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Impact of the Negative Resistance on the Characteristics of a Tunnel Diode-Inductive Circuit

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Abstract

The I-V diagram of a tunnel diode inherits a voltage range corresponding to a specific current domain with a negative slope. Within this range, the electric resistance is negatively impacting the characteristics of the electric circuits. One such circuit containing a tunnel diode in series with an inductor driven by a DC source is considered. The negative resistance significantly alters the characteristics of the circuit. In this research-oriented project, we unveil these characteristics comparing them to the classic inductive circuit with an ohmic resistor. This project stems from our previous work [1] and may be considered an application of the tunnel diode embodying unseen surprises. The circuit analysis is entirely based on utilizing a Computer Algebra System (CAS) specifically *Mathematica.* Without a CAS, the completion of the project wouldn't have been possible otherwise.

Keywords

Tunnel Diode, Negative Electric Resistance, Computer Algebra System, *Mathematica, Maple*

1. Introduction

We begin with a brief overview of the essential pointers of [1]. **Figure 1** is a duplicate of Fig 4 in [1]. On the left, it is the display of the I-V characteristics of a typical ohmic resistor; this is the general behavior of almost all electric resistors. It shows the voltage applied to the ends of a resistor associate current as a slated line with sustained positive constant slope indicative of the positive-valued resistance. The middle graph shows the behavior of a hypothetical scenario where for a certain voltage range its corresponding current exhibits a negative slope indicative of its negative-valued resistance. The other two voltage ranges, the one below and the other one above the shown cutoffs exhibit positive slopes alike in the shown far left diagram. The hypothetical scenario shown on the middle plate has an actual prototype replica with rounded edges shown on the far right plate. In this transition from a hypothetical to an actual case middle range, voltage sustains its negative slope character. The far right plate of **Figure 2** is the captured image of the I-V signal on the O-scope for the circuit shown on the far left of **Figure 2**. This is the character of the Esaki aka tunnel diode [2].

This brief review sets the forum for establishing the objectives of our project. We envision a "simple" electric circuit embodying a tunnel diode in series with an inductor. In the previous work, a circuit composed of an ohmic resistor, a capacitor, and a tunnel diode was analyzed [1]. The components of the circuit were selected such that its circuit equation however challenging was solvable. Our current proposed circuit has fewer components, actually two, and has the potential to reveal the impact of the negative resistance in a transparent way. To our surprise, the simplicity of the circuit leads to unexpected challenges. Solving the corresponding circuit equations and their interpretation establishes another pillar of our objectives.

This report is composed of three sections. In addition to the Introduction, Section 2 is the Procedure. Here for the proposed circuit, we craft its circuit equation. As expected, its ultimate non-approximated formal is a nonlinear differential equation leading to a non-analytic solution. After twice attempting to

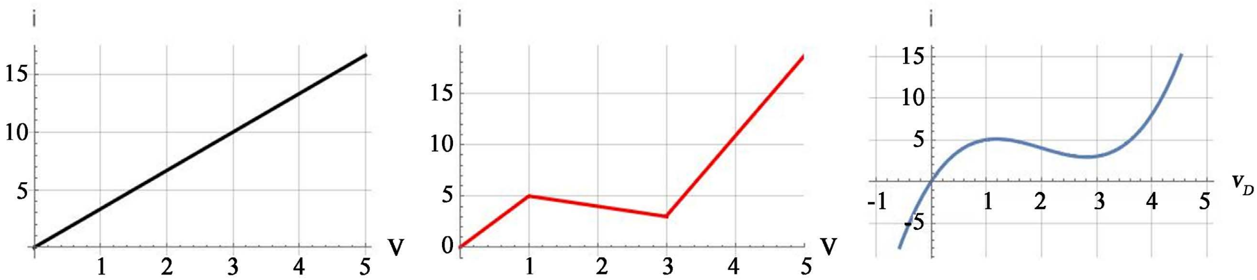

Figure 1. I-V diagrams of three different scenarios. The left corresponds to a typical ohmic resistor, the middle one is a hypothetical modified non-linear resistor, and the far-right diagram is the character of a tunnel diode, respectively.

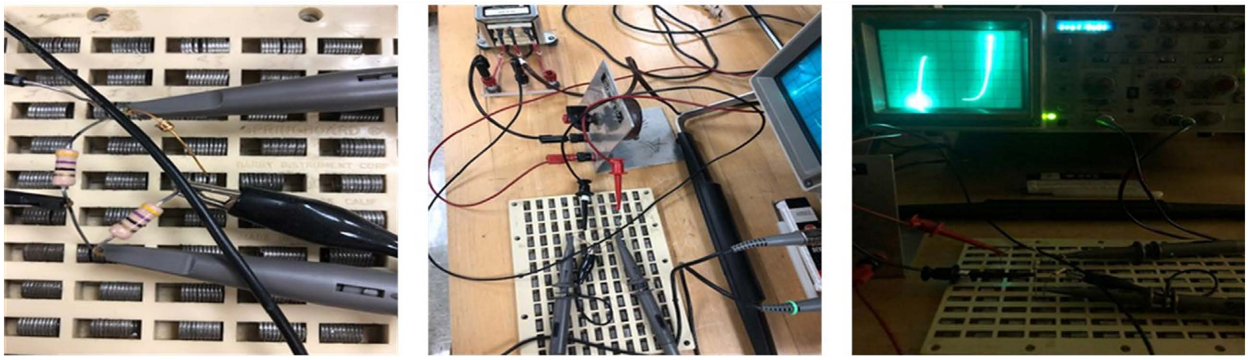

Figure 2. An actual laboratory setup of a circuit embodying a tunnel diode. The diode is the tiny golden cylindrical element shown at the one O'clock position in the left picture. The right photo is the captured I-V signal of the diode on the O-scope.

solve its approximated circuit equation leading to incomplete output utilizing *Mathematica* [3], we solve the exact circuit equation numerically. These yields fresh and never seen before needed to be interpreted as an outcome. The last section is the Conclusions summarizing topics learned, inviting the interested readers to voice their comments potentially extending the scope of the investigation.

2. Circuit Analysis

The I-V diagram of a tunnel diode shown on the far right of **Figure 1** is parametrized as,

$$i(v) = \alpha - \beta(v-\delta) + \gamma(v-\delta)^3, \tag{1}$$

where α, β, γ and δ being constants. By adjusting the numeric values of these four parameters character of the displayed figure such as coordinates of the extrema, the horizontal and vertical separation distance between the extrema, the coordinate of the inflection points, etc. may be adjusted to fit the character of an actual tunnel diode.

The schematic of a circuit composed of a diode in series with a self-inductor driven by a DC power supply is shown in **Figure 3(a)**. The self-inductor with inductance L responds by generating its opposing short-lived voltage against the supplied driver's voltage ϵ forming a circuit shown in **Figure 3(b)**.

Applying the loop theorem, a.k.a. Kirchhoff's 1st law yields the circuit equation,

$$\epsilon - L\frac{\mathrm{d}}{\mathrm{d}t}i = v \tag{2}$$

As given by (1), the current in the circuit, i is a function of voltage across the diode, differentiating (1) w/t and substituting the output in (2) gives,

$$\left[-\beta + 3\gamma(v-\delta)^2\right]v' = \frac{1}{L}(\epsilon - v) \tag{3}$$

where v' is the derivative w/t. This is the circuit equation. It looks like a trivial ODE with an implicit initial condition! An attempt was made to solve (3) analytically longhand. In doing so, by separating the variables and introducing a new variable $x = \epsilon - v$ yields,

$$\int_{\epsilon-v}^{\epsilon}\frac{1}{x}\left[-\beta + 3\gamma(x+\delta-\epsilon)^2\right]\mathrm{d}x = \frac{1}{L}t \tag{4}$$

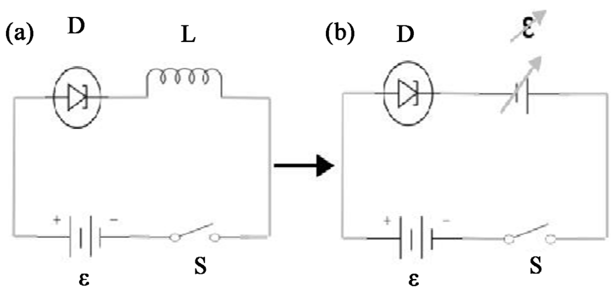

Figure 3. The left figure, (a) is the circuit of interest. The right plate, (b) is the scheme-tic of the actual function of the self-inductor in the intermediate stage while its impact tends to diminish.

Integrating (4) leads to an $\ln(1-v/\epsilon)$ term. Although in principle one might attempt to retain its functional form to obtain a "perfect" output because the voltage across the diode, v is always a fraction of the feeder voltage, ϵ we linearize $\ln(1-v/\epsilon)$. *i.e.* to the first-order approximation, $\ln(1-v/\epsilon)\sim-(v/\epsilon)$. This yields a quadratic equation *w*/ v these benefits gaining an insight into the competitive physical elements. With this substitution and some algebraic simplifications (4) yields,

$$\left(\frac{3}{2}\gamma\epsilon\right)v^2-\left(-\beta+3\gamma\delta^2\right)v+\frac{\epsilon}{L}t=0 \tag{5}$$

Equation (5) is a quadratic *w*/ v, its third term explicitly depends on time, t. This is the gained physical insight. *i.e.*, the time dependency of the voltage across the diode, $v(t)$ is given by a root(s) of a quadratic equation! The required positive value of the discriminant of (5) limits the time-scope,

$$t<\frac{L}{6\gamma}\left[\frac{1}{\epsilon}\left(-\beta+3\gamma\delta^2\right)\right]^2 \tag{6}$$

The value of t applying (6) for a set of parameters β, γ, and δ is used in [4] and e.g. L = 10. H, and ϵ = 10. V is 0 < t < 1.66 s. But what about the time exceeding this?

The answer is one rightfully might assume this narrow timespan (6) could have come about from the applied linearized logarithmic function. An attempt was made to curve this by including the second-order correction term. *i.e.* replacing $\ln(1-v/\epsilon)\sim-(v/\epsilon)-1/2(v/\epsilon)^2$. Inclusion of this term modifies (5) and still retains its quadratic format yielding,

$$\left\{-(3/2)\epsilon\gamma+\frac{1}{2\epsilon}\left[-\beta+3\gamma(\epsilon-\delta)^2\right]\right\}v^2+\left(-\beta+3\gamma\delta^2\right)v-\frac{\epsilon}{L}t=0 \tag{7}$$

The positiveness of its discernment modifies (6) confining the time-domain to,

$$t<L\frac{\left(-\beta+3\gamma\delta^2\right)^2}{6\epsilon^2\gamma-2\left[-\beta+3\gamma(\epsilon-\delta)^2\right]} \tag{8}$$

Applying the same set of parameters used in the previous paragraph, (8) stretches the upper limit of time to 0 < t < 4.54 s. Concluding, the inclusion of the higher-order corrections of the expanded logarithmic function boosts the time limit by 170%. This is an improvement in the right direction yet still the question poses "what about for time beyond this limit?".

From this mathematical exercise, we learn by including the higher-order terms in the expanded logarithmic function the upper limit of the associated time-span stretches. And therefore, ultimately, utilizing the non-approximated logarithmic function should be conducive to the longest possible timespan. To put this hypothesis in action the plot of t vs v after integrating (3) is shown in **Figure 4**.

The left panel shows the voltage across the diode on the horizontal and its associated time along the vertical. Should be noted the higher voltages across the diode occur earlier than the lower voltages at later times. This contrasts with the charter of an ohmic resistor. This graph also shows the multiple-voltage valued character of the diode, meaning their time instances that a chosen time corresponds to a multitude of voltages. For instance, according to the left depicted panel at $t = 0.1$ s a voltmeter would register three different voltages! This might be due to the very fast oscillations of the voltage. In the Conclusion section, this is posed as a challenging question to an interested reader. The right-side panel is the contour plot of the data shown on the left.

To put the impact of the diode in the circuit of interest depicted in Figure 3(a) in perspective the voltage across the ohmic resistor in an RL-series (Resistor-self Inductor) to the circuit DL-series (Diode-self Inductor) with compatible parameters is depicted in Figure 5. The blue curve is the classic character while the red signal is the one discussed in this report. Notably, the end tails of both signals as intuitively expected are comparable, while at the low end they are substantially

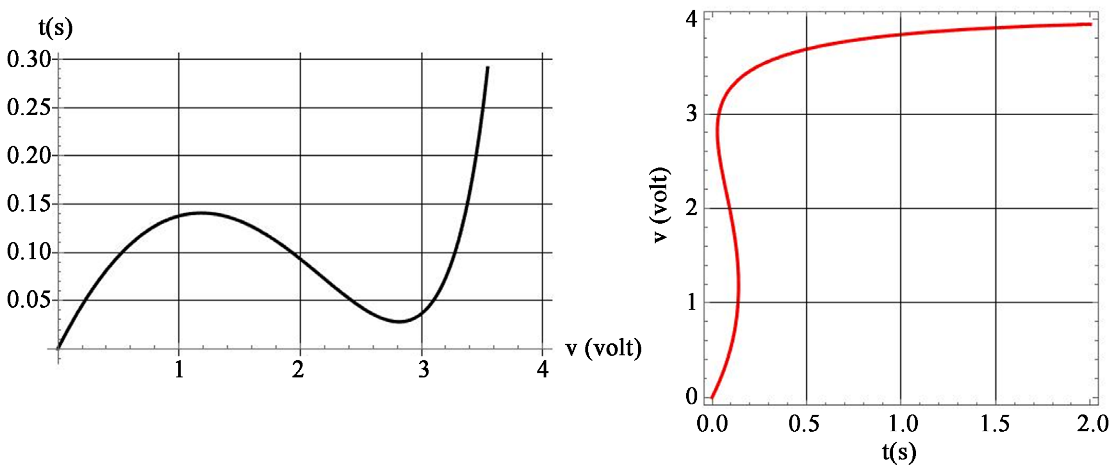

Figure 4. Display solution (3), t vs. v on the left. The interchanged character of the variables on the left graph is shown on the right plate.

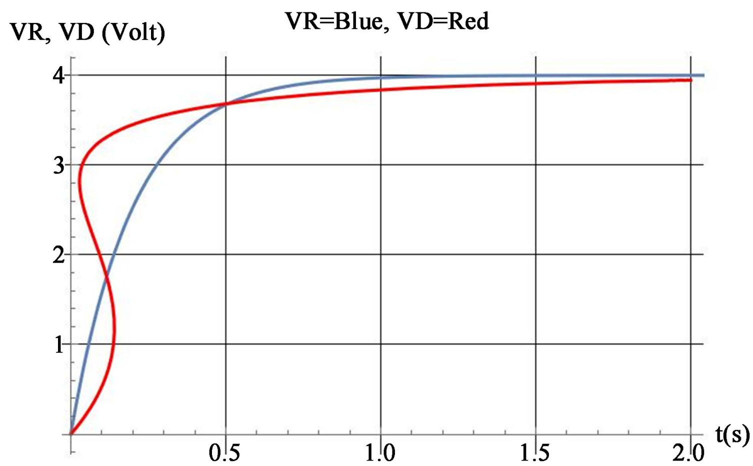

Figure 5. The blue curve is the transient character of the voltage across the resistor in an RL-series circuit. The red curve is the transient character of the voltage across a tunnel diode.

different. This highlights the impact of the negative resistance.

We extend the scope of our investigation by considering the characteristics of the current in the circuit. Specifically asking what is the impact of the mentioned multiple-valued voltage on the current. The answer is almost straightforward. Substituting the red voltage shown in **Figure 5** in (1) yields the answer. As shown in **Figure 5** the blue and red signals for times beyond 0.5 s are comparably possessing the expected characters. On the contrary, the low ends are different. The zoomed $i(t)$ character of the red signal is depicted in **Figure 6**. As noted earlier, this potentially stems from the very fast oscillation of the voltage within the negative resistance domain.

Moreover, curiously, if one plots the transient character of the voltage across the diode for times beyond the effective lifetime of the self-Inductor one would observe its "shivering" nature! C.F. the left panel of **Figure 7**. Its zoomed is

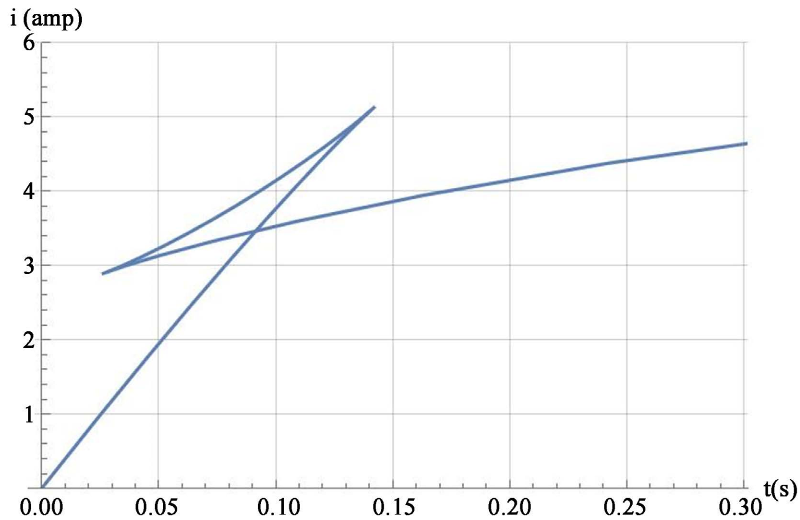

Figure 6. The zoomed display of the current, $i(t)$ in the DL-series circuit.

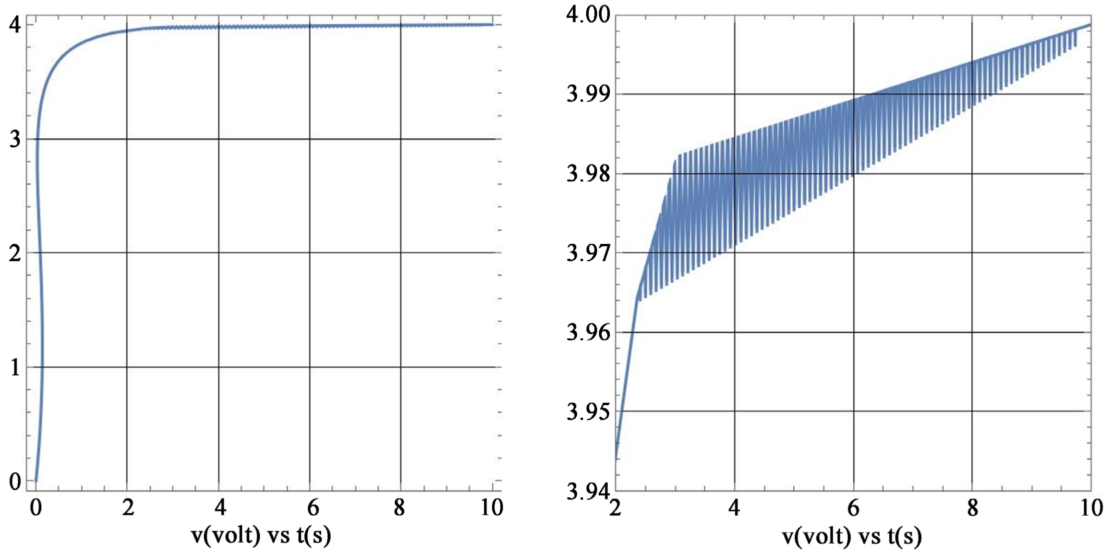

Figure 7. The left panel is the voltage across the diode vs. time. The right panel is zoomed in.

shown on the right. As shown, within 2.2 < t < 9.5 s the voltage fluctuates from within 0.02 Volts at the low end to gradually diminished vibrations at the high end. Increasing the accuracy of the computation and precession of the pixels aiming for a smoothened figure fails. This means, that this is not caused by the algorithm. This feature is not observed in the classic RL-series circuit. Conceptually, this observation reminds us of a similar scenario encountered in quantum physics referred to as "Zitterbewegung" literally translated as "quivering motion" [4].

3. Conclusion

The motivation of our current research-oriented report stems from the curious question about investigating the impact of the negative electric resistance on the classic electric circuits. Our previous investigation [1] concerned the same question is applied to a "similar" circuit embodying a diode-capacitor series circuit that produces understandable output with meaningful interpretation. Our careful analysis of the current proposed diode-inductor series circuit produces an unexpected curious output. The negative resistance of the tunnel diode causes multivalued voltages and currents in certain instances. These characters are potentially attributed to the fast oscillations of the voltage in the negative domain. By revealing these features, the author aims to stimulate a thought-provoking forum amongst the readers. We would also like to point out that utilizing *Mathematica* played an essential role in crafting this article. The interested reader will find [5] and [6] resourceful.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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