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A Short Review on the Modern Teaching of Electrical Machines Using MATLAB and Finite Elements

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Abstract

This review examines the role of simulation-based virtual laboratories in the teaching of electrical machines and electromagnetics, with particular attention to MATLAB and finite element analysis tools. Three guiding questions frame the discussion: 1) What strategies can mitigate the inherent limitations of these tools? 2) What practical challenges are associated with their implementation? and 3) What historical practices demonstrate their pedagogical value? The findings indicate that the effectiveness of simulation depends largely on the design of tailored instructional materials, which help overcome the constraints of verisimilitude and numerical approximations. Implementation challenges remain significant, especially in terms of licensing costs and institutional readiness, but the availability of student editions of commercial software provides affordable access and prepares graduates for both industry employment and self-employment. Historical evidence confirms that simulation tools enhance students' understanding by bridging abstract theory with visual and quantitative experimentation. Together, these insights highlight simulation-based laboratories as powerful complements to physical experimentation and as essential components of modern engineering education.

Keywords

Electrical Machines, Electromagnetic Engineering Education, Finite Element Analysis, Simulation Software

1. Introduction

Electrical machines and transformers represent a highly active area of research, and there are significant challenges in teaching the subject. Despite the increasing demand for engineers with experience and/or expertise in this field to work in industry, courses on electric machines suffer from a lack of student interest and, in some cases, are being temporarily withdrawn from the "menu" of offered subjects [1]. According to Molina *et al.*, the key points to respond to this demand for trained engineers are: 1) the modernization of both lectures and supporting laboratories; and 2) the use of extensive laboratory-based systems because students must become acquainted with or, at least, familiarized with the equipment and CAD systems they will meet in future employment [1].

In universities all over the world, surveys and other scientific publications on engineering education have been extremely useful to faculty members in their choices related to curriculum changes and the incorporation of innovative educational methods. A comprehensive study on changes in engineering practice in the new century may be found in the three reports of the US National Academy of Engineering, released between 2004 and 2007. A summary of these reports showing the major results and implications on engineering education may be found in [2].

In Europe, two major projects focusing on the ways universities contribute to educating creative engineers, namely the "Da Vinci Project" and "ELLI Project," have the task of bringing engineering students into focus and fostering their creativity [3]. The review paper of G. D. Kuh [4] on the substance and evolution of the US National Survey of Student Engagement and the work of J. J. Duderstadt [5] about engineering practice, research, and education are important contributions to the evolution of engineering education.

Research Method

The rapid advancement of computational tools offers promising avenues for enhancing electrical machine education. In fact, the achievements of electromagnetic CAD systems have surpassed the expectations discussed in review papers of the 1990s, with a focus on the future prospects of electromagnetic CAD. However, the benefits of using simulations based on finite element analysis and MATLAB cannot be overstated. It is therefore essential to explore some critical questions regarding the actual effectiveness of these tools in electrical machines education.

Based on this reflection, the review aims to answer the following questions: 1) What strategies can be employed to mitigate the inherent limitations of these simulation tools? 2) What are the practical challenges associated with their implementation? and 3) Which historical practices demonstrate that tools like MATLAB and finite element analysis enhance students' understanding of complex electromagnetic phenomena? Addressing these questions is crucial, as it provides educators and curriculum developers with insights into the pedagogical value and practical viability of integrating these technologies into engineering programs. These questions, in turn, define the framework and organization of the present review.

2. Finite Element Simulations: Capabilities and Limitations

This section outlines the major advantages and disadvantages of simulation tools, particularly in the context of using two-dimensional finite-element CAD in the teaching of electromagnetics and electrical machines.

2.1. Enhanced Capabilities of the Method

The main advantage of the finite element method is its enhanced capability to solve problems involving: 1) complex geometries; 2) non-linearities; 3) non-homogeneous media; and 4) time-dependent phenomena. The typical unstructured finite-element meshes allow good representation of curved objects, easy insertion of short gaps, and increased local resolution in regions wherein rapid variations in magnitude and/or direction of the numeric field solution are expected to occur. The methods employed to control the "grain" of the finite element mesh in these critical regions are discussed in [6].

2.2. Inherent Limitations of the Method

The main disadvantage of simulation tools is that they may give the appearance of verisimilitude while containing significant internal errors, potentially leading to incorrect interpretations. These tools produce visually appealing and smooth field lines, which suggest accurate results. Consequently, inexperienced users might misinterpret these results as accurate and reliable, leading to flawed design decisions and potential failures in real-world applications. The most common source of error in field solutions is the incorrect specification of boundary conditions, but inexperienced users tend to overlook critical problem specifications like the length unit of the numeric model and the depth of the device.

The difficulties related to the appearance of verisimilitude are better understood by giving a numerical example. Consider the flux distribution of a four-pole direct-current permanent-magnet motor, excited by four buried and radially oriented neodymium-iron-boron permanent magnets. In the magnetostatic field solution used to inspect cogging torque and magnetic saturation, electric currents in the stator slots are absent, and all ferromagnetic regions are "fully fluxed", *i.e.*, reaction fields are absent.

One of the most effective ways to develop a subjective understanding of a magnetic finite element solution is by inspecting the flux lines. Indeed, when a solution is retrieved, most CAD systems display the flux-density map together with flux lines by default, as these visualizations are popular among designers and engineers. However, this can be misleading in two-dimensional analyses, because the specification of the device depth does not alter the appearance of these plots. The flux distribution shown in **Figure 1(a)** appears perfectly periodic, with four identical regions where the flux circulates alternately in clockwise (cw) and counterclockwise (ccw) directions. The motor depth, which should be set to $z = 5.0$ cm as indicated in **Figure 1(b)**, has no visible impact on this graphical representation. If an inexperienced user overlooks this parameter, the program will instead assume the default unit value ($z = 1.0$ cm), as illustrated in **Figure 1(c)**. Although the plots for the two device depths look identical, the underlying electromagnetic global quantities, such as terminal inductance, stored energy, and torque, will be computed with the wrong scaling. In other words, all global quantities in the solution with $z = 1.0$ cm will be underestimated by a factor of five relative to those obtained with the correct depth specification.

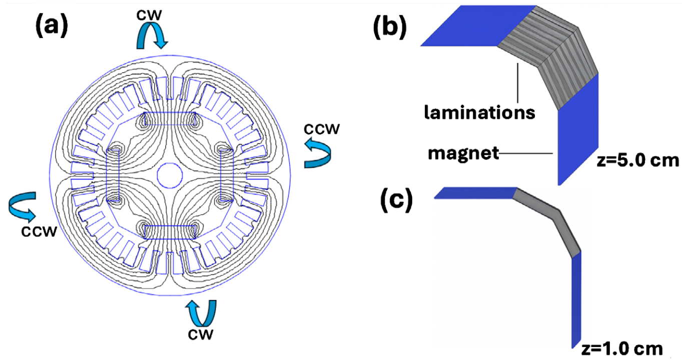

Figure 1. (a) Flux distribution; (b) Rotor surface when $z = 5.0$ cm; (c) Rotor surface when $z = 1.0$ cm.

Another disadvantage of finite-element simulators is the dependence on approximations, *i.e.*, simulation tools rely on mathematical approximations to model physical systems. If these approximations are not well understood, the results may be misleading. The construction of a finite-element model invariably involves several user-defined parameters and numerous choices that affect the accuracy and consistency of the finite element solutions. In other words, different numeric models for a given problem may lead to different results, so the quality of the finite element simulations relies heavily on the experience of the user in constructing a finite element model "adequate" to the problem. Besides, experience and judgment are needed to extract and examine the results.

Following the same strategy used to illustrate the limitations related to the appearance of verisimilitude in certain finite element solutions, it is natural to turn to a numeric example that highlights the second inherent limitation of finite element simulations: the reliance on approximations. The problem chosen for this purpose consists of using the $\boldsymbol{J} \times \boldsymbol{B}$—Lorentz force method—to compute the repulsive force between two long and parallel busbars carrying currents in opposite directions. This problem has several attractive features: 1) It is excellent to introduce the class of electromagnetic field problems classified as open boundary problems, and the need for open boundaries when analyzing them, since the FEM is essentially a finite-domain method; 2) Its major advantage is that the analytical result is well documented and easily applicable; 3) Whenever applicable, the Lorentz force method tend to provide very accurate results [7]; and 4) When using the $\boldsymbol{J} \times \boldsymbol{B}$ method, the current density $\boldsymbol{J}$ is defined exactly, so any numerical errors originate from the approximations involved in computing the vector potentials $\boldsymbol{A}$ and the resulting $\boldsymbol{B}$-field distribution.

This open-boundary problem is artificially closed by introducing a square "far" boundary where the condition $\boldsymbol{A} = 0$ is prescribed. When this boundary is placed too close to the conductors, the computed force is significantly underestimated. Students then experiment with progressively increasing the boundary distance, observing how the spurious reduction in force caused by the truncated boundary gradually disappears. Finally, the Kelvin transformation is applied, and the resulting force agrees very closely with the analytical solution. This problem is therefore highly effective in demonstrating the impact of approximations in FEM simulations, while also highlighting the value of truncation-free techniques such as the Kelvin transformation [8].

Both limitations of finite-element simulators should be tackled at an early stage of the virtual laboratory training program. As simulation software increasingly appears realistic, it is crucial to implement robust verification and validation techniques to prevent misconceptions. Verification involves ensuring that the simulation model has been correctly implemented and is free from errors such as incorrect boundary conditions or parameter setups. This step is vital to avoid producing results that are numerically accurate but unrelated to the actual device, like in the simulation exercise of the DCPM motor illustrated in **Figure 1**. It is the domain of instructors and educators to provide case studies and examples where simulations have led to incorrect conclusions and discuss how these errors could have been identified and mitigated.

Validation entails comparing simulation results with previously validated or well-established data to ensure their correctness and accuracy. Benchmark problems play a vital role in assessing the precision, computational efficiency, and robustness of finite element analyses. Generally, there are three types of benchmark problems used to evaluate finite element-based simulations, distinguished by the approach used to establish the benchmark: 1) Analytical benchmarks, which rely on exact solutions derived from simplified models; 2) Empirical benchmarks, based on experimental measurements and real-world data; and 3) Numerical benchmarks, involving standardized computational problems with known solutions or results from high-fidelity simulations [9]. Given the role of effective benchmarking in teaching and validating finite element models, it appears useful to indicate a selective set of references [10]-[14] that exemplify the three main categories of benchmark problems, as summarized in **Table 1**.

Table 1. Representative set of benchmark problems for teaching and validating finite element models.

Type[a]	Title of the Publication	Reference
A/C	Induction Motor Analysis: International T. E. A. M.[b] Workshop Problem 30	[10]
B	Description of T. E. A. M. Problem 32: A Test-Case for Validation of Magnetic Field Analysis with Vector Hysteresis	[11]

Continued

C	An Introductory Note on Finite Element Problems Based on the Eddy Current Testing Approach	[12]
C	A Comprehensive Benchmark of Different Motor Topologies for High-Performance 2-Wheelers Application	[13]
C	Benchmarking of Electric and Hybrid Vehicle Electric Machines, Power Electronics, and Batteries	[14]

a. Type of the benchmark problem: A: Analytical; B: Empirical; C: Numerical. b. T. E. A. M.: Abbreviation for Testing Electromagnetic Analysis Methods.

3. Practical Challenges of Engineering Virtual Laboratories

The adoption of engineering virtual laboratories has grown rapidly in recent years, yet their integration into curricula continues to face significant practical challenges. A recent meta-analysis by Li and Liang [15] shows that, while virtual laboratories enhance accessibility and support the development of specific cognitive skills, their overall effectiveness depends heavily on course design and alignment with learning objectives. These findings indicate that pedagogical strategy—rather than technology alone—plays a decisive role in determining the success of virtual laboratories. Beyond questions of effectiveness, educators also confront institutional and logistical barriers. A recent state-of-the-art evaluation emphasizes the difficulties associated with the high cost of software licenses and the requirement for reliable infrastructure [16]. These constraints often determine whether institutions can sustain the long-term use of virtual laboratories, particularly when adoption must be scaled across multiple courses or departments.

Further evidence of these difficulties is provided by May *et al.* [17], who examined the rapid transition of traditionally hands-on laboratories to online formats. Their study identifies significant gaps in preparedness, training, and resource allocation, which limited the effectiveness of the transition and revealed the vulnerability of institutions lacking robust digital strategies. These findings reinforce the view that the sustainable integration of virtual laboratories requires not only appropriate software and infrastructure but also systematic planning and institutional support.

Taken together, these perspectives provide a foundation for the detailed discussion that follows. Similar issues have been examined in previous works [18]-[20]. **Table 2** summarizes the main categories of practical challenges, which are then explored in greater detail in the subsequent sections.

Table 2. Practical challenges in implementing engineering virtual laboratories.

1) Duration of Skill Acquisition
• Students and faculty members require extensive training to use the software effectively. • Training resources and time can be barriers, especially for less experienced users.

Continued

2) Variability in Hardware and Software Access
• Many institutions do not have easy access to high-performance computers and/or licensed software. • This issue is particularly severe in developing countries where budgets are limited.
3) Disconnect from Real-World Experience
• Real-world challenges are often absent in virtual laboratories. • Over-reliance on simulations might cause one to overlook the importance of hands-on, physical experimentation.

A thorough evaluation of these challenges is essential, especially in developing countries where limited financial resources pose a significant barrier. Unlike universities in wealthier nations, where establishing state-of-the-art laboratories is often considered a responsibility of institutional leadership, educators in developing regions must exert considerable effort to justify and secure investments. Future sponsored projects submitted to government agencies must be meticulously prepared, clearly demonstrating the educational benefits and cost-effectiveness of integrating the proposed hardware and/or software facilities.

3.1. Duration of Skill Acquisition

For courses in electromagnetics, including electromechanical energy conversion and electrical machines, it is recommended that simulated laboratory activities be incorporated as a core element of the curriculum. These subjects are inherently abstract, with theoretical backgrounds dominated by vector calculus and field theory. Finite-element CAD systems provide visual and interactive aids that transform these abstractions into tangible learning experiences.

In a conventional laboratory, much of the work involves measuring terminal quantities—currents, voltages, power, and torque—using transducers. Many topics on energy conversion can be taught by combining electric or magnetic equivalent circuits and measurements, as the latter are obtained straightforwardly. However, this approach overlooks phenomena that are central to understanding the behavior of electromechanical devices. Important topics like energy storage in millimetric air gaps, force distributions in magnetized bodies, and the effects of eddy currents in conductive media cannot be properly addressed without analyzing devices from the perspective of electromagnetic fields. Finite-element simulations bridge this gap, allowing students to visualize and make a quantitative analysis of these effects.

Given the well-known advantages of combining numerical, analytical, and experimental approaches, it becomes essential to design laboratory experiments that intentionally address these conceptual challenges. Abstract topics are then transformed into tangible learning opportunities where students are guided through carefully crafted case studies and simulation-based tutorials. Tailored instructional materials play a decisive role here: they not only bridge the gap between theory and practice, but they also enable students to benefit from instructional tools that make possible "observation" and quantitative evaluation of imaginary, idealized line plots and density plots conceived to explain phenomena that otherwise remain hidden.

Instead of discussing abstract procedures for preparing such instructional resources, it seems more effective to present concrete samples of instructional materials. By examining the actual laboratory guides presented in **Section 4**, educators and instructors can immediately perceive how theoretical concepts are translated into structured learning tasks, without the distraction of general guidelines on material design.

Even when supported by high-quality instructional materials, students must dedicate a substantial number of hours to hands-on interaction with electromagnetic CAD systems, whether in the college laboratory or at home. For this training to be effective, two conditions deserve particular attention: first, the proposed tasks must offer an adequate level of challenge. If they are too easy, students will quickly lose interest and redirect their efforts to other subjects; if they are too demanding, frustration may prevail, and the training will no longer seem personally worthwhile. Second, the number of experiments must strike a careful balance—neither too few, which would confine the training to a narrow set of problems, nor too many, which could overwhelm students and dilute the learning outcomes. In this regard, well-designed home activities provide a valuable complement to structured laboratory sessions. It should also be noted that, despite careful planning, undergraduate and graduate students alike face considerable challenges when attempting to model laminated structures employed in transformers and electrical machines—a difficulty that will be further examined in a later section.

3.2. Software Access

Licensed software, such as MATLAB and Simulink, plays a significant role in engineering education. The licensing options vary, with costs depending on factors like the license type and scope. A Campus-Wide License (CWL) provides access to the entire academic community, though its price is not publicly specified and would require inquiry. Alternatively, licenses for a single user cost approximately USD 458.00 per year, while a group license for five users reduces the cost to about USD 291.00 per user annually. Students may obtain either the MATLAB and SIMULINK student suite or the standalone MATLAB student version via the purchase link indicated in [21]. These licensing models offer flexibility but also highlight the importance of budget considerations when integrating such tools into educational programs.

A second family of tools relevant for engineering virtual laboratories comprises electromagnetic field simulators. Commercial-grade simulators offer different editions aimed at specific user groups: 1) Student edition: free of charge, but with significant restrictions on the problem size; 2) Lite edition: a low-cost option that removes some constraints of the student's version but still maintains caps on model size and types of problems; and 3) Professional edition: the complete version, with full solver capabilities, materials libraries, and support for coupled physics.

It is worth noting that the licensing cost of the professional edition is prohibitive for many teaching institutions, and can only be justified when a department adopts a long-term strategy to integrate simulation-based activities across multiple courses and, in developing regions, through collaborative arrangements in which a single high-cost resource, such as a high-performance computer or a professional software license, is shared among multiple universities.

Most reported experiences with the finite element method as a complementary teaching tool have relied on general-purpose electromagnetic CAD systems. For instance, Calado *et al.* [22] employed the two-dimensional module of Flux2D to strengthen students' skills in calculating electromagnetic global quantities, such as co-energy and magnetic forces, in electrovalves and linear switched reluctance actuators. Similarly, Prasad *et al.* [23] used the design and optimization tools of Maxwell/ANSYS to analyze the electromechanical forces produced in linear switched reluctance motors (LSRMs). Additional experiences based on other electromagnetic CAD environments, specifically FEMM [24], Magnet [25], and COMSOL Multiphysics [26], are summarized in **Table 3**.

Table 3. Reported teaching experiences using electromagnetic CAD systems.

Title of the Publication	System
Using Finite Element Based Software to Teach Electrical Machines—The Linear Switched Reluctance Actuator[a]	Flux2D
An Educational Tool Based on Finite Element Method for Electromagnetic Study[b]	ANSYS
Simulated Experiments for Teaching Mutually Coupled Circuits CAD Techniques Using Analytic and Finite Element Solutions	FEMM
Orbital Performance Simulation of Ring Type Electro Motor Compared with Radial Motor Using Magnet Software	Magnet
Finite Element Method as an Aid to Machine Design: A Computational Tool	COMSOL

a, b: Includes evaluation of the learning experience.

3.3. Disconnect from Real-World Experience

The limited exposure students have to real-world data provided by industrial suppliers is a matter of considerable concern. The approximate nature of information regarding geometric dimensions, material properties, and driving sources of actual devices reduces the accuracy of numerical models, thereby limiting the reliability of experimental benchmarks. Consequently, students often have access to only a few experiments that effectively combine simulation with hands-on, experimental work.

A practical way to mitigate the disconnect from real-world experience is to encourage students to interact directly with manufacturers' manuals and data sheets. This practice not only develops their ability to handle technical documentation but also familiarizes them with the challenges of retrieving reliable information in real engineering contexts. Yet, searching for such data is a notoriously difficult task. On one side, students often encounter marketing brochures—scanned and repackaged—to promote products rather than provide precise technical content. On the other extreme, they may find scanned copies of books long out of print. Even when relevant documents are available, extracting a simple parameter such as the electrical conductivity of a laminated steel may require navigating several tables where thermal and mechanical properties are prioritized over electromagnetic data.

Electromagnetic field simulators contain a built-in library to facilitate access to materials data files. The user can access and expand this library by importing materials data files from other materials libraries. The data file for a soft magnetic material may contain all necessary information to fully characterize the ferromagnetic core of an electric power equipment, *i.e.*, the 1st quadrant DC magnetization curve or the relative permeabilities in the easy and hard directions of magnetization, electric conductivity of laminations, lamination thickness, and lamination fill factor. See **Table 4**.

Table 4. Properties for soft magnetic materials.

Provided Information	Symbol	In-Plane Value	Through-Plane Value
Relative permeability[a] (p. u.)	μ_r	$\mu_{r,\parallel}$	$\mu_{r,\perp}$
Electric conductivity[b] (MS/m)	σ	σ	σ
Special Attributes[c]			
Orientation of the lamination[d]	-	-	-
Lamination thickness (mm)	t	-	-
Lamination fill factor (p. u.)	c	-	-
Hysteresis lag angle[e] (degree)	ϕ_h	$\phi_\parallel$	$\phi_\perp$

a. In nonlinear analysis, permeabilities differ in the directions parallel to and across the lamination planes; b. Most problems in magnetics assume that the electric conductivity has the same value in all directions; c. This dialog box only becomes enabled when the material represents an actual laminated structure; d. Possible orientations for a lamination: i) in the x-y plane); ii) in the y-z plane; and iii) in the x-z plane; e. Hysteresis lag angle: either determined experimentally or obtained from technical literature.

In the FEMM CAD system, the library for problems of magnetism contains property files for conductors, as well as hard and soft magnetic materials, and these files are organized into separate directories [27]. Currently, property files for sixty different soft magnetic materials may be found in two main directories: 1) Metals Handbook DC Magnetization Curves; and 2) Soft Magnetic Materials. The organization of these two directories is illustrated in **Figure 2**.

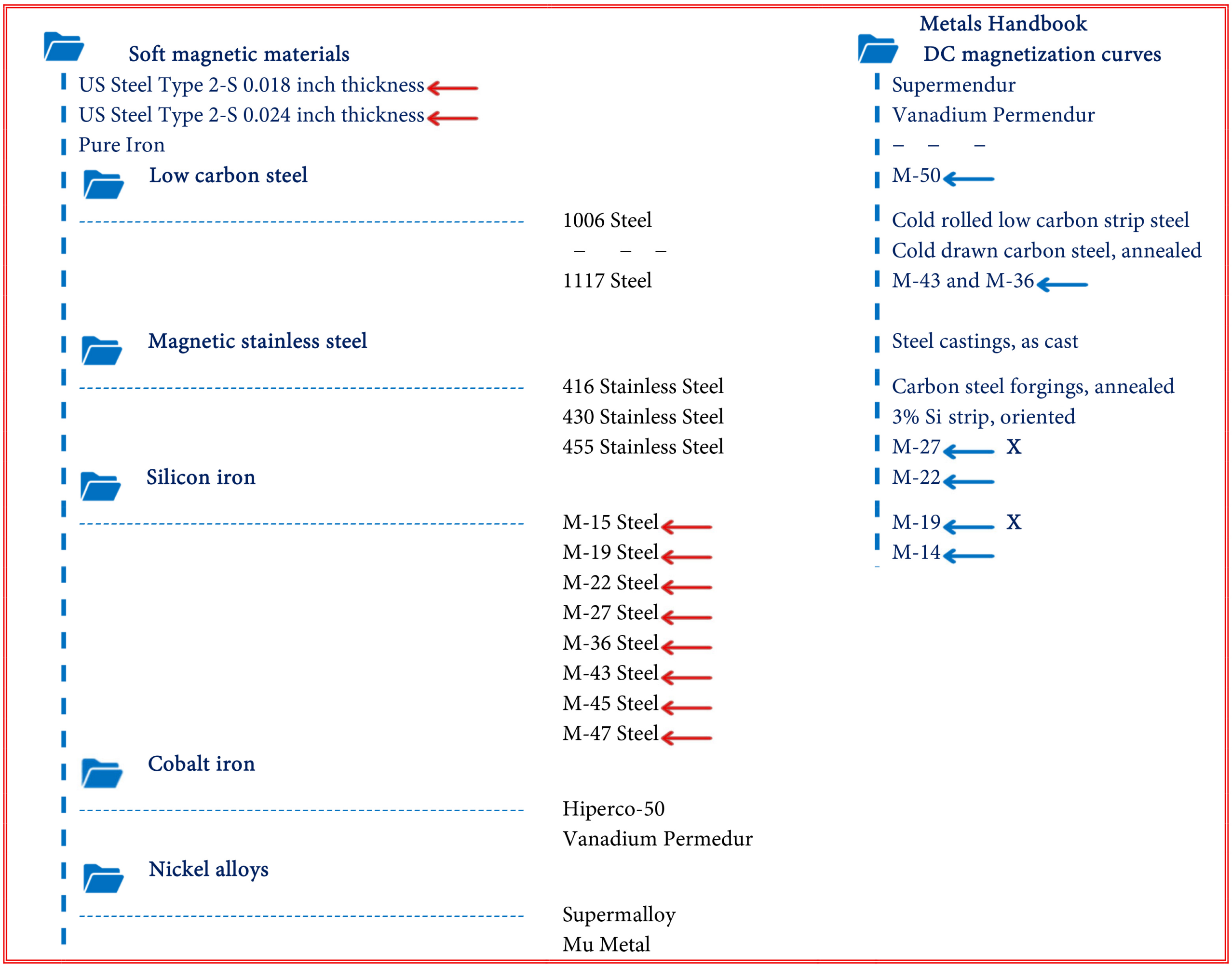

Figure 2. File directories for soft magnetic materials: FEMM CAD system.

Among the practical skills that require careful integration into the learning process, the numeric modeling of laminated magnetic cores in finite-element CAD systems deserves particular attention. Unlike abstract exercises, this task involves reconciling the simplifications inherent in virtual laboratories with the complex physical behavior of real laminated steels. Students must learn not only how to navigate software libraries and define material properties, but also how to make informed decisions about approximations on the modeling of: 1) magnetic linearity; 2) anisotropy; and 3) magnetic loss. Introducing these challenges in a structured manner creates a natural transition from general instructional tasks to the more specialized problem of representing laminated cores in simulation environments.

3.4. Magnetic Nonlinearity of Steel Laminations

The mathematical modeling of laminated cores is inherently complex and constitutes a central aspect of DC and AC magnetics problems in transformers and electrical machines. In most practical problems of magnetism, one must decide whether to treat the laminated material as magnetically linear or to include nonlinear effects, since neglecting nonlinearity can compromise the accuracy of key quantities of interest, particularly computed core losses. The solver directives associated with these assumptions are often software-specific, and their correct application requires practical experience in handling the materials data libraries provided by the simulation environment.

Micro and macro-eddy currents present in the magnetic core of electric equipment produce ohmic loss and their own magnetic field—known as the reaction field—that opposes the flux change that originates from alternating excitations [28]. Magnetic loss makes the ferromagnetic material medium less permeable, and this adverse effect is expressed numerically by a change in the complex-valued magnetic permeability μ of the ferromagnetic core. In a magnetically linear time-harmonic solution, the permeability μ of the core is complex-valued and has the same value in the whole volume occupied by the laminated core. In nonlinear problems, the complex value of the permeability varies from point to point in the region occupied by the laminated core.

Explicit control over magnetic linearity, however, typically arises only at the stage of defining material properties within each "block property" dialog. At this point, when a new material is added to the materials library or an existing one is modified, the user assigns a descriptive name and selects the magnetic characterization from the *B*-*H* curve drop-down list. This selection—linear or nonlinear—determines which additional parameters become available, allowing the user either to retain the default values or to modify them.

Laboratory activities on the numerical modeling of steel laminations should begin by highlighting the interdisciplinary nature of the subject, which combines magnetic circuit analysis with materials science concepts such as crystallographic texture, grain orientation, and processing-induced anisotropy. For users of the FEMM CAD system, a quick inspection of the built-in library of soft magnetic materials highlights the need for its continuous expansion and maintenance. At present, the database offers only a limited selection of silicon-iron materials, and the specifications available for modeling steel laminations are restricted to sheets with a thickness of 0.635 mm and a fill factor of 0.98. The "Soft Magnetic Materials" directory contains sixty property files organized into subdirectories for low-carbon steel, magnetic stainless steel, silicon-iron, cobalt-iron, and nickel alloys. However, only ten property files are specifically devoted to steel laminations—two for US Steel type 2-S at the root level, and eight under the silicon-iron subdirectory. In **Figure 2**, these ten files are marked with red arrows for illustration.

Under the directory entitled "Metals Handbook DC Magnetization Curves," the seven property files devoted to steel laminations specify the electrical conductivity of the material medium but provide no information regarding the thickness and fill factor of the lamination sheets. In **Figure 2**, these seven files are indicated by blue arrows for illustration. Moreover, the "X" marks next to the files for laminations of grades M-19 and M-27 indicate that the corresponding nonlinear *B*-*H* curve data are missing.

These shortcomings underscore once again the importance of maintaining and updating CAD systems' built-in libraries. At the same time, they provide instructors and educators with an opportunity to design laboratory guides that expose students to the practical challenges of interpreting and supplementing incomplete technical documentation.

3.5. Anisotropy of Steel Laminations

Within a two-dimensional finite-element framework, laminations are usually not modeled individually. Instead, the volume occupied by the lamination stack is represented as a homogeneous anisotropic material. A hypothetical magnetic characteristic described by two distinct "effective" permeabilities is then assigned to this equivalent homogenous medium. These permeabilities allow the numerical modeling of the stack's behavior according to the direction the magnetic flux flows through the pack of laminations: parallel to the lamination planes or across them. These two cases are illustrated in **Figure 3(a)** and **Figure 3(b)**, respectively. When the externally applied flux density $\boldsymbol{B}$ is oriented parallel to the lamination planes, along the easy direction, the bulk permeability for this section of the magnetic path is given by

$$\mu_{\parallel} = c\mu_r\mu_0, \tag{1}$$

where c is the fill factor of the pack of laminations, μ_r is the magnetostatic relative permeability, and μ_0 is the permeability of free space. Conversely, when $\boldsymbol{B}$ crosses the lamination planes, along the hard direction, the corresponding bulk permeability is given by

$$\mu_{\perp} = \frac{\mu_r\mu_0}{c + (1-c)\mu_r}. \tag{2}$$

To account for the geometric anisotropy of laminations, it is necessary to replace the scalar description of permeability with a tensor representation. In this way, the permeability tensor $\boldsymbol{\mu}$ takes the form:

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_{\parallel} & 0 \\ 0 & \mu_{\perp} \end{bmatrix}. \tag{3}$$

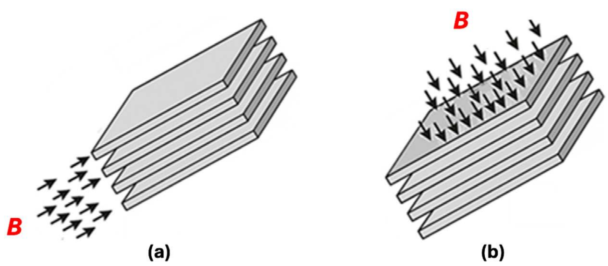

Figure 3. (a) Flux travelling parallel to the lamination planes; (b) Flux travelling across the lamination planes.

The off-diagonal terms of the tensor $\boldsymbol{\mu}$ are taken as zero for simplicity. This assumption is supported by the fact that the insulating coating of laminations suppresses the flow of eddy currents between adjacent sheets. It is important to note that the permeabilities $\mu_{\parallel}$ and $\mu_{\perp}$ as expressed in Equations (1) and (2), are real-valued quantities and are therefore applicable only to magnetostatic problems. In AC problems, the two diagonal elements of the permeability tensor $\boldsymbol{\mu}$, as expressed in Equation (3), become complex-valued and, in the case of nonlinear analysis, are also frequency-dependent.

In AC problems formulated with the $\boldsymbol{H}$- and $\boldsymbol{B}$-fields expressed in terms of phasor vector quantities, two distinct situations should be considered: 1) the effect of magnetic loss is negligible and, as illustrated in Figure 4(a), vectors $\boldsymbol{H}$- and $\boldsymbol{B}$- possess the same phase angle; and 2) the effect of magnetic loss is considerable, and can be included in the analysis by introducing a time-phase difference between the $\boldsymbol{H}$- and $\boldsymbol{B}$-fields, defined as the hysteresis angle ϕ_h of the magnetic material. This effect is known as hysteresis-induced lag and is illustrated in Figure 4(b). In this situation, the $\boldsymbol{B}$-field lags the $\boldsymbol{H}$-field by a fixed phase angle, the magnetic permeability μ becomes a complex-valued quantity, and the corresponding B-H characteristic becomes elliptical. A brief discussion on the definitions of complex-valued permeabilities in time-harmonic problems is presented in the **Appendix**.

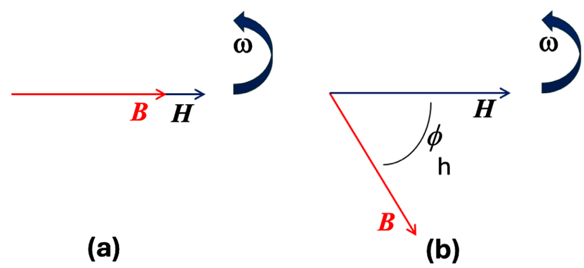

Figure 4. (a) The hysteresis-induced lag is negligible; (b) The hysteresis-induced lag is considerable.

The determination of the approximate elliptical B-H loop of laminated cores involves subtleties that are not always evident to inexperienced users. In a linear time-harmonic analysis, the solver must account for the reaction field inherently produced by the induced currents themselves, and this is done by adjusting the user-defined hysteresis angle. An illustrative example is provided in [29].

4. Sample of Instructional Materials

This section presents two representative laboratory guides that exemplify how abstract electromagnetic phenomena can be translated into structured learning tasks within a virtual laboratory environment. The description of the guides is very concise, and the proposed tasks are itemized in two sections: 1) the preparatory step; and 2) the lab processing tasks. In the first part of the laboratory guide, students are encouraged to consult the CAD system manual and the technical documentation provided by manufacturers, such as datasheets and application notes, in order to answer preliminary questions. This stage is not a mere formality: it equips learners with the practical knowledge needed to handle modeling artifices like the treatment of go-and-return windings as separate regions of the shading rings analyzed in the first experiment. Usually, the preparatory tasks also involve constructing simple electric or magnetic equivalent circuits based on experimental work. These circuits provide a first analytical assessment and bring considerable insight into the subject.

A common source of confusion in electromagnetic CAD systems is the choice between peak and RMS values in AC analysis. Unlike the convention adopted in manufacturers' manuals, where voltages and currents are usually given as RMS values, all electromagnetic quantities in tools such as FEMM—including driving currents and voltages, as well as computed potentials and field distributions—must be specified and interpreted using their peak values. Unfortunately, this requirement is not always documented in the user's manuals, leaving many users to discover it only after consulting community support forums. Indeed, online support groups have evolved into a dynamic component of traditional technical documentation, transforming user feedback into both FAQ updates and in-depth technical notes.

Based on these considerations, laboratory guides should be accompanied by a lab report template, often referred to by students as the "anti-trap guide". This document includes: 1) warnings regarding problem specifications; 2) caption templates for figures and tables that reinforce the adopted conventions for curve amplitudes (e.g., peak or rms) and require this information to be explicitly included in the tables; and 3) reminders about the expected level of numerical error and the importance of checking for gross errors in the problem setup.

The first laboratory guide examines the function of shading rings in AC contactors, enabling students to visualize how harmonic solutions capture the redistribution of magnetic flux in shaded and unshaded regions. The second guide investigates the occurrence of eddy currents in an *H*-shaped busbar, emphasizing how the skin effect leads to a progressive exclusion of magnetic flux from the center of the busbar as the operating frequency increases.

4.1. Experiment 1: Shading Rings in AC Contactors

This experiment involves a real industrial AC contactor. A cross-sectional view of the AC contactor is presented in **Figure 5**, and a detailed analysis of the device can be found in [30]. For the closed-core configuration of the AC contactor, the terminal current of the driving coil is specified in terms of its peak value, $I_{\text{peak}} = 71.42$ mA. The most important feature of this problem is the definition of three independent circuit properties: one for the driving coil and one for each of the two copper rings.

This problem is of particular interest to educators who wish to introduce verification techniques into their laboratory sessions. An instructive exercise can be designed by intentionally misconfiguring the model: if the four rectangular regions representing the shading rings are specified as parallel-connected but only a single circuit property with zero net current is assigned to them, the resulting harmonic solution shows the magnetic flux being diverted exclusively toward the innermost unshaded regions. By comparing this incorrect outcome with the correct harmonic solution—where the flux is symmetrically diverted toward both adjacent unshaded regions—students can clearly see how improper circuit property assignment leads to physically incorrect results. This approach provides a valuable opportunity for reinforcing good modeling practices and fostering critical analysis of simulation outcomes.

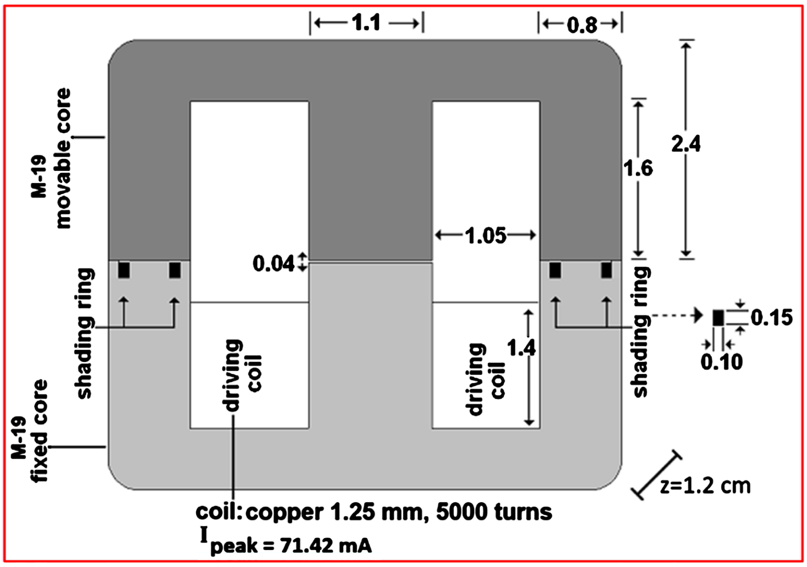

Figure 5. Cross-sectional view of the AC contactor; dimensions in centimeter.

Laboratory Guide: Industrial 220 V/60Hz AC Contactor

Description: The main effect of the shading rings is the circulation of an additional magnetic flux out of phase with the flux produced by the driving coil. As a result, the net flux crossing the interfaces between the movable and fixed cores never falls to zero. This ensures that the holding force—proportional to the square of the flux magnitude—remains continuously positive, thereby preventing undesired vibration or chatter in the closed-core configuration.

Objective: To illustrate the function of shading rings in AC contactors.

Preparatory step:

- In the AC contactor manual, the rated driving current for the closed-core configuration is specified by its RMS value. Check the CAD system's manual to determine whether electromagnetic quantities should be entered as RMS or peak values.
- In the model, the stranded coil that carries the driving current is represented by two non-contiguous regions. Which key parameters must be defined in order to assign the appropriate circuit property to these regions?
- Each pair of regions representing the two sides of a single-turn shading coil must be defined as parallel-connected. Which circuit property must be applied to this parallel-connected circuit to guarantee that the "computed" total current in one region will return through the other region?
- Concerning the configuration where the movable and fixed cores of the contactor are in contact, use the measured values of the terminal quantities to derive the equivalent electrical circuit. Values of measured reactive power Q and derived magnetizing inductance L_m should be compared to those computed numerically.
- It is advisable to begin the laboratory session with a linear time-harmonic solution in which the effect of the shading rings is deliberately omitted. This can be achieved by modifying the block label of the four rectangular regions representing the end sides of the shading rings. In the block properties dialog, select "air" instead of "copper", and remove the two previously assigned circuit properties. This harmonic solution will be referred to as a harmonic solution with shading rings omitted.

Continued

Lab Tasks:

- Launch the solver and obtain the time-harmonic solution with shading rings enabled.
- Determine the circuit properties of the driving coil and compare the numerical values of the reactive power Q and the magnetizing inductance L_m with corresponding measurements from the conventional laboratory. The maximum expected level of numerical error is 4.0%.
- Select the innermost rectangular region of the shaded pole present in the right limb and determine the value of the induced current, say I_1. Repeat for the outermost rectangular region to obtain the value of current I_2. Does the mathematical operation $I_1 = -I_2$ hold true? Indicate other post-processing operations that can be used to check the correctness and consistency of the field solution.
- Consider the shading ring in the left limb and compute the total current and the resistive loss in each rectangular region. Then, comment on the results.
- The combined effect of the magnetomotive forces of the main coil and shading rings leads to: 1) a decrease of the flux crossing the shading regions; 2) a substantial increase of the flux crossing the unshaded regions; and 3) a decrease of the flux crossing the central limb. Which post-processing operation yields a flux plot that illustrates these effects? Show this flux plot.
- The comparison of two harmonic solutions—one with shading rings and one without—highlights how each shading ring diverts the magnetic flux from the shaded region to the adjacent unshaded ones. Define a straight line inspection contour that intersects one of the lateral limbs, positioned just above the two rectangular regions representing the shading rings.
- In a single graph, plot the variation of $|\boldsymbol{B}|$ along the inspection contour for both harmonic solutions—one with shading rings and one without.
- For the harmonic solution with shading rings, compute the mean value of the magnetic flux crossing the shaded and adjacent unshaded regions, respectively.
- The computed induced current in each shading ring may reach values up to 1000 times greater than that of the driving coil. How can this apparently exaggerated result be explained? Provide a physical interpretation for such high current values.

4.2. Experiment 2: Eddy Currents on an *H*-Shaped Busbar

This linear time-harmonic eddy-current problem is described in the textbook by Silvester and Ferrari [31]. The formulation specifies the geometric dimensions and boundary conditions needed to model only one-quarter of the problem space. In problems with longitudinal symmetry, particular care must be taken when defining the "depth" parameter. This parameter, expressed in the same length unit used for the model's geometry, represents the extent of the device in the "into-the-page" direction. In the present case, the device depth is set to $z = 100$ mm. In the copper region, the z-directed driving current $I_{(\text{peak})} = 700$ A is assumed to be uniformly distributed, and this corresponds to a current density $J_s = 1.0 \times 10^6$ A/mm^2. In the original formulation of the problem, the analysis at 50 Hz reveals the expected deviation of the current density $\boldsymbol{J}$ from the driving current value J_s, both at the center and at the outer extremities of the busbar cross section.

This problem is of particular interest to students and self-employed engineers who rely on the student versions of commercial-grade CAD systems. It can be adequately modeled using the 500 to 600 first-order triangular elements that define the mesh size limit in recently released students' editions [32] [33]. Practical considerations in mesh construction still apply: 1) only one-quarter of the copper busbar needs to be modeled; 2) the conductive region should be discretized with a relatively uniform mesh of small, nearly equilateral triangles; and 3) the surrounding air region, which artificially closes the domain of analysis, can be modeled by a coarser mesh containing relatively larger triangles.

Laboratory Guide: *H*-Shaped Copper Busbar

Description: An *H*-shaped copper busbar carries a uniform driving current J_s at frequencies of 5 Hz, 50 Hz, and 200 Hz. Three linear time-harmonic analyses are used to obtain the distribution of the induced currents $\boldsymbol{J}$ and the corresponding values of dissipated power in the conductive region.

Objectives: To evaluate the effects of an increasing frequency on the spatial distribution of induced currents and corresponding copper loss.

Preparatory step:

- An important aspect of the eddy current analysis is the inspection of the orthogonality between pairs of vector quantities. Initially, consider the vectors $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{E}$, and $\boldsymbol{J}$, where $\boldsymbol{J}$ is the total current density in the conducting media. Answer the following questions: 1) In the problem setup, which quantities can be assumed to have only a longitudinal, *z*-directed component? 2) Which quantity can provide visual information about the magnetic flux? and 3) By considering that the source current and the induced currents are superimposed, is it possible to compute only the currents that are actually induced in the copper busbar?
- In time-harmonic problems, the longitudinal component of the vector potential, $\boldsymbol{A}_z$, is a complex-valued quantity, and this complicates post-processing procedures. Try to identify which visualization tools better convey visual information about the spatial distribution of the current density $\boldsymbol{J}$.
- To facilitate the analysis of results, it is advisable to begin the laboratory session with a magnetostatic field solution. Comparing the spatial distribution of magnetic flux in the static and time-varying cases allows for a clearer understanding of phenomena that arise exclusively under time-varying operation.

Lab Tasks:

- Plot the flux-line distribution corresponding to the field solutions at 0, 5, 50, and 200 Hz. Use your own words to explain what happens with the magnetic flux in the busbar when the frequency increases.
- For each time-harmonic solution, determine the power loss in the whole device.
- Produce density plots of $\boldsymbol{J}$ for the three harmonic solutions. Viewed as successive time snapshots, similar to movie frames, they illustrate how the distribution of current density changes with frequency. Describe your own observations.
- For the frequency of 50 Hz, determine the deviation in the total current density $\boldsymbol{J}$ from the uniform driving-current value J_s along a line segment connecting the right edge of the busbar to its center. Plot the characteristic that represents the deviation $\Delta\boldsymbol{J}$ versus position along that line segment, and compute the mean value of the dataset. In your laboratory report, write one or two paragraphs in your own words that include: 1) a brief description of the plotted characteristic; 2) the mean value of the dataset; and 3) a cause-and-effect explanation of why the deviation $\Delta\boldsymbol{J}$ is much smaller at the center of the cross-section than at the right edge of the busbar.

5. Conclusions

This review has examined strategies to mitigate the inherent limitations of simulation tools, the practical challenges of their implementation, and historical evidence of their pedagogical value in teaching electromagnetics. A recurring theme is that tailored instructional materials are essential to circumvent the inherent limitations of verisimilitude and the approximations of numerical models. Well-designed laboratory guides allow students to engage with simplified yet representative problems, ensuring that the focus remains on conceptual understanding rather than on the constraints of the software.

The practical challenges of adoption are equally significant. Licensing costs, infrastructure requirements, and institutional readiness frequently determine whether simulation-based laboratories can be sustained. In developing regions, resource sharing across institutions has proven a viable solution. Moreover, the widespread availability of student editions of commercial software deserves emphasis, as these versions not only provide affordable access during academic training but also prepare students who may later pursue self-employment to work confidently with industry-standard tools.

Finally, historical practices clearly demonstrate that tools such as MATLAB and finite element analysis have enriched the teaching of complex electromagnetic phenomena by bridging abstract theory with visual and quantitative experimentation. These tools should therefore be seen not as replacements for physical laboratories but as powerful complements that, when supported by tailored materials and institutional planning, significantly enhance the versatility and depth of engineering education.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix: Hysteresis and Complex Permeability

In time-harmonic problems, the complex permeability can be defined in different ways. In the following, two different definitions are briefly discussed.

A.1. The Complex Permeability Is Independent of the Frequency

In one of the definitions of complex permeability, it is assumed that magnetic hysteresis creates a "fixed" time phase difference, ϕ_h, between vectors $\boldsymbol{B}$ and $\boldsymbol{H}$ that is independent of the frequency of excitation. This situation is illustrated in **Figure 4(b)**. According to that illustration, the $\boldsymbol{B}$-field lags the $\boldsymbol{H}$-field by an angle ϕ_h, known as the hysteresis angle of the magnetic material. The complex-valued magnetic permeability, μ_h, is then defined by

$$\mu_h = \frac{|B|}{|H|} e^{-j\phi_h} = \mu_r \mu_0 e^{-j\phi_h}, \tag{A.1}$$

where μ_r is the magnetostatic relative permeability and μ_0 is the permeability of free space. In this approximate model, the hysteresis loop becomes an ellipse with its major axis making an angle of ϕ_h radians with the H-axis.

A.2. The Complex Permeability Is Frequency-Dependent

In time-harmonic problems involving thin laminations, the effect of eddy currents and hysteresis can be encapsulated in the effective, frequency-dependent permeability defined by

$$\mu_{eff} = \frac{\mu_r e^{\frac{-j\phi_h}{2}} \tanh\left[\frac{-j\phi_h}{2}\sqrt{j\omega\sigma\mu_r\mu_0}\,\frac{d}{2}\right]}{\sqrt{j\omega\sigma\mu_r\mu_0}\,\frac{d}{2}}, \tag{A.2}$$

where ϕ_h represents a constant phase lag between the $\boldsymbol{H}$- and $\boldsymbol{B}$-fields, σ is the electric conductivity of the laminations, d is the thickness of individual laminations, and ω is the angular frequency of excitation in rad/s. The term "$\omega\sigma\mu_r\mu_0$" present on the RHS of (A.2) reveals that the calculation of the effective permeability considers the attenuation of the $\boldsymbol{H}$-field below the surface of laminations due to the action of induced eddy currents.