Journal of Financial Risk Management, 2026, 15(4), 484-494
https://www.scirp.org/journal/jfrm
ISSN Online: 2167-9541
ISSN Print: 2167-9533

# Systemic Modeling of Supply and Demand Functions in Light Industry: A Lagrangian Extension via the CS-LR Model

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**How to cite this paper:** Gueye, M. S. (2026). Systemic Modeling of Supply and Demand Functions in Light Industry: A Lagrangian Extension via the CS-LR Model. *Journal of Financial Risk Management, 15,* 484-494.
https://doi.org/10.4236/jfrm.2026.154026

**Received:** September 16, 2026
**Accepted:** November 8, 2026
**Published:** November 11, 2026

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## Abstract

In this paper, we challenge the boundaries of standard microeconomics to analyze the supply function, demand function, and short-run profit maximization in West African light industries (mechanics, sheet metal, and electrical workshops) (Gueye, 2026a). Classical models assume perfect instrumental rationality and a transparent market, while we introduce managerial and consumer complexity by means of the CS-LR model (Confidence, Satisfaction, Loyalty, Risks) (Gueye, 2023; Gueye et al., 2020a). This paper proposes a theoretical framework where market equilibrium is modeled as an emergent property, structurally dependent on discrepancies between managers' declarative data and the psychosocial variables observed on the shop floor, by formalizing the producer's and consumer's optimization frameworks in a Lagrangian extension. The double-loop feedback mechanism points to the didactic and linguistic mediation based on the semantic plasticity of the national languages as a means of reducing informational uncertainty (in economic terms, the entropy of the system) and organizational risks. This model is characterized by the redefinition of the firm's shutdown point from a psychosocial perspective and the formalization of the relational resonance proposition. This purely conceptual framework aims at posing formal conjectures on relational resilience, constituting a theoretical basis waiting for subsequent econometric validations.

## Keywords

Complex Systems, CS-LR Model, Supply Function, Lagrange Multiplier, Light Industry, Professional Didactics, West Africa

## 1. Introduction

In today's world of globalization and fragmented value chains, emerging econo-

mies—particularly in West Africa—confront the critical challenge of modernizing their industrial base. In this ecosystem, light industry (assembly units, precision mechanics, technical maintenance, electricity, sheet metal fabrication) (Gueye, 2026a) can play an important strategic role in terms of job creation and local human capital development. The regional sector data indicate that mechanical repair crafts and light industries account for nearly 15 per cent of non-agricultural GDP in West Africa and employ a large share of urban informal workers (Chandra et al., 2012; ILO, 2024). However, the management of these organizations is marked by a systemic complexity that traditional economic tools find difficult to model exhaustively.

Classical microeconomics presents a mechanical and instrumental definition of the firm, assuming that the supply function and short-term profit maximization derive from the equality between the market price and the marginal cost (P = Cm). In this paradigm, the firm is a black box in a transparent environment. Agents have perfect rationality and faithfully carry out directives. But the empirical reality of workshops in West Africa is more complex than this simple model. In terms of cognitive psychology, it seems that individuals act rather intelligently. To do this, they use highly operational and pragmatic techniques in their workplaces to cope with all the current surrounding contingencies (Vergnaud, 1996). The real operation of light industrial structures reveals permanent frictions, strong social embeddedness and a structural gap between what governance plans (the declarative) and what actors concretely achieve on the ground (the observation). From the point of view of epistemological science, it then appears that a decoupling resonates with the founding principles of professional didactics. These principles take the form of an inherent divergence between scheduled and ordered tasks and actual work activity in the workshops (Pastré, 2011).

To overcome these epistemological boundaries, this article relies on complexity sciences, building on the framework developed at the École Polytechnique de Thiès. The CS-LR model (Gueye, 2026b) provides a rigorous interdisciplinary reading grid that allows the endogenization of four critical variables that are too often exogenized or relegated to qualitative status by neoclassical theory: Confidence (C), Satisfaction (S), Loyalty (L), and the dynamic management of internal and external Risks (R).

Stress that this mathematical modeling is strictly within the domain of pure economic theory: it seeks to examine the internal logical consistency of our axioms and to offer formalized theoretical conjectures, rather than to derive immediate empirical laws without prior econometric validation. We develop a double-loop mechanism using an optimization framework based on Lagrangian calculus to incorporate the feedback between short-term accounting performance and long-term human resilience of the system.

## 2. The Supply Function and Profit Maximization: Contractual Extensions and Mechanistic Foundations

### 2.1. The "Black Box" Firm and Instrumental Rationality

Traditionally, the modeling of industrial supply has been based on the neoclassical

paradigm of the firm, conceived as a rational automaton with the sole aim of maximizing short-term profits. Within this simple theoretical framework, the supply function is not the result of a complex human or organizational dynamic, but a simple mechanical and quantitative response of an economic agent to price signals emitted by the market.

In standard microeconomic analysis, the firm is treated as a "black box" (Demsetz, 1983). The model assumes that the information is completely transparent, where the entrepreneur is a price taker working under the framework of Perfect Competition. The market atomicity implies that the light industry manufacturer has an infinitely elastic demand curve for a given market price ($P$).

The classical optimization function maximizes the short-run economic profit ($\Pi$), unlike the standard remuneration of fixed capital:

$$\max_q \Pi(q) = Pq - CT(q)$$

where $q$ is the volume of output and $CT(q)$ is the total cost. The firm chooses the optimal quantity to supply by comparing the marginal revenue of producing one more unit (which is the selling price $P$) with the marginal cost, $Cm$. The first-order condition (FOC) has strict equality:

$$\frac{\partial \Pi}{\partial q} = 0 \Rightarrow P = Cm(q), \text{ avec } Cm'(q) > 0$$

The common theory is that any variance from this mathematical balance results in a loss of earnings. Thus, the short-run supply curve coincides with the increasing portion of the marginal cost curve.

Standard theory breaks the viability of the supplier through two precise mathematical milestones, determined by the intersection of marginal cost with average costs:

- **The Break-Even Point:** It corresponds to the lowest Average Cost (CM). At the tangency point ($P$ = min(CM)), there is no economic profit, but the factors of production are paid their marginal productivity. It is the entry and/or maintenance on a sustainable basis in the market.
- **Shutdown Point:** This is the point at which the minimum Average Variable Cost (AVC) is reached. In the short term, the firm faces incompressible fixed costs (CF). When the market price is lower than average cost but higher than the unit variable cost $P >$ min(CVM) standard rationality says it is preferable to continue producing. The revenue covers all variable costs (raw materials, direct labor) and cushions part of fixed costs, minimizing total loss.

Only if the price falls below the shutdown point $P <$ min(CVM), is it mathematically necessary to cease production immediately. Then, the firm shuts down all operations and only bears its latent fixed costs.

## 2.2. Other Theories of the Firm: Transaction Costs and Information Asymmetries

The New Institutional Economics has proposed an important extension of the an-

alytical framework of the firm in an effort to overcome the reductionism of the standard neoclassical model. Extended theories, mainly driven by transaction cost theory (Coase, 1937; Williamson, 1985) and agency theory (Jensen & Meckling, 1976), open the "black box" of the firm by removing the assumption of perfect market transparency. They introduce key concepts that are central to understanding the emergence of the CS-LR model, that is, opportunistic behavior, informational asymmetry, and the risk of contractual breach.

Williamson adds two realistic behavioral assumptions to this view: bounded rationality (agents have limited cognitive capacities in the face of environmental complexity) and opportunism (some agents may act strategically by hiding or distorting reality). In the light industry sector, these notions directly shed light on the gap between the declarative and the observed. The declarative (formalized activity reports) often hides operational frictions that can only be revealed by direct observation of the work process.

Agency theory (Jensen & Meckling, 1976) formalizes the link between a principal (the order-giver, the workshop owner) and an agent (the qualified artisan, the apprentice). In the presence of information asymmetries, this relation creates a risk of ex-ante adverse selection and ex-post moral hazard. The principal incurs agency costs (monitoring costs, bonding costs and residual loss) to align the agent's effort with the goal. Then, the standard theory cost function ($CT_{std}$) is discounted by a contractual friction variable $C_{\text{agency}}$.

$$CT(q) = CT_{std}(q) + C_{\text{agency}}(R_{\text{interne}})$$

where $R_{\text{interne}}$ is the internal Risk of opportunism or failure of behavior. More suspicion in the climate increases Cagency. This increases marginal cost and reduces the supply of the firm. Extended theories, while incorporating the concept of Risk ($R$), are based on a defensive view of human relations, not being able to model how positive variables such as Confidence ($C$), Satisfaction ($S$), or Loyalty ($L$) can endogenize costs spontaneously and neutralize them.

## 3. The Non-Standard Paradigm and the CS-LR Model: Formalization of Relational Capital

### 3.1. Non-Standard Foundations: From Situated Rationality to Managerial Capitalism

Extended theories analyze the organizational "black box" by means of control and incentives. The non-standard paradigm, drawing on economic sociology and managerial theories of the firm, fundamentally disputes the universality of mechanical profit maximization. Weber (1922) in his foundational economic sociology analyzes, demonstrated that individual utilitarianism is geographically and culturally situated. By pointing to the embeddedness of economic action in ethical and cultural value structures, Weber reminds us that producer motives are responsive to dynamics of legitimacy and community recognition.

Galbraith (1967) theorizes the historical divorce between capital ownership and the effective management of firms at the scale of large organizations. The firm's mo-

tives change when the locus of decision-making shifts from shareholders to the technostructure (managers, engineers, technicians). The objective is no longer to maximize profit, but rather to satisfy a survival constraint. The real objective of the technostructure is to make the system more sustainable, to reduce risk, and to expand the organization.

In this sense, Chandler (1990) confirms the rise of managerial capitalism where professional managers substitute the classic entrepreneurs. These actors rule light industry not by instantaneous marginal trade-offs, but by long-term strategic investments that attempt to consolidate the dynamic and relational capabilities of the organization.

### 3.2. The CS-LR Model for Reducing Informational Uncertainty

The CS-LR (Gueye et al., 2026a) model formalizes and quantifies this enlarged managerial rationality. It assumes that the supply performance of a light industry (SME) is not determined only by the material inputs but also by the state of a complex system controlled by four interrelated dimensions: Confidence ($C$) (Gueye, 2022), Satisfaction ($S$) (Gueye et al., 2020b; Gueye et al., 2020c), Loyalty ($L$), and Risk ($R$).

- Confidence ($C$): The level of pre-existing alignment and shared belief between management, technical workshop people, and clients.
- Satisfaction ($S$): The degree of correspondence between the effort exerted by agents and the rewards perceived (social climate, work environment).
- Loyalty ($L$): The degree of long-term commitment of actors, acting as a reducer of staff turnover and as an obstacle to skills flight.
- Risk ($R$): It is the sum of internal vulnerabilities (handling errors, absenteeism, breakdowns) and external vulnerabilities (market fluctuations, logistical disruptions).

The informational entropy ($E_s$) is given by: within the framework of economic organizations, we define entropy in a strict sense as the measure of dispersion, degradation, and asymmetry of the useful information inside the firm, quantifying the amplification of internal friction costs and the loss of productive coordination mathematically.

Mathematically, it is described as the inverse function of the observed psychosocial index $\Psi_{obs}$. An increase in entropy ($E_s \propto 1/\Psi_{obs}$) mimics the geometric amplification of internal friction costs and agency costs. The efficiency of the system is given by a state function $\Psi(C, S, L, R)$.

The originality of the model is the confrontation between the internal perception of management, the declarative $\Psi_{decl}$ and real indicators measured on production lines or in workshops, the observation variable $\Psi_{obs}$. If management overestimates Confidence or Satisfaction ($\Psi_{decl} > \Psi_{obs}$), the system goes into a high entropy (informational dispersion) phase and Risk ($R$) grows exponentially. In contrast, the permanent adjustment in the field with a double-loop feedback mechanism allows the firm to reach an optimal steady state, in which the internal cohesion compensates for the information asymmetry.

## 4. Producer Behavior: Lagrangian Optimization and the Extended Supply Function

### 4.1. Specification of State Function and Supplier Constraints

To confirm the embedding of the CS-LR model in the microeconomic theory, we define the observed systemic efficiency function of the producer, denoted as $\Psi_{obs}^{P}$, as the ratio between the product of the cohesion variables and the level of internal or external risk:

$$\Psi_{obs}^{P}\left(C,S,L,R\right)=\frac{C_{obs}^{P}S_{obs}^{P}L_{obs}^{P}}{R_{obs}^{P}}$$

The maximum supply capacity of the firm is found in the difference between the efficiency level expected or declared by the management (the declarative, $\Psi_{decl}^{P}$) and the operational reality of the field (the observation, $\Psi_{obs}^{P}$).

The short-run production frontier constraint is given by:

$$q\le\overline{q}\left(\frac{\Psi_{obs}^{P}}{\Psi_{decl}^{P}}\right)-\theta^{P}$$

where $\overline{q}$ is the nominal engineering capacity of the workshop and θ is the parameter of didactic learning and linguistic mediation in the production space.

### 4.2. Solving the Producer's Problem Using the Lagrangian

The firm maximizes its operating profit Π, taking into account the usual technical variable cost (here, the quadratic function $v{\cdot}q^{2}$, where $v>0$), the fixed costs $CF$, and the direct monetary penalty due to observed risk ($\gamma R_{obs}^{P}$, where $\gamma>0$).

The Lagrangian ($\mathcal{L}^{P}$) is constructed as:

$$\mathcal{L}^{P}\left(q,\mu^{P}\right)=Pq-CF-vq^{2}-\gamma R_{obs}^{P}+\mu^{P}\left[\overline{q}\left(\frac{C_{obs}^{P}S_{obs}^{P}L_{obs}^{P}}{R_{obs}^{P}\Psi_{decl}^{P}}\right)-\theta^{P}-q\right]$$

where $\mu^{P}$ is the Lagrange multiplier. In mathematical economics, $\mu^{P}$ is interpreted as the "shadow price" of organizational harmony: it represents the marginal increase in theoretical profit the firm could achieve if it managed to reduce internal asymmetry by one unit.

The FOC requires the first-order partial derivatives to be zero:

$$\frac{\partial\mathcal{L}^{P}}{\partial q}=0\Rightarrow P-2v\cdot q-\mu^{P}=0\Rightarrow P=Cm_{std}\left(q\right)+\mu^{P}$$

$$\frac{\partial\mathcal{L}^{P}}{\partial\mu^{P}}=0\Rightarrow q^{*}=\overline{q}\left(\frac{C_{obs}^{P}\cdot S_{obs}^{P}\cdot L_{obs}^{P}}{R_{obs}^{P}\cdot\Psi_{decl}^{P}}\right)-\theta^{P}$$

Isolating $\mu^{P}$ from the first condition ($\mu^{P}=P-2vq$) and injecting the optimal theoretical production volume $q^{*}$ from the second condition, we obtain the analytical formulation of the CS-LR Supply Function:

$$P=2v\left[\overline{q}\left(\frac{C_{obs}^{P}S_{obs}^{P}L_{obs}^{P}}{R_{obs}^{P}\Psi_{decl}^{P}}\right)-\theta^{P}\right]+\mu^{P}$$

### 4.3. Dynamics of the Feedback Double Loop of the Producer

Unlike static approaches, the model has a dynamic double-loop architecture:

- **The Simple Loop (Price and volume adjustment):** Short-run changes in market price $P$ lead the firm to adjust the quantity it supplies by changing its use of its physical capacity ( $q^*$ ). The multiplier $\mu^P$ accounts for the instantaneous frictions between the declarative and the observation.
- **Double Loop (Adjustment by Learning and Didactic Restructuring):** In the long run, economic performance results in re-evaluation of psychosocial variables. In the event that the observed production is lower than the expected declarative data ( $q_{obs} < q_{decl}$ ), the agents trigger the pedagogical learning mechanisms, driven by the parameter $\theta^P$. This variable refers, from the viewpoint of professional didactics (Pastré et al., 2006), to the restructuring of action schemes (e.g., technical verbalization in national languages in the workshop). This didactic mediation decreases risk ( $R^P_{obs} \to \min$ ), closes the gap between the declarative and the observed, and permanently shifts down the marginal cost curve.

## 5. Consumer Behavior: Psychosocial Utility Maximization and Extended Demand Function

**Consumer CS-LR Utility Function Specification and Elasticity Analysis**

This is the case of light industry and local craftsmanship, with high asymmetries of the ex-ante and ex-post information. The utility of the demander results from a permanent trade-off between the materiality of the technical object and the quality of the psychosocial structure surrounding this object.

The observed real utility function, $U_{obs}$, is modeled as a modified Cobb-Douglas type function which can be expressed analytically as

$$U_{obs}(q,C,S,L,R) = q^{\alpha}\left[\frac{C_{obs}\cdot S_{obs}\cdot L_{obs}}{R_{obs}}\right]^{\beta}$$

where $q$ is the quantity or intrinsic technical quality level of the industrial service, and the block in brackets formalizes the consumer's semantic and relational efficiency index ( $\Psi^C_{obs}$ ).

The parameters $\alpha$ and $\beta$ are the consumer's partial utility elasticities:

- **Technical Elasticity Alpha ($\alpha$):** Responsiveness to Asset Materiality: The coefficient $\alpha$ measures the relative variation in consumer utility due to a marginal variation in the physical quality or quantity of the service rendered ($q$):

$$\alpha = \frac{\partial U_{obs}}{\partial q}\frac{q}{U_{obs}}$$

We assume that $0 < \alpha < 1$. In sheet metal or maintenance workshops, $\alpha$ is the client's functional requirement.

- **The Psychosocial Embeddedness Coefficient:** The Relational Elasticity The $\beta$ (Beta) Coefficient: The $\beta$ coefficient is a utility elasticity coefficient in relation to the CS-LR behavioral block. It measures the consumer's sensitivity to changes

in his/her perception of Confidence, Satisfaction, and Loyalty, weighted by the perceived Risk:

$$\beta = \frac{\partial U_{obs}}{\partial \Psi^{C}_{obs}} \cdot \frac{\Psi^{C}_{obs}}{U_{obs}}$$

We take $\beta > 0$. This parameter is the theoretical measure of social embeddedness. Perceived risk ( $R_{obs}$ ) in the denominator is a utility reducer.

## 6. General Market Equilibrium: Convergence and Relational Homeostasis

### 6.1. Analytical Condition for the Equalization of General Market Equilibrium

Homeostatic Adaptive Equilibrium Definition: This notion refers to a dynamic steady state where the market stabilizes not by a mechanical ex-post adjustment with prices only but through non-market feedback flows (learning, linguistic adjustment) that stabilize transaction volumes under exogenous cost shocks.

The conditions of this equilibrium are formalized by equating the short-run Lagrangian supply function of the producer with the demand function of the consumer. At equilibrium $\left(P^{*}, q^{*}\right)$, the coupled system of equations is:

$$\mu^{P} + \theta^{P} - 2v\left[\overline{q} \cdot \left(\frac{C^{P}_{obs} \cdot S^{P}_{obs} \cdot L^{P}_{obs}}{R^{P}_{obs} \cdot \Psi^{P}_{decl}}\right) - \theta^{P}\right] = \frac{\alpha}{\lambda} \cdot \left(q^{*}\right)^{\alpha-1} \cdot \left(\frac{C^{C}_{obs} \cdot S^{C}_{obs} \cdot L^{C}_{obs}}{R^{C}_{obs}}\right)^{\beta}$$

The global steady-state equilibrium quantity $q^{*}$ is given by the following micro-systemic unification formula by inverting the exponent $(\alpha - 1)$:

$$q^{*} = \left(\frac{\lambda}{\alpha} \cdot \left[\Psi^{C}_{obs}\right]^{-\beta} \cdot \left[Cm_{std}\left(q^{*}\right) + \mu^{P}\right]\right)^{\frac{1}{\alpha-1}} \text{ or in extension:}$$

$$q^{*} = \left[\frac{\lambda}{\alpha} \cdot \left(\Psi^{C}_{obs}\right)^{-\beta} \cdot \left[Cm_{std}\left(q^{*}\right) + \mu^{P}\right]\right]^{\frac{1}{\alpha-1}}$$

$$Cm_{std}\left(q^{*}\right) = 2v\left[\overline{q} \cdot \left(\frac{\Psi^{P}_{obs}}{\Psi^{P}_{decl}}\right) - \theta^{P}\right]$$

### 6.2. Theoretical Proposition of Relational Resonance: Formal Axiomatic Demonstration

To meet editorial constraints, we rephrase the physical metaphors in rigorous axiomatic terms, formalizing this behavioral property:

- Axiom 1: Let the consumer's psychosocial state be denoted by a vector: $\boldsymbol{V^{C}} = \left[C^{C}, S^{C}, L^{C}, 1/R^{C}\right]^{\mathrm{T}} \in \mathbb{R}^{4}_{+}$. The aggregated efficiency index is given by a continuous and differentiable relational production function $\Psi^{C}_{obs} = f\left(\boldsymbol{V^{C}}\right)$ such that $\forall i$, $\frac{\partial f}{\partial V^{C}_{i}} > 0$.
- Axiom 2: Assume the vector of productive cohesion of the producer is

$\boldsymbol{V^P} = \left[\left[C^P, S^P, L^P, 1/R^P\right]^{\mathrm{T}}\right]' \in \mathbb{R}_+^4$. The producer's efficiency index is denoted by $\Psi_{obs}^P = g\left(\boldsymbol{V^P}\right)$, where $\forall i$, $\frac{\partial g}{\partial V_i^P} > 0$.

- Axiom 3 (Interrelational Coupling): There is a psychosocial transaction matrix $\boldsymbol{M} \in \mathbb{R}^{4\times 4}$ with strictly positive coefficients ($m_{ij} > 0$) that represents the social embeddedness of the proximity market, and such that the state vectors mutually adjust at equilibrium according to the simultaneous equation system: $\boldsymbol{V^C} = \boldsymbol{MV^P}$ and $\boldsymbol{V^P} = \boldsymbol{M'V^C}$.

Formal Consistency Proof:

Due to the direct matrix coupling introduced by Axiom 3, the change in the efficiency index of the producer is transmitted to the consumer's vector. The consumer's index of efficiency can be differentiated relative to the producer's index by the chain rule as follows:

$$\frac{\partial \Psi_{obs}^C}{\partial \Psi_{obs}^P} = \sum_{i=1}^{4} \frac{\partial f\left(\boldsymbol{V^C}\right)}{\partial V_i^C} \cdot \frac{\partial V_i^C}{\partial \Psi_{obs}^P}$$

By Axiom 1, $\frac{\partial f}{\partial V_i^C} > 0$. Moreover, Axiom 3 is linearly coupled, which means that:

$$\frac{\partial V_i^C}{\partial \Psi_{obs}^P} = \sum_{j=1}^{4} m_{ij} \frac{\partial V_j^P}{\partial \Psi_{obs}^P}$$

Since $\Psi_{obs}^P = g\left(\boldsymbol{V^P}\right)$ and $\frac{\partial g}{\partial V_j^P} > 0$, the inverse function theorem ensures that $\frac{\partial V_j^P}{\partial \Psi_{obs}^P} > 0$. By hypothesis, all the coefficients $m_{ij}$ are strictly positive. Hence, $\frac{\partial V_i^C}{\partial \Psi_{obs}^P}$ is strictly positive. Mathematically, the strict monotonic increase of the coupling in the model is a consequence of the fact that each term in the sum is a product of two strictly positive terms: $\frac{\partial \Psi_{obs}^C}{\partial \Psi_{obs}^P} > 0$ and, by strict algebraic symmetry, $\frac{\partial \Psi_{obs}^P}{\partial \Psi_{obs}^C} > 0$ ■

### 6.3. Psychosocial Shutdown Point (PSP) & Client Rupture Threshold (CRT)

Shared Psychosocial Surplus Analysis (Graph Interpretation):

The market does not collapse in the face of an exogenous inflation shock because the Client Rupture Threshold (CRT) increases (the client accepts paying a relational premium in an effort to keep trust) and the Psychosocial Shutdown Point (PSP) decreases (the artisan accepts temporarily a reduced accounting margin in an effort to keep their collective and network of didactic transmission). This common excess creates an immaterial buffer zone which allows relational homeostasis to maintain transaction stability.

## 7. Conclusion

This article aimed to deconstruct the mechanistic approach of standard microeconomics to model short-run supply and demand functions within light industries. We were able to propose an extended Lagrangian framework with the developments we brought in the introduction of human and didactic dynamics using the CS-LR model.

Again, it should be repeated that this pure mathematical model is not a substitute for direct empirical proof, but shows the internal logical consistency of our assumptions, according to the epistemological requirements of economic science. Our model formalizes the hypothesis that relational capital generates a psychosocial negotiation space, a collective relational excess that can buffer the economic breaking point, and account for the theoretical possibility of such markets in the face of exogenous shocks.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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