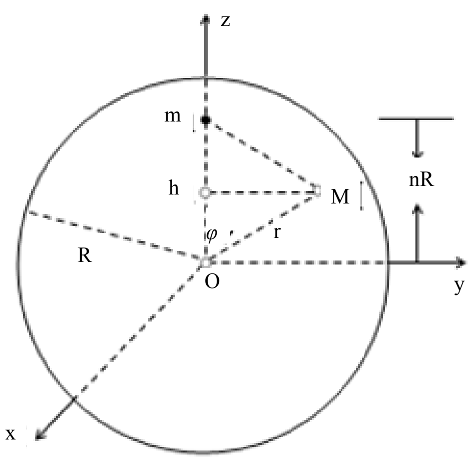


Figure 1. Sketch map of a particle (*m*), which with a distance, *nR*, from the center of the solid sphere.

$$
\begin{aligned}
a &= \int_{\Omega} \mathrm{d}a = \int_{\Omega} \frac{G}{r^2}\mathrm{d}M = \iiint_{\Omega} G\rho' \frac{\overline{mh}}{\overline{mM}^3}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z = G\rho' \iiint_{\Omega} \frac{nR - z}{\left(x^2 + y^2 + \left(nR - z\right)^2\right)^{3/2}}\mathrm{d}x\mathrm{d}y\mathrm{d}z \\
&= 4Gk\rho\int_0^R \mathrm{d}x\int_0^R \mathrm{d}y\int_{(n+1)R}^{(n-1)R} \mathrm{d}t\mathrm{d}\frac{-t}{\left(x^2 + y^2 + t^2\right)^{3/2}} \xrightarrow{t \;=\; nR - z} \\
&= 4Gk\rho\int_0^R \mathrm{d}x\int_0^R \mathrm{d}y\left(x^2 + y^2 + t^2\right)^{-1/2}\Big|_{(n+1)R}^{(n-1)R} \\
&= 4Gk\rho\int_0^R \mathrm{d}x\int_0^R \mathrm{d}y\left(\left(x^2 + y^2 + \left(n-1\right)^2 R^2\right)^{-1/2} - \left(x^2 + y^2 + \left(n+1\right)^2 R^2\right)^{-1/2}\right) \\
&= 4Gk\rho\int_0^R \mathrm{d}x\left(\ln\left(y + \sqrt{x^2 + y^2 + \left(n-1\right)^2 R^2}\right)\ln\left(y + \sqrt{x^2 + y^2 + \left(n+1\right)^2 R^2}\right)\right) \\
a &= 4Gk\rho\int_0^R \mathrm{d}x(\ln\left(R + \sqrt{x^2 + R^2 + \left(n-1\right)^2 R^2}\right) - \ln\left(R + \sqrt{x^2 + R^2 + \left(n+1\right)^2 R^2}\right) \\
&\quad + \left(\ln\left(\sqrt{x^2 + \left(n+1\right)^2 R^2}\right) - \ln\left(\sqrt{x^2 + \left(n-1\right)^2 R^2}\right)\right)
\end{aligned} \tag{1}
$$

where, "M" is the arbitrary point in the solid sphere, "h" the point corresponding to "M" horizontally. "r" is the distance of the point "M" from original point "o", ρ' is ideally homogeneous mass density of a solid sphere, ρ the average one. k stands for the calibration factor of average mass density.

For a general integral

$$
\begin{aligned}
&\int_0^R \mathrm{d}x\ln\left(\sqrt{x^2 + C}\right), \left(C = \left(n\pm1\right)^2 R^2\right) = x\ln\left(\sqrt{x^2 + C}\right)\Big|_0^R \int_0^R \mathrm{d}x\frac{x^2}{x^2 + C} \\
&= R\ln\left(R\sqrt{1 + C}\right) = R + \sqrt{C}\ \mathrm{arc\ tg}\frac{R}{\sqrt{C}}
\end{aligned} \tag{2}
$$

For integral

$$
\begin{aligned}
&\int_0^R \mathrm{d}x\ln\left(R + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}\right) = x\ln\left(R + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}\right)\Big|_0^R \\
&-\int_0^R \mathrm{d}x\frac{x^2}{R + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}}\frac{1}{\sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}} = x\ln\left(R + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}\right)\Big|_0^R \\
&-\int_0^R \mathrm{d}x\left(1 + \frac{\left(n\pm1\right)^2 R}{R + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}} - \frac{\left(1 + \left(n\pm1\right)^2\right)R}{\sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}}\right) \\
&= R\ln\left(R + \sqrt{R^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}\right) - R + \left(1 + \left(n\pm1\right)^2\right)R\ln\left(x + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}\right)\Big|_0^R \\
&-\int_0^R \mathrm{d}x\frac{\left(n\pm1\right)^2 R}{R + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}}\int_0^R \mathrm{d}x\ln\left(R + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}\right) \\
&= R\ln\left(R\left(1 + \sqrt{\left(2 + \left(n\pm1\right)^2\right)}\right)\right) - R + \left(1 + \left(n\pm1\right)^2\right)R\ln\frac{1 + \sqrt{2 + \left(n\pm1\right)^2}}{\sqrt{1 + \left(n\pm1\right)^2}} \\
&-\int_0^R \mathrm{d}x\frac{\left(n\pm1\right)^2 R}{R + \sqrt{x^2 + \left(1 + \left(n\pm1\right)^2\right)R^2}}.
\end{aligned} \tag{3}
$$

For general integral

$$\int_0^R \mathrm{d}x \frac{(n\pm1)^2 R}{A+\sqrt{x^2+B}}, \left(A=R, B=\left(1+(n\pm1)^2\right)R^2\right)$$

Let $f = \left(A+\sqrt{x^2+B}\right)^{-1}$

$$f(0)=\frac{1/R}{1+\sqrt{1+(n\pm1)^2}}.$$

$$f'=-\left(A+\sqrt{x^2+B}\right)^{-2} x\left(x^2+B\right)^{-1/2}.$$

$$f''=2x^2\left(x^2+B\right)^{-1}\left(A+\sqrt{x^2+B}\right)^{-3}-\left(A+\sqrt{x^2+B}\right)^{-2}\left(\left(x^2+B\right)^{-1/2}-x^2\left(x^2+B\right)^{-3/2}\right).$$

$$f''(0)=\frac{1}{R^3}\frac{1}{\sqrt{1+(n\pm1)^2}\left(1+\sqrt{1+(n\pm1)^2}\right)^2}.$$

$$\begin{aligned}f'''=&-6x^3\left(x^2+B\right)^{-3/2}\left(A+\sqrt{x^2+B}\right)^{-4}+2\left(A+\sqrt{x^2+B}\right)^{-3}\left(2x\left(x^2+B\right)^{-1}-2x^3\left(x^2+B\right)^{-2}\right)\\&-\left(A+\sqrt{x^2+B}\right)^{-2}-\left(3x\left(x^2+B\right)^{-3/2}+3x^3\left(x^2+B\right)^{-5/2}\right)\\&+2\left(\left(x^2+B\right)^{-1/2}-x^2\left(x^2+B\right)^{-3/2}\right)\left(A+\sqrt{x^2+B}\right)^{-3}x\left(x^2+B\right)^{-1/2}.\end{aligned}$$

$$\begin{aligned}f''''=&\,24x^4\left(x^2+B\right)^{-4/2}\left(A+\sqrt{x^2+B}\right)^{-5}\\&-6\left(A+\sqrt{x^2+B}\right)^{-4}\left(3x^2\left(x^2+B\right)^{-3/2}-3x^4\left(x^2+B\right)^{-5/2}\right)+2\left(A+\sqrt{x^2+B}\right)^{-3}\\&\left(2\left(x^2+B\right)^{-1}-4x^2\left(x^2+B\right)^{-2}-6x^2\left(x^2+B\right)^{-2}+8x^4\left(x^2+B\right)^{-3}\right)\\&-\left(A+\sqrt{x^2+B}\right)^{-2}\left(-3\left(x^2+B\right)^{-3/2}+9x^2\left(x^2+B\right)^{-5/2}-9x^2\left(x^2+B\right)^{-5/2}-15x^4\left(x^2+B\right)^{-7/2}\right)\\&+6x^2\left(x^2+B\right)^{-1/2}\left(A+\sqrt{x^2+B}\right)^{-3}\left(x^2+B\right)^{-3/2}+3x^3\left(x^2+B\right)^{-5/2}\Big)+2\left(\left(x^2+B\right)^{-1/2}-x^2\left(x^2+B\right)^{-3/2}\right)\\&\left(-3x^2\left(x^2+B\right)^{-2/2}\left(A+\sqrt{x^2+B}\right)^{-4}+\left(A+\sqrt{x^2+B}\right)^{-3}\left(\left(x^2+B\right)^{-1/2}-x^2\left(x^2+B\right)^{-3/2}\right)\right)\\&+2\left(A+\sqrt{x^2+B}\right)^{-3}x\left(x^2+B\right)^{-1/2}\times\left(-x(x^2+B)^{-3/2}-2x(x^2+B)^{-3/2}+3x\left(x^2+B\right)^{-5/2}\right).\end{aligned}$$

$$f''''(0)=\frac{1}{R^5}\left(\frac{6}{\left(1+(n\pm1)^2\right)\left(1+\sqrt{1+(n\pm1)^2}\right)^3}+\frac{3}{\left(1+(n\pm1)^2\right)^{3/2}\left(1+\sqrt{1+(n\pm1)^2}\right)^2}\right).$$

$$\int_0^R \mathrm{d}x \frac{(n\pm1)^2 R}{A+\sqrt{x^2+B}} = R'(n\pm1)^2 \int_0^R \mathrm{d}x\left(f(0)+\frac{f''(0)}{2!}x^2+\frac{f''''(0)}{4!}x^4+\cdots\right)$$

$$= R'(n\pm1)^2\left(\frac{1/R}{1+\sqrt{1+(n\pm1)^2}}R+\frac{1}{3\times2!}\frac{1}{R^3}\frac{1}{\sqrt{1+(n\pm1)^2}\left(1+\sqrt{1+(n\pm1)^2}\right)^2}R^3\right.$$

$$\left.+\frac{1}{5\times4!}\frac{1}{R^5}\left(\frac{6}{\left(1+(n\pm1)^2\right)\left(1+\sqrt{1+(n\pm1)^2}\right)^3}+\frac{3}{\left(1+(n\pm1)^2\right)^{3/2}\left(1+\sqrt{1+(n\pm1)^2}\right)^2}\right)R^5+\cdots\right)$$

$$= R'(n\pm1)^2\left(\frac{1}{1+\sqrt{1+(n\pm1)^2}}+\frac{1}{6}\frac{1}{\sqrt{1+(n\pm1)^2}\left(1+\sqrt{1+(n\pm1)^2}\right)^2}\right.$$

$$\left.+\frac{1}{40}\left(\frac{2}{\left(1+(n\pm1)^2\right)\left(1+\sqrt{1+(n\pm1)^2}\right)^3}+\frac{1}{\left(1+(n\pm1)^2\right)^{3/2}\left(1+\sqrt{1+(n\pm1)^2}\right)^2}\right)\right). \tag{4}$$

Substituted (3) with (4)

$$\int_0^R \mathrm{d}x \ln\left(R+\sqrt{x^2+\left(1+(n\pm1)^2\right)R^2}\right) = R\ln\left(R\left(1+\sqrt{\left(2+(n\pm1)^2\right)}\right)\right)$$

$$-R+\left(1+(n\pm1)^2\right)R\ln\frac{1+\sqrt{2+(n\pm1)^2}}{\sqrt{1+(n\pm1)^2}}-R'(n\pm1)^2\left(\frac{1}{1+\sqrt{1+(n\pm1)^2}}+\frac{1}{6}\frac{1}{\sqrt{1+(n\pm1)^2}\left(1+\sqrt{1+(n\pm1)^2}\right)^2}\right. \tag{5}$$

$$\left.+\frac{1}{40}\left(\frac{2}{\left(1+(n\pm1)^2\right)\left(1+\sqrt{1+(n\pm1)^2}\right)^3}+\frac{1}{\left(1+(n\pm1)^2\right)^{3/2}\left(1+\sqrt{1+(n\pm1)^2}\right)^2}\right)\right).$$

As the results of (2) and (5) expressed, thus, (1) becomes

$$a = 4Gk\rho R\left(\ln\left(R\left(1+\sqrt{2+(n-1)^2}\right)\right)-1\left(1+(n-1)^2\right)\ln\frac{1+\sqrt{2+(n-1)^2}}{\sqrt{1+(n-1)^2}}\right.$$

$$-(n-1)^2\left(\frac{1}{1+\sqrt{1+(n-1)^2}}+\frac{1}{6}\frac{1}{\sqrt{1+(n-1)^2}\left(1+\sqrt{1+(n-1)^2}\right)^2}\right.$$

$$\left.+\frac{1}{20}\frac{1}{\left(1+(n-1)^2\right)\left(1+\sqrt{1+(n-1)^2}\right)^3}+\frac{1}{40}\frac{1}{\left(1+(n-1)^2\right)^{3/2}\left(1+\sqrt{1+(n-1)^2}\right)^2}\right)$$

$$-\ln\left(R\left(1+\sqrt{2+(n-1)^2}\right)\right)+1-\left(1+(n-1)^2\right)\ln\frac{1+\sqrt{2+(n-1)^2}}{\sqrt{1+(n-1)^2}}$$

$$+\left(n-1\right)^2\left(\frac{1}{1+\sqrt{1+\left(n-1\right)^2}}+\frac{1}{6}\frac{1}{\sqrt{1+\left(n-1\right)^2}\left(1+\sqrt{1+\left(n-1\right)^2}\right)^2}\right.$$

$$\left.+\frac{1}{20}\frac{1}{\left(1+\left(n-1\right)^2\right)\left(1+\sqrt{1+\left(n-1\right)^2}\right)^3}+\frac{1}{40}\frac{1}{\left(1+\left(n+1\right)^2\right)^{3/2}\left(1+\sqrt{1+\left(n+1\right)^2}\right)^2}\right)$$

$$\left.+\ln\left(R\sqrt{1+\left(n+1\right)^2}\right)-1+\left(n+1\right)\text{arc tg}\frac{1}{\left(n+1\right)}-\ln\left(R\sqrt{1+\left(n-1\right)^2}\right)+1-\left|n-1\right|\text{arc tg}\frac{1}{\left|n-1\right|}+\cdots\right).$$

Let $a\left(n\right)=4Gk\rho RF\left(n\right)$, then,

$$F\left(n\right)=\left(\ln\frac{1+\sqrt{2+\left(n-1\right)^2}}{1+\sqrt{2+\left(n+1\right)^2}}+\left(1+\left(n-1\right)^2\right)\ln\frac{1+\sqrt{2+\left(n-1\right)^2}}{\sqrt{1+\left(n-1\right)^2}}-\left(1+\left(n+1\right)^2\right)\ln\frac{1+\sqrt{2+\left(n+1\right)^2}}{\sqrt{1+\left(n+1\right)^2}}\right.$$

$$-\left(n-1\right)^2\left(\frac{1}{1+\sqrt{1+\left(n-1\right)^2}}+\frac{1}{6}\frac{1}{\sqrt{1+\left(n-1\right)^2}\left(1+\sqrt{1+\left(n-1\right)^2}\right)^2}\right.$$

$$\left.+\frac{1}{20}\frac{1}{\left(1+\left(n-1\right)^2\right)\left(1+\sqrt{1+\left(n-1\right)^2}\right)^3}+\frac{1}{40}\frac{1}{\left(1+\left(n-1\right)^2\right)^{3/2}\left(1+\sqrt{1+\left(n-1\right)^2}\right)^2}\right)$$

$$+\left(n+1\right)^2\left(\frac{1}{1+\sqrt{1+\left(n+1\right)^2}}+\frac{1}{6}\frac{1}{\sqrt{1+\left(n+1\right)^2}\left(1+\sqrt{1+\left(n+1\right)^2}\right)^2}\right.$$

$$\left.+\frac{1}{20}\frac{1}{\left(1+\left(n+1\right)^2\right)\left(1+\sqrt{1+\left(n+1\right)^2}\right)^3}+\frac{1}{40}\frac{1}{\left(1+\left(n+1\right)^2\right)^{3/2}\left(1+\sqrt{1+\left(n+1\right)^2}\right)^2}\right)$$

$$+\ln\frac{\sqrt{1+\left(n+1\right)^2}}{\sqrt{1+\left(n-1\right)^2}}+\left|n+1\right|\text{arc tg}\frac{1}{\left|n+1\right|}-\left|n+1\right|\text{arc tg}\frac{1}{\left|n+1\right|}. \quad (6)$$

$F(n)$ is a function of n. The, $F-n$, diagram was showed in **Figure 2**.

For the earth, the average mass density ρ is $5.5153 \times 10^3\ \text{kg}\cdot\text{m}^{-3}$, the radius R is 6.356078×10^6 m. The gravitational acceleration on earth surface was calculated ($n = 1$) to be $12.68\ \text{m}\cdot\text{s}^{-2}$. However, the real value of gravitational acceleration on earth is $9.81\ \text{m}\cdot\text{s}^{-2}$. It is 0.7736 times than the calculated one. It is because of the earth that does not have an ideally homogeneous density used in above formula derivation process. The ratio can be used as calibration factor, k, of average density of a solid sphere ($\rho' = k\rho$. For earth, $k = 0.7736$; and it was supposed to be fit for any other celestial bodies).

2.3. Formula Derivation B—Self-Gravitational Pressure in Center of Solid Sphere

The gravitational pressure in center from solid sphere can be calculated according to Equation (6).

$\mathrm{d}F = a\mathrm{d}m = a\rho_1\mathrm{d}V = \rho_1 SRa\mathrm{d}n/3$, (where, $V = SnR/3$)

$dp = \rho_1 SRadn/3$

$$p=\int\mathrm{d}p=\frac{4}{3}\rho_1 Gk\rho_2 R^2\int_0^1 F\left(n\right)\mathrm{d}n. \quad (7)$$

The integral, $\int_0^1 F(n)\,\mathrm{d}n$, can be calculated diagrammatically. The diagram showed in **Figure 3**.

The area beneath the curve calculated by computer gave the result of 0.62129. Thus, $p = 0.62129 \times 4Gk\rho^2R^2/3$. Supposing the considered celestial body has the same value of calibration factor k of earth, then we obtained

$$p = 0.6408G\rho^2R^2. \tag{7.1}$$

2.4. Formula Derivation C—Gravitational Potential Energy from Solid Sphere

The gravitational potential energy, E_g, from a solid sphere can be calculated.

$$E_g = \int \mathrm{d}E_g = \int ma\mathrm{d}l = 4mGk\rho R^2 \int_n^\infty F(n)\,\mathrm{d}n. \tag{8}$$

According to relation (6), the integral, $\int_1^\infty F(n)\,\mathrm{d}n$, can be calculated diagrammatically. The diagram showed in **Figure 4**.

The area beneath the curve calculated by computer

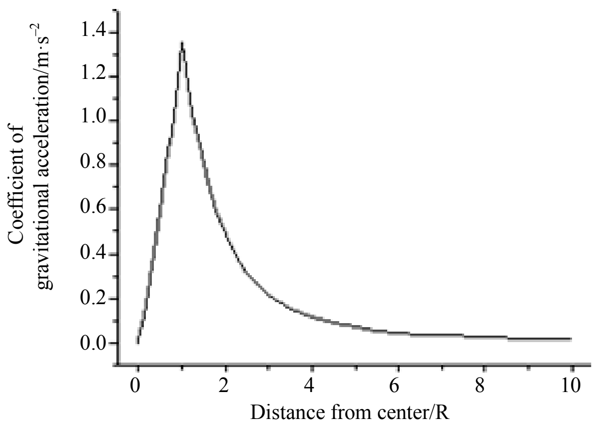


Figure 2. Diagram of function *F* (*n*).

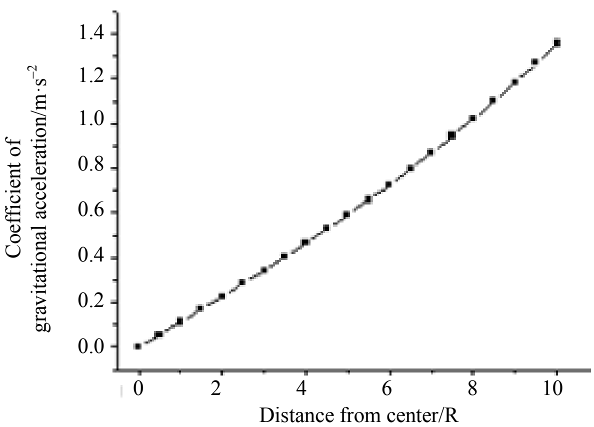


Figure 3. Partial diagram of function *F* (*n*). The variable *n* changes from zero to one.

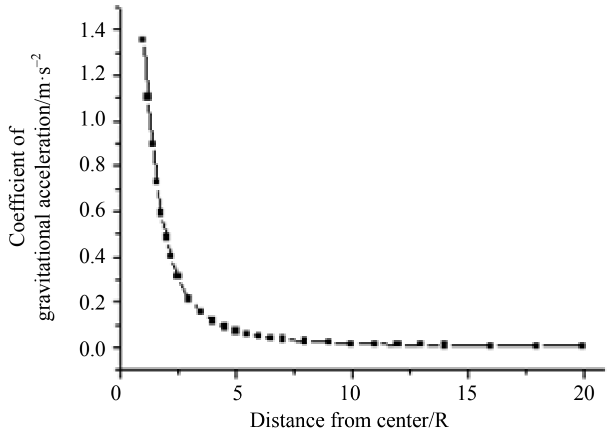


Figure 4. Partial diagram of function *F*(*n*). The variable *n* changes from 1 to ∞.

gave the result of 1.7193. Therefore, we have

$$E_g = 1.7193 \times 4Gk\rho mR^2 = 5.320G\rho mR^2. \tag{8.1}$$

3. Target Calculation

3.1. Maximal Radius of Celestial Bodies

Atoms have similar energy demand when one electron is compressed in nucleus forming a new atom (which has lower atomic number). Taking atom zinc as an example, when one electron of Zn is compressed in nucleus, the isotopic atom Cu is then formed. The energy demand, ΔE_a, can be calculated. According to quantum theory, the energy of an outer electron of a atom has the form E ≈ $-13.6Z^2/n^2$ (in ground state). Thus,

$$E_{Zn} = -13.6\frac{Z^2}{n^2} = -13.6\frac{30^2}{4^2} = -765\text{ eV}.$$

$$E_{cu} = -13.6\frac{Z^2}{n^2} = -13.6\frac{29^2}{4^2} = -715\text{ eV}.$$

$$\Delta E_1 = E_{cu} - E_{Zn} = 50\text{ eV}.$$

For an extreme hot electron of atom of hydrogen,

$$\Delta E_a = \Delta E_1 + 13.6 = 63.6\text{ eV} = 1.02 \times 10^{-17}\text{ J}$$

As the gravitational force increasing, many radioactive isotopes formed with the atomic number decreasing. Considering the atom of hydrogen, that has the maximum compression radius a_0, we guessed that the hydrogen atoms would be the last element to be compressed to nucleus. Then, the maximum pressure, $p_{\max,a}$, for compressing a hydrogen atom to neutron could be derived as:

$$\Delta E_a = F\Delta r = pS\Delta r = pSa_0.$$

$$p_{\max,a} = \Delta Ea/V = \frac{3\Delta E_a}{4\pi a_0^3} = \frac{3\times1.02\times10^{-17}}{3.14159\times4\times\left(5.29\times10^{-11}\right)^3} \tag{9}$$
$$= 1.64\times10^{13}\,\mathrm{Nm}^{-2}$$

where, Δr was replaced with the radius of hydrogen atom a_0.

The pressure comes from the gravitation of celestial body. Combining Equations (7.1) and (9), we have

$$0.6408G\rho^2R^2 = 1.64 \times 10^{13},$$

then,

$$R_{\max} = 5.07\times10^6 \frac{1}{\rho\sqrt{G}}.$$

If a solid celestial body has the same average mass density of earth (5.5153), then

$$R_{\max,solid} = 5.07\times10^6 \frac{1}{5.5153\times10^3\times\sqrt{6.670\times10^{-11}}} = 1.12\times10^8\,\text{m}.$$

To a hot gas celestial body, which has the same average mass density of sun (1.409), then, we have

$$R_{\max,gas} = 5.07\times10^6 \frac{1}{1.409\times10^3\times\sqrt{6.670\times10^{-11}}} = 4.41\times10^8\,\text{m}.$$

$$m_{\max} = \rho V = 5.06\times10^{29}\,\text{kg}.$$

3.2. Minimum Neutron Star

A model of neutron star was showed in **Figure 5**. A neutron body with the radius of R is in the center of the star. A general matter layer with the thickness of $(n - 1)R$ is around the neutron body.

The gravitational pressure on the surface of neutron body can be calculated. It contains two parts.

Equation (6) can express linearly in a short segment.

$$F(n) = 1.355571n, \ (n = 0 \to 1).$$

According to Equation (7),

$$p_i = \int \mathrm{d}p = \frac{4}{3}\rho_1 Gkp_2R^2\int F(n)\mathrm{d}n.$$

$$p_1 = \frac{4}{3}\rho_1 Gkp_2\int_1^n F(n)\,\mathrm{d}n = \frac{4}{3}\rho^2GkR^2 = \int_1^n 1.355571n\mathrm{d}n = 0.9037\rho^2kG\left(n^2-1\right)R^2.$$

$$p_2 = \frac{4}{3}\rho_1 Gk\rho_2R^2\int_1^n F(n)\,\mathrm{d}n = \frac{4}{3}\rho\rho_n kGR^2\int_1^n F(n)\,\mathrm{d}n.$$

$$p = p_1 + p_2.$$

where, ρ_n is the nuclear mass density, ρ the mass density layer matter. Substituting the parameters, we have

$$p = 1.1418\times10^{-3}\left(n^2-1\right)R^2 + 7.589\times10^{10}R^2\int_1^n F(n)\,\mathrm{d}n \tag{10}$$

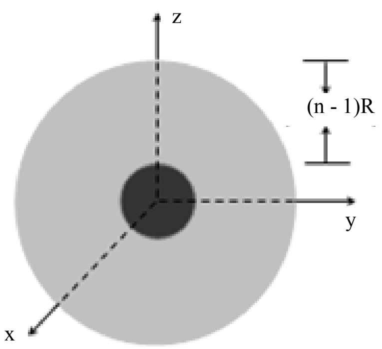


Figure 5. A model of neutron star.

Combining Equations (9) and (10), we have

$$1.1418\times10^{-3}\left(n^2-1\right)R^2 + 7.589\times10^{10}R^2\int_1^n F(n)\,\mathrm{d}n = 1.64\times10^{13}. \tag{11}$$

According to Equations (6) and (11), we obtained following results:

$R = 11.2$ m, $nR = 1121$ m

$R = 16$ m, $nR = 32$ m

$R = 261$ m, $nR = 261.6$ m

$R = 15257$ m, $nR = 15257$ m.

Combining Equations (7.1) and (13), we have

$$R = \sqrt{\frac{1.99\times10^{32}}{0.6408G\rho_n^{\ 2}}} = \sqrt{\frac{1.99\times10^{32}}{0.6408\times6.670\times10^{-11}\times\left(2\times10^{17}\right)^2}} = 15257\text{m}.$$

$$m_{\max} = p_nV = 2.98\times10^{30}\,\text{kg}.$$

3.3. Minimal Mass of Black Hole

When the radius of a neutron star achieved its maximal value, as the mass continuing accumulating, a graviton body began growing, the radius of neutron star then decreased correspondingly. When the mass of graviton body grew large enough, the neutron star began to have the ability to draw back a photon; we call it becoming a body of black hole. The maximum radius of black hole can be calculated by imitating the process of what done in paragraph 3.2. Just replacing ρ and ρ_n with the value of ρ_n and ρ_g (showed in Equation (20)), Equation (11) became Equation (12). Where ρ_g is stands for the mass density of graviton.

$$p = 1.865234\times10^{24}\left(n^2-1\right)R^2 + 8.682\times10^{24}R^2\int_1^n F(n)\,\mathrm{d}n. \tag{12}$$

According to Equations (12) and (13), following relations were obtained:

$R = 0.971$ m, $nR = 9714$ m, M = 7.679 ×10^{29} kg

$R = 9.714$ m, $nR = 9714$ m, M = 7.679 ×10^{29} kg

$R = 97.10$ m, $nR = 9710$ m, M = 7.671 ×10^{29} kg

$R = 1154$ m, $nR = 9279$ m, M = 6.722 ×10^{29} kg

$R = 4078$ m, $nR = 7340$ m, M = 4.537 ×10^{29} kg

$R = 10793$ m, $nR = 11873$ m, M = 3.672 ×10^{30} kg

$R = 14090$ m, $nR = 14908$ m, M = 7.825 ×10^{30} kg

$R = 42004$ m, $nR = 42273$ m, M = 1.971 ×10^{32} kg

$R = 74934$ m, $nR = 75084$ m, M = 1.114×10^{33} kg.

3.4. Calculation of Graviton

Nuclear has the combining energy of about 8 MeV (Wichmann 1971). Deducting the influence of proton, the maximum combining energy of nuclear would be 9.15 MeV, *i.e.*, 1.47 × 10^{-12} J. The parameter of the radius of a graviton, r_0, is the first needs to obtain. Depending on above suppositions described in Section 2.1, and the normal gravitational potential energy formula, $E = Gm_1m_2/r$, substituting m_1, m_2 with the mass of neutron, E with the combining energy in nucleus, then we have,

$$\begin{aligned} r_0 &= Gm_1m_2/E \\ &= 6.670\times10^{-11}\times\left(1.67495\times10^{-27}\right)^2/\left(1.47\times10^{-12}\right) \\ &= 1.273\times10^{-52}\,\text{m}. \end{aligned}$$

Supposing the energy demand for destroy a neutron, ΔE_n, is equal to its maximum combining energy, then we have

$$\Delta E_n = 1.47\times10^{-12}\ \text{J}.$$

Following the derivate process in Section 3.1, but replaced ΔE_a with ΔE_n, a_0 with r_n, Equation (9) becomes

$$\begin{aligned} p_{\max,n} &= \Delta En/V = \frac{3\Delta E_n}{4\pi r_n^{\ 3}} \\ &= \frac{3\times1.47\times10^{-12}}{4\times3.14159\times\left(1.2598\times10^{-15}\right)^3} \\ &= 1.76\times10^{32}\,\text{Nm}^{-2}. \end{aligned} \tag{13}$$

Following the derivate process in Section 3.2, but replaced ρ and ρ_n with ρ_n, and ρ_g, $p_{\max,a}$ with $p_{\max,n}$, Equation (11) became Equation (14).

$$\begin{aligned} &1.865\times10^{24}\left(n^2-1\right)R^2+1.376\times10^7\rho_gR^2\int_1^n F(n)\mathrm{d}n \\ &= 1.99\times10^{-32} \end{aligned} \tag{14}$$

$$\begin{aligned} R = \Big(&1.76\times10^{32}\Big/\Big(1.865\times10^{24}\left(n^2-1\right) \\ &+1.376\times10^7\rho_g\int_1^n F(n)\mathrm{d}n\Big)\Big)^{1/2}. \end{aligned} \tag{15}$$

A Smallest Known Black Hole (named XTE J1650-500) has the mass, M_b, of 7.558 × 10^{30} kg (Robert & Rob 2008). Perhaps it is not the minimal black hole theoretically; however, it was considered to be true here temporarily. According to the data and Equation (8), we have

$$\begin{aligned} E_g &= 1.7193\times4Gkp_nmn^2R^2 \\ &\quad+4mGk\left(\rho_g-\rho_n\right)R^2\int_n^\infty F(n)\mathrm{d}n \\ &= mc^2/2. \end{aligned}$$

$$3.4386\times10^{17}n^2R^2+\left(\rho_g-\rho_n\right)R^2\int_n^\infty(n)\mathrm{d}n = 2.180\times10^{26}$$

$$\begin{aligned} R = \Big(&2.180\times10^{26}\Big/\Big(3.4386\times10^{17} \\ &+\left(\rho_g-2\times10^{17}\right)\int_n^\infty F(n)\mathrm{d}n\Big)\Big). \end{aligned} \tag{16}$$

$$\begin{aligned} M_b &= 2\times10^{17}\times\left(4\pi n^3R^3/3\right)+\left(\rho_g-\rho_n\right)\times\left(4\pi R^3/3\right) \\ &= 7.558\times10^{30}. \end{aligned}$$

$$\begin{aligned} &2\times10^{17}\times\left(4\pi n^3R^3/3\right)+\left(\rho_g-\rho_n\right)\times\left(4\pi R^3/3\right) \\ &= 7.558\times10^{30}. \end{aligned}$$

$$\begin{aligned} &8.37766\times10^{17}n^3R^3+4.1888\left(\rho_g-\rho_n\right)R^3 \\ &= 7.558\times10^{30}. \end{aligned}$$

$$\begin{aligned} R = \Big(&7.558\times10^{30}\Big/\Big(8.3776\times10^{17}n^3 \\ &+4.1888\left(\rho_g-2\times10^{17}\right)\Big)\Big)^{1/3}. \end{aligned} \tag{17}$$

Combining Equations (15), (16) and (17), we have Equations (18) and (19).

$$\begin{aligned} R &= \Big(2.180\times10^{26}\Big/\Big(3.4386\times10^{17}n^2 \\ &\quad+\left(\rho_g-2\times10^{17}\right)\int_n^\infty(n)\mathrm{d}n\Big)\Big)^{1/2} \\ &= \Big(7.558\times10^{30}\Big/\Big(8.3776\times10^{17}n^3 \\ &\quad+4.1888.\left(\rho_g-2\times10^{17}\right)\Big)\Big)^{1/3} \end{aligned} \tag{18}$$

$$\begin{aligned} &\Big(2.180\times10^{26}\Big/\Big(3.4386\times10^{17}n^2 \\ &\quad+\left(\rho_g-2\times10^{17}\right)\int_n^\infty(n)\mathrm{d}n\Big)\Big)^{1/2} \\ &= \Big(1.76\times10^{32}\Big/\Big(1.865\times10^{24}\left(n^2-1\right) \\ &\quad+1.376\times10^7\rho_g\int_n^\infty(n)\mathrm{d}n\Big)\Big)^{1/2}. \end{aligned} \tag{19}$$

Combining Equations (18) and (19) diagrammatically, we obtained

$n = 1.0190$.

$R = 14090$ m.

$nR = 14358$ m.

$$\rho_g = 6.3096 \times 10^{17}\ \text{kg}\cdot\text{m}^{-3}. \tag{20}$$

$$m_g = \rho_g\left(4\pi r_0^3/3\right) = 6.3096\times10^{17} \times\left(4\pi\left(1.273\times10^{-52}\right)^3/3\right) = 5.452\times10^{-138}\,\text{kg}. \tag{21}$$

where m_g is stands for the mass of a graviton.

4. Evolvement of the Universe

The evolutive process of the universe was divided by us into six parts; and the process goes a circle (a → f →a).

a) Growing of normal celestial body. Preponderant normal celestial bodies will attract substances from its surroundings. It gradually grows as it accumulates the mass. It would have the maximum mass of 5.06×10^{29} kg, radius, 4.41×10^8 km.

b) Formation of neutron star. As the continuously accumulating of matter, the normal molecular celestial body begins to compress accompanied with neutron body engendered in the center. The compressing process looks like a process of phase variation. In the process, it will release large quantity of energy (lose of masses), become an extremely hot body and shine. A maximum neutron star has a maximum mass of 2.98×10^{30} kg, maximum radius of 15.3 km.

c) Growing of graviton star. With continuously compressing, a graviton body would begin to grow in the center. The graviton star has a critical mass of 7.558×10^{30} kg, radius of 14.9 km.

d) Formation of black hole. When graviton star grows to its critical value, it becomes a minimum black hole in the mean time. It can capture photons just near its surface. Afterward, it gradually grows in mass with the accumulating of matter. As the mass increasing, far more distant photons can be hold in. The compressing energy that released absorbed by itself. As an isolated system, the entropy of the black hole decreased spontaneously in its growing process.

e) Explosion of black hole. When the density of black hole increased large enough, the distance between gravitons going over a critical small value, the gravitational mechanism destroyed. The gravitation becomes repulsive force. Then, explosion occurred; the entropy increased sharply. Neutrons, protons, electrons, etc. particles and afterward atoms were produced subsequently in the explosion process. After explosion, this part of universe exists in expansion state in a unabiding period.

f) Constringency of partial universe. In the late period of explosion, the running matters slowed down, spread in wide space, mixed, and became part of cosmic dusts. Then, partial universe would exist in a contractive state in a long run. In intermediate constringency, the celestial bodies formed.

5. Discussion

When a celestial body has abundant material source in its surroundings, it will grow quickly and has a larger mass than we calculated. If the celestial body exists in process b, it would be a fixed star. The larger mass it has, the more quickly it will be compressed, and more efficiency the energy would release. *i.e.*, the larger mass it owns, the higher brightness it will has. In this way, a shining star could burn with no existence of special fuels of nuclear fusion, but merely existence of normal atoms.

According to the calculated results in this paper, it is evident that every galaxy will have a big black hole in its center.

The universe consists of vast cosmic dusts and galaxies. They go along circle evolutive processes described above. Occasionally a limit big black hole explodes, causing the expansion in a partial space. However, in a long time scale, most space contracted slowly. The universe exists in a mobile equilibrium state of explosion and constringency (Thomas & Hermann 1948). Those observed stars, which leave away at acceleratory velocity (Hubble 1929), might result in other reasons. It is by no means that the whole universe is expanding now. Reversely, our surrounding universe showed us its contractive views. We cannot imagine that the shape of our Milky Galaxy is formed in an expansion process.

6. Tags

A roughly information about graviton was derived in this paper. It is an exiting result to the researchers in fundamental physics. Future observation of existence of new smaller black hole is apparently meaningful.

REFERENCES

[1] V. L. Ginsburg, "On Physics and Astrophysics," Nauka Press, 1985.

[2] G. Erdei, "Principles of Structure of Matter," Mir Press, 1971.

[3] E. H. Wichmann, "Quantum Physics," *Berkeley Physics Course*, McGraw Hill, 1971.

[4] N. Robert and G. Rob, "NASA Scientists Identify Smallest Known Black Hole," *NASA's Goddard Space Flight Center*, 2008. http://www.nasa.gov/centers/goddard/news/topstory/2008/smallest_blackhole.html

[5] G. Thomas and B. Hermann, "The Steady-State Theory of the Expanding Universe," *Royal Astronomical Society*, Vol. 108, 1948, p. 252.

[6] E. Hubble, "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae," *Proceedings of the National Academy of Sciences*, Vol. 15, 1929, p. 16.

Journal of Modern Physics, **2012, 3, 1199-1210**
doi:10.4236/jmp.2012.329155 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

General Relativistic Treatment of the Pioneers Anomaly

Marcelo Samuel Berman, Fernando de Mello Gomide
Instituto Albert Einstein/Latinamerica, Av. Sete de Setembro 4500 # 101 80250-210, Curitiba, Brazil
Email: msberman@institutoalberteinstein.org, marsambe@yahoo.com, lf.gomide@hotmail.com

Received June 15, 2012; revised July 16, 2012; accepted July 24, 2012

ABSTRACT

We consider a General Relativistic generalized RWs metric, and find a field of Universal rotational global centripetal acceleration, numerically coincident with the value of the Pioneers Anomalous one. Related subjects are also treated. The rotation defined here is different from older frameworks, because we propose a Gaussian metric, whose tri-space rotates relative to the time orthogonal axis, globally.

Keywords: Cosmology; Einstein; Brans-Dicke; Pioneers Anomaly

1. Introduction

Detailed description of the subjects treated in this paper may be found in the two books recently published by Berman in 2012 [1,2]). Additional paper references are Berman in 2007 [3]; in 2011 [4] and [5]; in 2012 [6]) and with co-authors Costa, (Berman and Costa in 2012 [7]) and with Gomide (Berman and Gomide in 2012 [8] and in form as a Chapter in an edited book [9], by Berman and Gomide.

The subject treated in three papers by Marcelo Samuel Berman in this issue, two of them co-authored by Fernando de Mello Gomide (in 2012 [1]; and the present paper) and one co-authored by Newton C. A. da Costa (in 2012 [7]) are fully covered, along with all introductory material, in the books by Berman recently published (in 2012 [1,2]). Readers which are not familiar with the contents of the three papers in this issue of this Journal, may find relief by consulting those books.

Attempts to ascribe a rotational state to the Universe, were carefully described by Godlowski (in 2011 [10]). However, he confessed that there was no theoretical framework, within General Relativity, to guide the observations. In the present paper,such a mechanism is provided. The metric to be presented, makes the tri-dimensional space, globally rotate relative to the orthogonal time axis. We are now proposing a novel idea, a generalized Gaussian metric, which is minimally different from the Robertson-Walkers one. In Berman [11], a semi-relativistic treatment, based on the zero-total energy of the (rotating) Universe, made us conclude that the Pioneers anomalous deceleration, was a kind of peculiar centripetal effect of the rotation of the Universe, that could be observed by any cosmological observer. In the present paper, we prove the alleged zero-total energy of the rotating Universe, and supply the metric for such rotation with expansion. We keep a perfect fluid model, unlike Raychaudhuri's vorticities, and we also differ from the metrical rotational states, derived from non-diagonalized metrics. We shall find an energy-density solution, very similar to the Berman [11] solution. As Berman and Gomide (in 2012 [9]) have shown, by our framework, of a rotating Universe, we explain the three NASA anomalies, namely, the Pioneers linear deceleration, the spin-down of the spacecraft when they were undisturbed, and the fly-by. The present paper, yields a Machian solution, while the other one supplies a large class of general relativistic cosmological solutions with Universal rotation [8].

Ni [12,13], has reported observations on a possible rotation of the polarization of the cosmic background radiation, around 0.1 radians. As such radiation was originated at the inception of the Universe, we tried to estimate a possible angular speed or vorticity, by dividing 0.1 radians by the age of the Universe, obtaining about $10^{-19}\ \mathrm{rad\cdot s^{-1}}$.

The numerical result is very close to the theoretical estimate, by Berman (in 2007 [11]),

$$\omega \approx c/R = 3\times10^{-18}\ \mathrm{rad\cdot s^{-1}}$$

where c, R represent the speed of light in vacuum, and the radius of the causally related Universe.

We must remember, as Berman and Gomide [9] have pointed, that their calculation deals with material particles, or, in the language of General Relativity, non-null geodesics. The fact that the Universe may exhibit a rotating state, can be understood by a simple fine-tuning argument—it would be highly improbable that the Universe could keep since birth a state of no angular momentum at all.

The value of Bermans rotation, fits with the Pioneers anomaly, which consists on decelerations sufferred by Nasa space probes in non-closed curves, extending to outer space. Thermal emission was cited as resolving the Pioneers anomaly, but it does not explain the fly-bys, like Berman and Gomide [9] did through the present rotational theory. Worse, thermal emission is unable to explain why elliptical orbiters do not decelerate accordingly.

About this same numerical value of the angular speed is predicted also in Godel's rotational model, but it is not an expanding one (see Adler, Bazin and Schiffer [14]). In the next few years, the observational evidence may confirm or not such rotation .

Rotating metrics in General Relativity were first studied by Islam (in 1985 [15]), but Cosmology was not touched upon. However, it would be necessary an extreme perfect fine-tuning, in order to create the Universe without any angular-momentum. The primordial Quantum Universe, is characterized by dimensional combinations of the fundamental constants "*c*", "*h*" and "*G*" respectively the speed of light in vacuo, Planck's and Newton's gravitational constants. The natural angular momentum of Planck's Universe, as it is called, is, then, "*h*". It will be shown that the angular momentum grows with the expanding Universe, but the corresponding angular speed decreases with the scale-factor (or radius) of the Universe, such being the reason for the difficulty in detection of this speed with present technology. Notwithstanding, the so-called Pioneers' anomaly (Anderson *et al*., in 2002 [16]), which is a deceleration verified in the Pioneers space-probes launched by NASA more than thirty years ago, was attributed by Berman, to a "Machian" ubiquitous field of centripetal accelerations, due to the rotation of the Universe. Berman's calculation rested on the assumption that the zero-total energy of the Universe was a valid result for the rotating case, but the proof was not supplied in that paper (Berman, in 2007 [3]). By "proof", one thinks on the pseudotensor energy calculations of General Relativity—the best gravitational theory ever published.

In his three best-sellers Hawking (in 1996 [17]; 2001 [18]; 2003 [19]) describes inflation (Guth in 1981 [20] and in 1998 [21]), as an accelerated expansion of the Universe, immediately after the creation instant,while the Universe, as it expands,borrows energy from the gravitational field to create more matter. According to his description, the positive matter energy is exactly balanced by the negative gravitational energy, so that the total energy is zero, and that when the size of the Universe doubles, both the matter and gravitational energies also double, keeping the total energy zero (twice zero). Moreover, in the recent, next best-seller, Hawking and Mlodinow (in 2010) comment that if it were not for the gravity interaction, one could not validate a zero-energy Universe, and then, creation out of nothing would not have happened.

There are four methods, in GRT, to create rotations. Non-diagonal metrics, like Kerrs, is one. The adoption of an imperfect fluid model, with vorticities, as in Raychaudhuris equation, is second. Third, you may follow the Godlowski *et al*. (in 2004 [22]) idea, and add to the scale-factor s squared time derivative, $\dot{R}^2$ a rotational term $(\omega R)^2$. On the other hand, Berman (in 2008 [23,24]) has shown that Robertson-Walker's metric, is a particular, non-rotating case, of a general relativistic expanding and rotating metric first developed by Gomide and Uehara (in 1981 [25]). The peculiarity of the general metric is that instead of working with proper-time τ, one writes the field equations of General Relativity with a cosmic time *t* related by:

$$\mathrm{d}\tau = (g_{00})^{1/2}\,\mathrm{d}t \tag{1}$$

where,

$$g_{00} = g_{00}(r,\theta,\phi,t) \tag{2}$$

It was seen that when one introduces a metric temporal coefficient g_{00} which is not constant, the new metric includes rotational effects. In fact, we have a generalized Gaussian metric, because besides the fact that the tri-space is orthogonal to the time-axis, the spatial part of the metric, rotates as a whole, relative to this time axis. This is a new concept being introduced in the theory.

The present paper follows the steps of the semi-relativistic treatment by Berman (in 2007 [3]), but this time, it is General relativistic, and we shall find a Machian kind of solution. The general solution is to be found in Berman and Gomide (in 2012 [8]).

In a previous paper Berman (in 2009 [26]) has calculated the energy of the Friedman-Robertson-Walker's Universe, by means of pseudo-tensors, and found a zero-total energy. Our main task will be to show why the Universe is a zero-total-energy entity, by means of pseudo-tensors, even when one chooses a variable g_{00} such that the Universe also rotates, and then, to show how General Relativity predicts a universal angular speed, and a universal centripetal deceleration, numerically coincident with the observed deceleration of the Pioneers space-probes. The first calculation of this kind, with the Gomide-Uehara generalization of RWs metric, was undertaken by Berman (in 1981 [27]), in his M.Sc. thesis, advised by the present second author, but where the rotation of the Universe was not the scope of the thesis.

The pioneer works of Berman (in 1981 [27]), Nathan Rosen (in 1994 [28]), Cooperstock and Israelit, (in 1995 [29]), showing that the energy of the Universe is zero, by means of calculations involving pseudotensors, and Killing vectors, respectively, are here given a more simple approach. The energy of the (non-rotating) Robertson-Walker's Universe is zero, (Berman, in 2007 [11]; and in 2009 [26]). Berman (in 1981 [27]) was the first author to work, in pseudotensor calculations for the energy of Robertson-Walker's Universe. He made the calculations on which the present paper rest, and, explicitly obtained the zero-total energy for a closed Universe, by means of LL-pseudotensor, when Robertson-Walker's metric was generalised by the introduction of a temporal-time-varying metric coefficient. However, the present authors, were unaware, in the year 1981, of the exact significance of their findings.

The zero-total-energy of the Roberston-Walker's Universe, and of any Machian ones, have been shown by many authors (Berman in 2006 [30,31]; in 2007 [11]; 2007 [32]; 2007 [3]). It may be that the Universe might have originated from a vacuum quantum fluctuation. In support of this view, we shall show that the pseudotensor theory (Adler *et al*. in 1975 [14]) points out to a null-energy for a rotating Robertson-Walker's Universe. Some prior work is mentioned, (in 2006 [30]; 2006 [31]; in 2007 [11]; 2007 [32]; 2007 [3]; Rosen in 1995 [33]; York Jr in 1980 [34]; Cooperstock in 1994 [35]; Cooperstock and Israelit in 1995 [29]; Garecki in 1995 [36]; Johri *et al*. in 1995 [37]; Feng and Duan in 1996 [38]; Banerjee and Sen in 1997 [39]; Radinschi in 1999 [40]; Cooperstock and Faraoni in 2003 [41]). See also Katz (in 2006 [42], in 1985 [43]; Katz and Ori in 1990 [44]; Katz *et al*. in 1997 [45]). Recent developments include torsion models (So and Vargas in 2006 [46]), and, a paper by Xulu in 2000 [47].

The reason for the failure of non-Cartesian curvilinear coordinate energy calculations through pseudotensors, resides in that curvilinear coordinates carry non-null Christoffel symbols, even in Minkowski spacetime, thus introducing inertial or fictitious fields that are interpreted falsely as gravitational energy-carrying (false) fields.

Carmeli *et al*. in 1990 [48] listed four arguments against the use of Einstein's pseudotensor: 1) the energy integral defines only an affine vector; 2) no angular-momentum is available; 3) as it depends only on the metric tensor and its first derivatives, it vanishes locally in a geodesic system; 4) due to the existence of a super-potential, which is related to the total conserved pseudo-quadrimomentum, by means of a divergence, then the values of the metric tensor, and its first derivatives, only matter, on a surface around the volume of the mass-system.

We shall argue below that, for the Universe, local and global Physics blend together. The pseudo-momentum, is to be taken like the linear momentum vector of Special Relativity, *i.e.*, as an affine vector. In a previous paper (Berman in 2009 [26]), we stated that "if the Universe has some kind of rotation, the energy-momentum calculation refers to a co-rotating observer". Such being the case, we now go ahead for the actual calculations, involving rotation. Birch (in 1982 [49] and in 1983 [50]) cited inconclusive experimental data on a possible rotation of the Universe, which was followed by a paper written by Gomide, Berman and Garcia in 1986 [51].

2. Field Equations for the Rotating and Expanding Metric

Consider first a temporal metric coefficient which depends only on *t*. The line element becomes:

$$\mathrm{d}s^2 = -\frac{R^2(t)}{\left(1+kr^2/4\right)^2}\left[\mathrm{d}\sigma^2\right] + g_{00}(t)\,\mathrm{d}t^2 \tag{3}$$

The field equations, in General Relativity Theory (GRT) become:

$$3\dot{R}^2 = \kappa\left(\rho+\frac{\Lambda}{\kappa}\right)g_{00}R^2 - 3kg_{00} \tag{4}$$

and,

$$6\ddot{R} = -g_{00}\kappa\left(\rho+3p-2\frac{\Lambda}{\kappa}\right)R - 3g_{00}\dot{R}\dot{g}^{00} \tag{5}$$

Local inertial processes are observed through proper time, so that the four-force is given by:

$$F^{\alpha} = \frac{\mathrm{d}}{\mathrm{d}\tau}\left(mu^{\alpha}\right) = mg^{00}\ddot{x}^{\alpha} - \frac{1}{2}m\dot{x}^{\alpha}\left[\frac{\dot{g}_{00}}{g_{00}^2}\right] \tag{6}$$

Of course, when $g_{00}=1$, the above equations reproduce conventional Robertson-Walker's field equations.

We must mention that the idea behind Robertson-Walker's metric is the Gaussian coordinate system. Though the condition $g_{00}=1$ is usually adopted, we must remember that, the resulting time-coordinate is meant as representing proper time. If we want to use another coordinate time, we still keep the Gaussian coordinate properties.

From the energy-momentum conservation equation, in the case of a uniform Universe, we must have,

$$\frac{\partial}{\partial x^i}(\rho) = \frac{\partial}{\partial x^i}(p) = \frac{\partial}{\partial x^i}(g_{00}) = 0 \quad (i=1,2,3) \tag{7}$$

The above is necessary in the determination of cosmic time, for a commoving observer. We can see that the hypothesis (2)—that g_{00} is only time-varying—is now validated.

In order to understand Equation (6), it is convenient to relate the rest-mass *m*, to an inertial mass M_i, with:

$$M_i = \frac{m}{g_{00}} \tag{8}$$

It can be seen that M_i represents the inertia of a particle, when observed along cosmic time, *i.e.*, coordinate time. In this case, we observe that we have two acceleration terms, which we call,

$$a_1^\alpha = \ddot{x}^\alpha \tag{9}$$

and,

$$a_2^\alpha = -\frac{1}{2g_{00}}\left(\dot{x}^\alpha \dot{g}_{00}\right) \tag{10}$$

The first acceleration is linear; the second, resembles rotational motion, and depends on g_{00} and its time-derivative.

If we consider a_2^α a centripetal acceleration, we conclude that the angular speed ω is given by,

$$\omega = \frac{1}{2}\left(\frac{\dot{g}_{00}}{g_{00}}\right) \tag{11}$$

By comparison between the usual τ—metric, and the field equations in the t—metric, we are led to conclude that the conventional energy density ρ and cosmic pressure p are transformed into $\bar{\rho}$ and $\bar{p}$, where:

$$\bar{\rho} = g_{00}\left(\rho + \frac{\bar{\Lambda}}{\kappa}\right) \tag{12}$$

and,

$$\bar{p} = g_{00}\left(p - \frac{\bar{\Lambda}}{\kappa}\right) \tag{13}$$

We plug back into the field equations, and find,

$$\bar{\Lambda} = \Lambda - \frac{3}{2\kappa}\left(\frac{\dot{R}}{R}\right)\dot{g}^{00} \tag{14}$$

For a time-varying angular speed, considering an arc ϕ, so that,

$$\omega(t) = \frac{\mathrm{d}\phi}{\mathrm{d}t} = \dot{\phi} \tag{15}$$

we find, from (11),

$$g_{00} = Ce^{2\phi(t)} \quad (C = \text{constant}) \tag{16}$$

Returning to (14), we find,

$$\bar{\Lambda} = \Lambda + \frac{3}{\kappa C}\left(\frac{\dot{R}}{R}\right)\omega e^{-2\phi(t)} \tag{17}$$

This completes our solution.

The case where g_{00} depends also on r, θ and ϕ was considered also by Berman (in 2008 [24]) and does not differ qualitatively from the present analysis, so that, we refer the reader to that paper.

3. Energy of the Rotating Evolutionary Universe

Even in popular Science accounts (Hawking in 1996 [17]; in 2001 [18] and in 2003 [19]; Hawking and Moldinow in 2010; and Guth in 1998 [21]), it has been generally accepted that the Universe has zero-total energy. The first such claim, seems to be due to Feynman in 1962-3 [52]. Lately, Berman (in 2006 [30,31]) has proved this result by means of simple arguments involving Robertson-Walker's metric for any value of the tri-curvature $(0,-1,1)$.

The pseudotensor t_ν^μ, also called Einstein's pseudotensor, is such that, when summed with the energy-tensor of matter T_ν^μ, gives the following conservation law:

$$\left[\sqrt{-g}\left(T_\nu^\mu + t_\nu^\mu\right)\right]_{,\mu} = 0 \tag{18}$$

In such case, the quantity

$$P_\mu = \int\left\{\sqrt{-g}\left[T_\mu^0 + t_\mu^0\right]\right\}\mathrm{d}^3x \tag{19}$$

is called the general-relativistic generalization of the energy-momentum four-vector of special relativity (Adler *et al.* in 1975 [14]).

It can be proved that P_μ is conserved when:

a) $T_\nu^\mu \neq 0$ only in a finite part of space; and,

b) $g_{\mu\nu} \to \eta_{\mu\nu}$ when we approach infinity, where $\eta_{\mu\nu}$ is the Minkowski metric tensor.

However, there is no reason to doubt that, even if the above conditions were not fulfilled, we might eventually get a constant P_μ, because the above conditions are sufficient, but not strictly necessary. We hint on the plausibility of other conditions, instead of a) and b) above.

Such a case will occur, for instance, when we have the integral in (19) is equal to zero.

For our generalised metric, we get exactly this result, because, from Freud's (1939) formulae, there exists a super-potential, (Papapetrou in 1974 [54]):

$${}_F U_\lambda^{\mu\nu} = \frac{g_{\lambda\alpha}}{2\sqrt{-g}}\left(\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta} - \bar{g}^{\nu\alpha}\bar{g}^{\mu\beta}\right)_{,\beta}$$

where the bars over the metric coefficients imply that they are multiplied by $\sqrt{-g}$, and such that,

$$\kappa\sqrt{-g}\left(T_\lambda^\rho + t_\lambda^\rho\right) = {}_F U_\lambda^{\rho\sigma}{}_{,\sigma}$$

thus finding, after a brief calculation, for the rotating Robertson-Walker's metric,

$$P_\lambda = 0$$

The above result, with von Freud's superpotential, which yields Einstein's pseudotensorial results, points to a zero-total energy Universe, even when the metric is endowed with a varying metric temporal coefficient .

A similar result would be obtained from Landau-Lifshitz pseudotensor (Papapetrou in 1974 [54]), where we have:

$$P_{LL}^\nu = \int(-g)\left[T^{\nu 0} + t_L^{\nu 0}\right]\mathrm{d}^3x \tag{20}$$

where,

$$\kappa\sqrt{-g}\left(T^{\mu\rho}+\tilde{t}^{\mu\rho}\right)=\tilde{U}^{\mu\rho\sigma}{}_{,\sigma}$$

and,

$$\tilde{U}^{\mu\rho\sigma}=\bar{g}^{\lambda\mu}{}_{F}U_{\lambda}^{\rho\sigma}$$

A short calculation shows that, for the rotating metric, too, we keep valid the result,

$$P_{LL}^{\nu}=0\ \left(\nu=0,1,2,3\right) \tag{21}$$

Other superpotentials would also yield the same zero results. A useful source for the main superpotentials in the market, is the paper by Aguirregabiria *et al*. in 1996 [55].

The equivalence principle, says that at any location, spacetime is (locally) flat, and a geodesic coordinate system may be constructed, *where the Christoffel symbols are null. The pseudotensors are, then, at each point, null. But now remember that our old Cosmology requires a co-moving observer at each point. It is this co-motion that is associated with the geodesic system, and, as RWs metric is homogeneous and isotropic, for the co-moving observer, the zero-total energy density result, is repeated from point to point, all over spacetime. Cartesian coordinates are needed, too, because curvilinear coordinates are associated with fictitious or inertial forces, which would introduce inexistent accelerations that can be mistaken additional gravitational fields* (*i.e.*, *that add to the real energy*). *Choosing Cartesian coordinates is not analogous to the use of center of mass frame in New-tonian theory, but the null results for the spatial components of the pseudo-quadrimomentum show compatibility.*

4. An Alternative Derivation

Though so many researchers have dealt with the energy of the Universe, our present original solution involves rotation. We may paraphrase a previous calculation, provided that we work with proper time τ instead of coordinate time t (Berman in 2009 [26]). Then, the rotation of the Universe will be automatically included. We shall now consider, first, why the Minkowski metric represents a null energy Universe. Of course, it is empty. But, why it has zero-valued energy? We resort to the result of Schwarzschilds metric, (Adler *et al*. in 1975 [14]), whose total energy is,

$$E=Mc^2-\frac{GM^2}{2R}$$

If $M=0$, the energy is zero, too. But when we write Schwarzschilds metric, and make the mass become zero, we obtain Minkowski metric, so that we got the zero-energy result. Any flat RWs metric, can be reparametrized as Minkowskis; or, for closed and open Universes, a superposition of such cases (Cooperstock and Faraoni in 2003 [41]; Berman in 2006 [30,31]).

Now, the energy of the Universe, can be calculated at constant time coordinate τ. In particular, the result would be the same as when $\tau\to\infty$, or, even when $\tau\to 0$. Arguments for initial null energy come from Tryon (in 1973 [58]), and Albrow (in 1973 [59]). More recently, we recall the quantum fluctuations of Alan Guths inflationary scenario (Guth in 1981 [20] and 1998 [21]). Berman (see for instance [57]), gave the Machian picture of the Universe, as being that of a zero energy. Sciamas inertia theory results also in a zero-total energy Universe (Sciama in 1953 [58]; Berman in 2008 [59] and in 2009 [61]).

Consider the possible solution for the rotating case. We work with the τ-metric, so that we keep formally the RWs metric in an accelerating Universe. The scale-factor assumes a power-law, as in constant deceleration parameter models (Berman in 1983 [64]; and Berman and Gomide in 1988 [65]),

$$R=\left(mD\tau\right)^{1/m} \tag{22}$$

where, m, D = constants, and,

$$m=q+1>0 \tag{23}$$

where q is the deceleration parameter.

For a perfect fluid energy tensor, and a perfect gas equation of state, cosmic pressure and energy density obey the following energy-momentum conservation law, (Berman in 2007 [10,32]),

$$\dot{\rho}=-3H\left(\rho+p\right) \tag{24}$$

where, only in this Section, overdots stand for τ-derivatives. Let us have,

$$p=\alpha\rho\ \ (\alpha=\ \text{constant larger than}\ -1) \tag{25}$$

On solving the differential equation, we find, for any $k=0$, 1, -1, that,

$$\rho=\rho_0\tau^{-\frac{3(1+\alpha)}{m}}\ \ (\rho_0=\ \text{constant}) \tag{26}$$

When $\tau\to\infty$, from (26) we see that the energy density becomes zero, and we retrieve an "empty" Universe, or, say, again, the energy is zero. However, this energy density is for the matter portion, but nevertheless, as in this case, $R\to\infty$, all masses are infinitely far from each others, so that the gravitational inverse-square interaction is also null. The total energy density is null, and, so, the total energy. Notice that the energy-momentum conservation equation does not change even if we add a cosmological constant density, because we may subtract an equivalent amount in pressure, and Equation (24) remains the same. The constancy of the energy, leads us to consider the zero result at infinite time, also valid at any other instant.

We refer to Berman (in 2006 [30,31]) for another alternative proof of the zero-energy Universe. If we took τ instead of *t*, these references would provide the zero result also for the rotational case.

5. Pioneers Anomaly Revisited

Einstein's field Equations (4) and (5) above, can be obtained, when $g_{00} =$ constant, through the mere assumptions of conservation of energy (Equation (4)) and thermodynamical balance of energy (Equation (5)), as was pointed out by Barrow in 1988 [66]. The latter is also to be regarded as a definition of cosmic pressure, as the volume derivative of energy with negative sign

$$\left(p = -\frac{\mathrm{d}\left(\rho V\right)}{\mathrm{d}V}\right).$$

Now, let us consider a time-varying g_{00}. We may write the energy (in fact, the "energy-density")—equation, as follows:

$$\frac{3\dot{R}^2}{g_{00}} - \kappa\left(\rho + \frac{\Lambda}{\kappa}\right)R^2 = -3k = \text{constant} \tag{27}$$

The r.h.s. stands for a constant. We can regard the l.h.s. as the a sum of constant terms, thus finding a possible solution of the field equations, such that each term in the l.h.s. of (27) remains constant. For example, let us consider,

$$\rho = \rho_0 R^{-2} \tag{28}$$

$$\Lambda = \Lambda_0 R^{-2} \tag{29}$$

$$g_{00} = 3\gamma^{-1}\dot{R}^2 \tag{30}$$

where, ρ_0, Λ_0 and γ are non-zero constants. Relation (28) makes this solution practically of the Machian-type, similar to the semi-relativistic treatment by Berman (in 2007 [3]). More general solutions may be found also in the companion paper by Berman and Gomide (2012) [8] published in this issue of this Journal. See also Berman (in 2011 [4,5]; in 2012 [1,2,6]; Berman and Gomide in 2012 [9]).

When we plug the above solution to the cosmic pressure Equation (5), we find that it is automatically satisfied provided that the following conditions hold,

$$2\Lambda_0 = \kappa\rho_0\left(1+3\alpha\right) \tag{31}$$

$$p = \alpha\rho \quad \left(\alpha = \text{ constant}\right) \tag{32}$$

and,

$$\gamma = \kappa\rho_0 + \Lambda_0 - 3k \tag{32a}$$

As we found a general-relativistic solution, so far, we are entitled to the our previous general relativistic angular speed Formula (11), to which we plug our solution (30), to wit,

$$\omega = \frac{\ddot{R}}{\dot{R}} = H + \frac{\dot{H}}{H}$$

For the power-law solution of the last Section,

$$H = \frac{1}{mt}$$

so that,

$$\omega = -\frac{q}{mt} \approx t^{-1}$$

where we roughly estimated the present deceleration paramenter as $-1/2$, while, the centripetal acceleration,

$$a = -\omega^2 R \approx -t^{-2}R \simeq 8\times10^{-8}\ \text{cm}\cdot\text{s}^{-2}$$

Notice that the same result would follow from a scale-factor varying linearly with time. This is the sort of scale-factor associated with the Machian Universe. In fact,the field equations that we had (Equations (4) and (5)), were not enough in order to determine the exact form of the scale-factor, because we had an extra-unknown term, the temporal metric coefficient. When we advance a given equation of state, the original RWs field equations, with constant g_{00}, may determine the scale-factors formula. Just to remember, our solution is a particular one.

This is a general relativistic result. It matches Pioneers anomalous deceleration.

In an Appendix to this Section, we go ahead with the alternative calculation with a simple naive Special Relativistic-Machian analysis, as had been made in Berman (in 2007 [3]).

6. Final Comments and Discussion

Someone has made very important criticisms on our work. First, he says why do not the planets in the solar system show the calculated deceleration on the Pioneers? The reason is that elliptical orbits are closed, and localized. You do not feel the expansion of the universe in the sizes of the orbits either. In General Relativity books, authors make this explicit. You do not include Hubbles expansion in Schwarzschilds metric. But, those space probes that undergo hyperbolic motion, which orbits extend towards infinity, they acquire cosmological characteristics, like, the given P.A. deceleration. Second objection, there are important papers which resolve the P.A. with non-gravitational Physics. The answer—that is OK, we have now alternative explanations. This does not preclude ours. Third, cosmological reasons were discarded, including rotation of the Universe. The problem is that those discarded cosmologies, did not employ the correct metric. For instance, they discarded rotation by examining Godel model, which is non expanding, and with a strange metric. The kind of metric we employ now, or the one that we employed in the rotational case, were not discarded or discussed by the authors cited by this objecter. Then, the final question, is how come that a well respected author dismissed planetary Coriolis forces induced by rotation of distant masses, by means of the constraints in the solar system. Our answer is that, beside what we answered above, he needs to consider Machs Principle on one side, and the theoretical meaning of vorticities, because one is not speaking in a center or an axis of rotation or so. When we say, in Cosmology, that the Universe rotates, we mean that there is a field of vorticities,just that. The whole idea is that Cosmology does not enter the Solar System except for non-closed orbits that extend to outer space. We ask the reader to check Machs Principle, because in some formulations of this principle, rotation is in fact a *forbidden affaire*.

Another one pointed out a different "problem". He objects, that the angular speed formula of ours, is coordinate dependent. Now, when you choose a specific metric, you do it thinking about the kind of problem you have to tackle. After you choose the convenient metric, you forget tensor calculus, and you work with coordinate-dependent relations. They work only for the given metric, of course.

We have obtained a zero-total energy proof for a rotating expanding Universe. The zero result for the spatial components of the energy-momentum-pseudotensor calculation, are equivalent to the choice of a center of Mass reference system in Newtonian theory, likewise the use of comoving observers in Cosmology. It is with this idea in mind, that we are led to the energy calculation, yielding zero total energy, for the Universe, as an acceptable result: we are assured that we chose the correct reference system; this is a response to the criticism made by some scientists which argue that pseudotensor calculations depend on the reference system, and thus, those calculations are devoid of physical meaning.

Related conclusions by Berman should be consulted (see all Berman's references at the end of this article). As a bonus, we can assure that there was not an initial infinite energy density singularity, because attached to the zero-total energy conjecture, there is a zero-total energy-density result, as was pointed first by Berman elsewhere (Berman, for instance, see in 2012 [1,2]). The so-called total energy density of the Universe, which appears in some textbooks, corresponds only to the non-gravitational portion, and the zero-total energy density results when we subtract from the former, the opposite potential energy density.

As Berman(in 2009 [67,68]) shows, we may say that the Universe is *singularity-free*, and was created *abnihilo*, nor there is zero-time infinite energy-density singularity.

Paraphrasing Dicke (in 1964 [69,70]), it has been shown the many faces of Dirac's LNH, as many as there are about Mach's Principle. In face of modern Cosmology, the naif theory of Dirac is a foil for theoretical discussion on the foundations of this branch of Physical theory. The angular speed found by us, (Berman, in 2010 [68]; in 2009 [72]), matches results by Gödel (see Adler *et al*. in 1975 [14]), Sabbata and Gasperini (in 1979 [70]), and Berman (in 2007 [3], and in 2008 [24,74]).

Rotation of the Universe and zero-total energy were verified for Sciama's linear theory, which has been expanded, through the analysis of radiating processes, by one of the present authors (Berman in 2008 [59]; and in 2009 [60]).There,we found Larmor's power formula, in the gravitational version, leads to the correct constant power relation for the Machian Universe. However, we must remember that in local Physics, General Relativity deals with quadrupole radiation, while Larmor is a dipole formula; for the Machian Universe the resultant constant power is basically the same, either for our Machian analysis or for the Larmor and general relativistic formulae.

Referring to rotation, it could be argued that cosmic microwave background radiation deals with null geodesics, while Pioneers' anomaly, for instance, deals with time-like geodesics. In favor of evidence on rotation, we remark neutrinos' spin, parity violations, the asymmetry between matter and anti-matter, left-handed DNA-helices, the fact that humans and animals alike have not symmetric bodies, the same happening to molluscs. And, of course, the results of the rotation of the polarization of CMBR.

We predict that chaotic phenomena and fractals, rotations in galaxies and clusters, may provide clues on possible left handed preference through the Universe.

Berman and Trevisan (in 2010 [74]) have remarked that creation out-of-nothing seems to be supported by the zero-total energy calculations. Rotation was now included in the derivation of the zero result. We could think that the Universes are created in pairs, the first one (ours), has negative spin and positive matter; the second member of the pair, would have negative matter and positive spin: for the ensemble of the two Universes, the total mass would always be zero; the total spin, too. The total energy (twice zeros) is also zero. Our framework, is the only one to solve the fly-by anomaly altogether, and explains why elliptical orbiters do not decelerate.

For more details on the subjects treated here, the general recomendation is to refer the reader to both books published recently by Berman (in 2012 [72]).

7. Acknowledgements

One of the authors (MSB) thanks Marcelo Fermann Guimarães, Nelson Suga, Mauro Tonasse, Antonio F. da F. Teixeira, and for the important incentive offered by Miss Solange Lima Kaczyk, now, a brand new advocate, continued during the last five years of his research in Cosmology.

REFERENCES

[1] M. S. Berman, "General Relativity and the Pioneers Anomaly," Nova Science Publishers, New York, 2012.

[2] M. S. Berman, "Realization of Einstein's Machian Program," Nova Science Publishers, New York, 2012.

[3] M. S. Berman, "The Pioneer Anomaly and a Machian Universe," *Astrophysics and Space Science*, Vol. 312, No. 3-4, 2007, pp. 275-278.

[4] M. S. Berman, "The Two Pioneers Anomalies and Universal Rotation," *Astrophysics and Space Science*, Vol. 336, No. 2, 2011, pp. 337-339. doi:10.1007/s10509-011-0825-4

[5] M. S. Berman, "General Relativity with Variable Speed of Light and Pioneers Anomaly," *Astrophysics and Space Science*, Vol. 336, No. 2, 2011, pp. 327-329.

[6] M. S. Berman, "Realization of Einsteins Machian Program: The Pioneers and Fly-By Anomalies," *Astrophysics and Space Science*, Vol. 337, No. 1, 2012, pp. 477-481.

[7] M. S. Berman and N. C. A. da Costa, "On the Stability of Our Universe," *Journal of Modern Physics*, Vol. 3, 2012, pp. 1211-1215.

[8] M. S. Berman and F. M. Gomide, "Relativistic Cosmology and the Pioneers Anomaly," *Journal of Modern Physics*, Vol. 3, pp. 1178-1184.

[9] M. S. Berman and F. M. Gomide, "On the Rotation of the Zero-Energy Expanding Universe," In: J. R. O'Connell and A. L. Hale, Eds., *The Big-Bang-Theory*, *Assumptions and Problems*, Nova Science Publishers, New York, 2012, pp. 285-310.

[10] W. Godlowski, Los Alamos Archives, 2011. arxiv: 1103. 5786.

[11] M. S. Berman, "Introduction to General Relativity and the Cosmological Constant Problem," Nova Science, New York, 2007.

[12] W. T. Ni, "From Equivalence Principles to Cosmology: Cosmic Polarization Rotation, CMB Observation, Neutrino Number Asymmetry, Lorentz Invariance and CPT," *Progress of Theoretical Physics Supplement*, Vol. 172, 2008, pp. 49-60.

[13] W. T. Ni, "Cosmic Polarization Rotation, Cosmological Models, and the Detectability of Primordial Gravitational Waves," *International Journal of Modern Physics A*, Vol. 24, No. 18-19, 2009, pp. 3493-3500. doi:10.1142/S0217751X09047107

[14] R. J. Adler, M. Bazin and M. Schiffer, "Introduction to General Relativity," 2nd Edition, McGraw-Hill, New York, 1975.

[15] J. N. Islam, "Rotating Fields in General Relativity," Cambridge University Press, Cambridge, 1985. doi:10.1017/CBO9780511735738

[16] J. D. Anderson, *et al.*, "Study of the Anomalous Acceleration of Pioneer 10 and 11," *Physical Review D*, Vol. 65, No. 8, 2002, pp. 820041-8200450. doi:10.1103/PhysRevD.65.082004

[17] S. Hawking, "The Illustrated a Brief History of Time," Bantam Books, New York, 1996.

[18] S. Hawking, "The Universe in a Nutshell," Bantam Books, New York, 2001.

[19] S. Hawking, "The Illustrated Theory of Everything," Phoenix Books, Beverly Hills, 2003.

[20] A. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," *Physical Review D*, Vol. 23, No. 2, 1981, pp. 347-356. doi:10.1103/PhysRevD.23.347

[21] A. Guth, "The Inflationary Universe," Vintage, New York, 1998.

[22] W. Godlowski, *et al.*, *GERG Journal*, Vol. 35, 2003, p. 907. Los Alamos Archives, arxiv astroph-0404329.

[23] M. S. Berman, "A General Relativistic Rotating Evolutionary Universe," *Astrophysics and Space Science*, Vol. 314, No. 4, 2008, pp. 319-321. doi:10.1007/s10509-008-9772-0

[24] M. S. Berman, "A General Relativistic Rotating Evolutionary Universe—Part II," *Astrophysics and Space Science*, Vol. 315, No. 1-4, 2008, pp. 367-369. doi:10.1007/s10509-008-9830-7

[25] F. M. Gomide and M. Uehara, "Mach's Principle in Evolutionary Universes with Time Varying Temporal Metric Coefficient," *Astronomy and Astrophysics*, Vol. 95, No. 2, 1981, pp. 362-365.

[26] M. S. Berman, "On the Zero-Energy Universe," *International Journal of Theoretical Physics*, Vol. 48, No. 11, 2009, pp. 3278-3286. doi:10.1007/s10773-009-0125-8

[27] M. S. Berman, Master's Thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos, 1981.

[28] N. Rosen, "The Energy of the Universe," *General Relativity and Gravitation*, Vol. 26, No. 3, 1994, pp. 319-321. doi:10.1007/BF02108013

[29] F. I. Cooperstock and M. Israelit, "The Energy of the Universe," *Foundations of Physics*, Vol. 25, No. 4, 1995, pp. 631-635. doi:10.1007/BF02059009

[30] M. S. Berman, "Energy of Black-Holes and Hawkings Universe," In: P. V. Kreitler, Ed., *Trends in Black Hole Research*, Nova Science, New York, 2006.

[31] M. S. Berman, "Energy, Brief History of Black-Holes, and Hawkings Universe," In: P. V. Kreitler, Ed., *New Developments in Black Hole Research*, Nova Science, New York, 2006.

[32] M. S. Berman, "Introduction to General Relativistic and Scalar Tensor Cosmologies," Nova Science, New York, 2007.

[33] N. Rosen, "The Energy of the Universe," *General Relativity and Gravitation*, Vol. 26, No. 3, 1994, pp. 319-321.

[34] J. W. York Jr., "Energy and Momentum of the Gravitational Field, in a Festschrift for Abraham Taub," Academic Press, New York, 1980.

[35] F. I. Cooperstock, "Perspectives on the Energy of the Universe," *General Relativity and Gravitation*, Vol. 26, No. 3, 1994, pp. 323-327.

[36] J. Garecki, "Canonical Angular Supermomentum Tensors in General Relativity," *General Relativity and Gravitation*, Vol. 27, No. 1, 1995, pp. 55-64.

[37] V. B. Johri, *et al.*, "Gravitational Energy in the Expanding Universe," *General Relativity and Gravitation*, Vol. 27, No. 3, 1995, pp. 313-318.

[38] S. X. Feng and Y. S. Duan, "About the Energy of the University," *Chinese Physics Letters*, Vol. 13, No. 6, 1996, p. 409. doi:10.1088/0256-307X/13/6/003

[39] N. Banerjee and S. Sen, "Einstein Pseudotensor and Total Energy of the Universe," *Pramana—Journal of Physics*, Vol. 49, No. 6, 1997, pp. 609-615.

[40] I. Radinschi, "The Energy of a Dyonic Dilaton Black Hole," *Acta Physica Slovaca*, Vol. 49, No. 5, 1999, pp. 789-794.

[41] F. I. Cooperstock and V. Faraoni, "On the Total Energy of Open Friedmann-Robertson-Walker Universes," *The Astrophysical Journal*, Vol. 587, 2003, pp. 483-486.

[42] J. Katz, "Private Communication," 2006.

[43] J. Katz, "A Note on Komar's Anomalous Factor," *Classical and Quantum Gravity*, Vol. 2, No. 3, 1985, p. 423. doi:10.1088/0264-9381/2/3/018

[44] J. Katz and A. Ori, "Localisation of Field Energy," *Classical and Quantum Gravity*, Vol. 7, No. 5, 1990, p. 787. doi:10.1088/0264-9381/7/5/009

[45] J. Katz, J. Bicak and D. Lynden-Bell, "Relativistic Conservation Laws and Integral Constraints for Large Cosmological Perturbations," *Physical Review D*, Vol. 55, No. 10, 1997, pp. 5957-5969. doi:10.1103/PhysRevD.55.5957

[46] L. L. So and T. Vargas, Los Alamos Archives, 2006.

[47] S. Xulu, "Total Energy of the Bianchi Type I Universes," *International Journal of Theoretical Physics*, Vol. 39, No. 4, 2000, pp. 1153-1161.

[48] M. Carmeli, E. Leibowitz and N. Nissani, "Gravitation: SL (2, C) Gauge Theory and Conservation Laws," World Scientific, Singapore, 1990.

[49] P. Birch, "Is the Universe Rotating?" *Nature*, Vol. 298, No. 5873, 1982, pp. 451-454. doi:10.1038/298451a0

[50] E. S. Phinney and R. L. Webster, "Is There Evidence for Universal Rotation," *Nature*, Vol. 301, No. 5902, 1983, pp. 735-736.

[51] F. M. Gomide, M. S. Berman and R. L. Garcia, "A Cosmological Model with Cylindrical Symmetry," *Revista Mexicana de Astronomia y Astrofisica*, Vol. 12, 1986, pp. 46-48.

[52] R. P. Feynman, "Lectures on Gravitation," Addison-Wesley, Boston, 1962.

[53] M. S. Berman and L. A. Trevisan, Los Alamos Archives, 2001.

[54] A. Papapetrou, "Lectures on General Relativity," Reidel, Boston, 1974. doi:10.1007/978-94-010-2277-4

[55] J. M. Aguirregabiria, *et al.*, "Energy and Angular Momentum of Charged Rotating Black Holes," *General Relativity and Gravitation*, Vol. 28, No. 11, 1996, pp. 1393-1400.

[56] M. S. Berman and L. A. Trevisan, 2001. Los Alamos Archives. http://arxiv.org/abs/gr-qc/0111102

[57] M. S. Berman and L. A. Trevisan, 2001. Los Alamos Archives. http://arxiv.org/abs/gr-qc/0111101

[58] E. P. Tryon, "Is the Universe a Vacuum Fluctuation?" *Nature*, Vol. 246, No. 5433, 1973, pp. 396-397. doi:10.1038/246396a0

[59] M. G. Albrow, *Nature*, Vol. 241, 1973, p. 56.

[60] M. S. Berman, "A Primer in Black Holes, Mach's Principle and Gravitational Energy," Nova Science, New York, 2008.

[61] D. W. Sciama, "On the Origin of Inertia," *Monthly Notices of the Royal Astronomical Society*, Vol. 113, No. 1, 1953, pp. 34-42.

[62] M. S. Berman, "On the Machian Origin of Inertia," *Astrophysics Space Science*, Vol. 318, No. 3, 2008, pp. 269-272.

[63] M. S. Berman, "On Sciama's Machian Cosmology," *International Journal of Theoretical Physics*, Vol. 48, No. 2009, pp. 3257-3261. doi:10.1007/s10773-009-0112-0

[64] M. S. Berman, "Special Law of Variation for Hubbles Parameter," *IL Nuovo Cimento B*, Vol. 74, No. 2, 1983, pp. 182-186. doi:10.1007/BF02721676

[65] M. S. Berman and F. M. Gomide, "Cosmological Models with Constant Deceleration Parameter," *General Relativity and Gravitation*, Vol. 20, No. 2, 1988, pp. 191-198.

[66] J. D. Barrow, "The Inflationary Universe," In: R. Blin Stoyle and W. D. Hamilton, Eds., *Interactions and Structures in Nuclei*, Adam Hilger, Bristol, 1988, pp. 135-150.

[67] M. S. Berman, "Why the Initial Infinite Singularity of the Universe is not There," *International Journal of Theoretical Physics*, Vol. 48, No. 8, 2009, pp. 2253-2255. doi:10.1007/s10773-009-0007-0

[68] M. S. Berman, "General Relativistic Singularity-Free Cosmological Model," *Astrophysics and Space Science*, Vol. 321, 2009, p. 157.

[69] R. H. Dicke, "The Many Faces of Mach," In: H. Y. Chiu and W. F. Hoffmann, Eds., *Gravitation and Relativity*, Benjamin, New York, 1964, pp. 121-141.

[70] R. H. Dicke, "The Significance for the Solar System of Time-Varying Gravitation," In: H. Y. Chiu and W. F. Hoffmann, Eds., *Gravitation and Relativity*, Benjamin, New York, 1964.

[71] M. S. Berman, "Simple Model with Time-Varying Fine-Structure 'Constant'— Part II," *Revista Mexicana de Astronomia y Astrofisica*, Vol. 46, 2010, pp. 23-28.

[72] M. S. Berman, "Simple Model with Time-Varying Fine-Structure 'Constant'," *Revista Mexicana de Astronomia y Astrofisica*, Vol. 45, 2009, pp. 139-142.

[73] A. S. Eddington, "Expanding Universe," Cambridge University Press, Cambridge, 1933. doi:10.1017/CBO9780511564208

[74] M. S. Berman and L. A. Trevisan, "On the Creation of Universe out of Nothing," *International Journal of Modern Physics*, Vol. 19, No. 8-10, 2010, pp. 1309-1313.

[75] M. S. Berman and F. M. Gomide, 2010. http://arxiv.org/abs/1011.4627

[76] A. Albrecht and J. Magueijo, "A Time Varying Speed of Light as a Solution to Cosmological Puzzles," *Physical*

Review D, Vol. 59, No. 4, 1999, pp. 1-13.

[77] A. I. Arbab, "Quantum Universe and the Solution to the Cosmological Problems," *General Relativity and Gravitation*, Vol. 36, 2004, p. 3565.

[78] J. N. Bahcall and M. Schmidt, "The Decade of Discovery in Astronomy and Astrophysics," *Physical Review Letters*, Vol. 19, No. 22, 1967, pp. 1294-1295. doi:10.1103/PhysRevLett.19.1294

[79] J. D. Barrow, "Modern Cosmology in Retrospect," Cambridge University Press, Cambridge, 1990.

[80] J. D. Barrow, "Varying G and Other Constants," 1997.

[81] J. D. Barrow, In: K. Sato, T. Yanagida and T. Shiromizu, Eds., *Particle Cosmology*, Universal Academic Press, Tokyo, 1997, pp. 221-236.

[82] J. D. Barrow, "Cosmologies with Varying Light Speed," *Physical Review D*, Vol. 59, 1999.

[83] J. D. Barrow and J. Magueijo, "Solving the Flatness and Quasi-Flatness Problems in Brans-Dicke Cosmologies with Varying Light Speed," *Classical and Quantum Gravity*, Vol. 16, No. 4, 1999, pp. 1435-1454. doi:10.1088/0264-9381/16/4/030

[84] J. D. Bekenstein, "The Fine Structure Constant: Is it Really Constant," *Physical Review D*, Vol. 25, 1982, p. 1527. doi:10.1103/PhysRevD.25.1527

[85] M. S. Berman, "Large Number Hypothesis," *International Journal of Theoretical Physics*, Vol. 31, No. 8, 1992, pp. 1447-1450.

[86] M. S. Berman, "A Generalized Large Number Hypothesis," *International Journal of Theoretical Physics*, Vol. 31, No. 7, 1992, pp. 1217-1219. doi:10.1007/BF00673922

[87] M. S. Berman, "On a Generalized Large Number Hypothesis," *Astrophysics and Space Science*, Vol. 215, No. 1, 1994, pp. 135-136. doi:10.1007/BF00627466

[88] M. S. Berman, "Superinflation in G.R. and B.D. Theories: An Eternal Universe," *International Journal of Theoretical Physics*, Vol. 35, No. 8, 1996, pp. 1789-1792. doi:10.1007/BF02302271

[89] M. S. Berman, "Gravitomagnetism and Angular Momenta of Black-Holes," *Revista Mexicana de Astronomía y Astrofísica*, Vol. 43, No. 2, 2007, pp. 297-301.

[90] M. S. Berman, "Is the Universe a White-Hole," *Astrophysics and Space Science*, Vol. 311, No. 4, 2007, pp. 359-361. doi:10.1007/s10509-007-9515-7

[91] M. S. Berman, "General Relativistic Machian Universe," *Astrophysics and Space Science*, Vol. 318, No. 3-4, 2008, pp. 273-277. doi:10.1007/s10509-008-9929-x

[92] M. S. Berman, "Shear and Vorticity in a Combined Einstein-Cartan-Brans-Dicke Inflationary Lambda Universe," *Astrophysics and Space Science*, Vol. 314, No. 1-3, 2008, pp. 79-82.

[93] M. S. Berman, "Gravitons, Dark Matter, and Classical Gravitation," *AIP Conference Proceedings*, Sochi, 28 September-2 October 2009, pp. 1068-1071.

[94] M. S. Berman, "General Relativistic Singularity-Free Cosmological Model," *Astrophysics and Space Science*, Vol. 321, No. 3-4, 2009, pp. 157-160. doi:10.1007/s10509-009-0024-8

[95] M. S. Berman, "Entropy of the Universe," *International Journal of Theoretical Physics*, Vol. 48, No. 7, 2009, pp. 1933-1936.

[96] M. V. Berry, "Principles of Cosmology and Gravitation," Adam Hilger, Bristol, 1989.

[97] V. de Sabbata and M. Gasperini, *Lettere al Nuovo Cimento*, Vol. 25, 1979, p. 489.

[98] V. de Sabbata and C. Sivaram, "Spin and Torsion in Gravitation," World Scientific, Singapore, 1994. doi:10.1142/2358

[99] P. A. M. Dirac, "A New Basis for Cosmology," *Proceedings of the Royal Society A*, London, 5 April 1938, pp. 199-208.

[100] P. A. M. Dirac, "Cosmological Models and the Large Numbers Hypothesis," *Proceedings of the Royal Society A*, London, 14 July 1974, pp. 439-446. doi:10.1098/rspa.1974.0095

[101] A. S. Eddington, "New Pathways in Science," Cambridge University Press, Cambridge, 1935.

[102] A. S. Eddington, "The Cosmological Controversy," *Science Progress*, Vol. 34, 1939, pp. 225-236.

[103] P. H. Freud, "Uber Die Ausdrucke Der Gesamtenergie Und Des Gesamtimpulses Eines Materiellen Systems in Der Allgemeinen Relativitatstheorie," *The Annals of Mathematics*, Vol. 40, No. 2, 1939 pp. 417-419. doi:10.2307/1968929

[104] F. M. Gomide, *Letter Nuovo Cimento*, Vol. 15, 1976, p. 515.

[105] D. Halliday, R. Resnik and J. Walker, "Fundamentals of Physics," 8th Edition, Wiley, New York, 2008.

[106] L. Iorio, *Journal of Cosmology and Astroparticle Physics* Vol. 8, 2010.

[107] L. Landau and E. Lifshitz, "The Classical Theory of Fields," 4th Edition, Pergamon, Oxford, 1975.

[108] J. W. Moffat, "Possible Solution to the Initial Value Problem in Cosmology," *International Journal of Modern Physics D*, Vol. 2, No. 3, 1993, p. 351. doi:10.1142/S0218271893000246

[109] J. R. Reitz, F. J. Milford and R. W. Christy, "Foundations of Electromagnetic Theory," Addison-Wesley, Boston, 1979.

[110] V. de Sabbata and M. Gasperini, "A Semi-Minimal Coupling Principle for the Electromagnetic Field in a Space with Torsion," *Lettere al Nuovo Cimento*, Vol. 28, No. 7, 1980, pp. 229-233.

[111] B. P. Schmidt, "The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Curvature of the Universe Using Type Ia Supernovae," *The Astrophysical Journal*, Vol. 507, No. 1, 1998, pp. 46-63. doi:10.1086/306308

[112] B. Schwarzschild, "Cosmic Microwave Observations Yield More Evidence of Primordial Inflation," *Physics Today*, Vol. 54, No. 7, 2001, p. 16. doi:10.1063/1.1397385

[113] J. K. Webb, *et al.*, "Search for Time Variation of the Fine

Structure Constant," *Physical Review Letters*, Vol. 82, No. 5, 1999, pp. 884-887. doi:10.1103/PhysRevLett.82.884

[114] J. K. Webb, *et al.*, "Further Evidence for Cosmological Evolution of the Fine Structure Constant," *Physical Review Letters*, Vol. 87, No. 9, 2001, pp. 1-4. doi:10.1103/PhysRevLett.87.091301

[115] S. Weinberg, "Gravitation and Cosmology," Wiley, New York, 1972.

[116] C. Will, "300 Years of Gravitation," Cambridge University Press, Cambridge, 1987.

[117] C. Will, "General Relativity," *Proceedings of the 46th Scotish Universities Summer School in Physics*, Aberdeen, 16-29 July 1995.

[118] S. Hawking and L. Mlodinow, "The Grand Design," Bantam, New Yor, 2010.

Appendix to the Fifth Section

As we now have the pseudo-tensorial zero-total energy result, for rotation plus expansion, we might write in terms of elementary Physics, a possible energy of the Universe equation, composed of the inertial term of Special Relativity, Mc^2, the potential self-energy $-\frac{GM^2}{2R}$, and the cosmological "constant" energy, $\frac{\Lambda}{\kappa}\left(\frac{4}{3}\pi R^3\right)$, and not forgetting rotational energy, $\frac{1}{2}I\omega^2$, where I stands for the moment of inertia of a "sphere" of radius R and mass M. The energy equation is equated to zero, *i.e.*,

$$0 = Mc^2 - \frac{GM^2}{2R} + \frac{\Lambda}{\kappa}\left(\frac{4}{3}\pi R^3\right) + \frac{1}{2}I\omega^2 \quad (33)$$

It must be remembered that R is a time-increasing function, while the total-zero energy result must be time-invariant, so that the principle of energy conservation be valid. A close analysis shows that the above conditions can be met by solutions (28) and (29), which were derived or induced from the general relativistic equations. When we plug the inertia moment,

$$I = \frac{2}{5}MR^2 \quad (34)$$

we need also to consider the following Brans-Dicke generalised relations,

$$\frac{GM}{c^2R} = \Gamma = \text{constant} \quad (35)$$

and,

$$\omega = \frac{c}{R} \quad (36)$$

If we calculate the centripetal acceleration corresponding to the above angular speed, we find, for the present Universe, with $R \approx 10^{28}$ cm and $c \simeq 3.10^{10}$ cm·s^{-2}

$$a_{cp} = -\omega^2 R \cong -8\times10^{-8}\ \text{cm}/\text{s}^2 \quad (37)$$

This value matches the observed experimentally deceleration of the NASA Pioneers' space-probes.

We observe that the Machian picture above is understood to be valid for any observer in the Universe, *i.e.*, the center of the "ball" coincides with any observer; the "Machian" centripetal acceleration should be felt by any observed point in the Universe subject to observation from any other location.

We solve also other mystery concerning Pioneers anomaly. It has been verified experimentally, that those space-probes in closed (elliptical) orbits do not decelerate anomalously, but only those in hyperbolic flight. The solution of this other enigma is easy, according to our view. The elliptical orbiting trajectories are restricted to our local neighborhood, and do not acquire cosmological features, which are necessary to qualify for our Machian analysis, which centers on cosmological ground. But hyperbolic motion is not bound by the Solar system, and in fact those orbits extend to infinity, thus qualifying themselves to suffer the cosmological Machian deceleration. Thermal emission may solve the first Pioneer anomaly, but it does not solve the spin-down, nor the fly-bys in gravity assists. It is not clear why, thermal emission did not cause decelerations in elliptical orbiters. Rotation of the Universe solves all the three (Berman and Gomide in 2012 [8]).

Journal of Modern Physics, **2012, 3, 1211-1215**
doi:10.4236/jmp.2012.329156 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

On the Stability of Our Universe

Marcelo Samuel Berman, Newton C. A. da Costa
Instituto Albert Einstein/Latinamerica, Av. Sete de Setembro 4500 # 101 80250-210, Curitiba, Brazil
Email: msberman@institutoalberteinstein.org, marsambe@yahoo.com, ncacosta@institutoalberteinstein.org, ncacosta@usp.br, ncacosta@terra.com.br

Received June 16, 2012; revised July 20, 2012; accepted July 27, 2012

ABSTRACT

We argue that the Robertson-Walker's Universe is a zero-energy stable one, even though it may possess a rotational state besides expansion.

Keywords: Roberston-Walker's Universe; Rotation of the Universe; Stability

1. Introduction

The first pseudo-tensorial calculation of the energy of the Universe, has been made by Berman, in 1981 [1], in his Master of Science Thesis advised by F. M. Gomide. In his three best-sellers (Hawking, in 1996 [2]; in 2001 [3]; in 2003 [4]), Hawking describes inflation (Guth, in 1981 [5]; in 1998 [6]), as an accelerated expansion of the Universe, immediately after the creation instant, while the Universe, as it expands, borrows energy from the gravitational field to create more matter. According to his description, the positive matter energy is exactly balanced by the negative gravitational energy, so that the total energy is zero, and that when the size of the Universe doubles, both the matter and gravitational energies also double, keeping the total energy zero (twice zero). Moreover, in the recent, next best-seller, Hawking and Mlodinow in 2010 [7] comment that if it were not for the gravity interaction, one could not validate a zero-energy Universe, and then, creation out of nothing would not have happened.

In a previous paper Berman (2009 [8]) has calculated the energy of the Friedman-Robertson-Walker's Universe, by means of pseudo-tensors, and found a zero-total energy. Our main task will be to show that our possibly rotating Robertson-Walkers Universe is stable, in the sense that it has a reparametrized metric of Minkowski's, while the latter has been shown to be the ground state of energy level among possible universal metrics (see Witten, in 1981 [9]).

The zero-total-energy of the Roberston-Walker's Universe, and of any Machian ones, have been shown by many authors. It may be that the Universe might have originated from a vacuum quantum fluctuation. By "vacuum", we mean the spacetime of Minkowski. In support of this view, we shall show that the pseudotensor theory (Adler *et al.*, in 1975 [10]) points out to a null-energy for a rotating Robertson-Walker's Universe. Some prior work is mentioned: Tryon, in 1973 [11]; Berman (in 1981 [1]; in 2006 [12,13]; in 2007 [14,15], and [16]); Rosen (in 1994 [17], 1995 [18]); York Jr. in 1980 [19]; Cooperstock in 1994 [20]; Cooperstock and Israelit in 1995 [21]; Garecki in 1995 [22]; Johri *et al*. [23]; Feng and Duan in 1996 [24]; Banerjee and Sen in 1997 [25]; Radinschi, in 1999 [26]; Cooperstock and Faraoni, in (2003 [27]). See also Katz in 2006 [28], and 1985 [29]); Katz and Ori, in 1990[30]; and Katz *et al.* 1997 [31]. Recent developments include torsion models (So and Vargas, 2006 [32]), and, a paper by Xulu, in 2000 [33].

The reason for the failure of non-Cartesian curvilinear coordinate energy calculations through pseudotensors, resides in that curvilinear coordinates carry non-null Christoffel symbols, even in Minkowski spacetime, thus introducing inertial or fictitious fields that are interpreted falsely as gravitational energy-carrying (false) fields.

2. Reparametrization of Robertson-Walker's Metric

Consider first Robertson-Walker's metric, added by a temporal metric coefficient which depends only on *t*. The line element (Gomide and Uehara, 1981 [34]), becomes:

$$\mathrm{d}s^2 = -\frac{R^2(t)}{\left(1+kr^2/4\right)^2}\left[\mathrm{d}\sigma^2\right] + g_{00}(t)\,\mathrm{d}t^2 \qquad (1)$$

Of course, when g_{00} = *constant*, the above equations reproduce conventional Robertson-Walker's field equations.

We must mention that the idea behind Robertson-Walker's metric is the Gaussian coordinate system. Though the condition g_{00} = *constant*, is usually adopted, we must remember that, the resulting time-coordinate is meant as representing proper time. If we want to use another coordinate time, we still keep the Gaussian coordinate properties. Berman (2008 [35]) has interpreted the generalized metric as representing a rotating evolutionary model, with angular speed given by Berman (2011 [36]; 2011 [37]; 2012 [38-40]) and Berman and Gomide (2012 [41-43])

$$\omega = \frac{\dot{g}_{00}}{2g_{00}}$$

Consider the following reparametrization:

$$\mathrm{d}x'^2 \equiv \frac{R^2(t)}{\left(1+kr^2/4\right)^2}\mathrm{d}x^2 \tag{2}$$

$$\mathrm{d}y'^2 \equiv \frac{R^2(t)}{\left(1+kr^2/4\right)^2}\mathrm{d}y^2 \tag{3}$$

$$\mathrm{d}z'^2 \equiv \frac{R^2(t)}{\left(1+kr^2/4\right)^2}\mathrm{d}z^2 \tag{4}$$

$$\mathrm{d}t'^2 \equiv g_{00}(t)\,\mathrm{d}t^2 \tag{5}$$

In the new coordinates, the generalized RWs metric becomes:

$$\mathrm{d}s'^2 = \mathrm{d}t'^2 - \left[\mathrm{d}x'^2 + \mathrm{d}y'^2 + \mathrm{d}z'^2\right] \tag{6}$$

This is Minkowski's metric.

3. Energy and Stability of the Robertson-Walker's Metric

Even in popular Science accounts (Hawking, 1996 [2]; 2001 [3]; 2003 [4]; and Moldinow, 2010 [7]; Guth, 1981 and 1988 [5,6]), it has been generally accepted that the Universe has zero-total energy. The first such claim, seems to be due to Feynman, in years 1962-1963 [44]. Lately, Berman (2006 [12,13]) has proved this result by means of simple arguments involving Robertson-Walker's metric for any value of the tri-curvature ($0, -1, 1$).

Berman and Gomide (2012 [41-43]) has recently shown that the generalized Robertson-Walker's metric yielded a zero-energy pseudotensorial result. The same authors showed that the result applied in case of a rotating and expanding Universe.

The equivalence principle, says that at any location, spacetime is (locally) flat, and a geodesic coordinate system may be constructed, *where the Christoffel symbols are null*. *The pseudotensors are*, *then*, *at each point*, *null*. *But now remember that our old Cosmology requires a co-moving observer at each point. It is this co-motion that is associated with the geodesic system, and*, *as RWs metric is homogeneous and isotropic*, *for the co-moving observer*, *the zero-total energy density result*, *is repeated from point to point*, *all over spacetime*. *Cartesian coordinates are needed*, *too*, *because curvilinear coordinates are associated with fictitious or inertial forces*, *which would introduce inexistent accelerations that can be mistaken additional gravitational fields* (*i.e.*, *that add to the real energy*). *Choosing Cartesian coordinates is not analogous to the use of center of mass frame in New-tonian theory*, *but the null results for the spatial components of the pseudo-quadrimomentum show compatibility.*

Witten in 1981 [9], proved that within a semiclassical approach, Minkowski's space was in the ground state of energy, which was zero-valued. He also showed that in Classical General Relativity, this space also was the unique space of lowest energy. This last result was obtained with spinor calculus, and thus could be extended to higher dimensions whenever spinors existed. The proof was obtained through the study of the limit $h \to 0$ of a supergravity argument by Deser and Teitelboim, in 1977 [45], and by Grisaru, in 1978 [46], where h stands for Planck's constant.

The conclusion of Witten was that Minkowski's space was also stable, because perturbations in the form of gravitational waves should not decrease the total energy, because it is known that gravitational waves have positive energy. We now conclude that our Universe is also stable, due to the reparametrization above. But, first, let us deal with some conceptual issues.

We have three kinds of stability criteria: 1) Since a physical system shows a tendency to decay into its state of minimum energy, the criterion states that the system should not be able to collapse into a series of infinitely many possible negative levels of energy. There should be a minimum level, usually zero-valued, which is possible for the physical system; 2) The matter inside the system must not be possibly created out of nothing,or else, the bodies should have positive energy; 3) "Small" disturbances should not alter a state of equilibrium of the system (it tends to return to the original equilibrium state). In the case of the Universe, disturbances, of course, cannot be external.

According with our discussion, the rotating Robertson-Walkers Universe is locally and globally stable, whenever Classical Physics is concerned. Now, Berman and Trevisan (in 2010 [47]), have shown that Classical General Relativity can be used to describe the scalefactor of the Universe even inside Plancks zone, provided that we consider that the calculated scale-factor behaviour reflects an average of otherwise uncertain values, due to Quantum fluctuations, as Berman and Trevisan suggested in several papers at Los Alamos Archives, during the last decade, and in 2010, when it was published paper [47].

4. Final Comments and Conclusions

Berman and Gomide (2012 [41-43]) and Berman (2012

[38,39]) have obtained a zero-total energy proof for a rotating expanding Universe. The zero result for the spatial components of the energy-momentum-pseudotensor calculation, are equivalent to the choice of a center of Mass reference system in Newtonian theory, likewise the use of comoving observers in Cosmology. It is with this idea in mind, that we are led to the energy calculation, yielding zero total energy, for the Universe, as an acceptable result: we are assured that we chose the correct reference system; this is a response to the criticism made by some scientists which argue that pseudotensor calculations depend on the reference system, and thus, those calculations are devoid of physical meaning.

Related conclusions should be consulted (see all Berman's references and references therein). As a bonus, we can assure that there was not an initial infinite energy density singularity, because attached to the zero-total energy conjecture, there is a zero-total energy-density result, as was pointed by Berman elsewhere (see, for instance, Berman, in 2009 [48,49]). The so-called total energy density of the Universe, which appears in some textbooks, corresponds only to the non-gravitational portion, and the zero-total energy density results when we subtract from the former, the opposite potential energy density (Berman, 2012 [38,39]).

As Berman (2009 [49,50]) shows, we may say that the Universe is *singularity-free*, and was created *ab-nihilo*; in particular, there is no zero-time infinite energy-density singularity.

Rotation of the Universe and zero-total energy were verified for Sciama's linear theory, which has been expanded, through the analysis of radiating processes, by one of the present authors (Berman, 2008 [51]; 2009 [52]). There, Berman found Larmor's power formula, in the gravitational version,that leads to the correct constant power relation for the Machian Universe. However, we must remember that in local Physics, General Relativity deals with quadrupole radiation, while Larmor is a dipole formula; for the Machian Universe the resultant constant power is basically the same, either for our Machian analysis or for the Larmor and general relativistic formulae.

Referring to rotation, it could be argued that cosmic microwave background radiation should show evidence of quadrupole asymmetry and it does not, but one could argue that the angular speed of the present Universe is too small to be detected; also, we must remark that CMBR deals with null geodesics, while Pioneers' anomaly, for instance, deals with time-like geodesics. In favor of evidence on rotation, we remark neutrinos' spin, parity violations, the asymmetry between matter and antimatter, left-handed DNA-helices, the fact that humans and animals alike have not symmetric bodies, the same happening to molluscs.

We predict that chaotic phenomena and fractals, rotations in galaxies and clusters, may provide clues on possible left handed preference through the Universe.

Berman and Trevisan (2010 [47]) have remarked that creation out-of-nothing seems to be supported by the zero-total energy calculations. Rotation was included in the derivation of the zero result by Berman and Gomide (2012 [41-43]). We could think that the Universes are created in pairs, the first one (ours), has negative spin and positive matter; the second member of the pair, would have negative matter and positive spin: for the ensemble of the two Universes, the total mass would always be zero; the total spin, too. The total energy (twice zeros) is also zero.

Hawking and Mlodinow (2010 [7]) conclude their book with a remark on the fact that the Universe is locally stable, but globally unstable because spontaneous creation is the reason why the Universe exists, and new creations like this may still happen. Of course, this is a question of interpretation.

We now want to make a conjecture related to the stability criteria of last Section.

A physical system is not "chaotic", if small perturbations in its initial state do not originate "large" variations in its future behaviour. According to our discussion, the Robertson-Walkers Universe, with or without rotation, is locally and globally stable under the three criteria. As its total energy is zero, we conjecture that this type of Universe is not globally chaotic, and that the three criteria for stability imply that any such system cannot be globally chaotic altogether. We remark nevertheless, that because Einsteins field equations are non-linear, chaos is not forbidden in a local sense.

We regret that the name of a basic result in General Relativity Theory, is called "positive energy theorem" instead of the "non-negative energy theorem". Experimental observational evidence on the rotation of the Universe is dealt with, in the books by Berman (2012 [38, 39]), and references therein. Seminal papers on rotation evidence were due to Paul Birch in 1982, in the well-known *Nature* .

5. Acknowledgements

One of the authors (MSB) thanks Marcelo Fermann Guimarães, Nelson Suga, Mauro Tonasse, Antonio F. da F. Teixeira, and for the important incentive offered by Miss Solange Lima Kaczyk,now a brand new advocate, continued during the last five years of his research in Cosmology.

REFERENCES

[1] M. S. Berman, M.S. Thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos, 1981. http://www.sophia.bibl.ita.br/biblioteca/index.html

[2] S. W. Hawking, "The Illustrated a Brief History of Time," Bantam Books, New York, 1996, pp. 166-167.

[3] S. W. Hawking, "The Universe in a Nutshell," Bantam Books, New York, 2001, pp. 90-91.

[4] S. W. Hawking, "The Illustrated Theory of Everything," Phoenix Books, Beverly Hills, 2003, p. 74.

[5] A. Guth, Phys. Rev. D, Vol. 23, 1981, p. 347 .

[6] A. Guth, "The Inflationary Universe," Vintage, New York, 1998, p. 12.

[7] S. W. Hawking and L. Mlodinow, "The Grand Design," Bantam Books, New York, 2010.

[8] M. S. Berman, "On the Zero-Energy Universe," *International Journal of Theoretical Physics*, Vol. 48, No. 11, 2009, pp. 3278-3296. doi:10.1007/s10773-009-0125-8

[9] E. Witten, "A New Proof of the Positive Energy Theorem," *Communications in Mathematical Physics*, Vol. 80, No. 3, 1981, pp. 381-402. doi:10.1007/BF01208277

[10] R. J. Adler, M. Bazin and M. Schiffer, "Introduction to General Relativity," 2nd Edition, McGraw-Hill, New York, 1975.

[11] E. P. Tryon, "Is the Universe a Vacuum Fluctuation?" *Nature*, Vol. 246, No. 5433, 1973, pp. 396-397. doi:10.1038/246396a0

[12] M. S. Berman, "Energy of Black-Holes and Hawkings Universe," In: P. V. Kreitler, Ed., *Trends in Black Hole Research*, Nova Science, New York, 2006.

[13] M. S. Berman, "Energy, Brief History of Black-Holes, and Hawkings Universe," In: P. V. Kreitler, Ed., *New Developments in Black Hole Research*, Nova Science, New York, 2006.

[14] M. S. Berman, "Introduction to General Relativity and the Cosmological Constant Problem," Nova Science, New York, 2007.

[15] M. S. Berman, "Introduction to General Relativistic and Scalar Tensor Cosmologies," Nova Science, New York, 2007.

[16] M. S. Berman, "The Pioneer Anomaly and a Machian Universe," *Astrophysics and Space Science*, Vol. 312, 2007, p. 275. http://arxiv.org/abs/physics/0606117.

[17] N. Rosen, "The Energy of the Universe," *General Relativity and Gravitation*, Vol. 26, No. 3, 1994, pp. 319-321. doi:10.1007/BF02108013

[18] Rosen, N. GRG, Vol. 27, 1995, p. 313.

[19] J. W. York Jr, "Energy and Momentum of the Gravitational Field," In: F. J. Tipler, Ed., *A Festschrift for Abraham Taub*, Academic Press, New York, 1980.

[20] F. I. Cooperstock, - GRG Vol. 26, 1994, p. 323.

[21] F. I. Cooperstock and M. Israelit, "The Energy of the Universe," *Foundations of Physics*, Vol. 25, No. 4, 1995, pp. 631-635. doi:10.1007/BF02059009

[22] J. Garecki, GRG, Vol. 27, 1995, p. 55.

[23] V. B. Johri, *et al.*, GRG, Vol. 27, 1995, p. 313.

[24] S. Feng and Y. Duan, "About the Energy of the Universsity," *Chinese Physics Letters*, Vol. 13, No. 6, 1996, p. 409. doi:10.1088/0256-307X/13/6/003

[25] N. Banerjee and S. Sen, Pramana J.Phys., Vol. 49, 1997, p. 609.

[26] I. Radinschi, Acta Phys. Slov., Vol. 49, 1999, p. 789.

[27] F. I. Cooperstock and V. Faraoni, Ap.J. Vol. 587, 2003, p. 483.

[28] J. Katz, Private Communication, 2006.

[29] J. Katz, "A Note on Komar's Anomalous Factor," *Classical and Quantum Gravity*, Vol. 2, No. 3, 1985, p. 423. doi:10.1088/0264-9381/2/3/018

[30] J. Katz and A. Ori, "Localisation of Field Energy," *Classical and Quantum Gravity*, Vol. **7**, No. 5, 1990, p. 787. doi:10.1088/0264-9381/7/5/009

[31] J. Katz, J. Bicak and D. Lynden-Bell, *Physical Review D*, Vol. 55, 1997, p. 5957.

[32] L. L. So and T. Vargas, Los Alamos Archives, 2006.

[33] S. Xulu, *International Jounal of Theoretical Physics*, Vol. 39, 2000, p. 1153.

[34] F. M. Gomide and M. Uehara, *Astronomy and Astrophysics*, Vol. 95, 1981, p. 362.

[35] M. S. Berman, "A General Relativistic Rotating Evolutionary Universe," *Astrophysics and Space Science*, Vol. 314, No. 4, 2008, pp. 319-321. doi:10.1007/s10509-008-9772-0

[36] M. S. Berman, "The Two Pioneers Anomalies and Universal Rotation," *Astrophysics and Space Science*, Vol. 336, No. 2, 2011, pp. 337-339. doi:10.1007/s10509-011-0825-4

[37] M. S. Berman, "General Relativity with Variable Speed of Light and Pioneers Anomaly," *Astrophysics and Space Science*, Vol. 336, No. 2, 2011, pp. 327-329.

[38] M. S. Berman, "General Relativity and the Pioneers Anomaly," Nova Science Publishers, New York, 2012.

[39] M. S. Berman, "Realization of Einstein's Machian Program," Nova Science Publishers, New York, 2012.

[40] M. S. Berman, "Realization of Einsteins Machian Program: The Pioneers and Fly-By Anomalies," *Astrophysics and Space Science*, Vol. 337, No. 1, 2012, pp. 477-481.

[41] M. S. Berman and F. M. Gomide, "General Relativistic Treatment of Pioneers Anomaly," *Journal of Modern Physics*, *Special Issue on Gravitation*, *Astrophysics and Cosmology*, 2012.

[42] M. S. Berman and F. M. Gomide, "Relativistic Cosmology and the Pioneers Anomaly," *Journal of Modern Physics*, *Special Issue on Gravitation*, *Astrophysics and Cosmology*, 2012.

[43] M. S. Berman and F. de M. Gomide, "On the Rotation of the Zero-Energy Expanding Universe," In: J. R. O'Connell and A. L. Hale, Eds., *The Big-Bang-Theory*, *Assumptions and Problems*, Nova Science Publishers, New York, 2012.

[44] R. P. Feynman, "Lectures on Gravitation," Addison-Wesley, Reading, 1962-1963.

[45] S. Deser and C. Teitelboim, *Physical Review Letters*, Vol. 39, 1977, p. 249. doi:10.1103/PhysRevLett.39.249

[46] M. Grisaru, *Physical Letters*, Vol. 73B, 1978, p. 207.

[47] M. S. Berman and L. A. Trevisan, *International Journal of Modern Physics D*, Vol. 19, 2010, pp. 1309-1313. http://arxiv.org/abs/gr-qc/0104060

[48] M. S. Berman, "General Relativistic Singularity-Free Cosmological Model," *Astrophysics and Space Science*, Vol. 321, No. 3-4, 2009, pp. 157-160. doi:10.1007/s10509-009-0024-8

[49] M. S. Berman, "Why the Initial Infinite Singularity of the Universe Is Not There?" *International Journal of Theoretical Physics*, Vol. 48, No. 8, 2009, pp. 2253-2258. doi:10.1007/s10773-009-0007-0

[50] M. S. Berman, "General Relativistic Singularity-Free Cosmological Model," *Astrophysics and Space Science*, Vol. 321, 2009, p. 157. http://arxiv.org/abs/0904.3141

[51] M. S. Berman, "On the Machian Origin of Inertia," *Astrophysics Space Science*, Vol. 318, 2008, pp. 269-272. http://arxiv.org/abs/physics/0609026

[52] M. S. Berman, "On Sciama's Machian Cosmology," *International Journal of Theoretical Physics*, Vol. 48, No. 11, 2009, pp. 3257-3261. doi:10.1007/s10773-009-0112-0

[53] D. W. Sciama, M.N.R.A.S., Vol. 113, 1953, p. 34.

[54] M. S. Berman, "A Primer in Black Holes, Mach's Principle and Gravitational Energy," Nova Science, New York 2008.

Journal of Modern Physics, **2012, 3, 1216-1222**
doi:10.4236/jmp.2012.329157 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Confidence Level Estimator of Cosmological Parameters

G. Sironi
Physics Department, University of Milano Bicocca, Milano, Italy
Email: giorgio.sironi@unimib.it

Received June 21, 2012; revised July 24, 2012; accepted July 31, 2012

ABSTRACT

Cosmological Models frequently suggest the existence of physical, quantities, e.g. dark energy, we cannot yet observe and measure directly. Their values are obtained indirectly setting them equal to values and accuracy of the associated model parameters which best fit model and observation. Apparently results are so accurate that some researchers speak of precision cosmology. The accuracy attributed to these indirect values of the physical quantities however does not include the uncertainty of the model used to get them. We suggest a Confidence Level Estimator to be attached to these indirect measurements and apply it to current cosmological models.

Keywords: Cosmological Models; Cosmological Parameters

1. Introduction

Models of physical systems, including Cosmological Models, contain a number of free parameters associated to an equal number of measurable, independent, physical quantities ("observable" in the following) which characterize the system. Comparing measured values of the observables and allowed values of the parameters one can test a model, *i.e.* validate, improve or falsify it [1].

When a model introduces new parameters associated to observables previously ignored or never observed, searching and measuring the new observables is mandatory.

Some observables can be measured directly (e.g. galaxy redshift) or through a serie of definite, model independent, intermediate steps (e.g. object distance by parallax). Let's call them direct measurements.

Other observables (e.g. Dark Energy density we will discuss in the following), cannot yet be measured directly. We get their values looking to secondary observables linked in a way we presume we know to the primary observable we are interested in. Let's call them indirect measurements. The reliability of indirect measurements depends therefore on the accuracy of the link model, preferably an "ad hoc" model, with a reduced number of parameters, especially made for the particular observable we intend to measure, but in some cases it is the Model itself we want to test.

Present days cosmological models give a fair description of the birth and evolution of the Universe using six free parameters. Mostly of the associated observables are however measured indirectly.

In the following we discuss first the error bars associated to direct and indirect measurements of observables, then introduce an estimator (Confidence Level Estimator) to quantify the confidence we can attach to indirect measurements. We then briefly review present day most common Cosmological Models and apply to their parameters and observables our Confidence Level Estimator.

2. Observables: Expected and Measured Values

Results of independent direct measurements of an observable X give a serie $\{X\}$ of data which, analyzed by classical statistical methods (see for instance [2]) give mean value $\bar{X}$ and standard deviation σ_{me} of X. We call them *measured values* of X.

When X must be measured indirectly we collect by direct observation or from data in literature values of secondary observables associated to X, specify the model of the link between X and those secondaries and attach to X the value of the associated model parameter M_X which best fits model and values of the secondary observables. When considering Cosmological Models the best fitting procedure is usually made by Montecarlo methods: one repeats the evaluation of M_X with a random choice of the Model parameters and gets a distribution of M_X values around a value E_M, which optimizes the fit. We then set $\bar{X} \simeq E_M$ and attach to it a dispersion σ_{ex} equal to the width of the distribution of the M_X values around E_M which encompasses 68% of the values. We call them *expected values*. However σ_{ex} does not include information on how reliable is the model of the link between primary and secondary observables therefore is different from σ_{me}. If we are extremely confident in the model we can set $\sigma_{me} \simeq \sigma_{ex}$. If not we must write $\sigma_{me} \simeq (\sigma_{ex} + \sigma_{pr})$ where σ_{pr} is a sort of systematic error which accounts for Model uncertainty.

2.1. Single Parameter Model

Let's begin with a Model with just one parameter *M*. We call $P(X)$ the distribution, around $\bar{X}_M$, of the directly measured values *X* of the observable and $P(E)$ the distribution, around E_M, of the values attributed to parameter *M*. Most often $P(X)$ and $P(E)$ are gaussian

$$P(X) = \frac{1}{\sqrt{2\pi\sigma_{me}^2}} e^{-(X-\bar{X}_M)^2 / 2\sigma_{me}^2} \tag{1}$$

$$P(E) = \frac{1}{\sqrt{2\pi\sigma_{ex}^2}} e^{-(E-E_M)^2 / 2\sigma_{ex}^2} \tag{2}$$

when

$$\left|\bar{X}_M - E_M\right| \gg (\sigma_{me} + \sigma_{ex})$$

$P(X)$ and $P(E)$ overlap marginally and the Model does not describe properly the physical system associated to the observable.

When

$$\left|\bar{X}_M - E_M\right| \leq\sim 3(\sigma_{me} + \sigma_{ex})$$

$P(X)$ and $P(E)$ begin to overlap and the Model may be more descriptive of the physical system.

For models and priors[1] not too far from the real world

$$E_M \to \bar{X}_M \tag{3}$$

and differences between $P(X)$ and $P(E)$ can be entirely attributed to differences between σ_{ex}^M and σ_{me}^M, so we can write

$$\sigma_{me}^M = \sigma_{ex}^M + \sigma_{pr}^M$$

Evaluating σ_{pr} is far from obvious. We introduce instead

$$C_{l,M} = 1 - \frac{\sigma_{pr}^M}{\sigma_{me}^M} = \frac{\sigma_{ex}^M}{\sigma_{me}^M} \tag{4}$$

and call it Confidence Level Estimator of the observable associated to *M*.

For direct measurements $\sigma_{pr} = 0$ therefore $C_{l,M} = 1$ always. For secondary measurements $0 \leq C_{l,M} \leq 1$ because $\sigma_{ex} \leq \sigma_{me}$. $C_{l,M} \simeq 1$ suggests that Model, *priors* and observation do not conflict therefore the Model offer a convenient way for improving $\bar{X}^M$. When $C_{l,M} \to 0$ Model and *priors* cannot be used for improving $\bar{X}^M$ and very probably do not offer a completely correct description of the real world.

More formal derivations of $C_{l,M}$ are possible. In Appendix A we propose a derivation $C_{l,m}$ from the Bayes Theorem.

[1]In different applications meaning and content of *prior* may be different here and in the following *prior* will indicate just the ranges of allowed variability of the Model parameters.

2.2. Multiparameter Model

For a model with m independent parameters we can write:

$$C_l = \frac{1}{m} \sum_{i=1}^{m} C_{l,M^i} \tag{5}$$

as a collective estimator of the set of Model parameters. It is the average of the C_{l,M^i}, (see Equation (4)), associated to the Model m parameters. Its value is also indicative of Model and *priors* qualities.

Small values of Confidence Levels obtained by Equations (4) and (5) cannot be used to falsify the Model used to get them. Falsification in fact occurs only when the two probability distributions $P(X)$ and $P(E)$ do not overlap at all. When this is the case condition (3) and Equations (4) and (5) do not hold anymore.

3. Cosmological Models

The almost serendipitous discovery of the Cosmic Microwave Background in 1964 [3]: 1) marked the end of a famous revised version of the Steady State Model, proposed in 1948 by Bondi, Gold and Hoyle [4,5], in spite of its capability of preserving fundamental constants of physics and avoiding singularities during the Universe expansion; 2) boosted the class of the Big Bang Models (e.g. [6]). Difficulties of this class of models, like the initial singularity and the problem of "causal connections", were soon solved by the inclusion of the Inflation theory (see for instance [7]) with the additional bonus of gaining the possibility of estimating the spectrum of the primordial fluctuations, necessary to explain the birth of the matter condensations which characterize the present day Universe (for a review see for instance [8]); 3) triggered new cosmological observation of the CMB which, in about thirty years, confirmed that the CMB has: a) planckian spectrum ([9-11] and references therein); b) a small degree of anisotropy with a characteristic angular power spectrum ([12-14] and references therein); c) an even smaller degree of linear polarization ([15-18] and references therein).

So gradually the Standard Big Bang Model ([6]) emerged, whose more important parameters were: Hubble constant H_o, Universe matter density Ω (in units of critical density $\rho_c = 3H_o^2 / (8\pi G)$) and primordial Helium Hydrogen ratio.

In the same years other no-CMB based cosmological observations went on. They: 1) provided large samples of high redshift Supernovae (see for instance the Supernova Cosmological Project [19]), used to obtain better estimates of matter density and upper limit to the value of the cosmological constant); 2) showed the existence of dark matter at various astrophysical sites (for a review see for instance [20]); 3) detected Baryon Acoustic Oscillations (BAO) in the ordinary matter distribution with an angular power spectrum similar to the angular power spectrum of the CMB ([21] and references therein); 4) got the distances of objects at very large Z using new standard candles (SNIa and GRB) (e.g. [22,23] and references therein). Not to mention the results of numerical experiments (N-body simulations) on the formation and evolution of matter condensations (e.g. [24]).

The whole set of CMB and no-CMB observations suggests that: 1) the geometry of the Universe is euclidean (flat) or very close to it [25]; 2) recombination of nuclei and electrons at $Z \simeq 1000$ was followed by partial reionization of the matter when stable matter condensations formed (e.g. [26] and references therein); 3) after decelerating, the Universe is now going through re-acceleration (e.g. [27]).

To account for these effects new cosmological parameters were introduced: 1) Ω_b, Ω_{dm} and Ω_Λ, the abundances (in unit of critical density ρ_c) of barionic matter, dark matter and dark energy; 2) τ, the optical thickness of the Universe at reionization; 3) A_s and n_s, the amplitude and spectral index of the fluctuations; 4) σ_8 an indicator of the galactic matter distribution. Adding them to the Standard Big Bang Model the Concordance or Λ-CDM Model emerged [28,29]. It is characterized by six independent parameters plus a number of derived parameters, combinations of the independent ones. Usually H_o, Ω_b, Ω_{dm}, τ, A_s and n_s, are assumed as free parameters. Among the derived parameters are age of the Universe $T_{univ} \approx 1/H_o$, critical density ρ_c of matter-energy, Dark Energy density, Ω_Λ (in unit of ρ_c), reionization red shift Z_{ion} and σ_8.

Usually observation gives combinations of the observables associated to the above parameters. To disentangle them it is common practice to fit the Concordance Model to the full set of CMB and no-CMB data and extract the parameters values which best fit observation. Calculations are made by Montecarlo methods [30] using Markov chains to implement the stochastic procedure with the addition of *priors* which constrain the variability of the model parameters. Common choices are $\Omega = 1$ (perfectly euclidean Universe) and $\Omega = \Omega_b + \Omega_{dm} + \Omega_\Lambda$. By this method one gets the *expectation values* of the model parameters and their dispersion, (set equal to the width of the distribution of the calculated values E which encompasses 68% of the values) and attach them to the associated observables.

The procedure, now well established, is usually repeated whenever new data are added to the preexisting data base of observations. Very probably it will be repeated when the new CMB data presently being collected by the Planck mission will be released [31]. **Table 1** and **Table 2** show expectation values E_M and dispersion σ_{ex}^M of free and derived parameters *M* of the Concordance Model in literature [18,32,34]. Because different authors use different combinations of parameters and/or different units of measure, for uniformity of presentation in **Tables 1** and **2**, when necessary, the listed quantities have been obtained transforming the data in literature (preserving the published value of the combination).

In the same table are listed, when available, mean value $\bar{X}^M$ and standard deviation σ_{me}^M of direct measurements *X* of the observable associated to *M*.

It appears that for five, out of six, free parameters of the Concordance Model direct measurements of the associated observable are poor or not yet available. In particular observation gives only large intervals inside which measured values of the density of Dark Matter and Dark Energy can lay. These intervals coincide with the variability range of the parameters used in Monte Carlo studies of the Concordance Model [18]. The only exception is the Hubble constant for which accurate measurements now exist [33].

Table 1. Λ-CDM -Concordance Model: expectation values, measured values and confidence level of model parameters (from [34,18] see text).

Parameter/Observable		E_M Expec. Value	$\bar{X}_M$ Meas. Value	C_l Conf. Level
Hubble Constant (km/sec Mpc)	H_o	$70.4^{+1.3}_{-1.4}$	74.2 ± 3.6	0.38
Barionic Matter Density	Ω_b	0.0456 ± 0.0016	$0.005 - 0.1$	$< 2 10^{-2}$
Dark Matter Density	Ω_{dm}	0.227 ± 0.0014	$0.006 - 1$	$< 2 10^{-3}$
Optical Thickness at Reionization	τ	0.087 ± 0.0014	$0.01 - 0.80$	$< 2 10^{-3}$
Scalar Fluctuations Amplitude	A_s	$(2.441^{+0.088}_{-0.092} 10^{-9})$	?	?
Scalar Spectral Index	n_s	0.963 ± 0.012	$0.5 - 1.5$	$< 2 10^{-2}$

Table 2. Λ-CDM -Concordance Model: expectation values of derived parameters (from [18,34] see text).

Parameter/Observable		Expected Value
Dark Energy Density	Ω_Λ	$0.728^{+0.015}_{-0.016}$
Reionization Red Shift	Z_{ion}	10.4 ± 1.2
Galactic Fluctuations Amplitude	σ_8	0.809 ± 0.024
Universe Age (years)	t_o	$(13.75 \pm 0.11)10^9$

The above values of σ_{ex}^{M} are so small that today is common practice to speak of Precision Cosmology (e.g. [35,36]) and very probably they will be further reduced when the Planck results will appear. A caveat is however necessary: generally $\sigma_{ex}^{M} \le \sigma_{me}^{M}$ and in some cases $\sigma_{ex}^{M} \ll \sigma_{me}^{M}$. So when model assumptions fail, σ_{ex}^{M} might be optimistic and the stated precision of inference might understate the actual uncertainty of the observable.

4. Discussion

Analysis of cosmological observation and deduction of cosmological parameters in literature not always explicitly refers to Bayesian statistics so the language used is not necessarily the one which would be used by a Bayesian statistician (see [37] and references therein) Bayesian statistics however can provide useful hints at least about:

1) Dispersion of the *priors* (see for instance [38] and references therein). In its more common implementations the Concordance Model sets the very stringent limits $\Omega = \left(\Omega_b + \Omega_{dm} + \Omega_\Lambda\right) \simeq 1$. Assuming a Bayesian point of view there is therefore a risk that the *priors* on Ω, Ω_b, Ω_{dm} and Ω_Λ are underdispersed, undermining the validity the analysis results. Therefore these limits probably have to be relaxed.

2) Robustness of the results (see for instance [39] and references therein). The results so far published do not show evidence of oscillations of the values of the Concordance Model parameters around their expectation values, confirming that from a Bayesian point of view these results are robust.

But our Confidence Level Estimators hold also outside the borders of Bayesian Statistics. Expression (14) has been in fact obtained also on empirical basis (see Equation (4)).

So we will use our Estimators to evaluate the weight we can attach to observables and cosmological parameters provided by the Concordance Model, no matter if the procedures the authors ([18,32,34] and references therein) used to get them are fully Bayesian or not.

The last Column of **Table 1** shows the Confidence Levels of the Concordance Model free parameters, calculated by Equation (4) and approximation (3). For the whole Model, Equation (5) gives

$$C_l \sim 0.07$$

They do not falsify the Concordance Model (the distributions of the expected values of all the parameters of the Concordance Model are well inside the uncertainty intervals of the measured values).

However, with the exception of the Hubble constant, the differences $\sigma_{ex} - \sigma_{me}$ of the parameters are so large that σ_{pr} (see Section 2) and the role played by Model and *priors* can't be neglected. So we cannot assume the expectation values of some parameters of the Concordance Model as representative of the values of the associated observables, for instance when studying astrophysical situations where dark matter, dark energy, if present, are important.

This leaves open the possibility of considering other Models of the Universe different from the Concordance Model which is based on the strong double condition $\Omega = \Omega_b + \Omega_{dm} + \Omega_\Lambda = 1$. We can for instance keep $\Omega = 1$ and relax the condition $\Omega = \Omega_b + \Omega_{dm} + \Omega_\Lambda$, assuming a different recipe of the Universe composition, e.g. without or with a reduced quantity of Dark Energy, an exigency remarked also very recently (see for instance the comments by [40]). In fact: 1) no direct evidence for the existence of Dark Energy has been so far obtained; 2)in literature there are models which show the possibilities of producing effects similar to those attributed to the presence of Dark Energy, through inhomogeneities of the matter distribution (e.g. [41]). The work on these Models is still in progress. We cannot yet apply to them the same procedure used with the Concordance Model and extract expectation values for their parameters. Comparison of their Confidence Levels with the Confidence Levels of **Table 1** will probably become possible in the near future.

Meanwhile, it is necessary: 1) to improve direct measurements of all the observables associated to the Concordance Model parameters, aiming at $\bar{X}^i \simeq E_M^i$ and $\sigma_{me}^i \simeq \sigma_{ex}^i$ for all the parameters i, and/or 2) to get independent evidence of existence and weight of Ω_Λ, the Dark Energy density.

The above conclusions remain also if one adds to the set of preexisting data results of new indirect evaluations of the Cosmological Parameters more recently published (e.g. [34,42]). Probably they will not change until new direct measurements or indirect measurements based on other independent models will appear.

5. Conclusions

The use of Montecarlo methods and Bayesian Statistics to analyze the enormous quantity of data of cosmological interest which are continously poured by ground and space observations is almost unavoidable. However Montecarlo and Bayesian Methods are based on assumption (models and priors) whose statistical weight should be added, but rarely is added, to the the quoted accuracies of the parameter expectation values.

Researchers who currently use these methods are aware of that and warning has been already put forward (e.g. [36,43]). Unfortunately general public and professionals not involved in cosmological observations may be unaware of it, misinterpret the results of model simulations and attribute weights above their real values to models. Forgetting it may stop or reduce support to studies of other models not yet excluded by observation.

This situation is common to other fields of pure and applied research (e.g. unification of fundamental forces, string theories, elementary particle models, models of climate evolution and so on). The Confidence Level Estimator we propose can be used to avoid misunderstandings and preserve possibilities of pursuing alternatives lines of research also in these fields.

6. Acknowledgements

We acknowledge the support of MIUR (Italian Ministry of University and Research), CNR (Italian National Council of Research), PNRA (Italian Program for Antarctic Research) and (Italian Space Agency) to studies and observations of the CMB properties.

REFERENCES

[1] K. Popper, "Conjecture and Refutations: The Growth of Scientific Knowledge," Taylor and Francis, Abingdon, 1989.

[2] P. R. Bevington and K. D. Robinson, "Data Reduction and Error Analysis for the Physical Sciences," McGraw-Hill, London, 1992.

[3] A. A. Penzias and R. A. Wilson, "A Measurement of Excess Antenna Temperature at 4080 Mc/s," *Astrophysical Journal*, Vol. 142, No. 7, 1965, pp. 419-421. doi:10.1086/148307

[4] H. Bondi and T. Gold, "The Steady State Theory of the Expanding Universe," *Monthly Notices of the Royal Astronomical Society*, Vol. 108, No. 2, 1948, pp. 252-270.

[5] F. Hoyle, "A New Model of the Expanding Universe," *Monthly Notices of the Royal Astronomical Society*, Vol. 108, No. 3, 1948, pp. 372-382.

[6] P. J. E. Peebles, "Principles of Physical Cosmology," Princeton University Press, Princeton, 1993.

[7] A. H. Guth, "The Inflationary Universe," Perseus Book, Reading, 1997.

[8] R. B. Partridge, "3K: The Cosmic Microwave Background Radiation," Cambridge University Press, Cambridge, 1995. doi:10.1017/CBO9780511525070

[9] D. J. Fixsen and J. C. Mather, "The Spectral Results of the Far Infrared Absolute Spectrophotometer on COBE," *Astrophysical Journal*, Vol. 581, No. 12, 2002, pp. 817-822. doi:10.1086/344402

[10] G. F. Smoot, *et al.*, "Low Frequency Measurements of the Cosmic Bacground Radiation Spectrum," *The Astrophysical Journal Letters*, Vol. 291, No. 4, 1985, pp. L23-L27. doi:10.1086/184451

[11] M. Zannoni, *et al.*, "TRIS I: Absolute Measurements of the Sky Brightness Temperature at 0.6, 0.82 and 2.5 GHz," *Astrophysical Journal*, Vol. 688, No. 11, 2008, pp. 12-23. doi:10.1086/592133

[12] C. L. Bennett, *et al.*, "Four-Year COBE DMR Cosmic Microwave Background Observations: Maps and Basic Results," *Astrophysical Journal*, Vol. 464, No. 6, 1996, pp. L1-L4. doi:10.1086/310075

[13] D. Larson, *et al.*, "Seven-Years Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and WMAP-Derived Parameters," *The Astrophysical Journal Supplement*, Vol. 192, No. 16, 2011, pp. 1-19.

[14] R. B. Friedman, *et al.*, "Small Angular Scale Measurements of the Cosmic Microwave Background Temperature Power Spectrum from QUaD," *Astrophysical Journal*, Vol. 700, No. 8, 2009, pp. L187-L191. doi:10.1088/0004-637X/700/2/L187

[15] A. Kogut, *et al.*, "Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Temperature-Polarization Correlatioin," *The Astrophysical Journal Supplement*, Vol. 148, No. 9, 2003, pp. 161-173. doi:10.1086/377219

[16] F. Piacentini, *et al.*, "A Measurement of the Polarization-Temperature Angular Cross-Power Spectrum of the Cosmic Microwave Background from the 2003 Flight of BOOMERANG," *Astrophysical Journal*, Vol. 647, No. 8, 2006, pp. 833-839. doi:10.1086/505557

[17] G. Polenta, *et al.*, "The BRAIN CMB Polarization Experiment," *New Astronomy Reviews*, Vol. 51, No. 3, 2007, pp. 256-259. doi:10.1016/j.newar.2006.11.065

[18] M. L. Brown, *et al.*, "Improved Measurements of the Temperature and Polarization of the Cosmic Microwave Background from QUaD," *Astrophysical Journal*, Vol. 705, No. 4, 2009, pp. 978-999. doi:10.1088/0004-637X/705/1/978

[19] S. Perlmutter, *et al.*, "Measurements of Omega and Lambda from 42 High Redshift Supernovae," *Astrophysical Journal*, Vol. 517, No. 6, 1999, pp. 565-586. doi:10.1086/307221

[20] G. Bertone, D. Hooper and J. Silk, "Particle Dark Matter: Evidence, Candidates and Constraints," *Physics Reports*, Vol. 405, No. 1, 2005, pp. 279-390. doi:10.1016/j.physrep.2004.08.031

[21] W. J. Percival, *et al.*, "Baryon Acoustic Oscillations in the Sloan Digital Sky Survey Data Release 7 Galaxy Sample," *Monthly Notices of the Royal Astronomical Society*, Vol. 401, No. 2, 2010, pp. 2148-2168. doi:10.1111/j.1365-2966.2009.15812.x

[22] N. Panagia, "High Redshift Supernovare: Cosmological Implications," *Nuovo Cimento B*, Vol. 120, No. 6, 2005, pp. 667-680.

[23] G. Ghirlanda, G. Ghisellini and C. Firmani, "Gamma Ray Bursts as Standard Candles to Constrain the Cosmological Parameters," *New Jersey Postal History Society*, Vol. 8, No. 7, 2006, pp. 123-124.

[24] M. Macció, *et al.*, "Coupled Dark Energy: Constraints

from N-Body Simulations," *Physical Review D*, Vol. 69, No. 12, 2004, pp. 123516-123540. doi:10.1103/PhysRevD.69.123516

[25] P. de Bernardis, *et al*., "Multiple Peaks in the Angular Power Spectrum of the Cosmic Microwave Background Significance and Consequences for Cosmology," *Astrophysical Journal*, Vol. 564, No. 1, 2002, pp. 559-666. doi:10.1086/324298

[26] P. Valageas and J. Silk, "The Reheating an Reionization History of the Universe," *Astronomy & Astrophysics*, Vol. 347, No. 7, 1999, p. 20.

[27] A. G. Riess, *et al*., "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant," *The Astronomical Journal*, Vol. 116, No. 9, 1998, pp. 1009-1038.

[28] J. P. Ostriker and P. J. Steihardt, "Cosmic Concordance," 1995, arXiv:astro-ph/9505066v1.

[29] M. Kowalski, *et al*., "Improved Cosmological Constraints from New, Old and Combined Supernova Data Sets," *Astrophysical Journal*, Vol. 686, No. 10, 2008, pp. 749-778. doi:10.1086/589937

[30] D. P. Landau and K. Binder, "A Guide to Monte-Carlo Simulations in Statistical Physics," Cambridge University Press, Cambridge, 2009. doi:10.1017/CBO9780511994944

[31] Planck Science Team, "Planck Science Team Home," 2012. http://www.rssd.esa.int/index?project=Planck

[32] E. Komatsu, *et al*., "Five-Year Wilkinson Microwave Anisotropy Probe Observations: Cosmological Interpretation," *The Astrophysical Journal Supplement*, Vol. 180, No. 2, 2009, pp. 330-376. doi:10.1017/CBO9780511994944

[33] A.G. Riess *et al*., "A Redetermination of the Hubble Constant with the Hubble Space Telescope from a Differential Distance Ladder," *Astrophysical Journal*, Vol. 699, No. 7, 2009, pp. 539-563. doi:10.1088/0004-637X/699/1/539

[34] E. Komatsu, *et al*., "Seven-Year Wilkinson Microwave Anisotropt Probe (WMAP) Observations: Power Spectra and WMAP-Derived Parameters," *The Astrophysical Journal Supplement*, Vol. 192, No. 18, 2011, pp. 1-47.

[35] J. R. Primack, "Precision Cosmology," *New Astronomy Reviews*, Vol. 49, No. 5, 2005, pp. 25-34

[36] S. L. Bridle, O. Lahav and J. P. Ostriker, "Precision Cosmology? Not Just Yet ...," *Science*, Vol. 299, No. 3, 2003, pp. 1532-1533. doi:10.1126/science.1082158

[37] J. Joyce, "Bayes Theorem," In: E. N. Zalta, Ed., *The Stanford Encyclopedia of Philosophy*, The Metaphysics Research Lab, Stanford, 2008. http://plato.stanford.edu/entries/bayes-theorem/

[38] J. K. Ghosh, M. Delampady and T. Samanta, "An Introduction to Bayesian Analysis," Springer, New York, 2006.

[39] J. O. Berger, *et al*., "Bayesian Robustness," IMS, Hayward, 1996.

[40] A. Cho, "A Recipe for Cosmos," *Science*, Vol. 330, No. 12, 2010, pp. 1615-1616. doi:10.1126/science.330.6011.1615

[41] L. Amendola, R. Gannouji, D. Polarski and S. Tsyikawa, "Condition for the Cosmological Viability of f(R) Dark Energy Models," *Physical Review D*, Vol. 75, No. 8, 2007, pp. 83504-83560. doi:10.1103/PhysRevD.75.083504

[42] J. Dunkley, *et al*., "The Atacama Cosmology Telescope Cosmological Parameyters from the 2008 Power Spectrum," *Astrophysical Journal*, Vol. 739, No. 9, 2011, pp. 52-72. doi:10.1088/0004-637X/739/1/52

[43] R. G. Vishwakarma and J. V. Narlikar, "A Critique of Supernova Data Analysis in Cosmology," *Research in Astronomy and Astrophysics*, Vol. 10, No. 1, 2010, pp. 1195-1198.

[44] R. Swinburne, "Introduction to Bayes's Theorem," In: R. Swinburne, Ed., *Bayes's Theorem*, Oxford University Press, Oxford, 2002, pp. 1-55.

[45] S. J. Press, "Bayesian Statistics," Wiley, New York, 1989.

[46] AA.VV., "Bayes Theorem," In: R. K. Bock, K. Bos, S. Brandt, J. Myrheim and M. Regler, Eds., *Formulae and Methods in Experimental Data Evaluation*, EPS-CERN, Geneva, 1984, p. 7.

Appendix A: Derivation of $C_{l,M}$ from the Bayes Theorem

Let's assume (see Section 2.1):

1) $\{O\}$ set of "old" or preexisting, model independent, measurements of an observable;

2) $\{N\}$ set of "new", model dependent, measurements of parameter M associated by the model to the observable;

3) $P(O)$ likelihood function of the "old" measurements *O*;

4) $P(N)$ likelihood function of the "new" measurements *N*;

5) $P(O|N)$ *posterior* conditional probability of *O* given *N*;

6) $P(N|O)$ conditional probability of *N*, given *O*, produced by Model and *prior*.

These quantities are linked by the Bayes Theorem (see for instance [[38,44-46] and references therein) which reads:

$$P(O|N) * P(N) = P(N|O) * P(O) \tag{6}$$

We introduce

$$R(M) = \frac{P(O|N)}{P(N|O)} = \frac{P(O)}{P(N)} = \frac{P(O) * P(N)}{P^2(N)} \tag{7}$$

The numerical value of

$$I = \int_{-\infty}^{+\infty} R(M)\,\mathrm{d}M, \tag{8}$$

is proportional to the overlapping of $P(O)$ and $P(N)$, a measure of the correlation degree of direct and indirect measurements (when $P(O)$ and $P(N)$ do not overlap $I \to 0$). Properly normalized it gives a number $(0 \le C_{l,M} \le 1)$ we can assume as our confidence level indicator of the indirect measurements.

For gaussian distributions of $P(O)$ and $P(N)$

$$I = \frac{\sigma_{ex}}{\sigma_{me}} \cdot \int_{-\infty}^{+\infty} \exp\left\{ -\left[\left(\frac{\Delta M_{me}}{\sigma_{me}} \right)^2 - \left(\frac{\Delta M_{ex}}{\sigma_{ex}} \right)^2 \right] \Big/ 2 \right\} \mathrm{d}x$$

where $\Delta M_{me} = |M - \bar{X}_M|$, $\Delta M_{ex} = |M - E_M|$.

When

$$E_M \to \bar{X}_M, \tag{9}$$

$$R(M) \to \frac{\sigma_{ex}}{\sigma_{me}} \exp\left[-\frac{\Delta M^2}{2\sigma_{eq}^2} \right] \tag{10}$$

and

$$I \to \frac{\sigma_{ex}}{\sigma_{me}} \int_{-\infty}^{+\infty} g(M)\,\mathrm{d}M = \sqrt{2\pi}\,\frac{\sigma_{ex}}{\sigma_{me}}\sigma_{eq} \tag{11}$$

with $\Delta M \simeq \Delta M_{me} \simeq \Delta M_{ex}$

$$\sigma_{eq}^2 = \left| \frac{\sigma_{me}^2\,\sigma_{ex}^2}{\sigma_{ex}^2 - \sigma_{me}^2} \right|. \tag{12}$$

$$g(M) = \exp\left[-\frac{\Delta M^2}{2\sigma_{eq}^2} \right], \tag{13}$$

Normalization to the area $\sqrt{2\pi}\sigma_{eq}$ covered by $g(M)$, gives

$$C_{l,M} = \frac{I}{\sqrt{2\pi}\sigma_{eq}} = \frac{\sigma_{ex}}{\sigma_{me}} \tag{14}$$

identical to our empirical expression (4).

When $P(O)$ and $P(N)$ coincide $C_{l,M} = 1$. When $\sigma_{ex} \ll \sigma_{me}$ $C_{l,M} \to 0$. ($C_{l,M} > 1$, excluded because $\sigma_{ex} < \sigma_{me}$ (see Section 2), would imply models and priors which produce results worse than direct or preexisting measurements).

Journal of Modern Physics, 2012, 3, 1223-1230
doi:10.4236/jmp.2012.329158 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Does Gravitation Have an Influence on Electromagnetism?

Guido Zbiral
Konradtgasse 34, 3400 Klosterneuburg, Austria
Email: guido@zbiral.at

Received June 15, 2012; revised July 25, 2012; accepted August 1, 2012

ABSTRACT

For many years physicists have been engaged on research around the globe in fields such as the unification of gravitation and electromagnetism, and an explanation for dark matter and dark energy, etc., but so far to little avail. One is left with the impression that something might be fundamentally wrong with the premises underlying the doctrine of physics applicable today, which is preventing a solution of these problems from being found. As a possible cause, the author proposes that the gravitation of the photons is not so negligible that it can be completely ignored (although this assumption does not accord with the current state of physics). Departing therefore from the accepted doctrine, he assumes that gravitation might possess a hitherto unknown important influence on electromagnetism. This paper then examines the consequences of this assumption on physics. A precise analysis will lead to the insight that the gravitation of a photon is as dynamic as the photon itself, and therefore must be taken into account with all associated physical considerations. The hitherto accepted case of a static gravitation of photons, on the other hand, can be totally neglected, as it does not exist for photons. Of key importance is the statement that the gravitation of photons is produced by gravitational quanta, and thus appears in quantised form. It is therefore necessary to rethink the physics of photons. This leads to a number of other interesting insights, as will be borne out in the further course of this paper. In the event that the assumption of the influence of gravitation on electromagnetism turns out to be correct, then this would represent a major step in unravelling the still largely unknown nature of gravitation and its significance in the natural events of the microcosmos; furtheron it would be an important contribution regarding a "New Physics" and a "New Cosmology".

Keywords: Photon; Gravitation; Gravitational Quanta; Speed of Light; Maxwell's Theory

1. Introduction

Around the globe, physicists have long been engaged in research—albeit with little success—in the following special fields of physics:

Unifying the theories of General Relativity and Quantum Theory;

The nature of dark matter and dark energy;

The discovery of gravitons and evidence for the existence of cosmic gravitational waves.

All this gives the impression that at least one of the assumptions, which underlie either the current doctrine in physics or today's physical and/or cosmological model conceptions, must be wrong or incomplete—otherwise one or the other breakthroughs in these fields of physics would already have taken place.

I am convinced that a "radically new physical approach" is necessary, in order to introduce some movement into this current state of stagnation. It is clear that this new approach cannot completely accord with the conventional doctrine, otherwise everything would remain unchanged. A deliberately new way of thinking is called for. This does not represent a faulty way of thinking, but is a tactical necessity. I have therefore selected the following strategy for this paper:

I shall make an assumption, which is in contradiction with today's accepted doctrine in physics, but in my opinion is best suited for discovering new physical relationships:

1) I shall assume that there exists a hitherto unknown close relationship between gravitation and electromagnetism;

2) This assumption must be fully justified by physical arguments, which comply with today's doctrine;

3) In conclusion it will be determined, what the consequences of this assumption are for today's physics—in particular, whether it may result in new insights.

2. Arguments Pointing to a Possible Close Relationship between Gravitation and Electromagnetism

Remarks concerning static and dynamic gravitation:

Following the original creation of the cosmos consist-

ing only of radiation energy (*electromagnetic primordial photon radiation and superstrong gravitational radiation*), *no* "*static gravitation*" *could have existed in the* "*Big Bang*" *phase or shortly afterwards. In the quantum era, gravitation was to an overwhelming extent dynamic in nature and can therefore only have been created by means of gravitational quanta*! *The photons' dynamic gravitation remains conserved for the total lifetime of the photons, which are the oldest and longest elementary particles in existence. Their gravitation cannot be inherently static in nature.*

On the other hand, the gravitation of the baryonic elementary particles, which were only subsequently created from the highly energetic, dynamic radiation energy, as well as the gravitation of the non-baryonic "*dark matter*", *is indeed static in nature.*

The physical processes occurring in the earliest phase of the cosmos during the quantum era, are even today still taking place in the photons (*in qualitative terms*), *representing virtually a* "*remnant left over from that quantum era*".

Now to the Individual Arguments

It is possible to derive a possible physical relationship between the speed of light (c) and the Gravitational Constant (G) quite simply from G itself:

Taking the gravitational constant as $G = 6.67 \times 10^{-11}$ $(N \cdot m^2 \cdot kg^{-2})$ in consideration as well as the fact that 1 N is equal to 1 $(kg \cdot m \cdot s^{-2})$, then a physical dimension for G of $(m^3 \cdot kg^{-1} \cdot s^{-2})$ is arrived at. One can see, that the physical dimension of c^2 $(m^2 \cdot s^{-2})$ is contained in this dimension of G!

Thus the gravitational constant itself certainly has something in common with the speed of light. Without changing any of the contents, it is possible to convert the gravitational constant G by extrapolating c^2:
$G = c^2 \times 742 \times 10^{-28}$ $(m \cdot kg^{-1}) = c^2 \times$ const. or
$G/c^2 = 7.42 \times 10^{-28}$ $(m \cdot kg^{-1})$ = const.

This mathematical conversion merely represents an alternative notation of the gravitational constant G and therefore must also be physically valid.

Possible consequence: The possibility of the constancy of the speed of light being dependent on the constancy of the gravitational constant G cannot therefore be excluded. Any change in the numerical value of G would in this case also imply a change in the numerical value of c^2, in order to maintain the constancy of the ratio G/c^2. As long as G remains a constant value, then c also remains constant and the constancy of the speed of light remains unchanged in the entire cosmos. Nevertheless, according to Special Relativity this interpretation is not permitted, because c would not then be an "independent" universal constant of nature, but dependent on G and therefore not a "genuine" constant of nature. According to this, the speed of light is only constant if G is also constant!

The expression G/c^2 = const., on the other hand, could well be a "genuine" constant of nature, as it occurs several times in physics, e.g. in association with the "Schwarzschild Radius" $R_s = 2\,MG/c^2$.

Moreover, the (for me hitherto unknown) relationship

$$G/c^2 = l_{\text{Planck}}/m_{\text{Planck}} = \text{const.}\ (m/kg) \qquad [1]$$

also applies, which similarly reinforces my notion that G/c^2 = const. is an "original united constant of nature" that had applied as far back as the quantum era. Therefore for that quantum era, the above equation then becomes

$$G_{\text{super}}/c^2_{\text{super}} = \text{const.}$$

Since in the quantum era during the state of "supergravitation" (G_{super}), the value of G_{super} was many orders of magnitude higher than G, then the speed of light (c_{super}) at that time must also have been many orders of magnitude higher than c; c_{super} could then explain the super rapid expansion behaviour of the very early cosmos, as in this situation the extremely hot and dense cosmic radiation energy would have expanded explosively with a speed of c_{super} in the neighbouring cosmic space.

In this manner, we would have a physical justification for the super rapid expansion of the cosmos in its very early life—in contrast to today's usual, albeit completely arbitrarily constructed hypothesis of an exponentially generated "inflationary expansion".

The physical facts of the constancy of the speed of light independent of the energy of the photons as well as their wave propagation, are described in the Maxwell equations (refer to the internet link
http://www.mahag.com/srt/maxwell.php, respectively
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/maxeq.html), which are considered one of the most solid physical theories ever founded. However, Maxwell's equations cannot provide any statement concerning the reason for the constancy of the speed of light. There is also no directly recognisable connection to gravitation, as the gravitational constant G does not explicitly occur in them. According to this, it would seem that electromagnetism and the photons do not have anything to do with gravitation, as confirmed by today's accepted doctrine in physics. However, it is unexpectedly possible to establish a relationship between Maxwell's theory and gravitation by means of the following "detour":

As according to Maxwell's so called Fifth Equation, the following physical relationship between the electrical field constant ε_0, the magnetic field constant μ_0 and the speed of light c exists:

$\varepsilon_0 \cdot m_0 = 1/c^2$, then G/c^2 = const. as well as $G \cdot \varepsilon_0 \cdot \mu_0$. = const. can be written. This means that within Maxwell's Theory, not only the speed of light, but with it also electromagnetism itself, is linked via its field constants ε_0, μ_0 with the gravitational constant G. Are we to understand this merely as a random coincidence lacking any physical significance, or does this relationship in fact represent a very concrete relationship between these two forces of nature, as will be shown several times in the subsequent text? With regard to the relationships stated above, the expression G/c^2 = const. may possess a far greater physical significance than hitherto assumed.

Gravitation is known to be the weakest of the four elementary forces in the universe. Therefore the (static) gravitational force within an atomic structure between a proton and an electron is weaker than the electromagnetic force acting between them by a factor of 10^{-39}. For this reason, the gravitational force of the baryonic components of an atom among themselves within an atom can be completely ignored compared to the electromagnetic force between the proton and electron.

This fact forms the basis for the assumption of physicists that the gravitational force of a photon, compared to its far stronger electromagnetic force, can also be completely ignored, whereby always a static gravitation is assumed (but without using the word "static", since no dynamic gravitation is so far known).

Remark: *The possibility of dynamic gravitation in the form of high-frequency gravitational radiation for photons has not yet been considered, because taking gravity into account in relation to the* "*theory of special relativity*" *was never a subject of discussion*. *According to the* "*theory of general relativity*", *only low-frequency gravitational waves in the region of approx. ca.* 10^{-1} *to* 10^{4} *Hz are expected today in connection with spectacular cosmic events, although it has not yet been possible to prove their existence*.

The current school of thought in physics is therefore based on the assumption that (static) gravitation is also not a factor to be considered with photons or the speed of light and may be completely ignored, but: Static gravitation does not exist for photons!

The gravitation of the photons cannot be of a static nature; it must be as dynamic as the photons themselves. This opens up a completely new insight into the physics of photons!

In my opinion, the assumption underlying the current doctrine of a negligible gravitation of photons is an erroneous assumption!

The gravitation of a photon experiences the same speed or dynamic as the photon itself. Therefore the gravitation of a photon must appear like a high-frequency gravitational radiation and resonate with the same frequency as the electromagnetic radiation (it is, as it were, "clocked" by the latter). It remains associated with the photon until the photon is expended or transformed by performing work. It cannot rush away in order to become "externally" noticed; it only has a specific reaction on "its own" photon and as far as the outside world is concerned, it does not exist.

Furthermore, non-baryonic relativistic mass relationships exist at the speed of light for "free" photons as is the case for baryonic elementary particles "bound" by strong forces within the atom. These two completely different initial or environmental conditions are therefore not comparable! In contrast to the school of thought in physics stated above, I therefore hold the view that dynamic gravitation in the form of gravitational radiation must definitely be taken into account with photons, particularly as still further cosmological and physical connections exist between electromagnetism and gravitation:

Preliminary remarks on the next point, *which refer to the cosmos as a whole*:

"*A general consequence of Einstein's Theory of Gravitation (General Relativity) is that in a closed universe the total energy is always zero*" [2]. "*Positive energy*", *whose essence is in the expansion* (*electromagnetism*), *and* "*negative energy*", *whose essence is in the implosion or holding together* (*gravity*), *mutually cancel each other out exactly in a physically closed system. These two opposite forces of nature are equally strong in a physically closed system*, *therefore a stable state of equilibrium exists between them and the total energy is zero. The cosmos as a whole is a physically closed system*. *Thus in the Big Bang the sentence is not violated by the conservation of energy*.

At the same time (here we mean within the first Planck time of 10^{-43} secs) as the appearance of the expansionary radiation energy (of the primordial photon radiation) in the Big Bang, gravity also appeared in order to act as its "stabilising counter-force". As the contribution of gravity to the total energy according to Einstein is negative, the sum of the two components is then zero. From this it follows necessarily, that:

At no time did or does energy exist without gravitation; therefore, exactly as is the case with the primordial photon radiation, all its subsequent radiations and energy forms must be affected by gravitation!!!

Thus right at the beginning of the history of cosmic creation there exists a close connection between the primordial electrodynamic photon radiation and its dynamic gravitation e.g. gravitational radiation, whereby the two dynamic forces are of equal strength, but have an opposite effect on each other. Had gravity not appeared within the first Planck time as cosmic energy, but only a short time later, then the total positive energy would have immediately fizzled out without having any effect and the cosmos would not have come about.

Preliminary remarks on point 2.5, *which refer to each individual photon*:

The above sentence can also be applied to each indi-

vidual photon as the elementary fundamental component of the entire cosmos. Because the speed of light is constant, the forces influencing the photon must exist in equilibrium, with the result that no overall force influences the photon, which would otherwise accelerate or decelerate. At the speed of light the total energy of a photon is zero, therefore in this respect each photon corresponds to a "*physically closed system*".

The positive electromagnetic energy and the negative gravitational energy of each individual Photon are of equal strength and cancel each other out; a stable "equilibrium" exists between the two forces. There is no way for this equilibrium and hence the constancy of the speed of light to be achieved except by the dynamic gravitation of the photon. Electromagnetism is, on its own (without dynamic gravity), unable to produce a stable equilibrium. This is the evidence that each photon as a quantum of electromagnetic energy is connected with an equivalent quantum of gravitational energy.

The photon requires the full amount of its gravitational energy for its stabilisation, so there is nothing left which could penetrate to "the outside". Therefore it was erroneously assumed that photons have nothing to do with gravity. Only when the photon's electromagnetic energy has been expended or converted by performing work, does its gravitational energy appear "externally", albeit as a weak static gravitational force (a factor of 10^{-39}), as the dynamic drive provided by the photon has been lost.

Furthermore, included in the established state of knowledge in physics is the fact that each quantum—no matter how small—and each form of energy (e.g. mass) have a gravitational effect themselves (*i.e.*, they are "subject to gravity"), and therefore react to an external gravitational field. Since this also applies to photons, then they will be deflected when passing through a gravitational field, either gaining or losing energy depending on whether they are approaching or leaving the gravitational field—corresponding to a blue or red shift of a ray of light. There therefore exists an interaction between an external static gravitational field and the photon radiation passing through it.

All six criteria (taking each one into consideration separately) correspond to the certain state of knowledge in physics and therefore allow the conclusion to be made that the speed of light certainly does have something to do with the gravitational energy of photons. This very new physical insight also represents the "key fundamental prerequisite" for the explanation of dark matter.

3. Further Effects of Gravitation on Electromagnetism

Explanations of the terms used in the following section:

The "*quantum of gravitation*" *is always closely associated with its related photon*; *it adheres*, *as it were*, *to the photon, or manifests itself with its photon as well as interacting with it.*

This hitherto unknown "*gravitation quantum*" *might therefore turn out to be the elementary particle, which imparts the phenomenon of gravitation and gravitational mass* (*in the event that the* "*Higgs mechanism*" *does not work as expected, particularly in connection with relativistic mass of photons*).

The gravitation inherent in every elementary particle ("*intrinsic gravitation*") *always relates in a specific manner to individual elementary particles including photons, and is, in each case, the gravitation corresponding to the elementary particle's energy state, caused by its* "*gravitation quantum*". *No form of energy exists, which is not associated with the phenomenon of gravitation*.

The "*graviton*", *on the other hand*, *is a term reserved for the hypothetical* (*not yet proven*) *independent boson in a future quantum theory of gravitation and is the means of the gravitational interaction between elementary particles.*

From the six arguments made above, it follows that each photon must have emerged as a quantum of electromagnetic energy from the Big Bang with its quite specific (photon energy equivalent) gravitational quantum, with which it appears together and which causes the dynamic gravitation of the photon as well as its relativistic mass. What else could produce the photon's dynamic gravitation or relativistic mass? This gravitational quantum also appears due to its intimate meshing with the photon or electromagnetic photon radiation like radiation itself, namely as specific high-frequency gravitational radiation bound to the photon, whose speed of propagation is also equal to the speed of light.

A photon without dynamic gravitation does not exist. Therefore each photon consists of two components linked to each other, one expansive electromagnetic component and one gravitational radiation component acting in an opposite manner. These two physical quantities acting in opposite manner are ideally united in the photon. Therefore there must exist exactly the same number of gravitational quanta of the photons as there are of photons themselves, but their existence has not yet been noticed! This is why gravity is even today mostly still a mystery.

The hitherto unknown or unnoticed gravitational quantum therefore corresponds—as it were—to a "gravitational charge" of each single elementary particle, which for an agglomeration of elementary particles accordingly accumulates and produces a gravitational field, which cannot be compensated (in analogy to an electrical charge producing an electric field, which of course can be compensated).

The energy of photons is known to increase proportionally with its frequency ν in accordance with Planck's equation $E = h \cdot \nu$, where h is Planck's constant or "effect-quantum", a universal constant of nature. As electromagnetic radiation encompasses an extremely broad frequency spectrum (a ratio of up to 10^0 - 10^{25}), the energy of the photons also occupies this very wide range. Not-withstanding this, all photons always travel in a vacuum with the same constant speed of light, irrespective of the energy quantum that they transport with them. Each photon requires its entire positive electromagnetic energy for the purpose of maintaining the speed of light against the negative stabilising opposing force of gravitation.

The photon travelling at the speed of light does not have "free energy" available for any additional "external effect". The only external effect it manifests is the constancy of the speed of light.

Therefore the constant speed of light must be associated with the wide-ranging variable relativistic mass of photons in such a manner that the photon rich in energy must be subject to the stabilising opposing force of gravity to a greater extent than the photon low in energy.

As already determined in the above text, the constant speed of light can only represent a state of equilibrium between the expansive force of electromagnetic radiation imparted by the photon and the opposing force of gravitation, in each case. The greater the energy of the photons, the greater is the stabilising opposing force of gravitation.

The speed of light c in a vacuum is therefore constant 299,792,458 m/secs, since this is exactly the value, at which a stable state of equilibrium is established between the dynamic gravitation of the photons and their expansive electromagnetic force, thus cancelling out the two opposing forces. Why the state of equilibrium should be achieved precisely by this value for the speed of light, depends in connection with G/c^2 = const. on the value of the Gavitational Constant "G".

4. New Aspects Concerning Physical Events inside Photons

4.1. Regulation Process within Photons

What happens inside the photons, *i.e.*, what physical process must take place, in order for the speed of light to be constant irrespective of the energy of the photons, does not emerge from the Maxwell equations and has presumably not hitherto been investigated. These physical processes taking place inside the photons, which should be considered as the possible reason for the constancy of the speed of light, are dealt with below. The constancy of the speed of light itself is thus an effect of these internal processes.

After careful consideration the impression emerges that the speed of light is regulated to the constant value of c by a hitherto unnoticed "influencing factor" located within each individual photon. There is no other way to explain the constancy of the speed of light, which does not take into account of the intensity of the energy of its photons. There is obviously a typical regulation process here at work on the photons' constant velocity of propagation, which raises the following questions:

How does a "physical regulatory mechanism" work, acting on the extremely wide-ranging energy of photons in such a manner that results in a constant speed of light for all photons?

In particular, it is necessary to clarify whether there really is "an influencing factor" acting inside the photons, and if so, what its nature is, and how the regulation process of a constant speed of light actually functions. That is one of the important objectives of this paper.

As a "regulator" for the constancy of the speed of light the only candidate is thus the intrinsic gravitation of the photons themselves, since any change in energy of a photon simultaneously causes an equal change in its intrinsic gravitation as well as its relativistic mass. This change in intrinsic gravitation affects a photon to the extent that it is precisely the stabilising effect of gravitation corresponding to the existing energy state that is available out of all the energy states possible. Therefore the speed of light remains constant for all possible energy situations of a photon. There is no other possibility to act in a "stabilising" manner on a photon than by this standard physical process.

This feedback of a photon's stabilising intrinsic gravitation on its expansive electromagnetic force is the only identifiable physical cause for the constancy of the speed of light!

Note repeated here again: *Both the previous and the following considerations only apply under the assumption that the speed of light is not an independent constant of nature, but is dependent on the value of* "*G*". *If it is true, however, that the speed of light actually is an independent constant of nature, as insisted upon by an essential foundation of today*'*s modern physics, then neither a stabilising effect on the photons nor an internal control process is required in order to effect the constancy of the speed of light—in this case it is set at a fixed value predetermined by its origin.*

A photon's gravitation is a dynamic field just as much as its electromagnetic field, since it exhibits the characteristic of travelling together with the photon at the speed of light. In order for a state of equilibrium to result from the two dynamic forces acting on the photon, as required from the argument above, the gravitational field must exhibit behaviour precisely opposite to that of the electromagnetic field.

Therefore the physical principles of both fields must be in every respect completely equivalent but opposed to each other. It therefore follows that the same vectorial Maxwellian theory also underlies the gravitational radiation associated with the photon as the electromagnetic radiation, *i.e*. they also possess a common theoretical basis!

Remark: *It is highly remarkable*, *that the dynamic gravitation of the photons are described by the same theory as electrodynamics and electromagnetism*, *i.e.*, *that a close* "*family relationship*" *exists between the two physical forces. On this basis*, *it can be concluded that the same family relationship also exists between static gravitation and electrostatics*, *since they are* "*descended*" *in each case from their dynamic quantities*, *as reflected in the similar structure of their formulae for static forces*.

In addition, *Prof. K. O. Greulich* [3] *in a lecture on the occasion of the Congress of the German Physical Society in* 2012 *in Göttingen also establishes that electrostatics and* (*static*) *gravitation are alike from a physical perspective*!

Following the same logic, the gravitational radiation must possess transversal characteristics exactly as the electromagnetic radiation does, and it must also exhibit the same frequency as the electromagnetic radiation, as is indeed the case. It is only as a consequence of these conditions that there exists no resulting force on the photon, which could cause it to accelerate or decelerate. Therefore the photon travels in a completely uniform manner at the constant speed of light.

However, if the value of the gravitational constant G were different to the current value, then the effect of the photons' intrinsic gravitation would also differ from the current effect and the state of equilibrium between both natural forces and hence the constant value of the speed of light would also differ from the current value. This was, in fact, the case in the stage of "super gravitation" ($G = G_{super}$) and had, as a consequence, a value of the speed of light c_{super} many orders of magnitude greater than the current value.

The constancy of the speed of light is thus the result of an extremely finely-tuned interaction between the photons' intrinsic gravitation and their electromagnetism, from which it follows that at the speed of light there exists an extremely close association between these two fundamental forces of nature:

On one hand, the photons' intrinsic gravitation limits the speed of expansion of electromagnetic energy (by establishing the state of equilibrium between the two forces of nature)—and thus determining the value of the speed of light, while, on the other hand, maintaining it at this constant value!

The hitherto unknown precise "fine tuning" between the two opposite fundamental forces of nature within photons, to which we owe the whole of creation, is one of the greatest physical miracles of nature and has remained up to now beyond human imagination. It must be very difficult—as in numerous other cases—just to believe in a random coincidence!

4.2. The Quantum Structure of Gravitation inside Photons

Since the energy of the electromagnetic radiation is proportional to its frequency, it is necessary that the energy of the high-frequency gravitational radiation associated with a photon is also proportional to its frequency, whereby both frequencies are identical. This is necessary to achieve a stable state of equilibrium at the speed of light between the two physical quantities, which oppose each other by nature.

Therefore Planck's equation for the energy of the electromagnetic radiation of a photon $E = h \cdot \nu$ must also apply to the high-frequency gravitational radiation, which is associated with the photon and only selectively acts on it. It then follows that the energy of the gravitational radiation associated with each photon must similarly be quantised, as already mentioned on Section 2. The new term introduced in this paper in the above text "gravitation quantum" is therefore physically justified.

Therefore it is assumed that Planck's equation for the energy of the high-frequency gravitational radiation associated with the photon ($E = h \cdot \nu$) could also apply to cosmic low-frequency gravitational waves, should they actually exist (because two differing formulas are not possible for one physical phenomenon). If that were the case, then cosmic gravitational waves would also have Maxwell's theory as a physical foundation rather than Einstein`s General Relativity theory. And if that were true, cosmic gravitational waves could not be "perturbations of space-time", but would have to expand in exactly the same manner as electromagnetic waves. This may possibly be the reason that it had not been possible to detect cosmic gravitational waves up to now.

Remark: *In his book* "*The Structure of Physics*" (*Aufbau der Physik*), *C. F. von Weizsäcker* [4] *determined that all information relating to natural sciences—and particularly energy ≡ matter—is derived from captured* (*registered*) *photons. Every sensory perception is associated —in an intermediate step as a minimum—with photons. In other words*, *there can be no perception* (*information*) *without the electromagnetic interaction* (*photons*)*. The dimensionless variable* "*information*" *is therefore the most fundamental variable in physics.*

Due to the quantum nature of light, *it further follows that all physical information is digital*! *From this C. F. von Weizsäcker draws the following conclusion*: *If the whole of physics is based on quantised information*, *then similarly*, *the whole of physics can only consist of quantised variables*. (*This statement of C. F. von Weizsäcker also confirms that the hitherto unknown term* "*gravitational quantum*" *introduced here is perfectly justifiable*).

5. Unification of Gravitation and Electromagnetism

A photon's electromagnetic radiation forms a physical dualism with the gravitational radiation associated with it (complete equality of both components linked to each other) on the common basis of the same Maxwellian theory. It is not therefore surprising that the equation $E = h \cdot \nu$ applies equally both to the energy of the electromagnetic radiation and the high-frequency gravitational radiation associated with photons.

Although static gravitation is the weakest of the four fundamental forces in the universe, it is surprising to discover here that:

At the speed of light, the intrinsic gravitation of a photon as gravitational radiation or dynamic gravitation is selectively (related to its photon) just as strong as the electromagnetic radiation (of the photon), but acting in an opposing manner, so that both of these fundamental forces exist in a stable equilibrium and cancel each other out from an external perspective. Electromagnetism and its dynamic gravitation are perfectly united and unified in each photon!

Each photon carries with it its exactly specific gravitation quantum. Each photon forms an opposite interacting pair of forces with its gravitation quantum (precisely stated: they are mediators of two opposite interacting forces), which exists in a stable equilibrium precisely at the speed of light.

In the micro-world of photons, their dynamic gravitation is unambiguously a force and not a geometry (otherwise they couldn't get unified with electromagnetism), which at the speed of light precisely opposes the electromagnetic force and selectively "neutralises" it!

Remark: *In Einstein's Theory of General Relativity* (1915), *static gravitation is not a force but a property of space-time under the influence of cosmic* (*baryonic*) *masses*, *which is what corresponds to the geometry of the cosmos. On the other hand, there is no doubt that gravitation* (*or the force of gravity*) *is one of the four fundamental forces of the universe that causally determine the geometry of the cosmos and its structures*. *Gravitation is not*, *however*, *the geometry of the cosmos in itself*; *the geometry is rather an effect of the static gravitation of the cosmic agglomeration of masses* (*including the non-baryonic dark matter, which remained unknown until the* 1960*s*). *If cause and effect are interchanged*, *then this can result in a mistaken perception of reality*!

If the internal relationship between the photons' intrinsic gravitation and the speed of light described in these pages is correct, then at the speed of light the selective gravitational radiation of photons is only a mirror image and hence a facet of the electromagnetic radiation; thus the same vectorial Maxwellian theory applies in both cases!

"Albert Einstein is supposed to have spent 20 years attempting to describe electromagnetic interaction and gravitation as two aspects of a single higher-level interaction. All to no avail—because even today it is still not known whether it is possible to unify gravitation with other interactions" [5].

"Today's physicists are largely convinced that gravitation plays a key role not only in cosmology but in elementary particle physics as well (particularly applicable in the case of the photon), but this role is currently not yet properly understood" [5].

The considerations provided in this paper may possibly be of help in the hitherto unsuccessful search for a theory unifying gravitation with electromagnetism, because, at the level of photons travelling at the speed of light, a complete union and unification of these fundamental forces of nature, reduced to the "common basis $E = h \cdot \nu$" take place from a physical perspective. The union of gravitation with electromagnetism is also already clearly expressed in the equation G/c^2 = const. respectively $G \cdot \varepsilon_0 \cdot \mu_0$ = const.

6. Law of Conservation of Gravitation

A photon can never be made "gravity-free" (if this were possible it would no longer be a photon), because intrinsic gravity is a fundamental component of each photon just as of any other form of energy. As is the case for the photon itself, its gravitation quantum does not possess any baryonic rest mass, it is electrically neutral and cannot like energy be annihilated; it remains in existence forever, having a virtually unlimited lifetime. It therefore follows that in the event of energy transformation there must also exist—analogous to the "Law of Conservation of Energy"—a "Law of Conservation of Gravitation", which "expressis verbis" appears neither in my physics books nor in any cosmological book available to me, and which I have formulated as follows:

For every transformation of energy, the gravitation quantum corresponding to the converted amount of energy in each case is also transferred, so that the effect of the original gravitation quantum existing before the energy transformation is conserved overall. This conservation principle possesses the character of a law of nature, and it may possibly be the first time it has been mentioned.

As the cosmos at the time zero in the cosmic calendar contained a finite large and constant amount of energy ("pure" radiation energy, *i.e*. photons having the highest energy level in the history of creation), with this energy being subject to an ongoing transformation process, then the gravitation quantum appearing at the same time as—and associated with—this amount of energy must also be constant and remain conserved overall for the entire duration of existence. Therefore gravitation, as is the case with energy, can be neither created nor annihilated, thus also pointing to the same origin as energy at the same instant.

7. Conclusions

It is possible to derive the relationship G/c^2 = const. from the Gravitational Constant G itself; this infers that c would not actually be an "independent" universal constant of nature, but dependent on G and therefore not a "genuine" constant of nature. The expression G/c^2 = const., on the other hand, could well be both a "genuine" and "united" constant of nature, applicable as far back as the quantum era, because $G/c^2 = l_{Planck}/m_{Planck} = G_{super}/c^2_{super} = G \cdot \varepsilon_0 \cdot \mu_0$ = const. (m/kg)

The key insight derived in this paper is the fact that electromagnetism—and thus also photons—are associated with a hitherto unnoticed or unknown dynamic gravitation, equally strong as electromagnetism but acting with opposite effect, which has to be included in all relevant physical considerations. This gravitation of photons is therefore not static in nature, as previously assumed, but just as dynamic as the electromagnetic energy of photons. This consequently results in the now following new insights:

Each photon as a quantum of electromagnetic energy is closely connected with an equivalent quantum of gravitational energy. Due to its intimate meshing with the photon or its associated electromagnetic photon radiation, this gravitational quantum itself also appears as radiation, namely as specific high-frequency gravitational radiation bound to the photon. Therefore each photon consists of two components linked to each other, one expansive electromagnetic radiation component and one gravitational radiation component acting in an equal and opposite manner. Therefore there must exist exactly the same number of gravitational quanta of the photons as of photons themselves.

The speed of light c in a vacuum is therefore a constant 299,792,458 m/secs, since this is exactly the value, at which a stable state of equilibrium is established between the dynamic gravitation force of the photons and their expansive electromagnetic force, the two opposing forces thus cancelling each other out and resulting in a total zero energy of all photons.

The constancy of speed of light is achieved by a "physical regulatory mechanism". As a "regulator" for the constancy of the speed of light the only candidate is thus the intrinsic gravitation of the photons themselves, since any change in energy of a photon simultaneously causes an equal change in its intrinsic gravitation. Therefore the speed of light remains constant for all possible energy situations of a photon.

The physical principles of both the photon's electromagnetic radiation and its high-frequency gravitational radiation must be in every respect completely equivalent but opposed to each other. It therefore follows that the same vectorial Maxwellian theory also underlies both physical quantities, *i.e*. they also possess a common theoretical basis!

Planck's equation for the energy of the electromagnetic radiation of a photon $E = h \cdot \nu$ must also apply to the high-frequency gravitational radiation, which is associated with the photon and only selectively acts on it. It then follows that the energy of the gravitational radiation associated with each photon must similarly be quantised.

A photon's electromagnetic radiation forms a physical dualism with the gravitational radiation associated with it on the common basis of the same Maxwellian theory. Electromagnetism and its dynamic gravitation are perfectly united and unified in each photon!

Analogous to the "Law of Conservation of Energy" a "Law of Conservation of Gravitation" must also exist.

It should be noted that the dynamic gravitation of photons and their conversion into static gravitation following the energy transformation of photons may provide a plausible explanation for the mysterious "dark matter" and "dark energy". In view of the enormous significance of the "dark phenomena of the cosmos" for our world view from a natural science perspective, I have written a separate paper titled: "The hunt for dark matter and dark energy".

REFERENCES

[1] K. O. Greulich, "The Universality of the Specific Planck Charge and Its Role for Gravitation," *Lecture Presented at the German Physical Society*, Dresden, 2011.

[2] W. Thirring, "Baupläne der Schöpfung—Hat Die Welteinen Architekten? The Building Plans of Creation—Does the World Have an Architect?" Seifert Publishing House, Vienna, 2011, pp. 36-37.

[3] K. O. Greulich, "Constant Constants since the Big Bang?" *Lecture Presented at the German Physical Society*, Göttingen, 27 February-3 March 2012.

[4] C. F. von Weizsäcker and A. der Physik, "The Structure of Physics," Deutscher Taschenbuch Verlag, München, 2002.

[5] B. Povh, "Elementary Particle Physics and Cosmology," *Sterne und Weltraum*, No. 3, 2004, pp. 32-35.

Journal of Modern Physics, **2012, 3, 1231-1235**
doi:10.4236/jmp.2012.329159 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

The Generalized Newton's Law of Gravitation versus the General Theory of Relativity

Arbab Ibrahim Arbab
Department of Physics, Faculty of Science, University of Khartoum, Khartoum, Sudan
Email: aiarbab@uofk.edu

Received June 18, 2012; revised July 29, 2012; accepted August 5, 2012

ABSTRACT

Einstein general theory of relativity (GTR) accounted well for the precession of the perihelion of planets and binary pulsars. While the ordinary Newton law of gravitation failed, a generalized version yields similar results. We have shown here that these effects can be accounted for as due to the existence of gravitomagnetism only, and not necessarily due to the curvature of space time. Or alternatively, gravitomagnetism is equivalent to a curved space-time. The precession of the perihelion of planets and binary pulsars may be interpreted as due to the spin of the orbiting planet (m) about the Sun (M). The spin (S) of planets is found to be related to their orbital angular momentum (L) by a simple formula, *viz.*, $S \propto \frac{m}{M} L$.

Keywords: General Relativity; Gravitomagnetism; Perihelion Precession of Planets and Binary Pulsars; Origin of Planetary Spin

1. Introduction

We have recently introduced gravitomagnetism as a true cause of the precession of the perihelion of the orbit of planets and binary pulsars [1]. Einstein attributed these effects to the curvature of space-time. The effect of gravitomagnetism, in a similar manner to electromagnetism, is the Larmor precession of a gravitational moment in the gravitomagnetic field induced by the Sun on the planets.

Le Verrier discovered that the orbital precession of the planet Mercury was not quite what it should be; the ellipse of its orbit precesses by some minute value than the predicted by the Newtonian theory of gravitation, even after all the effects of the other planets had been accounted for [1]. This value amounts to 43 arcseconds per century. Several classical explanations were put forward, e.g., an interplanetary dust, an unobserved oblateness of the Sun, an undetected moon of Mercury, or a new planet named Vulcan. Others suggested that the Newton inverse-square law is not correct, and accordingly proposed a power law with an exponent that slightly differs from 2. Moreover, some authors argued in favor of a velocity-dependent potential (see [1] and references there in).

To resolve the above mentioned dilemmas, Einstein used a pseudo-Riemannian geometry to allow for the curvature of space-time that was necessary for the reconciliation of the observed gravitational phenomena. He concluded that the space-time should be curved in order to reproduce the observed physical laws of gravitation. Owing to Einstein's theory of general relativity, particles of negligible mass travel along geodesics in the space-time. An exact solution to the Einstein field equations is the Schwarzschild metric, which corresponds to the external gravitational field of a stationary, uncharged, non-rotating, spherically symmetric body of mass M. It is characterized by a length scale r_s, known as the Schwarzschild radius. The immediate solutions of the field equations explained the anomalous precession of Mercury, and predicted the observed bending of light, which were later confirm experimentally [2].

On the other hand, the theory of electromagnetic interaction is accomplished by Maxwell. This is coined in the four Maxwell equations relating the electric and magnetic fields to the electric charges and current. Lorentz then obtained the force experienced by a charged particle in electric and magnetic fields. Larmor has found that when an electron (magnetic moment) is placed in an external magnetic field, the magnetic moment precess about the magnetic field direction. This precession is due to the spin of the electron. This effect is prominent in the spin-orbit interaction exhibited by hydrogen atom [3,4].

If we now consider gravitation with some scrutiny, we will find that, unlike electromagnetism, moving mass doesn't create a magnetic-like field. Thus, Newton law of gravitation is not like Lorentz law of electromagnetism.

In this sense, gravity and electromagnetism are not analogous and can't be utterly compared with gravitation. To remedy this problem, we have to look for a gravitomagnetism counterpart of gravity. In this way, we can say gravity is analogous to electricity and gravitomagnetism is analogous to magnetism. The question is what is the gravitomagnetic field? By analogy, this should be obtained by looking at Biot-Savart law that defines the magnetic field of a uniformly moving charged particle in an electric field. To complete the analogy the charge of the particle should correspond to the mass of the particle. In this way, we may call the electric charge, the *electric mass* in contrast with the gravitational mass. This furnishes the complete analogy between gravitation and electromagnetism.

How we then avail the electrical phenomena and rules in one paradigm to interpret the other? To answer this question, we have to trust (beforehand) the existing analogies, and base all our new interpretation of the gravitational phenomena by explaining their corresponding ones. In this manner, the precession of the perihelion of the orbit of planets and binary pulsars is obtained from the precession of the electron (magnetic moment) in an external magnetic field. Planets and binary pulsars precess when they experience a gravitomagnetic filed (if any). In this case, we use the same laws holding for the counter (analogous) phenomenon, however.

Moreover, the deflection of light by the Sun is explained by using the laws governing the deflection of a charged particle (α-particle) by the nucleus [5]. If we continue in this manner, we may persuade our selves that, to every electromagnetic phenomenon there are gravitomagnetic counter-phenomena. Hence, electromagnetism and gravitomagnetism are same but different aspects of a unified origin.

In this respect, we will find our-selves distracted to interpret the gravitational phenomena as due to the curvature of space-time. We are then not abide by the GTR to interpret our physical world. Or alternatively, we treat the curvature and gravitomagnetism as a same object, or yield the same effects. This can be trusted if we are able to show that the term responsible for the curvature of space-time in Einstein field equations is the same as the that resulting from the influence of gravitomagnetism.

In this paper, we will show that the gravitomagnetism terms in the generalized Newton law of gravitation is the same as the one in the Einstein general field equations. In this way, we upgrade Newton law of gravitation to the general theory of gravitation, but with different predictions. Thus, the *correct* Newton law of gravitation still works finely, and expresses gravitational phenomena in accordance with observations. Hence, gravity and electromagnetism are governed by unified laws. In Section 2 we present the potential that gives rise to the precession of perihelion in the GTR. We compare this potential with that arising from the gravitomagnetic field. We find that the gravitomagnetic term is $\frac{\pi}{3}$ of the Einstein term (GTR). Einstein attributed this term to curvature of space.

Can we say that the gravitomagnetism is the cause of Einstein curvature?

Do we still adopt GTR that requires advanced mathematics, as the theory of gravitation and leave the simply-understood Newton's laws of gravitation aside? In effect, the gravitomagnetic theory (or Gravitational Lorentz force) is simple and can easily be handled without recourse to tensor (advanced mathematics) analysis to unravel gravitational phenomena. Besides, it is analogous to electromagnetic theory that is well understood and complies utterly with experimental facts. The idea of curvature of space is no longer adopted. Moreover, the Einstein's dream of unification of fundamental forces in nature will become imminent within this framework.

2. The General Theory of Relativity (GTR)

Einstein attributed the gravitational phenomena, now known, to the effect of the curvature of space-time induced by the presence of a massive object [2]. The effective gravitational potential of the object of mass m moving around a massive object of mass M takes the form [6]

$$U(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{c^2mr^3}, \qquad (1)$$

and the force, $F = -\frac{\partial U}{\partial r}$, can be written as

$$F(r) = -\frac{GMm}{r^2} + \frac{L^2}{mr^3} - \frac{3GML^2}{mc^2r^4}, \qquad (2)$$

where L is the orbital angular momentum of the mass m. This inverse-cubic energy term in Equation (1) causes elliptical orbits to precess gradually by an angle $\delta\varphi$ per revolution [2]

$$\delta\varphi = \frac{6\pi GM}{c^2a\left(1-e^2\right)}, \qquad (3)$$

where e and a are the eccentricity and semi-major axis of the elliptical orbit, respectively. This is known as the anomalous precession of the planet Mercury.

Another prediction famously used as evidence for GTR, is the bending of light in a gravitational field. The deflection angle is given by [2]

$$\delta\theta = \frac{4GM}{c^2b}, \qquad (4)$$

where b is the distance of closest approach of light ray to the massive object. Therefore, the gravitomagnetic force is equal to $\frac{\pi}{3}$ of the GTR force. Whether, the gravitational phenomena are in full agreement with our gravitomagnetic model or with GTR is a subject of the present and future observations. At any rate, we are lucky to have two complementary paradigms explaining the same effect in different ways. Can we deduce that it is the gravitomagnetic field that curves the space and *not* the Sun mass? Or can we say that it is the curvature that produces the gravitomagnetism?

3. The Generalized Newton Law of Gravitation

We have shown recently that Newton law of gravitation can be written, as a Lorentz-like law, as [7]

$$F(r) = m\boldsymbol{E}_g + mv\times\boldsymbol{B}_g, \quad \boldsymbol{E}_g = a = \frac{v^2}{r}, \tag{5}$$

where

$$\boldsymbol{B}_g = \frac{v\times\boldsymbol{E}_g}{c^2}. \tag{6}$$

Thomas introduced a factor $\frac{1}{2}$ to account for the spin-orbit interaction in hydrogen atom [8]. Here $\boldsymbol{B}_g$ is measured in s^{-1}. To convert it to rad/sec, we multiply it by 2π. Hence, the gravitomagnetic force becomes

$$F_m(r) = -\frac{\pi m v^4}{c^2 r}, \quad a = \frac{v^2}{r}, \quad v^2 = \frac{GM}{r}. \tag{7}$$

The gravitomagnetic field is divergenceless, since

$$\nabla\cdot\boldsymbol{B}_g = \frac{1}{c^2}\nabla\cdot\left(v\times\boldsymbol{E}_g\right)$$

$$\nabla\cdot\boldsymbol{B}_g = \frac{1}{c^2}\boldsymbol{E}_g\cdot\left(\nabla\times v\right) - \frac{1}{c^2}v\cdot\left(\nabla\times\boldsymbol{E}_g\right)$$

$$\nabla\cdot\boldsymbol{B}_g = -\frac{1}{c^2}v\cdot\frac{\partial\boldsymbol{B}_g}{\partial t} = -\frac{1}{c^2}\frac{\partial}{\partial t}\left(v\cdot\boldsymbol{B}_g\right) = 0.$$

This implies that the gravitomagnetic lines curl around the moving mass (gravitational current) creating it. This may also rule out the existence of negative mass. Therefore, as no magnetic monopole exits; no gravitomagnetic monopole (antigravity) exits. Thus, the search for magnetic monopole is tantamount to that of antigravity.

The angular momentum is defined by $L = mvr$, so that Equation (7) becomes

$$F_m(r) = -\frac{\pi GML^2}{mc^2r^4}. \tag{8}$$

The second term in Equation (2) is due to the centrifugal term arising from a central force field. In polar coordinates the force is written as

$$ma = m\left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta. \tag{9}$$

For a central force the second term vanishes. It yields, $\dot{\theta} = \frac{L}{mr^2}$, so that the first term becomes

$$ma_r = m\ddot{r} - \frac{L^2}{mr^3}. \tag{10}$$

Substituting Equation (10) in Equation (5) yields the full effective central force, owing to gravitomagnetism, as

$$F(r) = -\frac{GMm}{r^2} + \frac{L^2}{mr^3} - \frac{\pi GML^2}{mc^2r^4}. \tag{11}$$

The corresponding potential will be

$$U(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{\pi GML^2}{3mc^2r^3}.$$

Comparison of Equations (2) and (11) reveals that the gravitomagnetic force is equal to $\frac{\pi}{3}$ of the curvature force. Consequently, the generalized Newton law of gravitation and the general theory of relativity produce the same gravitational phenomena.

The gravitomagnetic force term, the last term in Equation (11), can be written as

$$\frac{\pi GML^2}{mc^2r^4} = \frac{\pi G^2M^2m}{c^2r^3}, \quad \text{where, } v^2 = \frac{GM}{r}. \tag{12}$$

Finally, Equation (11) can be written as

$$F(r) = -\frac{GMm}{r^2} + \frac{J_{\text{eff.}}^2}{mr^3}, \tag{13}$$

where

$$J_{\text{eff.}}^2 = L^2 - \left(\frac{\sqrt{\pi}GMm}{c}\right)^2. \tag{14}$$

4. Precession of Planets and Binary Pulsars

Owing to the above equivalence between gravitomagnetism and GTR, we interpret the precession of the perihelion of planets and binary pulsars as a Larmor-like precession, and not due to the GTR interpretation as due to the curvature of space-time. We may attribute this precession as due to the precession of gravitational moment (mass) in a gravitomagnetic field induced by the massive objects (Sun). In electromagnetism, the Larmor precession is defined by [4]

$$\omega = \frac{e}{2m}B, \tag{15}$$

while in gravitation (since $\boldsymbol{B}_g$ is in s^{-1} and $e \Leftrightarrow m$) it is defined as [1]

$$\omega_g = 2\pi\left(\frac{\boldsymbol{B}_g}{2}\right) = \frac{\pi v^3}{rc^2}, \quad \boldsymbol{B}_g = \frac{va}{c^2} = \frac{v^3}{rc^2}, \tag{16}$$

where (ω_g is in rad/seec) and

$$a = \frac{v^2}{r} \tag{17}$$

The precession rate in Equation (16) can be written as

$$\omega_g = \pi\left(\frac{2\pi GM}{Tc^2 r}\right) = \frac{\delta\phi_g}{T}, \tag{18}$$

where $T = \frac{2\pi r}{v}$ is the period of revolution. This corresponds to a precession angle of

$$\delta\varphi_g = \pi\left(\frac{2\pi GM}{c^2 r}\right) \text{ rad/s}, \tag{19}$$

that is equal to $\frac{\pi}{3}$ of the curvature effect, and for elliptical orbit $r = a\left(1-e^2\right)$.

5. Deflection of α-Particles by the Nucleus

We would like here to interpret the deflection of light by the Sun gravity in an analogous way to the deflection of α -particles by the nucleus, without resorting to the GTR calculation. The deflection angle of α -particles by a nucleus is given by [5]

$$\Delta\theta_e = \frac{4keQ}{mbv^2} \tag{20}$$

where Q is the nucleus charge, v the α -particle speed, k Coulomb constant, and b the impact factor. The corresponding gravitational analog for the deflection of light will be, $v \to c$, $e \to m$, $Q \to M$, $k \to G$, [9]

$$\Delta\theta_g = \frac{4GM}{bc^2} \tag{21}$$

without resorting to GTR calculation. Recall that, according to Equivalence Principle, all particles in gravity accelerate without reference to their mass (whether massive or massless). Therefore, it doesn't matter whether light has a mass or not. The relation in Equation (21) is the same as the relation obtained by GTR as in Equation (4). The minimum distance α particles can approach the nucleus is given by equating the kinetic energy and the Coulomb potential energy that yields the relation

$$b_e = \frac{2kq_1q_2}{mv^2}. \tag{22}$$

In gravitation and for light scattered by the Sun gravity, the above relation gives ($q_1 \to m$, $q_2 \to M$ and $k \to G$)

$$b_g = \frac{2GM}{c^2}. \tag{23}$$

This is nothing but the Schwarzschild distance that no particle can exceed. Therefore, the complete analogy between gravitation and electricity is thus realized. In this context, we have shown recently that the Larmor dipole radiation has a gravitational analogue [10]. Similarly, the same analogy exists between hydrodynamics and electromagnetism [11].

6. The Spin of Planets

The discovery of the spin of the electron by Goudsmit and Uhlenbeck in 1926 was crucial in understanding many physical phenomena that wouldn't have been explained without [12]. This spin is theoretically formulated by Dirac confirming the experimental finding. However, the spin of planets had been known since long time (1851) that was demonstrated by Foucault's pendulum. In a recent paper we have introduced the gravitomagnetism produced by moving planets as the magnetic field produced by moving charge [1]. We then obtained the gravitational Ampere's and Faraday's laws of gravitomagnetism. The gravitomagnetic moment of a planet due to its orbital motion is given by [1]

$$\mu_L = \frac{v^3 r^2}{2G}. \tag{24}$$

For circular orbit, Equation (24) yields

$$\mu_L = \left(\frac{M}{2m}\right)L. \tag{25}$$

In a similar manner the gravitomagnetic moment due to spin will be twice the above value (analogous to electromagnetism)

$$\mu_S = g_S\left(\frac{M}{2m}\right)S, \tag{26}$$

where g_S defines some gyro-gravitomagnetic ratio that is independent of the planet's mass. If we assume the precession of planets is a spin-orbit interaction, then we can equate $-\mu_S \boldsymbol{B}_g$ (assuming the angle to be zero) to the potential term arising from the gravitomagnetic force in Equation (11). This yields, for circular orbit,

$$S = \left(\frac{4\pi}{3g_S}\frac{m}{M}\right)L, \quad S = \left(\frac{4\pi}{3g_S}\frac{Gm^2}{v}\right). \tag{27}$$

This is a very interesting equation, since it determines the spin of planets from their orbital angular momentum. With the help of the above equation, the moment of inertia of planets can be precisely determined. It then follows that the spin and the geometrical form of planets is a consequence of its dynamics. Consequently, the spin

Table 1. The predicated values for spin and moment of inertia owing to Equation (27) with g_S = 57. Any deviation from known values that may appear could be attributed to the uncertainty in determining the radii of planets. Alternatively, the angle between *L* and *S* will be of importance.

Planet	Spin (J_s)	Moment of inertia (kg·m^2)
Mercury	1.12E+31	8.98E+36
Venus	3.31E+33	1.10E+40
Earth	5.84E+33	8.02E+37
Mars	8.32E+31	1.17E+36
Jupiter	1.35E+39	7.69E+42
Saturn	1.64E+38	1.00E+42
Uranus	5.44E+36	5.43E+40
Neptune	9.45E+36	8.12E+40

angular momentum is no longer an intrinsic property of the planet. The energy corresponding to this interaction may be converted into internal energy (heat) inside the planet.

Owing to Equation (27) we are entitled to say that any orbiting planet must spin! Thus, any gravitating object in curvilinear motion must spin. For consistency of the spin of the Earth with the present value with take $g_S = 57$. From this law the moment of inertia of all gravitation objects can be precisely determined. **Table 1** shows the anticipated values for the spin and the corresponding moment of inertia of the planetary system. Equation (27) can be used to estimate the hidden central mass around which another mass orbits. It can be generally useful in many astrophysical applications.

7. Conclusion

We have shown that the gravitomagnetism and the general theory of relativity are two theories of the same phenomenon. This entitles us to fully accept the analogy existing between electromagnetism and gravity. Hence, electromagnetism and gravity are unified phenomena. The precession of the perihelion of planets and binary pulsars may be interpreted as a spin-orbit interaction of gravitating objects. The spin of a planet is directly proportional to its orbital angular momentum and mass weighted by the Sun's mass. Alternatively, the spin is directly proportional to the square of the orbiting planet's mass and inversely proportional to its velocity.

8. Acknowledgements

I would like to thank Prof. G. Gadzirayi Nyambuyathe for critically revising the manuscript and drawing my attention to consider a formula for the spin of planets.

REFERENCES

[1] A. I. Arbab, "The Gravitomagnetism: A Novel Explanation of the Precession of Planets and Binary Pulsars," *Astrophysics and Space Science*, Vol. 330, No. 1, 2010, pp. 61-68. doi:10.1007/s10509-010-0353-7

[2] S. Weinberg, "Gravitation and Cosmology," John Wiley, New York, 1971.

[3] D. Griffiths, "Introduction to Electrodynamics," Prentice-Hall, Upper Saddle River, 1999.

[4] H. L. Malcolm, "Spin Dynamics," John Wiley, New York, 2000.

[5] E. Rutherford, "The Scattering of α and β Particles by Matter and the Structure of the Atom," *Philosophical Magazine*, Vol. 92, No. 4, 2012, pp. 379-398.

[6] T. Cheng, "Relativity, Gravitation, and Cosmology," Oxford University Press, Oxford, 2005, p. 108.

[7] A. I. Arbab, "The Generalized Newton's Law of Gravitation," *Astrophysics and Space Science*, Vol. 325, No. 1, 2010, pp. 37-40. doi:10.1007/s10509-009-0145-0

[8] L. H. Thomas, "The Motion of the Spinning Electron," *Nature*, Vol. 117, No. 2945, 1926, p. 514. doi:10.1038/117514a0

[9] A. I. Arbab, "A Phenomenological Model for the Precession of Planets and Bending of Light," *Astrophysics and Space Science*, Vol. 325, No. 1, 2010, pp. 41-45. doi:10.1007/s10509-009-0146-z

[10] A. I. Arbab, "On the Gravitational Radiation of Gravitating Objects," *Astrophysics and Space Science*, Vol. 323, No. 2, 2010, pp. 181-184. doi:10.1007/s10509-009-0058-y

[11] A. I. Arbab, "The Analogy between Electromagnetism and Hydrodynamics," *Physics Essays*, Vol. 24, No. 2, 2011, pp. 254-259. doi:10.4006/1.3570825

[12] S. Goudsmit and G. E. Uhlenbeck, "Over Het Roteerende Electron en de Structuur der Spectra," *Physica*, Vol. 6, 1926, pp. 273-290.

Journal of Modern Physics, **2012, 3, 1236-1246**
doi:10.4236/jmp.2012.329160 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Gravity Field Variations Associated with the Buried Geological Structures: San Marcos Fault (NE Mexico) Case Study

Vsevolod Yutsis, Yaneth Quintanilla-López, Konstantin Krivosheya, Juan Carlos Montalvo-Arrieta, Gabriel Chávez-Cabello
Faculty of Earth Sciences, Autonomous University of Nuevo Leon (UANL), Linares, Mexico
Email: vyutsis@hotmail.com

Received June 27, 2012; revised July 28, 2012; accepted August 4, 2012

ABSTRACT

Gravity data are sensitive to local vertical offsets across high-angle faults, where rocks with different densities are juxtaposed. Yet high densities in some Mesozoic sedimentary rocks just above the basement may smear out the subtle gravity signatures of basement faults. At this study the gravity data processing tends to avoid ill-described "black-box" techniques. The study area is situated in the Palomas site, Cuatrociénegas region, Coahuila, NE Mexico. The San Marcos Fault is at least 300 km long and has WNW-ESE trend from the central part of Nuevo León State through Coahuila, and finally to the eastern part of Chihuahua State. Gravimetric data shows that the lowest values of free air and Bouguer anomalies are in the southern part of the area, and the highest values are in the western and central part of the area. Between these parts exists a zone of high horizontal gravity gradient. Configuration of linear elements of gravity field (gradient zones) delimited the San Marcos Fault in the San Marcos valley below thickness of recent sedimentary cover. Two density models were carried out, which showed that the Cretaceous rocks are in discordant contact with the Paleozoic rocks that can be related to the San Marcos Fault. The density was determinate using to Nettleton's method, which results highlight the presence of the San Marcos Fault. Density models showed that the Quaternary sediments are in direct contact with the San Marcos Fault.

Keywords: Earth Gravity; Newton Potential; Geophysical Prospecting; Density Models; Free Air Anomaly; Gravity Field Variations

1. Introduction

Steep, straight faults are commonly expressed as subtle potential-field lineaments, which can be gradient zones, alignments of separate local anomalies of various types and shapes, aligned breaks or discontinuities in the anomaly pattern, and so on [1-4]. Many large magnetic and gravity anomalies represent the ductile, ancient, healed basement structures, obscuring the desirable subtle features [1-8].

Subtlety of the desirable lineaments necessitates detailed data processing, using a wide range of anomaly-enhancement techniques and display parameters. The final choice of processing steps depends on which aspects of the anomalies one aims to enhance, as well as on experimentation with various techniques [1,3,5,9,10].

Not a panacea, data processing is a necessary evil. Because the signal and noise anomaly characteristics commonly overlap, complete separation between them may be impossible. Noise artifacts may actually be introduced [3].

Unexpected consequences and processing side-effects are normal. As well, it may be hard to know in advance which of the many anomalies are desirable.

The processed and enhanced anomalies should ideally be easy to relate back to the original anomaly shapes. We kept the data processing to a minimum, avoided ill-described "black-box" techniques, and relied on mathematically simple and intuitive procedures [3,5-11].

The San Marcos Fault (SMF), was defined by Charleston [12,13], and is outcropping in Central Coahuila in northeast Mexico correspond to one of the three major zones of west-northwest trending lineaments interpreted as basement faults that controlled the tectono-sedimentary evolution since the Triassic-Jurassic in the region [14,15] (**Figures 1** and **2**), the others structural lineaments are: The La Babia fault (LBF) [12,13], located in Northern Coahuila; and the Mojave-Sonora Megashear (MSM) [14-16]. From these basement faults, only the SMF has a

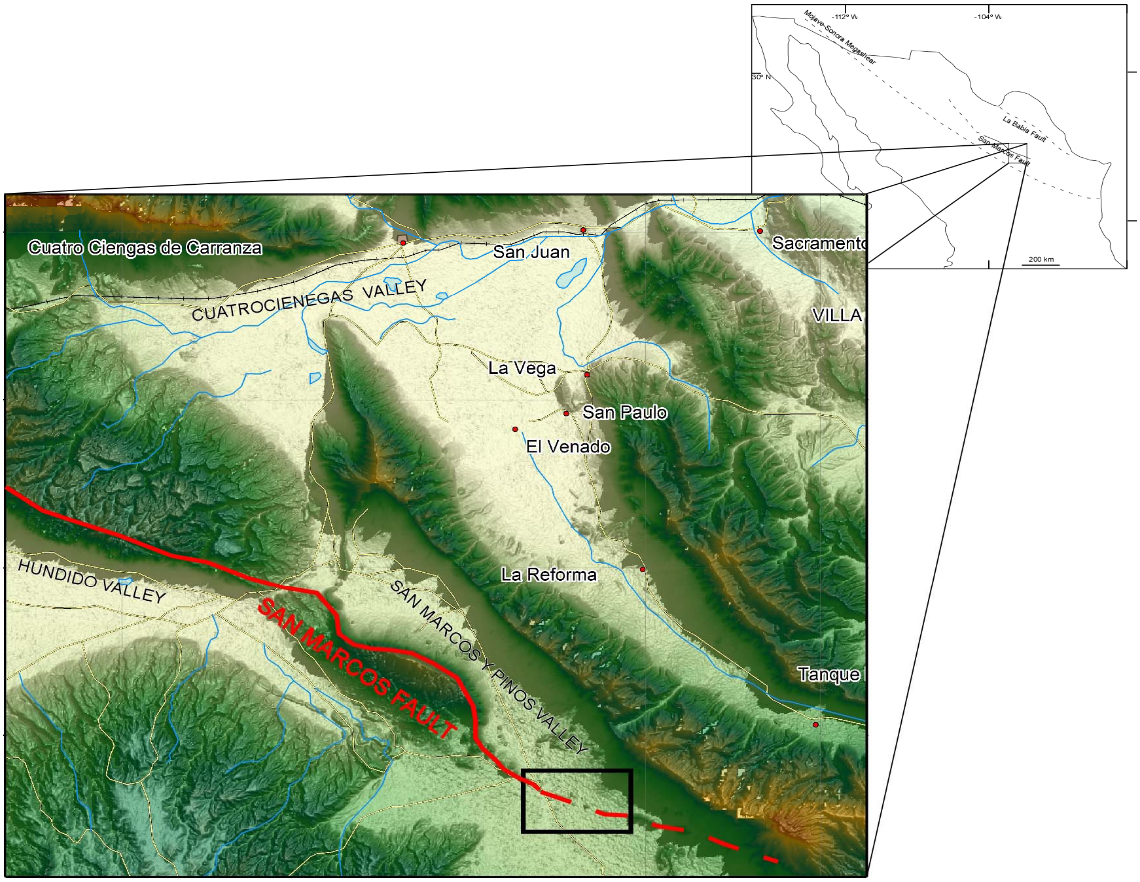


Figure 1. Position of San Marcos Fault in northern Mexico. Modified after McKee *et al.*, 1984 and 1990; Padilla y Sanchez, 1986; Aranda-Gómez *et al.*, 2005. Inset: Location of San Marcos Valley, Pinos Valley and the Palomas study area.

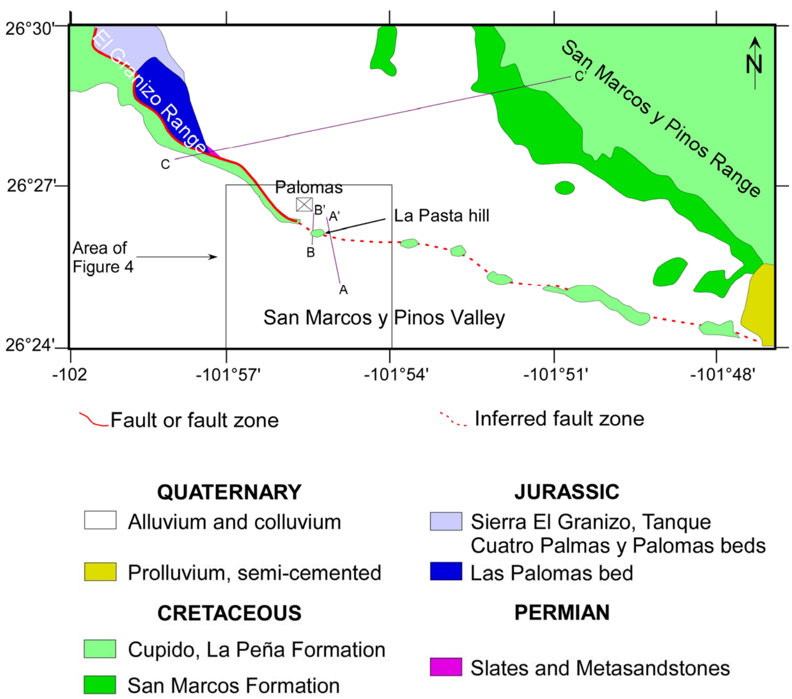


Figure 2. Geological map of the study area. A-A' and B-B' are the 2D density models; C-C' is the geological cross section by McKee *et al.*, 1990.

trace well defined from stratigraphic and structural evidence, which can be followed since Potrero La Gavia, through San Marcos Valley (SMV), to Potrero Colorado and to the Sierra Mojada [17-21]. These authors mentioned that SMF is distinguished by: contrasting structural styles between the Coahuila block and the Coahuila folded belt; the existence of a pre-Tithonian and Neocomian major sedimentary clastic wedge related to its activity and deposited in the hanging wall of the fault; and the existence of fault contacts between Permian, Jurassic, and Cretaceous rocks in the SMV and Potrero Colorado areas (**Figure 2**). In areas where the SMF is buried, such as the Camargo volcanic field has been inferred from structural features in the surface [22-24].

The SMF, LBF and the MSM play an important role to understand the tectonic and stratigraphic evolution of northeast Mexico. Several authors [14-16,23,25,26] propose that these fault systems (SMF and LBF) were reactivated at different times. McKee *et al*. (1984) found evidence of movement of the San Marcos fault during the Jurassic, Early Cretaceous, and Tertiary. Additionally, Aranda-Gómez *et al*. (2005) suggest that SMF have been reactivated during the Late Pliocene-Quaternary by normal faulting in the border of Chihuahua-Coahuila states. In central Coahuila not has been documented Quaternary activity, due to the works published have been focused mainly in the study of pre and syn-laramide reactivation of the fault. As mentioned before, the study of the SMF is related only with geologic information, geophysics data not are available concerning to characterization of this basement fault. One goal of interest about the possible reactivation of the SMF at Plio-Quaternary time is the seismic risk associated with potential earthquakes related to this fault. Northeast Mexico is characterized by low seismicity and a lack of strong ground motion records [27-30].

Large intraplate earthquakes in the relatively stable interior regions of continents are rare in comparison with those that originate in plate boundary regions. However, these occasional events can be extremely devastating, because cities located in continental interior regions are often built without seismic design criteria. Crone *et al*. (2003) mention that major intraplate earthquakes can cause widespread damage because the attenuation of seismic energy from large earthquakes is relatively low in plate interiors. Galván-Ramírez and Montalvo-Arrieta (2008) made a compilation of the historical seismicity in northeast Mexico; they found that some of the epicenters locations are overlaped or near to the traces of the major regional faults trending west-northwest postulated in northeast Mexico: LBF, SMF, and MMS. **Figure 3** shows the historic seismicity for northeast Mexico. These authors hypothesize that seismic activity reported for this region, which lies near or on the trace of these faults,

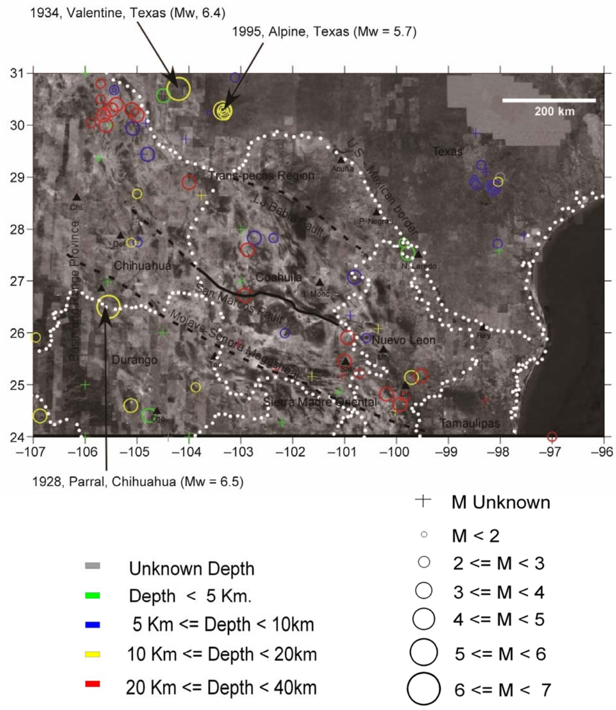


Figure 3. Historic seismicity of northeastern Mexico and southern Texas (1847-2005). The open circles represent the epicentral location of the earthquakes studied. The solid and dash lines depict the three general north-northwest trending lineaments and faults that have been identified or postulated in northeast Mexico, the La Babia fault, the San Marcos fault, and the Mojave-Sonora megashear (MSM). Triangles show largest cities (Acuña; Chi: Chihuahua; Del: Delicias; Parral; Tor: Torreón; Mon: Monclova; Sal: Saltillo; Mty: Monterrey; Lin: Linares; N. Laredo: Nuevo Laredo; P. Neg: Piedras Negras; Rey: Reynosa) (*Modified after* Galván-Ramírez and Montalvo-Arrieta, 2008).

could be related to reactivation of these major faults. However, to probe this hypothesis is necessary to install a seismic network to confirm or refute the presence of seismicity associated with these fault systems. Additionally, these authors mentioned that a possible critical scenario would represent a rupture ($M_W = 6.5$) in the south segment of the San Marcos fault in Central Coahuila. The importance of this scenario is the settlement of three of the most populated centers in northeast Mexico (Monterrey, Saltillo and Monclova with a total population of more than six million) located in a radius less than 150 km from the fault source.

Some evidence of recent seismicity could be obtained from visual observation of deformation of latest sediments covering fault zone. One way to identify buried faults systems in sedimentary valleys is by means of gravity data. The goal of this study is identifies the SMF zone in the SMV in central Coahuila from gravity data and have the first stage to identify if there is evidence of displacements in Quaternary sediments. To develop this, we obtain for first time precision gravity data for five north-south trending lines across the Palomas area (**Figure 4**) in SMV.

2. Geologic Framework

The Palomas area is located to the NW of San Marcos y Pinos valley and to the SE of Sierra El Granizo, between 26˚24'N and 26˚26'N, and 101˚50'W and 101˚57'W (**Figures 2** and **4**). In this area there are some isolate hills composed by limestone of Upper Tamaulipas Formation that can be associated to displacement of the SMF, we re-collected gravity data distributed in five profiles, and two of them cross the La Pasta hill (**Figure 4**).

SMV is located in central Coahuila and represents an area where there is excellent evidence of the SMF [14,19, 31].

The SMV are compose by pre-Jurassic outcrops composed by small lenses of granite rocks at NW of the valley and slates and metasandstones in fault contact with both Cretaceous and Jurassic rocks at northwest of the Las Palomas area. The Jurassic sediments composed by conglomeratic sandstone of Las Palomas beds and sandstone of the Sierra El Granizo beds are exposed along the foothills of the Sierra El Granizo and in the valley where a gradational contact with overlying Neocomian San Marcos Formation is exposed. Tanque Cuatro Palmas-beds consist of nearly 100 m of fine-grained marine sedimentary rocks without conglomeratic units [14]. The Cretaceous sediments are defined by the San Marcos Formation composed by continental alluvial deposits represented by conglomeratic units of pebbles or cobbles of volcanic rocks with subordinate quartz, and plutonic fragments, overlying in a discordant contact is limestone of the Cupido Formation, the sedimentary sequence are complete by shale and marble of the La Peña Formation, follow by limestone of Tamaulipas Superior Formation, overlying by limestone, shale and banded chert of Cuesta del Cura Formation and interbeded shales with marble and gysum veins of Indidura Formation, the marine sequence is overlie discordantly by Quaternary deposits [14, 19,31].

3. Geophysical Data Collection and Reduction

More than 150 gravity reading points grouped in 5 lines were carried out nearly perpendicular to the main trace of the SMF. Gravity stations spacing was from 15 - 20 up to 40 - 50 meters. Gravity meter Autograv CG-5 with 0.001 milligal (mGal) of reading resolution was used (1 Gal = 1 cm/s^2). Gravity reference station was established in the Central Place of Cuatrocienegas city.

The error of measurements (RMS error) did not exceed 5 microgal (μGal). Corrections were applied in real time for tilt errors, for long term drift, for the temperature of the sensor, and for earth tides. The seismic filter was applied too. The location of gravity stations was determined by a GPS Magellan Platinum with a horizontal accuracy of 5 - 7 m.

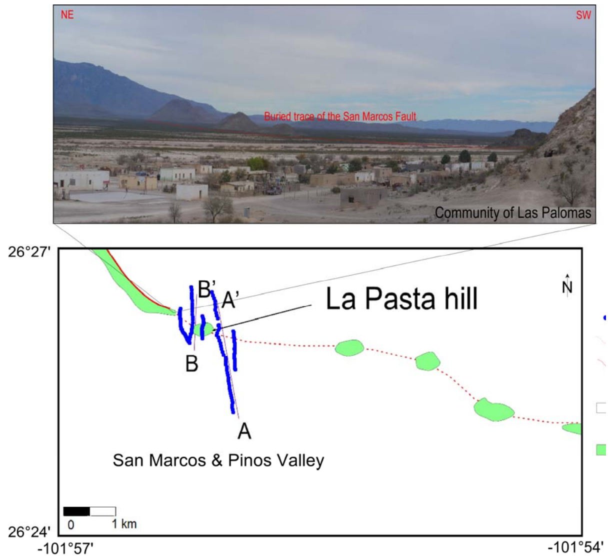


Figure 4. Panoramic view of the Palomas area. Geologic map and the distribution of gravity points over the study area. ***Modified after*** **Arvizu Gutiérrez (2006).**

Elevation of the gravity reading points was measured with a Total Station Nikon DTM-551 with precision of ±2 mm. The standard corrections, e.g. instrumental drift, latitude, elevation using new standards to reducing gravity data were applied to obtained measurements [32]. As a result of field data correction and processing free air and Bouguer gravity anomalies were obtained. Data interpretation procedures include Fourier transformation, wavelength filters, upward and downward continuation, vertical and horizontal derivates, etc. [1,5,9,11,33,34]. A series of maps (**Figures 5(a)-(d)**) and graphs (**Figures 6** and **7**) show gravity anomalies interpretation technique. 2D geological-geophysical models were elaborated (**Figures 8** and **9**).

A sea level datum NAD 83 and the standard density 2.3 g/cm^3 were used for the elevation correction. The terrain correction was calculated using the method by Hammer and was applied to each gravity station to obtain the complete Bouguer anomaly.

Gamma ray measurements were carried out additionally to gravity study. Gamma ray reading points were the same as gravity stations. GRS-500 Differential Gamma Ray Spectrometer/Scintillometer was used to measure the gamma radiation emitted by various daughter isotopes in the uranium decay series. Sample rate was selected as 10.0 seconds, and energy window was choosing detecting total count above 0.08 MeV. Observed count rates were corrected due to dead time (4 microseconds).

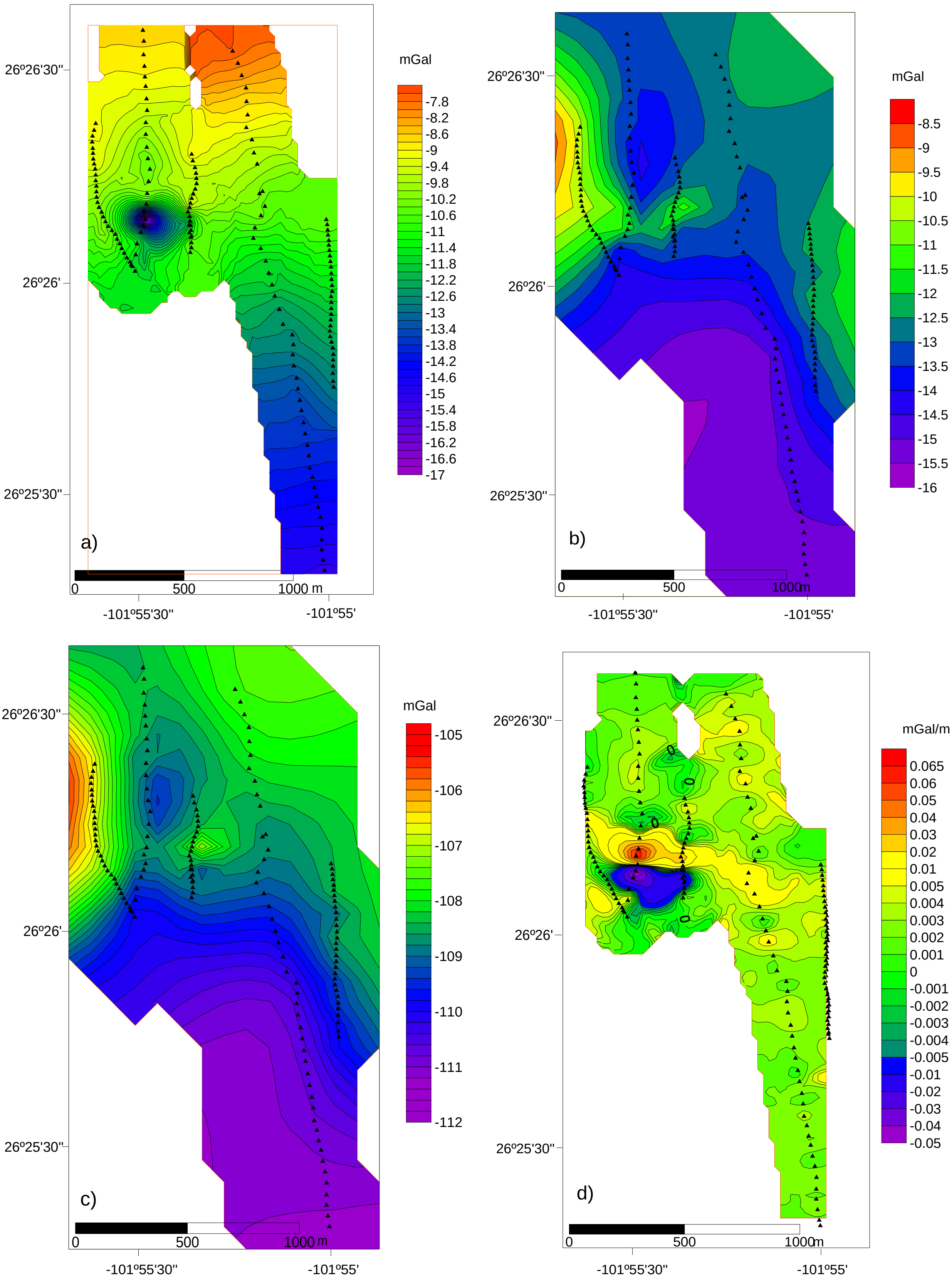


Figure 5. Gravity maps of study area: (a) Observed gravity; (b) Free air anomaly; (c) Bouguer gravity anomaly; (d) Horizontal gradient of gravity.

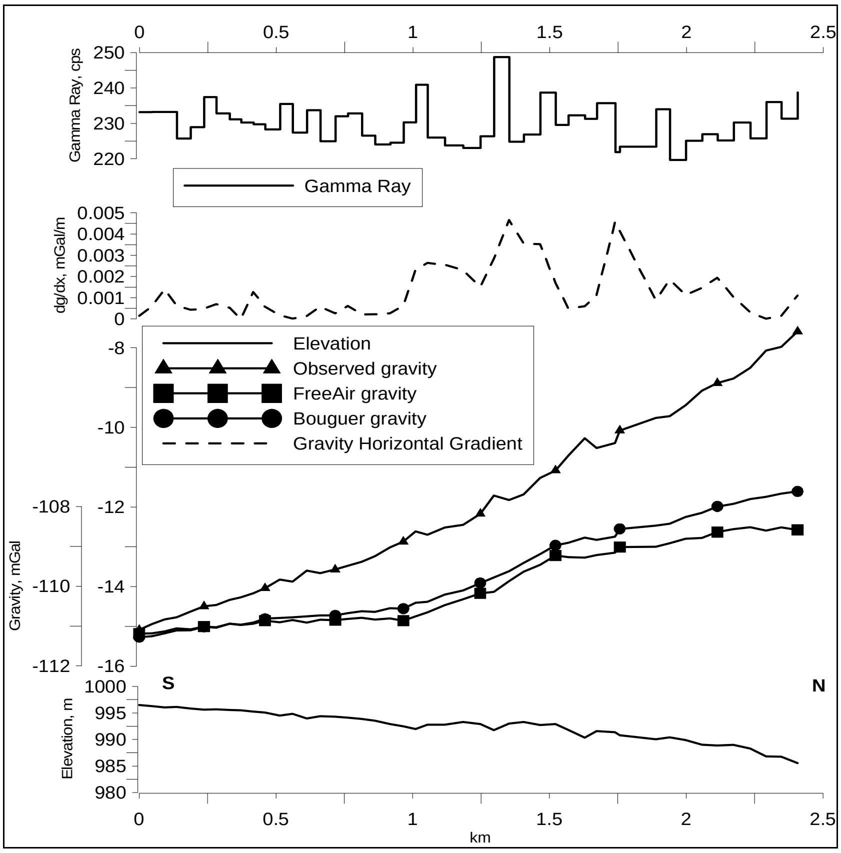


Figure 6. Geophysical curves along the profile 4. These graphs show strong coincidence between observed gravity, free air and Bouguer anomalies, horizontal gravity gradient, and also gamma ray measurements.

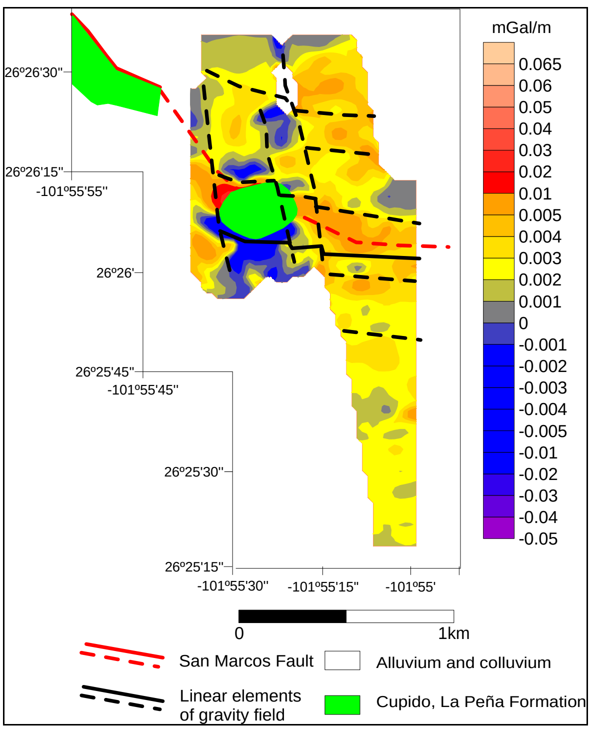


Figure 7. Delimitation of the San Marcos Fault in San Marcos Valley on the gravity data analysis.

4. Gravity and Gamma Ray Data Analysis

The observed gravity map (**Figure 5(a)**) shows an isolated gravity high in the NE part of study area. Southeast of this high, an extensive gravity low is observed. The amplitude of gravity field between these extreme parts reaches 24 milligals (mGal). A high horizontal gradient zone (up to 0.01 mGal/m) is shown here (**Figure 5(d)**).

The free air gravity map (**Figure 5(b)**) shows an anomaly that divide two big blocks. The southern block present the lowest values that varying between –15.5 to –14.5 mGal. The northern block shows high gravity values. Anomalies of –9.0 - 12.5 mGal are observed here. The Bouguer anomaly (**Figure 5(c)**) shows the same behavior that the free air gravity, where can see two main areas. The complete Bouguer gravity (**Figure 5(c)**) values in the southern part of the study area are in the range of –109 up to –112 mGal and show several weak local anomalies with an amplitude of ±0.2 mGal. Gravity high with a range of –105 up to –108 mGal is shown in the northern part of area. There is a relatively gravity low (up to –110 mGal) between two extreme parts of this positive zone. A strong gravity gradient zone marks a structural boundary between these blocks. It seems that this strong anomaly is related to the main trend of the fault zone in

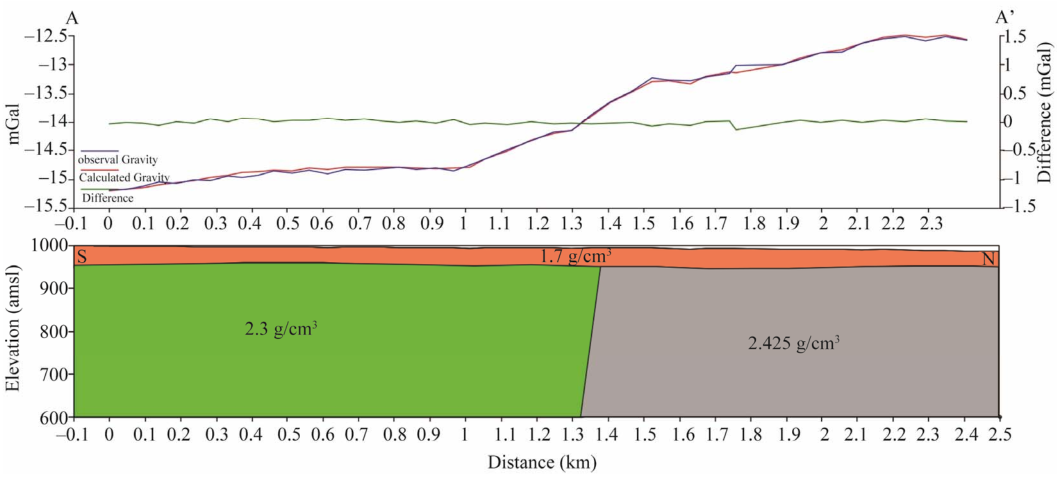


Figure 8. Results of gravity surveys for the profile A-A' (location shown in Figure 4) show residual Bouguer gravity anomaly and best-fit inverse density model at Palomas site, a normal buried fault is inferred.

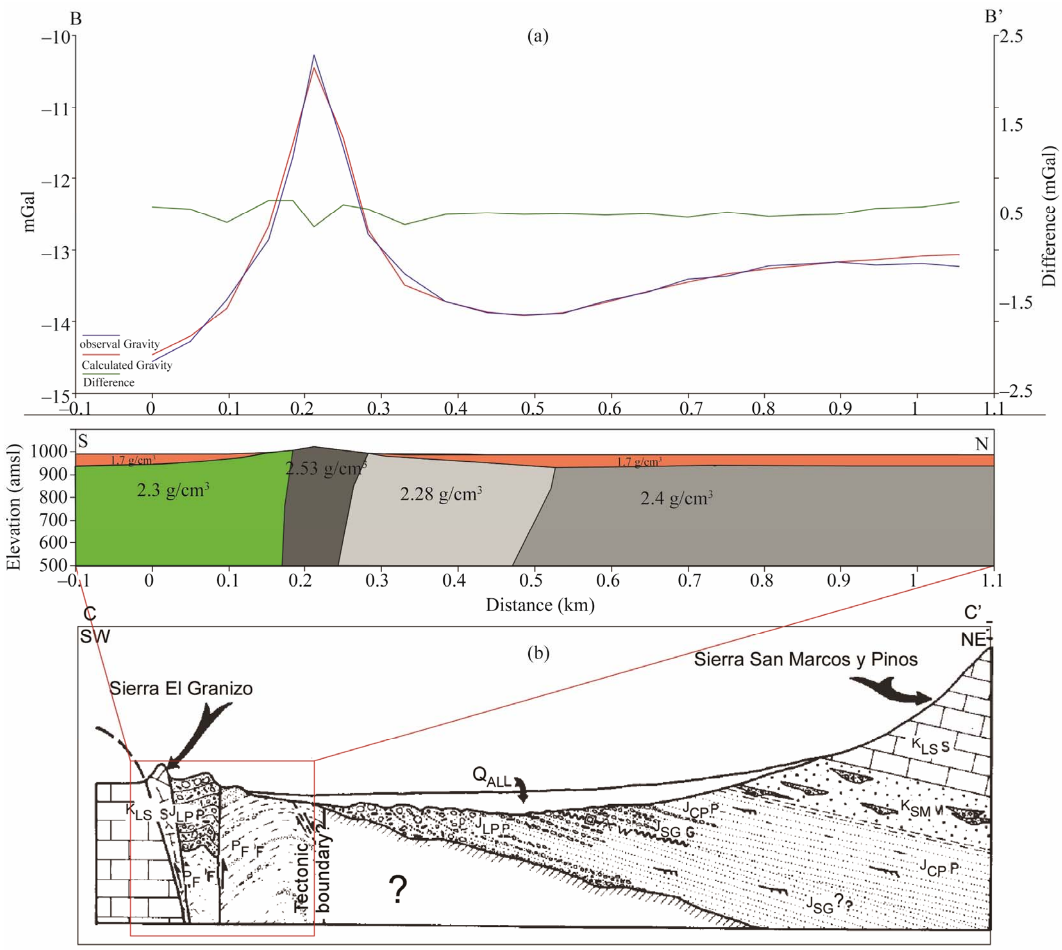


Figure 9. (a) Conceptual density model for the profile B-B' (location shown in Figure 4); and (b) Generalized geologic cross section extending from the Sierra El Granizo northeastward to the crest of Sierra San Marcos y Pinos when is localized the tectonic boundary (C-C'). PF, Paleozoic flysch and wildflysch; JLP, Las Palomas beds (Late Jurassic); JSG, Sierra El Granizo beds (Late Jurassic); JCP, Tanque Cuatro Palmas beds (Late Jurassic); KSM, San Marcos Formation (Early Cretaceous); KLS, Cretaceous limestone; Tamaulipas Superior, La Peña y Cupido Formations (Late Cretaceous); QAL, Alluvium (Quaternary). The box shows part of the cross section that can have relation with the density model for profile 5 (Modified after McKee *et al*., 1990).

the area. The values of horizontal gravity gradient here are about 0.0035 - 0.004 mGal/m (**Figure 6**) and reach 0.035 - 0.04 mGal/m near the main fault zone (**Figure 5(d)**). The same characteristics have the gamma ray curves, which show a close coincidence with gravity gradient zones. For example, **Figure 6** shows two extreme values of horizontal gradient 0.0024 mGal/m and 0.00450 mGal/m and two gamma ray peaks (240 cps and 250 cps) correspond them. As general, observed and transformed gravity anomalies was revealed trending northwest-southeast across the central part of Las Palomas area. An extensive gravity gradient zone trends in the same direction. This zone probably represents the San Marcos Fault, which crosses the area from northwest to southeast (**Figure 2**).

The gravity data suggest a complex subsurface structure in the San Marcos Valley area. It seems that two distinct systems of linear elements dominate in the basement as well as shallow structure (**Figure 7**). The first has a southeast direction and it coincides with the San Marcos Fault main direction, plotting along the general axe of the San Marcos Valley. The second linear system has a nearly perpendicular (north-northwest-south-southeast) direction. It seems that these faults divide the San Marcos Valley area to the several blocks of basement accordingly the main structure of the San Marcos Fault (**Figure 7**).

5. Modeling 2D of Gravity Data

Figure 7 shows the contour of the Bouguer map and the main geologic features in the study area. In this figure is depicted the fracture arrangement that we interpreted from gravity data too. Two sets of fractures can be interpreted from the gravity data: WNW-ESE and NNW-SSE (**Figure 7**). It is associated with the isolated calcareous hill (La Pasta) at center of figure, which can be correlated with the trace of the SMF (red discontinuous line).

We also interpreted secondary faults that have the same orientation that SMF and a systems of faults or fractures the cut the main trending system that corresponds to conjugate faults.

From gravity data we constructed two 2D cross sections models for profiles A-A’ and B-B’ (**Figure 4**). The first one (A-A’) shows the model located in the sedimentary valley (**Figure 8**). In this figure are identified three units with the follow density values: Red unit with a density of 1.7 g/cm^3 related to alluvial deposits. The green unit has a density value of 2.3 g/cm^3 correlate with calcareous materials that is in a strong discontinuity or fault contact with the grey unit that is related to material with an increase in density of 2.425 g/cm^3 that correspond to Paleozoic flysch. The same figure point up the correlation between the observed and calculate gravity data by the 2D model, which shows a step geometry of the gravitational anomaly; the interpreted source of this anomaly is due to the presence of the SMF. The difference of density for each block varies 0.125 g/cm^3. The denser rocks are in the handing-wall of the fault. In this profile the anomalies can be associated to: a) basement up-lift or b) density contrast between rocks. These two circumstances can be interconnected as follows: small negative anomalies can be caused by increase in the thickness of quaternary sediments in those places where the rocks are less strong and less dense; we observed local positives anomalies that can be related to denser rocks denser rocks which are less weathered and thicker than the surrounded county rocks.

Figure 9 depicts the profile B-B’, it cross-cut La Pasta hill which have a height about 40 m and where calcareous rocks are outcropped, this profile has a south-north direction. The observed and calculated gravity show a positive anomaly of 4 mGal in the La Pasta hill and a small negative anomaly nearby –0.5 mGal to the north from this hill which can be related to denser rocks (0.1 mGal). Positive anomaly in area of the La Pasta hill reflects a relative high effective density of 0.23 g/cm^3 (density of adjacent rocks is about 2.53 g/cm^3). Geologically, this anomaly can be correlated with stronger and denser blocks by mineralization or low grade metamorphism. Negative anomaly can be caused by a thicker package of quaternary sediments. We identified the follows density units (**Figure 9**): Red unit with a density of 1.7 g/cm^3 related to alluvial deposits, the green unit had a density value of 2.3 g/cm^3 correlate with calcareous materials, that is in a strong discontinuity or fault contact with the third unit with a density of 2.53 g/cm^3, that can be related with the main trace of the SMF that include conglomeratic sandstone of the Las Palomas beds and Paleozoic flysch. To the north of main fault exists another fault or discontinuity that separates the fourth and fifth units that have density values of 2.28 g/cm^3 and 2.4 g/cm^3 respectively.

6. Discussion

The Palomas area shows a clear evidence of the SMF documented by McKee *et al.*, (1990) and Chávez-Cabello *et al.*, (2007), in the Sierra El Granizo and La Pasta hill in SMV it is inferred because there are not geologic evidence of activity; in the Palomas area exist a isolate hills that can be associated with superficial expression of the SMF.

Gravity profiles ware carried out perpendicular to the main trace of the SMF in the SMV. The Bouguer anomaly shows maximum and minimum which can be used to define a system of fractures with a direction that can be correlated to the principal trending of the SMF in the

Sierra El Granizo and San Marcos y Pinos that have a NW-SE direction [14,19]; also, orthogonal faults were identified, this can be associated with the change of direction in the main trace of the SMF or the fault bend zone in the SMF.

From gravity 2D models, we observed that there are contrasts of density in both models that are associated with displacement of the SMF. The model A-A' (**Figure 8**) shows how the limestone are in discontinuity with the flysch and can be associated with the main traced of the SMF in this area where the fault had been inferred, in agreement with geological data [14,19] the fracture constrained with gravity data can be associate with the reverse reactivation of the SMF. On the other hand the Quaternary sediments could be cutting by recent activity of the SMF, this gravity study is the first geophysical study in the area that contemplate the mapping and found evidence if the SMF had had Quaternary activity such as suggested by Aranda-Gómez *et al*., (2005) in Chihuahua state, and its relation with seismic risk in northeast of Mexico.

The model B-B' (**Figure 9**) shows some discontinueties that constrain a fault system related with the main fault. This model is similar to the McKee *et al*. (1990) geological cross-section from Sierra El Granizo-Sierra San Marcos y Pinos (profile C-C', **Figure 2**). **Figure 8** shows the correlation between the gravity cross-section obtained in this study and the geological model by McKee *et al.* (1990). In the southern part of the gravity model there is a discontinuity between the second and third units that can be correlated with the SMF where the density values for de second unit (2.3 g/cm^3) could be associate with limestone of Upper Tamaulipas which is in reverse fault contact with the third unit, composed by a mixture of Paleozoic flysch and conglomeratic sandstone of the Las Palomas beds.

A second discontinuity north of the La Pasta hill involves a reverse fault with the third and fourth units, and correlate the Paleozoic flysch (unit fourth) and the shear zone composed by Las Palomas beds and the Paleozoic flysch [14,19].

A third discontinuity is formed by the fourth and fifth units, where the Paleozoic flysch (forth unit) is in reverse fault contact with an unknown material with a density value of 2.4 g/cm^3 (fifth unit). Finally, both profiles A-A' and B-B' have a fault that is in direct contact with Quaternary sediments (first unit) of the SMV and this sediments could be cross-cut by this fault.

7. Conclusions

This is the first study to detect a buried basement fault in the Palomas area; we used a precision gravity data to find density contrasts of rocks to map fractures or faults. We choose the Palomas area in the SMV because is around by Sierra El Granizo and Sierra San Marcos y Pinos in central Coahuila, where had been found clear geological evidence (stratigraphic and structural) of the San Marcos Fault.

Results of this study show that the Free Air and Residual Bouguer anomalies are separating two blocks related with the San Marcos Fault. By other hand, the Bouguer anomaly is dominated by a series of faults or a fault zone that is interconnected by a principal fault, this zone has a width around of 1000 m. This fault system can be separated by a trending fault with a NNW-SSE direction that is correlated with the strike of the San Marcos Fault identified in the foothills of Sierra El Granizo; the other faults system is conjugate to the main system and is related to the change of direction of the principal fault orientation proposed by some authors.

We presents two 2D gravity models where can define the San Marcos Fault, in the first profile (A-A') only can identify one discontinuity that is related with the princepal fault in agreement with geological data suggest the presence of a reverse fault. The second model (B-B') constrain a wide fault zone, this model can be compared with the geological cross-section published by McKee *et al.* (1990). This model show a fault zone where the limestone are outcropped (La Pasta hill), these rocks have a density value of 2.3 g/cm^3.

In both models, we can see that the Quaternary sediments are in direct contact with the fault; however, it is not enough evidence how this fault is cutting the Quaternary sediments. Other geophysical data (e.g., resistivity, magnetic and seismic methods) can be collected in this area to define if the San-Marcos Fault is cutting the Quaternary sediments.

8. Acknowledgements

This work was supported by the CONACYT Project 129550 "Evolución terciaria de cuencas continentales del norte de México: controles tectónicos heredados, pulsos de deformación, magmatismo y registro bioestratigráfico".

REFERENCES

[1] W. J. Hinze, C. Aiken, J. Brozena, B. Coakley, D. Dater, G. Flanaga, R. Forsberg, T. Hildenbrand, G. R. Keller, J. Kellogg, R. Kucks, X. Li, A. Mainville, R. Morin, M. Pilkington, D. Plouff, D. Ravat, D. Roman, J. Urrutia-Fucugauchi, M. Véronneau, M. Webring and D. Winester, "New Standards for Reducing Gravity Data: The North American Gravity Database," *Geophysics*, Vol. 70, No. 4, 2005, pp. J25-J32.

[2] C. Jorgensen, "Tectonic Interpretation Using Potential Field Data for the Sweetgrass Arch Area, Montana-Alberta, Saskatchewan. Rocky Mountain Section," *AAPG Meeting*, Denver, 9-11 August 2004.

[3] H. Lyatsky, D. P. Reg Olson and L. Godwin, “Detection of Subtle Basement Faults with Gravity and Magnetic Data in the Alberta Basin,” *The Leading Edge*, Vol. 24, No. 12, 2005, p. 1282.

[4] H. Lyatsky, D. P. R. Olson and L. Godwin, “Mapping of Basement Faults with Gravity and Magnetic Data in Northern Alberta,” 2005, 12 pages. http://www.cspg.org/documents/Conventions/Archives/Annual/035S0119.pdf

[5] P. V. Sharma, “Environmental and Engineering Geophysics,” Cambridge University Press, Cambridge, 2004.

[6] A. T. Ponce, V. Yutsis, E. R. H. Flores, A. A. Bulychev and K. Krivosheya, “Tectonic Features of La Popa Basin and Adjacent Areas in the NE Mexico Received with the Geophysical Potential Fields,” *Bulletin of the Mexican Geological Society*, Vol. 63, No. 2, 2011, pp. 271-283.

[7] V. V. Yutsis, U. Jenchen, H. de León-Gómez, F. Izaguirre Valdez and K. Krivosheya, “Paleogeographic Development of the Surroundings of Cerro Prieto Water Reservoir, Pablillo Basin, NE Mexico, and Geophysical Modeling of the Reservoir’s Subsurface,” *Neues Jahrbuch für Geologie und Paläontologie*, Vol. 253, No. 1, 2009, pp. 41-59. doi:10.1127/0077-7749/2009/0253-0041

[8] V. Yutsis, A. T. Ponce and K. Krivosheya, “Geophysical Modeling of the Surroundings of La Popa Basin, NE Mexico, with Gravity and Magnetic Data,” In: E. V. Sharkov, Ed., *New Frontiers in Tectonic Research—General Problems*, *Sedimentary Basins and Island Arcs*, InTech Publishing, Rijeka, 2011, pp. 125-142. doi.org/10.5772/18760

[9] M. Talwani, J. L. Worzel and M. Landisman, “Rapid Gravity Computations for Two-Dimensional Bodies with Application to the Mendocino Submarine Fracture Zone,” *Journal of Geophysical Research*, Vol. 64, No. 1, 1959, pp. 49-59. doi:10.1029/JZ064i001p00049

[10] W. M. Telford, L. P. Geldart and R. E. Sheriff, “Applied Geophysics,” Cambridge University Press, Cambridge, 1990. doi:10.1017/CBO9781139167932

[11] R. P. Y. Sánchez, “Post-Paleozoic Tectonics of Northeast Mexico and Its Role in the Evolution of the Gulf of Mexico,” *Geofísica Internacional*, Vol. 25, 1986, pp. 157-206.

[12] S. Charleston, “Stratigraphy, Tectonics and Hydrocarbon Potential of the Lower Cretaceous, Coahuila Series, Coahuila, Mexico,” Ph.D. Thesis, University of Michigan, Michigan, 1974.

[13] S. Charleston, “A Summary of the Structural Geology and Tectonics of the State of Coahuila, Mexico,” In: C. I. Schmidt and S. B. Katz, Eds., *Lower Cretaceous Stratigraphy and Structure*, *Northern Mexico*, Field Trip Guidebook, West Texas Geological Society, Midland, 1981.

[14] H. V. Lyatsky, D. I. Pana and M. Grobe, “Basement Structure in Central and Southern Alberta: Insights from Gravity and Magnetic Maps,” *EUB/AGS Special Report*, Vol. 72, 2005, pp. 1-83.

[15] J. W. McKee, N. W. Jones and L. E. Long, “Stratigraphy and Provenance of Strata along the San Marcos Fault, Central Coahuila, Mexico,” *Geological Society of America Bulletin*, Vol. 102, No. 5, 1990, pp. 593-614. doi:10.1130/0016-7606(1990)102<0593:SAPOSA>2.3.CO;2

[16] T. H. Anderson and V. A. Schmidt, “The Evolution of Middle America and the Gulf of Mexico-Caribbean Sea Region during Mesozoic Time,” *Geological Society of America Bulletin*, Vol. 94, No. 8, 1983, pp. 941-966. doi:10.1130/0016-7606(1983)94<941:TEOMAA>2.0.CO;2

[17] I. R. A. Gutiérrez, “Paleomagnetismo de Rocas Jurásicas y Cretácicas del Valle San Marcos, Coahuila, México,” Master Thesis, Universidad Nacional Autonoma de Mexico, Querétaro, 2006.

[18] G. Chávez-Cabello, J. J. Aranda-Gómez, R. S. Molina-Garza, T. Cossío-Torres, L. R. Arvizu-Gutiérrez and G. A. González-Naranjo, “The San Marcos Fault: A Jurassic Multireactived Basement Structure in Northeastern México,” *Geology of México*: *Celebrating the Centenary of the Geological Society of México*, Vol. 422, 2007, pp. 261-286.

[19] G. C. Cabello, “Deformación y Magmatismo de la Cuenca de Sabinas, Coahuila, México,” Ph.D. Thesis, Universidad Nacional Autonoma de Mexico, Centro de Geociencias, 2005.

[20] G. Chávez-Cabello, J. J. Aranda-Gómez, R. Molina-Garza, T. Cossío-Torres, I. R. Arvizu-Gutiérrez and y G. González-Naranjo, “The San Marcos Fault: A Jurassic Structure of Multi-Reactivated Basement of the North-East of Mexico,” In: S. A. Alaniz-Álvarez and A. F. Nieto-Samaniego, Eds., *Bulletin of the Mexican Geological Society*, *Commemorative Volume of the Centenary Big Tectonic Frontiers of Mexico*, Vol. LVII, No. 1, 2005, pp. 27-52.

[21] W. R. Muehlberger, R. C. Belcher and L. K. Goetz, “Quarternary Faulting in Trans-Pecos Texas,” *Geology*, Vol. 6, No. 6, 1978, pp. 337-340. doi:10.1130/0091-7613(1978)6<337:QFITT>2.0.CO;2

[22] J. J. Aranda-Gómez, J. F. Luhr, T. G. Housh, G. Valdez-Moreno and G. Chávez-Cabello, “Late Cenozoic Intraplate-Type Volcanism in Central and Northern México: A Review,” *Geology of México: Celebrating the Centenary of the Geological Society of México*, Vol. 422, 2007, pp. 93-128.

[23] J. J. Aranda-Gómez, J. F. Lhur, C. D. Henry, T. Becker and G. Chávez-Cabello, “Reactivation of the San Marcos Fault during Mid- to Late Tertiary Extension, Chihuahua, Mexico,” In: T. H. Anderson, J. A. Nourse, J. W. McKee and M. B. Steiner, Eds., *The Mojave-Sonora Megashear Hypothesis*: *Development*, *Assessment*, *and Alternatives*, Geological Society of America, Special Paper 393, 2005, pp. 481-521.

[24] J. Aranda-Gómez, J. Luhr, T. Housh, C. Cannor, T. Becker and C. Henry, “Synextensional Pliocene-Pleistocene Eruptive Activity in the Camargo Volcanic Field, Chihuahua, México,” *Geological Society of America Bulletin*, Vol. 115, No. 3, 2003, pp. 298-313.

[25] J. W. McKee, N. W. Jones and L. E. Long, “History of Recurrent Activity along a Major Fault in Northeastern Mexico,” *Geology*, Vol. 12 , No. 2, 1984, pp. 103-107. doi:10.1130/0091-7613(1984)12<103:HORAAA>2.0.CO;2

[26] R. S. Molina-Garza, I. R. Arvizu-Gutiérrez and y G. Chávez-Cabello, "Paleomagnetism of the Palomas Formation (Jurassic) and of the Permian-Triassic Granites in South Coahuila: Tectonic Implications," *GEOS*, Vol. 23, 2003, p. 112.

[27] A. J. Crone, P. M. De Martini, M. N. Machette, K. Okumura and J. Prescott, "Paleoseismicity of Two Historically Quiescent Faults in Australia: Implications for Fault Behavior in Stable Continental Regions," *Bulletin of the Seismological Society of America*, Vol. 93, No. 5, 2003, pp. 1913-1934. doi:10.1785/0120000094

[28] D. Doser, "The 16 August 1931 Valentine, Texas, Earthquake: Evidence for Normal Faulting in West Texas," *Bulletin of the Seismological Society of America*, Vol. 77, No. 6, 1987, pp. 2005-2017.

[29] D. Doser and J. Rodriguez, "The Seismicity of Chihuahua, Mexico, and the 1928 Parral Earthquake," *Physics of the Earth and Planetary Interiors*, Vol. 78, No. 1-2, 1993, pp. 97-104. doi:10.1016/0031-9201(93)90086-O

[30] I. N. Galván-Ramírez and J. C. Montalvo-Arrieta, "The Historical Seismicity and Prediction of Ground Motion in Northeast Mexico," *Journal of South American Earth Sciences*, Vol. 25, 2008, pp. 37-48. doi:10.1016/j.jsames.2007.07.004

[31] R. K. Goldhammer, "Mesozoic Sequence Stratigraphy and Paleogeographic Evolution of Northeast Mexico," *Geological Society of America*, Special Paper 340, 1999.

[32] T. Harding, "Seismic Characteristics and Identification of Negative Flower Strctures, Positive Flower Strctures, and Positive Structural Inversion," *Bulletin of the American Association of Petroleoum Geologists*, Vol. 69, No. 4, 1985, pp. 582-600.

[33] L. L. Nettleton, "Determination of Density for Reduction of Gravimeter Observations," *Geophysics*, Vol. 4, No. 3, 1939, pp. 176-183.

[34] L. L. Nettleton, "Elementary Gravity and Magnetics for Geologists and Seismologists," *Society of Exploration Geophysicists*, *Geophysical Monograph Series* No. 1, 1971.

Journal of Modern Physics, 2012, 3, 1247-1254
doi:10.4236/jmp.2012.329161 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Approximation Method for the Relaxed Covariant Form of the Gravitational Field Equations for Particles

Emanuel Gallo, Osvaldo M. Moreschi
FaMAF, Universidad Nacional de Córdoba, Instituto de Fsica Enrique Gaviola (IFEG), CONICET,
Ciudad Universitaria, Córdoba, Argentina
Email: emanuelgg@gmail.com

Received July 7, 2012; revised August 6, 2012; accepted August 14, 2012

ABSTRACT

We present a study of the so called relaxed field equations of general relativity in terms of a decomposition of the metric; which is designed to deal with the notion of particles. Several known results are generalized to a coordinate free covariant discussion. We apply our techniques to the study of a particle up to second order.

Keywords: General Relativity; Approximation Methods; Particles

1. Introduction

The notion of particle is fundamental to the Newtonian mechanics framework; in fact, this whole theoretical framework can be constructed in terms of the notion of test particles and massive particles. It is then natural to ask whether this notion can be translated to other frameworks, as is general relativity.

Within general relativity one understands Newtonian mechanics as the limit of weak field and slow motion. So we know that one can regain the notion of particle in this regime. Also in general relativity, the concept of test particle is a natural one, which allows to discuss several physically interesting situations.

At first sight it is not at all clear that one can extend the notion of particles (non-test) to the realm of general relativity. To begin with, if one imagines a process in which one shrinks the sizes of an object to obtain a point like object, one knows that at some moment in the process one would end up with the formation of a black hole, which has a characteristic size. However, the post-Newtonian approach to compact objects is frequently constructed in terms of the notion of particles; although post-Newtonian systems are normally required to have weak fields and slowly moving objects.

It is interesting to note that the most simple black hole, namely the one describing a vacuum spherically symmetric spacetime, can be expressed in terms of the so called Kerr-Schild decomposition. In this way, the Schwarzschild black hole, whose maximal analytic extension is described in terms of the well known causal conformal diagrams, when expressed in the Kerr-Schild decomposition shows a point like description in terms of the flat reference metric of the Kerr-Schild form.

This indicates that it might be possible to give a particle notion to a compact object in general relativity when expressed with respect to background reference metrics.

If one intends to study the problem of a systems composed of several compact objects, it appears as an appealing strategy to use approximation techniques for solving the field equations. Several problems are related to this.

In building approximation schemes for the study of the field equations in general relativity it is often useful to recur to the relaxed form of the field equations; that we recall below. Also, it frequently useful to decompose the physical metric in terms of a background metric. In this work we plan to study both techniques.

In the process of decomposing the metric a key issue is the notion of gauge, since in general one has more than one way to decompose the physical metric. In order to study this issue we bring the techniques used by Friedrich in his study of the hyperbolic nature of the gravitational field equations. We will present here a generalization of Friedrich's results that is convenient for our discussion.

Although we work with coordinate independent expressions, we also relate our work with the widely used harmonic gauge condition; and take the opportunity to restate Anderson's result in a coordinate independent fashion.

An approximation scheme is suggested in which the previous studies are taking into account.

We apply our techniques to the problem of a single particle up to the second order.

2. The Decomposition of the Metric

Let us express the metric g_{ab} of the spacetime M in terms of a reference metric η_{ab}, such that

$$g_{ab} = \eta_{ab} + h_{ab}. \tag{1}$$

Let ∂_a denote the torsion free metric connection of η_{ab} and ∇_a the torsion free metric connection of g_{ab}; then one can express the covariant derivative of an arbitrary vector v by

$$\nabla_a v^b = \partial_a v^b + \Gamma_{a\,c}^{\,b} v^c; \tag{2}$$

and one can prove that

$$\Gamma_{ab}^{\,c} = \frac{1}{2} g^{cd} \left(\partial_a h_{bd} + \partial_b h_{ad} - \partial_d h_{ab} \right) = \Gamma_{ba}^{\,c}. \tag{3}$$

Let us observe that

$$\begin{aligned} \Gamma_{a\,b}^{\,c} g_{ce} = \Gamma_{aeb} &= \frac{1}{2} g^{cd} g_{ce} \left(\partial_a h_{bd} + \partial_b h_{ad} - \partial_d h_{ab} \right) \\ &= \frac{1}{2} \left(\partial_a h_{be} + \partial_b h_{ae} - \partial_e h_{ab} \right). \end{aligned} \tag{4}$$

The relation between Γ and the curvature tensor can be calculated from

$$\begin{aligned} \left[\nabla_a, \nabla_b \right] v^d &= \left(\partial_a \Gamma_{b\,c}^{\,d} - \partial_b \Gamma_{a\,c}^{\,d} + \Gamma_{a\,e}^{\,d} \Gamma_{b\,c}^{\,e} - \Gamma_{b\,e}^{\,d} \Gamma_{a\,c}^{\,e} \right) v^c \\ &\quad + \Theta_{abc}{}^{d} v^c = R_{abc}{}^{d} v^c; \end{aligned} \tag{5}$$

where Θ is the curvature of the ∂_a connection. Then the Ricci tensor can be calculated from

$$R_{ac} \equiv R_{abc}{}^{b} = \Theta_{ac} + \partial_a \Gamma_{b\,c}^{\,b} - \partial_b \Gamma_{a\,c}^{\,b} + \Gamma_{a\,e}^{\,b} \Gamma_{b\,c}^{\,e} - \Gamma_{b\,e}^{\,b} \Gamma_{a\,c}^{\,e}; \tag{6}$$

where Θ_{ac} is the Ricci tensor of the connexion ∂_a.

3. Auxiliary Functions or Gauge Vector

Let us consider four independent auxiliary functions x^μ, with $\mu = 0,1,2,3$. Then let us observe that

$$g^{ab} \nabla_a \nabla_b x^\mu = g^{ab} \nabla_a \partial_b x^\mu = g^{ab} \partial_a \partial_b x^\mu - g^{ab} \Gamma_{ab}^{\,c} \partial_c x^\mu. \tag{7}$$

Then, if $I_\mu^{\,e}$ denotes the inverse of $\partial_c x^\mu$, which exists by assumption of the independence of the set x^μ, one has

$$g^{ab} \Gamma_{ab}^{\,c} = -\left(g^{ab} \nabla_a \nabla_b x^\mu - g^{ab} \partial_a \partial_b x^\mu \right) I_\mu^{\,c} = H^\mu I_\mu^{\,c} \tag{8}$$

where we are using

$$H^\mu = -g^{ab} \nabla_a \nabla_b x^\mu + g^{ab} \partial_a \partial_b x^\mu. \tag{9}$$

Alternatively, let us define the gauge vector $\mathcal{H}^c$

$$\mathcal{H}^c = H^\mu I_\mu^{\,c}; \tag{10}$$

which implies

$$H^\mu = \partial_c x^\mu \, \mathcal{H}^c = \mathcal{H}\left(x^\mu \right); \tag{11}$$

so that one also has

$$g^{ab} \Gamma_{a\,b}^{\,c} = \mathcal{H}^c. \tag{12}$$

These equations show the relation that exist between working with a coordinate system, given by the set of functions x^μ, and the gauge vector $\mathcal{H}^c$; which does not need any reference to coordinate systems at all. In what follows we will try to use the covariant approach that employs the use of the gauge vector $\mathcal{H}^c$. We emphasize that Latin indices are abstract; and therefore our expressions are coordinate independent and covariant.

Then, the Ricci tensor can be expressed by

$$\begin{aligned} R_{ac} &= \Theta_{ac} + \frac{1}{2} g^{bd} \left(\Theta_{bad}{}^{e} h_{ec} + \Theta_{bcd}{}^{e} h_{ea} + 2\Theta_{bca}{}^{e} h_{ed} \right) \\ &\quad + \frac{1}{2} g^{bd} \partial_b \partial_d h_{ac} - \partial_a \left(g_{ce} \mathcal{H}^e \right) \\ &\quad + g_{ed} \Gamma_{a\,c}^{\,e} \mathcal{H}^d - g^{bf} g_{ed} \Gamma_{a\,f}^{\,d} \Gamma_{b\,c}^{\,e} \\ &\quad - \frac{1}{2} \left(\Gamma_{a}^{\,bd} \Gamma_{bcd} + \Gamma_{c}^{\,bd} \Gamma_{bad} \right). \end{aligned} \tag{13}$$

Let us note that if the vector field $\mathcal{H}^c$ is given by (12), then for any function x^μ one has

$$-g^{ab} \nabla_a \nabla_b x^\mu + g^{ab} \partial_a \partial_b x^\mu = \mathcal{H}\left(x^\mu \right). \tag{14}$$

In the standard studies on approximations to the solution of the field equations, one frequently finds the choice of harmonic coordinates for the set of the x^μ's; however, in Equation (13) one can see that only the vector field $\mathcal{H}^c$ appears, without any reference to a choice of auxiliary functions. Therefore one could just refer to the gauge vector $\mathcal{H}^c$.

4. The Field Equations in Relaxed Covariant Form

Previous to the discussion of the relaxed covariant field equations, we would like to refer to the work of Friedrich [1] and its extension to this coordinate independent discussion.

4.1. Friedrich's Theorem without the Use of Coordinates

The field equations are

$$R_{ac} = -8\pi\kappa \left(T_{ac} - \frac{1}{2} g_{ac} g^{bd} T_{bd} \right). \tag{15}$$

Equation (3.22) in reference [1] can be obtained from (13) by expressing it in a coordinate frame and neglecting the Θ terms. In this way, one would obtain the analogous expression where all the appearance of ∂ derivatives are replaced by coordinate derivatives ∂_μ, the tensors Γ are replaced by the Chrirstoffel symbols and one uses $F^\varepsilon = -H^\varepsilon$; namely:

$$\frac{1}{2}g^{\beta\delta}\partial_\beta\partial_\delta g_{\alpha\sigma}+g_{\varepsilon(\sigma}\nabla_{\alpha)}F^{\varepsilon}-g^{\beta\varphi}g_{\varepsilon\delta}\Gamma^{\ \delta}_{\alpha\phi}\Gamma^{\ \varepsilon}_{\beta\ \sigma}$$
$$-\frac{1}{2}\left(\Gamma_{\alpha}^{\ \beta\delta}\Gamma_{\beta\sigma\delta}+\Gamma_{\sigma}^{\ \beta\delta}\Gamma_{\beta\alpha\delta}\right) \tag{16}$$
$$=-8\pi\kappa\left(T_{\alpha\sigma}-\frac{1}{2}g_{\alpha\sigma}g^{\beta\delta}T_{\beta\delta}\right)$$

Friedrich has studied [1] this system introducing the notion of "coordinate gauge source''

$$F^{\mu}=\nabla^{\alpha}\nabla_{\alpha}x^{\mu}. \tag{17}$$

Subsequently, Friedrich studied the case in which F^{μ} is given arbitrarily.

Then, we can rephrase Friedrich's theorem in the following form:

Theorem 4.1 *Let* x^{μ} *be four independent functions that are used as a coordinate system. If* $g_{\mu\nu}$ *is a solution of* (16) *together with the matter equations such that on the initial surface one has* $F^{\mu}=\nabla^{\nu}\nabla_{\nu}x^{\mu}$, $\nabla_{\beta}F^{\mu}=\nabla_{\beta}\nabla^{\nu}\nabla_{\nu}x^{\mu}$, *then* $g_{\mu\nu}$ *is in fact a solution of Einstein's field equations.*

This theorem can be understood in two ways. In one of them, we think that the four coordinates x^{μ} are given and then the theorem checks whether the F^{μ}'s satisfy the above equations. In the other way, one think that the F^{μ}'s are given and then the theorem checks whether there exists a coordinate system of x^{μ}'s such that the equations in the theorem are satisfied.

From the fact that $H^{\mu}=-F^{\mu}$, one deduces, using the same techniques as in [1], that the generalized Friedrich's theorem holds, namely, consider the four functions H^{μ} as given a priori, then:

Theorem 4.2 *If* g_{ab} *is a solution of* (15), *with the decomposition of the metric as in* (1) *and with the Ricci tensor as given by* (13) *with* $\mathcal{H}^{c}=H^{\mu}I_{\mu}^{\ c}$, *together with the matter equations such that on the initial surface one has* $H^{\mu}=-\left(g^{ab}\nabla_a\nabla_b x^{\mu}+g^{ab}\partial_a\partial_b x^{\mu}\right)$, $\nabla_c H^{\mu}=-\nabla_c\left(g^{ab}\nabla_a\nabla_b x^{\mu}-g^{ab}\partial_a\partial_b x^{\mu}\right)$, *where* x^{μ} *are four independent scalars, then* g_{ab} *is in fact a solution of Einstein's field equations.*

This result gives great freedom in the problem of finding solutions of the field equations in terms of a reference metric. Suppose that one solves (15) for a given vector field H^{μ}. Also assume that one can solve for the functions x^{μ} such that $g^{ab}\nabla_a\nabla_b x^{\mu}=-H^{\mu}$. Then, let us build a flat metric η so that $g^{ab}\partial_a\partial_b x^{\mu}=0$; which in particular can be satisfied if the x^{μ}s are thought as Cartesian coordinates of η. In this way one would obtain $H^{\mu}I_{\mu}^{e}=g^{ab}\Gamma_{ab}^{\ e}$, and so have a solution of the field equations.

It also might be of interest to researchers in numerical relativity, since it provides the possibility to use any coordinate system; *i.e.*, not necessarily an harmonic one.

Instead, one could have a proposition that does not refer to the auxiliary functions whatsoever; namely

Theorem 4.3 *If* g_{ab} *is a solution of* (15), *with the decomposition of the metric as in* (1) *and with the Ricci tensor as given by* (13), *together with the matter equations such that on the initial surface one has* $\mathcal{H}^{c}=g^{ab}\Gamma_{a\ b}^{\ c}$, $\nabla_d\mathcal{H}^{c}=\nabla_d\left(g^{ab}\Gamma_{a\ b}^{\ c}\right)$, *then* g_{ab} *is in fact a solution of Einstein's field equations.*

This theorem can be understood in two ways. In one of them, we think that the metric η_{ab} is given and then the theorem checks whether the vector $\mathcal{H}^{c}$ satisfies the above equations. In the other way, one think that the $\mathcal{H}$ is given and then the theorem checks whether there exists a metric η_{ab} such that the equations in the theorem are satisfied.

4.2. Relaxed Covariant Form of the Field Equations and a Generalization of Friedrich's Theorem

Alternatively one can use the form of the field equations in terms of a slight different logic.

When we use the expression of the Ricci tensor as given by (13) in (15), without assuming that $\mathcal{H}^{c}$ is $g^{ab}\Gamma_{ab}^{\ c}$, namely

$$\begin{aligned}
&\frac{1}{2}g^{bd}\partial_b\partial_d h_{ac}-\partial_a\left(g_{ce}\mathcal{H}^{e}\right)+g_{ed}\Gamma_{ac}^{\ d}\mathcal{H}^{e}+\Theta_{ac}\\
&+\frac{1}{2}g^{bd}\left(\Theta_{bad}^{\ \ \ e}h_{ec}+\Theta_{bcd}^{\ \ \ e}h_{ea}+2\Theta_{bca}^{\ \ \ e}h_{ed}\right)\\
&-g^{bf}g_{ed}\Gamma_{a\ f}^{\ d}\Gamma_{b\ c}^{\ e}-\frac{1}{2}\left(\Gamma_{a}^{\ bd}\Gamma_{bcd}+\Gamma_{c}^{\ bd}\Gamma_{bad}\right)\\
&=-8\pi\kappa\left(T_{ac}-\frac{1}{2}g_{ac}g^{bd}T_{bd}\right);
\end{aligned} \tag{18}$$

we will refer to these as the *relaxed field equations* [2].

Using the standard harmonic gauge technique, one would say: solve the relaxed field equation in the coordinate frame, with $H^{\mu}=0$, and then require the equation

$$g^{bd}\nabla_b\nabla_d x^{\mu}=0. \tag{19}$$

In the standard approach one makes use of coordinate basis; therefore the previous statement would be the complete story. However in our case, H^{μ} has a second term where two covariant derivatives of x^{μ} with respect to the metric η appears. At this point it is important to notice that if we have the solutions x^{μ} from (19) then, on constructing η with this as a Cartesian coordinate system, one would obtain $H^{\mu}=0$.

In some occasions it is preferable to work with a different set of equations. In this respect, several authors have indicated that actually to request Equation (19) is equivalent [2-4] to demand

$$g^{ab}\nabla_a T_{bc}=0. \tag{20}$$

When dealing with Einstein equations in the relaxed form, and treating the vacuum case, Equation (20) is understood as the condition that the divergence of the Einstein tensor must be zero (which of course is identically zero in the non relaxed form).

Let us study the relation between the divergence of the energy-momentum tensor and the vector $\mathcal{H}^e$. One can write the relaxed field Equations (18) in the usual form in which on the right hand side we have just the standard term $-8\pi\kappa T_{ac}$; and so on the left hand side, the terms involving $\mathcal{H}^e$ would be

$$-\nabla_a\left(g_{ce}\mathcal{H}^e\right)+\frac{1}{2}g_{ac}g^{ef}\nabla_e\mathcal{H}_f; \tag{21}$$

where we have used that the term $8\pi\kappa T$ contributes with the term

$$-g^{ef}\nabla_e\mathcal{H}_f. \tag{22}$$

Then in taking its divergence, on the left hand side, the terms involving $\mathcal{H}^e$ are

$$g^{ab}\nabla_a\left(-\nabla_a\left(g_{ce}\mathcal{H}^e\right)+\frac{1}{2}g_{ac}g^{ef}\nabla_e\mathcal{H}_f\right). \tag{23}$$

If we replace $\mathcal{H}^e$ by $\Gamma^e \equiv g^{ab}\Gamma^{\ e}_{ab}$, the divergence of the left hand side would be identically zero, since the Einstein tensor has divergence zero. Therefore we conclude that the divergence of the stress energy-momentum tensor is

$$\begin{aligned}-8\pi\kappa g^{ab}\nabla_a T_{bc} = g^{ab}\nabla_a &-\left(\nabla_b\left(g_{ce}\left(\mathcal{H}^e-\Gamma^e\right)\right)\right.\\ &\left.+\frac{1}{2}g_{bc}g^{de}\nabla_d\left(g_{ef}\left(\mathcal{H}^f-\Gamma^f\right)\right)\right).\end{aligned} \tag{24}$$

Therefore, the stress energy-momentum is conserved if and only if

$$\begin{aligned}&g^{ab}\nabla_a\left(-\nabla\left(g_{ce}\left(\mathcal{H}^e-\Gamma^e\right)\right)\right.\\ &\left.+\frac{1}{2}g_{bc}g^{de}\nabla_d\left(g_{ef}\left(\mathcal{H}^f-\Gamma^f\right)\right)\right)=0.\end{aligned} \tag{25}$$

Working out the relations, one finds that the previous equation can be expressed as

$$0=-\frac{1}{2}g_{ce}g^{ab}\nabla_a\nabla_b\left(\mathcal{H}^e-\Gamma^e\right)+\frac{1}{2}R_{ce}\left(\mathcal{H}^e-\Gamma^e\right). \tag{26}$$

Which coincides with Friedrich calculation.

It follows that if one solves Equation (26) such that on an initial hypersurface $\mathcal{H}^b=\Gamma^b$ and $\nabla_a\mathcal{H}^b=\nabla_a\Gamma^b$, then the energy-momentum tensor will be conserved in the evolution of the system. Furthermore, one also deduces that:

Theorem 4.4 *If, given the metric* η, *one solves the relaxed field equations for* h *together with the matter equations, which include the conservation of the energy-momentum tensor, such that* $\mathcal{H}^b=\Gamma^b$ *and* $\nabla_a\mathcal{H}^b=\nabla_a\Gamma^b$ *on an initial hypersurface, then* g_{ab} *is a solution of Einstein equations.*

This is a rephrasing of Friedrich's theorem applied to a decomposition of the metric and to its general relaxed covariant form of the field equations.

It is interesting to remark that Anderson [3], using a retarded integral expression for h, was able to prove the equivalence between the conservation of the energy-momentum tensor with the harmonic gauge condition. In relation to this let us remark that if the set of functions x^μ is obtained from the solutions of (19); and one uses them as harmonic coordinates of the metric η, then one deduces that $\Gamma^c=0$. And also, if $\Gamma^c=0$, then Cartesian coordinates of η are harmonic coordinates of g. This means that we can state Anderson's result in a coordinate independent way, namely:

Theorem 4.5 *Let* h *be the retarded solution, with respect to a flat metric* h *of the relaxed field equations together with the matter equations of state, such that* $\mathcal{H}^b=0$, *then the conservation of the energy-momentum tensor implies that* g_{ab} *is a solution of Einstein equations.*

5. The Approximation Method and the Treatment of Particles

The approximation method that we introduce below, is adapted to the treatment of particles; therefore, it is convenient to begin by treating the problem of one single particle in the context of linearized gravity, in order to clarify some of the techniques.

5.1. The Gravitational Field from One Particle in Linearized Gravity

5.1.1. The Description of a Particle

Let us consider a massive point particle with mass m_A describing, in a flat space-time $\left(M^0,\eta_{ab}\right)$, a curve C which in a Cartesian coordinate system x^a reads

$$x^\mu=z^\mu\left(\tau\right), \tag{27}$$

with τ meaning the proper time of the particle along C.

The unit tangent vector to C, with respect to the flat background metric is

$$u^\mu=\frac{dz^\mu}{d\tau}, \tag{28}$$

that is, $\eta\left(u,u\right)=1$. Now, for each point $Q=z\left(\tau\right)$ of C, we draw a future null cone $\mathfrak{C}_Q$ with vertex in Q. If we call x_P^μ the Minkowskian coordinates of an arbitrary point on the cone $\mathfrak{C}_Q$, then we can define the retarded radial distance on the null cone by

$$r = u_\mu \left(x_P^\mu - z^\mu(\tau) \right). \tag{29}$$

The energy momentum tensor $T_{ab}^{(0)}$ of a point particle is proportional to $m v_a v_b$; where m is the mass and v^a its four velocity. We are distinguishing between the unit tangent vector u^a and the four velocity vector v^a, because in future works we would like to consider the possibility to normalize the vector v with respect to a different metric. In order to represent a point particle $T_{ab}^{(0)}$ must also be proportional to a three dimensional delta function that has support on the world line of the particle.

We will suppose that the particle does not have multipolar structure. Then, given an arbitrary Minkowskian frame $\left(x^0, x^1, x^2, x^3\right)$, we will express the energy momentum by

$$\begin{aligned} &T^{(0)ab}\left(x^0 = z^0(\tau_0), x^1, x^2, x^3\right) \\ &= m_A v^a(\tau_0) v^b(\tau_0) \\ &\frac{\delta\left(x^1 - z^1(\tau_0)\right)\delta\left(x^2 - z^2(\tau_0)\right)\delta\left(x^3 - z^3(\tau_0)\right)}{u^0(\tau_0)}. \end{aligned} \tag{30}$$

5.1.2. The First Order Solution

The retarded solution, in terms of Green functions, for the relaxed field Equations (18) for particle A, in which we take $\mathcal{H}^b = 0$ and η a flat metric, is

$$h_{ab}^{(1)} = -4 m_A \frac{v_a v_b - \frac{1}{2}\eta_{ab}}{r}; \tag{31}$$

so that in general

$$g_{ab}^{(1)} = \left(1 + \frac{2m_A}{r}\right)\eta_{ab} - \frac{4m_A}{r} v_a v_b. \tag{32}$$

In these equations we have considered the definition

$$v_a \equiv \eta_{ab} v^b. \tag{33}$$

It is interesting to realize that the exact inverse of this metric is

$$g^{(1)ab} = \frac{1}{1 + \frac{2m_A}{r}}\eta^{ab} + \frac{\frac{4m_A}{r}}{1 - \left(\frac{2m_A}{r}\right)^2} v^a v^b. \tag{34}$$

Note that one can solve for h_{ab} for an arbitrary motion of the particle; however, the complete solution of the problem involves having to set also $\Gamma^b = 0$; which in terms of a coordinate frame treatment is equivalent to the harmonic condition. Then, recalling, as mentioned previously, that Anderson has proved [3] the equivalence between the harmonic condition and the divergence free condition on the energy-momentum tensor; one deduces from this, that for the case of the energy-momentum tensor of a particle it implies its geodesic motion.

5.2. Iterative Approximation Method

Now we present a general iterative method to solve the relaxed field equations.

First of all, let us note that given the decomposition (1) and defining the tensor $\tilde{h}^{ab}$ from

$$\tilde{h}^{ab} = \eta^{ac}\eta^{bd} h_{cd}, \tag{35}$$

where

$$\eta^{ab}\eta_{bc} = \delta_c^a, \tag{36}$$

that is η^{ab} is the inverse of η_{ab}, one can always express the inverse g^{ab} in the form

$$g^{ab} = \eta^{ab} - \tilde{h}^{ab} - d^{ab}. \tag{37}$$

Then making the contraction

$$g^{ab} g_{bc} \;\; = \delta_c^a - \tilde{h}^{ab} h_{bc} - d^{ab} g_{bc} = \delta_c^a; \tag{38}$$

one finds

$$d^{ab} = -\tilde{h}^{ad} h_{dc} g^{cb}; \tag{39}$$

which can be considered an implicit equation for d^{ab}; but it also shows explicitly that d is quadratic in terms of h.

This suggests the natural series $d_2, d_3, d_4, d_5, d_6, \cdots$ defined by

$$d_2^{ab} = -\tilde{h}^{ad} h_{dc}\left(\eta^{cb}\right), \tag{40}$$

$$d_3^{ab} = -\tilde{h}^{ad} h_{dc}\left(\eta^{cb} - \tilde{h}^{cb}\right); \tag{41}$$

$$d_n^{ab} = -\tilde{h}^{ad} h_{dc}\left(\eta^{cb} - \tilde{h}^{cb} - d_{(n-2)}^{cb}\right); \tag{42}$$

for natural numbers $n > 3$. It is clear that d_n is order $h^{(n)}$.

However, we have seen that in the first order solution for a single particle, the inverse of the metric has a term which is conformal to the flat metric η; which it will be convenient to take into account. For this reason we propose the following method of approximation where this issue is considered.

The idea is to express (18) and eventually (19) in the form

$$\varphi\, \eta^{ab}\partial_a\partial_b f = S(f); \tag{43}$$

where $\varphi\eta^{ab}$ is the term proportional to η^{ab} that is contained in g^{ab}; while the general case would be to consider just $\eta^{ab}\partial_a\partial_b f$ for the left hand side. This equation can also be expressed by

$$\eta^{ab}\partial_a\partial_b(\varphi f) = s(\varphi f) + S(f); \tag{44}$$

where

$$s(\varphi f) \equiv \eta^{ab}\partial_a\partial_b(\varphi f) - \varphi\eta^{ab}\partial_a\partial_b f \tag{45}$$

Now one would like to solve Equation (44) by iterations.

Let us define the sets $f^{(j)}$ such that for $j=0$, one takes $h=0$, x^{μ} to be harmonic functions of the metric η and $\varphi=1$; and for $j>0$, $f^{(j)}$ is the solution of

$$\eta^{ab}\partial_a\partial_b\left(\varphi^{(j-1)}\ f^{(j)}\right)=s\left(\varphi^{(j-1)}\ f^{(j-1)}\right)+S\left(f^{(j-1)}\right). \quad (46)$$

using the retarded Green function. As we have seen above, $\varphi^{(1)}$ clearly arises in the first order calculation.

The application of this method to the first order, for a single particle, reproduces the calculation explained in Subsection 5.1.2. Next we study this case at second order.

5.3. The Second Order Solution

Let us remark that the first order solution is stationary and spherically symmetric. This structure transports to the second order solution.

The equation for $h_{ab}^{(2)}$ is

$$\begin{aligned}\frac{1}{2}\eta^{bd}\partial_b\partial_d h_{ac}^{(2)} &= \partial_{(}\left(g_{ce}^{(1)}\mathcal{H}^e\right)+g_{ed}^{(1)}\Gamma_{a\,c}^{(1)e}\mathcal{H}^d\\ &-g^{(1)bf}g_{ed}^{(1)}\Gamma_{a\,f}^{(1)d}\Gamma_{b\,c}^{(1)e}\\ &+\frac{1}{2}\left(\Gamma_{a}^{(1)bd}\Gamma_{bcd}^{(1)}+\Gamma_{c}^{(1)bd}\Gamma_{bad}^{(1)}\right)\\ &-8\pi T_{ac}^{(1)}.\end{aligned} \quad (47)$$

We will call the right hand side, the tensor $\mathbb{T}$; which has the structure

$$\mathbb{T}=\alpha(r)dt^2-\beta(r)(d\boldsymbol{x})^2-\gamma(r)(\boldsymbol{x}\cdot d\boldsymbol{x})^2; \quad (48)$$

where we are using the three dimentional notation $\boldsymbol{x}=(x,y,z)$ and where

$$\alpha=\frac{2m^2}{(2m+r)^2(2m-r)r}-8\pi m\delta_3(\boldsymbol{x}), \quad (49)$$

$$\beta=\frac{-2m^2(4m+r)}{(2m+r)^2(2m-r)r^2}, \quad (50)$$

$$\gamma=\frac{8m^2(2m^2+mr-2r^2)}{(2m+r)^2(2m-r)^2r^4}. \quad (51)$$

Therefore one assumes for $h^{(2)}$ the same form, namely

$$h^{(2)}=A(r)dt^2-B(r)(d\boldsymbol{x})^2-C(r)(\boldsymbol{x}\cdot d\boldsymbol{x})^2. \quad (52)$$

In this way one has

$$h_{00}^{(2)}=A, \quad (53)$$

$$h_{ii}^{(2)}=-\left(B+\left(x^i\right)^2C\right), \quad (54)$$

$$h_{ij}^{(2)}=-x^ix^jC; \quad (55)$$

where the index $i,j=1,2,3$ denote spatial coordinates.

One can see then that the equations to solve are

$$\Box A=2\alpha, \quad (56)$$

$$\Box C-\frac{4}{r}\frac{dC}{dr}=2\gamma, \quad (57)$$

$$\Box B=2(\beta+C); \quad (58)$$

where we are using the symbol $\Box$ to denote $\eta^{bd}\partial_b\partial_d$.

We can solve these equations in two ways, either using Green function techniques, or, recalling the stationary nature of the solution, just integrating the Laplace operator. For this presentation we choose the second option. Let us note that for any function $f(r)$ one has that

$$\Box f(r)=-\nabla^2 f=-\left[\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}f}{\mathrm{d}r}\right)\right]. \quad (59)$$

Therefore one can find A by two consecutive integrations, obtaining

$$A(r)=\frac{1}{4}\left(1-\frac{2m}{r}\right)\ln\left(\frac{1-\frac{2m}{r}}{1+\frac{2m}{r}}\right)+ka_2-\frac{ka_1}{r}. \quad (60)$$

Similarly one can see that the function $C(r)$ satisfies

$$-\frac{1}{r^6}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^6\frac{\mathrm{d}C}{\mathrm{d}r}\right)=2\gamma; \quad (61)$$

which after integration gives

$$\begin{aligned}C(r)=\frac{1}{40m^2r^5}\Bigg(&-\left(r^5+288m^5\right)\ln\left(1-\frac{2m}{r}\right)\\ &+\left(416m^5-7r^5\right)\ln\left(1+\frac{2m}{r}\right)+128m^5\ln(r)\\ &-8kc_1m^2+40kc_2m^2r^5-352m^4r\\ &.+16m^3r^2-16m^2r^3+12mr^4\Bigg).\end{aligned} \quad (62)$$

Then the function $B(r)$ is given by

$$\begin{aligned}B(r)=\frac{1}{120m^2r^3}\Bigg(&\left(288m^5-20m^3r^2-30m^2r^3+r^5\right)\\ &\cdot\ln\left(1-\frac{2m}{r}\right)+\left(-416m^5+20m^3r^2\right.\\ &\left.-130m^2r^3+7r^5\right)\ln\left(1+\frac{2m}{r}\right)-128m^5\ln(r)\\ &-120kb_1m^2r^2+120kb_2m^2r^3+8kc_1m^2\\ &-40kc_2m^2r^5+352m^4r-1024m^3r^2-12mr^4\Bigg).\end{aligned} \quad (63)$$

Our choice for the integration constants is: $ka_1=m$, $ka_2=0$, $kb_1=-\frac{181}{15}m$, $kb_2=\frac{17}{15}$, $kc_1=-\frac{74}{5}m^3$ and $kc_2=0$. This choice is made taking into consideration the exact solution and the integration of the solution coming from a Green function approach; that we will not discuss here due to considerations of space. By the exact functions we mean the metric components of the Schwarzschild spacetime in harmonic coordinates; which are

$$A_{exact} = \frac{1-\frac{m}{r}}{1+\frac{m}{r}}, \quad (64)$$

$$B_{exact} = \left(1+\frac{m}{r}\right)^2, \quad (65)$$

$$C_{exact} = \left(\frac{1+\frac{m}{r}}{1-\frac{m}{r}}\right)\frac{m^2}{r^4}. \quad (66)$$

A graphical comparison with the exact functions of the Schwarzschild solution in harmonic coordinates are shown in **Figure 1**.

It can be observed that in second order one obtains anexcellent comparison of the solution with the exact values of the metric components; even for very small values of the radial coordinate. Although this comparison has limited value, it is in any case remarkable that it is only necessary to go only to second order to obtain such a good approximation.

6. Final Comments

We have presented an study of an approach to the gravitational field equation through the relaxed covariant form of them. The whole approach is intended to deal with the notion of compact objects.

The relaxed field equations was studied using Friedrich approach to the problem and we have also refer to Anderson's result in the field of harmonic conditions.

We have generalized Friedrich results to a covariant formulation in terms of a decomposition of the metric.

Anderson's result has been restated in a form that does not make reference to coordinate conditions.

We have presented an approximation method that can be applied to the notion of particles in general relativity; and which is successful in second order for the case of a solitary compact body.

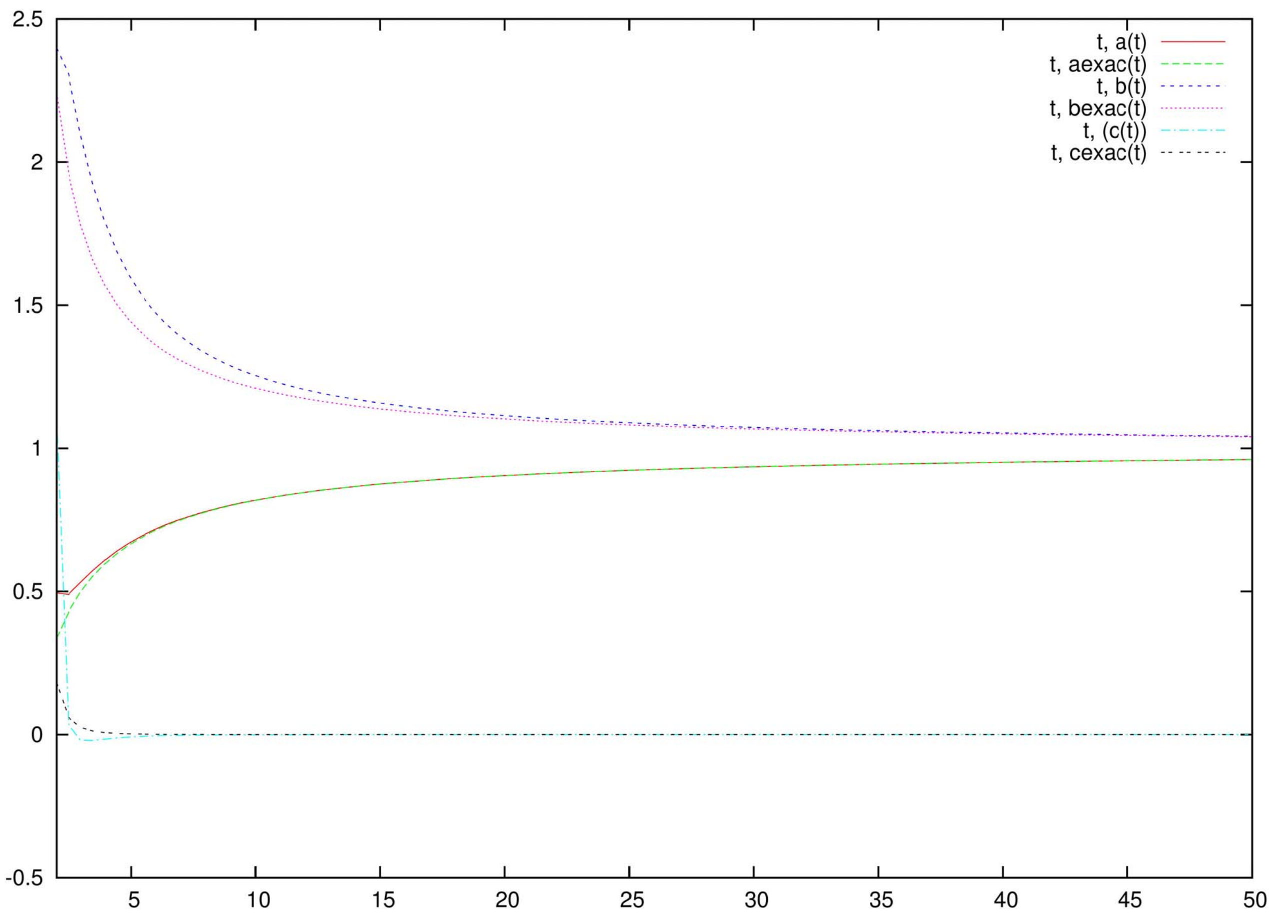


Figure 1. Comparison of the functions calculated in the second iteration with the exact values.

It is our intention to apply these techniques to the problem of a binary system in general relativity.

7. Acknowledgements

We acknowledge support from CONICET and SeCyT-UNC.

REFERENCES

[1] H. Friedrich, "On the Hyperbolicity of Einstein's and Other Gauge Field Equations," *Communications in Mathematical Physics*, Vol. 100, No. 4, 1985, pp. 525-543. doi:10.1007/BF01217728

[2] M. Walker and C. M. Will, "The Approximation of Radiative Effects in Relativistic Gravity-Gravitational Radiation Reaction and Energy Loss in Nearly Newtonian Systems," *Astrophysical Journal*, Vol. 242, 1980, pp. L129-L133. doi:10.1086/183417

[3] J. L. Anderson, "Satisfaction of deDonder and Trautman Conditions by Radiative Solutions of the Einstein Field Equations," *General Relativity and Gravitation*, Vol. 4, No. 4, 1973, pp. 289-297. doi:10.1007/BF00759848

[4] A. Einstein, L. Infeld and B. Hoffmann, "The Gravitational Equations and the Problem of Motion," *Annals of Mathematics*, Vol. 39, No. 1, 1938, pp. 65-100. doi:10.2307/1968714

Journal of Modern Physics, **2012, 3, 1255-1260**
doi:10.4236/jmp.2012.329162 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Can We Form Gravitinos by Something Other than a Higgs Boson in the Electro-Weak Era?

Andrew Beckwith
Physics Department, Chongqing University, Chongqing, China
Email: abeckwith@uh.edu

Received June 15, 2012; revised July 21, 2012; accepted July 31, 2012

ABSTRACT

What is the physical nature of gravitinos? As asked before, this question was the template of how to introduce Machian Physics as a way to link gravitinos in the electro weak era and gravitons as of the present. What we wish to do now is to ask how a flaw in the Higgs equation as brought up by Comay shows a branch off from orthodox quantum physics, leading to, with the Machs principle application done earlier a way to embed the beginning of the universe as a semi classical superstructure of which Quantum Mechanics is a subset of. We argue that this will necessitate a review of the Higgs equation of state for reasons stated in the manuscript. We also finally review a proprosal for another form of mass formation mechanism as a replacement for the Higgs mass as introduced by Glinka and Beckwith, 2012, with commentary as to how suitable it may be to get a gravitino mass in fidelity to the Machian proposal introduced by Beckwith previously, to get linkage between electroweak era gravitinos and present day gravitons.

Keywords: Machian Physics; Gravitinos; Higgs Mass Formation; Quantum Mechanics; Dirac Equation; Electro-Weak (EW) Era

1. Introduction

We will ask the question here. In an earlier document, the author presented an equivilence between Gravitinos in the electro weak era, and Gravitons today. The motivation of using two types of Machs principle, one for the Gravitinos in the electro weak era, and then the 2nd modern day Mach's principle, as organized by the author are as seen in [1]

$$\frac{GM_{electro-weak}\big|_{Super-partner}}{R_{electro-weak}c^2} \approx \frac{G\,M_{today}\big|_{Not-Super-Partner}}{R_0 c^2} \tag{1}$$

are really a statement of information conservation. What we now ask is if the Gravitino can be re stated in terms in fidelity to quantum mechanics, or if some other theoretical constuction must be used. The motivation for asking this question will be seen in examining if the Gravitino, as in the mass in the left hand side of Equation (1), as it materializes due to Comay's [2] presentation as to defects in the Higgs equation of state, is in fidelity with QM principles. If not, then what would replace it?

$$H_{Higgs-changed} = H_{Higgs} - \beta\cdot\dot{\phi}^{*}\cdot\dot{\phi} \tag{2}$$

And

$$i\frac{\mathrm{d}\phi_{Higgs-Gravitino}}{\mathrm{d}t} = H_{Higgs-changed}\,\phi_{Higgs-Gravitino} \tag{3}$$

In fidelity with the physics evolution of

$$i\frac{\mathrm{d}\phi_{Dirac}}{\mathrm{d}t} = H_{Dirac}\,\phi_{Dirac} \tag{4}$$

Whereas what is observed is, instead [2]

$$i\frac{\mathrm{d}\phi_{Higgs-Gravitino}}{\mathrm{d}t} \neq H_{Higgs}\,\phi_{Higgs-Gravitino} \tag{5}$$

To further elucidate this question, we will also ask if there is a way to encapsulate H_{Higgs} in Equation (2) above in the methodology of constucting QM within a larger, semi classical theory. As given in the 5th Dice 2010 work shop, as given by Elze, Gambarotta and Vallone [3] there is a speculated ensemble theory involving a "Liouville superator" $\hat{\varsigma}$ of

$$i\partial_t\,"state" = \hat{\varsigma}\,"state" \tag{6}$$

The end result is, after a Fourier transform re casting the Equation (6) in terms of a matrix equation looking like

$$\begin{aligned} i\cdot\partial_t\rho_{jk} &= \sum_{l,m}\varsigma_{jk,lm}\rho_{lm} \\ &= \sum_{l,m}\left[\varsigma_{jk,lm} = H_{jl}\delta_{mk} - \delta_{jl}H_{mk}\right]\rho_{lm} \end{aligned} \tag{7}$$

We will discuss Equation (7) in a generalized incantation in APPENDIX A which will show as that the quantum mechanics type interactions require a most specialized potential, as either a constant, or a Harmonic potential, with others not sutiable, if we wish to extract quantum mechanics from the results of Equation (7), and from there to comment upon candidate equations which may be a way to contain H_{Higgs} as far as a generalized theory which may contain QM (Dirac) type behavior. If not, then Equation (2) does not qualify as far as having H_{Higgs} reduced to a quantum mechanica subset and we must then go to the Comay description of the Higgs equation used to define the creation of/evolution of the Gravitino as faulty physics, needing an abrupt fix to reduce it to the form of Equation (2) to salvage quantum mechanics.

Appendix B brings up the relevance of the Dirac eqjatkl to the critique which Comay [2] brings to the discussion of a proper equation for a well behaved experimentally verified equation. We add an example of how early universe Lorentz violation is equivilent to the break up of the fidelity of the Higgs term, and in fact, the Equation (B12) presented in Appendix B is in its behavior (if it were 10 orders of magnitude larger, *i.e.* as a Torsion term added in) very similar to the problem outlined in equation(B6) in the Higgs potential, *i.e.* note in Equation (B3) with the unwanted $\dot{\phi}^{*}\dot{\phi}$ term which blocks the Higgs equation of state from having the good behavior postulated by Comay [2] in his Claims 1, 2 and 3 as given in Appendix B below. Note also that the problem as outlined in $\dot{\phi}^{*}\dot{\phi}$ term shows up in an even more glaring fashion in the incredibly complicated Lagrangian specified for the formation of Gravitinos in the early universe. We will get to that next. It is useful to compare these ideas with what J. Lee published recently [4].

2. Examining the Formation of Gravitinos in the Early Universe

In [5] the density is given by, if g_{*} is for early universe degrees of freedom

$$\rho = g_{*}\left[\frac{\pi^2}{30}\right]\cdot T^4 \tag{8}$$

With a resulting Hubble rate for the radiation era as written as for $H(T)$, radiation era, as

$$H(T) = \sqrt{\frac{g_{*}\cdot\pi^2}{90}}T^2 \tag{9}$$

The early Gravitino relic density is then given by an expression

$$\Omega_{\tilde{G}}^{TP}h^2 = \sum_{\alpha=1}^{3}\left(1+\frac{M_\alpha(T_R)^2}{3\cdot m_{\tilde{G}}^2}\right)\cdot\omega_\alpha\cdot g_\alpha(T_R)^2 \tag{10}$$

times $\ln\left(\frac{k_\alpha}{g_\alpha(T_R)}\right)\left(\frac{m_{\tilde{G}}}{100GeV}\right)\left(\frac{T_R}{10^{10}GeV}\right)$

This is, in terms of re heating temperature very close to linear in growth due to scaling with a re heating temperature T_R. One obtains an approximately linear growth rate in terms of gravitino density with a most complicated Lagrangian density function which is in the top of Section 2.2. of [5] is so complicated that one cannot, even in linear approximations of it get either a classical or a quantum analogy in terms easily identifiable terms of page 5 of this Ph.D. dissertation. We will review in Appendix A the DICE 2010 [3] article treatment of quantum mechanics in a larger non linear theory [3], and in Appendix B the Comay [2] treatment in terms of lagrangian density both for the Dirac Eq, and also for the Higgs, and then from there make the case necessary as to if the Gravititino is quantum mechanical in its construction or not.

3. Getting the Template as to Keeping Information Content Avaiable for Equation (10) Right and Its Implications for Equation (1) and Equation (4), and Equation (5). Yielding a New Expression of Gravitino Mass in the EW Regime?

The Machian hypothesis and actually Equation (10) are a way to address a serious issue, *i.e.* how to keep the consistency of physical law intact, in cosmological evolution [1]. Another significant issue is the following. How to reconcile the Comay hypothesis [2] and postulates, as given in Appendix B, and also the DICE 2010 delination of QM as in Appendix A either requiring a zero valued potential, a constant potential, or a potential with quadratic flavor to delineate clear quantum mechanical behavior [3]. If these potential field requirements are not met, as given by Appendix A [3], then one has to ask if a Higgs mechanism in fidelity with Appendix B [2] can be constructed for an allegedly optimal experimental modeling of mass formation.

Equation (10), which has neither a zero valued potential, a linear or a quadratic potential is clearly NOT in sync with the DICE 2010 Appendix A treatments leading to quantum mechanics, alone [3].

Equation (10) does NOT have fidelity with the sort of Comay criteria [2] as given in Appendix B as to a potential energy which is most likely to have optimal match up with experimental data as cited by the Dirac equation results as given in the Comay article.

Either Equation (10) signifies that there is no match up with the sort of evolution equation (for creation of a Gravitino in the electro weak era) as exemplified by the Dirac Equation which Comay likes so much [2], or we have to go to live with the results as given by Appendix

B, that what we are seeing in the Gravitino in the Electro weak era is quantum mechanics contained in a larger non linear theory, as Elze seem to be inferring [3] as brought up in Appendix A.

4. Another Approach. How about a New Method for Obtaining in the Electroweak Era Mass without the Higgs?

What we can look at is the Glinka-Beckwith [6] proposal as to a new mass formation process,which may show a different way to examine potential systems, as opposed to the either-or criteria as given by Appendix A [3] and Appendix B [2] below. To do so, note that the article as given in.

$$|m|^2 = \left(\frac{\hbar \cdot \omega}{c^2}\right)^2 - \left(\frac{\hbar \bar{k}}{c}\right)^2 \tag{11}$$

We can treat the k as a wave "vector", and look at the term $\hbar \cdot \omega$ as an energy term. Dependent upon how we interpret $\hbar \cdot \omega$, *i.e.* as a per unit interpretation of energy, we could reconcile a treatment of a physically averaged out quantity of the potential energy as given in [5] is contained via the correct effective Lagrangian for light gravitinos $\zeta^{(\alpha)}_{\psi,\mathrm{int}}$, which is Equation (2.82), page 22 of Pradler's dissertation [5] for obtaining gravitino interactions with ordinary matter fields.

We can, to first order model the at in the Gravitino-matter field interaction as [5]

$$V^{(\alpha)}_{\psi,\mathrm{int}} = i \cdot \bar{\psi}_\mu \cdot \left[\gamma^\rho, \gamma^\sigma\right] \cdot \gamma^\mu \cdot \lambda^{(\alpha)\cdot a} F^{(\alpha)\cdot a}_{\rho\cdot\sigma} \Big/ 8M_{Pl} \tag{12}$$

This Equation (12) is the potential energy term of Equation (2.82), page 22 of Josef Pradler's [5] dissertation, and we argue that the physics of the gravitino, as interacting with matter in the electro weak regime, can be to first order, averaged out to be an energy which can be then made equivilent to $\hbar \cdot \omega$ of Equation (11). We argue then that effectively, in early universe conditions that we are looking at, then [6],

$$\begin{aligned} &|m|^2\Big|_{EW} \sim \left(\frac{\hbar \cdot \omega}{c^2}\right)^2 \propto \left(\frac{\left\langle V^{(\alpha)}_{\psi,\mathrm{int}}\right\rangle}{c^2}\right)^2 + vanishingly \\ &-small - terms \\ &M = im \end{aligned} \tag{13}$$

Then, if we do Equation (13) in this spirit, we can then go to what Glinka-Beckwith wrote [6] and look at

$$M\Big|_{EW} \sim \left(\frac{\hbar \cdot \omega = E = \left\langle V^{(\alpha)}_{\psi,\mathrm{int}}\right\rangle}{c^2}\right)\Bigg|_{EW} \tag{14}$$

Terms such as

$$-\left(\hat{i} \cdot \frac{p_x}{c} + \hat{j} \cdot \frac{p_y}{c} + \hat{k} \cdot \frac{p_z}{c}\right) \tag{15}$$

vanish from Equation(14).

Ultimately, the analysis of terms as specified in a gravitino-EW "matter" regime would specify the exact particulars as to Equation (12). We will also venture a first order approximate description as to why the mass of the Graviton in the later regime of space time, near the present would be so much smaller than the Gravitino.

5. Conclusion

Via use of the Glinka-Beckwith approximation for the formation of Mass, we have come up with a criteria where the Gravitino interaction with space-time physics in the electro weak, as outlined above, can be construed as either embedded within a larger theory than QM, as suggested by Elze *et al.* [3], or a corrected Higgs mass formation [2], or something else, which has to be constructed. As outlined by Beckwith [1] there is room to delineate if such a gravitino, using some of the field theoretic construction as given by [5] will be either classically embedded, or something else. The formalism as to massive graviton distortion of early universe space time, as given in [7], and [8] needs to be developed more fully, and we hope we can experimentally test if t'Hoofts supposition about QM [9] is falisifiable experimentally, and analytically, in this early universe setting, as brought up by the author [1].

6. Acknowledgements

This work is supported in part by National Nature Science Foundation of China grant No.110752.

REFERENCES

[1] A. W. Beckwith, "Gravitons, Gravitinos, and Using Machs Principle to Make a Statement of Equivalent Information in a Gravitino-Graviton Correspondence of Critical Information Exchange from Electro-Weak to Present Era," http://vixra.org/abs/1204.0089

[2] E. Comay, "Physical Consequences of Mathematical Principles," *Progress in Physics*, Vol. 4, 2009, pp. 91-98.

[3] H.-T. Elze, G. Gambarotta and F. Vallone, "General Linear Dynamics-Quantum, Classical or Hybrid," *IOP Conference Proceedings of the DICE* 2010 *Meeting*, 2010. http://iopscience.iop.org/1742-6596/306/1/012010

[4] J.-W. Lee, "On the Origin of Entropic Gravity and Inertia," *Foundations of Physics*, Vol. 42, 2012. http://arxiv.org/abs/1003.4464

[5] J. Pradler, http://arxiv.org/pdf/0708.2786.pdf$$yThesis

[6] A. Beckwith and L. Glinka, "The Quaternionic Particle Mass," *Prespacetime Journal*, Vol. 3, No. 2, 2012, pp. 126-130.

[7] K. Hinterbichler, "Theoretical Aspects of Massive Gravity," http://arxiv.org/abs/1105.3735.pdf

[8] http://web.mit.edu/redingtn/www/netadv/Xgravitati.html

[9] G. t'Hooft, "Quantum Mechanics as a Dissipative Deterministic System." http://arxiv.org/PS_cache/gr-qc/pdf/9903/9903084v3.pdf

[10] J. Bjorken and S. Drell, "Relativisitic Quantum Mechanics," McGraw Hill, 1964.

[11] J. Bjorken, "Emergent Photons and Gravitons: The Problem of Vacuum Structure," arXIV 1008.0033 v1[hep-th], 2010.

Appendix A

Elze *et al.* DICE 2010 Summary as to Quantum Mechanics Embedded in a Larger Non Linear Classical Theory

This discussion serves to bring up a Quantum like version of the Liouville equation and to from there to also make sense of the given equation, as of the main text [3]:

$$i\partial_t \text{"}state\text{"} = \hat{\varsigma}\text{"}state\text{"} \tag{6}$$

To begin with, look at a generic Hamiltonian as given by

$$H(x,p) = \frac{1}{2}p^2 + V(x) \tag{A1}$$

This Hamiltonian is incorporated in the Lioville equation of motion

$$-\partial_t \rho = \frac{\partial H}{\partial p}\cdot\frac{\partial \rho}{\partial_x} - \frac{\partial H}{\partial x}\cdot\frac{\partial \rho}{\partial_p} = \left\{ p\cdot\partial_x - \frac{dV}{dx}\cdot\partial_p \right\}\rho \tag{A2}$$

The upshot if a Fourier transform is taken of Equation (A2) above, and the space like co-ordinates of

$$\begin{aligned} Q &:= x + y/2 \\ q &:= x - y/2 \end{aligned} \tag{A3}$$

Equation (A1) then becomes

$$i\partial_t \rho = \left\{ \hat{H}_Q - \hat{H}_q + \varepsilon(Q,q) \right\}\rho \tag{A4}$$

The term put in, namely $\varepsilon(Q,q)$ which retrieves if we have classical or quantum information, and also, note

$$\hat{H}_\chi = \frac{-1}{2}\partial_\chi^2 + V(\chi) \tag{A5}$$

And

$$\begin{aligned} \varepsilon(Q,q) &:= (Q-q)\cdot\left[\frac{d}{dt}V\left(\frac{Q+q}{2}\right)\right] \\ &\quad -V(Q) + V(q) = -\varepsilon(q,Q) \end{aligned} \tag{A6}$$

Then,

$$\begin{aligned} &\varepsilon(Q,q) := 0 \Leftrightarrow V = const, V \sim linear, \\ &V \sim harmonic \end{aligned} \tag{A7}$$

If so, then one can write

$$i\partial_t \rho = \left\{ \hat{H}_Q - \hat{H}_q \right\}\rho \tag{A8}$$

I.e. then we have that for the potentials represented by Equation (7), there is an overlap between classical and quantum versions of the Liouville equation as given by the Von Neuman equation as presented by

$$i\partial_t \hat{\rho} = \left[\hat{H}, \hat{\rho} \right] \tag{A9}$$

In so many words, we have a QM type situation guaranteed if Equation (A7) holds, whereas we can solve a more general theoretical construction in which there may be what is known as a super action given by

$$S := \int_{t_0}^{t} \mathrm{d}\tau \cdot \left(\frac{m}{2}\cdot\dot{Q}^2 - V(Q) - \left[\frac{m}{2}\cdot\dot{q}^2 - V(q)\right] - \varepsilon(Q,q) \right) \tag{A10}$$

We then will be stuck with working with Equation (A4).

When the super action is reduced to, with Equation (A7)

To

$$S := \int_{t_0}^{t} \mathrm{d}\tau \cdot \left(\frac{m}{2}\cdot\dot{Q}^2 - V(Q) - \left[\frac{m}{2}\cdot\dot{q}^2 - V(q)\right] \right) \tag{A11}$$

We recover Equation (A9).

In short, the restrictions on the potential energy, as given by Equation (A7) are essential for the formation of quantum mechanics for exact quantum mechanical Hilbert space operators, whereas more general cases with $\varepsilon(Q,q) :\neq 0$.

Embedd quatum mechanics into the semi classical equation regime, as was specified by Elze and others.

Appendix B

Problems with the Higgs Equation, Lectured Upon in Chongqing University, November 2011

We summarize the main point of Comay's article [2] in terms of their relationship to the Dirac equation and the question of what is the optimal form of a physics equation most in fidelity to experimental measurements.

The initial points of this borrowing from Comay have already been made in Equation (2) to Equation (5) so we will be discussing the action integral intepretation which Comay made, which was his primary way to differentiate between the faulty mathematics as he saw in the Higgs equation and the Dirac equation. We will reproduce his arguments as to that intepretation in this appendix.

$$S = \int \varsigma(\psi_1, \psi_2) \cdot \mathrm{d}^4 x \tag{B1}$$

Here, $\varsigma(\psi_1, \psi_2)$ is a Lagrangian density function which is a Lorentz scalar, so then Equation (B1) is a Lorentz scalar.

The consequences that equation (B1) is a Lorentz scalar lead to several claims by Comay to follow upon and to use.

CLAIM 1:

1) A relativistically consistent quantum theory may be derived from Lagrangian density $\varsigma(\psi_1, \psi_2)$ which is a Lorentz scalar.

2) An acceptable dimension for a Lagrangian density is of the form $\left[L^{-4}\right]$

3) A wave functional $\psi(x^\mu)$ for both $\varsigma(\psi_1, \psi_2)$ and S cannot define a composite particle **if** x^μ is for a single four dimensional point in space time

Sub claim to 3 above, and an effective re statement of 3 is: **If** $\psi(x^\mu)$ were for a single (not composite) particle, then

3*. A: $\psi(x^\mu)$ needs space time co-ordinates of its center of energy

3*. B: One needs additional co-ordinates for describing internal structure.

We shall then go to the next specific Comay Claim, namely

CLAIM 2

Use the following procedure to get consistency of a quantum (massive particle) theory with a classical (massive particle) particle theory, namely by using the following field equation, as given by

$$\frac{\partial}{\partial x^\mu} \cdot \frac{\partial \varsigma}{\partial\left(\dfrac{\partial \psi}{\partial x^\mu}\right)} - \frac{\partial \varsigma}{\partial x^\mu} = 0 \tag{B2}$$

For energy start off with the equation given by the 2nd order tensor, $T_{\mu\nu}$, with T_{00} the energy density, and $T_{\mu\nu}$ having L^{-4} dimensions, with

$$T_{\mu\nu} = \frac{\partial \varsigma}{\partial\left(\dfrac{\partial \psi}{\partial x^\mu}\right)} \cdot \frac{\partial \varsigma}{\partial x^\nu} - \varsigma g_{\mu\nu} \tag{B3}$$

Sub set of CLAIM 2. In QM, the Hamiltonian is equal to the total energy, so we can write T_{00} as the Hamiltonian density

$$T_{00} = H_{Hamiltonian-density} = \dot{\psi} \cdot \frac{\partial \varsigma}{\partial \dot{\psi}} - \varsigma \tag{B4}$$

Equation (B4) satisfies the continuity equation as given by

$$j^\mu_{,\mu} = 0 \tag{B5}$$

Then either of the two happen:

A. Hamiltionian density $H_{Hamiltonian-density}$ may be used to extract Hamiltonian H so that one can write a Hamiltonian H so that then the following happens: Energy E is an eignvalue of ψ

$$H\psi = E\psi \tag{B6}$$

And the De Broglie functions hold as given by

$$i\frac{\partial \psi}{\partial t} = E\psi \Rightarrow i\frac{\partial \psi}{\partial t} = H\psi \tag{B7}$$

So then the Hilbert space is formed using all ψ of H (completeness of the Hilbert space, using basis from ψ).

OR

B. Use expression for density to form inner product for inner product of ψ and construct an orthormal baisis set (often using Gram Schmitz orthoganization) for othnormal basis for corresponding Hilbert space.

Then, after B, to then look at a matrix equation given by

$$H_{i,j} = \int \mathrm{H}_{Hamiltonian-density}\left(\psi_i; \psi_{i,\mu}; \psi_j; \psi_{j,v}\right) \cdot \mathrm{d}x^3 \tag{B8}$$

Form a matrix from Equation (B8), and then diagonalize this matrix to get eignvalues ψ and ENERGY eignvectors .

ClAIM 3

Proceedures from CLAIM 1 and CLAIM 2, give the same eignvalues and eignvectors, SAME information.

CLAIM 4

The following Equations give almost the same information, one QM, and the other CM (Quantum versus Classical)

$$i\frac{\partial \psi}{\partial t} = E\psi \Rightarrow i\frac{\partial \psi}{\partial t} = H\psi \tag{B9}$$

$$\frac{\partial}{\partial x^\mu} \cdot \frac{\partial \varsigma}{\partial\left(\dfrac{\partial \psi}{\partial x^\mu}\right)} - \frac{\partial \varsigma}{\partial x^\mu} = 0 \tag{B10}$$

Applications of this formulation. See the Dirac Equation as given by Bjorken And Drell, [10], plus Comay [2].

This example works beautifully. Pion physics, Quark physics and more. There is an excellent match up with experiment.

Next application, Higgs equation, so that

$$\begin{aligned} E\big|_{Higgs} &\neq H_{Hamiltonian-density}\big|_{Higgs} \\ &= \dot{\phi} \cdot \frac{\partial \varsigma}{\partial \dot{\phi}} - \varsigma H \sim \{\ \} + \dot{\phi}^* \dot{\phi} \end{aligned} \tag{B11}$$

Here we see then that

$$i\frac{\partial\psi}{\partial t}=E\psi DOESNOT\Rightarrow i\frac{\partial\psi}{\partial t}=H\psi \tag{B12}$$

Specifically, for the Higgs, one has

$$\varsigma\left(Higgs\right)\equiv\phi^{*}_{,\mu}\phi_{,v}g_{uv}+L.O.T. \tag{B13}$$

Equation (B13) will then lead to a Higgs potential energy looking like, in simplest form. Where we only know the ratio of $\mu^{\otimes}/|\lambda|$.

$$V\left(Higgs\right)\left[\phi^{\dagger}\phi\right]=\mu^{\otimes}\left(\phi^{\dagger}\phi\right)+|\lambda|\left(\phi^{\dagger}\phi\right)^{2} \tag{B14}$$

And we get a vacuum state given by

$$\langle\phi\rangle_{0}=\begin{pmatrix}0\\ \sqrt{-\mu^{\otimes}/|\lambda|}\end{pmatrix} \tag{B15}$$

For the Higgs nucleation of mass, for a Graviton, we have a huge problem, *i.e*. many undetermined coefficients.

This is similar to what happens with Bjorken's work [11].

Let $H\left(DE\right)$ be the Hubble rate of expansion of the cosmos, and set a scale factor as

$$a\left(t\right)=\exp\left(-H\left(DE\right)\cdot t\right) \tag{B17}$$

Here we can re phrase H_{oc} as being the Hubble rate of expansion without torsion added in. Also

$$H\left(DE\right)=H_{oc}-\left[\left(4\pi\cdot\gamma\cdot\rho_{A}\right)^{2}\Big/M_{Pl}^{4}\cdot\left(1+\gamma^{2}\right)\right] \tag{B18}$$

If we go to the Zeldovich relationship

$$\begin{aligned}&\left[\left(4\pi\cdot\gamma\cdot\rho_{A}\right)\Big/M_{Pl}^{0}\cdot\left(1+\gamma^{2}\right)^{1/2}\right]\\&\sim\left(\Lambda_{QCD}=10^{-20}M_{Pl}\right)^{3}\end{aligned} \tag{B19}$$

Then we get a Lorentz violating "Lagrangian" added on term looking like, if

$$b_{\mu}=\frac{\eta_{\mu}2\pi\rho_{A}\gamma}{M_{Pl}\left(1+\gamma^{2}\right)}\leq10^{-33}eV \tag{B20}$$

$$L'=b_{\mu}\bar{\psi}\gamma^{\mu}\gamma_{5}\psi \tag{B21}$$

This Equation (B20) is a ten orders too small Lorentz violation term, in the Potential for a Lagrangian, for space-time emergence, but if it were larger, it would be similar in effect to the problem with the Higgs which Comay is outlining. Very close.

Journal of Modern Physics, **2012, 3, 1261-1271**
doi:10.4236/jmp.2012.329163 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

YinYang Bipolar Atom—An Eastern Road toward Quantum Gravity*

Wen-Ran Zhang
Department of Computer Science, Georgia Southern University, Statesboro, USA
Email: wrzhang@georgiasouthern.edu

Received June 19, 2012; revised July 22, 2012; accepted July 31, 2012

ABSTRACT

Based on bipolar dynamic logic and bipolar quantum linear algebra, a causal theory of YinYang bipolar atom is introduced in a completely background independent geometry that transcends spacetime. The causal theory leads to an equilibrium-based super symmetrical quantum cosmology of negative-positive energies. It is contended that the new theory has opened an Eastern road toward quantum gravity with bipolar logical unifications of particle and wave, matter and antimatter, relativity and quantum entanglement. Information recovery after a black hole is discussed. It is shown that not only can the new theory be applied in physical worlds but also in logical, mental, social and biological worlds. Falsifiability of the theory is discussed.

Keywords: YinYang Bipolar Atom; Bipolar Geometry; Quantum Cellular Automata; Matter and Antimatter; Information Recovery after a Black Hole; Real World Quantum Gravity

1. Introduction

Stephen Hawking's black hole theory originally suggested that the universe would ultimately disappear in a black hole without information preservation. This suggestion was criticized for violating the 2nd law of thermodynamics. To remedy the inconsistency, Hawking proposed black body evaporation [2] and then particle emission [3]. After then he held his position for three decades. In 2004, he finally conceded a bet and agreed that black hole emission does in fact preserve information. But so far it is unclear how to recover the information from the evaporation or particle emission and how the universe will evolve after a black hole. This uncertainty makes quantum theory incomplete and nihilism unavoidable. For instance, M-theory predicts that a great many universes were created out of nothing [4, p. 5].

Equilibrium is a well-known scientific concept that subsumes symmetry or broken symmetry. Since equilibrium is central in the 2nd law of thermodynamics—the paramount law of existence, energy, life, and information where bipolar equilibrium is a generic form, YinYang bipolar equilibrium-based approach to physics and science provides a fundamental super symmetrical alternative for scientific unification. (Note: Equilibrium subsumes equilibrium, non-equilibrium and quasi-equilibrium because local non-equilibriums can form global equilibrium or quasi-equilibrium.)

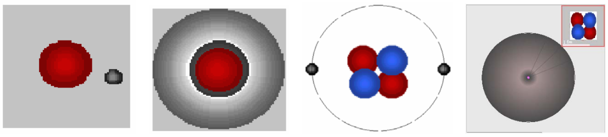
*The idea has been partially presented in Ref. [1].

Atom as a basic unit of matter should follow equilibrium or non-equilibrium conditions. It consists of a dense, central nucleus surrounded by a cloud of negatively charged electrons. The nucleus contains a mix of positively charged protons and electrically neutral neutrons (except in the case of hydrogen-1). The electrons of an atom are bound to the nucleus by the electromagnetic force. Likewise, a group of atoms can remain bound to each other, forming a molecule. In the case of antimatter atom, the cloud is formed with positively charged positrons and the atomic nucleus is negatively charged.

Molecule is an electrically neutral group of at least two atoms held together by covalent chemical bonds. A covalent bond is a form of chemical bonding that is characterized by the sharing of pairs of electrons between atoms. The stable balance of attractive and repulsive forces between atoms when they share electrons is known as covalent bonding.

An atom containing an equal number of protons and electrons is electrically neutral. Otherwise, it has a positive or negative charge. A positively or negatively charged atom is known as an ion. An atom is classified according to the number of protons and neutrons in its nucleus: the number of protons determines the chemical element and the number of neutrons determines the isotope of the element. **Figure 1** shows some examples.

Legendary Danish physicist Niels Bohr, a father figure of quantum mechanics, brought YinYang into quantum theory for his particle-wave complementarity principle.



(a) (b) (c) (d)

Figure 1. (a) Matter hydrogen atom; (b) Proton of a hydrogen surrounded by an electron cloud; (c) Matter helium atom with a nucleus (two protons and two neutrons) and two electrons; (d) Tiny nucleus of a helium atom is surrounded by electron cloud (Creative Commons: by User: Yzmo).

When he was awarded the Order of the Elephant by the Danish government in 1947, he designed his own coat of arms which featured in the center a YinYang logo (or Taiji symbol) with the Latin motto "contraria sunt complementa" or "opposites are complementary".

While quantum mechanics recognized particle-wave complementarity it stopped short of identifying the essence of YinYang bipolar coexistence. Without bipolarity any complementarity is less fundamental due to the missing "opposites". On the other hand, if bipolar equilibrium is the most fundamental form of equilibrium, any multidimensional model such as string, superstring or M-theory cannot be most fundamental. In brief, action-reaction forces, particle-antiparticle pairs, negative-positive energies, input and output, or the Yin and Yang in general are the most fundamental opposites of nature, but man and woman, space and time, particle and wave, truth and falsity are not exactly bipolar opposites. This could be the reason why Bohr believed that a causal description of a quantum process cannot be attained and we have to content ourselves with particle-wave complementary descriptions [5]. It may also be the reason why modern physics so far failed to find a definitive battleground for quantum gravity.

Einstein pointed out: "*For the time being we have to admit that we do not possess any general theoretical basis for physics which can be regarded as its logical foundation*." "*Physics constitutes a logical system of thought which is in a state of evolution*, *whose basis* (*principles*) *cannot be distilled*, *as it were, from experience by an inductive method*, *but can only be arrived at by free invention*."

In the above light, a causal theory of YinYang bipolar atom is introduced in this paper based on bipolar dynamic logic and bipolar quantum linear algebra [6-8]. The theory provides a springboard to an equilibrium-based logical unification of particle and wave, matter and antimatter, relativity and quantum theory, strings and reality as well as big bang and black hole. Information recovery after a black hole is discussed. The logical, physical, mental, biological and social implications of this work are formalized into a Q5 paradigm of quantum gravities [8].

This paper is organized into six sections. Following this introduction, a background review of the mathematical basis of this work is presented in Section 2. YinYang bipolar atom is presented in Section 3. Bipolar quantum cellular automata are introduced in Section 4. Section 5 presents the theory of YinYang bipolar quantum gravity. Section 6 draws a few conclusions as well as philosophical distinctions.

2. YinYang Bipolar Dynamic Logic and Quantum Linear Algebra

2.1. YinYang Bipolar Quantum Lattice and Bipolar Dynamic Logic (BDL)

Aristotle's causality principle became controversial in the 18th century after David Hume challenged it from an empirical perspective. Hume argued that causation is irreducible to pure regularity. YinYang bipolar dynamic logic (BDL) [6,8-10] has changed this situation in a fundamental way. BDL is defined on a bipolar quantum lattice $B_1 = \{-1, 0\} \times \{0, +1\} = \{(0,0), (0,1), (-1,0), (-1,1)\}$ in YinYang bipolar geometry as shown in **Figure 2**. The four values of B_1 form a bipolar set [8] which stand respectively for eternal equilibrium (0,0), non-equilibrium (–1,0), non-equilibrium (0,+1); equilibrium or harmony (–1,+1). Equation (1)-(12) in **Table 1** provide the basic operations of BDL. The laws in **Table 2** hold on BDL. Most interestingly, BUMP makes equilibrium-based bipolar quantum causality logically definable.

An axiomatization of BDL (**Table 3**) has been proven sound and complete [8]. A key element in the axiomatization is bipolar universal modus ponens (BUMP) (**Table 4**) which is a bipolar tautology, a non-linear bipolar dynamic generalization of classical modus ponens and a logical representation of bipolar quantum entanglement. Thus, BDL generalizes Boolean logic to a quantum logic where $\oplus$ and $\oplus^-$ are "balancers"; $\varnothing$ and $\otimes$ are intuitive "oscillators"; $\varnothing^-$ and $\otimes^-$ are counter-intuitive "oscillators"; & and $\&^-$ are "minimizers." The linear, cross-pole, bipolar fusion, oscillation, interaction and entanglement properties are depicted in **Figure 3**. Bipolar relations and equilibrium relations are defined in [6,8,11,12].

2.2. Bipolar Quantum Linear Algebra (BQLA)

The bipolar lattice $B_1 = \{-1,0\} \times \{0,1\}$ and bipolar fuzzy lattice $B_F = [-1,0] \times [0,1]$ can be naturally extended to the infinite bipolar lattice $B_\infty = [-\infty,0] \times [0,+\infty]$. While B_1 and B_F are bounded complemented unit square crisp/fuzzy lattices, respectively, B_∞ is unbounded. $\forall (x,y),(u,v) \in B_\infty$, Equations (13) and (14) define two major operations.

Tensor Bipolar Multiplication:

$$(x,y) \times (u,v) \equiv (xv+yu,\ xu+yv); \tag{13}$$

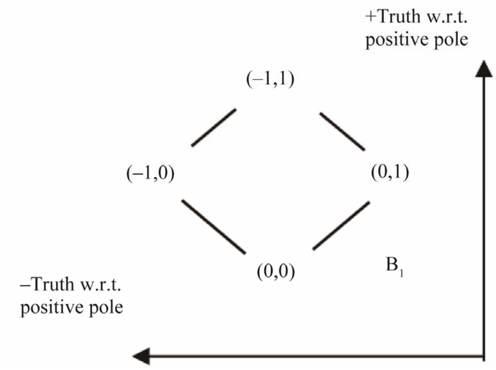


Figure 2. Hasse diagrams of B_1 in YinYang bipolar geometry.

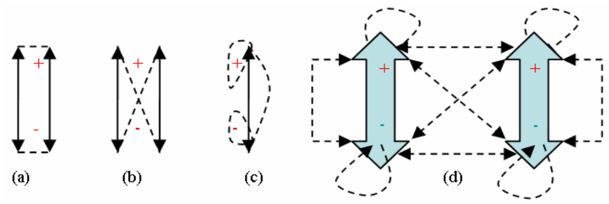


Figure 3. YinYang bipolar relativity: (a) Linear interaction; (b) Cross-pole non-linear interaction; (d) Oscillation; (e) Two entangled bipolar interactive variables.

Table 1. YinYang Bipolar Dynamic Logic (BDL). (Note: The use $|x|$ through this paper is for explicit bipolarity only).

Bipolar Partial Ordering: $(x,y) \geq\geq (u,v)$, *iff* $|x| \geq |u|$ *and* $y \geq v$; (1)

Complement: $\neg(x,y) \equiv (-1,1)-(x,y) \equiv (\neg x, \neg y) \equiv (-1-x, 1-y)$; (2)

Implication: $(x,y) \Rightarrow (u,v) \equiv (x \rightarrow u, y \rightarrow v) \equiv (\neg x \vee u), \neg y \vee v)$; (3)

Negation: $-(x,y) \equiv (-y, -x)$; (4)

Bipolar least upper bound (blub):

$blub((x,y),(u,v)) \equiv (x,y) \oplus (u,v) \equiv (-(|x| \vee |u|), y \vee v)$; (5)

Bipolar greatest lower bound (bglb):

$bglb((x,y),(u,v)) \equiv (x,y) \& (u,v) \equiv (-(|x| \wedge |u|), y \wedge v))$; (6)

-blub: $blub^{-}((x,y),(u,v)) \equiv (x,y) \oplus^{-} (u,v) \equiv (-(y \vee v), (|x| \vee |u|))$; (7)

-bglb: $bglb^{-}((x,y),(u,v)) \equiv (x,y) \&^{-} (u,v) \equiv (-(y \wedge v), (|x| \wedge |u|)))$; (8)

Cross-pole greatest lower bound (cglb):

$cglb((x,y),(u,v)) \equiv (x,y) \otimes (u,v) \equiv (-(|x| \wedge |v| \vee |y| \wedge |u|), (|x| \wedge |u| \vee |y| \wedge |v|))$; (9)

Cross-pole least upper bound (cglb):

$club((x,y),(u,v)) \equiv (x,y) \varnothing (u,v) \equiv (-1,1) - (-(x,y) \otimes \neg(u,v))$; (10)

-cglb: $cglb^{-}((x,y),(u,v)) \equiv (x,y) \otimes^{-} (u,v) \equiv -((x,y) \otimes (u,v))$; (11)

-club: $club^{-}((x,y),(u,v)) \equiv (x,y) \varnothing^{-} (u,v) \equiv -((x,y) \varnothing (u,v))$. (12)

Table 2. Bipolar laws.

Excluded Middle	$(x,y) \oplus \neg(x,y) \equiv (-1,1)$; $(x,y) \oplus \neg(x,y) \equiv (-1,1)$;
No Contradiction	$\neg((x,y) \& \neg(x,y)) \equiv (-1,1)$; $\neg((x,y) \&^{-} \neg(x,y)) \equiv (-1,1)$;
Linear Bipolar DeMorgan's Laws	$\neg((a,b) \& (c,d)) \equiv \neg(a,b) \oplus \neg(c,d)$; $\neg((a,b) \oplus (c,d)) \equiv \neg(a,b) \& \neg(c,d)$; $\neg((a,b) \&^{-} (c,d)) \equiv \neg(a,b) \oplus^{-} \neg(c,d)$; $\neg((a,b) \oplus^{-} (c,d)) \equiv \neg(a,b) \&^{-} \neg(c,d)$;
Non-Linear Bipolar DeMorgan's Laws	$\neg((a,b) \otimes (c,d)) \equiv \neg(a,b) \varnothing \neg(c,d)$; $\neg((a,b) \varnothing (c,d)) \equiv \neg((a,b) \otimes \neg(c,d)$; $\neg((a,b) \otimes^{-} (c,d)) \equiv \neg(a,b) \varnothing^{-} \neg(c,d)$; $\neg((a,b) \varnothing^{-} (c,d)) \equiv \neg(a,b) \otimes^{-} \neg(c,d)$

Table 3. Bipolar axiomatization

Bipolar Linear Axioms:

BA1: $(\phi^{-},\phi^{+}) \Rightarrow ((\varphi^{-},\varphi^{+}) \Rightarrow (\phi^{-},\phi^{+}))$;

BA2: $((\phi^{-},\phi^{+}) \Rightarrow ((\varphi^{-},\varphi^{+}) \Rightarrow (\chi^{-},\chi^{+}))) \Rightarrow$
$(((\phi^{-},\phi^{+}) \Rightarrow (\varphi^{-},\varphi^{+})) \Rightarrow ((\phi^{-},\phi^{+}) \Rightarrow (\chi^{-},\chi^{+})))$;

BA3: $(\neg(\phi^{-},\phi^{+}) \Rightarrow (\varphi^{-},\varphi^{+})) \Rightarrow ((\neg(\phi^{-},\phi^{+}) \Rightarrow \neg(\varphi^{-},\varphi^{+})) \Rightarrow (\phi^{-},\phi^{+}))$;

BA4: (a) $(\phi^{-},\phi^{+}) \& (\varphi^{-},\varphi^{+}) \Rightarrow (\phi^{-},\phi^{+})$;
(b) $(\phi^{-},\phi^{+}) \& (\varphi^{-},\varphi^{+}) \Rightarrow (\varphi^{-},\varphi^{+})$;

BA5: $(\phi^{-},\phi^{+}) \Rightarrow ((\varphi^{-},\varphi^{+}) \Rightarrow ((\phi^{-},\phi^{+}) \& (\varphi^{-},\varphi^{+})))$;

Non-Linear Bipolar Universal Modus Ponens (BUMP)

BR1: IF $((\phi^{-},\phi^{+}) * (\psi^{-},\psi^{+}))$, $[((\phi^{-},\phi^{+}) \Rightarrow (\varphi^{-},\varphi^{+})) \& ((\psi^{-},\psi^{+}) \Rightarrow (\chi^{-},\chi^{+}))]$,
THEN $[(\varphi^{-},\varphi^{+}) * (\chi^{-},\chi^{+})]$;

Bipolar Predicate Axioms and Rules of Inference

BA6: $\forall x, (\phi^{-}(x), \phi^{+}(x)) \Rightarrow (\phi^{-}(t), \phi^{+}(t))$;

BA7: $\forall x, ((\phi^{-},\phi^{+}) \Rightarrow (\varphi^{-},\varphi^{+})) \Rightarrow ((\phi^{-},\phi^{+}) \Rightarrow \forall x, (\varphi^{-},\varphi^{+})$;

BR2-Generalization: $(\phi^{-},\phi^{+}) \Rightarrow \forall x, (\phi^{-}(x), \phi^{+}(x))$

Table 4. Bipolar Universal Modus Ponens (BUMP).

$\forall \phi = (\phi^{-},\phi^{+})$, $\varphi = (\varphi^{-},\varphi^{+})$, $\psi = (\psi^{-},\psi^{+})$, and $\chi = (\chi^{-},\chi^{+}) \in B_1$,
$[(\phi \Rightarrow \varphi) \& (\psi \Rightarrow \chi)] \Rightarrow [(\phi * \psi) \Rightarrow (\varphi * \chi)]$.

Two-fold universal instantiation:

1) Operator instantiation: $*$ as a universal operator can be bound to $\&$, $\oplus$, $\&^{-}$, $\oplus^{-}$, $\otimes$, $\varnothing$, $\otimes^{-}$, $\varnothing^{-}$. $(\phi \Rightarrow \varphi)$ is designated bipolar true or (-1,+1); $((\phi^{-},\phi^{+}) * (\psi^{-},\psi^{+}))$ is undesignated.
2) Variable instantiation:
$\forall x, (\phi^{-},\phi^{+})(x) \Rightarrow (\varphi^{-},\varphi^{+})(x)$; $(\phi^{-},\phi^{+})(A)$; $\therefore (\varphi^{-},\varphi^{+})(A)$.

Bipolar Addition:

$$(x,y) + (u,v) \equiv (x+u, y+v) \tag{14}$$

In Equation (13), $\times$ is a cross-pole multiplication operator with the infused non-linear bipolar tensor semantics of --=+, -+=+-=1, and ++=+; + in Equation (14) is a linear bipolar addition or fusion operator. With the two basic operations, classical linear algebra is naturally extended to BQLA with bipolar fusion, diffusion, interaction, oscillation, and quantum entanglement properties. These properties enable physical or biological agents to interact through bipolar fields such as atom-atom, cell-cell, heart-heart, heart-brain, brain-brain, organ-organ, and genome-genome bio-electromagnetic quantum fields as well as biochemical pathways. Thus, the bipolar properties are suitable for equilibrium-based bipolar dynamic modeling with quantum aspects where one kind of equilibrium or non-equilibrium can have causal effect to another.

Given an input bipolar row vector matrix $E = [e_i] = [(e_i^{-}, e_i^{+})] \in B_{\infty}$, $I = 1, 2, \cdots, k$, and a bipolar connectivity matrix $M = [m_{ij}] = [(m_{ij}^{-}, m_{ij}^{+})]$, $i = 1, 2, \cdots, k$ and $j = 1, 2, \cdots, n$, *we have* $V = E \times M = [V_j] = [(v_i^{-}, v_i^{+})]$. While E is the input vector to a dynamic system characterized with the connectivity matrix M, V is the result row vector with n bipolar elements following Equation (15).

$$V = E \times M = \left[V_j\right] = \left[\left(v_j^{-}, v_j^{+}\right)\right]; \; V_j = \sum_{i=1}^{k}\left(e_j \times m_{ij}\right) \tag{15}$$

Equation (15) has the same form as in classical linear algebra except for: 1) e_j and m_{ij} are bipolar elements; 2) the multiplication operator is defined in Equation (13) on bipolar variables with bipolar (quantum) entanglement; and 3) the Σ operator is based on bipolar addition defined on bipolar variables in Equation (14).

BQLA provides a new mathematical tool for modeling YinYang-n-elements with explicit bipolar equilibrium, quasi- or non-equilibrium representation for energy and stability analysis. Energies in a row matrix can be considered as physical or biological energies of any agents such as quantum or cosmological negative and positive energies, repression and activation energies of regulator proteins. Energies embedded in a connectivity matrix can be deemed organizational energies that bind the agents together. The following laws hold for any physical or biological systems [7,8,13].

YinYang Bipolar Elementary Energy. Given a bipolar element $e = \left(e^{-}, e^{+}\right)$,

1) $\varepsilon^{-}(e) = e^{-}$ is the Yin or negative energy of e;

2) $\varepsilon^{+}(e) = e^{+}$ is the Yang or positive energy of e;

3) $\varepsilon(e) = (\varepsilon^{-}(e), \varepsilon^{+}(e)) = (e^{-}, e^{+})$ is the YinYang bipolar energy measure of e;

4) The absolute total $|\varepsilon|(e) = |\varepsilon^{-}|(e) + |\varepsilon^{+}|(e)$ is the total energy of e;

5) $\varepsilon_{\text{imb}}(e)=|\varepsilon^{+}|(e) - |\varepsilon^{-}|(e)$ is the imbalance of e;

6) $\text{EnergyBalance}(e) = (|\varepsilon|(e)-|\varepsilon_{\text{imb}}(e)|)/2.0 = \min(|e^{-}|, e^{+})$;

7) $\text{Harmony}(e) = \text{Balance}(e) = (|\varepsilon|(e) - |\varepsilon_{\text{imb}}(e)|)/|\varepsilon|(e)$.

YinYang Bipolar System Energy. Given an $k \times n$ bipolar matrix $\text{M} = [m_{ij}] = (\text{M}^{-},\text{M}^{+}) = (\left[m_{jj}^{-}\right], \left[m_{jj}^{+}\right])$, where M^{-} is the Yin half with all the negative elements and M^{+} is the Yang half with all the positive elements,

1) $\varepsilon^{-}\left(\text{M}\right) = \sum_{i=1}^{k}\sum_{j=1}^{n}\varepsilon_{ij}^{-} = \sum_{i=1}^{k}\sum_{j=1}^{n}m_{ij}^{-}$ is the negative or Yin energy of M;

2) $\varepsilon^{+}\left(\text{M}\right) = \sum_{i=1}^{k}\sum_{j=1}^{n}\varepsilon_{ij}^{+} = \sum_{i=1}^{k}\sum_{j=1}^{n}m_{ij}^{+}$ is the positive or Yang energy of M;

3) the polarized total, denoted $\varepsilon(\text{M}) = (\varepsilon^{-}(\text{M}), \varepsilon^{+}(\text{M}))$ is the YinYang bipolar energy of M of M;

4) the absolute total, denoted $|\varepsilon|(\text{M}) = |\varepsilon^{-}|(\text{M}) + |\varepsilon^{+}|(\text{M})$, is the total energy of M;

5) the energy subtotal for row i of M is denoted

$$|\varepsilon|\left(\text{M}_{i*}\right) = \left|\sum_{j=0}^{n}\varepsilon_{ij}\right|;$$

6) the energy subtotal for column j of M is denoted

$$|\varepsilon|\left(\text{M}_{j*}\right) = \left|\sum_{i=0}^{k}\varepsilon_{ij}\right|;$$

7) $\varepsilon_{\text{imb}}\left(\text{M}\right) = \sum_{i=1}^{k}\sum_{j=1}^{n}\varepsilon_{imp}\left(m_{ij}\right) = \sum_{i=1}^{k}\sum_{j=1}^{n}\left(m_{ij}^{+} - \left|m_{ij}^{-}\right|\right)$ is the YinYang imbalance of M;

8) balance or harmony or stability of M is defined as $\text{Harmony}(\text{M}) = \text{Balance}(\text{M}) = \text{Stability}(\text{M}) = (|\varepsilon|(\text{M}) - |\varepsilon_{\text{imb}}(\text{M})|)/|\varepsilon|(\text{M})$;

9) the average energy of M is measured as $h = (\varepsilon^{-}(\text{M})/(kn), \varepsilon^{+}(\text{M})/(kn))$ where $kn = k \times n$ is the total number of elements in M.

Law 1. Elementary Energy Equilibrium Law. $\forall(x,y) \in B_{\infty} = [-\infty, 0] \times [0, +\infty]$ and $\forall(u,v) \in B_F = [-1,0] \times [0,1]$, we have

a) $[|\varepsilon|(u,v) \equiv 1.0] \Rightarrow [|\varepsilon|((x,y)\times (u,v)) \equiv |\varepsilon|(x,y)]$;

b) $[|\varepsilon|(u,v)<1.0] \Rightarrow [|\varepsilon|((x,y)\times (u,v)) < |\varepsilon|(x,y)]$;

c) $[|\varepsilon|(u,v)>1.0] \Rightarrow [|\varepsilon|((x,y)\times (u,v)) > |\varepsilon|(x,y)]$.

Equilibrium/Non-Equilibrium System. A bipolar dynamic system S is said an equilibrium system if the system's total energy $|\varepsilon|\text{S}$ remains in an equilibrium state or $\text{d}(|\varepsilon|\text{S})/\text{d}t = 0$ without external disturbance. Otherwise it is said a non-equilibrium system. A non-equilibrium system is said a strengthening system if $\text{d}(|\varepsilon|\text{S})/\text{d}t > 0$; it is said a weakening system if $\text{d}(|\varepsilon|\text{S})/\text{d}t < 0$.

Law 2. Energy Transfer Equilibrium Law. Given an $n \times n$ input bipolar matrix $\text{E} = [e_{ik}] = [(e_{ik}^{-}, e_{ik}^{+})]$, $0 < i, k \le n$, an $n \times n$ bipolar connectivity matrix $\text{M} = [m_{kj}] = [(m_{kj}^{-}, m_{kj}^{+})]$, $0 < k, j \le n$, and $\text{V} = \text{E} \times \text{M} = [\text{V}_{ij}] = [(\text{v}_{ij}^{-}, \text{v}_{ij}^{+})]$, $\forall$k, j, let $|\varepsilon|(\text{M}_{k*})$ be the k-th row energy subtotal and let $|\varepsilon|(\text{M}_{*j})$ be the j-th column energy subtotal, we have, $\forall$k, j,

a) $[|\varepsilon|(\text{M}_{k*}) \equiv |\varepsilon|(\text{M}_{*j}) \equiv 1.0] \Rightarrow [|\varepsilon|(\text{V}) \equiv |\varepsilon|(\text{E})]$;

b) $[|\varepsilon|(\text{M}_{k*}) \equiv |\varepsilon|(\text{M}_{*j}) < 1.0] \Rightarrow [|\varepsilon|(\text{V}) < |\varepsilon|(\text{E})]$;

c) $[|\varepsilon|(\text{M}_{k*}) \equiv |\varepsilon|(\text{M}_{*j}) > 1.0] \Rightarrow [|\varepsilon|(\text{V}) > |\varepsilon|(\text{E})]$.

From the above, it is clear that without YinYang bipolarity, classical linear algebra cannot deal with the coexistence of the Yin and the Yang of nature and their causal interactions in bipolar quantum entanglement.

Law 3. Law of Energy Symmetry. Let $t = 0, 1, 2,\cdots$, $\text{Y}(t+1) = \text{Y}(t) \times \text{M}(t)$, $|\varepsilon|\text{Y}(t)$ be the total energy of an YinYang-N-Element vector $\text{Y}(t)$, $|\varepsilon|\text{M}(t)$ be the total energy of the connectivity matrix $\text{M}(t)$, $|\varepsilon|\text{M}_{i*}(t)$ be the energy subtotal of row i of $\text{M}(t)$, $|\varepsilon|\text{M}_{*j}(t)$ be the energy subtotal of column j of $\text{M}(t)$.

1) Regardless of the local YinYang balance or imbalance of the elements at any time point t, the system will remain a global energy equilibrium if, $\forall t$, $\text{d}(|\varepsilon|\text{Y}(t))/\text{d}t \equiv 0$, or (a)$\forall i,j$, $[|\varepsilon|(\text{M}_{i*}) \equiv |\varepsilon|(\text{M}_{*j}) \equiv 1.0]$ and (b) no external disturbance to the system occurs after the initial vector Y(0) is given.

2) Under the same conditions of (1), if, $\forall t$, $|\varepsilon^{-}(\text{M}_{*j})| > 0$ and $|\varepsilon^{+}(\text{M}_{*j}))| > 0$, all bipolar elements connected by M will eventually reach a local YinYang balance $(-|\varepsilon|\text{Y}(t)/(2\text{N}), |\varepsilon|\text{Y}(t)/(2\text{N}))$ at time t.

Law 4. Law of Broken Symmetry (Growing). For the same system with Law 3, if, $\forall i, j$, $|\varepsilon|(\text{M}_{i*}) \equiv |\varepsilon|(\text{M}_{*j}) > 1.0$, regardless of the local YinYang balance or imbalance of the elements at any time point t, the system energy will increase and eventually reach a bipolar infinite

($-\infty,\infty$) or fission state without external disturbance or we have, $\forall t$, $d(|\varepsilon|Y(t))/dt > 0$.

Law 5. Law of Broken Symmetry (Weakening). For the same system as for Law 3, if, $\forall i, j$, $|\varepsilon|(M_{i*}) \equiv |\varepsilon|(M_{*j}) < 1.0$, regardless of the local YinYang balance or imbalance of the elements at any time point t, the system energy will decrease and eventually reach a (0,0) or decayed state without external disturbance or we have, $\forall t$, $d(|\varepsilon|Y(t))/dt < 0$, until $|\varepsilon|Y(t) = 0$.

3. Bipolar Strings and Bipolar Atom

3.1. YinYang Bipolar Strings

Fundamentally different from the mainstream string theory or "theory of everything", BDL and BQLA provide the logical and physical bipolar bindings for the "strings" of reality but retain the open-world non-linear dynamic property of nature tailored for open-ended exploratory scientific discovery. While strings are far from observable reality, the non-linear dynamic property of BDL and BQLA do not compromise the law of excluded middle—a unique basis for a scalable and observable alternative bipolar string theory.

Since $(-1,0) \otimes (-1,0) = (-1,0)^2 = (0,1)$ and $(-1,1) \otimes (-1,1) = (-1,1)^2 = (-1,1)$, $(-1,0)^n$ defines an oscillatory non-equilibrium and $(-1,1)^n$ defines a non-linear dynamic equilibrium. Such properties provide a unifying logical representation for particle-wave duality. For instances, $\phi(P)(f) = (-1,0)^n (3 \times 10^{12})$ can denote that "particle P changes polarity three trillion times per second"; $\phi(P)(f) = (-1,1)^n (3 \times 10^{12})$ can denote that "The two poles of P interact three trillion times per second."

As strings can be one-dimensional oscillating lines or points, a bipolar string can be defined as an elementary bipolar variable or quantum agent $e = (-e,+e)$ and characterized as $\phi(e)(f)(m)$ where $\phi(e) \in B_1$ or B_∞, f is the frequency of bipolar interaction or oscillation, and m is mass. If e is massless we have m = 0. The two poles of e as negative and positive strings are non-exclusive, reciprocal, entangled, and inseparable. Thus, bipolar strings cannot be dichotomous and bipolar string theory is a non-linear dynamic unification of singularity, bipolarity, and particle-wave duality.

3.2. YinYang Bipolar Atom

Figure 4 shows a YinYang-n-element bipolar quantum cellular automaton (BQCA), where each link and each element is characterized with a bipolar value (n,p). A negative side n can indicate output of an element or repression of a link weight; a positive side p can indicate input of an element or activation of a link weight. A set of dynamic equations have been derived based BQLA for characterizing the cellular structure in **Figure 4**. The set

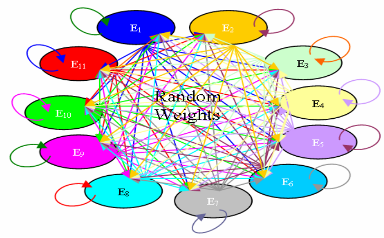


Figure 4. A YinYang-n-element cellular structure.

of equations can be simplified as $\mathbf{Y}(t+1) = Y(t) \times M(t)$, where $Y(t)$ is a bipolar vector at time t and $M(t)$ a connection matrix at time t. Now, our questions are:

1) How to use a YinYang-n-element cellular structure to describe and unify matter and antimatter atoms?

2) How to use a YinYang-n-element cellular structure to unify particle and wave?

3) How to use a YinYang-n-element cellular structure to describe and unify quantum theory and relativity?

4) How to integrate multiple YinYang-n-element cellular structures together?

5) How to use BDL, BQLA and BQCA to unify big bang and black hole as well as space and time?

Dramatically, BQLA and BQCA can be used for representing both matter and antimatter atoms as well as particles and waves. **Figure 5(a)** shows the bipolar representation of a hydrogen atom. **Figure 5(b)** is a redrawn of **Figure 4** by omitting connectivity. The positrons can be regrouped to the nucleus of a matter atom as shown in **Figure 5(c)**, where the negative signs can character electrons or electron cloud. Similarly, an antimatter atom is shown in **Figure 5(d)**. Thus, both matter and antimatter atoms can be characterized using Equation (15) in BQLA.

It is evident from **Figure 5** that YinYang bipolar atom has the potential to bridge a gap between black hole and big bang in a cyclic process model because it allows particles and antiparticles emitted from a black hole [2,3] to form matter and antimatter again. While Laws 1 - 5 provide the axiomatic conditions for energy equilibrium, growing, and degenerating, we introduce a new law of oscillation [1] in the following:

Law 6. Law of Oscillation. Let $t = 0, 1, 2, \cdots$, $Y(t+1) = Y(t) \times M(t)$, $|\varepsilon|Y(t)$ be the total energy of an YinYang-n-element vector $Y(t)$, $|\varepsilon|M(t)$ be the total energy of the connectivity matrix $M(t)$, if, $\forall i, j$, $|\varepsilon|(M_{i*})(t_k) \equiv |\varepsilon|(M_{*j})(t_k) > 1.0$ and $|\varepsilon|(M_{i*})(t_{k+1}) \equiv |\varepsilon|(M_{*j})(t_{k+1}) < 1.0$, the system's total energy will be alternatively increasing at time k and decreasing at time $k + 1$.

Evidently, any particle or wave form can be represented with Yin energy, Yang energy, or unified YinYang form. But without YinYang, the bipolar coexistence and interaction of the two poles can't be visualized. The four cases of equilibrium, growing, degeneration and oscillation are simulated in **Figures 6-9**.

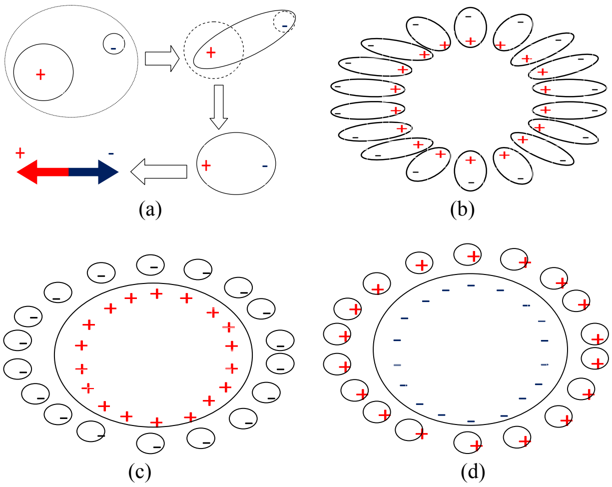


Figure 5. (a) Bipolar representation of a hydrogen; (b) Bipolar representation of YinYang-n-elements; (c) Matter atom; (d) Antimatter atom.

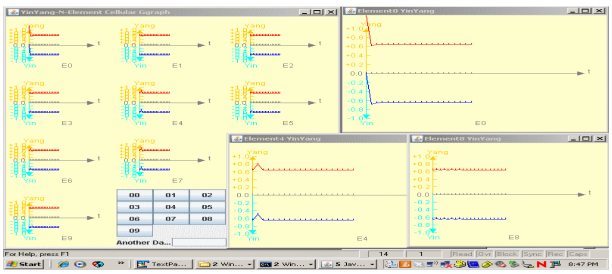


Figure 6. Bipolar energy rebalancing wave forms after a disturbance to one element [8].

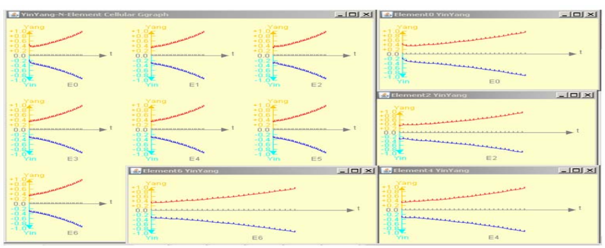


Figure 7. YinYang bipolar energy growing [8].

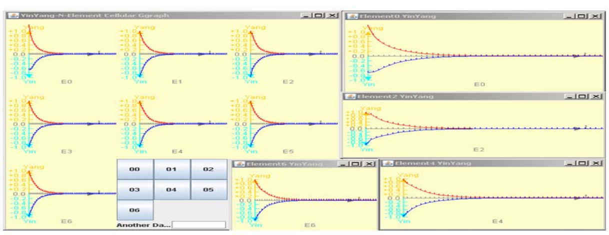


Figure 8. YinYang bipolar energy decreasing [8].

4. Bipolar Quantum Cellular Automata

YinYang bipolar atom leads to bipolar quantum cellular automata (BQCA) for advancing research in cosmological and molecular interactions. YinYang as the basis of

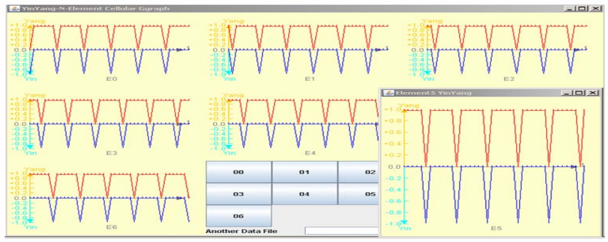


Figure 9. YinYang bipolar energy oscillation [8].

traditional Chinese medicine (TCM) has been in the dilemma of lacking a formal logical, mathematical, physical, and biological foundation. On the other hand, despite one insightful surprise after another the genome has yielded to biologists, the primary goal of the Human Genome Project—to ferret out the genetic roots of common diseases like cancer and Alzheimer's and then generate treatments—has been largely elusive. Although quantum mechanics provides a basis for chemistry and molecular biology, it so far has not found unification with Einstein's relativity theory. This situation provides an opportunity for YinYang to enter modern science and play a unifying role. For instance, given the cellular structures in **Figure 10**, we have the question: "How to model the integration, interaction, and equilibrium conditions?"

Law 7 (Following Law 3). Law of Integrated Energy Symmetry. Given **Figure 10**, let $t = 0, 1, 2, \cdots$, $Y(t+1) = Y(t) \times M(t)$, $|\varepsilon|Y(t)$ be the total energy of the integrated BQCA vector $Y(t)$, $|\varepsilon|M(t)$ be the total energy of the integrated connectivity matrix $M(t)$, $|\varepsilon|M_{i*}(t)$ be the energy subtotal of row i of $M(t)$, $|\varepsilon|M_{*j}(t)$ be the energy subtotal of column j of $M(t)$, the integrated BQCA can satisfy the following two global conditions:

1) Regardless of the local YinYang balance/imbalance of the subsystems at any time point t, the integrated system will remain a global energy equilibrium if, $\forall t$, $d(|\varepsilon|Y(t))/dt \equiv 0$, or

(a) $\forall i,j$, $[|\varepsilon|(M_{i*}) \equiv |\varepsilon|(M_{*j}) \equiv 1.0]$;

(b) no external disturbance or input/output to/from the system after the initial vector $Y(0)$ is given;

(c) no internal disturbance or energy creation and consumption in the system after the initial vector $Y(0)$ is given. That is, all the k component BQCA satisfy the condition, $\forall t$, $d(|\varepsilon|Y_k(t))/dt \equiv 0$, or, equivalently, $\forall i$, j, $[|\varepsilon_k|(M_{i*}) \equiv |\varepsilon_k|(M_{*j}) \equiv 1.0]$. Otherwise, there will be internal disturbance.

2) Under the conditions of (1), if, $\forall t$, $|\varepsilon^-(M_{*j})| > 0$ and $|\varepsilon^+(M_{*j}))| > 0$, all components connected by M will eventually reach a local YinYang balance $(-|\varepsilon|Y(t)/(2K), |\varepsilon|Y(t)/(2K))$ at certain time point t.

Law 8 (Following Law 4). Law of Integrated Energy Broken Symmetry (Growing). For the same integrated BQCA as for Law 7, if, (a) $\forall i, j$, $|\varepsilon|(M_{i*}) \equiv |\varepsilon|(M_{*j}) > 1.0$; (b) no external disturbance after the initial vector

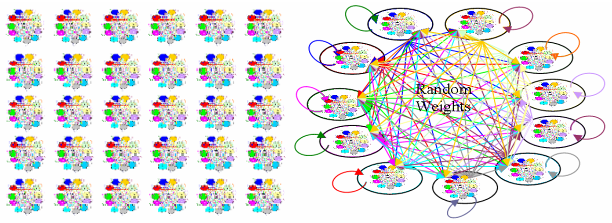


Figure 10. Integration of bipolar cellular subsystems.

Y(0) is given; (c) no internal disturbance or energy creation and consummation after the initial vector Y(0), regardless of the local YinYang balance or imbalance of its local component BQCAs at any time t, the system energy will increase and eventually reach a bipolar infinite $(-\infty,\infty)$ or $\forall t$, $d(|\varepsilon|Y(t))/dt > 0$.

Law 9 (Following Law 5). Law of Integrated Energy Broken Symmetry (Weakening). For the same system as for Law 7, if, (a) $\forall i,j$, $|\varepsilon|(M_{i*}) \equiv |\varepsilon|(M_{*j}) < 1.0$; (b) no external disturbance to the system after the initial vector Y(0) is given; (c) no internal disturbance or energy creation and energy consumption after the initial vector Y(0) is given, regardless of the local YinYang balance/imbalance of its local component BQCAs at any time t, the system energy will decrease and eventually reach an eternal equilibrium (–0,+0) state or, equivalently, $\forall t$, $d(|\varepsilon|Y(t))/dt < 0$, until $|\varepsilon|Y(t) = 0$.

Law 10 (Following Laws 3-9). Necessary and Sufficient Conditions for Collective Bipolar Adaptivity. The two conditions of Law 3 are necessary for collective bipolar adaptivity of any simple or integrated BQCA into equilibrium and symmetry; the two conditions are sufficient for collective bipolar adaptivity of any simple BQCA but not for integrated BQCAs; the two conditions in Law 7 are both necessary and sufficient for collective bipolar adaptivity of any simple BQCA or integrated BQCA into equilibrium and symmetry.

5. An Eastern Road to Quantum Gravity

5.1. Q5 Paradigm

Since acceleration is equivalent to gravitation under general relativity, any physical, socioeconomic, mental, and biological acceleration, growth, degeneration or aging are qualified to be a kind of quantum gravity. It can be further argued that as a most fundamental scientific unification not only can quantum gravity be applied in physical science, but also in computing science, social science, brain science, and life sciences as well. This argument leads to five sub-theories of a *Q5 paradigm* of quantum gravities: *physical quantum gravity*, *logical quantum gravity*, *mental quantum gravity*, *biological quantum gravity*, *and social quantum gravity* [8]. In the Q5 paradigm, the theory of physical quantum gravity is concerned with quantum physics; logical quantum gravity is focused on quantum computing; mental quantum gravity is focused on the interplay of quantum mechanics and brain dynamics; biological quantum gravity is focused on life sciences; social quantum gravity spans social sciences.

The Q5 paradigm may sound like a mission impossible. It actually follows a single undisputable observation and a single condition: 1) bipolar equilibrium or non-equilibrium is a generic form of any multidimensional equilibrium from which nothing can escape; 2) bipolar quantum entanglement is logically definable with BUMP that unifies truth, being and dynamic equilibrium with logically definable causality.

Roger Penrose described two mysteries of quantum entanglement [14, p. 591]. The first mystery is the phenomenon itself; the second one is: "*Why do these ubiquitous effects of entanglement not confront us at every turn*?" Penrose remarked: "*I do not believe that this second mystery has received nearly the attention that it deserves*." It is contended that YinYang bipolar quantum entanglement provides a resolution to the first mystery and the Q5 paradigm provides a resolution to the second.

Since the Yin and the Yang are two reciprocal opposite poles or energies that are completely background independent, YinYang bipolar geometry is fundamentally different from Euclidian, Hilbert, and spacetime geometries. With the background independent property, the new geometry makes quadrants irrelevant because bipolar identity, interaction, fusion, separation, and equilibrium can be accounted for in it even without quadrants (**Figure 11**).

Defined in YinYang bipolar geometry, BDL and BUMP make quantum causality logically definable as equilibrium-based quantum entanglement. It simply states: For all bipolar equilibrium functions ϕ, φ, ψ, and χ, IF $(\phi\Rightarrow\varphi)$ & $(\psi\Rightarrow\chi)$, THEN the bipolar interaction $(\phi*\psi)$ implies that of $(\varphi*\chi)$. With the emergence of space and time, BUMP leads to a completely background independent theory of YinYang bipolar relativity defined by Equation (16) [8]. $\forall$a,b,c,d,

$$
\begin{aligned}
&\Big[\psi\big(a(t_x,p_1)\big)\Rightarrow\chi\big(c(t_y,p_3)\big)\Big]\,\&\\
&\qquad\qquad\Big[\phi\big(b(t_x,p_2)\big)\Rightarrow\varphi\big(d(t_y,p_4)\big)\Big]\\
&\Rightarrow\Big[\psi\big(a(t_x,p_1)\big)*\phi\big(b(t_x,p_2)\big)\\
&\qquad\qquad\Rightarrow\chi\big(c(t_y,p_3)\big)*\varphi\big(d(t_y,p_4)\big)\Big]
\end{aligned}
\tag{16}
$$

In Equation (16), $a(t_1,p_1)$, $b(t_1,p_2)$, $c(t_2,p_3)$, $d(t_2,p_4)$ are any bipolar agents where $a(t,p)$ stands for "*agent a at time t and space p*" (t_x, t_y, p_x and p_y can be the same or different points in time and space). An agent without time and space is assumed at any time t and space p. An agent at time t and space p is therefore more specific.

The symmetrical property of YinYang bipolar geome-

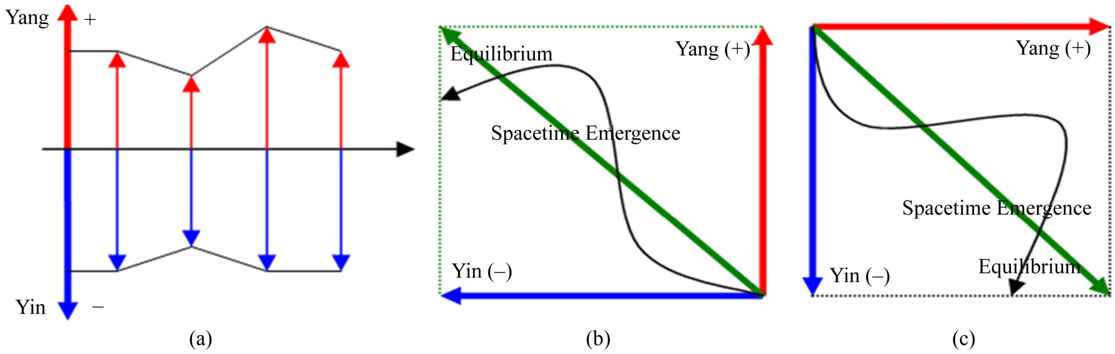


Figure 11. Background-Independent YinYang bipolar geometry: (a) Magnitudes of Yin and Yang; (b) Growing curve; (c) Quadrant irrelevant property.

try enables information to be passed through large or small scale quantum entanglement with or without passing observable energy or mass. When photon or electron is passed the speed is limited by the speed of light that has been proven in physics. Physicists have so far failed to experimentally verify the existence of graviton and the speed of gravity. If all action-reaction forces are fundamentally equilibrium-based and bipolar quantum entangled in nature, gravity would be logically unified with quantum mechanics in the form of Equation (16) [8].

For instance, based on general relativity, gravity "travels" at the speed of light and the effect of a disturbance to the Sun (S) could take 499 seconds to reach the Earth (E). Let $f(S) = f(E) = (-f,f)(S) = (-f,f)(E)$ be the gravitational (*reaction*, *action*) forces between S and E; let time t be in second; let p_1 and p_2 be points for S and E, respectively; let (0,0) (S) be the hypothetical Sun's vanishment or eternal equilibrium; we have

$$\left[f\left(S\left(t,p_1\right)\right)\Rightarrow f\left(E\left(t+499,p_2\right)\right)\right]$$
$$\Rightarrow\left[f\left(S\left(t,p_1\right)\right)\blacklozenge(0,0)\Rightarrow f\left(E\left(t+499,p_2\right)\right)\blacklozenge(0,0)\right] \tag{17a}$$

If $f()$ is normalized to a bipolar predicate, ♦ can be replaced with $*$, and the binding of &, $\&^-$, $\otimes$, $\otimes^-$, $\varnothing$, or $\varnothing^-$ to $*$ in Equation (17a) would lead to the vanishment of the Sun and then the disappearing of the Earth from its orbit after 499 seconds. Thus, bipolar quantum entanglement and general relativity are logically unified under equilibrium-based YinYang bipolar relativity [8]. Here bipolar relativity can host space and time emergence following agents' arrivals.

Equation (17a) assumes that the speed of gravity equals the speed of light based on general relativity. This assumption is actually questionable. If we assume gravitation is a kind of large scale quantum entanglement of action and reaction forces, gravity could have a minimum lower bound of 10,000 times the speed of light [15] and would travel from the sun to the Earth in less than 0.0499 second and we would have Equation (17b).

$$\left[f\left(S\left(t,p_1\right)\right)\Rightarrow f\left(E\left(t+0.0499,p_2\right)\right)\right]$$
$$\Rightarrow\left[f\left(S\left(t,p_1\right)\right)\blacklozenge(0,0)\Rightarrow f\left(E\left(t+0.0499,p_2\right)\right)\blacklozenge(0,0)\right] \tag{17b}$$

A comparison of Equations (17b) with (17a) reveals an equilibrium-based logical "*bridge*" from relativity to quantum mechanics—a bridge toward quantum gravity. Why cannot other logical and statistical systems be used for the above unification? The answer is that without bipolarity a truth value in {0,1} or a probability $p \in [0,1]$ is incapable of carrying any shred of direct physical syntax or semantics such as equilibrium (–1,+1), non-equilibrium (–1,0) or (0,+1), quasi-equilibrium (–0.9, +0.9), eternal equilibrium (0,0) and, therefore, unable to represent non-linear bipolar dynamic interactions such as bipolar fusion, fission, oscillation, quantum entanglement, and annihilation.

Bipolar relativity can also support causal reasoning with time reversal because the premise of Equation (16) could be a future event and the consequent a past one. Although time travel in physics and cosmology is highly speculative in nature, time reversal analysis has been proven very useful in many other scientific, technological, and engineering research and development.

The equilibrium-based interpretation leads to a number unifying features for particle-wave, matter-antimatter, strings and atom as well as black hole and big bang. Evidently, Law 6 provides the basic condition for both waves and particles; YinYang bipolar atom provides the unification for matter and antimatter. Since **Figures 5(a)-(d)** are redrawing of a bipolar representation like **Figure 8** (different only in the number of elements), BQLA, BQCA, and Laws 1 - 6 all apply to the unipolar representations of **Figures 5(c)** and **(d)**. Thus, BQCA presents a unifying mathematical model for matter and antimatter atoms as well as particles and waves. In turn, it makes the unification of black hole and big bang possible because the theory allows particles and antiparticles emitted from a black hole [2,3] to form matter and antimatter again. Thus, it bridges a major gap in quantum cosmology and set the stage for another cycle of a cyclic process model of the universe. The unifying features are made possible by the complete background independent property of YinYang bipolar geometry (**Figure 11**).

YinYang bipolar elements and sets [8] provide an alternative interpretation for strings as well. Different from mainstream string theory, bipolar strings are scalable and can be the makings of bipolar atoms (**Figure 5**). Thus, the alternative interpretation brings strings into the real world of matter and antimatter for the first time.

Since action and reaction or negative and positive energies can be electromagnetic or gravitational in nature, YinYang bipolar atom can serve as a basis for real world quantum gravity. If we treat the centrifugal and centripetal forces of a planet similarly as that of an electron (or positron) rotating around its nucleus, gravity can be a superposition on quantum interaction. In either case, since nothing can escape bipolar equilibrium or non-equilibrium, renormalization is made possible in equilibrium-based terms using BQLA and BQCA.

The YinYang negative-positive energies also provide a possible unification for the many universes in M-theory. It can be argued that the multiverses have to follow the same equilibrium or non-equilibrium conditions of the 2nd law of thermodynamics and become one universe. Otherwise, the two energies can't form the regulating force of the multiverses. Thus, the different laws followed by different universes as described in *The Grand Design* have to be unified under the same 2nd law of thermodynamics.

Different from other approaches to quantum gravity, the equilibrium-based approach is rooted in the real world. Due to YinYang bipolarity in mental health, bioinformatics, life and social sciences [6-13,17-23], physical and logical quantum gravity can be naturally extended to mental, biological and social quantum gravities [8]. Thus, it is contended that the new approach has opened an Eastern road toward quantum gravity.

5.2. Falsifiability

Falsifiability is a must for any viable physical theory. It is of course correct that bipolar quantum entanglement needs experimental verification. However, 1) bipolar atom finds its equivalent representation in classical atom theory (**Figure 5**); 2) bipolar quantum entanglement or BUMP is physical and logical; 3) unlike the predicted but unverified existence of monopoles in string theory, dipoles are everywhere. Thus, we have:

Postulate 1: *Bipolar quantum entanglement is the most fundamental entanglement in quantum gravity.*

Postulate 2: *YinYang bipolarity is the most fundamental property of the universe.*

The two postulates are actually logically provable axioms. For Postulate 1, if a bipolar element (**Figures 4** and **5**) characterizes the energy superposition of gravitational and quantum action-reaction, an atom would be a set of bipolar elements. As the total must be equal to the sum, without bipolar entanglement there would be no atom level entanglement. Postulate 2 follows Postulate 1.

Postulate 3: *YinYang bipolar atom is a bipolar set of quantum entangled particle and antiparticle pairs*.

Postulate 4: *Gravity is fundamentally large or small scale bipolar quantum entanglement*.

Postulate 5: *The speed of gravity is limited by the speed of quantum entanglement and not by that of light*.

According to Einstein, "*Evolution is proceeding in the direction of increasing simplicity of the logical basis* (*principles*)." "*We must always be ready to change these notions—that is to say*, *the axiomatic basis of physics—in order to do justice to perceived facts in the most perfect way logically*." While string and superstring theories up to 11 or more dimensions failed the simplicity measure, YinYang bipolar atom and bipolar quantum entanglement are simple and logically comprehendible with definable causality in BUMP. The bipolar quantum interpretation coincides with MIT Professor Seth Lloyd's startling thesis that the universe is itself a quantum computer [24]. According to Lloyd, the universe is all about quantum information processing. Once we understand the laws of physics completely, we will be able to use small-scale quantum computing to understand the universe completely as well. Could YinYang bipolar quantum entanglement or BUMP be such a basic law?

6. Conclusions

Based on YinYang bipolar dynamic logic and bipolar quantum linear algebra, a logically definable causal theory of YinYang bipolar atom has been introduced. The causal theory has led to an equilibrium-based super symmetrical quantum cosmology of negative-positive energies. It is contended that the new theory has opened an Eastern road toward quantum gravity with bipolar logical unifications of matter-antimatter, particle-wave, strings and reality, big bang and black hole, quantum entanglement and relativity. It has been shown that not only can the theory be applied in physical worlds but also provides a Q5 paradigm of physical, logical, mental, biological and social quantum gravities. Furthermore, it provides a logically consistent cyclic process model of the universe with information recovery after a black hole.

The strength of the equilibrium-based approach is its interpretation and unification aspects. The strength comes from the background-independent property of YinYang bipolar geometry that transcends spacetime. The strength would also be a weakness should YinYang be exclusive of spacetime geometry. Fortunately, the new geometry is not exclusive but inclusive. It promotes equilibrium, harmony and complementarity by hosting, regulating or integrating background-dependent models as emerging parameters for more challenging scientific explorations and unifications.

This work is limited to qualitative simulation, interpretation and unification. A major research topic is bipolar quantization and space emergence. The negativepositive energies of an electron-positron pair under certain condition provides a candidate bipolar unit for quantization with space emergence as a result of particle-antiparticle interaction.

Finally, the equilibrium-based approach to quantum gravity is fundamentally different from other approaches in philosophical basis. Since all beings must exist in certain equilibrium or non-equilibrium, a scientific reincarnation of philosophy is predicted [25].

REFERENCES

[1] W. R. Zhang, "YinYang Bipolar Atom and Quantum Cellular Automation," *Proceedings of* 2011 *IEEE International Conference on Bioinformatics and Bioengineering*, Atlanta, 12-15 November 2011, pp. 791-797.

[2] S. Hawking, "Black Hole Evaporation," *Nature*, Vol. 248, No. 5443, 1974, pp. 30-31. doi:10.1038/248030a0

[3] S. Hawking, "Particle Creation by Black Holes," *Communications in Mathematical Physics*, Vol. 43, No. 3, 1975, pp. 199-220. doi:10.1007/BF02345020

[4] S. Hawking and L. Mlodinow, "The Grand Design," Random House Digital, Inc., New York, 2010.

[5] N. Bohr, "On the Notions of Causality and Complementarity," *Dialectica*, Vol. 2, No. 3-4, 1948, pp. 312-319. doi:10.1111/j.1746-8361.1948.tb00703.x

[6] J. Gore and A. van Oudenaarden, "Synthetic Biology: The Yin and Yang of Nature," *Nature*, Vol. 457, No. 7227, 2009, pp. 271-272. doi:10.1038/457271a

[7] W. R. Zhang, H. J. Zhang, Y. Shi and S. S. Chen, "Bipolar Linear Algebra and YinYang-N-Element Cellular Networks for Equilibrium-Based Biosystem Simulation and Regulation," *Journal of Biological Systems*, Vol. 17, No. 4, 2009, pp. 547-576.

[8] W. R. Zhang, "YinYang Bipolar Relativity: A Unifying Theory of Nature, Agents and Causality with Applications in Quantum Computing, Cognitive Informatics and Life Sciences," IGI Global, Hershey, 2011. doi:10.4018/978-1-60960-525-4

[9] W. R. Zhang and L. Zhang, "YinYang Bipolar Logic and Bipolar Fuzzy Logic," *Information Sciences*, Vol. 165, No. 3-4, 2004, pp. 265-287. doi:10.1016/j.ins.2003.05.010

[10] W. R. Zhang, "YinYang Bipolar Lattices and L-Sets for Bipolar Knowledge Fusion, Visualization, and Decision," *International Journal of Information Technology and Decision Making*, Vol. 4, No. 4, 2005, pp. 621-645.

[11] W. R. Zhang, "Equilibrium Energy and Stability Measures for Bipolar Decision and Global Regulation," *International Journal of Fuzzy Systems*, Vol. 5, No. 2, 2003, pp. 114-122.

[12] W. R. Zhang, "YinYang Bipolar Fuzzy Sets and Fuzzy Equilibrium Relations for Bipolar Clustering, Optimization, and Global Regulation," *International Journal of Information Technology and Decision Making*, Vol. 5, No. 1, 2006, pp. 19-46.

[13] W. R. Zhang, A. Pandurangi and K. Peace, "YinYang Dynamic Neurobiological Modeling and Diagnostic Analysis of Major Depressive and Bipolar Disorders," *IEEE Transactions on Biomedical Engineering*, Vol. 54, No. 10, 2007, pp. 1729-1739. doi:10.1109/TBME.2007.894832

[14] R. Penrose, "The Road to Reality: A Complete Guide to the Laws of the Universe," Alfred A. Knopf, New York, 2005.

[15] D. Salart, A. Baas, C. Branciard, N. Gisin and H. Zbinden, "Testing the Speed of 'Spooky Action at a Distance'," *Nature*, Vol. 454, No. 7206, 2008, pp. 861-864. doi:10.1038/nature07121

[16] J. Gore and A. van Oudenaarden, "Synthetic Biology: The Yin and Yang of Nature," *Nature*, Vol. 457, No. 7227, 2009, pp. 271-272. doi:10.1038/457271a

[17] Y. Shi, E. Seto, L. S. Chang and T. Shenk, "Transcriptional Repression by YY1, a Human GLI-Kruppel Related Protein, and Relief of Repression by Adenovirus E1A Protein," *Cell*, Vol. 67, No. 2, 1981, pp. 377-388. doi:10.1016/0092-8674(91)90189-6

[18] S. Vasudevan, Y. Tong and J. A. Steitz, "Switching from Repression to Activation: Micro RNAs Can Up-Regulate Translation," *Science*, Vol. 318, No. 5858, 2007, pp. 1931-1934. doi:10.1126/science.1149460

[19] W. R. Zhang, S. Chen and J. C. Bezdek, "POOL2: A Generic System for Cognitive Map Development and Decision Analysis," *IEEE Transactions on Systems*, *Man*, *and Cybernetics*, Vol. 19, No. 1, 1989, pp. 31-39.

[20] W. R. Zhang, S. Chen, W. Wang and R. King, "A Cognitive Map Based Approach to the Coordination of Distributed Cooperative Agents," *IEEE Transactions on Systems*, *Man*, *and Cybernetics*, Vol. 22, No. 1, 1992, pp. 103-114.

[21] W. R. Zhang and S. S. Chen, "Equilibrium and Non-Equilibrium Modeling of YinYang WuXing for Diagnostic Decision Analysis in Traditional Chinese Medicine," *International Journal of Information Technology and Decision Making*, Vol. 8, No. 3, 2009, pp. 529-548.

[22] W. R. Zhang, "NPN Fuzzy Sets and NPN Qualitative Algebra: A Computational Framework for Bipolar Cognitive Modeling and Multiagent Decision Analysis," *IEEE Transactions on Systems*, *Man*, *and Cybernetics*, Vol. 26, No. 4, 1996, pp. 561-574.

[23] W. R. Zhang, K. A. Pandurangi, K. E. Peace, Y. Zhang and Z. Zhao, "Mental Squares—A Generic Bipolar Support Vector Machine for Psychiatric Disorder Classification, Diagnostic Analysis and Neurobiological Data Mining," *International Journal on Data Mining and Bioinformatics*, Vol. 17, No. 4, 2011, pp. 547-576.

[24] S. Lloyd, "Programming the Universe," Alfred A. Knopf, Inc., New York, 2006.

[25] W. R. Zhang, "Beyond Spacetime Geometry—The Death of Philosophy and Its Quantum Reincarnation," *Journal of Modern Physics*, Vol. 3, No. 29A, 2012, pp. 1272-1284. doi:10.4236/jmp.2012.329164

Journal of Modern Physics, **2012, 3, 1272-1284**
doi:10.4236/jmp.2012.329164 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Beyond Spacetime Geometry—The Death of Philosophy and Its Quantum Reincarnation*

Wen-Ran Zhang
Department of Computer Science, Georgia Southern University, Statesboro, USA
Email: wrzhang@georgiasouthern.edu

Received July 5, 2012; revised August 8, 2012; accepted August 16, 2012

ABSTRACT

Contrary to the "end" and "death" assertions on philosophy, this paper predicts an equilibrium-based and harmony-centered scientific reincarnation of philosophy. Logically, the reincarnation is backed by a formal system and a background independent geometry that transcends spacetime. Physically, it is supported by definable quantum causality and bipolar logical unifications of matter and antimatter, particle and wave, big bang and black hole, relativity and quantum entanglement. Philosophically, it is distinguished from Western metaphysics and dialectics as well as the Dao of Laozi. It is named a quantum reincarnation for its central claim that YinYang bipolar quantum entanglement is the source of causality for the Being of beings following the 2nd law of thermodynamics. Thus, it presents a modest unification of science and philosophy for their reciprocal interaction (Note: Equilibrium subsumes non-equilibrium and quasi—equilibrium as local non-equilibriums can form global equilibrium or quasi-equilibrium).

Keywords: End and Death of Philosophy; Bipolar Quantum Entanglement; YinYang Bipolar Geometry; Formal YinYang Cosmology; Nature of Time; Quantum Reincarnation of Philosophy

1. Introduction

The Dao in *Yi Jing* claims that everything has two opposites and all changes in nature are caused by the reciprocal interactions of the two sides. The two sides are the Yin and the Yang of nature. Thus, Dao subsumes an equilibrium-based and harmony-centered super symmetry of negative-positive energies. The words "historically long standing" and "broad and profound" are often used to describe the Chinese philosophical thinking.

Due to its lack of a formal logic, however, the Dao has never reached the status of science philosophy. This left room for a variety of scientific and unscientific interpretations. When French philosopher Jacques Derrida visited Shanghai in 2001 he reiterated Hegel's assertion that China has no philosophy but thoughts. In his 2004 speech at Beijing Nobel Laureate Chen Ning Yang attributed China's failure in becoming the cradle of modern science to *Yi Jing*. While some Chinese hailed Yang's speech, many bloggers were filled with righteous indignation by his comment and spoke out against it with excitement. But few pointed out the possibility of inventing a unique formal YinYang logic and geometry for scientific unification.

On the other hand, modern physics is in a different dilemma. The searches for ether and monad have found no result; the modern quest for monopoles and strings has turned out no concrete findings. As a basis of string theory, monopoles and strings are too far away from reality. For instance, it is not clear how monopoles and strings can form an atom with equilibrium or non-equilibrium. Notably, string theory is criticized as "The Trouble with Physics" [2] and "Not Even Wrong" [3] by insiders. (Remark: When the discovery of Higgs boson is being hailed, the century old quest for quantum gravity still finds no definitive battleground).

Whenever physics is in trouble, it needs new philosophical thinking. But philosophy or metaphysics is also in trouble. Despite the continuing debate on various theories regarding being, time, and truth, philosophy is being-centered and truth-based. Now, philosophy is faced with extinction. About two centuries ago, Hegel pronounced its end. He claimed that his truth-based and contradiction-centered dialectic logic had brought philosophy to its end and there should be no new philosophy after him. Anglo-American philosophers on the whole, however, found it hard to put up with contradiction while seeking truth. Indeed, contradiction is not a scientific concept and Hegel's *The Science of Logic* is not the logic of science as Einstein asserted later: "*For the time being we have to admit that we do not possess any general theoretical basis for*

*This work is based on a working book (Ref. [1]). By this note, there is no need for further copyright clearance for the book's publication. For a more thorough and complete coverage of the subject readers are referred to the book.

physics which can be regarded as its logical foundation."

While the end of philosophy was meant by Hegel to be the "top" or "apex", some scholars went one step further to proclaim the death of philosophy. In *The Grand Design*, Hawking and Mlodinow declared [4, p. 5]: "*Philosophy is dead*"; "*M-theory predicts that a great many universes were created out of nothing*"; "*Their creation does not require the intervention of some supernatural being or god.*" When they advocated M-theory, however, they also promoted the concept of negative and positive energies [4, pp. 179-180] but stopped short of pointing out the unavoidable consequence that the two energies are respectively the Yin and Yang of nature. And when they proclaimed the death of philosophy, they are calling back a different philosophy.

Hawking is renowned for his black hole theory. The theory originally suggested the universe's disappearance without information preservation. It was criticized for violating the 2nd law of thermodynamics. To remedy the inconsistency, Hawking proposed black body evaporation [5] and then particle emission [6]. After then, despite continuing criticism, he held up his theory for three decades. In 2004, he finally conceded. But so far it is logically unclear how the universe's information can be recovered from emitted particles after a black hole.

It seems that Hawking has introduced a similar paradox by advocating M-theory and negative-positive energies at the same time. If particles and antiparticles can survive a black hole and the two energies form the regulating force of the multiverses, YinYang bipolarity has to be the most fundamental property of the universe from which the information of the universe can be recovered and the multiverses have to be unified in a single equilibrium-based universe. Otherwise, they would be completely isolated and the negative-positive energies can't form regulating forces of them.

The quantum reincarnation of philosophy discussed in this work claims:

1) Being and truth are not the most fundamental properties of the universe; the most fundamental property of the universe should be YinYang bipolarity.

2) *The Science of Logic* is not the logic of science. To have the logic of science, contradiction has to be replaced with bipolar dynamic equilibrium.

This paper is organized in seven sections. Section 2 provides a background review. Section 3 presents a formal theory of YinYang bipolar cosmology. Section 4 predicts a reincarnation of philosophy. Section 5 argues that science cannot replace philosophy. Section 6 discusses the relation between equilibrium, harmony and Einstein's God. Section 7 draws a few conclusions.

2. Background

2.1. Philosophical Divide

Karl Popper is well-known for his positivist stance in science philosophy and his sharp criticism on dialectics. He stated: "*The whole development of dialectic should be a warning against the dangers inherent in philosophical system-building. It should remind us that philosophy should not be made a basis for any sort of scientific system and that philosophers should be much more modest in their claims. One task which they can fulfill quite usefully is the study of the critical methods of science.*"

Popper was right to criticize dialectics and to warn the world "*that philosophers should be much more modest in their claims*". However, it seems that he stopped short of pinpointing the crux of the problem in Hegel's dialectics. His stance against holism seems to be out of date due to the new phenomena of global warming, global economy and quantum entanglement. His firm support for Einstein against Bohr on quantum theory seems to be lopsided. He overlooked the importance of the mutually beneficial interactions between science and philosophy as well as the possibility of a scientific reincarnation of a modest philosophy just as he overlooked quantum entanglement. His warning "*that philosophy should not be made a basis for any sort of scientific system*" seems to be questionable. Otherwise, the equilibrium condition of the 2nd law of thermodynamics could be violated, YinYang should be long gone, and there should no science philosophy.

It is contended that the equilibrium-based philosophy discussed in this work can be regarded a scientific reincarnation. Evidently, anyone (good or bad) can claim having truth in his or her hand to start a contradiction, a conflict or even a world war by free will (as done by Hitler) but no one except God, if God exists, can claim the possession of global dynamic equilibrium that may well be the ultimate power for the creation, regulation and evolution of being and truth. Subsequently, the key for the reincarnation of philosophy is whether we can have an equilibrium-based quantum logic that is both scientific and philosophical and can reveal the ubiquitous effect of quantum entanglement with simple logically definable causality but contradiction-free.

2.2. YinYang Bipolar Geometry and Bipolar Dynamic Logic (BDL)

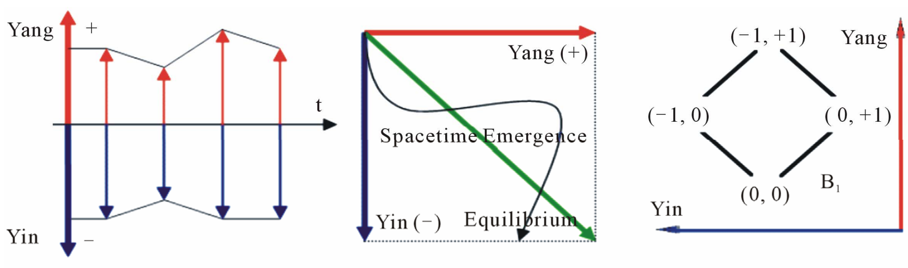
YinYang bipolar dynamic logic (BDL) [7-12] shows a number of distinguishing properties. First, it is a formal logic ever defined on a bipolar quantum lattice $B_1 = \{-1,0\} \times \{0,+1\}$ in a completely background independent YinYang geometry of negative and positive energies, where quadrant is made irrelevant and, therefore, it transcends being, truth and spacetime. This transcendence makes spacetime emergence possible—a desirable feature in quantum gravity (**Figure 1**). In B_1, $(0,0)$, $(0,1)$, $(-1,0)$, and $(-1,+1)$ stand, respectively, for eternal equilibrium, non-equilibrium, non-equilibrium, equilibrium or harmony.

BDL exhibits the properties for a scientific reincarnation of philosophy. First, its non-linear dynamic property doesn't compromise the law of excluded middle (LEM) (**Figure 2**) which makes BDL contradiction-free and leads to a sound axiomatization (**Figure 3**) [12]. Bipolar universal modus ponens (BUMP) presents an equilibrium-based non-linear bipolar dynamic generalization of classical modus ponens (MP) and provides logically definable quantum causality for bipolar quantum entanglement. Another distinguishing factor of BDL is its bipolar symmetrical property. This property makes super symmetrical bipolar fusion, fission, oscillation, interaction and quantum entanglement possible as depicted in **Figure 4**.

BUMP simply states that, for all bipolar equilibrium functions ϕ, φ, ψ, and χ, IF $(\phi \Rightarrow \varphi) \& (\psi \Rightarrow \chi)$, then the bipolar interaction $(\phi * \psi)$ implies that of $(\varphi * \chi)$. With the emergence of space and time, BUMP leads to a theory of YinYang bipolar relativity [12] characterized by Equation (1).

$$\begin{aligned}&\left[\psi\left(a\left(t_x,p_1\right)\right) \Rightarrow \chi\left(c\left(t_y,p_3\right)\right)\right] \& \\ &\qquad \left[\phi\left(b\left(t_x,p_2\right)\right) \Rightarrow \varphi\left(d\left(t_y,p_4\right)\right)\right] \\ &\Rightarrow \left[\psi\left(a\left(t_x,p_1\right)\right) * \phi\left(b\left(t_x,p_2\right)\right)\right. \\ &\qquad \left.\Rightarrow \chi\left(c\left(t_y,p_3\right)\right) * \varphi\left(d\left(t_y,p_4\right)\right)\right].\end{aligned} \tag{1}$$

In Equation (1), $a(t_1,p_1)$, $b(t_1,p_2)$, $c(t_2,p_3)$, $d(t_2,p_4)$ are any bipolar agents or celestial entity where $a(t,p)$ stands for "*agent a at time t and space p*" (t_x, t_y, p_x and p_y can be the same or different points in time



(a) (b) (c)

Figure 1. YinYang bipolar geometry and Hasse diagram of $B_1 = \{-1,0\} \times \{0,1\}$.

Excluded middle	$(x,y) \oplus \neg(x,y) \equiv (-1,1)$;	$(x,y) \oplus^- \neg(x,y) \equiv (-1,1)$;
No contradiction	$\neg((x,y) \& \neg(x,y)) \equiv (-1,1)$;	$\neg((x,y) \&^- \neg(x,y)) \equiv (-1,1)$.

Figure 2. Bipolar laws.

Bipolar Linear Axioms:

BA1: $(\phi^-,\phi^+) \Rightarrow ((\varphi^-,\varphi^+) \Rightarrow (\phi^-,\phi^+))$;

BA2: $((\phi^-,\phi^+) \Rightarrow ((\varphi^-,\varphi^+) \Rightarrow (\chi^-,\chi^+))) \Rightarrow (((\phi^-,\phi^+) \Rightarrow (\varphi^-,\varphi^+)) \Rightarrow ((\phi^-,\phi^+) \Rightarrow (\chi^-,\chi^+)))$;

BA3: $(\neg(\phi^-,\phi^+) \Rightarrow (\varphi^-,\varphi^+)) \Rightarrow ((\neg(\phi^-,\phi^+) \Rightarrow (\varphi^-,\varphi^+)) \Rightarrow (\phi^-,\phi^+))$;

BA4: a) $(\phi^-,\phi^+) \& (\varphi^-,\varphi^+) \Rightarrow (\phi^-,\phi^+)$; b) $(\phi^-,\phi^+) \& (\varphi^-,\varphi^+) \Rightarrow (\varphi^-,\varphi^+)$;

BA5: $(\phi^-,\phi^+) \Rightarrow ((\varphi^-,\varphi^+) \Rightarrow ((\phi^-,\phi^+) \& (\varphi^-,\varphi^+)))$;

Bipolar Universal Modus Ponens (BUMP)

BR1: IF $((\phi^-,\phi^+) * (\psi^-,\psi^+))$, $[((\phi^-,\phi^+) \Rightarrow (\varphi^-,\varphi^+)) \& ((\psi^-,\psi^+) \Rightarrow (\chi^-,\chi^+))]$,

THEN $[(\varphi^-,\varphi^+) * (\chi^-,\chi^+)]$;

Bipolar Predicate axioms and Rules of inference

BA6: $\forall x, (\phi^-(x),\phi^+(x)) \Rightarrow (\phi^-(t),\phi^+(t))$;

BA7: $\forall x, ((\phi^-,\phi^+) \Rightarrow (\varphi^-,\varphi^+)) \Rightarrow ((\phi^-,\phi^+) \Rightarrow \forall x, (\varphi^-,\varphi^+))$;

BR2-Generalization: $(\phi^-,\phi^+) \Rightarrow \forall x, (\phi^-(x),\phi^+(x))$

Figure 3. Bipolar axiomatization.

or space, respectively). An agent without time or space is assumed at any time or space.

2.3. Bipolar Quantum Linear Algebra (BQLA) and Bipolar Atom

The bipolar lattices $B_1 = \{-1,0\} \times \{0,1\}$ and $B_F = [-1,0] \times [0,1]$ have been extended to the infinite bipolar lattice $B_\infty = [-\infty,0] \times [0,+\infty]$. With B_∞, BDL has been extended to a bipolar quantum linear algebra (BQLA) [11,12] for modeling the equilibrium, non-equilibrium and harmony properties of the negative-positive energies of Yin-Yang-n-element bipolar quantum cellular automata and bipolar atoms (**Figures 5** and **6**).

Interestingly, a bipolar quantum cellular automaton can be as small as an atom or as large as the universe or multiverse, where all elements have negative and positive energies connected by a bipolar link matrix [11-16]. **Figure 6** (adapted from [14]) provides a matter and antimatter unification with the same quantum cellular structure in **Figure 5**. Dynamic equations using BQLA and their wave forms have been presented in [11-16]. Since all types of action-reaction energies are fundamentally bipolar in nature. Not only can bipolar cellular automata be applied in biological world [15,16] but also be a logical and algebraic candidate for quantum gravity [11].

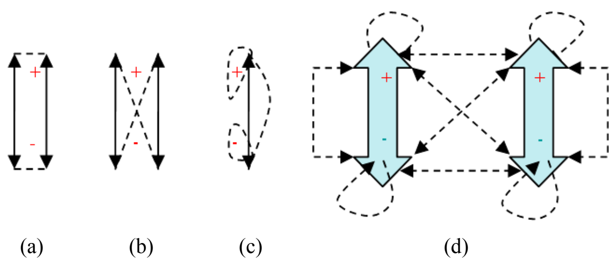


Figure 4. Bipolar relativity: (a) Linear interaction; (b) Cross-pole non-linear interaction; (c) Oscillation; (d) Two entangled bipolar interactive variables.

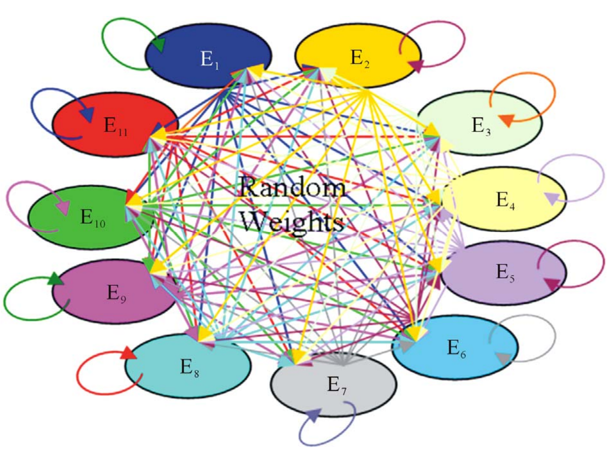


Figure 5. YinYang-n-element bipolar cellular automaton [16].

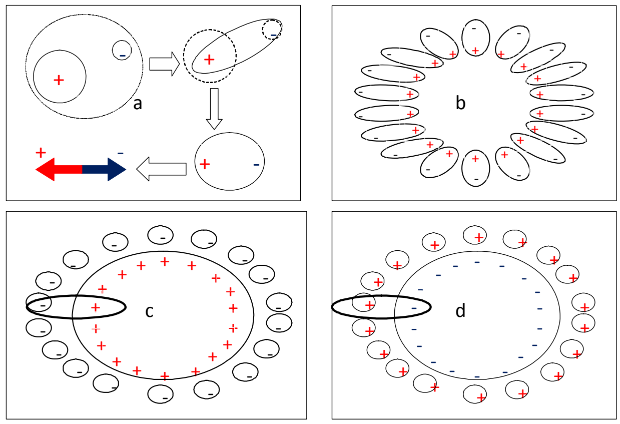


Figure 6. (a) Bipolar representation of hydrogen atom; (b) Yin-Yang-n-elements; (c) Matter atom; (d) Antimatter atom.

Moreover, it provides a basis for particles and antiparticles emitted from black holes to form matter or antimatter again to bridge a major gap in quantum cosmology.

2.4. Bipolar Equilibrium and Harmony

With BDL and BQLA, YinYang bipolar energy equilibrium and harmony of being can be logically and mathematically unified in YinYang bipolar geometry. The geometry has a Yin dimension, a Yang dimension and an equilibrium dimension (**Figure 1**). Harmony is defined as the reciprocal interaction of two direct opposites of one being in quasi-equilibrium with suitable oscillation amplitudes and frequencies [1,11].

3. Formal YinYang Bipolar Cosmology

3.1. A Process Model of Space and Time

The nature of space and time has long been a matter of debate in the history of philosophy. The subject focuses on a number of basic issues, including but not limited to whether or not time and space exist independently of the mind, whether they exist independently of one another, what accounts for time's apparently unidirectional flow, whether time other than the present moment exist, and what is the nature of space and time.

Most notably, Newtonian space provided the absolute frame of reference for the motion of objects; Einstein proposed the principle of relativity. The latter holds that light propagates at the same speed in all reference frames; no speed can exceed the speed of light; force felt by an observer in a given gravitational field and that felt by an observer in an accelerating frame of reference are indistinguishable. This led to the conclusion that the mass of an object warps the geometry of the spacetime surrounding it, as described in Einstein's field equations.

The physical theory of YinYang bipolar atom suggests a cyclic process model of the cosmos. Since the particles and antiparticles emitted from a black hole [5,6] can form matter and antimatter again (**Figure 6**), it sets the stage for another cycle of the cyclic process. Thus, Yin-Yang bipolar atom may lead to the unification of black hole and big bang.

While previous theories on space and time have not been supported by a quantum logic, the cyclic YinYang dynamic process model of space and time depicted in **Figures 7** and **8** is supported by BDL with simple logically definable quantum causality (Note: Einstein and others proposed different cyclic universe models). Based on YinYang bipolar causality, time is not like a unidirectional river but like a go-go train on its ring-shaped railways. One cycle of the ring is depicted in **Figure 7** and infinite cycles of the train are depicted in **Figure 8**. The train on one ring can cross to another by a random probability.

As indicated in **Figure 7**, each cycle of the railway has two major stations—big bang and the black. The big bang station can be marked as a Yang state or $(0,+1)$; The black hole station can be marked as a Yin state or $(-1,0)$. Traditionally, time is said to start from a big bang and stop at a black hole. But that seems to be just an illusion. Interestingly, equilibrium can result from the fusion of the Yin and Yang as $(-1,0)\oplus(0,+1)=(-1,+1)$. Here, eternal equilibrium or death state $(0,0)$ characterizes the illusive disappearance of the universe into a black hole. Moreover, the transition from Yin to Yang (or from black hole to big bang) can be characterized with the logical equation $(-1,0)\otimes(-1,0)=(-1,0)^2=(0,+1)$.

Remarkably, the four different logical values $(0,0)$ $(-1,0)$ $(0,+1)$ $(-1,+1)$ form the YinYang bipolar lattice (**Figure 1**)—a logical or mathematical structure. On the one hand, the lattice is defined in a completely background independent YinYang bipolar geometry. On the other hand, it is actually a mathematical version of Yin-Yang 4-images. Evidently, physics and philosophy are brought together by the logical structure.

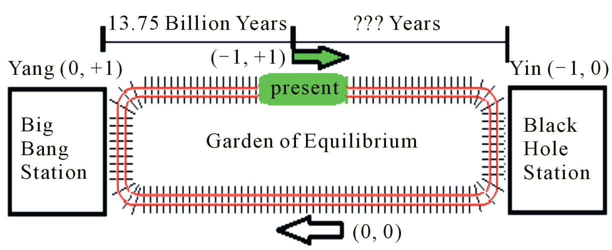


Figure 7. A cyclic process model of space and time [1].

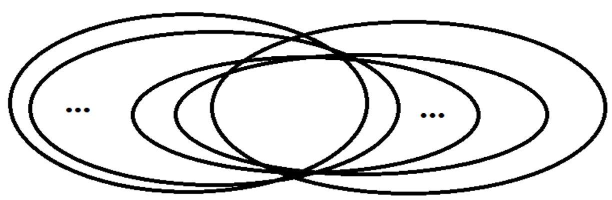


Figure 8. Infinite cycles of the time train [1].

The time train interpretation gives a logical account for time's cyclic flow instead of unidirectional flow. Just like Aristotle claimed the Earth was the static and stationary center of the universe, human beings couldn't sense the curvature of a huge ring-shaped time cycle.

Here we are focused on the equilibrium-based and harmony-centered philosophy. The philosophical guidance is evidently logical, systematic and rational. Since YinYang bipolar logic as a formal system has been proven a sound and complete non-linear bipolar dynamic generalization of truth-based thinking, truth, equilibrium, metaphysics and dialectics are corrected and unified into one philosophy. On the other hand, with logically definable quantum causality, relativity and quantum theory as well as philosophy and science are all brought together.

3.2. Eastern and Western Metaphysics

The Chinese Dao as Eastern metaphysics is defined as YinYang in *Yi Jing*. But Western metaphysics was later matched to the Dao; YinYang as the essence of the Dao was left out of the big picture by Western as well as Chinese modern philosophers. Two major reasons for this are: 1) YinYang lacked a formal logical basis for thousands of years; 2) being-centered and truth-based thinking has been proven most effective until mankind encountered quantum entanglement, global warming, global economic overheat and recession. With BDL, the Eastern and Western metaphysics can be distinguished and unified under dynamic equilibrium for dealing with unsolved scientific and social problems.

Equilibrium-based philosophical thinking is central in both science and philosophy. Without equilibrium-based thinking, the being-centered and truth-based Western tradition can't connect the metaphysical Being to the physical beings because truth as a static concept can't provide the ultimate Being a dynamic definition for revealing and regulating all beings. Consequently, after searching for more than two thousand years, Western philosophers failed to give a clear definition to Being. With the equilibrium-based thinking, all beings are revealed and regulated by the Dao of YinYang and Being as well as all beings including the universe itself finally finds its home in dynamic equilibrium (**Figure 9**).

3.3. Cosmological Predictions

YinYang bipolar philosophy has led to the theory of bipolar relativity which reveals the ubiquitous effects of quantum entanglement with a rich set of predictions [12].

Prediction 1. The bipolar axiomatization (**Figure 3**) is the most primitive (with minimal semantics) and most

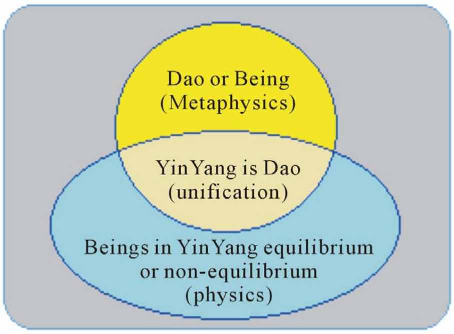


Figure 9. A scientific unification [1].

general (domain independent) equilibrium-based axiomatization of physics, life sciences and socioeconomics; any other less primitive axiomatization with added semantics (such as space, time, mass, and energy) must necessarily be less general (or more domain-specific).

Prediction 2. Let $\psi=\left(\psi^{-},\psi^{+}\right)$ = (repression, activation) be a bipolar predicate for the abilities of regulator genomic agents [17] such as YY1 [18]; let $\phi=\left(\phi^{-},\phi^{+}\right)$ = (repressability, activatability) be a predicate for the bipolar capacities of regulated agents; let $\left(\chi^{-},\chi^{+}\right)$ and $\left(\varphi^{-},\varphi^{+}\right)$ be any bipolar predicates; let *a*, *b*, *c*, *d* be any agents. YinYang bipolar quantum entanglement or BUMP is a fundamental law for equilibrium-based regulation of gene expression, mutation, and molecular interaction in bioinformatics. $\forall a, b, c, d$ we have ($*$ and $\blacklozenge$ can be bound to any logical or physical bipolar operator):

1) $$\left[\psi\left(a\left(t_x,p_1\right)\right)\Rightarrow\phi\left(c\left(t_y,p_3\right)\right)\right]\&\left[\psi\left(b\left(t_x,p_2\right)\right)\Rightarrow\phi\left(d\left(t_y,p_4\right)\right)\right]$$
$$\Rightarrow\left[\psi\left(a\left(t_x,p_1\right)\right)*\psi\left(b\left(t_x,p_2\right)\right)\Rightarrow\phi\left(c\left(t_y,p_3\right)\right)*\phi\left(d\left(t_y,p_4\right)\right)\right]$$
$$\Rightarrow\left[\psi\left(a\left(t_x,p_1\right)\blacklozenge b\left(t_x,p_2\right)\right)\Rightarrow\phi\left(c\left(t_y,p_3\right)\blacklozenge d\left(t_y,p_4\right)\right)\right];$$

2) $$\left[\psi\left(a\left(t_x,p_1\right)\right)\Rightarrow\psi\left(c\left(t_y,p_3\right)\right)\right]\&\left[\phi\left(b\left(t_x,p_2\right)\right)\Rightarrow\phi\left(d\left(t_y,p_4\right)\right)\right]$$
$$\Rightarrow\left[\psi\left(a\left(t_x,p_1\right)\right)*\phi\left(b\left(t_x,p_2\right)\right)\Rightarrow\psi\left(c\left(t_y,p_3\right)\right)*\phi\left(d\left(t_y,p_4\right)\right)\right]$$

Prediction 3. Let $\psi=\left(\psi^{-},\psi^{+}\right)$ = (self-negation, self-assertion) be a bipolar predicate for the mental equilibrium measures of a patient set P at the neurophysiologic level; let $\left(\chi^{-},\chi^{+}\right)$ be that of the set P at the mood or behavior level; let $\phi=\left(\phi^{-},\phi^{+}\right)$ = (negative, positive) be a bipolar predicate for the biochemical capacities of a medicine set M for bipolar disorders; let $\left(\varphi^{-},\varphi^{+}\right)$ = (un-excite, un-depress) be that for the effects of M at the mental level. $\forall a, b, a\in P$ and $b\in M$,

$$\left[\left(\psi\left(a\left(t_x\right)\right)\Rightarrow\chi\left(a\left(t_y\right)\right)\right)\right]\&\left[\left(\phi\left(b\left(t_x\right)\right)\Rightarrow\varphi\left(b\left(t_y\right)\right)\right)\right]$$
$$\Rightarrow\left[\left(\psi\left(a\left(t_x\right)\right)*\phi\left(b\left(t_x\right)\right)\right)\Rightarrow\left(\chi\left(a\left(t_y\right)\right)*\varphi\left(b\left(t_y\right)\right)\right)\right]$$

is a fundamental law of equilibrium-based brain and behavior, which can be applied in nanobiomedicine for psychiatric mood regulation on an individual and/or a cohort of mental disorder patients.

Prediction 4. Let $\psi=\left(\psi^{-},\psi^{+}\right)$ = (negative, positive) be a bipolar predicate and *a*, *b*, *c*, *d* be any four antimatter and/or matter bindings or couplings, bipolar quantum entanglement or BUMP is an equilibrium-based fundamental law for scientific discovery in astrophysics or particle physics.

The next prediction is on bipolar twistor space. Roger Penrose introduced a twistor theory [19] but "*No one yet knows what a quantum twistor space looks alike.*" [2, p. 244].

Prediction 5. YinYang bipolar geometrical space is a minimal but most general quantum twistor space; bipolar universal modus ponens (BUMP) or bipolar quantum entanglement is a minimal but most general quantum twistor; bipolar causality and bipolar relativity is a minimal but most general twistor theory.

Prediction 6. Black hole is to a galaxy (or universe) as bipolar depression is to a dysfunctional brain; big bang is to a galaxy (or universe) as bipolar mania is to a dysfunctional brain; wormhole is to a galaxy (or universe) as bipolar mental equilibrium is to a functional brain.

Prediction 7. If the universe had been created by a big bang, the big bang must have been caused by the equilibrium or non-equilibrium of negative and positive energies of the cosmos.

Prediction 8. YinYang bipolar relativity is the simplest mathematically conceivable cosmological order—a non-linear bipolar dynamic fusion of 1) a unipolar truth-based explicate order and 2) an equilibrium-based implicate order with bipolar quantum entanglement that regulates the evolution of the explicit order.

Prediction 9. When observable mass or energy is propagated through bipolar relativity or causality the speed of the propagation is limited by the speed of light (e.g. the propagation of photon or electron); when information is propagated without passing observable mass or energy the speed of the propagation is not limited by the speed of light but by the "speed" of equilibrium-based quantum non-local connection or bipolar quantum entanglement.

Prediction 10. YinYang bipolar quantum entanglement is the source of causality for the Being of beings. All physical, social, mental, biological action and reaction are fundamentally different forms of bipolar quantum entanglement in large or small scales.

Falsifiability is a must for scientific predictions. It may be argued that YinYang bipolar quantum entanglement needs experimental verification or falsification. That is, of course, correct. However, YinYang bipolar quantum entanglement or BUMP is logical. Unlike the predicted existence of monopoles in string theory, dipoles are proven physical reality. Furthermore, from **Figure 6**, it is evident that without bipolar quantum entanglement there would be no atom-atom quantum entanglement. Thus, we can assert that YinYang bipolar quantum entanglement is the most fundamental form of any quantum entanglement [11,12].

4. Quantum Reincarnation of Philosophy

4.1. Meaning of Reincarnation

Seeking the ultimate truth from the universe is not easy because the universe is not completely truthful and the ultimate Being of beings is unreachable with the truth-based approach. A key element in Leibniz truth-based metaphysics is a kind of soul-like monad through which it is said God ordained harmony into the universe. But Leibniz did not figure out why being is there but nothingness is not and he could not avoid the trap of nihilism.

While Hegel failed to provide a formal logic to back up his truth-based and contradiction-centered dialectics, he named his system *The Science of Logic* and described it verbally without forms. Nevertheless, he claimed he brought philosophy to the end with a circle. His circle, however, has no logically definable causality and his science of logic is not the logic of science.

While Steven Hawking advocated negative-positive energies he did not see the potential of YinYang bipolar dynamic equilibrium as a unifying philosophical basis. When he needed a new philosophy, he declared the death of philosophy.

Is philosophy really ended by Hegel? Evidently that is not true because, even if the truth-based, being- or contradiction-centered Western metaphysics were really ended, Eastern equilibrium-based and harmony-centered metaphysics is still in hibernation. As a key concept in *Yi Jing* and a doctrine of Daoism, Buddhism and Confucianism, harmony is advocated in both of the East and the West by friends and adversaries. For instances, following Confucianism, President Hu Jintao of China advocates social harmony; following Buddhism, Dalai Lama, the spiritual leader of Tibet in exile, advocates religious harmony for which he was awarded 2012 Templeton Prize; legendary German mathematician, logician, philosopher and sinologist Leibniz advocated harmony ordained by God through monad; Einstein believed a nature God "*who reveals himself in the orderly harmony of what exists*"; all democratic systems are formed with checks and balances for social stability or harmony.

Evidently, regardless of their proper or improper interpretations and usage, equilibrium and harmony are the ultimate desire of human civilization. Since any being must exist in and be revealed by dynamic equilibrium, the essence of being is not truth as Aristotle and Heidegger once claimed but equilibrium and harmony whose bipolarity provides the defining property.

Indeed, even after the whole world is unified into a single democratic society, mankind will still need equilibrium and harmony. Notably, contrary to Sir Karl Popper's stance against holism in the 20th century, Prince Charles has become a strongest advocate for holism and nature-man harmony in the 21st century. The prince starts with these alarming words for his 2010 book titled *Harmony* [20, p. 3]: "*This is a call to revolution. The Earth is under threat. It cannot cope with all that we demand of it. It is losing its balance and we humans are causing this to happen.*"

The prince has challenged the readers to reconsider the assumptions that determine how we live, and how we might change in seeking a more durable future for humankind. For the first time, he shows how the solutions to problems like climate change lie not only in technology but in our ability to change the way we view the modern world. In brief, we need new philosophical thinking to deal with the problem.

Is philosophy really dead? Evidently that is not true. When Hawking declared the death of philosophy, his many universes are still truth-based and being-centered. Since being and nothingness share the same philosophical root, Hawking like Leibniz cannot avoid nihilism. That led to his assertion "*a great many universes were created out of nothing.*" Moreover, the negative and positive energies or the Yin and Yang seem to be unavoidable for the regulation and unification of the multiverses.

To be fair, Western science and philosophy are not to blame for overlooking the Yin and Yang of nature. Chinese philosophy itself is to blame because, for thousands of years, it failed to provide a formal logical and geometric foundation to back up its cosmological claims, that also prompted Chen Ning Yang to blame *Yi Jing* for the failure of China to become the cradle of modern science.

Although philosophy perhaps will never end or die, it is a fact that no new philosophy was born since Hegel. Due to the widely accepted end or death, now we have to talk about its reincarnation.

4.2. Overcome Metaphysics

When Heidegger tried to overcome Aristotle's "being qua being" metaphysics he could not figure out what is the ultimate Being that reveals all beings. His search once reached the Dao [21,22] but he returned empty handed because he didn't understand what *Yi Jing* claimed "*One Yin and one Yang are called the Dao.*" He intended to escape Western nihilism but in the end was trapped in an Eastern version of nihilism: "*The Dao that can be told is not the eternal Dao*; *the name that can be named is not the eternal name.*" Although the word of Laozi has been oft-quoted and widely loved, it is actually an obliteration of YinYang, a retreat from *Yi Jing*, and a mystification of the Dao. Should Laozi be correct, Einstein's grand unification would be meaningless.

To Heidegger, even though philosophy deviated from its primordial state, it still thinks. But, according to him, science does not think. Similarly, to Him logic does not think either. In his writings Heidegger often denounced logic in a sarcastic tone. According to him, in logical reasoning, the laws of thought are replaced by the laws of logical expression. He claimed that logic was invented by teachers and not by philosophers. In his observation, Leibniz, Cant and Hegel—the three greatest German thinkers—had tried to avoid the old logical tradition but, unfortunately, they often became sacrifices of it. He claimed that thought was still stipulated by proposition in Hegelian dialectics and logic was completely framed into technology. He called the logic-only thinkers the lowest point of thinking (cf. [23]).

Unfortunately, Heidegger himself failed to overcome metaphysics. When he denounced science and logic, he stopped short of going beyond truth-based thinking. He did not realize that the crux of the problem is not science but exactly the truth-based reasoning that dominated human's positive thinking. Since truth is unipolar and static, it can't provide logically definable causality and the dynamics for the ultimate metaphysical Being to reveal all beings.

If you are not for truth you would be called a liar. Besides Richard Rorty (1931-2007) few dared to go beyond truth. In front of truth, even Heidegger had to surrender. Ironically, when Heidegger tried to overcome Aristotle's metaphysics, he reasserted truth as the essence of being following Aristotle. He evidently didn't realize that beyond truth there is still equilibrium—the only dynamic concept that can regulate the mighty universe including all the beings and truths in it. Moreover, since equilibrium is central in the 2nd law of thermodynamics, it is scientific and could be the Being of revealing.

4.3. Possibility of Scientific Reincarnation

In the money-driven technology-dominated modern world, it is extremely difficult to talk about a reincarnation of philosophy. In a typical modern university, philosophy curriculum has been replaced with truth-based logic teaching. But Karl Popper claimed that we can never prove something true, we can only show that it is false. He deemed it the current state of science and physics, which is founded on uncertain induction from empirical facts rather than certain logical deduction from principles which correctly describe reality. However, Popper's empirical positivist view didn't go beyond truth-based thinking because any empirical certainty factor or probability is a degree of truth. As a realist, Popper took Einstein's side and firmly opposed Niels Bohr's interpretation of quantum entanglement. He overlooked the possibility of logically definable causality. Without causality any scientific view is incomplete. In contrast, Einstein believed that a logical axiomatization of physics is possible and famously stated: "*Evolution is proceeding in the direction of increasing simplicity of the logical basis* (*principles*)"; "*We must always be ready to change these notions—that is to say*, *the axiomatic basis of physics—in order to do justice to perceived facts in the most perfect way logically*"; "*Pure thought can grasp reality*, *as the ancients dreamed*"; "*Nature is the realization of the simplest conceivable mathematical ideas*."

Now, the phenomenon of quantum entanglement has been repeatedly demonstrated through experiments. Even though Einstein was right on a possible axiomatization of physics, it is now believed by many that Bohr came out the winner of the historical Einstein-Bohr debate on quantum entanglement—a new phenomenon that entails new philosophical thinking.

Remarkably, Hawking and Mlodinow opposed Einstein and Popper's quantum realism. They stated [4, p. 44]: "*Though realism may be a tempting viewpoint*, $\cdots$, *what we know about modern physics makes it a difficult one to defend*." They also quoted quantum physics: "*For example*, *according to the principles of quantum physics*, *which is an accurate description of nature*, *a particle has neither a definite position nor a definite velocity unless and until those quantities are measured by an observer*." They concluded: "*In fact*, *in some cases individual objects don't even have an independent existence but rather exist only as part of an ensemble of many*."

What Hawking and Mlodinow are saying is that, in the quantum world, a being A may not be A or $A \neq A$ because its identity may depend on B or C. Logically speaking, the identity law $A = A$, the most fundamental law in Western philosophy and science, is shattered by quantum mechanics. Truth-based logical tradition, however, can't make any sense from this quantum phenomenon other than characterizing it as a contradiction.

Niels Bohr was the first one to bring YinYang into the center of quantum mechanics for particle-wave complementarity. When he was awarded the Order of the Elephant in 1947, he designed his own coat of arms which featured a YinYang logo (or Taiji symbol) in the center with the Latin motto "*contraria sunt complementa*" or "*opposites are complementary*".

Now we need to revisit the primordial meaning of YinYang complementarity. Such a revisit reveals that YinYang complementarity can be between any two sides of one subject. However, the essence of YinYang is that the two sides should be opposite but reciprocal poles or energies. While Bohr's particle-wave complementarity is central in quantum theory, atomic physics and chemistry, particle-wave as well as man-woman, space-time and truth-falsity are not direct opposites in the most fundamental way. Instead, particle-antiparticle and action-reaction forces are the most fundamental opposites of nature. Without the most fundamental bipolar opposites any complementarity is less fundamental. That could be why Bohr deemed quantum causality unattainable [24]. Dramatically, bipolar quantum causality is logically definable (Equation (1)).

Based on de Broglie's work, Einstein's former associate David Bohm proposed a causal interpretation of quantum mechanics [25]. Central in Bohm's interpretation is a wave function. Bohm's causal interpretation was not well received at the beginning. It was branded as "meta-physical" and "ideological" (cf. [26], p. 340). Einstein had initially encouraged Bohm on his work. Without a deep philosophical basis and a new logic, Bohm's wave function was later dismissed by Einstein as "too cheap".

With BDL, the shattered identity law $A \neq A$ is reformulated as BUMP or bipolar quantum entanglement that provides logically definable quantum causality. It states: Equilibrium variables A and B are bipolar quantum entangled if A is bipolar equivalent to B or $A \Leftrightarrow B$ in bipolar geometry. With bipolar quantum entanglement, it is natural for A and B not to have independent existence as:

1) YinYang bipolar geometry is equilibrium-based that transcends spacetime, being, and truth;

2) Any being A may exist in the same bipolar equilibrium or entanglement with another being B in another side of the universe;

3) The negative-positive energies of nature are bipolar quantum entangled from which nothing can escape.

Dramatically, a contradiction in the truth-based spacetime geometry has become a sound new law in equilibrium-based YinYang bipolar geometry. The new law provides logically definable causality for the first time ever based on bipolar equilibrium, non-equilibrium, symmetry and non-symmetry. It has made bipolar quantum gravity hopeful and ubiquitous logically, physically, biologically, socially and mentally [1,7-18,27-32].

4.4. Equilibrium and Harmony vs Fire and War

Someone may quote Heraclitus and argue against YinYang equilibrium and harmony. Heraclitus famously claimed that everything is in a state of flux, nothing stays still; fire is the most fundamental element and war is father of all, king of all. He believed that fire gave rise to everything. He regarded soul a mixture of fire and water, with fire being the noble part and water the ignoble part. He believed that worldly pleasures made the soul "moist" and a soul should, therefore, be purified to a "dry" state.

Following Hegel's suggestion, Heraclitus is widely recognized as the founding father of dialectical thinking. His prominence is partly due to the view that his prediction of fire being the most fundamental element of everything is corroborated by the big bang theory or even verified by particle-antiparticle annihilation.

Although Heraclitus is right in claiming a forever changing world, a re-examination of his fire-based philosophy can reveal its unscientific nature as we have:

1) Modern science has proven that fire is not the most fundamental element and thus Heraclitus' prediction has been falsified;

2) Fire is subject to equilibrium condition of the 2nd law of thermodynamics; the big bang has to be caused by dynamic equilibrium or non-equilibrium where fire is effect but not cause;

3) YinYang bipolarity survived the big bang and can also survive a black hole due to particle or antiparticle emission from a black hole;

4) With logically definable causality, bipolar dynamic equilibrium has to be more fundamental than fire.

Mentally speaking, it is misleading to say that soul is a mixture of fire and water, with fire being the noble part and water the ignoble. Here the founding father of dialectics seemed to have once fallen into a being-centered metaphysical dichotomy. Regardless of good or bad, the balance of self-negation and self-assertion abilities of a person is a key for mental equilibrium. Without mental equilibrium, there would be no healthy genetic mutation, no mind, no being, and no truth.

Socially speaking, although war and fire are occasionally unavoidable, the advancement of humanity is mainly due to (mankind should also strive for) bipolar reciprocal and peaceful development in equilibrium, complementarity, and harmony, not mainly due to war and fire. This is a fundamental difference between the Eastern YinYang and the Western dialectics. In another word, war and fire cannot escape global equilibrium. It is needless to say that peace and harmony are not for war and fire, but war and fire are for peace and harmony. Interestingly, Zhuang Zi (4th century BCE) said: *Fish thrive in water*, *man thrives in the Dao*; Heidegger sighed: *For a long time now*, *all too long*, *thinking* (*like a fish*) *has been stranded on dry land*.

5. Science Cannot Replace Philosophy

5.1. Being, Truth, Equilibrium and Harmony

Aristotle's "*being qua being*" has been central in science as well as in philosophy. Following the metaphysical doctrine, ancient people assumed that the Earth was a static and stationary being centered in the universe; God was an exemplary being who created everything; air, water, fire, earth and ether were the fundamental beings of the universe; mental disorder like bipolar disorder or schizophrenia were caused by some ghostly beings.

Nevertheless, under the guidance of being-centered philosophy, truth-based Western science and technology have made glorious achievements. It brought mankind wealth and health. It improved the peoples' living standards in many countries. Unfortunately, the glory is associated with side effects. Global warming is believed by many a deadly one threatening the very existence of mankind. Therefore, the problem is the being-centered and truth-based positive thinking itself.

Until this day, modern science is still strictly following the "being qua being" doctrine. For instance, the many decade searches for quantum gravity resulted in strings and superstrings—imaginary fundamental beings. Despite the sharp criticisms such as "*The Trouble with Physics*" and "*Not Even Wrong*" in physics as well as "ended" or "*dead*" in philosophy, science is still being-centered and truth-based. Even some scientists who proclaimed "*philosophy is dead*" failed to realize that the multiverses in M-theory are still under the guidance of the "dead" philosophy.

Einstein asserted that "*Physics constitutes a logical system of thought.*" and "*the axiomatic basis of theoretical physics cannot be extracted from experience but must be freely invented*". Presumably, the light at the end of the quantum tunnel is not likely to come nearer until someone grasps the reality with pure thought to invent a philosophically different new logic with definable causality. Is that possible at all? Einstein thought so. He asserted that "*nature is the realization of the simplest conceivable mathematical ideas*"; "*pure thought can grasp reality*".

Why can't science be equilibrium-based? The answer is simple: equilibrium is not "*being qua being*". Even though no being can exist beyond equilibrium or non-equilibrium, even though the truth-based philosophical tradition has been proven inadequate for furthering scientific explorations, even though philosophy as metaphysics has been proclaimed "*ended*" or "*dead*," even though equilibrium is central in the 2nd law of thermodynamics, the truth-based and being-centered philosophical thinking is and will still be the only major guiding light as well as a major barrier of science until the reincarnation of an alternative or complementary philosophy.

5.2. M-Theory and Philosophy

Hawking and Mlodinow argued in their book *The Grand Design* [4] that, because philosophy has not kept up with developments in modern science, particularly physics, as a result, scientists have become the bearers of the torch in humans' quest for knowledge. Therefore, they claim: "*philosophy is dead*."

When they made their above argument, they evidently overlooked the chilly fact that modern physics is now in urgent need for the guidance of new philosophical thinking. If string theory is not testable how could the M-theory of superstrings be testable? If no geometry can go beyond spacetime, how could an M-theory of multiverses be scientific? If Einstein believed "*evolution is proceeding in the direction of increasing simplicity of the logical basis* (*principles*)" for physics, how could an M-theory with eleven or more dimensions be simple? Where is the definitive battleground of quantum gravity? How is quantum entanglement related to the real world? Where is the simple logical foundation that provides logically definable causality? Where is the deeper theory that can transcend spacetime, relativity, quantum mechanics and the multiverses?

As a matter of fact, the science of *The Grand Design* did not avoid the guidance of philosophical thinking. For instance, on Page 154 of the book the authors wrote cautiously: "*Though it may sound like philosophy*, *the weak anthropic principle can be used to make scientific prediction.*" On Page 162, they unconsciously wrote: "*It cannot be so easily explained*, *and has far deeper physical and philosophical implications.*" Evidently, when the authors needed to recall philosophy back into physics they forgot that they already pronounced its death on Page 5 of the same book.

Consequently, leaving God alone, *The Grand Design* stopped short of answering two deeper questions:

1) Do the multiverses in M-theory need to follow the same equilibrium condition of the 2nd law of thermodynamics?

2) Can all the truth-based and being-centered multiverses be unified under a single equilibrium-based and harmony-centered universe?

5.3. Can Science Replace Philosophy?

When new philosophical thinking is badly needed as an alternative guidance for science, especially physics, it seems that science has no way to replace philosophy:

1) Without new philosophical thinking we cannot have significant scientific invention.

2) Sometimes philosophy leads to new scientific discoveries; sometimes new scientific discoveries lead to new philosophical thinking.

3) Existing philosophical thinking is inadequate for solving unanswered scientific problems.

4) The truth-based and being-centered intensive searches

for ether, strings and monopoles have got no concrete result so far but dipoles are everywhere.

5) Physicists so far failed to apply string theory in the real world to reveal the ubiquitous effect of quantum entanglement. New philosophical thinking is needed for a new formal logical and mathematical system. BDL is the first step forward.

6) Even if the God pillar of metaphysics were indeed broken, it does not mean Western philosophy as metaphysics is completely dead because the truth-based logical reasoning of metaphysics continues to be a major pillar of modern science and even equilibrium can be regarded as holistic truth.

7) Although evolution has been proven true in biology, we still need to find out what is the driving power of mutation, natural selection and evolution. Could it be dynamic equilibrium or non-equilibrium?

8) Even if the truth-based and being-centered Western philosophy were really dead, the Eastern equilibrium-based and harmony-centered YinYang philosophy is still underdeveloped.

9) YinYang bipolar quantum entanglement can be an equilibrium-based ubiquitous regulating power of spacetime, being, truth, science and philosophy.

10) Particle-antiparticle bipolarity can survive big bang and black hole, but all beings and truths are subject to observation and limited to certain spacetime.

Evidently, philosophy like science will never end or die as long as mankind is faced with unsolved problems. What it may do is hibernation followed by reincarnation or awakening. Thus, science can't replace philosophy.

6. Harmony and Einstein's God

This paper is about scientific reincarnation of philosophy, not theology. It should be remarked, however, that Spinoza (1632-1677) defined God as nature. Spinoza's God provided a living natural God. On the other hand, Leibniz claimed that things cause one another because God ordained a pre-established divine harmony among everything in the universe. Clearly, equilibrium, harmony and Spinoza's God are related.

Einstein combined the above views and famously stated: "*I believe in Spinoza's God who reveals himself in the orderly harmony of what exists, not in a God who concerns himself with the fates and actions of human beings.*" Einstein also said: "*Everyone who is seriously involved in the pursuit of science becomes convinced a spirit is manifest in the laws of the Universe—a spirit vastly superior to that of man, and one in the face of which we with our modest powers must feel humble. In this way the pursuit of science leads to a religious feeling of a special sort, which is indeed quite different from the religiosity of someone more naive.*"

Now, with YinYang bipolar relativity and bipolar quantum entanglement, equilibrium-based harmony can be logically and mathematically defined and revealed. Logically speaking, theology might need to follow Einstein and elevate God from the heaven of being and truth to the heaven of nature's equilibrium and harmony. Such an elevation would position God on a unifying higher ground above science and philosophy. That seems necessary for the returning of divinity to the *technology-dominated Godless society.* Otherwise, we human beings as a curious species may always wonder whether God as an exemplar being has to be subject to equilibrium or harmony such as mental equilibrium or harmony?

7. Conclusions

A brief review on the end and death assertions of philosophy has been presented. YinYang bipolar dynamic logic (BDL) has been introduced as an equilibrium-based logic of science (vs *The Science of Logic*). The logic is then used for an equilibrium-based cosmological interpretation and unification of matter, antimatter, space, time, big bang and black hole, relativity and quantum mechanics. With the new interpretation, a scientific reincarnation of philosophy has been predicted. Due to its central claim that YinYang bipolar quantum entanglement is the source of causality for the Being of beings, the reincarnation is named a quantum reincarnation.

The possibility and unavoidability of the reincarnation has been discussed. Symbolically speaking, if contradiction in Western dialectics is replaced with bipolar equilibrium, Being in Western metaphysics is replaced with harmony, and God is elevated from the heaven of being and truth to the heaven of nature's equilibrium and harmony, the science of logic by Hegel can be replaced with BDL—an equilibrium-based logic of science. Hegel's circle can then be replaced with the Taiji symbol (**Figure 10**), where the non-isomorphic coexistence of the Yin and the Yang of nature has been proven by dipoles and CP-violation. An immediate consequence is the falsification of Hegel's assertion of the end of philosophy.

Another consequence is that the negative and positive energies of the many universes in M-theory have to follow the same equilibrium condition of the 2nd law of thermodynamics and become one universe. This tells us that science can't replace philosophy. Subsequently, the different laws followed by the multiverses as described

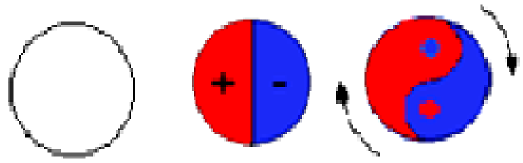


Figure 10. From Hegelian circle to YinYang logo.

in *The Grand Design* have to be unified under the same equilibrium or non-equilibrium condition. Thus, the reincarnation discussed in this work presents a modest unification of science and philosophy within equilibrium based YinYang bipolar geometry.

Although the reincarnation of philosophy is a modest result originated from YinYang bipolar quantum entanglement, its social implication could be profound and far reaching. Hopefully, the reincarnation would bring philosophy from the unbalanced ground where Thales once set his feet 2500 year ago to the firmer and balanced ground where mankind will set feet in the next 1000 years. Thales should have avoided his embarrassing fall into a well and being mocked by a servant girl. Now mankind should maintain nature-man harmony to avoid potential falls. Indeed, even after the world is unified into a single democratic society, mankind will still need social, natural, and nature-man equilibrium and harmony.

The Grand Design is concluded with these words [4, p. 181]: "*M-theory is the unified theory Einstein was hoping to find. The fact that we human beings—who are ourselves mere collections of fundamental particles of nature—have been able to come close to an understanding of the laws governing us and our universe is a great triumph. But perhaps the true miracle is that abstract considerations of logic lead to a unique theory that predicts and describes a vast universe full of the amazing variety that we see. If the theory is confirmed by observation, it will be the successful conclusion of a search going back more than* 3000 *years. We will have found the grand design*."

In contrast to the above assertions, the author concludes this paper with a balanced set of words: "*It might be questionable whether M-theory is indeed the unified theory Einstein was hoping to find unless the multiverses are unified into a single one under equilibrium or non-equilibrium following the* 2^{nd} *law of thermodynamics. The fact that we human beings—who are ourselves mere collections of negative-positive particles of nature with self-negation and self-assertion bipolar mental equilibrium or disorder—have revealed the bipolar nature of the laws governing us and our universe is an overlooked great discovery. But the true miracle is perhaps that YinYang bipolar dynamic logic has led us to an equilibrium based and harmony-centered unification of science and philosophy. If the unification is accepted as an alternative guiding light for mutually beneficial reciprocal interaction, it may enhance our mental equilibrium, social stability, nature-man harmony, and one day lead us to the unified theory Einstein was hoping to find. Otherwise, the vast universe full of the amazing variety that we see is likely to be forever a mystery world with God ordained equilibrium and harmony*." [1].

REFERENCES

[1] W.-R. Zhang, "A Quantum Dream of Red Mansion: The End of Philosophy and Its Reincarnation," Working Book.

[2] L. Smolin, "The Trouble with Physics: The Rise of String Theory, the Fall of a Science, and What Comes Next?" Houghton Mifflin Harcourt, New York, 2006.

[3] P. Woit, "Not Even Wrong: The Failure of String Theory and the Search for Unity in Physical Law," Basic Book, New York, 2006.

[4] S. Hawking and L. Mlodinow, "The Grand Design," Random House Digital, Inc., New York, 2010.

[5] S. Hawking, "Black Hole Evaporation," *Nature*, Vol. 248, No. 5443, 1974, pp. 30-31. doi:10.1038/248030a0

[6] S. Hawking, "Particle Creation by Black Holes," *Communications in Mathematical Physics*, Vol. 43, No. 3, 1975, pp. 199-220. doi:10.1007/BF02345020

[7] W.-R. Zhang, "Equilibrium Relations and Bipolar Cognitive Mapping for Online Analytical Processing with Applications in International Relations and Strategic Decision Support," *IEEE Transactions on Systems*, *Man*, *and Cybernetics*, *Part B*, Vol. 33. No. 2, 2003, pp. 295-307. doi:10.1109/TSMCB.2003.810444

[8] W.-R. Zhang and L. Zhang, "YinYang Bipolar Logic and Bipolar Fuzzy Logic," *Information Sciences*, Vol. 165, No. 3-4, 2004, pp. 265-287. doi:10.1016/j.ins.2003.05.010

[9] W.-R. Zhang, "YinYang Bipolar Lattices and L-Sets for Bipolar Knowledge Fusion, Visualization, and Decision," *International Journal of Information Technology & Decision Making*, Vol. 4, No. 4, 2005, pp. 621-645. doi:10.1142/S0219622005001763

[10] W.-R. Zhang, A. Pandurangi and K. Peace, "YinYang Dynamic Neurobiological Modeling and Diagnostic Analysis of Major Depressive and Bipolar Disorders," *IEEE Transactions on Biomedical Engineering*, Vol. 54, No. 10, 2007, pp. 1729-1739. doi:10.1109/TBME.2007.894832

[11] W.-R. Zhang, "YinYang Bipolar Atom—An Eastern Road toward Quantum Gravity," *Journal of Modern Physics*, Vol. 3, Same Issue, 2012, pp. 1261-1271.

[12] W.-R. Zhang, "YinYang Bipolar Relativity: A Unifying Theory of Nature, Agents and Causality with Applications in Quantum Computing, Cognitive Informatics and Life Sciences," IGI Global, Hershey, 2011. doi:10.4018/978-1-60960-525-4

[13] W.-R. Zhang, "Quantum Cellular Combinatorics—An Equilibrium or Non-Equilibrium Approach," *Proceedings of the* 2012 *International Conference on Bioinformatics & Computational Biology* (*BIOCOMP*-2012), Las Vegas, 2012, pp. 312-318.

[14] W.-R. Zhang, "YinYang Bipolar Atom and Quantum Cellular Automation," 2011 *IEEE International Conference on Bioinformatics and Biomedicine Workshops* (*BIBMW*), Atlanta, 12-15 November 2011, pp. 791-797.

[15] W.-R. Zhang and S. S. Chen, "Equilibrium and Non-Equilibrium Modeling of YinYang Wuxing for Diagnostic Decision Analysis in Traditional Chinese Medicine," *International Journal of Information Technology & Decision Making*, Vol. 8, No. 3, 2009, pp. 529-548.

doi:10.1142/S0219622009003521

[16] W.-R. Zhang, H. J. Zhang, Y. Shi and S. S. Chen, "Bipolar Linear Algebra and YinYang-N-Element Cellular Networks for Equilibrium-Based Biosystem Simulation and Regulation," *Journal of Biological Systems*, Vol. 17, No. 4, 2009, pp. 547-576. doi:10.1142/S0218339009002958

[17] S. Vasudevan, Y. Tong and J. A. Steitz, "Switching from Repression to Activation: MicroRNAs Can Up-Regulate Translation," *Science*, Vol. 318, No. 5858, 2007, pp. 1931-1934. doi:10.1126/science.1149460

[18] Y. Shi, E. Seto, L.-S. Chang and T. Shenk, "Transcriptional Repression by YY1, a Human GLI-Kruppel-Related Protein, and Relief of Repression by Adenovirus E1A Protein," *Cell*, Vol. 67, No. 2, 1981, pp. 377-388. doi:10.1016/0092-8674(91)90189-6

[19] R. Penrose, "The Road to Reality: A Complete Guide to the Laws of the Universe," Alfred A. Knopf, New York, 2005.

[20] Charles HRH The Prince of Wales, T. Juniper and I. Skelly, "Harmony: A New Way of Looking at Our World," Harper Collins Publishers, New York, 2010.

[21] E. Q. Wang, "Time, History, and Dao: Zhang Xuecheng and Martin Heidegger," *Dao*: *A Journal of Comparative Philosophy*, Vol. 1, No. 2, 2002, pp. 251-276.

[22] X. Zhang, "Heidegger and Taoism on Humanism," Asian Humanities Pr., 2003.

[23] J. Chen, "Introduction to Heidegger's Philosophy," Joint Publishing Co Ltd., 1995.

[24] N. Bohr, "On the Notions of Causality and Complementarity," *Dialectica*, Vol. 2, No. 3-4, 1948, pp. 312-319. doi:10.1111/j.1746-8361.1948.tb00703.x

[25] D. Bohm, "Causality and Chance in Modern Physics," 1961 Harper Edition Reprinted, University of Pennsylvania Press, Philadelphia, 1980.

[26] M. Kumar, "Quantum: Einstein, Bohr, and the Great Debate about the Nature of Reality," W. W. Norton & Company, New York, 2008.

[27] "Fermi National Accelerator Laboratory," Press Release 06-19, 25 September 2006.

[28] J. Gore and A. van Oudenaarden, "Synthetic Biology: The Yin and Yang of Nature," *Nature*, Vol. 457, No. 7227, 2009, pp. 271-272. doi:10.1038/457271a

[29] W.-R. Zhang, S. Chen and J. C. Bezdek, "POOL2: A Generic System for Cognitive Map Development and Decision Analysis," *IEEE Transactions on Systems*, *Man*, *and Cybernetics*, Vol. 19, No. 1, 1989, pp. 31-39. doi:10.1109/21.24529

[30] W.-R. Zhang, S. Chen, W. Wang and R. King, "A Cognitive Map Based Approach to the Coordination of Distributed Cooperative Agents," *IEEE Transactions on Systems*, *Man*, *and Cybernetics*, Vol. 22, No. 1, 1992, pp. 103-114. doi:10.1109/21.141315

[31] W.-R. Zhang, "YinYang Bipolar Quantum Entanglement—Toward a Complete Quantum Theory," *ICQNM'*10. 4*th International Conference on Quantum*, *Nano and Micro Technologies*, St. Maarten, 10-16 February 2010, pp. 77-82.

[32] W.-R. Zhang, K. A. Pandurangi, K. E. Peace, Y. Zhang and Z. Zhao, "MentalSquares—A Generic Bipolar Support Vector Machine for Psychiatric Disorder Classification, Diagnostic Analysis and Neurobiological Data Mining," *International Journal of Data Mining and Bioinformatics*, Vol. 17, No. 4, 2011, pp. 547-576.

Journal of Modern Physics, 2012, 3, 1285-1288
doi:10.4236/jmp.2012.329165 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Investigating Initial Conditions of the WdW Equation in Flat Space in a Transition from the Pre-Planckian Physics Era to the Electroweak Regime of Space-Time

Andrew W. Beckwith
Physics Department, Chongqing University, Chongqing, China
Email: abeckwith@uh.edu

Received June 5, 2012; revised July 30, 2012; accepted August 6, 2012

ABSTRACT

This document is due to reviewing an article by Maydanyuk and Olkhovsky, of a Nova Science conpendium as of "The big bang, theory assumptions and Problems", as of 2012, which uses the Wheeler De Witt equation as an evolution equation assuming a closed universe. Having the value of *k*, not as the closed universe, but nearly zero of a nearly flat universe, which leads to serious problems of interpretation of what initial conditions are. These problems of interpretations of initial conditions tie in with difficulties in using QM as an initial driver of inflation. And argue in favor of using a different procedure as far as forming a wave function of the universe initially. The author wishes to thank Abhay Ashtekar for his well thought out criticism but asserts that limitations in space-time geometry largely due to when $\hbar$ is formed from semi classical reasoning, *i.e.* Maxwell's equation involving a close boundary value regime between Octonionic geometry and flat space non Octonionic geometry is a datum which Abhay Ashekhar may wish to consider in his quantum bounce model and in loop quantum gravity in the future.

Keywords: Wheeler De Witt Equation; Planck's Constant; Wavefunction of the Universe; Octonionic Geometry; Quantum Mechanics

1. Introduction

What we are looking at, in Maydanyuk and Olkhovsky [1], is a way to define the initial Wheeler De Witt equation, not as what they did, for a closed universe, but to get to the actual nearly flat space Euclidian universe conditions which suggest that quantum mechanics will not work well as to initial conditions, and that a different procedure than what was done for closed universe conditions [1] needs to be considered for the start of cosmological evolution. Note that the difficulty in initial conditions has startling similarities as to the problem with gravitions having mass as noted by Maggiorie [2] which specifically delineated for non zero graviton mass, where $h \equiv \eta^{uv} h_{uv} = \text{Trace} \cdot \left(h_{uv}\right)$ and $T = \text{Trace} \cdot \left(T^{uv}\right)$ that

$$-3m_{\text{graviton}}^2 h = \kappa T/2 \tag{1}$$

As noted by Maggiore, one gets into serious analytical difficulties from the beginning, with (1) and the reader is invited to look at his massive Graviton section [2] which delineates some of the problems. In a similar manner, the closed universe analysis done in [1] encounters serious problems in initial conditions if we used flat space in the onset which sheds light upon the vulnerabilities of quantum mechanics in forming appropriate initial conditions, which we will comment upon and offer a solution for.

2. Looking at the Way to Form a Wheeler De Witt Equation via a Nearly Flat Space Model

The author is quite aware of work discussed with him in conferences, noticiably Rencontres De Moriond, in the experimental gravity conference, which alledges that from the initial conditions that inflation mandated almost completely flat space. For the sake of argument in this work, we will work with flat space, and will commence a derivation which shows serious issues with the Wheeler De Witt analysis of Quantum space time offered in [1] which works passably well in a closed universe condition.

To do this, we will reproduce, using instead of $k = 1$ (closed universe), $k \cong \varepsilon^+ \sim 0^+$, and use that to reproduce the Wheeler De Witt argument and wave functions in [1], designating what we think are serious initial condition problems inherient in the $k \cong \varepsilon^+ \sim 0^+$ nearly flat space conditions, so as to look at first the mini super space Langrangian. This document is due to reviewing an article by Maydanyuk and Olkhovsky, of a Nova Science conpendium as of "The big bang, theory assumptions and Problems", as of 2012, which uses the Wheeler De Witt equation as an evolution equation assuming a closed universe. Having the value of *k*, not as the closed universe, but nearly zero of a nearly flat universe, which leads to serious problems of interpretation of what initial conditions are. These problems of interpretations of initial conditions tie in with difficulties in using QM as an initial driver of inflation. And argue in favor of using a different proceedure as far as forming a wave function of the universe initially, which is written in [1] as for a mini superspace lagrangian

$$L(a,\dot{a}) = (3a/8\pi G)\cdot\left(-\dot{a}^2 + \left[k \cong \varepsilon^+ \sim 0^+\right]\right) - (3a/8\pi G)\cdot(8\pi G/3)\cdot a^2\rho(a) \tag{2}$$

A Chapylgin gas equation of state was used, in working with Equation (2) using $0 < \alpha < 1$ so that

$$p_{\text{Chapyglin}} = -A/\rho^{\alpha}_{\text{Chapyglin}} \tag{3}$$

And, in conditions which specify $A = \rho_\Lambda$ and $B = \rho_{Dust}$

$$\rho_{\text{Chapyglin}}(a) = \left(A + B/a^{3\cdot(1+\alpha)}\right) \xrightarrow[\alpha\to 0]{} \rho_{\text{Dust}} + \rho_\Lambda \tag{4}$$

and a general density equation we will write up as

$$\rho(a) = \left(\rho_\Lambda + \rho_{\text{Dust}}/a^{3\cdot(1+\alpha)}\right)^{(1/1+\alpha)} + \rho_{\text{Radiation}}/a^4 \tag{5}$$

The end result as given is that [3] one has a S.E. with a wavefunction $\phi(a)$

$$\left\{-\frac{\partial^2}{\partial a^2} + V(a)\right\}\phi(a) = E_{\text{Radiation}}\phi(a) \tag{6}$$

with

$$V(a) = \left(\frac{3}{4\pi G}\right)^2\cdot\left(k \cong \varepsilon^+ \sim 0^+\right)\cdot a^2 - \frac{3}{2\pi G}\cdot a^4\cdot\left(\rho_\Lambda + \rho_{\text{Dust}}/a^{3\cdot(1+\alpha)}\right)^{(1/3+\alpha)} \tag{7}$$

The difficulty in the change of variables comes next and is attributed to $k \cong \varepsilon^+$. Set $8\pi G = M^{-2} = 1$, and then the Equation (7) becomes, instead, if $E_{\text{radiation}} = 12\rho_{\text{radiation}}$

$$V(a) = 36\cdot\left(k \cong \varepsilon^+\right)\cdot a^2 - 12\cdot a^4\cdot\left(\rho_\Lambda + \rho_{\text{Dust}}/a^{3\cdot(1+\alpha)}\right)^{(1/3+\alpha)} \tag{8}$$

This potential is almost identitcal to what was done in [1] but the term $k \cong \varepsilon^+$ is what creates initial conditions which simply do not work out and are to be commented upon directly. If one does an expansion of Equation (8) as given above by $q = a - \bar{a}$ then by [1]

$$V_{\text{Chapyglin}}(q) = V_0 - V_1 q \tag{9}$$

$$V_0 = V_{\text{Chapyglin}}(a = \bar{a});$$

$$V_1 = 72\cdot\left[k = \varepsilon^+\right]\cdot a + 12\cdot\left\{-4\Lambda - \rho_{\text{Dust}}/a^{3\cdot(1+\alpha)}\right\}\cdot\left(\Lambda + \rho_{\text{Dust}}/a^{3\cdot(1+\alpha)}\right)^{-(\alpha/1+\alpha)} \tag{10}$$

Then Equation (6) becomes, with $\phi(q)$ a wave function of the universe for $q = a - \bar{a}$

$$\left\{-\frac{\mathrm{d}^2}{\mathrm{d}q^2} + \left(V_0 - E_{\text{radiation}} + V_1\cdot q\right)\right\}\phi(q) = 0 \tag{11}$$

The following change of variables is where the problem in the Planckian regime becomes acute. *i.e.* set

$$\xi = \frac{V_0}{|V_1|^{2/3}} - \frac{V_1}{|V_1|^{2/3}}\cdot q \tag{12}$$

Then, Equation (11) become an Airy style differential equation with

$$\frac{\mathrm{d}^2\phi(\xi)}{\mathrm{d}\xi^2} + \xi\cdot\phi(\xi) = 0 \tag{13}$$

The following change of variables is where the problem in the Planckian regime becomes acute. Equation (13) above becomes undefinable, in the Planck regime of space time due to working with

$$\xi\big|_{\text{Planck-regime}} \sim \frac{\left[E_{\text{radiation}} - V_0\right]}{\left[\varepsilon^+ \sim 0^+\right]^{2/3}} \tag{14}$$

In this case, the $\varepsilon^+ \sim 10^{-33}$ centimeters is so small, that it is next to impossible to define Equation (14) , with a solution as given in [1] via

$$\phi(\xi) \equiv T\cdot\psi^+(\xi);$$

$$\psi^+(\xi) = \int_0^{\mu\max} \exp i\cdot\left[-\frac{\mu^3}{3} + f(\xi)\cdot\mu\right]\cdot\mathrm{d}\mu \tag{15}$$

If we do a power series expansion of the function $f(\xi)$, [1] asserts that Equation (15) becomes proportional to an airy function with $Ai(z); Bi(z)$, provided $f_0 = 0; f_1 = 1$.

3. Criticism of the Above Methodology by Abhay Ashtekar

We introduce several criticisms of the above methodology leading to what was said about Equation (14) by Abhay Ashtekar, in private communication with the author [4].

"There are several technical problems. For instance, the substitution from (11)-(13), introducing (12), seems to overlook the fact that the new variable *xi* in (12) depends on q or a not just by the explicit factor but also via the potentials. And even if there is a coefficient dividing by a small epsilon (related to k), this value is not zero and there is no problem with well-defined equations. One would simply make a poor choice of variables in which some coefficients are unnaturally large (After all, a flat universe with $k = 0$ has a well-defined formulation)".

4. The Author's Answer to Abhay Ashtekar

First of all the author wishes to thank Abhay Astekar for his direct communications to correct what he perceived as sloppy thinking. The first place to start is to look at (12) above again, and to ask what is possibly driving

$$\xi = \frac{V_0}{|V_1|^{2/3}} - \frac{V_1}{|V_1|^{2/3}} \cdot q \\ \to \xi \Big|_{\text{Planck-regime}} \sim \frac{\left[E_{\text{radiation}} - V_0\right]}{\left[\varepsilon^+ \sim 0^+\right]^{2/3}} \tag{16}$$

Recall Equation (11)

$$\left\{-\frac{\mathrm{d}^2}{\mathrm{d}q^2} + \left(V_0 - E_{\text{radiation}} + V_1 \cdot q\right)\right\}\phi(q) = 0 \tag{11}$$

This presumably would happen when $q = a - \bar{a}$, and then we would be really looking at

$$\left\{-\frac{\mathrm{d}^2}{\mathrm{d}q^2} + \left(V_0 - E_{\text{radiation}} + V_1 \cdot q\right)\right\}\phi(q) = 0 \\ \to \left\{-\frac{\mathrm{d}^2}{\mathrm{d}q^2} + \left(V_0 - E_{\text{radiation}}\right)\right\}\phi(q) = 0 \tag{17}$$

The transition from the left to the right hand side in Equation (20) above is tandem to what was said by Beckwith [5,6] as to formation of Planck's constant.

5. Criticism of Forming Wave Function of Equation (15) if an Airy Function, with Using Equation (14)

We assert that in the Planck regime of space time, that Equation (14) is in reality undefinable due to the denominator of $k \cong \varepsilon^+ \sim 0^+$ at or below 10 ^ - 33 centimeters of space time. The value of this parameter is so small, in fact, that what really needs to be addressed, to make any sense out of how small Equation (14) really is, is the following observation. Namely in looking at an evolution of a Wheeler De Witt equation of space time, that we can define a spatial evolution, via expansion of the scale factor a, as in Equation (11), but we have to put in by hand the initial time step *i.e.* the exact same problem shows up in Loop quantum gravity. In the case of scale factor $a(t)$, the spatial evolution is amendable by QM, but there is no idea as to how to get about putting in "by hand" the initial time step, which we presume would be a Planck time interval.

6. So If a Domain Wall Enters the Picture, Then What Does This Do to Structure Formation and also Plank's Constant?

In [5] we are stuck with how a semi classical argument can be used to construct **Table 1** above. In particular, we look at how Planck's constant is derived, as in the electroweak regime of space time, namely that given the prime in both Equations (16) and (17) is for a total derivative [7,8]

$$E_y = \frac{\partial A_y}{\partial t} = \omega \cdot A'_y\left(\omega \cdot (t - x)\right) \tag{18}$$

Similarly [15]

$$E_y = \frac{\partial A_y}{\partial t} = \omega \cdot A'_y\left(\omega \cdot (t - x)\right) \tag{19}$$

The A field so given would be part of the Maxwell's equations given by [7] as, when $[\ ]$ represents a D'Albertain operator, that in a vacuum, one would have for an A field [7,8]

$$[\ ]A = 0 \tag{20}$$

And for a scalar field ϕ

$$[\ ]\phi = 0 \tag{21}$$

Following this line of thought we then would have an energy density given by, if ε_0 is the early universe permeability [7]

$$\eta = \frac{\varepsilon_0}{2} \cdot \left(E_y^2 + B_z^2\right) = \omega^2 \cdot \varepsilon_0 \cdot A'^2_y\left(\omega \cdot (t - x)\right) \tag{22}$$

Table 1. Organizing WdW evolution.

Time Interval Consequences	Dynamical Consequencs	Does QM/WdW Apply?
Just before Electroweak Era	Form $\hbar$ from early E & M fields, and use Maxwell's Equations with necessary to implement boundary conditions created from change from Octonionic geometry to flat space	NO
Electro-Weak Era	$\hbar$ kept constant due to Machian relations	YES
Post Electro-Weak Era to Today	$\hbar$ kept constant due to Machian relations	YES Wave Function of Universe

We integrate (20) over a specified E and M boundary, so that, then we can write the following condition namely [7,8].

$$\begin{aligned}&\iiint \eta \mathrm{d}(t-x)\mathrm{dydz} \\ &= \omega\varepsilon_o \iiint A_y'^2(\omega\cdot(t-x))\mathrm{d}(t-x)\mathrm{dydz}\end{aligned} \tag{23}$$

(21) would be integrated over the boundary regime from the transition from the Octonionic regime of space time, to the non Octonionic regime, assuming an abrupt transition occurs, and we can write, the volume integral as representing [7,8]

$$E_{\text{gravitational-energy}} = \hbar\cdot\omega \tag{24}$$

Our contention for the rest of this paper, is that Mach's principle will be necessary as an information storage container so as to keep the following, *i.e.* having no variation in the Planck's parameter after its formation from electrodynamics considerations as in (21) and (22). Then by applying [7,8]

$$\hbar(t) \xrightarrow[\text{Apply-Machs-Relations}]{} \hbar \quad \text{(Constant value)} \tag{25}$$

7. Conclusions. We Need to Reconsider the Role of Quantum Gravity Models at the Onset of Inflation

We are stuck in all Quantum gravity models as of putting in an initial time step "by hand" so to speak which raises fundamental issues of what would form an initial time step in Quantum gravity. How the transtion from the left to the right hand side of Equation (17) occurs is crucial and it comes about because of a transition from Octonionic geometry to quantum accessible and analyzable flat space geometry. The key equation to understand is Equation (17) which delineates how one can have Equation (11), Equation (16), and Equation (17) happen. This indeterminate nature of time, itself, at the onset of Quantum gravity models of space time may be seen as a fundamental defect killing off all initial QM influences at the start of inflation. The other way to look at the role of an undefined initial starting point for time, which we put in by "hand" is that the special nature of time itself may be if experimentally verified, via observations, the best hope we have of falsifiable measurements of t'Hoofts conjecture [9,10] that quantum mechanics is embedded within a classical physics frame work which we have yet to fully develop. To do that would also, if the Gravition exists with initial measurements, such as given by

$$\begin{aligned}&m_{\text{graviton}}\Big|_{\text{relativistic}} < 4.4\times10^{-22}\,\mathrm{h}^{-1}\mathrm{eV}/\mathrm{c}^2 \\ &\Leftrightarrow \lambda_{\text{graviton}} \equiv \frac{\hbar}{m_{\text{graviton}}\cdot c} < 2.8\times10^{-8}\,\text{meters}\end{aligned} \tag{26}$$

Perhaps lead to signals from early universe gravitational waves which may confirm or falsify the role of quantum mechanics in initial univese conditions. As well as the role that set as a working approximation [6].

$$\begin{aligned}&v_S^2k^2\delta - 4\pi G\rho_b\delta \\ &\equiv\left[v_S^2k^2 - 4\pi G\cdot\left(\rho_b = T_i^i - \lambda\right)\right] = \text{constant}\end{aligned} \tag{27}$$

Affects the formation of baryonic matter fluctuations, which may play a role in the formation of **Table 1**.

8. Acknowledgements

This work is supported in part by National Nature Science Foundation of China grant No.110752.

REFERENCES

[1] S. Maydanyuk and V. Olkhovsky, "Fully Quantum Study of the FRW Model with Radiation and Chaplygin Gas," In: J. O'Connell and A. Hale, Eds., *The Big Bang, Theory, Assumptions and Problems*, Nova Science Publishers, New York, 2012, pp. 185-196.

[2] M. Maggiore, "Gravitational Waves, Volume 1: Theory and Experiment," Oxford University Press, Oxford, 2008.

[3] G. A. Monerat, *et al.*, "Dynamics of the Early Universe and the Initial Conditions for Inflation in a Model with Radiation and a Chaplygin Gas," *Physical Review D*, Vol. 76, No. 2, 2007, Article ID: 024017. doi:10.1103/PhysRevD.76.024017

[4] A. Ashtekar, "Private Communications with the Author," Monday, 9 July 2012.

[5] A. Beckwith, "Is Quantum Mechanics Involved at the Start of Cosmological Evolution? Does a Machian Relationship between Gravitons and Gravitinos Answer This Question?" 2012. http://vixra.org/abs/1206.0023

[6] A. W. Beckwith, "Is There a Change in the Baryonic Structure Formation if Quark Strings and Domain Walls Exist at about the Electro-Weak Era?" 2012. http://vixra.org/abs/1207.0034

[7] U. Bruchholz, "Derivation of Planck's Constant from Maxwell's Electrodynamics," *Progress in Physics*, Vol. 4, 2009, p. 67.

[8] U. Bruzchholz, "Key Notes on a Geometric Theory of Fields," *Progress in Physics*, Vol. 2, 2009, pp. 107-113.

[9] G. 't Hooft, "Determinism beneath Quantum Mechanics," In: *Beyond the Quantum*, Th. M. Nieuwenhuizen, *et al.*, Eds., World Press Scientific, Singapore, 2002. http://arxiv.org/PS_cache/quant-ph/pdf/0212/0212095v1.pdf

[10] G. 't Hooft, "The Mathematical Basis for Deterministic Quantum Mechanics," In: Th. M. Nieuwenhuizen, *et al.*, Eds., *Beyond the Quantum*, World Press Scientific, Singapore, 2006. http://arxiv.org/PS_cache/quant-ph/pdf/0604/0604008v2.pdf

Journal of Modern Physics, **2012, 3, 1289-1297**
doi:10.4236/jmp.2012.329166 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Time Evolution of Horizons

Arundhati Dasgupta
Department of Physics and Astronomy, University of Lethbridge, Lethbridge, Canada
Email: arundhati.dasgupta@uleth.ca

Received June 24, 2012; revised July 25, 2012; accepted August 2, 2012

ABSTRACT

Finding the origin of Hawking radiation has been a puzzle to researchers. Using a loop quantum gravity description of a black hole slice, a density matrix is defined using coherent states for space-times with apparent horizons. Evolving the density matrix using a semi-classical Hamiltonian in the frame of an observer outside the horizon gives the origin of Hawking radiation.

Keywords: Black Hole Physics; Quantum Gravity; Hawking Radiation; Loop Quantum Gravity; Coherent States

1. Introduction

A new theory is expected to take over at Planck distances as "quantum effects" of gravity start dominating. One of the promising approaches to the theory of quantum gravity is the theory of Loop Quantum Gravity (LQG), which is by formulation non-perturbative and background independent [1-3]. LQG has a well defined kinematical Hilbert space, and though the Hamiltonian constraint remains unsolved, the theory allows for a semiclassical sector of the theory. This includes "coherent states" [4,5] which are peaked at classical phase space elements. Using these as a starting point, I defined in a series of papers [6-8] coherent states for the Schwarzschild spacetime, and derived an origin of entropy using quantum mechanical definition of entropy from density matrices. The exact entropy is a function of the graph used to obtain the LQG phase space variables [9]. The zeroeth order term is proportional to the area of the horizon signifying a universality of the Bekenstein-Hawking entropy. The proportionality constant and the correction terms bring out the details of the graph [8].

In this paper we take this new way of finding the origin of entropy a step further by evolving the spatial slice in time [10], and observing the evolution of the density matrix in the process. This state as of now does not satisfy the Hamiltonian constraint, but one is allowed to take an arbitrary initial state, or a wavepacket with appropriate properties, representing a macroscopic configuration. The evolution discussed in this paper is semi-classical, *i.e*. no attempt is made to use the full Hamiltonian.

The quasilocal energy (QLE) of an outside observer, defined in [11] is used as the Hamiltonian to evolve the system. As the time clicks in the observers clock, the Hamiltonian evolves the coherent state such that the area of the horizon remains the same as predicted by classical physics. However, classically forbidden regions become accessible quantum mechanically, and vertices of the graph hidden behind the horizon in one slice emerge outside the horizon in the next slice. This gives a net change in area, and the mass deficit is emitted from the black hole. This evolution is not unitary, and the quasi-local energy which is used to evolve the slice is not mapped to a Hermitian operator. When matter is coupled to the gravitational system, a net flux emerges causing a decay of the horizon.

In Section 2 we introduce the formalism by describing the coherent state, the black hole time slice, the apparent horizon equation, and the density matrix. Section 3 describes the time evolution of the system and gives a derivation of the change in entropy. In Section 4 we give a description of a matter current emergent from behind the horizon. Finally in the concluding section we include a discussion about the implications of the non-Hermitian evolution.

2. The Coherent State in LQG

For gravity, finding the canonical variables which describe the physical phase space is an odd task as there is no unique time. Nevertheless a fiducial time coordinate can be chosen, which breaks the manifest diffeomorphism invariance, restored in the Hilbert space of states by imposing constraints.

The constant time slices are described by the intrinsic metric q_{ab} and the extrinsic curvature K_{ab} $(a,b = 1,2,3)$. The theory can be formulated in terms of the square root of the metric, the triads e^I_a defined thus:

$$e_a^I e_b^I = q_{ab} \tag{1}$$

where I represents the internal index for the rotation group SO(3) of the tangent space and $a, b = 1,2,3$. The internal group is taken to be SU(2), as this is locally isomorphic to SO(3). The theory is then defined in terms of the "spin connection" $\Gamma_a^I = \varepsilon^{IJK} e_J^b \nabla_a e_{bK}$ and the triads. However, a redefinition of the variables in terms of tangent space densitised triads E_a^I and a corresponding gauge connection A_a^I where I represents the SU(2) index simplifies the quantisation considerably.

$$A_a^I = \Gamma_a^I - \beta K_{ab} e^{Ib} \quad E_I^a = \frac{1}{\beta}(\det e) e_I^a \tag{2}$$

(e_a^I are the usual triads, K_{ab} is the extrinsic curvature, Γ_a^I the associated spin connection, β the one parameter ambiguity which remains named as the Immirzi parameter). The quantisation of the Poisson algebra of these variables is done by smearing the connection along one dimensional edges e of length δ_e of a graph Γ to get holonomies $h_e(A)$. The triads are smeared in a set of 2-surface decomposition of the three dimensional spatial slice to get the corresponding momentum P_e^I. The algebra is then represented in a kinematic "Hilbert space", in which the physical constraints have been "formally" realised [12]. Once the phase space variables have been identified, one can write a coherent state for these [4] *i.e.* minimum uncertainty states peaked at classical values of h_e, P_e^I. In analogy with the harmonic oscillator coherent states, where the coherent state is a function of the complexified phase space element $x - ip$, the SU(2) coherent states are peaked at the complexified phase space element $g_e = e^{iT^I P^I/2} h_e$. These g_e are thus elements in the complexification of SU(2) as $e^{iT^I P^I/2}$ (T^I being the generator matrices of SU(2)) is a Hermitian matrix and h_e is the unitary SU(2) matrix. Whether these are physical coherent states, or have appropriate behavior under the action of the constraints has to be examined carefully [13]. The coherent state in the momentum representation for one edge is defined to be

$$\left|\psi^t(g_e)\right> = \sum_{jmn} e^{-tj(j+1)/2} \pi_j(g_e)\left| jmn\right> \tag{3}$$

In the above g_e is a complexified classical phase space element $e^{iT^I P_e^{Icl}/2} h_e^{cl}$, (the P_e^{Icl} and the h_e^{cl} represent classical momenta and holonomy obtained by embedding the edge in the classical metric). The $\left|jmn\right>$ are the usual basis spin network states given by $\pi_j(h)_{mn}$, which is the jth representation of the SU(2) element h_e. Similarly, $(2j+1)\times(2j+1)$ dimensional representations of the 2×2 matrix g_e are denoted as $\pi_j(g_e)_{mn}$. The j is the quantum number of the SU(2) Casimir operator in that representation, and m, n represent azimuthal quantum numbers which run from $-j..j$. The coherent state is precisely peaked with maximum probability at the h_e^{cl} for the variable h_e as well as the classical momentum P_e^{Icl} for the variable P_e^I. The fluctuations about the classical value are controlled by the parameter t (the semiclassicality parameter). This parameter is given by l_p^2/a where l_p is Planck's constant and a a dimensional constant which characterises the system. The coherent state for an entire slice can be obtained by taking the tensor product of the coherent state for each edge which form a graph Γ,

$$\Psi_\Gamma = \prod_e \psi_e^t. \tag{4}$$

In [7] the g_e was evaluated for the Schwarzschild black hole by embedding a graph on a spatial slice with zero intrinsic curvature. The particular graph which was used had the edges along the coordinate lines of a sphere. This simplistic graph, was very useful in obtaining the description of the space-time in terms of discretised holonomy and momenta. A particularly interesting consequence of this was that the phase space variables were finite and well defined even at the singularity.

Given that the area of a surface in gravity is measured as the integral of the square root of the metric over the surface, the area operator can be written simply as $\hat{A} = \sqrt{\hat{P}_e^I \hat{P}_e^I}$. The expectation value of the area operator in the coherent state emerges as [9]

$$\left\langle \psi \middle| \hat{A} \middle| \psi \right\rangle = \left(j + \frac{1}{2} \right) t \tag{5}$$

Thus we are considering a semiclassical state, which is a state such that expectation values of operators are closest to their classical values. The information of the classical phase space variables are encoded in the complexified SU(2) elements labeled as g_e. The fluctuations over the classical values are controlled by the semiclassical parameter t.

The density matrix which describes the entire black hole slice is obtained as

$$\rho^{\text{Total}} = \left|\Psi_\Gamma\right>\left<\Psi_\Gamma\right| \tag{6}$$

where $\left|\Psi_\Gamma\right>$ is the coherent state wavefunction for the entire slice, a tensor product of coherent state for each edge.

2.1. Apparent Horizons

We concentrate on the coherent state near the apparent horizon contained in the spatial slice. We find that motivated from the apparent horizon equation the graph across the horizon can be taken to be populated by radial edges, linking vertices outside and inside the horizon. One then traces over the coherent state within the horizon. Initially we take a particular time slicing of the black hole, which has the spatial slices with zero intrinsic curvature [7]. One such metric which has the time slices as flat is the Lemaitre metric

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \frac{\mathrm{d}R^2}{\left[\frac{3}{2r_g}(R-\tau)\right]^{2/3}} + \left[\frac{3}{2}(R-\tau)\right]^{4/3} r_g^{2/3}\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right). \tag{7}$$

The $r_g = 2GM$, (in units of c = 1) and in the $\tau =$ constant slices one can define the induced metric in terms of a "r" coordinate defined as

$$\mathrm{d}r = \mathrm{d}R \Big/ \left[3/2r_g\left(R-\tau_c\right)\right]^{1/3}$$

($\tau = \tau_c$) on the slice. One gets the metric of the three slice to be

$$\mathrm{d}s_3^2 = \mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right). \tag{8}$$

The entire curvature of the space-time metric is contained in the extrinsic curvature or $K_{\mu\nu} = \frac{1}{2}\partial_\tau g_{\mu\nu}$ tensor of the $\tau =$ constant slices. Now if there exists an apparent horizon somewhere in the above spatial slice, then that is located as a solution to the equation

$$\nabla_a S^a + K_{ab} S^a S^b - K = 0 \tag{9}$$

where S^a, ($(a,b = 1,2,3)$ denote the spatial indices) is the normal to the horizon, K_{ab} the extrinsic curvature in the induced coordinates of the slice, and K the trace of the extrinsic curvature. If the horizon is chosen to be the 2-sphere, then in the coordinates of (8), $S^a \equiv (1,0,0)$, the apparent horizon equation as a function of the metric reduces to:

$$K_{rr}\left(1-q^{rr}\right) - K_{\phi\phi}q^{\phi\phi} - \Gamma^{\phi}_{\phi r} - K_{\theta\theta}q^{\theta\theta} - \Gamma^{\theta}_{\theta r} = 0 \tag{10}$$

Note that the first term of the equation disappears trivially as $1 = q^{rr}$ for any point in the spatial slice. Even at the operator level the q^{rr} can be set to the identity operator in the first approximation, as $q^{rr} = P_{e_r}P_{e_r} \big/ V^2$ ($\hat{V}$ being the volume operator) upto normalisations, and in the spherically symmetric metric $V = P_{e_r}\delta_{e_r}$ (upto discretisation constants). Thus the operators in the numerator and denominator cancel and the normalisation conspire, leaving $\hat{q}^{rr} = I$. To understand the rest of the equation in terms of the holonomy and momentum variables of LQG, which are classically measured in the same metric as (8), we use the following regularisation

$$K_{\theta(\phi),\theta(\phi)} = e^I_{\theta(\phi)} K^I_{\theta(\phi)},\ q_{\theta(\phi)\theta(\phi)} = e^I_{\theta(\phi)} e^I_{\theta(\phi)} \tag{11}$$

$$e^I_{\theta,(\phi)} \equiv N\ \mathrm{Tr}\left[T^I h^{-1}_{e_{\theta(\phi)}}\left\{h_{e_{\theta(\phi)}}, V\right\}\right] \tag{12}$$

(N is a constant, a function of the edge lengths and the area bits of the discretisation) and V is the volume operator.

$$K^I_{\theta(\phi)} = \frac{1}{\delta_{e_\theta}}\mathrm{Tr}\left[h^{-1}_{e_\theta} T^I \beta \frac{\partial}{\partial\beta} h_{e_\theta}\right] \tag{13}$$

Here β has been used as a parameter to identify the K^I_a operator, and this is mainly a trick. In the continuum limit

$$h_e\left(A^I_a\right) = \lim\mathrm{it}_{\delta_{e_a}\to 0} \mathrm{e}^{\int A_a \mathrm{d}x^a} = \left(I + A^I_a T^I \delta_{e_a}\right) \tag{14}$$

As the gauge connection is a function of the Immirzi parameter due to (2), the expectation value of this operator in a coherent state will be a function of the Immirzi parameter. By taking the derivative wrt to the Immirzi parameter we are giving the same status to the parameter as is given to "dimension" in a dimensional regularisation of Feynman diagrams. We let the parameter vary by an infinitesimal amount from its value in the particular quantisation sector, take the derivative, and put its original value in the final answer for the K^I_a operator. The Formula (13) is facilitated by the fact that the dependence of A^I_a on the β is linear. One way to check whether this gives the proper answer is to take a solved quantum mechanical system and use a similar method there. The most useful example is the Harmonic Oscillator Hamiltonian, which can be written as

$$H = \frac{p^2}{2m} + \frac{1}{2}m\,\omega^2 x^2 \tag{15}$$

The ground state is a coherent state, so we take that as an example. We define the operator

$$x^2 = \frac{2}{m}\frac{\partial H}{\partial\omega^2} \tag{16}$$

Thus

$$\left\langle x^2 \right\rangle = \frac{2}{m}\left\langle \frac{\partial H}{\partial\omega^2}\right\rangle = \frac{2}{m}\frac{\partial}{\partial\omega^2}\left(\frac{\hbar\omega}{2}\right) = \frac{\hbar}{2m\omega} \tag{17}$$

The regularisation (13) is thus an allowed approximation.

The terms involving the Christoffel connections like $\Gamma^\theta_{\theta r}$ include derivatives in the regularised version, the derivatives appear as difference of triads across two vertices. Thus

$$\Gamma^\theta_{\theta r} = e^\theta_I e^\theta_I \frac{1}{\delta_{e_r}}\left(e^J_\theta\left(v_1\right) - e^J_\theta\left(v_2\right)\right) e^J_\theta\left(v_1\right) \tag{18}$$

As a result of this if we impose restrictions on the Christoffel connections and one of the vertices v_1 is within the horizon, whereas v_2 is outside the horizon, there will be correlations across the horizon.

If one evaluates the expectation value of the apparent horizon equation using the regularised variables in the coherent states, then one would obtain

$$\begin{aligned} &4\left\langle \psi \left| P_{e_\theta}^2 \left[\mathrm{Tr}\left(T^J h_{e_\theta}^{-1} V^{1/2} h_{e_\theta}\right)_{v_1} - \mathrm{Tr}\left(T^J h_{e_\theta}^{-1} V^{1/2} h_{e_\theta}\right)_{v_2}\right] \right.\right. \\ &\quad \left.\left. \times \mathrm{Tr}\left(T^J h_{e_\theta}^{-1} V^{1/2} h_{e_\theta}\right)_{v_2} \right| \psi \right\rangle \\ &\quad - N' \left\langle \psi \left| \mathrm{Tr}\left(h_e^{-1} T^I \beta \frac{\partial}{\partial\beta} h_{e_\theta}\right) P^I_{e_\theta\, v_1} \right| \psi\right\rangle = 0 \end{aligned} \tag{19}$$

(N' is a constant).

2.2. Density Matrix

The density matrix is obtained as

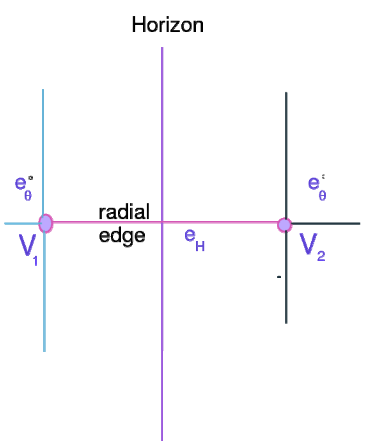
$$\rho^{\mathrm{Total}} = |\Psi_\Gamma >< \Psi_\Gamma| \tag{20}$$

where $|\Psi_\Gamma>$ is the coherent state wavefunction for the entire slice, a tensor product of coherent state for each edge.

But given this, we concentrate in a "local" region to see the behavior of the horizon

$$\rho^{\mathrm{Total}} = \rho^{\mathrm{outside}} \rho^{\mathrm{local}} \rho^{\mathrm{inside}} \tag{21}$$

where ρ^{local} covers a band of vertices surrounding the horizon one set on a sphere at radius $r_g - \delta_{e_r}/2$ and one set on a sphere at radius $r_g + \delta_{e_r}/2$ within the horizon, as described in [9], and in the figure enclosed. This local density matrix and the correlations due to the apparent horizon equation (19) was used to derive entropy [6]. This entropy counts the number of ways to induce the horizon area using the spin networks, though the constraints have not been appropriately imposed as was obtained using a Chern-Simons theory in [14]. However, the entropy calculation using the coherent states provides a tracing mechanism, and a method to obtain correlations across the horizon which are gravitational in origin. We will henceforth deal with ρ^{local}, but we will drop the local label for brevity.



3. Time Evolution

In physical systems, the Hamiltonian generates time evolution, but in General Theory of Relativity, the Hamiltonian is a constraint and generates diffeomorphisms in the time direction. So the question is, what is physical time, and if that exists, what would be the operator evolving the system in that direction? In case of space-times with time like Killing vectors, notion of time can be identified with the Killing direction, and a notion of "quasilocal energy" (QLE) defined using the same. The QLE then generates translations in the Killing time. In case of the Schwarzschild space-time, the QLE has been defined in [11]. We build the Hamiltonian which evolves the horizon from one time slice to the next by appropriately regularising the QLE. Note the "Killing time" and QLE are classical concepts, and thus regularising QLE gives us a "semiclassical" Hamiltonian.

3.1. Change in Entropy

Before we get into the analysis of what QLE evolution means, we take a simple system made up of two subsystems, and examine the consequences of a Hamiltonian evolution. Let the density matrix be defined for a system whose states are given in the tensor product Hilbert space $H_1 \otimes H_2$ and given by

$$|\psi> = \sum_{ij} \mathrm{d}_{ij} |i> |j> \tag{22}$$

where $|i>$ is the basis in H_1 and $|j>$ is the basis in H_2 and d_{ij} are the non-factorisable coefficients of the wavefunction in this basis. Let us label the wavefunction at time $t=0$ to be given by the coefficients d_{ij}^0. The density matrix is

$$\rho^0 = \sum_{iji'j'} \mathrm{d}_{i'j'}^{0*} \mathrm{d}_{ij}^0 |i>|j><j'|<i'| \tag{23}$$

The reduced density matrix if one traces over H_2 is:

$$\mathrm{Tr}_2 \rho^0 = \sum_{ii'} \sum_j \mathrm{d}_{i'j}^{0*} \mathrm{d}_{ij}^0 |i><i'| \tag{24}$$

We now evolve the system using a Hamiltonian which has the matrix elements $H_{iji'j'} |i>|j><j'|<i'|$, we assume that the Hamiltonian does not factorise, that is there exists interaction terms between the two Hilbert spaces. The evolution equation is:

$$i\hbar \frac{\partial \rho}{\partial \tau} = [H, \rho] \tag{25}$$

which in this particular basis gives the density matrix elements at a infinitesimally nearby slice to be

$$\mathrm{d}_{i'j'}^{\delta\tau *} \mathrm{d}_{ij}^{\delta\tau} = \mathrm{d}_{i'j'}^{0*} \mathrm{d}_{ij}^0 - \frac{i}{\hbar} \delta\tau \left[\sum_{kl} \left(H_{ijkl} \mathrm{d}_{kl}^0 \mathrm{d}_{i'j'}^{0*} - \mathrm{d}_{ij}^0 \mathrm{d}_{kl}^{0*} H_{kli'j'} \right) \right] \tag{26}$$

Thus we evolve the "unreduced" density matrix and then trace over the H_2 in the evolved slice. The reduced density matrix in the evolved slice is:

$$\sum_j \mathrm{d}_{i'j}^{\delta\tau *}\mathrm{d}_{ij}^{\delta\tau} = \sum_j \mathrm{d}_{i'j}^{0*}\mathrm{d}_{ij}^{0} - \frac{i}{\hbar}\delta\tau\left[\sum_{klj}\left(H_{ijkl}\mathrm{d}_{kl}^{0}\mathrm{d}_{i'j}^{0*} - \mathrm{d}_{ij}^{0}\mathrm{d}_{kl}^{0*}H_{kli'j}\right)\right]. \tag{27}$$

This gives:

$$\rho^{\delta\tau} = \rho^0 - \frac{i}{\hbar}\delta\tau A \tag{28}$$

where A represents the commutator. Clearly the entropy in the evolved slice evaluated as

$$S_{\mathrm{BH}}^{\delta\tau} = -\mathrm{Tr}\left(\rho\ln\rho\right)$$

can be found as

$$S_{\mathrm{BH}}^{\delta\tau} = S_{\mathrm{BH}}^{0} + \frac{i}{\hbar}\delta\tau\left[\mathrm{Tr}A\ln\rho^0 + \mathrm{Tr}\rho^0\rho^{0\,-1}A\right] \tag{29}$$

Given the definition of A_{ii}, one gets

$$A_{ii} = \sum_{jkl}\left[\rho_{ijkl}^0 H_{klij} - H_{ijkl}\rho_{klij}^0\right] \tag{30}$$

In case both the Hamiltonian and the density operator are Hermitian, one obtains

$$\sum_j A_{jj} = 2\imath\,\mathrm{ImTr}\left(\rho^0 H\right) \tag{31}$$

This is clearly calculable, and gives the change in entropy ΔS_{BH}. The $\ln\rho^0$ term yields corrections, and we ignore it in the first approximation.

3.2. The Hamiltonian

To trace the origin of Horizon fluctuations, we must take an observer who is stationed outside the horizon, or in other words is not a freely falling observer. The quasilocal energy is defined using a "surface" integral of the extrinsic curvature with which the surface is embedded in three space. In our case, we take the bounding surface to be the horizon and the quasilocal energy is given by the surface term [11,15].

$$\tilde{H} = \frac{1}{\kappa}\int \mathrm{d}^2x\sqrt{\sigma}k \tag{32}$$

where k is the extrinsic curvature with which the 2-surface, which in this case is the horizon S^2 is embedded in the spatial 3-slices, and σ is the determinant of the two metric $\sigma_{\mu\nu}$ defined on the 2-surface. This "quasilocal energy" is measured with reference to a background metric. Thus $H = \tilde{H} - H_o$. We concentrate on the physics observed in an observer stationed at a r = constant sphere.

The metric in static $r = \mathrm{const}$ observer's frame is

$$\mathrm{d}s^2 = -f^2\mathrm{d}t^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right). \tag{33}$$

The $f = \sqrt{1 - r_g/r}$ where r_g is the Schwarzschild radius. If we take n_μ to be the space-like vector, normal to the 2-surface, then the extrinsic curvature is given by:

$$k_{\mu\nu} = \sigma_\mu^\alpha\nabla_\alpha n_\nu \tag{34}$$

and the trace is obviously

$$k = \nabla^\alpha n_\alpha \tag{35}$$

In the special slicing of the of the stationary observer the normal to the horizon 2-surface is given by $\left(0, f\left(r\right), 0, 0\right)$. However, we built the coherent state on the Lemaitre slice. The Lemaitre and the Schwarzschild observer's coordinates are related by the following coordinate transformations,

$$\sqrt{\frac{r}{r_g}}\mathrm{d}r = \left(\mathrm{d}R - \mathrm{d}\tau\right)$$

$$\mathrm{d}t = \frac{1}{1-f'}\left(\mathrm{d}\tau - f'\mathrm{d}R\right) \quad f' = \frac{r_g}{r} \tag{36}$$

The r = const cylinder of the Schwarzschild coordinate corresponds to $\mathrm{d}R = \mathrm{d}\tau$ of the Lemaitre coordinates, and for these $\mathrm{d}t = \mathrm{d}\tau$. Thus unit translation in the t coordinate coincides with unit translation in the τ coordinate. Further, the intersection of the r = constant cylinder with a t = constant surface coincides with the intersection of r = constant and the τ = constant surface. Thus in the initial slice, the QLE Hamiltonian can be written as

$$H = \frac{1}{2\kappa}\int \mathrm{d}\theta\mathrm{d}\phi\sqrt{g_{\theta\theta}g_{\phi\phi}}\left[-g^{\theta\theta}\frac{\partial g_{\theta\theta}}{\partial r} - g^{\phi\phi}\frac{\partial g_{\phi\phi}}{\partial r}\right]f\left(r\right) - H_0 \tag{37}$$

The reference frames' quasilocal energy is a number, it just defines the zero point Hamiltonian. Thus, we replace the classical expressions by operators evaluated at the τ = constant slice. In the first approximation we simply take the $f\left(r\right)$ as classical

$$\sqrt{1 - r_g/r} = \sqrt{\delta_{e_r}/2r_g} = \varepsilon,$$

as this arises due to the coordinate transformation and the norm of the vector n_r in the previous frame. In the rewriting of (37) in regularised LQG variables the Hamiltonian appears rather complicated.

One can rewrite these in a much simpler form, using the apparent horizon equation. Since the Hamiltonian is an integral over the horizon, the variables will satisfy the apparent horizon Equation (10) upto quantum fluctuations. Thus the Hamiltonian operator is then re-written as

$$H_{\mathrm{horizon}} = \frac{\varepsilon}{\kappa}\int \mathrm{d}\theta\mathrm{d}\phi\sqrt{g_{\theta\theta}g_{\phi\phi}}\left[K_\theta^I e^{I\theta} + K_\phi^I e^{I\phi}\right] \tag{38}$$

where we have used the classical apparent horizon equation (10) (with $q^{rr} = 1$).

$$H_{\text{Horizon}} = \frac{Ca\varepsilon}{2\kappa\delta_{e_\theta} s_{e_\theta}} \sum_{v_1} \text{Tr}\left[h_{e_\theta}^{-1} T^I \beta \frac{\partial}{\partial\beta} h_{e_\theta}\right] P_{e_\theta I} + h.c. + (\theta \to \phi) \tag{39}$$

where C consists of some dimensionless constants s_{e_θ} is the 2-dimensional area bit over which E_I^θ is smeared, a is a dimensionfull constant which appears to get the $P_{e_\theta I}$ dimension less. δ_{e_θ} is the length for the angular edge e_θ over which the gauge connection is integrated to obtain the holonomy. The sum over v_1 is the set of vertices immediately outside the horizon. The (39) can then be lifted to an operator.

This regularised expression for QLE is for the horizon 2-surface only and would not apply for any other spherical surface in the Schwarzschild space-time.

3.3. U(1) Case

Let us take the U(1) case to make the calculations easier and observe the action of the QLE Hamiltonian on the evolution of the coherent state. The spin network states are replaced by $|n\rangle = e^{\iota n\zeta}$, $0 < \zeta < 2\pi$, n is an integer and the coherent states are:

$$\psi^t(g_e) = \sum_n e^{-(tn^2)/2} e^{in(\chi_e - ip_e)} e^{-\iota n\zeta} \tag{40}$$

$g_{ne} = e^{in(\chi_e - ip_e)}$ is the complexified phase space element in the "*n*-th" representation.

The QLE operator also takes the simplified form

$$H_{\text{Horizon}}^{U(1)} = -\frac{1}{2} C' \iota \hat{h}_e^{-1} \beta \frac{\partial}{\partial\beta} \hat{h}_e \hat{p}_e + \frac{1}{2} C' \iota \hat{p}_e \beta \frac{\partial \hat{h}_e^{-1}}{\partial\beta} \hat{h}_e \tag{41}$$

The prefactors have been clubbed into C'.

In the calculation of the matrix elements, we drop the label of the edges e for the Hamiltonian.

$$\left\langle m \middle| \hat{H}_{\text{Horizon}}^{U(1)} \middle| n \right\rangle = \int e^{-\iota m\zeta} H_{\text{Horizon}}^{U(1)} e^{\iota n\zeta} \mathrm{d}\zeta \tag{42}$$

This calculation can be done by putting an assumption that the $\zeta = \zeta_1 + \beta\zeta_2$. In this ζ_1, ζ_2 are completely independent of β. It is an allowed assumption, and identifies the β dependence of the operator matrix elements, which are otherwise "hidden". The calculation however introduces an arbitrariness in the formula, which can be fixed by requiring that the expectation value of the Hamiltonian agrees with the classical QLE [10]. However, in this paper we use the "annihilation" operators defined in [16].

This is done by observing that the U(1) coherent states are eigenstates of an annihilation operator defined thus:

$$\hat{g}_e = e^{t/2} e^{\hat{p}_e} \hat{h}_e \quad \hat{g}_e |\psi\rangle = g_e |\psi\rangle \tag{43}$$

The holonomy operator can thus be written as

$$\hat{h}_e = e^{-t/2} e^{-\hat{p}_e} \hat{g}_e \tag{44}$$

And the derivative wrt Immirzi parameter of the holonomy which appears in the definition of the Hamiltonian replaced by

$$\begin{aligned} \beta \frac{\partial \hat{h}_e}{\partial\beta} &= e^{-t/2}\left[-\beta \frac{\partial \hat{p}}{\partial\beta} e^{-\hat{p}_e} \hat{g}_e + e^{-\hat{p}_e} \beta \frac{\partial \hat{g}_e}{\partial\beta}\right] \\ &= e^{-t/2}\left[\hat{p}_e e^{-p_e} \hat{g}_e + e^{-\hat{p}} \beta \frac{\partial \hat{g}_e}{\partial\beta}\right] \end{aligned} \tag{45}$$

The dependence of the operator p on the Immirzi parameter is known (2), and thus we could evaluate the derivative

$$\left(\beta\partial_\beta \; p_e(\beta) = \beta\partial_\beta \left(p_e(1)/\beta\right) = -p_e(\beta)\right)$$

The term

$$\text{Tr}\left(\rho^0 H_{\text{Horizon}}^{\text{U(1)}}\right) \tag{46}$$

is then computable. Let us take the first term of (41) and find (46). As $\rho^0 = |\psi\rangle\langle\psi|$, (46) gives simply (we drop the "*e*" label for brevity)

$$\begin{aligned} \left\langle \psi \middle| H_{\text{Horizon}}^{\text{U(1)}} \middle| \psi \right\rangle &= \left\langle \psi \middle| -\frac{1}{2} C' \iota \hat{h}^{-1} \beta \frac{\partial}{\partial\beta} \hat{h} \; \hat{p} \middle| \psi \right\rangle \\ &\quad + \left\langle \psi | h.c. | \psi \right\rangle \\ &= -\frac{1}{2} \iota C' \left\langle \psi \middle| \hat{g}^\dagger e^{-t/2} e^{-\hat{p}} e^{-t/2} \left[\hat{p}\, e^{-\hat{p}} \hat{g} + e^{-\hat{p}} \beta \frac{\partial \hat{g}}{\partial\beta}\right] \hat{p} \middle| \psi \right\rangle \\ &\quad + \left\langle \psi | h.c. | \psi \right\rangle \\ &= -\frac{1}{2} \iota C' e^{-t} g^* \left[\left\langle \psi \middle| e^{-\hat{p}} \; \hat{p}\, e^{-\hat{p}} \hat{p} \middle| \psi \right\rangle g \right. \\ &\quad \left. + \left\langle \psi \middle| e^{-2\hat{p}} \; \beta \frac{\partial \hat{g}}{\partial\beta} \hat{p} \middle| \psi \right\rangle\right] \\ &\quad + \left\langle \psi | h.c. | \psi \right\rangle \end{aligned}$$

We then concentrate on the 2nd term of the above

$$\begin{aligned} &\left\langle \psi \middle| e^{-2\hat{p}} \beta \frac{\partial \hat{g}}{\partial\beta} \hat{p} \middle| \psi \right\rangle \\ &= \left\langle \psi \middle| e^{-2\hat{p}} \beta \frac{\partial \hat{g}}{\partial\beta} \int \mathrm{d}\nu(g') \middle| \psi' \right\rangle \left\langle \psi' | \hat{p} | \psi \right\rangle \\ &= \int \mathrm{d}\nu(g') \beta \frac{\partial g'}{\partial\beta} \left\langle \psi \middle| e^{-2\hat{p}} \middle| \psi' \right\rangle \left\langle \psi' | \hat{p} | \psi \right\rangle \end{aligned} \tag{47}$$

where we have used the fact that coherent states resolve unity. It can be shown that the expectation value of the operators in the $t \to 0$ collapses the integral to $g' = g$ point [16]. Thus one obtains from the above

$$\begin{aligned}&\mathrm{Tr}\left(\rho^0 H^{U(1)}_{\text{Horizon}}\right)\\&=-\frac{1}{2}\iota C' e^{-t/2}\left[p+g^* e^{-2p}\ \beta\frac{\partial g}{\partial\beta}\right]p+h.c.\\&=C'\beta\frac{\partial\chi}{\partial\beta}p\end{aligned} \tag{48}$$

which is real, and thus

$$\Delta S_{\text{BH}}=0 \tag{49}$$

this is actually the classical QLE as it should be from $\mathrm{Tr}\left(\rho^0 H_{\text{Horizon}}\right)$.

This is obvious, as the way the Hamiltonian is defined, this is simply a function of the Hilbert space outside the horizon, and the matrix elements of this will not yield anything new. We approximated the horizon sphere by summing over v_1 vertices immediately outside the horizon. We could do the same by summing over v_2 vertices immediately within the horizon. For the Lemaitre slice, the metric is smooth at the horizon, and one can take the "quantum operators" evaluated at the vertex v_2. In this case however, as the region is within the classical horizon, the norm of the Killing vector is negative, and n_r has components which are imaginary. The $\varepsilon\to\pm\iota\varepsilon$. Thus $H_{\text{Horizon}}=\frac{1}{2}\left[\sum_{v_1}H_{v_1}+\sum_{v_2}H_{v_2}\right]$. In the evaluation of the QLE, the energy would emerge correct in the $\delta_{e_r}\to 0$ limit as $\varepsilon\to 0$ The regularised Hamiltonian is not Hermitian, and the evolution equation is

$$\iota\hbar\frac{\partial\rho}{\partial\tau}=H\rho-\rho H^{\dagger} \tag{50}$$

And thus the operator which appears in the change of entropy equation is

$$\Delta S_{\text{BH}}=\frac{\iota\delta\tau}{\hbar}\mathrm{Tr}\left[H\rho^0-\rho^0 H^{?}\right] \tag{51}$$

$$\Delta S_{\text{BH}}=\mp\frac{\delta\tau}{\hbar}C'\beta\frac{\partial\chi}{\partial\beta}p \tag{52}$$

The "rate of change" of entropy is thus

$$\dot{\Delta S}_{\text{BH}}=\mp\frac{\tilde{C}}{l_p^2}\beta\frac{\partial\chi}{\partial\beta}p \tag{53}$$

we extracted the κ from C' to get l_p^2 and rewrote the rest of the constants as $\tilde{C}$.

Thus there is a net change in entropy, but, to see if this is Hawking radiation, we have to couple matter to the system.

3.4. SU(2) Case

The SU(2) case is easily reduced to the U(1) case in the actual calculation due to the gauge fixing. This is achieved by making the following observations: To retain the metric as in the same form as the classical metric, we impose the conditions at the operator level

$$P_{e_a}\cdot P_{e_b}=0 \tag{54}$$

such that the corresponding metric has only the diagonal terms as non-zero. With these additional "constraints" on the operators, we can put the $P^I_{e_a}$ such that each has only one component surviving, let's say $P^I_{e_\theta}=\delta^I_3 P_{e_\theta}$. This also makes the holonomy restricted to the U(1) case, as the gauge connection $A^I_{e_\theta}$ gets restricted to the $I=3$ and other directions can be put to zero. Thus we can take the holonomy to be diagonal. If the holonomy matrix is off-diagonal the U(1) projection still works out to be the same

$$h_e=\begin{pmatrix}e^{\iota\zeta} & 0\\ 0 & e^{-\iota\zeta}\end{pmatrix} \tag{55}$$

The operator is then obtained as

$$\begin{aligned}H&=\mathrm{Tr}\left[h_e^{-1}T^I\beta\frac{\partial h_e}{\partial\beta}\right]\\&=\mathrm{Tr}\left[h_e^{-1}T^3\beta\frac{\partial h_e}{\partial\beta}\right]\\&=\beta\frac{\partial\zeta}{\partial\beta}P_e^3\end{aligned} \tag{56}$$

This is same as the U(1) Hamiltonian (upto normalisations). The spin network states also project on to U(1) subgroup, thus giving us the same techniques to use in the calculation of the U(1) states as for this one. To observe this, the non-zero elements for the holonomy matrix

$$h=\begin{pmatrix}a & b\\ -\bar{b} & \bar{a}\end{pmatrix} \tag{57}$$

in the *j*-th representation is given by:

$$\pi_j(h)_{mn}=\sum_l\frac{\sqrt{(j-m)!(j+m)!(j-n)!(j+n)!}}{(j-m-l)!(j-n-l)!(m-n-l)!l!}\times a^{j-n+l}\bar{a}^{j-m+l}b^{m-n-l}\bar{b}^{l} \tag{58}$$

Clearly in the particular case we are considering, the $b=0$, and *m*, *n* = –*j* and *j*. Thus the two non-zero elements are

$$\pi_j(h)_{jj}=e^{2j\zeta}\quad \pi_j(h)_{-j-j}=e^{-2j\zeta} \tag{59}$$

The sum over *j* in the $\mathrm{Tr}\left(\rho^0 H_{\text{Horizon}}\right)$ with the coherent state defined in (3) thus reduces to the U(1) case in the computation of the change in entropy. Thus the rate of change in entropy of a classically spherically symmetric black hole is given by

$$\Delta S_{\text{BH}}=\mp\frac{\tilde{C}\delta\tau}{l_p^2}\sum_v\beta\left[\frac{\partial\chi_{e_\theta}}{\partial\beta}P_{e_\theta}+\frac{\partial\chi_{e_\varphi}}{\partial\beta}P_{e_\phi}\right] \tag{60}$$

where the classical holonomies $h_{e_{\theta(\phi)}} = e^{\iota\chi_{e_{\theta(\phi)}}}$. If we plug in the actual values, we get this to be

$$\Delta S_{\mathrm{BH}} = \pm\frac{2C\varepsilon\delta\tau}{l_p^2}\sum_v \mathrm{d}A_v\beta r_g \tag{61}$$

where $\mathrm{d}A_v$ the area element at vertex v on the sphere. This change in entropy is totally gravitational in origin, and seems to signify the emergence of "geometry" from within the horizon.

In fact, if we some over the area, we get the $\Delta S_{\mathrm{BH}} = \pm\frac{8\pi\varepsilon\delta\tau}{l_p^2}r_g$ (if we set $C = 1/\beta$), which would be the change in entropy when the radius of the horizon changes by $\delta r_g = \varepsilon\delta\tau$!

4. Outgoing Flux of Radiation

In the previous section we found that as the system evolves in time, the horizon fluctuates and the area decreases. But is this Hawking radiation? Adding matter to a "coherent state" description of semiclassical gravity has been discussed [17]. Thus, given a massless scalar field Lagrangian coupled to gravity, whose Hamiltonian is given by

$$H_{\mathrm{sc}} = \int \mathrm{d}^3x\left[\frac{\pi^2}{\sqrt{q}} + \left(\nabla\phi\right)^2\right], \tag{62}$$

the "gravity" in the Hamiltonian can be regularised in terms of the h_e, P_e^I operators in the coherent state formalism. The integral over the three volume gets converted to a sum over the vertices dotting the region. Thus

$$H_{\mathrm{sc}}^v = \sum_v H_v\left(h_e, P_e^I, V\right) \tag{63}$$

This Hamiltonian is an operator, and one evaluates an expectation value of the Hamiltonian in the reduced density matrix of the initial slice, to find the classical behavior of the scalar field as observed by an observer outside the horizon. Thus

$$\mathrm{Tr}\left(\rho^\tau H_{\mathrm{sc}}^\tau\right) \tag{64}$$

This Hamiltonian and the density matrix are then both evolved according to the time-like observers frame. One gets

$$i\hbar\frac{\partial H_{\mathrm{sc}}}{\partial\tau} = \left[H, H_{\mathrm{sc}}\right] \tag{65}$$

This gives

$$\begin{aligned}&\mathrm{Tr}\left(\rho^{\tau+\delta\tau}H_{\mathrm{sc}}^{\tau+\delta\tau}\right) - \mathrm{Tr}\left(\rho^\tau H_{\mathrm{sc}}^\tau\right)\\ &= -\left(\delta\tau\right)^2\mathrm{Tr}\left\{\left[H,\rho^\tau\right]\left[H, H_{\mathrm{sc}}^\tau\right]\right\}\end{aligned} \tag{66}$$

It is very clear thus that the order $\delta\tau$ terms are zero for this. However, allowing for the non-unitary evolution using the non-Hermitian Hamiltonian, the $\delta\tau$ terms survive. In fact the terms are

$$\begin{aligned}&-\frac{\iota\delta\tau}{\hbar}\mathrm{Tr}\left[\left(H\rho - \rho H^\dagger\right)H_{\mathrm{sc}}\right]\\ &-\frac{\iota\delta\tau}{\hbar}\mathrm{Tr}\left[\rho\left(HH_{\mathrm{sc}} - H_{\mathrm{sc}}H^\dagger\right)\right]\end{aligned} \tag{67}$$

The first term is remarkable, it shows that the term giving rise to entropy change teams up with the expectation value of the scalar Hamiltonian. The second term yields corrections, so we ignore it in the first approximation. The exact details of the computation have to be obtained using the coherent state of the matter and gravity coupled system [17,18]. If one simple takes the matter + gravity system in a tensor product form, and one has matter quanta of energy ω sitting at one vertex, then the first term would give new matter in the evolved slice as $\Delta S_{\mathrm{BH}}\,\omega$. The "rate" of particle creation thus has the form $-2\,\varepsilon\omega/T_H$ where T_H is the Hawking temperature for the signs $+(-)\iota\varepsilon$ and negative (positive) ω.

Thus from the above it seems

1) One has found emission of matter quanta from a black hole but from a "semiclassical" description rooted in a theory of quantum gravity, beyond quantum fields in curved space-time.

2) The results indicate a non-unitary evolution which allows space to emerge from within the horizon.

3) The emission is perceived by a static or an accelerating observer as anticipated, and the non-unitary flow might be due to the semiclassical approximations. A quantum evolution using the quantum Hamiltonian might still be unitary.

The above derivation seems to be a "quantum gravity" description of the tunneling mechanism for describing Hawking radiation [19]. However, the results are preliminary and further investigation has to be done.

5. Conclusion

In this paper we showed a method to obtain the origin of Hawking radiation using a coherent state description of a black hole space-time. We took a quantum wavefunction defined on an initial slice, peaked with maximum probability at classical phase space-variables. We then evolved the slice using a Hamiltonian, which is the "quasilocal energy" at the horizon. This QLE evolved the system in the time and the entropy was shown to change, indicating a change in black hole mass and hence an emergence of interesting non-unitary dynamics. One of course has to investigate further to see what is the endpoint of this time evolution. The time flow indicates one might have to formulate quantum theory of gravity rooted in irreversible physics. The presence of additional degrees of freedom in the form of "graphs" also indicates that the classical phase space might not be described by deterministic physics, but by distributions, a manifestation of microscopic irreversible physics in complex systems.

6. Acknowledgements

This research is supported by NSERC; research funds of University of Lethbridge. I would like to thank B. Dittrich for useful discussion; J. Supina for proofreading the manuscript.

REFERENCES

[1] A. Ashtekar, "New Perspectives in Canonical Gravity," Bibliopolis, Napoli, 1988.

[2] C. Rovelli, "Quantum Gravity," Cambridge University Press, Cambridge, 2004. doi:10.1017/CBO9780511755804

[3] T. Thiemann, "Introduction to Modern Canonical Quantum General Relativity," Cambridge University Press, Cambridge, 2007. doi:10.1017/CBO9780511755682

[4] B. Hall, "The Segal-Bargmann 'Coherent State' Transform for Compact Lie Groups," *Journal of Functional Analysis*, Vol. 122, No. 1, 1994, pp. 103-151. doi:10.1006/jfan.1994.1064

[5] T. Thiemann, "Gauge Field Theory Coherent States (GCS): 1. General Properties," *Classical and Quantum Gravity*, Vol. 18, No. 1, 2001, pp. 2025-2064. doi:10.1088/0264-9381/18/11/304

[6] A. Dasgupta, "Semiclassical Quantisation of Space-Times with Apparent Horizons," *Classical and Quantum Gravity*, Vol. 23, No. 3, 2006, pp. 635-671. doi:10.1088/0264-9381/23/3/007

[7] A. Dasgupta, "Coherent States for Black Holes," *Journal of Cosmology and Astroparticle Physics*, Vol. 3, No. 8, 2004, pp. 1-36.

[8] A. Dasgupta, "Semiclassical Horizons," *Canadian Journal of Physics*, Vol. 86, No. 4, 2008, pp. 659-662.

[9] A. Dasgupta and H. Thomas, "Correcting Gravitational Entropy of Spherically Symmetric Horizons," 2006. arXiv:gr-qc/0602006

[10] A. Dasgupta, "Entropic Origin of Hawking Radiation," *Proceedings of* 12*th Marcel Grossman Meeting*, Paris, July 2009.

[11] J. Brown and J. York, "Quasilocal Energy and Conserved Charges Derived from the Gravitational Action," *Physical Review D*, Vol. 47, No. 4, 1993, pp. 1407-1419. doi:10.1103/PhysRevD.47.1407

[12] B. Dittrich and T. Thiemann, "Testing the Master Constraint Program for Loop Quantum Gravity: I. General Framework Class," *Classical and Quantum Gravity*, Vol. 23, No. 4, 2006, pp. 1025-1066. doi:10.1088/0264-9381/23/4/001

[13] B. Bahr and T. Thiemann, "Gauge-Invariant Coherent States for Loop Quantum Gravity. II. Non-Abelian Gauge Groups," *Classical and Quantum Gravity*, Vol. 26, No. 4, 2009, Article ID: 045012.

[14] A. Ashtekar, J. Baez, K. Krasnov and A. Corichi, "Quantum Geometry and Black Hole Entropy," *Physical Review Letters*, Vol. 80, No. 5, 1998, pp. 904-907. doi:10.1103/PhysRevLett.80.904

[15] S. W. Hawking and G. Horowitz, "The Gravitational Hamiltonian, Action, Entropy and Surface Terms," *Classical and Quantum Gravity*, Vol. 13, No. 6, 1996, pp. 1487-1498. doi:10.1088/0264-9381/13/6/017

[16] T. Thiemann and O. Winkler, "Gauge Field Theory Coherent States (GCS) III: Ehrenfest Theorems," *Classical and Quantum Gravity*, Vol. 18, No. 21, 2001, pp. 4629-46841. doi:10.1088/0264-9381/18/21/315

[17] H. Sahlmann and T. Thiemann, "Towards the QFT on Curved Space-Time Limit of QGR. 1. A General Scheme," *Classical and Quantum Gravity*, Vol. 23, No. 3, 2006, pp. 867-908. doi:10.1088/0264-9381/23/3/019

[18] H. Sahlmann and T. Thiemann, "Towards the QFT on Curved Space-Time Limit of QGR. 2. A Concrete Implementation," *Classical and Quantum Gravity*, Vol. 23, No. 3, 2006, pp. 909-954. doi:10.1088/0264-9381/23/3/020

[19] M. K. Parikh and F. Wilczek, "Hawking Radiation as Tunneling," *Physical Review Letters*, Vol. 85, No. 24, 2000, pp. 5042-5045. doi:10.1103/PhysRevLett.85.5042

Journal of Modern Physics, **2012, 3, 1298-1300**
doi:10.4236/jmp.2012.329167 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

The Machian Origin of the Centrifugal Force

Lorenzo Barattini, Paolo Christillin
Department of Physics, University of Pisa and I.N.F.N., Pisa, Italy
Email: christ@df.unipi.it

Received June 20, 2012; revised July 19, 2012; accepted July 28, 2012

ABSTRACT

A derivation of the centrifugal force from an effective vector formulation of gravitation is attempted. The centrifugal force appears to be due to a relativistic effect of the counter-rotating Universe. Gravitomagnetic energy effects, a million times stronger than the self-energy effects responsible for curvature in the GR language, would thus produce the centrifugal acceleration. The Machian picture, already successful in the case of the Coriolis force, gets an additional circumstantial support.

Keywords: Centrifugal Force; Mach's Principle

1. Introduction

In spite of centennial speculations [1,2], a satisfactory, at least semiquantitative, solution of the problem of relative rotational motion is, in our opinion, still lacking.

Is the rotation of, say, the earth with respect to the rest of the Universe equivalent to a counter rotation of the latter?

Further arguments in favor of this logically stringent position have been put forward more recently by Sciama [3], who however has not gone farther than stressing the analogy of gravitomagnetic with magnetic forces, thus making *plausible* such an origin in the case of the Coriolis force.

This has been proven by us in [4].

General relativity (GR) does not address the problem at all, since in its privileged frame of reference ("the precession takes place with respect to the inertial frame, which is generally believed to be defined by the distant extragalactic nebulae, the so called "fixed stars" [5]) no mention is made of the rest of the Universe.

According to [6] also "Ironically, though GR was intended to be based on relational concepts, contrary to its name it still contains absolute elements. This is already expressed in the calculation of the advance of Mercury's perihelion, which is referred to a coordinate system."

The aim of the present paper is to extend the considerations already used in [4] to account for the Coriolis force, to predict unavoidably the *form* of the centrifugal force and to show that its coefficient is, within the present Universe estimates, compatible with the canonical value.

The essential points will be:

1) the proportionality between the gravitomagnetic field of a rotating mass distribution and its angular velocity and their dimensional equivalence;

2) the expression of the gravitomagnetic energy density;

3) the kinematical relation among quantities in inertial and rotating frames by which the centrifugal acceleration can be linked to the gravitomagnetic field (our final equation).

2. The Centrifugal Force from the Counter Rotating Universe

In two recent works of ours [4,7], a set of effective vector equations for low velocity weak field gravitation has been *derived* from special relativity and shown to predict in simple terms the quadrupole gravitational radiation as well as geodetic precession, frame dragging and the gravitational clock effect.

Numerous NR reductions of GR for the same conditions have been recently appeared [11-15] confirming the soundness of such an approach.

Most important, in respect to the matter we are addressing here, the Coriolis force (since the equivalence principle is explicitly used we will speak indifferently of force and acceleration) has been shown to play a crucial role in the abovementioned stationary processes and the role of the (rest of the) Universe to be crucial in explaining the observed effects.

Indeed the gravitomagnetic (GM) force of a rotating mass M at a distance R on a test mass m reads

$$\boldsymbol{F}_{GM} = m\boldsymbol{v}\times\left(\frac{2GM}{c^2R}\boldsymbol{\omega}\right) = 2m\boldsymbol{v}\times\left(\frac{GM}{c^2R}\right)\boldsymbol{\omega} \quad (1)$$

which can be compared to the expression for the Coriolis force

$$\boldsymbol{F}_{Cor} = 2m\boldsymbol{v}\times\boldsymbol{\omega} \tag{2}$$

Thus, when applied to the Universe, if

$$\frac{GM_U}{c^2 R_U} = 1 \tag{3}$$

and this relation compares favourably with present day estimates as well as with other theoretical considerations [9,10] , it follows that

$$\boldsymbol{F}_{Cor} = \boldsymbol{F}_{GM} \tag{4}$$

The relevant point in this argument is that in the relative rotation, the magnetic field generated by distant layers of matter goes as $1/R$ *i.e.* the same behaviour of radiation, rather than the usual $1/R^2$ of Newtonian forces. Therefore a relative more important role even of distant stars is a matter of fact.

Thus the physical origin of the Coriolis force seems to get a semiquantitative confirmation.

Let us pass over to the centrifugal force with some additional remarks.

Now whereas a gravitomagnetic origin of a Coriolis force might seem reasonable (effect of counterrotating masses on a moving one), at first sight it might seem puzzling the effect of the same counterrotating masses on a mass in its rest frame. As it has been pedagogically underlined in [7] however a mass at rest experiences a force from the relativistic effects (*i.e.* $\mathbf{O}(\boldsymbol{v}^2/\boldsymbol{c}^2)$) of moving ones (even if this is customarily expressed as magnetic force).

And indeed the relativistic origin of the effect is evident from the proportionality coefficient $\frac{GM}{c^2 R}$!

The essential point in the previous considerations is that a rotating matter distribution produces a gravitomagnetic field $\boldsymbol{h}$ proportional to the angular velocity of rotation $\boldsymbol{\omega}$

$$\boldsymbol{h} \propto \boldsymbol{\omega} \tag{5}$$

the proportionality coefficient depending of course on the geometry (loop, spherical shell, etc.). In other words a gravitomagnetic field produced by moving masses is dimensionally equivalent to an angular velocity.

This has a profound physical meaning. We know that the $T \neq 0$ cosmic background radiation, essentially coincident with the fixed stars system, represents *the* privileged inertial reference frame. However in terms of relative motion the fact that the rotation of the Universe, as seen from us, be determined by the properties of the other masses (M_U and R_U) renders physical what seemed just a kinematical affair.

Therefore if the previous relation between $\boldsymbol{h}$ and $\boldsymbol{\omega}$ holds true, just a two-fold application of the kinematical relation for operators

$$\left(\frac{\mathrm{d}}{\mathrm{d}t_{(I)}}\right) = \left(\boldsymbol{\omega}\times\right) + \left(\frac{\mathrm{d}}{\mathrm{d}t_{(R)}}\right) \tag{6}$$

(where the suffixes refer respectively to the inertial (I) and rotating (R) frames) yields for the acceleration of the radius vector $\boldsymbol{r}$ the additional centrifugal acceleration.

Let us give some additional arguments.

Consider a symmetric spherical rotating shell.

Its mass (energy and mass are used indifferently) density reads [7]

$$\rho_h = -\frac{1}{4\pi G}\frac{h^2}{2} \tag{7}$$

The Coriolis force has been accounted for by a gravitomagnetic field where for the contribution of the Universe the same expression obtained for a mass loop (the orbiting earth) where $\boldsymbol{h} = \frac{2GM}{c^2 R}\boldsymbol{\omega}$ has been used. On the contrary if one considers spherical symmetry, in the interior the *constant* gravitomagnetic field (see e.g. [8]) is $\boldsymbol{h} = \frac{4}{3}\frac{GM}{c^2 R}\boldsymbol{\omega}$. If we use the value 2 which reproduces the Coriolis force, then from the expression of the field energy $U = \frac{4\pi r^3 \rho_h}{3\boldsymbol{r}}$ one gets

$$\boldsymbol{F}_C = \frac{2}{3}m\boldsymbol{\omega}\times\left(\boldsymbol{\omega}\times\boldsymbol{r}\right) \tag{8}$$

a *centrifugal* force due to the *negative* energy density.

This result is noteworthy in many respects.

First *the centrifugal force is a relativistic effect*!

Second, the correct dimensional requirement for the acceleration comes from a (subtle?) interplay between the expression for the mass density and that for the field, which makes the desired ω^2 factor unavoidable. Moreover the gravitational constant G only enters through the standard weak field formula in brackets.

The coefficients, upon whose evaluation many criticisms might apply, is remarkably close to one.

In this respect let us once more underline how even two drastically different density expressions like $\rho \approx$ constant and $\rho \approx 1/r^2$ which implements the black hole possibility, yield for the self energy the two very close coefficients 3/5 and 1 respectively. Thus even if our evaluation of the total Universe contribution by simply substituting its values is surely questionable, the semiquantitative agreement can hardly be regarded as fortuitous.

The reason why only gravitomagnetic forces act is obvious: within a symmetric spherical shell the static gravitoelectric effects cancel out because of the symmetry, whereas the magnetic ones, constant in R, are different from zero and along $\boldsymbol{\omega}$.

The fact that no retardation for magnetic terms is present, depends on our choice of the gauge, as explained in [7], see also [16].

3. Conclusions

The fact that only *relative rotations* have a physical significance has thus been substantiated, both as regards the expression of the centrifugal force as well as its actual value.

Some more comments are in order.

It is not superfluous to underline the similarities and differences with the case of orbiting satellites [4]. There for the gyroscopes in free fall around the earth the effect of the Universe rotation provided only part of the effect (essentially 1/2) the other being due to the earth rotation. Here of course only the former contributes both for moving objects (Coriolis on the earth) and for masses at rest (centrifugal). Thus this double constraint gives us some more confidence in a non accidental agreement.

Therefore it is really rewarding to have such an interesting link between local and global properties of the Universe and probably a deeper understanding of gravitoelectric effects (self energy or space time curvature where only the earth constituents are involved) and of the gravitomagnetic ones (much bigger centrifugal acceleration determined by the Universe).

In conclusion Berkeley-Mach's [1,2] thinking enters quite rightly our picture of the Universe through the prediction, in addition to the Coriolis, also of the "fictitious" centrifugal force as "real ones"!

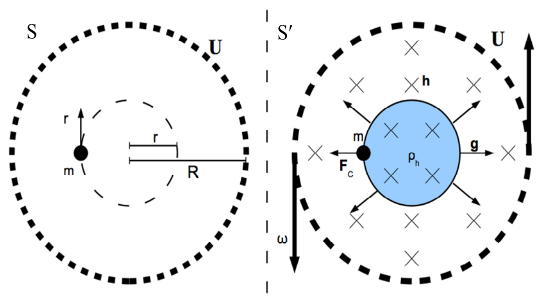


Figure 1. A mass m rotates in the fixed Universe frame S at a distance r from the center. From the mass rest frame S' the Universe is seen to counter rotate, generating a gravitomagnetic field $h \propto \omega$ and a gravitomagnetic field density which causes the "fictitious" centrifugal force. As a particular case our mass is at the surface of the earth and the whole Universe contributes to the repulsion. Thus for the Earth one has the fascinating fact of a gravitoelectric self energy (space curvature) effect of $O(10^{-9})$ with respect to g, and of a much bigger gravitomagnetic influence, due to its rotation (or better to the counter rotation of the Universe), of $O(10^{-3})$. Also in the former case self energy acts "centrifugally" so as to diminish g.

4. Acknowledgements

It is a pleasure to thank C. Bonati, G. Cicogna and G. Morchio for a critical reading of the manuscript and for helpful comments.

REFERENCES

[1] B. De Motu, "Berkeley's Philosophical Writings," Collier, New York, 1974.

[2] E. Mach, "Space and Geometry," Open Court Publishing, Chicago, 1906.

[3] D. W. Sciama, "The Unity of the Universe," Faber and Faber, London, 1959.

[4] P. Christillin and L. Barattini, "The Machian Contribution of the Universe to Geodetic Precession, Frame Dragging and Gravitational Clock Effect," arXiv:1206.4593v1 [physics.gen-ph], 2012.

[5] L. I. Schiff, "Motion of a Gyroscope According to Einstein's Theory," *Proceedings of the National Academy of Sciences*, Vol. 46, 1960, pp. 871-882. doi:10.1073/pnas.46.6.871

[6] H. Lichtenegger and B. Mashoon, "Mach's Principle," arXiv:physics/0407078 [physics.hist-ph].

[7] P. Christillin and L. Barattini, "Gravitomagnetic Forces and Quadrupole Gravitational Radiation from Special Relativity," arXiv:1205.3514v1, 2012.

[8] D. J. Griffiths, "Introduction to Electrodynamics," Pearson, San Francisco, 2008, p. 234

[9] P. Christillin, "Spooky Black Holes: Gravitational Self-Energy and Space Curvature," *EPJ Plus*, Vol. 126, 2011, p. 48.

[10] P. Christillin, "Speakable and Unspeakable in Cosmology: Dark Matter vs. Gravitational Self-Energies. Hubble's Constant the Cosmological Term and All That," *EPJ Plus*, Vol. 126, 2011, p. 88.

[11] R. P. Lano "Gravitational Meissner Effect," 1996. arXiv:hep-th/9603077

[12] D. Bedford and P. Krumm, "On Relativistic Gravitation," *American Journal of Physic*, Vol. 53, No. 9, 1985, p. 889. doi:10.1119/1.14358

[13] H. Kolbenstvedt, "Gravomagnetism in Special Relativity," *American Journal of Physics*, Vol. 56, No. 6, 1988, p. 523. doi:10.1119/1.15544

[14] B. Mashhoon, *et al.*, "Gravitomagnetism and the Clock Effect," arXiv:gr-qc/991202, 1999.

[15] M. Agop *et al.*, "On Gravitational Shielding in Electromagnetic Fields," arXiv:gr-qc/9911011.

[16] R. P. Feynman, "Lectures on Gravitation," Penguin Books, London, p. 40.

Journal of Modern Physics, **2012, 3, 1301-1310**
doi:10.4236/jmp.2012.329168 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

A Global Solution of the Einstein-Maxwell Field Equations for Rotating Charged Matter

Andreas Georgiou
School of Physics Astronomy and Mathematics, University of Hertfordshire, Hatfield, UK
Email: a.georgiou@herts.ac.uk

Received June 27, 2012; revised July 31, 2012; accepted August 9, 2012

ABSTRACT

A stationary axially symmetric exterior electrovacuum solution of the Einstein-Maxwell field equations was obtained. An interior solution for rotating charged dust with vanishing Lorentz force was also obtained. The two spacetimes are separated by a boundary which is a surface layer with surface stress-energy tensor and surface electric 4-current. The layer is the spherical surface bounding the charged matter. It was further shown, that all the exterior physical quantities vanished at the asymptotic spatial infinity where spacetime was shown to be flat. There are two different sets of junction conditions: the electromagnetic junction conditions, which were expressed in the traditional 3-dimensional form of classical electromagnetic theory; and the considerably more complicated gravitational junction conditions. It was shown that both—the electromagnetic and gravitational junction conditions—were satisfied. The mass, charge and angular momentum were determined from the metric. Exact analytical formulae for the dipole moment and gyromagnetic ratio were also derived. The conditions, under which the latter formulae gave Blackett's empirical result for rotating stars, were investigated.

Keywords: Gravitation; Exact Solutions; Einstein-Maxwell Equations; Rotation; Charged Dust

1. Introduction

There are difficulties in finding exact solutions of the Einstein or of the Einstein-Maxwell field equations for a volume distribution of rotating bounded matter [1]. Such solutions should consist of an interior filled with matter and an asymptotically flat vacuum or electrovacuum exterior, these being separated by a surface on which appropriate boundary conditions should be satisfied. The main aim of this work is to obtain an exterior and matching interior solution of the Einstein-Maxwell field equations with finite bounded rotating charged matter as a source of the spacetime. Due to the rotation, the boundary will actually be an oblate spheroid, but it is assumed that it is a spherical surface with equation $r = a$. The main objective and emphasis after all, is to see how far the attempt at finding a solution can be taken—a solution with finite bounded rotating matter as a source of the spacetime. The additional complication of spheroidal co-or-dinates is avoided, in a problem which is already enormously complicated.

Most of the equations and expressions for the various physical quantities are difficult to derive and they require involved and lengthy analysis. It is not therefore possible or desirable to include these calculations in the paper, but directions in which to proceed are indicated.

2. The Einstein-Maxwell Field Equations

Consider electrically charged pressure-free matter (charged dust) bounded by the hypersurface $r = a$ and rotating with constant angular velocity about the polar axis $\theta = 0$ under zero Lorentz force. It is assumed that the current is carried by the dust. The transformed expression (2.1) in [2] for the Weyl-Lewis-Papapetrou metric for a stationary axially symmetric spacetime V is

$$\begin{aligned}\mathrm{d}s^2 &= -e^{\mu}\left(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2\right)\\ &\quad -F^{-1}\left(r^2\sin^2\theta - K^2\right)\mathrm{d}\phi^2 - 2K\mathrm{d}\phi\mathrm{d}t + F\mathrm{d}t^2\end{aligned} \tag{1}$$

where we have taken the signature of the spacetime metric tensor $g_{\lambda\mu}$ to be -2. It is implicit in the form (1) of the metric that we have assumed, without loss of generality, that $LF + K^2 = r^2\sin^2\theta$ and so the component g_{33} of $g_{\lambda\mu}$ is $g_{33} = -L$. We shall use units $c = G = 1$ where c is the vacuum speed of light and G the Newtonian gravitational constant. Unless otherwise specified, we shall adopt the convention in which Roman indices take the values 1, 2, 3 for the space coordinates (r,θ,ϕ) which are spherical polar coordinates co-moving with the dust, and Greek indices take the values $(1,2,3,4)$ for the spacetime coordinates (r,θ,ϕ,t). Semicolons and commas indicate covariant and partial derivatives respectively, and the suffixes r and θ denote partial differentiation with respect to r and θ. All the functions are assumed to depend on r and θ only, or they are constant.

The results to be used in this work may be found in a number of different publications [2,3] but we shall use [2] where all the necessary equations have been collected together and written in terms of the cylindrical polar coordinates and time (z,ρ,ϕ,t). We shall transform those equations in [2] that are required here, to the spherical polar coordinates and time (r,θ,ϕ,t) with $r=\left(\rho^2+z^2\right)^{1/2}$, $\tan\theta=\rho/z$, $\phi=\phi$, $z=r\cos\theta$, $\rho=r\sin\theta$ and $t=t$.

The contravariant and covariant forms u^λ and u_λ of the 4-velocity are

$$u^\lambda=\delta_4^\lambda F^{-1/2} \qquad u_\lambda=F^{1/2}\left(-w\delta_\lambda^3+\delta_\lambda^4\right). \tag{2}$$

The electric 4-current J^λ, the electromagnetic 4-potential A_λ and the Faraday tensor $F_{\lambda\mu}$, are

$$\begin{aligned} J^\lambda &= \sigma u^\lambda \\ A_\lambda &= A_3\delta_\lambda^3+A_4\delta_\lambda^4 \\ F_{\lambda\mu} &= A_{\lambda,\mu}-A_{\mu,\lambda}. \end{aligned} \tag{3}$$

where σ is the electric charge density. The Einstein-Maxwell field equations for charged dust are

$$G_\mu^\lambda=-8\pi T_\mu^\lambda \tag{4}$$

$$\begin{aligned} &F_{\lambda\mu,\nu}+F_{\mu\nu,\lambda}+F_{\nu\lambda,\mu}=0 \\ &\frac{1}{\sqrt{-g}}\left(\sqrt{-g}F^{\lambda\nu}\right)_{,\nu}=4\pi J^\lambda. \end{aligned} \tag{5}$$

Here, G_μ^λ is the Einstein tensor

$$G_\mu^\lambda := R_\mu^\lambda-\frac{1}{2}\delta_\mu^\lambda R \tag{6}$$

where R_μ^λ is the Ricci tensor of the spacetime defined by its fully covariant form as

$$R_\mu^\lambda := \Gamma^\beta_{\lambda\beta,\mu}-\Gamma^\beta_{\lambda\mu,\beta}+\Gamma^\gamma_{\lambda\beta}\Gamma^\beta_{\mu\gamma}-\Gamma^\beta_{\lambda\mu}\Gamma^\gamma_{\beta\gamma} \tag{7}$$

with $\Gamma^\nu_{\lambda\mu}$ the Christoffel symbols of the second kind based on the metric of V in Equation (1), $R=g^{\beta\gamma}R_{\beta\gamma}$ is the spacetime scalar curvature invariant and g is the determinant of $g_{\lambda\mu}$. The total stress-energy tensor T_μ^λ is

$$T_\mu^\lambda=M_\mu^\lambda+E_\mu^\lambda \tag{8}$$

where

$$M_\mu^\lambda=\rho u^\lambda u_\mu \tag{9}$$

$$E_\mu^\lambda=\frac{1}{4\pi}\left(\frac{1}{4}\delta_\mu^\lambda F^{\beta\gamma}F_{\beta\gamma}-F^{\lambda\beta}F_{\mu\beta}\right) \tag{10}$$

are, respectively, the matter and electromagnetic stress-energy tensors and ρ is the mass density.

Instead of expressing the electromagnetic field equations in 4-dimensional form as in Equations (5), we shall use the Maxwell form (Maxwell's equations), because we can make direct comparisons with the results from classical electromagnetic theory. The electric and magnetic intensities and corresponding inductions in 3-vector form, are [4,5]

$$\begin{aligned} &\boldsymbol{E}=E_a=F_{a4} \qquad \boldsymbol{D}=D^a=-\sqrt{F}F^{a4}, \\ &\boldsymbol{H}=H_a=\frac{1}{2}\sqrt{F}\ e_{akp}F^{kp} \quad \boldsymbol{B}=B^a=\frac{1}{2}e^{akp}F_{kp} \end{aligned} \tag{11}$$

where $e_{akp}=\sqrt{-\gamma}\ \varepsilon_{akp}$, $e^{akp}=\varepsilon_{akp}/\sqrt{-\gamma}$ are the completely antisymmetric permutation tensors, $\sqrt{-\gamma}=\sqrt{-g}/\sqrt{F}$, γ being the determinant of the spatial metric tensor γ_{ab} which is given by $\gamma_{ab}=g_{ab}+\gamma_a\gamma_b$ with $\gamma_a=g_{a4}/\sqrt{F}$, and ε_{akp} is the Levi-Civita symbol. It is easy to show that $\sqrt{-\gamma}=F^{-3/2}r^2\sin\theta$.

The transformed equations (2.14) and (2.13) of [2] may be written as

$$\begin{aligned} 4\pi r^2\sin^2\theta e^\mu J^3 &= K\nabla^{*2}A_4+F\nabla^{*2}A_3 \\ &+K_rA_{4r}+r^{-2}K_\theta A_{4\theta}+F_rA_{3r}+r^{-2}F_\theta A_{3\theta} \end{aligned} \tag{12}$$

$$\begin{aligned} 4\pi r^2\sin^2\theta e^\mu J^4 &= -L\nabla^{*2}A_4+K\nabla^{*2}A_3 \\ &-L_rA_{4r}-r^{-2}L_\theta A_{4\theta}+K_rA_{3r}+r^{-2}K_\theta A_{3\theta} \end{aligned} \tag{13}$$

where the operators ∇^2 and ∇^{*2} are defined by

$$\nabla^2\equiv\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{2}{r}\frac{\partial}{\partial r}+\frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta} \quad 14)$$

$$\nabla^{*2}\equiv\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}-\frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}. \tag{15}$$

Equations (12) and (13) are the detailed form of the source-containing Maxwell equations given in the second of (5).

The non-zero components of the Ricci tensor obtained from the transformed Equations (2.16)-(2.21) of [2] are:

$$\begin{aligned} R_3^3=-R_4^4=\frac{1}{2}e^{-\mu}\Bigg\{&\frac{\nabla^2F}{F}-\frac{1}{F^2}\left(F_r^2+\frac{F_\theta^2}{r^2}\right) \\ &+\frac{F^2}{r^2\sin^2\theta}\left(w\nabla^{*2}w+w_r^2+\frac{w_\theta^2}{r^2}\right) \\ &+\frac{2Fw}{r^2\sin^2\theta}\left(F_rw_r+\frac{F_\theta w_\theta}{r^2}\right)\Bigg\} \end{aligned} \tag{16}$$

$$\begin{aligned} R_1^1=-\frac{1}{2}e^{-\mu}\Bigg\{&\mu_{rr}+\frac{\mu_{\theta\theta}}{r^2}+\frac{\mu_\theta\cot\theta}{r^2} \\ &+\frac{F_r^2}{F^2}-\frac{2F_r}{rF}-\frac{F^2w_r^2}{r^2\sin^2\theta}\Bigg\} \end{aligned} \tag{17}$$

$$R_2^2 = -\frac{1}{2}e^{-\mu}\left\{\mu_{rr} + \frac{2\mu_r}{r} + \frac{\mu_{\theta\theta}}{r^2} - \frac{\mu_\theta \cot\theta}{r^2} + \frac{F_\theta^2}{r^2F^2} - \frac{2F_\theta\cot\theta}{r^2F} - \frac{F^2w_\theta^2}{r^4\sin^2\theta}\right\} \quad (18)$$

$$R_4^3 = -\frac{e^{-\mu}}{2r^2\sin^2\theta}\left\{F\nabla^{*2}w + 2F\left(F_r w_r + \frac{F_\theta w_\theta}{r^2}\right)\right\} \quad (19)$$

$$R_3^4 = \frac{2KR_3^3}{F} - \frac{\left(r^2\sin^2\theta - K^2\right)R_4^3}{F^2} \quad (20)$$

$$R_1^2 = \frac{R_2^1}{r^2} = \frac{e^{-\mu}}{2r^2}\left\{-\frac{F_rF_\vartheta}{F^2} + \frac{1}{F\sin\theta}\left(F_r\cos\theta + \frac{F_\theta\sin\theta}{r}\right) + \frac{1}{\sin\theta}\left(\mu_r\cos\theta + \frac{\mu_\theta\sin\theta}{r}\right) + \frac{F^2w_rw_\theta}{r^2\sin^2\theta}\right\} \quad (21)$$

$$R = -e^{-\mu}\left\{\mu_{rr} + \frac{\mu_{\theta\theta}}{r^2} + \frac{\mu_r}{r} + \frac{1}{2F^2}\left(F_r^2 + \frac{F_\theta^2}{r^2}\right) - \frac{1}{rF}\left(F_r + \frac{F_\theta\cot\theta}{r}\right) - \frac{F^2}{2r^2\sin^2\theta}\left(w_r^2 + \frac{w_\theta^2}{r^2}\right)\right\} \quad (22)$$

The entire Riemannian spacetime V, will be separated into the following 4-dimensional manifolds: the hypersurface Σ with equation $r = a$ separates V into the interior $V^-\left(0 \le r \le a\right)$ and exterior $V^+\left(a \le r < \infty\right)$ spacetimes. We shall use the + and – signs to denote quantities in V^+ and V^- whenever it is necessary to do so. Quantities without the + or – indicators, may be associated either with V^+ or with V^-.

3. The Exterior Solution

In accordance with the formalism in [2], we first form the complex function

$$\Phi = \eta + i\zeta \quad (23)$$

where η and ζ are harmonic functions. With a star denoting complex conjugation, the metric functions F^+ and μ^+ are then given by

$$\left(F^+\right)^{-1} = \exp\left(\mu^+\right) = \Phi\Phi^* = \eta^2 + \zeta^2. \quad (24)$$

If we denote the real and imaginary parts of Φ^{-1} by φ and ψ, then

$$\varphi = \frac{\eta}{\eta^2+\zeta^2} \qquad \psi = -\frac{\zeta}{\eta^2+\zeta^2}. \quad (25)$$

We now choose the functions η and ζ as follows:

$$\eta = bC\left(r\right) \qquad C\left(r\right) = 1 + \frac{m}{r} \qquad a \le r < \infty \qquad 0 \le \theta \le \pi \quad (26)$$

$$\zeta\left(r,\theta\right) = \begin{cases} b\left\{-\lambda C\left(r\right) + \sum_{n=1}^{\infty} A_{2n-1}a^{2n}r^{-2n}P_{2n-1}\left(\cos\theta\right)\right\} \\ a \le r < \infty \quad 0 \le \theta < \pi/2 \\ 0 \\ a \le r < \infty \qquad \theta = \pi/2 \\ b\left\{+\lambda C(r) + \sum_{n=1}^{\infty} A_{2n-1}a^{2n}r^{-2n}P_{2n-1}\left(\cos\theta\right)\right\} \\ a \le r < \infty \quad \pi/2 < \theta \le \pi \end{cases} \quad (27)$$

where b, m and λ are constants whose significance will emerge later. From now on we shall omit writing the argument $\cos\theta$ of the Legendre polynomials and we shall write, for example, P_{2n-1} instead of $P_{2n-1}\left(\cos\theta\right)$. We note the significant fact that at $\theta = \pi/2$, $P_{2n-1} = 0$; this enables us to set $\zeta\left(r,\theta\right) = 0$ at $\theta = \pi/2$ $a \le r < \infty$ as in (27).

The function $w^+ := K^+/F^+$ and the electromagnetic 4-potential $A_\lambda^+ = A_3^+\delta_\lambda^3 + A_4^+\delta_\lambda^4$ in the exterior are obtained from

$$w_r^+ = 2\eta\zeta_\theta\sin\theta \qquad w_\theta^+ = 2r^2\sin\theta\left(\zeta\eta_r - \eta\zeta_r\right) \quad (28)$$

$$A_{3r}^+ = -w^+\varphi_r + \left(F^+\right)^{-1}\psi_\theta\sin\theta \quad (29)$$

$$A_{3\theta}^+ = -w^+\varphi_\theta - \left(F^+\right)^{-1}\psi_r r^2\sin\theta \quad (30)$$

$$A_4^+ = -b + \varphi \quad (31)$$

where an arbitrary constant in A_4^+ was set equal to b in order to satisfy the continuity condition of A_4. Note that the full expression for w_r^+ in the first of (28) is $w_r^+ = -2\sin\theta\left(\zeta\eta_\theta - \eta\zeta_\theta\right)$, but by (26), $\eta_\theta = 0$.

From Equations (24) and (28)-(31), we obtain the following expressions for $\left(F^+\right)^{-1}$, $\exp\left(\mu^+\right)$, w^+, A_3^+ and A_4^+:

$$\left(F^+\right)^{-1} = \exp\left(\mu^+\right) = b^2C^2\left(r\right) + b^2\left\{\mp\lambda C\left(r\right) + \sum_{n=1}^{\infty} A_{2n-1}a^{2n}r^{-2n}P_{2n-1}\right\}^2 \quad (32)$$

$$w^+ \equiv \frac{K^+}{F^+} = 2b^2\sum_{n=1}^{\infty}\frac{A_{2n-1}}{4n-1}a^{2n} \times\left\{2nC\left(r\right)r^{1-2n} - mr^{-2n}\right\}\left(P_{2n-2} - P_{2n}\right) \quad (33)$$

$$A_3^+ = -bC(r)F^+w^+ + b^2\sum_{n=1}^{\infty}\frac{A_{2n-1}}{4n-1}\left\{2nC\left(a\right)a^{2n}r^{1-2n} - m\right\} \times\left(P_{2n-2} - P_{2n}\right) \quad (34)$$

$$A_4^+ = bF^+C\left(r\right) - b. \quad (35)$$

It is a little difficult to solve the two equations in (28) to find w^+ in (33). It is even more difficult to solve the two Equations (29)-(30) to find A_3^+ in (34) and complete details of the calculation are not given. Whenever there are two signs in a term, the upper sign gives the expression in $0 \le \theta < \pi/2$ and the lower sign the expression in $\pi/2 < \theta \le \pi$, as in Equation (32).

The function B defined by

$$B = \begin{cases} +\lambda C(a) & 0 \le \theta < \pi/2 \\ 0 & \theta = \pi/2 \\ -\lambda C(a) & \pi/2 < \theta \le \pi \end{cases} \tag{36}$$

has Legendre polynomial expansion of the form

$$\begin{aligned} B &= \sum_{n=1}^{\infty} A_{2n-1} P_{2n-1} \\ A_{2n-1} &= \left(\frac{4n-1}{2}\right) \int_{-1}^{1} B P_{2n-1}(x) \mathrm{d}x \end{aligned} \tag{37}$$

where $A_{2n-2} = 0 \quad n = 1, 2, \cdots$ It therefore follows from (27), that $\zeta(a, \theta) = 0$. The function B in (36) satisfies the conditions for such an expansion [6] and we have for the odd coefficients A_{2n-1}

$$A_{2n-1} = \frac{(-1)^{n-1}(4n-1)(2n-2)!n}{2^{2n-1}(n!)^2} \lambda C(a), \quad n=1, 2, \cdots \tag{38}$$

With $\zeta(a, \theta) = 0$, by Equations (24) and (26), the metric function F^+ at $r = a$ becomes $F^+(a, \theta) = 1/\eta^2 = 1/b^2 C^2(a)$. It will be shown in Section 4, that $F^-(r, \theta) = 1$ everywhere in the interior. In order to satisfy the junction condition at $r = a$ therefore, we must have $F^+(a, \theta) = 1/b^2 C^2(a) = 1$. It is easily seen that, as $r \to \infty$, $F(r, \theta) \to 1/b^2(1 + \lambda^2)$, which is a constant. If we take this to be equal to 1, we obtain $b^2(1 + \lambda^2) = 1$ and collecting these relationships together we have

$$\begin{aligned} b^2(1+\lambda^2) = 1 \quad b^2 = \frac{1}{C^2(a)} \\ \lambda^2 = \frac{2m}{a} + \left(\frac{m}{a}\right)^2. \end{aligned} \tag{39}$$

The third of Equation (39) is the result of substituting the second of these equations into the first, bearing in mind the second of Equation (26) for $r = a$.

For the calculations that follow the functions X and Y defined by

$$X = \sum_{n=1}^{\infty} A_{2n-1} 2n a^{2n} r^{-2n-1} P_{2n-1} \tag{40}$$

$$Y = \sum_{n=1}^{\infty} \frac{A_{2n-1}}{4n-1} 2n(2n-1) a^{2n} r^{-2n} (P_{2n-2} - P_{2n}) \tag{41}$$

will be required. We express η_r, ζ_r and ζ_θ as

$$\eta_r = -\frac{bm}{r^2} \quad \zeta_r = \pm\frac{b\lambda m}{r^2} - bX \quad \zeta_\theta = -\frac{bY}{\sin\theta}. \tag{42}$$

The components of G_μ^λ are therefore calculated using the exterior functions (32)-(35) with Equations (16)-(22) and, whenever necessary, bearing in mind the first of Equation (39). The calculations give the following nonzero components:

$$\begin{aligned} {}^+G_1^1 &= -\left({}^+G_2^2\right) = -8\pi\, {}^+E_1^1 \\ &= \left(F^+\right)^2 \left\{-\frac{m^2}{r^4} \pm \frac{2b^2\lambda m X}{r^2} - b^2 X^2 + \frac{b^2 Y^2}{r^2 \sin^2\theta}\right\} \end{aligned} \tag{43}$$

$$\begin{aligned} {}^+G_2^1 &= r^2\left({}^+G_1^2\right) = -8\pi\, {}^+E_2^1 \\ &= 2b^2\left(F^+\right)^2 \left(\pm\frac{\lambda m}{r^2} - X\right) \frac{Y}{\sin\theta} \end{aligned} \tag{44}$$

$${}^+G_4^3 = -8\pi\, {}^+E_4^3 = \frac{2b^2 m \left(F^+\right)^3 Y}{r^4 \sin^2\theta} \tag{45}$$

$$\begin{aligned} {}^+G_3^4 &= -8\pi\, {}^+E_3^4 \\ &= 2\left\{\left(F^+\right)^2 w^+ \left(\frac{m^2}{r^4} \mp \frac{2b^2\lambda m X}{r^2} + b^2 X\right)\right. \\ &\quad - \frac{b^2 F^+ m Y}{r^2} + \frac{b^2\left(F^+\right)^2 w^+ Y^2}{r^2 \sin^2\theta} \\ &\quad \left. - \frac{b^2\left(F^+\right)^3\left(w^+\right)^2 m Y}{r^4 \sin^2\theta}\right\} \end{aligned} \tag{46}$$

$$\begin{aligned} {}^+G_4^4 &= -\left({}^+G_3^3\right) = -8\pi\, {}^+E_4^4 \\ &= -\left(F^+\right)^2 \left\{\frac{m^2}{r^4} \mp \frac{2b^2\lambda m X}{r^2}\right. \\ &\quad \left. + b^2 X^2 + \frac{b^2 Y^2}{r^2 \sin^2\theta} - \frac{b^2\left(F^+\right) m w^+ Y}{r^4 \sin^2\theta}\right\}. \end{aligned} \tag{47}$$

Here, ${}^+E_\mu^\lambda$ are the nonzero components of the electromagnetic energy tensor. The components of ${}^+E_\mu^\lambda$ were obtained from (10) the third of (3) for $F_{\lambda\mu}$, the exterior electromagnetic potentials in (34) and (35). Equation (22) gives $R = 0$ in V^+ and so by (6), $G_\mu^\lambda = R_\mu^\lambda$ whether λ is equal to μ or not. Another consequence of the result $R = 0$, is that the matter energy tensor M_μ^λ will be null as should be the case in the electrovac V^+.

The sourceless Maxwell equations in the first of (5) give $F_{41,\theta}^+ - F_{42,r}^+ = 0$ and $F_{23,r}^+ + F_{31,\theta}^+ = 0$. By the third of (3) and with A_3^+ and A_4^+ given by (34) and (35), these become $-A_{3,\theta r}^+ + A_{3,r\theta}^+ = 0$, $A_{4,r\theta}^+ - A_{4,\theta r}^+ = 0$ which are trivially satisfied. The source-containing Maxwell Equations (12) and (13) with A_3^+ and A_4^+ given by Equations (34) and (35) will give $J^3 = 0$ and $J^4 = 0$ and so the 4-current is null in the electrovac V^+.

4. The Interior Solution

In accordance with the results of [2], the functions F^-, μ^- and A_4^- are constant which we shall take as

$$F^- = 1 \quad \mu^- = 0 \quad A_4^- = 1 - b \tag{48}$$

The functions K^- and A_3^- satisfy an equation of the form $\nabla^{*2}\Psi = 0$ with ∇^{*2} given by (15). This implies that K^- for example, is obtained from

$$K^- = r\sin\theta\left(\xi_r \sin\theta + r^{-1}\xi_\theta \cos\theta\right) \tag{49}$$

where ξ is a harmonic function, which therefore satisfies Laplace's equation $\nabla^2\xi = 0$ with ∇^2 given by (14).

We choose ξ as

$$\xi = \pm\sum_{n=1}^{\infty} D_{2n-1} r^{2n} P_{2n} \tag{50}$$

where the constants D_{2n-1} are determined from the junction condition $K^-(a,\theta) = K^+(a,\theta)$. We use Equation (49) for K^- with ξ given by (50) to find

$$K^- = \sum_{n=1}^{\infty} D_{2n-1} 2nr^{2n}\left[\left(1-\cos^2\theta\right)P_{2n} + \frac{2n+1}{4n+1}\left(P_{2n+1} - P_{2n-1}\right)\cos\theta\right].$$

We can further show that

$$\left(1-\cos^2\theta\right)P_{2n} = P_{2n} - \frac{\cos\theta}{4n+1}\left(2nP_{2n-1} + (2n+1)P_{2n+1}\right)$$

and with this, the above expression for K^- becomes

$$K^- = \sum_{n=1}^{\infty} D_{2n-1} 2nr^{2n}\left(P_{2n} - \cos\theta P_{2n-1}\right).$$

Finally, after a little manipulation, the above expression for K^- becomes

$$K^- = -\sum_{n=1}^{\infty} D_{2n-1}\frac{2n(2n-1)}{4n-1}\times r^{2n}\left(P_{2n-2} - P_{2n}\right). \tag{51}$$

The junction condition for the continuity of K implies that on $r = a$, $K^-(a,\theta) = K^+(a,\theta)$. Using the expression (33) for w^+ and bearing in mind that $F^+(a,\theta) = 1$, we have $K^+(a,\theta) = w^+(a,\theta)$. It therefore follows from (33) and (51), that the constants D_{2n-1} are given by

$$-D_{2n-1} 2n(2n-1)a^{2n} = 2b^2 A_{2n-1}\left(2naC(a) - m\right).$$

This implies that K^-, but also w^- and A_3^- are given by

$$K^- = w^- = -2A_3^- = 2b^2\sum_{n=1}^{\infty}\frac{A_{2n-1}}{4n-1}a^{-2n}r^{2n} \times\left\{2naC(a) - m\right\}\left(P_{2n-2} - P_{2n}\right). \tag{52}$$

The functions Z and U defined by

$$Z = \sum_{n=1}^{\infty}\frac{A_{2n-1}}{4n-1}\left(2naC(a) - m\right) \times 2na^{-2n}r^{2n-1}\left(P_{2n-2} - P_{2n}\right) \tag{53}$$

$$U = \sum_{n=1}^{\infty} A_{2n-1}\left(2naC(a) - m\right)a^{-2n}r^{2n}P_{2n-1} \tag{54}$$

will be required to simplify the components of the Einstein tensor.

The components of ${}^-G_\mu^\lambda$ are calculated using the interior functions (48) and (52) with Equations (16)-(22) and, whenever necessary, bearing in mind the first of Equation (39). The calculations give the following nonzero components:

$${}^-G_1^1 = -\left({}^-G_2^2\right) = -8\pi\,{}^-E_1^1 = -b^4\left(\frac{U^2}{r^4} - \frac{Z^2}{r^2\sin^2\theta}\right) \tag{55}$$

$${}^-G_3^3 = -8\pi\,{}^-E_3^3 = b^4\left(\frac{U^2}{r^4} + \frac{Z^2}{r^2\sin^2\theta}\right) \tag{56}$$

$${}^-G_4^4 = -8\pi\left({}^-M_4^4 + {}^-E_4^4\right) = -3b^4\left(\frac{U^2}{r^4} + \frac{Z^2}{r^2\sin^2\theta}\right) \tag{57}$$

$${}^-G_3^4 = -8\pi\left({}^-M_3^4 + {}^-E_3^4\right) = 4b^4\left(\frac{U^2}{r^4} + \frac{Z^2}{r^2\sin^2\theta}\right)w^- \tag{58}$$

$${}^-G_2^1 = r^2\left({}^-G_1^2\right) = -8\pi\,{}^-E_2^1 = \frac{2b^4UZ}{r^2\sin\theta}. \tag{59}$$

Here, ${}^-E_\mu^\lambda$ and ${}^-M_\mu^\lambda$ are the nonzero components of the electromagnetic and mass energy tensors respectively. The components of ${}^-E_\mu^\lambda$ were obtained from (10) the third of (3) for $F_{\lambda\mu}$, the interior functions in (48) and (52). The components of ${}^-M_\mu^\lambda$ were obtained from Equations (3) and (9) together with the interior functions (48) and (52). Equations (55)-(59) state that Einstein's Field Equations are satisfied in V^-. The sourceless Maxwell equations in the first of (5) give $F_{23,r}^- + F_{31,\theta}^- = 0$. By the third of (3) and with A_3^- given in Equation (52), this becomes $-A_{3,\theta r}^- + A_{3,r\theta}^- = 0$ which is trivially satisfied. The source-containing Maxwell Equations (12) and (13) with A_4^- and A_3^- as in (48) and (52) respectively, will give

$${}^-J^3 = 0 \quad {}^-J^4 = \sigma^- = -\frac{b^4}{2\pi}\left(\frac{U^2}{r^4} + \frac{Z^2}{r^2\sin^2\theta}\right). \tag{60}$$

It is easily seen that

$$8\pi^{-}M_4^4 = 8\pi\rho^{-} = 2b^2\left(\frac{U^2}{r^4} + \frac{Z^2}{r^2\sin^2\theta}\right).$$

It follows from this and Equation (60) that $\sigma^{-} = -2\rho^{-}$ or, in dimensional units, $\sigma^{-} = -2\sqrt{G}\rho^{-}$.

If N is any function in V, we write

$$\begin{aligned}&[N] := N^{+}(a,\theta) - N^{-}(a,\theta)\\ &N^{+}(a,\theta) := \lim_{\varepsilon\to 0} N(a+\varepsilon,\theta)\\ &N^{-}(a,\theta) := \lim_{\varepsilon\to 0} N(a-\varepsilon,\theta)\end{aligned} \tag{61}$$

where the second and third of Equation (61), represent the values of N on the V^{+} and V^{-} sides of Σ.

It follows from Equations (32)-(35), (48) and (52) that $[g_{\alpha\beta}] = 0$ and $[A_\lambda] = 0$. The functions $g_{\alpha\beta}$ and A_λ are therefore continuous across Σ, but one degree of smoothness is lost because the first order partial r-derivatives of these functions are discontinuous on Σ. It follows that the ordinary junction conditions requiring the continuity of the directional derivatives of these functions normal to Σ, cannot be applied. The discontinueties of these normal derivatives will generate a surface layer on Σ with surface stress-energy tensor and surface 4-current and a more complicated set of junction conditions will apply. The Equations (12) and (13) for J^3 and J^4 will give rise to expressions with factors of delta-functions and first order partial r-derivatives which are discontinuous on Σ. We shall denote these terms by Gothic symbols, and we find from (12) and (13) that these are

$$\begin{aligned}&4\pi r^2\sin^2\theta e^{\mu}\mathfrak{J}^3\\ &=\left\{KA_{4r}^{+} + F\left(A_{3r}^{+} - A_{3r}^{-}\right)\right\}\times\delta(r-a)\end{aligned} \tag{62}$$

$$\begin{aligned}&4\pi r^2\sin^2\theta e^{\mu}\mathfrak{J}^4\\ &=\left\{-LA_{4r}^{+} + K\left(A_{3r}^{+} - A_{3r}^{-}\right)\right\}\times\delta(r-a)\end{aligned} \tag{63}$$

where A_3^{+}, A_4^{+} and A_3^{-}, are given in (34), (35) and (52) respectively. To obtain the surface 4-current $\boldsymbol{s}$ and s^4, we form the integrals of J^3 and J^4 with respect to proper distance measured perpendicularly through Σ from $r = a-\varepsilon$ to $r = a+\varepsilon$ and then find the limits as $\varepsilon \to 0$. There are no sign indicators with the metric functions K, F and L in (62) and (63) because their values in both, V^{+} and V^{-}, are required in these integrations, where the only nonzero contributions will arise from the delta-function parts $\mathfrak{J}^3$ and $\mathfrak{J}^4$ of J^3 and J^4 in Equations (62) and (63). With $\hat{\boldsymbol{\varphi}}$ the unit vector in the ϕ direction, this gives

$$\boldsymbol{s} = s^3\hat{\boldsymbol{\varphi}} = \frac{b^3}{2\pi a^2\sin^2\theta}\left\{Y(a,\theta) + bZ(a,\theta)\right\}\hat{\boldsymbol{\varphi}}$$

$$\boldsymbol{s}^4 = -\frac{bm}{4\pi a^2} + \frac{b^3}{2\pi a^2\sin^2\theta}w(a,\theta)\left\{Y(a,\theta) + bZ(a,\theta)\right\}$$

The electromagnetic junction conditions are

$$\begin{aligned}[\boldsymbol{D}\cdot\hat{\boldsymbol{n}}] &= 4\pi\boldsymbol{s}^4\\ &= -\frac{bm}{a^2} + \frac{2\mathrm{b}^3}{a^2\sin^2\theta}w^{-}(a,\theta)\left\{Y(a,\theta) + bZ(a,\theta)\right\}\end{aligned}$$

$$[\boldsymbol{H}] = 4\pi\boldsymbol{s}\times\hat{\boldsymbol{n}} = \frac{2b^3}{\sin\theta}\left\{Y(a,\theta) + bZ(a,\theta)\right\}\hat{\boldsymbol{\theta}}$$

where $\hat{\boldsymbol{n}}$ is the unit normal to the sphere and $\hat{\boldsymbol{\theta}}$ is the unit vector in the θ direction. In these equations, the contravariant component D^1 of $\boldsymbol{D}$ and the covariant component H_2 of $\boldsymbol{H}$ from the second and third of Equation (11) were used.

The Equations (16)-(22) for R_μ^λ and R, will give rise to terms with factors of delta-functions and first order partial r-derivatives which are discontinuous on Σ.

Denoting these terms by Gothic symbols, the Einstein tensor $\mathfrak{G}_\mu^\lambda$ and the associated matter stress-energy tensor $\mathfrak{M}_\mu^\lambda$ are connected through the field equations, and so on Σ we have

$$\mathfrak{G}_\mu^\lambda := \mathfrak{R}_\mu^\lambda - \frac{1}{2}\delta_\mu^\lambda\mathfrak{R} \qquad \mathfrak{G}_\mu^\lambda = -8\pi\mathfrak{M}_\mu^\lambda \tag{64}$$

Bearing in mind that $\exp(-\mu) = F$ and that in V^{-}, $\exp(-\mu^{-}) = F^{-} = 1$, we display below the components $\mathfrak{R}_1^1$ and $\mathfrak{R}_2^2$ as examples:

$$\mathfrak{R}_1^1 = \mathfrak{R}_2^2 = -\frac{1}{2}\delta(r-a)F\mu_r^{+}.$$

The surface stress-energy tensor S_μ^λ is expressed in terms of the limits as $\varepsilon \to 0$ of the integrals of $\mathfrak{M}_\mu^\lambda e^{\mu/2}$ with respect to r from $r = a-\varepsilon$ to $r = a+\varepsilon$ and with $\mathfrak{M}_\mu^\lambda$ given in Equations (64). The junction conditions on Σ are [2,7]

$$\begin{aligned}&-2G_1^1 = {}^{(3)}R + k_a^b k_b^a - k^2\\ &G_2^1 n_1 = k_{2;b}^b - k_{;2}\\ &8\pi S_a^b = \left[k_a^b\right] - \delta_a^b[k]\\ &S_{2;b}^b + \left[T_2^1\right]n_1 = 0.\end{aligned} \tag{65}$$

Here, k_{ab} is the extrinsic curvature tensor of Σ defined by $k_{ab} = -n_{a;b}$, where the covariant differentiation is connected with the metric of V. Since $\mu = 0$ on Σ this gives $k_{ab} = g_{ab,1}/2$.

The hypersurface scalar curvature invariant of Σ is ${}^{(3)}R := {}^{(3)}R_{ab}g^{ab}$ where the Ricci tensor ${}^{(3)}R_{ab}$ is given by

$${}^{(3)}R_{ab} = \Gamma_{ad,b}^d - \Gamma_{ab,d}^d + \Gamma_{ad}^n\Gamma_{bn}^d - \Gamma_{ab}^d\Gamma_{dn}^n,$$

Γ_{ab}^d being the Christoffel symbols of the second kind based on the metric of Σ. With these, all the elements in the junction conditions (65) may be calculated and these conditions may be shown to be valid.

5. Mass, Charge, Angular Momentum and the Magnetic Dipole Moment

The mass, charge and angular momentum are defined by their imprints on the spacetime geometry far from the source. To obtain the gravitational mass and electric charge therefore, we expand the exterior metric function F^+ up to the term $3m^2/r^2$. Bearing in mind the first of (39), we then obtain from (32)

$$F^+ = \frac{1}{C^2(r)} = \left(1+\frac{m}{r}\right)^{-2} = 1-\frac{2m}{r}+\frac{3m^2}{r^2}. \quad (66)$$

We may transform F^+ in (66) to the F_{RN} of the Reissner-Nordstrom solution, by the transformation $r=\overline{r}-m$ giving $F_{RN}=1-\frac{2m}{\overline{r}}+\frac{q^2}{\overline{r}^2}$ with $q=\pm m$ [8], or in physical units, $F_{RN}=1-\frac{2Gm}{c^2\overline{r}}+\frac{G^2m^2}{c^4\overline{r}^2}$ with $q=\pm\sqrt{G}m$. This expression therefore implies that the gravitational mass is m and the electric charge is q and these are connected by [8]

$$q=\pm\sqrt{G}m \quad (67)$$

If we now expand K^+ to $\mathrm{O}(1/r)$ we have

$$K^+ = \frac{2b^2A_1a^2\sin^2\theta}{r} \quad (68)$$

where A_1 is obtained from (38) by setting $n=1$, which will then give, bearing in mind (39)

$$A_1 = \frac{3\lambda C(a)}{2} = \frac{3\lambda}{2b}. \quad (69)$$

If J is the total angular momentum, we have [9]

$$K^+ = \frac{2J\sin^2\theta}{r} \quad (70)$$

From (68) and (70), we then obtain $J=b^2a^2A_1$ and on using (69), this gives

$$J=\frac{3}{2}b\lambda a^2. \quad (71)$$

The dipole field is the part of the magnetic field $\boldsymbol{H}$ whose physical components H_r and H_θ contain the factors $r^{-3}\cos\theta$ and $r^{-3}\sin\theta$ respectively. Since only the r^{-3} power is required, we only need the $n=1$ mode of the third of the expressions in (11) for $\boldsymbol{H}$. We find that these components are

$$\begin{aligned} H_r &= \frac{2b^3A_1a^2\left(1-\lambda^2\right)\cos\theta}{r^3} \\ H_\theta &= \frac{b^3A_1a^2\left(1-\lambda^2\right)\sin\theta}{r^3} \end{aligned} . \quad (72)$$

With these, the magnetic dipole moment is therefore, $P=b^3A_1a^2\left(1-\lambda^2\right)$ and on using (69), this gives

$$P=\frac{3}{2}b^3A_1a^2\left(1-\lambda^2\right) \quad (73)$$

From (71) and (73), we deduce that the gyromagnetic ratio is

$$\frac{P}{J}=b\left(1-\lambda^2\right). \quad (74)$$

In physical units Equations (71), (73), (74) and the third of (39), become

$$J=\frac{3}{2}b\lambda a^2\left(\frac{c^3}{G}\right) \quad (75)$$

$$P=\frac{3}{2}b^2a^2\lambda\left(1-\lambda^2\right)\frac{c^2}{\sqrt{G}} \quad (76)$$

$$\frac{P}{J}=b\left(1-\lambda^2\right)\frac{\sqrt{G}}{c} \quad (77)$$

$$\lambda^2 = 2\frac{Gm}{ac^2}+\left(\frac{Gm}{ac^2}\right)^2 \quad (78)$$

It may be shown that the units of J and P are $[J]=M\,L^2\,T^{-1}$ and $[P]=M^{1/2}\,L^{5/2}\,T^{-1}$ respectively, which are the units of angular momentum and magnetic dipole moment. We also find from the second of (26) and the second of (39) that

$$C(a)=1+\frac{Gm}{ac^2} \qquad b=\frac{1}{C(a)} \quad (79)$$

We stress the fact that all the above formulae are for an electrically charged sphere whose mass m and charge q are related by Equation (67). We note from (75) and (76) that the angular momentum J and dipole moment P depend on a^2 but also in a somewhat more subtle way, on the mass to radius ratio through the quantities λ and b. The analytical Formula (77) may be applied to a number of different objects. We note that there exists a formula for the gyromagnetic ratio of stars known as Blackett's empirical Formulas [10-12], which reads

$$\frac{P}{J}=\beta\frac{\sqrt{G}}{c} \quad (80)$$

where β is a constant of the order of unity so that (80) becomes

$$\frac{P}{J}=\left(\frac{\sqrt{G}}{c}\right). \quad (81)$$

Blackett suggested that an explanation of this relation "must be sought in a new fundamental property of matter not contained within the structure of present day physical theory." We note in this connection that the factor

$\sqrt{G}/c$, occurs in both our analytical Formula (77) and in Blackett's empirical Formula (80). The explanation for the presence of this factor in the analytical Formula (77) however is implicit in its derivation. Furthermore, the coefficient of $\sqrt{G}/c$ in this formula is $b\left(1-\lambda^2\right)$, and in Blackett's Formula (80), it is a constant equal to 1, or approximately equal to 1. The quantity $b\left(1-\lambda^2\right)$ with λ and b given by (78) and (79) respectively, is expected to vary from star to star, but β in Blackett's Formula (80) is a constant equal to 1 for all stars, an assertion that seems improbable. In the context of our solution, it is difficult to see why different objects which can be as diverse as the Earth and the Sun, will conform to such a requirement as implied by Blackett's empirical Formula (81). Although the "new physics" idea was subsequently abandoned, it is nevertheless of interest to investigate further under what circumstances, if any, our exact analytical Formula (77) reduces to Blackett's empirical Formula (81).

In order to gain an insight into the relation between the analytical Formula (77) and Blackett's empirical Formula (81), we shall consider three cases with different numerical values for the radius a and gravitational mass m of the sphere. We shall then proceed to calculate the corresponding quantities in λ^2, λ, b and P/J in (78), (79) and (77):

$$\begin{gathered} m=1.989\times10^{33}\text{ g} \quad a=6.9599\times10^{10}\text{ cm} \\ \lambda^2=4.243406362\times10^{-6} \quad \lambda=2.059953\times10^{-3} \\ b=0.999997878 \quad \frac{P}{J}=0.999993635\frac{\sqrt{G}}{c} \end{gathered} \tag{82}$$

$$\begin{gathered} m=4.33602\times10^{33}\text{ g} \quad a=1.4337394\times10^{11}\text{ cm} \\ \lambda^2=4.493\times10^{-6} \quad \lambda=2.119669786493\times10^{-3} \\ b=0.999997755 \quad \frac{P}{J}=0.999727877\frac{\sqrt{G}}{c} \end{gathered} \tag{83}$$

$$\begin{gathered} m=5.976\times10^{27}\text{ g} \quad a=6.3675\times10^{8}\text{ cm} \\ \lambda^2=1.393554681\times10^{-9} \quad \lambda=3.733034531\times10^{-5} \\ b=0.999999999 \quad \frac{P}{J}=0.999999998\frac{\sqrt{G}}{c}. \end{gathered} \tag{84}$$

The above masses and radii were deliberately chosen to be numerically equal to those of the Sun, 78 Virginis and the Earth. These correspond to the three astronomical objects that are quoted in the literature by later authors in connection with Blackett's empirical Formula (80) [10]. It is seen from the numerical results in (82)-(84), that in the case of our electrically charged spheres, the coefficient of $\sqrt{G}/c$ is very nearly equal to 1 in every case. We must conclude that in situations where the ratio m/a is such that $b\left(1-\lambda^2\right)$ is approximately equal to 1, our analytical Formula (77) will give Blackett's empirical Formula (81). These reductions however, are only possible in the cases where, $b\left(1-\lambda^2\right)=1$. Thus, if we consider a typical neutron star as a fourth case we have

$$\begin{gathered} m=1.4M_s=2.7846\times10^{33}\text{ g} \quad a=10^{6}\text{ cm} \\ \lambda^2=0.4562170327 \quad \lambda=0.6754384003 \\ b=0.959011213 \quad \frac{P}{J}=0.521493962\frac{\sqrt{G}}{c} \end{gathered} \tag{85}$$

where M_S is the mass of the Sun.

It is seen that $P/J\neq\sqrt{G}/c$ and this is because $b\left(1-\lambda^2\right)=0.521493962\neq1$. In the context of our equations, we found the precise condition under which our analytical Formula (77) will give Blackett's empirical Formula (81). Again, in the context of our equations, this provides a full explanation why Blackett's formula is sometimes valid and why this occurs only for a range of objects. Our formula for the gyromagnetic ratio P/J is not empirical, but an exact analytical formula which is a consequence of the equations derived from the exact global solution of the Einstein-Maxwell field equations found here. It does not require any new fundamental properties of matter or any new physics and it is valid for all values of the ratio m/a.

We note that Wilson [12,13] observed that in the case of the Earth and the Sun, the Formula (80) can be accounted for, if we assume that a rotating mass m has the same effect as a rotating electrical charge q where m and q are connected by Equation (67). It is a little puzzling that our electrically charged spheres charged as they are in accordance with Equation (67), seem to echo the above observation by Wilson. In our case however, m and q are connected by Equation (67) in reality. The quantity of charge required is quite small. As noted by Bonnor [8], if the mass m and charge q are related by Equation (67), then if in a sphere of neutral hydrogen one atom in 10^{18} had lost its electron, this would be sufficient.

6. Discussion and Conclusions

Exact exterior and interior solutions of the Einstein-Maxwell field equations for rigidly rotating pressure-free matter were obtained. The exterior and interior spacetimes are separated by a boundary which is a surface layer with surface stress-energy tensor and surface electric 4-current.

Perhaps one of the most important aspects of this work is that the source of spacetime, is rotating charged matter bounded by a closed surface. As far as we know, a global solution with a volume distribution of finite bounded rotating matter as a source of the spacetime, does not exist in the literature, although flat disk solutions do indeed exist [1]. Another important outcome of this work is the derivation of analytical formulae for the angular momentum, dipole moment and gyromagnetic ratio of a rotating sphere based on general relativistic equations.

The mass, charge, angular momentum and the magnetic dipole moment were determined in Section 5. In particular, we derived the analytical Formula (77) for the gyromagnetic ratio and discussed special cases to establish the facts regarding the connection between the analytical Formula (77) and Blackett's empirical for Formula (80) the conditions under which the analytical Formula (77) reduces to Blackett's empirical formula, were obtained. No new properties of matter and no new physics was required. Perhaps the analytical Formula (77) is valid for all rotating objects and in particular for stars, but we have no data to demonstrate this, except for the cases of the Sun, 78 Virginis and the Earth.

All the physical quantities of interest in the interior and exterior were calculated as well as those associated with the spherical surface layer. In this problem, the ordinary gravitational junction conditions are inappropriate. In fact there are two sets of junction conditions, the electromagnetic and the gravitational ones. The former were expressed in the familiar form of classical electromagnetic theory. The gravitational junction conditions in this problem are more complicated than the usual ones, because of the surface layer. These were clearly stated, although no detailed formulae were displayed.

This solution permits a reversal of the signs of A_3^+ and A_4^+ in (34) and (35) [14], which will cause a reversal of the signs of A_3^- and A_4^- in (52) and (48). If we replace the harmonic functions η and ζ in (26) and (27) by

$$\eta = C(r) = 1 + \frac{m}{r} \qquad \zeta = \frac{J\cos\theta}{r^2}$$

then, instead of the metric functions in (32) and (33), we shall have

$$F^+ = \left\{\left(1+\frac{m}{r}\right)^2 + \left(\frac{J\cos\theta}{r^2}\right)^2\right\}^{-1}$$

$$w^+ = \frac{J\sin^2\theta}{r}\left(2+\frac{m}{r}\right)$$

with appropriate modifications to the remaining functions in (32)-(34). Our exterior solution, given by these equations, reduces to the solution obtained by Perjes [14].

To find the limit of the exterior solution (32)-(35) when the angular momentum J is reduced to zero, we replace the harmonic function ζ in (27) by zero, choose $b = 1$, and base the solution on the single harmonic function $\eta = C(r)$ in (26). This leads to

$$\left(F^+\right)^{-1} = \exp\left(\mu^+\right) = C^2(r) \qquad K^+ = w^+ = 0$$

$$A_4^+ = \frac{1}{C(r)} - 1 \qquad A_3^+ = 0$$

which is the Papapetrou solution [15] for which Bonnor has found a matching interior solution [8].

Referring to the surface layer that occurs in our solution, we note the result obtained by Ruffini and Treves in a non-relativistic treatment, in which they had shown that a magnetized rotating object has surface charge and current densities; it is also endowed with a net electric charge [16]. This agrees with our results and in particular, it confirms the existence of a surface layer with 4-current and stress-energy tensor on the boundary $r = a$.

The mass, charge, angular momentum and the magnetic dipole moment were determined in Section 5. In particular, we derived the analytical Formula (77) for the gyromagnetic ratio and discussed special cases to establish the facts regarding the connection between the analytical Formula (77) and Blackett's empirical Formula (80). The conditions under which the analytical Formula (77) reduces to Blackett's empirical formula, were obtained. No new properties of matter and no new physics were required. Perhaps the analytical Formula (77), is valid for all rotating objects and in particular for stars, but we have no data to demonstrate this, except for the cases of the Sun, 78 Virginis and the Earth.

REFERENCES

[1] M. A. H. MacCallum, M. Mars and P. Vera, "Stationary Axisymmetric Exteriors for Perturbations of Isolated Bodies in General Relativity, to Second Order," *Physical Review D*, Vol. 75, 2007, Article ID: 024017.

[2] A. Georgiou, "Rotating Einstein-Maxwell Fields: Smoothly Matched Exterior and Interior Spacetimes with Charged Dust and Surface Layers," *Classical and Quantum Grav*ity, Vol. 11, No. 1, 1994, pp. 167-186. doi:10.1088/0264-9381/11/1/018

[3] W. Bonnor, "Rotating Charged Dust in General Relativity," *Journal of Physics A*: *Mathematical and General*, Vol. 13, 1980, pp. 3465-3477.

[4] A. Georgiou, "Comments on Maxwell's Equations," *International Journal of Mathematical Education in Science and Technology*, Vol. 13, No. 1, 1982, pp. 43-47. doi:10.180/0020739820130106

[5] C. Moller, "The Theory of Relativity," 2nd Edition, Oxford University Press, Oxford, 1972.

[6] E. T. Whittaker and G. N. Watson, "A Course of Modern Analysis," 4th Edition, Cambridge University Press, Cambridge, 1950.

[7] W. Israel, "Singular Hypersurfaces and Thin Shells in General Relativity," *Nuovo Cimento B*, Vol. 48, No. 2, 1966, p. 463. doi:10.1007/BF02712210

[8] W. Bonnor, "The Equilibrium of a Charged Sphere," *Monthly Notices of the Royal Astronomical Society*, Vol. 129, 1965, p. 433. http://adsabs.harvard.edu/abs/1965MNRAS.129.443B

[9] J. N. Islam, "Rotating Fields in General Relativity," Cambridge University Press, Cambridge, 1985.

doi:10.1017/CB09780511735738

[10] P. M. S. Blackett, "The Magnetic Field of Massive Rotating Bodies," *Nature*, Vol. 159, No. 4046, 1947, pp. 658-666.

[11] D. V. Ahluwalia and T. Y. Wu, "On the Magnetic Field of Cosmological Bodies," *Lettere Al Nuovo Cimento*, Vol. 23, No. 11, 1978, pp. 406-408. doi:10.1007/BF02786999

[12] J. A. Nieto, "Gravitational Magnetism and General Relativity," *Revista Mexicana de Fisica*, Vol. 34, No. 4, 1988, pp. 571-576.

[13] H. A. Wilson, "An Experiment on the Origin of the Earth's Magnetic Field," *Proceedings of the Royal Society*, Vol. 104A, 1923, p. 451. http://www.jstor.org/stable/94216

[14] Z. Perjes, "Solutions of the Coupled Electromagnetic Equations Representing the Fields of Spinning Sources," *Physical Review Letters*, Vol. 27, 1971, p. 1668.

[15] A. Papapetrou, "A Static Solution of the Equations of the Gravitational Field for an Arbitrary Charge-Distribution," *Proceedings of the Royal Irish Academy*, Vol. 51, 1945-1948, p. 191.

[16] R. Ruffini and A. Treves, "On a Magnetized Rotating Sphere," *Astrophysical Journal Letters*, Vol. 13, 1973, p. 109.

Journal of Modern Physics, **2012, 3, 1311-1313**
doi:10.4236/jmp.2012.329169 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

General Relation Connecting the Fundamental Fields

Mukul Chandra Das[1*], Rampada Misra[2]
[1]Singhania University, Jhunjhunu, India
[2]Department of Electronics, Vidyasagar University, Midnapore, West Bengal, India
Email: *mukuldas.100@gmail.com

Received June 3, 2012; revised July 18, 2012; accepted July 25, 2012

ABSTRACT

There are four fundamental forces: gravitational force, electromagnetic force, strong force and weak force, in the well known physics. The unified field theory considers the constructive relations among these forces or fields. In the present work the fundamental relations have been studied and trial has been made to derive more significant relations among the known fields. This gives out a generalized unification.

Keywords: Fundamental Force; Unified Field; Generalized Relation

1. Introduction

According to Newton's law, two bodies of mass m_1 and m_2 attract one another with gravitational force whose magnitude is $F_{grav} = \frac{Gm_1m_2}{r^2}$. But Einstein's general relativity does not consider gravity as a force rather it is a space-time curvature. As in [1] Newtonian field equation is $\nabla^2\Phi = 4\pi G\mu$, but in general relativity the Einstein equation is $R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = 8\pi GT_{\alpha\beta}$. On the other hand Maxwell equations [2] are the field equations of electromagnetism that relate the electromagnetic field to its source-charge and current. But Einstein's equation relates the space-time curvature to its source- the mass-energy of matter. The well known unified electromagnetic field Equations [2] are $\boldsymbol{E} = \gamma\left(\boldsymbol{E}' + \boldsymbol{v}\times\boldsymbol{B}'\right)$ and $\boldsymbol{B} = \gamma\left(\boldsymbol{B}' + \frac{\boldsymbol{v}\times\boldsymbol{E}'}{c^2}\right)$. These imply that one observer's electric field is another's magnetic field and that depends on the relativity. In 1935, H. Yukawa proposed a theory on generation of strong force [3] which deals with particle physics. This theory implies a relation between electromagnetic field and strong field. After a long year of this contribution, the weak force and the electromagnetic force were unified in a theory presented independently by A. Salam, Weinberg and Glashow [4-6]. Afterwards a lot of papers, regarding unified field theory, have been published. However, in [7,8], trial have been made to deduce relations among the known fields (*i.e.* gravitational field, electromagnetic field, strong field) following a constructive method, which may satisfy the dream of Einstein's fields unification. The present work is the modified formulation of unified field equations as discussed in [7,8].

*Corresponding author.

2. Modified Relation among the Fields

The well known relations between electric field and magnetic field are

$$\boldsymbol{E} = \boldsymbol{v}\times\boldsymbol{B} \tag{1}$$

$$\boldsymbol{B} = \frac{\boldsymbol{v}\times\boldsymbol{E}}{c^2} \tag{2}$$

From (1) and (2) we shall have the matrix form of these field transformation as

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = k_1 v_{ij} \begin{pmatrix} B'_x \\ B'_y \\ B'_z \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = k_2 v_{ij} \begin{pmatrix} E'_x \\ E'_y \\ E'_z \end{pmatrix} \tag{4}$$

where $v_{ij} = \begin{pmatrix} v_{xx} & v_{xy} & v_{xz} \\ v_{yx} & v_{yy} & v_{yz} \\ v_{zx} & v_{zy} & v_{zz} \end{pmatrix}$, k_1 and k_2 are two constants. Again, we would obtain from relativistic electrodynamics [2] the relations

$$\boldsymbol{E} = \gamma\boldsymbol{v}\times\boldsymbol{B} \tag{5}$$

$$\boldsymbol{B} = \gamma\frac{\boldsymbol{v}\times\boldsymbol{E}}{c^2} \tag{6}$$

where, $\gamma\boldsymbol{v} = \boldsymbol{V}$ is the proper velocity. So, using (3) and

(4) we get from (5) and (6)

$$\begin{pmatrix} E_x \\ E_y \\ E_z \\ E_t \end{pmatrix} = \chi_1 V_{ij} \begin{pmatrix} B'_x \\ B'_y \\ B'_z \\ B'_t \end{pmatrix} \quad (7)$$

$$\begin{pmatrix} B_x \\ B_y \\ B_z \\ B_t \end{pmatrix} = \chi_2 V_{ij} \begin{pmatrix} E'_x \\ E'_y \\ E'_z \\ E'_t \end{pmatrix} \quad (8)$$

χ_1 and χ_2 are also two constants.

where, $V_{ij} = \begin{pmatrix} V_{xx} & V_{xy} & V_{xz} & V_{xt} \\ V_{yx} & V_{yy} & V_{yz} & V_{yt} \\ V_{zx} & V_{zy} & V_{zz} & V_{zt} \\ V_{tx} & V_{ty} & V_{tz} & V_{tt} \end{pmatrix}$

But, $\boldsymbol{E}$ and $\boldsymbol{B}$ are not separate. These are included in a field which is called electromagnetic field. According to [9,10] electromagnetic field function $\boldsymbol{\psi} = \boldsymbol{E} + i\boldsymbol{B}$. So, from (7) and (8) we get a generalized relation

$$\psi_\alpha\left(\boldsymbol{E},\boldsymbol{B}\right) = \Phi V_{ij} \psi'_\alpha\left(E',B'\right) \quad (9)$$

where, $\psi_\alpha(\boldsymbol{E},\boldsymbol{B}) = \begin{pmatrix} \psi_x \\ \psi_y \\ \psi_z \\ \psi_t \end{pmatrix}$, $\psi'_\alpha(\boldsymbol{E},\boldsymbol{B}) = \begin{pmatrix} \psi'_x \\ \psi'_y \\ \psi'_z \\ \psi'_t \end{pmatrix}$

This means that B' and E' in ψ'_α transfer to E and B respectively in ψ_α . In [7] it reveals that through two simultaneous superimposed motions gravitational field transfers to electromagnetic field and the relation is

$$\psi_\alpha = \Upsilon\, w_{ij} G'_\alpha \quad (10)$$

where $G'_\alpha = \begin{pmatrix} G'_x \\ G'_y \\ G'_z \\ G'_t \end{pmatrix}$, $w_{ij} = \begin{pmatrix} w_{xx} & w_{xy} & w_{xz} & w_{xt} \\ w_{yx} & w_{yy} & w_{yz} & w_{yt} \\ w_{zx} & w_{zy} & w_{zz} & w_{zt} \\ w_{tx} & w_{ty} & w_{tz} & w_{tt} \end{pmatrix}$, and

$\boldsymbol{w} = \boldsymbol{a} + \frac{i}{c}\boldsymbol{b}$ as in [7]. Again in [8] relation between strong field and electromagnetic field is given by

$$G_\alpha = \mathrm{K} w_{ij} \psi'_\alpha \quad (11)$$

This leads to a relation between strong gravitational field (strong field) and weak gravitational field $\left(\boldsymbol{G}'_\alpha\right)$ which is

$$G_\alpha = \Gamma w_{ij} w'_{ij} G' \quad (12)$$

Equations (7), (8), (10) and (11) are analogous. So, following (5) and (6) we can write the relations in vectorial form as

$$\psi_\alpha\left(\boldsymbol{E},\boldsymbol{B},\right) = \Phi_1 \boldsymbol{w} \times \boldsymbol{G}'_\alpha \quad (13)$$

$$\boldsymbol{G}_\alpha = \Phi_2 \boldsymbol{w} \times \psi'_\alpha\left(E',B',\right) \quad (14)$$

where, $\boldsymbol{G}'_\alpha$ in (13) represents weak gravitational field and $\boldsymbol{G}_\alpha$ in (14) represents strong gravitational field or strong field. $\boldsymbol{w}$ is the composed velocity as in [7] as well as four-velocity. In (13) and (14) Φ_1 and Φ_2 are two constants.

Again from (12), (13) and (14) we can consider the vector relation between strong field and weak gravitational field which would give

$$\boldsymbol{G}_\alpha = \Phi_3 \boldsymbol{w} \times \left(\boldsymbol{w}' \times \boldsymbol{G}'_\alpha\right) \quad (15)$$

where, Φ_3 is a constant like Φ_1 and Φ_2

3. Conclusion

In this work a constructive vector relation among the fields has been deduced. Equations (13)-(15) represent such relations which can clear the concepts of fields transformations. These also imply that field transformations are associated with relativistic phenomenon in different frames.

4. Acknowledgements

Author thanks the authorities and staff of Satmile High School, Satmile-721452, West Bengal, India for their continuous encouragements.

REFERENCES

[1] J. B. Hartle, "Gravity. An Introduction to Einstein's General Relativity," 3rd Edition, Pearson Education Inc., India, 2009, p. 510.

[2] D. J. Griffiths, "Introduction to Electrodynamics," 7th Edition, Pearson Education Inc., India, 2009, pp. 549-550.

[3] S. N. Ghoshal, "Nuclear Physics," S. Chand & Company Limited, New Delhi, 2006, pp. 365-430, 881-889.

[4] L. Glashow, "Towards a Unified Theory: Threads in a Tapestry," *Reviews of Modern Physics*, Vol. 52, No. 3, 1980, pp. 539-543. doi:10.1103/RevModPhys.52.539

[5] A. Salam, "Gauge Unification of Fundamental Force," *Reviews of Modern Physics*, Vol. 52, No. 3, 1980, pp. 525-538. doi:10.1103/RevModPhys.52.525

[6] S. Weinberg, "Conceptual Foundations of the Unified Theory of Weak and Electromagnetic Interaction," *Reviews of Modern Physics*, Vol. 52, No. 3, 1980, pp. 515-523. doi:10.1103/RevModPhys.52.515

[7] M. C. Das and R. Misra, "Fundamental Way of Charge Formation and Relation between Electromagnetic Field and Gravitational Field," *International Journal of Astronomy and Astrophysics*, Vol. 2, No. 2, 2012, pp. 97-100.

[8] M. C. Das and R. Misra, "A Short Note on Alternative Strong Force," *International Journal of Astronomy and Astrophysics*, Vol. 2, No. 2, 2012, pp. 74-75.

[9] I. I. Popescu, P. Sterian and M. Dobre, "The Photon Wave Function and the Fresnel Formulas," *Romanian Reports in Physics*, Vol. 62, No. 2, 2010, pp. 360-368.

[10] R. E. Nistor, "Quantum Aspects of Photon Propagation in Transparent Infinite Homogeneous Media," *Romanian Reports in Physics*, Vol. 60, No. 3, 2008, pp. 471-491.

Journal of Modern Physics, 2012, 3, 1314-1335
doi:10.4236/jmp.2012.329170 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Geometrical Models of the Locally Anisotropic Space-Time

V. Balan[1], G. Yu. Bogoslovsky[2*], S. S. Kokarev[3], D. G. Pavlov[3], S. V. Siparov[4], N. Voicu[5]
[1]University Politehnica of Bucharest, Bucharest, Romania
[2]Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia
[3]Institute of Hypercomplex Systems in Geometry and Physics, Fryazino, Russia
[4]State University of Civil Aviation, St. Petersburg, Russia
[5]"Transilvania" University of Braşov, Braşov, Romania
Email: *bogoslov@theory.sinp.msu.ru

Received June 16, 2012; revised July 23, 2012; accepted August 3, 2012

ABSTRACT

Along with the construction of non-Lorentz-invariant effective field theories, recent studies which are based on geometric models of Finsler space-time become more and more popular. In this respect, the Finslerian approach to the problem of Lorentz symmetry violation is characterized by the fact that the violation of Lorentz symmetry is not accompanied by a violation of relativistic symmetry. That means, in particular, that preservation of relativistic symmetry can be considered as a rigorous criterion of the viability for any non-Lorentz-invariant effective field theory. Although this paper has a review character, it contains (with few exceptions) only those results on Finsler extensions of relativity theory, that were obtained by the authors.

Keywords: Lorentz-, Poincare- and Gauge Symmetry; Spontaneous Symmetry Breaking; Alternative Gravity Theories; Space-Time Anisotropy; Finsler Differential Geometry

1. Introduction

Nowadays, the program of geometrization and algebraization of the fundamental laws of nature which was formulated at the early stage of GR development is still not fulfilled. Every step in realization of this program suggests partial or complete reconsideration of the common notions and of the properties of the corresponding to them physical objects. Many basic concepts of the modern physics and mathematics are expressed in terms of the notion of manifold, which allows possibility of universal arythmetization of the events of the physical world and of the relations between them; the notion of manifold is also a symbiosis of geometric and algebraic ideas.

Despite the abstract character of the manifolds studied in modern physics and mathematics and of a lot of additional structures which geometrically describe the laws of nature, some of these structures still remain rather conservative. First of all, we mention the manifolds endowed with metrics, while the majority of modern geometrical models deal with the metric tensor as a function on the tangent bundle. In every coordinate chart, the metric tensor field depends on the coordinates of the base in an arbitrary smooth way, and it depends bi-linearly on the coordinates of the fiber. Despite the fact that the most natural generalization of this construction is known to the mathematicians for a very long time within Finsler geometry [1-3] which describes the locally anisotropic spaces, the first viable model of Finsler space-time [4] and the based on it special relativistic theory of locally anisotropic space-time [5-7] were promoted not long ago. These works were motivated by the suggested at that time and now popular idea [8,9] of Lorentz symmetry violation, which means that the "true" metric of the flat space-time deviates from the Minkowski metric.

Generally speaking, the discussion on space-time anisotropy needs to clarify first two issues: 1) why this should be done, *i.e.* what are its physical premises and 2) what does the suggested anisotropy mean. The second question implies that geometry in mathematics corresponds to the theory of measurements in physics, that is, when we speak of, say, space-time curvature, we presume that it will show itself in measurements. If we have in mind the physical applications of the geometrical constructions, the same must be true for anisotropy. Notice that to speak of the curvature or anisotropy of the empty space is possible only when we don't deal with experimental science at all, and if we do deal with it, the characteristic scale for the possible applications of the theoretical speculations must

*Corresponding author.

be provided. The last means that when the necessity to study the space-time anisotropy occurs, one should suggest its local source.

The answer to the first question is less obvious and is rather vast from the point of view of the analysis of the situation in physics (see [10-15] and additional reasoning below). The result of this analysis has both general and concrete aspects. The general conclusion is that the gravitation theory, *i.e*. GR, was developed for and successfully applied at the scale of planetary systems. When applied to cosmological (galactic) scales in the way in which this is done now, it demands the introduction of corrections that are 25 times larger than the value of mass of the observable Universe and which are related to the existence of the new (still unknown) substances—dark matter and dark energy, which were not supposed to be present in the initial theory. Obviously, alongside with their tracking, one should make sure that the theoretical models are valid. These models are: the so called simplest scalar used in the expression for the Hilbert-Einstein action, *i.e*. scalar curvature; the geometry used for the space-time description, *i.e*. Riemann geometry; and the 4D space-time used for the description of the physical reality itself.

Important observations that make simple sense, have sufficient value and statistical validity, but contradict classical GR, are the rotation curves of spiral galaxies. The attempts to modify the theory in order to describe them in an adequate way based on increase of complexity [16,17] or change [18] of the simplest scalar, or on the modification of the metric [19], appeared to be either not consistent enough—f(R)-theories, or imposed as well to introduce a new unknown scalar field or some new unknown interaction. The phenomenological MOND theory [20] required either an arbitrary change of the dynamics law, or an arbitrary change of the expression of the gravitation force, in order to provide an acceptable description of the phenomena observed at galactic scales. Its covariant generalization [21] also leads to the introduction of the new scalar field.

The concrete consequence of the analysis is that there is a necessity to make the next step and to study the possibility to use a new geometry to interpret the observations. The natural generalizations of Riemann geometry are Finsler and Lagrange geometries, both taking into account the dependence of the metric tensor on direction at the given point. This direction can be global—which corresponds to one of the geometries constructed on the commutative-associative algebra, namely, to Berwald-Moor geometry. If we use the Berwald-Moor metric to interpret the gravitation theory, there appear a fixed number of stationary global sources of gravitation whose nature is unclear. This direction could be local—and then the interpretation might correspond to the motion of the local sources of curvature. The last one seems well-grounded, since the common features of the gravitation theory and of electrodynamics from the point of view of Lorentz invariance and of the inverse square law were long ago noticed. The corresponding attempts to generalize the theory with the help of the notion of mass currents were undertaken in [22,23], and the common geometrical background of both theories was discussed in [12]. Nevertheless, the gravito-electromagnetism [23] doesn't seem to be self-consistent enough, because one cannot deal with the gravitation charges in the same way as with electric charges: the first are sources of curvature, while the second are not. Instead of the introduction of Lorentz force according to a formal analogy, one should require that, in the case of gravitation, the metric becomes anisotropic. This would lead to the gravitational force dependence on the velocity of the test particle and on the vector field corresponding to the motion of the sources of curvature. The literal meaning of the equivalence principle suggests the same: the inertial forces might depend on velocities and have large values, while the usual relativistic corrections interpreted as the force dependence on velocities, are small. In this case the application of the Schwarzschild type solutions to the problems stated at galaxy scale is not appropriate, and cannot be used to describe the spiral galaxies dynamics which is revealed by observations.

Turning back to the motivation of the research which deals with Finsler geometric structures of space-time, one should notice that the whole variety of astrophysical data including the anisotropy of the acceleration of the Universe expansion and the anisotropy of relic radiation, points at the anisotropy of space-time only in an indirect way. The same can be said about the baryonic asymmetry problem, a breaking of the discrete space-time symmetries in weak interactions, the problem of anomalous magnetic moment of muon, etc. This emphasizes the significance of new results obtained in the two independent experiments which show directly the existence of the space-time anisotropy.

In the first of them [24], the precise atomic interfereometer was used to measure the phase shifts of the freely falling atoms. The local Lorentz symmetry break larger than 2 standard errors was found, which means that there exists an anisotropic condensate of unknown nature, and that, this interacts with the gravitation field in such a way, that the central symmetry of the gravitational potential is broken. Consider now the second experiment.

Recently, at Large Hadronic Collider (LHC) there was found a new phenomenon [25] which is now known as Ridge/CMS-effect (CMS stands for Compact Muon Solenoid which is both the detector and the name of the corresponding research collaboration). One of the features of the effect consists in the following. If the proton-proton collisions with the full energy 7 TeV produce more than

100 particles, the planes corresponding to the tracks of every pair of the produced charged particles are oriented in such a way that a significant part of them has a common cross-line coinciding with the initial protons collision axis. This resembles the situation with the elastic scattering of a moving particle on a particle at rest: due to the momentum conservation (the momentum is equal to the flying particle momentum), all the planes to which the tracks of the two particles belong after scattering, have the common cross-line which coincides with the track of the initial flying particle. But contrary to the elastic scattering on the particle at rest, the total momentum of the colliding protons at LHC is equal to zero. This fact and also the fact that the Ridge/CMS-effect is characteristic only to the high multiplicity events are hard to explain by regular considerations: the physical origin of the appearance of the preferred direction coinciding with the protons collision axis when a hundred or more particles are emitted, remains unclear.

High multiplicity events take place in case of the central collision of the initial protons. Then the energy density at the moment of the collision is comparable to the energy density shortly after the Big Bang, when instead of hadrons there was quark-gluon plasma. It is clear that dealing with the high multiplicity events in the proton-proton collisions, one should account for the phase transitions corresponding to the high gauge symmetries violations that are accompanied by the vacuum rearrangement. The condensate appearing during such rearrangement is locally isotropic (Higgs type) only in frames of the usual relativistic theory. In the relativistic theory with Lorentz symmetry violation, or, in other words, in the anisotropic theory of relativity, which will be discussed below, the role of the Higgs condensate is played by the axially symmetric anisotropic fermion-antifermion condensate. Besides, when rapid cooling and hadronization of quark-gluon plasma takes place, an entirely anisotropic three-gluon condensate can appear. On the one hand, quantum-field vacuum, that includes the anisotropic condensate, is the physical carrier of the local anisotropy of space-time, and it can be regarded as an anisotropic quintessence, on the other—it imparts all the particles the properties of quasi-particles in the crystalline environment. In particular, apart from the rest energy, the particles obtain a rest momentum. With regard to the Ridge/CMS-effect, this means that in the reference frame coinciding with the center of masses of the colliding protons, (relative to the laboratory), the total momentum of the appearing primordial plasma differs from zero and lies on the collision axis (this is due to the anisotropy of the condensate, which arises spontaneously along the collision axis). This is why the correlation of paired tracks in the CMS experiment has turned out such that the planes to which the tracks belong cross mostly on the axis of proton collisions.

Thus, the Ridge/CMS-effect directly demonstrates that in the early Universe there spontaneously emerged the axially symmetric local anisotropy of space-time with a group $DISIM_b(2)$ as an inhomogeneous group of local relativistic symmetry and the corresponding Finsler metric. As for the possibility of spontaneous emergence of the complete local anisotropy of space-time with the Abelian homogeneous group of local relativistic symmetry and the corresponding generalized Finslerian Berwald-Moor metric, the answer to this question will depend on the three-particle correlation function, whose measurement is already planned by the CMS collaboration.

In Section 2, we consider the relativistic Finslerian $DISIM_b(2)$-invariant model of a flat space-time with partially broken isotropy in the 3D space. It will be shown that in this model the physical carrier of the anisotropy of flat space-time is axially symmetric neutrino-antineutrino condensate, and the model itself underlies the anisotropic special theory of relativity and admits a natural generalization to the case of curved space-time and the Finslerian extension of GR. The mentioned above Finsler extensions of general relativity necessarily leads to the existence of, at least, one gauge vector field and of its interaction with the conserved current of the rest mass.

A number of astrophysical effects of this interaction were studied in detail in the framework of the approach proposed by S. V. Siparov. It is suggested to model the physical real world by the 8-dimensional phase space-time, one of the coordinates of which appears to have a constant value. The discussion of this approach, of its origination and of the corresponding calculated and observed effects is given in Section 3 of this review.

As to Section 2, in addition to the flat space-time with partially broken isotropy of the 3D space, it contains a brief review of a three-parameter family of flat Finsler spaces with entirely broken 3D isotropy and with Abelian three-parametric group of relativistic symmetry. The Abelian group structure of the relativistic symmetry was the starting point for a deeper study of Finsler Berwald-Moor space, which for the four-dimensional case belongs to the specified family.

In Section 4 we consider the geometric, algebraic, and physical aspects of the commutative associative algebras and Berwald-Moor geometries of various dimensions associated with them. In recent years, studies of this kind were also conducted within the framework of international cooperation between the Romanian Academy and the Academy of Sciences of the Russian Federation. In particular, thanks to the work of Romanian geometers led by V. Balan, the results concerning the algebraic side of the theory of Berwald-Moor metrics for various dimensions were complemented by the specific results originating from the modern differential geometry of Finsler spaces. Their description in a concentrate form can be found in Section 5 of this review.

2. Relativistically Invariant Finslerian Spaces with Local Lorentz Symmetry Violation

As it is known, space-time is Riemannian within the framework of GR, and the distribution and motion of matter only determines the local curvature of space-time without affecting the geometry of the tangent spaces. In other words, regardless of the properties of the material medium which fills the Riemannian space-time, any flat tangent space-time remains the space of events of SR, *i.e.* the Minkowski space with its Lorentz symmetry, which is usually identified with the relativistic symmetry.

However, in recent literature there is an increasing interest in the problem of violation of Lorentz symmetry. Particularly, the string-motivated approach to this problem is widely discussed.

The point is that even if the original unified theory of interactions possesses Lorentz symmetry up to the most fundamental level, this symmetry can be spontaneously broken due to the emergence of the condensate of vector or tensor field. The appearance of such a condensate, or of a constant classical field on the background of Minkowski space, implies that it can affect the dynamics of the fundamental fields and thereby modify the Standard Model of strong, weak and electromagnetic interactions. Since the constant classical field is transformed by the passive Lorentz transformations as a Lorentz vector or tensor, its influence on the dynamics of fundamental fields of the Standard Model is described by the introduction of the additional terms representing all possible Lorentz-covariant convolutions of the condensate with the Standard fundamental fields into the Standard Lagrangian. The phenomenological theory, based on such a Lorentz-covariant modification of the Standard model is called the Standard Model Extension (SME) [26-32].

By design, the phenomenological SME theory is not Lorentz-invariant, since its Lagrangian is not invariant under active Lorentz transformations of the fundamental fields against the background of fixed condensate. In addition, in the context of SME, a violation of Lorentz symmetry also involves the violation of relativistic symmetry, since the presence of non-invariant condensate breaks the physical equivalence of the different inertial reference systems.

It should be added that in the low-energy limit of gravitation theories with broken Lorentz and relativistic symmetries, there appears an unlimited number of possibilities to build a variety of effective field theories, each of which could potentially explain at least some of the recently discovered astrophysical phenomena (see, e.g., [33]).

The very existence of the Finsler geometric models of space-time within which a violation of Lorentz symmetry occurs without the violation of relativistic symmetry strongly constrains the possible effective field theories with broken Lorentz symmetry: in order to be viable, such theories, in spite of the presence of Lorentz violation, should have the property of relativistic invariance.

Since only two types of Finsler spaces with broken Lorentz symmetry are relativistic invariant [34], we first consider the Finsler spaces of the first type.

2.1. The Relativistically Invariant Finslerian Spaces with Partially Broken 3D Isotropy

The metric of such spaces suggested in [4] has the following form

$$\mathrm{d}s^2 = \left[\frac{\left(\mathrm{d}x_0 - \boldsymbol{\nu}\mathrm{d}\mathbf{x}\right)^2}{\mathrm{d}x_0^2 - \mathrm{d}\mathbf{x}^2}\right]^r \left(\mathrm{d}x_0^2 - \mathrm{d}\mathbf{x}^2\right) \tag{1}$$

This metric depends on two constant parameters r and $\boldsymbol{\nu}$, and generalizes the Minkowski metric, where r determines the spatial anisotropy, characterizing, thus, the degree of deviation of (1) from the Minkowski metric. Instead of the 3-parametric group of rotations of Minkowski space, Finsler spaces (1) can have only an 1-parametric group of rotations around the unit vector $\boldsymbol{\nu}$, which presents a physically preferred direction in the 3D space. The translational symmetry suffers no change: space-time translations preserve metric (1) invariant (in this sense, it is natural to consider the family of spaces (1) as a family of flat Finsler spaces. With regard to the transformations connecting different inertial reference frames, the usual Lorentz boosts conformally modify metric (1). Therefore, they do not belong to a group of isometries of this metric. However, by using them, we can construct such transformations [5] which belong to the group of isometries of metric (1). The corresponding generalized Lorentz transformations (generalized Lorentz boosts) are as follows

$$x'^i = D\left(\mathbf{v},\boldsymbol{\nu}\right) R_j^i\left(\mathbf{v},\boldsymbol{\nu}\right) L_k^j\left(\mathbf{v}\right) x^k \tag{2}$$

where $\mathbf{v}$ stands for the velocities of the moving (primed) reference frames, the matrices $L_k^j\left(\mathbf{v}\right)$ are the usual Lorentz boosts, the matrices $R_j^i\left(\mathbf{v},\boldsymbol{\nu}\right)$ are the additional rotations of the spatial axes of the moving systems around vectors $\left[\mathbf{v}\boldsymbol{\nu}\right]$ at angles

$$\varphi = \arccos\left\{1 - \frac{(1-\sqrt{1-\mathbf{v}^2/c^2})\left[\mathbf{v}\boldsymbol{\nu}\right]^2}{\left(1-\mathbf{v}\boldsymbol{\nu}/c\right)\mathbf{v}^2}\right\}$$

corresponding to the relativistic aberration of vector $\boldsymbol{\nu}$, and, finally, the diagonal matrices

$$D\left(\mathbf{v},\boldsymbol{\nu}\right) = \left(\frac{1-\mathbf{v}\boldsymbol{\nu}/c}{\sqrt{1-\mathbf{v}^2/c^2}}\right)^r I$$

present the additional dilatational transformations of the coordinates of events.

In contrast to the usual Lorentz boosts, the generalized boosts (2) determine a 3-parametric non-compact group with generators X_1, X_2, X_3 . Thus, with inclusion of 1-parameter group of rotations around the preferred direction $\mathbf{v}$ and 4-parameter translation group, the inhomogeneous group of isometries, or in other words, inhomogeneous group of relativistic symmetry of flat Finsler spaces (1) appears to have 8-parameters. To obtain the simplest representation for its generators, it is enough to send the third spatial axis along $\mathbf{v}$ and rewrite the transformation (2) in the infinitesimal form. As a result, we come to the following eight generators

$$
\begin{aligned}
X_1 &= -\left(x^1 p_0 + x^0 p_1\right) - \left(x^1 p_3 - x^3 p_1\right),\\
X_2 &= -\left(x^2 p_0 + x^0 p_2\right) + \left(x^3 p_2 - x^2 p_3\right),\\
X_3 &= -r x^i p_i - \left(x^3 p_0 + x^0 p_3\right),\\
R_3 &= x^2 p_1 - x^1 p_2; \qquad p_i = \partial/\partial x^i
\end{aligned} \tag{3}
$$

According to [5], these generators satisfy the commutation relations

$$
\begin{aligned}
&[X_1 X_2] = 0 && [R_3 X_3] = 0\\
&[X_3 X_1] = X_1, && [R_3 X_1] = X_2,\\
&[X_3 X_2] = X_2, && [R_3 X_2] = -X_1;\\
&[p_i p_j] = 0; &&\\
&[X_1 p_0] = p_1, && [X_2 p_0] = p_2,\\
&[X_1 p_1] = p_0 + p_3, && [X_2 p_1] = 0,\\
&[X_1 p_2] = 0, && [X_2 p_2] = p_0 + p_3,\\
&[X_1 p_3] = -p_1, && [X_2 p_3] = -p_2,\\
&[X_3 p_0] = r p_0 + p_3, && [R_3 p_0] = 0,\\
&[X_3 p_1] = r p_1, && [R_3 p_1] = p_2,\\
&[X_3 p_2] = r p_2, && [R_3 p_2] = -p_1,\\
&[X_3 p_3] = r p_3 + p_0, && [R_3 p_3] = 0.
\end{aligned} \tag{4}
$$

This shows that the homogeneous isometry group of flat Finsler spaces with partially broken 3D isotropy contains four parameters (generators X_1, X_2, X_3 and R_3). It is a subgroup of the 11-parametric Weyl group [35], and it is isomorphic to the corresponding 4-parametric subgroup (with generators $X_1, X_2, X_3|_{r=0}$ and R_3) of the homogeneous Lorentz group. Since the 6-parametric homogeneous Lorentz group does not have any 5-parametric subgroup, while its 4-parametric subgroup is unique up to isomorphisms [36], the passage from Minkowski space to Finsler spaces (1) implies a minimum possible violation of Lorentz symmetry. With this, the relativistic symmetry represented now by the generalized Lorentz boosts (2) remains valid [37].

Here it is worth noting the following. Despite the fact that at $r = 0$ the Finsler metric (1) reduces to Minkowski metric $\mathrm{d}s^2 = \mathrm{d}x_0^2 - \mathrm{d}\mathbf{x}^2$, the 3-parametric non-compact transformations (2) that serve as the homogeneous relativistic symmetry transformations for Finsler metric (1) don't reduce to the usual Lorentz boosts $x'^i = L^i_k(\mathbf{v})x^k$ but reduce to the transformations

$$
x'^i = R^i_j(\mathbf{v}, \mathbf{v}) L^j_k(\mathbf{v}) x^k \tag{5}
$$

that differ by additional rotations $x'^i = R^i_k(\mathbf{v}, \mathbf{v})x^k$ of the space axes. These rotations are designed so that if a light-beam in one inertial frame has the direction of $\mathbf{v}$, then it will have the same direction in all inertial frames.

Thus, at $r = 0$ *i.e.* in frames of the usual SR, the transformations (5) are the alternative to the Lorentz boosts, however, in contrast to the Lorentz boosts, for any value of $\mathbf{v}$, they present a 3-parameter non-compact subgroup of the 6-parametric homogeneous Lorentz group. As it was noted in [37], in order to realize these transformations physically, it is enough to choose $\mathbf{v}$ as a direction at any star and then perform an arbitrary Lorentz boost, supplementing it with such rotation of spatial axes that in the new reference system, the direction at the star does not change. Taken together, these transformations form the specified subgroup (5) of the 6-parametric homogeneous Lorentz group. As a result, we can say that within the framework of SR, $\mathbf{v}$ has no physical meaning and serves to the relativistically invariant calibration of the directions of spatial axes of inertial frames.

In connection with the last statement, it is necessary to make another important remark concerning the 3-parametric non-compact group of homogeneous transformations (5). If one complements this group by the 1-parametric group of rotations around $\mathbf{v}$ and 4-parametric translation group, the result is an 8-parameter subgroup of the Poincare group, whose generators and Lie algebra have in our basis the forms (3) and (4) assuming that $r = 0$ For such a group the name *ISIM*(2) is now used, and its homogeneous 4-parametric subgroup *SIM*(2), which includes (5) and the rotations around $\mathbf{v}$, is the basis for the so-called Very Special Relativity (VSR) [38]. According to VSR, *SIM*(2) symmetry suggests a more fundamental local space-time symmetry than the local Lorentz symmetry. In particular, the requirement of *SIM*(2) symmetry was sufficient to show [39] that neutrinos may have mass along with the lepton number conservation, and it is important that this result can not be obtained within the framework of Lorentz-invariant approach without introducing sterile neutrinos. However, a significant drawback of the VSR is that $\mathbf{v}$ is regarded only as a phenomenological parameter and VSR can not say anything of its physical nature.

Much more meaningful from the physical point of view is the special relativistic theory of the locally anisotropic space-time [5-7], based on Finsler metric (1), which describes a family of flat relativistically invariant spaces of events with partially broken 3D isotropy, and hence with broken Lorentz symmetry. Most of the results obtained under such a theory have been reproduced in [40] using alternative methods (see also [41]). In particular, the inhomogeneous 8-parametric group of relativistic symmetry of metric (1) with its Lie algebra (4) were obtained using the method of continuous deformations of algebra *ISIM*(2). As a result, the corresponding symmetry is more frequently called $DISIM_b(2)$ symmetry (where b is the new designation of the parameter r), and the theory itself [5-7] is more frequently called General Very Special Relativity (GVSR).

2.1.1. The Rest Momentum in Addition to the Rest Energy

In order to modify the usual relativistic mechanics in accordance with the requirement of invariance with respect to $DISIM_b(2)$ it is enough to replace the Minkowski line element $\mathrm{d}s = \sqrt{\mathrm{d}x_0^2 - \mathrm{d}\mathbf{x}^2}$ in the integral of action

$$S = -mc\int_a^b \mathrm{d}s \tag{6}$$

by the Finsler line element (1). As a result, the Lagrange function corresponding to a relativistic particle in a locally anisotropic space (1), is the following

$$L = -mc^2\left(\frac{1-\mathbf{v}\mathbf{v}/c}{\sqrt{1-\mathbf{v}^2/c^2}}\right)^r\sqrt{1-\mathbf{v}^2/c^2}. \tag{7}$$

With this, one can get the expression for the energy E and momentum $\mathbf{p}$ of the relativistic particle [6]:

$$E = \frac{mc^2}{\sqrt{1-\mathbf{v}^2/c^2}}\left(\frac{1-\mathbf{v}\mathbf{v}/c}{\sqrt{1-\mathbf{v}^2/c^2}}\right)^r\left[1-r+r\frac{1-\mathbf{v}^2/c^2}{1-\mathbf{v}\mathbf{v}/c}\right] \tag{8}$$

$$\begin{aligned}\mathbf{p} &= \frac{mc}{\sqrt{1-\mathbf{v}^2/c^2}}\left(\frac{1-\mathbf{v}\mathbf{v}/c}{\sqrt{1-\mathbf{v}^2/c^2}}\right)^r\\ &\times\left[(1-r)\mathbf{v}/c + r\boldsymbol{\nu}\frac{1-\mathbf{v}^2/c^2}{1-\mathbf{v}\mathbf{v}/c}\right]\end{aligned} \tag{9}$$

According to (8), the particle energy E reaches its absolute minimum $E = mc^2$ at $\mathbf{v} = 0$. As for the momentum $\mathbf{p}$, then according to (9), at $\mathbf{v} = 0$ it takes the value $\mathbf{p} = mcr\boldsymbol{\nu}$. Thus, in the anisotropic space with metric (1), in addition to the rest energy $E = mc^2$, any massive particle obtains another observable parameter—the rest momentum $\mathbf{p} = mcr\boldsymbol{\nu}$. Note also that as shown in [6], the 4-momentum $p^i = \left(p^0 = E/c, \mathbf{p}\right)$ satisfies the $DISIM_b(2)$-invariant dispersion relation, which we give here in the form:

$$\begin{aligned}&\left(p_0^2 - \mathbf{p}^2\right)\\ &= (mc)^2(1-r)^{(1-r)}(1+r)^{(1+r)}\left[\frac{\left(p_0 - \mathbf{p}\boldsymbol{\nu}\right)^2}{p_0^2 - \mathbf{p}^2}\right]^r.\end{aligned} \tag{10}$$

In the non-relativistic limit, the Lagrange function (7) has the following form

$$L = -mc^2 + mcr(\boldsymbol{\nu}\mathbf{v}) + (1-r)\frac{m\mathbf{v}^2}{2} + r(1-r)\frac{m(\boldsymbol{\nu}\mathbf{v})^2}{2}.$$

Since this expression $-mc^2 + mcr(\boldsymbol{\nu}\mathbf{v})$ which is present here is the total derivative over time, it can be omitted. As a result, we see that the kinetic energy and momentum

$$T = \frac{1}{2}m_{\alpha\beta}\mathrm{v}^\alpha\mathrm{v}^\beta,$$
$$p_\alpha = m_{\alpha\beta}\mathrm{v}^\beta,$$
$$(\alpha, \beta = 1, 2, 3)$$

of the non-relativistic particle in the anisotropic space (1) are determined by the tensor of the inertial mass [34]:

$$m_{\alpha\beta} = m(1-r)\left(\delta_{\alpha\beta} + r\nu_\alpha\nu_\beta\right). \tag{11}$$

Let us now rewrite the Finsler metric (1) so that it is expressed through the four-dimensional quantities:

$$\begin{aligned}\mathrm{d}s &= \left[\frac{\left(\mathrm{d}x_0 - \boldsymbol{\nu}\mathrm{d}\mathbf{x}\right)^2}{\mathrm{d}x_0^2 - \mathrm{d}\mathbf{x}^2}\right]^{r/2}\sqrt{\mathrm{d}x_0^2 - \mathrm{d}\mathbf{x}^2}\\ &= \left[\frac{\left(\nu_i\mathrm{d}x^i\right)^2}{\eta_{ik}\mathrm{d}x^i\mathrm{d}x^k}\right]^{r/2}\sqrt{\eta_{ik}\mathrm{d}x^i\mathrm{d}x^k}\end{aligned} \tag{12}$$

Since $\boldsymbol{\nu}^2 = 1$, it is clear that here we have

$$\nu_i = \{1, -\boldsymbol{\nu}\}, \quad \eta_{ik} = diag\{1, -1, -1, -1\},$$
$$\nu^i = \{1, \boldsymbol{\nu}\}, \quad \nu_i\nu^i = 0.$$

Finally, we present the physical carrier of the anisotropy of the flat space of events (12) and outline a plan for the further development of the theory. In order to do this, we first turn our attention to the unique property of Finsler metric (12). On the one hand, for $r = 0$, this turns into Minkowski metric, on the other, at $r = 1$, it transforms into the total differential $\mathrm{d}s = \nu_i\mathrm{d}x^i$. The latter means that in this case the action (6) does not depend on the shape of the world line connecting the points a and b In other words, the space-time loses such a physical characteristics as spatial extension, and only a temporal duration which represents the absolute time interval $\mathrm{d}s = \nu_i\mathrm{d}x^i$ is left. Moreover, according to (11), the inertial masses of all particles also vanish, and ν_i becomes not the spurionic vector field but is transformed into a covariant constant vector field defined on this degenerate (from the metric point of view) space-time manifold. Incidentally, we note that it is on the space-time manifold, and not on the Minkowski space-time, that the massless fundamental fields (for example, those of the Standard Model) are introduced before spontaneous violation of the initial gauge symmetry and before the appearance of masses of the initially massless particles. It is clear due to the fact that in the massless world there are no inertial reference systems, with their mandatory attribute—the reference stick.

In accordance with (6) and (12), a constant non-zero field *r* defines the specific inseparable interaction of the constant spurionic field ν_i with massive particles. The effect of this interaction is that the particles obtain—according to (11), the properties of quasi-particles in an axially symmetric crystalline medium. The complex of constant fields containing the scalar field *r* and the spurionic field ν_i is, thus, the physical carrier of the anisotropy of the flat space of events (12). As it turned out, the null-vector spurionic field ν_i presents a neutrino-antineutrino condensate constructed out of constant Weyl spinors. Such spinors are an exact solution of the $DISIM_b(2)$-invariant generalized massive Dirac equation [42], whose Lagrangian has the form

$$L = \frac{i}{2}\left(\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \partial_{\mu}\bar{\psi}\gamma^{\mu}\psi\right) - m\left[\left(\frac{\nu_{\mu}\bar{\psi}\gamma^{\mu}\psi}{\bar{\psi}\psi}\right)^{2}\right]^{r/2}\bar{\psi}\psi \tag{13}$$

If the constant scalar field *r* is set to zero, the $DISIM_b(2)$-invariant generalized massive Dirac equation becomes the standard massive Dirac equation, which does not have any solution in the form of constant spinors. However, the Weyl equations, arising from the standard massless Dirac equation have solutions in the form of constant spinors, and they provide the possibility to build a constant null-vector spurionic field ν_i. But physically, it would be unobservable, since at $r = 0$ we come back to the framework of SR: the Finsler metric (12) becomes the Minkowski metric, the rest momentum $\mathbf{p} = mcr\boldsymbol{\nu}$ disappears, the tensor of inertial mass (11) ceases to be a tensor and becomes a scalar *m*. Accordingly, all the other effects of spatial anisotropy discussed in [43] will be lost.

2.1.2. On the Problem of Construction of the Finslerian GR Based on the Group $DISIM_b(2)$

As noted in the Introduction, the Ridge/CMS-effect which was observed at the LHC, directly suggests that in the early Universe the axially symmetric anisotropy of space-time spontaneously arose, and it had the $DISIM_b(2)$ group as an inhomogeneous group of local relativistic symmetry and the respective Finsler metric (12). This is the first and most important reason to regard the problem of constructing a Finslerian general relativity based on the group $DISIM_b(2)$ Needless to speak about the complexity of such a problem, especially because it involves the answers to the questions concerning the nature of the dark matter and dark energy. Despite some advances in this direction, this problem is still not completely solved. So, in the end of Section 2.1, we suggest a possible way the progress on which is likely to lead to the planned purpose.

The key point in the generalization of the flat $DISIM_b(2)$-invariant Finsler metric (12) to a Finsler metric, which describes the corresponding curved locally anisotropic space-time is the following. If the constant values on which the metric (12) depends, namely a scalar r, the spurion null-vector ν_i and the spurion tensor $\eta_{ik} = diag\{1,-1,-1,-1\}$ are replaced by the corresponding conventional fields defined on the space-time manifold, *i.e.* in the metric (12) the substitutions $r \to r(x)$, $\nu_i \to \nu_i(x), \eta_{ik} \to g_{ik}(x)$ are performed, then the result will be the curved Finsler metric of the following form (see [44,45])

$$\mathrm{d}s = \left[\frac{\left(\nu_i \mathrm{d}x^i\right)^2}{g_{ik}\mathrm{d}x^i\mathrm{d}x^k}\right]^{r/2}\sqrt{g_{ik}\mathrm{d}x^i\mathrm{d}x^k}\,, \tag{14}$$

where $g_{ik} = g_{ik}(x)$ is the Riemannian metric tensor associated with the gravitational field, $r = r(x)$ is a scalar field, which characterizes the magnitude of the local space-time anisotropy and $\nu_i = \nu_i(x)$ is a null-vector field that indicates the locally preferred directions in the space-time.

At any point of the curved Finsler space (14), the corresponding flat tangent Finsler space (12) has its own values of the parameters *r* and $\mathbf{v}$. These values are nothing but the values of the fields $r(x)$ and $\mathbf{v}(x)$ at the point of tangency.

Obviously, the dynamics of a Finsler space (14) is completely determined by the dynamics of the interacting fields $g_{ik}(x)$, $r(x)$, $\nu_i(x)$, and these fields together with fields of matter form a unified dynamic system. Therefore, in contrast to the existing purely geometric approaches to the Finsler generalization of Einstein's equations, our approach [44,45] to this problem is based on the use of methods of the conventional theory of interacting fields.

The fact that during the transition from a flat $DISIM_b(2)$-invariant Finsler metric (12) to a curved Finsler metric (14), we replaced the spurion tensor $\eta_{ik} = diag\{1,-1,-1,-1\}$ and the spurion null-vector ν_i by the conventional fields, became the property of metric (14) invariance with regard to the following local transformations

$$\begin{aligned} &g_{ik} \to e^{2\sigma(x)} g_{ik}, \\ &\nu_i \to e^{(r-1)\sigma(x)/r} \nu_i, \\ &r \to r, \end{aligned} \tag{15}$$

where $\sigma(x)$ is an arbitrary function.

In addition to metric (14), the local transformations (15) leave invariant all the observables. Therefore, in the theory of gravitation based on the group $DISIM_b(2)$ the transformations (15) have the meaning of local gauge transformations. For example, the action

$$S = -\frac{1}{c}\int \mu^* \left(\frac{\nu_i \nu^i}{\sqrt{g_{ik}\nu^i \nu^k}}\right)^{4r} \sqrt{-g}\, \mathrm{d}^4 x$$

for a compressible fluid in a Finsler space (14) is gauge invariant. In this formula, μ^* is the invariant energy density of the liquid, $v^i = \mathrm{d}x^i/\mathrm{d}s$, and ds is metric (14).

In connection with the above-mentioned local gauge invariance, the dynamical system consisting of the fields g_{ik}, r, ν_i and a compressible fluid must be supplemented by two vector gauge fields A_i and B_i, that under local transformations (15) are transformed in the corresponding gradient manner. The A_i field for a certain class of problems is a pure gauge field, and the B_i field, whose gauge transformation has the form

$$B_i \to B_i + b\left[(r-1)\sigma(x)/r\right]_{;i},$$

where b is a constant with the dimensionality of length, interacts with the conserved rest mass current j^i, adding the term proportional to $B_i j^i$ to the full gauge invariant Lagrangian.

2.2. The Relativistically Invariant Finslerian Spaces with Entirely Broken 3D Isotropy

In general case, the metric of relativistically invariant Finslerian spaces with entirely broken 3D isotropy [46,47] is:

$$\begin{aligned} \mathrm{d}s = &\left(\mathrm{d}x_0 - \mathrm{d}x_1 - \mathrm{d}x_2 - \mathrm{d}x_3\right)^{(1+r_1+r_2+r_3)/4} \\ &\times\left(\mathrm{d}x_0 - \mathrm{d}x_1 + \mathrm{d}x_2 + \mathrm{d}x_3\right)^{(1+r_1-r_2-r_3)/4} \\ &\times\left(\mathrm{d}x_0 + \mathrm{d}x_1 - \mathrm{d}x_2 + \mathrm{d}x_3\right)^{(1-r_1+r_2-r_3)/4} \\ &\times\left(\mathrm{d}x_0 + \mathrm{d}x_1 + \mathrm{d}x_2 - \mathrm{d}x_3\right)^{(1-r_1-r_2+r_3)/4}. \end{aligned} \tag{16}$$

The three parameters (r_1, r_2 and r_3) characterize the anisotropy of spaces (16) and have the following restrictions

$$\begin{aligned} &1 + r_1 + r_2 + r_3 \ge 0, \qquad 1 + r_1 - r_2 - r_3 \ge 0, \\ &1 - r_1 + r_2 - r_3 \ge 0, \qquad 1 - r_1 - r_2 + r_3 \ge 0. \end{aligned}$$

It should be noted that if $r_1 = r_2 = r_3 = 0$, then the metric (16) becomes the fourth power root of the product of four 1-forms

$$\begin{aligned} &\mathrm{d}s_{B-M} \\ &= \left[\left(\mathrm{d}x_0 - \mathrm{d}x_1 - \mathrm{d}x_2 - \mathrm{d}x_3\right)\left(\mathrm{d}x_0 - \mathrm{d}x_1 + \mathrm{d}x_2 + \mathrm{d}x_3\right)\right. \\ &\left.\times\left(\mathrm{d}x_0 + \mathrm{d}x_1 - \mathrm{d}x_2 + \mathrm{d}x_3\right)\left(\mathrm{d}x_0 + \mathrm{d}x_1 + \mathrm{d}x_2 - \mathrm{d}x_3\right)\right]^{1/4} \end{aligned}$$

Thus, in this particular case, we obtain the well-known Berwald-Moor metric, but written in the basis, which was introduced in [46].

Now consider the group of isometries of flat Finsler space (16). The homogeneous 3-parametric non-compact group of isometries, *i.e.* the group of the relativistic symmetry of space-time (16) appears to be Abelian, and the transformations belonging to such a group have the same meaning as the ordinary Lorentz boosts. The explicit form of these transformations is

$$x_i' = DL_{ik} x_k \tag{17}$$

where

$$D = e^{-(r_1\alpha_1 + r_2\alpha_2 + r_3\alpha_3)}$$

L_{ik} are the unimodular matrices that are given by the formulas

$$L_{ik} = \begin{pmatrix} A & -B & -C & -D \\ -B & A & D & C \\ -C & D & A & B \\ -D & C & B & A \end{pmatrix} \tag{18}$$

$$\begin{aligned} &A = \cosh\alpha_1 \cosh\alpha_2 \cosh\alpha_3 + \sinh\alpha_1 \sinh\alpha_2 \sinh\alpha_3, \\ &B = \cosh\alpha_1 \sinh\alpha_2 \sinh\alpha_3 + \sinh\alpha_1 \cosh\alpha_2 \cosh\alpha_3, \\ &C = \cosh\alpha_1 \sinh\alpha_2 \cosh\alpha_3 + \sinh\alpha_1 \cosh\alpha_2 \sinh\alpha_3, \\ &D = \cosh\alpha_1 \cosh\alpha_2 \sinh\alpha_3 + \sinh\alpha_1 \sinh\alpha_2 \cosh\alpha_3, \end{aligned}$$

α_1, α_2, α_3 are the parameters of the group. Along with the parameters α_i, the components $\mathrm{v}_i = \mathrm{d}x_i/\mathrm{d}x_0$ of the coordinate velocity of the primed reference frame can also be used as group parameters. The parameters v_i and α_i are related by

$$\begin{aligned} &\mathrm{v}_1 = \frac{\left(\tanh\alpha_1 - \tanh\alpha_2 \tanh\alpha_3\right)}{\left(1 - \tanh\alpha_1 \tanh\alpha_2 \tanh\alpha_3\right)} \\ &\mathrm{v}_2 = \frac{\left(\tanh\alpha_2 - \tanh\alpha_1 \tanh\alpha_3\right)}{\left(1 - \tanh\alpha_1 \tanh\alpha_2 \tanh\alpha_3\right)} \\ &\mathrm{v}_3 = \frac{\left(\tanh\alpha_3 - \tanh\alpha_1 \tanh\alpha_2\right)}{\left(1 - \tanh\alpha_1 \tanh\alpha_2 \tanh\alpha_3\right)} \end{aligned} \tag{19}$$

The reverse relations have the form

$$\begin{aligned} &\alpha_1 = \frac{1}{4}\ln\frac{\left(1+\mathrm{v}_1-\mathrm{v}_2+\mathrm{v}_3\right)\left(1+\mathrm{v}_1+\mathrm{v}_2-\mathrm{v}_3\right)}{\left(1-\mathrm{v}_1-\mathrm{v}_2-\mathrm{v}_3\right)\left(1-\mathrm{v}_1+\mathrm{v}_2+\mathrm{v}_3\right)}, \\ &\alpha_2 = \frac{1}{4}\ln\frac{\left(1-\mathrm{v}_1+\mathrm{v}_2+\mathrm{v}_3\right)\left(1+\mathrm{v}_1+\mathrm{v}_2-\mathrm{v}_3\right)}{\left(1-\mathrm{v}_1-\mathrm{v}_2-\mathrm{v}_3\right)\left(1+\mathrm{v}_1-\mathrm{v}_2+\mathrm{v}_3\right)}, \\ &\alpha_3 = \frac{1}{4}\ln\frac{\left(1-\mathrm{v}_1+\mathrm{v}_2+\mathrm{v}_3\right)\left(1+\mathrm{v}_1-\mathrm{v}_2+\mathrm{v}_3\right)}{\left(1-\mathrm{v}_1-\mathrm{v}_2-\mathrm{v}_3\right)\left(1+\mathrm{v}_1+\mathrm{v}_2-\mathrm{v}_3\right)}. \end{aligned} \tag{20}$$

As for the generators X_i of the homogeneous 3-parametric group of isometries (17) of the space-time (16), they can be represented as follows

$$X_1 = -r_1 x_\alpha p_\alpha - (x_1 p_0 + x_0 p_1) + (x_2 p_3 + x_3 p_2),$$
$$X_2 = -r_2 x_\alpha p_\alpha - (x_2 p_0 + x_0 p_2) + (x_1 p_3 + x_3 p_1),$$
$$X_3 = -r_3 x_\alpha p_\alpha - (x_3 p_0 + x_0 p_3) + (x_1 p_2 + x_2 p_1),$$

where $p_\alpha = \partial/\partial x_\alpha$ are the generators of the 4-parametric group of translations. Thus, with inclusion of the latter, a inhomogeneous group of isometries of the entirely anisotropic Finsler space of events (16) is a 7-parametric group. As to its generators, they satisfy the commutation relations

$$\begin{aligned}
&[X_i X_j] = 0, && [p_\alpha p_\beta] = 0,\\
&[X_1 p_0] = r_1 p_0 + p_1, && [X_2 p_0] = r_2 p_0 + p_2,\\
&[X_1 p_1] = r_1 p_1 + p_0, && [X_2 p_1] = r_2 p_1 - p_3,\\
&[X_1 p_2] = r_1 p_2 - p_3, && [X_2 p_2] = r_2 p_2 + p_0,\\
&[X_1 p_3] = r_1 p_3 - p_2, && [X_2 p_3] = r_2 p_3 - p_1,\\
&[X_3 p_0] = r_3 p_0 + p_3, && [X_3 p_1] = r_3 p_1 - p_2,\\
&[X_3 p_2] = r_3 p_2 - p_1, && [X_3 p_3] = r_3 p_3 + p_0.
\end{aligned}$$

3. Modeling Real World by the Phase Space-Time and Physical Results Obtained on This Way

The theory and results briefly given below are discussed in detail in the monograph [83].

Let $M = \mathbb{R}^4$ be a differentiable 4-dimensional manifold of class C^∞. Let *TM* be its tangent bundle with coordinates $(x, y) = (x^i, y^i)$; $i = 0,1,2,3$. If *c* is a parametrizable curve on *M*, $c:[a,b] \to M$, $t \to (x^i(t))$, then its natural extension on *TM* is $c:[a,b] \to TM$, $t \to (x^i(t), y^i(t))$, where $y^i = \mathrm{d}x^i/\mathrm{d}t$. The arc length *s*, usually chosen as the natural parameter on the curve is thus equal to $s = \int_0^t \sqrt{g_{ij} y^i y^j}\,\mathrm{d}\tau$; $i, j = 0,1,2,3$. Suppose that the metric introduced above depends on *y*, *i.e.* $g_{ij} = g_{ij}(x, y)$. In general, this metric corresponds to the generalized Lagrange geometry, $g_{ij}(x, y)$ is a twice covariant symmetric tensor on *TM* with the only restrictions: a) $det(g_{ij}) \neq 0$ for any (x, y) on *TM* and b) when the coordinates on *TM* change in the way corresponding to the change of coordinates on *M*, the components of the metric vary in the same way as the components of the (0,2)-tensor on the main manifold *M*. This means that *TM* is an 8-dimensional Riemannian manifold, analogous to the 6-dimensional phase space well-known in physics. Its geometry is quite complicated and uses such concepts as nonlinear connection (Ehresmann). But if we limit ourselves to the case of linear coordinate transformations with constant coefficients and of weak gravitational field, *i.e.*

$$g_{ij}(x, y) = \eta_{ij} + \varepsilon_{ij}(x, y);$$
$$\eta_{ij} = diag\{1, -1, -1, -1\};$$
$$\varepsilon_{ij}(x, y) = \chi \tilde{\varepsilon}_{ij}(x, y);\ \chi \ll 1,$$

the geometry essentially simplifies, and the definition of y^i makes it possible to use the Sasaki lift for raising and lowering indices on the vertical and horizontal components of the bundle, that is use the same metric tensor. The tensor g_{ij} is a zero-order homogeneous in *y* tensor, *i.e.* the metric depends only on the direction of *y*, but not on its value. This is expressed by the relation $(\partial g_{ij}/\partial y^k) y^k = 0$. If there also holds the condition $(\partial g_{ij}/\partial y^k) y^j = 0$, then this metric becomes the usual Finsler one [1], but in this approach this is not assumed.

The described formalism means that alongside with the use of a new geometry for the modeling of phenomena in the physical world, instead of the space and time of Newton or of the Minkowski space-time, the 8-dimensional phase space-time is introduced. The character of its extra dimensions is not formal, but they have clear physical meaning, due to the used approach. Clearly, the correspondence between the Lagrangian and Hamiltonian formalism now obtains a new dimension. It should be noted that similarly to the situation when the transition from Newton's time and space to the Minkowski space-time took place and the fundamental constant *c* with the dimension of speed was demanded, the transition from Minkowski space-time to the 8-dimensional phase space-time demands another fundamental constant, *l*, this time with the dimension of length. One can associate it with the fundamental speed and take $l = c/H$, then *H* will be a new constant which has the dimension s^{-1}. This suggests that in the interpretations, the following correspondence

$$(x^0, x^1, x^2, x^3, y^0, y^1, y^2, y^3) \leftrightarrow$$
$$(ct, x, y, z, c/H, v_x/H, v_y/H, v_z/H)$$

should be borne in mind. One should also pay attention to the fact that all the events would take place in the 7-dimensional subspace of the 8-dimensional phase space-time, one of the coordinates of which is constant according to construction. The symmetry groups corresponding to this space will be the generalized Lorentz group and de Sitter group. The last can be contracted and be used in the Carroll space and in the Newton-Hooke space that are of interest for the astronomical applications. The possibility of separating the resulting space into such parts as (x, y, z) and $(ct, c/H, v_x/H, v_y/H, v_z/H)$ allows the use of Lobachevsky geometry to describe the space of velocities, this geometry was previously used only in the theory of high-energy particles.

Preserving only linear terms proportional to $\partial\varepsilon_{ij}/\partial x^k$, $\partial\varepsilon_{ij}/\partial y^k$ and $\partial^2\varepsilon_{ij}/\partial x^k\partial y^l$, one can obtain the generalized geodesics similarly to [48-50] in the following form

$$\frac{\mathrm{d}y^i}{\mathrm{d}s}+\left(\Gamma^i{}_{lk}+\frac{1}{2}\eta^{it}\frac{\partial^2\varepsilon_{kl}}{\partial x^j\partial y^t}y^j\right)y^ky^l=0 \qquad (21)$$

where $\Gamma^i{}_{jk}=\frac{1}{2}\eta^{ih}\left(\partial\varepsilon_{hj}/\partial x^k+\partial\varepsilon_{hk}/\partial x^j-\partial\varepsilon_{jk}/\partial x^h\right)$ is the Christoffel symbol depending on y. Thus, in order to obtain the equations of motion (dynamic equations) in the weak field limit in the anisotropic space, one should use (21), but not geodesic equation $\mathrm{d}y^i/\mathrm{d}s+\Gamma^i{}_{lk}y^ly^k=0$, which is appropriate in the same approximation only in a space with Riemann geometry. As a result, after certain simplifications and extraction of the anti-symmetric part of the auxiliary tensor introduced in [10-15], the equation of motion obtained from the geodesic (21) and applied to the spatial cross-section of the space takes the form

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=\frac{c^2}{2}\left\{-\nabla\varepsilon_{00}+\left[\mathbf{v},rot\frac{\partial\varepsilon_{00}}{\partial\mathbf{v}}\right]+\nabla\left(\mathbf{v},\frac{\partial\varepsilon_{00}}{\partial\mathbf{v}}\right)\right\} \qquad (22)$$

where ε_{00} is the only (temporal) component of the metric tensor, which remains in the equation of motion in the approximation of the weak field. Regarding (22) as the equation of dynamics, we obtain the expression for the generalized gravitational force depending on velocities [10-15]

$$\mathbf{F}^{(g)}=\frac{mc^2}{2}\left\{-\nabla\varepsilon_{00}+\left[\mathbf{v},rot\frac{\partial\varepsilon_{00}}{\partial\mathbf{v}}\right]+\nabla\left(\mathbf{v},\frac{\partial\varepsilon_{00}}{\partial\mathbf{v}}\right)\right\} \qquad (23)$$

The last two equations are obtained from the geodesics corresponding to the field equations for an anisotropic metric. They do not require a special choice of the energy-momentum tensor, and any additional a priori assumptions. The field equations in the anisotropic space in the linear approximation for weak fields retain their form [51], although their terms may now depend on y.

To study the dynamics of spiral galaxies, one could choose

$$\mathbf{u}\equiv\frac{c^2}{4}\frac{\partial\varepsilon_{00}}{\partial\mathbf{v}}\equiv[\mathbf{\Omega},\mathbf{r}],$$

where

$$\mathbf{\Omega}=\frac{c^2}{4}rot\frac{\partial\varepsilon_{00}}{\partial\mathbf{v}}$$

and

$$\mathbf{\Omega}=rot\left(\int\frac{j^{(m)}(r)}{|r-r_0|}\mathrm{d}V\right)$$

where $j^{(m)}(r)$ is the mass current density, and r_0 corresponds to the observer. Then the equation for the gravitational force obtains the form

$$\mathbf{F}^{(g)}=\frac{mc^2}{2}\nabla\left\{-\varepsilon_{00}+\frac{2}{c^2}\cdot 4(\mathbf{u},\mathbf{v})\right\} \qquad (24)$$

If we demand the existence of limit transition to the usual GRT, then

$$\mathbf{F}^{(g)}=\frac{mc^2}{2}\nabla\left\{-\sum_n\frac{r_{n,s}}{r_n}+\frac{2}{c^2}\cdot 4(\mathbf{u},\mathbf{v})\right\} \qquad (25)$$

and it can be shown that the second term under the gradient has the same order of magnitude as the first one at distances of the order of a galaxy radius. It is this that prevents the vanishing of orbital velocity required by the general relativity. At the same time, the motion in the galactic plane and perpendicular to this plane is now described by the different laws, which removes the well-known paradox [52] in the observations of motion of stellar globular clusters.

In the framework of the suggested approach—anisotropic geometrodynamics (AGD)—the notion of a point mass is not sufficient to model the elementary (effective) source of gravitation, and one should use a system of "center plus current" which represents a gravitational analogue of the circular coil with current around the central charge. The use of such a system for simulation of a spiral galaxy, leads to the expression $v_{orb}\sim const$ for the orbital velocity corresponding to the observed flat rotation curve, and to the empirical Tully-Fisher law $v_{orb}\sim L_{lum}^{1/4}$, which has no explanation in general relativity. The same model can explain the observed substantial excess of deflection in some gravitational lenses over the theoretical calculations, which appears to be due to the internal motions of the masses in the galaxy-lens. It has been also shown that in addition to the known convex gravitational lenses, in the AGD there exist concave gravitational lenses. This can lead to the incorrect determination of distances to the sources compared to "standard candles". And this can account for another interpretation of the data which led to the idea of the acceleration of the Universe expansion and to the notion of dark energy.

Calculation of the explosion of the central body in the "center plus current" model, resulting in the release of the two equal masses in opposite directions in the plane of the coil leads to trajectories that resemble the well-known observations obtained by the HUBBLE telescope (compare **Figures 1(a)** and **(b)**).

Besides, there are also the images received recently by the space observatory HERSCHEL [84] (compare **Figure 1(b)** and **Figure 2**) when photographing the center of our galaxy. Thus, there could be a new approach to the study of the origins of the arms and bars, characteristic of most spiral galaxies.

The theory presented in this section is based on the new notion, which serves the basis for the description of physical reality—the phase space-time admitting the use

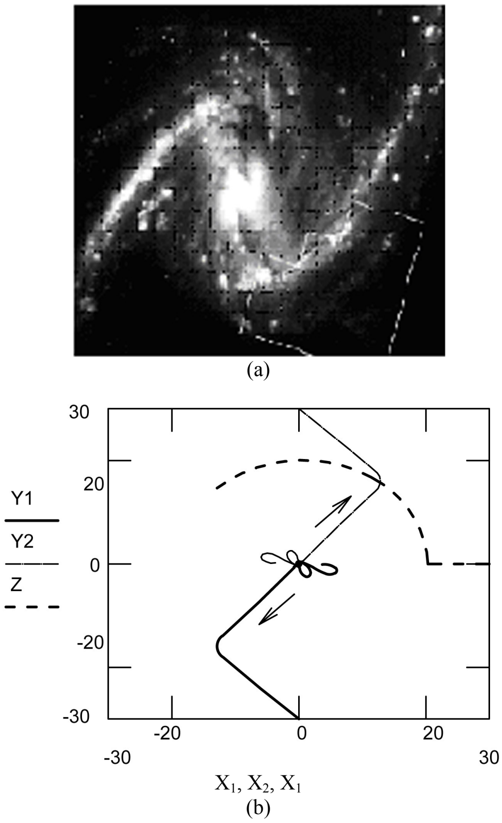


Figure 1. (a) Galaxy NGC-1365 (Hubble telescope image, NASA/ESA); (b) Numerical calculation based on the "center plus current" model in the framework of AGD (the exact view of the central details depends on the step of the calculation but they remain always present).

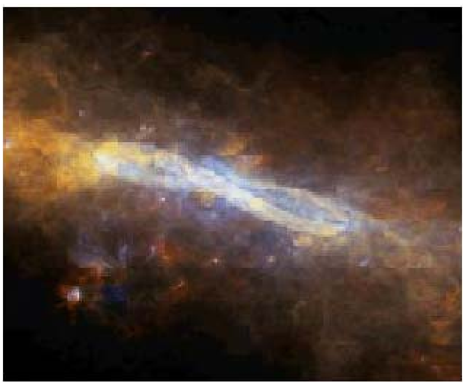


Figure 2. Details discovered by Herschel orbital observatory in the center of milky way.

of various geometries to describe its subspaces. The AGD approach is consistent with observations at the galactic scale, and does not require the introduction of dark matter. Besides, it includes a new (or additional) interpretation of the Hubble law, which takes into account not the radial expansion of the Universe but the various tangential motions of its distant parts. The last being regarded in the framework of AGD leads to a linear decrease of frequency with distance to the radiation sources explained by the *gravitational* red shift. The observations of extremely high tangent velocities of the distant quasars present an indirect evidence supporting this idea.

4. Mathematical, Physical and Geometric Aspects of Hyper-Complex Numbers Algebra

The natural basis for Finsler geometries of special type (the so-called Berwald-Moor spaces $\mathcal{H}_n$ with metric

$$ {}^nG = \hat{S}\left(\mathrm{d}x^1 \otimes \mathrm{d}x^2 \cdots \otimes \mathrm{d}x^n\right), \tag{26}$$

where $\hat{S}$ is the symmetrization operator (without the numerical factor)) represent the well-known associative-commutative algebras P_n.

This section of the review is devoted to presenting the geometrical, algebraic and physical results obtained in the study of poly-numbers associative-commutative algebras and Berwald-Moor geometries of various dimensions related to them.

4.1. Conformal Gauges and Non-Linear Symmetries

It is well known that the Finslerian Berwald-Moor space $\mathcal{H}_n$ have a rich (infinite) group of conformal symmetries $\mathcal{CH}_n$. We denote by $\mathcal{H}_n^f$ the Berwald-Moor manifold in a *special conformal gauge*, which can be obtained from $\mathcal{H}_n$ by the action of some $f \in \mathcal{CH}_n$. Instead of the transformations of the manifold $\mathcal{H}_n$ belonging to the group $\mathrm{Iso}\mathcal{H}_n$, we now have the transformations of the manifold $\mathcal{H}_n^f$ belonging to the group $\left(\mathrm{Iso}\mathcal{H}_n\right)^f < \mathcal{CH}_n^f$, whose elements ι^f are defined by the formula $\iota^f \equiv f \circ \iota \circ f^{-1}$. The action of the group $\left(\mathrm{Iso}\mathcal{H}_n\right)^f$ in the coordinate space of the manifold $\mathcal{H}_n^f$ in general case is described by nonlinear functions, so this group is naturally called *the nonlinear* f *-representation of the group* $\mathrm{Iso}\mathcal{H}_n$. In general, the group $\left(\mathrm{Iso}\mathcal{H}_n\right)^f$ can always be regarded as a (generally nonlinear) group of isometries $\mathrm{Iso}\mathcal{H}_n'^f$ of a manifold $\mathcal{H}_n'^f$, which differs from $\mathcal{H}_n^f$, only by its metric. The form of this metric depends on the type of the gauge function f.

In [53,54] there are concrete examples that illustrate the fact that the isometry group and the group of conformal symmetries of the Berwald-Moor metric can interact with each other in a non-trivial way leading to nonlinear symmetries of the known geometries.

4.2. Osculating Riemannian Metrics

With the disposal of the metric (26) and vector fields of Lie algebras of the groups $\mathrm{Iso}\mathcal{H}_n$ (and $\mathcal{CH}_n$), one can naturally obtain an infinite number of Riemannian metrics out of the metric (26) with the help of the following general technique. Consider the "incomplete" scalar poly-product of the form:

$$g = {}^{n}G\left(X_{(1)}, X_{(2)}, \cdots, X_{(n-2)}, , \right),$$

where $X_{(j)}$ are the elements of Lie algebras of the groups $\mathrm{Iso}\mathcal{H}_n$ and (or) $\mathcal{CH}_n$, that are for convenience numbered by the indices corresponding to their places as the arguments of the Berwald-Moor metric. It is obvious that g is a (pseudo-) Riemannian metric, depending on the chosen fields $X_{(j)}$. The described method leads to a generalization of the concept of "Riemannian metric osculating to a given Finslerian metric", discussed in [1].

Consider as a reference vector field a common element of the Lie algebra of the subgroup of the uni-modular dilations $\mathrm{Iso}_D\mathcal{H}_3$ of the complete group $\mathrm{Iso}\mathcal{H}_3$, which has the form:

$$X = b_1 D_1 + b_2 D_2 = b_1 x^1 \partial_1 + \left(b_2 - b_1\right) x^2 \partial_2 - b_2 x^3 \partial_3,$$

where b_1, b_2 are arbitrary real parameters. The Riemannian metric osculating along this field has the form:

$$\begin{aligned} g = {} & b_1 x^1 \left(\mathrm{d}x^2 \otimes \mathrm{d}x^3 + \mathrm{d}x^3 \otimes \mathrm{d}x^2\right) \\ & + \left(b_2 - b_1\right) x^2 \left(\mathrm{d}x^1 \otimes \mathrm{d}x^3 + \mathrm{d}x^3 \otimes \mathrm{d}x^1\right) \\ & - b_2 x^3 \left(\mathrm{d}x^1 \otimes \mathrm{d}x^2 + \mathrm{d}x^2 \otimes \mathrm{d}x^1\right). \end{aligned} \tag{27}$$

This metric is generally not flat. Its determinant defining a local volume element is given by:

$$\det\left(g\right) = 2 b_1 b_2 \left(b_2 - b_1\right) x^1 x^2 x^3.$$

One can see that the metric (27) is nonsingular only if all of the conditions: $b_1 \neq 0, \quad b_2 \neq 0, \quad b_1 \neq b_2$ are fulfilled simultaneously. The standard study of isometries and conformal symmetries of this metric reveals the fact that this metric has a 3-dimensional algebra of isometries and 10-dimensional algebra of conformal symmetries. Such a rich algebra of conformal symmetries is a residual "track" of the infinite-dimensional algebra of conformal symmetries of the original Berwald-Moor metric (26) for $n = 3$.

The study of residual symmetries of the Riemannian metrics osculating to the Berwald-Moor metrics admits a more general setting in which we obtain the following basic relations:

$$\begin{aligned} L_{X_{(i)}} g_{(j)} &= L_{X_{(i)}} G\left(X_{(j)}, , \right) \\ &= G\left(\left[X_{(i)}, X_{(j)}\right], \ , \right) = c_{ij}^{k} g_{(k)} \end{aligned} \tag{28}$$

and

$$\begin{aligned} L_{\bar{X}_{(i)}} \bar{g}_{(j)} &= L_{\bar{X}_{(i)}} G\left(\bar{X}_{(j)}, , \right) \\ &= \phi_i \bar{g}_{(j)} + \bar{c}_{ij}^{k} \bar{g}_{(k)} = \left(\phi_i \delta_j^k + \bar{c}_{ij}^{k}\right) \bar{g}_{(k)}, \end{aligned} \tag{29}$$

where $X_{(j)}$ is an element of the Lie algebra of the group $\mathrm{Iso}\mathcal{H}_3$, $\bar{X}_{(j)}$ is an element of the Lie algebra of the conformal group $\mathcal{CH}_3$, c_{ij}^k and $\bar{c}_{ij}^k$ are the structural constants or the structural functions of the Lie algebra of the groups $\mathrm{Iso}\mathcal{H}_3$ and $\mathcal{CH}_3$ respectively.

Thus, *the families of metrics* $\left\{g_{(j)}\right\}$ *and* $\left\{\bar{g}_{(j)}\right\}$ *form differential ideals with respect to their Lie differentiation along the families of fields* $\left\{X_{(j)}\right\}$ *and* $\left\{\bar{X}_{(j)}\right\}$ *respectively*. This is a general property and it allows us to formulate some general theorems concerning the symmetry of Riemannian metrics $g_{(j)}$ and $\bar{g}_{(j)}$ [55].

4.3. Metric Bingles in $\mathcal{H}_3$

Studying the properties of angles in Finsler geometry is of particular interest for its physical applications. One of the approaches to the problem of constructing of additive poly-angles (e.g. bingles and tringles) is to formulate and solve the corresponding functional equations that satisfy the additivity condition [56]. Instead of solving the functional differential equations in the space of basic conformal invariants of B-M geometry, one can from the very beginning relate all the types of poly-angles with notions additive by their definition, such as lengths, areas or volumes, calculated on the unit sphere (indicatrix) of the B-M geometry.

It turns out that for any pair of non-isotropic vectors *A* and *B* one can introduce two types of bingles—mutual and relative. The expression for the mutual bingle has the following form:

$$\phi\left[A, B\right] = \left|A^{\flat} - B^{\flat}\right| \tag{30}$$

where $\flat$ is a bi-projection operation in $\mathcal{H}_3$, which acts on an arbitrary element $X \in \mathcal{H}_3$ according to the rule:

$$\left(X^{\flat}\right)^{i} = \ln \frac{X^{i}}{\left|X\right|}.$$

The norm in (30) is calculated with the help of the standard Berwald-Moor metric in the isotropic coordinates. The bingle defined by (30) is additive by definition, *i.e.* for any triplet of the "coplanar" vectors A, B, C there is a condition which is analogous to the Euclidean one:

$$\phi\left[A, C\right] = \phi\left[A, B\right] + \phi\left[B, C\right]. \tag{31}$$

The condition of coplanarity of the vectors A, B, C has the form of the condition of collinearity of the corresponding $\flat$-images:

$$\left(A^{\flat} - B^{\flat}\right) \wedge \left(A^{\flat} - C^{\flat}\right) = 0. \tag{32}$$

Expressions for the second (mutual) bingle (there may be three types of it, depending on the mutual orientation of vectors *A* and *B*) have the form:

$$\operatorname{cfh}\psi_i\left[A,B\right]=e^{\left(B^{bb}\right)_i-\left(A^{bb}\right)_i},\ i=1,2,3, \tag{33}$$

and the function, which is inverse to the Finsler-hyperbolic cosine cfh, is defined by the integral:

$$\operatorname{arccfh}(\xi)\equiv -\frac{1}{2}\int_{2^{-1/3}}^{\xi}\left(\left(x^2-\sqrt{x\left(x^3+4\right)}\right)\left(3x^2+\sqrt{x\left(x^3+4\right)}\right)\times\frac{\left(\sqrt{x(x^3+4)}+x^3-2\right)}{x^4(x^3+4)}\right)^{1/3}\mathrm{d}x. \tag{34}$$

Finally, the expression for the value analogous to the solid angle on the vectors *A, B* and *C* is given by the following integral:

$$\begin{aligned}\sum\left(A,B,C\right)=&\frac{3}{2}\phi^2\left[A,B\right]\left(\operatorname{cfh}\psi_1\left[B,C\right]\operatorname{cfh}\psi_1\left[A,B\right]-\operatorname{cfh}\psi_2\left[B,C\right]\operatorname{cfh}\psi_2\left[A,B\right]\right)^2\\&\int_{\operatorname{cfh}\psi_1[A,B]}^{\operatorname{cfh}\psi_1[A,C]}\left(\frac{1}{\sqrt{x^4+4x}}\times\frac{\mathrm{d}x}{\left(x/\operatorname{cfh}\psi_1\left[B,C\right]-\left(\sqrt{x^4+4x}-x^2\right)\Big/\left(2x\operatorname{cfh}\psi_2\left[B,C\right]\right)\right)^2}\right)\end{aligned} \tag{35}$$

The exact formulations, proofs and illustrations can be found in [57].

4.4. Classification of Homogeneous Cubic Metrics

Symmetry analysis of geometric objects is a key means of study of their internal invariant properties (*i.e.* being independent on the coordinates). In order to understand the place of Berwald-Moor metric among other related cubic metrics, the study of the isometry group of the general homogeneous cubic form

$$G=G_{\alpha\beta\gamma}\mathrm{d}x^{\alpha}\otimes\mathrm{d}x^{\beta}\otimes\mathrm{d}x^{\gamma} \tag{36}$$

was undertaken. Here $G_{\alpha\beta\gamma}$ are the constant real components of the cubic form. The results of the study are summarized in **Table 1**.

This proves that the symmetry analysis reveals 6 different symmetry classes (7th class is empty, and the 6th coincides with the 5th), the previously known 13 projective classes [58] are distributed amongst them. The Berwald-Moor metric falls into the 1st symmetry class. One of the important findings of this study is the conclusion on the incompleteness of the classification of cubic homogeneous metrics according to their isometry algebras [59].

4.5. *h*-Holomorphic Functions of a Double Variable

For the interpretation of $\mathbb{R}^2$ as a plane of a double variable $\mathcal{H}$, it is natural to consider only the maps that preserve the hyperbolic complex structure of the plane, *i.e.* by the maps $\mathcal{H}\to\mathcal{H}$ of the form: $h\mapsto s=F\left(h\right)$. The differentiable functions $\mathbb{R}^2\to\mathbb{R}^2$ which satisfy the condition: $F_{,\bar{h}}=0$, are called h-holomorphic functions of double variable *h*.

Let us formulate some important properties of the *h*-holomorphic functions as theorems.

Theorem 1. *Any* h*-holomorphic function maps zero divisors into zero divisors.*

Theorem 2. *The components* U *and* V *of the* h*-holomorphic function* $F=U+jV$ *satisfy the hyperbolic Cauchy-Riemann conditions:* $U_{,t}=V_{,x}$; $U_{,x}=V_{,t}$.

Theorem 3. *For any* h*-holomorphic function* F *in* D, *there holds the integral Cauchy Theorem*:

$$\oint_{\Gamma}F\left(h\right)\mathrm{d}h=0,$$

where Γ *is a simple closed piecewise smooth contour which has no isotropic elements and lies entirely in* D.

Theorem 4. *For any* h*-holomorphic function* F *in* D, *there holds true the integral Cauchy formula*:

$$\oint_{\Gamma}\frac{F\left(h\right)}{h-h_0}\mathrm{d}h=0,$$

where Γ *is a simple closed piecewise smooth contour which has no isotropic elements, lies entirely in* D. *and encloses the point* h_0.

Other versions of Cauchy's integral formula are given in [60].

Theorem 5. *For a simple closed piecewise smooth contour* Γ *that has no isotropic elements and encloses the point* h_0, *we have the formula*:

$$\oint_{\Gamma}\left(h-h_0\right)^{\alpha}\mathrm{d}h=\begin{cases}0, & \alpha\neq-1;\\ j\ell_H, & \alpha=-1.\end{cases} \tag{37}$$

Table 1. Projective and symmetry classes of 3D cubic metrics.

Symmetry classes	1	2	3	4	5	6	7	8
Projective classes	III, XII	V	1):VIII, 2):VI,XIII, 3): VII	IV	II, X, XI	?	—	Gen., I, IX

where α *is any real number,* ℓ_H *is the improper "fundamental constant" in the plane of double variable that determines the amount of space of the hyperbolic angles (analogous to the constant* 2π *in the complex plane).*

Theorem 6. *The pseudo-Euclidean metric* $\eta = \mathrm{Re}\left(\mathrm{d}h \otimes \mathrm{d}\bar{h}\right)$ *is conformal relative to an arbitrary h-holomorphic mapping of the plane of the double variable.*

In [60] the properties of the basic elementary *h*-holomorphic functions of the double variable were studied in detail.

4.6. Hyperbolic Field Theory on the $\mathcal{H}_2$ Plane

We consider an arbitrary *h*-holomorphic function $F(h) = U + jV$ as complex *h*-potential of a certain 2-dimensional vector field (*h-field*) in the plane of double variable. The real part U of this function we associate with the potential of the field (h*-potential function*) and the imaginary part *V* we associate with *the strength function* of this field. We define the strength, $\mathcal{E}$ of the h-field by the formula:

$$\mathcal{E} = \mathcal{E}_t + j\mathcal{E}_x = -\overline{\frac{\mathrm{d}F}{\mathrm{d}h}} = -\frac{\mathrm{d}\bar{F}}{\mathrm{d}\bar{h}} = -U_{,t} + jU_{,x}, \tag{38}$$

which can be regarded as a double form of representation for the vector field of the gradient of the function *U* with respect to the pseudo-Euclidean metric. Equation (38) is obtained taking into account the hyperbolic Cauchy-Riemann conditions.

In view of the relation $\mathcal{E} = \mathcal{E}(\bar{z})$ (antiholomorphicity of strength), arising from the definition (38), we obtain the following identity:

$$\frac{\partial \mathcal{E}}{\partial h} = \frac{1}{2}\left[\mathcal{E}_{t,t} + \mathcal{E}_{x,x} + j\left(\mathcal{E}_{t,x} + \mathcal{E}_{x,t}\right)\right] = 0, \tag{39}$$

which is equivalent to two identities:

$$\begin{aligned} &\mathrm{divh}\,\mathcal{E} \equiv \mathcal{E}_{t,t} + \mathcal{E}_{x,x} = 0; \\ &\mathrm{roth}\,\mathcal{E} \equiv \mathcal{E}_{t,x} + \mathcal{E}_{x,t} = 0, \end{aligned} \tag{40}$$

expressing, respectively, *the solenoidal* and h*-potential* properties of the electrostatic field[1].

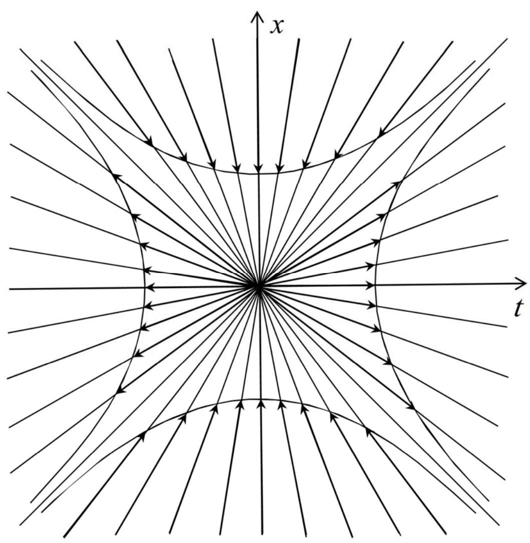
As an example, consider the h-potential of the form

$$F(h) = -q \ln h, \tag{41}$$

which is obviously the hyperbolic generalization of the Coulomb potential. The corresponding field strength is given by (38) and has the form:

$$\mathcal{E} = \frac{q}{\bar{h}} = \frac{qh}{|h|^2} = q\left(\frac{t}{t^2 - x^2} + j\frac{x}{t^2 - x^2}\right). \tag{42}$$

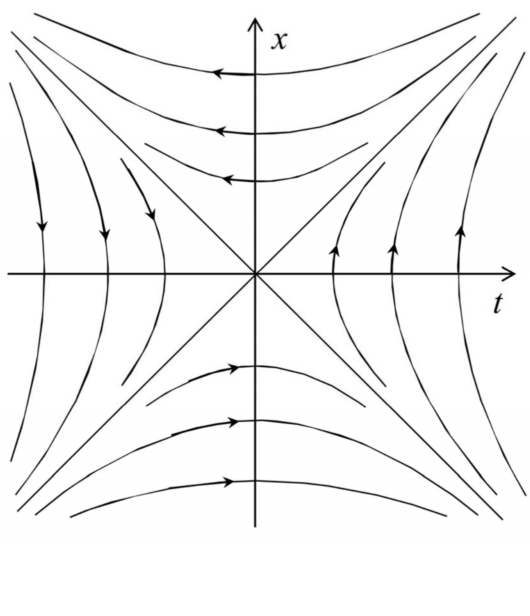
The field lines of a hyperbolic point source are the radial lines with $\psi = \text{const},$ and the equipotential lines are the hyperbolas $\varrho = \text{const}.$ The picture of the field lines in all 4 wedges is shown on the figure:



The dual interpretation of the hyperbolic point source is obtained by passing from the potential $F(h)$ in (41) to the potential $jF(h)$ At the same time for a new dual field $\mathcal{B}$ we get the following expression:

$$\mathcal{B} = j\frac{\mathrm{d}\bar{F}}{\mathrm{d}\bar{h}} = -\frac{qj}{\bar{h}} = -q\frac{x + jt}{t^2 - x^2}. \tag{43}$$

Field $\mathcal{B}$ is a hyperbolic analogue of a point vortex. Its lines of force are shown in the picture and present the hyperbolas:



[1]Notice that the divergence of the vector field is defined in the same way in the complex and hyperbolic cases, as opposed to the operation of the curl of a vector field, which includes the symmetric combination of partial derivatives in the hyperbolic case.

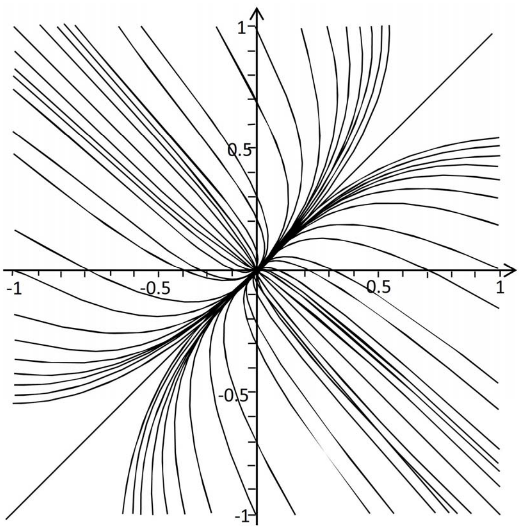
By analogy with the complex case, it is possible to combine the above two situations into one introducing the concept of *hyperbolic vortex-source* with the complex charge $\mathcal{Q} = q - jm$. Then the potential takes the form:

$$\begin{aligned} F(z) &= -\mathcal{Q}\ln h \\ &= -q\ln\varrho + m\psi - j\left(-m\ln\varrho + q\psi\right). \end{aligned} \tag{44}$$

Such a potential can be most naturally interpreted in the framework of dual-symmetric hyperbolic field theory in which the hyperbolic electric and magnetic charges and currents are present on "equal footing". The equation for the field lines of such field is obtained from (44) by equating the imaginary part to a constant:

$$(t+x)^{1-\alpha}(t-x)^{1+\alpha} = \text{const}, \tag{45}$$

where $\alpha = q/m$. The picture of the field lines for $\alpha = -2$ is shown on the figure:



For physical applications it is necessary to generalize the concept of the *h*-field for the case of commutative and associative algebras of higher dimensions. In what follows we illustrate the idea of such a generalization by the example of the algebra of 3-numbers P_3.

We start with an isotropic basis in the P_3, in which the h-holomorphic function has the following representation:

$$F(h) = F(\xi_1)e_1 + F(\xi_2)e_2 + F(\xi_3)e_3. \tag{46}$$

The operators of differentiation with respect to the independent variables $h, h^\dagger, h^\ddagger$ have the following form:

$$\begin{aligned} \frac{\partial}{\partial h} &= e_1\frac{\partial}{\partial \xi_1} + e_2\frac{\partial}{\partial \xi_2} + e_3\frac{\partial}{\partial \xi_3}; \\ \frac{\partial}{\partial h^\dagger} &= e_1\frac{\partial}{\partial \xi_3} + e_2\frac{\partial}{\partial \xi_1} + e_3\frac{\partial}{\partial \xi_2}; \\ \frac{\partial}{\partial h^\ddagger} &= e_1\frac{\partial}{\partial \xi_2} + e_2\frac{\partial}{\partial \xi_3} + e_3\frac{\partial}{\partial \xi_1}. \end{aligned} \tag{47}$$

With the help of (47), one can easily verify the validity of equations for function F in the form of (46) by the direct calculation in components

$$\begin{aligned} &\frac{\partial F}{\partial h^\dagger} = \frac{\partial F}{\partial h^\ddagger} = 0; \\ &\frac{\partial F}{\partial h} = \frac{\partial F_1}{\partial \xi_1}e_1 + \frac{\partial F_2}{\partial \xi_2}e_2 + \frac{\partial F_3}{\partial \xi_3}e_3, \end{aligned} \tag{48}$$

where here and further $F_i \equiv F(\xi_i)$ is the same function of various isotropic variables.

The conditions of holomorphicity (multidimensional analogue of the standard Cauchy-Riemann conditions), in symmetric non-isotropic basis $\{j_1, j_2, j_3\}$ which is defined by:

$$\begin{aligned} j_1 &= e_1 - e_2 - e_3; \\ j_2 &= -e_1 + e_2 - e_3; \\ j_3 &= -e_1 - e_2 + e_3 \end{aligned} \tag{49}$$

and by the rules of multiplication:

$$\begin{aligned} &j_i^2 = -(j_1 + j_2 + j_3); \\ &j_i \cdot j_k = j_l \quad (j \neq k \neq l), \end{aligned} \tag{50}$$

have the form of matrix differential equations:

$$\begin{pmatrix} -(\bar\partial + \partial_3) & \partial_{2-1} & \partial_{1-2} \\ \partial_{2-3} & -(\bar\partial + \partial_1) & \partial_{3-2} \\ \partial_{1-3} & \partial_{3-1} & -(\bar\partial + \partial_2) \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = 0, \tag{51}$$

$$\begin{pmatrix} -(\bar\partial + \partial_2) & \partial_{1-3} & \partial_{3-1} \\ \partial_{1-2} & -(\bar\partial + \partial_3) & \partial_{2-1} \\ \partial_{3-2} & \partial_{2-3} & -(\bar\partial + \partial_1) \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = 0 \tag{52}$$

for every *h*-holomorphic function $\underline{F}(h) = U_1 j_1 + U_2 j_2 + U_3 j_3$. Here $\partial_{i-j} \equiv \partial_i - \partial_j$, $\bar\partial = \partial_1 + \partial_2 + \partial_3$. Due to the invariance properties of the *h*-holomorphy with respect to the choice of the algebra basis, we can say that the general solution of (51) and (52) is written by representing U_i in terms of F_i (components in the isotropic basis) expressed in terms of *x*-coordinates:

$$\begin{aligned} U_1 &= F(x_2 - x_1 - x_3) + F(x_3 - x_1 - x_2); \\ U_2 &= F(x_1 - x_2 - x_3) + F(x_3 - x_1 - x_2); \\ U_3 &= F(x_1 - x_2 - x_3) + F(x_2 - x_1 - x_3). \end{aligned} \tag{53}$$

This fact can be verified by direct substitution of (53) into (51) and (52). The combinations of coordinates in the arguments of *F* present the higher analogues of retarded and advanced arguments in the double plane.

The third-order operator

$$\Delta^{(3)} \equiv \frac{\partial}{\partial h}\frac{\partial}{\partial h^\dagger}\frac{\partial}{\partial h^\ddagger} = (e_1 + e_2 + e_3)\frac{\partial}{\partial \xi_1}\frac{\partial}{\partial \xi_2}\frac{\partial}{\partial \xi_3} \tag{54}$$

is proportional to the algebraic unity, so for every smooth function $F\left(h,h^{\dagger},h^{\ddagger}\right)$

$$\Delta^{(3)}F=\left(\Delta^{(3)}U_1\right)j_1+\left(\Delta^{(3)}U_2\right)j_2+\left(\Delta^{(3)}U_3\right)j_3. \quad (55)$$

If the function F is h-holomorphic, then because the operator $\Delta^{(3)}$ contains differentiation with respect to $h^{\dagger}$ and $h^{\ddagger}$, there holds the relation $\Delta^{(3)}F\equiv 0$, which is equivalent to its three components:

$$\Delta^{(3)}U_i\equiv 0 \quad (i=1,2,3). \quad (56)$$

Equation (56) is a 3-dimensional analogue of the harmonicity conditions or the hyperbolic harmonicity conditions which are identically satisfied by holomorphic functions of complex or of double variable, respectively.

The discussion and development of these ideas can be found in [61].

4.7. Conformal Two-Dimensional Theory of Relativity

We extend the Poincare group acting on the two-dimensional space-time $\mathcal{M}_2$ to a group of arbitrary h-holomorphic transformations that operate on points-events of space-time as on the elements of $\mathcal{H}_2$. Using the exponential representation for the derivative of F':

$$F'(h)=\epsilon\left|F'\right|(t,x)e^{j\psi(t,x)}, \quad (57)$$

we conclude that locally h-holomorphic transformations implement not only reflections and boosts known in the theory of relativity but also the extension of lengths of the vectors (scalar factor $\left|F'\right|(t,x)$). Let us consider the function $F=U+jV$ as the complex potential of the reference vector field of the 2-velocity or *the reference field of the proper time*. The field of 2-velocity u is determined by the formula:

$$u=\frac{\mathrm{d}F}{\mathrm{d}h}=\frac{\partial U}{\partial t}+j\frac{\partial U}{\partial x}, \quad (58)$$

which uses the definition of the operator of complex differentiation and the hyperbolic Cauchy-Riemann conditions. The square of the modulus of the 2-velocity is

$$|u|^2=(\nabla U)^2=(\nabla V)^2=\left|F'\right|^2. \quad (59)$$

"The velocity field" of the proper time for any integral curve Γ of this field is given by:

$$\frac{\mathrm{d}\tau}{\mathrm{d}s}=\left|F'\right|. \quad (60)$$

Now in the h-holomorphic theory of relativity under consideration, the intervals of the pseudo-Euclidean length and time become different and the relationship between them at each point is governed by the hyper-complex potential F.

The integral curves of the field ∇V are spatial sections of 2-dimensional space-time, orthogonal to the lines of time at every point. Thus, the scale factor "governs" both the course of the proper time and spatial distances.

For arbitrary motions of test particles, the length and time intervals are calculated as:

$$\frac{\mathrm{d}\tau}{\mathrm{d}s}=\eta(\nabla U,w);\quad \frac{\mathrm{d}\ell}{\mathrm{d}s}=\eta(\nabla V,w), \quad (61)$$

where w is the standard vector of the 2-velocity of the test particle $(|w|=1)$.

The simplest version of the variational principle of the dynamics theory of the hyperbolic field that takes into account the non-holomorphy of the hyperbolic potential inside the sources is determined by the action of the form:

$$\mathcal{S}\left[F,\bar{F}\right]=\alpha\int_{\mathcal{H}_2}\left\{\left|F_{,h}\right|^2-\mathcal{U}\left(\left|F_{,\bar{h}}\right|^2\right)\right\}\mathrm{d}h\wedge\mathrm{d}\bar{h}, \quad (62)$$

where the first term under the integral is a hyperbolic "kinetic term". It is responsible for the dynamics of the hyperbolic potential in vacuum. The second term represents a hyperbolic "potential term" and is responsible for the properties and for the contribution of sources. This last term depends only on the hyperbolic modulus of the magnitude of non-holomorphy, and in the region outside the sources, where the non-holomorphy becomes equal to zero, it defines (in the action) a certain "full divergence" that does not give any contribution to the equations of motion. The standard procedure of varying the action (62) over the field variables $\bar{F},F$ leads to the following field equations:

$$\frac{1}{4}\Box F=\left(\mathcal{U}'F_{,\bar{h}}\right)_{,h}. \quad (63)$$

This expression is the inhomogeneous wave equation with a source on the right-hand side, depending only on the non-holomorphy of F. As expected, the field equations are nonlinear, since the field F, as follows from the principles of the theory, describes its own sources through effective self-interaction. In this sense the developed theory is adjacent to the versions of the unified field theory by Mie.

A remarkable feature of Equation (63) is the existence (regardless of the specific form of the potential function $\mathcal{U}$) of the first integral

$$F_{,\bar{h}}\left(1-\mathcal{U}'\right)=\varphi\left(\bar{h}\right) \quad (64)$$

containing an arbitrary function $\varphi\left(\bar{h}\right)$.

The explicit expressions for the energy density of the algebraidized matter ε and its pressure p, obtained using the standard formalism of the field theory (Noether's theorem), have the form:

$$\varepsilon = \mathcal{U} - \mathcal{U}'X + \left(1-\mathcal{U}'\right)\sqrt{XY};$$
$$p = \mathcal{U} - \mathcal{U}'X - \left(1-\mathcal{U}'\right)\sqrt{XY}, \tag{65}$$

where $Y \equiv \left|F_{,h}\right|^2$.

With the help of the super-variational principle introduced in [62], it appears possible to calculate the general form of the potential $\mathcal{U}$ in the theory presented here

$$\mathcal{U}\left(X\right) = 3X + U_0 + 2U_1 \ln\left|1-\frac{X}{U_1}\right|, \tag{66}$$

where U_0 and U_1 are two fundamental constants of the theory.

5. Differential-Geometric Aspects of the Theory of Berwald-Moor Type Finsler Spaces of Various Dimensions

In order to find out the fundamental relations between the scalar poly-product and geometric objects induced by it, as a continuation of research on these interrelations (which started by the works of M.Matsumoto, H. Shimada, S.Numata, K.Okubo and of Romanian geometers [70, refs. [29-32] and [43]] and [75, refs.[3,4]]), new correlations were obtained between the Berwald-Moor *m*-th root pseudo-norms and geometric objects from the classical Finslerian context ([75, §2 and 5-7]). Such relations were investigated in [72] and [74]. Their role is a methodological one: they enhance the process of deriving properties of certain structures (e.g., projective ones, [75]), or passing from algebraic aspects of *m*-th root metric theory to differential geometry specific aspects from the theory of Finsler spaces.

Description of the Obtained Results

The study of connections which are compatible with remarkable geometric structures was performed in [72], where, for specific connections from Finsler geometry (e.g., for Cartan, Barthel and Miron connections), the authors point out the properties of induced connections on hypersurfaces, as a necessary step in the study of mean *Y*-curvature within the *N*-extremality framework. In this study, the authors propose an original software for the calculation and the use of Finslerian geometric objects specific to the study of *y*-minimal submanifolds. With the help of Maple symbolic calculations, they determine the coefficients of these geometric objects for low-dimensional manifolds equipped with 3-rd and 4-th root metrics and with Berwald-Moor conformal metrics. The underlying algorithm of this Maple software, was introduced by M. Matsumoto ([72, refs. [39,40]]) and is likely to provide a wide range of applications in the study of anisotropic media.

Moreover, in the paper [75,§3], there are indicated the essential connections to be used in determining whether an *m*-th root metric space is of Berwald or of Douglas type, and are derived original results concerning spaces with Berwald-Moor type metrics.

On the other side, in [68], Landsberg spaces are characterized by means of classical connections (Vasiman and Levi-Civita) and the connections which are subsequently induced into the structural and transversal fibers of the bundle, are indicated.

The work [63,§2-3] performs a preliminary study of the cotangent bundle of spaces endowed with Berwald-Moor metric; in the cited paper, one defines the *v*-curvature tensors and the *T*-tensor of the Berwald-Moor space, for the case of Shimada-type *m*-th root metrics; the results are specialized for the case of dual *m*-th root metric spaces having the indicatrix given by the product of the momentum components. In this case, the classical results regarding the vanishing of the torsion tensor and of the T-tensor, obtained by M. Matsumoto and H. Shimada ([63, refs.[5,11]]) and also the property of $S3$-likeness for the Berwald-Moor space, were obtained for the first time for the dual case.

The determining of connections and of induced geometric objects on submanifolds of *m*-th root metric spaces was carried out in the paper [72], by summing up known results and by original implementation of their construction into Maple code, using macros and supplementary procedures which simplify the use of the code and allows to extend the results specific to the study of the indicatrix.

The procedure for obtaining the mean curvature and minimal (*Y*-extremal) surfaces/hypersurfaces and corresponding computer simulation are presented in [72], which embeds two addenda devoted to the 4-dimensional case. Here, the mean curvature and the equations of Y-extremal (hyper-)surfaces are explicitly obtained by the use of symbolic software, and the calculation of the explicit form of the normal field to a submanifold (theoretically described in [72, ref. [40]]), represents a concrete application of software procedures in solving nonlinear equations. Also, the mean curvature – depending on the energy of a space-like or light-like normal vector field, is obtained by using specific procedures of the relativistic pseudo-Finslerian approach. This approach imposes restrictions on the submanifolds for the indicated practical applications, aiming to find solutions of the equations of *Y*-extremal submanifolds.

Another aim of an earlier planned research ([68, ref. [2]]) relates to determining specific types of cohomology in *m*-th root metric spaces ([68,77]), including the cohomologies of certain Berwald-Moor Finslerian spaces. These papers present new results concerning fibered structures of Finsler type: in [77], it is proved the existence of a diffeomorphism between the 2-jet vertical bundle induced by the canonical bundle and the product of the horizontal, vertical and mixed subbundles of 2-jets induced by the second order tangent bundle. In [68], it is introduced the Vaisman connection and it is proven that the pair Levi-Civita connection—Vaisman connection induces a pair of connections of the same type as the initial ones in the structural bundle, only if the base manifold M is a Landsberg space, and that the slit tangent bundle (the tangent bundle without the image of its null section, denoted as TM^0) is a Reinhart space if and only if the base manifold is Riemannian. Further, the 2-leaf jets on TM^0 are studied, it is obtained a decomposition of this space, and the 1-dimenional Cech cohomology group with coefficients from the sheaf of basic functions is constructed in terms of fields on leaves of 2-jets. It is defined the Mastrogiacomo cohology group with respect to the connection on the structure sheaf induced by a connection on the manifold TM^0 and it is proven that the associated cohomology group is isomorphic to the 1-dimensional Cech cohomology group on the manifold TM^0, having as coefficients germs of functions on TM^0, which are related to the induced connection; in particular, for 4-dimensional *m*-th root spaces, it is proven that this sheaf is isomorphic to the sheaf of basic functions on TM^0.

In [78,79], the $HC(n)$ Halphen-Castelnuovo problem for smooth curves is split into two parts: the study of the lacunary and of the non-lacunary domain. The latter one is studied: existence obstructions are determined and examples of curves on rational surfaces and irrational scrolls are built. There are studied for the first time the existence of smooth irreducible non-degenerate curves of degree d and genus g from the projective space (the Halphen-Castelnuovo $HC(n)$ problem). For the domains D_1^n and D_2^n, built in the plane ($d; g$), it is shown that D_1^n is simply connected, by using curves which are displaced on rational surfaces related to hyper-elliptic type sections, and are presented well known theorems of Ciliberto, Sernesi and Pasarescu. As well, using results of Horrowitz, Ciliberto, Harris and Eisenbud, it is conjectured that D_n is exactly the targeted lacunary domain.

Geodesics and Jacobi fields are investigated in [75,§2] and [74,§3], where geodesic equations and spray coefficients are introduced and studied for conformal flag metrics. In [75], the *hv*-curvature tensor is determined for arbitrary *m*-th root structures; this result is further used in determining the specific characterizations of Landsberg and Berwald-type *m*-th root metric spaces. Relations between the coefficients of two sprays for non-decomposable metrics are obtained in an explicit form, for cubic metrics in [74,§6]. All these results complete known results obtained for *m*-th root metric spaces by M. Matsumoto and H. Shimada. It is emphasized the role of flag curvature, which is a key one in describing the behavior of geodesics. This is continued in [73], where there are described the geodesic equations perturbed by the presence of an electromagnetic field. In Finsler spaces, the 4-potential 1-form is anisotropic, and represents a horizontal 1-form on the tangent bundle, having specific properties.

Relativistic models based on 4-dimensional *m*-th root metric are subject of intensive research. In [48], original results were obtained for the OMPR (*optic-metrical parametric resonance*) effect, with applications to relativity theory and to experimental physics (detection of gravitational waves). This work has been carried out in collaboration with the Russian physicist S. Siparov. The influence of a weak deformation of a flat pseudo-Finslerian metric upon the electromagnetic field tensor is studied in [73] and, in particular—for the case of *m*-th root metrics of Berwald-Moor type. The generalized geometric models are obtained and the physical meaning of such a generalization, together with its role in the equations of electromagnetism in Finsler spaces is pointed out. Geodesics and Jacobi fields are also studied in [70,§1-2] in the context of structural stability of second order differential equations, where the authors obtain original results for sheaves of curves and for the forces which deviate trajectories from geodesics in the case of conformal deformations of *m*-th root structures or locally Minkowski metrics. With the help of supplementary software designed to determine geodesics by means of the computer, original procedures for defining the invariants which characterize the stability of structures were derived.

In [75,§3-6], there are studied *m*-th root Berwald and Landsberg spaces and projectively flat spaces and, in the work [74,§4]—cubic spaces. In [75, Th.17 and Th.18], there are investigated *m*-th root projective spaces and, in particular, Riemann-projective spaces with *m*-th root metrics [75, Th.19, 20 and Prop.22]. All these results are original and they complete, in the case of *m*-th root metrics, known results obtained by S. Bacso, Zs.Szabo, L. Tamassy and Cs. Vincze.

In [70], the authors present the basic notions from the theory of structural stability (KCC—Kosambi-Cartan-Chern)—created and developed by P. L.Antonelli, I. Bucataru and S. Sabau [70, refs. [1-8, 48-49]] and carried out by V. Balan and I. R. Nicola for biological and ecological models ([70, refs. [10-13]]). The five KCC-invariants are described and in the Appendix, original Maple programs for determining the invariants describing Jacobi stability of dynamical systems associated with the Finslerian approach, are presented. Sections 4 and 5 contain original results for the case of conformal deformations of *m*-th root metrics and describe the properties of the associated deformation algebras.

The papers [64,66] extend known results for symmetric positive definite tensors for Z, H and E-spectra of Berwald-Moor and Chernov tensors in 4-dimensional spaces. The algebraic properties of these tensors induce geometrical properties: by spectral techniques it is shown that the indicatrices of the associated Finsler metric are not ruled and compact, and that the problem of the minimal distance between the origin of the frame and the indicatrix has a solution depending on the *Z*-eigenvalue with maximal absolute value and with the corresponding direction given by the generating *Z*-eigenvector. The problem of asymptotic behavior of the indicatrix is solved by means of spectral properties of the symmetric tensor associated to the fundamental Finsler function. There are defined: recession vectors, degenerate vectors, singular points of the indicatrix and the best first-order approximation. The qualitative description of the three types of eigenvalues is obtained in [71] with the help of the theory of resolvents for the cubic Berwald-Moor metric.

Hamilton equations, Legendre duality and physical models associated with *m*-th root metric spaces on the tangent or on the cotangent bundle are studied in [67], where it is indicated an essential parallelism between different transformations with physical meaning, and it is emphasized a Legendre-type relation between the Lagrangian formalism and the Hamiltonian one. It is investigated the alternative given by the use of the Rashevskii transformation commonly used in mechanics and its degenerate nature. The hyperbolic character of the Finsler and Cartan functions is emphasized and the correspondence between the basic geometric objects and their duals given by Legendre-Finsler duality for general Berwald-Moor metrics of arbitrary dimension is described.

In [51,80-82], the authors build models for the gravitational and for the electromagnetic fields, based on generalized Lagrange metrics and in particular, on the locally Minkowskian Berwald-Moor metric.

In [65], it is studied the geometry of submanifolds in *m*-th root metric spaces and in [67] the hyperbolic character of the Berwald-Moor metric is emphasized; in [65], the author proposes a pseudo-Finslerian formalism for Finsler metrics of locally Minkowski type, including the Berwald-Moor metric; a special attention is paid to the objects which allow to characterize minimal surfaces. Linear and nonlinear Cartan connections are studied and Gauss-Weingarten, Gauss-Codazzi, Peterson-Mainardi and Ricci-Kuhn equations are obtained. In [64,66], geometric properties of the Berwald-Moor indicatrix are obtained by means of spectral theory associated to a supersymmetric tensor; the spectra are obtained with the help of the Maple software.

The study of cohomology classes for *m*-th root metric spaces and the study of associated bundles extend known results by addressing to the initial context of Finsler spaces (in particular, for locally Minkowskian metrics). The results include an explicit description of the Vaisman connection for a vertical fiber with respect to the verical bundle and a proof of the fact that its leaves are Reinhart spaces [76]. Further, in [77], in a vertical fiber of a Finsler manifold it is defined an adapted basis to the Liouville fibration, the vertical bundle of 2-jets and the leaves of vertical bundles of 2-jets of transversal and mixed types. It is proven that there exists a canonical diffeomorphism between the total space of the vertical bundle of 2-jets and one of the product bundles of vertical, transversal and mixed leaves of 2-jets.

Specific variational features of the energy in *m*-th root metric spaces and the behavior of geodesics is studied in [81,82]; the authors investigate extensions of Lorentz geodesics to generalized Lagrange relativistic models obtained for small deformations of Berwald-Moor and locally Minkowski metrics. In [82], it is described a class of solutions of the Einstein field equations for such models. In [69], a system of second order differential equations is considered as an extension of geodesic equations and it is investigated by means of the KCC approach.

The study of constant scalar curvature and constant flag curvature is continued in [65]; there, it is investigated the horizontal curvature of a pseudo-Finsler manifold. It is proven that, in the Berwald-Moor case, the horizontal and the flag curvature of the space vanish, while the induced curvatures on submanifolds are generally nontrivial.

The investigation of rheonomic KCC models is continued in the work [69]; the autonomous case is extended to the rheonomic one by means of a geometrization of classical KCC theory on first order jet spaces. Here, the authors study the relations between spatial and time semisprays and define a nonlinear connection on the 1-jet space. They find the five invariants of the theory and point out the differences between the rheonomic case and the autonomous one, considering the geometric objects related to induced connections and KCC invariants.

6. Acknowledgements

The last author was partially supported by the Sectorial Operational Program Human Resources Development (SOP HRD), financed from the European Social Fund and by the Romanian Government under the Project number POSDRU/89/1.5/S/59323.

REFERENCES

[1] H. Rund, "Differential Geometry of Finsler Spaces," Springer, Berlin, 1959.

[2] D. Bao, S. S. Chern and Z. Shen, "An Introduction to Finsler Geometry," Springer-Verlag, New York, 2000. doi:10.1007/978-1-4612-1268-3

[3] G. I. Garas'ko, "Elements of Finsler Geometry for Physicists," Tetru, Moscow, 2009.

[4] G. Yu. Bogoslovsky, "On a Special Relativistic Theory of Anisotropic Space-Time," *Doklady Akademii Nauk SSSR*, Vol. 213, No. 5, 1973, pp. 1055-1058.

[5] G. Yu. Bogoslovsky, "A Special Relativistic Theory of

the Locally Anisotropic Space-Time, Part I. The Metric and Group of Motions of the Anisotropic Space of Events," *Nuovo Cimento B*, Vol. 40, No. 1, 1977, pp. 99-115. doi:10.1007/BF02739183

[6] G. Yu. Bogoslovsky, "A Special Relativistic Theory of the Locally Anisotropic Space-Time, Part II. Mechanics and Electrodynamics in the Anisotropic Space," *Nuovo Cimento B*, Vol. 40, No. 1, 1977, pp. 116-134. doi:10.1007/BF02739184

[7] G. Yu. Bogoslovsky, "Theory of Locally Anisotropic Space-Time," Moscow State University Press, Moscow, 1992.

[8] D. A. Kirzhnits and V. A. Chechin, "Ultra-High Energy Cosmic Rays and Possible Generalization of the Relativistic Theory," *Yadernaya Fizika*, Vol. 15, No. 5, 1972, pp. 1051-1059.

[9] T. G. Pavlopoulos, "Breakdown of Lorentz Invariance," *Physical Review*, Vol. 159, No. 5, 1967, pp. 1106-1110. doi:10.1103/PhysRev.159.1106

[10] S. V. Siparov, "Theory of Zero Order Effect That Can Be Used to Investigate the Space-Time Geometrical Properties," *Hypercomplex Numbers in Geometry and Physics*, Vol. 3, No. 2, 2006, pp. 155-173.

[11] S. V. Siparov, "Theory of Zero Order Effect Suitable to Investigate the Space-Time Geometrical Properties," *Acta Mathematica APN*, Vol. 24, No. 1, 2008, pp. 135-144.

[12] S. V. Siparov, "On the Problem of Anisotropy in Geometrodynamics," *Hypercomplex Numbers in Geometry and Physics*, Vol. 5, No. 2, 2008, pp. 64-74.

[13] S. V. Siparov, "Gravitation Law and Source Model in the Anisotropic Geometrodynamics," *Hypercomplex Numbers in Geometry and Physics*, Vol. 6, No. 2, 2009, pp. 140-161.

[14] S. V. Siparov, "Anisotropic Metric for the Gravitation Theory: New Ways to Interpret the Classical GRT Tests," In: S. V. Siparov, Ed., *BSG Proceedings*, Geometry Balkan Press, Bucharest, 2010, pp. 205-218.

[15] S. V. Siparov, "Anisotropic Geometrodynamics in Cosmological Problems," In: S. V. Siparov, Ed., *AIP Conference Proceedings*, Melville, New York, 2010, pp. 222-231.

[16] B. S. DeWitt, "Relativity, Groups and Topology," Gordon and Breach, New York, 1964.

[17] C. Brans and R. H. Dicke, "Mach's Principle and a Relativistic Theory of Gravitation," *Physical Review*, Vol. 124, No. 3, 1961, pp. 925-935. doi:10.1103/PhysRev.124.925

[18] P. D. Mannheim and D. Kazanas, "Newtonian Limit of Conformal Gravity and the Lack of Necessity of the Second Order Poisson Equation," *General Relativity and Gravitation*, Vol. 26, No. 4, 1994, pp. 337-345. doi:10.1007/BF02105226

[19] J. W. Moffat, "Nonsymmetric Gravitational Theory," *Physics Review Letters B*, Vol. 355, No. 3-4, 1995, pp. 447-452. doi:10.1016/0370-2693(95)00670-G

[20] M. Milgrom, "A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis," *Astrophysical Journal*, Vol. 270, 1983, pp. 365-370. doi:10.1086/161130

[21] J. D. Bekenstein, "Relativistic Gravitation Theory for the Modified Newtonian Dynamics Paradigm," *Physics Review D*, Vol. 70, No. 8, 2004, pp. 1-28.

[22] J. Lense and H. Thirring, "On the Influence of the Proper Rotation of Central Bodies on the Motions of Planets and Moons According to Einstein's Theory of Gravitation," *Physikalische Zeitschrift/Physical Journal*, Vol. 19, 1918, pp. 156-163.

[23] M. L. Ruggiero and A. Tartaglia, "Gravitomagnetic Effects," Preprint, 2002. arXiv:gr-qc/0207065v2

[24] H. Muller, S. W. Chiow, S. Herrmann, S. Chu and K. Y. Chung, "Atom Interferometry Tests of the Isotropy of Post-Newtonian Gravity," *Physics Review Letters*, Vol. 100, No. 3, 2008, pp. 1-4.

[25] The CMS Collaboration, "Observation of Long-Range Near-Side Angular Correlations in Proton-Proton Collisions at the LHC," *JHEP*, Vol. 9, 2010, pp. 1-37.

[26] D. Colladay and V. A. Kostelecky, "Lorentz-Violating Extension of the Standard Model," *Physics Review D*, Vol. 58, No. 11, 1998, pp. 1-23.

[27] Q. G. Bailey and A. Kostelecky, "Signals for Lorentz Violation in Post-Newtonian Gravity," *Physics Review D*, Vol. 74, No. 4, 2006, pp. 1-46.

[28] V. A. Kostelecky, "CPT and Lorentz Symmetry," World Scientific, Singapore, 1999.

[29] V. A. Kostelecky, "CPT and Lorentz Symmetry II," World Scientific, Singapore, 2002.

[30] V. A. Kostelecky, "CPT and Lorentz Symmetry III," World Scientific, Singapore, 2005.

[31] V. A. Kostelecky, "CPT and Lorentz Symmetry IV," World Scientific, Singapore, 2008.

[32] V. A. Kostelecky, "CPT and Lorentz Symmetry V," World Scientific, Singapore, 2011.

[33] D. Blas and S. Sibiryakov, "Technically Natural Dark Energy from Lorentz Breaking," Preprint, 2011. arXiv:1104.3579v1 [hep-th]

[34] G. Yu. Bogoslovsky and H. F. Goenner, "Finslerian Spaces Possessing Local Relativistic Symmetry," *General Relativity and Gravitation*, Vol. 31, No. 10, 1999, pp. 1565-1603. doi:10.1023/A:1026786505326

[35] J. Patera, P. Winternitz and H. Zassenhaus, "Continuous Subgroups of the Fundamental Groups of Physics. II. The Similitude Group," *Journal of Mathematical Physics*, Vol. 16, No. 8, 1975, pp. 1615-1624. doi:10.1063/1.522730

[36] P. Winternitz and I. Fris, "Invariant Expansions of Relativistic Amplitudes and Subgroups of the Proper Lorentz Group," *Yadernaya Fizika*, Vol. 1, No. 5, 1965, pp. 889-901.

[37] G. Yu. Bogoslovsky, "Lorentz Symmetry Violation without Violation of Relativistic Symmetry," *Physics Letters A*, Vol. 350, No. 1-2, 2006, pp. 5-10. doi:10.1016/j.physleta.2005.11.007

[38] A. G. Cohen and S. L. Glashow, "Very Special Relativity," *Physics Review Letters*, Vol. 97, No. 2, 2006, pp. 1-3.

[39] A. G. Cohen and S. L. Glashow, "A Lorentz-Violating Origin of Neutrino Mass?" Preprint, 2006. arXiv:hep-ph/0605036v1.

[40] G. W. Gibbons, J. Gomis and C. N. Pope, "General Very Special Relativity is Finsler Geometry," *Physics Review D*, Vol. 76, No. 8, 2007, pp. 1-5.

[41] G. W. Gibbons, J. Gomis and C. N. Pope, "Deforming the Maxwell-Sim Algebra," *Physics Review D*, Vol. 82, No. 6, 2010, pp. 1-15.

[42] G. Yu. Bogoslovsky and H. F. Goenner, "Concerning the Generalized Lorentz Symmetry and the Generalization of the Dirac Equation," *Physics Letters A*, Vol. 323, No. 1-2, 2004, pp. 40-47. doi:10.1016/j.physleta.2004.01.040

[43] G. Yu. Bogoslovsky, "Some Physical Displays of the Space Anisotropy Relevant to the Feasibility of Its Being Detected at a Laboratory," Preprint, 2007. arXiv:0706.2621v1 [gr-qc]

[44] G. Yu. Bogoslovsky, "Finsler Model of Space-Time," *Physics of Particles and Nuclei*, Vol. 24, No. 3, 1993, pp. 354-379.

[45] G. Yu. Bogoslovsky, "A Viable Model of Locally Anisotropic Space-Time and the Finslerian Generalization of the Relativity Theory," *Fortschritte der Physik/Progress of Physics*, Vol. 42, No. 2, 1994, pp. 143-193. doi:10.1002/prop.2190420203

[46] G. Yu. Bogoslovsky and H. F. Goenner, "On the Possibility of Phase Transitions in the Geometric Structure of Space-Time," *Physics Letters A*, Vol. 244, 1998, pp. 222-228.

[47] G. Yu. Bogoslovsky, "Dynamic Rearrangement of Vacuum and the Phase Transitions in the Geometric Structure of Space-Time," *International Journal of Geometric Methods in Modern Physics*, Vol. 9, No. 1, 2012, pp. 1-20. doi:10.1142/S0219887812500077

[48] S. V. Siparov and N. Brinzei, "Space-Time Anisotropy: Theoretical Issues and the Possibility of an Observational Test," Preprint, 2008. arXiv:0806.3066 [gr-qc]

[49] N. Brinzei and S. V. Siparov, "On the Possibility of the OMPR Effect in Spaces with Finsler Geometry, Part I," *Hypercomplex Numbers in Geometry and Physics*, Vol. 4, No. 2, 2007, pp. 41-53.

[50] S. V. Siparov and N. Brinzei, "On the Possibility of the OMPR Effect in Spaces with Finsler Geometry, Part II," *Hypercomplex Numbers in Geometry and Physics*, Vol. 5, No. 2, 2008, pp. 56-64.

[51] N. Voicu, "New Considerations on Einstein Equations in Anisotropic Spaces," In: N. Voicu, Ed., *AIP Conference Proceedings*, Melville, New York, 2010, pp. 249-258.

[52] A. Aguirre, C. P. Burgess, A. Friedland and D. Nolte, "Astrophysical Constraints on Modifying Gravity at Large Distances," *Classical and Quantum Gravity*, Vol. 18, No. 23, 2001, pp. R223-R232.

[53] D. G. Pavlov and S. S. Kokarev, "Conformal Gauges of the Berwald-Moor Geometry and Nonlinear Symmetries Induced by Them," *Hypercomplex Numbers in Geometry and Physics*, Vol. 5, No. 2, 2008, pp. 3-14.

[54] S. S. Kokarev, "Is It Really That Different Geometries Are Different?" In: M. C. Duffy, V. O. Gladyshev, A. N. Morozov and P. Rowlands, Eds., *Proceedings of XV International Meeting PIRT*-2009, BMSTU Press, Moscow, 2009, pp. 116-123.

[55] D. G. Pavlov and S. S. Kokarev, "Riemannian Metrics Osculating to the 3-Dimentional Berwald-Moor Finsler metric," *Hypercomplex Numbers in Geometry and Physics*, Vol. 5, No. 2, 2008, pp. 15-24.

[56] D. G. Pavlov and S. S. Kokarev, "Additive Angles in H_3," *Hypercomplex Numbers in Geometry and Physics*, Vol. 5, No. 2, 2008, pp. 25-43.

[57] D. G. Pavlov and S. S. Kokarev, "Polyangles and Their Symmetries in H_3," *Hypercomplex Numbers in Geometry and Physics*, Vol. 6, No. 1, 2009, pp. 42-67.

[58] N. P. Sokolov, "Spatial Matrices and Their Applications," GIFML, Moscow, 1960 (in Russian).

[59] S. S. Kokarev, "Isometry Classification of Cubic Homogeneous 3-Dimensional Forms," *Symmetry*: *Culture and Science*, *Symmetry Festival*, Vol. 20, No. 1-4, 2009, pp. 371-391.

[60] D. G. Pavlov and S. S. Kokarev, "h-Holomorphic Functions of Double Variable and Their Applications," *Hypercomplex Numbers in Geometry and Physics*, Vol. 7, No. 1, 2010, pp. 44-77.

[61] D. G. Pavlov and S. S. Kokarev, "Hyperbolic Theory of Field on the Plane of Double Variable," *Hypercomplex Numbers in Geometry and Physics*, Vol. 7, No. 1, 2010, pp. 78-126.

[62] D. G. Pavlov and S. S. Kokarev, "Algebraic Unified Theory of Space-Time and Matter on the Plane of Double Variable," *Hypercomplex Numbers in Geometry and Physics*, Vol. 7, No. 2, 2010, pp. 11-37.

[63] Gh. Atanasiu and M. Neagu, "On Cartan Spaces with the *m*-th Root Metric $\sqrt[m]{a^{i_1 i_2 \dots i_m} p_{i_1} p_{i_2} \dots p_{i_m}}$," *Hypercomplex Numbers in Geometry and Physics*, Vol. 6, No. 2, 2009, pp. 67-73.

[64] V. Balan, "Spectral Properties and Applications of Numerical Multilinear Algebra of m-Root Structures," *Hypercomplex Numbers in Geometry and Physics*, Vol. 5, No. 2, 2008, pp. 101-108.

[65] V. Balan, "Notable Submanifolds in Berwald-Moor Spaces," *BSG Proceedings* of 17th (*ISI Proceedings*), Geometry Balkan Press, Bucharest, 2010, pp. 21-30.

[66] V. Balan, "Numerical Multilinear Algebra of Symmetric m-Root Structures. Spectral Properties and Applications," *Symmetry*: *Culture and Science*, *Symmetry Festival* 2009, *Symmetry in the History of Science, Art and Technology*. *Part* 2. *Geometric Approaches to Symmetry*, Vol. 21, No. 1-3, 2010, pp. 119-131.

[67] V. Balan and S. Lebedev, "On the Legendre Transform and Hamiltonian Formalism in Berwald-Moor Geometry," *Differential Geometry-Dynamical Systems*, Vol. 12, No. 1, 2010, pp. 4-11.

[68] V. Balan and A. Manea, "Leafwise 2-Jet Cohomology on Foliated Finsler Manifolds," *BSG Proceedings* 16 (*ISI Proceedings*), Geometry Balkan Press, Bucharest, 2009, pp. 28-41.

[69] V. Balan and M. Neagu, "Jet Geometrical Extension of the KCC-Invariants," *Balkan Journal of Geometry and Its Applications*, Vol. 15, No. 1, 2009, pp. 8-16.

[70] V. Balan and I.-R. Nicola, "Berwald-Moor Metrics and Structural Stability of Conformally-Deformed Geodesic SODE," *Differential Geometry-Dynamical Systems*, Vol. 11, 2009, pp. 19-34.

[71] V. Balan and N. Perminov, "Applications of Resultants in the Spectral m-Root Framework," *Applied Science*, Vol. 12, 2010, pp. 20-29.

[72] V. Balan and A. Pitea, "Symbolic Software for Y-Energy Extremal Finsler Submanifolds," *Applied Science*, Vol. 1, 2009, pp. 41-53.

[73] N. Brinzei (Voicu) and S. V. Siparov, "Equations of Electromagnetism in Some Special Anisotropic Spaces," *Hypercomplex Numbers in Geometry and Physics*, Vol. 5, No. 2, 2008, pp. 45-55.

[74] N. Brinzei (Voicu), "On Cubic Berwald Spaces," *Bulletin of the Calcutta Mathematical Society*, Vol. 17, No. 1-2, 2009, pp. 75-84.

[75] N. Brinzei (Voicu), "Projective Relations for m-th Root Metric Spaces," *Journal of the Calcutta Mathematical Society*, Vol. 5, No. 1-2, 2009, pp. 21-35.

[76] A. Manea, "The Vaisman Connection of the Vertical Bundle of a Finsler Manifold," *Bulletin of the Transilvania University of Brasov*, *Series* III, Vol. 2, No. 51, 2009, pp. 199-206.

[77] A. Manea, "Some New Types of Vertical 2-Jets on the Tangent Bundle of a Finsler Manifold," *Scientific Bulletin—University Politehnica of Bucharest*, *Series A*, Vol. 172, No. 1, 2010, pp. 177-194.

[78] O. Pasarescu, "Curves on Some Irrational Scrolls," Romanian Academy of Sciences, GAR 4/2009.

[79] O. Pasarescu, "Curves on Rational Surfaces with Hyperelliptic Hyperplane Sections," in press. arXiv:1101.0577v1 [math.AG].

[80] N. Voicu (Brinzei), "Anisotropy and Analogies between Gravity and Electromagnetism," In: M. C. Duffy, V. O. Gladyshev, A. N. Morozov and P. Rowlands, Eds., *Proceedings of XV International Meeting PIRT*-2009, BMSTU Press, Moscow, 2009, pp. 124-132.

[81] N. Voicu-Brinzei and S. Siparov, "A New Approach to Electromagnetism in Anisotropic Spaces," *BSG Proceedinds* 17*th International Conference on Differential Geometry and Dynamical Systems*, Bucharest, 2009, pp. 235-245.

[82] N. Voicu-Brinzei and S. Siparov, "Space-Time Anisotropy—Mathematical Formalism and the Possibility of an Experimental Test," *AIP Proceedings of* 4*th Gamow International Conference on Astrophysics and Cosmology after Gamow*: *Recent Progress and New Horizons*, Odessa, 2009, pp. 152-162.

[83] S. Siparov, "Introduction to the Anisotropic Geometrodynamics," World Scientific, Singapore, 2011.

[84] S. Molinari, *et al*., "A 100-Parsec Elliptical and Twisted Ring of Cold and Dense Molecular Clouds Revealed by HERSCHEL around the Galactic Center," Preprint, 2011. arXiv:1105.5486v1, [astro-ph.GA].

Journal of Modern Physics, **2012, 3, 1336-1341**
doi:10.4236/jmp.2012.329171 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Variation of Vacuum Energy if Scale Factor Becomes Infinitely Small, with Fixed Entropy Due to a Non Pathological Big Bang Singularity Accessible to Modified Einstein Equations

Andrew Beckwith
Department of Physics, Chongqing University, Chongqing, China
Email: abeckwith@uh.edu

Received June 28, 2012; revised July 30, 2012; accepted August 8, 2012

ABSTRACT

When initial radius $R_{initial} \to 0$ if Stoica actually derived Einstein equations in a formalism which remove the big bang singularity pathology, then the reason for Planck length no longer holds. The implications of $R_{initial} \to 0$ are the first part of this manuscript. Then the resolution is alluded to by work from Muller and Lousto, as to implications of entanglement entropy. We present entanglement entropy in the early universe with a steadily shrinking scale factor, due to work from Muller and Lousto, and show that there are consequences due to initial entanged $S_{entropy} = 0.3 r_H^2 / a^2$ for a time dependent horizon radius r_H in cosmology, with for flat space conditions $r_H = \eta$ for conformal time. In the case of a curved, but not flat space version of entropy, we look at vacuum energy as proportional to the inverse of scale factor squared times the inverse of initial entropy, effectively when there is no initial time in line with $\rho \sim H^2/G \Leftrightarrow H \approx a^{-1}$. The consequences for this initial entropy being entangled are elaborated in this manuscript. No matter how small the length gets, $S_{entropy}$ if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, t, re scaled does not go to zero. Even if the length goes to zero. This preserves a minimum non zero Λ vacuum energy, and in doing so keep the bits, for computational bits cosmological evolution even if $R_{initial} \to 0$.

Keywords: Fjortoft Theorem; Thermodynamic Potential; Matter Creation; Vacuum Energy Non Pathological Singularity Affecting Einstein Equations; Planck Length; Braneworlds

1. Introduction

This article is to investigate what happens physically if there is a non pathological singularity at the start of space-time, *i.e.* no reason to have a minimium nonzero length. The reasons for such a proposal come from [1] by Stoica who may have removed the reason for the development of Planck's length as a minimum safety net to remove what appears to be unadvoidable pathologies at the start of applying the Einstein equations at a space-time singularity, and are commented upon in this article. $\rho \sim H^2/G \Leftrightarrow H \approx a^{-1}$ in particular is remarked upon. This is a counter part to *Fjortoft* theorem in **Appendix I** below.

Note a change in entropy formula given by Lee [3] about the inter relationship between energy, entropy and temperature as given by

$$m \cdot c^2 = \Delta E = T_U \cdot \Delta S = \frac{\hbar \cdot a}{2\pi \cdot c \cdot k_B} \cdot \Delta S \tag{1}$$

Lee's formula is crucial for what we will bring up in the latter part of this document. Namely that changes in initial energy could effectively vanish if [1] is right, *i.e.* Stoica removing the non pathological nature of a big bang singularity.

If the mass m, *i.e.* for gravitons is set by acceleration (of the net universe) and a change in enthropy $\Delta S \sim 10^{38}$ between the electroweak regime and the final entropy value of, if $a \cong \frac{c^2}{\Delta x}$ for acceleration is used, so then we obtain

$$S_{Today} \sim 10^{88} \tag{2}$$

Then we are really forced to look at (1) as a paring between gravitons (today) and gravitinos (electro weak) in the sense of preservation of information.

Having said this note by extention $\rho \sim H^2/G \Leftrightarrow H \approx a^{-1}$. As ρ changes due to $\rho \sim H^2/G$ and $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$, then a is also altered *i.e.* goes to zero.

What will determine the answer to this question is if $\Delta E_{initial}$ goes to zero if $R_{initial} \to 0$ which happens if there is no minimum distance mandated to avoid the pathology of singularity behavior at the heart of the Einstein equations. In doing this, we avoid using the $E \to 0^+$ situation, and instead refer to a nonzero energy, with $\Delta E_{initial}$ instead vanishing. In particular, the Entanglement entropy concept as presented by Muller and Lousto [4] is presented toward the end of this manuscript as a partial resolution of some of the pathologies brought up in this article before the entanglement entropy section. No matter how small the length gets, $S_{entropy}$ if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, *t*, re scaled goes to zero. This preserves a minimum non zero. Λ vacuum energy, and in doing so keep the bits, for computational bits cosmological evolution even if $R_{initial} \to 0$.

Before doing that, we review Ng [5-7] and his quantum foam hypothesis to give conceptual underpinnings as to why we later even review the implications of entanglement. Entropy, *i.e.* the concept of bits and computations is brought up because of applying energy uncertainty, as given by [3] and the Margolis theorem appears to indicate that the universe could not possibly evolve if [1] is applied, in a 4 dimensional closed universe. This bottle neck as indicated by Ng's [4] formalism is even more striking in its proof of the necessity of using entanglement entropy in lieu of the conclusion involving entanglement entropy, which can be non zero, even if $R_{initial} \to 0$.

2. Review of Ng [5-7] with Comments

First of all, Ng refers to the Margolus-Levitin theorem with the rate of operations $< E/\hbar \Rightarrow \#operations < E/\hbar \times time = \frac{Mc^2}{\hbar} \cdot \frac{l}{c}$. Ng wishes to avoid black-hole formation $\Rightarrow M \le \frac{lc^2}{G}$. This last step is not important to our view point, but we refer to it to keep fidelity to what Ng brought up in his presentation. Later on, Ng refers to the $\#operations \le (R_H/l_P)^2 \sim 10^{123}$ with R_H the Hubble radius. Next Ng refers to the $\#bits \propto [\#operations]^{3/4}$. Each bit energy is $1/R_H$ with $R_H \sim l_P \cdot 10^{123/2}$

The key point as seen by Ng [4] and the author is in

$$\#bits \sim \left[\frac{E}{\hbar} \cdot \frac{l}{c}\right]^{3/4} \approx \left[\frac{Mc^2}{\hbar} \cdot \frac{l}{c}\right]^{3/4} \tag{3}$$

Assuming that *E* of the universe is not set equal to zero, which the author views as impossible, the above equation says that the number of available bits goes down dramatically if one sets $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$? Also Ng writes entropy *S* as proportional to a particle count via *N*.

$$S \sim N \cong [R_H/l_P]^2 \tag{4}$$

We rescale R_H to be

$$R_H\big|_{rescale} \sim \frac{l_{Ng}}{\#} \cdot 10^{123/2} \tag{5}$$

The upshot is that the entropy, in terms of the number of available particles drops dramatically if # becomes larger.

So, as $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$ grows smaller, as # becomes larger.

a) The initial entropy drops;

b) The nunber of bits initially available also drops.

The limiting case of (4) and (5) in a closed universe, with no higher dimensional embedding is that both would vanish, *i.e.* appear to go to zero if # becomes very much larger.

3. Does It Make Sense to Talk of Vacuum Energy if $R_{initial} \neq 0$ Is Changed to $R_{initial} \to 0$? Only Answerable Straightforwardly if an Embedding Superstructure Is Assigned. Otherwise Difficult. Unless One Is Using Entanglement Entropy which Is Non Zero Even if $R_{initial} \to 0$

We summarize what may be the high lights of this inquiry leading to the present paper as follows.

a) One could have the situation if $R_{initial} \to 0$ of an infinite point mass, if there is an initial nonzero energy in the case of just four dimensions and no higher dimensional embedding even if [1] goes through verbatim. The author sees this as unlikely. But is prepared to be wrong. The infinite point mass construction is verbatim if one assumes a closed universe, with no embedding superstructure. Note this appears to nullify the parallel brane world construction author, in lieu of the manuscript sees no reason as to what would perturb this infinite point structure, so as to be able to enter in a big bang era. In such a situation, one would not have vacuum energy. That is unless one has a non zero entanglement entropy [4] present even if $R_{initial} \to 0$. See [8] for a smilar argument.

b) The most problematic scenario. $R_{initial} \to 0$ and no initial cosmological energy, *i.e.* this in a 4 dimensional closed universe. Then there would be no vacuum energy at all.initially. A literal completely empty initial state, which is not held to be viable by Volovik [9].

c) Finding that additional dimensions are involved, than just 4 dimensions may give credence to the authors speculation as to initial degrees of freedom reaching up to 1000, and the nature of a phase transition from essentially very low degrees of freedom, to over 1000 maybe in fact a chaotic mapping as speculated by the author in 2010 [10].

d) What the author would be particularly interested in knowing would be if actual semiclassical reasoning could be used to get to an initial prequantum cosmological state. This would be akin to using [11], but even more to the point, using [12] and [13], with both these last references relevant to forming Planck's constant from electromagnetic wave equations. The author points to the enormous Electromagnetic fields in the electroweak era as perhaps being part of the background necessary for such a semi-classical derivation, plus a possible Octonionic space-time regime, as before inflation flattens space-time, as forming a boundary condition for such constructions to occur [14].

The relevant template for examinging such questions is given in the following **Table 1**.

e) The meaning of Octonionic geometry prior to the introduction of quantum physics presupposes a form of embedding geometry and in many ways is similar to Penrose's cyclic conformal cosmology speculation. Note the following argument, as:

f) We are stuck with how a semiclassical argument can be used to construct **Table 1** below. In particular, we look at how Planck's constant is derived, as in the electroweak regime of space-time, for a total derivative [12, 13]

$$E_y = \frac{\partial A_y}{\partial t} = \omega \cdot A'_y\left(\omega \cdot (t-x)\right) \quad (6)$$

Similarly [12,13]

$$B_z = -\frac{\partial A_y}{\partial x} = \omega \cdot A'_y\left(\omega \cdot (t-x)\right) \quad (7)$$

The *A* field so given would be part of the Maxwell's equations given by [11] as, when $[\ ]$ represents a D'Albertain operator, that in a vacuum, one would have for an A field [12,13]

$$[\ ]A = 0 \quad (8)$$

And for a scalar field ϕ

$$[\ ]\phi = 0 \quad (9)$$

Following this line of thought we then would have an energy density given by, if ε_0 is the early universe permeability [12]

$$\eta = \frac{\varepsilon_0}{2} \cdot \left(E_y^2 + B_z^2\right) = \omega^2 \cdot \varepsilon_0 \cdot A_y'^2\left(\omega \cdot (t-x)\right) \quad (10)$$

We integrate (10) over a specified E and M boundary, so that, then we can write the following condition namely [12,13].

$$\begin{aligned} &\iiint \eta \mathrm{d}(t-x)\mathrm{d}y\mathrm{d}z \\ &= \omega\varepsilon_0 \iiint A_y'^2\left(\omega \cdot (t-x)\right)\mathrm{d}(t-x)\mathrm{d}y\mathrm{d}z \end{aligned} \quad (11)$$

(11) would be integrated over the boundary regime from the transition from the Octonionic regime of space time, to the non Octonionic regime, assuming an abrupt transition occurs, and we can write, the volume integral as representing [12,13]

$$E_{gravitational-energy} = \hbar \cdot \omega \quad (12)$$

Our contention for the rest of this paper, is that Mach's principle will be necessary as an information storage container so as to keep the following, *i.e.* having no variation in the Planck's parameter after its formation from electrodynamics considerations as in (11) and (12). Then by applying [12,13] we get $\hbar$ formed by semi-classical reasons and need to have Machs principle (1) to have the same value up to the present era. In semi classical reasoning similar to [11]

$$\hbar(t) \xrightarrow[Apply-Machs-\mathrm{Re}lations]{} \hbar \ \text{(Constant value)} \quad (13)$$

The question we can ask, is that can we have a pre-quantum regime commencing for (11) and (12) for $\hbar$ if $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$? And a closed 4 dimensional universe? If so, then what is the necessary geometrial regime of space-time so that the integration performed in (11) can commence properly? Also, what can we say about the formation of (12) above, as a number, # gets larger and larger, effectively leading to. Also,with an Octonionic geometry regime which is a pre quantum state [14].

Table 1. Time interval dynamical consequences does qm/wdw apply?

Just before Electroweak Era	Form $\hbar$ from early E & M fields, and use Maxwell's Equations with necessary to implement boundary conditions created from change from Octonionic geometry to flat space	No
Electro-Weak Era	$\hbar$ kept constant due to Machian relations	Yes
Post Electro-Weak Era to Today	$\hbar$ kept constant due to Machian relations	Yes Wave function of Universe

In so many words, the formation period for $\hbar$ is our pre-quantum regime. This **Table 1** could even hold if $R_{initial} \to 0$ but that the 4 dimensional space-time exhibiting such behavior is embedded in a higher dimensional template. That due to $R_{initial} \to 0$ not removing entanglement entropy as is discussed near the end of this article.

4. If $R_{initial} \to 0$, Then if There Is an Isolated, Closed Universe, There Is a Disaster Unless One Uses Entanglement Entropy

One does not have initial entropy, and the number of bits initially disappears.

Abandoning the idea of a completely empty universe, this unperturbed point of matter-energy appears to be a recipede for a static point with no perturbation, as may be the end result of applying Fjortoft theorem [15] to the thermodynamic potential as given in [16], *i.e.* the non definitive anwer for fufillment of criteria of instability by applying Fjortoft's theorem [15] to the potential [16] leading to no instability as given by the potential given in [16] may lead to a point of space-time with no change, *i.e.* a singular point with "infinite" mass which does not change at all.

5. Can an Alternative to a Minimum Length Be Put in? Consider the Example of Planck Time as the Minimal Component, Not Planck Length

From J. Dickau, the following was given to the author, as a counter part as to how to view threshholds as to how a Mandelbrot set may pre select for critical behavior different from what is being pre supposed in this manuscript. [17].

Dickau writes:

"*If we examine the Mandelbrot Set along the Real axis, it informs us about behaviors that also pertain in the Quaternion and Octonic case-because the real axis is invariant over the number types. If numbers larger than* 0.25 *are squared and summed recursively* (*i.e.* –z = z^2 + c) *the result will blow up, but numbers below this threshold never get to infinity, no matter how many times they are iterated. But once space-like dimensions are added, i.e. an imaginary compoent—the equation blows up exponentially, faser than when iterated*".

Dickau concludes:

"*Anyhow there may be a minimum* (*space-time length*) *involved but it is probably in the time direction*".

This is a counter pose to the idea of minimum length, *i.e.* the idea being a replacement for what the author put in here: looking at a beginning situation with a crucial parameter $R_{initial}$ even if the initial time step is "put in by hand". First of all, look at [4], if E is M, due to setting c = 1, then

$$\Delta E_{initial} \approx 4\pi\rho\left(R_{initial}\right)^2 \Delta R_{initial} \tag{14}$$

Everything depends upon the parameter $R_{initial}$ which can go to zero. The choice as to $R_{initial}$ going to zero, or not going to zero will be conclusion of our article.

We have to look at what (14) tells us, even if we have an initial time step for which time is initially indeterminate, as given by a redoing of Mitra's g_{00} formula [8] which we put in to establish the indeterminacy of the initial time step if quantum processes hold.

$$\left(g_{00} = \exp\left[\frac{-2}{1+\left(\rho(t)/p(t)\right)}\right]\right) \xrightarrow[\rho+p=0]{} 0 \tag{15}$$

What Dickau is promoting is, that the Mandelbrot set, if applicable to early universe geometry, that what the author wrote, with

$R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck} \xrightarrow[\#\neq\infty]{} small - value$ potentially going to zero, is less important than a minimum time length. To which the author states, if Dickau is correct as to applicability of the Mandelbrot set, that he, the author is happily corrected, but he also thinks that the Mandelbrot set is a beautiful example of the fungability of space-time metrics used, *i.e.* how one sets the initial space-time potential is to determine the correctness of the Mandelbrot set, *i.e.* the [16] reference, as given, by Thanu Padmanabhan appears not to have a Mandelbrot set, in its thermodynamic potential. The instability issue is reviewed in **Appendix II**. For those who are interested in the author's views as to lack proof of instability. It uses [16] which the author views as THE reference as far as thermodynamic potentials and the early universe.

6. Muller and Lousto Early Universe Entanglement Entropy, and Its Implications. Solving the Spatial Length Issue, Provided a Minimum Time Step Is Preserved in the Cosmos, in Line with Dickau's Suggestion

We look at [4]

$$S_{entropy} = 0.3 r_H^2 / a^2 \quad \text{for a time dependent horizon radius } r_H \text{ in cosmology} \tag{16}$$

Equation (16) above was shown by the author to be fully equivalent to

$$S_{entropy} = 0.3 r_H^2 / a^2 \sim \frac{0.3}{a^2}\exp\left[-t\cdot\sqrt{\frac{\Lambda}{3}}\right] \tag{17}$$

i.e.

$$\left[-t\sqrt{\frac{\Lambda}{3}}\right] \sim \ln_e\left(\frac{a^2}{0.3}\cdot S_{entropy}\right) \tag{18}$$

So, then one has

$$\Lambda \approx \frac{3}{t^2} \cdot \left[\ln_e \left(\frac{a^2}{0.3} \cdot S_{entropy} \right) \right]^2 \qquad (19)$$

No matter how small the length gets, $S_{entropy}$ if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, *t*, re scaled does not go to zero. *This preserves a minimum non zero* Λ *vacuum energy*, *and in doing so keep the bits, for computational bits even if* $R_{initial} \to 0$.

7. Conclusions

a) The universe if $R_{initial} \to 0$ [1] and if it is an isolated system, i.e. not as embedded in higher dimensions as referred to in [18] may have no bits, or computations as thought of by Ng [5-7]. This would be in tandem with the authors conclusion that one would have an initial infinite point mass and no evolution. And no generation of entropy. The only way about this, as indicated in section 6 would be to use entanglement entropy, [4] and to keep the minimum time step from going to zero.

b) If $R_{initial} \to 0$ [1] but the universe is embedded in a higher dimensional system, as given by [19], then there is no reason to say there are no bits, or computations, and the universe will continue to evolve with entropy as a by product of that evolution.

The future endeavor to investigate, is if entanglement entropy can be set up so as to have Vacuum energy no matter what in terms of (19). Satisfying this will make $R_{initial} \to 0$ a tractable cosmological problem, and [1] very useful [4].

8. Acknowledgements

This work is supported in part by National Nature Science Foundation of China grant No. 110752. The author particularly appreciates the Mandelbrot example given to him by J. Dickau as a legitimate research inquiry question which he will pursue in future work.

REFERENCES

[1] C. Stoica, "Beyond the FRWL Big Bang Singularity." http://arxiv.org/pdf/1203.1819.pdf

[2] A. W. Beckwith, "Is Quantum Mechanics Involved at the Start of Cosmological Evolution? Does a Machian Relationship between Gravitons and Gravitinos Answer This Question?" http://vixra.org/abs/1206.0023

[3] J.-W. Lee, "On the Origin of Entropic Gravity and Inertia," 2012. http://arxiv.org/abs/1003.4464; Foundations of physics,

[4] R. Muller and A. C. Lousto, "Entanglement Entropy in Curved Space-Times with Event Horizons," Vol. 1, 28 April, 1995. arXIV gr-qc/9504049

[5] Y. J. Ng, "Spacetime Foam: From Entropy and Holography to Infinite Statistics and Nonlocality," *Entropy*, Vol. 10, No. 4, 2008, pp. 441-461. doi:10.3390/e10040441

[6] Y. J. Ng and H. van Dam, "Measuring the Foaminess of Space-Time with Gravity-Wave Interferometers," *Foundations of Physics*, Vol. 30, No. 5, 2000, pp. 795-805.

[7] Y. J. Ng, "Quantum Foam and Dark Energy," *Conference International Work Shop on the Dark Side of the Universe*. doi:10.1023/A:1003745212871

[8] A. Mitra, "Why the Big Bang Model Cannot Describe the Observed Universe having Pressure and Radiation", *Journal of Modern Physics*, Vol. 2, 2011, pp. 1436-1442.

[9] G. E. Volovik, "Vacuum Energy: Myths and Realities," arXIV: gr-qc/0604062 v2, April 16, 2006.

[10] A. Beckwith, "How to Use the Cosmological Schwinger Principle for Energy Flux, Entropy, and 'Atoms of Space-Time' to Create a Thermodynamic Space-Time and Multiverse", *5th International Workshop DICE*2010: *Space-Time-Matter—Current Issues in Quantum Mechanics and Beyond*, Tuscany, 13-17 September 2010. http://iopscience.iop.org/1742-6596/306/1;jsessionid=A05372A78C18D970BF35F40A9A863B51.c2

[11] M. Gryzinski, "Classical Theory of Electronic and Ionic Inelastic Collisions," *Physical Review*, Vol. 115, No. 2, 1959, pp. 374-383

[12] U. Bruchholz, "Derivation of Planck's Constant from Maxwell's Electrodynamics," *Progress in Physics*, Vol. 4, 2009, p. 67. http://www.ptep-online.com/index_files/2009/PP-19-07.PDF

[13] U. Bruzchholz, "Key Notes on a Geometric Theory of Fields," *Progress in Physics*, Vol. 2, 2009, pp. 107-113.

[14] A. Beckwith, "Octonionic Gravity Formation, Its Connections to Micro Physics," *Open Journal of Microphysics*, Vol. 1 No. 1, 2011, pp. 13-18. doi:10.4236/ojm.2011.11002

[15] J. Pringle and A. King, "Astrophysical Flows," Cambridge University Press, New York, 2007. doi:10.1017/CBO9780511802201

[16] T. Padmanabhan, "Lessons from Classical Gravity about the Quantum Structure of Space-Time," *Journal of Physics*: *Conference Series*, Vol. 306, No. 1, Article ID: 012001. http://iopscience.iop.org/1742-6596/306/1/012001

[17] J. Dickau, "Private Communication to the Author as to Minimum Space-Time 'Length'," 7 August 2012.

[18] A. K. Avessian, "Plancks Constant Evolution as a Cosmological Evolution Test for the Early Universe", *Gravitation and Cosmology*, Vol. 15, No. 1, 2009, pp. 10-12

[19] R. Durrer, *et al*., "Dynamical Casmir Effect for Gravitons in Bouncing Braneworlds," 2009. http://theory.physics.unige.ch/~durrer/papers/casimir.pdf; Earlier version http://www.scribd.com/doc/79001898/Ruth-Durrer-and-Macus-Ruser-The-dynamical-Casimir-effect-in-braneworlds;Version put into PRD Marcus Ruser, Ruth Durrer, "Dynamical Casimir Effect for Gravitons in Bouncing Braneworlds," http://arxiv.org/abs/0704.0790.

Physical Review D, Vol. 76, 2007, Article ID: 104014. doi:10.1103/PhysRevD.76.104014

[20] S. Surya, "In Search of a Covariant Quantum Measure," 2010. http://iopscience.iop.org/1742-6596/306/1/012018

[21] A. W. Beckwith, "Identifying a Kaluza Klein Treatment of a Graviton Permitting a Deceleration Parameter Q (Z) as an Alternative to Standard DE," *Journal of Cosmology*, Vol. 13, 2011. http://journalofcosmology.com/BeckwithGraviton.pdf

Appendix I. Fjortoft Theorem

A **necessary condition for instability** is that if z_* is a point in spacetime for which $\frac{\mathrm{d}^2 U}{\mathrm{d}z^2} = 0$ for any given potential U, then there must be some value z_0 in the range $z_1 < z_0 < z_2$ such that

$$\left.\frac{\mathrm{d}^2 U}{\mathrm{d}z^2}\right|_{z_0} \cdot \left[U(z_0) - U(z_*)\right] < 0 \tag{1}$$

For the proof, see [13] and also consider that the main discussion is to find instability in a physical system which will be described by a given potential U. Next, we will construct in the boundary of the EW era, a way to come up with an optimal description for U.

Appendix II. Constructing an Appropriate Potential for Using Fjortoft Theorem in Cosmology for the Early Universe Cannot Be Done. We Show Why

To do this, we will look at Padamanabhan [16] and his construction of (in Dice 2010) of thermodynamic potentials he used to have another construction of the Einstein GR equations. To start, Padamanabhan [16] wrote

If P_{cd}^{ab} is a so called Lovelock entropy tensor, and T_{ab} a stress energy tensor

$$\begin{aligned} U(\eta^a) &= -4 \cdot P_{ab}^{cd} \nabla_c \eta^a \nabla_d \eta^b + T_{ab}\eta^a\eta^b \\ &\quad + \lambda(x) g_{ab}\eta^a\eta^b \\ &= U_{gravity}(\eta^a) + U_{matter}(\eta^a) + \lambda(x) g_{ab}\eta^a\eta^b \\ &\Leftrightarrow U_{matter}(\eta^a) = T_{ab}\eta^a\eta^b ; U_{gravity}(\eta^a) \\ &= -4 \cdot P_{ab}^{cd} \nabla_c \eta^a \nabla_d \eta^b \end{aligned} \tag{1}$$

We now will look at

$$\begin{aligned} U_{matter}(\eta^a) &= T_{ab}\eta^a\eta^b \\ U_{gravity}(\eta^a) &= -4 \cdot P_{ab}^{cd} \nabla_c \eta^a \nabla_d \eta^b \end{aligned} \tag{2}$$

So happens that in terms of looking at the partial derivative of the top (1) equation, we are looking at

$$\frac{\partial^2 U}{\partial(\eta^a)^2} = T_{aa} + \lambda(x) g_{aa} \tag{3}$$

Thus, we then will be looking at if there is a specified η_*^a for which the following holds.

$$\begin{aligned} &\left[\frac{\partial^2 U}{\partial(\eta^a)^2} = T_{aa} + \lambda(x) g_{aa}\right]_{\eta_0^a} \\ &* \Big[-4 \cdot P_{ab}^{cd}\left(\nabla_c \eta_0^a \nabla_d \eta_0^b - \nabla_c \eta_*^a \nabla_d \eta_*^b\right) \\ &\quad + T_{ab} \cdot \left[\eta_0^a \eta_0^b - \eta_*^a \eta_*^b\right] + \lambda(x) g_{ab} \cdot \left[\eta_0^a \eta_0^b - \eta_*^a \eta_*^b\right]\Big] \\ &< 0 \end{aligned} \tag{4}$$

What this is saying is that there is no unique point, using this η_*^a for which (4) holds. Therefore, we say there is no official point of instability of η_*^a due to (3). The Lagrangian structure of what can be built up by the potentials given in (3) with respect to η_*^a mean that we cannot expect an inflection point with respect to a 2nd derivative of a potential system. Such an inflection point designating a speed up of acceleration due to DE exists a billion years ago [21]. Also note that the reason for the failure for (4) to be congruent to Fjoroft's theorem is due to

$$\left[\frac{\partial^2 U}{\partial(\eta^a)^2} = T_{aa} + \lambda(x) g_{aa}\right] \neq 0, \text{for } \forall \eta_*^a \text{ choices} \tag{5}$$

Journal of Modern Physics, **2012, 3, 1342-1355**
doi:10.4236/jmp.2012.329172 Published Online September 2012 (http://www.SciRP.org/journal/jmp)

Geometrization of Radial Particles in Non-Empty Space Complies with Tests of General Relativity

I. E. Bulyzhenkov
Moscow Institute of Physics and Technology, Moscow, Russia
Email: inter@mipt.ru

Received June 25, 2012; revised July 25, 2012; accepted August 2, 2012

ABSTRACT

Curved space-time 4-interval of any probe particle does not contradict to flat non-empty 3-space which, in turn, assumes the global material overlap of elementary continuous particles or the nonlocal Universe with universal Euclidean geometry. Relativistic particle's time is the chain function of particles speed and this time differs from the proper time of a motionless local observer. Equal passive and active relativistic energy-charges are employed to match the universal free fall and the Principle of Equivalence in non-empty (material) space, where continuous radial densities of elementary energy-charges obey local superpositions and mutual penetrations. The known planetary perihelion precession, the radar echo delay, and the gravitational light bending can be explained quantitatively by the singularity-free metric without departure from Euclidean spatial geometry. The flatspace precession of non-point orbiting gyroscopes is non-Newtonian one due to the Einstein dilation of local time within the Earth's radial energy-charge rather than due to unphysical warping of Euclidean space.

Keywords: Euclidean Material Space; Metric Four-Potentials; Radial Masses; Energy-to-Energy Gravitation; Nonlocal Universe

1. Introduction

The ideal penetration of a static superfluid medium through a rotating drag one was observed in He3-He4 experiments well before the distributed Cooper pair explained the nonlocal nature of superconductivity. But does spatial distribution of paired superelectrons mean that two nonlocal carriers move one through another as overlapping continuous distributions of mass-densities or do these densities bypass each other separately without mutual penetrations? Is there a principle difference between the superfluid motion of two paired electrons and the free, geodesic motion of any normal electron between drag collisions with energy exchanges?

During the last fifty years the celebrated Aharonov-Bohm effect is trying to dismiss doubts regarding the nonlocal nature of the electron, while the Classical Theory of Fields is persisting to accept a self-coherent analytical distribution of the charged elementary density (instead of the point particle approximation for the electron). Fermions take different energies and, therefore, cannot exhibit one net phase even under the ideal (without dissipation) motion. At the same time, each distributed electron may have a self-coherent state of its own matter. Particles motion with drag collisions and heat release represents much more complicated physics than the superfluid motion with a self-coherent state of distributed elementary mass. Such a nonlocal superfluid state is free from energy and information exchanges. Charged densities of drag and superconducting electrons in the same spatial point can move even in opposite directions, for example under thermoelectric phenomena where nonequilibrium superconductors exhibit up to five [1] different ways for heat release/absorption. Such a steady countermotion of drag and superfluid densities of electrons may be a laboratory guiding for theories toward the global counterbalance of all material flows in the nonlocal Universe with local energy dissipation. However, if there is a mutual penetration of extended bodies (with or without dissipation), then how can General Relativity (GR) address the laboratory nonlocality of each electron in order to incorporate the material spatial overlap of distributed carriers of mass-energy? Below we accept the ideal global overlap of all elementary energy flows in all points of their joint 3D space, which is associated with the superposition of flat material 3-sections of curved elementary 4D manifolds. We shall rely on superfluid, self-coherent states of extended elementary particle (called the astroparticle due to its infinite spatial distribution [2]) between drag collisions and dissipation events. At the same time, 3D overlap of such self-coherent radial distributions can rarely exhibit, due to drag collisions, summary superfluid states of identical bosons, while 3D energy ensembles of extended fermions can exhibit only ideal summary flows without joint coherent properties.

It is important to emphasize that strict spatial flatness is principal for reasonable Quantum Mechanics, say for the Bohr-Sommerfeld quantization, and for reasonable Electrodynamics, which is based on constant Gauss flux through any closed surface. The author does not see clear experimental reasons why one should redesign Classical Electrodynamics for a curved-space laboratory in question. On the contrary, due to well established measurements of magnetic flux quantization in superconducting rings, one may insist that would gravity contribute to length of superconducting contours, then SQUIDs could not be explained satisfactorily, for example [3]. Indeed, would spatial intervals depend on gravity or acceleration, working SQUID accelerometers were already created. In such a view, Einsteins metric gravitation, which started from the very beneficial 1913 idea of 4D geometrization of fields, should double-check its wide opportunities and overcome the current phase with unphysical metric singularities. There are no sharp material densities in reality like Dirac operator delta-densities and relativistic physics should try continuous particles prior to declare singularities and black holes in physical space. One may expect that advanced metric gravitation should be a self-cotained theory of continuous energy flows which ought to derive analytical components of the metric tensor $g_{\mu\nu}$ for space-time dynamics of distributed astroparticles without references on the point matter paradigm in question and the Newtonian limit for point masses. Advanced GR solutions for mass-energy densities of moving material space should provide Lorentz force analogs even in the non-relativistic limit. Newtons gravitation cannot satisfactorily describe this limit for moving sources and, therefore, should not be used for relevant gravitational references for a rotating galaxy (that raised the dark matter problem).

Recall that in 1913 Einstein and Grossmann published their *Entwurf* metric formalism for the geodesic motion of a passive material point in a gravitational field [4]. In October 1915, Einstein's field equation [5] and the Hilbert variational approach to independent field and particle densities [6] were proposed in Berlin and Gottingen, respectively, for geometrization of gravitational fields "generated" by the energy-momentum density of Mies continuous matter [7], which later failed to replace point masses of the pre-quantum Universe. This metric theory of gravitational fields around still localized particles, known today as General Relativity, can operate fluently with curved spatial displacement $\mathrm{d}l_N = \sqrt{\gamma_{ij}^N \mathrm{d}x^i \mathrm{d}x^j}$ of a point mass m_N by accepting the Schwarzschild or Droste empty-space solutions [8] without specific restrictions on the space metric tensor

$$\gamma_{ij}^N \equiv g_{oi}^N g_{oj}^N \left(g_{oo}^N\right)^{-1} - g_{ij}^N .$$

GR solutions for dynamics of the considered probe particle N are related to its space-time interval,

$$\mathrm{d}s_N^2 \equiv g_{\mu\nu}^N \mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}\tau_N^2 - \mathrm{d}l_N^2 ,$$

where the time element

$$\mathrm{d}\tau_N \equiv \left[g_{oo}^N \left(\mathrm{d}x^o + g^{-1}_{N\,oo} g_{oi}^N \mathrm{d}x^i \right)^2 \right]^{1/2}$$

depends on the local pseudo-Riemannian metric tensor $g_{\mu\nu}^N$ and, consequently, on local gravitational fields. Hereinafter, $i = 1,2,3$, $\mu = 0,1,2,3$, and the speed of light $c = 1$ in the most of equations.

The author intends to revisit time, $\mathrm{d}\tau_N$, and space, $\mathrm{d}l_N \equiv \sqrt{\gamma_{ij}^N \mathrm{d}x^i \mathrm{d}x^j}$, elements within the conventional GR four-interval $\mathrm{d}s \equiv \sqrt{g_{\mu\nu}^N \mathrm{d}x^\mu \mathrm{d}x^\nu}$ in order to prove that the time element of the freely moving mass m_N depends not only on the world time differential $\mathrm{d}t$ (with $\mathrm{d}t \equiv \sqrt{\delta_{oo} \mathrm{d}x^o \mathrm{d}x^o} = \left|\mathrm{d}x^o\right| > 0$) and gravitation, but also on space differentials or matter displacements $\mathrm{d}x^i$ in gravitational fields. Then the ratio $\mathrm{d}l_N / \mathrm{d}\tau_N \equiv v$, called the physical speed in Special Relativity (SR), should non-linearly depend on spatial displacement $\mathrm{d}l_N \equiv \sqrt{\gamma_{ij}^N \mathrm{d}x^i \mathrm{d}x^j}$, called the space interval in SR. Nonlinear field contributions to such an anisotropic (Finsler-type) time element $\mathrm{d}\tau_N(x, \mathrm{d}x)$ within the four-interval $\mathrm{d}s^2 = \mathrm{d}\tau^2(x, \mathrm{d}x) - \mathrm{d}l^2(x)$ of Einstein's Relativity may modify Schwarzschild-type metric solutions based on curved three-space around non-physical point singularities for GR energy-sources [2]. Moreover, the calculated ratio $\mathrm{d}l_N / \mathrm{d}\tau_N(v) = v$ may differ from a real speed $\mathrm{d}l_N / \mathrm{d}\tau_o$ measured by a motionless observer with local proper-time $\mathrm{d}\tau_o(\mathrm{d}l = 0) \neq \mathrm{d}\tau_N(v)$. This metric-type anisotropy of measured time rate was already confirmed by observations of the gravitational Sagnac effect when $g_{oi} \mathrm{d}x^i / \mathrm{d}\tau \neq 0$. Rigorous consideration of anisotropic physical time $\mathrm{d}\tau(x, \mathrm{d}x) \equiv \mathrm{d}\tau(v)$ of each moving particle may preserve universal flatness of its 3-space element $\mathrm{d}l$. We shall start from the 1913 *Entwurf* metric formalism for the geodesic motion of passive masses. Then, we shall employ the tetrad approach and analyze nonlinear relations in the anisotropic relativistic time for a passive mass under the geodesic motion. This will suggest to keep for physical reality Euclidean 3D sub-intervals in curved 4D intervals of moving probe particles.

The first attempt to interpret GR in parallel terms of curved and flat spaces was made by Rosen [9], Einstein's co-author of the unpublished 1936 paper about the non-existence of plane metric waves from line singularities of cylindrical sources. Later, Sommerfeld, Schwinger, Brillouin and many other theorists tried to justify Euclidean space for better modern physics. Moreover, the original proposal of Grossmann (to use 4D Riemannian geometry for geometrization of gravitational fields in the 1913 *Entwurf* version of GR) relied exclusively on 3D Euclidean sub-space. Grossmann did not join further GR metric developments with curved 3D intervals. In 1913 Einstein clearly underlined that space cannot exist without matter in the *Entwurf* geometrization of fields. However, at that pre-quantum time there were not many options for geometrization of particles, because all (but Mie) considered them localized entities for local events. This might be the reason why in January 1916 Einstein promptly accepted Schwarzschild's warping of 3D space around the point particle. Nonetheless, in 1939 Einstein finally rejected Schwarzschild metric singularities for physical reality. The well derived Schwarzschild's solution has no mathematical errors in the empty-space paradigm. However, we tend to use the non-empty-space paradigm for the global superfluid overlap of self-coherent elementary particles, when each continuous particle is distributed over the entire Universe together with the elementary field. This nonlocal approach to matter can avoid difficulties of the *Entwurf* geometrization of fields, proposed in 1913 without geometrization of particles, and, ultimately, can avoid non-physical warping of the universal spatial ruler, which becomes the same for all local observers in the flat Universe.

Contrary to non-metric approaches to gravitation with spatial flatness, for example [10], we shall comply with the Einstein-Grossmann extension of Special Relativity (SR) to gravitation through warped space-time with non-Euclidean pseudo-geometry, founded by Lobachevsky, Bolyai and Riemann [11]. Inertia and gravitation keep the same metric nature in our reiteration of the Einstein-Grossmann approach. The proposed 4D geometrization of matter together with fields will be made under six metric bounds for $g_{\mu\nu}$ (called sometime intrinsic metric symmetries) in the GR tensor formalism for every physical object. In other words, the author is planning to revise neither Einstein's Principle of Relativity nor the GR geometrization concept. On the contrary, I am planning further GR geometrization of continuous particles together with the already available geometrization of gravitational fields. Local nullification of the Einstein tensor curvature for paired densities of the distributed astroparticle and its field will be requested in their rest frame of references. I intend to prove, for example, that Schwarzschild's solution for a central field is not "the only rotationally invariant GR metric extension of the SR interval". One should admit non-empty (material) space or Newtonian stresses of the material medium-aether associated with continuous very low dense distributions of non-local gravitation/inertial mass-energies. Then bound ensembles of elementary radial energies form so called "macroscopic" bodies with sharp visual boundaries (observed exclusively due to experimental restrictions to measure fine energy densities).

First, we discuss a local time element, $d\tau(v) \equiv d\tau(dl)$, which should be considered as a chain function of speed $v = dl/d\tau$ or spatial displacement dl of a passive material point in external gravitational field. Then, we discuss the electric Weber-type potential energy

$$U_o^W = U_o\sqrt{1-v^2}\Big/m_N = U_o P_o^{-1}\Big/\left(1-U_o P_o^{-1}\right)$$

for a point planet with mass m_N and relativistic energy $P_o = m_N V_o$ in the Sun's static field generated by the active energy-charge E_M. Ultimately, this paper presents the self-contained GR scheme with the energy-to-energy interaction potential $U_o/P_o = -G E_M/r$ for Machian mechanics of nonlocal astroparticles with an analytical radial density $n(r) = r_o\big/4\pi r^2\left(r + r_o\right)^2$ instead of the Dirac delta density $\delta(r)$. One should see arguments for the singularity-free gravitational contribution U_o/P_o to the smooth metric tensor component $g_{oo} = \left(1 - U_o/P_o\right)^{-2}$. The main challenge here was to keep the free fall universality and the GR Principle of Equivalence for all carriers of probe (passive, inertial) energies P_o in radial fields of the Sun's gravitational (active) energy E_M.

In the speed-dependent time approach, the warped GR four-interval $ds\left[d\tau(dl), dl\right]$ cannot be approximated in weak fields by pure time and pure space subintervals, like in Schwarzschild-type solutions [8] with their formal time and space metric split without chain relations. In order to justify the indivisible non-linear involvement of space displacements into physical time $d\tau(dl)$ of a probe particle under the the geodesic motion, one should clarify how the already known gravitational tests of GR can be explained quantitatively without departure from spatial flatness. Then we discuss our energy-to-energy attraction under the Einstein-Grossmann geodesic motion in metric fields with flat 3-section (*i.e.* without Schwarzschild singularities). The author also accepts the Einstein-Infeld-Hoffmann approach (but under flat 3-space) to non-point slow-moving gyroscopes in order to describe the Gravity Probe B quantitatively.

In 1913, Einstein and Grossmann put weak Newtonian field only into the temporal part of the *Entwurf* 4D interval. Today, one tends to justify that strong-field GR metric may also admit for reality six metric bounds $\gamma_{ij}^N = \delta_{ij}$ which preserve universal 3D interval in specifically curved space-time for any elementary particle N. Then the metric tensor $g_{\mu\nu}$ for curved 4D with flat 3-section depends on four gravitational potentials $G_\mu \equiv U_\mu/P_o$ for the particle energy-charge

$$P_o = m_N\sqrt{g_{oo}}\Big/\sqrt{1-v^2}\,.$$

This finding matches 6 metric bounds for spatial flatness under any gravitational fields and their gauges. Since 2000, this post-Entwurf metric scheme with warped space-time, but strictly flat three-space, became consistent with the observed Universe's large-scale flatness, confirmed at first by balloon measurements of the Cosmic Microwave Background and then by all ongoing Wilkinson Microwave Anisotropy Probe (WMAP) Observations of the flat Universe [12]. This new reading of curved 4D geometry with non-linearly dilated anisotropic time and flat non-empty space, explains quantitatively all GR tests, the known planet perihelion precession, the radar echo delay, and the gravitational light bending, for example [13].

Speed-dependent time corrections to post-Newtonian dynamics in Sun's flat material space lead to computation results similar to numerical computations of other authors who traditionally correct Newton in empty, but curved 3-space. Observable dynamics of matter in moderate and strong static fields provides, in principle, an opportunity to distinguish our metric solutions with isotropic flat space and speed-dependent time from Schwarzschild's solutions, based on curved 3-space and dilated time. Alternative empty-space and non-empty space paradigms can also be distinguished through different probe body dynamics in stationary fields of rotating astrophysical objects.

2. Warped Four-Space with Intrinsic Metric Symmetries for Flat Three-Space

To begin, we employ the GR tetrad formalism, for example [14,15], in covariant expressions for an elementary rest-mass m_N in order to justify the mathematical opportunity to keep a flat 3D subspace x_N^i in curved four-space x_N^μ with a pseudo-Riemannian metric tensor $g_{\mu\nu}^N = g_{\mu\nu}$ (for short). First, we rewrite the curved four-interval,

$$\begin{aligned} \mathrm{d}s_N^2 &\equiv g_{\mu\nu}^N \mathrm{d}x_N^\mu \mathrm{d}x_N^\nu \equiv g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\ &\equiv \eta_{\alpha\beta} e_\mu^{(\alpha)} e_\nu^{(\beta)} \mathrm{d}x^\mu \mathrm{d}x^\nu \\ &\equiv \eta_{\alpha\beta}\mathrm{d}x^{(\alpha)}\mathrm{d}x^{(\beta)}, \end{aligned}$$

in plane coordinates $\mathrm{d}x^{(\alpha)} \equiv e_\mu^{(\alpha)}\mathrm{d}x^\mu$ and $\mathrm{d}x^{(\beta)} \equiv e_\nu^{(\beta)}\mathrm{d}x^\nu$, with

$$\eta_{\alpha\beta} = \mathrm{diag}\left(+1,-1,-1,-1\right).$$

One can find $e_\mu^{(o)} = \left\{\sqrt{g_{oo}};-\sqrt{g_{oo}}\,g_i\right\}$ and $e_\mu^{(b)} = \left\{0, e_i^{(b)}\right\}$ from the equality

$$\mathrm{d}s^2 \equiv \left[\sqrt{g_{oo}}\left(\mathrm{d}x^o - g_i \mathrm{d}x^i\right)\right]^2 - \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$
$$g_i \equiv -g_{oi}/g_{oo}.$$

At first glance, the spatial triad $e_i^{(b)} \equiv e_{N^i}^{(b)}$ (a, b = 1,2,3 and α, β = 0,1,2,3) should always depend essentially on the gravitational fields of other particles because this triad is related to components of $g_{\mu\nu}^N$. However, this might not be the case when there are internal metric relations or bounds in the general pseudo-Riemannian metric with the warped tensor $g_{\mu\nu}^N$. Shortly, a curved mathematical 4D manifold does not necessarily mean a curved 3D section for real matter (warped 2D paper in 3D trash, for example, keeps parallel Euclidean lines due to steady metric relations between neighboring points of paper).

It is not obvious that physical restrictions for four-velocities of real matter, like $g^{\mu\nu}V_\mu V_\nu = 1$, might require to keep flat 3D sections of curved pseudo-Riemannian 4D manifolds. Therefore, let us look at three spatial components V_i of the four-vector $V_\mu \equiv g_{\mu\nu}\,\mathrm{d}x^\nu/\mathrm{d}s$ by using the conventional tetrad formalism,

$$\begin{aligned} &-\left(\sqrt{g_{oo}}\,g_i + v_i\right)\left(1 - v_i v^i\right)^{-1/2} \\ &\equiv V_i \equiv e_i^{(\beta)}V_{(\beta)} \equiv e_i^{(o)}V_{(o)} + e_i^{(b)}V_{(b)} \\ &\equiv -\left(\sqrt{g_{oo}}\,g_i + e_i^{(b)}v_{(b)}\right)\left(1 - v_{(b)}v^{(b)}\right)^{-1/2}. \end{aligned}$$

Here, we used $e_i^{(o)} = -\sqrt{g_{oo}}\,g_i$ and

$$V_{(\beta)} = \left\{\left(1 - v_{(b)}v^{(b)}\right)^{-1/2}; -v_{(b)}\left(1 - v_{(b)}v^{(b)}\right)^{-1/2}\right\}.$$

Now one can trace that the considered equalities $V_i \equiv e_i^{(\beta)}V_{(\beta)}$ admit trivial relations $v_i v^i = v_{(b)}v^{(b)}$ and $v_i = e_i^{(b)}v_{(b)} = \delta_i^{(b)}v_{(b)}$ between the curved velocities,

$$v_i \equiv \gamma_{ij}\mathrm{d}x^j \Big/ \sqrt{g_{oo}}\left(\mathrm{d}x^o - g_i \mathrm{d}x^i\right) \equiv \gamma_{ij}\mathrm{d}x^j/\mathrm{d}\tau,$$

and the plane velocities, $v_{(b)} = \delta_{ab}\,\mathrm{d}x^{(a)}/\mathrm{d}\tau$. All spatial triads for these "trivial" relations may be considered as universal Kronecker delta symbols, $e_{N^i}^{(b)} = \delta_i^{(b)}$, and, consequently, the three-space metric tensor is irrelevant to gravitation fields, *i.e.*

$$\delta_{ij} = g_{oi}g_{oj}g_{oo}^{-1} - g_{ij} \equiv \gamma_{ij} = \gamma_{ij}^N = \gamma_{ij}^K.$$

All components $g_{\mu\nu}$, involved in these six relations, may depend on gravitations fields or system accelerations but their combination should always keep spatial flatness under admissible coordinate transformations. One could, surely, ignore flat 3-space option within curved 4D manifold, as was suggested by the above tetrad analysis, by trying curved 3D solutions in V_i when $e_{N^i}^{(b)} \neq \delta_i^{(b)}$. But we do not see much physical sense in such complications and, therefore, restrict GR geometrical constructions by a partial case with six metric relations $g_{oi}g_{oj}g_{oo}^{-1} = g_{ij} + \delta_{ij}$. Applications of pseudo-Riemannian space-time with flat 3-sections will quantitatively describe all known gravitational experiments plus magnetic flux quantization. The latter and the Aharonov-Bohm effect require only flat 3-space for satisfactory interpretations.

Again, we shall read $g^K_{\mu\nu} \equiv \eta_{\alpha\beta} e^{(\alpha)}_{\mu} e^{(\beta)}_{\nu}$ though

$$e^{(o)}_{\mu} = \left\{ \sqrt{g_{oo}}; -\sqrt{g_{oo}} g_i \right\}$$

and

$$e^{(b)}_{\mu} = \left\{ 0, \delta^{(b)}_{i} \right\} \equiv \delta^{(b)}_{\mu}$$

for all physical cases we are interested in describing. This means for our consideration that

$$g_{oo} \equiv e^{(o)}_{o} e^{(o)}_{o}, g_{oi} \equiv e^{(o)}_{o} e^{(o)}_{i},$$

and

$$g_{ij} \equiv e^{(o)}_{i} e^{(o)}_{j} - \delta_{ab} e^{(a)}_{i} e^{(b)}_{j} = e^{(o)}_{i} e^{(o)}_{j} - \delta_{ij}.$$

And Euclidean spatial geometry,

$$\mathrm{d}l^2_K \equiv \gamma^K_{ij} \mathrm{d}x^i \mathrm{d}x^j = \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j,$$

will be applied to pseudo-Riemannian 4-intervals of all particles (due to intrinsic metric relations $g^K_{oi} g^K_{oj} \left(g^K_{oo} \right)^{-1} - g^K_{ij} \equiv \delta_{ij}$).

Contrary to universal spatial displacements $\mathrm{d}l$, invariant four-intervals have differently warped metrics for particles *K* and *N*, because $g^N_{\mu\nu} \neq g^K_{\mu\nu}$ and $\mathrm{d}s_k \neq \mathrm{d}s_N$ in different external fields (for example, in the two-body problem). The GR four-interval for a selected mass-energy carrier,

$$\begin{aligned} \mathrm{d}s^2 &\equiv \mathrm{d}\tau^2 - \mathrm{d}l^2 \\ &= \left(\sqrt{g_{oo}} \mathrm{d}x^o + g_{oi} \mathrm{d}x^i \sqrt{g_{oo}} \right)^2 - \gamma_{ij} \mathrm{d}x^i \mathrm{d}x^j, \end{aligned} \tag{1}$$

is defined for only one selected probe mass m_N despite notifications $\mathrm{d}s_N \equiv \mathrm{d}s$ and $\mathrm{d}x_N \equiv \mathrm{d}x$ are regularly used for brevity. This geometrical 4-interval should be physically commented in terms of time $\mathrm{d}\tau^2(\mathrm{d}x)$ and space $\mathrm{d}l^2 \equiv \gamma_{ij} \mathrm{d}x^i \mathrm{d}x^j = \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j$ elements, albeit 3-space differentials $\mathrm{d}x^i$ contribute to particle's physical time $\mathrm{d}\tau(\mathrm{d}x)$. We prove below that particles proper time $\mathrm{d}\tau$ depends on $\mathrm{d}l$ even in constant gravitational fields (where there is a first integral of motion $P_o = \mathrm{const}$). Such an anisotropic time element

$$\mathrm{d}\tau_N(\mathrm{d}x) \equiv \sqrt{g_{oo}(x)} \left(\mathrm{d}x^o - g_i \mathrm{d}x^i \right)$$

of the moving mass m_N always counts its spatial displacement $\mathrm{d}l$ in a oriented gravitational field, despite the fact that it is not immediately obvious from the physical time definition for metrics with $g_{oi} = 0$. This post-Newtonian phenomenon, related to the energy nature of anisotropic time, appears in nonlinear gravitational equations through the energy(velocity)-dependent potentials. Our interpretation of the warped four-interval (1), based on warped anisotropic time in isotropic non-empty flat-space rather than in empty warped space, may be considered as a prospective way for further developments of the 1913 metric gravitation through joint geometrization of distributed fields and distributed elementary particles.

Now we return to components of the four-vector $V^N_{\mu} = g^N_{\mu\nu} \mathrm{d}x^{\nu} / \mathrm{d}s$. Notice that

$$\begin{aligned} V_{\mu} &= e^{(\alpha)}_{\mu} V_{(\alpha)} = \left(e^{(b)}_{\mu} V_{(b)} + e^{(o)}_{\mu} V_{(o)} \right) \\ &= \left(e^{(b)}_{\mu} V_{(b)} + \delta^{(o)}_{\mu} V_{(o)} \right) + \left(e^{(o)}_{\mu} - \delta^{(o)}_{\mu} \right) V_{(o)} \\ &\equiv \mathcal{V}_{\mu} + m_N^{-1} U_{\mu}, \end{aligned}$$

with the four-velocity

$$\mathcal{V}_{\mu} \equiv e^{(b)}_{\mu} V_{(b)} + \delta^{(o)}_{\mu} V_{(o)} = \delta^{\alpha}_{\mu} V_{\alpha},$$

because $e^{(b)}_{o} = 0$ and $e^{(b)}_{i} = \delta^{(b)}_{i}$. Flat three-space geometry is a promising way to introduce gauge invariant gravitational potentials,

$$G_{\mu} \equiv U_{\mu} / P_o = G'_{\mu} + \partial_{\mu} \varphi_N$$

with

$$U_{\mu} \equiv \left(e^o_{\mu} - \delta^o_{\mu} \right) m_N V_o = U'_{\mu} + P_o \partial_{\mu} \phi_N,$$

for the passive (probe) mass m_N, in close analogy to four-component electromagnetic potentials for the classical electric charge. The point is that a four-momentum $P^N_{\mu} \equiv m_N V^N_{\mu}$ of the selected scalar mass m_N (without rotation) can be rigorously decomposed into mechanical, K^N_{μ}, and gravitational, U^N_{μ}, parts only under strict spatial flatness,

$$\begin{aligned} P^N_{\mu} &= \left\{ m_N \sqrt{1-v^2}; -m_N v_i \sqrt{1-v^2} \right\} \\ &\quad + \left\{ m_N \left(\sqrt{g_{oo}} - 1 \right) \sqrt{1-v^2}; -m_N g_i \sqrt{g_{oo}} \sqrt{1-v^2} \right\} \\ &\equiv K^N_{\mu} + U^N_{\mu}, \end{aligned} \tag{2}$$

where

$$\begin{aligned} &v_i \equiv \gamma_{ij} v^j, v^2 \equiv v_i v^i, v^i \equiv \mathrm{d}x^i / \mathrm{d}\tau, ds = \left(\mathrm{d}x_{\mu} \mathrm{d}x^{\mu} \right)^{1/2}, \\ &\mathrm{d}x_{\mu} = g_{\mu\nu} \mathrm{d}x^{\nu}, \mathrm{d}x^{\mu} \equiv \mathrm{d}x^{\mu}_N, g_i \equiv -g_{oi} / g_{oo}; \\ &\gamma_{ij} \equiv g_i g_j g_{oo} - g_{ij} = \delta_{ij} = -\eta_{ij}. \end{aligned}$$

Again, we use a time-like worldline with $\mathrm{d}t = \mathrm{d}x^o > 0$ and $\mathrm{d}\tau = +g^{1/2}_{oo} \left(\mathrm{d}x^o - g_i \mathrm{d}x^i \right) > 0$ for the passive-inertial $m_N > 0$. The gravitational energy-momentum part U_{μ} is defined in (2) for a selected mass m_N and its positively defined passive energy $P_o = m_N V_o > 0$, associated with the global distribution of all other masses m_K. This gravitational part, $U_{\mu} \equiv G_{\mu} P_o$, is not a full four-vector in pseudo-Riemannian space-time, like P^N_{μ}, nor is the mechanical summand $K_{\mu} \equiv m_N \mathcal{V}_{\mu}$.

Because $e^{(b)}_{\mu} = \left\{ 0, \delta^{(b)}_{i} \right\} = \delta^{(b)}_{\mu}$ and $\mathrm{d}x_{\mu} = e^{(\beta)}_{\mu} \mathrm{d}x_{(\beta)}$, the tetrad with the zero (*i.e.* time) label takes the following components from (2):

$$e_\mu^{(o)} = \left\{1+\sqrt{1-v^2}U_o m_N^{-1}; \sqrt{1-v^2}U_i m_N^{-1}\right\}$$
$$= \delta_\mu^{(o)} + \sqrt{1-v^2}U_\mu m_N^{-1}.$$

Ultimately, the tetrad $e_\mu^{(\beta)}$ for the selected particle N and the metric tensor $g_{\mu\nu}^N \equiv \eta_{\alpha\beta} e_\mu^{(\alpha)} e_\nu^{(\beta)}$, with $g_{\mu\nu} g^{\mu\lambda} = \delta_\nu^\lambda$, depends in Cartesian coordinates only on the gravitational four-potential $U_\mu / P_o \equiv G_\mu$ (introduced for the relativistic energy-charge $cP_o \equiv cP_{o_N}$ [16]),

$$\begin{cases} e_\mu^{(\beta)} = \delta_\mu^{(\beta)} + \delta_o^{(\beta)}\sqrt{1-v^2}U_\mu m_N^{-1} \\ \quad = \delta_\mu^{(\beta)} + \delta_o^{(\beta)} U_\mu P_o^{-1} \Big/ \left(1-U_o P_o^{-1}\right) \\ g_{oo}^N \equiv e_o^{(o)} e_o^{(o)} = \left(1+\sqrt{1-v^2}U_o m_N^{-1}\right)^2 \\ \quad = 1\Big/\left(1-U_o P_o^{-1}\right)^2 \\ g_{oi}^N \equiv e_o^{(o)} e_i^{(o)} = (1+\sqrt{1-v^2}U_o m_N^{-1})\sqrt{1-v^2}U_i m_N^{-1} \\ \quad = g_{oo}^N U_i P_o^{-1} \\ g_{ij}^N \equiv e_i^{(o)} e_j^{(o)} - \delta_{ab} e_i^{(a)} e_j^{(b)} = \left(1-v^2\right) U_i U_j m_N^{-2} - \delta_{ij} \\ \quad = g_{oo}^N U_i U_j P_o^{-2} - \delta_{ij} \\ g_N^{oo} = (1-U_o P_o^{-1})^2 - \delta^{ij} U_i U_j P_o^{-2}, \\ g_N^{oi} = U_i P_o^{-1},\ g_N^{ij} = -\delta^{ij}, \end{cases} \tag{3}$$

where we used $g_{oo} \equiv e_o^{(o)} e_o^{(o)} = \left(1+\sqrt{1-v^2}U_o\right)^2$ and $V_o^2 = g_{oo} \big/ \left(1-v^2\right)$ to prove that

$$\sqrt{g_{oo}} = 1 + \sqrt{g_{oo}} U_o P_o^{-1} = 1\Big/\left(1-U_o P_o^{-1}\right).$$

Therefore, the passive-inertial GR energy,

$$P_o = m\sqrt{g_{oo}} \Big/ \sqrt{1-v^2} = m\Big/\sqrt{1-v^2}\left(1-U_o P_o^{-1}\right)$$
$$= \left(m\Big/\sqrt{1-v^2}\right) + U_o,$$

takes a linear superposition of kinetic and potential energies in all points of pseudo-Riemannian space-time warped by strong external fields. Note that we did not assign spin S_μ or internal angular mechanical momentum to the Einstein-Grossmann "material point" or the probe mass m_N with the energy-momentum (2). The affine connections for the metric tensor (3) depend only on four gravitational potentials U_μ / P_o in our space-time geometry, which is not relevant to warped manifolds with asymmetrical connections and torsion fields, for example [17].

Every component of the metric tensor in (3) depends on the gravitational part $U_\mu \equiv m_N V_\mu - m_N V_\mu \equiv G_\mu P_o$ of the probe carrier energy-momentum P_μ. At the same time, all the components of the three-space metric tensor, $\gamma_{ij} \equiv g_{oi} g_{oj} g_{oo}^{-1} - g_{ij} = \delta_{ij}$, are always independent from the gravitational potential $G_\mu = U_\mu / P_o$ or its gauge. Such inherent metric symmetries for 3D subspace may be verified directly from (3). In fact, our tetrad, and the metric tensor, depends formally on the inharmonic Weber-type potentials,

$$U_\mu \sqrt{1-v^2} \Big/ m_N = U_\mu P_o^{-1} \Big/ \left(1-U_o P_o^{-1}\right),$$

associated with the particle speed $v^2 = \mathrm{d}l^2 / \mathrm{d}\tau^2$. In 1848 Weber introduced [18] the non-Coulomb potential $q_1 q_2 \left(1-v_{12}^2/2\right)/r_{12}$ based on lab measurements of accelerating forces between moving charges q_1 and q_2 with the relative radial velocity $v_{12}^2 \ll 1$. This might was the first experimental finding that mechanical inertia and acceleration depend on the kinetic energy or speed of interacting bodies.

By substituting the metric tensor (3) into the interval $\mathrm{d}s^2 \equiv g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}^2\tau - \mathrm{d}l^2$, one can rewrite (1) and submit the chain relation for the proper time $\mathrm{d}\tau = \mathrm{d}\tau_N$ of the probe mass-energy carrier N in external gravitational fields,

$$\mathrm{d}\tau(\mathrm{d}l) \equiv \left[g_{oo}^N\left(\mathrm{d}x^o + g_N^{-1} g_{oi}^N \mathrm{d}x^i\right)^2\right]^{1/2} \equiv e_\mu^{(o)} \mathrm{d}x^\mu$$
$$= \mathrm{d}x^o + \mathrm{d}x^\mu U_\mu^N m_N \sqrt{1-\mathrm{d}l^2 \mathrm{d}\tau^2(\mathrm{d}l)}. \tag{4}$$

Notice that the proper-time differential,

$$\mathrm{d}\tau_O = \mathrm{d}x^o\left(1+U_o^K m_K^{-1}\right),$$

of the local observer K, with $\mathrm{d}x_K^i = 0$ and $\mathrm{d}l_K = 0$, differs from the time element (4) of the moving mass m with the GR energy-charge $P_o = m\sqrt{g_{oo}} \big/ \sqrt{1-v^2}$. The proper interval $\mathrm{d}s$ of the moving mass and its proper time element (4) depends, in general, on all four components of U_μ. Therefore, the observable three-speed $\mathrm{d}l / \mathrm{d}\tau_O$, of a moving particle always differs in relativistic gravito-mechanics from the non-linear ratio $\mathrm{d}l / \mathrm{d}\tau(\mathrm{d}l) \equiv v$, called the particle's physical speed (1). The chain relation $\mathrm{d}\tau = f(\mathrm{d}\tau)$ in the physical time (4) of a moving particle changes the GR interpretation of the geodesic motion and allows to apply flat 3D space for gravitational tests.

The metric tensor (3), the interval (1), and the local time element (4) are associated with warped space-time specified by external fields for one selected mass m_N or, to be precise, for the passive energy-charge P_o^N. We may employ common three-space for all elementary particles (due to universal Euclidean geometry for their spatial displacements), but we should specify warped space-times with differently dilated times for the mutual motion of gravitational partners. The particle's time element $\mathrm{d}\tau \equiv \mathrm{d}\tau_N(\mathrm{d}l, v)$ in (4) may depend on the particles velocity or displacement. Ultimately, a non-linear time rate $\dot{\tau} = e_\mu^{(o)} \mathrm{d}x^\mu \big/ \mathrm{d}x^o$ (hereinafter $\mathrm{d}f/\mathrm{d}t \equiv \dot{f}, \mathrm{d}t \equiv \mathrm{d}x^o/c$) of moving material objects in (4) depends on the ratio $\dot{l}^2 / \dot{\tau}^2 = v^2$. This non-linear chain relation can be simplified in several subsequent steps through the following equalities to (4):

$$\begin{aligned} \mathrm{d}\tau &\equiv \mathrm{d}t 1 + U_o m_N^{-1}\sqrt{1-v^2}\, 1 - v^i U_i m_N^{-1}\sqrt{1-v^2} \\ &\equiv \mathrm{d}t 1 - U_o P_o^{-1} - P_o^{-1} U_i v^i \\ &\equiv \mathrm{d}t 1 + U_i P_o^{-1}\dot{x}^i 1 - U_o P_o^{-1}. \end{aligned} \tag{5}$$

Such anisotropic time dilatation in (5) by the external four-potential $G_\mu^N = U_o^N / P_o^N$ results in the gravitational Sagnac effect when an observer compares the dynamics of different elementary energy-charges P_o in fields with $U_i \neq 0$.

Now, one may conclude that the anisotropic time element $\mathrm{d}\tau$ in the metric interval (1) and, consequently, in the physical speed $v = \mathrm{d}l/\mathrm{d}\tau$, depends only on universal four potentials G_μ for positive probe charges $P_o > 0$. The potential energy part $m_N U_\mu \equiv \left(P_{N^\mu} - m_N V_\mu\right)$ contributes to GR energy-momentum of the probe body and, therefore, to its passive energy-charge, $m_N V_o = P_o$. The universal ratio U_μ / P_o should be tried in Einstein's gravitation as a metric field four-potential (which is not a covariant four-vector) of active gravitational charges for passive energy-charges. Contrary to Newton's gravitation for masses, Einstein's gravitation is the metric theory for interacting energies. The static Sun, with the active energy-charge $E_M = Mc^2$, keeps the universal potential $U_\mu / E_m = \left\{-GE_M r^{-1}; 0\right\}$ in the Sun's frame of reference for the passive, inertial energy content $cP_o = E_m = \text{const} \neq mc^2$ of the probe mass m_N. Below, we employ the universality of the Sun's potential, $U_o^N / P_o^N = -GE_M / r = -r_o / r$, for all planets in our computations for gravitational tests of General Relativity with dilated time (4)-(5) and flat material space filled everywhere by r^{-2} gravitational fields and the r^{-4} extended masses.

3. Flatspace for the Planetary Perihelion Precession

Now we consider the metric tensor (3) for a central gravitational field with a static four-potential, $U_i P_o^{-1} = 0$, $U_o P_o^{-1} = -GE_M r^{-1}$, where $E_M = Mc^2 \equiv r_o / G = \text{const}$ is the active gravitational energy of the 'motionless' Sun (in the moving Solar system). We use Euclidean geometry for the radial distance $r \equiv u^{-1}$ from the Sun's center of spherical symmetry in agreement with spatial flatness maintained by (3) for any gravitational four-potential G_μ and its gauge $\partial_\mu \phi$. Let us denote the energy content of a probe mass m in the static central field as a passive energy-charge

$$P_o = m_N V_o = m_N \sqrt{g_{oo} / \left(1 - v^2\right)} = E_m .$$

Then, the interval (1) for the passive energy carrier in a central field with $U_i = 0$ takes two equivalent presentations due to (4) and (5),

$$\begin{aligned} \mathrm{d}s^2 &= \left(1 - GE_M E_m rm \sqrt{1 - \mathrm{d}l^2 \mathrm{d}\tau^2\left(\mathrm{d}l\right)}\right)^2 \mathrm{d}t^2 - \mathrm{d}l^2 \\ &\equiv \mathrm{d}t^2 \left(1 + GE_M r\right)^{-2} - \mathrm{d}l^2, \end{aligned} \tag{6}$$

where iterations

$$\mathrm{d}t^2 \left[1 - \left(GE_M E_m / rm\right)\sqrt{1 - \mathrm{d}l^2 / \mathrm{d}\tau^2\left(\mathrm{d}l\right)}\right]^2 = \mathrm{d}\tau^2\left(\mathrm{d}l\right)$$

over the chain function $\mathrm{d}\tau^2\left(\mathrm{d}l\right)$ in the Lorentz factor result in $\mathrm{d}t^2 / \left[1 + \left(GE_M / r\right)\right]^2$ for the Sun-Mercury potential energy $U_o = -GE_M E_m / r$. In other words, the specific, Weber velocity-dependent potentials exhibit after chain iterations common for all probe particles local time, $\sqrt{\mathrm{d}s^2 + \mathrm{d}l^2} = \mathrm{d}\tau_N = \mathrm{d}\tau_K$, in static fields. Spherical coordinates can be equally used in (6) for the Euclidean element $\mathrm{d}l^2 = \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2 = \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$ in flat laboratory space.

The static metric solution (6) for probe elementary energy-charges in non-empty space of the radial energy-charge does not coincide with the Schwarzschild metric [8] in empty space. Therefore, the Schwarzschild extension of the SR interval is not the only rotationally invariant solution which GR's tensor formalism can propose for tests of space-time-energy self-organizations. Ultra-relativistic velocities, $v \equiv \mathrm{d}l/\mathrm{d}\tau \to 1$ and $\sqrt{1-v^2} \to 0$, in the Weber-type energy-to-energy interaction in (6) revise the Schwarzschild singularity. The latter is not expected at the finite radius in the energy-charge formalism of Einstein's gravitation. Einstein, "the reluctant father of black holes", very strictly expressed his final opinion regarding the Schwarzschild solution: 'The essential result of this investigation is a clear understanding as to why Schwarzschild singularities do not exist in physical reality' [19]. In authors view, Schwarzschild's metric solution, and all Birkhoff class solutions for the empty space dogma, originates with *ad hoc* modeling of matter in the 1915 Einstein equation in terms of point particles. However, Einstein anticipated extended sources for his equation and for physical reality. Below, we prove that the static metric (6) corresponds to the r^{-4} radial energy-charge or the extended source of gravity. Therefore, our analysis denies the empty space paradigm. Non-empty material space is in full agreement with Einstein's idea of continuous sources and Newton's "absurd" interpretation of distant attractions through stresses in an invisible material ether (called in 1686 as "God's sensorium").

Our next task is to derive integrals of motion for the passive (probe) mass-energy in a strong central field from the geodesic equations

$$\mathrm{d}^2 x^\mu / \mathrm{d}p^2 = -\Gamma^\mu_{\nu\lambda}\mathrm{d}x^\nu \mathrm{d}x^\lambda / \mathrm{d}p^2 .$$

Nonzero affine connections $\Gamma^\mu_{\nu\lambda}$ for the metric (6) take the following components:

$$\mathrm{d}^2x^\mu/\mathrm{d}p^2 = \Gamma^r_{\theta\theta} = -r, \Gamma^r_{\varphi\varphi} = -r\sin^2\theta,$$
$$\Gamma^r_{tt} = \mathrm{d}g_{oo}/2\mathrm{d}r, -\Gamma^\mu_{\nu\lambda}\mathrm{d}x^\nu\mathrm{d}x^\lambda/\mathrm{d}p^2$$
$$\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \Gamma^\varphi_{\varphi r} = \Gamma^\phi_{r\varphi} = 1/r,$$
$$\Gamma^\theta_{\varphi\varphi} = -\sin\theta\cos\theta, \Gamma^\varphi_{\varphi\theta} = \Gamma^\varphi_{\theta\varphi} = \mathrm{ctg}\theta,$$

and $\Gamma^t_{tr} = \Gamma^t_{rt} = \mathrm{d}g_{oo}/2g_{oo}\mathrm{d}r$, where g_{oo} is the function next to $\mathrm{d}t^2$ in the interval (6), $\mathrm{d}s^2 = g_{oo}\mathrm{d}t^2 - \mathrm{d}l^2$.

By following the verified approach with $\theta = \pi/2 = \mathrm{const}$ for the isotropic central field, for example [15], and by substituting flatspace connections $\Gamma^\mu_{\nu\lambda}$ into GR's geodesic equations, one can define the parametric differential $\mathrm{d}p$ and write the following gravitational relations,

$$\begin{aligned} &g_{oo}\,\mathrm{d}t/\mathrm{d}p = 1, \mathrm{d}p/\mathrm{d}s = g_{oo}\,\mathrm{d}t/\mathrm{d}s \\ &= E_m/m = \mathrm{const}\; r^2\,\mathrm{d}\varphi/\mathrm{d}p = J_\varphi \\ &= \mathrm{const}, r^2\,\mathrm{d}\varphi/\mathrm{d}s = J_\varphi\, E_m/m \equiv L \\ &= \mathrm{const}\left(\mathrm{d}r/\mathrm{d}p\right)^2 + \left(J_\varphi/r\right)^2 - g^{-1}_{oo} \\ &= \mathrm{const}\left(-m^2/E^2_m\right)\left(\mathrm{d}r/\mathrm{d}s\right)^2 \\ &+\left(r\mathrm{d}\varphi/\mathrm{d}s\right)^2 - E^2_m/m^2\, g_{oo} = -1, \end{aligned} \tag{7}$$

with the first integrals $E_m, m,$ and J_φ of the relativistic motion in strong static fields.

The last line in (7) is the interval equation $\mathrm{d}s^2 = g_{oo}\mathrm{d}t^2 - \mathrm{d}l^2$ with two integrals of motion $E^2_m/m^2 = g^2_{oo}\mathrm{d}t^2/\mathrm{d}s^2$ and $\theta = \pi/2$. Therefore, the scalar invariant (6) is actually the equation of motion for the constant energy charge $E_m = \mathrm{const}$ in a central field with the static Weber-type potential

$$\begin{aligned} U^W_o &= \left(U_o/m\right)\sqrt{1-v^2} \equiv U_o/\left(E_m - U_o\right) \\ &= -GE_M/\left(r + GE_M\right), \end{aligned}$$

which is inharmonic for the Laplacian, $\nabla^2 U^W_o \neq 0$. Recall that Schwarzschild's curved 3D solution not only differs from (6), but results in conceptual inconsistencies [20] for the Einstein equation. We can use (6) and (7) for relativistic motion in strong central fields in order to reinforce the ignored statement of Einstein that Schwarzschild singularities do not exist in physical reality. There are no grounds for metric singularities either in the interval (6), or in the radial potential $U^W_o(r)$ for $r \to 0$, because $\mathrm{d}\tau/\mathrm{d}t = \sqrt{g_{oo}} = r/\left(r + GM\right)$ is a smooth function. One can verify that the non-empty space metric tensor (3), as well as $\nabla^2 U^W_o \neq 0$, does correspond to the continuous energy-source in the 1915 Einstein equation.

The strong field relations (6) and (7) can be used, for example, for computations of planetary perihelion precession in the solar system. The planet's gravitational energy for the GR energy-to-energy attraction, $U_o = -GE_M E_m r^{-1} = -r_o E_m u$, where $r_o \equiv GM/c^2 = \mathrm{const}$ and $u \equiv 1/r$, is small compared to the planet's energy, $|U_o| \ll E_m = \mathrm{const}$, that corresponds to the non-relativistic motion of a planet N (with $E_m/m = \mathrm{const} \approx 1$, $E_m \ll E_M$, and $v^2 \equiv \mathrm{d}l^2/\mathrm{d}\tau^2 \ll 1$) in the Sun's rest frame, with $U_i = 0$. The GR time element for the planet reads from (6) or (7) as

$$\begin{aligned} &\mathrm{d}s^2 - \mathrm{d}l^2 \equiv \mathrm{d}\tau^2\left(\mathrm{d}l\right) \\ &= \mathrm{d}t^2\left(1 - r_o u E_m m\sqrt{1 - \mathrm{d}l^2\mathrm{d}\tau^2\left(\mathrm{d}l\right)}\right)^2 \\ &\approx \left(1 - 2r_o u\right)\mathrm{d}t^2 + r_o u\,\mathrm{d}l^2, \end{aligned} \tag{8}$$

where we set $r_o u \ll 1$, $E_m/m = 1$, $\mathrm{d}l^2 \ll \mathrm{d}\tau^2\left(\mathrm{d}l\right)$, and $\mathrm{d}t^2 - \mathrm{d}\tau^2\left(\mathrm{d}l\right) \ll \mathrm{d}t^2$.

The field term with spatial displacement $r_o u\mathrm{d}l^2$ on the right hand side of (8) belongs to the physical time element within the invariant $\mathrm{d}s^2$. This displacement corresponds to the non-linear chain nature of anisotropic time $\mathrm{d}\tau\left(\mathrm{d}l\right) = f\left(\mathrm{d}l/\mathrm{d}\tau\right)$, originating from the Weber-type energy potential $U_\mu\sqrt{1-v^2}/m$ in (3). There is no departure from Euclidean space geometry with the flat metric

$$\mathrm{d}l^2\left(\theta = \pi/2\right) = \mathrm{d}r^2 + r^2\mathrm{d}\varphi^2 = u^{-4}\mathrm{d}u^2 + u^{-2}\mathrm{d}\varphi^2$$

in the chain reading of geometrical intervals (6) or (8). Again, a particle's non-linear time with chain spatial displacement $\mathrm{d}\tau\left(\mathrm{d}l\right)$ differs in (8) from the proper-time $\mathrm{d}\tau_O = \left(1 - 2r_o u\right)^{1/2}\mathrm{d}t$ of the local (motionless) observer. Displacement corrections, $r_o u\mathrm{d}l^2/\mathrm{d}t^2$, for the non-relativistic limit are very small compared to the main gravitational corrections, $\left(-2r_o u\right)$, to Newtonian time rate $\dot{t}^2 \equiv 1 \gg 2r_o u \gg r_o u\mathrm{d}l^2/\mathrm{d}t^2$. However, the chain dependence of a particle's time element $\mathrm{d}\tau^2$ from spatial displacement $\mathrm{d}l^2$ accounts for the reverse value of this time element, $r_o u\mathrm{d}l^2/\mathrm{d}\tau^2$, that is ultimately a way to restore strict spatial flatness at all orders of Einstein's metric gravitation. Here there is some kind of analogy with electrodynamics where small contributions of Maxwell's displacement currents restore the strict charge conservation in Ampere's quasi-stationary magnetic law.

Two integrals of motion $\left(1 - 2r_o u\right)\mathrm{d}t/\mathrm{d}s = E_m/m$ and $r^2\mathrm{d}\varphi/\mathrm{d}s = L$ result from (7) and (8) for weak fields in a rosette motion of planets,

$$\left(1 - 2r_o u\right)L^{-2} + \left(1 - 3r_o u\right)\left(u'^2 + u^2\right) = E^2 L^{-2} m^{-2}, \tag{9}$$

where $u' \equiv du/d\varphi$ and $r_o u \ll 1$. Indeed, (9) may be differentiated with respect to the polar angle φ,

$$u'' + u - r_o L^2 = 92 r_o u^2 + 3r_o u'' u + 32 r_o u'^2, \tag{10}$$

by keeping only the largest gravitational terms. This equation may be solved in two steps when a non-corrected Newtonian solution, $u_o = r_o L^{-2}\left(1 + \varepsilon\cos\varphi\right)$, is substituted into the GR correction terms at the right hand side of (10).

The most important correction (which is summed over century rotations of the planets) is related to the "resonance" (proportional to $\varepsilon \cos\varphi$) GR terms. Therefore, one may ignore in (10) all corrections apart from $u^2 \sim 2\mu^2 L^{-4}\varepsilon\cos\varphi$ and $u''u \sim -r_o^2 L^{-4}\varepsilon\cos\varphi$. Then the approximate equation for the rosette motion, $u'' + u - r_o L^{-2} \approx 6r_o^3 L^{-4}\varepsilon\cos\varphi$, leads to the well known perihelion precession $\Delta\varphi = 6\pi r_o^2 L^{-2} \equiv 6\pi r_o / a\left(1-\varepsilon^2\right)$, which may also be derived through Schwarzschild's metric approximations with warped three-space, as in [13-15].

It is important to emphasize that the observed result for a planet perihelion precession $\Delta\varphi$ (in the Solar nonempty flatspace with dilated time by Sun's energy densities) has been derived here from the invariant four-interval (1) under flat three-space, $\gamma_{ij} = \delta_{ij}$, rather than under empty but curved three-space.

4. The Radar Echo Delay in Flatspace

The gravitational redshift of light frequency ω can be considered a direct confirmation that gravity couples to the energy content of matter, including the massless photon's energy E_γ, rather than to the scalar mass of the particle. Indeed, Einstein's direct statement $E = mc^2$ for all rest-mass particles is well proved, but the inverse reading, $m = E/c^2$, does not work for electromagnetic waves (with $m = 0$) and requires a new notion, the wave energy-charge $E_\gamma \equiv m_\gamma c^2 \neq 0$ or the relativistic mass $m_\gamma \neq 0$.

In 1907, Einstein introduced the Principle of Equivalence for a uniformly accelerated body and concluded that its potential energy depends on the gravitationally passive ("heavy") mass associated with the inertial mass [21]. This correct conclusion of Einstein was generalized in a wrong way that any energy, including light, has a "relativistic mass" (the gravitational energy-charge in our terminology) for Newtons mechanics. Proponents of this generalization in question proposed that photon's "relativistic mass" is attracted by the Sun's mass *M* in agreement with the measured redshift

$$\Delta\omega/\omega = \Delta E_\gamma / E_\gamma = \Delta\left(-m_\gamma G M R_S^{-1}\right) / m_\gamma c^2 .$$

Nonetheless, the coherent application (in the absence of the correct EM wave equations in gravitational fields) of the "relativistic mass" to zero-mass waves promptly re- sulted in the underestimated light deflection, $\varphi = -2GM/R_S c^2 \equiv -2r_o/R_S$, for the "mechanical free fall" of photons in the Sun's gravitational field [22]. In 1917, when Schwarzschild's option [8] for spatial curvature had been tried for all GR solutions, the new non-Newtonian light deflection, $\varphi = -4r_o/R_S$, had been predicted due to additional contributions from the supposed spatial curvature in question. Later, all measurements supported this curve-space modification for the "relativistic mass" deflection by the Sun that provided false "experimental evidences" of non-Euclidean three-space in contemporary developments of metric gravitation.

Below, we prove that Einstein's GR for the Maxwell wave equation firmly maintains the flatspace concept for interpretation of light phenomena in gravitational fields if one coherently couples the Sun's rest energy to the photon's wave energy E_γ. We consider both the radar echo delay and the gravitational deflection of light by coupling its energy-charge with local gravitational potentials. Our purpose is to verify that Euclidean space can match the known measurements [13,23,24] of light phenomena in the Solar system. Let us consider a static gravitational field ($g_i = 0$, for simplicity), where the physical slowness of photons, $n^{-1} \equiv v/c$, can be derived directly from the covariant Maxwell equations [14], $n^{-1} = \sqrt{\tilde{\varepsilon}\tilde{\mu}} = \sqrt{g_{oo}}$. Recall that a motionless local observer associates g_{oo} with the gravitational potential U_o/P_o at a given point. The light velocity $v = \mathrm{d}l/\mathrm{d}\tau_O$, measured by this observer, as well as the observed light frequency $\omega = \omega_o \,\mathrm{d}t/\mathrm{d}\tau_O$, is to be specified with respect to the observer's time rate $\mathrm{d}\tau_O = \sqrt{g_{oo}}\mathrm{d}t$. This consideration complies with Einsteins approach, where the light's redshift is associated with different clock rates (of local observers) in the Sun's gravitational potential [21].

Compared to the physical speed of light, $v = \mathrm{d}l/\mathrm{d}\tau_o = cn^{-1}$, its coordinate speed

$$\begin{aligned} \dot{l} &\equiv \mathrm{d}l\mathrm{d}\tau_O \times \mathrm{d}\tau_O \mathrm{d}t = cn \times \sqrt{g_{oo}} = cg_{oo} \\ &\equiv c\left(1+r_o r\right)^{-2} \approx c\left(1-2r_o r\right) \end{aligned} \tag{11}$$

is double-shifted by the gravitational potential $U_o/P_o = -r_o/r$, where $r_o = GM_S/c^2 = 1.48\ \mathrm{km}$ and $r_o \ll r \approx R_S$. Notice that both the local physical slowness $n^{-1} = \sqrt{g_{oo}}$ and the observer time dilation $\mathrm{d}\tau_O/\mathrm{d}t = \sqrt{g_{oo}}$ are responsible for the double slowness of the coordinate velocity (11), which is relevant to observations of light coordinates or rays under gravitational tests.

A world time delay of Mercury's radar echo reads through relation (11) as

$$\begin{aligned} \Delta t &= 2\int_{l_E}^{l_M} \mathrm{d}l\left(1\dot{l} - 1c\right) \approx 2c\int_{x_E}^{x_M} 2r_o \mathrm{d}x\sqrt{x^2+y^2} \\ &\approx 4r_o c ln 4 r_{MS} r_{ES} R_S^2 = 220\ \mu\mathrm{s}, \end{aligned} \tag{12}$$

where $y \approx R_S = 0.7\times10^6$ km is the radius of the Sun, while $r_{ES} = 149.5\times10^6$ km and $r_{MS} = 57.9\times10^6$ km are the Earth-Sun and Mercury-Sun distances, respectively. Notice that in flat space we use the Euclidean metric for spatial distance, $r = \left(x^2+y^2\right)^{1/2}$, between the Sun's center (0,0) and any point (*x*, *y*) on the photonic ray. One can measure in the Earth's laboratory only the physical time delay $\Delta\tau_E = \sqrt{g_{oo}^E}\Delta t$, which practically coincides with the world time delay Δt in the Earth's weak field, *i.e.* $\Delta\tau_E \approx \Delta t = 220\ \mu s$. From here, the known experimental results [13,24] correspond to the radar echo delay (12), based on strictly flat three-space and dilated time as in 1913 *Entwurf* metric scheme.

5. Gravitational Light Bending in Non-Empty Flatspace

A coordinate angular deflection $\varphi = \varphi_\infty - \varphi_{-\infty}$ of a light wave front in the Sun's gravitational field can be promptly derived in flat space geometry by using the coordinate velocity (11) for observations,

$$\begin{aligned}\varphi &= -2\int_0^\infty \mathrm{d}l\partial\partial y\left(\dot{l}c\right) \approx -2\int_0^\infty \mathrm{d}x\partial\partial y 2r_o\sqrt{x^2+y^2} \\ &\approx -4r_o\int_0^\infty R_S\mathrm{d}x\left(x^2+R_S^2\right)^{3/2} = -4r_oR_S = -1.75''.\end{aligned} \tag{13}$$

The most rigorous classical procedure to derive the ray deflection (13) is to apply the verified Fermat principle to light waves. This basic principle of physics should also justify spatial flatness under suitable applications [25].

In agreement with Einstein's original consideration [21], one may relate the vector component K_o in the scalar wave equation $K_\mu K^\mu = 0$ to the measured (physical) energy-frequency $\hbar\omega$ of the photon $cK_o = E = \hbar\omega = \hbar\omega_o\,\mathrm{d}t/\mathrm{d}\tau_o, \hbar\omega_o = \text{const}$). Recall that P_o is also the measured particle's energy in the similar equation, $P_\mu P^\mu = m^2c^4$, for a rest-mass particle. The scalar wave equation $K_\mu K^\mu = g_N^{\mu\nu}K_\mu K_\nu = 0$ has the following solution for the electromagnetic wave,

$$\begin{aligned}K_o &\equiv \hbar\omega_o\,\mathrm{d}t/c\mathrm{d}\tau = g_{oo}\left(K^o - g_iK^i\right)\gamma_{ij}K^iK^j \\ &= g_{oo}\left(K^o - g_iK^i\right)^2 = K_o^2/g_{oo} \\ &= \hbar^2\omega_o^2\,\mathrm{d}t^2/c^2\,g_{oo}\mathrm{d}\tau^2K^i \\ &= \hbar\omega_o\,\mathrm{d}t\mathrm{d}x^i/c\sqrt{g_{oo}}\,\mathrm{d}\tau\mathrm{d}lK_i \\ &= -\left[\left(\mathrm{d}x_i/\mathrm{d}l\right) + \sqrt{g_{oo}}\,g_i\right]\hbar\omega_o\,\mathrm{d}t/c\sqrt{g_{oo}}\,\mathrm{d}\tau,\end{aligned} \tag{14}$$

with $K_\mu = \left\{E, -E\left[\left(\delta_{ij}\mathrm{d}x^j/\sqrt{g_{oo}}\mathrm{d}l\right) + g_i\right]\right\}c^{-1}$.

The Fermat-type variations with respect to $\delta\varphi$ and δu ($r \equiv u^{-1}, \varphi$, and $\vartheta = \pi/2$ are the spherical coordinates) for photons in a static gravitational field are

$$\begin{aligned}\delta\int K_i\mathrm{d}x^i &= -\delta\int\hbar\omega_o\gamma_{ij}\mathrm{d}x^jcg_{oo}\mathrm{d}l\mathrm{d}x^i \\ &= -\hbar\omega_oc\delta\int\sqrt{\mathrm{d}u^2 + u^2\mathrm{d}\varphi^2}\left(1+r_ou\right)^2u^2 = 0,\end{aligned} \tag{15}$$

(where $g_{oo} = \left(1+r_ou\right)^{-2}$, $g_i = 0$, $\gamma_{ij} = \delta_{ij}$, $\mathrm{d}l = \sqrt{\delta_{ij}x^ix^j} = \sqrt{\mathrm{d}r^2 + r^2\mathrm{d}\varphi^2}$) resulting in a couple of light ray equations for $r_ou \ll 1$,

$$\left\{\left(1-4r_ou\right)\left[\left(u'_\varphi\right)^2 + u^2\right] = u_o^2 = \text{const}\ u''_{\varphi\varphi} + u = 2r_ou_o^2 \right. \tag{16}$$

Solutions of (16),

$$u \equiv r^{-1} = u_o\sin\varphi + 2r_ou_o^2\left(1+\cos\varphi\right)$$

and $r_ou_o \approx r_o/R_S \ll 1$, may be used for the Sun's weak field. The propagation of light from $r(-\infty) = \infty, \varphi(-\infty) = \pi$ to $r(+\infty) \to \infty, \varphi(+\infty) \to \varphi_\infty$ corresponds to the angular deflection

$$\begin{aligned}\varphi_\infty &= \arcsin\left[-4r_oR_S^{-1}\left(1+\cos\varphi_\infty\right)\right] \\ &\approx -4r_o/R_S = -1.75''\end{aligned}$$

from the light's initial direction. This deflection coincides with (13) and is in agreement with the known measurements $-1.66'' \pm 0.18''$, for example [13].

We may conclude that there is no need to warp Euclidean three-space for the explanation of the "non-Newtonian" light deflections if one strictly follows Einstein's original approach to light in gravitational fields [21]. In fact, the massless electromagnetic energy exhibits an inhomogeneous slowness of its physical velocity, $v \equiv \mathrm{d}l/\mathrm{d}\tau_o = c\sqrt{g_{oo}}$, and, therefore, a double slowness of the coordinate velocity, $\mathrm{d}l/\mathrm{d}t = cg_{oo}$. This coordinate velocity slowness is related to the coordinate bending of light measured by observers. In closing, the variational Fermat's principle supports *Entwurf* physics of Einstein and Grossmann with dilated time and strict spatial flatness for light in the Solar system.

6. Geodetic and Frame-Dragging Precessions of Orbiting Gyroscopes

Precession of the orbiting gyroscopes in the Gravity Probe B Experiment [26] has been compared only with Schiffs formula [27] based on the Schwarzschild-type metric for curved and empty 3D space. Here the author plans to criticize the point spin model for GP-B computations in favor of the regular Einstein-Infeld-Hoffman approach to slowly rotating distributions of masses. This original GR approach practically coincides in the weak Earths field with our flatspace reading of Einstein's physics. Recall that our *Entwurf*-type space interval is strictly flat due to the intrinsic metric bounds in the GR four-interval (1) with the metric tensor (3). However, the GR tensor formalism can be universally applied to any warped space-time manifold with or without intrinsic metric bounds.

By following Schiff and many other point particle proponents in gravitation, one has to assume for a moment that the vector geodesic equation,

$$\mathrm{d}S_\mu/\mathrm{d}p = \Gamma^\lambda_{\mu\nu}S_\lambda\,\mathrm{d}x^\nu/\mathrm{d}p,$$

in pseudo-Riemannian four- space with only symmetrical connections, $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$, may be applied to the point spin "four-vector" S_μ with "invariant" bounds $V^\mu S_\mu = 0$ or $S_o = -\dot{x}^iS_i$ for orthonormal four-vectors,

$$\begin{aligned} \mathrm{d}S_i \mathrm{d}t &= \Gamma_{i\nu}^{o} S_o \dot{x}^{\nu} + \Gamma_{i\nu}^{j} S_j \dot{x}^{\nu} \\ &= \left(-\Gamma_{io}^{o} \dot{x}^{j} - \Gamma_{ik}^{o} \dot{x}^{k} \dot{x}^{j} + \Gamma_{io}^{j} + \Gamma_{ik}^{j} \dot{x}^{k}\right) S_j. \end{aligned} \tag{17}$$

Our flat-space for a strong static field with (3) and $g^{oi} = 0, g^{oo} = \left(1 - U_o P_o^{-1}\right)^2 = 1/g_{oo}$, and $g^{ij} = -\delta^{ij}$, would formally maintain an inertial conservation,

$$\begin{aligned} g^{\mu\nu} S_\mu S_\nu &= \left(S_o S_o / g_{oo}\right) - \delta^{ij} S_i S_j \\ &= \left(\dot{x}^i / \sqrt{g_{oo}}\right)\left(\dot{x}^j / \sqrt{g_{oo}}\right) S_i S_j \end{aligned}$$

$$-S^i S_i = (vS)^2 - S^2 = \text{const},$$

in agreement with Einstein's teaching for a free-falling body. At the same time, Schwarzschild's metric option (curved space) tends to suggest [15,27] the non-compensated Newtonian potential $\phi = -GM/r$ even in the "free fall" equation,

$$\text{const} = g_{Sch}^{\mu\nu} S_\mu S_\nu = (vS)^2 - S^2 (1 + 2\phi).$$

Therefore, formal applications of the Einstein-Grossmann geodesic relations (derived for spatial translations of material points) to localized spins S_μ (which are not four-vectors in 4D manifolds with symmetrical affine connections) contradict the spirit of GR inertial motion and, ultimately, the Principle of Equivalence.

Our affine connections $\Gamma_{\mu\nu}^{\lambda} = \Gamma_{\nu\mu}^{\lambda}$, related to the metric tensor (3), depend only on four field potentials $G_\mu \equiv U_\mu P_o^{-1} = \left\{U_o P_o^{-1}, U_i P_o^{-1}\right\}$. This post-*Entwurf* metric tensor has been introduced for the local energy-momentum (2) without any rotational or spin components. Moreover, neither the mechanical part, K_μ, nor the gravitational part, $P_o G_\mu$, in (2) are separately covariant four-vectors in warped space-time with the metric tensor (3). Therefore, there are no optimistic grounds to believe that four spin components S_μ might accidentally form a covariant four vector in space-time with symmetrical connections for translation of the energy-momentum four-vector, $P_\mu \equiv K_\mu + P_o G_\mu$. Nonetheless, we try by chance these symmetrical connections for the point spin avenue (17) in question in constant fields (when $\partial_o g_{\mu\nu} = 0$, for simplicity),

$$\begin{aligned} 2\Gamma_{io}^{j} &= U_j P_o^{-1} \partial_i g_{oo} + \partial_j \left(U_i P_o^{-1} g_{oo}\right) - \partial_i \left(U_i P_o^{-1} g_{oo}\right) \\ 2\Gamma_{io}^{j} &= \left[\left(1 - U_o P_o^{-1}\right)^2 - U_i^2 P_o^{-2}\right] \partial_i g_{oo} + P_o^{-1} U_i \left[\partial_j \left(U_i P_o^{-1} g_{oo} - \partial_i \left(U_j P_o^{-1} g_{oo}\right)\right)\right] \\ 2\Gamma_{ik}^{j} &= \partial_j \left(U_i U_k P_o^{-2} g_{oo}\right) - U_k P_o^{-1} g_{oo} \partial_i \left(U_j P_o^{-1}\right) - U_i P_o^{-1} g_{oo} \partial_k \left(U_i P_o^{-1} g_{oo}\right) \\ 2\Gamma_{ik}^{o} &= \left[\left(1 - U_o P_o^{-1}\right)^2 \delta^{ij} U_i U_j P_o^{-2}\right]\left[\partial_k \left(U_k P_o^{-1} g_{oo}\right) + \delta_k \left(U_i P_o^{-1} g_{oo}\right)\right] \\ &\quad + U_o P_o^{-1} \left[\partial_i \left(U_j U_k P_o^{-2} g_{oo}\right) + \delta_k \left(U_j U_k P_o^{-2} g_{oo}\right) - \partial_j \left(U_j U_k P_o^{-2} g_{oo}\right)\right] \end{aligned} \tag{18}$$

One could start with $U_o P_o^{-1} = -G E_M r^{-1}$ and

$$U_i P_o^{-1} = 2GIr^{-3} \left[r \times \omega\right]_i$$

for the homogeneous spherical mass *M* rotating with low angular velocity, *i.e.* $\omega r \ll 1$, $U_i U_i / P_o^2 \ll 1$, $E_M \approx M$, and $I = \sum_n m_n x_n \times v_n \approx 2MR_E^2/5$ for $R_E < r$ [14]. Then, by keeping only linear terms with respect to U_i/P_o, one can rewrite (17) for a slowly rotating gravitational field:

$$\begin{aligned} \mathrm{d}S_i \mathrm{d}t \approx &-S_j \dot{x}^j \partial_i ln\sqrt{g_{oo}} - \delta^{jk} S_j \partial_i \left(U_k P_o^{-1} g_{oo}\right) - \partial_k \left(U_i P_o^{-1} g_{oo}\right) 2 + S_j U_j P_o \partial_i g_{oo} \\ &-\dot{x}^j \dot{x}^k S_j \partial_i \left(U_k P_o^{-1} g_{oo}\right) + \partial_k \left(U_i P_o^{-1} g_{oo}\right) 2 g_{oo}. \end{aligned} \tag{19}$$

The last three terms on the right-hand side of (19) are responsible for frame rotation and frame dragging, which vanish for non-rotating centers when $\omega \to 0$ and $U_i/P_o \to 0$. Precessions of the constant magnitude vector $J = S - (vS)\left(v + 2UP_o^{-1}\right)/2$, obtained for the weak-field limit of

$$g^{\mu\nu} S_\mu S_\nu = \left[\left(1 - U_o P_o^{-1}\right)^2 - U_i U_i P_o^{-2}\right]\left(\dot{x}^j S_j\right)^2 + 2U_j P_o^{-1} S_j \left(\dot{x}^i S_i\right) - \delta^{ij} S_i S_j \equiv \delta^{ij} J_i J_j = \text{const},$$

when $\left(-U_o/P_o\right) \ll 1$, $\dot{x}^i \dot{x}^i \ll 1$, and $\dot{x}^i \approx v^i \approx -\partial_i U_o P_o^{-1}$ in (19),

$$\begin{aligned} \mathrm{d}J_i \mathrm{d}t \approx &-J_j 2\left[v^j \partial_i \left(U_o P_o^{-1}\right) - v^i \partial_j \left(U_o P_o^{-1}\right)\right] - J_j \delta^{jk} 2\left(\partial_i U_k P_o^{-1} - \partial_k U_i P_o^{-1}\right) \\ &+ J_j \left[U_j P_o^{-1} \partial_i \left(U_o P_o^{-1}\right) - U_i P_o^{-1} \partial_j \left(U_o P_o^{-1}\right)\right], \end{aligned} \tag{20}$$

may be compared with Schiff's non-relativistic prediction $\mathrm{d}J/\mathrm{d}t = \left(\Omega_{geo} + \Omega_{fd}\right) \times J$ for Gravity Probe B. The second summand at the right hand side of (20), $-J_j \delta^{jk}\left(\partial_i U_k P_o^{-1} - \partial_k U_i P_o^{-1}\right)/2 \equiv \left(\Omega_{fd} \times J\right)_i$, takes exactly Schiff's answer [27] for the frame-dragging precession,

$$\Omega_{fd} \approx -12\nabla \times \left(\frac{2GIr}{r^3} \times \omega\right) = GIr^3\left(\frac{3r(\omega \cdot r)}{r^2} - \omega\right). \quad (21)$$

The first and third precession terms in (20) depend on the Earth's radial field $\partial_i\left(U_o P_o^{-1}\right)$ and they count together geodetic and frame phenomena. These terms provide $\Omega_{gf} = -\left(2^{-1}v - UP_o^{-1}\right) \times \nabla U_o P_o^{-1}$. Such a precession for a point spin model, formally borrowed from the Einstein-Grossmann theory for the probe mass without rotation, fails to reiterate the already well verified de Sitter geodetic precession,

$$\Omega_{geo} = -(3/2) v \times \nabla U_o P_o^{-1} = 3GM\left(r \times v\right)/2r^3 ,$$

of the Earth-Moon gyroscope in the Sun's field, where $U \equiv \{U_1, U_2, U_2\} = 0$. Why does the Einstein-Grossmann geodesic point mass fail for physics of spins and mass rotations?

First of all, there is a clear mathematical reason to reject point spins from the Einstein-Grossman metric formalism. The point spin approach to GR matter cannot justify that S_μ is a covariant four-vector in pseudo-Riemannian space-time where the metric tensor is defined exclusively for matter without self-rotations or for the four-momentum of a probe particle without spin. Therefore, one cannot place S_μ into the Einstein-Grossmann geodesic equation with symmetrical connections. Riemann-Cartan geometries with the affine torsion and asymmerical connection [17] are still under discussions for proper applications.

In 1938 Einstein already answered the point spin question by developing with Infeld and Hoffmann relativistic dynamics of slowly moving distributions of active and passive masses. It is well known (Weyl in 1923 and Einstein-Infeld-Hoffmann in 1938 for example [14]) that the inhomogeneous GR time dilation (or inhomogeneous $g_{oo}(r)$ for mass elements rotating over a joint axis) defines a relativistic Lagrangian for the classical non-point gyroscope. Therefore, Einstein's relativity quantitatively explains the de Sitter precession through local non-Newtonian time rates for distributed rotating systems. The non-Newtonian (three-times enhanced) precession originates exclusively from different GR time rates in neighboring material points, rather than from a local space curvature in question for the ill-defined GR spin of a point mass. The author does not understand Schiffs reasons to ignore Einstein-Infeld-Hoffmann physics and Weyl results for relativistic gyroscopes prior to testing General Relativity through rotation of masses.

The Einstein-Hilbert tensor formalism for energy densities of a gravitational source (rather than for a point source) requires non-Schwarzschildian interpretation of all gravitational tests, including Lunar-Laser-Ranging and Gravity Probe B data. In authors view, the 1913 Einstein-Grossmann geodesic motion in pseudo-Riemannian space-time with flat space can provide a physical basis for translational dynamics of only point particles, but not for self-rotations of distributed relativistic matter. Point spin models for geodetic and frame-dragging angular drifts of free-falling gyroscopes cannot be reasonable for GR physics even under formal success of point-spin approximations for the observable geodetic precession. Possible speculations that the de Sitter geodetic precession of the Earth-Moon gyroscope or that the Mercury perihelion precession have already confirmed non-Euclidean space geometry are against proper applications of the well-tested GR time dilation by gravitational fields, and, therefore, against Einstein-Infeld-Hoffmann's physics of slowly rotating systems having finite active/passive masses at finite dimensions. In fact, the available GP-B releases (einstein.stanford.edu) of the processed geodetic precession data perfectly confirmed time dilatation for Einstein-Infeld-Hoffmann rotating distributions of masses. Lunar laser ranging of the Earth-Moon gyroscope and the GP-B geodetic precession are irrelevant, in fact, to experimental proofs of space warping by the missing inch. These tests are equally irrelevant to experimental proves of black holes existence. On the contrary, all known precision measure- ments in gravitation confirms the strong-field metric (3) with time dilation and continuous gravitational masses in nonempty Euclidean 3-space.

7. Conclusions

There are a lot of disputes in modern gravitation and astroparticle physics. Our main goal was to reinforce spatial flatness for real, non-point matter in a line of the original *Entwurf* geometrization of fields, rather than to discuss other consequences of the selfcontained SR-GR metric scheme [2,16]. In order to achieve this main goal, we derived quantitative geodesic predictions for Mercury's perihelion precession, Mercury's radar echo delay, and the gravitational light deflection by the Sun in strictly flat three-space without references on the 1915 GR equations at all. The numerical results are well known from the Schwarzschild empty-space approximation of reality. Recall that the conventional interpretation of post-Newtonian corrections relies on space warping around the localized gravitational source (including the 'point' Sun). On the contrary, our chain analysis of particles physical time allows us to infer that curved 4-interval can keep strict spatial flatness and the *Entwurf* metric scheme for strong-field gravitation. The GR displacement $\mathrm{d}l$ may be referred as a space interval (like in Special Relativity) in flatspace relativity of nonlocal superfluid masses with mutual spatial penetrations. Consequently, the integral $\int \mathrm{d}l$ along a space curve does not depend anymore on gravitational fields and takes a well-defined meaning. Such a Machian-type nonlocality of superfluid astroparticles reconciles 3D space properties with the relativistic Sommerfield quantization along a line contour. Indeed, these are no reasonable explanations for quantized magnetic flux in laboratory SQUIDs, unless one accepts 3D spatial flatness for any 2D surface [3].

GR physics may attach all field corrections within the GR invariant $\mathrm{d}s^2$ to the time element $\mathrm{d}\tau^2(\mathrm{d}l)$ with chain relations. Gravity indeed curves elementary space-time intervals (therefore $\mathrm{d}\tau$ and $\mathrm{d}s$ are specific for each moving particle), but their space sub-intervals $\mathrm{d}l$ are always flat or universal for all particles and observers. It is not surprising that our approach to relativistic corrections, based on the strong-field equations (7), resulted in Schwarzschild-type estimations, which are based on very close integrals of motion in the Sun's weak field. However, strong fields in (7) will not lead to further coincidences with empty-space Schwarzschild-type solutions for dynamics of probe particles.

Both the Euclidean space interval $\mathrm{d}l = \sqrt{\delta_{i\nu}\mathrm{d}x^i\mathrm{d}x^\nu} > 0$ and the Newtonian time interval

$$\mathrm{d}t = \sqrt{\delta_{o\nu}\mathrm{d}x^o\mathrm{d}x^\nu} = \left|\mathrm{d}x^o\right| > 0$$

are independent from local fields and proper parameters of elementary particles. This absolute universality of world space and time rulers is a mandatory requirement for these notions in their applications to different particles and their ensembles. Otherwise, there would be no way to introduce for different observers one universal ruler to measure three-intervals and to compare dynamics of particles in common 3-space under the common time parameter. For example, it is impossible to measure or to compare differently warped four-intervals $\mathrm{d}s_N = \sqrt{g^N_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu}$ of different particles. In other words, there is no universal, non-specific pseudo-Riemannian geometry for all world matter. Therefore, joint evolution of energy carriers can be observed only in common sub-spaces when they maintain universal (for all matter) sub-metrics.

Space-time-energy self-organization of extended matter can be well described without 3D metric ripples, which have no much sense in strictly flat material space. Laboratory search of observable chiral phenomena for paired vector interactions in flat material space is worth to be performed before expansive projects to find 3D metric ripples in cosmic space. Record measurements of flat material space beyond the present limit $10^{-18}\,\mathrm{m}$ might not be required for confirmation of the residual EM nature of elementary masses under their Einstein-type geometrization. Once chiral symmetry for hadrons was violated at $10^{-15}\,m$, then this mass-forming symmetry was equally violated in the entire nonlocal structure of the superfluid astroparticle [2] or in its infinite material space. Non-empty Euclidean 3-space does match curved 4D space-time in metric gravitation. Such a matching allows the extended radial electron to move (both in theory and in practice) without spatial splits of mass and electric charge densities. Strict spatial flatness is a real way for quantization of elementary fields and for unified geometrization of extended gravitational and electric charges.

8. Acknowledgements

I acknowledge useful discussions with Yu. S. Vladimirov regarding relation nature for binary interactions of elementary matter and for the non-empty space concept.

REFERENCES

[1] I. E. Bulyzhenkov, "Thermoelectric Flux in Superconducting Hollow Cylinders," *Physical Review B*, Vol. 51, No. 2, 1995, pp. 1137-1139. doi:10.1103/PhysRevB.51.1137

[2] I. E. Bulyzhenkov, "Superfluid Mass-Energy Densities of Nonlocal Particle and Gravitational Field," *Journal of Superconductivity and Novel Magnetism*, Vol. 22, No. 8, 2009, pp. 723-727. doi:10.1007/s10948-009-0583-5

[3] I. E. Bulyzhenkov, "Relativistic Quantization of Cooper Pairs and Distributed Electrons in Rotating Superconductors," *Journal of Superconductivity and Novel Magnetism*, Vol. 22, No. 7, 2009, pp. 627-629. doi:10.1007/s10948-009-0510-9

[4] A. Einstein and M. Grossmann, "Entwurf Einer Verallgemeinerten Relativitatstheorie und Einer," *Zeitschrift für Mathematik und Physik*, Vol. 62, 1913, pp. 225-261.

[5] A. Einstein, "Sitzungsber. Preuss. Akad. Wiss. Berlin," *Mathematical Physics*, 1915, pp. 778, 799, 831, 844.

[6] A. Einstein, "Die Grundlage der Allgemeinen Relativitätstheorie," *Annalen der Physik*, Vol. 354, No. 7, 1916, pp. 769-822. doi:10.1002/andp.19163540702

[7] D. Hilbert, "Die Grundlagen der Physik," Nachrichten K. Gesellschaft Wiss. Gøttingen, Math-Phys. Klasse, Heft 3, 1915, S. 395.

[8] G. Mie, "Grundlagen einer Theorie der Materie," *Annalen der Physik*, Vol. 344, No. 11, 1912, pp. 1-40. doi:10.1002/andp.19123441102

[9] K. Schwarzschild, "Uber das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie," *Sitzungsberichte der Königlich-Preussischen Akademie der Wissenschaften*, Vol. 3, 1916, pp. 189-196.

[10] J. Droste, "The Field of a Single Centre in ElNSTEIN's Theory of Gravitation," *Proceedings of the Koninklijke Nederlandse Akademie van Wetenschappen*, Vol. 19,

1916, p. 197.

[11] N. Rosen, "General Relativity and Flat Space. I," *Physical Review*, Vol. 57, No. 2, 1940, pp. 147-150. doi:10.1103/PhysRev.57.147

[12] A. A. Logunov, "The Theory of Gravitational Field," Nauka, Moscow, 2001.

[13] W. Petry, "Gravitation in Flat Space-Time," *General Relativity and Gravitation*, Vol. 13, No. 9, 1981, p. 865.

[14] N. I. Lobachevsky, "A Concise Outline of the Foundations of Geometry," University of Kazan Messenger, Kazan, 1829.

[15] J. Bolyai, "Appendix Explaining the Absolutely True Science of Space," In: F. Bolyai, Ed., *Tentamen*, Transylvania, 1832.

[16] B. Riemann, "On the Hypotheses That Form the Foundations of Geometry," 1854 Lecture (Nachrichten K. Gesellschaft Wiss, Gottingen, 1868.

[17] P. De Bernardis, *et al.*, "A Flat Universe from High-Resolution Maps of the Cosmic Microwave Background Radiation," *Nature*, Vol. 404, No. 6781, 2000, pp. 955-959. doi:10.1038/35010035

[18] A. Lange, *et al.*, "Cosmological Parameters from the First Results of Boomerang," *Physical Review D*, Vol. 63, No. 4, 2001, Article ID: 042001. doi:10.1103/PhysRevD.63.042001

[19] S. Hanany, *et al.*, "MAXIMA-1: A Measurement of the Cosmic Microwave Background Anisotropy on Angular Scales of 10' - 5°," *The Astrophysical Journal Letters*, Vol. 545, No. 1, 2001, pp. L5-L9. doi:10.1086/317322

[20] E. Komatsu, *et al.*, "Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation," *The Astrophysical Journal Supplement Series*, Vol. 180, No. 2, 2009, pp. 330-376. doi:10.1088/0067-0049/180/2/330

[21] C. W. Misner, K. S. Thorne and J. A. Wheeler, "Gravitation," Freeman, San Francisco, 1973.

[22] R. Wald, "General Relativity," University of Chicago Press, Chicago, 1984.

[23] C. M. Will, "Theory and Experiment in Gravitational Physics," Cambridge University Press, Cambridge, 1981.

[24] L. D. Landau and E. M. Lifshitz, "The Classical Theory of Fields," Pergamon, Oxford, 1971.

[25] S. Weinberg, "Gravitation and Cosmology," John Wiley and Sons, New York, 1972.

[26] I. E. Bulyzhenkov, "Einstein's Gravitation for Machian Relativism of Nonlocal Energy-Charges," *International Journal of Theoretical Physics*, Vol. 47, No. 5, 2008, pp. 1261-1269. doi:10.1007/s10773-007-9559-z

[27] R. Weitzenbock, "Invariantentheorie," Noordhoff, Groningen, 1923.

[28] G. Vitali, *Atti Soc. Ligust. Sci. Lett.*, Vol. 11, 1924, p. 248.

[29] E. Cartan and J. Schouten, "On Riemannian Geometries admitting an Absolute Parallelism," *Proceedings Knkl. Neder. Akad.* Vol. 28, 1926, p. 400.

[30] W. Weber, "On the Electromagnetic and Electrostatic Units of Current and the Meaning of the Absolute System of Units," *Annalen der Physik*, Vol. 73, 1848, p. 193.

[31] A. Einstein, "On a Stationary System with Spherical Symmetry Consisting of Many Gravitating Masses," *Annals of Mathematics*, Vol. 40, No. 4, 1939, pp. 922-936. doi:10.2307/1968902

[32] J. V. Narlikar, "A Random Walk in General Relativity and Cosmology," Wiley Eastern, New Delhy 1985, p. 171.

[33] J. V. Narlikar and T. Padmanabhan, "The Schwarzschild Solution: Some Conceptual Difficulties," *Foundation of Physics*, Vol. 18, No. 6, 1988, pp. 659-668.

[34] A. Einstein, "Über das Relativitätsprinzip und die aus Demselben Gezogenen Folgerungen," *Jahrbuch der Radioaktivitaet und Elektronik*, Vol. 4, 1907, p. 411.

[35] A. Einstein, "Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes," *Annalen der Physik*, Vol. 340, No. 10, 1911, pp. 898-908. doi:10.1002/andp.19113401005

[36] R. V. Pound and G. A. Rebka, "Apparent Weight of Photons," *Physical Review Letters*, Vol. 4, No. 7, 1960, pp. 337-341. doi:10.1103/PhysRevLett.4.337

[37] I. I. Shapiro, "Fourth Test of General Relativity," *Physical Review Letters*, Vol. 13, No. 26, 1964, pp. 789-791. doi:10.1103/PhysRevLett.13.789

[38] R. d'E. Atkinson, "General relativity in Euclidean Terms," *Proceedings of the Royal Society A*, Vol. 272, No. 1348, 1963, pp. 60-78.

[39] F. H. J. Cornish, "General Relativity in Terms of a Background Space," *Proceedings of the Royal Society A*, Vol. 376, No. 1366, 1963, pp. 413-417. doi:10.1098/rspa.1963.0214

[40] C. Møller, "The Theory of Relativity," Oxford University Press, Oxford, 1952, p. 308.

[41] C. W. F. Everitt, "The Stanford Relativity Gyroscope Experiment: History and Overview," In: J. D. Fairbank, *et al.*, Eds., *Near Zero*: *New Frontiers of Physics*, Freeman and Co., New York, 1988, p. 587.

[42] L. I. Schiff, "Motion of a Gyroscope According to Einstein's Theory of Gravitation," *Proceedings of the National Academy of Sciences of the United States of America*, Vol. 46, No. 6, 1960, pp. 871-882. doi:10.1073/pnas.46.6.871

[43] L. I. Schiff, "Possible New Experimental Test of General Relativity Theory," *Physical Review Letters*, Vol. 4, No. 5, 1960, pp. 215-217. doi:10.1103/PhysRevLett.4.215

Journal of Modern Physics, 2012, 3, 1097-1355
Published Online September 2012 in SciRes (http://www.SciRP.org/journal/jmp).

TABLE OF CONTENTS

Volume 3 Special Issue September 2012

