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Solving the Problem of Relativistic Thermodynamics in the Special and General Cases According to the Inverse Relativity Model

Michael Girgis

Independent Researcher, Menia, Egypt
Email: michael.g.fahim@gmail.com

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Abstract

In this paper, we will try to solve the problem of relativistic thermodynamics and explain why the special theory of relativity has failed in this matter for almost 120 years. We will do this through a new model called inverse relativity, derived from special and general relativity, and not as a replacement for either of them. This model is based on dividing total spacetime into positive and negative subspaces. In the special case, the paper includes transformations of some thermal quantities, such as volume, pressure, temperature, probability density, etc. in each subspace-time. The positive subspace-time, as a space of causality, represents the internal structure space of the thermodynamic system, where we obtain the same concepts and mathematical formulas for the laws of thermodynamics at both the microscopic and macroscopic levels. Negative subspace-time, as a space devoid of causality where the concepts and laws of thermodynamics break down, represents the space of the system's motion. The paper also includes zero-point energy, its nature and importance, and macroscopic quantum tunneling, both of which are related to energy fluctuations between subspaces, as well as the quantizability of time, and the possibility of a thermodynamic system reaching the speed of light without infinite energy. In general case, the paper includes new types of local energy-momentum tensors for a perfect fluid in each subspace-time. This gives us an understanding of relativistic thermodynamics on the surfaces of stars and black holes, where uncertainties in escape velocity, black hole radius, and the disappearance of infinite energy density arise in negative subspace-time, while uncertainties in thermal quantities arise in positive subspace-time. Finally, we propose three distinct tests of some of the results of our model.

Keywords

Relativistic Gas, Temperature Transformation, Inverse Heisenberg Principle, Super Time, Time Quantization, Gravity and Thermodynamics, Local Tensors, Inverse Length Relativity, Subspace-Times, Reverse Subspace-Time, Black Holes, Dark Matter, Inverse Theory of Relativity

1. Introduction

For 119 years, special relativity has not accepted being combined with thermodynamics into just one agreed model, but has given us three conflicting viewpoints [1] without even knowing why. The first viewpoint emerged early, shortly after the publication of special relativity by Einstein and Planck separately in 1907. Einstein believed that a moving body (system) would appear cooler to a stationary observer. Max Planck also reached the same conclusion. Both proposed the same transformation set shown in the table below. The Planck-Einstein model prevailed for a long time, but later Einstein expressed doubts about the results he had obtained in his correspondence with von Laue [2]. The second viewpoint emerged in 1963 with Ott, who believed that a moving body would appear hotter relative to a stationary observer and arrived at the transformation set shown in the table below [3]. The third viewpoint was with Landsberg in 1966 and 1970, who believed that the body's temperature remained constant relative to a stationary observer and arrived at his own transformation set; see the table as well [4].

Transformations of thermal quantities	Volume	Pressure	Temperature
Planck-Einstein	$V_{ol} = V_{ol}^{`}\gamma^{-1}$	$P = P^{`}$	$T = T^{`}\gamma^{-1}$
Out	$V_{ol} = V_{ol}^{`}\gamma^{-1}$	$P = P^{`}\gamma^{2}$	$T = T^{`}\gamma$
Landsberg	$V_{ol} = V_{ol}^{`}$	$P = P^{`}$	$T = T^{`}$
	$\gamma = 1\Big/\sqrt{\left(1-\frac{V_S^2}{c^2}\right)}$		

Even today, there is no agreement on which viewpoint is correct and why. There is a lot of literature review on this subject, but it is all confined to the context of the previous viewpoints. Some believe that the reason for the existence of different viewpoints is due to the difference in the initial assumptions of each author or in the different definitions of thermal quantities (temperature - heat transfer - work) [4]. But I wonder why there are any initial assumptions at all. Wouldn't it be more logical for the transformations of thermal quantities to be directly and smoothly derived from the laws of thermodynamics, according to the principle of relativity? Furthermore, shouldn't the different definitions of thermal quantities be considered equivalent? Because they are derived from the same laws of thermodynamics, they should all lead to the same results—I mean the transformations. I believe the main reason for the problem is special relativity itself. We can deduce this from a simple example. If we have a moving thermodynamic system to be a container containing an ideal gas, then the temperature with respect to a moving observer with the container is a measure of the average velocity of the gas particles [5] with respect to the center of mass of the container or with respect to the frame of reference of the container and the observer, as both definitions are equivalent here. As for a stationary observer, temperature is a measure of the average velocity of the gas particles relative to the center of mass of the moving container and not relative to the observer's frame of reference. That is, the stationary observer must distinguish in his frame of reference between the velocity of the particles inside the container and the velocity of the particles with the motion of the container. The problem here lies in the fact that velocity transformations for any particle from one frame of reference to another in special relativity are of the particle's total velocity. There is no difference here between the velocity of the particle around the container's center of mass and the velocity of the particle with the container's center of mass for a stationary observer. The same applies to energy transformations of the particle in special relativity, as they depend on mass, not velocity. This makes it even more difficult to distinguish the energy associated with these types of velocities. Furthermore, the concept of time in special relativity is incompatible with the thermodynamic concept of time, and the volume transformation under Lorentz contraction gives rise to many paradoxes. However, these limitations do not constitute a flaw in special relativity, but it only reveals its limitations as a physical model, such as Newtonian mechanics at high speeds or subatomic scales. Some researchers have recognized these limitations in special relativity but have circumvented them by analyzing the relativistic total energy of the system at the macroscopic level to obtain the thermal component, although this is a physically valid approach. But the problem everyone overlooked is that analyzing energy at the macroscopic level implicitly means analyzing energy at the microscopic level; therefore, energy analysis must be fundamentally at the microscopic level. Herein lies the catastrophe, because analyzing the kinetic energy of a particle in a container necessarily requires analyzing the force vector, momentum, velocity, and 4D displacement of that particle, which will lead us to a completely new relativistic mechanics. Furthermore, the analysis of the 4D displacement vector will necessarily also lead to new transformations of the space-time coordinates specific to each displacement vector resulting from the analysis. These transformations will describe new spacetimes with new properties of space and time, which are sub-space-times of Minkowski's total spacetime, representing a complete departure from special relativity. This is what we did in the three previous papers [6]-[8]. This is the reason for the emergence of the inverse relativity model. The primary purpose of establishing the inverse relativity model was to seek a solution to the problem of relativistic thermodynamics. However, this model does not represent a replacement for special and standard general relativity, but rather stems from both as an analytical model. Will the inverse relativity model be able to establish a single, universally agreed-upon model of relativistic thermodynamics? Is this model testable and distinct from all previous models of relativistic thermodynamics? This is what we will try to present in this paper.

2. Methods

2.1. Transformation of Volume in the Positive Subspace

Suppose we have two inertial frames S and S' [9] from Cartesian coordinate systems, each with an observer at the origin point O and O', and that frame S' is moving at a constant velocity V_S relative to frame S in the positive x-direction. We also assume that in frame S' we have a fixed thermodynamic system (a cubic container of monatomic ideal gas [10]) in thermodynamic equilibrium, as shown in the following **Figure 1**.

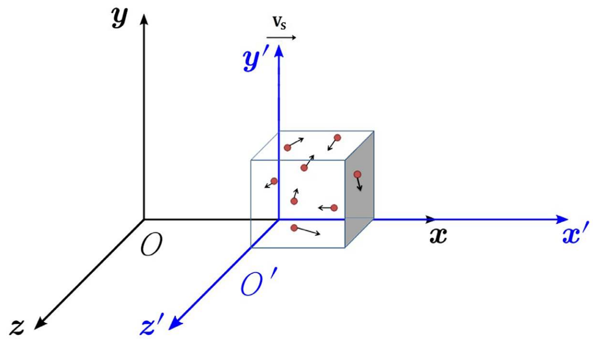

Figure 1. A thermodynamic system belonging to a frame of reference S' and moving at a constant speed relative to the frame of reference S.

As we know, the volume of a gas is one of the macroscopic properties of a thermodynamic system, but at the microscopic level, according to the kinetic molecular theory of gases, the volume of a gas represents the volume of the space in which the gas particles move and spread within the system, *i.e.* the volume of the gas is equivalent to the volume of the thermodynamic system (the container) ([11] pag, 3), while the volume of the gas particles is negligible. The observer O' measures the volume of the gas $\grave{V}_{ol}$ relative to the frame S' by multiplying the geometric dimensions of the thermodynamic system, Because the container is cubic, as shown in **Figure 1**, then, $\grave{V}_{ol} = \Delta x` \cdot \Delta y` \cdot \Delta z`$. As for the observer O in the positive subspace, the volume of the gas $\tilde{V}_{ol}$ is equal to, $\tilde{V}_{ol} = \Delta\tilde{x} \cdot \Delta\tilde{y} \cdot \Delta\tilde{z}$. Since the structure of the positive subspace is symmetrical for both observers according to the positive inverse modified Lorentz transformations shown in the first paper, Section 2.3 (see also Equations (11.1), (12.1), (13.1) [6]), the volume of the gas remains constant under the transformation from the frame S' to S.

$$\tilde{V}_{ol} = \grave{V}_{ol} \implies d\tilde{V}_{ol} = d\grave{V}_{ol} \tag{1.4}$$

Also as a result of the symmetry of the spatial structure between the observers in this subspace, all the displacement vectors that the particles of the system take at a given moment with respect to observer O', we have displacement vectors that are identical in length and direction with respect to observer O in the positive subspace. This means that the particles of the system in the positive subspace have the same random motion, and therefore the positive volume here represents the space in which the gas particles move randomly, *i.e.* the positive volume is considered a thermodynamic variable of the system. Although the emergence of random motion of the system's particles relative to the frame S is self-evident, we wanted to illustrate this result in the positive subspace, as the situation would be entirely different in the negative subspace. The volume transformation here agrees with the volume transformation in the Landsberg model, thus resolving the paradoxes arising from volume contraction in other models.

2.2. Transformation of Temperature in the Positive Subspace

Temperature is also one of the macroscopic properties of the thermodynamic system. But at the microscopic level, according to the kinetic theory of gases, it represents a measure of the average speed and kinetic energy of the particles ([11] pag, 3, 60) [12]. As a result of the random motion and collisions of gas particles, they have different speed values or speed distributions. Because the gas here is ideal and at a high temperature, we can therefore describe this distribution as a Maxwell-Boltzmann distribution function [13] [14] with respect to the observer O′. By integrating this function for all values of the velocity multiplied by the square of the velocity $\langle u^{`2} \rangle = \int_0^\infty u^{`2} f(u^`)$, the observer O' can obtain the average value of the squared resultant velocity $\langle u^{`2} \rangle$ of the particles of the system with respect to the frame S', in terms of the temperature of the gas $T^`$, the mass of the gas particle $m^`$ and the Boltzmann constant k_B according to the following formula.

$$\langle u^{`2} \rangle = \frac{3k_B T^`}{m^`} \implies T^` = \frac{m^` \langle u^{`2} \rangle}{3k_B} \tag{2.4}$$

As for the observer O in the positive subspace, from the definition of this subspace shown in the first paper [6], Item 2.3, it is the space of causality or collision, where every collision that occurs between any two particles with respect to the observer O', also occurs between the same two particles with respect to the observer O in the positive subspace. See the example shown in the first paper. Also, from the definition of positive relativistic mechanics explained in the second paper, section on discussions, physical quantities such as the speed of a particle relative to the observer O in positive subspace change with the change of their counterparts relative to the observer O' as a result of any causation that occurs to the particle as a collision with another particle. That is, the positive relativistic mechanics of the particles here is linked to the causality that occurs in this subspace, which here represents the collision of gas particles with each other. Therefore, for every value of the velocity that the particles take at a given moment of entry into the system relative to the observer O', we also have corresponding values in the positive subspace. By squaring the equations for transforming the velocity components of a particle in the positive subspace shown in the second paper [7], Set No. 15.2, and by summing all the possible values of the square of each velocity component that the particles take at a given moment within the system, and by dividing both sides of each resulting equation by the number of particles, We obtain transformation equations for the average of the squared components of the particles velocity in the positive subspace.

$$\langle \tilde{u}_x^2 \rangle = \langle u`_x^2 \rangle \gamma^{-2}$$
$$\langle \tilde{u}_y^2 \rangle = \langle u`_y^2 \rangle \gamma^{-2}$$
$$\langle \tilde{u}_z^2 \rangle = \langle u`_z^2 \rangle \gamma^{-2} \tag{3.4}$$

Because there is no advantage to one velocity direction over another within the system with respect to the observer O', therefore the velocity distribution here is symmetrical around the center of mass of the system, *i.e.* the sum of the velocity components in the positive and negative directions on each axis with respect to the frame S' equals zero. By substituting that into the previous set of equations, we obtain the same result for frame S in the positive subspace. This means that the velocity distribution in the positive subspace is also symmetrical about the system's center of mass relative to the observer O. By summing the average of the squared components, we obtain a transformation equation for the average of the squared net velocity of the particles in the system.

$$\sum_{i=3}^{3} \langle \tilde{u}_i^2 \rangle = \gamma^{-2} \sum_{i=3}^{3} \langle u`_i^2 \rangle \implies \langle \tilde{u}^2 \rangle = \langle u`^2 \rangle \gamma^{-2} \tag{4.4}$$

Equation (4.4) shows us that the average of the squared net velocity of the particles decreases relative to the observer O in the positive subspace as the velocity of the frame of reference (container) increases. By taking the square root of both sides of the equation, we obtain a transformation equation for the root mean square net velocity. $\tilde{u}_{rms} = u`_{rms} \gamma^{-1}$. Following the same steps as before, but first summing the possible values of the velocity components, then dividing by the number of particles and squaring the result. We can obtain the transformation of the average resultant velocity, $\langle \tilde{u} \rangle = \langle u` \rangle \gamma^{-1}$, and also the most probable velocity of the particles in the system $\tilde{u}_p = u`_p \gamma^{-1}$. By substituting from the velocity transformation Equation (4.4), and also by substituting from the mass transformation equation according to special relativity $m = m`\gamma$ (where m represents the relativistic mass of the particle of the gas relative to the observer O in total space), in Equation (2.4) we obtain:

$$T` = \frac{m \langle \tilde{u}^2 \rangle}{3k_B} \gamma \gamma^{-2} \tag{5.4}$$

The right side of the equation represents the same mathematical formula for calculating temperature, but in the positive subspace, *i.e.*, $m\langle \tilde{u}^2 \rangle / 3k_B = \tilde{T}$, substituting this into Equation (5.4) and rearranging the equation, we get.

$$\tilde{T} = T` \gamma^{-1} \implies d\tilde{T} = dT` \gamma^{-1} \tag{6.4}$$

Equation (6.4) shows that the temperature of the gas decreases relative to the observer O in the positive subspace as the speed of the reference frame (container) increases. To understand why the temperature decreases despite taking into account the increase in mass according to special relativity, it is because the effect of time dilation, represented by the decrease in speed $\tilde{u}$, is greater, and therefore the kinetic energy of the particle decreases (see Paper 3, Equation (39.3)), which leads to a decrease in temperature, which is a measure of the average kinetic energy in this subspace. When the frame of reference reaches the speed of light (theoretically only), the system's temperature reaches absolute zero, which represents a breakdown in the transformation equation. However, when the equation is quantized according to the energy fluctuation hypothesis between subspaces, as explained in Paper 3 [8], Section 2.7, as an interpretation of Heisenberg's principle in positive subspace, and considering that the energy fluctuations between subspaces occurring for all particles in the system are not necessarily simultaneous. Therefore, we take the average values that exist in most of the particles of the system for both energy and time according to Heisenberg's principle [15] $\langle\Delta\tilde{t}_{un}\rangle\langle\Delta\tilde{E}_{un}\rangle \geq \frac{1}{2}\hbar$. So when the average uncertainty in time $\langle\Delta\tilde{t}_{un}\rangle = 1$, then the average uncertainty in the positive energy of the particles $\langle\Delta\tilde{E}_{un}\rangle = \frac{1}{2}\hbar$ even when $V_S = \mathrm{c}$, Because the infinite dilation in uncertainty time is canceled out by the infinite contraction in uncertainty time associated with the energy transfer from the negative to the positive subspace. Therefore, the quantity $\frac{1}{2}\hbar$ remains constant under the transformation, as explained in Paper 3. Consequently, the transformation equation under quantization becomes the following inequality.

$$\tilde{T} \geq T^{`}\gamma^{-1} + T^{`}_{\hbar} \implies \tilde{T} \geq T^{`}_{\hbar} \quad V_S = \mathrm{c} \tag{7.4}$$

where $T^{`}_{\hbar}$ represents the temperature of the system's particles at an average kinetic energy of the particles equal to the reduced Planck energy $\langle kE^{`}\rangle = \frac{1}{2}\hbar$, it represents the lowest possible temperature of the gas in the system. When substituting into the previous inequality, $V_S = \mathrm{c}$, we find that we have doubt about the existence of a minimum gas temperature in the system. In other words, we cannot confirm the disappearance of the system temperature in the positive subspace when the frame speed reaches the speed of light, due to the oscillation of the minimum average energy between the subspaces. As for the transformation of the probability density of particles at a certain speed ([11] pag, 147) with respect to the frame of reference S', it is determined by the observer O' according to the Maxwell-Boltzmann velocity distribution function.

$$f(u^{`}) = 4\pi\left(\frac{m^{`}}{2\pi k_B T^{`}}\right)^{\frac{3}{2}} + u^{`2}\exp\left(-\frac{E^{`}}{k_B T^{`}}\right) \tag{8.4}$$

where $f(u^{`})$ represents the probability of a particle existing in the speed range between $u^{`}$ and $u^{`} + du^{`}$, $E^{`} \approx kE^{`}$ is the total relativistic energy of the particle, which is equivalent in value to the relativistic kinetic energy because the gas is at a high temperature or a relativistic gas. Comparing the temperature transformation equation in the positive subspace shown above (No. 6.4) with the relativistic kinetic energy or relativistic total energy transformation equation in the positive subspace shown in Paper 3 (Equation (39.3)) and Paper 2 (Equation (32.2)), respectively [7] [8], we find that the Boltzmann coefficient is constant under the transformation.

$$\exp\left(-\frac{\tilde{E}}{k_B\tilde{T}}\right) = \exp\left(-\frac{E^{`}}{k_B T^{`}}\right) \tag{9.4}$$

By substituting the value of each of the following values: Boltzmann's coefficient from Equation (9.4), the value of $m`$ from the mass transformation equation according to special relativity, the value of $T`$ from Equation (6.4), and the value of $u`^2$ according to the formula $\tilde{u}^2 = u`^2\gamma^{-2}$ in Equation (8.4)

$$f(u`) = 4\pi\left(\frac{m\gamma^{-1}}{2\pi k_B\tilde{T}\gamma}\right)^{\frac{3}{2}}\tilde{u}^2\gamma^2\exp\left(-\frac{\tilde{E}}{k_B\tilde{T}}\right) \tag{10.4}$$

By reducing γ

$$f(u`) = \gamma^{-1}4\pi\left(\frac{m}{2\pi k_B\tilde{T}}\right)^{\frac{3}{2}}\tilde{u}^2\exp\left(-\frac{\tilde{E}}{k_B\tilde{T}}\right) \tag{11.4}$$

The right side of the equation represents the same mathematical formula for the Boltzmann-Maxwell distribution, but in the positive subspace. Therefore, it is equal to $f(\tilde{u})$, which is the probability of a particle existing in the speed range between $\tilde{u}$ and $\tilde{u}+d\tilde{u}$. Substituting this into (11.4) and rearranging the equation, we obtain.

$$f(\tilde{u}) = f(u`)\gamma \tag{12.4}$$

Equation (12.4) shows that the probability density of particles at a given velocity increases relative to the observer O in the positive subspace as the velocity of the reference frame (container) increases. This is a logical result because it corresponds to the decrease in temperature in this subspace. We can represent the transformation of the Maxwell-Boltzmann distribution in the positive subspace graphically according to the diagram shown in Figure 2. Here, the statistics for each type of speed decrease with temperature, while the probability density increases with increasing velocity of the reference frame (container).

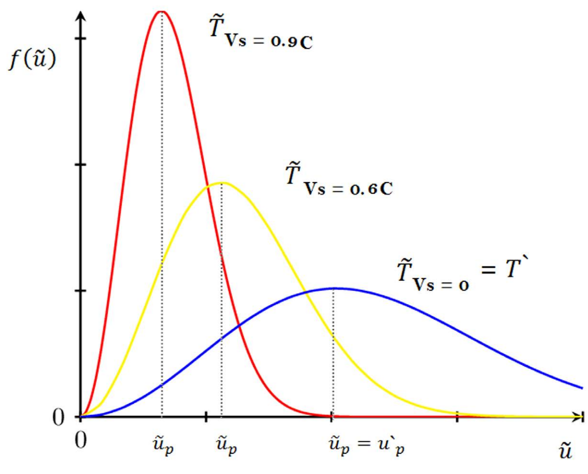

Figure 2. The velocity and probability density distribution of particles in the positive subspace at each value of the reference frame velocity.

Inverse relativity is characterized by this result. Special relativity fails to find a probability density transformation for particles at a given speed for two reasons.

The first reason is that the velocity transformations in special relativity, as we mentioned in the introduction, represent transformations for the sum of the velocities (total velocity) of the particle on each axis [16]. It does not distinguish between the particle's velocity relative to the system's center of mass (*i.e.*, inside the container) and the particle's velocity with the system's center of mass (*i.e.*, with the container's velocity). As for the velocity transformations in inverse relativity, especially in this subspace, they represent the velocity of the particles relative to the center of mass of the system. We have already explained this above. They also explain the effect of the motion of the system on this type of velocity. The second reason is that the relativistic kinetic energy of a particle in special relativity increases with the increase in the relativistic mass of the particle or with the motion of the frame of reference (container). Therefore, a decrease in the relativistic kinetic energy or relativistic total energy of a particle cannot be justified by an increase in its relativistic mass. This result can only be achieved in a subspace, as in inverse relativity.

2.3. Transformation of Pressure in the Positive Subspace

Gas pressure is also a macroscopic variable of the thermodynamic system. However, at the microscopic level, according to the kinetic theory of gases, gas pressure arises from the collision of gas particles on the container walls as a result of the random motion of particles within the system. Therefore, the gas pressure relative to the observer O' depends mainly on the average force $\bar{F}^{`}$ exerted by each particle, the number of particles $N$, and the total surface area of the system walls $A^{`}$ ([11] pag, 3, 59, 60) relative to the frame of reference S'.

$$P^{`} = \frac{N\bar{F}^{`}}{A^{`}} \tag{13.4}$$

As for the observer O in the positive subspace, as we mentioned above, every collision that occurs has a corresponding collision in this subspace. This means that in the positive subspace, we will also have collisions between particles and the wall. As mentioned above, in positive relativistic mechanics, physical quantities such as particle momentum here, relative to observer O in the positive subspace, change with the change of their counterparts relative to the observer O', due to causality (particle collisions with the wall). Therefore, for every value of force that the particles exert at a given moment on the wall of the system relative to the observer O', we also have corresponding values in the positive subspace. From the equations for transforming the relativistic force components particle in the positive subspace shown in Paper 2 [7], Set No. 40.2, we obtain the transformation of all possible values of each force component that each particle exerts on the wall of the system at a given moment. By summing all possible values of each force component, dividing both sides of each resulting equation by the number of particles, and squaring them, we obtain transformation equations for the square of the average force components for each particle in the system.

$$\bar{\tilde{F}}_x^2 = \bar{F}_x^{`2}\gamma^{-2}$$

$$\tilde{\bar{F}}_y^2 = \bar{F}_y^{`2}\gamma^{-2}$$
$$\tilde{\bar{F}}_z^2 = \bar{F}_z^{`2}\gamma^{-2} \tag{14.4}$$

Because the thermodynamic system is static with respect to frame S', the force distribution here is symmetrical around the system's center of mass. Substituting zero into the previous set of equations, we obtain the same result with respect to frame S in the positive subspace. This means that the force distribution in the positive subspace is also symmetrical about the system's center of mass with respect to the observer O. By summing the square of the average force components in each direction, we obtain a transformation equation for the square of the average net force exerted by each particle at a point on the walls of the system.

$$\sum_{i=3}^{3} \tilde{\bar{F}}_i^2 = \gamma^{-2} \sum_{i=3}^{3} \bar{F}_i^{`2} \implies \tilde{\bar{F}}^2 = \bar{F}^{`2}\gamma^{-2} \implies \tilde{\bar{F}} = \bar{F}^{`}\gamma^{-1} \tag{15.4}$$

Substituting from Equation (15.4) into (13.4), and also substituting for $\tilde{A} = A^{`}$ where the surface area is constant under the transformation from frame S' to S as mentioned above in the section on volume transformation.

$$P^{`} = \frac{N\tilde{\bar{F}}}{\tilde{A}}\gamma \tag{16.4}$$

The right side of the equation represents the same mathematical formula for calculating pressure, but in the positive subspace, where, $N\tilde{\bar{F}}/\tilde{A} = \tilde{P}$. Substituting this into (16.4) and rearranging the equation, we obtain

$$\tilde{P} = P^{`}\gamma^{-1} \tag{17.4}$$

Equation (17.4) shows us that the relativistic gas pressure decreases relative to the observer O in the positive subspace as the velocity of the frame of reference (container) increases. when $V_S = \mathrm{c}$, theoretically only, the gas pressure in the positive space reaches zero, which represents a breakdown of the pressure transformation equation. However, when the equation is quantized according to the hypothesis of energy fluctuations between subspaces as explained in the previous section. The equation changes to the following inequality.

$$\tilde{P} \geq P^{`}\gamma^{-1} + P_{\hbar}^{`} \implies \tilde{P} \geq P_{\hbar}^{`} \tag{18.4}$$

where $P_{\hbar}^{`} = Nk_B T_{\hbar}^{`}/V_{ol}^{`}$ represents the gas pressure in the system at the reduced Planck temperature, $T_{\hbar}^{`}$, it represents the minimum possible gas pressure at a specific volume. Substituting this into the previous inequality, $V_S = \mathrm{c}$, we find that there is some doubt about the existence of a minimum gas pressure in the system; that is, we cannot confirm the disappearance of pressure, which represents one of the macroscopic Information of the thermodynamic system in the positive subspace.

2.4. Transformation of Internal Energy in the Positive Subspace

According to the kinetic theory of gases, the concept of the internal energy of a thermodynamic system is limited at the microscopic level ([11], p. 91) [17] to the sum of the kinetic energy of the particles of the system, whether their motion is translational, rotational or vibrational. Because the gas used in the system is a monatomic gas, both rotational and vibrational energy are zero here, and the only energy that the particles possess in this case is relativistic translational kinetic energy. Because the gas is relativistic, *i.e.*, at a high temperature, $kE^{`} \approx E^{`}$ as mentioned previously. Therefore, the internal energy of the system here represents the sum of the relativistic energy of all the particles in the system with respect to the frame S'. Assuming here that the total number of particles N is distributed across a number of energy levels j where $j = 0, 1,2,3$ then the sum of the relativistic energy of all the particles in the system according to statistical mechanics equals ([18]. pag, 130).

$$U^{`} = \sum_j E_j^{`} p^{`}{}_j \tag{19.4}$$

where $E_j^{`}$ represents the energy of level $j$ in the system, and $p^{`}{}_j$, is the probability of n_j particles occupying this energy level. From the statistical formula, we find that the energy transformation in the positive subspace depends on the relativistic energy and the probability. Using the equation for the total relativistic energy transformation of a particle in the positive subspace, shown in Paper 2 [7], Equation (32.2), we obtain a transformation equation for the sum of the energy levels of the system in the positive subspace.

$$\sum_j \tilde{E}_j = \gamma^{-1} \sum_j E_j^{`} \implies \sum_j d\tilde{E}_j = \gamma^{-1} \sum_j dE_j^{`} \tag{20.4}$$

As for the probability transformation according to the Boltzmann distribution, it is equal to $p^{`}{}_j = \frac{1}{q^{`}} e^{E_j^{`}/k_B T^{`}}$, where $e^{E_j^{`}/k_B T^{`}}$ is the Boltzmann coefficient and is a constant under transformation in the positive subspace according to Equation (9.4), while $q^{`} = \sum_j e^{E_j^{`}/k_B T^{`}}$ is the Boltzmann partition function and is also a constant under transformation in the positive subspace because it represents the sum of the Boltzmann coefficients ([18], pag, 136). Therefore, the probability $p^{`}{}_j$ is also a constant under transformation in this subspace.

$$\tilde{p}_j = p^{`}{}_j \implies d\tilde{p}_j = dp^{`}{}_j \tag{21.4}$$

By substituting from (20.4) and (21.4) into (19.4)

$$U^{`} = \gamma \sum_j \tilde{E}_j \tilde{p}_j \tag{22.4}$$

The right side of the equation represents the same mathematical formula for calculating internal energy, but in the positive subspace, *i.e.*, $\sum_j \tilde{E}_j \tilde{p}_j = \tilde{U}$, substituting this into (22.4) and rearranging the equation, we obtain

$$\tilde{U} = U^{`} \gamma^{-1} \implies d\tilde{U} = dU^{`} \gamma^{-1} \tag{23.4}$$

The previous equation shows us that the internal energy of the system decreases relative to the observer O in the positive subspace as the speed of the frame of reference (container) increases. We can also obtain the internal energy transformation at the macroscopic level, where, $U^{`} = 3Nk_B T^{`}$. Substituting from the temperature transformation equation in Equation (6.4), we obtain the same mathematical formula for the law in the positive subspace and the same result for the internal energy transformation. When the frame of reference theoretically reaches the speed of light, the internal energy equals zero, or the transformation equation collapses. However, when the equation is quantized according to the energy oscillation hypothesis between subspaces, as previously described. The equation transforms into the following inequality.

$$\widetilde{U} \geq \grave{U}\gamma^{-1} + \grave{U}_{\hbar} \implies \widetilde{U} \geq \grave{U}_{\hbar} \quad V_S = \mathrm{c} \tag{24.4}$$

where $\grave{U}_{\hbar} = \frac{1}{2}N\hbar$ represents the internal energy of the particles of the system at an average particle energy equal to the reduced Planck energy $\frac{1}{2}\hbar$, so it represents the lowest possible internal energy of the gas in the positive subspace, and when substituting in the previous inequality $V_S = \mathrm{c}$ we find that we have doubt about the existence of a minimum limit of internal energy in the system, *i.e.* we cannot confirm the disappearance of the internal energy of the system in the positive subspace, which represents one of the characteristics of the macroscopicity of the system. Assuming that the observer O' cooled the system until the system's temperature theoretically reached absolute zero, according to Heisenberg's principle in quantum mechanics, the system retains internal energy at that point, known as the zero-point energy $\grave{U}_{0k}$, and the observer O' cannot justify the source of this energy. This is substituted into inequality (24.4), and $V_S = \mathrm{c}$

$$\widetilde{U} \geq \grave{U}_{0k}\gamma^{-1} + \grave{U}_{\hbar} \implies \widetilde{U}_{un} \geq \grave{U}_{\hbar}, \; V_S = \mathrm{c} \implies \widetilde{U}_{un} = \grave{U}_{0k} \tag{25.4}$$

We find that the observer O in the positive subspace observes an energy uncertainty close to the zero-point energy of the system $\widetilde{U}_{un} \geq \grave{U}_{\hbar}$, and as the uncertainty in time decreases, the uncertainty in energy or amount of the energy oscillation from the negative to the positive subspace increases until $\widetilde{U}_{un}$ equals $\grave{U}_{0k}$ [19]. This means that the energy source $\grave{U}_{0k}$ is justified for the observer O at the level of total space. But only at the level of positive subspace we can't justify the existence of this amount of energy, except that it is created from nothing and destroyed into nothingness. We can also conclude from this that the system at that point has a temperature in the range of the reduced Planck temperature, and the particles of the system at that point have a motion known as zero-point motion. This result is consistent with some experiments [20]. This does not contradict the nature of the zero point, because this energy, as single-level energy, oscillates simultaneously between the subspaces. Therefore, its presence in the positive subspace is temporary. At the moment that energy disappears from the positive subspace, the system actually reaches absolute zero, *i.e.*, no temperature and no motion [21]. This means that a pure crystal can reach absolute zero without requiring an infinite number of thermodynamic processes, but only temporarily or momentarily. It is sufficient for the system's temperature to reach the reduced Planck temperature or zero-point energy; then the system loses energy from that point for a short period of time due to its transition into negative subspace. Thus, we find that the inverse relativity model can provide a logical explanation for the nature and properties of zero-point energy. This explanation violates the classical law of conservation of energy in the positive subspace, which represents the space of physical laws, but on the other hand, it preserves the laws of quantum mechanics or Heisenberg's principle in the positive subspace.

2.5. Transformation of Heat in the Positive Subspace

If the thermodynamic system, with respect to the frame of reference S', is an unisolated system placed in a water tank, then at the macroscopic level the system can lose or gain a quantity of heat $dQ^{`}$ to or from the tank in a thermodynamic process [22]. At the microscopic level of the system, the quantity of heat is the quantity of energy transferred from the particles of the system to the particles of the water tank or vice versa through indirect collisions (*i.e.*, via the wall particles). This leads to a change in the internal energy of the system. Differentiating Equation (19.4), we obtain an infinitesimal change in the internal energy of the system at the microscopic level, *i.e.*, in terms of both $E_j^{`}$ and $p^{`}{}_j$.

$$dU^{`} = \sum_j E_j^{`}\, dp^{`}{}_j + \sum_j dE_j^{`}\, p^{`}{}_j \tag{26.4}$$

The first term on the left side of the previous equation represents an infinitesimal change in the internal energy of the system due to a change in the probability, without changing the energy levels. According to the Boltzmann partition function shown above, the change in probability at constant energy levels is due to heating; therefore, the first term represents the amount of heating energy [23] lost or gained by the system at the microscopic level in the positive subspace.

$$dQ^{`} = \sum_j E_j^{`}\, dp^{`}{}_j \tag{27.4}$$

By substituting from (20.4) and (21.4) into (27.4)

$$dQ^{`} = \gamma \sum_j \tilde{E}_j\, d\tilde{p}_j \tag{28.4}$$

The right side of the equation represents the same mathematical formula for calculating the quantity of heat, but in the positive subspace, *i.e.*, $\sum_j \tilde{E}_j\, d\tilde{p}_j = d\tilde{Q}$, by substituting this into (28.4) and rearranging the equation.

$$d\tilde{Q} = dQ^{`}\gamma^{-1} \;\;\Rightarrow\;\; \Delta\tilde{Q} = \Delta Q^{`}\gamma^{-1} \tag{29.4}$$

The previous equation shows us that the quantity of heat lost or gained by the system in a thermodynamic process decreases relative to the observer O in the positive subspace as the speed of the reference frame (container) increases. We can also obtain the transformation of the quantity of heat at the macroscopic level according to the law, $dQ^{`} = C_V dT^{`}$, where C_V or C_P is the heat capacity of the gas with respect to a constant volume or pressure, respectively, and it is constant under the transformation. By substituting from the transformation equation for the change of temperature No. 6.4, we obtain the same mathematical formula for the previous law in positive space and the same result of the transformation of the quantity of heat. When the frame of reference theoretically reaches the speed of light, the value of the heat becomes zero, or the transformation equation breaks down, as we have seen. However, when the equation is quantized according to the hypothesis of energy fluctuations between subspaces, it transforms into the following inequality.

$$d\tilde{Q} \geq dQ^{`}\gamma^{-1} + dQ^{`}_{\hbar} \implies d\tilde{Q} \geq dQ^{`}_{\hbar} \quad V_S = \mathrm{c} \tag{30.4}$$

where $dQ^{`}_{\hbar} = C_{P\backslash V} dT^{`}_{\hbar}$ represents the amount of heat lost or gained by the system when its temperature changes by the reduced Planck temperature. Therefore, it represents the minimum amount of heat that can be lost or added to the system. When substituting into the previous inequality, $V_S = \mathrm{c}$, we find that we have doubt about the existence of the minimum heat limit for the gas in the system. In other words, we cannot confirm the disappearance of heat and the thermodynamic processes associated with the heat in the system, but rather we have doubts about the existence of all this information.

2.6. Transformation of Mechanical Work in the Positive Subspace

If a thermodynamic system in a frame of reference S' has a freely moving piston, then at the macroscopic level the system can do mechanical work on or by the gas in a thermodynamic process [22] [24]. At the microscopic level, the mechanical work done by the piston represents the amount of energy transferred to or from the particles of the system by the force acting on the piston. According to quantum mechanics, the change in volume resulting from the movement of the piston leads to a change in the energy levels of the system. Therefore, the second quantity on the left side of Equation (26.4) represents an infinitesimal amount of change in the internal energy of the system by the mechanical work of the piston [23] in the positive subspace.

$$dW^{`} = \sum_j dE^{`}_j p^{`}_j \tag{31.4}$$

By substituting from (20.4) and (21.4) into (31.4)

$$dW^{`} = \gamma \sum_j d\tilde{E}_j \tilde{p}_j \tag{32.4}$$

The right side of the equation represents the same mathematical formula for calculating the work done, but in the positive subspace, *i.e.*, $\sum_j d\tilde{E}_j \tilde{p}_j = d\widetilde{W}$, by substituting this into (32.4) and rearranging the equation

$$d\widetilde{W} = dW^{`}\gamma^{-1} \implies \Delta\widetilde{W} = \Delta W^{`}\gamma^{-1} \tag{33.4}$$

The previous equation shows us that the mechanical work done on or by the system in a thermodynamic process decreases relative to the observer O in the positive subspace as the speed of the reference frame (container) increases. We can also obtain the transformation of the mechanical work of the piston at the macroscopic level, where, $dW^{`} = P^{`} dV^{`}_{ol}$. By substituting from the pressure transformation Equation (17.4) and the volume change Equation (1.4), we obtain the same mathematical formula for the law in positive space and the same result of the transformation of the mechanical work. When the frame of reference theoretically reaches the speed of light, the work done equals zero, or the work transformation equation collapses. However, when the equation is quantized, it becomes the following inequality.

$$d\widetilde{W} \geq dW^{`}\gamma^{-1} + dW^{`}_{\hbar} \implies d\widetilde{W} \geq dW^{`}_{\hbar} \qquad V_S = \mathrm{c} \tag{34.4}$$

where $dW^{`}_{\hbar} = \frac{1}{2}\hbar p^{`}{}_{j}$ represents the mechanical work done by the piston when the energy levels of the system change by the amount of the reduced Planck energy, therefore it represents the least possible mechanical work done in the system. When substituting into the previous inequality $V_S = \mathrm{c}$, we find that we have doubt about the existence of a minimum amount of mechanical work for the gas in the system. In other words, we cannot confirm the disappearance of mechanical work, piston motion, and the thermodynamic processes associated with the piston of the system in the positive subspace.

2.7. Transformation of Entropy in the Positive Subspace

When a thermodynamic system in a frame of reference S' transitions from one state to another in an irreversible thermodynamic process, the system's entropy changes or increases. At the macroscopic level, an observer O' can detect or measure the change in the system's entropy relative to the frame of reference S' using the second law of thermodynamics. At the microscopic level, the absolute entropy represents a measure of the degree of randomness or disorder in the system and is determined by the well-known Boltzmann equation ([18], pg, 138).

$$S^{`}_{th} = k_B \ln \Omega^{`} \tag{35.4}$$

where $S^{`}_{th}$ is the absolute thermodynamic entropy of the system, $\Omega^{`}$ represents the total number of microscopic states of the system, which is constant under the transformation from frame S' to S in the positive subspace, because it depends on the total number of particles N, and also on the number of possible particles in each energy level n_j, and each of them is constant under transformation.

$$\widetilde{\Omega} = \Omega^{`} = \frac{N!}{n_0!\, n_1! \ldots n_j!} \tag{36.4}$$

By substituting from (36.4) to (35.4)

$$S^{`}_{th} = k_B \ln \widetilde{\Omega} \tag{37.4}$$

The right side of the equation represents the same mathematical formula for calculating the absolute entropy of the system, but in the positive subspace, *i.e.* $k_B \ln \widetilde{\Omega} = \tilde{S}_{th}$, by substituting this into (37.4) and rearranging the equation.

$$\tilde{S}_{th} = S^{`}_{th} \implies d\tilde{S}_{th} = dS^{`}_{th} \tag{38.4}$$

The previous equation shows us that the absolute entropy, or the change in the entropy of a system in a thermodynamic process, is constant under transformation relative to the observer O in the positive subspace as the velocity of the frame of reference (container) increases. We can also obtain the entropy transformation at the macroscopic level, where, $dS^{`}_{th} = dQ^{`} \backslash T^{`}$. Substituting from the temperature transformation equation No. 6.4 and the change in heat quantity equation No. 29.4, we obtain the same mathematical formula for the law in the positive space and the same result for the entropy transformation. Dividing the previous equation by the time dilation equation in the positive subspace shown in

Paper 1 [6] No. 15.1, we obtain the same result.

$$\frac{d\tilde{S}_{th}}{d\tilde{t}} = \frac{dS^{`}_{th}}{dt^{`}}\gamma^{-1} \tag{39.4}$$

The previous equation shows us that the entropy per unit of time decreases in the positive subspace relative to the observer O as the speed of the frame of reference (container) increases, but its direction is constant. In other words, the direction of the arrow of time is constant, but the rate of flow of time in the same direction varies from one observer to another. This result differs from the nature of time in special relativity, which states that time is an illusion and that there is no arrow of time, but it is consistent with the nature of time in inverse relativity, which we arrived at in the third paper, where simultaneity is absolute, and the moment of the past precedes the moment of the present, and the moment of the present precedes the moment of the future, and time has a real direction [25] [26]. When the frame of reference theoretically reaches the speed of light, the entropy per unit of time equals zero. In other words, when time dilates to infinity, random motion, and consequently information disorder, ceases entirely within the system at the microscopic level, and the system has only one microscopic state, resulting in zero entropy. This contradicts the (38.4) transformation equation, which states that entropy remains constant. At the macroscopic level, we also find that the system temperature reaches absolute zero according to Equation (6.4), and the change in heat quantity also reaches zero according to Equation (29.4). Consequently, the entropy of the system in this case is undefined, which means a breakdown in the entropy transformation equation at both the microscopic and macroscopic levels. However, as we know, the previous transformation equations are classical equations. When we quantize these equations according to inequalities (7.4) and (30.4), we obtain

$$\frac{d\tilde{S}_{th}}{d\tilde{t}} \geq \frac{dS^{`}_{th}}{dt^{`}}\gamma^{-1} + \frac{dS^{`}_{\hbar}}{dt^{`}} \tag{40.4}$$

where, $dS^{`}_{\hbar} = dQ^{`}_{\hbar} \backslash T^{`}_{\hbar}$, represents the minimum entropy per unit time. Substituting this into the previous inequality, $V_S = \mathrm{c}$, We find that we have doubts about the existence of a minimum entropy flow in the system. That is, over relatively long periods of time, the rate of entropy flow is infinitesimally small and fluctuates, which means that time itself in this case is very slow or almost standing still.

$$\frac{d\tilde{S}_{th}}{d\tilde{t}} \geq \frac{dS^{`}_{\hbar}}{dt^{`}} \qquad V_S = \mathrm{c} \tag{41.4}$$

However, the previous result is only obtained when $\langle \Delta\tilde{t}_{un} \rangle = 1$ as we explained in Section 2-2. The lower the average uncertainty in the time period, the higher the average uncertainty in the positive energy until it reaches the maximum possible value in the system or equals the average actual energy of the particles in the system, *i.e.*, $\langle \Delta\tilde{E}_{un} \rangle_{max} = \langle E^{`} \rangle$. In this case, the average uncertainty in the time represents the shortest possible time period for the system, $\langle \Delta\tilde{t}_{un} \rangle_{min}$. During this period we have a doubt about the existence of a temperature equivalent to the actual temperature $\tilde{T}_{un} = T^{`}$, a doubt about the existence of a quantity of heat equivalent to the actual quantity of heat $d\tilde{Q}_{un} = dQ^{`}$, and therefore a doubt about the existence of an entropy equivalent to the actual entropy.

$$d\tilde{S}_{un} = dS^{`}_{th} \qquad V_S = \mathrm{c} \tag{42.4}$$

Assuming here, $dt^{`} = \langle d\tilde{t}_{un}\rangle_{min}$, by dividing that by the previous equation.

$$\frac{d\tilde{S}_{un}}{\langle d\tilde{t}_{un}\rangle} = \frac{dS^{`}_{th}}{dt^{`}} \quad V_S = \mathrm{c}, \quad dt^{`} = \langle d\tilde{t}_{un}\rangle_{min} \tag{43.4}$$

The last equation shows that during the period $\langle d\tilde{t}_{un}\rangle_{min}$ we have a doubt that the flow of entropy is equal for both observers in the positive subspace. This means that over very short time intervals, the rate of entropy flow is symmetrical or equal but also fluctuates. This implies that time itself is symmetrical or equal for both observers for a very short or period temporary. From all of the above, we conclude that over relatively long time periods, we have doubts about the existence of a minimum limit of thermal quantities, and over infinitesimally short time periods, we have doubts about the existence of a maximum limit of thermal quantities. That is, we have two scales for thermal quantities in the positive subspace when the system reaches the speed of light, depending on the time interval during which the observer O observes this system.

Multiplying both sides of the Equation (41.4) by (43.4)

$$\frac{d\tilde{S}_{un}}{\langle d\tilde{t}_{un}\rangle} \cdot \frac{d\tilde{S}_{th}}{d\tilde{t}} = \frac{dS^{`}_{th}}{\langle d\tilde{t}_{un}\rangle} \cdot \frac{dS^{`}_{\hbar}}{dt^{`}} \tag{44.4}$$

When rearranging the denominators, the right side of the previous equation becomes $\frac{dS^{`}_{th}}{dt^{`}} \cdot \frac{dS^{`}_{\hbar}}{\langle d\tilde{t}_{un}\rangle}$. We can assume here that the minimum entropy in the shortest period of time is equivalent to the maximum entropy in the longest period of time. Therefore, the product of the previous quantities equals, $\left(\frac{d\tilde{S}_{th}}{d\tilde{t}}\right)^2$. Substituting this into the previous equation, and also substituting from Equation (42.4) into Equation (44.4)

$$d\tilde{t}\langle d\tilde{t}_{un}\rangle = dt^{`2} \implies d\tilde{t}_{hol} = \frac{1}{\langle d\tilde{t}_{un}\rangle_{min}} \quad dt^{`} = 1 \tag{45.4}$$

The last equation shows us that the greater the time dilation $d\tilde{t}$ resulting from the increased speed of the reference frame (container), the lower the average time uncertainty $\langle d\tilde{t}_{un}\rangle$. When the system (container) theoretically reaches the speed of light, assuming that, $dt^{`} = 1$, the dilation per unit of time reaches its maximum value, or time is almost at a standstill. In contrast, the average uncertainty per unit of time is reduced to its minimum value $\langle d\tilde{t}_{un}\rangle_{min}$, where time is the same between observers. This means that the pauses of time are not continuous, but are separated by periods of time symmetry. Therefore, these time pauses represent gaps or holes $d\tilde{t}_{hol}$ in the timeline. In other words, the passage of causal time, or time associated with thermodynamic entropy in positive subspace, is not continuous or uninterrupted time, but rather discontinuous time or in the form of an infinitesimally small, quantized time periods (Because it represents the minimum time in the system) that are symmetrical between observers, separated by large gaps of time dilation or pauses. In this way, the inverse relativity model reveals to us the quantization of time, where time in special relativity is continuous and it stops permanently when the container theoretically reaches the speed of light.

Equation (42.4) shows us that the entropy of energy fluctuations in the positive subspace relative to observer O is equivalent to the thermodynamic entropy of the system relative to observer O'. This means that a decrease in thermodynamic entropy due to a decrease in the system's temperature or cooling relative to observer O', will appear relative to observer O in the positive subspace, as a decrease in the entropy of energy fluctuations. This leads to the synchronization or unification of energy fluctuations between the subspaces of most of the system's particles, and consequently, the behavior of the particles within the system becomes uniform or as a single macroscopic particle [27]. That is, when the system is cooled, energy fluctuations at the macroscopic level of the system appear over relatively long periods in the form of the system's zero-point energy, and over infinitesimal periods in the form of macroscopic quantum tunneling [28].

2.8. Transformation of Volume in the Negative Subspace

The volume of the thermodynamic system (container) with respect to the observer O in the negative subspace $\check{V}_{ol}$ is also determined by multiplying the geometric dimensions in the negative subspace $\check{V}_{ol} = \Delta\check{x} \cdot \Delta\check{y} \cdot \Delta\check{z}$. According to the inverse negative modified Lorentz transformations shown in Paper 1 [6], Section 2.4, Nos. (38.1), (39.1), (40.1), we find that $\Delta\check{y} = \Delta\check{z} = 0$, therefore the volume of the system in the negative subspace is zero $\check{V}_{ol} = 0$. But this result is only obtained when the relative motion between the frames of reference is on a single axis, such as $xx`$. However, when the motion of the frames relative to each other is on three axes, then both $\Delta\check{y} \neq 0, \Delta\check{z} \neq 0$. In this case, we can write the volume transformation in the negative subspace in terms of both the total relativistic volume V_{ol} (*i.e.*, in the total space) and the positive volume $\tilde{V}_{ol}$ according to the following equation: $\check{V}_{ol} = V_{ol} - \tilde{V}_{ol}$, Where, the transformation of the total relativistic volume is according to Lorentz contraction or special relativity transformations $V_{ol} = V`_{ol}\,\gamma^{-1}$, while the transformation of the positive volume is according to the symmetry of the spatial structure $\tilde{V}_{ol} = V`_{ol}$. By substituting the transformation of each volume into the negative volume equation, we obtain the volume transformation in the negative subspace according to the following equation.

$$\check{V}_{ol} = V_{ol}^{`}(\gamma^{-1} - 1) \tag{46.4}$$

The previous equation shows us that the volume of the container increases or expands from zero to its original value, but with a negative sign relative to the observer O in the negative subspace, as the speed of the frame of reference (container) increases. When the speed of the frame of reference reaches the speed of light, theoretically we find that, $\check{V}_{ol} = -V_{ol}^{`}$. But how do particles move in a negative volume? It must be taken into account here that the speed of gas particles in negative subspace is uniform in magnitude., which is the magnitude of the speed of the frame of reference, and also uniform in direction, which is the direction of the motion of the frame of reference according to the velocity transformations shown in Paper 2 [7], Equation (53.2), *i.e.* there is no random motion of the particles in this subspace. Therefore, the negative volume of the system here does not represent a space for the random motion of gas particles; in other words, the negative volume of the container cannot be considered a thermodynamic variable in the system. It must also be taken into account that the container will appear to be moving relative to the observer O in the negative subspace, and therefore when measuring the geometric dimensions of the container, such as the length dimension, the measurement of the two ends of the length must be done at the same instant, as we did when measuring the length of the rod in the negative subspace shown in Paper 3, Section 2.1 of Inverse Relativity (or as in Special Relativity [29]). But at this moment the system will appear stationary relative to the observer O, and consequently the particles of the system are also stationary because they have the same speed as the system in this subspace as we mentioned above. This means that at the moment of measuring or observing the negative volume, the particles of the system are stationary relative to the observer O, and therefore we cannot consider the negative volume to be even a space for the uniform motion of the particles of the system, but rather it appears as a space for the position of the particles. Therefore, when the system reaches the speed of light, the negative volume here represents a reflection of the particle's position or direction of motion. So, if the value of this volume is zero or negative, this does not contradict the particle's motion, because it does not represent the actual space in which the particles move. Furthermore, the particle's motion in this space is uniform and not random, as we mentioned.

2.9. Transformation of Temperature in the Negative Subspace

As mentioned earlier in section 2-2, temperature is a measure of the average speed and kinetic energy of the particles within the system relative to the observer O'. But what about the temperature transformation of the system relative to the observer O in a negative subspace? According to the definition of this subspace, as explained in Paper 1 [6], Section 2.4, it is a space without causality or without collisions, meaning there are no collisions occurring between the gas particles. Also, from the definition of negative relativistic mechanics explained in paper 2 [7], the section on discussions, where physical quantities such as velocity with respect to the observer O are constant when the velocity of the frame of reference is constant, *i.e.*, they do not change with changes in their counterparts with respect to the observer O'. This means that the physical quantities of the particle here are not related to any causal event that occurs to the particle as a collision with another particle. By squaring the equations for transforming the relativistic velocity components of a particle in the negative subspace shown in Paper 2, Set No. 53.2, and by summing all the possible values of the square of each velocity component that the particles take within the system at a given moment, and by dividing both sides of each resulting equation by the number of particles, we obtain transformation equations for the average of the squared components of the particles velocity in negative subspace.

$$\langle \breve{u}_x^2 \rangle = V_S^2$$
$$\langle \breve{u}_y^2 \rangle = 0$$
$$\langle \breve{u}_z^2 \rangle = 0 \tag{47.4}$$

The previous set of equations shows that the velocity transformations of the particles in this subspace are uniform in magnitude and direction, which is the velocity and direction of the frame of reference. Therefore, they represent the motion of the system's particles with the system's center of mass (the container). By summing the average of the squared components, we obtain a transformation equation for the average of the squared net velocity of the particles in the system.

$$\sum_{i=3}^{3} \langle \breve{u}_i^2 \rangle = V_S^2 \implies \langle \breve{u}^2 \rangle = V_S^2 \tag{48.4}$$

The previous equation shows us that the average of the squared net velocity of the particles relative to the observer O in the negative subspace increases with the increasing velocity of the frame of reference (container). Taking the square root of both sides of the equation, we obtain a transformation equation for the root mean square resultant velocity, $\breve{u}_{rms} = V_S$. By following the same steps as before, but with the sum of possible values of the components first, then dividing by the number of particles and squaring the result, we can obtain the transformation of the average resultant velocity of the particles $\langle \breve{u} \rangle = V_S$ and the most probable velocity of the particles $\breve{u}_p = V_S$ in the negative subspace. From all of the above, we conclude that, $\breve{u}_{rms} = \langle \breve{u} \rangle = \breve{u}_p$. Therefore, when, $\breve{u} \neq V_S$, the probability of finding particles at that speed in this subspace is zero, *i.e.*, $f(\breve{u}) = 0$. And when, $\breve{u} = V_S$, the probability of finding particles at that speed is equal to its maximum value, $f(\breve{u}) = \infty$. This means that the velocity distribution of the particles of the system in the negative subspace does not follow the Maxwell-Boltzmann distribution function, but a new distribution function, which is the shifted Dirac delta function, [30] written in this form.

$$\delta(\breve{u} - V_S) = \begin{cases} 0 & \breve{u} \neq V_S \\ \infty & \breve{u} = V_S \end{cases} \tag{49.4}$$

As we know, the integral of this function is equal to one, while the average of the squared resultant velocity of the particles in the negative subspace is equal to the integral of the function for all speed values $\breve{u}$ multiplied by $\breve{u}^2$.

$$\langle \breve{u}^2 \rangle = \int_0^\infty \breve{u}^2 \delta(\breve{u} - V_S)\, d\breve{u} = V_S^2 \tag{50.4}$$

When the Maxwell-Boltzmann distribution function $f(\breve{u})$ is arbitrarily imposed in negative space

$$f(\breve{u}) = 4\pi \left(\frac{m}{2\pi k_B \breve{T}} \right)^{\frac{3}{2}} \breve{u}^2 \exp\left(-\frac{\breve{E}}{k_B \breve{T}} \right) \tag{51.4}$$

where $f(\breve{u})$ is the probability of a particle existing in the speed range between $\breve{u}$ and $\breve{u} + d\breve{u}$., $\breve{E} \approx k\breve{E}$ is the relativistic energy of the particle in negative subspace, and m represents the total relativistic mass according to special relativity. However, to make $f(\breve{u}) = \infty$ when, $\breve{u} = V_S$, must be $\breve{T} = 0\ \text{K}$.

$$\left(\frac{m}{2\pi k_B \breve{T}}\right)^{\frac{3}{2}} = \infty, \quad \exp\left(-\frac{\breve{E}}{k_B \breve{T}}\right) = \frac{1}{0} = \infty, \quad \breve{T} = 0\ \text{K} \tag{52.4}$$

This means that in order for the Maxwell-Boltzmann function to collapse in the negative subspace to a Dirac delta function, the temperature of the gas must always be absolute zero in that subspace; that is

$$f(\breve{u}) = \delta(\breve{u} - V_S) \ \text{when}\ \breve{T} = 0\ \text{K} \tag{53.4}$$

We conclude. Therefore, that the temperature of the system relative to the observer O in the negative subspace is always equal to absolute zero, regardless of the temperature of the system relative to the observer O' or the velocity of the frame of reference. In other words, the concept of temperature disappears or does not exist in this subspace.

$$\breve{T} = 0 \implies d\breve{T} = 0 \tag{54.4}$$

We can graphically represent the shifted Dirac delta distribution in the negative subspace according to the diagram shown in **Figure 3**.

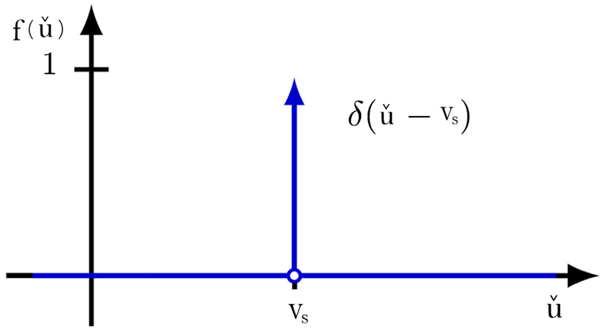

Figure 3. The velocity and probability density distribution of particles in the negative subspace at a given velocity of the reference frame.

2.10. Transformation of Pressure in the Negative Subspace

We explained above in section 3.2 that pressure depends on the average of the relativistic forces exerted by the particles when they collide with the container wall, relative to observer O' ([11], pgs, 3, 59, 60). But what about the gas pressure relative to observer O in negative subspace? As we mentioned above, this subspace is a space without causality or collision, and therefore there are no collisions between the wall and particles here, and the physical quantities with respect to the observer O are constant with the constant speed of the frame of reference S'. For example, the momentum of the particle here remains constant with respect to the observer O, regardless of the change in the momentum of the same particle with respect to the observer O'. From the equations for transforming the relativistic force components of a particle in the negative subspace shown in Paper 2 [7], Set

No. 77.2, we obtain the transformation of all possible values of each force component that each particle exerts on the wall of the system at a given moment. By summing all possible values of each force component, dividing both sides of each resulting equation by the number of particles, and squaring them.

We obtain transformation equations for the square of the average force components for each particle in the system

$$\breve{\bar{F}}_x^2 = 0$$
$$\breve{\bar{F}}_y^2 = 0$$
$$\breve{\bar{F}}_z^2 = 0 \tag{55.4}$$

The previous set of equations shows that, the square of the average force components are zero in all directions. This means that the center of mass of the system (container) is either stationary or moving at a constant speed in this subspace. By summing the square of the average force components in each direction, we obtain a transformation equation for the square of the average net force exerted by each particle at a point on the walls of the system.

$$\sum_{i=3}^{3} \breve{\bar{F}}_i^2 = 0 \implies \breve{\bar{F}}^2 = 0 \implies \breve{\bar{F}} = 0 \tag{56.4}$$

By multiplying both sides of the equation by the number of particles N, we obtain the sum of the average forces exerted by all the particles of the system on the inner wall of the container.

$$N\breve{\bar{F}} = 0 \tag{57.4}$$

Dividing both sides of the equation by the surface area of the inner wall of the container, taking into account here that the areas of the interior surfaces under the transformation from frame S' to frame S in the negative subspace do not necessarily equal zero as we mentioned above in the negative volume section.

$$\frac{N\breve{\bar{F}}}{\breve{A}} = 0 \implies \breve{P} = 0 \tag{58.4}$$

where, $\breve{P}$ is the relativistic gas pressure in negative subspace, the previous equation shows us that the gas pressure is always zero with respect to observer O in the negative subspace at any value of the velocity of reference frame (container). This means that the concept of thermodynamic pressure disappears or does not exist in this subspace.

2.11. Transformation of Internal Energy in the Negative Subspace

Just as we limited the concept of internal energy in the positive subspace to the kinetic energy of the system's particles, we will also limit the concept of internal energy in the negative subspace to the relativistic kinetic energy, which is equivalent in value to the total relativistic energy of the particle $k\breve{E} \approx \breve{E}$, since the gas is relativistic as mentioned above. Thus, the internal energy of the system becomes, according to statistical mechanics ([18], pg. 130).

$$\breve{U} = \sum_j \breve{E}_j \breve{p}_j \tag{59.4}$$

where, $\breve{U}$ is the internal energy of the system, $\breve{E}_j$ represents the energy of level j in the system, and $\breve{p}_j$ is the probability of having n_j of particles occupying this energy level. From the second formula of the equation for the transformation of relativistic kinetic energy or the total relativistic energy of a particle in the negative subspace shown in Paper 3, Equation (42.3) and Paper 2, Equation (71.2) respectively [7] [8], we obtain a transformation equation for the sum of the energy levels of the system in the negative subspace. Note: We do not use the first formula of each equation, because the momentum $p`_x$ in the first formula of both equations represents the momentum of the particle moving with the system and not inside the system, and because the system is stationary with respect to the frame S', therefore $p`_x = 0$, each equation reduces to the second formula.

$$\sum_j \breve{E}_j = \frac{V_s}{c}(\gamma - 1)\sum_j E`_j \Rightarrow \sum_i \left(d\breve{E}_j\right)_u = \frac{V_s}{c}(\gamma - 1)\sum_j dE`_j \tag{60.4}$$

The previous equation represents a change in the energy of the particle at a constant speed, because the speed of the particles in this subspace is constant as we mentioned above, and therefore the change in the energy of the particle here depends only on the change in mass. See Equation (37.3) on Paper 3 [8], where every change in the speed of the particle relative to the observer O' corresponds to a change in the mass of the particle relative to the observer O in negative space. In other words, every change in the mass of the particle resulting from a change in the speed of the particle relative to the observer O' corresponds to a change in the mass of the particle relative to the observer O in the negative subspace. This means that the negative subspace contains a copy of the system's microscopic information preserved in particle masses. This explains how the microscopic and macroscopic properties of the system are restored when energy oscillates from the negative subspace to the positive. However, the change in the relativistic energy of a particle with respect to speed or at a constant mass is zero.

$$\sum_j \left(d\breve{E}_j\right)_m = 0 \tag{61.4}$$

As for the probability transformation here, when the Boltzmann function is applied arbitrarily in the negative subspace, and by substituting the value of temperature $\breve{T} = 0$ into the function, the function collapses and we obtain $\breve{p}_j = 1$ for the system in the negative space.

$$\breve{p}_j = 1 \Rightarrow d\breve{p}_j = 0 \tag{62.4}$$

By substituting from (62.4) and (60.4) to (59.4)

$$\breve{U} = \frac{V_s}{c}(\gamma - 1)\sum_j E`_j \tag{63.4}$$

Multiplying both sides of the equation $p`_j$

$$\breve{U}p`_{j} = \frac{V_s}{c}(\gamma - 1)\sum_{j} E_j` p`_{j} \tag{64.4}$$

The right side represents the internal energy of the system.

$$\breve{U} = U` \frac{V_s}{cp`_{j}}(\gamma - 1) \;\Rightarrow\; d\breve{U} = dU` \frac{V_s}{cdp`_{j}}(\gamma - 1) \tag{65.4}$$

The previous equation shows us that the internal energy increases relative to the observer O in the negative subspace as the velocity of the frame of reference (the container) increases. When the velocity of the frame of reference is zero, the value of the internal energy is also zero. However, when the equation is quantized according to the hypothesis of energy oscillation between subspaces, as an explanation of the inverse Heisenberg principle explained in Paper 3 [7], Section 2.7, the equation becomes an inequality.

$$\breve{U} \le U` \frac{V_s}{cdp`_{j}}(\gamma - 1) + U_{\hbar}` \;\Rightarrow\; \breve{U} \le U_{\hbar} \quad V_S = 0 \tag{66.4}$$

When substituting into the previous inequality $V_S = 0$, we find that we have doubt about the existence of a minimum internal energy of the system in the negative subspace, *i.e.*, we cannot confirm the disappearance of the internal energy of the system in the negative subspace even when there is no motion of the system. However, this result is only obtained when $\langle \Delta\check{t}_{un} \rangle = 1$. As the average uncertainty in the time period of each particle decreases until it reaches the smallest possible time period of the system $\langle \Delta\check{t}_{un} \rangle_{min}$,the average uncertainty in the energy of each particle increases until it reaches the maximum possible value in the system, which here equals the zero point energy. *i.e.*, $N\langle \Delta\breve{E}_{un} \rangle_{max} = NE_0`$ or $\breve{U}_{un} = U`_{0k}$. But $\breve{U}_{un} = Nm_0 \langle \Delta\check{V}_{un}^2 \rangle_{max}$. Therefore, we have doubt about the existence of self- acceleration produced by the energy of the zero point for each particle in the system or for the system as a whole (the container), because $\Delta\check{V}_{un} = \Delta V_S$, the doubt here is due to the oscillation of that energy all at once. This does not represent a violation of the principle of special relativity because the acceleration is infinitesimally small.

$$Nm_0 \Delta\check{V}_{un}^2 = NE_0` \;\Rightarrow\; \Delta\check{V}_{un} = \sqrt{\frac{\hbar\omega`_0}{2m_0}} \tag{67.4}$$

Accordingly, it is not necessary to do infinite mechanical work to move the system (container) to the speed of light. It is sufficient to do mechanical work to bring the system to the critical speed $\check{V}_c$, which is less than the speed of light by $\Delta\check{V}_{un}$, and then it accelerates itself through the zero point energy to reach the speed of light.

$$\check{V}_c = c - \Delta\check{V}_{un} \tag{68.4}$$

By substituting from (67.4) to (68.4)

$$\check{V}_c = c - \sqrt{\frac{\hbar\omega`_0}{2m_0}} \tag{69.4}$$

So, reaching the speed of light is no longer a theoretical matter now, but has become practically possible without infinite energy. We explained this result in Paper 3 [8], but for a single particle or a dust field without the zero-point energy term because there is no thermal component for either. If we assume that $\omega`_0 = 1$ in the previous equation, we obtain the same Equation (74.3) shown in Paper 3. Here, we can write the internal energy of the system in the total space U, or according to special relativity, as a sum of both the internal energy in the positive and negative subspaces

$$U = \breve{U} + \tilde{U} \tag{70.4}$$

We can also rewrite the previous equation in the following form

$$U = N\langle \breve{E} \rangle + N\langle \tilde{E} \rangle \tag{71.4}$$

where, $\langle \breve{E} \rangle$ and $\langle \tilde{E} \rangle$ are the average of relativistic energy of the particles with respect to the observer O in the negative and positive subspaces, respectively. Since the energy distribution in the positive subspace follows the Maxwell-Boltzmann energy distribution, we can obtain the relation, $\langle \tilde{E} \rangle = 3k_B\tilde{T}$. The energy distribution in the negative subspace follows the energy-shifted Dirac delta distribution, and by integrating this function in the negative subspace, we obtain $\langle \breve{E} \rangle = \langle m \rangle V_S^2$. By substituting this into the Equation (71.4)

$$U = N\langle m \rangle V_S^2 + 3Nk_B\tilde{T} \tag{72.4}$$

The previous equation shows us that the internal energy of the system relative to the observer O in the positive subspace $3Nk_B\tilde{T}$ is dynamic thermal energy of the system because it depends on temperature. However, in the negative subspace, $N\langle m \rangle V_S^2$, it is kinetic energy with the system, because it depends on the velocity of the frame of reference (container). This means that the internal energy of a thermodynamic system in total space, or according to special relativity, is not entirely thermal energy; part of it is thermal energy, and the other part is non-thermal, but kinetic. In classical physics, the non-thermal or kinetic part is not considered internal energy of the system, but here, as a result, the energy oscillates between positive and negative subspaces. There is no clear boundary between the thermal energy of the system and the kinetic energy of the system at the quantum level, as is commonly understood in classical physics.

2.12. Transformation of Heat in the Negative Subspace

To obtain the heat transformation in the negative subspace, as we mentioned above, the heat at the microscopic level represents the energy transferred from the particles of the system to the particles of the water tank or vice versa through mutual collisions, which leads to a change in the internal energy of the system [22]. By differentiating Equation (59.4), we obtain an infinitesimal change in the internal energy of the system in the negative subspace at the microscopic level, *i.e.*, in terms of both $\breve{E}_j$ and $\breve{p}_j$.

$$d\breve{U} = \sum_j \breve{E}_j \, d\breve{p}_j + \sum_j d\breve{E}_j \, \breve{p}_j \tag{73.4}$$

As we have already mentioned, the infinitesimal change in the internal energy of the system through the change in probability only represents the amount of heat that the system loses or gains at the microscopic level [23], *i.e.*, the first quantity on the left side of the equation represents an infinitesimal amount of heat energy in the negative subspace.

$$d\breve{Q} = \sum_j \breve{E}_j \, d\breve{p}_j \tag{74.4}$$

By substituting from (60.4) to (62.4), in (74.4)

$$d\breve{Q} = 0 \implies \Delta\breve{Q} = 0 \tag{75.4}$$

The previous equation shows us that the amount of heat lost or gained by the system in a thermodynamic process is zero relative to the observer O in the negative subspace at any value of the frame of reference velocity. This represents a breakdown of the law of heat energy in the negative subspace at the microscopic level, and also at the macroscopic level, $d\breve{Q} = C_V d\breve{T}$, where $d\breve{T} = 0$ according to Equation (54.4). This is due to the absence of mutual collisions between particles; therefore, heat transfer cannot occur here. This means that the concept of heat also disappears in the negative subspace.

2.13. Transformation of Mechanical Work in the Negative Subspace

To obtain the transformation of the mechanical work of the piston in the negative subspace, as we mentioned above, the mechanical work at the microscopic level of the system represents the energy transferred to or from the system by the force acting on the piston, which leads to a change in the energy levels of the system. Therefore, the second quantity from the left side of Equation (73.4) represents an infinitesimal amount of the change in the internal energy of the system by the mechanical work of the piston [23] in the negative subspace.

$$d\breve{W} = \sum_j d\breve{E}_j \, \breve{p}_j \tag{76.4}$$

Because the change in the relativistic energy of a particle in thermodynamics depends primarily on its velocity and not it's mass. Therefore, we substitute from Equation (61.4) and (62.4) into (76.4).

$$d\breve{W} = 0 \implies \Delta\breve{W} = 0 \tag{77.4}$$

The previous equation shows us that the mechanical work done on or by the system in a thermodynamic process is zero relative to the observer O in the negative subspace at any value of the reference frame speed (the container). This represents a breakdown of the law of mechanical work in the negative subspace at the microscopic level, and also at the macroscopic level $d\breve{W} = \breve{P} d\breve{V}_{ol}$, where $\breve{P} = 0$ according to Equation (58.4). This is due to the absence of collisions between the particles and the piston; therefore, no movement of the piston can be observed here. This means that the concept of work also disappears in the negative subspace.

2.14. Transformation of Entropy in the Negative Subspace

As mentioned above, the thermodynamic entropy at the microscopic level of a system is a measure of the degree of randomness or disorder resulting from the random motion of the system's particles, according to Boltzmann equation ([18], pg, 138). By arbitrarily rejecting the same mathematical formula in a negative subspace, we obtain

$$\check{S}_{th} = k_B \ln \breve{\Omega} \tag{78.4}$$

where $\breve{\Omega}$ is the total number of microscopic states of the system in the negative subspace, because there is only one thermodynamic probability for the system in the negative subspace according to Equation (62.4), therefore we have only one microscopic state, *i.e.*, $\breve{\Omega} = 1$. Substituting this into the previous equation, we get

$$\check{S}_{th} = 0 \implies d\check{S}_{th} = 0 \tag{79.4}$$

The previous equation shows us that the absolute entropy, or the change in entropy of a system in a thermodynamic process, is zero relative to the observer O in the negative subspace at any value of the velocity of reference frame (the container). This represents a breakdown of the Boltzmann equation for entropy. In paper 3, we explained, based on Minkowski's spacetime diagrams, that when a particle reaches the speed of light due to self-acceleration by reduced Planck energy, the total spacetime structure collapses according to special relativity. While the negative subspace-time of the particle transforms into inverted spacetime (a reversal of both space and time), the particle returns to its position in the past, or in other words, the particle's time becomes the opposite of the stationary observer's time. Because the thermodynamic system can also reach the speed of light as a result of self-acceleration by the zero-point energy of the system, as we explained above, therefore the negative subspace-time of the thermodynamic system also turns into inverted spacetime. The reversal of spacetime here does not mean a reversal of the arrow of time or a reversal of thermodynamic processes within the system. As mentioned in previous sections, the entropy of the system in negative subspace is zero, and the concepts of heat, work, and thermodynamic processes do not exist. Rather, the reversal of spacetime here means a reversal of the motion of the frame of reference (the motion of the container). The observer O, in his future, observes the container's past or its return to its past position without any reversal or change in the system's information at the microscopic and macroscopic levels. This reversal is temporary, resulting from fluctuations in zero point energy. We can also understand the reversal of spacetime from another perspective. Where the infinite contraction in uncertainty time is canceled out by the infinite dilation in uncertainty time associated with the zero-point energy transfer from the positive to the negative subspace. Thus, time becomes symmetrical or equal between the reference frames in the negative subspace, but only temporarily or for a short period. Since negative volume represents the position space of the system's particles, as explained above, the reversal of volume represents a reversal in position space, that is, a reversal in the motion of the reference frame (the container) or in the direction of time itself. Therefore, time in this case appears symmetrical but inverted.

2.15. Local Energy-Momentum Tensor of a Fluid on Total Spacetime in the General Case

In the second paper of the inverse relativity model [7], we analyzed energy-momentum tensor of a dust field on the total spacetime fabric, whether in the special or general case locally, to obtain the energy-momentum tensor specific to each subspace-time. But, the dust field is a non-thermal field because it does not contain thermal components such as pressure and temperature. Therefore, in this paper we will analyze the energy-momentum tensor of a fluid moving with acceleration on the total spacetime with respect to two different observers, but the analysis in the general case will be local, *i.e.*, at a point on the total spacetime where the fluid has an instantaneous velocity, which is equivalent to the special case. We use locality here for the purpose of getting rid of the uniform acceleration of the particles with the center of mass of the fluid (Acceleration of the reference frame), so that we are left with the random acceleration of particles which is symmetrical around the center of mass of the fluid with respect to both observers. In other words, we get rid of the fluid pressure on spacetime and obtain only the thermodynamic pressure of the fluid with respect to both frames. We did not explain this matter in the second paper because the gas field does not have a thermodynamic pressure. The fluid has a proper density, $\rho_0 = N\,m_0/(V_{ol})_0$, where m_0 is the uniform mass of the particles, but it is not necessarily the rest mass of the particles if the particles have a relativistic random velocity inside the fluid, N is the number of particles in the fluid, $(V_{ol})_0$ is the proper volume of the fluid, but besides the proper density, the fluid also has thermal components such as the proper pressure P_0. If we want to describe both the energy density and the relativistic pressure of the fluid on the total spacetime with respect to each observer, it is done through the traditional energy-momentum tensor of the fluid ([31], pgs, 209, 210). With respect to observer O', it is according to the following equation.

$$T^{`kl} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^{`k} u^{`l} - P_0 \eta^{`kl}, \;\; k = l = 0,1,2,3 \tag{80.4}$$

where $T^{`kl}$ is the energy-momentum tensor, c is the speed of light constant, $u^{`k}u^{`l}$ here represent the contravariant components of the 4D velocity vector of the fluid at the macroscopic level (At the microscopic level, it is the velocity of particles with the center of mass, not around the center of mass of the fluid) on the total spacetime and they are instantaneous vectors because the fluid is accelerating., $\eta^{`kl}$ is the metric tensor of the total spacetime, which is locally flat with respect to the observer O' ([31], pag, 112) [32], and is written in terms of Cartesian coordinates according to the following formula

$$\eta^{`kl} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{81.4}$$

As we know, the 4D velocity vectors $u`^{k}$ or $u`^{l}$ are the differential of the 4D displacement vector with respect to the frame S' in terms of cartesian coordinates with respect to the infinitesimal proper time $d\tau$. ([31], pag, 138). Since the time $d\tau$ is the time with respect to the fluid, the time transformation from the fluid's frame of reference to the S' frame is given by the following equation: $dt` = d\tau\,\gamma`_{u}$, where $\gamma`_{u}$ depends on the fluid velocity $u`$ with respect to frame S'

$$u`^{k} = \gamma`_{u}\left(\frac{c}{dt`}, \frac{dx`}{dt`}, \frac{dy`}{dt`}, \frac{dz`}{dt`}\right) \tag{82.4}$$

Assuming here that the observer O' observes the fluid when the velocity of the frame of reference S' is equal to the instantaneous velocity of the fluid, *i.e.*, at the instant when the velocity of the fluid with respect to frame S' is zero $u` = 0$, and therefore $\gamma`_{u} = 1$, in this case it will appear to the observer O' that the fluid is moving only in the dimension of time and that all components of the velocity in space with respect to its frame of reference are equal to zero, so the velocity vector with respect to it is according to the following formula.

$$u`^{k} = (c, u`^{x}, u`^{y}, u`^{z}) = (c, 0,0,0) \tag{83.4}$$

It is the same formula used in paper 2 [7], Equation (85.2) for the 4D velocity vector, with the difference that the instantaneous velocity here represents the velocity of the fluid, not the dust. As explained in Paper 2, the tensor consists of the following matrix

$$T`^{kl} = \begin{bmatrix} T`^{00} & T`^{01} & T`^{02} & T`^{03} \\ T`^{10} & T`^{11} & T`^{12} & T`^{13} \\ T`^{20} & T`^{21} & T`^{22} & T`^{23} \\ T`^{30} & T`^{31} & T`^{32} & T`^{33} \end{bmatrix} \tag{84.4}$$

To obtain the value of the first component, $T`^{00}$, which here represents the relativistic total energy density of the fluid relative to the observer O', we substitute into Equation (80.4) the value of the velocity component $u`^{0}$ from Equation (83.4), and the value of the matrix tensor component $\eta`^{00}$ from Equation (81.4).

$$T`^{00} = \left(\rho_0 + \frac{P_0}{c^2}\right)c^2 - P_0 = \rho_0 c^2 \tag{85.4}$$

We find that the energy density of the fluid relative to the observer O' is equal to the proper energy density. Following the same steps of the solution with the following components $T`^{11}, T`^{22}, T`^{33}$ which represent the relativistic pressure of the fluid in each of the three directions, we obtain the value of those components.

$$\begin{aligned} T`^{11} &= 0 - P_0(-1) = P_0 \\ T`^{22} &= 0 - P_0(-1) = P_0 \\ T`^{33} &= 0 - P_0(-1) = P_0 \end{aligned} \tag{86.4}$$

Following the same solution steps with components, $T`^{10}, T`^{20}, T`^{30}$, and the components $T`^{21}, T`^{31}, T`^{32}$, which represent the energy and momentum flow of the fluid in each direction respectively, we find its values equal to zero. This indicates that the fluid is in thermodynamic equilibrium. Following the same solution steps also with components, $T`^{01}, T`^{02}, T`^{03}$, and the components $T`^{12}, T`^{13}, T`^{23}$, which represent the momentum density and shear stress (viscosity) of the fluid respectively, also yields values of zero. This indicates that we are describing a perfect fluid [33]. By substituting all the values of the tensor components that we obtained in equation No. 84.4, we obtain the energy-momentum tensor matrix of a perfect fluid in thermodynamic equilibrium in the general case locally with respect to the observer O'.

$$T^{`kl} = \begin{bmatrix} \rho_0 c^2 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{bmatrix} \tag{87.4}$$

In the case where the fluid pressure is zero, Substituting this into the previous matrix, we obtain the same formula for the dust field matrix in the special case that we obtained in Paper 2 [7], Section 2.10, Equation (87.2), because zero pressure means the disappearance of the random motion of the fluid particles, where the fluid becomes like dust, and the general case locally is equivalent to the special case. As for the observer O in the frame of reference S, the fluid will also appear to be moving with acceleration on the total spacetime fabric, but at the previous moment of observation, the fluid also has an instantaneous velocity u with respect to the frame of reference S. We write the energy-momentum tensor of the fluid with respect to the frame of reference S according to the following equation.

$$T^{\mu\nu} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^\mu u^\nu - P_0 \eta^{\mu\nu}, \quad \mu = \nu = 0,1,2,3 \tag{88.4}$$

where $T^{\mu\nu}$ is the energy-momentum tensor of the fluid with respect to observer O, u^μ, u^ν are the contravariant components of the 4D instantaneous velocity vector of the fluid, $\eta^{\mu\nu}$ is the metric tensor of a locally flat spacetime with respect to observer O, *i.e.*, $\eta^{\mu\nu} = \eta^{`kl}$. The 4D velocity vector u^μ or u^ν is also written in the same form as above in Equation (82.4), but the time transformation here is from the fluid's frame of reference to the frame S is given by the following equation: $dt = d\tau\,\gamma_u$, where γ_u depends on the fluid velocity u with respect to S.

$$u^\mu = \gamma_u (c, u^x, u^y, u^z) \tag{89.4}$$

But at the instant the fluid's instantaneous velocity relative to frame S' is zero, the fluid's instantaneous velocity with respect to frame S is the same as the velocity of frame S', *i.e.*, $u = V_s$. Therefore, $\gamma_u = \gamma$. Substituting this into Equation (89.4), we obtain

$$u^\mu = \gamma (c, u^x, u^y, u^z) \tag{90.4}$$

The energy-momentum tensor matrix on total spacetime with respect to the observer O is written in the following form.

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix} \tag{91.4}$$

To obtain the value of the first component, T^{00}, which here represents the total relativistic energy density of the fluid relative to the observer O, we substitute into Equation (88.4) the value of the velocity component u^0 from Equation (90.4), and the value of the matrix tensor component η^{00} from Equation (81.4), because $\eta^{\mu\nu} = \eta^{`kl}$, we obtain

$$T^{00} = \rho_0 c^2 \gamma^2 + P_0(\gamma^2 - 1) \quad (92.4)$$

Because the fluid is ideal, we can substitute the proper pressure in terms of the proper density and temperature, the universal constant for gases, according to the equation of state, where $P_0 = \rho_0 R T_0$

$$T^{00} = \rho_0 c^2 \gamma^2 + \rho_0 RT(\gamma^2 - 1) \quad (93.4)$$

Taking $\rho_0 c^2 \gamma^2$ as a common factor

$$T^{00} = \rho_0 c^2 \gamma^2 \left(1 + \frac{RT_0}{c^2} - \frac{RT_0}{c^2 \gamma^2}\right) \quad (94.4)$$

But in most cases, we find, $\frac{RT_0}{c^2} - \frac{RT_0}{c^2\gamma^2} \ll 1$, so we can ignore this quantity

$$T^{00} \approx \rho_0 c^2 \gamma^2 \quad (95.4)$$

Following the same solution steps with components $T^{10} = T^{20} = T^{30}$, which represent the energy flow in the three directions, we obtain the value of each of the previous components as shown below on the left side. By substituting the fluid velocity in each direction in terms of the distance and proper time, we obtain the value of each component shown below on the right side.

$$\begin{gathered} T^{10} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^x c \gamma^2 + 0 \implies T^{10} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dxc\gamma \\ T^{20} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^y c \gamma^2 + 0 \implies T^{20} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dyc\gamma \\ T^{30} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^z c \gamma^2 + 0 \implies T^{30} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dzc\gamma \end{gathered} \quad (96.4)$$

Because the fluid is in thermodynamic equilibrium. Therefore, the differential of density and proper pressure with respect to proper time equals zero. Substituting this into the previous set of equations, we find that the energy flow in each direction relative to observer O is zero. This is because the instantaneous velocity of the fluid (the velocity of the particles with the center of mass) at the moment of observation is constant, and therefore the instantaneous energy flux of the fluid is zero with respect to the observer O. As for the energy flux of the particles around the center of mass, it is symmetrical in all directions, and therefore its resultant in each direction is also zero.

$$T^{10} = T^{20} = T^{30} = 0 \quad (97.4)$$

Because the energy-momentum tensor is a symmetric tensor, where $T^{\mu\nu} = T^{\nu\mu}$, therefore the following quantities also equal zero.

$$T^{01} = T^{02} = T^{03} = 0 \quad (98.4)$$

This means that the momentum density in each direction is zero, although the fluid has instantaneous momentum here, but the volume transformation in special relativity does not represent the space of motion of the fluid particles with the center of mass, but around the center of mass, and this type of motion and the associated momentum is symmetrical in all directions of space, *i.e.*, its resultant is zero. As for the following components, $T^{21} = T^{31} = T^{32}$, which represent the momentum flow of the fluid in the three directions, by following the same previous solution steps we obtain the values of those components.

$$T^{21} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^y u^x \gamma^2 + 0 \implies T^{21} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dx u^y \gamma$$
$$T^{31} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^z u^x \gamma^2 + 0 \implies T^{31} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dz u^x \gamma$$
$$T^{32} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^z u^y \gamma^2 + 0 \implies T^{32} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dy u^z \gamma \quad (99.4)$$

By differentiating the proper density and pressure with respect to the proper time. Substituting this into the previous set of equations, we find that the momentum flux in each direction with respect to the observer O is zero. This is because the instantaneous velocity of the fluid (the velocity of the particles with the center of mass) at the moment of observation is constant, and therefore the instantaneous momentum of the fluid is also constant at that moment with respect to the observer O. As for the momentum of the particles around the center of mass, it is symmetrical in all directions, and therefore its resultant in any direction is also zero.

$$T^{21} = T^{31} = T^{32} = 0 \quad (100.4)$$

Through tensor symmetry, we also find that the values of the following components equal zero.

$$T^{12} = T^{13} = T^{23} = 0 \quad (101.4)$$

This means that the shear stress or viscosity in every direction is zero, because the fluid velocity at the moment of observation is constant, and therefore the tangential forces to the surfaces in any direction are zero with respect to the observer O. As for the particle forces around the center of mass, it is equal to zero in all directions because it is a perfect fluid. As for the components $T^{11} = T^{22} = T^{33}$, which represents the relativistic pressure of the fluid in the three directions, we obtain the values of those components by following the same previous solution steps.

$$T^{11} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^x u^x \gamma^2 + P_0 \implies T^{11} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dx u^x \gamma + P_0$$
$$T^{22} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^y u^y \gamma^2 + P_0 \implies T^{22} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dy u^y \gamma + P_0$$
$$T^{33} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^z u^z \gamma^2 + P_0 \implies T^{33} = \frac{1}{d\tau}\left(\rho_0 + \frac{P_0}{c^2}\right) dz u^z \gamma + P_0 \quad (102.4)$$

Substituting here for the differential density and pressure relative to the proper time, which equals zero in each direction, we obtain a pressure value in each direction relative to the observer O that is not zero, even though the instantaneous velocity of the fluid is constant and the fluid acceleration (the acceleration of the particles with the center of mass) is locally zero. However, the particles have symmetric acceleration around the fluid's center of mass, so the resulting pressure here does not represent fluid pressure on spacetime, but rather thermodynamic pressure.

$$T^{11} = T^{22} = T^{33} = P_0 \quad (103.4)$$

By substituting all the components of the tensor that we obtained from equations

(95.4), (97.4), (98.4), (100.4), (101.4), (103.4), we obtain the energy-momentum tensor matrix of a fluid in the general case locally with respect to the observer O.

$$T^{\mu\nu} = \begin{bmatrix} \rho_0 c^2 \gamma^2 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{bmatrix} \tag{104.4}$$

The previous matrix represents a different solution from the traditional solution for the energy-momentum tensor of an ideal fluid, which yields physically ambiguous components [33] due to the confusion between special and general states, between the acceleration of particles with center of mass and around the center of mass of the fluid. For example, the fluid pressure on spacetime is written in terms of the relativistic thermodynamic pressure, and this has no significance or meaning in the transformations of thermodynamics, so it was eliminated as we explained above. Equation (88.4) shows us that, the energy- momentum tensor of the fluid, whether in the special or general case locally, it depends on the 4D velocity and the metric tensor. Therefore, the metric tensor analysis of total spacetime shown in Paper 1 [6], and the 4D velocity analysis shown in Paper 2 [7], will necessarily lead to the energy-momentum tensor analysis of the fluid. Consequently, we will also have energy-momentum tensors for the fluid in positive and negative subspace-times in the general case, but locally, because the analysis is for instantaneous velocity or at a point on total spacetime.

2.16. Local Energy-Momentum Tensor of a Fluid in Positive Subspace-Time

We can obtain the energy-momentum tensor of the fluid on the positive subspace-time in the special or general case locally with respect to the observer O through the proper density, proper pressure and the 4D instantaneous velocity vector of the fluid resulting from the analysis of the 4D instantaneous resultant vector of the velocity, and also the metric tensor of the positive subspace-time resulting from the analysis of the metric tensor of the total spacetime in the special or general case locally according to the following equation.

$$\tilde{T}^{\epsilon\lambda} = \left(\rho_0 + \frac{P_0}{c^2}\right)\tilde{u}^{\epsilon}\tilde{u}^{\lambda} - P_0\tilde{\eta}^{\epsilon\lambda},\ \ \epsilon = \lambda = 0,1,2,3 \tag{105.4}$$

where $\tilde{T}^{\epsilon\lambda}$ is the energy-momentum tensor of the fluid on the positive subspace-time, $\tilde{u}^{\epsilon}, \tilde{u}^{\lambda}$ are the contravariant components of the 4D instantaneous velocity vector of the fluid on the positive subspace-time, $\tilde{\eta}^{\epsilon\lambda}$ is the metric tensor of the positive subspace-time, locally flat or in the special case in terms of the Cartesian coordinates with respect to the observer O, *i.e.* $\tilde{\eta}^{\epsilon\lambda} = \eta^{`kl}$. We use the formula shown in the first paper [6] not in the third, because it depends on the analysis, Equation (62.1) (with a difference in the compounds from covariant to contravariant and the notation from (1, 2, 3, 4) to (0, 1, 2, 3) and also the notation of λ instead of τ). We also write the 4D velocity vector $\tilde{u}^{\epsilon}$ or $\tilde{u}^{\lambda}$ in the same form as above in Equation (82.4). Since the time $d\tilde{t}$ and dt belong to the same frame of reference, the transformation of the proper time to the time $d\tilde{t}$ has the same transformation as before, where, $d\tilde{t} = d\tau\gamma_u$. However, $\gamma_u = \gamma$ at the previous observational moment, so we write the vector in the following form

$$\tilde{u}^{\epsilon} = \gamma\left(\tilde{V}, \tilde{u}^x, \tilde{u}^y, \tilde{u}^z\right) \tag{106.4}$$

As for the transformation of the 4D velocity components constant under the transformation in positive subspace-time, this is also explained in Paper 2 [7]. See the set No. 14.2 in the special case which is equivalent to the general case locally. Therefore, to observer O, it will appear that the fluid in positive subspace-time is moving only in the time dimension, and that all its velocity components in positive subspace are zero. Substituting this into Equation (106.4), we obtain

$$\tilde{u}^{\epsilon} = \gamma\left(\tilde{V}, \tilde{u}^x, \tilde{u}^y, \tilde{u}^z\right) = \gamma\left(\tilde{V}, 0,0,0\right) \tag{107.4}$$

It is also the same formula used in the second paper for the 4D velocity vector in positive subspace-time, Equation (96.2, with the difference that the instantaneous velocity here represents the velocity of the fluid and not the dust. Here, we can also obtain the matrix of this tensor by calculating each of its components. To obtain the value of the first component, $\tilde{T}^{00}$, which represents the positive energy density of the fluid, we substitute the value of the velocity component $\tilde{u}^0$ from Equation (107.4) and the value of the matrix tensor component $\tilde{\eta}^{00}$ according to Equation (81.4). into Equation (105.4).

$$\tilde{T}^{00} = \left(\rho_0 + \frac{P_0}{c^2}\right)\tilde{V}^2\gamma^2 - P_0 \tag{108.4}$$

Substituting $\tilde{V}^2 = c^2\gamma^{-2}$ according to Equation (23.1) shown in Paper 1 [6] into the previous equation

$$\tilde{T}^{00} = \rho_0\tilde{V}^2\gamma^2 \tag{109.4}$$

By substituting the proper density in terms of both the proper mass and the proper volume, where the mass transformation here is according to special relativity $m = m_0\gamma$

$$\tilde{T}^{00} = \rho_0\tilde{V}^2\gamma^2 = \frac{Nm_0\gamma}{(V_{ol})_0}\tilde{V}^2\gamma = \frac{Nm}{(V_{ol})_0}\tilde{V}^2\gamma \tag{110.4}$$

As for the volume transformation in the positive subspace, as we mentioned above, the spatial structure is symmetrical with respect to both observers, therefore the volume transformation here is constant, *i.e.*, $\tilde{V}_{ol} = (V_{ol})_0$

$$\tilde{T}^{00} = \frac{Nm}{\tilde{V}_{ol}}\tilde{V}^2\gamma = \tilde{\rho}\tilde{V}^2\gamma \tag{111.4}$$

where, $\tilde{\rho}\tilde{V}^2$ represents the positive energy density of the fluid. Following the same solution steps as in Equation (105.4), with the components $\tilde{T}^{11}, \tilde{T}^{22}, \tilde{T}^{33}$ representing the relativistic pressure of the fluid in each direction of the positive subspace, we obtain

$$\tilde{T}^{11} = \tilde{T}^{22} = \tilde{T}^{33} = P_0 \tag{112.4}$$

Here we find that the energy-momentum tensor of the fluid in the general case locally contains pressure, despite the absence of fluid acceleration; therefore, it is also a thermodynamic pressure. Since the general case is locally equivalent to the special case, the thermodynamic pressure transformation in the general case here is according to Equation (17.4), *i.e.*, $\tilde{P}\gamma = P_0$. substituting this into the previous equation.

$$\tilde{T}^{11} = \tilde{T}^{22} = \tilde{T}^{33} = \tilde{P}\gamma \tag{113.4}$$

Following the same steps, the solution with the following components $\tilde{T}^{10}, \tilde{T}^{20}, \tilde{T}^{30}$ and the components, $\tilde{T}^{21}, \tilde{T}^{31}, \tilde{T}^{32}$, which represent the energy and momentum flow of the fluid in each direction, respectively, is found to have values equal to zero as well. Following also the same steps with the components $\tilde{T}^{01}, \tilde{T}^{02}, \tilde{T}^{03}$ and the components, $\tilde{T}^{12}, \tilde{T}^{13}, \tilde{T}^{23}$, which represent the momentum density and shear stress of the fluid, respectively. We find their values equal to zero. By substituting all the values of the tensor components that we obtained, we obtain the energy-momentum tensor matrix of a fluid in the general case locally with respect to the observer O in positive subspace-time.

$$\tilde{T}^{\epsilon\lambda} = \begin{bmatrix} \rho_0 \tilde{V}^2\gamma^2 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{bmatrix} \text{ or } \begin{bmatrix} \tilde{\rho}\tilde{V}^2\gamma & 0 & 0 & 0 \\ 0 & \tilde{P}\gamma & 0 & 0 \\ 0 & 0 & \tilde{P}\gamma & 0 \\ 0 & 0 & 0 & \tilde{P}\gamma \end{bmatrix} \tag{114.4}$$

The previous equation shows us that this tensor is a thermal tensor because it contains a thermal component, which is the thermodynamic pressure. When the pressure equals zero, by substituting this into the previous matrix, we obtain the same formula for the dust field matrix in the special case that we obtained in Paper 2 [7] Section 2.11, Equation (101.2) where the fluid becomes like dust. The transformation of the energy-momentum tensor of a fluid from one frame of reference to another in positive subspace-time, whether in the special or general case locally, is achieved through the transformation of the 4D velocity vectors and the metric tensor of the positive subspace-time. As mentioned above in the special case the 4D velocity vectors are constant under the transformation, and the metric tensor of the positive subspace-time is locally flat.

$$\tilde{u}^{\epsilon} = u^{`k}, \ \ \tilde{u}^{\lambda} = u^{`l}, \ \ \tilde{\eta}^{\epsilon\lambda} = \eta^{`kl} \tag{115.4}$$

By substituting from (115.4) to (105.4)

$$\tilde{T}^{\epsilon\lambda} = \left(\rho_0 + \frac{P_0}{c^2}\right) u^{`k} u^{`l} - P_0 \eta^{`kl} \tag{116.4}$$

By substituting from (80.4) to (116.4)

$$\left(\tilde{T}^{\epsilon\lambda}\right)_{loc} = \left(T^{`kl}\right)_{loc} \tag{117.4}$$

That is, the energy-momentum tensor of the fluid in positive subspace-time is constant under transformation in the special or general case locally, which is the same result we obtained for the dust field in the special case in Paper 2 [7] Equation (103.2). By substituting the value of each component we obtained from the previous tensor components in the following equation, we obtain the transformation of both energy density and pressure in the positive subspace with respect to the observer O.

$$\tilde{T}^{00} = T^{`00} \ \Rightarrow \ \tilde{\rho}\tilde{V}^2\gamma = \rho_0 c^2 \ \Rightarrow \ \tilde{\rho}\tilde{V}^2 = \rho_0 c^2 \gamma^{-1} \tag{118.4}$$

$$\tilde{T}^{11} = T^{`11} \implies \tilde{P}\gamma = P_0 \implies \tilde{P} = P_0\gamma^{-1} \quad (119.4)$$

Because the fluid is ideal, we can apply the universal gas equation here, where the pressure P_0 on the right side of the equation can be replaced by the temperature, T_0, the volume $(V_{ol})_0$, the number of moles n_{mol}, and the universal gas constant R. The same applies to the left side of each equation according to the principle of inverse relativity (which is an application of the principle of special relativity in the positive subspace only).

$$\tilde{T}^{11} = T^{`11} \implies \frac{n_{mol}R\tilde{T}}{\tilde{V}_{lo}}\gamma = \frac{n_{mol}RT_0}{(V_{ol})_0} \implies \tilde{T} = T_0\gamma^{-1} \quad (120.4)$$

The three previous equations show us that the energy density, thermodynamic pressure, and relativistic temperature of the fluid all decrease with respect to the observer O in positive subspace-time as the velocity of the frame of reference (fluid velocity) increases.

2.17. Local Energy-Momentum Tensor of a Fluid in Negative Subspace-Time

We can also obtain the energy-momentum tensor of the ideal fluid on the negative subspace-time in the special or general case locally with respect to the observer O through the proper density, proper pressure and the 4D instantaneous velocity vector of the fluid resulting from the analysis of the 4D instantaneous resultant vector of the velocity and also the metric tensor of the negative subspace-time resulting from the analysis of the metric tensor of the total spacetime in the special or general case locally according to the following equation.

$$\check{T}^{\sigma\delta} = \left(\rho_0 + \frac{P_0}{c^2}\right)\check{u}^{\sigma}\check{u}^{\delta} - P_0\check{\eta}^{\sigma\delta},\ \ \sigma = \delta = 0,1,2,3 \quad (121.4)$$

where $\check{T}^{\sigma\delta}$ is the energy-momentum tensor of the fluid on the negative subspace-time, $\check{u}^{\sigma},\check{u}^{\delta}$ are the contravariant components of the 4D instantaneous velocity vector of the fluid on the negative subspace-time, $\check{\eta}^{\sigma\delta}$ is the metric tensor of the negative subspace-time, locally flat or in the special case in terms of the Cartesian coordinates with respect to the observer O. We use the formula shown in the first paper [6] not the third, because it depends on the analysis, Equation (74.1) (with a difference in the compounds from covariant to contravariant and the notation from (1, 2, 3, 4) to (0, 1, 2, 3) and also the notation of δ instead of ρ). Because the components $\check{u}^{\sigma},\check{u}^{\delta}$ are the result of analyzing the components u^{μ},u^{ν} on total spacetime, we can write them in terms of both the vectors of total spacetime and positive subspace, according to the following equations, $\check{u}^{\sigma} = u^{\mu} - \tilde{u}^{\epsilon}$ and $\check{u}^{\delta} = u^{\nu} - \tilde{u}^{\lambda}$, and then calculate the product of those components.

$$(u^{\mu} - \tilde{u}^{\epsilon}) \cdot (u^{\nu} - \tilde{u}^{\lambda}) = (u^{\mu}u^{\nu} - u^{\mu}\tilde{u}^{\lambda} - \tilde{u}^{\epsilon}u^{\nu} + \tilde{u}^{\epsilon}\tilde{u}^{\lambda}) \quad (122.4)$$

Because it's a dot multiplication, we can simplify to the following formula.

$$-u^{\mu}\tilde{u}^{\lambda} = -\tilde{u}^{\epsilon}u^{\nu} = -2(\tilde{u}^{\epsilon}u^{\nu})\cos\theta \quad (123.4)$$

where θ represents the angle between the tangent vectors $\tilde{u}^{\epsilon},\check{u}^{\sigma}$, substituting from (123.4) to (122.4)

$$\left(u^{\mu}-\tilde{u}^{\epsilon}\right)\cdot\left(u^{\nu}-\tilde{u}^{\lambda}\right)=\left[u^{\mu}u^{\nu}+\tilde{u}^{\epsilon}\tilde{u}^{\lambda}-2\left(\tilde{u}^{\epsilon}u^{\nu}\right)\cos\theta\right] \tag{124.4}$$

Also, the metric tensor $\check{\eta}^{\sigma\delta}$ can be written in terms of the metric tensor of total spacetime and positive subspace-time according to the following equation

$$\check{\eta}^{\sigma\delta}=\eta^{\mu\nu}+\tilde{\eta}^{\epsilon\lambda}-2\bar{\eta}^{\epsilon\nu}\cos\theta \tag{125.4}$$

By substituting from (124.4) and (125.4) to (121.4)

$$\check{T}^{\sigma\delta}=\left[\left(\rho_0+\frac{P_0}{c^2}\right)\left(u^{\mu}u^{\nu}+\tilde{u}^{\epsilon}\tilde{u}^{\lambda}-2\left(\tilde{u}^{\epsilon}u^{\nu}\right)\cos\theta\right)\right]$$
$$-\left[P_0\left(\eta^{\mu\nu}+\tilde{\eta}^{\epsilon\lambda}-2\bar{\eta}^{\epsilon\nu}\cos\theta\right)\right] \tag{126.4}$$

By multiplying the outside term by each term inside, and rearranging the equation

$$\check{T}^{\sigma\delta}=\left[\left(\rho_0+\frac{P_0}{c^2}\right)u^{\mu}u^{\nu}-P_0\eta^{\mu\nu}\right]+\left[\left(\rho_0+\frac{P_0}{c^2}\right)\tilde{u}^{\epsilon}\tilde{u}^{\lambda}-P_0\tilde{\eta}^{\epsilon\lambda}\right]$$
$$-\left[\left(\rho_0+\frac{P_0}{c^2}\right)2\left(\tilde{u}^{\epsilon}u^{\nu}\right)\cos\theta-2P_0\bar{\eta}^{\epsilon\nu}\cos\theta\right] \tag{127.4}$$

The first bracket on the right side of the equation represents the local energy-momentum tensor on total spacetime, $T^{\mu\nu}$, while the second bracket represents the local energy-momentum tensor on positive subspace-time $\tilde{T}^{\epsilon\lambda}$. The third bracket represents the local energy-momentum tensor of the fluid composed of both vectors $\tilde{u}^{\epsilon}$ and u^{ν}, and a composite metric tensor $\bar{T}^{\epsilon\nu}$, By substituting all of that into the previous equation.

$$\left(\check{T}^{\sigma\delta}\right)_{loc}=\left(T^{\mu\nu}+\tilde{T}^{\epsilon\lambda}-2\bar{T}^{\epsilon\nu}\cos\theta\right)_{loc} \tag{128.4}$$

In the third paper [7], we assumed $\theta=90$ to eliminate the composite tensor $T^{\sigma\epsilon}$, since we cannot determine its value without knowing the components of $\check{T}^{\sigma\rho}$. However, here we analyzed the negative subspace-time tensor instead of the total tensor, thus obtaining the composite tensor $\bar{T}^{\epsilon\nu}$, composed of the total and positive tensors, allowing us to determine its components. We assume here, that the angle equals zero in order to obtain a linear analysis of the tensor, where $\cos 0=1$. By substituting in the previous equation

$$\left(\check{T}^{\sigma\delta}\right)_{loc}=\left(T^{\mu\nu}+\tilde{T}^{\epsilon\lambda}-2\bar{T}^{\epsilon\nu}\right)_{loc} \tag{129.4}$$

As for determining the components of the composite tensor $\bar{T}^{\epsilon\nu}$, according to the following equation

$$\bar{T}^{\epsilon\nu}=\Lambda_k^{\epsilon}\Lambda_l^{\nu}T^{`kl} \tag{130.4}$$

Because $\Lambda_k^{\epsilon}\Lambda_l^{\nu}$ are diagonal transformation matrices, we can therefore write each one as the square root of the matrix multiplied by itself.

$$\bar{T}^{\epsilon\nu}=\sqrt{\Lambda_k^{\epsilon}\Lambda_l^{\lambda`}}\sqrt{\Lambda_k^{\mu}\Lambda_l^{\nu}}T^{`kl} \tag{131.4}$$

But $\tilde{T}^{\epsilon\lambda}=\Lambda_k^{\epsilon}\Lambda_l^{\lambda`}T^{`kl}$ and also, $T^{\mu\nu}=\Lambda_k^{\mu}\Lambda_l^{\nu}T^{`kl}$, substituting that into the previous equation

$$\bar{T}^{\epsilon\nu}=\frac{\sqrt{\tilde{T}^{\epsilon\lambda}}}{\sqrt{T^{`kl}}}\frac{\sqrt{T^{\mu\nu}}}{\sqrt{T^{`kl}}}T^{`kl} \tag{132.4}$$

From it we obtain

$$\bar{T}^{\epsilon\nu} = \sqrt{T^{\mu\nu}}.\sqrt{\tilde{T}^{\epsilon\lambda}} \tag{133.4}$$

By substituting from (133.4) to (129.4)

$$\left(\check{T}^{\sigma\delta}\right)_{loc} = \left(T^{\mu\nu} + \tilde{T}^{\epsilon\lambda} - 2\sqrt{T^{\mu\nu}}\sqrt{\tilde{T}^{\epsilon\lambda}}\right)_{loc} \tag{134.4}$$

By substituting each tensor with the matrix we obtained in the general case locally, Equations (114.4), (104.4), we obtain

$$\left(\check{T}^{\sigma\delta}\right)_{loc} = \begin{bmatrix} \rho_0 c^2\gamma^2 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{bmatrix} + \begin{bmatrix} \rho_0 \tilde{V}^2\gamma^2 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{bmatrix}$$
$$-2\begin{bmatrix} \rho_0 \tilde{V} c\gamma^2 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{bmatrix} \tag{135.4}$$

$$\left(\check{T}^{\sigma\delta}\right)_{loc} = \begin{bmatrix} \rho_0\left(c^2 + \tilde{V}^2 - 2c\tilde{V}\right)\gamma^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{136.4}$$

The previous equation represents the energy-momentum tensor matrix of a fluid in the general case locally with respect to the observer O in negative subspace-time. The matrix shows a local distribution for only one component, which is the energy density of the fluid, and the value is equal to

$$\check{T}^{00} = \rho_0\left(c^2 + \tilde{V}^2 - 2c\tilde{V}\right)\gamma^2 \tag{137.4}$$

where, $\check{V}^2 = \left(c^2 + \tilde{V}^2 - 2c\tilde{V}\right)$ substituting this, and also substituting the proper density in terms of both the proper mass and the proper volume, where the mass transformation is also according to the transformation followed above, $m = m_0\gamma$, we obtain the following result

$$\check{T}^{00} = \rho_0\check{V}^2\gamma^2 = \frac{Nm_0\gamma}{(V_{ol})_0}\gamma\check{V}^2 = \frac{Nm}{(V_{ol})_0}\gamma\check{V}^2 \tag{138.4}$$

As for the volume transformation in the negative subspace, as we explained in the Section 2-8, that the volume transformation when the motion of the reference frames relative to each other is on three axes, the transformation is according to the equation $\check{V}_{ol} = (V_{ol})_0(\gamma^{-1} - 1)$, by adding the estimated $(\gamma^{-1} - 1)$ in the numerator and denominator of the equation we obtain

$$\check{T}^{00} = \frac{Nm}{(V_{ol})_0(\gamma^{-1} - 1)}\gamma(\gamma^{-1} - 1)\check{V}^2 = \check{\rho}\check{V}^2(1 - \gamma) \tag{139.4}$$

where $\check{\rho}\check{V}^2$ represents the negative energy density of the fluid, and $\check{\rho}$ is the relativistic mass density in negative subspace-time, substituting from (139.4) into (136.4)

$$\left(\check{T}^{\sigma\delta}\right)_{loc} = \begin{bmatrix} \check{T}^{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{or} \begin{bmatrix} \check{\rho}\check{V}^2(1-\gamma) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{140.4}$$

We obtain the same matrix as in Paper 2, Section 2.12, Equation (118.2) for the special case of the dust field, with no clear distinction here between the fluid and the dust, even though the fluid contains thermal components. This means that the local energy-momentum tensor in the negative subspace-time is a non-thermal tensor.

By substituting the value of $\breve{T}^{00}$ from Equation (139.4) into Equation (137.4)

$$\breve{\rho}\breve{V}^2(1-\gamma) = \rho_0\gamma^2\left(c^2 + \tilde{V}^2 - 2c\tilde{V}\right) \tag{141.4}$$

Substituting the value of $\tilde{V} = c\gamma$ from Equation (23.1) shown on the first Paper into Equation (141.4)

$$\breve{\rho}\breve{V}^2(1-\gamma) = \rho_0 c^2\gamma^2 + \rho_0 c^2 - 2\rho_0\gamma^2 cc\gamma \tag{142.4}$$

Taking $\rho_0 c^2$ as a common factor

$$\breve{\rho}\breve{V}^2 = \rho_0 c^2\frac{(\gamma^2 + 1 - 2\gamma)}{(1-\gamma)} \tag{143.4}$$

$$\breve{\rho}\breve{V}^2 = \rho_0 c^2\frac{(1-\gamma)^2}{(1-\gamma)} \implies \breve{\rho}\breve{V}^2 = \rho_0 c^2(1-\gamma) \implies \breve{\rho}\breve{V}^2 \approx -\rho_0 c^2\gamma \tag{144.4}$$

The previous equation shows us that the energy density of the fluid increases with the increasing velocity of the frame of reference (the fluid velocity) relative to the observer O in the negative subspace-time, but with a negative value, which is the same result we obtained in the second paper for the density of the dust field, and the negative sign here is for the negative volume as we explained in item 2 - 8 above.

2.18. A Perfect Fluid Sphere in Positive and Negative Subspace-Time

Now let's assume that the liquid we have has a huge mass, is spherically symmetrical in shape, and is in a state of hydrostatic equilibrium, that is, it is not collapsing due to gravity [34] (the fluid sphere here can be a star). In this case, the acceleration of the fluid is under the influence of the gravity of its mass and not due to the presence of an external factor. According to the equivalence principle of general relativity, the non-inertial acceleration of a fluid is equal to the gravitational acceleration of the fluid, and the total curved spacetime is also locally flat. Therefore, we can use the local energy-momentum tensor of an accelerated fluid in both the positive and negative subspace-times shown above, with the fluid accelerating under the influence of its own mass's gravity. Because these tensors are local, they must be applied at every point on the total spacetime within the fluid sphere, where we consider the energy density, thermodynamic pressure, and fluid temperature at each point on the total spacetime within the fluid sphere as proper energy density, pressure, and temperature. Because those quantities change from point to point inside the fluid sphere according to the radius, therefore they are a function of the radius of the fluid sphere. As for the gamma coefficient γ_g here, it depends on the escape velocity from gravity, as we previously explained in the first paper [6] Equation (93.1), that the speed of any particle in the negative curved subspace under the influence of a mass is always equal to the escape velocity from the gravity of the mass V_{esc}. Therefore, the instantaneous velocity of the fluid at any point on negative subspace-time, is always equal to the escape velocity at that point.

Since the escape velocity from gravity also varies from point to point within the fluid sphere according to the radius. Therefore, it is also a function of the radius of the liquid sphere. $V_{esc}(r) = \frac{2M(r)G}{r}$. Therefore, we write the transformations of energy density, pressure, and proper temperature at each point inside the fluid sphere in positive subspace-time with respect to an observer far from the sphere's gravitational source according to the following formula.

$$\begin{aligned} \tilde{\rho}\tilde{V}^2(r) &= \rho_0(r)c^2\gamma_g^{-1}(r) \\ \tilde{P}(r) &= P_0(r)\gamma_g^{-1}(r) \\ \tilde{T}(r) &= T_0(r)\gamma_g^{-1}(r) \end{aligned} \tag{145.4}$$

Likewise, the transformations of energy density, pressure, and proper temperature at each point inside the fluid sphere on the negative subspace-time, also with respect to an observer far from the source of gravity of the sphere, are according to the following formula.

$$\begin{aligned} \check{\rho}\check{V}^2(r) &= \rho_0(r)c^2\left[1 - \gamma_g(r)\right] \\ \check{P}(r) &= 0 \\ \check{T}(r) &= 0 \end{aligned} \tag{146.4}$$

Because the fluid sphere is spherically symmetric, any point on the surface of the fluid sphere will have the same density, pressure, and proper temperature. Also, the gamma coefficient or escape velocity at the surface of the sphere depends here on the total mass of the fluid sphere and the radius of the sphere. Therefore, we can write the previous thermal transformations in positive subspace at the surface of the fluid sphere according to the following formula

$$\begin{aligned} \tilde{\rho}\tilde{V}^2 &= \rho_0 c^2\sqrt{1 - \frac{2MG}{rc^2}} \\ \tilde{P} &= P_0\sqrt{1 - \frac{2MG}{rc^2}} \\ \tilde{T} &= T_0\sqrt{1 - \frac{2MG}{rc^2}} \quad \frac{2MG}{c^2} = r_s \end{aligned} \tag{147.4}$$

where, M is the total mass of the fluid sphere, r is the radius of the fluid sphere, and r_s is the Schwarzschild radius. The previous set of equations represents the relativistic thermodynamics at the surface of the fluid sphere (which here represents the surface of the star) in positive subspace-time. This shows us that the energy density, thermodynamic pressure, and temperature of the fluid on the surface of the sphere decrease in positive subspace, smaller the radius of the fluid sphere, or the greater the total mass of the fluid sphere relative to an observer farther from the source of gravity (The density transformation equation here is the same as the density transformation equation for the dust field at a point on the positive subspace-time under the influence of gravity, as shown in Papers 2 and 3 [7] [8], but without specifying that that point is located on the surface of the dust sphere). This is an important result because it means that an observer far from the gravitational sphere of a star, for example, cannot observe the star's actual temperature but will always observe a temperature lower than the actual temperature according to the ratio M/r. When the radius of the fluid sphere reaches the Schwarzschild radius (where the fluid sphere then becomes a black hole), the values of energy density, pressure, and temperature become zero, or the transformation equations collapse. However, when these equations are quantized, each equation transforms into the following inequality.

$$\begin{aligned}
&\tilde{\rho}\tilde{V}^2 \geq \rho_0 c^2 \left(1-\frac{r_s}{r}\right)^{\frac{1}{2}} + \rho_\hbar c^2 \qquad && \tilde{\rho}\tilde{V}^2 \geq \rho_\hbar c^2 \qquad && r = r_s \\
&\tilde{P} \geq P_0 \left(1-\frac{r_s}{r}\right)^{\frac{1}{2}} + P_\hbar && \tilde{P} \geq P_\hbar && r = r_s \\
&\tilde{T} \geq T_0 \left(1-\frac{r_s}{r}\right)^{\frac{1}{2}} + T_\hbar && \tilde{T} \geq T_\hbar && r = r_s
\end{aligned} \tag{148.4}$$

Substituting $r = r_s$ into the inequalities, we find that the thermal quantities in the positive subspace-time at the event horizon of the black hole do not equal zero or disappear. Rather, there is uncertainty about the existence of a minimum for all the above thermal quantities at relatively long timescales; but at infinitesimally small timescales, there is uncertainty about the existence of a maximum for all the above thermal quantities. That is, the event horizon of the black hole has two scales for the thermal quantities, depending on the time period of observation. This is a result of energy fluctuations between subspaces, or time quantization. This represents a solution to the information paradox [35]. As for the transformations of the above thermal quantities on the surface of the fluid sphere (which represents the surface of the star) in the negative subspace, they are according to the following formula

$$\check{\rho}\check{V}^2 = \rho_0 c^2 \left[1-\left(1-\frac{2MG}{rc^2}\right)^{-\frac{1}{2}}\right] \tag{149.4}$$

This shows us that the energy density of the fluid on the surface of a fluid sphere (the star) increases in negative subspace as the radius of the fluid sphere decreases relative to an observer farther from the sphere's gravitational source, and vice versa. When the radius of the fluid sphere reaches infinity, the energy density equals zero (which is the same as the density transformation equation for the dust field at a point in positive subspace under the influence of gravity, as shown in Papers 2 and 3, but without specifying that point is located on the surface of the dust sphere). This represents a breakdown in the transformation equation. When the equation is quantized according to the subspace energy oscillation hypothesis or Heisenberg's inverse principle, as explained earlier in the special case, the equation becomes an inequality

$$\check{\rho}\check{V}^2 = \rho_0 c^2 \left[1-\left(1-\frac{r_s}{r}\right)^{-\frac{1}{2}}\right] + \grave{\rho}_\hbar c^2 \implies \check{\rho}\check{V}^2 \leq \grave{\rho}_\hbar c^2, \qquad r = \infty \tag{150.4}$$

where, $\grave{\rho}_\hbar c^2$ represents the reduced Planck energy density. Substituting $r = \infty$

into the previous inequality, we find that there is uncertainty regarding the existence of a minimum energy density in negative space, even though gravity is negligible here, due to the fluctuation of the minimum energy. We explained earlier that this result is obtained when, $\langle \Delta\check{t}_{un}\rangle = 1$. As the time uncertainty decreases, the energy uncertainty increases, and consequently, the energy density increases until it reaches the zero-point energy density of the fluid. Therefore, we also have doubts about the existence of self-acceleration, *i.e.*, without gravity, produced by the zero-point energy of the fluid. And because $\Delta\check{V}_{un} = \Delta V_{esc}$ in the general case. Accordingly, it is not necessary for the fluid sphere to reach the Schwarzschild radius in order for the escape velocity in negative spacetime to equal the speed of light c. It is sufficient for the fluid sphere to reach a critical radius $\check{r}_c$ where the escape velocity equals the critical velocity $\check{V}_C$, which is less than the speed of light by $\Delta\check{V}_{un}$ (we have already calculated its value in Equation (67.4)), and then the fluid accelerates itself without gravity to reach the speed of light. The critical radius is determined by $\check{r}_c$, according to the critical escape velocity equation.

$$\check{r}_c = \frac{2MG}{V_c^2}, \quad \check{r}_c > r_s \tag{151.4}$$

By substituting from (69.4) into (151.4)

$$\check{r}_c = 2MG\left(\mathrm{c} - \sqrt{\frac{\hbar\omega_0}{2m_0}}\right)^{-2} \tag{152.4}$$

Here we find that the critical radius $\check{r}_c$ is slightly larger than Schwarzschild's radius, and it depends on the quantum properties of the matter, such as the mass of the fluid particles and the zero-point energy of the fluid. Therefore, it represents the quantized radius of the black hole. Multiplying both sides of the Equation (67.4) by Nm_0, we obtain the uncertainty in the uniform momentum of each particle on the surface of the fluid sphere in negative subspace.

$$Nm_0\Delta\check{V}_{un} = N\sqrt{\frac{\hbar\omega_0 m_0^2}{2m_0}} \implies \Delta\check{p}_{un} = \sqrt{\tfrac{1}{2}\hbar\omega_0 m_0} \tag{153.4}$$

By substituting the value of $\Delta\check{p}_{un}$ into Heisenberg's inverse principle shown in Paper 3, ([8], pag. 1913), and changing the $\check{x}$-axis to $\check{r}$, according to the following formula $\Delta\check{r}_{un}\Delta\check{p}_{un} \le \frac{1}{2}\hbar$, we obtain

$$\Delta\check{r}_{un} \le \frac{\hbar}{2\sqrt{\frac{1}{2}\hbar\omega_0 m_0}} \tag{154.4}$$

By squaring both sides of the equation and taking the square root

$$\Delta\check{r}_{un} \le \sqrt{\frac{\hbar}{2\omega_0 m_0}} \tag{155.4}$$

We obtain the amount of the uncertainty in the uniform position of each particle on the surface of the fluid sphere on the $\check{r}$ coordinate or in the radius of the sphere (Black hole) in general $\Delta\check{\mathrm{r}}_{un} \le \sqrt{\hbar/2\omega m_0}$ (In the third paper, $\Delta\check{\mathrm{r}}_{un}$ was calculated incorrectly). From the above, we conclude that when the radius of the black hole is set to equal the critical radius, we have an uncertainty in the escape velocity, which is an infinitesimal amount resulting from the zero-point energy. When the escape velocity is set to equal the critical velocity, we have an uncertainty in the radius of the black hole $\Delta\check{r}_{un}$. This means that we cannot determine the radius of the black hole and the escape velocity at the event horizon with absolute precision simultaneously. When the radius is determined with extreme precision, it equals half of Schwarzschild, *i.e.*, $\Delta\check{r}_{un} = 0$. Substituting this into Heisenberg's inverse principle, we obtain infinite uncertainty in determining the escape velocity $\Delta\check{V}_{un} \leq \infty$. And when the escape velocity is determined with extreme precision to be equal to the speed of light, *i.e.*, $\Delta\check{V}_{un} = 0$, and by substituting this into the inverse Heisenberg principle, we obtain infinite uncertainty in determining the radius of the black hole $\Delta\check{r}_{un} \leq \infty$. We also conclude that there is no infinite energy density here [36], since the maximum energy density that a fluid can reach in negative subspace-time is the energy density at the smallest radius, which is the critical radius. Therefore, it is the critical density.

3. Results

We can limit the results we obtained in the special case. From the positive and negative modified Lorentz transformations, we obtained the volume transformations, where the volume remains constant in the positive space and is a thermodynamic variable, while the volume expands from a zero value to the original volume in the negative subspace as a non-thermodynamic variable with the frame motion. From the velocity transformations in each subspace, we obtain the transformations of temperature and probability density of the Maxwell-Boltzmann distribution, where the temperature decreases and the probability density increases in the positive space with the frame motion, while in the negative subspace, the Maxwell-Boltzmann distribution collapses into a shifted Dirac delta distribution and the concept of temperature disappears. From the transformations of force in each subspace, we obtain the transformation of pressure, where, the pressure decreases in positive subspace with the frame motion, while the concept of pressure disappears in negative subspace. From the transformations energy in each subspace, we obtain the transformation of energy forms. Internal energy decreases in the positive subspace, and it is energy of a thermodynamic nature, while internal energy increases in negative subspace with the frame motion, and it is energy of a kinetic nature. The heat and mechanical work also decrease in positive subspace with the frame motion, and the concepts of heat and mechanical work disappear in negative subspace. From the transformations of microscopic states in each subspace, we obtained constant entropy under transformation in the positive subspace with the frame motion, while it equals zero or the concept of thermodynamic entropy disappears in the negative subspace (see comparison **Table 1**). In the general case, through a local energy-momentum tensor in each subspace, we obtained transformations of each of some thermodynamic quantities on the surface of the fluid sphere (the star), where the energy density, pressure, and temperature of the fluid decrease in the positive subspace as the radius of the fluid sphere decreases or its mass increases, while the energy density increases and the concept of pressure and temperature of the fluid disappears in the negative subspace as the radius of the fluid sphere decreases or its mass increases relative to an observer far from the surface of the fluid sphere. See comparison Table 2.

Table 1. Comparison table between relativistic thermodynamic transformations in Positive and negative subspace in the special case.

Thermodynamic Transformations	Positive Subspace	Negative Subspace
Volume	$\tilde{V}_{ol} = \grave{V}_{ol}$	$\check{V}_{ol} = \grave{V}_{ol}(\gamma^{-1} - 1)$
Pressure	$\tilde{P} = \grave{P}\gamma^{-1}$	$\check{P} = 0$
Temperature	$\tilde{T} = \grave{T}\gamma^{-1}$	$\check{T} = 0$
Probability Density	$f(\tilde{u}) = f(\grave{u})\gamma$	$f(\check{u}) = \delta(\check{u} - V_S)$
Internal Energy	$d\tilde{U} = d\grave{U}\gamma^{-1}$	$d\check{U} = d\grave{U}\frac{V_S}{c\grave{p}_j}(\gamma - 1)$
Heat	$d\tilde{Q} = d\grave{Q}\gamma^{-1}$	$d\check{Q} = 0$
Mechanical Work	$d\tilde{W} = d\grave{W}\gamma^{-1}$	$d\check{W} = 0$
Thermodynamic Entropy	$d\tilde{S}_{th} = d\grave{S}_{th}$	$d\check{S}_{th} = 0$

Table 2. Comparison table between relativistic thermodynamic transformations in Positive and negative subspace in general case locally.

Thermodynamic Transformations	Positive Subspace	Negative Subspace
Local tensor	$\left(\tilde{T}^{\epsilon\lambda}\right)_{loc} = \left(\grave{T}^{kl}\right)_{loc}$	$\left(\check{T}^{\sigma\delta}\right)_{loc} = \left(T^{\mu\nu} + \tilde{T}^{\epsilon\lambda} - 2\bar{T}^{\mu\epsilon}\cos\theta\right)_{loc}$
Energy Density	$\tilde{\rho}\tilde{V}^2 = \rho_0 c^2 \left(1 - \frac{r_s}{r}\right)^{\frac{1}{2}}$	$\check{\rho}\check{V}^2 = \rho_0 c^2 \left[1 - \left(1 - \frac{r_s}{r}\right)^{-\frac{1}{2}}\right]$
Thermodynamic Pressure	$\tilde{P} = P_0 \left(1 - \frac{r_s}{r}\right)^{\frac{1}{2}}$	$\check{P} = 0$
Temperature	$\tilde{T} = T_0 \left(1 - \frac{r_s}{r}\right)^{\frac{1}{2}}$ $r_s = \frac{2MG}{c^2}$	$\check{T} = 0$

4. Discussions

Through the inverse relativity model, we were able to solve the problem of relativistic thermodynamics and explain the reasons for the failure of special relativity in this matter, through the model's ability to distinguish between physical quantities associated with motion around the center of mass and motion with the center of mass of the thermodynamic system, and to represent them in positive and negative subspaces. Based on this, we obtained transformations of thermal quantities in positive subspace that preserve the mathematical formulas of the laws of thermodynamics at the microscopic and macroscopic levels. This is a subject that has been neglected by some special relativity literature when combined with thermodynamics in the total space, which confirms the validity of the principle of inverse relativity or that the application of the principle of special relativity should only be done in the positive subspace. Because it is the space of physical causality and the laws of physics related to causality; in other words, the space of the internal structure of the thermodynamic system. We also obtained, through the hypothesis of energy fluctuations between subspaces, the non-collapse of the transformation equations and the preservation of information at the micro and macroscopic levels of the system when the speed of the reference frame reaches the speed of light, something that most, if not all, of the literature of special relativity has also failed to achieve when combined with thermodynamics. As for the negative subspace, despite the arbitrary imposition of mathematical formulas for the laws of thermodynamics, we find that the transformations of thermal quantities ultimately collapse. This confirms that the negative subspace is a space of non-causality, where the laws of physics associated with causality disappear. This space also reveals to us the existence of a kinetic component of the system's internal energy at the microscopic and macroscopic levels spontaneously, without any arbitrary imposition of this conception. In other words, it appears to us as a space for the motion of the thermodynamic system.

We were also able, through the inverse relativity model, to explain some quantum properties of the thermodynamic system, such as zero-point energy and its importance in positive space, where a pure crystal can temporarily reach absolute zero, and in negative subspace where the system can temporarily reach the speed of light without infinite energy. We also explained macroscopic quantum tunneling as a result of the reduced entropy of energy fluctuations between subspaces. Also, the quantization of time, which is the fourth property of time, was revealed. In the third paper, in the section on the inverse relativity of simultaneity, we revealed the first property, which is the absolute simultaneity of events, and the elimination of the concept of the illusion of time, and the confirmation of the reality of the arrow of time in the positive subspace. Here in the fourth paper, we confirm this concept through the constancy of entropy under transformation. And in the section on the inverse relativity of time, we revealed the second property, which is time contraction or the concept of supertime in negative subspace. In the section on the reversal of negative subspace-time, we revealed the third property of time, which is time with a negative sign or time reversal, but only for a single particle. And here, we generalized time reversal to a vast number of particles without reversal of the time arrow, since the reversal occurs in negative space where thermodynamic entropy disappears or is always zero. Therefore, the quantization of time in this paper represents the fourth property of time (I hope this is the last one), which maintains the dilation of time in positive subspace alongside the contraction of time (supertime) associated with energy fluctuations, When one cancels the other out, it is for an infinitesimally small period (quantum time) followed by a long period of dilation or cessation (time gap), the opposite is true in negative subspace-time.

4.1. The First Test

The thermodynamic pressure transformation in the positive subspace in the special case is a distinctive result of our model, as it differs from the pressure transformation in the three proposed models of relativistic thermodynamics (Planck, Ott, Landsberg). See the comparison table shown in the introduction. Therefore, the positive pressure test represents a key test for our model. Although we cannot conduct experiments in the special case, we can in the general case. Just as a star's gravity affects the gas pressure on its surface, Earth's gravity can also affect the gas pressure in a container at different distances from the Earth's surface. Assuming we have a container filled with an ideal gas, thermally insulated from the surrounding environment, connected to it is a high-precision pressure measuring device, such as Hot Cathode Gauges or similar [37]. When the container is placed at the closest possible level to the Earth's surface (sea level), we obtain a pressure transformation according to our model

$$\tilde{P}_E = P_0 \sqrt{\left(1 - \frac{2M_E G}{r_E\, c^2}\right)} \tag{156.4}$$

where, M_E is the mass of the Earth, r_E is the radius of the Earth, P_0 is the proper pressure of gas, and $\tilde{P}_E$ is the gas pressure at the initial level or position of the container but in positive space relative to an observer far from the source of gravity. When the container is moved to the second level at a height h above sea level, the distance of the container from the center of the Earth changes, the distance becomes $r_2 = r_e + h$ while the mass M_E remains constant because the container is moving upwards. When the container is as far from the Earth's surface as possible, where $r \approx \infty$, the pressure is at its maximum. We obtain the pressure transformation according to our model

$$\tilde{P}_2 = P_0 \sqrt{\left(1 - \frac{2M_e G}{(r_e + h)c^2}\right)} \Rightarrow \tilde{P}_\infty = P_0 \tag{157.4}$$

By Substituting from (157.4) into (156.4)

$$\tilde{P}_E = \tilde{P}_\infty \sqrt{\left(1 - \frac{2M_E G}{r_E\, c^2}\right)} \tag{158.4}$$

The last equation tells us that the gas pressure in the container varies from one position to another in positive subspace relative to an observer far from gravity. We can formulate the equation inversely relative to an observer close to the container.

$$P_\infty = \frac{P_0}{\sqrt{\left(1 - \frac{2M_E G}{r_E\, c^2}\right)}} \tag{159.4}$$

As for an observer close to the container, the gas pressure that he measures and records, which is the proper pressure, will appear to vary from one position to another inside the container, and will increase as the container and the observer move further away from the source of gravity. If experimentally recorded gas pressure measurements at the aforementioned position agree with the equation above, this is evidence that the thermodynamic pressure increases as we move further away from the source of gravity, and vice versa, that our model is correct.

4.2. The Second Test

We can follow the same steps of the previous experiment, replacing the pressure gauge with a high-precision temperature gauge, and by following the same previous calculations, we arrive at the following equation

$$\tilde{T}_E = \tilde{T}_\infty \sqrt{\left(1 - \frac{2M_E G}{r_E\, c^2}\right)} \tag{160.4}$$

Here we also find that the temperature of the gas in the container also changes from one position to another in positive subspace relative to an observer far from gravity. We can also formulate the equation inversely relative to an observer close to the container.

$$T_\infty = \frac{T_0}{\sqrt{\left(1 - \frac{2M_E G}{r_E\, c^2}\right)}} \tag{161.4}$$

As for an observer close to the container, the gas temperature that he measures and records, which is the proper temperature, will appear to vary from one position to another inside the container, and will increase as the container and the observer move further away from the source of gravity. If the temperature measurements recorded at the previous location experimentally agree with the previous equation, this is evidence that the temperature increases as we move away from the source of gravity, and vice versa. Although this result is not unique to our model, as some previous models have acknowledged it in the special case, it is sufficient to solve the problem experimentally between the three viewpoints explained in the introduction.

4.3. The Third Test

The self-acceleration of a system in negative space, whether in the special or general case, is also a characteristic result of our model, as it depends on the inverse of Heisenberg's principle and occurs at the macroscopic level. Therefore, the self-acceleration test is a characteristic test of our model. The following diagram illustrates a macroscopic pendulum made of a pure crystal with a large mass M, suspended by a string of length l inside a vacuum chamber to eliminate resistance during the pendulum's motion, see Figure 4.

In the first step of the experiment, we use an uncooled pendulum. Because the pendulum is uncooled, the entropy of energy fluctuations of the particles inside the pendulum (crystal) is high; in other words, they are asynchronous and cancel

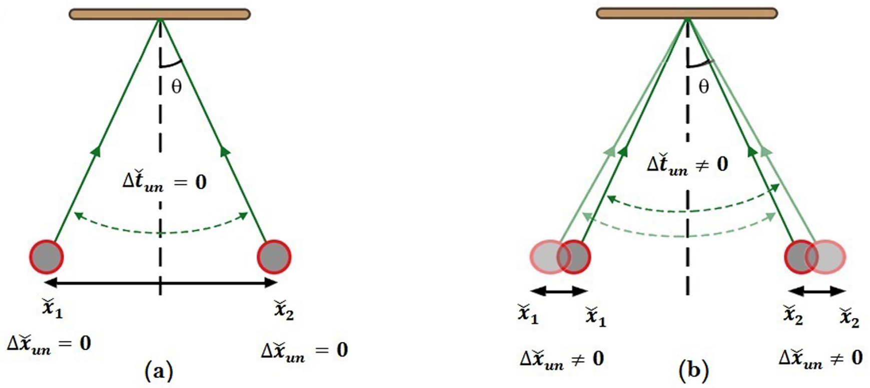

Figure 4. The difference in the motion of a macroscopic pendulum at normal and low temperatures near absolute zero.

each other out. Therefore, the motion of the pendulum (position x and Periodic time t) is determined with extreme precision see **Figure 4(a)**. Consequently, we can also measure the acceleration due to gravity g with extreme precision, according to the well-known classical equation. $g = 4\pi^2 \, l/t^2$ [38]. In the second step of the experiment, we cool the pendulum to very low temperatures close to absolute zero. As a result, the entropy of the energy fluctuations of the pendulum crystal particles decreases, or in other words, the energy fluctuations synchronize, where the lowest energy (zero point energy) oscillates from the positive subspace (the internal structure space of the pendulum crystal) to the negative subspace (the motion space of the pendulum crystal) as the maximum energy. This amount of energy leads to a uniform uncertainty in the velocity of the particles of the pendulum crystal, *i.e.*, in the velocity of the pendulum as a whole, $\Delta\check{V}_{un}$, the value of which we have already calculated in equation number (67.4). Accordingly, when determining the initial and final position of the pendulum for each cycle, we have uncertainty in the position of the pendulum by the amount of $2\Delta\check{x}_{un}$. See **Figure 4(b)**. We have also previously calculated its value, but in terms of the $\check{r}$ coordinate, as shown above in Equation (155.4). Likewise, when determining the initial and final time of the pendulum for each cycle, we have an uncertainty in the time of the pendulum cycle, *i.e.*, in the periodic time, by an amount of $2\Delta\check{t}_{un}$, the value of which is calculated according to the following equation

$$\Delta\check{t}_{un} = \frac{\Delta\check{x}_{un}}{\Delta\check{V}_{un}} \tag{162.4}$$

By substituting from (67.4) and (155.4) into (162.4)

$$\Delta\check{t}_{un} \leq \sqrt{\frac{\hbar}{2\,\omega_0\, m_0}} \cdot \sqrt{\frac{2\, m_0}{\hbar\omega_0}} \tag{163.4}$$

$$\Delta\check{t}_{un} \leq \frac{1}{\omega_0} \tag{164.4}$$

The previous equation and also Equation (155.4), shows us that the uncertainty in the periodic time and the position of the cooled pendulum depends on the angular frequency of the crystal particles in the ground energy level. Therefore, we can control its value by the type of crystal. This uncertainty in the period leads to an uncertainty in the Earth's gravitational acceleration, according to the previous equation

$$\Delta\breve{g}_{un} \leq \frac{4\pi^2 l}{4\Delta\breve{t}_{un}^2} \tag{165.4}$$

By substituting from (164.4) to (165.4)

$$\Delta\breve{g}_{un} \leq \pi^2 l\, \omega_0^2 \tag{166.4}$$

where, $\Delta\breve{g}_{un}$ represents the uncertainty in the acceleration due to gravity. It is the same as the amount of the self-acceleration of the oscillating pendulum. If we obtain the amount of the periodic time uncertainty of the cooling pendulum at the macroscopic level experimentally by modern methods [39] that is less than or equal to the theoretically calculated value on the right-hand side of Equation (164.4), this is evidence of the inverse Heisenberg principle and of the existence of an oscillating self-acceleration of the pendulum, and that the source of this acceleration is the zero-point energy of the pendulum crystal.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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