

A Combinatorial-Geometric Conjecture on the Origin of Dark Matter and Dark Energy in Higher-Dimensional Spacetime

Zhuoxiao Yao 

School of Mechanical Engineering, Tianjin University, Tianjin, China

Email: yaozhuoxiao@tju.edu.cn

How to cite this paper: Yao, Z.X. (2026) A Combinatorial-Geometric Conjecture on the Origin of Dark Matter and Dark Energy in Higher-Dimensional Spacetime. *Journal of High Energy Physics, Gravitation and Cosmology*, 12, 2174-2180.

<https://doi.org/10.4236/jhepgc.2026.124108>

Received: April 28, 2026

Accepted: October 5, 2026

Published: October 8, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

The cosmic dark matter density parameter $\Omega_c \approx 0.268$ and dark energy density parameter $\Omega_\Lambda \approx 0.684$ still lack a unified theoretical understanding. String/M-theory requires the number of spatial dimensions to be $N = 9$ or $N = 10$. We explore a phenomenological hypothesis: if all orientations of 3-dimensional subspaces (3-branes) were produced with equal a priori probability in the early universe, and each brane carries ordinary matter, then branes that share no spatial dimensions with the visible brane interact only via higher-dimensional gravity, behaving as cold dark matter. Its abundance is given by the purely combinatorial factor $f_{\text{DM}} = C(N-3, 3)/C(N, 3)$: for $N = 9$ it is $\approx 23.8\%$, and for $N = 10 \approx 29.2\%$, which bracket the Planck value of 26.8% . The remaining branes naturally split into a unity-sum classification: the visible brane ($\approx 1\%$), fully invisible branes (dark matter), and partially overlapping branes (sharing 1 or 2 dimensions). The statistical weight of the partially overlapping class is 75.0% ($N = 9$) and 70.0% ($N = 10$), of the same order as $\Omega_\Lambda \approx 68.4\%$. We propose a geometric interpretation in which gravitational contraction of matter on partially overlapping branes projects as an effective accelerating expansion on the visible brane, and illustrate it with a 5-dimensional toy model. Under the natural distinction between branes sharing one vs. two dimensions, the model yields a two-component effective dark energy: a transient early-dark-energy-like contribution from two-dimensional overlaps, and a slowly evolving late-time component from one-dimensional overlaps. This framework provides a statistical origin for the “coincidence problem” and offers testable predictions for the dark energy equation of state evolution, which can be probed by DESI, Euclid, and other surveys.

Keywords

Dark Matter, Extra Dimensions, String Theory, Brane World, Combinatorial Statistics

1. Introduction

The existence of dark matter is firmly established, yet its cosmic abundance $\Omega_c = 0.268 \pm 0.012$ [1] and the nature of dark energy ($\Omega_\Lambda \approx 0.684$) remain central open problems. String theory and M-theory require, for self-consistency, 10 or 11 spacetime dimensions, leaving $N = 9$ or $N = 10$ spatial dimensions after compactification [2] [3]. Several works have explored dark matter candidates from extra dimensions, such as Kaluza-Klein dark matter [4] [5] and mirror matter in brane-world scenarios [6] [7]. However, a direct link between the number of extra dimensions and the observed cosmic fractions is still missing.

For completeness, the dark matter problem can also, in principle, be addressed within extended theories of gravity. As discussed by Corda [8], modifications of the gravitational sector, including $f(R)$ and scalar-tensor theories, provide an alternative framework in which effects conventionally attributed to dark matter and dark energy may be accounted for by modified gravitational dynamics. The present conjecture instead attributes the dark matter contribution to matter on other branes coupled through higher-dimensional gravity; its combinatorial estimate does not exclude extended-gravity explanations.

In this study, we adopt a simplified statistical approach based on the possible orientations of 3-branes in a higher-dimensional bulk. If, during the quantum gravity era, 3-branes of different orientations emerge with nearly equal a priori probability, and if Standard Model fields are confined to our visible 3-brane, then matter on other 3-branes that share no spatial dimensions with the visible sector will interact only through higher-dimensional gravity. Their abundance becomes a purely combinatorial quantity, independent of detailed particle physics. We call this the “Combinatorial Brane Statistics” (CBS) hypothesis.

We will show that for $N = 9$ and $N = 10$, the combinatorial factor yields a dark matter fraction of 23.8% and 29.2%, respectively, which numerically bracket the Planck value. The threefold classification of 3-branes—fully visible, fully invisible, and partially overlapping—is mutually exclusive and exhaustive, automatically satisfying $\Omega_{\text{total}} = 1$ and thus consistent with the observed spatial flatness of the Universe [1]. Notably, the statistical weight of partially overlapping branes (75.0% for $N = 9$, 70.0% for $N = 10$) is of the same order as Ω_Λ , suggesting a possible geometric connection. We explore a mechanism in which gravitational contraction on partially overlapping branes projects as accelerated expansion on the visible brane, and we distinguish between overlaps of one and two spatial dimensions. This distinction naturally leads to a two-component effective dark energy, with the two-dimensional overlap providing a transient early contribution and the one-dimensional overlap driving the late-time acceleration. The resulting dark energy equation of state $w(z)$ can evolve, offering a testable prediction for upcoming surveys.

2. Combinatorial Model of 3-Branes

We assume a $D = (1 + N)$ -dimensional bulk spacetime with one time dimension

and N spatial dimensions. A 3-brane is a hypersurface spanned by three out of the N spatial coordinates:

$$\begin{aligned} M_3^{(i)} &= \text{span}(x^{a_1}, x^{a_2}, x^{a_3}) \\ \{a_1, a_2, a_3\} &\in \{1, 2, 3, \dots, N\} \end{aligned} \quad (1)$$

Gauge interactions are confined to the brane, while gravity propagates in the full bulk. We assume that in the extremely early universe, quantum gravitational processes produce 3-branes of all possible orientations with approximately uniform probability. The total number of 3-branes is then $C(N, 3)$. Fixing the visible brane along dimensions $\{1, 2, 3\}$, all other 3-branes fall into three mutually exclusive classes:

- Fully visible brane: all three dimensions belong to $\{1, 2, 3\}$. There is exactly 1 such brane (our own).
- Partially overlapping branes: one or two dimensions belong to $\{1, 2, 3\}$, the rest lie in the extra dimensions $\{4, \dots, N\}$.
- Fully invisible branes: all three dimensions are taken from $\{4, \dots, N\}$. They share no spatial dimensions with the visible brane.

Matter on fully invisible branes has no gauge interaction with the visible sector, only higher-dimensional gravity. It therefore behaves as cold, collisionless dark matter. The number of such branes is $C(N-3, 3)$. If each 3-brane carries, on average, roughly the same initial matter energy density, and if no substantial energy exchange occurs between branes, the fraction of dark matter in the total cosmic energy budget is

$$f_{\text{DM}}(N) = \frac{C_{N-3}^3}{C_N^3} = \frac{(N-3)(N-4)(N-5)}{N(N-1)(N-2)} \quad (2)$$

For the cases relevant to string/M-theory:

- $N=9$: $C(9, 3) = 84$, $C(6, 3) = 20 \rightarrow f_{\text{DM}}(9) = 20/84 \approx 0.238$.
- $N=10$: $C(10, 3) = 120$, $C(7, 3) = 35 \rightarrow f_{\text{DM}}(10) = 35/120 \approx 0.292$.

The Planck 2018 measurement gives $\Omega_c \approx 0.268$. These two values bracket it, with relative deviations of $\sim 11\%$ and $\sim 9\%$, without any adjustable parameters.

3. Completeness and Numerical Hint for Dark Energy

The three classes are exhaustive and mutually exclusive, so their statistical weights sum to exactly 1:

$$f_{\text{visible}} + f_{\text{overlap}} + f_{\text{DM}} = 1 \quad (3)$$

Table 1 lists the numbers and weights for $N=9$ and $N=10$. The completeness automatically yields $\Omega_{\text{total}} = 1$, consistent with the observed flatness $\Omega_k \approx 0.0007 \pm 0.0019$ [1].

Remarkably, the weight of partially overlapping branes (75.0% for $N=9$, 70.0% for $N=10$) is of the same order as $\Omega_\Lambda \approx 68.4\%$. This numerical proximity suggests that partially overlapping branes might be the geometrical origin of dark energy.

Table 1. Combinatorial classification and statistical weights for $N=9$ and $N=10$.

Brane Type	Number $N=9$	Weight $N=9$	Number $N=10$	Weight $N=10$
Fully Visible Brane	1	1.2%	1	0.8%
Fully Invisible Branes	20	23.8%	35	29.2%
Partially Overlapping Branes	63	75.0%	84	70.0%
Total	84	100%	120	100%

4. Toward a Geometric Interpretation of Dark Energy

In general relativity, tidal forces demonstrate that compression in one direction can be accompanied by expansion in orthogonal directions. We propose that matter on partially overlapping branes undergoes gravitational contraction; through the shared dimensions, this contraction is geometrically projected onto the visible brane. Along the shared directions, the projection may manifest as an attractive acceleration, while along the non-shared directions it appears as an effective expansion of space. Thus, the late-time accelerated expansion on the visible brane would not originate from a true cosmological constant but from the inter-brane geometric relationship (see **Figure 1**).

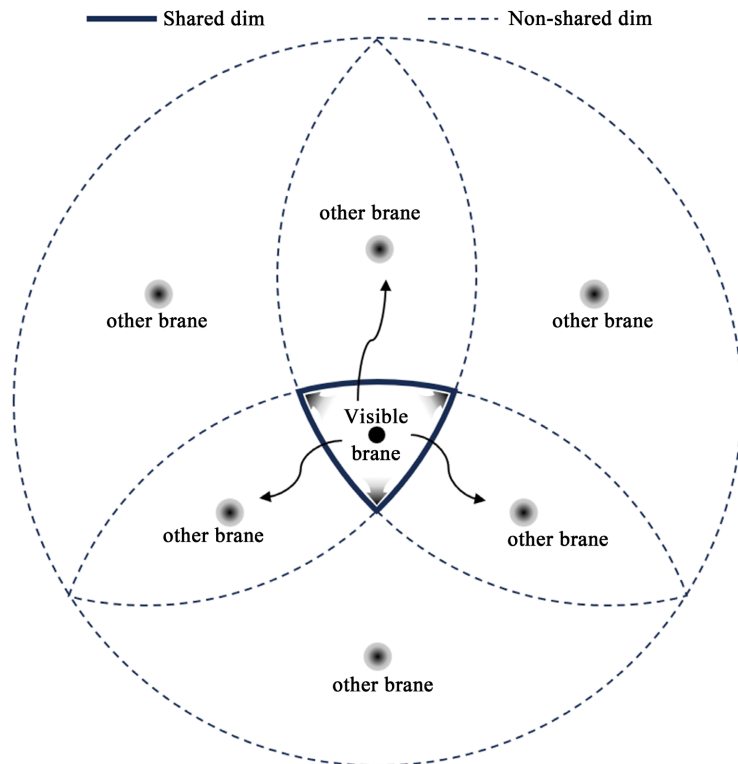


Figure 1. Schematic illustration of 3-brane interactions in 1 overlapping dimension. Each enclosed region represents a 3-brane. Shared edges indicate dimensions common to adjacent branes. The central region corresponds to the visible brane, surrounded by partially overlapping branes.

To illustrate the mechanism, consider a 5-dimensional bulk ($N = 4$) with coordinates (t, x, y, z, w) . The visible Brane A is at $w = 0$, extended along $\{x, y, z\}$. A partially overlapping Brane B is at $z = 0$, extended along $\{x, y, w\}$, sharing two dimensions $\{x, y\}$. Assume a Kasner-type metric ansatz:

$$ds^2 = -dt^2 + a_1(t)^2 dx^2 + a_2(t)^2 dy^2 + a_3(t)^2 dz^2 + a_4(t)^2 dw^2 \quad (4)$$

If gravitational contraction on Brane B causes a_4 to contract, the vacuum Einstein equations admit solutions where a_3 expands exponentially while a_4 contracts, preserving the total volume element. The visible brane thus experiences an effective Hubble parameter H_{eff} along its exclusive direction z , sourced entirely by the ordinary matter density $\rho_{(B)}$ on Brane B:

$$H_{\text{eff}} \sim \frac{8\pi G_5}{3} \rho_{(B)} \quad (5)$$

This toy model demonstrates the core idea: contraction in overlapping extra dimensions can project as expansion in the visible non-shared dimensions. Generalization to $N = 9$ or 10 and inclusion of moduli stabilization remain for future work.

5. Discussion and Observational Prospects

The partially overlapping class splits further into branes that share one spatial dimension with the visible brane and those that share two. For $N = 10$, for example, there are 63 one-dimension-sharing branes and 21 two-dimension-sharing branes. Their geometric coupling to the visible brane is expected to differ in strength and distance dependence. Let the coupling strengths be $\kappa_1(d) \propto d^{-\alpha_1}$ and $\kappa_2(d) \propto d^{-\alpha_2}$, where $d(a) \propto a^\beta$ denotes the typical scale of the extra dimensions as a function of the cosmic scale factor a , and $\alpha_2 < \alpha_1$ is expected. The matter density on all branes redshifts as $\rho_{(B)} \propto a^{-3}$. The effective dark energy density induced on the visible brane from the two subclasses is then

$$\rho_{\text{DE}}^{(1)}(a) = A_1 \cdot \rho_{(B)}(a) \cdot \kappa_1(a) \propto a^{-3-\alpha_1\beta} \quad (6a)$$

$$\rho_{\text{DE}}^{(2)}(a) = A_2 \cdot \rho_{(B)}(a) \cdot \kappa_2(a) \propto a^{-3-\alpha_2\beta} \quad (6b)$$

where A_1, A_2 are proportional to the respective brane numbers and their average matter density. The total effective dark energy density is

$$\rho_{\text{DE}}(a) = \rho_{\text{DE}}^{(1)}(a) + \rho_{\text{DE}}^{(2)}(a) \quad (7)$$

The effective equation-of-state parameter follows from the continuity equation:

$$\begin{aligned} w(a) &= -1 - \frac{1}{3} \frac{d \ln \rho_{\text{DE}}}{d \ln a} \\ &= -1 + \frac{1}{3} \frac{(3 + \alpha_1\beta) \rho_{\text{DE}}^{(1)}(a) + (3 + \alpha_2\beta) \rho_{\text{DE}}^{(2)}(a)}{\rho_{\text{DE}}^{(1)}(a) + \rho_{\text{DE}}^{(2)}(a)} \end{aligned} \quad (8)$$

Because $\rho_{\text{DE}}^{(2)}$ has a larger initial coupling but decays more rapidly ($\alpha_2 < \alpha_1$), it

dominates at early times and is a natural candidate for Early Dark Energy (EDE), potentially easing the Hubble tension. $\rho_{\text{DE}}^{(1)}$ becomes dominant at late times, driving the present-day acceleration. Depending on the sign of β and the α -values, $w(z)$ can exhibit either thawing ($w > -1$) or phantom ($w < -1$) evolution. Recent DESI 2024 results, which hint at $w_0 \approx -0.85$ in a $w_0 w_a$ CDM parametrization, are qualitatively compatible with a thawing behavior from the one-dimension-sharing component with a small positive $\alpha_1 \beta$.

The observed macroscopic space is three-dimensional. In string theory, D3-branes are stable objects supporting four-dimensional gauge theories. The current model faces two primary theoretical challenges: the lack of a dynamical origin for the equal a priori probability hypothesis, and the absence of a full quantitative derivation of the dark energy geometric mechanism for the physically relevant cases $N = 9, 10$. Nevertheless, the framework offers several testable signatures.

Most importantly, the two-component dark energy derived predicts an evolving equation of state $w(z)$. The one-dimension-overlap component produces a slowly evolving late-time w , while the two-dimension-overlap component yields a transient early dark energy signature. Upcoming surveys such as DESI, Euclid, and LSST will map $w(z)$ with high precision. A detection of $w(z)$ deviating from -1 , especially with a past EDE-like bump, would strongly motivate a geometric dark energy picture of the type proposed here. Conversely, a persistent $w = -1$ at all redshifts would rule out the simplest versions of this scenario.

Traces of EDE could also be imprinted in the CMB and the measurement of H_0 . A full confrontation with cosmological data, combined with a derivation of the coupling parameters from a fundamental compactification, is left for future work.

6. Conclusions

Starting from the statistical distribution of 3-brane orientations in higher dimensions, we have provided a phenomenological framework that addresses the cosmic dark matter abundance and the dark energy fraction simultaneously. The dark matter fraction is purely combinatorial, $f_{\text{DM}} = C(N-3, 3)/C(N, 3)$, giving 23.8% ($N = 9$) and 29.2% ($N = 10$) with no free parameters. The mutually exclusive and exhaustive classification naturally enforces a flat Universe. The statistical weight of partially overlapping branes is of the same order as Ω_Λ , and we explored a geometric interpretation where gravitational contraction on these branes projects as dark energy. Distinguishing between overlaps of one and two spatial dimensions leads to a two-component effective dark energy that can produce a transient early dark energy signal and a thawing late-time equation of state. While a complete dynamical foundation is still missing, the numerical coincidences and the testable predictions for $w(z)$ make this framework a worthwhile stimulus for further research on the connection between extra dimensions and cosmological parameters.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Planck Collaboration, Aghanim, N., Akrami, Y., *et al.* (2020) Planck 2018 Results. VI. Cosmological Parameters. *Astronomy & Astrophysics*, **641**, A6.
- [2] Polchinski, J. (1998) String Theory. Vol. I: An Introduction to the Bosonic String. Cambridge University Press. <https://doi.org/10.1017/cbo9780511618123>
- [3] Becker, K., Becker, M. and Schwarz, J.H. (2006) String Theory and M-Theory. Cambridge University Press. <https://doi.org/10.1017/cbo9780511616086>
- [4] Servant, G. and Tait, T.M.P. (2003) Is the Lightest Kaluza-Klein Particle a Viable Dark Matter Candidate? *Nuclear Physics B*, **650**, 391-419. [https://doi.org/10.1016/s0550-3213\(02\)00102-x](https://doi.org/10.1016/s0550-3213(02)00102-x)
- [5] Cheng, H.C., Feng, J.L. and Matchev, K.T. (2002) Kaluza-Klein Dark Matter. *Physical Review Letters*, **89**, Article 211301. <https://doi.org/10.1103/physrevlett.89.211301>
- [6] Kolb, E.W., Chung, D.J.H. and Riotto, A. (1999) Gravitational Production of Super-heavy Dark Matter. *Physical Review D*, **64**, Article 043503. <https://journals.aps.org/prd/abstract/10.1103/PhysRevD.64.043503>
- [7] Berezhiani, Z., Comelli, D. and Villante, F.L. (2001) The Early Mirror Universe: Inflation, Baryogenesis, Nucleosynthesis and Dark Matter. *Physics Letters B*, **503**, 362-375. [https://doi.org/10.1016/s0370-2693\(01\)00217-9](https://doi.org/10.1016/s0370-2693(01)00217-9)
- [8] Corda, C. (2009) Interferometric Detection of Gravitational Waves: The Definitive Test for General Relativity. *International Journal of Modern Physics D*, **18**, 2275-2282. <https://doi.org/10.1142/s0218271809015904>