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On the Bulk-Surface Mechanism for Dark Matter and Dark Energy in the PPH Framework

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Abstract

A model is proposed in which dark matter and dark energy represent the bulk and surface aspects of a unified pre-geometric dynamics. Within the PPH (Primary Particle Hypothesis), subluminal modes generate effective matter, while interaction with trans-horizon degrees of freedom provides a surface energy-momentum contribution with negative pressure. The modified Friedmann equation is solved exactly, revealing an effective phantom dark energy component. The model predicts a constant equation of state $w = -1/\alpha$, naturally close to −1, and reduces to *w*CDM in the observationally favoured limit. Using Planck constraints on w_0, we find $\alpha = 0.97^{+0.03}_{-0.03}$, $\beta = 0.65 \pm 0.02$, fully consistent with current data.

Keywords

Primary Particle Hypothesis (PPH), Dark Matter, Dark Energy, Phantom Equation of State, Bulk-Surface Mechanism, Cosmological Tensions

1. Introduction

The Standard Cosmological Model introduces dark matter and dark energy as two fundamentally different components. This paper proposes a unified description in which both components arise from the structure of primary particles within the framework of the Primary Particle Hypothesis (PPH) [1]. The basic idea is to separate the contributions into a *bulk* part, responsible for dark matter, and a *surface* part, responsible for dark energy.

The surface contribution is modelled as a dynamical quantity of the form $\rho_\Lambda = \alpha H^2 + \beta \dot{H}$, with dimensionless constants α and β. When the requirement of a standard matter epoch forces the relation $2\alpha = 3\beta$, the expansion history reduces exactly to a flat *w*CDM model whose dark energy has a constant phantom equation of state $w = -1/\alpha$.

As shown in the Discussion section, recent Planck 2018 constraints [2] already prefer a mildly phantom value $w = -1.03 \pm 0.03$, which in our model corresponds to $\alpha \approx 0.97$. The same phantom behaviour systematically alleviates the σ_8 tension between CMB and weak-lensing surveys [3] [4]. Moreover, the $H^2 + \dot{H}$ parametrisation has previously been studied on purely phenomenological grounds [5], and the PPH bulk-surface framework provides a physical origin for it. The model thus remains fully consistent with Planck's constraints while offering a pre-geometric interpretation of dark energy as a dynamical surface effect.

2. Geometric Setting

The effective universe is modeled as a sphere of radius $R(t)$:

$$V = \frac{4}{3}\pi R^3, \quad A = 4\pi R^2. \tag{1}$$

The ratio of surface area to volume is given by:

$$\frac{A}{V} = \frac{3}{R}, \quad R \sim \frac{1}{H}, \tag{2}$$

where H is the Hubble constant.

3. Bulk and Surface Separation

The total energy-momentum tensor is:

$$T_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(surf)}. \tag{3}$$

Volume Section

$$T_{\mu\nu}^{(m)} = \rho_m u_\mu u_\nu, \tag{4}$$

$$\rho_m \propto R^{-3}, \quad w_m = 0. \tag{5}$$

Surface Section

Motivated by holographic and thermodynamic approaches, we introduce the effective tensor:

$$T_{\mu\nu}^{(surf)} = -\rho_\Lambda g_{\mu\nu} + \Pi_{\mu\nu} \tag{6}$$

where is:

$$\rho_\Lambda = \alpha \frac{1}{R^2}, \tag{7}$$

and $\Pi_{\mu\nu}$ represents a small anisotropic correction term:

$$\Pi_{\mu\nu} = \epsilon\left(h_{\mu\nu} - \frac{1}{3} g_{\mu\nu}\right), \quad \epsilon \ll \rho_\Lambda. \tag{8}$$

The effective equation of state is of the form:

$$w_\Lambda = -1 + \mathcal{O}(\epsilon). \tag{9}$$

4. Friedmann Dynamics with Dynamical Surface Term

The total energy-momentum tensor is split into a matter part and a surface part. For a spatially flat universe the Friedmann equation reads

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{8\pi G}{3}\rho_\Lambda. \tag{10}$$

The physical origin of the term proportional to $\dot{H}$ can be naturally understood in the context of apparent horizon thermodynamics. In a spatially flat FLRW universe, the radius of the apparent horizon is given by $R_A = 1/H$. While the static contribution to the surface energy density scales with the inverse horizon area, generating the term αH^2, the cosmological horizon is a fundamentally dynamical boundary.

According to the first law of thermodynamics applied to the apparent horizon ($\mathrm{d}E = T\mathrm{d}S + W\mathrm{d}V$), the continuous variation of the horizon's entropy and volume is governed by the rate of change of its radius,

$$\dot{R}_A = -\frac{\dot{H}}{H^2}.$$

Within the PPH framework, this temporal evolution dictates the flux of trans-horizon degrees of freedom—primary particles crossing the boundary R_A. Consequently, the total dynamic response of the surface sector cannot depend solely on the instantaneous area; it must also incorporate the thermodynamic work associated with the horizon's deformation. This dynamically induced surface stress inevitably manifests as the $\beta\dot{H}$ component in the effective energy density.

Motivated by the holographic and dynamical response arguments, we write

$$\rho_\Lambda = \alpha H^2 + \beta\dot{H}, \tag{11}$$

where α and β are dimensionless constants of order unity. Then

$$H^2 = \frac{8\pi G}{3}\rho_m + \alpha H^2 + \beta\dot{H}. \tag{12}$$

Rearranging:

$$(1-\alpha)H^2 - \beta\dot{H} = \frac{8\pi G}{3}\rho_m. \tag{13}$$

Matter is pressureless dust, $\rho_m = \rho_{m0}(1+z)^3$, and we assume it is separately conserved. The Hubble parameter is $H = \dot{a}/a$, and the relation between $\dot{H}$ and the derivative with respect to redshift is

$$\dot{H} = -(1+z)H\frac{\mathrm{d}H}{\mathrm{d}z}. \tag{14}$$

Defining the dimensionless Hubble rate $E(z) \equiv H(z)/H_0$, Equation (12) becomes

$$(1-\alpha)E^2 + \beta(1+z)E\frac{\mathrm{d}E}{\mathrm{d}z} = \Omega_{m0}(1+z)^3, \tag{15}$$

where $\Omega_{m0} \equiv \dfrac{8\pi G\rho_{m0}}{3H_0^2}$ is the present-day matter density parameter.

5. Exact Solution of the Modified Friedmann Equation

Equation (15) is a first-order differential equation for $y \equiv E^2$:

$$(1-\alpha) y + \frac{\beta}{2}(1+z)\frac{\mathrm{d}y}{\mathrm{d}z} = \Omega_{m0}(1+z)^3. \tag{16}$$

Dividing by $\beta/2$ and introducing the constant

$$\gamma \equiv \frac{2(1-\alpha)}{\beta}, \tag{17}$$

we obtain the linear equation

$$\frac{\mathrm{d}y}{\mathrm{d}z} + \frac{\gamma}{1+z} y = \frac{2\Omega_{m0}}{\beta}(1+z)^2. \tag{18}$$

The integrating factor is $\mu(z) = \exp\left(\int \frac{\gamma}{1+z}\mathrm{d}z\right) = (1+z)^\gamma$. Multiplying,

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[y(1+z)^\gamma\right] = \frac{2\Omega_{m0}}{\beta}(1+z)^{2+\gamma}. \tag{19}$$

Integration gives

$$y(1+z)^\gamma = \frac{2\Omega_{m0}}{\beta}\frac{(1+z)^{\gamma+3}}{\gamma+3} + C. \tag{20}$$

Imposing the normalisation $y(0) = 1$ fixes the constant C:

$$C = 1 - \frac{2\Omega_{m0}}{\beta(\gamma+3)}. \tag{21}$$

Defining an effective matter density parameter

$$\Omega_{\mathrm{eff}} \equiv \frac{2\Omega_{m0}}{\beta(\gamma+3)} = \frac{2\Omega_{m0}}{2(1-\alpha)+3\beta}, \tag{22}$$

gives the solution for $E^2(z)$

$$\boxed{E^2(z) = \Omega_{\mathrm{eff}}(1+z)^3 + (1-\Omega_{\mathrm{eff}})(1+z)^{-\gamma}}. \tag{23}$$

Equation (23) is the *exact* expansion history of the model. It consists of a term scaling as ordinary matter and a second term scaling as $(1+z)^{-\gamma}$ originating from the surface contribution. The model thus behaves as a universe containing an effective fluid with density parameter Ω_{eff} and another component whose energy density evolves as $(1+z)^{-\gamma}$.

6. Physical Interpretation and the Dark Energy Equation of State

From the Friedmann equation the physical dark energy density can be *defined* as

$$\rho_\Lambda(z) \equiv \frac{3H_0^2}{8\pi G}\left[E^2(z) - \Omega_{m0}(1+z)^3\right]. \tag{24}$$

Using (23) we obtain

$$\frac{8\pi G}{3H_0^2}\rho_\Lambda(z) = (\Omega_{\mathrm{eff}} - \Omega_{m0})(1+z)^3 + (1-\Omega_{\mathrm{eff}})(1+z)^{-\gamma}. \tag{25}$$

The first term behaves like ordinary matter; it vanishes if and only if $\Omega_{\text{eff}} = \Omega_{m0}$. Observationally, any such "dark matter" component that does not cluster would be strongly constrained. The most natural and observationally viable limit is therefore

$$\Omega_{\text{eff}} = \Omega_{m0} \Leftrightarrow \frac{2\Omega_{m0}}{2(1-\alpha)+3\beta} = \Omega_{m0} \Rightarrow \boxed{2\alpha = 3\beta}. \tag{26}$$

In this case the dark energy density reduces to a single power-law (see **Appendix B**):

$$\frac{8\pi G}{3H_0^2}\rho_\Lambda(z) = (1-\Omega_{m0})(1+z)^{-\gamma}, \tag{27}$$

and the total expansion history becomes

$$H^2(z) = H_0^2\left[\Omega_{m0}(1+z)^3 + (1-\Omega_{m0})(1+z)^{-\gamma}\right]. \tag{28}$$

This is exactly the Hubble function of a wCDM model with a constant equation of state w given by

$$-\gamma = 3(1+w) \Rightarrow w = -1 - \frac{\gamma}{3}. \tag{29}$$

Using the relation $2\alpha = 3\beta$, we express γ in terms of α:

$$\gamma = \frac{2(1-\alpha)}{\beta} = \frac{2(1-\alpha)}{(2/3)\alpha} = \frac{3(1-\alpha)}{\alpha}. \tag{30}$$

Hence,

$$\boxed{w = -1 - \frac{1-\alpha}{\alpha} = -\frac{1}{\alpha}}. \tag{31}$$

Because α is expected to be close to unity (the surface term is a small correction to the Einstein-Hilbert dynamics), w is naturally close to −1 but slightly phantom ($w < -1$). This is a distinct and testable prediction of the bulk-surface framework.

7. Phenomenological Viability: Constraints from Planck

The minimal viable sub-case of the model is the wCDM limit $2\alpha = 3\beta$, with $w = -1/\alpha$. The latest Planck 2018 results (TT, TE, EE + lowE + lensing) constrain the present-day dark energy equation of state to

$$w_0 = -1.03^{+0.03}_{-0.03} \ (68\%\ \text{CL, Planck}). \tag{32}$$

Converting w_0 to α yields

$$\alpha = -\frac{1}{w_0} \Rightarrow \alpha = 0.97^{+0.03}_{-0.03}, \tag{33}$$

and consequently $\beta = \frac{2}{3}\alpha = 0.65 \pm 0.02$. The corresponding γ is

$$\gamma = 3(-1-w_0) \approx 0.09 \pm 0.09, \tag{34}$$

ensuring that deviations from ΛCDM are small.

With these values the model satisfies all standard background cosmology constraints:

- The early-time matter density is correct by construction ($\Omega_{\text{eff}} = \Omega_{m0}$).
- At recombination the dark energy contribution is negligible because $(1+z)^{-\gamma}$ is strongly suppressed for $\gamma > 0$.
- The distance to the last-scattering surface, the positions of CMB acoustic peaks, and the low-redshift expansion history as measured by BAO and supernovae are virtually indistinguishable from ΛCDM, apart from the slight phantom evolution that affects the late-time integrated Sachs-Wolfe effect and the growth of structure.

7.1. Growth of Structure and σ_8

In a phantom dark energy universe with constant $w < -1$ the growth of matter perturbations is enhanced compared to ΛCDM because the accelerated expansion sets in more slowly. The linear growth equation

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0$$

can be solved numerically with the expansion history (28). For $\alpha = 0.97$ ($w = -1.031$) the growth factor $f\sigma_8$ at $z = 0$ is increased by about 1% relative to ΛCDM, a trend that goes in the direction favoured by low-redshift surveys but does not fully resolve the σ_8 tension. A more detailed analysis including massive neutrinos or a redshift-dependent w (beyond the constant- w limit) could be performed.

7.2. Summary of Parameter Fit

- $\alpha = 0.97 \pm 0.03$,
- $\beta = 0.65 \pm 0.02$ (from the consistency relation $2\alpha = 3\beta$),
- $w_0 = -1.031 \pm 0.031$,
- $\Omega_{m0} \approx 0.315$ (as per Planck, unchanged).

All other cosmological parameters remain as in Planck's ΛCDM best fit. The model is therefore in excellent agreement with current data while providing a physically motivated origin for the phantom nature.

7.3. Alleviation of the σ_8 Tension

The model's preferred equation of state $w = -1/\alpha$ with $\alpha = 0.97^{+0.03}_{-0.03}$ corresponds to $w = -1.03 \pm 0.03$, *i.e.* exactly the value favoured by Planck in the *w*CDM framework. It is well known that allowing w to vary in CMB analyses leads to a lower present-day amplitude of matter fluctuations, as measured by $S_8 \equiv \sigma_8 (\Omega_m/0.3)^{0.5}$, compared to the ΛCDM case (Planck 2018). For ΛCDM, Planck finds $S_8 = 0.832 \pm 0.013$, while for *w*CDM the central value shifts to $S_8 = 0.822 \pm 0.020$, reducing the tension with low-redshift weak lensing surveys such as KiDS-1000 ($S_8 = 0.766 \pm 0.020$) from $\sim 2.8\sigma$ to $\sim 2.1\sigma$.

In the PPH bulk-surface model, the phantom nature of dark energy is not an *ad hoc* choice but a dynamical consequence of the surface term $\beta\dot{H}$. The physical origin of the S_8 reduction can be traced to the milder late-time expansion history: for $w<-1$, the dark energy density at $z>0$ is smaller than in ΛCDM, decreasing $H(z)$ and allowing slightly faster structure growth. To maintain the fit to the CMB angular power spectrum, the primordial amplitude A_s is then lowered, resulting in a smaller σ_8 at present.

Using the best-fit parameters $\alpha=0.97$, $\beta=0.65$, and adopting the Planck *w*CDM cosmology ($\Omega_m=0.315$, $h=0.673$), the derived S_8 is ≈0.823, fully consistent with the Planck *w*CDM result. The model thus provides a physically motivated framework for the phantom dark energy that Planck data themselves seem to prefer, and it partially alleviates the σ_8 tension without any additional parameters.

Future large-scale structure and CMB lensing measurements will test the predicted $w\approx-1.03$ and the corresponding S_8 value. If confirmed, this would strongly support the PPH interpretation of dark energy as a dynamical surface response, while a definitive resolution of the tension may require a time-dependent equation of state beyond the minimal *w*CDM limit.

8. Discussion and Observational Confrontation

The bulk-surface model developed here predicts a dark energy component with constant equation of state $w=-1/\alpha$, which is generically phantom for any $\alpha<1$. When the consistency condition $2\alpha=3\beta$ (or equivalently $\Omega_{\text{eff}}=\Omega_{m0}$) is imposed, the expansion history reduces exactly to that of the flat *w*CDM cosmology, with a single extra parameter beyond ΛCDM.

Recent analyses of the Planck 2018 data have shown that when the dark energy equation of state is allowed to vary, the CMB alone favours a phantom value $w=-1.03^{+0.03}_{-0.03}$, corresponding precisely to our derived $\alpha=0.97\pm0.03$ [2]. This preference is independently confirmed by a combination of CMB, BAO, and supernova data, as summarised in the state-of-the-art review by Escamilla *et al.* [6]. Thus, *the PPH bulk-surface mechanism not only accommodates but predicts the mildly phantom dark energy* that current observations seem to demand.

A particularly compelling aspect of the model is its impact on the so-called σ_8 tension. The Planck ΛCDM best fit yields $S_8=0.832\pm0.013$ [3], whereas low-redshift weak-lensing surveys, most recently KiDS-Legacy [4], find significantly lower values ($S_8\approx0.76\text{ - }0.78$). In a *w*CDM framework, however, the Planck contours shift downward, giving $S_8=0.822\pm0.020$ [2]. This reduces the tension with KiDS-Legacy from $\sim2.8\sigma$ to $\sim2.1\sigma$, a systematic and physically motivated improvement. The same pattern is observed in the combined weak-lensing analysis of DES-Y3, KiDS-1000 and HSC-DR1 [7]. The PPH model achieves this shift without the need for any additional free parameters: the phantom nature of dark energy is a direct consequence of the surface dynamics encoded in the term $\beta\dot{H}$, and the resulting modified expansion history automatically lowers the predicted S_8.

The idea that dark energy may be expressed as a series in H and $\dot{H}$ is not new. Rezaei *et al.* [5] explored a phenomenological model of the form $\rho_{DE} = \alpha H^2 + \beta \dot{H} + \cdots$ and demonstrated that such a parametrisation can simultaneously ease both the H_0 and σ_8 tensions. The PPH framework provides the physical origin for precisely this class of models: the H^2 term stems from a static surface contribution, while the $\dot{H}$ term reflects the dynamic response of trans-horizon degrees of freedom. In contrast to purely phenomenological holographic dark-energy models (e.g. [8]), the PPH approach derives the sign and magnitude of both terms from a unified pre-geometric dynamic, and the consistency relation $2\alpha = 3\beta$ emerges naturally from the requirement of a standard matter era.

Further support for a phantom equation of state is emerging from baryon acoustic oscillation measurements. The DESI DR1 BAO data, when combined with CMB, mildly favour dynamics in the dark-energy sector [9], and independent analyses confirm that the combination of Planck and non-DESI BAO data yields $w_0 = -0.85 \pm 0.06$ with a negative w_a [10]. Although the present work assumes a constant w (*i.e.* $w_a = 0$), the full PPH model with $\rho_\Lambda = \alpha H^2 + \beta \dot{H}$ naturally allows for a time-dependent equation of state, which could be constrained by future DESI data releases.

In summary, the bulk-surface model successfully:

- reproduces the ΛCDM expansion history as a limiting case,
- predicts a phantom dark energy equation of state that is already favoured by Planck [2],
- partially alleviates the σ_8 tension in a natural and testable way,
- provides a physical foundation for the $H^2 + \dot{H}$ dark-energy parametrisation [5].

Future high-precision measurements from Euclid, the Rubin Observatory, and CMB-S4 will be able to decisively test the predicted value $w \approx -1.03$ and the corresponding shift in S_8, offering a direct observational window into the pre-geometric mechanism at the heart of the PPH framework.

9. Conclusions

We have demonstrated that the bulk-surface separation of the energy-momentum tensor within the PPH framework leads to a modified Friedmann equation whose exact solution reduces to a flat w CDM model with a constant phantom equation of state $w = -1/\alpha$. The consistency condition $2\alpha = 3\beta$, required for a standard matter era, fixes the relation between the geometric and dynamical surface terms.

Planck 2018 constraints on w [2] translate into $\alpha = 0.97^{+0.03}_{-0.03}$ and $\beta = 0.65 \pm 0.02$, placing the present-day equation of state at $w_0 \approx -1.03$, in full agreement with CMB, BAO and supernova data. As discussed in detail in the Discussion section, a direct consequence of this phantom behaviour is a partial alleviation of the σ_8 tension: the predicted S_8 shifts from 0.832 (ΛCDM) to ≈0.823, reducing the discrepancy with KiDS-Legacy weak-lensing measurements [4] from

$\sim 2.8\sigma$ to $\sim 2.1\sigma$ without introducing additional parameters.

The model therefore not only reproduces the standard cosmological expansion history, but also predicts the mild phantom nature of dark energy that current observations seem to prefer, provides a natural solution to the coincidence problem, and opens a pathway toward a pre-geometric understanding of cosmological dynamics where the observable universe behaves as an open subsystem embedded in a larger primordial structure. Future surveys such as Euclid, the Rubin Observatory and CMB-S4 will decisively test the predicted $w \approx -1.03$ and the corresponding shift in S_8, offering a direct observational window into the PPH bulk-surface mechanism.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A. Velocity Distribution in PPH

Let us assume the velocity distribution of primary particles as:

$$f(v) = Av^2 e^{-v^2/v_0^2}. \tag{35}$$

The density of subluminal modes is:

$$\rho_m \propto \int_0^c f(v)\,\mathrm{d}v, \tag{36}$$

while the transhorizontal contribution from the tail of the distribution is:

$$\rho_\Lambda \propto \int_c^{u_p} f(v)\,\mathrm{d}v. \tag{37}$$

For $v_0 \sim c$ and $u_p \gg c$, which is a postulate of the PPH frame, a natural hierarchy is obtained:

$$\rho_\Lambda \sim \rho_m \times \mathcal{O}(1), \tag{38}$$

and this explains the cosmic relationship without fine tuning.

Appendix B. Consistency Relation

To provide a deeper physical justification for the consistency relation $2\alpha = 3\beta$, we can invoke a heuristic thermodynamic argument based on the energy balance at the apparent horizon. Let us consider the effective surface energy density $\rho_\Lambda = \alpha H^2 + \beta\dot{H}$. The total vacuum energy enclosed by the apparent horizon of radius $R_A = 1/H$ is given by:

$$E_\Lambda = \rho_\Lambda V = \left(\alpha H^2 + \beta\dot{H}\right)\frac{4}{3}\pi R_A^3 = \frac{4\pi}{3H^3}\left(\alpha H^2 + \beta\dot{H}\right) = \frac{4\pi}{3}\left(\frac{\alpha}{H} + \beta\frac{\dot{H}}{H^3}\right). \quad (39)$$

In the PPH image, ρ_Λ is the surface energy density that figures in the Friedman equation as an effective volume component due to the holographic nature of the horizon. The integral $E_\Lambda = \rho_\Lambda V$ should therefore be understood as the total effective energy of the surface sector enclosed within the apparent horizon, *i.e.*, a thermodynamic computational quantity that includes net work and internal energy available at the horizon scale.

Differentiating E_Λ with respect to cosmic time t yields the rate of change of the enclosed surface energy:

$$\dot{E}_\Lambda = \frac{4\pi}{3}\left(-\alpha\frac{\dot{H}}{H^2} + \beta\frac{\ddot{H}H^3 - 3H^2\dot{H}^2}{H^6}\right). \quad (40)$$

During the matter-dominated era, the universe expands according to the standard power-law $a(t) \propto t^{2/3}$, which implies $H = \frac{2}{3t}$, $\dot{H} = -\frac{2}{3t^2} = -\frac{3}{2}H^2$, and $\ddot{H} = \frac{4}{3t^3} = \frac{9}{2}H^3$. Substituting these matter-dominated scaling relations into the expression for $\dot{E}_\Lambda$, the higher-derivative terms simplify drastically:

$$\frac{\dot{H}}{H^2} = -\frac{3}{2},\ \frac{\ddot{H}}{H^3} = \frac{9}{2},\ \frac{\dot{H}^2}{H^4} = \frac{9}{4}. \quad (41)$$

Consequently, the energy variation rate reduces to:

$$\dot{E}_\Lambda = \frac{4\pi}{3}\left[\frac{3}{2}\alpha + \beta\left(\frac{9}{2} - 3\left(\frac{9}{4}\right)\right)\right] = \frac{4\pi}{3}\left(\frac{3}{2}\alpha - \frac{9}{4}\beta\right). \quad (42)$$

On the other hand, the first law of thermodynamics applied to the apparent horizon, $\mathrm{d}Q = \mathrm{d}E_\Lambda + W\mathrm{d}V$, implies that in a state of local thermodynamic equilibrium with pressureless dust during the matter era, the net heat flux across the horizon must vanish ($\mathrm{d}Q = 0$), meaning the work done by the dynamic surface pressure must balance the internal energy change. For the surface component to act as a clean cosmological constant-like background without continuously draining energy from or injecting it into the clustering dark matter sector (which would violate the separate conservation law $\rho_m \propto a^{-3}$), the net change of this enclosed background energy $\dot{E}_\Lambda$ must be zero. Setting $\dot{E}_\Lambda = 0$ immediately demands:

$$\frac{3}{2}\alpha - \frac{9}{4}\beta = 0 \Rightarrow \boxed{2\alpha = 3\beta}. \quad (43)$$

This demonstrates that the consistency relation is not merely an empirical adjustment to fit the wCDM expansion history, but a thermodynamically motivated consistency condition for the thermodynamic stability of the PPH surface sector during the cosmic spin-down of the matter epoch.

B.1 Microscopic Origin of α and Stability of the Phantom Regime

The bulk-surface parameter α can be expressed directly through the moments of the velocity distribution (**Appendix A**). In the PPH framework the effective surface pressure and energy density arise from the superluminal tail $v > c$, while the bulk matter is carried by the subluminal modes $v < c$. A kinetic analysis of the primary-particle gas yields for the equation of state of the surface component

$$w_\Lambda = -\frac{c^2}{v_0^2}, \tag{44}$$

where v_0 is the characteristic velocity of the distribution $f(v) \propto v^2 e^{-v^2/v_0^2}$. (Intuitively, the ratio c^2/v_0^2 controls the relative weight of the kinetic and potential contributions to the surface stress-energy tensor.)

Comparing $w_\Lambda = -1/\alpha$ with (44) immediately gives the identification

$$\boxed{\alpha = \frac{v_0^2}{c^2}}. \tag{45}$$

The observationally favoured value $\alpha = 0.97$ therefore corresponds to

$$v_0 = c\sqrt{0.97} \simeq 0.985c, \tag{46}$$

i.e. the characteristic speed of primary particles is only ~1.5% below the limiting speed c. This small offset is the direct origin of the mildly phantom equation of state $w \approx -1.03$.

With $v_0 = 0.985c$ we can evaluate the ratio of the trans-horizon and sub-horizon populations from the integrals of **Appendix A**:

$$R \equiv \frac{\int_c^{v_p} f(v)\,\mathrm{d}v}{\int_0^c f(v)\,\mathrm{d}v} \approx 1.27, \tag{47}$$

which is of order unity. Consequently, the energy density of the superluminal component, and therefore the dark energy density ρ_Λ, is naturally comparable to the matter density today, offering a dynamical resolution of the coincidence problem.

The specific value $\alpha = 0.97$ is not fine-tuned; it emerges from a v_0 just fractionally below c. In the PPH framework the number of effective trans-horizon degrees of freedom has been estimated as $N_{\text{eff}} \sim 10^{10}$ [1]. Such a large but finite reservoir implies that the mean-square velocity of primary particles receives a negative finite-size correction $\langle v^2 \rangle = v_0^2 \left(1 - c/N_{\text{eff}}^{1/2} + \cdots\right)$, naturally shifting α below unity.

Finally, we note that the phantom regime $w < -1$ does not suffer from the usual instabilities in the PPH approach. The surface energy is not a fundamental scalar field but an emergent thermodynamic response of the trans-horizon particle reservoir. The total Hamiltonian of the bulk-plus-surface system is bounded from below, which explicitly indicates that the usual phantom ghost instability is absent, and the finite number of horizon degrees of freedom prevents runaway behaviour. Thus, the mildly phantom dark energy predicted by the bulk-surface mechanism is both physically consistent and stable.