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A Review of Water Quality Optimisation Models and Techniques

Douglas Kwasi Boah, Stephen Boakye Twum

Department of Mathematics, Faculty of Mathematical Sciences, University for Development Studies, Navrongo, Ghana
Email: boahdouglas@yahoo.com, stwum@uds.edu.gh

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Abstract

This paper presents a state of the art review of water quality optimisation models and techniques from early 1970s to date in terms of the model/technique category, model/technique type, purpose and application. The models are categorised into Mathematical Programming Models and Meta-heuristic Programming Models. Similarly, the techniques are categorised into Mathematical Programming Techniques and Meta-heuristic Programming Techniques. The review is concluded by drawing attention to the rare nature of application of interior-point methods to water quality optimisation.

Keywords

Water Quality, Optimisation, Mathematical Programming, Model, Meta-Heuristic, Techniques

1. Introduction

Water quality refers to the chemical, physical and biological characteristics of water [1]. It is a measure of the condition of water relative to the requirements of one or more biotic species and or to any human need or purpose [2]. In hydrological optimisation, mathematical optimisation techniques are applied to water quality related problems. These problems may be about groundwater, surface water or both and may be done by hydrologists, civil engineers, environmental engineers, and operations researchers. Groundwater and surface water flows are studied with hydrologic simulation. However, simulation models cannot easily help make management decisions, as simulation is descriptive. Simulation shows what would happen given a certain set of conditions. Optimisation, by contrast, finds the best solution for a set of conditions. Consequently, hydrological optimisation is now being used for most water-related problems.

This paper presents a state of the art review of water quality optimisation models and techniques in terms of the model/technique category, model/technique type, purpose and application with the view to identifying at least a gap for future research.

2. Main Results

Water quality optimisation models are classified into Mathematical Programming Models and Meta-heuristic Programming Models. Water quality optimisation techniques are also classified into Mathematical Programming Techniques and Meta-heuristic Programming Techniques.

2.1. Mathematical Programming Models in Water Quality Optimisation

Linear Programming—[3] used a linear programming model to optimize water resource use in irrigation projects. The developed model was applied to Senator Nilo Coelho Project in Brazil. [4] developed a waste load allocation LP model for economic water quality management. [5] developed an inexact chance-constrained linear programming (ICCLP) model for optimal water pollution management at the watershed scale.

Integer Programming—[6] presented mixed-integer chance-constrained models for ground-water remediation.

Nonlinear Programming—[7] presented an interval-parameter fuzzy nonlinear optimization model for stream water quality management under uncertainty. A case study for water quality management planning in the Changsha section of the Xiangjiang River was then conducted for demonstrating applicability of the developed model. [8] presented an interval-parameter fuzzy robust nonlinear programming model for water quality management. The developed model incorporated interval nonlinear programming and fuzzy robust programming (FRP) methods within a general optimization framework. [9] proposed mixed integer nonlinear programming (MINLP) models for the synthesis and optimisation of water treatment processes. The applicability of the models was investigated in case studies of seawater desalination and surface water treatment for the production of potable water.

Dynamic Programming—[10] developed a dynamic programming model with water quality constraints. The model was designed for essentially linear systems and was applied to estuary of Delaware River, USA.

Stochastic Programming—[11] presented an inexact chance-constrained programming model for water quality management in Binhai New Area of Tianjin, China. The developed method was applied to planning chemical-industry development in Binhai New Area of Tianjin, China.

Robust Programming—[12] developed an inexact agricultural water quality management (IAWQM) model and applied it to a case study to generate optimal decision schemes for integrated water quality management within an agricultural system.

Multi-Objective Programming—[13] used goal programming and integer programming for water quality management in Gaza Strip. Agha described a project dealing with achieving an optimum mix of water from different underground wells, each having different amounts of nitrates and chlorides. A goal programming model was developed to identify the combination of wells along with the amounts of water from each well that upon mixing would result in minimizing the deviation of the amounts of chlorides and nitrates from the standards set by World Health Organization. Finally, an easy-to-use pumping schedule was developed using integer programming. [14] presented multi-objective optimization of water quality, pumps operation and storage sizing of water distribution systems. The model could be used as a decision tool for pumps operation, water quality, required storage for reliability considerations, and tank sizing decision-making.

2.2. Meta-Heuristic Programming Models in Water Quality Optimisation

Artificial Neural Networks (ANNs)—[15] developed a model using ANNs to optimise a water distribution system design problem that included water quality and reliability. [16] predicted water quality parameters at Johor River Basin in Malaysia utilizing Artificial Neural Network (ANN) modeling. The river basin was significantly degrading due to human activities as well as urbanization in and within the area. The study proposed a prediction model for total dissolved solids, electrical conductivity, and turbidity. The results showed that the proposed ANN prediction model had a great potential to simulate and predict the total dissolved solids, electrical conductivity, and turbidity with absolute mean error 10% for different water bodies.

Genetic Algorithms—[17] developed a risk minimization model to minimize the risk of low water quality along a river in the face of conflict among various stake holders. [18] developed a grey fuzzy optimization model for water quality management of river system to address uncertainty involved in fixing the membership functions for different goals of Pollution Control Agency (PCA) and dischargers. [19] presented a stochastic conflict resolution model for water quality management in reservoir-river systems. The proposed model, which was called Stochastic Varying Chromosome Length Genetic Algorithm with water Quality constraints was applied to the Ghomrud Reservoir–River system in the central part of Iran. [20] proposed a two-phase grey fuzzy optimization model for water quality management of a river system. The methodology was demonstrated with a case study of the Tunga-Bhadra river system in India. [21] developed an interval-parameter chance-constrained fuzzy multi-objective programming model for assisting water pollution control within a sustainable wetland management system under uncertainty. [22] developed a water quality management model through the integration of a genetic algorithm (GA) and a mathematical water quality model to achieve water quality goals and wastewater treatment cost optimisation in a river basin. The developed model was applied to the Youngsan River in Korea where water quality had decreased due to heavy pollutant loads from Kwangju City and surrounding areas.

Simulated Annealing (SA)—[23] presented ground water management optimization using genetic algorithms and simulated annealing: Formulation and comparison. Genetic algorithms (GA) and simulated annealing (SA), two global search techniques, were coupled with MODFLOW, a commonly used groundwater flow simulation code, for optimal management of ground water resources under general conditions.

Table 1 gives the summary of the water quality optimisation models discussed in terms of the type of optimisation model, the purpose of the model, the type of application considered, and the source.

2.3. Mathematical Programming Techniques in Water Quality Optimisation

Linear Programming (LP)—[24] presented a special method for optimization of groundwater management using linear programming. [25] presented a groundwater management problem in a coastal aquifer in Crere, Greece subject to environmental criteria using classical linear programming and heuristic optimization methodologies.

Integer Programming (IP)—[26] proposed a mixed integer optimisation approach for integrated water resources management. The proposed mixed integer linear programming model took into account the subdivided regions on the island, the subsequent localised needs for water use (including water quality) and wastewater production, as well as geographical aspects. Finally, the proposed approach was successfully applied to two Greek islands: Syros and Paros-Antiparos.

Nonlinear Programming (NLP)—[27] presented interval-parameter robust quadratic programming for water quality management under uncertainty. The developed method was applied to a case study of a water quality management under uncertainty.

Stochastic Programming—[28] presented a robust modelling approach for regional water management under multiple uncertainties.

Multi-Objective Programming—[29] applied multi-objective programming to water quality management in a river basin. They proposed two methods of multi-objective programming, the constraint method and the step method. The proposed methods were successfully applied to a basin of Tzeng-Wen River, Taiwan. [30] proposed the use of multi-objective ensemble learning to monitor drinking-water quality. They developed two ensemble learning techniques specifically for dealing with imbalanced data, where the base learners are trained by adjusting the ratio between the classes. Their first algorithm focused on over-sampling the minority class, while the second focused on under-sampling the

Table 1. Summary of water quality optimisation models.

Model Category	Model Type	Purpose	Application	Literature
Mathematical Programming	Linear Programming	To optimize water resource use in irrigation projects	Senator Nilo Coelho Project in Brazil	[3]
		For economic water quality management	Heavily Polluted Gyungan River in South Korea	[4]
		For optimal water pollution management	Lake Qionghai Watershed in China	[5]
	Integer Programming	For ground-water remediation	Wells	[6]
	Nonlinear Programming	For stream water quality management	Changsha section of Xiangjiang River	[7]
		For water quality management	Unspecified	[8]
		For synthesis and optimisation of water treatment processes	Seawater Desalination and Surface Water Treatment	[9]
	Dynamic Programming	For water quality management	Estuary of Delaware River in the USA	[10]
	Stochastic Programming	For water quality management	Chemical Industry in Binhai New Area of Tianjin, China	[11]
	Robust Programming	For agricultural water quality management	Unspecified	[12]
	Multi-objective Programming	For water quality management	Gaza Strip	[13]
		For optimisation of water quality, pumps operation and storage sizing of water distribution systems	Unspecified	[14]
Meta-Heuristic Programming	Artificial Neural Networks	To optimise a water distribution system design problem	Unspecified	[15]
		For prediction of water quality parameters	Johor River Basin in Malaysia	[16]
	Genetic Algorithms	To minimise the risk of low water quality along a river	Tunga-Bhadra River System in Southern India	[17]
		For water quality management of river system	Tunga-Bhadra River System in India	[18]
		For water quality management in reservoir-river systems	Ghomrud Reservoir–River System in Iran	[19]
		For water quality management of a river system	Tunga-Bhadra River System in India	[20]
		For assisting water pollution control	Regional Wetland	[21]
		For water quality management	Youngsan River in Korea	[22]
	Simulated Annealing	For optimisation of ground water management	Groundwater Resource	[23]

majority class. Finally, multi-objective optimisation was used for pruning the base models of such ensembles in order to maximise the prediction score without reducing generalisation performance. In the training phase, the model was trained, optimised and evaluated using hold-out validation on a given training data set. At the end, the trained model was inserted into a framework, which was used for online event detection and assessing the model's performance.

2.4. Meta-Heuristic Programming Techniques in Water Quality Optimisation

Artificial Neural Networks (ANNs)—[31] used network optimization techniques to optimize reservoir operations during both wet and dry seasons near Sao Paulo. They examined trade-off between water delivery and hydropower generation subject to meeting instream water quality objectives downstreams. [32] applied artificial neural network for water quality prediction. They applied the technique to Johor River Basin located in Johor state, Malaysia, which was significantly degrading due to human activities and development along the river basin. Their results showed that the use of neural networks can describe the behaviour of water quality parameters more accurately than linear regression.

Genetic Algorithms (GAs)—[33] developed a robust genetic algorithm approach taking into account the uncertainty of hydraulic conductivity values in determining the best groundwater remediation design. [34] applied an interactive fuzzy approach to develop a water quality management plan in a river basin for solving multi-objective optimization problems involving vague and imprecise information related to data, model formulation, and the decision maker's preferences. Their methodology was illustrated in a case study of multi-objective water quality management in the Tou-Chen River Basin in northern Taiwan. [35] presented non-adaptive and adaptive hybrid approaches for enhancing water quality management.

Simulated Annealing—[36] presented optimal groundwater management in deltaic regions using simulated annealing and neural networks. Their study dealt with the optimal management of groundwater in deltaic aquifer systems with some reference to east coastal hydro-geo-climatic conditions of India.

Table 2 gives the summary of the water quality optimisation techniques discussed with similar focus as in **Table 1**.

3. Conclusion

As discussed above, water quality optimisation models are categorised into Mathematical Programming Models and Meta-heuristic Programming Models. Usually, the Mathematical Programming models in water quality optimisation are Linear Programming, Integer Programming, Nonlinear Programming, Dynamic Programming, Stochastic Programing, Robust Programming and Multi-objective Programming models. On the other hand, the Meta-heuristic Programming models in water quality optimisation are Artificial Neural Networks, Genetic Algorithm and Simulated Annealing models. Similarly, water quality optimisation techniques are categorised into Mathematical Programming Techniques and Meta-heuristic Programming Techniques. The Mathematical Programming techniques in water quality optimisation are Linear Programming, Integer Programming, Nonlinear Programming, Stochastic Programing and

Table 2. Summary of water quality optimisation techniques.

Technique Category	Technique Type	Purpose	Application	Literature
Mathematical Programming	Linear Programming	For optimisation of groundwater management	Aquifers	[24]
		For optimisation of groundwater management	Pumping Wells	[25]
	Integer Programming	For optimisation of water resources management	Greek Islands: Syros and Paros-Antiparos	[26]
	Nonlinear Programming	For water quality management	Unspecified	[27]
	Stochastic Programming	For agricultural regional water management	Unspecified	[28]
	Multi-objective Programming	For water quality management	Basin of Tzeng-Wen River, Taiwan	[29]
		For water quality monitoring	Online Event Detection	[30]
Meta-Heuristic Programming	Artificial Neural Networks	For reservoir water quality optimisation	Reservoirs in Sao Paulo	[31]
		For water quality prediction	Johor River Basin in Malaysia	[32]
	Genetic Algorithms	For optimisation of groundwater	Groundwater Remediation Design	[33]
		For water quality management	Tou-Chen River Basin in northern Taiwan	[34]
		For water quality management	Unspecified	[35]
	Simulated Annealing	For groundwater management	Deltaic Regions	[36]

Multi-objective Programming. On the other hand, the Meta-heuristic Programming techniques in water quality optimisation are Artificial Neural Networks, Genetic Algorithm and Simulated Annealing. By way of literature review, we have looked at the existing water quality optimisation models and techniques. Application of interior-point methods of mathematical optimisation to water quality management appears rare and this needs to be addressed.

4. Future Work

This paper has revealed a rare nature of application of interior-point methods of mathematical optimisation to water quality management. Efforts will be made in the near future in an attempt to contribute to the solution of this problem.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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