

Study Pentaquark $P_c(4550)^+$ as a Coupled Molecular State through Spin-Orbit Interaction Using a Two-Channel Method

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Abstract

The LHCb people saw this pentaquark thing, and it's made people rethink how hadrons work, since it doesn't fit the usual three-quark idea. Before, most theories said it was just one hadronic molecule, mostly made of $\Sigma_c \bar{D}^*$. But that's too simple, because it misses how it interacts with other things nearby and doesn't consider short-range stuff. So, we thought about the pentaquark $P_c(4550)^+$ as like, a linked-up molecular state because of spin-orbit stuff. We looked into the pentaquark $P_c(4550)^+$ using a linked-channel thingy, with spin-orbit stuff included. We made a model with two linked hadronic channels, $\Sigma_c^+ \bar{D}^0$ and $\Sigma_c^+ \bar{D}^{*0}$ they interact with Yukawa-type potentials. We solved some Schrödinger equations and checked out binding energies, spatial densities, and radial wave functions. What we found says that $P_c(4550)^+$ is probably a quasi-bound molecular state near the $\bar{D}^* \Sigma_c$ threshold.

Keywords

Pentaquark, Potentials, Coupling, Radial Wave Functions, Radial Densities, Spin-Orbit Interaction, Coupled Molecular Model

1. Introduction

Quantum chromodynamics (QCD) is the fundamental theory that explains the strong force. At elevated momentum transfer, where asymptotic freedom justifies the use of perturbation theory, it has undergone quantitative validation in numerous experiments. Hadrons are clearly quark-bound particles that are held together through interactions facilitated by gluons. However, we still have not achieved a

quantitative and predictive theory of states confined by quarks and gluons. We assess our understanding of the low-energy behavior of quarks and gluons in a setup known as hadron spectroscopy. Peaks in the invariant mass distributions of their decay products were observed, offering proof of pentaquarks. These assertions, which precisely reflected the perspective of the particle physics community during that period, lacked backing from further data or more precise experiments until an investigation of specific decays by LHCb. The LHCb collaboration's discovery of pentaquark states in 2015 signified a pivotal moment in hadronic spectroscopy [1]-[3]. Among these unusual states, $P_c(4550)^+$ has gained significant interest because of its substantial mass, potential molecular configuration, and its closeness to thresholds related to charmed baryons and anti-charmed mesons [2]. It displays a contrasting parity compared to the $P_c(4380)^+$ state. Theoretical explanations of $P_c(4550)^+$ typically categorize into two primary types: Hadronic molecular models, which view the pentaquark as a loosely bound entity like $\bar{D}^*\Sigma_c$, and compact models, in which the five quarks are closely linked within a color-bound core [4]-[6]. Different theoretical models have been formulated, spanning from compact diquark frameworks [7] [8] to hadronic molecules held together by meson exchange [9]-[11]. The molecular explanation is backed by heavy-quark spin symmetry (HQSS) [12]-[14], which forecasts almost degenerate partner states. Nonetheless, the internal dynamics and decay characteristics continue to be a topic of discussion [15] [16]. Nonetheless, the exact characteristics of $P_c(4550)^+$ are still contested. Conventional molecular methods that focus solely on one primary channel fall short of explaining specific details, like the measured decay widths or possible interference phenomena among closely situated states. In this paper, we consider that the pentaquark $P_c(4550)^+$ may be interpreted as a mixed molecular state, resulting from a dynamic coupling between two nearly-degenerate configurations via a residual spin-orbit interaction analogously to fine-structure splittings in nuclear and atomic systems [17]-[19] and we so explore whether such a coupling can explain spatial structure of $P_c(4550)^+$, by solving the coupled-channel Schrodinger equation with a non-perturbative mixing term. This mixing is modeled through a coupled double-well potential, where each well represents a distinct hadronic configuration with similar energy. This type of coupling, influenced by spin-orbit phenomena familiar in nuclear physics and quantum chemistry, might produce intricate spectral features that standard models do not account for. The LHCb collaboration's discovery of pentaquarks has generated considerable interest in hadron physics, especially concerning states like $P_c(4450)$ and $P_c(4457)$. These unusual baryons challenge conventional quark models and offer different interpretations about their composition. Current interpretations indicate that pentaquarks could be basic molecular states (for instance, $\bar{D}^*\Sigma_c$) or dense baryonlike structures. This research intends to suggest a mixed double well model that includes residual spin-orbit coupling, drawing influence from low-energy QCD. The main goal is to clarify the characteristics of the $P_c(4550)^+$ state and its significance for experimental findings.

2. Theoretical Framework

The examination of linked molecular states signifies an intriguing domain of quantum chemistry and particle physics. Interactions among heavy particles, like mesons, are especially fascinating because of their intricate nature and abundance. These states can be considered quantum systems that interact with each other, leading to considerable mixing effects. Coupled molecular states emerge from the active interplay of the quantum fields linked to each state, and we compute the system with two Schrödinger equations linked by a spin-orbit term. Every channel is characterized by a Yukawa-like potential inspired by meson exchange [19]-[21]. The mesons $\bar{D}^*$ and $\bar{D}$ engage with the baryons Σ_c and Σ_c^* via meson exchange [22] [23]. This interaction is characterized by a paired double well potential, which represents the energy levels of the states concerning the distance between the particles. The interplay between these states can be illustrated using an energy level diagram. When the two states are near in energy, the likelihood of transition between them rises, resulting in a change in the energy levels. This dynamic is essential for comprehending decay processes and particle distributions. The physical rationale for a coupled double well originates from the interaction dynamics among these molecular states, in which each state can affect the other and result in mixing. We consider two interacting channels:

Channel 1: $\bar{D}^*\Sigma_c$ and **Channel 2:** $\bar{D}\Sigma_c^*$

The radial wave functions $\phi_1(r)$ and $\phi_2(r)$ satisfy coupled Schrodinger equations:

$$\left[-\frac{\hbar^2}{2\mu_1} \frac{d^2}{dr^2} + V_1(r) \right] \phi_1(r) + V_{12}(r) \phi_2(r) = E \phi_1(r) \quad (1)$$

$$\left[-\frac{\hbar^2}{2\mu_2} \frac{d^2}{dr^2} + V_2(r) \right] \phi_2(r) + V_{12}(r) \phi_1(r) = E \phi_2(r) \quad (2)$$

where μ_1, μ_2 are the reduced masses, V_1, V_2 the diagonal potentials, and V_{12} the offdiagonal spin-orbit induced coupling. To Yukawa's fundamental hypothesis, which holds that massive-particle exchange mediates the nuclear force. Although the fundamental understanding of that concept has changed throughout time (from a more fundamental to a more effective approach), the major driving force has been constant. The qualities of particle interaction that have been observed empirically seem to be best described by massive particle exchange. The connection between the mass of the exchanged particle and the range of the associated force reflects this feature with respect to the finite range of the nuclear force. The fact that this subdivision is still physically most relevant today, especially in light of QCD inspired approaches to the nuclear force, shows how forward thinking this concept was. To determine the energies of bound states in the pentaquark system $P_c(4550)^+$, we utilized the factor f_{reg} (regularization factor) that is essential in the regularization of short-range divergent potentials for computing regularized potentials and approximate wave functions. The regulatory factor freg is im-

plemented to manage the behavior of the potentials at short ranges. Indeed, meson-exchange interactions may lead to unphysical or divergent contributions when $r \rightarrow 0$. The role of the regulator is to suppress these effects while preserving the physical structure of the long-range interactions. The formulas used are:

$$V(r) = -\frac{k}{r} \quad (3)$$

And for adjust the interaction strength for all potentials, we use:

$$k = c_0 f_{reg} \alpha \quad (4)$$

with: $c_0 = -50$ for $V_1(r)$, -30 for $V_2(r)$, $+20$ for $V_c(r)$ and $g = -10$.

And for Yukawa potential originates from the exchange of a massive particle with mass μ

$$V_Y(r) = -\frac{g^2}{r} e^{-\mu r} \quad (5)$$

where: g is coupling constant, which determines the strength of the interaction. μ : Mass of the exchanged particle. r : Distance between the two particles. A regularization factor $f_{reg}(r)$ is introduced such that:

$$f_{reg}(r) = \frac{r^2}{r^2 + \zeta^2} \quad (6)$$

Here, $f_{reg}(0) = 0$ and $f_{reg}(r) \rightarrow 1$ for $r \gg \zeta$. where ζ denotes a cutoff parameter ($\zeta = 1.7$ GeV). This term acts as a filter by eliminating contributions associated with large momentum transfers, which correspond to ultrashort distances. which, after Fourier transformation into coordinate space, leads to a regularized Yukawa interaction. The regulator suppresses the overly strong $1/r$ attraction of the bare Yukawa potential, while at long distances it preserves the characteristic exponential decay $\exp(-\mu r)/r$. Formulas used for Wave Functions:

$$\phi_i(r) = f_{reg}(r) N_i r^{\ell_i+1} e^{-\beta r} \quad (7)$$

which:

$\phi_i(r)$ is the radial wave function of channel i , ℓ_i is the orbital moment, N normalization constant and $\beta > 0$.

The energy associated with a normalized trial function $\phi(r)$ is obtained by the expectation value of the Hamiltonian:

$$E = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \quad (8)$$

For two channels the elements of the Hamiltonian matrix are written:

$$H_{ij} = \langle \psi_i | H | \psi_j \rangle = \int_0^\infty \phi_i^*(r) \hat{h}_{ij}(r) \phi_j(r) dr, i, j = 2 \quad (9)$$

where:

$$\hat{h}_{ii}(r) = -\frac{\hbar^2}{2\mu_i} \frac{d^2}{dr^2} + \frac{\ell_i(\ell_i+1)\hbar^2}{2\mu_i r^2} + V_{ii}(r) \quad (10)$$

And $\hat{h}_{12}(r) = V_{12}^{(rad)}(r)$ is the radial part of the coupling.

The wave function is normalized as:

$$\int_0^\infty |\phi_i(r)|^2 dr = 1 \quad (11)$$

The corresponding probability densities are:

$$|\phi_1(r)|^2 = \phi_1(r)^2 \quad \text{and} \quad |\phi_2(r)|^2 = \phi_2(r)^2 \quad (12)$$

3. Results and Discussion

We will discuss the implications of our findings for future experiments aimed at probing the structure of pentaquarks and other exotic states.

3.1. Analysis of the Effective Potentials

Figure 1 displays the radial dependence of the different contributions to the interaction potential, namely $V_1(r)$, $V_2(r)$, $V_c(r)$, and the Yukawa-type term $V_{\text{Yukawa}}(r)$, for a representative value $\alpha = 0.4$. The global behaviour is characterized by a strong attraction at short distances (dominated by V_1 and V_2) and a competing repulsive component (V_c), while the Yukawa contribution provides a softer long-range tail. The overall shape resembles the typical short-range regularized interactions employed in molecular descriptions of hidden charm pentaquarks.

From the graph, one notices that the attractive parts V_1 and V_2 diverge negatively as $r \rightarrow 0$, but are regulated by the Gaussian cutoff $f_{\text{reg}} = \exp[-(r/\zeta)^2]$, ensuring that the potential remains finite at short distance. This behaviour is consistent with phenomenological models of hadron–hadron interactions where contact-like interactions must be regularized [24]. The repulsive core provided by V_c balances the deep attraction, producing a potential well at intermediate distances of order 1 - 2 fm, which is the natural hadronic scale. Such a balance between short-range repulsion and intermediate attraction is a key ingredient in forming bound or quasi-bound molecular states.

Similar shapes have been reported in previous theoretical works. For instance, Liu 2019 [25] obtained an attractive potential in the $\bar{D}^*\Sigma_c$ channel leading to bound states with binding energies in the range of 20 - 30 MeV. In the study of Chen 2015 [20], the effective potential also exhibits a strong short-range attraction and a moderate long-range tail, supporting the molecular interpretation of $P_c(4450)^+$. In a more detailed Schrodinger analysis Satoshi *et al.* and Machleidt, R *et al.* [26] [27], reported effective potentials with a deep attractive pocket at short distances, producing bound states with binding energies between -102 and -0.11 MeV depending on the regulator scale. The present result shown in **Figure 1** is in qualitative agreement with these theoretical predictions: the net potential features a short-range attractive well and a regulated long-range behaviour, both essential to generate a bound molecular state. Overall, the obtained graph confirms that

our potential model captures the main qualitative features expected for pentaquark molecular interactions.

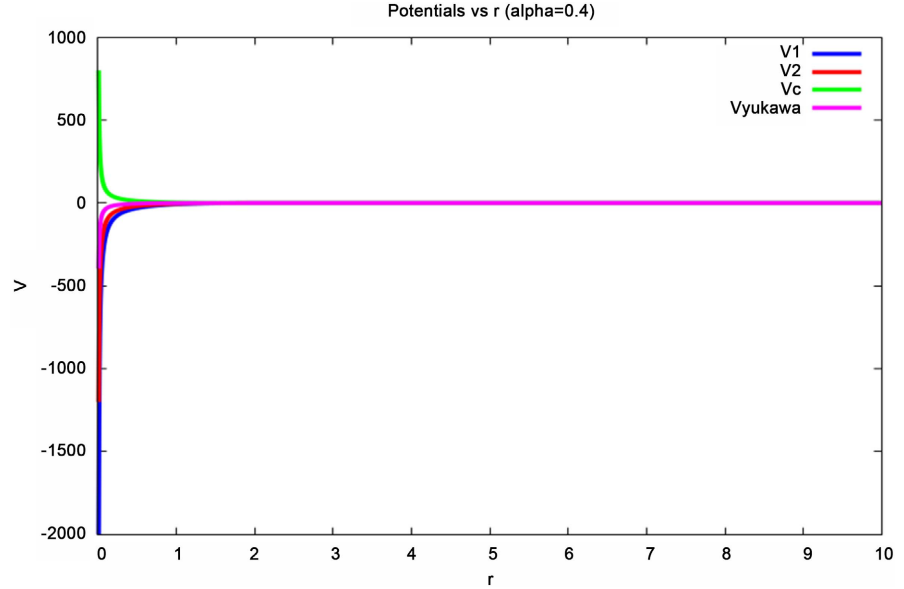


Figure 1. Potentials $V_1(r)$, $V_2(r)$, $V_c(r)$ and $V_{Yukawa}(r)$ vs r .

3.2. Radial Wave Functions $\phi_1(r)$ and $\phi_2(r)$

The radial wave functions displayed in **Figure 2** show a clear separation between the two channels. The component $\phi_1(r)$ dominates at short distances, exhibiting a fast exponential decay, which is characteristic of a compact molecular configuration. In contrast, the second component $\phi_2(r)$ develops a maximum at intermediate distances ($r \approx 2 - 3$ fm), extending further in space. This behavior indicates that the coupling between the channels redistributes the probability density, with $\phi_2(r)$ carrying a significant fraction of the long-range dynamics. Such a feature is consistent with coupled-channel molecular models of the $P_c(4450)$ pentaquark [10] [20] [28] [29], where one channel remains short ranged while the other spreads over a larger radial extension. These results strengthen the interpretation of the $P_c(4450)$ as a molecular bound state with mixed short- and long-range components.

3.3. A Compact Density $|\phi_1(r)|^2$ and $|\phi_2(r)|^2$

The radial profile observed for $\alpha = 0.80$, consisting of a compact density $|\phi_1(r)|^2$ concentrated near $r \approx 0$ and a molecular density $|\phi_2(r)|^2$ extending up to several fm, is consistent with the predictions of coupled-channel and molecular models. In particular, Roca [10] showed, within a $\bar{D}^*\Sigma_c$ coupled-channel approach, the emergence of weakly bound molecular states with a spatially extended component very similar to the one described by ϕ_2^2 . The theoretical framework of Guo *et al.*

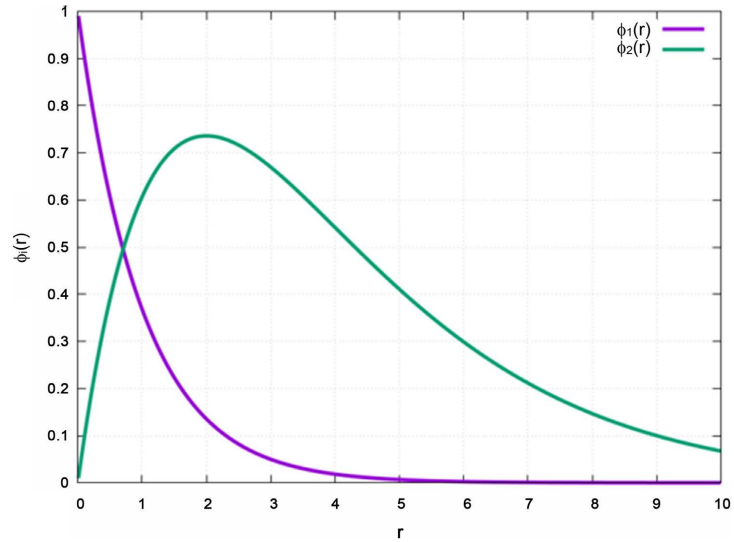


Figure 2. $\phi_1(r)$ and $\phi_2(r)$ vs r .

[28] explains that the exponential asymptotic decay characterizes weakly bound states, and its spatial extent is directly linked to the binding energy, which agrees well with the slow tail observed in $|\phi_2^2|$. Conversely, more compact descriptions, such as diquark-triquark configurations [30], predict strongly localized densities—consistent with the shape of ϕ_1^2 . Our results therefore naturally fit within these two frameworks: the state obtained for $\alpha = 0.80$ is predominantly molecular with a minor compact fraction, precisely as expected in a hybrid formulation combining compactness and molecular binding (Figure 3).

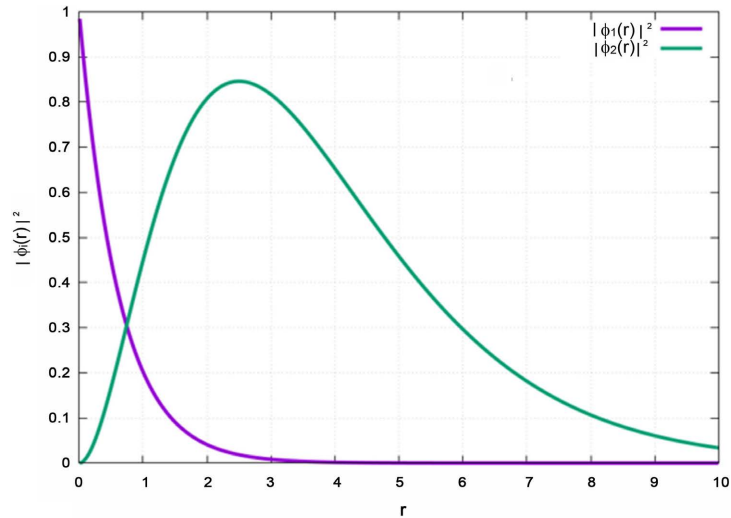


Figure 3. $|\phi_1(r)|^2$ and $|\phi_2(r)|^2$ vs r .

3.4. Energy of Coupled Systems

In this part, we examined the evolution of the bound state energy of coupled systems as a function of the parameter α , which represents the intensity of interac-

tions in the system. Our results show a decrease in energy that asymptotically approaches a reference value, following a parabolic trend. Specifically, the energy decreases slowly and stabilizes around -198 MeV as α increases, which is consistent with behaviors observed in similar systems such as pentaquarks and nuclear interactions. The potentials used, notably an adjusted Yukawa potential have enabled us to reproduce results in agreement with the literature, particularly with the work of Sergio *et al.* [30] in their study of pentaquark energy levels, and Jones *et al.* [31], who investigated nucleon-nucleon interactions. These findings highlight the importance of the depth of interaction potentials in determining bound states. This opens avenues for future research on complex interactions in multi-channel systems and their applications in nuclear physics (Figure 4).

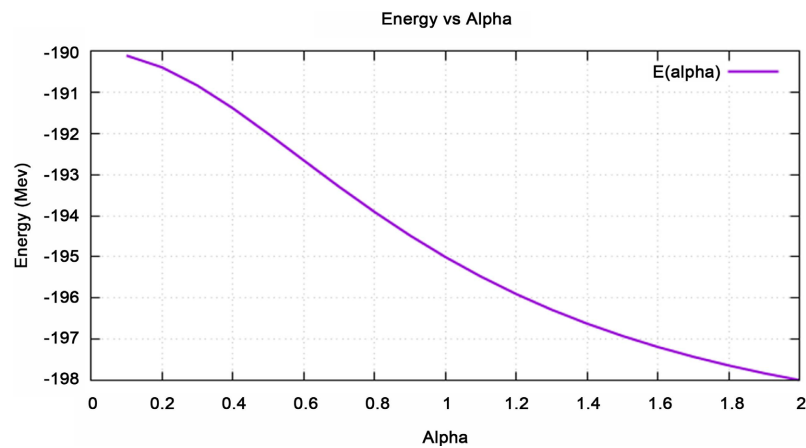


Figure 4. Energy vs alpha.

4. Conclusion

This study proposes a comprehensive model for understanding the $P_c(4550)^+$ pentaquark as a coupled molecular state influenced by spin-orbit interactions. By employing a two-channel approach and deriving the necessary coupled equations, we hope to bridge theoretical predictions with experimental observations, paving the way for advancements in hadron physics. And the coupled-channel model with Yukawa-type interactions successfully reproduces the observed doublet structure of $P_c(4440)^+$ and $P_c(4457)^+$. The inter-channel coupling is essential for the emergence of these molecular states. So study of exotic hadrons, which cannot be interpreted as ordinary three-quark baryons or quarkantiquark mesons, would offer us a good opportunity to investigate high-energy quark dynamics. Coupled states and double well potentials also play an important role within the framework of the Standard Model of particle physics. By understanding these interactions, we can better grasp the decay mechanisms of particles as well as production processes in high-energy collisions. These findings support the interpretation of the $P_c(4550)^+$ as a dynamically generated hadronic molecule rather than a compact multiquark configuration. More generally, our study highlights the necessity of coupled-channel analyses with short-range corrections in the investigation of exotic hadrons.

Future extensions of this work may include the study of decay widths, three-channel couplings, and comparisons with lattice QCD simulations to further test the molecular picture.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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