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Pulsating Vacuum States

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Abstract

A function in projective space is introduced for describing matter states. Spacetime is explained by additions on elliptic curves and chaotic one-dimensional quadratic maps. The introduced vacuum state is like a paired superfluid state of a non-Hermitian Hamiltonian. Doubly-periodic stable orbits are investigated in a quadratic surrounding of nontrivial zeros of the zeta function. The relevant partition function indicates phase transitions and predicts various vacuum states in dependence on processing. Partition function zeros quadratic in two complex masses and two complex curvatures of spacetime are predicted phase transitions which are statistically dominating instead of being exceptional.

Keywords

Complex Lagrangian, Quadratic Newtonian Root Finding, Minkowski Spacetime, Green's Function Singularity, Bifurcation, Elliptic Curves, Pulsating Vacuum State, Phase Transition

1. Introduction

The cosmological constant problem (CCP) questions unique vacuum stress-energy density due to discrepancies up to hundreds of orders of magnitude for various interactions [1]. Field equations require unique vacuum energy density ρ_{vac} and a real Langrangian. Thermodynamic laws ρ_{vac} concern a volume $\mathrm{vol}(\mathcal{M})$ of a closed system in Minkowski spacetime $\mathcal{M}$ with well-defined temperature. Models on pulsating or oscillating universes propose that space and matter are dynamic, oscillating, or pulsating fields [2]-[5]. Spacetime can be explained by additions on elliptic curves and chaotic one-dimensional quadratic maps [6] [7]. The fine structure constant can be explained by Feigenbaum renormalization [8]. Covariant coordinates can be defined by rational triangles [6]. The paper continues definition of $\mathcal{M}$, mass and charge by the hyperbolic border between the Julia set and the Fatou set $\mathcal{J}(N_q)-\mathcal{F}(N_q)$ of a quadratic map [7] [9]. Rotations by $\pm\pi$ in interval $[0,1]$ are capable to explain wave vectors $\boldsymbol{k}_\mu$ on Mandelstam plane M_{stu} and Feigenbaum renormalization on complex plane [7]. A quadruple q of algorithmic period-3 steps $q \simeq k+3 \in k, k+1, k+2$ is under constrained with respect to two steps k and $k+1$. Section 2 introduces a vacuum state which covers quantum statistics (QS) and general relativity (GR). Section 2, 3, 4, 5, 6 show that ground states are paired superfluid unified respiratory-vacuum states with quadratic in-mass fluctuations and doubly-periodic non-Hermitian interactions. Elliptic symmetry explains that lower energies ρ_{vac} can be obtained in QS and GR. Addition on elliptic curves obeys high complexity used in cryptography which would indicate a low statistical weight of cyclic orbits. However, equivalent elliptic curves obey symmetries by permutations of quartic roots x_i. Two-component ± rotations create 2^{2^k} wave vectors $\boldsymbol{k}_\mu$ for cyclotomic units $\zeta^{(2)}$ and $\zeta^{(3)}$ with $mod\,2$ and $mod\,3$ congruences explaining matter as a quasi-continuum of k-steps.

2. Partition Function and Vacuum States

The partition function $\zeta(z,\mathcal{Z}[g_2,g_3])$ for a complex Lagrangian $\mathcal{L}$ is investigated for logarithmic singularities of an elliptic integral of the third kind. Symplectic structures of $\mathcal{L}$ are suspected only in the immediate vicinity of zeros for entire transcendent function $\phi^{(\infty)}(z)$. This algorithmically accessible region on complex plane requires a unique factorization domain (UFD). The non-Hermitian Lagrangian $\mathcal{L}$ restricts to a quadratic root finding process $N_q(z)$ of $q(z)=(z-z_1)(z-z_2)$ where $z \simeq \phi^{(\infty)}(z) \simeq \zeta(z) \simeq \mathcal{Z}$ compares to zeta function $\zeta(z)$ and partition function $\mathcal{Z}$ which is denoted by $\zeta(z,\mathcal{Z})$. Starting from homographies in projective space $\mathbb{P}^3$ induced currents are defined by the Legendre module $\lambda=(x_ix_j)(x_kx_l)/(x_ix_k)(x_jx_l)$ as cross-ratios of four points x_i which are projectible to complex plane $\mathbb{C}$. The reduced vector of points i,j

$$\phi_{vac}(\lambda_{ij})=M_{ij}(z) \tag{1}$$

consists of cross-ratios $\lambda_{ij}=M_{ij}(z)=\dfrac{z-z_i}{z-z_j}$. λ_{ij} pairs i,j are pinned to planar rational triangles $T(z_q)$ as $\mathcal{M}$-invariant currents on complex plane $\mathbb{C}$ where

$$T(z,z_i,z_j)=T\left(z=z_0,z_i,z_j,z_3=\frac{1}{2}(z_i+z_j)\right)=T(z_q)$$

The aim of the present paper is to show that (1) is a vacuum state which allows equivalent doubly-periodic states explaining the CCP. Processing two simultaneous currents is a unified respiratory vacuum in a breathing pulsating universe. $N_q(z)$ is conjugate to z^2 by a Moebius substitution $M_{ij}(z_k)=M_{ij}(z_{k+1})^2$ [10]. One-periodic vacuum states $\phi_{vac}(\lambda_{ij}=0,1)$ enable rational z-values in $T(z_q)$. Homogeneous quartic roots $(x_ix_j)=\dfrac{1}{2i}\psi_i\wedge\bar{\psi}_j$ depend on a four-component complex $\psi_i = x_{i1} + ix_{i2}$ on Gaussian plane which are $4 \cdot 4$ component Grassmann variable pairs $(c_1, \bar{c}_2)$ and $(\bar{c}_1, c_2)$ in a forthcoming paper. The product of (1) with $(x_i x_k)(x_i x_l)$ yields the paired state (4) of a renormalization group (RG) flow where the R_Δ derivative $\frac{\mathrm{d}}{\mathrm{d}\ln(z-e_i)}$ is the fermion number Operator N_Δ. In case $\lambda_{ij} = a^+ a = 0,1$ functions are rewritable in terms of Fermion creation a^+ and annihilation a operator subjected to a $2 \cdot 2$ substitution in a forthcoming paper. Doubly-periodic functions $\phi_{vac}(\lambda_{ij})$ consist of k-components which are viewed as particle pairs. A particle is an invariant image reflected by epipolar, trifocal and quadrifocal matrices. Their two-dimensional minors are binary invariant on Poincare' upper half plane $\mathcal{H}^2$. The entire transcendent polynomial $\phi^{(\infty)}(z)$

$$\phi^{(\infty)}(z) = \xi(z) = \frac{z(z-1)}{2} \pi^{-\frac{z}{2}} \Gamma\left(\frac{z}{2}\right) \zeta(z) = \prod_n \left(1 - \frac{z}{z_{nt,n}}\right) \tag{2}$$

is regarded an eigenvalue equation of a non-Hermitian matrix $H^\infty(z)_{nn'} = \delta_{nn'} z_{nt} - z_0$ which only requires a Hilbert-Schmidt norm $N^{(\infty)} = \sqrt{\mathrm{tr} H^{(\infty)} H^{(\infty)+}}$. The state $\phi_{vac}(\lambda_{ij})$ of triangles $z_0 + z_i + z_j = 0$ approximates only a definite zero $z_{nt,n}$. The determination of a specific transcendent value of $z_{nt,n}$ is left open. Accordingly, the zeta function is a partition function

$$\mathcal{Z}\left[g_2(\omega), g_3(\omega)\right] \simeq \zeta(z - z_0) = \prod_p \left(1 - p^{-(z-z_0)}\right)^{-1} \Bigg|_{z - z_0 \in \phi_{vac}(\lambda_{ij})} \tag{3}$$

where $\zeta(z, \mathcal{Z})$ is created by non-Hermitian Lagrangian $\mathcal{L}^{(\infty)}$. The present paper investigates only an encircling of $z_{nt,n}$ by fractional substitutions of $\gamma^{(3)} \circ z$ [6]

$$z - e_i = \gamma^{(3)} \circ x = (x_i x_k)(x_i x_l) M_{j,i}(x) \to \bar{\psi}_i \bar{\psi}_j \Gamma_{ijkl} \psi_k \psi_l \tag{4}$$

This quadratic map is a permutation of quartic roots [11]. However, $\gamma^{(3)}$ maps the complex plane to a cubic number field with half-differentials for λ-invariant λ_{ij} states in a Newtonian root finding process

$$z_{k+1} \leftarrow N_q(z_k) = z_k - \frac{q(z)}{\partial_z q(z)}\Bigg|_{z_k} \simeq F^{(3)}(w, z_k) = \gamma^{(3)}(w) \circ z_k \tag{5}$$

The rational substation (18) implies a symbolic cubic power integral base $w^k \to w_k$ by $\{w\} = \{w_0 = 1, w_1, w_2\}$. The algorithmic advantage is that (18) obeys a transvectant giving the invariant equation $\phi^{(3)}(z) \to 4z^3 - g_2 z - g_3 = 0$ with elliptic invariants g_2, g_3. Accordingly, phases on complex plane $L(w,z) = \ln(w - z)$ allow to define binary invariant Green's function $G(w,z) = \partial_z L(w,z) = \frac{1}{z_{k+1} - z_k}$ by using a singularity $\delta(\varphi_q - \varphi_{q'})$ in interval $[0,1]$. The physical origin of the Dirac delta function $\delta(\varphi_q - \varphi_{q'})$ is a source in heat equation because the string (11) enters theta constants via cubic roots. A time-interval average $\left\langle (t+\Delta t)^2 \right\rangle \simeq t\Delta t$ connects the heat equation with the wave equation [12]. This average is crucial by defining spacetime as an average over step quadruples q where dynamics enters as a uniform drift-diffusion process. One has in conjunction to (36)

$$\int_0^z G(z)\mathrm{d}z = \int_0^{z_k} \frac{\mathrm{d}z}{z_{k+1}-z} = \ln q(z_k) \to \int_0^{\Omega_k} \mathrm{d}\Omega \tag{6}$$

where a γ iterated quadratic form $q(z)$ has different representations

$$q(z) = \mathrm{e}^{\int_0^z \frac{\mathrm{d}z}{F(w,z)-z}} = \mathrm{e}^{-\int_0^z G(z)\mathrm{d}z} = \mathrm{e}^{-\int_0^{\Omega_k}\mathrm{d}\Omega} \simeq G^{-1}(z) G^{-1}(z) \tag{7}$$

which are compared to a holomorphic

$$\xi(z) = \prod_n \mathrm{e}^{L(z_{nt}-z_0,z)} = \mathrm{e}^{\sum_n \int_{C_n}^{z/z_{nt}} \frac{\mathrm{d}z}{z-1}} \tag{8}$$

for contours C_n around z_{nt}. Various representations of $q(z)$-iterates apply to mean values in QS or as a product of inverse functions G^{-1} in GR in the vicinity of a definite $\xi(z)$ zero. Mathematically, the exponent in (18) indicates a relation to poles of L-functions and a proportionality to the regulator R_Δ of algebraic units. For r-dimensional units in (15) R_Δ is determined by feasible units ε which maximize the ε- density. These optimal states $\varphi_q^2 \simeq (\varphi_q+\rho_0)^2 \simeq \mathrm{e}^{\varphi_q}$ are realizable by *mod* 2 and *mod* 3 congruent cyclotomic units $\zeta_r^{(2)}$ and $\zeta_r^{(3)}$ in definite circles of radius ρ_0 on complex plane [7]. *mod* 2 congruences are used to define fermions. The inverse Green's function G_0^{-1} measures the distance ρ_0 from a definite point on complex plane. Optimal states are expansion series of self-energy $\Sigma \simeq GG$ into the Green's function on complex plane for a one-dimensional singularity $\delta(\varphi_q - \varphi_q')$ of $\pm\pi$ rotations on real interval $[0,1]$. Therefore, optimal (feasible) units ε are realized for discrete *mod* 2 and *mod* 3 congruences in $G(z) \simeq \ln(G_0^{-1} - \Sigma)$. Zeros of (8) are also zeros of $\mathcal{Z}[g_2, g_3]$ which are discussed in conjunction with topological phase transitions [13]. Topological phase transitions induced by permutations of quartic roots x_i are plausible on a circle $q(z)$ travelling around a circulating string in $\mathcal{R}_L$. This logarithmic singularity in a non-Hermitian $\mathcal{L}$ is a complex non-dissipative state as an exact elliptic integral of the third kind. For rational z the envelope polynomial of (1) is $\phi^{(3)}(z) = z^3 - g_2 z - g_3$ because a rational substitution $z \to \gamma^{(n)} \circ z$ yields invariants g_2, g_3 resulting in $z \simeq f(\omega) = \mathrm{e}^{\frac{-i\pi\omega}{24}} \prod_{n=1}^{\infty} \left(1+\mathrm{e}^{i\pi\omega(2n-1)}\right)$. For invariant $z_{k+1} \leftarrow F^{(3)}(w, z_k) = w_0\left(4z_k^2 + \frac{2}{3}g_2\right) + w_1 z_k$ a Mandelbrot map with $c = \frac{2}{3}g_2$ is very close to a field density. Introducing field strength $D_{\mu\nu} = 2\mathfrak{Re} z_k \to \left[\boldsymbol{k}_\mu, \boldsymbol{k}_\nu\right]_-$, current density $I_{\mu\nu} = 2\mathfrak{Im} z_k$, field densities $\mathcal{F} = \frac{1}{2}\mathcal{R}(c - z_{k+1})$ and

$\mathcal{G}=\frac{1}{2}\Im\left(c-z_{k+1}\right)$ the map (5) reads $D_{\mu\nu}^4+2\mathcal{F}D_{\mu\nu}^2-\mathcal{G}^2=0$. This can be rewritten as a density $\mathcal{F}+i\mathcal{G}=D_{\mu\nu}^2+2iD_{\mu\nu}I_{\mu\nu}-I_{\mu\nu}^2$ in the presence of a current. The invariant $g_2=3\left(\mathcal{F}+i\mathcal{G}\right)+\frac{3}{2}z_{k+1}$ in (5) is like an energy density writing $D_{\mu\nu}+D_{\mu\nu}^*=\boldsymbol{X}=\sqrt{\mathcal{F}+i\mathcal{G}}\simeq\boldsymbol{E}+i\boldsymbol{B}$ for an electromagnetic field [4] [14].

3. Logarithmic Riemann Surface $\mathcal{R}_L$

Minkowski spacetime $\mathcal{M}$ is seen as a subset of rational coordinates of a under constrained complex Riemann surface $\mathcal{R}_L$ of bifurcating elliptic curve fragments. Iterates (4) and (5) create an under constrained Riemann surface $\mathcal{R}_L$ in the immediate vicinity of a nontrivial zero $z_{nt}=\frac{1}{2}+im_n$ of a transcendent entire polynomial. Under conformal transformations $\gamma\circ z$ creates a half-differential dz on $\mathcal{R}_L$ [15]. For invariant quadruples q an expansion up to the discretized second derivative is sufficient. One has $z_{k+2}-z_{k+1}=\delta_F\left(z_{k+1}-z_k\right)$ with Feigenbaum constant δ_F. The set $\mathcal{J}\left(N_q\right)-\mathcal{F}\left(N_q\right)$ are discrete additions on elliptic curves E_λ. Under γ the diameter (Δz) depends on q-invariants with half-differentials $\sqrt{\mathrm{dz}}$. The adiabatic approximation $\left(g_2,g_3=const\right)$ for $\phi^{(3)}\left(z\right)$ consists of two concentric balls $L_1=L_2=L_3$ in an infinite string texture $L_4\to\infty$ in $\mathcal{R}_L$. Their Hausdorff measure is the string volume $\mathrm{vol}_q\left(\mathcal{R}_L\right)$ connected with the Lebesgue measure of $\mathrm{dz}\simeq\sqrt{\left(\Delta z\right)^2}$ by a ball of volume $\mathrm{vol}_{k,k+1}\left(\mathcal{R}_L\right)$

$$\begin{aligned}&\left(\varphi_k\exp\left(\boldsymbol{k}_{\mu,k}\sigma^\mu\right)\right)^{\mathrm{T}}\exp\left(\boldsymbol{k}_{\nu,k+1}\sigma^\nu\right)\varphi_{k+1}\\&\simeq\left(\varphi_k\right)^{\mathrm{T}}\exp\left(D\right)\varphi_{k+1}\simeq\sqrt{\left(z_{k+2}-z_{k+1}\right)\left(z_{k+1}-z_k\right)}\end{aligned}\tag{9}$$

Wave vectors $\boldsymbol{k}_\mu$ are simply discrete φ_q sequences. Two φ_k pairs constitute a quadruple φ_q of pairs with $\pm\pi$ rotations leading to a coordinate-spin-quadruple φ_q with $D=D_{\mu\nu}\left[\gamma^\mu,\gamma^\nu\right]$ with Dirac matrices γ^ν. The diameter of (9) is

$$L_q=\ln\left(\sqrt{g_2/3}+\frac{1}{2}\varphi_0\right)=\frac{1}{3}\ln\left(\sqrt{\Delta}+\sqrt{g_2^3+\Delta}\right)-\frac{1}{2}\ln 3\tag{10}$$

with complex angle φ_0 defined by $\Delta=g_2^3\sin^2\varphi_0$. Chaotic dynamics is on circles of radius L_q with discrete angles

$$\varphi_{q,l}=\frac{1}{3}\left(\pm\pi+2n\pi+\ln\zeta_l^{(2)}+\ln\zeta_l^{(3)}\right)\tag{11}$$

are chosen centered around z_{nt} on $\mathcal{R}_L$. k-components of γ correspond to cubic roots

$$\begin{aligned}x_i&\simeq\mathrm{e}^{L_q+i\varphi_q}\simeq\left\{\pm\infty+\pm i\infty,e_i\right\}:e_i\\&=\sqrt{g_2/3}\left(\cos\left(\left(\varphi_0-\pi\right)/3\right),\cos\left(\left(\varphi_0+\pi\right)/3\right),\cos\left(\varphi_0/3\right)\right)\end{aligned}\tag{12}$$

leading to the 12-component string φ_q. A φ_q vector winds into the half-differentials dz on $\mathcal{R}_L$

$$\mathrm{d}z \simeq \sqrt{(\Delta z)^2} \simeq \int \mathrm{d}\boldsymbol{k}_{4,k}\mathrm{d}\boldsymbol{k}_{4,k+1}\mathcal{L}(z_k, z_{k+1}) \tag{13}$$

with Lagrangian $\mathcal{L}(z_k, z_{k+1})$ two-step density. $\varphi_q \simeq \varphi_\mu$ coefficients are wave vectors $\boldsymbol{k}_\mu \simeq 2\pi/L_q$. The surface $\mathcal{R}_L$ is defined by covariant substitutions $\boldsymbol{k}_\mu = e_\mu^\nu \boldsymbol{k}_\nu$. The vierbein e_μ^ν introduces differentials $\partial^\mu = e_\nu^\mu \partial^\nu$. Shifting a triangle $T(z_q)$ by dz yields again a median of triangle $T(z_q)$ with four squares to ensure rationality. The phase $\mathrm{d}\varphi_q$ is the arc length $\mathrm{d}\varphi_q \simeq \mathrm{d}s$ of a circle around $T(z_q)$. From one-periodic wave vectors $\boldsymbol{k}_\mu$ one can conclude to rational coordinates x_μ. An optimal phase $\varphi_q^2 \simeq \mathrm{e}^{\varphi_q}$ requires to expand complex angles $i\varphi_q + L_q$ around self-consistent circles for $mod\,2$ and $mod\,3$ congruences. It is claimed that the square of the arc length is metric in $\mathcal{M}$

$$\varphi_q^2 \simeq \mathrm{d}s^2 \simeq g_{\mu\nu}\mathrm{d}x_\mu \mathrm{d}x_\nu \tag{14}$$

with tensor $g_{\mu\nu} = e_\mu^\alpha e_\nu^\beta \eta_{\alpha\beta}$ and signature $\eta_{\mu\nu} = (-1,1,1,1)$ due to $L_1 = L_2 = L_3 \neq L_4$ with $L_4 \to \infty$ in the adiabatic approximation. This model uses degrees of freedom by x_i - permutations (4). This period-fluctuating cubic field can also be described by a real unit ε with $\sqrt{\varepsilon} = \prod_r \zeta_r^{(2)}\zeta_r^{(3)}\rho_r$ $mod\,2$ and mod 3 congruent cyclotomic units $\zeta_r^{(2)}$ and $\zeta_r^{(3)}$. Another representation of cubic roots

$$e_i(\varepsilon, \varphi) = \varepsilon^{-1}, \sqrt{\varepsilon}\mathrm{e}^{i\varphi/3}, \sqrt{\varepsilon}\mathrm{e}^{-i\varphi/3} \tag{15}$$

and discriminant [16]

$$\sqrt{\Delta(\rho_r, \varphi)} = 2i\left(\varepsilon^{3/2} + \varepsilon^{-3/2} - 2\cos(\varphi/3)\right)\sin(\varphi/3) \tag{16}$$

is conceivable through fluctuating units ρ_l in the region of phases φ. The volume $\mathrm{vol}_{k,k+1}(R_L)$ of the ball is calculated by the density of ideals of units which is the limit of the number of ideals T in (17) per its norm t [16]. This proves that the half-differential dz vicinity of z_{nt} depends on a Lagrangian $\mathcal{L}(z_k, z_{k+1})$ defined by the circulant regulator index $R_\Delta = \ln\varepsilon$ [17]. A semi-quantitative calculation up to constants would yield [18]

$$\begin{aligned}\mathrm{vol}_{k,k+1}(\mathcal{R}_L) &\simeq \lim_{t\to\infty}\frac{T}{t} \simeq \int \mathrm{d}\varphi_1 \cdots \int \mathrm{d}\varphi_r \simeq \lim_{z\to 1}(z-1)\zeta(z, \mathbb{K}) \\ &\simeq \frac{\det \ln\varepsilon}{\sqrt{\Delta}} \simeq \mathcal{L}(z_k, z_{k+1})\end{aligned} \tag{17}$$

Summarizing, the first non- trivial case of a quadratic $q(z) = (z - z_1)(z - z_2) \simeq G^{-1}(z)G^{-1}(z)$ yields a product of inverse Green's functions $G^{-1}(z)$ whereas QS is linear in G^{-1}. For $k \to \infty$ (4) and (5) describe two hyperbolic regions with focal points z_1, z_2 provided it is a UFD. An expansion into $\partial_z N_q$ converges for $\partial_z N_q < 2$ into $\sqrt{1 + \frac{1}{2}\partial_z N_q}$. A UFD allows to relate ∂_z to Δz and vice versa. Rational coordinates require a UFD for rational

$z \in \mathcal{Q}$ with $\Delta z[\partial_z]$ and $\partial_z[\Delta z]$. Rational iterated variable z

$$F^{(3)}(w,z) = \frac{\phi^{(3)}(w)}{w-z} - \frac{1}{3}\partial_w \phi_w^{(3)} \tag{18}$$

are cubic roots concentric around L_q. The adiabatic approach with four quartic roots spins a 4-component thread ψ_q of 4-component ribbons shown in **Figure 1** including the spectator root at infinity around $f(\omega) \simeq \zeta^{(12)} e^{\frac{-i\pi\omega}{24}}$. It is noted that the UFD derivative

$$\partial_z N_q(z) = \partial_z F^{(3)}(w,z) = 8w_0 z + 4w_1 \simeq \Gamma_{ijkl} \simeq \Gamma[D_{\mu\nu}] \simeq \Gamma\left[\left[A_\mu, A_\nu\right]_-\right] \tag{19}$$

fluctuates with z around a cubic base component w_1. Accordingly, the expansion of $q(z)$ is into the vertex $\Gamma[D_{\mu\nu}]$.

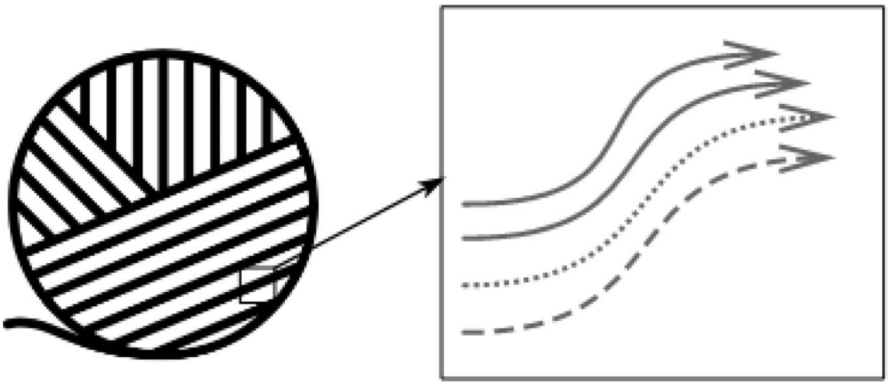

Figure 1. A ball of strings $(\varphi_k)^{\mathrm{T}} \exp(D)\varphi_{k+1} \simeq (\Delta z)^2$. Pair of $\pm\pi$ rotations in $[0,1]$ (solid), spectator root in $x_i = \{\pm\infty + \pm i\infty, e_i\}$ (dotted), infinite shift (dash-dotted).

4. Complex Non-Hermitian Field Equations

The estimated phase volume $\frac{\det \ln \varepsilon}{\sqrt{\Delta}}$ is a determinant of a circulant matrix giving $\det \ln \varepsilon = \prod_{j=0}^{r-1} \phi^{(r)}\left(\zeta^{(r)j}\right)$ with coefficients ε_j in polynomial $\phi^{(m)}$ for various cyclotomic generators $\zeta^{(r)}$, $r = 2,3,4,6,12,24$. The under constrained Riemann surface $\mathcal{R}_L$ is thought as bifurcating line bundles which are regular and invertible for a UFD. This constrains the minimum of $\frac{\det \ln \varepsilon}{\sqrt{\Delta}}$ to a limited number of fundamental units ε. However, the complex phase $L(w,z)$ in $\mathcal{R}_L$ fluctuates in degrees of freedom of discriminant (13) and (14). The cubic field has cylindrical orbits of complex phase $l = \ln \varepsilon = (\rho, \varphi)$. Feigenbaum renormalization $z \to g(z)$ with respect to variable φ is set in context to direct and exchange diagrams in QS with respect to the vertex Γ. The one-dimensional renormalized function $g(\varphi)$ or $g(\Gamma)$ produces a single maximum or twin peak maxima. It is claimed that a quadratic expansion in φ or Γ with three conformal steps is sufficient for γ-invariance [15]. In this approximation the Lagrangian is quadratic in N_Δ [17]

$$\ln\varepsilon \simeq \sum_{q=k,k+1,k+2,k+3} 1-\Delta\boldsymbol{l}\partial\boldsymbol{l}-\frac{1}{2}\left(\Delta\boldsymbol{l}\partial\boldsymbol{l}\right)\left(\Delta\boldsymbol{l}'\partial\boldsymbol{l}'\right)\ln\varepsilon \tag{20}$$

Then γ-invariance of orbits at the minimum of the circulant $\prod_{j=0}^{r-1}\phi^{(r)}\left(\zeta^{(r)j}\right)$ yields a linear and a quadratic condition

$$\begin{aligned}\sum_{q=k,k+1,k+2,k+3}\Delta\boldsymbol{l}\partial_{\boldsymbol{l}}\ln\varepsilon &= -2\sum\Delta\boldsymbol{l}\partial_{\boldsymbol{l}}\left(\ln\zeta^{(2)}+\ln\rho_r\right)\\ &= \sum_{q=k,k+1,k+2,k+3}\frac{1}{\rho}\left(\Delta\rho\partial_{\ln\rho}+\Delta\varphi\partial_{\varphi}\right)\ln\varepsilon = 0\end{aligned} \tag{21}$$

where $\ln\varepsilon \simeq \int_0^z G(z)\mathrm{d}z \to \oint \mathrm{d}\nu\left(\varphi_q+\frac{1}{2}(\ln g_2-\ln 3)\right)$. This proves a relation between the *mod* 2 field $\zeta^{(2)}$ and the Green's function $G(z)$. Locally the linear term in (20) with (13) and (18) reads for $\varphi_q^2 \simeq \rho\mathrm{e}^{\varphi_q}$, $\Delta\varphi_q^2 \simeq \Delta\rho\mathrm{e}^{\varphi_q} \simeq g_{\mu\nu}\mathrm{d}x_\mu\mathrm{d}x_\nu$ and normal vector $\mathrm{d}x_\mu\mathrm{d}x^\nu$

$$\begin{aligned}&\sum_q\frac{1}{\rho}\left(\Delta\rho\partial_{\ln\rho}+\Delta\varphi^2\partial_{\varphi^2}\right)\ln\varepsilon \simeq \sum_q\frac{-2\Delta\rho}{\rho}+\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}\left(z_k,z_{k+1}\right)\right)}{\delta g_{\mu\nu}}\\ &= \delta g_{\mu\nu}\left[-R_{\mu\nu}+\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}\left(z_k,z_{k+1}\right)\right)}{\delta g_{\mu\nu}}\right] = 0\end{aligned} \tag{22}$$

where $-2\frac{\delta\left(\frac{\Delta\rho}{\rho}\right)}{\delta g_{\mu\nu}} = -R_{\mu\nu}$ is a curvature tensor. The second term is γ-invariant

$$\left(\Delta\boldsymbol{l}\partial\boldsymbol{l}\right)\left(\Delta\boldsymbol{l}'\partial\boldsymbol{l}'\right)\ln\varepsilon \tag{23}$$

and should be written in terms of the Schwarzian derivative $\{F,z\} = \frac{\dddot{F}}{\dot{F}}-\frac{3}{2}\left(\frac{\ddot{F}}{\dot{F}}\right)^2$ where $\dot{F} = \partial_z F$

$$\sum_{q,q'}\left\{z_q,z'_q\right\}\mathrm{d}z_q\mathrm{d}z'_q = \sum_{q,q'}\delta\left(\varphi_q-\varphi'_q\right)\mathrm{d}\varphi_q\mathrm{d}\varphi'_q \tag{24}$$

The singularity $\delta\left(\varphi_q-\varphi'_q\right)$ yields the RG flow stress-energy $\left\{z_\mu,z_\nu\right\} = \gamma^\nu\partial_\mu G^{\pm}_{ss'}$. Further investigation is needed to show that radial and tangential derivatives on a circle on complex plane become curvature tensor and stress-energy.

5. Phase Transition by Logarithmic Singularity

For $\phi_{vac}\left(\lambda_{ij}=0,1\right)$ the discriminant Δ vanishes which yields for frequency $\nu_0 = \sqrt{9g_3/2g_2}$.

$$\frac{\Delta\rho}{\rho} = \frac{\Delta\sigma(u,\omega)}{\sigma(u,\omega)} \to \nu_0\cot\left(u\nu_0\right)+\frac{1}{3}\nu_0^2 \tag{25}$$

a one-periodic behavior of $\mathcal{L}$ and energy (21) [19]. The period rectangle ω changes into a line. For arbitrary g_2, g_3 the Lagrangian

$$\mathcal{L}\left(z_k, z_{k+1}\right) \simeq \frac{\det \ln \varepsilon}{\sqrt{\Delta}} \ln \varepsilon \simeq \frac{\Delta \rho}{\rho} = \frac{\Delta \sigma(u,\omega)}{\sigma(u,\omega)} \tag{26}$$

can be written in terms of changes of topological entropy h_t due to additions on equivalent elliptic curves. Topological entropy

$$h_t(N) = \lim_{N \to \infty} \frac{C(N, g_2, g_3)}{N} \tag{27}$$

is defined by the specific cardinality $C(N, g_2, g_3)$ for indistinguishable orbits [20]. On universal covering on complex plane $C(N, g_2, g_3)$ can be related to n^{th} order functions [19]

$$C(N, g_2, g_3) = \ln \frac{\prod_{i,k} \sigma(u - u_i, \omega_k)}{\prod_{i,k} \sigma(u - v_i, \omega_k)} = \ln \prod_k \det_{ij} \wp^{(i)}(u_j, \omega_k) \tag{28}$$

defined by Weierstrass σ-functions and the i^{th} derivative of the Weierstrass function $\wp^{(i)}$. Then the cardinality $C(N, g_2, g_3)$ is an elliptic integral of the third kind. This integral is related to the two-dimensional Green's function

$$G^{(2)}(z_q, z) \simeq \ln(z - z_q)(\bar{z} - \bar{z}_q) \simeq L(z_q, z) + L(\bar{z}_q, \bar{z}) \tag{29}$$

which is based on the one-dimension Dirac delta function $\delta(\varphi_q - \varphi_q')$. Due to (19) a phase $L(w, z)$ is reformulable as an integral over a vertex Γ

$$\frac{\mathrm{d}\Gamma}{z - \Gamma} = \frac{\mathrm{d}\mu}{z\mathrm{e}^{-\mu} - 1} = \mathrm{d}L(\Gamma, z) \tag{30}$$

or an integral over the electro-chemical potential μ with fugacity RG flow $\Gamma = \mathrm{e}^{\mu} \simeq z$. The squared Dirac equation for $\psi_s \simeq \psi_q$

$$\left[\left(i\hbar\partial_\mu - eA_\mu\right)^2 - m_n^2 - ieF_{\mu\nu}\left[\gamma_\mu, \gamma_\nu\right]_-\right]\psi_s = 0 \tag{31}$$

m_n^2 operates on the Mandelstam plane M_{stu} with $s,t,u \simeq \varphi_q^2 \simeq \left(\boldsymbol{k}_\mu + \boldsymbol{k}_\nu\right)^2 \simeq m_n^2$. As known (31) implies negative mass densities ρ_{vac}. Next it is shown that the logarithmic singularity in (25)-(29) yields equivalent $\zeta\left(z, \mathcal{Z}[g_2, g_3]\right)$ minima

$$\zeta\left(z, \mathcal{Z}[g_2, g_3]\right) \simeq (z - z_{nt}) \simeq \gamma \circ \omega \tag{32}$$

6. Doubly-Periodic Processing

Constrained one-periodic systems of length L have energy $\rho_{vac} \simeq \sum_n \omega_n \simeq -\sum_n \frac{2\pi}{L} n \simeq -\frac{2\pi}{L} \zeta(-1)$ which can be related by renormalization (Casimir effect) to the zeta function at argument $z = -1$ [21]. Confinement replaces a unit volume by the lower mean of a standing wave $\sin^2(\omega_n)$ with real ω_n which is a small correction of ρ_{vac}. Doubly-periodic ω imply a self-consistent confinement by means of non-dissipative damped exponential tails. Whereas in QS a complex energy induces damping the non-Hermitian $\mathcal{L}$ of (1) induces a superfluid pairing of charges [22] [23]. Invariant quadratic root finding (5) creates the state (1) and induces a complex period ω. It is shown that this simple model simulates phase transitions at zeros of $\zeta\left(z, \mathcal{Z}[g_2, g_3]\right)$ where iterates z_k become periods with complex multiplication. In the cubic case z_k is a transvectant which tends to the Weber invariant $f(\omega)$ with $ff_1 f_2 = \sqrt{2}$ [11]

$$\varepsilon\omega\overline{\varepsilon\omega} \sim \prod_i \frac{\eta^2(\omega_i)}{\eta^2(\omega_{i-1})} \overline{\prod_l \frac{\eta^2(\omega_l)}{\eta^2(\omega_{l-1})}} \sim f_1 f / f_2^2 \tag{33}$$

where bars denote conjugated units $\varepsilon \sim f_1 / f_2$, $\bar{\varepsilon} \sim f / f_2$ of a cubic normal field $\mathbb{K} \cdot \mathbb{K}' \cdot \mathbb{K}''$ of discriminant Δ and Dedekind eta function $\eta(\omega)$. Logarithmic singularities in $\sigma(u - u_i, \omega_k)$

$$L(w,u) \simeq \int \mathrm{d}u \zeta(u - u_i, \omega_k) \tag{34}$$

are proportional to $\frac{\Delta\rho}{\rho} \simeq \prod_{i,k} \sigma(u - u_i, \omega_k) \simeq \mathrm{e}^{L(w,u)} \simeq \sum \varepsilon\omega$. Nontrivial zeros become approximated by mean values of periods

$$z_{nt} = \frac{1}{2} + im_n \simeq \sum \gamma \circ \omega_k \tag{35}$$

The vacuum state (1) is capable to resolve an algorithmic step quadruple q whereas $\mathcal{M}$ is not. Processing in particle accelerators, fusion reactors and artificial photosynthesis is mainly sequential steps of one-periodic interactions. This classifies unique vacuum energy, binding energy, inverse temperature β as a mean thermodynamic energy Ω which reads in QS

$$\delta\Omega \simeq \sum \int \frac{\mathrm{d}\tau}{\tau} G_0^{-1}(G - G_0) \to \Omega = -\frac{1}{\beta} \ln \mathcal{Z}[g_2, g_3] \tag{36}$$

without having logarithmic singularities [24]. Doubly-periodic processing consists in infinitely many simultaneous changes of at least two different parameters like a breathing process. Vacuum polarization in QS is one-periodic virtual scattering and a one-periodic chemical potential ν with occupation number N_Δ for $\lambda_{ij} = a^+ a = 0,1$ in $\phi_{vac}(\lambda_{ij} = 0,1)$. Accordingly, Feynman diagrams sum direct and exchange scattering. QS proves this behavior by Γ-linear and Γ- quadratic scattering amplitudes. Both terms are statistically equally weighted over smoothed out $\mathcal{L}$ - singularities. The QS time interval of the measurement is large as compared to internal frequencies. The Feigenbaum renormalized $\zeta z, \mathcal{Z}[g_2, g_3]$ receives either a single maximum or two maxima. Accordingly, the logarithmic singularity in

$$\zeta(z, \mathcal{Z}[g_2, g_3]) = \int \mathcal{D}\Gamma(z_k) \mathcal{D}\Gamma(z_{k+1}) \mathrm{e}^{\mathrm{d}\boldsymbol{k}_{4,k} \mathrm{d}\boldsymbol{k}_{4,k+1} \mathcal{L}(z_k, z_{k+1})} \tag{37}$$

is a complex chemical potential of an eternal process of pair creation and topological phase transition. This process traverses a zero of the partition function for $\phi_{vac}(\lambda_{ij})$ with arbitrary λ_{ij} around a phase transition on a circle quadratic in two complex masses and two complex curvatures of spacetime which is felt as a drift-diffusion process with two velocities of light c_l. It is argued that in a spacetime volume $\mathrm{vol}(\mathcal{M})$ both processes are averaged. The standard spacetime is sequences of k-component states with lower energy in dependence on γ-processing.

7. Conclusions

The doubly-periodic paired vacuum state (1) is a quasi-stationary state which encounters phase transitions by travelling in the neighborhood of zeros of zeta functions and partition functions. Therefore, the unique vacuum state of a real Lagrangian e.g. with $\zeta(z=-1)$ at the Casimir effect can be undercut by quasi-stationary continued fractions $\gamma \circ z$. A physical realization would be smart technology by correlated one-periodic processing. This not exceptional process is a precursor for stable spacetime $\mathcal{M}$. A quadratic amplitude amplified Carnot cycle is proposed for changing correlated both topological entropy h_t and temperature β^{-1}. This replaces a rectangular entropy-temperature cycle h_t, β^{-1} by a circular-like cycle of an open system where β^{-1} is not well defined. Whereas for closed systems temperature is well defined, open systems depend on temperature fluctuations. The dimensionless interaction state (1) should hold for all physical interactions. A forthcoming work aims to show that permutations (4) on complex plane relate the shifted ground state $(z-e_i)M_{ij}(z)$ to a paired superfluid state comparable to a BCS-state with non-Hermitian Lagrangian of a renormalization group flow [22] [25]. Invariant quadratic root finding on complex plane is used as a precondition for covariance which results in two different curvatures and two masses in each spacetime point [26].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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