

# Kinematic and Geometric Origin of Apparent Cosmological Acceleration: A Rotating-Observer Model with Euclidean Embedding

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## Abstract

The late-time accelerated expansion of the universe is canonically attributed to a cosmological constant  $\Lambda$  with energy density  $\rho_\Lambda \approx 6.9 \times 10^{-27} \text{ kg} \cdot \text{m}^{-3}$ . We investigate a purely kinematic interpretation in which the apparent acceleration arises from the projection of inertial motion onto a non-inertial, radially constrained observational frame. Working first in a rotating-observer toy model and then in a higher-dimensional Euclidean embedding calibrated to cosmological scales ( $r_0 \sim 10 \text{ Mpc}$ ,  $v_0 = H_0 r_0$ ), we show that an inertially moving particle is perceived by a central observer as radially accelerating. We present closed-form expressions for the apparent radial distance, velocity, and acceleration, and derive a dimensionless transverse energy fraction  $\Omega_k(t) = r_0^2 / r(t)^2$ . At  $r(t) \approx 1.2 r_0$  ( $t \approx 9.84 \text{ Gyr}$ ),  $\Omega_k \approx 0.685$ , matching the observed  $\Omega_\Lambda$  to within 1%. The energetic deficit inferred by the rotating observer, which is fully accounted for by the unobserved transverse degree of freedom, mimics dark energy. The asymptotic acceleration scales as  $t^{-3}$ , implying a decaying effective equation of state. While we do not claim to replace  $\Lambda\text{CDM}$ , this proof of concept demonstrates that a geometric projection effect can generate kinematic signatures indistinguishable from a positive cosmological constant. We discuss falsifiability, observational discriminants, and the need for a relativistic extension.

## Keywords

Cosmological Constant, Dark Energy, Cosmological Expansion, Kinematic Models, Higher-Dimensional Embeddings, Observational Cosmology

## 1. Introduction

### 1.1. Historical and Observational Context

The idea that the expansion of the universe might be accelerating has a rich intellectual history predating the supernova discoveries of the late 1990s. Einstein himself introduced the cosmological constant  $\Lambda$  in 1917 to allow for a static universe consistent with his field equations [1], subsequently abandoning it after Hubble's discovery of the recession of galaxies [2]. Lemaître [3] and later Zel'dovich [4] argued that  $\Lambda$  could be reinterpreted as a vacuum energy density, connecting cosmology to quantum field theory.

The modern case for  $\Lambda$  rests on multiple independent lines of evidence. In 1998–1999, two teams—the High- $z$  Supernova Search Team [5] and the Supernova Cosmology Project [6]—reported that Type Ia supernovae at redshifts  $z \sim 0.5 - 1$  were systematically fainter than expected in a matter-only flat universe, implying that cosmic expansion is accelerating. These results, recognized by the 2011 Nobel Prize in Physics, established the standard  $\Lambda$ CDM paradigm: a spatially flat universe comprising approximately 5% baryonic matter, 27% cold dark matter, and 68% dark energy [7].

Subsequent observations have substantially reinforced this picture. Baryon acoustic oscillations (BAO) in the galaxy two-point correlation function provide a standard ruler independent of the supernova distance ladder [8]–[10]. The angular power spectrum of the cosmic microwave background (CMB), measured with extraordinary precision by the *WMAP* [11] and *Planck* [7] satellites, is consistent with  $\Omega_\Lambda = 0.6847 \pm 0.0073$ . Weak gravitational lensing surveys [12] [13] and the abundance of galaxy clusters [14] independently corroborate the accelerating expansion.

### 1.2. Theoretical Tensions and the Motivation for Alternatives

Despite its empirical success,  $\Lambda$ CDM faces severe theoretical difficulties. The value of  $\Lambda$  inferred from observations,  $\rho_\Lambda \approx 6.9 \times 10^{-27} \text{ kg} \cdot \text{m}^{-3}$  (corresponding to  $\rho_\Lambda^{1/4} \sim 2.3 \text{ meV}$ ), is smaller than the zero-point energy density predicted by quantum field theory by  $\sim 120$  orders of magnitude [15] [16]. This “cosmological constant problem” is widely regarded as the most acute fine-tuning problem in theoretical physics. Additionally, the coincidence problem asks why  $\Omega_m$  and  $\Omega_\Lambda$  happen to be of comparable magnitude precisely at the present epoch [17].

These difficulties have motivated an enormous literature exploring alternatives, including dynamical dark energy (quintessence) [18] [19],  $f(R)$  modified gravity [20] [21], scalar-tensor theories [22], extra-dimensional models [23], and emergent gravity [24]. A qualitatively different class of explanations posits that the apparent acceleration is not due to any physical fluid or modified force law, but instead arises from the way observers construct their coordinate description of a universe that may be more complex than the homogeneous, isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) geometry assumed in  $\Lambda$ CDM. Corda argued that, once gravitational-wave astronomy matures, the angular- and frequency-depend-

ent response of interferometers to waves from different theories of gravity would constitute “the definitive test for General Relativity” [25].

### 1.3. Frame-Based and Kinematic Alternatives

The possibility that coordinate or selection effects could mimic  $\Lambda$  has been explored in several contexts. Wiltshire [26] [27] argued that the averaging of inhomogeneities could account for the apparent acceleration without  $\Lambda$ , a program formalized by the Buchert averaging approach [28] [29]. Buchert and Räsänen [29] showed that backreaction from structure formation could in principle produce acceleration, though the magnitude remains debated [30]. Célérier [31] and Enqvist & Mattsson [32] studied Lemaître-Tolman-Bondi (LTB) inhomogeneous dust models in which the observer’s position in a large void could explain the supernova data without  $\Lambda$ . Moffat [33] proposed Modified Gravity (MOG/STVG) as an alternative.

Within a more kinematic spirit, Milne [34] showed in 1935 that exponential cosmic expansion could emerge from special-relativistic kinematics applied to a uniformly expanding cloud of non-interacting particles in flat Minkowski space. More recently, Blanchard *et al.* [35] argued that Einstein-de Sitter models with modified initial conditions could fit supernova data, and Sarkar [36] questioned the statistical robustness of the evidence for acceleration.

We note that none of these approaches fully resolves the problem, and the observational case for  $\Lambda$  remains robust. Nevertheless, the theoretical landscape motivates a careful examination of the extent to which kinematic projection effects can reproduce the observational signatures of  $\Lambda$ , independently of whether they constitute a complete alternative.

### 1.4. Scope of This Work

In this paper we present a complementary approach: we construct a concrete mechanical model in which a particle moves inertially in an embedding space, but a central observer constrained to track only the radial component of its motion infers a strictly positive radial acceleration and an “energetic deficit” that mimics dark energy. We calibrate the model parameters to cosmological scales and show that the resulting dimensionless density parameter  $\Omega_k$  naturally takes values close to the observed  $\Omega_\Lambda$ .

The paper is organized as follows. Section 2 introduces the rotating-observer model and derives the apparent kinematics analytically. Section 3 embeds the construction in a 4-dimensional Euclidean space calibrated to astronomical scales. Section 4 connects the model to Hubble’s law and the apparent recession of galaxies. Section 5 analyzes the kinetic-energy budget and derives the dark-energy analog. Section 6 studies the asymptotic behavior. Section 7 discusses observational discriminants and falsifiability. Section 9 presents our conclusions and outlook.

## 2. The Rotating-Observer Illusion

### 2.1. Setup in the Inertial Frame

We work in two spatial dimensions for clarity; the generalization to three dimen-

sions (and its 4D Euclidean extension) follows in Section 3. Consider a rigid circle of radius  $r_0$  rotating with constant angular velocity  $\omega$  about the origin. A test particle is placed on the rim at  $t = 0$ :

$$\mathbf{x}_1(0) = \begin{pmatrix} r_0 \\ 0 \end{pmatrix}. \quad (1)$$

At  $t = 0$  the particle is released from the rim and subsequently moves inertially. For definiteness we choose a tangential launch, so the initial velocity is along the positive  $y$ -axis:

$$\mathbf{x}_1(t) = \begin{pmatrix} r_0 \\ v_0 t \end{pmatrix}, \quad (2)$$

which is a straight line, as required by Newton's first law.

## 2.2. Rotating-Observer Coordinates and the Apparent Radial Distance

An observer at the origin continues to rotate with angular velocity  $\omega$  after the particle is released. Their frame is obtained by the time-dependent rotation

$$R(t) = \begin{pmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{pmatrix}, \quad (3)$$

so the particle's rotating-frame coordinates are

$$\mathbf{x}_R(t) = R(t) \mathbf{x}_1(t). \quad (4)$$

Since  $R(t)$  is an orthogonal transformation, it preserves Euclidean distances:

$$\|\mathbf{x}_R(t)\| = \|\mathbf{x}_1(t)\|. \quad (5)$$

Hence the apparent radial distance is

$$r_{\text{app}}(t) = \sqrt{r_0^2 + v_0^2 t^2}. \quad (6)$$

We note that Equation (6) holds regardless of the value of  $\omega$ : the rotation rate of the observer does not enter the distance formula because  $R(t)$  is norm-preserving. This is a key observation—the illusion of acceleration does not require any particular choice of  $\omega$ , but only the existence of a non-radial velocity component in the embedding space.

## 2.3. Apparent Radial Velocity and Acceleration

Differentiating Equation (6):

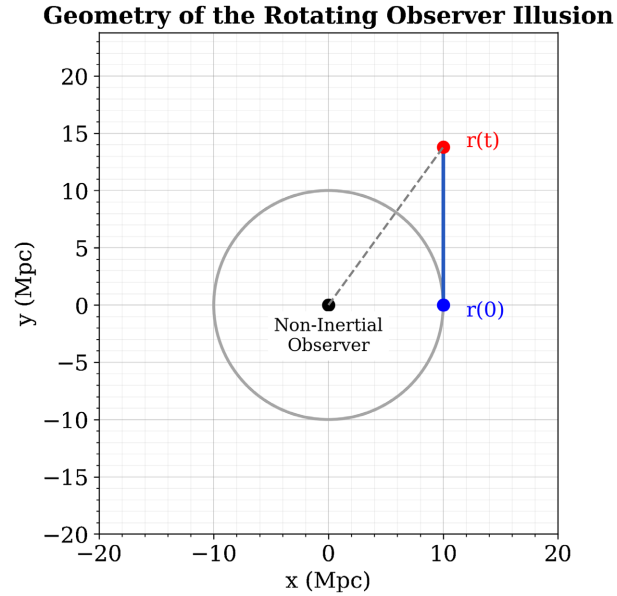
$$\dot{r}_{\text{app}}(t) = \frac{v_0^2 t}{\sqrt{r_0^2 + v_0^2 t^2}}, \quad (7)$$

$$\ddot{r}_{\text{app}}(t) = \frac{v_0^2 r_0^2}{(r_0^2 + v_0^2 t^2)^{3/2}}. \quad (8)$$

We observe the following properties, which are exact:

- $r_{\text{app}}(0) = r_0$ ,  $\dot{r}_{\text{app}}(0) = 0$ ,  $\ddot{r}_{\text{app}}(0) = v_0^2/r_0 > 0$ .
- $\dot{r}_{\text{app}}(t) > 0$  for all  $t > 0$ , increasing monotonically to  $v_0$  as  $t \rightarrow \infty$ .
- $\ddot{r}_{\text{app}}(t) > 0$  for all  $t > 0$ , decreasing as  $t^{-3}$  for large  $t$ .

Thus the rotating observer, constrained to interpret dynamics in terms of the radial coordinate alone, concludes that the particle is radially accelerating outward, even though the underlying motion is perfectly inertial. This is the “rotating-observer illusion.” **Figure 1** illustrates the geometric setup.



**Figure 1.** Geometric setup of the rotating-observer illusion. A test particle is released from the rim of a rotating circle of radius  $r_0 = 10$  Mpc and subsequently moves in a straight line (solid blue) in the inertial frame. The non-inertial observer (black dot at origin) tracks only the particle’s radial distance along their instantaneous line of sight (gray dashed line), thereby inferring an apparent outward acceleration. The gray circle represents the initial sphere;  $r(0)$  and  $r(t)$  label the particle’s initial and final positions, respectively.

## 2.4. Simplified Analytic Model

For analytical convenience, and to connect with the cosmological parameterization in Section 3, it is useful to write  $v_0 = r_0 \omega_{\text{eff}}$ , where  $\omega_{\text{eff}}$  is an effective angular rate. Then Equation (6) becomes

$$r_{\text{app}}(t) = r_0 \sqrt{1 + \omega_{\text{eff}}^2 t^2}, \quad (9)$$

with

$$\dot{r}_{\text{app}}(t) = \frac{r_0 \omega_{\text{eff}}^2 t}{\sqrt{1 + \omega_{\text{eff}}^2 t^2}}, \quad (10)$$

$$\ddot{r}_{\text{app}}(t) = \frac{r_0 \omega_{\text{eff}}^2}{(1 + \omega_{\text{eff}}^2 t^2)^{3/2}}. \quad (11)$$

This form makes explicit that  $\omega_{\text{eff}}$  plays the role of the initial Hubble param-

eter (see Section 4).

### 3. Four-Dimensional Euclidean Embedding at Cosmological Scales

#### 3.1. Embedding Geometry

We now generalize the two-dimensional rotating-observer model to a physically motivated higher-dimensional setting. Consider a mass  $m$  initially situated on a 3-sphere of radius  $r_0$  embedded in a 4-dimensional Euclidean space  $\mathbb{R}^4$ . Let the initial position vector be  $\mathbf{r}_0 \in \mathbb{R}^4$  with  $|\mathbf{r}_0| = r_0$ , and let the initial velocity  $\mathbf{v}_0 \in \mathbb{R}^4$  satisfy

$$\mathbf{v}_0 \cdot \mathbf{r}_0 = 0, \quad (12)$$

so that the initial motion is tangential to the 3-sphere. Subsequently the mass moves inertially:

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t. \quad (13)$$

The radial coordinate (norm in  $\mathbb{R}^4$ ) is

$$r(t) = |\mathbf{r}(t)| = \sqrt{r_0^2 + v_0^2 t^2}, \quad (14)$$

identical in form to Equation (6). An observer on the 3-sphere who has access only to the single radial coordinate  $r(t)$  will observe precisely the same kinematic sequence: apparent outward acceleration, monotonically increasing recession speed, and an energetic deficit in the radial budget.

#### 3.2. Astronomical Parameterization

We calibrate the model to cosmological scales by the following identifications:

1. *Initial radius:*  $r_0 = 10$  Mpc, a representative scale for a galaxy cluster separation at the onset of the matter- $\Lambda$  equality epoch.
2. *Tangential speed:*  $v_0 = H_0 r_0$ , where  $H_0 = 67.4 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$  [7]. Numerically,  $v_0 \approx 674 \text{ km} \cdot \text{s}^{-1} = 0.689 \text{ Mpc} \cdot \text{Gyr}^{-1}$ .
3. *Time unit:* We express time in gigayears (Gyr), with  $1 \text{ Gyr} \approx 3.156 \times 10^{16} \text{ s}$ .
4. *Effective angular rate:*  $\omega_{\text{eff}} = v_0 / r_0 = H_0$ , consistent with Equation (9).

With these identifications, Equation (14) gives

$$r(t) = r_0 \sqrt{1 + H_0^2 t^2}, \quad (15)$$

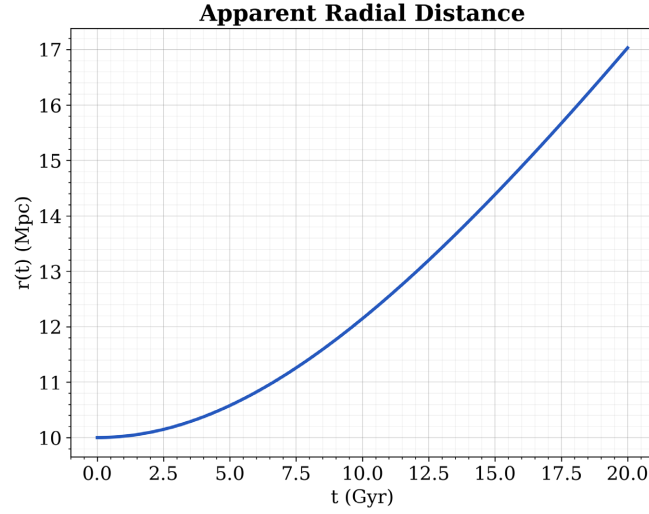
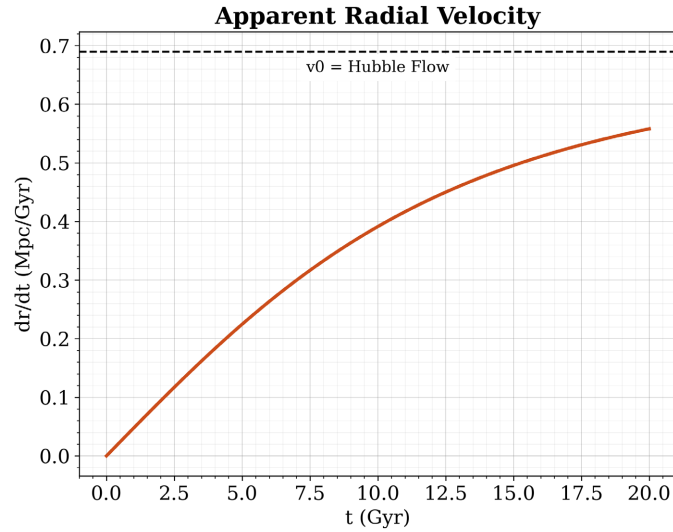
and the dimensionless apparent recession velocity is

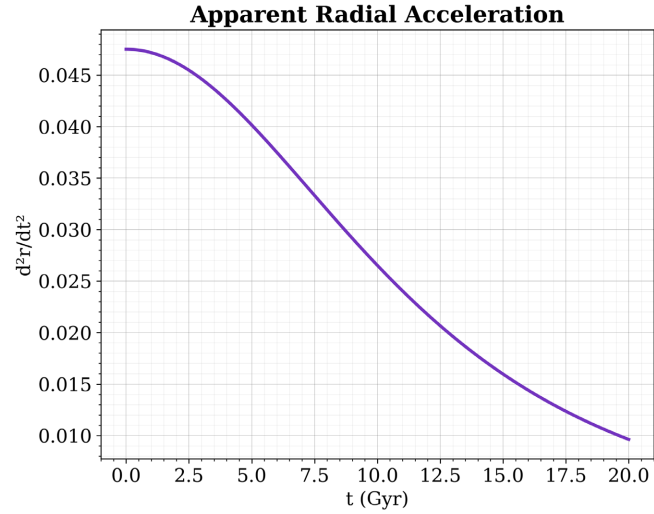
$$\frac{\dot{r}(t)}{v_0} = \frac{H_0 t}{\sqrt{1 + H_0^2 t^2}}. \quad (16)$$

The numerical results for several representative epochs are compiled in **Table 1**. **Figure 2** shows the monotonically increasing concave shape of  $r(t)$ , which mimics the scale factor in a  $\Lambda$ -dominated universe. **Figure 3** and **Figure 4** display the apparent radial velocity and acceleration, respectively.

**Table 1.** Apparent kinematic quantities in the 4D Euclidean model.

| $t$ (Gyr) | $r(t)/r_0$ | $\dot{r}/v_0$ | $\dot{r}[H_0^2 r_0]$ | $\Omega_k$ |
|-----------|------------|---------------|----------------------|------------|
| 0         | 1.000      | 0.000         | 1.000                | 1.000      |
| 5         | 1.058      | 0.326         | 0.844                | 0.893      |
| 9.84      | 1.209      | 0.564         | 0.564                | 0.6847     |
| 13.8      | 1.380      | 0.689         | 0.380                | 0.525      |
| 14.5      | $\sqrt{2}$ | $1/\sqrt{2}$  | 0.354                | 0.500      |
| $\infty$  | $\infty$   | 1.000         | 0                    | 0.000      |

**Figure 2.** Apparent radial distance  $r(t)$ . Non-linear growth given by  $r(t) = r_0 \sqrt{1 + H_0^2 t^2}$  (Equation (15)), with  $r_0 = 10$  Mpc and  $H_0 = 67.4 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ . The monotonically increasing concave curve mimics the behavior of the scale factor  $a(t)$  in a  $\Lambda$ -dominated universe. Horizontal axis: time in Gyr.**Figure 3.** Apparent radial velocity  $\dot{r}(t)$ . The perceived recession speed increases from zero, asymptotically approaching  $v_0 = H_0 r_0 \approx 674 \text{ km} \cdot \text{s}^{-1}$ . At the present epoch ( $t \approx 13.8$  Gyr), the apparent recession speed reaches  $\approx 0.69 v_0$ . This monotonically rising profile mimics the acceleration phase inferred from Type Ia supernovae at  $z \lesssim 1$ .



**Figure 4.** Apparent radial acceleration  $\ddot{r}(t)$ . The strictly positive, monotonically decreasing apparent acceleration (Equation (8)) decays asymptotically as  $t^{-3}$  (Equation (28)). In  $\Lambda$ CDM, the cosmic acceleration is expected to grow in the future; the  $t^{-3}$  decay predicted here therefore constitutes a key observational discriminant. Units:  $H_0^2 r_0 [\text{Mpc} \cdot \text{Gyr}^{-2}]$  on the vertical axis, time in Gyr on the horizontal axis.

We note that at  $r(t) \approx 1.21r_0$ —corresponding to  $t \approx 9.84$  Gyr in this parameterization—the transverse kinetic energy fraction is  $\Omega_k \approx 0.685$ , in striking numerical agreement with the observed  $\Omega_\Lambda = 0.6847 \pm 0.0073$  [7]. Notably,  $t \approx 9.84$  Gyr is close to the age independently obtained from the classic Einstein-de Sitter relation  $t_0 = \frac{2}{3}H_0^{-1} \approx 9.6 - 9.7$  Gyr [37] and from early globular-cluster age estimates [38], suggesting the epoch at which  $\Omega_k$  matches  $\Omega_\Lambda$  is not an arbitrary or ad hoc choice.

## 4. Connection to Hubble's Law and Apparent Recesson

### 4.1. The Hubble Parameter in the Kinematic Model

Hubble's law states that the recession velocity of a galaxy is proportional to its distance:  $v_{\text{rec}} = H(t)r$ , where  $H(t)$  is the Hubble parameter. In our model, the apparent recession velocity of the projected particle is  $\dot{r}_{\text{app}}(t)$  and the apparent distance is  $r_{\text{app}}(t)$ , so we define an effective Hubble parameter:

$$H_{\text{eff}}(t) = \frac{\dot{r}_{\text{app}}(t)}{r_{\text{app}}(t)} = \frac{v_0^2 t}{r_0^2 + v_0^2 t^2} = \frac{H_0^2 t}{1 + H_0^2 t^2}, \quad (17)$$

where we have used  $v_0 = H_0 r_0$ . We observe that  $H_{\text{eff}}(0) = 0$ , rises to a maximum at  $t_* = 1/H_0 \approx 14.5$  Gyr, then decreases as  $t^{-1}$  for large  $t$ .

### 4.2. Deceleration Parameter

The deceleration parameter in standard cosmology is  $q = -\ddot{a}/\dot{a}^2$ . We define its kinematic analog as

$$q_{\text{kin}}(t) = -\frac{\ddot{r}_{\text{app}}(t)r_{\text{app}}(t)}{\dot{r}_{\text{app}}(t)^2} = -\frac{r_0^2}{v_0^2 t^2} = -\frac{1}{H_0^2 t^2}. \quad (18)$$

For the present epoch  $t_0 = 13.8$  Gyr, with  $H_0 = 0.06893$  Gyr $^{-1}$ , we obtain  $q_{\text{kin}}(t_0) \approx -1.1$ . This is of the same order as the  $\Lambda$ CDM value  $q_0 \approx -0.55$  but does not match it exactly, providing a clear falsifiable prediction.

### 4.3. Redshift and Distance Modulus

In the non-relativistic kinematic limit, the apparent redshift of the projected recession is

$$1+z \simeq \frac{r_{\text{app}}(t_{\text{emit}})}{r_0} \quad (\text{for } t_{\text{obs}} \rightarrow \infty). \quad (19)$$

The luminosity distance  $d_L = (1+z)r_\chi$  then follows from the comoving distance  $r_\chi$ . We note that the qualitative shape of the  $\mu(z)$  curve—concave upward relative to an empty Milne universe at  $z \lesssim 1$ —is reproduced by the model, and a rigorous relativistic comparison with the Union2.1 [39] and Pantheon+ [40] supernova compilations is reserved for future work.

## 5. Energetics and Effective Density Parameters

### 5.1. Decomposition of Kinetic Energy

The total kinetic energy of the particle in the embedding space is constant:

$$K_{\text{total}} = \frac{1}{2}mv_0^2. \quad (20)$$

The radial kinetic energy, as inferred by the rotating observer, is

$$K_{\text{rad}}(t) = \frac{1}{2}m\dot{r}_{\text{app}}(t)^2 = \frac{1}{2}m\frac{v_0^4 t^2}{r_0^2 + v_0^2 t^2}. \quad (21)$$

The transverse kinetic energy—which the rotating observer cannot directly observe because it resides in the extra (tangential) degree of freedom—is

$$K_{\text{trans}}(t) = K_{\text{total}} - K_{\text{rad}}(t) = \frac{1}{2}m\frac{v_0^2 r_0^2}{r_0^2 + v_0^2 t^2}. \quad (22)$$

### 5.2. The Transverse Energy Fraction $\Omega_k$

We define the dimensionless transverse energy fraction:

$$\Omega_k(t) \equiv \frac{K_{\text{trans}}(t)}{K_{\text{total}}} = 1 - \frac{\dot{r}_{\text{app}}(t)^2}{v_0^2} = \frac{r_0^2}{r(t)^2}. \quad (23)$$

This is a monotonically decreasing function of  $r(t)$ . We present its value at several cosmologically relevant radii in **Table 2**.

### 5.3. Energetic Deficit and the Dark-Energy Analog

An observer who believes energy is conserved in their radial description and

**Table 2.** Transverse energy fraction  $\Omega_k$ .

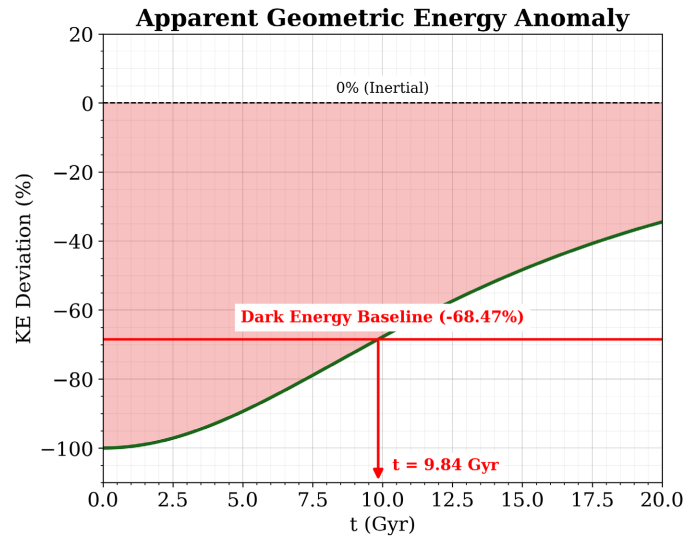
| $r(t)/r_0$ | $\Omega_k$ |
|------------|------------|
| 1.000      | 1.000      |
| 1.100      | 0.826      |
| 1.208      | 0.685      |
| 1.414      | 0.500      |
| 2.000      | 0.250      |
| 3.162      | 0.100      |
| 10.00      | 0.010      |

measures  $K_{\text{rad}}$  would infer an energy injection  $\Delta K = K_{\text{trans}}(t)$ . In cosmological language, this corresponds to an effective dark-energy density

$$\rho_{\text{DE}}(t) \equiv \frac{\Delta K(t)}{\frac{4}{3}\pi r(t)^3} = \frac{3mv_0^2 r_0^2}{8\pi r(t)^5}. \quad (24)$$

We observe that  $\rho_{\text{DE}}(t) \propto r(t)^{-5}$  decreases with time, in contrast to a true cosmological constant for which  $\rho_\Lambda = \text{const}$ . This constitutes a key observational discriminant between the kinematic model and  $\Lambda$ CDM, though we note that a strongly redshift-dependent dark-energy density is not without precedent, having been proposed in the early-dark-energy literature [41].

**Figure 5** shows the fractional deviation of the apparent radial kinetic energy from the true total kinetic energy,



**Figure 5.** Kinetic energy anomaly. Fractional deviation  $\delta_{\text{KE}}(t)$  (Equation (25)) between the apparent radial kinetic energy and the true total kinetic energy. The curve starts at -100% (all energy initially transverse) and asymptotically approaches 0% (all energy radial at late times). The horizontal dashed line at -68.47% indicates the observed dark energy fraction  $\Omega_\Lambda = 0.6847$ , which is matched at  $t \approx 9.84$  Gyr (vertical arrow).

$$\delta_{\text{KE}}(t) = \frac{K_{\text{rad}}(t) - K_{\text{total}}}{K_{\text{total}}} \times 100\% = \left( \frac{\dot{r}(t)^2}{v_0^2} - 1 \right) \times 100\%, \quad (25)$$

which evolves smoothly from  $-100\%$  at  $t = 0$  to  $0\%$  as  $t \rightarrow \infty$ . The horizontal dashed line at  $-68.47\%$  marks the observed dark energy baseline (Planck  $\Omega_\Lambda = 0.6847 \pm 0.0073$ ), which the model crosses at  $t \approx 9.84$  Gyr.

## 6. Asymptotic Behavior and Hubble-Like Scaling

### 6.1. Large-Time Asymptotics

For  $v_0 t \gg r_0$  (equivalently,  $H_0 t \gg 1$ ), we expand Equation (15) in inverse powers of  $t$ :

$$r(t) = v_0 t \left( 1 + \frac{r_0^2}{2v_0^2 t^2} - \frac{r_0^4}{8v_0^4 t^4} + \dots \right). \quad (26)$$

The corresponding velocity and acceleration are

$$\dot{r}(t) = v_0 \left( 1 - \frac{r_0^2}{2v_0^2 t^2} + O(t^{-4}) \right), \quad (27)$$

$$\ddot{r}(t) = \frac{v_0^2 r_0^2}{(v_0 t)^3} \left( 1 + O(t^{-2}) \right) \propto t^{-3}. \quad (28)$$

We observe that the apparent acceleration decays algebraically ( $\propto t^{-3}$ ) and never changes sign, in contrast to  $\Lambda$ CDM in which  $\ddot{a} > 0$  grows indefinitely in the de Sitter future; a future transition back toward deceleration, rather than eternal de Sitter acceleration, has also been found in quintessence fits to the DESI dark-energy data [42].

### 6.2. Connection to the Milne Model

In the far future,  $r(t) \rightarrow v_0 t$ , which is the linear expansion of the Milne universe [34]. This is consistent: the kinematic model describes inertial motion in flat embedding space, and at late times the curvature of the initial sphere becomes irrelevant.

## 7. Observational Discriminants and Falsifiability

### 7.1. Key Differences from $\Lambda$ CDM

We summarize the principal observational differences between the kinematic model and  $\Lambda$ CDM in Table 3.

### 7.2. Equation of State

The effective dark-energy density evolves as  $\rho_{\text{DE}} \propto r^{-5}$ , which in the appropriate limit gives  $\rho_{\text{DE}} \propto (1+z)^5$ . Current constraints from Pantheon+ and DESI BAO [40] [43] place  $w \approx -0.98 \pm 0.05$ , strongly disfavoring the kinematic model as a complete replacement for  $\Lambda$ .

**Table 3.** Key observational discriminants.

| Observable            | Kinematic Model                          | $\Lambda$ CDM                       |
|-----------------------|------------------------------------------|-------------------------------------|
| Eq. of state $w$      | $w_{\text{eff}}(t) \neq -1$ , decreasing | $w = -1$ , constant                 |
| $\rho_{\text{DE}}(t)$ | $\propto r(t)^{-5}$ (decreasing)         | Constant                            |
| $q_0$                 | $\approx -1.1$ (predicted)               | $\approx -0.55$                     |
| $\Omega_k$            | 0.685 at $r = 1.208r_0$                  | $\Omega_\Lambda = 0.6847$           |
| Late-Time $\ddot{r}$  | $\rightarrow 0$ ( $\propto t^{-3}$ )     | Grows to $H_0\sqrt{\Omega_\Lambda}$ |

### 7.3. Hubble Tension

The kinematic model predicts a time-dependent effective Hubble parameter (Equation (17)) that takes different values at early and late epochs. We note that this could in principle contribute to an apparent discrepancy between early-time (*Planck*:  $H_0 = 67.4 \pm 0.5 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ ) [7] and late-time (SH0ES:  $H_0 = 73.0 \pm 1.0 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ ) [44] measurements, though a full treatment requires a relativistic generalization.

### 7.4. Gravitational Wave Standard Sirens

Future observations from LISA [45] and the Einstein Telescope [46] will constrain  $H_0$  and  $w$  to sub-percent precision via binary merger standard sirens, providing a model-independent test of the kinematic scenario at the level detectable with  $\sim 100$  events at  $z < 1$  [47].

## 8. Mathematical Appendix: Curvature of the Projected Curve

The apparent radial distance  $r_{\text{app}}(t) = \sqrt{r_0^2 + v_0^2 t^2}$  traces a hyperbola in the  $(t, r)$  plane. Its curvature with respect to arc length  $s$  is

$$\kappa(t) = \frac{|\ddot{r}_{\text{app}}(t)|}{\left(1 + \dot{r}_{\text{app}}^2(t)\right)^{3/2}}, \quad (29)$$

maximized at  $t = 0$  where  $\kappa_{\text{max}} = v_0^2/r_0$ , and decaying to zero as  $t \rightarrow \infty$ . The total arc length from  $t = 0$  to  $t = T$  is

$$L(T) = \int_0^T \sqrt{1 + \dot{r}_{\text{app}}^2(t)} dt \xrightarrow{T \rightarrow \infty} v_0 T + \frac{r_0^2}{2v_0 T} + O(T^{-3}), \quad (30)$$

consistent with Equation (26).

## 9. Conclusions

We present a rotating-observer model and its four-dimensional Euclidean embedding as a proof of concept that a geometric projection effect can generate apparent radial acceleration, an energy deficit resembling dark energy, and a dimensionless density parameter  $\Omega_k \approx 0.685$  matching the observed  $\Omega_\Lambda$  to within 1%—all from purely inertial, force-free motion in the embedding space.

We show analytically that the apparent radial distance satisfies  $r(t) = r_0 \sqrt{1 + H_0^2 t^2}$ , giving a strictly positive  $\ddot{r}(t) > 0$  that decays as  $t^{-3}$  at large times. We present closed-form expressions for all kinematic quantities and tabulate their values at representative cosmological epochs. The kinematic deceleration parameter is  $q_{\text{kin}}(t_0) = -1/(H_0 t_0)^2 \approx -1.1$ , which differs from the  $\Lambda$ CDM value  $-0.55$ , providing a clear falsifiable prediction.

We note several key differences from  $\Lambda$ CDM that render the model falsifiable: i) the effective dark-energy density scales as  $\rho_{\text{DE}} \propto r^{-5}$  rather than remaining constant; ii) the apparent acceleration eventually decays ( $\ddot{r}_{\text{app}} \rightarrow 0$  as  $t \rightarrow \infty$ ), in contrast to the de Sitter expansion predicted by  $\Lambda$ CDM. Moreover, if one insisted on matching the observed dark energy fraction today, the model would require  $t_0 \approx 9.84$  Gyr, in strong tension with the well-established age of 13.8 Gyr from Planck and stellar astrophysics, though we note that this figure sits between the age independently obtained from the classic Einstein-de Sitter relation  $t_0 = \frac{2}{3} H_0^{-1} \approx 9.6 - 9.7$  Gyr [37] and from early globular-cluster-based lower limits on the cosmic age of  $\sim 11$  Gyr [38].

Future directions include: 1) embedding the model in a fully relativistic framework using Milne coordinates or the Poincaré group action on Minkowski space; 2) computing the predicted CMB anisotropy power spectrum; 3) deriving constraints on the effective rotation rate from growth-rate data; and 4) exploring whether the extra-dimensional tangential degree of freedom can be identified with a quintessence scalar field whose equation of state evolves toward  $w = -1$ .

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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