

Design and Optimization of a Variable-Divergence Galilean Laser Optical System Using a Kerr-Based Nonlinear Aspherical Lens

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Abstract

This paper presents the design and optimization of a Galilean laser optical system incorporating a Kerr-based nonlinear aspherical lens (NAL) for variable beam divergence control. The system retains the classical axial-shift zoom principle while replacing the conventional precision-polished positive aspherical lens with a Kerr nonlinear element modeled as an equivalent aspherical surface. The Kerr effect introduces an intensity-dependent refractive index modulation that compensates for spherical aberrations, thereby improving wavefront quality. The optical design was developed and optimized using Zemax in multi-configuration mode. A compact afocal beam-expanding and collimating system is proposed for a 1064 nm laser source with an initial divergence of approximately 5 mrad and an input beam diameter of 1 mm. The system provides a continuously tunable expansion ratio from 4× to 20×, reducing the output divergence to as low as 0.25 mrad. Across all configurations, the optimized design achieves a root means square (RMS) wavefront aberration below $\lambda/14$, reaching the diffraction limit. Compared with conventional zoom systems, the proposed configuration offers comparable optical performance with reduced alignment sensitivity and improved manufacturability, demonstrating the practical potential of Kerr-based nonlinear aspherical lenses for variable-divergence laser applications.

Keywords

Nonlinear Aspherical Lens, Zemax Software, Kerr Medium, Optical System, Footprint

1. Introduction

The divergence angle of a laser beam is a fundamental optical parameter that plays a critical role in various modern laser-based systems, including laser ranging, laser cutting, and free-space optical communication [1]-[8]. The ability to flexibly control the divergence allows a single optical system to operate effectively under different working conditions [1]-[3] [9]. Conventional fixed-divergence optics, however, can only provide a single beam divergence state, thereby limiting operational adaptability [1]-[3] [10]. Consequently, the design of a compact, continuously tunable, and diffraction-limited variable-divergence system has become an important research direction in modern optical engineering. Among existing approaches, Galilean type afocal zoom systems have gained widespread use due to their compact structure, high transmission efficiency, and the absence of intermediate focal planes [1] [2]. By axially translating internal lens groups, a Galilean zoom system can continuously adjust the angular magnification while maintaining the afocal condition. Nevertheless, achieving high wavefront quality and practical manufacturability simultaneously remains challenging. Traditional zoom systems often require precision-fabricated aspherical lenses to correct spherical aberration, leading to high production costs and stringent assembly tolerances [2]. These limitations restrict the integration of Galilean zoom optics into compact electro-optical devices. To overcome these challenges, this study proposes a variable-divergence Galilean optical system incorporating a NAL as the first optical element [11] [12]. The nonlinear lens behaves as an equivalent aspherical element formed through an intensity-dependent refractive index distribution within the Kerr medium, thereby compensating spherical aberration and improving beam quality while reducing the number of physically aspherical surfaces. The optical configuration maintains the classical three group structure, in which the zoom and compensating groups translate axially according to a motion law derived from the afocal condition. The nonlinear lens simultaneously serves as both a focusing and aberration correcting element, enhancing output beam quality without increasing mechanical complexity. The optical system was designed and optimized using Zemax software in multi-configuration mode [1]-[4] [11] [13]. The results demonstrate that nonlinear optical materials can be effectively applied to complex optical systems to achieve high performance and manufacturable design.

2. Theoretical Background

2.1. Principle of the Variable Divergence Galilean Afocal System

The variable-divergence Galilean optical system is an afocal configuration capable of continuously adjusting the divergence angle of a laser beam through axial translation of its internal lens groups [1] [2] [14]. The classical Galilean architecture typically consists of three optical groups arranged in either a negative-negative-positive or positive-negative-positive configuration [1] [2] [15]. In this study, the latter configuration is adopted, consisting of three lens groups defined as follows:

L_1 —the front fixed positive group, L_2 —the movable negative zoom group, and L_3 —the positive compensating group. The effective focal lengths of the three groups are denoted as f'_1 , f'_2 , and f'_3 , respectively, while the initial axial separations between them are represented by d_{12} and d_{23} .

During operation, the system continuously maintains the afocal condition, meaning that both the incident and emergent beams remain collimated, while the angular magnification varies with the axial displacement of the internal lens groups. When the zoom group L_2 is translated by an axial distance Δd_2 , the location of its image plane shifts accordingly, which may cause the system to deviate from its afocal state [1] [2]. To restore this condition, the compensating group L_3 must be translated by a corresponding amount Δd_3 (as illustrated in **Figure 1(b)**), determined by the lateral magnification $m_2(\Delta d_2)$ of the zoom group [1] [2]:

$$m_2(\Delta d_2) = \frac{f'_2}{f'_1 + f'_2 - (d_{12} + \Delta d_2)} \quad (1)$$

$$\Delta d_3 = \frac{f'_1 f'_2 - (d_{12} + \Delta d_2) f'_2}{f'_1 + f'_2 - (d_{12} + \Delta d_2)} + f'_3 - d_{23} + \Delta d_2 \quad (2)$$

The beam divergence ratio M and the corresponding relationship of the divergence angles are defined as follows [1] [2]:

$$M = \frac{h_3}{h_1} = -\frac{f'_3}{f'_1} \cdot \frac{1}{m_2(\Delta d_2)} \quad (3)$$

$$\theta_{out} = \frac{\theta_{in}}{M} \quad (4)$$

where h_1 and h_3 represent the beam heights at the entrance and exit, respectively.

The set of Equations (1)-(4) describes the axial displacement laws of Δd_2 and Δd_3 , which ensure that the system remains in the afocal state throughout all zoom configurations.

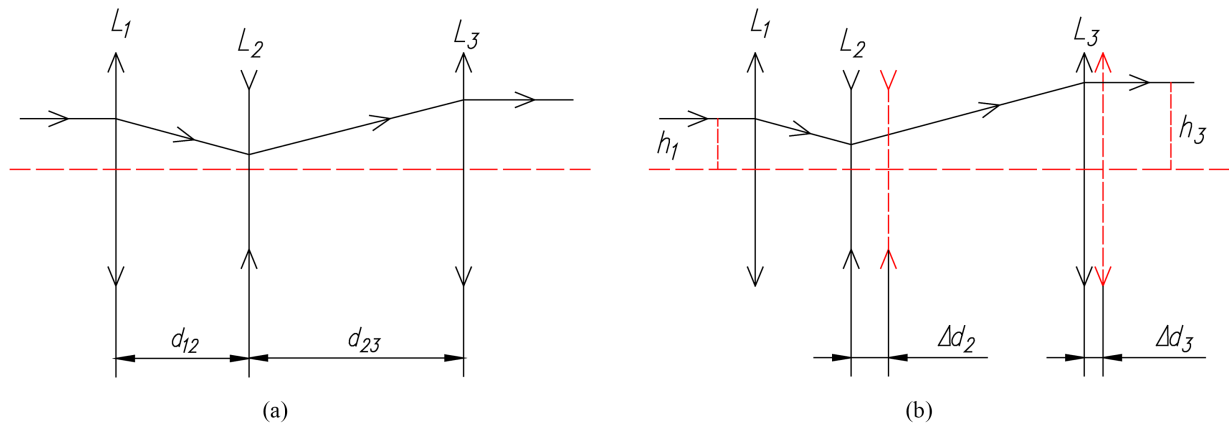


Figure 1. Principle layout of the afocal zoom beam expander. (a) System with fixed focus; (b) System with variable focus [1] [2].

This analytical model enables rapid determination of the axial positions of the lens groups for any desired magnification and serves as the theoretical foundation for optical optimization in Zemax software. To further enhance the optical performance and manufacturability of the system, the first lens group L_1 is replaced by a NAL.

2.2. Model of NAL

We consider a thin Kerr medium layer of thickness d , characterized by a linear refractive index n_0 and a nonlinear refractive index coefficient n_2 . When a Gaussian laser beam (TEM₀₀) with peak intensity I_0 and waist radius W_0 propagates through this medium, the refractive index distribution across the transverse (radial) coordinate can be expressed as follows [11]-[13] [16]-[20]:

$$n(\rho) = n_0 + n_2 I \exp\left(-\frac{2\rho^2}{W_0^2}\right) \quad (5)$$

As is well known, the radial refractive index distribution within the Kerr medium induces an optical path difference (OPD) that is equivalent to that produced by an aspherical surface. By expanding the OPD expression and comparing it with the sag function of a rotationally symmetric aspherical surface, the equivalent radius of curvature R at the vertex and the conic coefficient k of the NAL can be derived [11] [12].

$$\begin{cases} R = \frac{W_0^2}{4n_2 I_0 d_{phys}} \\ k = -1 - \frac{W_0^2}{4(n_2 I_0 d_{phys})^2} \end{cases} \quad (6)$$

where, λ denotes the operating wavelength. Accordingly, the NAL can be modeled in Zemax software as an aspherical surface, enabling accurate simulation of light propagation through the nonlinear medium. When the NAL replaces the first group L_1 , the system preserves the classical afocal operating mechanism described by Equations (1)-(4). It should be noted that the focal length f' of a conventional lens depends on the surface curvatures R_1, R_2 and the refractive index n , and is determined by the following relationship [21]-[25]:

$$\frac{1}{f'} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (7)$$

Therefore, for the NAL, which is equivalent to an aspherical lens consisting of one planar surface and one aspherical surface, the focal length of the NAL corresponding to the effective focal length f'_1 of the first group L_1 is determined as follows:

$$f'_1(I_0) = \frac{R}{n(I_0) - 1} \quad (8)$$

It is evident that when the laser intensity I_0 varies, the effective focal length

$f_1'(I_0)$ of the NAL changes accordingly, together with the radius of curvature R and the conic coefficient k . This nonlinear response modifies the equivalent aspherical profile of the NAL and enables compensation of spherical aberration.

As a result, the incorporation of the NAL improves wavefront quality and helps maintain stable optical performance over the entire zoom range. The combination of axial mechanical translation and nonlinear aberration compensation leads to a high-performance optical system suitable for practical implementation. This configuration demonstrates the feasibility of integrating nonlinear materials into the design of variable divergence laser optical systems, paving the way for the development of adaptive and industrially manufacturable optical architectures.

3. Design and Optimization

3.1. The System Configuration and Design Target

Based on the theoretical framework discussed above, a variable-divergence Galilean optical system integrating a NAL is proposed and designed according to the optical layout illustrated in **Figure 2**. Using Zemax software, the system parameters and output beam quality were optimized. The system adheres to the conventional axial-translation principle of the Galilean afocal configuration, as employed in previous designs, but replaces the first lens with a nonlinear element to enhance wavefront quality and reduce manufacturing complexity and cost.

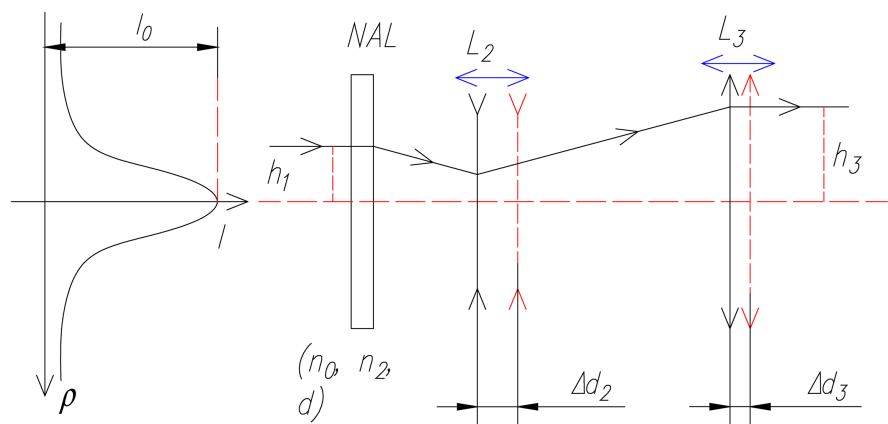


Figure 2. The sketch of afocal zoom beam expander with NAL.

The optical configuration consists of three lens groups: the NAL; L_2 , a movable negative group used to control the beam divergence; and L_3 , a positive compensating group that maintains the afocal condition across the entire zoom range. The designed optical system is required to satisfy the following design specifications [1] [2]:

- Input beam: Gaussian beam, 1 mm diameter, wavelength 1064 nm;
- Output beam: continuously variable diameter from 4 mm to 20 mm;
- Optical quality: wavefront aberration $\text{RMS} < \lambda / 14$, free from vignetting;
- Mechanical structure: total system length approximately 150 mm.

The initial system configuration was constructed based on a conventional variable Galilean design employing spherical lenses. The initial spacings d_{12} and d_{23} , as well as the displacement values Δd_2 and Δd_3 , were determined using Equations (1)-(4). The system was modeled in multi-configuration mode in Zemax software, where d_{12} and d_{23} were varied to control the relative positions of the lens groups during zooming $(\Delta d_2, \Delta d_3)$. The “*Afocal Image Space*” mode was activated to ensure that the output beam remained collimated along the optical axis.

The optical design was performed using sequential ray tracing in Zemax OpticStudio. The first lens L_1 was replaced with a NAL, modeled as an equivalent even aspherical surface with curvature radius R and conic coefficient k calculated from Equation (6). The nonlinear medium selected was a thin film of Oil Red O (ORO) [20] [26], characterized by a nonlinear refractive index coefficient $n_2 = 10^{-6} \text{ cm}^2/\text{W}$, a linear refractive index $n = 1.50$, a linear absorption coefficient $\beta \approx 10^{-4} \text{ W/cm}$ and a film thickness $d = 1 \text{ mm}$. The laser beam waist $W_0 = 0.5 \text{ mm}$ and peak intensity $I_0 = 10^5 \text{ W/cm}^2$ were also defined for the simulation. All initial design parameters are summarized in Table 1.

Table 1. Design parameters of initial proposal optical system.

Surf: Type	Comment	Radius (mm)	Thickness (mm)	Glass	Semi-Diameter (mm)	Conic
1 Standard	Input beam	Infinity	Infinity		Infinity	0.000
2 Standard	expander	Infinity	1.000 V	1.50, 0.0	0.5 U	0.000
3 Even Asphere		-0.625 V	48.188 V		0.5 U	-7.250 V
4 Standard		-4.051 V	8.256 V	K8	4 U	0.000
5 Even Asphere		13.835 V	44.426 V		1 U	560.913 V
6 Standard		-67.450 V	5.269 V	K8	1 U	0.000
7 Standard		-41.300 V	36.779 V		1 U	0.000
8 Standard		-262.666 V	9.373 V	K8	1 U	0.000
9 Standard		-57.018 V	10.000		1 U	0.000
10 Standard		Infinity	-		1 U	0.000

Table 2. Laser beam opening angle constraints.

Type	Surface	Wave	Hx	Hy	Px	Py	Target	Weight
Beam expansion constraints								
CONF	1							
REAY	10	1	0.000	0.000	0.000	1.000	2.000	1.000
CONF	2							
REAY	10	1	0.000	0.000	0.000	1.000	5.000	1.000
CONF	3							
REAY	10	1	0.000	0.000	0.000	1.000	8.000	1.000
CONF	4							
REAY	10	1	0.000	0.000	0.000	1.000	10.000	1.000

The optimization process was carried out in the multi-configuration mode of Zemax software, where different zoom states were simulated by varying the axial separations between lens groups. The main variables optimized include radii of curvature, conic coefficients, and inter-element spacings.

Four representative configurations were defined to cover the entire zoom range of the system, corresponding to output beam diameters of 2 mm, 5 mm, 8 mm, and 10 mm. The merit function was constructed using REAY operands to control the output beam size, together with constraints on wavefront aberration and afocality. The Physical Optics Propagation (POP) module was subsequently used to verify the wavefront quality. Each REAY term defines the vertical coordinate of the chief ray ($Py = 1.0$) for a given configuration, corresponding to the required output beam radius. Target values of 2 mm, 5 mm, 8 mm, and 10 mm were assigned respectively to configurations 1-4, ensuring that the system achieves the precise beam divergence across the full zoom range (see **Table 2**). In addition to the beam-divergence constraints, the merit function also included conditions for controlling the wavefront aberration ($RMS < \lambda/14$) and maintaining the afocal condition by enforcing the chief rays to exit the system parallel to the optical axis [9] [10] [23]. Further constraints were imposed on lens thicknesses and inter-lens spacings to ensure compliance with fabrication tolerances and practical requirements.

3.2. Results and Discussion

The optimization process was iterative and progressively convergent, performed through multiple refinement cycles to ensure that all system parameters satisfied the design requirements for beam divergence, wavefront aberration, and overall optical performance. The resulting variable divergence Galilean optical system integrating the NAL was successfully optimized. The final optical configuration after optimization is illustrated in **Figure 3**, and the principal optical parameters of the system including lens curvatures, thicknesses, and group separations are summarized in **Table 3**.

Table 3. Parameters of optical system after optimization.

Surf:	Type	Comment	Radius (mm)		Thickness (mm)		Glass	Semi-Diameter (mm)		Conic
1	Standard	Input beam	Infinity		Infinity			Infinity		0.000
2	Standard	expander	Infinity		1.000	V	1.50, 0.0	0.5	U	0.000
3	Even Asphere		-23.260	V	32.607	V		0.5	U	-2886.00 V
4	Standard		-109.695	V	10.166	V	K8	0.27	U	0.000
5	Even Asphere		2.417	V	48.232	V		0.231	U	83.231 V
6	Standard		-6.200	V	7.151	V	K8	3.550	U	0.000
7	Standard		-10.209	V	41.430	V		5.361	U	0.000
8	Standard		-210.909	V	3.001	V	K8	9.903	U	0.000
9	Standard		-38.654	V	10.000			10.067	U	0.000
10	Standard		Infinity		-			10.000	U	0.000

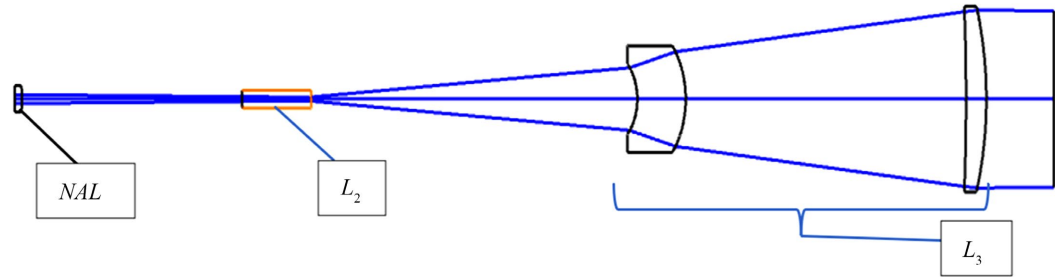


Figure 3. Optical system configuration after optimization.

From **Table 3**, it can be seen that the total optical length of the system is less than 150 mm, ensuring that the beam divergence varies with the relative axial displacement between L_2 and L_3 . The NAL functions as an aberration correcting element, effectively reducing the overall optical aberrations of the system. The optimization results indicate that the optical system consists of two aspherical lenses: the NAL and the movable negative lens L_2 . The parameters of both lenses are listed in **Table 4**. The axial separation d_{12} between the NAL and L_2 , and d_{23} between L_2 and L_3 , for four different configurations are summarized in **Table 5**.

The optimized system can vary the diameter of the output laser beam from 4 mm, 10 mm, 16 mm to 20 mm, corresponding to expansion ratios of 4 $\times$, 10 $\times$, 16 $\times$, and 20 $\times$, as illustrated in the footprint diagram (**Figure 4**). The RMS wavefront aberration for the four configurations is below $\lambda/14$, with values of 0.060λ , 0.070λ , 0.036λ , and 0.033λ , respectively. The peak-to-valley (PV) wavefront aberration is also below $\lambda/4$, with corresponding values of 0.245λ , 0.211λ , 0.126λ , and 0.242λ . These results demonstrate that the optical performance of the optimized system approaches the diffraction limit, confirming that the design meets the desired optical quality.

Table 4. Parameters of aspherical lenses.

	<i>Surface Type</i>	<i>Conic coefficient</i> k	<i>Coefficient</i> α_4	<i>Coefficient</i> α_6
3	Even Asphere	-2886		
5	Even Asphere	83.231	0.69	-21.498

Table 5. Thickness between neighbor moving components.

<i>Thickness</i>	<i>Config 1</i>	<i>Config 2</i>	<i>Config 3</i>	<i>Config 4</i>
d_{12}	50.085	40.521	34.493	32.607
d_{23}	30.556	40.467	46.343	48.232

The obtained results are in good agreement with those of previous systems [1] [2] [7] [15], which employed three aspherical lenses, and even show slightly better performance compared with systems that use two aspherical lenses combined with additional spherical surfaces [1] [2] [15].

Accordingly, the optimized optical system satisfies the design requirements in terms of both beam divergence and optical quality. This confirms that the incorporation of a nonlinear aspherical lens (NAL) into the variable-divergence Galilean system fully meets the criteria of a high-performance optical design. In this configuration, the NAL consists of one planar surface and one aspherical surface with a curvature radius of $R = -23.26$ mm (the negative sign indicates that the center of curvature lies to the left of the surface). For the optimized case, the aspherical surface of the NAL has an equivalent curvature radius of $R = 23.26$ mm and a conic coefficient of $k = -2886$. Based on these parameters, the corresponding input conditions of the laser beam and the nonlinear medium can be determined as follows: $W_0 \approx 0.5$ mm, $I_0 = 6.64$ W/cm², $n_2 \approx 10^{-6}$ cm²/W.

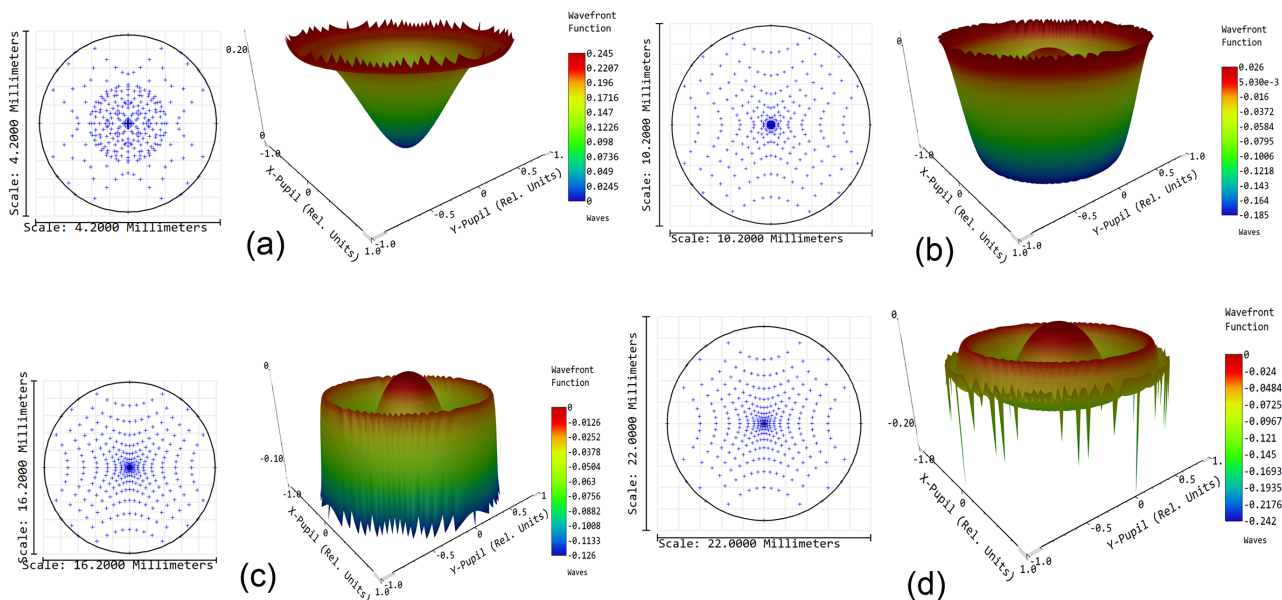


Figure 4. 2D—footprint diagrams (left) and 3D—wavefront aberration (right) at different magnification. (a) $M = 4$; (b) $M = 10$; (c) $M = 16$; (d) $M = 20$.

The obtained value of the nonlinear refractive index coefficient n_2 falls within the typical range of organic dye materials, such as ORO, which exhibits a nonlinear coefficient of the same order of magnitude and a comparable linear refractive index [8]. Therefore, the use of organic dye materials (in the form of dry thin films) as the NAL in the Galilean system is entirely feasible. The selected nonlinear material exhibits relatively low absorption ($\beta \approx 10^{-4}$ W/cm) at the operating wavelength and a fast response time on the order of picoseconds. In addition, the use of a continuous wave laser helps reduce thermal effects under moderate power levels [21] [27]. However, at high intensities, thermal accumulation and potential optical damage may occur, which should be considered negligible in practical implementations [26].

For comparison, a conventional Galilean beam expander reported in Ref. [9] exhibits a maximum RMS wavefront aberration of 0.1769λ . In contrast, the pro-

posed system achieves significantly lower RMS values, with a maximum of 0.070λ across all configurations, indicating a substantial improvement in wavefront quality. Furthermore, unlike conventional designs that rely on precision-fabricated aspherical lenses, the nonlinear aspherical lens employed in this work does not require complex surface machining, thereby avoiding fabrication-induced errors and offering improved manufacturability while maintaining high optical performance.

4. Conclusion

A Galilean zoom optical system employing a nonlinear aspherical lens (NAL) to replace conventional aspherical lenses has been proposed, designed, and optimized using Zemax software. The system consists of three classical lens groups, where the divergence angle of the laser beam is controlled through the coordinated axial translation of the zoom and compensating groups. By substituting the first lens with a NAL modeled as an equivalent aspherical surface, the proposed design effectively corrects spherical aberration while simplifying fabrication and improving flexibility for practical applications. Optimization using the POP model in Zemax software demonstrates that the system can continuously vary the output beam diameter from 4 mm to 20 mm while maintaining diffraction limited performance (wavefront aberration $\text{RMS} < \lambda/14$) for all configurations. The first nonlinear lens, modeled according to the thin Kerr-layer approximation, exhibits an equivalent curvature radius of $R = 23.26 \text{ mm}$ and a conic coefficient of $k = -2886$, corresponding to an effective nonlinear refractive index of $n_2 \approx 10^{-6} \text{ cm}^2/\text{W}$. This value aligns well with the optical characteristics of ORO, confirming the suitability of this material for realizing Kerr-based nonlinear aspherical lenses. These results not only verify the feasibility of integrating nonlinear materials into adaptive laser optical systems but also introduce a new design approach for compact, high-performance, and manufacturable beam-expanding optics. Such systems hold significant potential for applications in electro-optical detection, beam control, and precision alignment. In future work, the research team aims to fabricate the nonlinear element experimentally, evaluate the system performance under different power levels, and extend the study to Kerr materials with higher stability and broader spectral response.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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