Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2026, 14(11), 4312-4334
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Information-Theoretic Criterion for Physical Model Selection: Mathematical Foundations and Independent Experimental Tests

Boris Menin

Refrigeration Expert, Beer-Sheva, Israel
Email: meninbm@gmail.com

How to cite this paper: Menin, B. (2026) Information-Theoretic Criterion for Physical Model Selection: Mathematical Foundations and Independent Experimental Tests. *Journal of Applied Mathematics and Physics*, **14**, 4312-4334.
https://doi.org/10.4236/jamp.2026.1411211

Received: October 5, 2026
Accepted: November 7, 2026
Published: November 10, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

When constructing a physical model, the researcher inevitably limits the number of quantities used to describe the phenomenon under study. This paper develops the Finite Information Quantity (FIQ) approach, which allows for the quantitative assessment of the information limitation arising from such a choice. A physical model is considered a finite representation of the system under study, the information content of which is determined by the set of physical quantities used. Based on this, an expression for comparative uncertainty is derived and the value of ε_{opt} is determined. This value is calculated directly from the information structure of the model before the experiment and without using measurement results. The equiprobability of informationally indistinguishable FIQs is justified by the principle of maximum entropy; the invariance of the criterion under changes in units of measurement and informationally equivalent representations of the model is also examined. For experimental verification, a procedure is proposed for independently determining experimental comparative uncertainty ε_{exp} from published data and comparing it with ε_{opt} without adjusting FIQ parameters. The method was applied to measurements of liquid zirconium density, heat transfer in a bulk layer, and two independent measurements of the gravitational constant. The resulting $\varepsilon_{exp}/\varepsilon_{opt}$ ratios were ≈2.92 for zirconium density and ≈4.44 and 4.43 for the two gravitational-constant measurements. Analysis of the heat transfer data revealed the need to independently determine the experimental uncertainty and the range of possible values of the measured quantity. The results demonstrate that the FIQ defines a priori computable benchmark for the information content of a physical model, comparable with the achieved experimental uncertainty without adjusting FIQ parameters. ε_{opt} is treated as a descriptive balance benchmark, not as a lower bound.

Keywords

Finite Information Quantity (FIQ), Physical Model Selection, Information Theory, Comparative Uncertainty, Model Information Content, Maximum Entropy, Measurement Uncertainty, Experimental Verification

1. Introduction

A mathematical model is one of the fundamental tools for studying the physical world. It describes the phenomenon under study using a limited set of physical quantities, relationships, and parameters selected by the researcher. Therefore, constructing a model inevitably involves a tradeoff between simplicity and comprehensiveness of description. A model that is too simple may fail to capture essential properties of the object, while excessive complexity increases the number of parameters, the volume of experimental data, and calculations, without necessarily improving the description of new data.

A number of well-established methods have been developed for selecting the optimal model complexity. The Akaike Information Criterion (AIC) considers the goodness of fit of the model to observations and imposes a penalty for the number of estimated parameters [1]. The Bayesian Information Criterion (BIC) uses a similar principle, but the magnitude of the penalty also depends on sample size [2]. The Minimum Description Length (MDL) principle links the quality of a model to the length of the combined description of the model and the data [3]. Cross-validation evaluates a model's ability to describe data not used to adjust its parameters; under certain conditions, it is asymptotically related to the AIC [4]. Regularization methods address a related problem: ridge regression reduces the instability of parameter estimates when input variables depend on each other [5], while LASSO can simultaneously constrain coefficients and exclude some variables from the model [6].

Despite their differences, these approaches primarily address the same problem: determining which of the models under consideration best fits the existing data or best predicts new data. A different question is addressed here: is it possible to quantify the limitation imposed by the choice of physical quantities included in the model before analyzing the experimental data? This question pertains not to the statistical quality of the existing model, but to its physical structure.

In metrology, this distinction is particularly significant. The Guide to the Expression of Uncertainty in Measurement (GUM) establishes a procedure for estimating measurement uncertainty based on the measurement model, the uncertainties of the input quantities, and their propagation to the output quantity [7]. JCGM 101 extends this procedure by using Monte Carlo propagation of probability distributions through the model [8]. However, in both cases, the measurement model and its constituent quantities must be specified before estimating uncertainty. Consequently, these methods characterize the uncertainty within an al-

ready chosen model but do not answer the question of whether the choice of its physical quantities imposes a quantifiable information constraint.

Information theory allows us to pose this question differently. Shannon linked the quantity of information to the probabilities of possible states of a system [9]. Jaynes showed that, under incomplete information, the principle of maximum entropy allows one to obtain the least biased probabilistic description consistent with known constraints [10]. Brillouin developed the application of information representations to physical systems, linking information, entropy, and the physical constraints of the observation process [11]. These ideas allow us to consider a physical model not only as a system of equations whose parameters are determined from data, but also as a finite information representation of the object under study.

This approach was developed in the Finite Information Quantity (FIQ) method. Its original formulation introduced comparative uncertainty, intended to quantify the discrepancy between the object under study and its model depending on the qualitative and quantitative composition of the physical quantities used [12]. A related concept of finite physically accessible information was considered by Del Santo and Gisin, who introduced finite-information quantities in their interpretation of classical physics [13]. Subsequently, the physical model was considered as an information channel between the phenomenon under study and the observer, which made it possible to distinguish between the uncertainty arising in the choice of the model structure and the uncertainty of subsequent measurements and data processing [14]. Subsequent work applied FIQ to the analysis of the simplicity of physical laws [15] and to the choice of physical models [16].

However, the successful application of the method to individual problems does not yet establish the generality of its physical conclusions. This requires a clear distinction between the initial assumptions of the FIQ, the results following from information theory, and their physical interpretation. Specifically, it is necessary to establish the conditions for the existence of an optimal value of comparative uncertainty, justify the probabilistic representation of the FIQ used, and determine which transformations of the physical model representation do not alter the resulting criterion. It is equally important to test this representation using experimental data not used in the development of the method.

In this paper, this problem is addressed in two stages. First, an information-theoretical description of the final physical model is constructed, and the ε_{opt} benchmark is derived, determined by its information structure before the use of experimental results. The conditions for the equiprobability of informationally indistinguishable FIQs based on the principle of maximum entropy and the invariance of the criterion under changes in units of measurement and informationally equivalent representations of the model are considered separately. ε_{opt} is then independently compared with the experimental comparative uncertainty ε_{exp}, calculated from published experimental data. The model structure and the quantities required for calculating the FIQ are established before this comparison, eliminat-

ing subsequent selection to bring ε_{exp} closer to ε_{opt}.

Three independent examples not involved in the development of the method are used for experimental verification: a measurement of the density of liquid zirconium under microgravity conditions, an experimental study of heat transfer and flow in a bulk layer, and two independent methods for measuring the gravitational constant. These examples differ significantly in their physical nature and the structure of the measurement model and allow us to verify not only the numerical relationship between ε_{exp} and ε_{opt} but also the conditions under which such a comparison is generally valid. Thus, the aim of the work is not to postulate a predetermined universal limit of accuracy, but to examine whether the finite information content of a physical model determines, before the analysis of experimental uncertainty, a calculated value of comparative uncertainty ε_{opt}, and how the independently determined ε_{exp} is positioned relative to it. In this paper ε_{opt} is interpreted as a descriptive balance benchmark: the point of equality of the two normalized information contributions to ε for the chosen GOP. It is not a hypothesized physical lower bound on experimental uncertainty, and the comparisons in Section 5 do not test such a bound.

2. Finite Information Representation of a Physical Model

Any physical model is constructed using a specific set of physical quantities describing the properties of the object under study and the relationships between them. The choice of these quantities determines not only the mathematical form of the model but also the amount of information about the object that can be represented within its framework. However, a physical quantity cannot contain an infinite amount of physically accessible information. A related idea of finite information content in physical quantities was considered by Del Santo and Gisin [13]. The present FIQ approach to physical-model construction and comparison was developed in [12] [14]-[16].

For quantitative application of FIQ, it is necessary to define the system of quantities within which the model is constructed. In the present approach, the initial structure is the System of Primary Quantities (SPQ; denoted SPV in [12] [14]-[16]). SPQ is understood as a selected system of independent primary quantities through which the dimensions of the physical quantities of a model are represented. SPQ should be distinguished from the concept of a system of quantities adopted in metrology. According to the International Vocabulary of Metrology (VIM), a system of quantities is a set of quantities together with consistent equations relating to these quantities [17]. The International System of Quantities (ISQ) is based on seven base quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity [17], while the International System of Units (SI) is a system of units based on the ISQ [18].

FIQ is not limited to the SI. In physics, the CGS and its electrostatic, electromagnetic, and Gaussian variants, atomic and natural units are also used; NIST, in particular, considers CGS units and their relationships with the SI [19]. The SI is

then used as the most common system in modern science, engineering, and metrology. This ensures a unified quantitative representation without limiting the FIQ mathematical scheme exclusively to the SI.

For the SI, the set of primary quantities used in the FIQ is written as

$$\mathrm{L},\mathrm{M},\mathrm{T},\mathrm{I},\Theta,\mathrm{J},\mathrm{F}, \tag{1}$$

where L denotes length, M denotes mass, T denotes time, I denotes electric current, Θ denotes thermodynamic temperature, J denotes luminous intensity, and F denotes the amount of substance [12]. The notation F is retained in accordance with the original FIQ notation and ensures unambiguous reference to previous publications.

The dimension of any quantity q representable in the chosen SPQ can be written as

$$\left[q\right]=\mathrm{L}^{l},\mathrm{M}^{m},\mathrm{T}^{t},\mathrm{I}^{i},\Theta^{\theta},\mathrm{J}^{j},\mathrm{F}^{f}, \tag{2}$$

where l, m, t, i, θ, j, and f are the exponents of the corresponding primary quantities. This representation corresponds to the usual definition of dimension as the product of powers of the dimensions of the base quantities [17].

Each dimension corresponds to an ordered set of seven integer exponents (l, m, t, i, θ, j, f). The set of all such ordered 7-tuples is denoted by $\mathbb{Z}^7$. For example, the dimension of velocity LT^{-1} corresponds to (1, 0, −1, 0, 0, 0, 0), and the dimension of force LMT^{-2} corresponds to (1, 1, −2, 0, 0, 0, 0). When multiplying physical quantities, the exponents are added, when dividing, they are subtracted, and the transition to the reciprocal corresponds to a change in the signs of all exponents. These operations have the properties of an Abelian group; Therefore, the dimensional system built on the seven SI primary quantities can be represented as a free Abelian group Z^7 [20] [21].

Within the FIQ framework, the finite information space is constructed not from all mathematically possible products of powers of the primary quantities, but from the dimensions of established physical quantities used in scientific and engineering practice. The variables of a physical model represent measurable or computable physical quantities, rather than arbitrary algebraic combinations of dimensions. In the SI, the dimension of a derived quantity is represented as a product of powers of the seven base dimensions; the corresponding dimensional exponents can be positive, negative, or zero and, as noted in the SI Brochure, are typically small integers [18].

An initial assessment of the FIQ information space was made based on an analysis of the dimensions of physical quantities presented in the CRC Handbook of Chemistry and Physics [22]. For each primary quantity, a range of exponents occurring in the considered set of physical quantities was determined. As a result, the following working intervals were adopted:

$$\begin{aligned} &-3\le l\le 3,-1\le m\le 1,-4\le t\le 4,-2\le i\le 2,\\ &-4\le \theta\le 4,-1\le j\le 1,-1\le f\le 1. \end{aligned} \tag{3}$$

These intervals are not the mathematical limits of all dimensions that can be

formed in the SI. They define a finite set of dimensions of physical quantities accepted in the FIQ. Formally, higher-order derived quantities may have exponents outside these intervals, but this does not expand the accepted set of FIQ unless such quantities are included in the set of independent physical quantities used in constructing models.

Each interval (3) contains a finite number of admissible exponent values. We denote them by dimensional cardinalities:

$$e_l = 7, e_m = 3, e_t = 9, e_i = 5, e_\theta = 9, e_j = 3, e_f = 3 \tag{4}$$

The total number of formally possible combinations of dimensions within a given finite space is determined by the product

$$N = 7 \times 3 \times 9 \times 5 \times 9 \times 3 \times 3 = 76545. \tag{5}$$

This set excludes the combination in which all the parameters are equal to zero:

$$N - 1 = 76544. \tag{6}$$

Dimensional combinations that differ by simultaneous reversal of the signs of all dimensional exponents correspond to mutually inverse quantities. In the informational account adopted in FIQ, such a pair is considered a single independent combination. Therefore:

$$(N-1)/2 = 38272. \tag{7}$$

After eliminating the seven primary quantities, we obtain

$$\mu_{\text{si}} = (N-1)/2 - 7 = 38265 \tag{8}$$

Therefore,

$$\mu_{\text{si}} = 38265 \tag{9}$$

where μ_{SI} represents the total number of available FIQs available in the SI representation under consideration [12] [16]. The value $\mu_{\text{SI}} = 38265$ is not an empirical or universal physical constant: it follows from the adopted procedure for constructing a finite space of dimensional combinations and characterizes the information capacity of the chosen SPQ.

Counting protocol. The dimensions of physical quantities used to establish the working intervals (3) are taken from the CRC Handbook of Chemistry and Physics, 97th Edition [22]. For each primary quantity, the working interval is the range of exponents occurring in this set. Each dimension corresponds to an ordered set of seven integer exponents (l, m, t, i, θ, j, f); physical quantities with identical dimensions (for example, work and torque, both L^2MT^{-2}) correspond to the same ordered set, are informationally indistinguishable, and are counted as one FIQ. This procedure is not tied to the SI. For another SPQ (for example, CGS), the same steps applied to its primary quantities and its working intervals give another value of μ, while the mathematical structure of the FIQ and the subsequent relations are formulated for a general μ. The catalogue is used to establish the working exponent intervals, rather than to enumerate every FIQ individually. The finite FIQ space is subsequently constructed from all dimensional combinations within these

intervals according to Equations (4)-(8), including combinations not explicitly listed in the catalogue.

For a different SPQ, the number of dimensional combinations and the corresponding μ will generally be different. However, the mathematical structure of the FIQ does not depend on the specific value of μ: the subsequent relations are formulated for a general μ, and $\mu_{SI} = 38265$ is its particular value for the adopted SI representation [14]-[16].

A real physical model never utilizes the entire set μ of possible FIQs. The nature of the phenomenon being studied determines which primary quantities are needed to describe it. In FIQ, the corresponding set of primary quantities is called a Group of Phenomena (GOP). For example, mechanical processes can be represented by a GOP LMT; the description of thermal processes additionally requires temperature Θ, and electromagnetic processes require electric current I. Thus, the choice of a GOP is the first constraint on the full information space of the SPQ.

If z' denotes the total number of dimensional variants allowed within the chosen GOP, and β' is the number of primary quantities excluded when forming independent dimensionless complexes, then the information capacity of the GOP is defined as

$$\gamma_{\text{GOP}} = z' - \beta' \tag{10}$$

For example, the GOP≡LMT uses the following ranges of dimensional exponents for L, M, and T:

$$z' = \left(e_l \cdot e_m \cdot e_t - 1\right)/2 = \left(7 \cdot 3 \cdot 9 - 1\right)/2 = 94, \tag{11}$$

$$\beta' = 3 \tag{12}$$

Therefore,

$$\gamma_{\text{GOP}} = z' - \beta' = 94 - 3 = 91 \tag{13}$$

Thus, from the total space of $\mu_{SI} = 38265$, $\gamma_{GOP} = 91$ informationally admissible combinations remain for a mechanical phenomenon. This reduction is not related to the quality of the experimental data. It occurs earlier—at the moment when the researcher defines the physical class of the phenomenon and thereby limits the set of quantities that can participate in its description.

The next level of limitation arises when constructing a specific model. Even within a chosen GOP, the model does not use all admissible FIQs, but only a subset of them. Let z'' denote the total number of quantities considered at the level of a specific model, and β'' the number of primary quantities included in its description. Then the number of FIQs used by the model is determined as

$$\gamma_{\text{mod}} = z'' - \beta''. \tag{14}$$

Thus, the FIQ structure is hierarchical:

$$\text{SPQ} \rightarrow \text{GOP} \rightarrow \text{model}, \tag{15}$$

or in terms of the number of available FIQs:

$$\mu \rightarrow \gamma_{\text{GOP}} \rightarrow \gamma_{\text{mod}}. \tag{16}$$

moreover,

$$\gamma_{\text{mod}} < \gamma_{\text{GOP}} < \mu. \tag{17}$$

This hierarchy reflects a sequential restriction of the set of available FIQs: from the full space defined by the chosen SPQ, to the space of the chosen GOP, and then to the set of FIQs used by a specific model. Therefore, for further analysis, not only the numbers μ, γ_{GOP}, and γ_{mod} are important, but also the ratios

$$\gamma_{\text{GOP}} / \mu. \tag{18}$$

$$\gamma_{\text{mod}} / \gamma_{\text{GOP}}. \tag{19}$$

The first expresses the fraction of the FIQ of the selected GOP relative to the total FIQ space, and the second expresses the fraction of FIQs used by a particular model relative to the space of the selected GOP. These two dimensionless ratios are used in the next section to derive comparative uncertainty.

The introduction of a finite information space does not mean that a physical phenomenon itself contains only μ possible characteristics. The limitation concerns its representation within the chosen system of primary quantities and the accepted range of dimensions. This distinction is significant: the FIQ characterizes not the "finiteness of nature," but the finiteness of the physical-mathematical description available to the observer. Therefore, changing the experimental technique can reduce measurement uncertainty, and expanding or modifying the structure of a model can change its information content; however, these two processes relate to different levels of description and should not be confused. This formulation creates the basis for the next step—a quantitative determination of the uncertainty that arises precisely as a result of the sequential limitation of information during the transitions SPQ → GOP and GOP → model.

3. Information-Theoretical Derivation of Comparative Uncertainty

In the previous section, the physical model was presented as the result of two successive constraints on the information space: from a full SPQ containing μ admissible FIQs, to a selected GOP with γ_{GOP} FIQs, and then to a specific model using γ_{mod} FIQs. These transitions correspond to the ratios γ_{GOP}/μ and $\gamma_{\text{mod}}/\gamma_{\text{GOP}}$. Let us establish their relationship with comparative uncertainty.

We will consider FIQs to be equally probable if there is no additional information that allows us to a priori prefer one over the other. For a finite set of N such elements, the entropy has the form [11] [12]

$$H = k \cdot \ln(N). \tag{20}$$

where k determines the selected unit of information. The information-theoretical justification for the equiprobability of FIQs is presented independently in Section 4.

When moving from the full SPQ space to the selected GOP, the reduction in the number of available FIQs from μ to γ_{GOP} corresponds to an information difference

$$\Delta A' = k \ln\left(\mu/\gamma_{\mathrm{GOP}}\right) \tag{21}$$

According to the Brillouin relationship between the information difference and the uncertainty contribution used in FIQ [11] [12], the first normalized contribution is written as

$$\Delta'/S = \exp\left(\Delta A'/k\right) = \gamma_{\mathrm{GOP}}/\mu \tag{22}$$

where S is the interval of variation of the variable under study. This contribution is determined by the relative number of FIQs remaining after selecting the GOP.

When moving from the GOP to a specific model, the number of available FIQs decreases from γ_{GOP} to γ_{mod}. Similarly,

$$\Delta A'' = k \cdot \ln\left(\gamma_{\mathrm{GOP}}/\gamma_{\mathrm{mod}}\right) \tag{23}$$

and

$$\Delta''/S = \exp\left(-\Delta A''/k\right) = \gamma_{\mathrm{mod}}/\gamma_{\mathrm{GOP}} \tag{24}$$

The connection between finite model information and comparative uncertainty rests on two distinct elements. The first is the Brillouin information-theoretical relationship [11], according to which the reduction in the number of equiprobable possibilities from μ to γ_{GOP} and from γ_{GOP} to γ_{mod} determines the information differences (21) and (23). The second is the physical modeling assumption introduced in the original FIQ formulation [12] and further developed through the interpretation of a physical model as an information channel [14]. Within this framework, the normalized fractions of FIQs retained at each stage are interpreted as contributions to the comparative uncertainty, representing the discrepancy between the physical system and its model relative to the interval S of the measurand. This interpretation is specific to FIQ and does not follow from information theory alone. The maximum-entropy principle (Section 4) justifies equiprobability in the absence of discriminating information, yielding $H = k\ln(N)$, but does not independently establish the relationship between dimensional information and measurement uncertainty.

In the two-stage FIQ construction, these contributions correspond to two consecutive constraints in the construction of the model. According to the composition rule adopted in the original formulation of FIQ [12] [14], they are summed:

$$\Delta = \Delta' + \Delta'' \tag{25}$$

Therefore, the comparative uncertainty $\varepsilon = \Delta/S$ has the form

$$\varepsilon = \gamma_{\mathrm{GOP}}/\mu + \gamma_{\mathrm{mod}}/\gamma_{\mathrm{GOP}} \tag{26}$$

Formula (26) is the central expression of the FIQ. The first term characterizes the limitation of the information space during the transition from SPQ to GOP, the second-during the transition from GOP to a specific model.

Adding the information differences (21) and (23) yields

$$\Delta A' + \Delta A'' = k \ln\left(\mu/\gamma_{\mathrm{mod}}\right) \tag{27}$$

The total logarithmic information difference between the original SPQ space

and the final model depends only on μ and γ_{mod}; the intermediate value γ_{GOP} cancels out. Therefore, for fixed μ and γ_{mod}, the value of γ_{GOP} determines the distribution of the information difference between two successive transitions without changing its overall value.

The comparative uncertainty in (26), however, is determined by the sum of the corresponding normalized contributions. For fixed μ and γ_{mod}, for two positive terms in (26), the inequality between the arithmetic mean and the geometric mean yields

$$\varepsilon \geq 2 \cdot \left[\left(\gamma_{\text{GOP}} / \mu \right) \cdot \left(\gamma_{\text{mod}} / \gamma_{\text{GOP}} \right) \right]^{1/2} = 2 \cdot \left(\gamma_{\text{mod}} / \mu \right)^{1/2} \tag{28}$$

Hence,

$$\varepsilon \geq 2 \cdot \left(\gamma_{\text{mod}} / \mu \right)^{1/2} \tag{29}$$

Equality in (29) is achieved if and only if the two normalized information contributions are equal:

$$\gamma_{\text{GOP}} / \mu = \gamma_{\text{mod}} / \gamma_{\text{GOP}} \ , \tag{30}$$

Whence

$$\left(\gamma_{\text{GOP}} \right)^2 = \mu \cdot \gamma_{\text{mod}} \tag{31}$$

Thus, for fixed μ and γ_{mod}, the expression on the right-hand side of (29) represents the minimum possible value of ε for a continuous change in γ_{GOP}, with the minimum being achieved when (31) is satisfied.

In a real physical problem, GOP is determined by a set of primary quantities necessary to describe the phenomenon under study, so γ_{GOP} is usually not a free variable. For given μ and γ_{GOP}, condition (31) determines the value of γ_{mod} at which the two normalized contributions to (26) are equal:

$$\gamma_{\text{mod,opt}} = \left(\gamma_{\text{GOP}} \right)^2 / \mu \tag{32}$$

The corresponding value of comparative uncertainty is

$$\varepsilon_{\text{opt}} = 2 \cdot \gamma_{\text{GOP}} / \mu = 2 \cdot \left(\gamma_{\text{mod,opt}} / \mu \right)^{1/2} \tag{33}$$

The index "opt" in (32) and (33) requires careful interpretation. For fixed μ and γ_{mod}, the minimum in (29) is achieved relative to γ_{GOP} when (31) holds. If the GOP is physically specified, (32) is not the result of minimizing ε relative to γ_{mod}: it follows from (26) that, for fixed μ and γ_{GOP}, ε monotonically increases with γ_{mod}. Therefore, $\gamma_{\text{mod,opt}}$ denotes the value of γ_{mod} at which the two normalized information contributions are equal, and ε_{opt} is the corresponding value of comparative uncertainty. Accordingly, ε_{opt} is used in this paper as a descriptive balance benchmark for the chosen GOP, not as a minimum attainable by a model and not as a lower bound on experimental uncertainty.

For a specific model with an actual γ_{mod}, its comparative uncertainty is determined directly from (26). For a given GOP, in contrast, the quantities γ_{mod},opt and ε_{opt} are determined from (32) and (33) without using experimental uncertainty data. For the mechanical GOP LMT, where $\gamma_{\text{GOP}} = 91$ and $\mu_{\text{SI}} = 38265$, we obtain

$$\gamma_{\mathrm{mod,opt}} = 91^2/38265 = 0.2164$$
$$\varepsilon_{\mathrm{opt}} = 2\cdot 91/38265 = 0.004756 \tag{34}$$

For GOP≡LMT, $\gamma_{\mathrm{mod,opt}} = 0.2164$ is a formal balance point and cannot be attained by a physical model with an integer number of FIQs. Since Equation (26) increases monotonically with γ_{mod} for fixed μ and γ_{GOP}, the minimum admissible value occurs at $\gamma_{\mathrm{mod}} = 1$, giving $\varepsilon_{\min} = 91/38265 + 1/91 \approx 0.01337$. Thus, $\varepsilon_{\mathrm{opt}} = 0.004756$ represents the formal equality of the two normalized information contributions, rather than the attainable minimum comparative uncertainty. This distinction does not affect the use of $\varepsilon_{\mathrm{opt}}$ as a priori information-balance benchmark.

For GOP≡LMTΘ, with $\gamma_{\mathrm{GOP}} = 846$, respectively

$$\gamma_{\mathrm{mod,opt}} = 846^2/38265 = 18.71 \approx 19$$
$$\varepsilon_{\mathrm{opt}} = 2\cdot 846/38265 = 0.04421 \tag{35}$$

In this case, $\gamma_{\mathrm{mod,opt}} = 18.71$ is close to the permissible integer value $\gamma_{\mathrm{mod}} = 19$. The obtained values correspond to previously published FIQ results [12] [16], but the presented derivation clarifies their mathematical interpretation.

Thus, it is necessary to distinguish three quantities: ε of a specific model, calculated from (26) with its actual γ_{mod}; $\varepsilon_{\mathrm{opt}}$, corresponding to the equality of two normalized information contributions for a given GOP; and $\varepsilon_{\mathrm{exp}}$, determined independently from experimental data using the appropriate interval S. In an independent experimental test, μ and γ_{GOP}, and therefore $\varepsilon_{\mathrm{opt}}$, are established before using experimental uncertainty. Then, $\varepsilon_{\mathrm{exp}}$ is independently calculated and the ratio $\varepsilon_{\mathrm{exp}}/\varepsilon_{\mathrm{opt}}$ is examined. The actual γ_{mod} of a specific model can be analyzed additionally using (26), but is not used to fit $\varepsilon_{\mathrm{opt}}$ to the experiment.

4. Information-Theoretical Basis for Equiprobability of FIQs

In Section 3, the condition of equiprobability of FIQs was used to derive comparative uncertainty if there were no additional information that would allow one to a priori prefer one element over another. We will show that under this condition, equiprobability follows from the principle of maximum entropy and does not require a separate postulate.

Let a finite set contain N admissible FIQs with probabilities $p_i \geq 0$ and $\sum_{i=1}^{N} p_i = 1$. The entropy of the distribution P is determined by Shannon's expression [9]

$$H(P) = -k\cdot\sum_{i=1}^{N} p_i \cdot \ln p_i \tag{36}$$

Suppose that the available information does not provide grounds for distinguishing the prior probabilities of individual FIQs. According to the principle of maximum entropy [10], one should choose the distribution that maximizes (36) under the normalization condition.

Let U denote the uniform distribution on the same set, $u_i = 1/N$. For an arbitrary distribution P, the non-negativity of the Kullback-Leibler divergence [23] yields

$$D(P\|U)=\sum_{i=1}^{N}p_i\cdot\frac{\ln p_i}{u_i}=\sum_{i=1}^{N}p_i\cdot\ln(Np_i)$$
$$=\ln(N)+\sum_{i=1}^{N}p_i\cdot\ln(p_i)=\ln N-H(P)/k \tag{37}$$

where $u_i=1/N$, the normalization condition, and the definition of entropy (36) are used. Since $D(P\|U)\geq 0$, it follows directly

$$H(P)\leq k\cdot\ln(N), \tag{38}$$

Equality is achieved if and only if $D(P\|U)=0$, that is, $P=U$. Therefore,

$$p_i=1/N, i=1,\cdots,N \tag{39}$$

Thus, for a finite set of informationally indistinguishable FIQs, the uniform distribution is the unique distribution of maximum entropy. Therefore, the expression $H = k\ln(N)$ used in Section 3 represents the maximum entropy value in the absence of information that would allow a priori discrimination between the elements of the set.

This result does not imply that different physical quantities are equally frequent in nature, have the same physical significance, or are included with equal probability in a particular model. Uniform probability pertains only to the a priori information weight of the elements of an already defined feasible set. In the absence of discriminating information, assigning them different a priori probabilities would introduce a difference not contained in the original description.

This condition is consistent with the SPQ → GOP → model hierarchy introduced in Section 2. At each level, a finite set of FIQs is considered: μ for the SPQ, γ_{GOP} for the chosen GOP, and γ_{mod} for the specific model. The transition to the next level reduces the number of admissible FIQs, but does not in itself determine differences between the a priori probabilities of the remaining elements. In the absence of additional discriminating information, the maximum-entropy distribution of the corresponding set is uniform. Thus, the informational differences used in Section 3 for the SPQ → GOP and GOP → model transitions receive an information-theoretical justification.

However, this justification is valid only under the stated condition. If there is additional physical information distinguishing admissible FIQs or imposing constraints on their probabilities, entropy maximization must take these constraints into account, and the resulting distribution will generally not be uniform.

Consequently, equiprobability is not a universal property of physical quantities and is not introduced into FIQ as an independent physical postulate. It is a maximum-entropy assignment for a finite set of FIQs in the absence of information allowing a priori discrimination of its elements. Under this condition, the expression $H = k\ln(N)$, used in Section 3, receives information-theoretical justification.

5. Independent Applications and Comparison

Experimental comparison with the FIQ benchmark requires distinguishing between the relative uncertainty reported by the experimental authors and the ex-

perimental comparative uncertainty ε_{exp} used in the present approach. The latter is typically not reported in the original publication and must be calculated from the published data independently of ε_{opt}.

The comparisons below are descriptive: they show where the experimental comparative uncertainty ε_{exp} lies relative to the benchmark ε_{opt} and do not test ε_{opt} as a lower bound. For each study, two independent procedures are applied. First, the GOP is established using the published measurement model and γ_{GOP} is determined. On this basis, without using experimental uncertainty, ε_{opt} is calculated according to Section 3.

Then, the published experimental data are used. If the relative uncertainty r_{exp} is reported for the measured value z, the corresponding absolute uncertainty is determined as

$$\Delta_{exp} = z \cdot r_{exp} . \tag{40}$$

Experimental comparative uncertainty is calculated as

$$\varepsilon_{exp} = \Delta_{exp} / S , \tag{41}$$

where S is a physically justified interval of possible values ¯of the quantity under study. Thus, ε_{opt} is compared to ε_{exp}, not the published relative uncertainty r_{exp}.

This difference is significant: relative uncertainty is normalized to the measured value z, while comparative uncertainty is normalized to the interval S. Therefore, a direct comparison of r_{exp} with ε_{opt} is generally incorrect.

In the IARU (Information Approach-Relative Uncertainty) procedure, the interval S is established from independent published measurements of the same measurand under physically comparable conditions. Alternatively, S may be defined as the full width of the 95% uncertainty interval provided by an independent critical evaluation (IUPAC/IATP), when available:

$$S = z_{max} - z_{min} . \tag{42}$$

The reference data used to establish S must be independent of the experiment under examination and selected without regard to the expected $\varepsilon_{exp}/\varepsilon_{opt}$ ratio.

After independently determining ε_{exp} and ε_{opt}, the ratio

$$R = \varepsilon_{exp} / \varepsilon_{opt} . \tag{43}$$

The value of R indicates the position of the experimentally achieved comparative uncertainty relative to ε_{opt}. For $R \to 1$, the values ¯converge, while $R > 1$ means $\varepsilon_{exp} > \varepsilon_{opt}$. R is reported as a descriptive measure; $R < 1$ would not contradict the framework but would indicate that the achieved comparative uncertainty is below the balance point of the two information contributions for the given GOP.

5.1. Independent Application: Measuring the Density of Liquid Zirconium in Microgravity

As a first independent test, we consider the experiment by Zhong *et al.* [24], performed on the Electrostatic Levitation Furnace (ELF) of the Chinese Space Station. This work is not related to the development of FIQ and allows the method to be

tested using experimental data obtained independently of its theoretical assumptions. The authors measured the density, surface tension, and viscosity of molten zirconium in microgravity. Here, only the density measurement is considered.

In the range of 1800-2220 K, Zhong *et al.* obtained the dependence

$$\rho(T) = 6489 - 0.25 \cdot (T - T_m), \mathrm{kg \cdot m^{-3}}, \tag{44}$$

where T_m is the melting point of Zr. The relative standard combined uncertainty of the density measurement is 0.51%. This value is not $\varepsilon_{\mathrm{exp}}$ in the FIQ sense and is used only as initial experimental information for its calculation.

Since density is studied as a function of temperature, the measurement model refers to the GOP≡LMTΘ. For this model, $\gamma_{\mathrm{GOP}} = 846$ and, according to (33), $\varepsilon_{\mathrm{opt}} = 0.04421$. Thus, $\varepsilon_{\mathrm{opt}}$ is determined by the GOP information structure and is independent of the experimental results of Zhong *et al.*

The IARU procedure is used to determine the experimental comparative uncertainty. Since published values ¯of the density of liquid Zr depend on the measurement method, the interval S is formed from comparable modern containerless measurements normalized to the melting point. Earlier measurements using crucibles are excluded, since for chemically active liquid Zr, interaction with the container material can create additional systematic discrepancies [25]. The selection criterion is the physical comparability of the experiments, not the resulting $\varepsilon_{\mathrm{exp}}/\varepsilon_{\mathrm{opt}}$ value.

The IARU procedure is applied using an independent reference correlation for the density of liquid zirconium established by Ntonti *et al.* [26]. Their critical evaluation of published experimental data provides a reference density of approximately 6100.0 kg m^{-3} at the melting temperature $T_m = 2128$ K, with a relative expanded uncertainty of 2.1% at the 95% confidence level. The evaluated data set does not include the result of Zhong *et al.* [24].

In the present comparison, S_ρ is defined as the full width of this independently established 95% reference interval, rather than the difference between the extreme experimental values:

$$S_\rho = 2 \cdot 0.021 \cdot 6100 = 256.2 \ \mathrm{kg \cdot m^{-3}}. \tag{45}$$

The primary experimental data underlying the reference correlation are identified in Table 1 of Ntonti *et al.* [26].

The absolute standard uncertainty of Zhong *et al.*'s measurement is calculated independently:

$$\Delta\rho_{\mathrm{exp}} = \rho \cdot r_\rho = 6489 \cdot 0.0051 = 33.09 \ \mathrm{kg \cdot m^{-3}}. \tag{46}$$

Therefore,

$$\varepsilon_{\mathrm{exp}} = \Delta\rho_{\mathrm{exp}} / S_\rho = 33.09/256.2 = 0.1292. \tag{47}$$

For GOP≡LMTΘ, $\varepsilon_{\mathrm{opt}} = 0.04421$, giving

$$R = \varepsilon_{\mathrm{exp}} / \varepsilon_{\mathrm{opt}} = 0.1292/0.04421 \approx 2.92. \tag{48}$$

The measured density of Zhong *et al.* lies outside the independently established

reference interval. This discrepancy does not affect the independence of S_ρ, which was determined without using their result. The comparison employs the published standard uncertainty of Zhong *et al.* and the full width of the independently evaluated reference interval. If an expanded uncertainty with coverage factor $k = 2$ is used for the Zhong *et al.* measurement, the ratio becomes approximately 5.84. Thus, the numerical comparison depends on the uncertainty convention, while $\varepsilon_{\mathrm{exp}}$ remains above the FIQ balance benchmark under both conventions.

The obtained ratio shows that the experimental comparative uncertainty in this experiment is higher than the $\varepsilon_{\mathrm{opt}}$ calculated for GOP≡LMTΘ. However, neither the experimental uncertainty nor the measured density values ¯were used to determine $\varepsilon_{\mathrm{opt}}$, so there is no fitting of the FIQ criterion to the experimental result.

$\varepsilon_{\mathrm{opt}}$ should also be distinguished from the characteristics of a specific measurement model. In Section 3, $\gamma_{\mathrm{mod,opt}}$ was defined as the γ_{mod} value at which the two normalized information contributions are equal. This does not mean that the independently constructed model by Zhong *et al.* must contain exactly this FIQ number. Therefore, the actual γ_{mod} of this model is not used as an adjustable parameter when calculating ratio (48).

The numerical value of $\varepsilon_{\mathrm{exp}}$ depends on the choice of interval S. Including historical measurements of liquid Zr, performed by other methods and subject to melt-container interaction, would change this interval. Therefore, the rule for forming the comparison set must be established before calculating $\varepsilon_{\mathrm{exp}}$ and based on the physical comparability of the measurements.

The independent reference interval yields $\varepsilon_{\mathrm{exp}}/\varepsilon_{\mathrm{opt}} \approx 2.92$ for liquid zirconium. This result does not establish equality between experimental comparative uncertainty and the FIQ benchmark; rather, it provides a quantitative comparison without using the tested measurement to construct the reference interval.

5.2. Independent Application: Heat Transfer in a Shallow Bulk Bed

As a second independent test, we consider the data of Weber *et al.* [27] on transient heat transfer and air flow in a shallow bulk bed at low Reynolds numbers. This work was performed independently of the FIQ and is of particular interest because the authors published the original data and conducted replicated experiments to assess reproducibility. A total of nine experiments were performed; three replicates were conducted for cooling air flow rates of 15 and 38 $\mathrm{Nm^3/h}$. In each repetition, the bed was rebuilt, while the heater and insulation configuration remained the same. Temperature was recorded by more than 40 thermocouples at a rate of approximately 60 measurements per second [27].

Weber *et al.* report two different types of metrological information: the accuracy of the sensors used and the scatter of the results of the replicate experiments. Neither of these characteristics is directly related to $\varepsilon_{\mathrm{exp}}$ in the FIQ sense. The measurement model includes geometric parameters, mass, time, air movement, pressure, and temperature, and therefore relates to GOP≡LMTΘ. For this model,

$\gamma_{GOP} = 846$ and $\varepsilon_{opt} = 0.04421$. As in the previous example, ε_{opt} is set before analyzing the experimental scatter.

For repeated experiments, the authors synchronized the time series and calculated the standard deviation of the corresponding temperature measurements using numpy.std [27]. However, the numerical time series of these standard deviations are not tabulated in the paper. Therefore, extracting individual σ values from the plots was not used, as this would introduce additional uncertainty.

The range S cannot be determined from the same three repeated measurements used to calculate σ. If we take

$$\Delta z_{exp} = \sigma, S = z_{max} - z_{min}, \tag{49}$$

where z_{max} and z_{min} refer to the same three values, the numerator and denominator are statistically dependent. For any three values, the ratio of the population standard deviation to their range satisfies the condition

$$0.40825 \le \sigma / \left(z_{max} - z_{min} \right) \le 0.47140 . \tag{50}$$

Considering $\varepsilon_{opt} = 0.04421$ from (50) it follows

$$9.23 \le \varepsilon_{exp} / \varepsilon_{opt} \le 10.66 . \tag{51}$$

However, (51) is not an experimental result of the comparison of ε_{exp} with ε_{opt}. The obtained interval is due to the mathematical dependence of σ and S, calculated using the same three-point set. Therefore, such a definition of S does not satisfy the IARU requirement for independent range formation.

Additional information is provided by the sensor characteristics. Weber *et al.* report an accuracy of ±4.57 K in the range of 323 - 523 K and ±2.28 K in the range of 573 - 823 K for calibrated type N thermocouples, and ±2.5 K in the range of 233 - 606 K for a control type K thermocouple [27]. These values cannot be identified with the experimental uncertainty Δz_{exp}, but they can be used for an auxiliary scale check.

When normalizing the published accuracy to the corresponding instrument range, we obtain

$$\begin{aligned} &4.57/(523-323) = 0.02285, \\ &2.28/(823-573) = 0.00912, \\ &2.50/(606-233) = 0.0067, \end{aligned} \tag{52}$$

and relatively $\varepsilon_{opt} = 0.04421$

$$R^* = 0.517, 0.206, 0.152 , \tag{53}$$

where R^* denotes an auxiliary ratio, not the main ratio $\varepsilon_{exp}/\varepsilon_{opt}$, since the instrument range is used, not the physically established interval S for the measured value.

To test sensitivity, we also consider the interpretation of $\pm a$ as the half-width of a rectangular distribution:

$$u = a/(3)^{1/2} . \tag{54}$$

When using u, the R^* values are 0.298, 0.119, and 0.088, respectively, while for

$U = 2u$, they are 0.597, 0.238, and 0.175. Thus, none of the considered interpretations of instrument accuracy yields $R^* \approx 1$; the maximum value is approximately 0.60.

A reverse check can also be performed. If we formally require $R^* = 1$, the corresponding range is determined by the condition

$$S_{\mathrm{FIQ}} = \Delta^* / \varepsilon_{\mathrm{opt}}, \tag{55}$$

from where for the three characteristics of the devices we obtain

$$S_{\mathrm{FIQ}} = 103.35\ \mathrm{K}, 51.56\ \mathrm{K}, 56.55\ \mathrm{K}. \tag{56}$$

These values are 103.35, 51.56, and 56.55 K, respectively, and are significantly smaller than the instrument ranges of 200, 250, and 373 K. However, using the obtained values ⁻for subsequent selection of S would be a post-hoc adjustment: the range would be selected after calculating $\varepsilon_{\mathrm{opt}}$ in order to obtain $R^* = 1$.

Thus, unlike Zhong *et al.* [24], the data of Weber *et al.* [27] do not allow one to determine a strict $\varepsilon_{\mathrm{exp}}/\varepsilon_{\mathrm{opt}}$ ratio directly from the published article. Using the same three replicate measurements to determine Δz_{exp} and S creates a statistical dependence, whereas instrument accuracy and sensor range provide only an auxiliary indicator of R^*, not $\varepsilon_{\mathrm{exp}}/\varepsilon_{\mathrm{opt}}$.

This example therefore does not serve as a confirmation or refutation of the FIQ, but as a boundary test of the applicability of the procedure. For a valid independent test, experimental uncertainty Δz_{exp} and a physically justified interval S, related to a single measurement and determined independently of each other and of $\varepsilon_{\mathrm{opt}}$, are required. If these conditions are not met, the ratio $\varepsilon_{\mathrm{exp}}/\varepsilon_{\mathrm{opt}}$ should not be used for comparison with the benchmark $\varepsilon_{\mathrm{opt}}$.

5.3. Independent Application: Two Precision Measurements of the Gravitational Constant

The third independent test is based on measurements of the Newtonian gravitational constant G published by Li *et al.* [28]. The authors used two different torsional pendulum methods—time-of-swing (TOS) and angular-acceleration-feedback (AAF)—implemented on different experimental setups. This allows comparison of the results of two independent determinations of the same physical quantity.

For the TOS method, $G_{\mathrm{TOS}} = 6.674184 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$ was obtained with a relative standard uncertainty of 11.64 ppm, while for the AAF method, $G_{\mathrm{AAF}} = 6.674484 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$ was obtained with an uncertainty of 11.61 ppm [28].

To determine S independently of the two measurements under examination, we use the published compilation of gravitational-constant measurements [29]. The extreme values are $G_{\mathrm{min}} = 6.67191 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$ (LENS-14, Table 3, entry 10) and $G_{\mathrm{max}} = 6.67559 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$ (BIPM-01, entry 2). After excluding these extrema, the next limiting values are 6.67222×10^{-11} (HUST-05, entry 5) and 6.67554×10^{-11} (BIPM-14, entry 9). All four measurements are independent of the Li *et al.* results evaluated here

$$S_G = G_{\max} - G_{\min} = (6.67559 - 6.67191)\times 10^{-11} = 3.680\times 10^{-14}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}. \quad (57)$$

The SG range is thus determined independently of subsequent comparison with the FIQ. The dimension of the gravitational constant $[G]$ is $L^3M^{-1}T^{-2}$. The measurement models under consideration are based on the mechanical interaction of masses and therefore relate to GOP≡LMT. For this model, $\gamma_{\mathrm{GOP}} = 91$ and $\varepsilon_{\mathrm{opt}} = 0.004756$.

$$\gamma_{\mathrm{GOP}} = 91, \varepsilon_{\mathrm{opt}} = 2\cdot 91/38265 = 0.004756. \quad (58)$$

For the TOS measurement, the absolute experimental uncertainty calculated from the published relative standard uncertainty is:

$$\Delta G_{\mathrm{Gexp,TOS}} = G_{\mathrm{TOS}}\cdot r_{\mathrm{TOS}} = 6.674184\times 10^{-11}\times 11.64\times 10^{-6} = 7.769\times 10^{-16}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}} \quad (59)$$

Therefore,

$$\varepsilon_{\mathrm{exp,TOS}} = \Delta G_{\mathrm{exp,TOS}} / S_G = 7.769\times 10^{-16} / \left(3.680\times 10^{-14}\right) = 0.02111. \quad (60)$$

and

$$\varepsilon_{\mathrm{exp,TOS}} / \varepsilon_{\mathrm{opt}} = 0.02111/0.004756 \approx 4.44. \quad (61)$$

For the independent AAF method we similarly obtain

$$\Delta G_{\mathrm{exp,AAF}} = 6.674484\times 10^{-11}\times 11.61\times 10^{-6} = 7.749\times 10^{-16}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}, \quad (62)$$

$$\varepsilon_{\mathrm{exp,AAF}} = 7.749\times 10^{-16} / \left(3.680\times 10^{-14}\right) = 0.02106, \quad (63)$$

and

$$\varepsilon_{\mathrm{exp,AAF}} / \varepsilon_{\mathrm{opt}} = 0.02106 / 0.004756 \approx 4.43 \quad (64)$$

Thus, the two independent methods yield virtually identical values of $\varepsilon_{\mathrm{exp}}$ and the $\varepsilon_{\mathrm{exp}}/\varepsilon_{\mathrm{opt}}$ ratio: 4.44 and 4.43, respectively. In both cases, the experimental comparative uncertainty significantly exceeds $\varepsilon_{\mathrm{opt}}$.

To test the sensitivity of the result, we simultaneously exclude the maximum and minimum values from the original population [29]. The following limits are approximately 6.67222×10^{-11} and $6.67554\times 10^{-11}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}$, therefore

$$S_{G'} = 3.320\times 10^{-14}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}, \quad (65)$$

and the corresponding relations are equal

$$\varepsilon_{\mathrm{exp,TOS}} / \varepsilon_{\mathrm{opt}} \approx 4.92, \varepsilon_{\mathrm{exp,AAF}} / \varepsilon_{\mathrm{opt}} \approx 4.91. \quad (66)$$

Excluding the extreme values changes the ratios by approximately 11%, to 4.92 and 4.91, but does not change the main result: $\varepsilon_{\mathrm{exp}}$ remains several times higher than $\varepsilon_{\mathrm{opt}}$.

If, instead of an independent population, we define S as the difference between the two results of Li *et al.*, we obtain:

$$S_{\mathrm{HUST}} = \left|G_{\mathrm{AAF}} - G_{\mathrm{TOS}}\right| = 3.00\times 10^{-15}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}} \quad (67)$$

and $\varepsilon_{\mathrm{exp}}/\varepsilon_{\mathrm{opt}}$ will increase to approximately 54. However, this option is not used, since S would be formed from the same results whose uncertainty is being tested. As in the case of Weber *et al.* [27], this would violate the requirement for an inde-

pendent determination of the comparison interval.

The scatter of modern G measurements is a known metrological problem unrelated to FIQ. CODATA also notes the significant inconsistency of G measurement results and the need to expand the initial uncertainties when reconciling them [30]. Therefore, the S_G range reflects the existing scatter of independent experimental results.

Thus, the TOS and AAF methods yield virtually identical ratios $\varepsilon_{exp}/\varepsilon_{opt} \approx 4.4$, robust to a moderate change in the comparison set. They do not confirm the equality $\varepsilon_{exp} = \varepsilon_{opt}$: the experimental comparative uncertainty in both cases is significantly higher than ε_{opt}. At the same time, the example shows that with independent determination of experimental uncertainty and physically based S, the IARU procedure yields a reproducible quantitative result.

6. Discussion

These results allow us to clarify the physical meaning of ε_{opt} and the limits of its experimental interpretation. In FIQ, ε_{opt} is determined by the information structure of the selected GOP and is independent of the subsequent experimental result. In contrast, the experimental comparative uncertainty ε_{exp} is determined by the experimental uncertainty Δ_{exp} and the physically justified range S of a particular measurement. Therefore, the published relative uncertainty itself is not ε_{exp} and should not be directly compared with ε_{opt}.

Experimental verification requires that the GOP, Δ_{exp}, and the formation rule S be established independently before calculating $\varepsilon_{exp}/\varepsilon_{opt}$. Changing the GOP or S after obtaining the result would turn the verification into a fitting.

Three independent applications in Section 5 demonstrate different aspects of this requirement. For Zhong *et al.* [24], the published relative standard combined uncertainty of the density of liquid zirconium and independent containerless measurements [26] allow us to separately determine $\Delta\rho_{exp}$ and S_ρ. The obtained $\varepsilon_{exp}/\varepsilon_{opt} \approx 2.92$ shows that the experimental comparative uncertainty is higher than ε_{opt} for the GOP≡LMTΘ.

The example of Weber *et al.* [27] reveals a limitation of the verification procedure. The published data are insufficient to independently determine both Δ_{exp} and the physically based S of a single measurement. Using the same small set of replicate measurements to determine both parameters creates a statistical dependence, and the sensor range is not the physical range of the measurand. Therefore, a strict $\varepsilon_{exp}/\varepsilon_{opt}$ ratio cannot be determined for this experiment.

In Li *et al.* [28]'s measurements of the gravitational constant, the independent TOS and AAF methods, when using a comparison set of modern definitions of G [29], yield virtually identical ratios of $\varepsilon_{exp}/\varepsilon_{opt} \approx 4.44$ and 4.43. After excluding the extreme values, these ratios change to 4.92 and 4.91, preserving the main result. The known inconsistency of high-precision G measurements is also noted by CODATA [30].

Thus, the results reviewed do not demonstrate a universal tendency for $\varepsilon_{exp}/\varepsilon_{opt}$

to approach unity. In two cases that allow full quantitative application of the IARU, $\varepsilon_{exp} > \varepsilon_{opt}$ was obtained; the example of Weber *et al.* demonstrates conditions under which this ratio should not be calculated at all. In this paper ε_{opt} is used as a descriptive balance benchmark for the chosen GOP, not as a lower bound on experimental comparative uncertainty; the ratios R describe the position of the achieved ε_{exp} relative to it.

The critical element of verification is the choice of S. It should relate to the same measurand, be based on physically comparable measurements, and be determined independently of the expected $\varepsilon_{exp}/\varepsilon_{opt}$. Using a range of different operating conditions, the instrument's rated range, or the same small sample from which Δ_{exp} is determined changes the meaning of comparative uncertainty and does not provide an independent verification. Therefore, to experimentally verify the FIQ, it is necessary to determine the measurement model and GOP a priori, independently establish Δ_{exp} and the physically justified S, and either exclude or explicitly account for the statistical dependence between them. Under these conditions the comparison is reproducible and involves no fitting: the experimental value need not approach ε_{opt}.

Further testing should be based on pre-defined series of independent measurements for different GOPs with full uncertainty budgets and an independent rule for forming S. Such studies will make it possible to establish whether ε_{exp} changes systematically with ε_{opt} when passing from one GOP to another; this is the test that would give the benchmark a predictive meaning.

7. Conclusions

This paper proposes an information-based procedure for selecting a physical model, in which the comparative uncertainty is determined by two contributions: the information space of the selected GOP and the number of FIQs used by a particular model within this GOP. For a finite FIQ space, a value of ε_{opt} corresponding to the balance of these contributions is obtained, which is used as a descriptive balance benchmark.

The value of ε_{opt} is determined by the GOP structure independently of the results of a particular experiment. This allows it to be compared with the experimental comparative uncertainty ε_{exp}, independently calculated from experimental data using IARU, and thus allows ε_{opt} to be compared with experiment without adjusting the FIQ parameters.

Three independent applications yielded different results. For measuring the density of liquid zirconium, Zhong *et al.* [24] obtained $\varepsilon_{exp}/\varepsilon_{opt} \approx 2.92$. The data of Weber *et al.* [27] showed that the correct ratio cannot be determined without independently establishing Δ_{exp} and S. For two independent measurements of the gravitational constant, Li *et al.* [28] obtained $\varepsilon_{exp}/\varepsilon_{opt} \approx 4.44$ and 4.43; the result persists when changing the comparison set.

These comparisons demonstrate the applicability of the proposed normalization procedure but neither test a physical lower bound nor establish the predictive

validity of the FIQ benchmark. Assessing its predictive value requires further independent experiments across different GOPs, with predefined rules for selecting the measurement model, evaluating experimental uncertainty, and determining the reference interval S.

The comparisons presented do not test a lower bound. Whether the benchmark has predictive value requires a more extensive independent verification for different GOPs under pre-established rules for determining the measurement model, experimental uncertainty, and range S.

The main result of the work is a priori computable benchmark ε_{opt}, determined by the information structure of the model, which can be compared with the experimental comparative uncertainty without adjusting the parameters to the measurement results.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Akaike, H. (1974) A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control*, **19**, 716-723. https://doi.org/10.1109/tac.1974.1100705

[2] Schwarz, G. (1978) Estimating the Dimension of a Model. *The Annals of Statistics*, **6**, 461-464. https://doi.org/10.1214/aos/1176344136

[3] Rissanen, J. (1978) Modeling by Shortest Data Description. *Automatica*, **14**, 465-471. https://doi.org/10.1016/0005-1098(78)90005-5

[4] Stone, M. (1977) An Asymptotic Equivalence of Choice of Model by Cross-Validation and Akaike's Criterion. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, **39**, 44-47. https://doi.org/10.1111/j.2517-6161.1977.tb01603.x

[5] Hoerl, A.E. and Kennard, R.W. (1970) Ridge Regression: Biased Estimation for Nonorthogonal Problems. *Technometrics*, **12**, 55-67. https://doi.org/10.1080/00401706.1970.10488634

[6] Tibshirani, R. (1996) Regression Shrinkage and Selection via the Lasso. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, **58**, 267-288. https://doi.org/10.1111/j.2517-6161.1996.tb02080.x

[7] Joint Committee for Guides in Metrology (2008) Evaluation of Measurement Data—Guide to the Expression of Uncertainty in Measurement. JCGM 100:2008. https://ncc.nesdis.noaa.gov/documents/documentation/JCGM_100_2008_E.pdf

[8] Joint Committee for Guides in Metrology (2008) Evaluation of Measurement Data—Supplement 1 to the "Guide to the Expression of Uncertainty in Measurement"—Propagation of Distributions Using a Monte Carlo Method. JCGM 101:2008. https://www.bipm.org/en/doi/10.59161/jcgm101-2008

[9] Shannon, C.E. (1948) A Mathematical Theory of Communication. *Bell System Technical Journal*, **27**, 623-656. https://doi.org/10.1002/j.1538-7305.1948.tb00917.x

[10] Jaynes, E.T. (1957) Information Theory and Statistical Mechanics. *Physical Review*, **106**, 620-630. https://doi.org/10.1103/physrev.106.620

[11] Brillouin, L. (1962) Science and Information Theory. 2nd Edition, Academic Press. https://archive.org/details/scienceinformati0000bril

[12] Menin, B. (2017) Information Measure Approach for Calculating Model Uncertainty of Physical Phenomena. *American Journal of Computational and Applied Mathematics*, **7**, 11-24. http://article.sapub.org/10.5923.j.ajcam.20170701.02.html

[13] Del Santo, F. and Gisin, N. (2019) Physics without Determinism: Alternative Interpretations of Classical Physics. *Physical Review A*, **100**, Article 062107. https://doi.org/10.1103/physreva.100.062107

[14] Menin, B. (2021) Construction of a Model as an Information Channel between the Physical Phenomenon and Observer. *Journal of the Association for Information Science and Technology*, **72**, 1198-1210. https://doi.org/10.1002/asi.24473

[15] Menin, B.M. (2022) Simplicity of Physical Laws: Informational-Theoretical Limits. *IEEE Access*, **10**, 56711-56719. https://doi.org/10.1109/access.2022.3177274

[16] Menin, B. (2024) Objective Model Selection in Physics: Exploring the Finite Information Quantity Approach. *Journal of Applied Mathematics and Physics*, **12**, 1848-1889. https://doi.org/10.4236/jamp.2024.125115

[17] Joint Committee for Guides in Metrology (2012) International Vocabulary of Metrology—Basic and General Concepts and Associated Terms (VIM). 3rd Edition, JCGM 200:2012. https://www.bipm.org/documents/20126/2071204/JCGM_200_2012.pdf

[18] Bureau International des Poids et Mesures (2019) The International System of Units (SI). 9th Edition, BIPM, Sèvres.

[19] Thompson, A. and Taylor, B.N. (2008) Guide for the Use of the International System of Units (SI). NIST Special Publication 811, National Institute of Standards and Technology, Gaithersburg.

[20] László, A. (1964) Systematization of Dimensionless Quantities by Group Theory. *International Journal of Heat and Mass Transfer*, **7**, 423-430. https://doi.org/10.1016/0017-9310(64)90134-6

[21] Jonsson, D. (2024) On Augmented Dimensional Analysis. *Acta Applicandae Mathematicae*, **192**, Article No. 10. https://doi.org/10.1007/s10440-024-00673-y

[22] Haynes, W.M. (2016) CRC Handbook of Chemistry and Physics. 97th Edition, CRC Press. https://ia800704.us.archive.org/27/items/CRCHandbookOfChemistryAndPhysics97thEdition2016/CRC%20Handbook%20of%20Chemistry%20and%20Physics%20-%2097th%20Edition%20%282016%29.pdf

[23] Kullback, S. and Leibler, R.A. (1951) On Information and Sufficiency. *The Annals of Mathematical Statistics*, **22**, 79-86. https://doi.org/10.1214/aoms/1177729694

[24] Zhong, Q., Yang, L., Tao, Y., Li, H., Yu, J., Zhong, H., *et al.* (2024) Non-Destructive Investigation of Thermophysical Properties on the China's Space Station: In-Orbit Experiment Measurements and Analysis. *NDT & E International*, **143**, Article 103046. https://doi.org/10.1016/j.ndteint.2024.103046

[25] Paradis, P.F. and Rhim, W.K. (1999) Thermophysical Properties of Zirconium at High Temperature. *Journal of Materials Research*, **14**, 3713-3719. https://doi.org/10.1557/jmr.1999.0501

[26] Ntonti, E., Sotiriadou, S., Assael, M.J., Huber, M.L., Wilthan, B. and Watanabe, M. (2024) Reference Correlations for the Density and Thermal Conductivity, and Review of the Viscosity Measurements, of Liquid Titanium, Zirconium, Hafnium, Vanadium, Niobium, Tantalum, Chromium, Molybdenum, and Tungsten. *International Journal of Thermophysics*, **45**, Article No. 18. https://doi.org/10.1007/s10765-023-03305-z

[27] Weber, A., Schürg, S., Roeder, T., Grobbel, J., Puttkamer, M.N. and Sattler, C. (2025)

Experimental Dataset of Fluid Flow and Heat Transfer in a Shallow Packed Bed at Low Reynolds Numbers. *Data in Brief*, **61**, Article 111743. https://doi.org/10.1016/j.dib.2025.111743

[28] Li, Q., Xue, C., Liu, J.P., Wu, J.F., Yang, S.Q., *et al.* (2018) Measurements of the Gravitational Constant Using Two Independent Methods. *Nature*, **560**, 582-588. https://doi.org/10.1038/s41586-018-0431-5

[29] Xue, C., Liu, J.P., Li, Q., Wu, J.F., Yang, S.Q., *et al.* (2020) Precision Measurement of the Newtonian Gravitational Constant. *National Science Review*, **7**, 1803-1817. https://doi.org/10.1093/nsr/nwaa165

[30] Mohr, P.J., Newell, D.B., Taylor, B.N. and Tiesinga, E. (2025) CODATA Recommended Values of the Fundamental Physical Constants: 2022. *Reviews of Modern Physics*, **97**, Article 025002. https://doi.org/10.1103/revmodphys.97.025002