

Intrinsic Supersymmetry Breaking in a Composite Model

Risto Raitio 

Helsinki Institute of Physics, University of Helsinki, Helsinki, Finland

Email: risto.rautio@gmail.com

How to cite this paper: Raitio, R. (2026) Intrinsic Supersymmetry Breaking in a Composite Model. *Journal of High Energy Physics, Gravitation and Cosmology*, 12, 1377-1386.
<https://doi.org/10.4236/jhepgc.2026.123069>

Received: March 3, 2026

Accepted: June 7, 2026

Published: June 10, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc. This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

We analyze the global symmetry structure of a phenomenological supersymmetric preon model in which vector and chiral supermultiplets already contain all elementary degrees of freedom, including fermionic superpartners, like the photino and the gluino. The role of preon symmetries in earlier formulations of the model is clarified, and a mechanism for supersymmetry breaking arising intrinsically from compositeness is identified. Gauge anomaly conditions associated with a $U(1) \times SU(3)$ symmetry are examined, and a minimal anomaly-free preon spectrum is obtained. The analysis is independent of detailed binding dynamics and focuses on structural consistency requirements relevant for preon-based extensions of the Standard Model. A brief comparison with the Minimal Supersymmetric Standard Model is presented. A consistent hyperunified theory group is pointed out.

Keywords

Composite Particles, Gauge Anomalies, Supersymmetry, Supersymmetry Breaking

1. Introduction

Preon models provide a possible extension of the Standard Model to shorter distance scales. In this paper, we revisit the supersymmetric preon model introduced in [1] with particular emphasis on its internal symmetry structure. We clarify the role of preon global symmetries and examine gauge anomaly cancellation and 't Hooft consistency conditions. Gauge anomaly free preon model is ensured. Within this framework, supersymmetry breaking arises as a necessary consequence of compositeness.

The analysis is independent of detailed assumptions about the binding dynamics responsible for preon confinement. Preons are treated as algebraic objects de-

fined by their transformation properties under the internal symmetry group, in analogy with early quark-model descriptions prior to the establishment of QCD.

A central assumption of the model is, however, that spacetime supersymmetry is primary, while internal particle symmetries are secondary. Accordingly, particles are embedded into supersymmetric multiplets from the outset, rather than introducing supersymmetry by doubling the Standard Model particle content.

The paper is organized as follows. In Section 2, we introduce the preon supermultiplets. Section 3 discusses the internal $U(1) \times SU(3)$ symmetry and the associated gauge consistency conditions. Supersymmetry breaking is addressed in Section 4. Outlook for extending the internal symmetry and conclusions are presented in Section 5, with additional technical details collected in the appendices.

2. Preon Supermultiplets

In the particle picture, the preons could be free particles above the critical scale $\Lambda_{cr} \sim 10^{12} - 10^{16}$ GeV, close to the standard cosmology reheating scale T_R and the Grand Unified Theory (GUT) scale. Below Λ_{cr} , preons would form composite states by a binding or confining interaction, which is unknown. Therefore, we consider preons as objects having only algebraic properties.

To determine the low-energy spectrum, preons are grouped into vector and chiral supermultiplets [1]. **Table 1** summarizes the superparticle content of the model. The m 's are fermions, with the superscript indicating their charge in units of one-third electron charge, and the subscript indicating color (R, G, B). The s and σ are scalars. The γ and g_i are the familiar gauge bosons of the SM. Their superpartners m^0 and m_i^0 are usually known as the photino and gluino, respectively.

Table 1. Superscripts: $U(1)$ charge (in units of $\frac{e}{3}$), subscripts: $SU(3)$ color index.

| | |
|--------|-------------------------------------|
| Chiral | $m^-, s^-; m_i^0, \sigma_i^0; n, a$ |
| Vector | $\gamma, m^0; g_i, m_i^0$ |

The Standard Model particles and dark matter are composed of preon composites in the very early universe, at a temperature approximately equal to the reheating value T_R . Numerically, inflation begins at about 10^{-36} s and ends at 10^{-32} s, corresponding to energies 10^{13} to 10^{16} GeV, depending on the model. Towards the end of inflation, all fermionic preons are arranged by a binding/confining force into composite states. After 10^{-32} s, reheating begins in the standard way with inflaton producing the Minimal Supersymmetric Standard Model (MSSM) particles (with some model-dependent differences, see Section 5). For reference, the Electroweak Phase Transition (EWPT) occurs at about 10^{-11} s after inflation ends.

3. Preon Internal Symmetry

3.1. Finite-Dimensional Three-Particle Representations

The key property of the model is the product group

$$G = U(1) \times SU(3). \quad (1)$$

The abelian group $U(1)$ acts via complex phases $e^{i\theta q}$, where q is the electric charge. The fundamental representation of $SU(3)$ is denoted $\mathbf{3}$ with basis vectors labeled by color (R, G, B) .

Each preon transforms under one of two representations:

$$\psi_0 \sim (0, \mathbf{3}) \text{ neutral, color-triplet preon,} \quad (2)$$

$$\psi_1 \sim \left(\frac{1}{3}, \mathbf{1}\right) \text{ charged, color-singlet preon.} \quad (3)$$

Three-body composites are built as follows: $U(1)$ charges are additive and the $SU(3)$ tensor product composition is $\mathbf{1} \otimes \mathbf{3} = \mathbf{3}$, $\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \bar{\mathbf{3}}$, $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$. Each composite state is of the form (charge, color). Composites containing $SU(3)$ $\mathbf{6}$, $\mathbf{8}$, or $\mathbf{10}$ irreducible representations are so far unobserved exotics.

3.2. Gauge Anomalies and Composite Consistency

Before turning to the composite spectrum, it is useful to briefly connect the general discussion of gauge anomalies [2] to the structure of the present preon model. Since the symmetry group $U(1) \times SU(3)$ is gauged at the ultraviolet level, all gauge anomalies must cancel already in the fundamental preon theory. This requirement places nontrivial constraints on the allowed chiral preon content, independently of any assumptions about infrared dynamics.

In particular, the non-abelian $SU(3)^3$ anomaly measures the net chirality of preons transforming under the color group. Because the fundamental representation of $SU(3)$ is complex, quantum consistency requires equal net contributions from $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations. This condition motivates the inclusion of conjugate color preons in the ultraviolet spectrum. Similarly, the absence of the cubic and gravitational $U(1)$ anomalies requires that the abelian charges of the preons satisfy appropriate cancellation conditions.

An important structural feature of the model is that preons carrying $SU(3)$ color are neutral under the abelian factor, while $U(1)$ -charged preons are singlets of $SU(3)$. As a result, the mixed $U(1) - SU(3)^2$ anomaly vanishes identically, ensuring compatibility between the confining dynamics and the abelian charge assignments.

Once these ultraviolet gauge consistency conditions are satisfied, no further gauge anomaly constraints arise at the composite level. Gauge invariance of the infrared theory is then automatic, while additional constraints on the composite spectrum follow only from 't Hooft anomaly matching for exact global symmetries. A detailed discussion of the gauge consistency conditions and their minimal satisfaction in the present model is given in **Appendices**.

For completeness, we note that quantum consistency of the ultraviolet theory requires an anomaly-free chiral fermion content. In addition to the preons

$$\psi_0 \sim (0, \mathbf{3}) \text{ and } \psi_1 \sim \left(\frac{1}{3}, \mathbf{1}\right),$$

this is achieved by including a conjugate color preon $\tilde{\psi}_0 \sim (0, \bar{\mathbf{3}})$ together with an oppositely charged singlet $\psi_{-1} \sim \left(-\frac{1}{3}, \mathbf{1}\right)$.

With this minimal completion, all gauge anomalies associated with $U(1) \times SU(3)$ cancel identically in the ultraviolet.

Once gauge consistency is ensured at the preon level, no further gauge anomaly constraints arise for the composite spectrum. Additional restrictions from ‘t Hooft anomaly matching apply only to exact global symmetries that remain unbroken in the infrared. In the absence of a detailed specification of the full infrared spectrum, these matching conditions are therefore understood as consistency checks, rather than as rigid constraints on the model.

3.3. Heavy Particle Decoupling

The infrared spectrum of a field theory can be studied using the Appelquist-Carazzone Theorem [3]. It states that in a renormalizable field theory, in physical processes at momenta (k) much smaller than the mass (m) of a heavy particle ($k \ll m$), the heavy particle essentially disappears from the dynamics. The only lasting signal the heavy particles leave behind is a modification of the values of the constants in the theory. They contribute only to the renormalization of the coupling constants and field strengths of the remaining light particles. Consequently, the low-energy physics can be accurately described by an effective renormalizable Lagrangian that contains only the light fields. Any direct effects from the heavy fields are suppressed by powers of the small ratio k/m .

3.4. Physical Picture

The correspondence between the representations and SM particles is shown in **Table 2**. The state $(0, 3)$ is an uncharged, color-triplet quark, a good candidate for dark matter. The fractionally charged color-singlet states $(\frac{1}{3}, 1)$ have not been observed. They would either be forbidden by integer charge quantization or indicate lepto-quark properties [4].

Table 2. Physical picture.

| Representation | Charge | Color | Particle |
|---|----------------|---------|------------|
| $(0, \mathbf{1})$ | 0 | singlet | neutrino |
| $(-1, \mathbf{1})$ | -1 | singlet | electron |
| $\left(\frac{2}{3}, \mathbf{3}\right)$ | $\frac{2}{3}$ | R/G/B | up quark |
| $\left(\frac{-1}{3}, \mathbf{3}\right)$ | $-\frac{1}{3}$ | R/G/B | down quark |

The SM fermions as composites of preons and their superpartners are shown in **Table 2**. SM superpartners are formed by the transition preon \rightarrow spreon in **Table 1**.

Table 3. First-generation composite particles. The quark indices follow the cyclic permutation (e.g., d_r is composed of preons with colors G and B).

| SM fermion | Preon composition | Sfermion/spreon composition |
|-------------|-------------------------------|---|
| ν_e | $m_R^0 m_G^0 m_B^0$ | $\tilde{\nu}_e : \sigma_R^0 \sigma_G^0 \sigma_B^0$ |
| $u_{R/G/B}$ | $m^+ m^+ m_{R/G/B}^0$ | $\tilde{u}_{R/G/B} : s^+ s^+ \sigma_{R/G/B}^0$ |
| $d_{R/G/B}$ | $m^- m_{G/B/R}^0 m_{B/R/G}^0$ | $\tilde{d}_{R/G/B} : s^- \sigma_{G/B/R}^0 \sigma_{B/R/G}^0$ |
| e^- | $m^- m^- m^-$ | $\tilde{e}^- : s^- s^- s^-$ |

It is seen that quarks and leptons, as well as all the superpartners, are of the same, interconnected structure.¹

4. Supersymmetry Breaking

In this framework, squarks (sleptons) are $SU(3)$ triplet (singlet) composite states of charge $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$ ($0, \pm 1$), as seen in **Table 3**. Superpartners in our scenario are composites of scalar bosons. Therefore, the superpartners are qualitatively different by nature from the fermionic SM particles. Superpartner masses are expected to have different mass scale(s) than particles due to different statistics. Accordingly, they have different vacuum energies, and supersymmetry is likely strongly broken below Λ_{cr} . This high scale translates to the lack of observation of superpartners at the LHC. On the other hand, the different qualitative nature of spin 0 spreons makes spreon matter a candidate of matter elsewhere, like dark matter [5] and little red dots [6].

In general, bosonic states include laboratory-observable particles, boson stars [7] and Bose-Einstein condensate fluids [8]. The chromodynamic force could bind hadronic-scale particle states, but they have not been detected at the LHC [9] [10]. Ultralight particles are being intensively searched for, with possible masses down to 10^{-33} eV [11].

Explicit supersymmetry breaking can strengthen the inherent breaking. The quadratic sensitivity to M_{UV} —the scale at which new physics modifies the loop integrations—is absent in SUSY theories with the following SUSY-breaking terms added (for a review, see [12]):

$$\mathcal{L}_{soft} = -\frac{1}{2} \left(M_a \lambda^a \lambda^a + c.c. \right) - \left(m^2 \right)_j^i \phi^{*j} \phi_i - \left(\frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + c.c. \right) \tag{4}$$

¹We also find in **Table 2** that a hypercolor force is needed for preon confinement. This is left for future study.

where M_a is the gaugino, or m_i^0 , mass and $(m^2)_j^i$, are scalar mass and couplings, respectively.

All non-gauge masses and interactions are determined by the superpotential W

$$W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k \tag{5}$$

From **Table 2**, we see two scalars, $\tilde{\nu}_e$ and \tilde{e}^- . Across three generations there are six scalars. There may be more than one axion. The masses and couplings of the gauginos and sleptons are, in principle, calculable in the present composite model, where supersymmetry is broken internally. Concrete calculations are beyond the scope of this short note.

5. Outlook and Conclusions

Incorporating the weak interactions into the present framework is conceptually straightforward. Once quarks and leptons emerge as composite states, the standard electroweak structure can be introduced in the usual way. This includes the weak isospin gauge group $SU(2)$, with gauge bosons W_1, W_2, W_3 that combine to form the physical W^\pm and Z^0 , as well as the weak hypercharge $U(1)_Y$, associated with the B boson and mixing with W_3 to yield the photon and the Z^0 . Spontaneous symmetry breaking then proceeds via the Higgs mechanism, providing masses to the weak gauge bosons. In some cosmological settings, the Higgs field may also play the role of an inflaton and could therefore be relevant for supersymmetry in the early universe.

What could be the emerging, $U(1) \times SU(3)$ consistent full group on quark-lepton level? In [13] the starting point is that in the SM eight fundamental quantum numbers are observed. These are the two isospins, three colors and three families. This suggests to introduce the local metaflavor symmetry $SU(8)_{MF}$ as the internal symmetry group at small distances. This line of argumentation looks promising, but showing its applicability in our scenario is subject for another project.

Table 4. Feature comparison MSSM vs. preons.

| Feature | MSSM | Preon model |
|-----------------|----------------------------------|------------------------------|
| Basic idea | SM particle doubling | Supermultiplets |
| Gauge Group | $SU(3) \times SU(2) \times U(1)$ | $U(1) \times SU(3)$ |
| Gauge anomalies | None | None |
| Higgs Sector | Two doublet Higgs super fields | Std. Higgs structure of SM |
| Matter | Fermions and bosons have B, L | SM particles, unified struct |
| SUSY breaking | Unsolved | Automatic |
| No. parameters | 100+ | 1 (the fermion/boson ratio) |

We briefly compare our model to the mainstream Minimal Supersymmetric Standard Model [14]-[16]. Some of the important properties of both models are

shown in **Table 4**. It is seen that the problems of parametrization and supersymmetry breaking seem to be easier to solve in the preon model.² For experimental searches, both models have the same (s)particles. The Higgs particle in the preon model is obviously a composite particle [17].

The main result of this note is that preons transforming under a global $U(1) \times SU(3)$ symmetry can be embedded economically into a supersymmetric framework. Fermionic superpartners, e.g. the photino (m^0) and gauginos (m_i^0) are already present at the preonic level, without the need for additional assumptions. Within this setting, the compositeness of Standard Model particles leads naturally to supersymmetry breaking. The associated supersymmetry-breaking parameters are, in principle, calculable. Phenomenologically, one may take the ratio of fermionic to bosonic amplitude, or wave function, as the free parameter of the model. Additional symmetry breaking terms may be introduced if required for phenomenological applications.

These observations suggest that compositeness and supersymmetry breaking may be closely linked. Our scenario provides a coherent setting for further exploration of preon-based extensions, e.g. $SU(8)$, of the Standard Model.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Raitio, R. (2018) Supersymmetric Preons and the Standard Model. *Nuclear Physics B*, **931**, 283-290. <https://doi.org/10.1016/j.nuclphysb.2018.04.021>
- [2] Weinberg, S. (1996) *The Quantum Theory of Fields, Volume II: Modern Applications*. Cambridge University Press.
- [3] Appelquist, T. and Carazzone, J. (1975) Infrared Singularities and Massive Fields. *Physical Review D*, **11**, 2856-2861. <https://doi.org/10.1103/physrevd.11.2856>
- [4] Adler, S.L. (1996) Frustrated $SU(4)$ as the Preonic Precursor of the Standard Model. arXiv: hep-th/9610190. <https://arxiv.org/pdf/hep-th/9610190.pdf>
- [5] Bozorgnia, N., Bramante, J., Cline, J.M., Curtin, D., McKeen, D., Morrissey, D.E., Ritz, A., Viel, S., Vincent, A.C. and Zhang, Y. (2025) Dark Matter Candidates and Searches. *Canadian Journal of Physics*, **103**, 671-703. <https://doi.org/10.1139/cjp-2024-0128>
- [6] Juodzbališ, I., *et al.* (2025) A Direct Black Hole Mass Measurement in a Little Red Dot at the Epoch of Reionization. arXiv: 2508.21748. <https://arxiv.org/pdf/2508.21748.pdf>
- [7] Liebling, S.L. and Palenzuela, C. (2023) Dynamical Boson Stars. *Living Reviews in Relativity*, **26**, Article No. 1. <https://doi.org/10.1007/s41114-023-00043-4>
- [8] Berezhiani, L., Cintia, G., De Luca, V. and Khoury, J. (2012) Superfluid Dark Matter. arXiv: 1202.5809. <https://arxiv.org/pdf/1202.5809.pdf>
- [9] Olsen, S.L., Skwarnicki, T. and Zieminska, D. (2018) Nonstandard Heavy Mesons and Baryons: Experimental Evidence. *Reviews of Modern Physics*, **90**, Article ID: 015003.

²In **Table 4**, the bottom line numbers of parameters may not be exactly comparable if SUSY-breaking (5) is included.

- <https://doi.org/10.1103/revmodphys.90.015003>
- [10] Constantin, L., Kraml, S. and Mahmoudi, F. (2025) The LHC Has Ruled Out Supersymmetry—Really? arXiv: 2505.11251. <https://arxiv.org/pdf/2505.11251.pdf>
- [11] Ringwald, A. (2025) Ultralight Bosonic Dark Matter. *QTFP Meeting*, Glasgow, 21-22 January 2025.
- [12] Dine, M. and Mason, J.D. (2011) Supersymmetry and Its Dynamical Breaking. *Reports on Progress in Physics*, **74**, Article ID: 056201. <https://doi.org/10.1088/0034-4885/74/5/056201>
- [13] Chkareuli, J.L. (2019) The SU(8) GUT with Composite Quarks and Leptons. *Nuclear Physics B*, **941**, 425-457. <https://doi.org/10.1016/j.nuclphysb.2019.02.009>
- [14] Fayet, P. (1975) Supergauge Invariant Extension of the Higgs Mechanism and a Model for the Electron and Its Neutrino. *Nuclear Physics B*, **90**, 104-124. [https://doi.org/10.1016/0550-3213\(75\)90636-7](https://doi.org/10.1016/0550-3213(75)90636-7)
- [15] Fayet, P. (2001) About the Origins of the Supersymmetric Standard Model. *Nuclear Physics B - Proceedings Supplements*, **101**, 81-98. [https://doi.org/10.1016/s0920-5632\(01\)01495-5](https://doi.org/10.1016/s0920-5632(01)01495-5)
- [16] Fayet, P. (2016) The Supersymmetric Standard Model. In: Fritzsche, H., *et al.*, Eds., *Advanced Series on Directions in High Energy Physics*, World Scientific, 397-454. https://doi.org/10.1142/9789814733519_0020
- [17] Banerjee, A., Merchand, M. and Nałęcz, I. (2024) Phase Transition and Gravitational Waves in Maximally Symmetric Composite Higgs Model. *Journal of High Energy Physics*, **2024**, Article No. 106. [https://doi.org/10.1007/jhep10\(2024\)106](https://doi.org/10.1007/jhep10(2024)106)
- [18] Adler, S.L. (1969) Axial-Vector Vertex in Spinor Electrodynamics. *Physical Review*, **177**, 2426-2438. <https://doi.org/10.1103/physrev.177.2426>
- [19] Bilal, A. (2008) Lectures on Anomalies. arXiv: 0802.0634. <https://arxiv.org/pdf/0802.0634.pdf>
- [20] Bell, J.S. and Jackiw, R. (1969) A PCAC Puzzle: $\pi^0 \rightarrow \gamma\gamma$ in the σ -Model. *Il Nuovo Cimento A*, **60**, 47-61. <https://doi.org/10.1007/bf02823296>
- [21] Alvarez-Gaumé, L. and Witten, E. (1984) Gravitational Anomalies. *Nuclear Physics B*, **234**, 269-330. [https://doi.org/10.1016/0550-3213\(84\)90066-x](https://doi.org/10.1016/0550-3213(84)90066-x)
- [22] Peskin, M.E. and Schroeder, D.V. (1995) *An Introduction to Quantum Field Theory*. Addison-Wesley.
- [23] Hooft, G. (1980) Naturalness, Chiral Symmetry, and Spontaneous Chiral Symmetry Breaking. In: Hooft, G., *et al.*, Eds., *Recent Developments in Gauge Theories*, Springer, 135-157. https://doi.org/10.1007/978-1-4684-7571-5_9
- [24] Źenczykowski, P. (2008) The Harari-Shupe Preon Model and Nonrelativistic Quantum Phase Space. *Physics Letters B*, **660**, 567-572. <https://doi.org/10.1016/j.physletb.2008.01.045>

Appendices

Appendix A: Gauge Consistency Conditions in the $U(1) \times SU(3)$ Preon Framework

Gauge anomalies provide general and well-established consistency conditions for four-dimensional chiral gauge theories [2] [18] [19]. In the present work, these conditions are not imposed as phenomenological constraints a posteriori, but are instead understood as structural compatibility requirements for the assumed ultraviolet preon framework. It is essential that they are realized in a quantum-consistent manner. All anomaly considerations are therefore formulated at the level of elementary preons, independently of the details of their binding dynamics.

A.1 Preon Representations

The fermionic preons introduced in the main text transform under the internal symmetry group

$$U(1) \times SU(3)$$

according to

$$\psi_0 \sim (0, \mathbf{3}), \quad (6)$$

$$\psi_1 \sim \left(\frac{1}{3}, \mathbf{1}\right), \quad (7)$$

where the first entry denotes the abelian charge and the second the representation of $SU(3)$.

A.2 Gauge Consistency Conditions

Gauge anomalies arise from triangle diagrams involving chiral fermions and signal a quantum violation of classical gauge invariance. In four dimensions, only chiral fermions contribute; vector-like fermion pairs are anomaly-free. Quantum consistency of chiral gauge theories requires the cancellation of all gauge anomalies [18] [20]. For the symmetry structure considered here, this includes the non-abelian cubic anomaly $SU(3)^3$, the mixed anomaly $U(1) - SU(3)^2$, as well as the cubic and gravitational anomalies associated with the abelian factor.

The $SU(3)^3$ anomaly measures the net chirality of preons transforming under the color group. Since the fundamental representation of $SU(3)$ is complex, consistency requires that the total contributions from $\mathbf{3}$ and $\bar{\mathbf{3}}$ representations balance [2] [21]. This observation motivates the inclusion of conjugate color representations in the ultraviolet spectrum.

The mixed anomaly $U(1) - SU(3)^2$ measures whether non-abelian gauge configurations carry abelian charge. It is proportional to the sum of abelian charges weighted by Dynkin indices of the corresponding $SU(3)$ representations [2] [19]. The mixed $U(1) - SU(3)^2$ anomaly is proportional to

$$\mathcal{A}_{U(1)-SU(3)^2} = \sum_i q_i T(R_i),$$

where q_i denotes the $U(1)$ charge and $T(R)$ the Dynkin index [2] [19]. In

the present framework, preons carrying $SU(3)$ color are neutral under the abelian factor, while $U(1)$ -charged preons are $SU(3)$ singlets. As a result, this mixed anomaly vanishes identically, independently of further assumptions.

Finally, the cubic and gravitational abelian anomalies constrain the overall distribution of $U(1)$ charges. Together, these conditions restrict the admissible chiral fermion content and play a central role in the construction of consistent preon models and their composite realizations. The cubic and gravitational abelian anomalies,

$$\mathcal{A}_{U(1)^3} = \sum_i q_i^3, \quad (8)$$

$$\mathcal{A}_{U(1)\text{-grav}} = \sum_i q_i, \quad (9)$$

impose additional constraints on the allowed charge assignments [19] [22]. Their cancellation can be achieved by a minimal extension of the preon spectrum that preserves the overall economy of the construction.

A.3 Minimal Completion

Taken together, the above considerations indicate that a simple and natural completion of the fermionic preon content suffices to satisfy all gauge consistency conditions. This completion does not rely on the introduction of additional confining gauge interactions and does not alter the qualitative features of the composite spectrum discussed in the main text. Rather, it ensures that the assumed internal symmetries can coexist consistently with the primacy of spacetime supersymmetry.

A.4 Remarks on 't Hooft Anomaly Matching

Once gauge anomalies are cancelled at the preon level, further constraints on the infrared description arise only from exact global symmetries. For such symmetries, the associated anomalies must be matched between the ultraviolet and infrared theories, as emphasized by 't Hooft [23]. In the present context, these matching conditions provide a useful consistency check on possible massless composite fermions, without requiring a detailed specification of the full infrared spectrum.

Appendix B: Preons in Non-Relativistic Quantum Phase Space

We present a different perspective to our choice of the groups $U(1)$ and $SU(3)$ [24]. The global symmetry group $U(1) \times SU(3)$ is derived using a non-relativistic quantum phase space model.

Symmetry between position and momentum implies that the combination $x^2 + p^2$ is a proper invariant in phase space with the invariance group $O(6)$. If one takes into account that the standard commutation relations stay invariant we get a subgroup of $O(6)$, namely the group $U(1) \times SU(3)$. The $U(1)$ factor corresponds to the transformations $x' = -p$, $p' = +x$. $SU(3)$ is a generalization of the rotation group $SO(3)$. The generator of $U(1)$ in phase space is

$$R^{(x,p)} = x^2 + p^2. \quad (10)$$