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Mathematical Modeling of Leptospirosis Dynamics and Control in an Urban Environment

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Abstract

In Brazil, leptospirosis has a high incidence due to poor environmental sanitation conditions, which facilitate rodent reinfestation even after the application of chemical control measures. To reduce the risk of infection in humans, a mathematical model of leptospirosis was developed to propose strategies that combine different control measures. A sensitivity analysis was conducted to investigate different scenarios involving rodent and human populations. Four control measures were implemented: habitat improvement, chemical control, surface disinfection and reduction of environmental transmission. The implementation of control measures resulted in a significant reduction in the proportion of infected rodents and humans. The implementation of chemical control to reduce the rodent population led to a drastic reduction in that population. However, if additional measures are not implemented to permanently alter the suitability of the rodent habitat, the population tends to recover and grow until it reaches its original carrying capacity. Sensitivity analysis of control measures identified that reducing environmental transmission is the most efficient strategy to decrease the number of infected humans. In conclusion, to minimize infection in humans and rodents, it is necessary to implement strategies that reduce environmental transmission, limit rodent population growth and increase the mortality of both rodents and leptospires.

Keywords

Leptospirosis, Control, Mathematical Model

1. Introduction

Leptospirosis is a potentially fatal zoonotic disease, widely distributed worldwide,

tending to become endemic in tropical regions and triggering epidemics after periods of heavy rain and floods [1] [2]. In Brazil, this disease is neglected by public health policies and mainly affects regions with limited resources and poor sanitation infrastructure [3].

Leptospira reservoirs include domestic and wild synanthropic animals, which can become long-term asymptomatic carriers of the etiological agent. When leptospires are excreted in urine, they contaminate the external environment (water, soil or food). In this way, humans are only accidental and terminal hosts within the transmission chain [4]-[6].

The population of rodents infected with *Leptospira spp* bacteria, including *Rattus norvegicus*, *Rattus rattus* and *Mus musculus*, plays a crucial role in the spread of leptospirosis in urban areas, especially in areas with inadequate sanitation. Effective control of these populations is vital to reduce the disease incidence, although it is important to avoid inadequate measures that may result in the so-called "boomerang effect", in which the rodent population quickly recovers after temporary interventions [7] [8].

Furthermore, leptospirosis is also an occupational disease, affecting workers in various occupations, from those involved in sewage cleaning to veterinarians and farmers. In urban slums in developing countries, the rapid expansion of precarious housing has created favorable conditions for the transmission of the bacteria by rodents, making it a serious public health problem in these areas [7].

The data indicates the severity problem. Between 2004 and 2008, 17,416 cases of leptospirosis were confirmed in Brazil, with an average case fatality rate of 10.6%. In the period from 2009 to 2019, there was a significant increase, with 41,602 confirmed cases throughout the country and a fatality rate of 8.6% [5] [9]. For a more in-depth understanding of the epidemiology of infectious diseases such as leptospirosis, mathematical modeling stands out as a powerful tool, allowing the description of the disease transmission process and the evaluation of the effectiveness of control measures in infected populations [10].

The modeling dynamics for leptospirosis are similar to the modeling dynamics of a disease transmitted by mosquito, because both have a transmission agent. Therefore, rodent-borne zoonoses models follow the standard model for all vector-borne diseases [10].

Mathematical models have been used to study the spread of leptospirosis, with rodents as the main reservoir of leptospires [11]-[13]. Thus, various studies in mathematical modeling have been required to identify trends and make predictions of leptospirosis prevalence.

This study is justified by the high incidence of leptospirosis in Brazil, especially during rainy periods in areas with socioenvironmental vulnerability and inadequate sanitation infrastructure, which facilitate rodent infestation and, consequently, disease transmission. Therefore, the development of a mathematical model of leptospirosis emerges as an essential tool for simulate different scenarios and evaluating the effectiveness of interventions that can assist health authorities

in planning actions to contain the leptospirosis transmission in humans. The aim of this study is to develop a mathematical model capable of analyzing the dynamics and control of leptospirosis in humans and rodents, in order to simulate and evaluate the impact of rodent control strategies on the transmission of this disease.

2. Methods

2.1. Mathematical Model for Uncontrolled Leptospirosis

Based on a review of leptospirosis models found in the literature [11]-[15], as well as the mode of transmission of leptospirosis described in the Health Surveillance Guide, the Rodent Control Manual [7] and the Leptospirosis and Rodent Surveillance and Control Program of the municipality of São Paulo [16], we have developed a mathematical model to simulate the spread of this disease in the population of rodents and humans. The dynamics of this model are based on the following assumptions:

1) Infected rodents shed leptospires in the environment through their urine.

2) Humans become infected through direct contact with leptospires in the environment.

3) When direct contact occurs between leptospires and susceptible humans, the humans acquire the infection immediately and, shortly after antibiotics treatment, become recovered. Subsequently, they lose immunity and become susceptible again.

4) Infection of leptospirosis in susceptible rodents occurs through direct contact with leptospires in the environment, with other infected rodents and through vertical transmission.

5) The environmental infection rate, both in rodents and humans, follows a seasonal pattern, being minimal in June and maximal in January, trying to reflect the rainfall pattern and consequent floods in the southeastern region of Brazil as an example.

6) When contact occurs between infected and susceptible rodents, the susceptible rodents acquire the infection immediately and remain in this state throughout their lives.

The human population at time t is divided into three groups: $H_S(t)$ (susceptible), $H_I(t)$ (infected) and $H_R(t)$ (recovered). The rodent population is divided into two groups: $R_S(t)$ (susceptible), $R_I(t)$ (infected) and $L(t)$ represents the number of leptospires in the environment at time t. The transmission dynamics is shown in **Figure 1** and the transition rates are shown in **Table 1**.

This model is described by the system of differential equations shown below:

$$\begin{aligned}\frac{\mathrm{d}R_S(t)}{\mathrm{d}t} &= r_R\left(1-\frac{R_S(t)+R_I(t)}{K}\right)\left(R_S(t)+(1-V_1)R_I(t)\right)\\ &\quad -V_2\frac{R_S(t)R_I(t)}{R_S(t)+R_I(t)}-R_S(t)\frac{V_3(t)L(t)}{L(t)+h}-\mu_R R_S(t)\end{aligned} \tag{1}$$

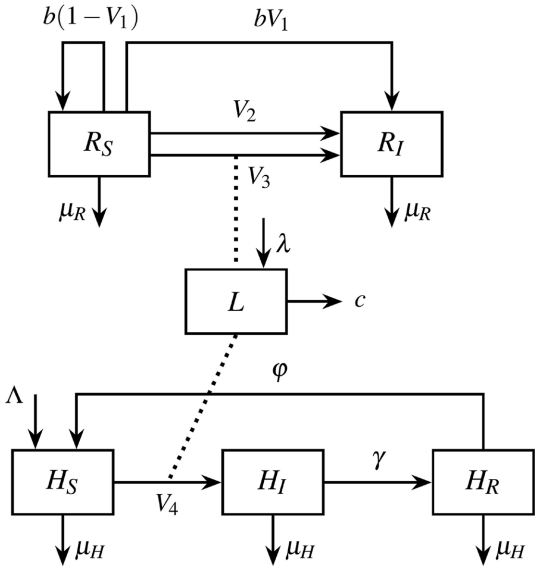


Figure 1. Compartmental scheme of the transmission dynamics of leptospirosis in the human and rodent populations.

Table 1. Model parameters.

Parameter	Meaning
μ_H	Human mortality rate
b	Rodent birth rate
μ_R	Rodent mortality rate
c	Leptospira death rate
λ	Leptospira shedding rate into environment
V_1	Proportion of vertical transmission in rodents
V_2	Horizontal transmission rate in rodents
$V_3(t)$	Environmental transmission rate in rodents
$V_4(t)$	Environmental transmission rate in humans
r_R	Rodent intrinsic growth rate
h	Number of leptospires at which environmental transmission rate is half its maximum
$\Lambda(t)$	Human birth rate
γ	Recovery rate of infected humans
φ	Rate of loss of immunity in recovered humans
K	Rodent population carrying capacity

$$\begin{aligned}\frac{\mathrm{d}R_I(t)}{\mathrm{d}t} &= r_R\left(1-\frac{R_S(t)+R_I(t)}{K}\right)V_1R_I(t)\\ &+V_2\frac{R_S(t)R_I(t)}{R_S(t)+R_I(t)}+R_S(t)\frac{V_3(t)L(t)}{L(t)+h}-\mu_RR_I(t)\end{aligned} \tag{2}$$

$$\frac{\mathrm{d}L(t)}{\mathrm{d}t} = \lambda R_I(t) - cL(t) \tag{3}$$

$$\frac{\mathrm{d}H_S(t)}{\mathrm{d}t} = \Lambda(t) + \varphi H_R(t) - H_S(t)\frac{V_4(t)L(t)}{L(t)+h} - \mu_H H_S(t) \tag{4}$$

$$\frac{\mathrm{d}H_I(t)}{\mathrm{d}t} = H_S(t)\frac{V_4(t)L(t)}{L(t)+h} - (\mu_H + \gamma)H_I(t) \tag{5}$$

$$\frac{\mathrm{d}H_R(t)}{\mathrm{d}t} = \gamma H_I(t) - (\mu_H + \varphi)H_R(t) \tag{6}$$

where

$$V_3(t) = \frac{V_3^*}{2}\left(1 + \left(\cos\frac{\pi t}{180}\right)\right) \tag{7}$$

and

$$V_4(t) = \frac{V_4^*}{2}\left(1 + \left(\cos\frac{\pi t}{180}\right)\right) \tag{8}$$

The first term in Equation (1), $r_R\left(1 - \frac{R_S(t)+R_I(t)}{K}\right)\left(R_S(t) + (1-V_1)R_I(t)\right)$ is related to the logistic growth of the rodent population, in which the population grows at a rate of r_R until it reaches the carrying capacity K. Only the proportion of individuals not infected through vertical transmission V_1 enters the susceptible compartment. These individuals die of natural causes at a rate of μ_R. On the other hand, individuals infected through vertical transmission V_1 (Equation (2)), enter the infected compartment. This modeling of the infection in offspring during breastfeeding was also used in the works by Minter *et al.* [13] [17].

The second term in Equations (1) and (2), $V_2\frac{R_S(t)R_I(t)}{R_S(t)+R_I(t)}$ is associated with infection through direct contact between the population of infected rodent and susceptible rodents (V_2). The third term, $R_S(t)\frac{V_3(t)L(t)}{L(t)+h}$, is related to infection through contact with an environment contaminated by leptospires ($V_3(t)$), where the constant h represents the leptospira load present in the environment, which is part of the saturation term $\frac{L(t)}{L(t)+h}$. This term was used in the numerical model for leptospirosis in rodents in Tanzania by Holt *et al.* [12].

It is important to distinguish between $L(t)$ and h: $L(t)$ is the state variable representing the number of leptospires in the environment at time t, whereas h is the number of leptospires in the environment at which the environmental transmission rate reaches half of its maximum value ($V_3(t)$ or $V_4(t)$), following the same definition used by Holt *et al.* [12]. As $L(t)$ increases relative to h, the term $L(t)/(L(t)+h)$ approaches 1, so that the transmission rate saturates and becomes essentially independent of further increases in environmental leptospire abundance. It should be noted that the value of h used here (10^6) was originally

estimated by Holt *et al.* [12] as a density (leptospires·ha^{-1}) in the absence of precise quantitative data, and was adopted here as an absolute value without rescaling to the study area; this substitution represents a simplification whose implications are discussed further in the limitations of this study.

In Equation (3), the compartment of leptospira is given by the rate at which rodents shed leptospires into the environment λ times the number of infected rodents, minus leptospiral mortality.

The human population is divided into three groups: susceptible, infected and recovered (Equations (4), (5) and (6)). The susceptible population grows at a birth rate Λ and increases with individuals recovering from infection. The susceptible population also decreases as individuals interact with the environment contaminated by leptospires ($V_4(t)$), in the same proportion that the infected population increases due to environmental infection. On the other hand, the population of infectious individuals decreases as some individuals recover from infection. Susceptible, infected, and recovered individuals can also die from natural causes at a rate of μ_H.

2.2. Estimating the System Parameters

The parameter values were selected from wide ranges to encompass a broad diversity of transmission scenarios. **Table 2** shows the description, units and values of the parameters described above.

2.2.1. Parameters for Transmitters

To estimate the intrinsic population growth rate of the rodent ($r_R = b - \mu_R$), it is necessary to know the birth rate (b) and rodent mortality (μ_R). The mortality rate μ_R is inversely associated with the life expectancy of rodents. One of the main species of rodent found in urban environments is the brown rat (*Rattus norvegicus*), with an average lifespan of 24 months [7]. Therefore, the calculated mortality rate is 0.00139 day^{-1}.

However, the mortality rate can be extremely high among commensal rodents, with only 5% of a rodent population living for a year. This reality also applies to domestic rodents, being particularly significant among recently weaned offspring. The high mortality rate among young individuals can result in rapid population decline [18]. Therefore, for sensitivity analysis, it is assumed that the rodent mortality rate can vary from 0.00139 day^{-1} to 0.125 day^{-1}, based on the high mortality rate of juvenile rodents found in the model by Minter *et al.* [17].

To calculate the rodent birth rate b, the study by Panti-May *et al.* [19] on the reproductive parameters of *Rattus norvegicus* in an urban slum in Salvador, Brazil, was used. In that study, the average sexually mature female gave birth to a viable offspring every 4.6 days, which corresponds to 79 viable offspring per year per sexually mature female [19]. Dividing this value by 365 days gives a birth rate of 0.216 day^{-1} and, consequently, an intrinsic growth rate of 0.215 day^{-1} (value assumed for a mortality of 0.00139 day^{-1}). Because this estimate refers to sexually mature females, its direct use as the per capita birth rate of the whole rodent pop-

ulation implicitly assumes that all individuals reproduce at this rate; therefore, it represents an upper bound for b. In the same study, females accounted for 49.6% of the captured rats (393 of 793), and 57.7% of the females showed at least one characteristic of sexual activity [19]. Weighting the birth rate by these proportions gives a population-level per capita birth rate of approximately $0.216 \times 0.496 \times 0.577 \approx 0.062$ day^{-1}, which corresponds to an intrinsic growth rate of approximately 0.060 day^{-1}. For this reason, in the sensitivity analysis, the intrinsic growth rate of rodents was varied from 0 day^{-1} to 0.215 day^{-1}, a range that includes both the population-level estimate and the upper bound.

The proportion of infected rodents will give rise to infected offspring through vertical transmission (V_1). It is assumed that, once infected, a susceptible rodent becomes infected instantly, without the need for an incubation time for the development of the infectious agents (leptospirosis). Once infected, rodents are infected for life. The estimated proportion of offspring infected through breastfeeding and born infected is 0.25 [13] [20].

In the model by Minter *et al.* [13], it is assumed that the maximum capacity of rodents that a valley (environment) can support will be 200 rodents. For sensitivity analysis, the rodent carrying capacity will vary from 200 to 2000 rodents.

In references [17] and [21], estimated daily loads of bacteria (leptospires) shed in the urine by *Rattus norvegicus* can be found. This value varies from 1.6×10^7 to 8.1×10^8 leptospires shed in urine per day. The survival time of leptospira bacteria depends on the environment where they are located. Viable leptospires have been observed to persist in water for a little 6 hours up to 180 days, so the mortality rate per day of leptospires varies from 1/180 to 4 [22]. Direct transmission between susceptible and infected rodents can occur through sexual contact, direct contact with urine or some other direct mechanism, such as bites. This horizontal transmission rate is estimated to be 0.0001 day^{-1} [17] [20]. The rate of rodents becoming infected when they come into contact with free leptospires in the environment is 0.005 day^{-1} [12].

2.2.2. Parameters for Humans

It is assumed that the human population at time t, $H(t)$, will remain constant over 365 days, and to achieve this, Equations (4), (5) and (6) are added.

$$\frac{\mathrm{d}H(t)}{\mathrm{d}t} = \frac{\mathrm{d}H_S(t)}{\mathrm{d}t} + \frac{\mathrm{d}H_I(t)}{\mathrm{d}t} + \frac{\mathrm{d}H_R(t)}{\mathrm{d}t} \tag{9}$$

$$\frac{\mathrm{d}H(t)}{\mathrm{d}t} = \Lambda - \mu_H\left(H_S(t) + H_I(t) + H_R(t)\right) \tag{10}$$

For a constant population, $\frac{\mathrm{d}H(t)}{\mathrm{d}t} = 0$. Therefore, the birth rate of humans is expressed as:

$$\Lambda(t) = \mu_H\left(H_S(t) + H_I(t) + H_R(t)\right). \tag{11}$$

Assuming that the human mortality rate is inversely proportional to life expectancy, the life expectancy in the state of São Paulo in 2022, provided by the Foun-

dation for the State System of Data Analysis (SEADE) [23], is 75.8 years. Based on this information, the human mortality would be $1/(75.8 \times 365) = 3.61 \times 10^{-5}$ per day.

The rate at which infected individuals become immune is estimated as the inverse of the infectious period. The leptospirosis infection period lasts for about 15 days and shortly after this time, the individual recovers or dies [11].

The duration of human immunity is around 360 days, with the rate of immunity loss for recovered humans returning to the susceptible state being 0.00278 per day [11].

To estimate the rate of human infection upon contact free leptospires in the environment, it is considered that humans live in places near water bodies, such as lakes, rivers, streams, etc., where leptospires can proliferate. It is assumed that the number of contacts between humans and bacteria varies between 3 and 6 days and that the effectiveness of contacts is 10^{-3} [22]. **Table 2** shows the notation, description, values, units and source of the parameters mentioned above.

Table 2. Notation, description, values, and source of the leptospirosis mathematical model parameters.

Notation	Description	Values	Source
μ_H	Human mortality rate	3.61×10^{-5} day^{-1}	[23]
μ_R	Rodent mortality rate	0.00139 day^{-1}	[7]
h	Number of leptospires at which environmental transmission rate is half its maximum	1,000,000	[12]
K	Rodent population carrying capacity	200	[13]
c	Leptospira death rate	1/180 day^{-1}	[22]
λ	Leptospira shedding rate into environment	8.1×10^{8} day^{-1}	[17] [21]
V_1	Proportion of vertical transmission in rodents	0.25	[13] [20]
V_2	Horizontal transmission rate in rodents	0.0001 day^{-1}	[17] [20]
V_3^*	Environmental transmission rate in rodents	0.005 day^{-1}	[12]
V_4^*	Environmental transmission rate in humans	0.006 day^{-1}	[22]
r_R	Rodent intrinsic growth rate	0.215 day^{-1}	[7] [19]
γ	Recovery rate of infected humans	1/15 day^{-1}	[11]
φ	Rate of loss of immunity in recovered humans	0.00278 day^{-1}	[11]

2.3. Mathematical Model for Leptospirosis with Control

In order to analyze and compare the effects of leptospirosis transmission when

implementing control strategies, four control measures were incorporated into the mathematical model. These measures were based on the results of the sensitivity analysis conducted in the section for the mathematical model for leptospirosis without control, as well as on the Health Surveillance Guide, the Rodent Control Manual [7] and the Leptospirosis and Rodent Surveillance and Control Program of the São Paulo Municipality [16]. These measures were applied to the infection dynamics of rodents, humans and the environmental burden of leptospires, with the aim of reducing the transmission of the disease.

The dynamics of this model include the following four control measures applied:

1) Habitat improvement: This control involves a set of preventive measures, which include:

Proper waste management: it is essential to correctly store waste using suitable containers that are protected against rodents. Furthermore, it is important to ensure protected disposal and transportation, preventing rodents from having access to waste.

Removal of debris and unserviceable materials: it is necessary to remove any debris or material that could serve as a shelter for rodents. This includes regularly cleaning areas prone to the accumulation of debris, thus preventing them from becoming havens for rodents.

Proper food storage: Residential, commercial or rural property owners must store food in a way that prevents rodents from accessing it. Furthermore, it is important to keep properties free of debris, construction materials or disused objects that could serve as shelter for rodents. Garbage cans must be properly sealed, and their contents must be sent to the public collection service.

Proper treatment of solid waste: solid waste must be collected, packaged and sent to the storage and treatment points defined by the competent body. In urban areas, special attention must be paid to the storage and correct disposal of household waste, as this is the main source of food for rodents in these areas.

Land maintenance: public or private lands must be closed, walled, clean and free of weeds and debris, avoiding controlled conditions for the installation and supervision of rodents.

These preventive measures aim to reduce the suitability of the habitat for rodents. These actions limit the availability of food and shelter for rodents, making their reproduction and population growth more difficult.

In the mathematical model, this control is implemented through a reduction in the intrinsic growth rate of the rodent population (r_R), in the proportion of $1-U_R$. The carrying capacity K is kept constant in all simulations, so that this intervention acts on the growth rate of the rodent population and not directly on K . To perform sensitivity analysis, the U_R value varies from 0 to 0.9.

2) Chemical control: To model the chemical control of rodents, the study carried out by Masi [24] was used. The study evaluated the effectiveness of chemical control measures implemented in Jardim Comercial, São Paulo, Brazil. Chemical

control consists of reducing the rodent population through the application of rodenticides, resulting in mortality through contact.

The mortality rate resulting from poison administration is calculated by the following equation

$$\theta(t) = A_1\left(Q + (1-Q)\mathrm{e}^{-A_2 t}\right) \tag{12}$$

The above equation, formulated by Gentry [25], describes the rate at which rodents die from rodenticide, represented by $\theta(t)$. It takes into account the fraction of rodents that would be poisoned if the rodents were unable to distinguish that they were being poisoned, represented by A_1. Furthermore, the equation considers the fraction of rodents that do not learn to distinguish the rodenticide, represented by Q. A_2 represents how quickly rodents learn to distinguish the rodenticide.

For the chemical rodent control carried out in the Jardim Comercial [24], only rodenticides with anticoagulant effects based on hydroxycoumarin were used. In Jardim Comercial, single-dose second-generation rodenticide formulations were used, as well as first-generation contact powder. However, as there are no estimates of the parameters of the Gentry equation available in the literature for this specific type of rodenticide, sensitivity analyzes were carried out in this work during the simulations to investigate the effect of these parameters on the model output. This approach was adopted as a way to deal with this limitation.

To carry out the simulation of the proposed model, it is assumed that chemical control is applied continuously 30 days after implementing measures to improve the habitat and reduce the rate of environmental transmission. This application is maintained for a maximum period of 30 days.

3) Surface disinfection: By introducing the term (d) to reduce the concentration of leptospires in the differential equations, it is possible to simulate the effect of applying sodium hypochlorite in cleaning mud resulting from floods, as well as disinfecting residential areas that have recently been flooded. This control is applied continuously 30 days after the implementation of measures to improve the habitat and reduce the rate of environmental transmission. This application is maintained for a maximum period of 30 days.

4) Reduction in the rate of environmental transmission: This control involves a set of preventive measures [7] [16], which include:

Improve urban infrastructure to reduce flooding.

Implement efficient rainwater drainage systems.

Improve drainage in construction areas.

Repair of structural damage that could provide access for rodents.

Construction of rodent-proof buildings, that is, construction in such a way that it is practically impossible for rodents to actively penetrate these facilities.

Application of defenses to support structures, such as pilotis and roof beams, and also to overhead wiring that reaches buildings. These defenses consist of tin discs shaped like a "Chinese hat" which, when placed around columns and beams,

prevent rodents from passing through, as long as they are positioned at a minimum height of 1.50 m from the ground. Flat tin discs, with a minimum radius of 40 cm, are used as insurmountable barriers for rodents in general around wires and cables.

Implementation of physical barriers in underground water, sewage, rainwater or cabling galleries.

Use of unidirectional devices in the first segment of the pipe connected to the toilet, which prevent rodents from accessing this route.

Use of metal drains fixed to the floor with permanent grates. Installation of 6 mm thick metal screens to seal vents, especially those in basements, and the nozzles of gutters and rainwater conduits.

These preventive measures aim to reduce the environmental transmission of leptospires, promoting urban infrastructure with a lower incidence of flooding and greater protection for homes.

In the mathematical model, this control is implemented through a reduction in environmental transmission ($V_4(t)$), in the proportion of $1-U_H$. To perform sensitivity analysis, the U_H value varies from 0 to 0.9.

This model is described by the system of differential equations shown below:

$$\begin{aligned}\frac{\mathrm{d}R_S(t)}{\mathrm{d}t} &= r_R(1-U_R)\left(1-\frac{R_S(t)+R_I(t)}{K}\right)\left(R_S(t)+(1-V_1)R_I(t)\right)\\ &\quad -V_2\frac{R_S(t)R_I(t)}{R_S(t)+R_I(t)}-R_S(t)\frac{V_3(t)L(t)}{L(t)+h}-\mu_R R_S(t)-\theta R_S(t)\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\mathrm{d}R_I(t)}{\mathrm{d}t} &= r_R(1-U_R)\left(1-\frac{R_S(t)+R_I(t)}{K}\right)V_1R_I(t)\\ &\quad +V_2\frac{R_S(t)R_I(t)}{R_S(t)+R_I(t)}+R_S(t)\frac{V_3(t)L(t)}{L(t)+h}-\mu_R R_I(t)-\theta R_I(t)\end{aligned} \tag{14}$$

$$\frac{\mathrm{d}L(t)}{\mathrm{d}t} = \lambda R_I(t)-(c+d)L(t) \tag{15}$$

$$\frac{\mathrm{d}H_S(t)}{\mathrm{d}t} = \Lambda(t)+\varphi H_R(t)-H_S(t)\frac{V_4(t)(1-U_H)L(t)}{L(t)+h}-\mu_H H_S(t) \tag{16}$$

$$\frac{\mathrm{d}H_I(t)}{\mathrm{d}t} = H_S(t)\frac{V_4(t)(1-U_H)L(t)}{L(t)+h}-(\mu_H+\gamma)H_I(t) \tag{17}$$

$$\frac{\mathrm{d}H_R(t)}{\mathrm{d}t} = \gamma H_I(t)-(\mu_H+\varphi)H_R(t) \tag{18}$$

where

$$V_3(t) = \frac{V_3^*}{2}\left(1+\left(\cos\frac{\pi t}{180}\right)\right) \tag{19}$$

and

$$V_4(t) = \frac{V_4^*}{2}\left(1+\left(\cos\frac{\pi t}{180}\right)\right) \tag{20}$$

The values of the control parameters shown in Table 3 were chosen arbitrarily, dividing them into two sets of values for each control parameter, except for the parameter A_2 which remains constant in both scenarios. This choice is intended to represent two different scenarios:

1) In the first "Intense Control" scenario, higher values were adopted for the parameters, showing the implementation of control measures in a more incisive and rigorous manner.

2) In the second "Moderate Control" scenario, reduced values were used for the same parameters, outlining a context in which control measures are less intense.

The purpose of these scenarios is to analyze how the population of rodents and leptospires, as well as their interaction with the human population, respond to different values of control parameters.

Table 3. Notation, description, and values of the control parameters of the leptospirosis mathematical model for the intense and moderate control scenarios.

Notation	Description	Values	
		Intense control	Moderate control
U_R	Proportion of reduction in intrinsic growth of rodents	0.9	0.4
U_H	Proportion of reduction in environmental transmission	0.9	0.4
A_1	Fraction of rodents that would be poisoned	0.8	0.4
Q	Fraction of rodents that do not learn to distinguish poison	0.2	0.1
A_2	How quickly rodents learn to distinguish poison	0.02 day^{-1}	0.02 day^{-1}
d	Rate of reduction in leptospira concentration	0.9 day^{-1}	0.4 day^{-1}

2.4. Numerical Implementation and Sensitivity Analysis

All simulations and analyses were performed in R, version 4.5.2 [26]. The systems of ordinary differential equations of the model without control (Equations (1)-(8)) and with control (Equations (13)-(20)) were solved numerically with the package deSolve [27], using the Runge-Kutta method of fourth and fifth order.

The sensitivity analysis was carried out by means of partial rank correlation coefficients (PRCC), calculated with the package epiR [28]. The PRCC measures the strength of the monotonic association between each input parameter and the model output after removing the effects of the remaining input parameters, and its sign indicates whether the output increases (positive PRCC) or decreases (neg-

ative PRCC) as the parameter increases. The outputs considered were the cumulative numbers of new infections in humans and in rodents over 365 days. The input parameters were varied within the ranges described in Sections 2.2 and 2.3, which were obtained from the literature. In the sensitivity analysis of the model without control, all control parameters were set to zero.

3. Results

3.1. Simulation of the Mathematical Model for Uncontrolled Leptospirosis

Figures 2-4 show the computer simulation of the dynamics of the leptospirosis model for the case of persistence of the infectious disease in human, rodent and bacterial populations.

For consistency with the parameters h and K adopted from Holt *et al.* [12] and Minter *et al.* [13], respectively—both originally defined as densities per unit area—the state variables and initial conditions used in the simulations (e.g., $R_S = 150$, $R_I = 1$, $H_S = 1000$) are assumed to refer to a reference area of 1 hectare, matching the spatial scale implicit in these literature-derived parameters. This assumption is adopted here as a practical simplification, without requiring a full reformulation of the model in density terms, and is discussed further as a limitation of the study.

The highest incidence in humans occurs in the first few months and then shows an annual seasonality, with high peaks at the beginning of the year (Figure 2). In the case of rodents (Figure 3), there is an oscillation in the number of infected over time, with a significant increase. As for the leptospira population (Figure 4), there is a slight oscillation, but a significant increase.

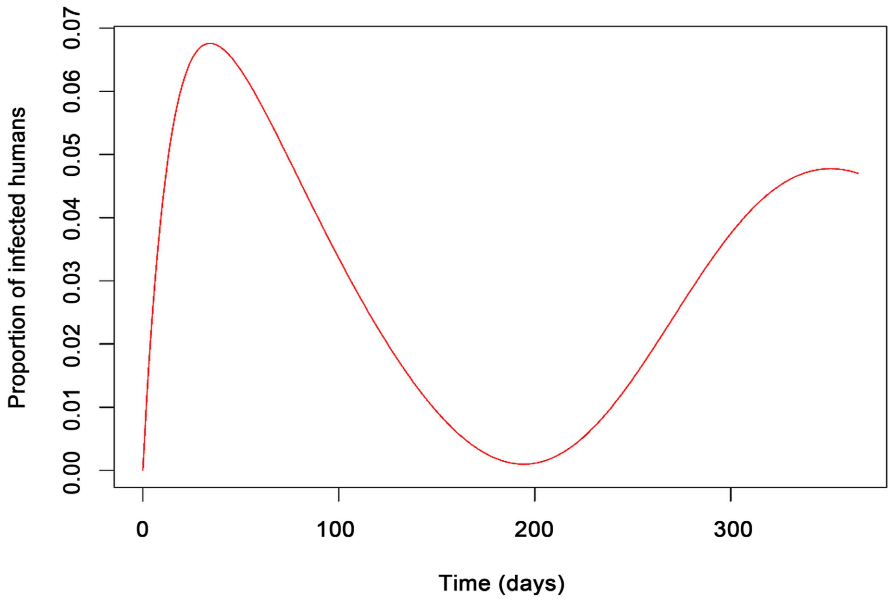


Figure 2. Proportion of infected humans versus time in the model without control, with initial conditions $R_S = 150$, $R_I = 1$, $H_S = 1000$, $H_I = 0$, $H_R = 0$ and $L = 0$.

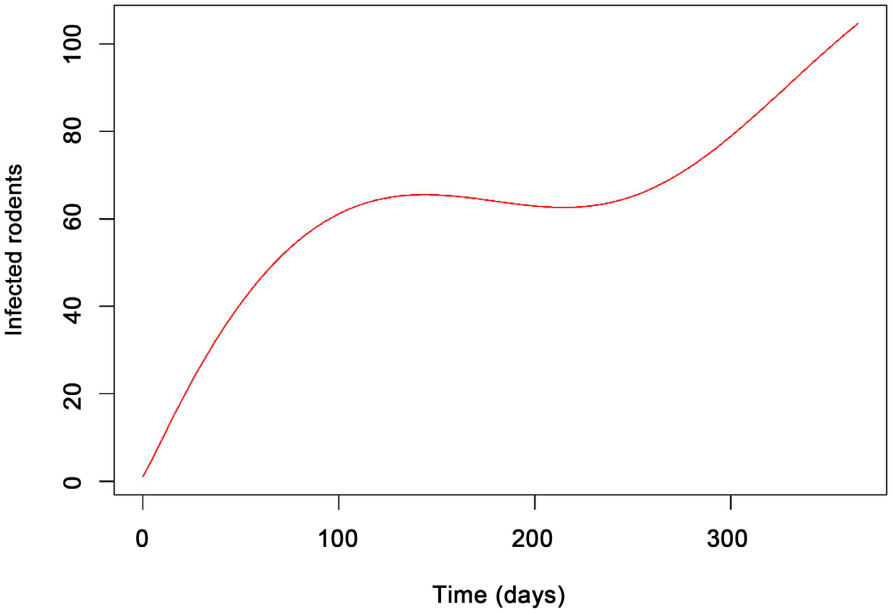


Figure 3. Number of infected rodents versus time in the model without control, with initial conditions $R_S = 150$, $R_I = 1$, $H_S = 1000$, $H_I = 0$, $H_R = 0$ and $L = 0$.

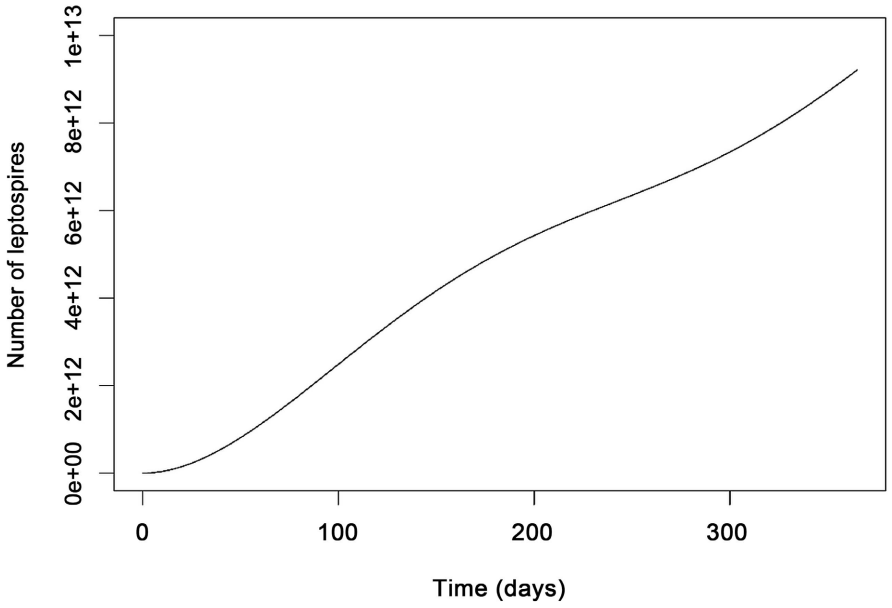


Figure 4. Number of leptospires versus time with initial conditions $R_S = 150$, $R_I = 1$, $H_S = 1000$, $H_I = 0$, $H_R = 0$ and $L = 0$.

Sensitivity Analysis for the Model without Control

The cumulative number of new infections was numerically calculated over 365 days. **Figure 5** shows that the number of cumulative cases of infected humans decreases as mouse mortality increases (μ_R) or as leptospira mortality increases (c). This is indicated by the negative sign of the Partial Rank Correlation Coefficient (PRCC), which qualitatively reflects the negative correlation between the input parameter and the output variable.

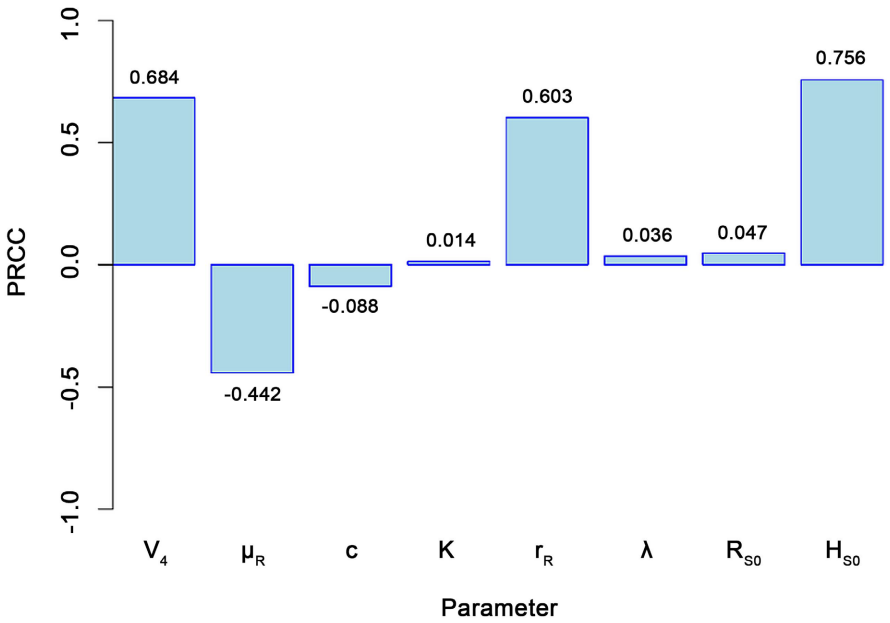


Figure 5. Partial Rank Correlation Coefficients (PRCC) for the cumulative number of new infections in humans, considering the following input variables: the environmental transmission rate in humans (V_4), the rodent mortality rate (μ_R), the leptospira death rate (c), the leptospira shedding rate in the environment (λ), the carrying capacity (K), the intrinsic growth rate of rodents (r_R), the initial number of susceptible rodents (R_{S0} , min = 50, max = 150), and the initial number of susceptible humans (H_{S0} , min = 500, max = 1000). All control parameters mentioned in Table 3 were set to zero.

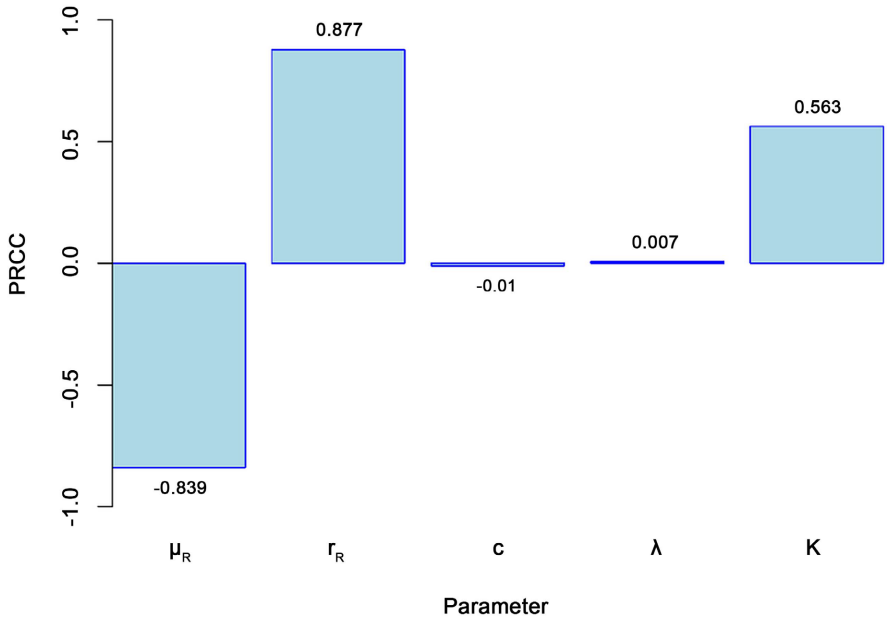


Figure 6. Partial Rank Correlation Coefficients (PRCC) for the cumulative number of new infections in rodents, considering the following input variables: the rodent mortality rate (μ_R), the intrinsic growth rate of rodents (r_R), the leptospira death rate (c), the leptospira shedding rate in the environment (λ), and the carrying capacity (K). All control parameters mentioned in Table 3 were set to zero.

On the other hand, when the values of the environmental transmission rate in humans (V_4), the excretion rate of leptospires in the environment (λ), the carrying capacity of the rodent population (K), the intrinsic growth rate of rodents (r_R), the initial number of susceptible rodents (R_{S0}) and the initial number of susceptible humans H_{S0} increase, the number of cumulative cases of infected humans also increases.

Figure 6 shows the PRCC in relation to the number of cumulative cases of infected rodents. The sign of the PRCC was negative for rodent mortality (μ_R) and slightly so for leptospira mortality (c). It was positive for the rate at which rodents eliminate leptospires into the environment (λ), the carrying capacity K and intrinsic growth rate of rodents (r_R).

When examining the PRCC values, it can be seen that decreasing the intrinsic growth rate of rodents (r_R) and increasing rodent mortality (μ_R) reduce the number of infected individuals in both populations. In contrast, reducing the rate of environmental transmission to humans (V_4) decreases only the number of infected humans, because the model contains no feedback from humans to rodent transmission.

3.2. Simulation of the Mathematical Model for Leptospirosis with Control

The values of the moderate control and intense control parameters, as highlighted in Table 3, were applied to Figure 7 and Figure 8. There was a considerable reduction in both the infected rodent population and the proportion of infected humans when the control measures were adopted. The prevalence of infection in humans fell by 24.05% after one year, when comparing the model with moderate control to the model without interventions. Similarly, the prevalence in humans fell by 82.49% after one year when comparing the model with intense control to the model without interventions.

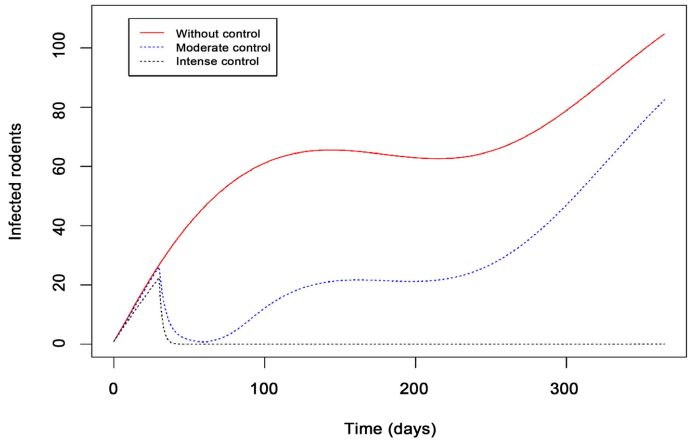


Figure 7. Number of infected rodents versus time in the scenarios without control, with moderate control and with intense control (control parameters in Table 3), with initial conditions $R_S = 150$, $R_I = 1$, $H_S = 1000$, $H_I = 0$, $H_R = 0$ and $L = 0$.

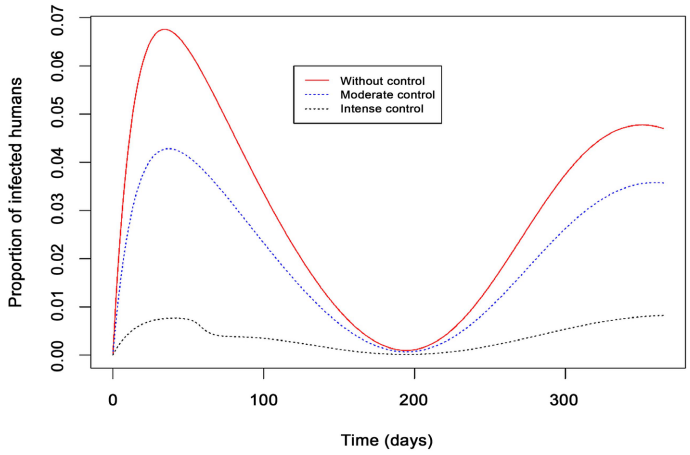


Figure 8. Proportion of infected humans versus time in the scenarios without control, with moderate control and with intense control (control parameters in **Table 3**), with initial conditions $R_S = 150$, $R_I = 1$, $H_S = 1000$, $H_I = 0$, $H_R = 0$ and $L = 0$.

When analyzing the number of infected rodents between the model with moderate control and the model without control, there was a 21.33% reduction after one year. When comparing the number of infected rodents between the model with intense control and the model without control, there was a highly significant reduction of 99.92% after one year.

In **Figure 9**, the strategies previously described in section 2.3 were implemented, including chemical control and habitat improvement. Looking at the same figure when U_R is zero, there is a significant decrease in the rodent population at the beginning. However, after chemical control is completed, the population grows again until it reaches its carrying capacity again. This suggests that chemical control alone is not enough to keep the rodent population down in the long term.

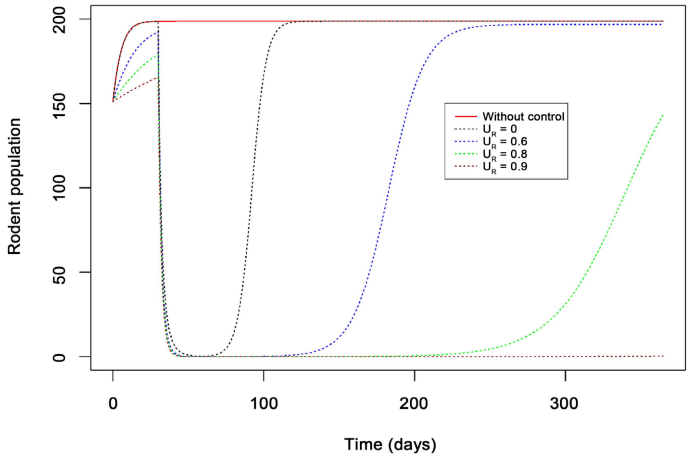


Figure 9. Rodent population versus time with initial conditions: $R_S = 150$, $R_I = 1$, $H_S = 1000$, $H_I = 0$, $H_R = 0$ and $L = 0$. The chemical control parameter values were: $A_1 = 0.8$, $A_2 = 0.02$ and $Q = 0.2$.

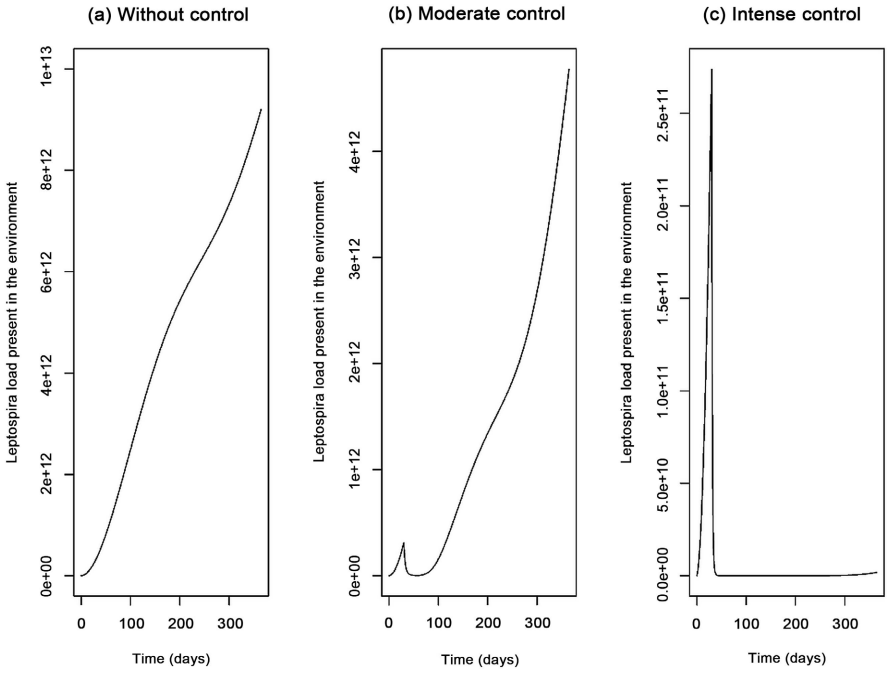


Figure 10. *Leptospira* load versus time with initial conditions $R_S = 150$, $R_I = 1$, $H_S = 1000$, $H_I = 0$, $H_R = 0$ and $L = 0$.

As the control parameter U_R increases, the rodent population decreases and takes longer to recover after chemical control. This implies that increasing the intensity of habitat improvement strategies is associated with a longer reduction in the rodent population. Consequently, when U_R is equal to 0.9, a near-zero rodent abundance is observed during the simulated period. This result does not establish permanent elimination of the rodent population, particularly because reinvasion from surrounding areas lies outside the scope of the model.

Figure 10 also shows a decrease in the leptospira load present in the environment when control measures are applied. The leptospira population without any control is 9.21×10^{12}. However, by applying moderate control, this load is reduced to 4.78×10^{12}, and with intense control, the presence of leptospires is practically eliminated, reaching close to zero.

Sensitivity Analysis for the Model with Control

Figure 11 shows that the cumulative number of infected human cases decreases as most control parameters increase, as indicated by their negative PRCC values. This reduction is especially pronounced for the proportion of reduction in environmental transmission (U_H), followed by the fraction of poisoned rodents (A_1), the rate of reduction in leptospire concentration (d) and the proportion of reduction in intrinsic rodent growth (U_R). In contrast, the PRCC of the speed at which rodents learn to distinguish the poison (A_2) is positive (+0.231). A higher value of A_2 means that rodents learn to avoid the rodenticide more quickly, which reduces the mortality caused by chemical control (Equation (12)) and, consequently, increases the number of infected humans. Therefore, A_2 does not

represent the intensity of an intervention, but a characteristic of the rodent population that reduces the effectiveness of chemical control. These results show that reducing the suitability of habitat for rodents, improving urban infrastructure to minimize flooding, establishing more protected housing, cleaning surfaces that have suffered flooding and implementing chemical control measures to reduce the rodent population would be effective actions to reduce the number of infected humans.

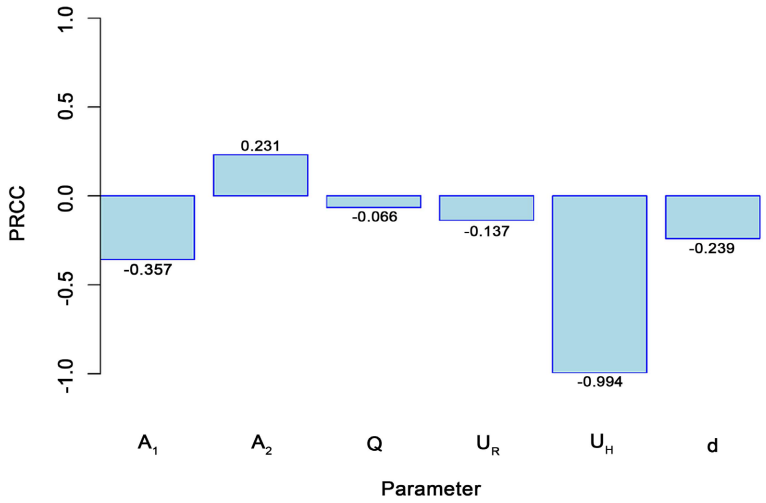


Figure 11. Partial Rank Correlation Coefficients (PRCC) for the cumulative number of new infections in humans, considering the following input variables: the proportions of reduction in intrinsic rodent growth (U_R) and environmental transmission (U_H), the fractions of rodents poisoned (A_1) and those that do not learn to distinguish poison (Q), the speed of rodent learning to distinguish poison (A_2), and the rate of reduction in leptospira concentration (d).

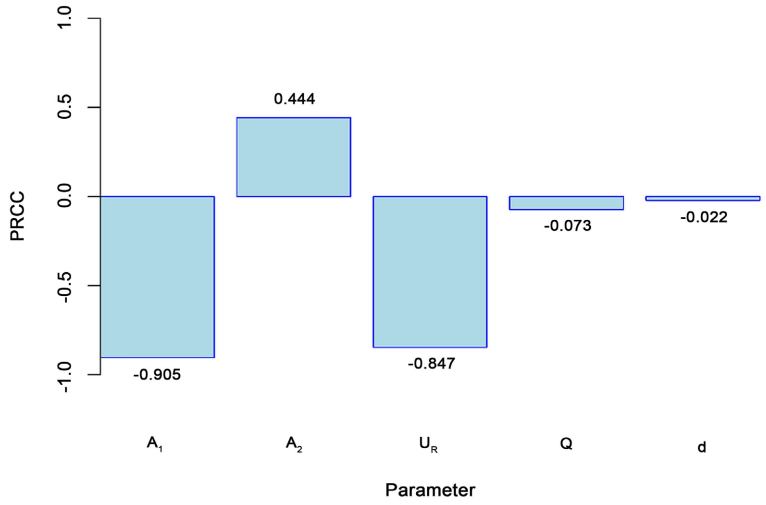


Figure 12. Partial Rank Correlation Coefficients (PRCC) for the cumulative number of new infections in rodents, considering the following input variables: the proportion of reduction in intrinsic rodent growth (U_R), the fractions of rodents poisoned (A_1) and those that do not learn to distinguish poison (Q), the speed of rodent learning to distinguish poison (A_2), and the rate of reduction in leptospira concentration (d).

Figure 12 also shows that the negative PRCC values are associated with a decrease in the number of cumulative cases of infected rodents as the control parameters are increased, especially the fraction of poisoned rodents (A_1) and the proportion of reduction in intrinsic rodent growth (U_R). As observed for humans, the PRCC of A_2 is positive (+0.444), indicating that faster learning to avoid the rodenticide reduces the effectiveness of chemical control and increases the number of infected rodents. These results show that the combination of chemical control with habitat improvement measures that reduce the growth rate of the rodent population would be ideal for reducing the population of infected rodents.

4. Discussion

Based on the results presented above, we will now discuss the main factors that influence the dynamics of the spread of leptospirosis in the human and rodent population.

The four routes of transmission of leptospirosis (V_1 , V_2 , V_3 and V_4) are responsible for the spread of this infection in animals and humans. It is important to note that environmental transmission in rodents is not solely responsible for the endemic occurrence of the infection in this species, but can contribute when associated with vertical and horizontal transmission. However, in the dynamics of transmission in humans, environmental transmission is solely responsible for the spread of the infection in the population.

The mathematical model proposed here showed the dynamics of the spread of leptospirosis in the human and rodent population, with the highest peaks of infection in humans occurring at the beginning of the year (this would correspond to times of heavy rain in São Paulo) and its minimum value occurring in the middle of the year, as shown in Figure 2 and Figure 3.

The sensitivity analysis of the uncontrolled model (Figure 5 and Figure 6) suggests that different measures act on different components of the transmission cycle. Reducing the rate of environmental transmission to susceptible humans (V_4) decreases infection in humans only, because humans are accidental and terminal hosts and the model contains no feedback from humans to rodent transmission. In contrast, decreasing the intrinsic growth rate of rodents (r_R) and increasing the rodent mortality rate (μ_R) reduce infection in rodents and, consequently, the environmental load of leptospires and the infection in humans. Increasing the mortality rate of leptospires (c) mainly reduces infection in humans (Figure 5), with only a slight effect on infection in rodents (Figure 6).

These results reinforce the recommended practices in integrated rodent management, which involves combining chemical control measures, such as deratization, with the adoption of preventive and corrective measures in the environment. These actions aim to prevent the establishment and growth of new rodent colonies.

Figures 7-10 show that control measures (habitat improvement, chemical control, surface disinfection and reduction in the rate of environmental transmission) are effective in reducing the number of infected humans, the number of infected

rodents and the number of leptospires present in the environment. However, in **Figure 9**, when the proportion of reduction in intrinsic rodent growth (U_R) is zero, there is an existing problem in rodent control actions, which is the re-establishment or even an increase in the rodent population, since the rodenticide has no lasting effects and mortality decreases over time. In other words, these results show that chemical control alone is not enough to keep the rodent population at low levels. Furthermore, in **Figure 9**, a near-zero rodent abundance during the simulated period is observed when the proportion of reduction in intrinsic rodent growth (U_R) reaches a value of 0.9. In the model, this result reflects a strong and sustained reduction in the growth rate of the rodent population, representing intense habitat improvement; it does not establish permanent elimination, since re-invasion is not represented in the model.

These findings are in line with the results presented in the study carried out by Masi [24], in which it was observed that six months after the implementation of chemical control measures, there was a rapid recovery in building infestation rates. The author reports that reinfestation by rodents was similar to that found in other studies, in which the average time for rodent populations to recover after chemical control ranged from six months to one year. This recovery is probably related to the persistence of favorable environmental conditions, which results in the continued availability of food and shelter for the rodents. These factors favor population growth again. Based on these results, it is clear that, in order to effectively reduce the transmission of leptospirosis, rodent control actions must be carried out continuously, without temporary interruptions, covering the entire proposed area [29].

In **Figure 11** and **Figure 12** it was possible to quantitatively analyze which of the control measures was most effective in reducing the number of infected humans and rodents. It is notable that the most effective control measure for reducing the number of infected humans is reducing environmental transmission (U_H). In addition, reducing the intrinsic growth of rodents (U_R) and chemical control have also shown significant efficiency in reducing leptospirosis transmission. The positive PRCC values of A_2 indicate that the effectiveness of chemical control decreases when rodents quickly learn to avoid the rodenticide.

However, although in this study it was possible to determine the efficiency of control measures in reducing the transmission of leptospirosis through simulation and sensitivity analysis of the control measures tested, there are few empirical studies that analyze the effectiveness of rodent control programs used in the eradication of zoonoses - something that shows that this is a complex subject and still little explored in the academic literature. This fact is corroborated by Graco [30] who highlights the need for more research to understand how control programs can impact the incidence of rodent-borne zoonoses, with a view to reducing or even eliminating this transmission.

It is important to note that most studies on rodent control programs used to eradicate zoonoses have not evaluated the effectiveness of the measures adopted.

In this study, assessing effectiveness was only possible by simulating a mathematical model for each control method. Sensitivity analysis revealed which measures are significant in reducing the number of infected individuals. This highlights the importance of mathematical models as valuable tools for epidemiology. These models make it possible to simulate the behavior of the variables involved and identify those that have the greatest impact on the spread of leptospirosis.

This study has some limitations that should be acknowledged. Several key parameters, including h and the environmental transmission rate in rodents (V_3^*), were adapted from a model originally developed for a rodent species (*Mastomys natalensis*) in Tanzania [12], and the rodent carrying capacity (K) was obtained from a model developed for a distinct urban slum environment [13]. These parameters were not recalibrated for the specific ecological and urban conditions of São Paulo, and, as discussed in Section 3.1, their use assumes an implicit reference area consistent with the spatial scale of the original studies. In particular, the value adopted for h was itself derived by its original authors in the absence of precise quantitative data [12]. Although the wide parameter ranges used in the sensitivity analysis were intended to encompass part of this uncertainty, direct calibration or validation of the model against Brazilian leptospirosis surveillance data, such as the case counts and case-fatality rates cited in the Introduction, was beyond the scope of this study and represents an important direction for future work.

Additionally, the values used to define the intense and moderate control scenarios (**Table 3**), as well as the parameters of the Gentry equation governing chemical control, were chosen to illustrate plausible ranges of intervention effort rather than derived from empirical field data specific to rodent control programs in urban Brazil, given the scarcity of published estimates for these parameters. Consequently, the quantitative reductions reported here, such as the percentage decrease in human and rodent infection under each scenario, should be interpreted as indicative of the relative effectiveness of different control strategies rather than as precise predictions. Future studies incorporating field-validated control parameters and locally calibrated transmission and environmental parameters would strengthen the applicability of this model to public health decision-making in Brazilian urban settings.

5. Conclusions

The main objective of this work was to develop and implement a mathematical model to analyze the dynamics and control of leptospirosis in human and rodent populations. To achieve this, differential equations were used to describe the transmission of the disease and sensitivity analysis, specifically partial rank correlation coefficients (PRCC), to investigate the effects of control strategies on both populations.

The mathematical model of leptospirosis allowed for a comprehensive description and analysis of the transmission dynamics between rodent populations, humans and leptospires. As well as considering direct transmission between rodents,

the model also incorporated environmental (indirect) transmission in the human and rodent population.

The sensitivity analysis of the uncontrolled model suggested that limiting rodent population growth and increasing rodent mortality reduce infection in both rodents and humans, whereas reducing environmental transmission to humans and increasing the mortality of leptospires mainly reduce infection in humans.

The analysis of the process of leptospirosis transmission between humans and rodents using the mathematical model with control showed computationally that chemical control alone is not enough to keep the rodent population at low levels, since the population recovers after the end of the application. It is therefore essential to adopt continuous control measures, constantly evaluating the effectiveness of these actions.

The results found in the simulation of this mathematical model offer a valuable insight into the complexity of leptospirosis transmission and highlight the importance of continuously implementing and evaluating control measures to effectively combat the disease.

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Author Contributions

C.G.A.L.: conceptualization, methodology, software, formal analysis, writing—original draft. R.A.D. and O.S.B.: methodology, writing—review and editing. M.A.: conceptualization, methodology, supervision, writing—review and editing.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Brazil, Ministry of Health (2010) Infectious and Parasitic Diseases: Pocket Guide. 8th Edition, Ministry of Health.

[2] Levett, P.N. (2005) Leptospirosis. In: Mandell, G.L., Bennett, J.E. and Dolin, R., Eds., *Principles and Practice of Infectious Diseases* (6*th Edition*), Elsevier, 2789-2795.

[3] Martins, M.H.D.M. and Spink, M.J.P. (2020) A Leptospirose Humana Como Doença Duplamente Negligenciada No Brasil. *Ciência & Saúde Coletiva*, **25**, 919-928. https://doi.org/10.1590/1413-81232020253.16442018

[4] Andersen-Ranberg, E.U., Pipper, C. and Jensen, P.M. (2016) Global Patterns of Leptospira Prevalence in Vertebrate Reservoir Hosts. *Journal of Wildlife Diseases*, **52**, 468. https://doi.org/10.7589/2014-10-245

[5] State of Bahia, Department of Health (2021) Boletim Epidemiológico: Leptospirose no Estado da Bahia, No. 01/2021. Secretaria da Saúde do Estado da Bahia, Salvador. https://www.ba.gov.br/saude/sites/site-sesab/files/migracao_2024/arquivos/wp-con-

tent/uploads/2017/11/boletimLepto2021_no01.pdf

[6] Mwachui, M.A., Crump, L., Hartskeerl, R., Zinsstag, J. and Hattendorf, J. (2015) Environmental and Behavioural Determinants of Leptospirosis Transmission: A Systematic Review. *PLOS Neglected Tropical Diseases*, **9**, e0003843. https://doi.org/10.1371/journal.pntd.0003843

[7] Brazil, Ministry of Health, National Health Foundation (2002) Rodent Control Manual. Ministry of Health, Brasília.

[8] Buckle, A.P. and Smith, R.H. (2015) Rodent Pests and Their Control. CABI.

[9] Brazil, Ministry of Health (2022) Guia de Vigilância Epidemiológica. 5th Edition, Ministry of Health, Brasília.

[10] Keeling, M.J. and Rohani, P. (2008) Modeling Infectious Diseases in Humans and Animals. Princeton University Press.

[11] Triampo, W. (2007) A Simple Deterministic Model for the Spread of Leptospirosis in Thailand. *International Journal of Biomedical Sciences*, **2**, 22-26.

[12] Holt, J., Davis, S. and Leirs, H. (2006) A Model of Leptospirosis Infection in an African Rodent to Determine Risk to Humans: Seasonal Fluctuations and the Impact of Rodent Control. *Acta Tropica*, **99**, 218-225. https://doi.org/10.1016/j.actatropica.2006.08.003

[13] Minter, A., Diggle, P.J., Costa, F., Childs, J., Ko, A.I. and Begon, M. (2018) A Model for Leptospire Dynamics and Control in the Norway Rat (*Rattus norvegicus*) the Reservoir Host in Urban Slum Environments. *Epidemics*, **25**, 26-34. https://doi.org/10.1016/j.epidem.2018.05.002

[14] Zaman, G., Khan, M.A., Islam, S., Chohan, M.I. and Jung, I.H. (2012) Modeling Dynamical Interactions between Leptospirosis Infected Vector and Human Population. *Applied Mathematical Sciences*, **6**, 1287-1302.

[15] Pongsumpun, P. (2012) Mathematical Model for the Transmission of Leptospirosis in Juvenile and Adults Humans. *International Journal of Mathematical and Computational Sciences*, **6**, 1639-1644.

[16] São Paulo, Municipal Health Department, Health Surveillance Coordination (2013) Surveillance and Control Program for Leptospirosis and Rodents in the Municipality of São Paulo. Municipal Health Department.

[17] Minter, A., Costa, F., Khalil, H., Childs, J., Diggle, P., Ko, A.I., *et al.* (2019) Optimal Control of Rat-Borne Leptospirosis in an Urban Environment. *Frontiers in Ecology and Evolution*, **7**, Article 209. https://doi.org/10.3389/fevo.2019.00209

[18] Brooks, J.E. and Jackson, W.B. (1973) A Review of Commensal Rodents and Their Control. *C R C Critical Reviews in Environmental Control*, **3**, 405-453. https://doi.org/10.1080/10643387309381607

[19] Panti-May, J.A., Carvalho-Pereira, T.S.A., Serrano, S., Pedra, G.G., Taylor, J., Pertile, A.C., *et al.* (2016) A Two-Year Ecological Study of Norway Rats (*Rattus norvegicus*) in a Brazilian Urban Slum. *PLOS ONE*, **11**, e0152511. https://doi.org/10.1371/journal.pone.0152511

[20] Minter, A., Diggle, P.J., Costa, F., Childs, J., Ko, A.I. and Begon, M. (2017) Evidence of Multiple Intraspecific Transmission Routes for *Leptospira* Acquisition in Norway Rats (*Rattus norvegicus*). *Epidemiology and Infection*, **145**, 3438-3448. https://doi.org/10.1017/s0950268817002539

[21] Costa, F., Wunder, E.A., De Oliveira, D., Bisht, V., Rodrigues, G., Reis, M.G., *et al.* (2015) Patterns in Leptospira Shedding in Norway Rats (*Rattus norvegicus*) from

Brazilian Slum Communities at High Risk of Disease Transmission. *PLOS Neglected Tropical Diseases*, **9**, e0003819. https://doi.org/10.1371/journal.pntd.0003819

[22] Bacca, C.M.C.D. (2014) Modelación matemática de la Leptospirosis e estruturas de R0 in analogous models. Ph.D. Thesis, Universidad de Sonora.

[23] Fundação Sistema Estadual de Análise de Dados (SEADE) (2023) Esperança de vida volta a crescer no Estado de São Paulo em 2022. SP Demográfico, Ano 23, No. 01, Fundação Seade, São Paulo. https://mortalidade.seade.gov.br/integra/?analise=esperanca-vida-volta-crescer-estado-sao-paulo-2022

[24] Masi, E. (2006) Avaliação da efetividade das ações de controle de roedores na região do Campo Limpo, Município de São Paulo/SP. Ph.D. Thesis, Universidade de São Paulo.

[25] Gentry, J.W. (1971) Evaluation of Rat Eradication Programs. *Environmental Science & Technology*, **5**, 704-709. https://doi.org/10.1021/es60055a005

[26] R Core Team (2025) R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing. https://www.R-project.org/

[27] Soetaert, K., Petzoldt, T. and Setzer, R.W. (2010) Solving Differential Equations in *R*: Package Desolve. *Journal of Statistical Software*, **33**, 1-25. https://doi.org/10.18637/jss.v033.i09

[28] Stevenson, M. and Sergeant, E. (2026) epiR: Tools for the Analysis of Epidemiological Data. https://doi.org/10.32614/CRAN.package.epiR

[29] São Paulo, Municipal Health Department, Health Surveillance Coordination (COVISA) (2005) Programa de Vigilância e Controle de Roedores: Sistematização de Procedimentos de Campo. Prefeitura do Município de São Paulo, São Paulo. https://docs.bvsalud.org/biblioref/sms-sp/2012/sms-6617/sms-6617-3062.pdf

[30] Graco, C.Z. (2021) Ecologia de populações de Rattus norvegicus urbanos: Ecologia como base para one health. Ph.D. Thesis, Universidade Federal da Bahia.