

Introducing a Huge Flux of Energy in Inflation? *i.e.* Use of Hup and QM Number N to Unify Black Holes, with a QM Universe via Entanglement Arguments and Torsion and Its Links to Quantum Number n

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Abstract

First, we consider if a generalized HUP set greater than or equal to Planck's constant divided by the square of a scale factor as well as an inflation field, yields the result that ΔE times Δt is embedded in a 5-dimensional field which is within a deterministic structure. Our proof ends with Δt as of Planck time yielding an enormous potential energy, Second, we tie this energy to black hole physics and the early universe. *i.e.*, our idea for black hole physics being used for GW generation, is using Torsion to form a cosmological constant. Planck sized black holes allow for a spin density term linked to Torsion. The relationship of entanglement and initial frequency are also raised. Half of the manuscript is to set the template as to why we reference entanglement including the work of Nye as to his version of the HUP.

Keywords

Torsion, Entanglement, Initial GW Frequencies, HUP, Black Holes

1. Introduction

As indicated, there are several states in this presentation. First is the idea of using the HUP from a deterministic embedding in 5 dim, as given in Equation (1), and Equation (2), where we have then [1] [2].

$$|dp_\alpha dx^\alpha| \approx \frac{L}{l} \cdot \frac{h}{c} \cdot \left[\frac{dl}{l} \right]^2 \quad (1)$$

$$\sqrt{\Lambda} = \frac{k_B E}{\hbar c S_{\text{entropy}}} \quad (2)$$

$$S_{\text{entropy}} = k_B N_{\text{particles}}$$

The upshot is that we will be changing the cosmological constant from

$$\left(\frac{8\pi G}{3}\right) \cdot \left[-\frac{2\pi G \sigma^2}{3c^4}\right] + \frac{\Lambda c^2}{3} = 0 \quad (3)$$

What we are arguing is that instead, one is seeing, instead

$$\left(\frac{8\pi G}{3}\right) \cdot \left[-\frac{2\pi G \sigma^2}{3c^4}\right] + \frac{\Lambda_{pl} c^2}{3} \approx 10^{-122} \times \left(\frac{\Lambda_{pl} c^2}{3}\right) \quad (4)$$

Our timing as to Equation (4) is to unleash a Planck time interval t about 10^{-43} seconds. As to Equation (3) versus Equation (4), the creation of the torsion term is due to a presumed particle spin density of $n_{pl} \approx 10^{98} \text{ cm}^{-3}$.

2. Energy Flux at the Start of Inflation?

Space constraints do not allow us to explain more than this other than to bring up that the Torsion argument is to buttress the following for energy density which we give as [1] [2].

We lower the energy used in Equation (6) by lowering the upper bound to the integral by use of

$$(3 \times 10^{19} \text{ GeV}) \cdot \frac{1}{n^\beta} \propto 3 \times 10^{-11} \text{ GeV} \quad (5)$$

Leading to a nucleated variable $n^\beta \approx 10^{30}$ for particle count and quenching

$$\rho_\Lambda c^2 = \int_0^{E_{\text{Plank}}/c} \frac{4\pi p^2 dp}{(2\pi\hbar)^3} \cdot \left(\frac{1}{2} \cdot \sqrt{p^2 c^2 + m^2 c^4}\right) \approx \frac{(3 \times 10^{19} \text{ GeV})^4}{(2\pi\hbar)^3} \quad (6)$$

$$\xrightarrow{E_{\text{Plank}}/c \rightarrow 10^{-30}} \frac{(2.5 \times 10^{-11} \text{ GeV})^4}{(2\pi\hbar)^3}$$

This means shifting the energy level of the Equation (6) downward by 10^{-30} , *i.e.* the top value energy becomes a down scale of Planck energy times 10^{-30} . We argue that the topping off of this integral is dependent upon Equation (5) with respect to black holes, and that the quantum number comes from [1] [2]

$$M_{BH} \approx \sqrt{N_{\text{gravitons}}} M_P$$

$$\Rightarrow (1/M_{BH})^{1/2} \approx \frac{n_{\text{quantum}}}{2} \approx \frac{1}{(N_{\text{gravitons}})^{1/4}} \quad (7)$$

$$\Rightarrow n_{\text{quantum}} \approx \frac{2}{(N_{\text{gravitons}})^{1/4}}$$

Finally this also leads to a huge energy flux we can write for Potential energy initially [1] [2]

$$V_0 = \left(\sqrt{\frac{v(3v-1)}{8\pi}} + \sqrt{2 \cdot (3v-1)} \cdot \frac{a_{\text{init}}^2 \cdot (l_2 - l_1)^2}{l^3(c) \cdot (3\Lambda)^{1/2}} \right)^2 \approx \Delta E \quad (8)$$

This uses a scale factor we write as $a(t) \approx a_{init} t^v$ and also a linkage to a 5 dimensional line element, Λ and time which we write as [1] [2] as well as Planck time which we give below in Equation (10) which is linked to Equation (1) as discussed in [2]. This leads to Planck time. Also we use a 5 dimensional wave number as $K_1 = 1/l$

$$dS_{5-d}^2 = \frac{L^2}{l^2} dS_{4-d}^2 - \frac{L^4}{l^4} dl^2 \quad (9)$$

$$t = t_{\text{planck}} \rightarrow 1 = \sqrt{\frac{v(3v-1)}{8\pi V_0}} + \sqrt{\frac{2 \cdot (3v-1)}{V_0}} \cdot \frac{a_{init}^2 \cdot (l_2 - l_1)^2}{l^3(c) \cdot (3\Lambda)^{1/2}} \quad (10)$$

Before proceeding, we strongly advise the readers to look at [1]-[5] in order to make sense of what is going on with the rest of the paper. And this leads us to consider initial GW frequencies.

3. What about Initial Frequencies? At Start of Inflation?

From [1] and [2] we will be treating a start of the inflation Potential well along the lines of the discussion given in [1] and [2].

We should before proceeding also note that we would also be utilizing having $\Delta E \equiv \frac{d(\text{dim})}{2} \cdot k_B \cdot T_{\text{universe}}$ Where the so that we have, T (universe) as at least Planck temperature in value so then we would have at the start of inflation a quantum number n as defined by

$$n^2 (\text{quantum number}) \approx \frac{2d(\text{dim}) \cdot V_{\text{space volume}} m_g \cdot (R_{\text{radius well}})^2}{\pi^2 \hbar^2} \quad (11)$$

Here we assume the rest mass of a graviton would be likely less than 10^{-62} grams, or about 1.23 times 10^{-23} eV, where we are assuming having an almost one to one connection between g_* and $d(\text{dim})$

$$\rho = \frac{E(\text{cylindrical well energy})}{V_{\text{volume}}} = \frac{n^2 (\text{quantum number}) \cdot \pi^2 \hbar^2}{V_{\text{volume}} \cdot m_g \cdot (r_{\text{radius well}})^2} \quad (12)$$

For there to be an equality, which would be a necessary condition for having a correspondence principle in Cosmology, *i.e.* to have quantum effects for high numbers, *i.e.* $n_{\text{quantum number}}^2 \gg 1$, one would likely have, even if we state g_* is a degree of freedom, would be that the stated dimensional values of inputs into a very large value for d_{dim} for inputs into the Pre Planckian state, prior to emergence into Planckian cosmology conditions would have to be an extremely large number. *i.e.* we would be looking for conditions in the pre Planckian space time for which $n_{\text{quantum number}}^2 \gg 1$ due to an enormous value for d_{dim} .

If so then we also reference the following arguments as to tying in the following *i.e.* from [1] and [2] $V(\text{volume}) = \text{cube of planck length} \times 0.27002422918$. Therefore if we write in the following units Planck length $= L_{\text{planck}} = 1 = \hbar = m_g$, we have

$$+ p_{\text{Bose fluid}} (\text{Pressure}) = \frac{1}{2\hbar^3} \cdot \left(E_{\text{Planck } 2}^3 E_{\text{Planck } 1} - \frac{16}{15\pi^2} E_{\text{Planck } 1}^4 \right) \quad (13)$$

This expression becomes negative if $E_{\text{Planck}2}^3 E_{\text{Planck}1} < \frac{16}{15\pi^2} E_{\text{Planck}1}^4$ and if so we have negative pressure. *i.e.* For our problem if we configure the initial contents of the “well” we assume for having a near singularity, for space-time expansion start we can have $n(\text{Particle density}) = N/V(\text{volume})$, with N as the number of would be “gravitons”, and $V(\text{volume})$ being the “Volume of space-time for our evaluation”. Whereas $m = m(\text{mass}) = N(\text{number gravitons}) \times m_g$ with the use of the massive graviton value $m_g = \text{mass}(\text{heavy gravity graviton})$. If so a simple calculation for this problem would have, then a negative value for pressure if we have the following, namely

$$\frac{\hbar c \cdot \sqrt[3]{V_{\text{Volume} <}}}{(N_{\text{graviton number}} m_g)^{1/3}} \leq \sqrt[3]{\frac{16}{15\pi^2}} \times N_{\text{graviton number}} m_g c^2 \quad (14)$$

Equation (14) re written as

$$\left(\frac{15\pi^2}{16}\right)^{1/3} \times 0.270024 \leq (.5 \times 10^{-58} m_{Pl})^{4/3} N_{\text{graviton}}^{4/3} \quad (15)$$

Or roughly

$$\left(\frac{15\pi^2}{16}\right)^{1/3} \times 0.270024 \leq 10^{-77} N_{\text{graviton}}^{4/3} \quad (16)$$

$$\text{Leading to } 10^{77} \leq N_{\text{graviton}}^{4/3} \quad (17)$$

Or an upper bound of say for graviton mass of 10^{-62} grams, we have that we have negative pressure in our system for the number of gravitons being less than 10^{58} , in a volume about 0.27 times the cube of Planck length. Therefore we have the number of gravitons, initially about 10^{58} , whereas by [1]-[3] we have

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \quad (18)$$

The long and short of it is, to tie this value of the cosmological constant, and the production of gravitons due to early universe conditions, to a relationship between De Broglie wavelength, Planck length, and if the velocity v gets to a partial value close to the speed of light, that, we have, say by using [4] as given by Diosi, in Dice (2018) for quantum systems, if we have instead of a velocity much smaller than the speed of light, a situation where the particle moves very quickly (a fraction of the speed of light) that instead of the slow massive particle postulated in [4]

$$\lambda_{\text{De-Broglie}} \approx \frac{2 \cdot \pi \cdot \hbar}{m_g v(\text{velocity})} \cdot \sqrt{1 - \left(\frac{v(\text{velocity})}{c}\right)^2} \ell_{\text{Planck}} \quad (19)$$

$$\ell_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^3}} \quad (20)$$

If the velocity of particle is just under the speed of light, *i.e.* $v(\text{velocity}) \rightarrow c - \delta^+$ one then would have per Graviton particle, say an upper

bound of per graviton if moving near the speed of light of

$$\varepsilon(\text{Particle energy}) \approx E_{\text{Planck}}(\text{Planck energy}) \quad (21)$$

whereas the energy of all the gravitons *i.e.* 10^{58} of them would be of the order of Equation (8) *i.e.* the enormous value of

$$V_0 = \left(\sqrt{\frac{v(3v-1)}{8\pi}} + \sqrt{2 \cdot (3v-1)} \cdot \frac{a_{\text{init}}^2 \cdot (l_2 - l_1)^2}{l^3(c) \cdot (3\Lambda)^{1/2}} \right)^2 \approx \Delta E$$

At the start of inflation, the universe was in a high-energy state known as the GUT (Grand Unified Theory) scale, with an energy density estimated around 10^{16} GeV This would lead to an approximate value of a temperature as given by Eq. (21) specifying a temperature on the order of Planck Temperature, initially.

With a convenient treatment of [5]

$$\begin{aligned} a &\approx a_{\text{min}} t^\gamma \\ \Delta\phi &\approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left\{ \sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma-1)}} \cdot t \right\} \\ \Delta V &\approx V_0 \cdot \exp \left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\} \end{aligned} \quad (22)$$

This could, in the minimum radii expected by a quantum induced scale factor, a_{min} [6] [7], as well as the details of [8] as to how to form g_u , lead to in time to, if we take the minimum uncertainty given by. Equation (1) lead to making the following equivalence, *i.e.*

$$\begin{aligned} \Delta E &\approx \frac{\hbar}{g_u \Delta t} \approx \frac{k_B}{2} \cdot T_{\text{initial}} \\ \Delta T_{\text{initial}}(\text{initial temperature}) &\approx \frac{2\hbar}{k_B \cdot g_u \cdot \Delta t} \end{aligned} \quad (23)$$

And [9]

$$\begin{aligned} \Delta E \Delta t &\geq \frac{\hbar}{g_u} \\ \& \ g_u &= a_{\text{min}}^2 \phi_{\text{initial-inf}} \end{aligned} \quad (24)$$

This leads to an initial frequency as given below.

To begin with. Look at how to construct entropy for black holes and the early universe.

Note that for gravity one has, if k is Boltzmann's constant, and N the number of Microstates. Note that formula 1 turns to formula 2 if N is large [10]

$$S = k \ln N \quad (25)$$

Now, by Muller and Luosto [11] as well as Crowell [12] one can write for the early universe:

$$S = kA/4l_p^2 \quad (26)$$

What if one looks at a treatment of black holes?

The area A is such, that by Crowell [12] we can write this area as, for a black

hole of mass M

$$A = 16\pi M^2 \quad (27)$$

For a string theory treatment of black holes we will write [12]

$$A = 16\pi\alpha \sum_{i=1}^N n_i \quad (28)$$

So what is α ?

If what Ng writes for Quantum infinite statistics [10] is true, then

$$E = \alpha E_p \sqrt{n} \Leftrightarrow \alpha = \frac{1}{2} \frac{\sqrt{\ln 2}}{\pi} \quad (29)$$

Now let us access a Partition function treatment of black holes [12].

Crowell wrote having a partition function for Black holes defined by

$$Z = \sum_n \exp[4\pi\omega_n] \cdot \exp[-\beta\alpha\sqrt{n}] \quad (30)$$

This was achieved by normal modes for black holes, of mass M which was of the form [12]

$$\omega_n = \alpha^2 = \frac{\ln 3}{8\pi M} + \frac{i}{4M} \cdot \left(n + \frac{1}{2}\right) \quad (31)$$

M in this case would be about 10^{58} times the rest mass of a graviton, giving the real part of Equation (31) a value of about 10^{40} Hz.

4. What about the Issue of Entanglement from a Prior to the Present Universe?

First of all before we get to this, it is important to look at an alternative to traditional CCC cosmology as given by Penrose. This leads to prior to present universe conditions.

Penrose CCC Cosmology with a Multiverse Interpretation

We now outline the generalization for Penrose CCC (Cosmology) before inflation which we state we are extending Penrose's suggestion of cyclic universes, black hole evaporation, and the embedding structure our universe is contained within. This multiverse has BHs and may resolve what appears to be an impossible dichotomy. The following is largely from [13] [14] and has serious relevance to the final part of the conclusion. That there are N universes undergoing Penrose "infinite expansion" (Penrose) [13] contained in a mega universe structure [14] Furthermore, each of the N universes has black hole evaporation, which is with Hawking radiation from decaying black holes. If each of the N universes is defined by a partition function, called $\{\Xi_i\}_{i=1}^{i=N}$, then there exist an information ensemble of mixed minimum information correlated about $10^7 - 10^8$ bits of information per partition function in the set $\left\{\Xi_i\right\}_{i=1}^{i=N}\Big|_{\text{before}}$, so minimum information is conserved between a set of partition functions per universe [14].

$$\left\{\Xi_i\right\}_{i=1}^{i=N}\Big|_{\text{before}} \equiv \left\{\Xi_i\right\}_{i=1}^{i=N}\Big|_{\text{after}} \quad (32)$$

However, there is non-uniqueness of information put into individual partition function $\{\Xi_i\}_{i=1}^{i=N}$. Also Hawking radiation from black holes is collated via a strange attractor collection in the mega universe structure to form a new inflationary regime for each of the N universes represented.

Our idea is to use what is known as CCC cosmology [13], which can be thought of as the following [14]. First. Have a big bang (initial expansion) for the universe which is represented by $\{\Xi_i\}_{i=1}^{i=N}$. Verification of this mega structure compression and expansion of information with stated non-uniqueness of information placed in each of the N universes favors ergodic mixing of initial values for each of N universes expanding from a singularity beginning. The n_f stated value, will be using (Ng, 2008) $S_{\text{entropy}} \sim n_f$. [10]. How to tie in this energy expression, will be to look at the formation of a nontrivial gravitational measure as a new big bang for each of the N universes as by $n(E_i)$ the density of states at energy E_i for partition function [14].

$$\{\Xi_i\}_{i=1}^{i=N} \propto \left\{ \int_0^\infty dE_i \cdot n(E_i) \cdot e^{-E_i} \right\}_{i=1}^{i=N} \quad (33)$$

Each of E identified with Equation (33) above, are with the iteration for N universes [13] and [14] (Penrose, 2006) Then the following holds, by asserting the following claim to the universe, as a mixed state, with black holes playing a major part, *i.e.*

Claim 1

See the below [14] representation of mixing for assorted N partition functions per CCC cycle

$$\frac{1}{N} \cdot \sum_{j=1}^N \Xi_j \Big|_{j \text{ before nucleation regime}} \xrightarrow{\text{vacuum nucleation transfer}} \Xi_i \Big|_{i \text{ fixed after nucleation regime}} \quad (34)$$

For N number of universes, with each $\Xi_j \Big|_{j \text{ before nucleation regime}}$ for $j=1$ to N being the partition function of each universe just before the blend into the RHS of Equation (34) above for our present universe. Also, each independent universe as given by $\Xi_j \Big|_{j \text{ before nucleation regime}}$ is constructed *by the absorption of one to ten million black holes taking in energy. i.e. (Penrose) [14]*. Furthermore, the main point is done in [14] in terms of general ergodic mixing [15].

Claim 2

$$\Xi_j \Big|_{j \text{ before nucleation regime}} \approx \sum_{k=1}^{\text{Max}} \tilde{\Xi}_k \Big|_{\text{black holes ith universe}} \quad (35)$$

What is done in **Claims 1 and 2** [14] is to come up as to how a multi dimensional representation of black hole physics enables continual mixing of spacetime [14] as well as employ [15] largely as a way to avoid the Anthropic principle [16], as to a preferred set of initial conditions.

5. Why We Are Looking at the Modification of the Penrose CCC (Cosmology)

We argue this modification is mandated by having the initial DE wavefunction set

as having a wave length as stated by

$$\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}} \quad (36)$$

This will be used to make sense of the presentation given in [17] as well as rigorous data analysis of CMBR data and in all of this, we will be making the scale factor approximation in macro scale *i.e.*

We have that for a scale factor expansion of the universe, that.

$$a(t) = a_0 \left\{ \frac{1}{2\Omega_\Lambda} [\cosh(3\Lambda t) - 1] \right\}^{1/3} \xrightarrow{t \rightarrow \text{Large}} \exp\left(\sqrt{\frac{\Lambda}{3}} t\right) \quad (37)$$

The substitution of Equation (37) is in its large time t limit form relevant to **Figure 1** whereas the earlier before time t is large value of the scale factor is more relevant to the Planckian physics, whereas we still can in very small be using the scale factor as proportional to t First of all is the old standby namely in the onset of inflation, there would be a huge speed of inflationary expansion with the coefficient of scale factor given as [5] *i.e.* this is looking at the coefficient showing up in scale factor expansion, that if we go to Equation (38)

$$v \xrightarrow{\text{Planck normalization}} 4\pi \times (\omega_{gw})^{12} \times \frac{\zeta^4}{\tilde{\beta}^2} \quad (38)$$

For mass greater than Planck mass, namely $M_{\text{mass}} \approx \zeta m_p$, with m_p for Planck Mass. We refer to [3], in that this is for the mass of a physical system, *i.e.* M_{mass} of an object which in its physical configuration is generating gravitational waves, ω_{gw} and we find that in the Planckian regime, $\tilde{\beta}$ is a coefficient connected to a fifth force argument due to reasoning from [3].

This leads to the following *i.e.* in [3] which is reproduced here, In addition after approximating $\langle r^2 \rangle^2 \approx \ell_p^4$, *i.e.* Planck length to the fourth power and $r \langle r^2 \rangle^2 \approx \ell_p^5$ e power of Equation (37), *i.e.* a very large number in the Planckian regime.

So, can we ascertain the GW radiation of pre Universe stars getting into the present universe? *i.e.* keep in mind any suggestion as to cosmology will have to satisfy the following diagram.

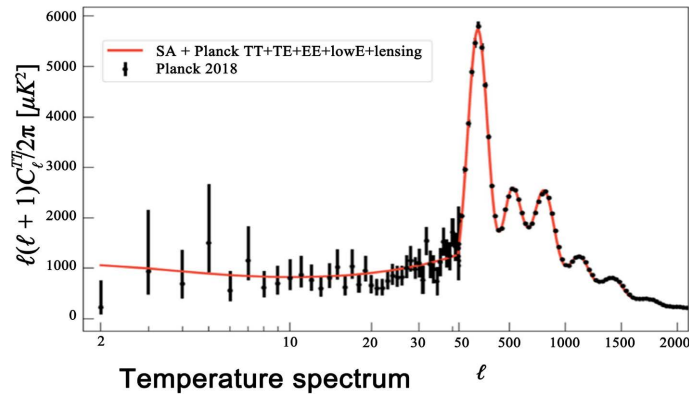


Figure 1. According to the physics of the CMB, as given in [18] [49] Abhay Ashtekar in Zeldovich4. On September 7, 2020 [11].

In our **Figure 1**, we copy what was done by Ashtekar, in Zelsovich4 as to what was part of anisotropic fits to the E and B polarization, as given we argue that this realistically means quantum entanglement, and this leads to our multiverse generalization of the Penrose suggestion.

6. Examining Pre Planckian to Planckian Distribution of Black Holes

To do this, note that Figure one has to be getting **Figure 1** satisfied in the present universe, but to do that we look at

Table 1. From [19] [20] assuming Penrose recycling of the Universe as stated in that document.

End of Prior Universe time frame	Mass (black hole): super massive end of time BH 1.98910^{+41} to about 10^{44} grams	Number (black holes) 10^6 to 10^9 of them usually from center of galaxies
Planck era Black hole formation Assuming start of merging of micro black hole pairs	Mass (black hole) 10^{-5} to 10^{-4} grams (an order of magnitude of the Planck mass value)	Number (black holes) 10^{40} to about 10^{45} , assuming that there was not too much destruction of matter-energy from the Pre Planck conditions to Planck conditions
Post Planck era black holes with the possibility of using Equation (1) and Equation (2) to have say 10^{10} gravitons/second released per black hole	Mass (black hole) 10 grams to say 10^6 grams per black hole	Number (black holes) Due to repeated Black hole pair forming a single black hole multiple time. 10^{20} to at most 10^{25}

As to **Table 1**, we obtain, due to the quantum number n , per black hole.

Table 1 data will be connected to the following given consideration of spin density, as to Planck sized black holes, *i.e.* we will be considering the role of entanglement in connection of the Pre Planck to Planck data sets as represented in **Table 1** above. Keep in mind that the black holes form spin density as defined in [21] which was used initially to cancel the cosmological constant, whereas we applied modifications of it, to obtain the actual cosmological constant via [1] [2] [19] [20].

First of all we will go back to the idea of use of the HUP to connect between the Pre Planckian regime of space time as seen in **Table 1** to Planckian space time, and from there reference how information from prior black holes as given in **Table 1** may be transported to our present universe. To do this, start first with the following considerations.

From the following model we may have a series of wormholes from a prior to the present universe to connect pre big bang black holes, to filling in the Planckian relic black holes created the present universe.

Future project as to explicitly working in prior Universe white hole linked to present universe black hole, via a special wormhole, for each wormhole linking prior to present universes.

In doing this we should note that we are assuming as a future work that there would be black holes, in our initial configuration, plus a white hole in the immediate pre inflationary regime. Likely in a recycled universe. Reference [22] is what we will start off with its given metric as far as a black hole to white hole solution. *i.e.*

$$dS^2 = -A(r, a)dt^2 + B(r, a)^{-1}dr^2 + g^2(r, a)d\Omega^2 \quad (39)$$

We can perform a major simplification by setting, then

$$A(r, a) = B(r, a) = f(r, a) \quad (40)$$

In doing so, [22] gives us the following stress energy tensor values as give

$$\begin{aligned} T_t^t &= \frac{1}{8\pi} \cdot \left(\frac{1}{g} \cdot (fg' + 2fg'') - \frac{1}{g^2} \cdot (1 - fg'^2) \right) \\ T_r^r &= \frac{1}{8\pi} \cdot \left(\frac{1}{g} \cdot (fg') - \frac{1}{g^2} \cdot (1 - fg'^2) \right) \\ T_\theta^\theta = T_\phi^\phi &= \frac{1}{8\pi} \cdot \left(\frac{1}{g} \cdot (fg' + fg'') + \frac{1}{2} \cdot (f'') \right) \end{aligned} \quad (41)$$

In doing this, we will choose the primed coordinate as representing a derivative with respect to r . Also in the case of black hole to white hole joining, we will be looking at a gluing surface as to the worm hole joining a black hole to white hole given as with regards to a gluing surface connecting a black hole to a white hole which we give as ξ . And $\tilde{n}$ is a quantum gravity index. Note that in [22] the authors often set it at 3, if so then for a black hole, to white hole to worm hole configuration they give

$$g(r, a) = \begin{cases} r^2 + a^2 \left(1 - \frac{r^2}{\xi^2} \right)^{\tilde{n}}, & \text{when } (r \leq \xi) \\ r^2, & \text{when } (r > \rho) \end{cases} \quad (42)$$

In practicality this usually means

$$g(r, a) \equiv r^2 \quad (43)$$

We then make the following connection to energy density in a black hole to white hole system, *i.e.*

$$\begin{aligned} \rho_{\text{black hole white hole wormhole}} &\equiv -T_r^r \\ &\approx \hbar \omega_{\text{black hole white hole wormhole}} \tilde{n}_{\text{black hole white hole wormhole}} \end{aligned} \quad (44)$$

This will lead to, if we use Planck units where we normalize $\hbar$ to being 1, of

$$\begin{aligned} &\tilde{n}_{\text{black hole white hole wormhole}} \\ &= \frac{1}{8\pi} \cdot \left(\frac{1}{g} \cdot (fg') - \frac{1}{g^2} \cdot (1 - fg'^2) \right) \cdot \frac{1}{\omega_{\text{black hole white hole wormhole}}} \end{aligned} \quad (45)$$

Mind you this is a way of stating there would be grounds for a linkage between the different components of prior universes and present universes.

We could choose the exact line element chosen for Equation (39) and my take is that the frequency in the denominator would likely be about 10^{44} Hertz. Placing

an enormous premium on very large Equation (40) values whereas there would be constraints placed upon functions g , as chosen to this analysis.

Having said that, let us delve into the possible HUP applications for a flux between prior to present universes.

Equation (8) and Equation (9) can be reviewed, with Equation (9) being contrasted with Equation (39) and Equation (40)

$$dS^2 = -A(r, a) dt^2 + B(r, a)^{-1} dr^2 + g^2(r, a) d\Omega^2 \Leftrightarrow dS_{5-d}^2 = \frac{L^2}{l^2} dS_{4-d}^2 - \frac{L^4}{l^4} dl^2 \quad (46)$$

The RHS of Equation (46) has a term dS_{4-d}^2 corresponding to the LHS of Equation (46). *i.e.* Equation (39) and Equation (40) correspond to the RHS of Equation (46), *i.e.* via dS_{4-d}^2 which says something very direct about Equation (45). *i.e.* Equation (45) is a 3 + 1 space to time decomposition which is placed within the 4 + 1 decomposition of Equation (9). In other words, the HUP will be deterministically embedded within dS_{5-d}^2 while we examine dS_{4-d}^2 as linked to Equation (39) and Equation (40).

Let us now do a review of candidates for the dS_{4-d}^2 HUP first and its linkages to say **Table 1** information.

We will now give a first order estimate as to calculation of $\hbar$ bar, *i.e.* isolate the actual spatial length, for the creation of a present day $\hbar$ bar Planck's constant. To do this look at

$$\Delta x \Delta p \geq \hbar + \frac{l_{\text{Planck}}^2}{\hbar} \cdot (\Delta p)^2 \quad (47)$$

Then THE FOLLOWING ARE EQUIVALENT by [1] [2] [23].

The idea would be that the Planck constant, $\hbar$ bar would be formulated as of the present day value. Also, the modification for the string length, would have

$\Delta x|_{\min} \sim 10^\beta l_{\text{Planck}}$, so then

$$\begin{aligned} \& \Delta x|_{\min} \Delta p \approx \hbar + \frac{l_{\text{Planck}}^2}{\hbar} \cdot (\Delta p)^2 \\ \& \hbar^2 - \hbar \Delta x|_{\min} \Delta p + l_{\text{Planck}}^2 \cdot (\Delta p)^2 \approx 0 \\ \hbar & \approx \frac{\Delta x|_{\min} \Delta p}{2} \cdot \left(1 + \sqrt{1 - 4 \frac{l_{\text{Planck}}^2}{(\Delta x|_{\min})^2}} \right) \\ \hbar & \approx \frac{\Delta x|_{\min} \Delta p}{2} \cdot \left(1 + \sqrt{1 - 4 \cdot 10^{-2\beta}} \right) \\ & \approx \Delta x|_{\min} \Delta p \cdot \left(1 - \frac{2}{10^{2\beta}} \right) \end{aligned} \quad (48)$$

Then,

$$\begin{aligned} \text{if } \Delta p & \sim N_{\text{graviton}} \cdot m_{\text{graviton}} \cdot c \\ \hbar & \approx \Delta x|_{\min} \cdot N_{\text{graviton}} \cdot m_{\text{graviton}} \cdot c \cdot \left(1 - \frac{2}{10^{2\beta}} \right) \\ \Delta x|_{\min} & \approx \frac{\hbar}{N_{\text{graviton}} \cdot m_{\text{graviton}} \cdot c \cdot \left(1 - \frac{2}{10^{2\beta}} \right)} \end{aligned} \quad (49)$$

This should be greater than a Plank length, mainly due to the situation of

$$\left(1 - \frac{2}{10^{2\beta}}\right)^{-1} \sim 1 + \frac{2}{10^{2\beta}} \quad (50)$$

We assume, here that this will be occurring in an interval of time approximately the value of Planck time given by

$$t(\text{initial}) \sim \hbar / \rho(\text{initial}) \cdot V(\text{initial}) \sim \frac{\hbar}{\left(\frac{m_{\text{Planck}}}{l_{\text{Planck}}^3}\right)} \left(\frac{\hbar}{N_{\text{graviton}} \cdot m_{\text{graviton}} \cdot c \cdot \left(1 - \frac{2}{10^{2\beta}}\right)} \right)^{-3} \quad (51)$$

We argue this time is equivalent to Planck time. A time unit which can be scaled to 1.

This leads us to the precursor of entanglement *i.e.* HUP arguments for a bound to initial energy as given below.

7. Final Application of the Uncertainty Principle to Consider. FWIW *i.e.* Information Transfer from a Prior to a Present Universe

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early universe will keep

$$\delta g_{tt} \neq g_{tt} = 1 \quad (52)$$

In fact, we have that from Giovannini [8], that if ϕ is a scalar function, and $a^2(t) \sim 10^{-110}$, then if

$$\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1 \quad (53)$$

Then, there is no way that an early universe HUP is going to come close to $\delta t \Delta E \geq \frac{\hbar}{2}$. *i.e.* it depends assuming time is for all purposes fixed at about Planck time to isolate V_0 .

I.e. for the sake of argument, in the near Planckian regime, we can figure that Equation (53) will have as far as evaluation of the argument the following configuration, *i.e.*

$$a(t) \approx a_{\text{initial}} \cdot \left(t/t_p\right)^v \quad (54)$$

Given this we will be looking at

$$\Delta x \Delta p \geq \frac{\hbar}{\delta g_{tt} = \left[a_{\text{initial}} \cdot \left(\frac{t}{t_p}\right)^v \right]^2 \left[\ln \left(\sqrt{\frac{8\pi G V_0}{v \cdot (3v-1)}} \cdot t \right)^{\sqrt{\frac{v}{16\pi G}}} \right]} \quad (55)$$

Then eventually we obtain by comparing our Equation (55) with the HUP given in [3] and [23] and [24] that we have

$$V_0 \cong \left[\frac{v \cdot (3v-1)}{8\pi} \right] \cdot \left[\exp \left(\frac{16\sqrt{\pi}}{\sqrt{v}} \cdot \frac{1}{a_{\min}^2 \cdot (t/t_p)^2} \right) \right] \cdot \left(\frac{1}{1+2\tilde{\gamma} \frac{\partial C}{\partial V}} \right)^2 \quad (56)$$

So then we are now doing an Evaluation of Equation (56) if we are near Planck time. Two limits 1st, what if we have expansion of the scale factor initially at greater than the speed of light?

Set $v \approx 10^{88}$ and then we can obtain if we are just starting off inflation say $a_{\min}^2 \approx 10^{-44}$. Then using [1] [2] and [23] which references Nye [24]

$$V_0 \cong [10^{176}] \cdot [\exp(16\sqrt{\pi})] \cdot \left(\frac{1}{1+2\tilde{\gamma} \frac{\partial C}{\partial V}} \right)^2 \quad (57)$$

If we wish to have a Planck energy magnitude of the V_0 term, we will then be observing

$$V_0 \cong [10^{176}] \cdot [\exp(16\sqrt{\pi})] \cdot \left(\frac{1}{1+2\tilde{\gamma} \frac{\partial C}{\partial V}} \right)^2 \xrightarrow{2\tilde{\gamma} \frac{\partial C}{\partial V} \approx [10^{88}]} o(1) \quad (58)$$

I.e. the system complexity will become effectively almost infinite, and this will be explained in the conclusion by use of [23] [24]

$$2\tilde{\gamma} \frac{\partial C}{\partial V} \approx [10^{88}] \Rightarrow V_0 \cong o(1) \quad (59)$$

On the other hand, if there is a very small value for $2\tilde{\gamma} \frac{\partial C}{\partial V}$ we can see the following behavior for Equation (57), namely

$$2\tilde{\gamma} \frac{\partial C}{\partial V} \approx o(1) \Rightarrow V_0 \cong [10^{176}] \quad (60)$$

i.e. low complexity in the measurement process will then imply an enormous initial inflaton potential energy.

Secondly, now what if we have instead $v \approx 1$

$$V_0 \cong \left[\frac{1}{4\pi} \right] \cdot \left[\exp \left(\frac{16\sqrt{\pi}}{a_{\min}^2 \cdot (t/t_p)^2} \right) \right] \cdot \left(\frac{1}{1+2\tilde{\gamma} \frac{\partial C}{\partial V}} \right)^2 \quad (61)$$

The threshold if $2\tilde{\gamma} \frac{\partial C}{\partial V} \approx [10^{88}]$ *i.e.* a huge value for initial complexity would be effectively made insignificant in cutting down the initial inflaton lead to

$$\exp \left(\frac{16\sqrt{\pi}}{\sqrt{v}} \cdot \frac{1}{a_{\min}^2 \cdot (t/t_p)^2} \right) \xrightarrow{a_{\min}^2 \approx 10^{-88}} V_0 \cong \exp(10^{88}) \quad (62)$$

i.e. we come to the seemingly counter Intuitive expression that the initial inflaton potential would still be infinite if we used Equation (61) in Equation (57)

Having said that does it make sense to ascertain the following as far as early universe geometry? *i.e.* we say its not so simple. *i.e.*

The value of Equation (59) and Equation (62) will be commensurate with Equation (8). *i.e.* two very different values where we should treat Equation (59) value of 1 as being commensurate with 1 being Planck energy value, as a normalization factor, whereas Equation (62) will be Planck energy times $\exp(10^{88})$ which is unimaginably huge.

Now for the questions this raises. Assuming scaling Planck time to be of the value of 1, using Planck Units, we then see that we are really looking at a HUP [23]

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \quad (63)$$

Unless $\delta g_{tt} \sim O(1)$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early universe will keep

$$\delta g_{tt} \neq g_{tt} = 1 \quad (64)$$

In fact, we have that from Giovannini [8], that if ϕ is a scalar function, as given in [5] and $a^2(t) \sim 10^{-110}$, then if we use [1] [2] [23]

$$\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1 \quad (65)$$

We then have after assuming Planck unit normalization of Planck time, an ENORMOUS value for initial energy which gives credence to the idea of quantum entanglement between a prior to the present universe between pre universe ensemble of black holes to relic black holes in the present universe.

8. Now for Entanglement of Prior Universe Black Holes to Primordial Black Holes in the Present Universe

We begin first with a BEC condensate given by [19] [23] [25] as to BEC treatment of black holes, and entropy

$$\begin{aligned} m &\approx \frac{M_p}{\sqrt{N_{\text{gravitons}}}} \\ M_{BH} &\approx \sqrt{N_{\text{gravitons}}} \cdot M_p \\ R_{BH} &\approx \sqrt{N_{\text{gravitons}}} \cdot l_p \\ S_{BH} &\approx k_B \cdot N_{\text{gravitons}} \\ T_{BH} &\approx \frac{T_p}{\sqrt{N_{\text{gravitons}}}} \end{aligned} \quad (66)$$

This will lead to the following as to [26]

$$S_{BH} \propto 2n \cdot \log 2 - \frac{3}{2} \cdot \log(n) \propto k_B N_{\text{Gravitons}} \quad (67)$$

n here is the number of qubits, whereas $N_{\text{Gravitons}}$ is the number of gravitons per primordial black hole meaning that there are approximately 2 qubits per grav-

iton, if n and N are large.

So then that we get a representation of a graviton, in terms of wave functions. To do this, look at [27] with say

$$\Psi_{BB} \doteq \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} \quad (68)$$

$$\psi_+ = \sqrt{\frac{4\pi\alpha}{\hbar c}} \cdot e_+(k) g(2k) \cdot \exp(-i\omega t) \quad (69)$$

Similarly

$$\psi_- = \sqrt{\frac{4\pi\alpha}{\hbar c}} \cdot e_-(k) g(-2k) \cdot \exp(-i\omega t) \quad (70)$$

Then

$$\Psi_{BB} \doteq \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} \doteq \psi_+ \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \psi_- \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (71)$$

Using [28], page 35 the term $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ has a probability of measurement of $|\psi_+|^2$,

whereas $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ has a probability of measurement of $|\psi_-|^2$, and this can be compared with the expression given in [29] where we look at a total entropy as stated as [29] [30] [31] where we have in Loop quantum gravity

$$S_{BH} = \frac{k_B \ln \mathbb{N}(A, \gamma)}{\beta_{B1}} \quad (72)$$

where $\mathbb{N}(A, \gamma)$ is the number of Microstates having the Horizon area A (black hole) and β_{B1} is the Barbero-Immirzi parameter given in [30] and [31]. To first approximation we can assert the following equivalence as given by the following rough equivalence, *i.e.*

$$k_B \ln \mathbb{N}(A, \gamma) = k_B N_{\text{gravitons}} \quad (73)$$

This gives a rough equivalence between Equation (72) and Equation (66) where we are assuming that we have wave function for a single graviton with an integrand expression of Equation (71), so let us examine if this is in any way in reference to an entangled state.

First of all, in the language of Fuzzballs as given by [32] we have the following relations, *i.e.*

$$\begin{aligned} N_{\text{states}}(M) &\sim \exp(S_{BH}) \\ \text{i.e.} \\ \text{if } \ln(N_{\text{states}}(M)) &= N_{\text{gravitons}} \\ S_{BH} &\approx \ln(N_{\text{states}}(M)) \\ \xrightarrow{\ln(N_{\text{states}}(M)) = N_{\text{gravitons}}} &N_{\text{gravitons}} \end{aligned} \quad (74)$$

Before proceeding it is important to look at [33] and [34] to put in context what is said next *i.e.* especially when it comes to the matter of entanglement.

If so then we need to examine the behavior of individual gravitons given by Equation (71) if we wish to examine if we have entanglement. So how do we bridge between Equation (71) and the Qubit treatment of a black hole which may have thousands of gravitons on its surface A of an Event Horizon?

To do so we consider a bridge between a two-qubit system which may be appropriate between a graviton, and then the three-qubit system for a black hole,

Going back to [26], we have the following state as given as a reference for an entangled state, *i.e.*

Linking a two-qubit state to a three-qubit state typically involves utilizing a controlled-NOT (CNOT) gate to entangle an existing pair with a third, initialized qubit. This process expands the system's dimensionality, transforming a two-qubit state (e.g., a Bell pair) into a three-qubit state (e.g., a GHZ or W state).

Reference [26] has the following as the simplest example of a two-qubit entangled state

$$\Psi = \frac{1}{\sqrt{2}} \cdot \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \quad (75)$$

whereas we can look at teleportation of an entangled state from a source A to a source, B as given by [33] and [34] and [35] and [36] we have the following.

So what can we say about the black holes given in **Table 1** which we wish to ship from a Pre Planckian Universe, to our present universe?

1) The process involves linking a two-qubit state, with respect to Gravitons, to a three bit state involving a black hole. The states are assumed to be entangled, and we wish to use quantum Teleportation, in cosmology to dump information as to the physical states we wish to analyze.

2) We reference [35] as to teleportation on Photons. *i.e.* this is a tutorial as to the very complicated Quantum teleportation of information given in the black hole case

3) Black hole information teleportation is cited in [36] and is commensurate as to the following from [36]

$$i\hbar\partial_t\Psi_{BH} = H\Psi_{BH} \quad (76)$$

This is where

$$H = H_{\text{fields}} + H_{QMM} + H_{\text{int}} \quad (77)$$

Here

$$\Psi_{BH}(0) = |\Psi_{\text{fields}}(0)\rangle \otimes |\Psi_{QMM}(0)\rangle = \sum_n c_n |n\rangle_{\text{fields}} \otimes |0\rangle_{QMM} \quad (78)$$

Also

$$\sum_n |c_n|^2 = 1 \quad (79)$$

Keep in mind that Equation (13) and Equation (14) of reference [36] are crucially important and that

$$\Psi_{BH}(t) \approx \left(1 - \frac{i}{\hbar} \cdot H \right) \Psi_{BH}(0) \quad (80)$$

Here is the point As to Equation (77) From [36], page 10 of the article
Quote

The QMM hypothesis provides a structured framework for understanding how information is preserved during black hole formation and evolution. In this section, we formalize the information encoding process that occurs during black hole absorption, illustrating it with mathematically detailed models. Consider a black hole formed by the collapse of matter represented by a real scalar field $\hat{\phi}(x)$. As matter collapses, intense gravitational effects near the event horizon generate strong interactions between the scalar field and the QMM. According to the QMM hypothesis, these interactions lead to quantum imprints embedded within the space-time quanta at or near the event horizon. To mathematically model this, we represent the imprint left on the QMM by an operator I_x dependent on the scalar field

$$I_x = g \cdot \hat{\phi}(x) \quad (81)$$

End of quote

This will engage some of the material seen in [33] [34] [35] [36] and also [37].

This leads to an interaction hypothesis of a Hamiltonian term shaped when the scalar fields are really valued as of the form

$$H_{\text{int}} = 2g \sum_x \hat{\phi}(x) \otimes \hat{\phi}(x) \quad (82)$$

where g is a coupling constant defining the interaction strength between the field and the QMM at point x . The Hamiltonian governing the interaction between the scalar field and the QMM is H_{int} , which determines the information exchange and can be expressed as in Equation (82).

Whereas H_{fields} is seen in the following

$$\begin{aligned} H_{\text{fields}} &= \int d^3x \cdot \left[\frac{1}{2} \cdot \left[\left(\partial_x \hat{\phi}(x) \right)^2 + \left(\nabla \hat{\phi}(x) \right)^2 + 2 \cdot V_{\text{Potential}} \left(\hat{\phi}(x) \right) \right] \right] \\ \partial_x \hat{\phi}(x) &= \text{conjugate momentum} \\ V_{\text{Potential}} \left(\hat{\phi}(x) \right) &= \text{Potential energy density} \end{aligned} \quad (83)$$

Also

$$\begin{aligned} H_{\text{QMM}} &= \sum_x \hat{h}_x + \sum_{x,y} \hat{h}_{x,y} \\ \hat{h}_x &= \text{operates on individual quanta } H_x \\ \hat{h}_{x,y} &= \text{interaction between neighboring quanta } H_x \text{ and } H_y \end{aligned} \quad (84)$$

There are many more than three qubits which are necessary as to making teleportation possible between black holes in a prior to black holes in a present universe, *i.e.* and the reason for this is in the representation of Equation (77). In particular looking at just the black hole qubit representation. In particular from [37] we have the usual simplified version of two and three qubits as given in [37] *i.e.* as given as

One Qubit = A line

Two Qubits= A square

Three Qubits, = A cube

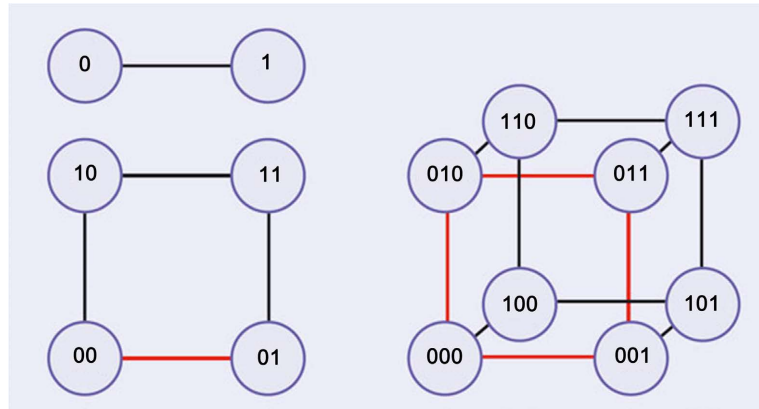


Figure 2. From [37] with the qubits in line with string theory representation of black holes.

Here note that [37] has its **Table 1** which we reproduce as our **Table 2**, which is in [37] represented as

Table 2. A GHZ state corresponds to a four-charge black hole in 4-dimensional space-time x_0, x_1, x_2, x_3 coming from four D3-branes each wrapping three of the six extra dimensions $(x_4, x_6, x_8), (x_4, x_7, x_9), (x_5, x_6, x_9)$ and (x_5, x_7, x_8) .

x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	brane	ABC
x	0	0	0	x	0	x	0	x	0	D3	000
x	0	0	0	x	0	0	x	0	x	D3	011
x	0	0	0	0	x	x	0	0	x	D3	101
x	0	0	0	0	x	0	x	x	0	D3	110

This is in [37] whereas it is for a four Qubit representation for a Black hole. This is interesting but in my view highly oversimplified.

I have nothing particular against this, but in point of fact, I prefer the methodology of [36] which when coupled with the Two qubit representation of a graviton as given in Equation (67) to Equation (69) seems to point to a black hole as on its horizon surface characterized by Equation (67) to Equation (69) as referencing to an ENSEMBLE of gravitons on the surface of a black hole *i.e.* the event Horizon of a black hole, as providing information which may be teleported by referencing a black hole in the Pre Planckian state of the universe as given in **Table 1** of our article.

The entanglement of an ensemble of graviton states on the event horizon of a primordial black hole, may be entangled by the implied rules given in Equation (76) to Equation (84).

Recall what we wrote earlier

Quote

Linking a two-qubit state to a three-qubit state typically involves utilizing a controlled-NOT (CNOT) gate to entangle an existing pair with a third, initialized

qubit. This process expands the system's dimensionality, transforming a two-qubit state (e.g., a Bell pair) into a three-qubit state (e.g., a GHZ or W state).

End of quote

We would do the same, *i.e.* give more additional higher qubit states, from say a three Qubit state as given in **Figure 2**, in a cube, to a (higher dimensional) qubit state, while trying to adhere to the simplicity in a GHZ state initially.

We have referenced [38] in particular, and this article, furthermore convinces the author that it is best to reference the black hole as an ensemble of gravitons, while referencing each Graviton as emitted from a candidate black hole emitter, as a quantum computing problem, which is teleported from a prior to the present universe via the input of **Table 1**, as given earlier in our article.

In particular, we can use the [22] reference for a black hole to white hole transformation for coming up with the initial frequency associated with early universe primordial black holes, to set conditions for energy frequency spectrum for emitted into our Plackian era teleported information of gravitons associate from a prior universe to be emitted by primordial black holes in the early present universe, according, once again by

Quote

$$\tilde{n}_{\text{black hole white hole wormhole}} = \frac{1}{8\pi} \cdot \left(\frac{1}{g} \cdot (fg') - \frac{1}{g^2} \cdot (1 - fg'^2) \right) \cdot \frac{1}{\omega_{\text{black hole white hole wormhole}}} \quad (85)$$

Mind you this is a way of stating there would be grounds for a linkage between the different components of prior universes and present universes.

Understanding this thoroughly may allow for a great refinement of entropy associated by string theory arguments for black holes, as given in [39].

I.E. MORAL of story, if one is examining black holes from a prior to present universe, it is probably better to refer to the information encoded in gravitons, in entangled qubits as a bridge between prior to present universes, as in **Table 1**, where we can roughly track the number of black holes as an ensemble of gravitons via the methodology implied by my Equation (85) which I link to reference [22] *i.e.* to keep the worm hole business of transfer of black hole information separate initially to the qubit interpretation of gravitons. With the gravitons in question associated with black holes, before the present universe, to the early Planck universe.

End of quote

9. Examination of Toroidal Geometry in Terms of Measurement of Gravitons

What we are doing is to enfold this linkage of graviton production with toroidal geometry and in doing so in [39] we come up with the following.

Then let's go to calculating for both the Toroidal geometry and the Friedman Universe *i.e.* if $T_{00} \neq 0$ then by Hooper, [39] as well as [40] we have then that if

we assume [41]-[44]

$$\begin{aligned}
 T_{00} &= \frac{M_{pl}^2}{8\pi} \cdot \left(R_{00} - \frac{R_s \cdot g_{00}}{2} \right) \\
 \text{which} &= \frac{M_{pl}^2}{8\pi} \cdot \left(-\frac{R_s}{2} \right) \\
 = (\text{Toroid}) &= \frac{M_{pl}^2}{8\pi} \left(\frac{\cos \phi}{\alpha (\beta + \alpha \cos \phi)} \right) \\
 = (\text{Toroid}) &= \frac{M_{pl}^2}{8\pi} \left(\frac{\cos \phi}{\alpha (\beta + \alpha \cos \phi)} \right) \\
 \approx (\text{Friedman}) &= \frac{3M_{pl}^2}{8\pi} \left(\frac{1.66\sqrt{g_*} \cdot T_{vac}^2}{M_{pl}} \right)^2
 \end{aligned} \tag{86}$$

This is with regards to the following geometry from [45] (Figure 3, Figure 4).

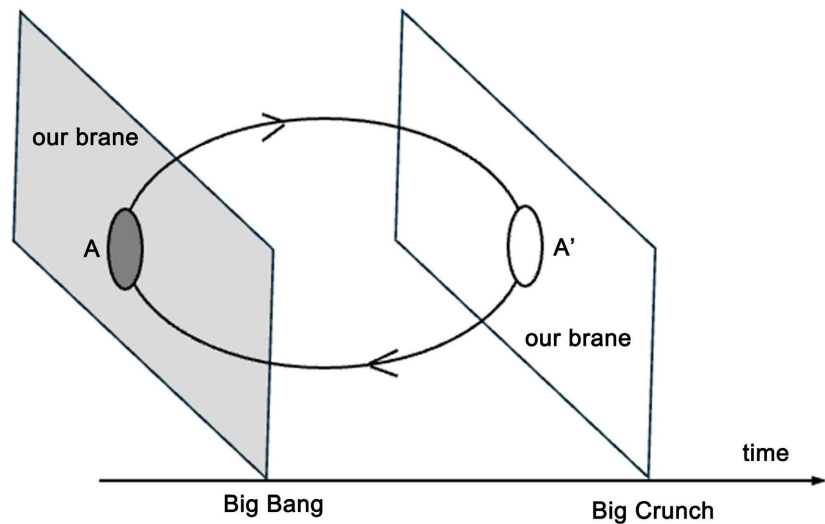


Figure 3. *i.e.* Brane cosmology used as far as toroidal geometry, in [41]-[44].

As well as using from [41]-[44].

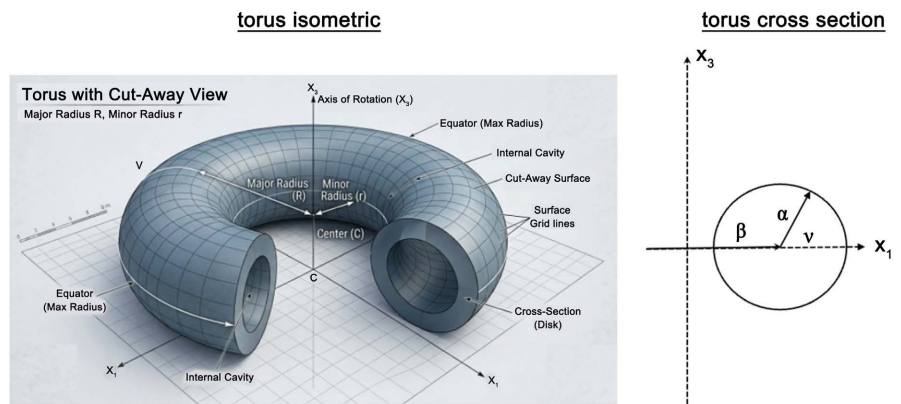


Figure 4. Toroidal geometry as to universe, in [41] [44].

We will use this in future geometry interpretation of Graviton production in prior to present universe construction to give experimental credence to the ideas given in this document.

In particular, the temperature T , as given is linkable to Equation (66) which we wish to seek to obtain experimental verification of in future publications.

We will for completeness of this record include a dictionary as to entropy and black holes. One of our further projects will be to incorporate the linkage of black hole entropy, in its complexity, with the material we have outlined as to entanglement and also the quantum number n . If we can integrate this thoroughly, we believe it will be highly relevant to Tokamak machine work we do later.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A. Further Thoughts on Entropy

How we can interpret this paper as to black holes and entropy, *i.e.* this is our methodology. This is a dimensional scaling argument. And is how we introduce quantum number n , rather than particle count, N (number of gravitons?) as a way to delineate.

Keep in mind that in this segment of the text, we have n as a quantum number.

How we wish to interpret how to interpret the rise of entropy from a black hole and entropy of the early universe. Note that [11] has an alternative expression for the early universe which can be written as, if α is the scale factor, of radii r_H for a horizon radius, with

$$S = \frac{0.3r_H^2}{a^2} \quad (A1)$$

And [11]

$$r_H = \sqrt{\frac{3}{\Lambda}} \quad (A2)$$

Here, the cosmological constant as given by [45] by Park, *et al.* is of the form with T the background temperature, as given by

$$\Lambda \propto T^{\tilde{\beta}} \sqrt{\frac{3}{\Lambda}} \Rightarrow r_H = \sqrt{\frac{3}{\Lambda}} \cong \sqrt{3} T^{-\tilde{\beta}/2} \Rightarrow S \approx 0.3 \cdot 3 T^{-\tilde{\beta}} / a^2 \quad (A3)$$

Above almost scales exactly as having the universe with entropy proportional to one over the temperature to the minus beta power times one over the square of the scale factor for early universe conditions.

To make it more revealing, note from [11] that one can write

$$S_{\text{Early Universe}} \sim 16\pi\alpha^2 n \quad (A4)$$

Here also, from [11] we have an energy expression from as well as employing the string theory result of

$$\begin{aligned} S_{\text{Early Universe}} &\sim 16\pi\alpha^2 n \sim T^{-\tilde{\beta}} / a^2 \Rightarrow T^{-\tilde{\beta}} \propto 16\pi\alpha^2 n a^2 \\ &\Rightarrow T \approx \frac{1}{(16\pi\alpha^2 n a^2)^{\tilde{\beta}}} \end{aligned} \quad (A5)$$

Assuming we have a condition for which is in a short period of time a constant in the early universe and that we have for H the initial Hubble expansion parameter, and the time, then if what is below, is

$$a \sim a_0 \exp(H \cdot t) \sim a_0 (\text{Planck time}) \quad (A6)$$

Then in the regime of Planck time we are looking at a quantum excitation temperature we write as

$$T \approx \frac{1}{(16\pi\alpha^2 n a^2)^{\tilde{\beta}}} \sim \left[\frac{(1-H \cdot t)^{\tilde{\beta}}}{a_0^{\tilde{\beta}}} \right] \cdot \frac{1}{n^{\tilde{\beta}}} \propto \frac{1}{n^{\tilde{\beta}}} \quad (A7)$$

The proportionality of temperature, T , in the Planck time regime is saying that

as n is “nucleated” quantum number n or created, that the temperature scales down. Note that beyond the Planck interval of time, one will be beginning to look at a time dependence, according to the coefficient $\left[\frac{(1-H \cdot t)^\beta}{a_0^\beta} \right]$ with H a constant. Before then the dominant effect of scaling down will be on the creation of $\frac{1}{n^\beta}$ contributions to dropping of the temperature.

In doing so we establish a relationship between n and initial spatial regions of Planckian space we can write as given in the following quantity as given in [1] [2] that as for a huge initial degree of freedom value of $g_* \approx 100$ for pre Planckian to Planckian transitions showing large quantum number values so that the correspondence principle in cosmology would hold would be to have using energy as given in E

$$n^2 (\text{quantum number}) = \frac{2 \cdot V_{\text{space volume}} \cdot m_g \left(1.66 \times \sqrt{g_*} \cdot T_{\text{max temperature}}^2 \right)^2}{\pi^2 \hbar^2} \quad (\text{A8})$$

where we are assuming having an almost one to one connection between g_* and $d(\text{dim})$.

$$\rho = \frac{E(\text{cylindrical well energy})}{V_{\text{Volume}}} = \frac{n^2 (\text{quantum number}) \cdot \pi^2 \hbar^2}{V_{\text{Volume}} \cdot m_g \cdot (r_{\text{radius well}})^2} \quad (\text{A9})$$

For there to be an equality, which would be a necessary condition for having a correspondence principle in Cosmology, *i.e.* to have quantum effects for high numbers, *i.e.* $n_{\text{quantum number}}^2 \gg 1$ one would likely have, even if we state g_* is a degree of freedom, would be that the stated dimensional values of inputs into a very large value for d_{dim} for inputs into the Pre Planckian state, prior to emergence into Planckian cosmology conditions would have to be an extremely large number. *i.e.* we would be looking for conditions in the pre Planckian space time for which $n_{\text{quantum number}}^2 \gg 1$ due to an enormous value for d_{dim} .

Having said that, we will also use the following for black holes, numbered as N , as opposed to quantum number n .

We begin first with a BEC condensate given by [1] [2] [25] as to BEC treatment of black holes, and entropy

$$\begin{aligned} m &\approx \frac{M_P}{\sqrt{N_{\text{gravitons}}}} \\ M_{BH} &\approx \sqrt{N_{\text{gravitons}}} \cdot M_P \\ R_{BH} &\approx \sqrt{N_{\text{gravitons}}} \cdot l_P \\ S_{BH} &\approx k_B \cdot N_{\text{gravitons}} \\ T_{BH} &\approx \frac{T_P}{\sqrt{N_{\text{gravitons}}}} \end{aligned} \quad (\text{A10})$$

This will lead to the following as to [1] [2] that for individual black holes

$$S_{BH} \propto 2n \cdot \log 2 - \frac{3}{2} \cdot \log(n) \propto k_B N_{\text{Gravitons}} \quad (\text{A11})$$

We assert that the quantum number n given above, would have to tie into Eq. (9), and that we can consider for individual black holes, a very complicated by the following value of

$$n^2 (\text{quantum number}) = \frac{2 \cdot V_{\text{space volume}} \cdot m_g \left(1.66 \times \sqrt{g_*} \cdot T_{\text{max temperature}}^2 \right)^2}{\pi^2 \hbar^2}$$

Furthermore, in terms of black holes we may have an energy relationship given as follows:

I wish to Thank Christian Corda for bringing this question to my attention. The answer is maybe, but if we do that we can assume that the modeling of E , as a function of temperature T may be commensurate for the energy levels of a spherical infinite square well, *i.e.* see this, [1] [2] We will assume the spherical, zero angular momentum case if we do this, so then we have if the radius of the well has zero inside the well and an infinite potential barrier value just outside, that to first approximation we have that. By [1] [2] we have an absolute magnitude of a pre Planckian energy value which may be thought of as

$$E_{n,0} = \frac{n_{\text{quantum}}^2 \pi^2 \hbar^2}{2m \cdot r_{\text{spherical well}}^2} \quad (\text{A12})$$

We state categorically that this is an absolute magnitude approximation and that in actuality it may be negative, in sign.

Here, we are making the approximation that m , in this last set of calculation is the same as the mass of a graviton, and that the term a , as given above is less than or equal to Planck length, if the resulting n , as used in Equation (13) is large, with that temperature dependence, we may see the start of classical to quantum correspondence, for large n , and a tie in that way to the Weak correspondence principle. What we can do is to look also at a relation given by Kerson Huang, in [46], as well as page 481 of the Hubble parameter given in [40] where we have normalized the Planck mass to have a value of 1. If so then in the Pre Planckian to Planckian regime of space-time we may have [1] [2]

$$\frac{8\pi G \rho}{3\tilde{H} (\text{Hubble parameter})^2} - \frac{\kappa_{\text{Flatness parameter}}}{\tilde{H} (\text{Hubble parameter})^2 \cdot a(t)^2} = 1 \quad (\text{A13})$$

Doing this would lead to if we say the space volume is proportional to Planck length cubed a phenomenological linkage to n , quantum number, and N

Furthermore if we have a NEGATIVE initial energy for a pre Planckian state of the universe, we can make the following deductions

This leads to the open question we frame as follows.

Can this tie in with early universe e folds? Here e folds are between 55 to 60 in value

E folds in cosmology are a way of delineating if we have enough expansion of

the universe is in line with inflation. in order to solve the most important cosmological problems. As seen in [11] we can have inflation. in order to solve the most important cosmological problems. As seen in [1] [2] we can have

$$\mathbb{N}(\text{e-folding}) = -\int dt H(\cos m) \quad (\text{A14})$$

Here, $H(\cos m)$ a value of the Friedman equation, and be defined via that the potential energy, V , of initial inflation is initially over shadowed by the contributions of the Friedman equation, H , at the onset of inflation. Then

$$\mathbb{N}(\text{e-folding}) = 55 - 60 \quad (\text{A15})$$

What we wish to explore will be if Equation (16) above is consistent with

$$N(\text{Entropy}) \approx 10^{58} \equiv 10^{\mathbb{N}} \Leftrightarrow \mathbb{N} \approx 58 \quad (\text{A16})$$

Doing so may involve use of the Corda articles, as given in [47] [48]

Now for foundational treatment as to if we may have an influence of the 5th dimension in our problem.

Wesson, [49] has a procedure as far as a five-dimensional uncertainty principle which is written as, if $n \sim L/l$. Where L is for 4th dimensions, and l is a five dimensional representation, so we have

$$dS_5^2 = n^2 ds_4^2 - n^4 dl^2 = \left[\frac{L}{l} \right]^2 ds_4^2 - \left[\frac{L}{l} \right]^4 dl^2 \quad (\text{A17})$$

Then we have an uncertainty principle in 5 dimensions as by Wesson [49] for which we can do if we look at the zeroth contribution as given in the deterministic structure [1] [2] [49]

$$|dp_\alpha dx^\alpha| \equiv \frac{dn^2}{n} \xrightarrow{\alpha \rightarrow 0} |dEdt| \approx |\Delta E \Delta t| \equiv \frac{dn^2}{n} \equiv n \cdot \ln n \quad (\text{A18})$$

Using a numerical expansion of the form from CRC tables [50]

$$n \ln n = n \cdot (n-1) - \frac{n}{2} \cdot (n-1)^2 + \frac{n}{3} \cdot (n-1)^3 + \dots \quad (\text{A19})$$

Up to cubic roots we obtain one real root and 2 conjugate complex roots of, if we use minimum uncertainty of $\Delta E \Delta t = \hbar = 1$ and set $c = 1$, we have then one real root, and two conjugate complex roots, so that

$$n_1 \approx 1.54715 \quad (\text{as real root for a cubic equation for } n) \quad (\text{A20})$$

$$n_{2,3} \approx 0.426413 \pm 1.2242i \quad (\text{as two complex conjugate roots for } n) \quad (\text{A21})$$

If so for the real case, of n , we have about the Planckian regime we look at

$$l = \frac{l_{\text{Planck}}}{1.54715} \quad (\text{A22})$$

We will then look at the consequences of the real root, first, in terms of variation of minimum time step before going to other cases, but for the record, we have then the weird case of, for real root n in Equation (22) that to other cases, but for the record, we have then the weird case of, for real root n in Equation (22) that

$$\Delta t \approx -\frac{0.845184}{\Delta E} \quad (\text{A23})$$

Having said, this we are assuming that the ΔE term is negative, in line with assuming we are working with a potential well.