

Galactic Rotation Curves in the 4DEU Framework: Explicit Derivation of the Extra-Velocity Law from 3D Spatial Curvature Induced by the Gravitational Constraint

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Abstract

The present work serves as a mathematical addendum to the previous analysis of galactic rotation curves within the Four-Dimensional Electromagnetic Universe (4DEU) framework by Maglione, D. (*Journal of High Energy Physics, Gravitation and Cosmology*, 12, 431-470, 2026), in which the extra velocity component was fitted through the law $v_{cve}(r) = V_0 \left(r/R_{ref} \right)^{\epsilon/2}$ (Equation (12) therein). In 4DEU, this extra component is interpreted not as dark matter, but as the effect of purely three-dimensional spatial curvature induced by the gravitational constraint that locally blocks the 3D-only cosmic expansion in gravitationally bound systems. Although the kinematic law, its physical interpretation, and the corresponding effective-density/curvature profiles were already presented in the above-mentioned 4DEU rotation-curve paper by Maglione, the reverse analytical connection—namely, how the same law can be recovered starting from the outer-curvature behavior—was not written out there in full mathematical detail, in order to avoid excessively burdening the paper. In the present addendum, that derivation is given explicitly. Starting from the local relation between the three-dimensional Ricci scalar ${}^{(3)}R$ and the effective density and combining it with the standard expression linking effective density to the circular velocity profile, it is shown step by step how the curvature-induced term leads to the same kinematic law previously adopted in the fits. The derivation therefore makes explicit that, within the 4DEU framework, the law used to reproduce the extra-velocity component of galactic rotation curves follows from the 3D spatial curvature generated by the gravitational constraint.

Keywords

Four-Dimensional Electromagnetic Universe (4DEU), Galactic Rotation

1. Introduction

The present work is directly connected to Maglione's previous 4DEU rotation-curve analysis [1]. Galactic rotation curves are one of the main observational manifestations of the broader dark matter problem [2] [3]. Their observed near-flat behavior in the outer regions of galaxies has long been interpreted as evidence that the gravitational field inferred from visible baryonic matter alone is insufficient, within the standard framework, to account for the measured orbital velocities [2] [3]. This broader context is supported not only by rotation-curve observations, but also by independent lines of evidence, such as gravitational lensing in galaxy clusters [4] and cosmological constraints from the cosmic microwave background [5].

Various alternative approaches have been proposed in the literature to address this discrepancy without introducing standard dark matter halos, including modified dynamics and extended-gravity frameworks [6] [7]. In the broader context of alternative gravitational perspectives and tests of the standard framework, see also [8]. In this broader landscape, the present manuscript is not intended as a general review of all such approaches, but only as a mathematical addendum aimed at providing the explicit derivation of the specific extra-velocity law previously used in the 4DEU rotation-curve analysis [1].

In the previous work devoted to galactic rotation curves within the framework of the Four-Dimensional Electromagnetic Universe (4DEU) theory [1], it was shown that the observed kinematic behavior in the outer regions of galaxies can be reproduced without introducing dark matter by interpreting the extra component of the circular velocity as the effect of purely three-dimensional spatial curvature induced by the gravitational constraint that locally blocks the 3D-only cosmic expansion inside gravitationally bound systems. Accordingly, the extra component was modeled through the law:

$$v_{cve}(r) = V_0 \left(\frac{r}{R_{ref}} \right)^{\varepsilon/2}$$

corresponding to Equation (12) in [1], and this expression was successfully used to fit the rotation curves of a large sample of galaxies from the SPARC database.

In [1], this kinematic law was introduced starting from the equivalent constant-logarithmic-slope form

$$\frac{d \ln v_{cve}^2}{d \ln r} = \varepsilon$$

corresponding to Equation (6) in [1]. This relation was adopted as an operational ansatz on the outer radial domain, where the extra component dominates over the baryonic contribution. Although this formulation proved empirically effective and

consistent with the physical interpretation proposed within the 4DEU framework, and although [1] already presented both the fitted kinematic law and the corresponding effective-density and curvature profiles associated with it, the reverse mathematical route linking the outer-curvature behavior to the final law used in the fits was not fully developed there. The reason was simply to avoid overloading a paper primarily focused on the observational and statistical analysis of galactic rotation curves.

For this reason, the present addendum is intentionally limited in scope: it is not meant to provide a general review of the dark matter problem or of all alternative approaches proposed in the literature, but only to make explicit the derivational step that remained implicit in [1].

The present work is motivated precisely by this point and should be regarded as a theoretical and mathematical addendum to [1]. Its aim is neither to repeat the observational analysis nor to modify the numerical results already obtained. Rather, it is to make explicit the inverse analytical route that remained implicit in the physical interpretation and mathematical structure adopted in [1]. In particular, the paper shows how the same kinematic law of the extra velocity component previously used in the rotation-curve fits can be recovered starting from three-dimensional spatial curvature, described through the Ricci scalar ${}^{(3)}R$, in the nearly flat outer regime. In this sense, the present study provides an explicit analytical clarification of a connection already contained in the framework adopted in [1], reconstructing the reverse path from the outer-curvature behavior to the same power-law expression previously used in the fits.

More specifically, the starting point is the local relation, in the static-slicing limit, between the three-dimensional Ricci scalar and the effective density, together with the standard relation linking the effective density to the circular velocity profile under weak-field spherical symmetry. From these relations, one first recovers, in the limit of a perfectly flat outer rotation curve, the isothermal-like behavior $\rho_{\text{eff}}(r) \propto r^{-2}$. The crucial step is then to introduce the simplest local generalization of this law, capable of describing small but finite departures from exact flatness in the outer radial domain. It is shown that this extension leads directly, through explicit integration, to the form

$$v_{\text{cve}}^2(r) = V_0^2 \left(\frac{r}{R_{\text{ref}}} \right)^\epsilon$$

which is equivalent to the law already employed in [1]. Here, V_0 denotes the normalization of the curvature-induced extra velocity component at the reference radius R_{ref} , the same reference radius adopted in [1], chosen as the geometric mean of the fitted radii.

It is important to stress that the present work does not introduce a new physical mechanism beyond that already proposed in [1]. Rather, it makes explicit the mathematical passage required to connect that mechanism to the fitted law. Its purpose is therefore twofold: first, to clarify the analytical foundation of the kinematic law adopted for the fitting of galactic rotation curves; and second, to

strengthen the internal consistency of the 4DEU interpretation, according to which the observed extra component does not correspond to dark matter but to three-dimensional spatial curvature induced by the gravitational constraint.

The next section presents the full derivation step by step, starting from the relation between spatial curvature and effective density and ending with the final extra-velocity law used in the previous work.

2. Derivation of the 4DEU Extra-Velocity Law from 3D Spatial Curvature Induced by the Gravitational Constraint

In the previous 4DEU analysis of galactic rotation curves [1], the curvature-induced extra velocity component was fitted through the law:

$$v_{cve}(r) = V_0 \left(\frac{r}{R_{ref}} \right)^{\varepsilon/2} \quad (1)$$

which corresponds to Equation (12) in [1]. In that work, the equivalent logarithmic-slope form

$$\frac{d \ln v_{cve}^2}{d \ln r} = \varepsilon \quad (2)$$

was adopted as the fitting ansatz on the outer, extra-dominated radial domain (Equation (6) in [1]). The purpose of the present section is to provide the explicit reverse mathematical derivation of this kinematic law from the underlying 4DEU physical hypothesis. Specifically, the extra component of the rotation curves is interpreted as arising from purely three-dimensional spatial curvature induced by the gravitational constraint that locally blocks the 3D-only cosmic expansion in gravitationally bound systems.

Within the 4DEU framework, in the local static-slicing limit, the Hamiltonian constraint is considered in the standard 3+1 geometric language of spatial slices and intrinsic curvature [9]. In this limit, it reduces to the relation:

$${}^{(3)}R(r) = \frac{16\pi G}{c^2} \rho_{eff}(r) \quad (3)$$

which is Equation (18) in [1]. Here, ${}^{(3)}R(r)$ denotes the three-dimensional Ricci scalar, *i.e.* the intrinsic scalar curvature of the three-dimensional spatial slice [9]; $\rho_{eff}(r)$ is the effective density associated with the local spatial curvature, G is the gravitational constant, and c is the speed of light. In 4DEU, this effective density does not represent an additional material component, but rather the purely spatial curvature generated by the baryonic contribution together with the local gravitational constraint.

Under weak-field conditions and effective spherical symmetry, the effective density can be written in terms of the circular velocity profile as

$$\rho_{eff}(r) = \frac{1}{4\pi G r^2} \frac{d}{dr} [r v^2(r)] \quad (4)$$

which is Equation (3) in [1]. Expanding the derivative gives:

$$\rho_{\text{eff}}(r) = \frac{1}{4\pi G r^2} \left[v^2(r) + r \frac{dv^2(r)}{dr} \right] \quad (5)$$

corresponding to Equation (3f) in [1]. In these expressions, $v(r)$ denotes the total circular velocity profile.

In the outer galactic region, where the observed rotation curve becomes nearly flat, one has approximately:

$$v(r) \simeq V_{0,\text{tot}} \Rightarrow \frac{dv(r)}{dr} \simeq 0 \quad (6)$$

where $V_{0,\text{tot}}$ is the asymptotic plateau of the total observed velocity profile. Therefore:

$$\frac{dv^2(r)}{dr} \simeq 0 \quad (7)$$

Substituting Equation (7) into Equation (5) yields:

$$\rho_{\text{eff}}(r) \simeq \frac{V_{0,\text{tot}}^2}{4\pi G r^2} \quad (8)$$

which is Equation (4) in [1]. Therefore, in the exactly flat limit, the effective density follows the isothermal-like radial behavior

$$\rho_{\text{eff}}(r) \propto r^{-2} \quad (9)$$

This result provides the starting point for the derivation.

The total observed circular speed is decomposed in [1] as

$$v_{\text{obs}}^2(r) = v_{\text{bulge}}^2(r) + v_{\text{disk}}^2(r) + v_{\text{gas}}^2(r) + v_{\text{extra}}^2(r) \quad (10)$$

which is Equation (5) in [1]. Within the 4DEU interpretation, the extra component is identified with the curvature-induced contribution generated by the gravitational constraint. Accordingly, we set:

$$v_{\text{extra}}(r) \equiv v_{\text{cve}}(r) \quad (11)$$

where the subscript *cve* denotes the Constraint-Velocity contribution to cosmic Expansion, following the notation introduced in [1]. In the outer fitted domain, where this component dominates, the effective density associated with the gravitational constraint can be treated as the leading contribution to the extra-curvature term.

Up to this point, the derivation directly recovers only the limiting isothermal behavior of Equation (9). To describe outer segments that are not exactly flat but only nearly flat, a minimal local generalization of the outer-curvature law is required. This step should be understood as a local closure ansatz for the nearly flat outer regime, rather than as a new physical ingredient beyond the framework already adopted in [1]. Accordingly, we introduce:

$$\rho_{\text{cve}}(r) \propto r^{-2+\varepsilon} \quad (12)$$

where $\rho_{\text{cve}}(r)$ denotes the effective density associated with the curvature induced by the gravitational constraint, and ε is a local radial-flexibility parameter.

ter defined on the fitted radial interval, consistently with the interpretation already adopted in [1]. Equation (12) reduces to the isothermal case when $\varepsilon = 0$, while allowing for small departures from exact flatness when $\varepsilon \neq 0$.

In other words, Equation (12) expresses the working assumption that, in the nearly flat outer regime, the curvature-induced effective density still decreases approximately as r^{-2} , but with a small local power-law deformation quantified by ε ; the corresponding kinematic law then follows from the same weak-field spherical relation used above.

This is the only additional modeling step needed to reconstruct, in explicit analytical form, the reverse connection between the limiting outer-curvature behavior and the fitted velocity law adopted in [1].

Equation (12) means that there exists a constant C such that:

$$\rho_{cve}(r) = C r^{-2+\varepsilon} \quad (13)$$

Introducing the reference radius R_{ref} , chosen in [1] as the geometric mean of the fitted radii (Equation (29) in [1]), one may rewrite:

$$r^{-2+\varepsilon} = R_{ref}^{-2+\varepsilon} \left(\frac{r}{R_{ref}} \right)^{-2+\varepsilon} \quad (14)$$

Substituting Equation (14) into Equation (13) gives:

$$\rho_{cve}(r) = C R_{ref}^{-2+\varepsilon} \left(\frac{r}{R_{ref}} \right)^{-2+\varepsilon} \quad (15)$$

Defining

$$\rho_0 \equiv C R_{ref}^{-2+\varepsilon} \quad (16)$$

the previous expression becomes:

$$\rho_{cve}(r) = \rho_0 \left(\frac{r}{R_{ref}} \right)^{-2+\varepsilon} \quad (17)$$

where $\rho_0 = \rho_{cve}(R_{ref})$ is the value of the curvature-induced effective density at the reference radius R_{ref} .

The next step is to relate this density profile to the extra-velocity component. Applying the same weak-field spherical relation used in Equation (4), but now to the curvature-induced contribution alone, one obtains:

$$\rho_{cve}(r) = \frac{1}{4\pi G r^2} \frac{d}{dr} [r v_{cve}^2(r)] \quad (18)$$

Substituting Equation (17) into Equation (18) yields:

$$\frac{1}{4\pi G r^2} \frac{d}{dr} [r v_{cve}^2(r)] = \rho_0 \left(\frac{r}{R_{ref}} \right)^{-2+\varepsilon} \quad (19)$$

Multiplying both sides by $4\pi G r^2$, one obtains:

$$\frac{d}{dr} [r v_{cve}^2(r)] = 4\pi G \rho_0 r^2 \left(\frac{r}{R_{ref}} \right)^{-2+\varepsilon} \quad (20)$$

Since

$$\left(\frac{r}{R_{ref}}\right)^{-2+\varepsilon} = r^{-2+\varepsilon} R_{ref}^{2-\varepsilon} \quad (21)$$

it follows that

$$r^2 \left(\frac{r}{R_{ref}}\right)^{-2+\varepsilon} = R_{ref}^{2-\varepsilon} r^\varepsilon \quad (22)$$

Equation (20) then becomes:

$$\frac{d}{dr} [r v_{cve}^2(r)] = 4\pi G \rho_0 R_{ref}^{2-\varepsilon} r^\varepsilon \quad (23)$$

Integrating with respect to r :

$$\int \frac{d}{dr} [r v_{cve}^2(r)] dr = 4\pi G \rho_0 R_{ref}^{2-\varepsilon} \int r^\varepsilon dr$$

one obtains:

$$r v_{cve}^2(r) = \frac{4\pi G \rho_0 R_{ref}^{2-\varepsilon}}{1+\varepsilon} r^{1+\varepsilon} + C_1, \quad \varepsilon \neq -1 \quad (24)$$

where C_1 is the integration constant. Dividing by r , one finds:

$$v_{cve}^2(r) = \frac{4\pi G \rho_0 R_{ref}^{2-\varepsilon}}{1+\varepsilon} r^\varepsilon + \frac{C_1}{r} \quad (25)$$

The second term has the form of a residual Keplerian contribution. However, the present derivation is restricted to the curvature-induced extra component alone, consistently with the decomposition adopted in [1], in which the extra term is separated from the baryonic contribution. Therefore, within this restricted extra-component analysis, the residual Keplerian term must vanish:

$$C_1 = 0 \quad (26)$$

Equation (25) then reduces to:

$$v_{cve}^2(r) = A r^\varepsilon \quad (27)$$

where

$$A \equiv \frac{4\pi G \rho_0 R_{ref}^{2-\varepsilon}}{1+\varepsilon} \quad (28)$$

To normalize the solution, we define:

$$V_0^2 \equiv v_{cve}^2(R_{ref}) \quad (29)$$

Using Equation (27) at $r = R_{ref}$, one gets:

$$V_0^2 = A R_{ref}^\varepsilon \quad (30)$$

Hence,

$$A = \frac{V_0^2}{R_{ref}^\varepsilon} \quad (31)$$

It should be stressed that Equation (31) does not introduce a new constant. The

quantity A is the same normalization constant already defined in Equation (28); Equation (28) expresses it in terms of ρ_0 , whereas Equation (31) rewrites the same constant in terms of V_0 by using the normalization condition at R_{ref} .

Substituting Equation (31) into Equation (27), one finally obtains:

$$v_{cve}^2(r) = V_0^2 \left(\frac{r}{R_{ref}} \right)^\varepsilon \quad (32)$$

or equivalently:

$$v_{cve}(r) = V_0 \left(\frac{r}{R_{ref}} \right)^{\varepsilon/2} \quad (33)$$

Equation (33) is exactly the kinematic law used in the previous rotation-curve analysis, *i.e.* Equation (12) in [1]. As a consistency check, taking the logarithm of Equation (32) gives:

$$\ln v_{cve}^2(r) = \ln V_0^2 + \varepsilon \cdot \ln \left(\frac{r}{R_{ref}} \right) \quad (34)$$

and differentiation with respect to $\ln r$ immediately yields:

$$\frac{d \ln v_{cve}^2}{d \ln r} = \varepsilon \quad (35)$$

which recovers Equation (6) in [1]. Thus, the power-law form previously adopted as a fitting ansatz for the extra-velocity component is shown to follow explicitly from the three-dimensional spatial curvature induced by the gravitational constraint, once the nearly flat outer-curvature regime is locally generalized as in Equation (12).

3. Concluding Remarks

The present work should be regarded as a mathematical addendum to the previous 4DEU analysis of galactic rotation curves [1]. Its purpose has not been to revise the results already obtained, nor to introduce any new physical mechanism, but to make explicit the reverse analytical passage that remained implicit in the physical interpretation and mathematical structure adopted in [1]. In the original paper, the kinematic law and its associated effective-density/curvature interpretation were already presented, but the reverse derivational route from the outer-curvature behavior to that same law was not written out in full intermediate detail, in order to avoid excessively burdening a work primarily focused on the observational and statistical analysis of galactic rotation curves.

Starting from the local relation between the three-dimensional Ricci scalar ${}^{(3)}R$ and the effective density, and combining it with the standard weak-field expression linking effective density to the circular velocity profile, we have shown explicitly that, once the nearly flat outer-curvature regime is generalized through the minimal local power-law ansatz introduced here, the curvature-induced contribution associated with the gravitational constraint recovers the law:

$$v_{cve}(r) = V_0 \left(\frac{r}{R_{ref}} \right)^{\varepsilon/2}$$

This is the same law that was previously used to reproduce the outer galactic rotation curves in [1]. Within the 4DEU framework, the present derivation shows that this law for the extra-velocity component is explicitly recovered from the three-dimensional spatial curvature induced by the gravitational constraint in gravitationally bound systems.

The result therefore strengthens the internal analytical consistency of the 4DEU interpretation of galactic rotation curves while leaving unchanged the empirical results previously obtained from the SPARC sample. In this sense, the present addendum provides the explicit mathematical formulation of a connection already contained, though not fully developed step by step, in the framework adopted in [1].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Maglione, D. (2026) Galactic Rotation Curves Explained in the 4DEU Framework by 3D Spatial Curvature from Local Gravitational Blocking of Cosmic Expansion: No Need for Dark Matter. *Journal of High Energy Physics, Gravitation and Cosmology*, **12**, 431-470. <https://doi.org/10.4236/jhepgc.2026.121025>
- [2] Sofue, Y. and Rubin, V. (2001) Rotation Curves of Spiral Galaxies. *Annual Review of Astronomy and Astrophysics*, **39**, 137-174. <https://doi.org/10.1146/annurev.astro.39.1.137>
- [3] Lelli, F., McGaugh, S.S. and Schombert, J.M. (2016) Sparc: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves. *The Astronomical Journal*, **152**, Article No. 157. <https://doi.org/10.3847/0004-6256/152/6/157>
- [4] Clowe, D., Bradač, M., Gonzalez, A.H., Markevitch, M., Randall, S.W., Jones, C., et al. (2006) A Direct Empirical Proof of the Existence of Dark Matter. *The Astrophysical Journal*, **648**, L109-L113. <https://doi.org/10.1086/508162>
- [5] Planck Collaboration, Aghanim, N., Akrami, Y., Ashdown, M., et al. (2020) Planck 2018 Results. VI. Cosmological Parameters. *Astronomy & Astrophysics*, **641**, A6. <https://doi.org/10.1051/0004-6361/201833910>
- [6] Milgrom, M. (1983) A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis. *The Astrophysical Journal*, **270**, 365-370. <https://doi.org/10.1086/161130>
- [7] Capozziello, S., Cardone, V.F. and Troisi, A. (2007) Low Surface Brightness Galaxy Rotation Curves in the Low Energy Limit of R^n Gravity: No Need for Dark Matter? *Monthly Notices of the Royal Astronomical Society*, **375**, 1423-1440. <https://doi.org/10.1111/j.1365-2966.2007.11401.x>
- [8] Corda, C. (2009) Interferometric Detection of Gravitational Waves: The Definitive Test for General Relativity. *International Journal of Modern Physics D*, **18**, 2275-2282. <https://doi.org/10.1142/s0218271809015904>
- [9] Arnowitt, R., Deser, S. and Misner, C.W. (2008) Republication of: The Dynamics of General Relativity. *General Relativity and Gravitation*, **40**, 1997-2027. <https://doi.org/10.1007/s10714-008-0661-1>