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Modeling the Random Failure of the B3 Jaw Crusher Using the Euler-Maruyama Method: Determination of the Critical Intervention Threshold at KTC

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Abstract

The progressive functional degradation of the B3 jaw crusher, a critical asset in the mineral processing circuit of the Kamoto Concentrator (KTC, Kamoto Copper Company), causes random shutdowns that threaten production continuity. This study proposes a stochastic model to determine when the crusher should be serviced. Functional performance is described by an Itô-type stochastic differential equation with a degradation drift proportional to the current state and additive Brownian noise, solved with the Euler-Maruyama method and coupled with Monte Carlo simulation. The degradation rate is calibrated so that the expected time to functional failure matches the mean time between failures observed over one year of operation (11 major failures), and the condition of the crusher is checked at weekly inspections. The intervention threshold is optimized by minimizing the long-run average direct maintenance cost per operating day. The optimal threshold is $C^* = 0.40$, giving a mean cycle of 17.7 operating days, a preventive share of 95% and an average direct cost of 312 USD per operating day, against 609 USD for the observed run-to-failure practice—a direct saving of about 80,000 USD per year. The optimum is stable with respect to the time step and moderately sensitive to volatility, degradation rate and inspection interval; with monthly inspections, most interventions become corrective, which explains the current situation. The approach provides a basis for condition-based maintenance of the crusher within a sensor-fed Computerized Maintenance Management System (CMMS).

Keywords

Random Failure, Optimization, Critical Intervention Threshold,

Euler-Maruyama Method, Predictive Maintenance, Monte Carlo Simulation

1. Introduction

1.1. Industrial and Scientific Background

Kamoto Copper Company (KCC) is a major producer of copper and cobalt in the Democratic Republic of the Congo, with its operations located in Kolwezi, Lualaba Province. KCC's assets include two open-pit mines (KOV and Mashamba East), one underground mine (Kamoto), the Kamoto Concentrator (KTC) and the Luilu refinery, which has an annual production capacity of 300,000 tonnes of copper cathode and 40,000 tonnes of cobalt hydroxide.

The KTC concentrator comprises seven primary crushers and six ball mills. Each grinding line requires individualized control, in which the grinding feed-rate ratio is optimized for milling performance. Ore feeding the mills must first pass through the B3 jaw crusher to reach a particle size suitable for downstream processing. The B3 jaw crusher is therefore an indispensable link, whose availability conditions that of the entire ore-processing chain.

Over several production campaigns, the plant has experienced sudden shutdowns linked to the advanced degradation of the B3 crusher. The usual operational response has been to address outlying chain elements on a case-by-case basis to preserve site reliability and availability. However, even when operating conditions have been improved, shutdowns continue to occur unpredictably. The equipment's current level of deterioration brings it close to a near-random failure regime, which compromises its performance. In the absence of any possibility of immediate replacement, operators are compelled to keep running the same crusher under close monitoring, in order to avoid a definitive failure that would bring the entire plant to a halt.

This situation calls for a method for monitoring the crusher's degradation state during operation. The preferred approach is predictive maintenance, which consists of tracking a machine's health to determine the appropriate moment for intervention, so as to avoid not only the failure of the equipment but also its consequences and collateral damage.

1.2. Problem Statement

The aim of this work is to design a solution that favors both production continuity and the minimization of downtime frequency for the B3 jaw crusher. Random failure, although independent of age, generally concerns equipment that is already in an advanced state of degradation. The central question thus becomes: at what precise moment does intervention become necessary?

Several subsidiary questions guide the reflection: which tools are relevant for monitoring equipment condition? which data-collection strategy is most reliable?

which measurable parameters best capture the B3 crusher's degradation state? which failure model applies to this specific case? and which mathematical approach allows the wear law to be traced rigorously?

1.3. Objectives of the Study

The general objective is to build a stochastic mathematical model of the machine's in-service behavior, capable of describing the time evolution of its degradation state, predicting the probability of functional failure, and determining the best moment for intervention.

The specific objectives are to: a) identify the mechanical constraints (jaw-plate wear, bearing and bolt fatigue), the random constraints (nature of the crushed material, feed-rate variations, input particle size) and the indirect parameters (vibration, heating, energy performance); b) establish a differential equation describing progressive degradation; c) simulate the time evolution of the degradation state; d) estimate the failure probability as a function of time; and e) determine the optimal intervention threshold by minimizing a cost function that integrates the direct costs of preventive and corrective interventions.

1.4. Hypotheses and Overall Methodology

Three hypotheses guide this study. First, the work falls within preventive maintenance in its predictive (condition-based) form, in line with standard NF EN 13306 [1]. Second, numerical methods are best suited to a context of uncertain data; among them, the Euler-Maruyama method [2] is retained for its simplicity of implementation. Third, between the degradation model and the catalectic (sudden-failure) model, the former is judged compatible with the observed behavior of the B3 crusher.

Failure data are processed by work-order analysis, which plots the cumulative duration of interventions (an unavailability indicator), the number of failures per family (an unreliability indicator) and the average duration of interventions (an unmaintainability indicator). The model is solved numerically with the Euler-Maruyama method and Monte Carlo simulation.

2. Theoretical and Conceptual Framework

2.1. Reliability Fundamentals

AFNOR standard X 60-500 defines reliability as the ability of a piece of equipment to perform a required function under given conditions for a specified period [3]. Reliability is most commonly derived from the probability of correct operation over a given time span, the failure rate, and the mean lifetime [4].

The survival function $R(t)$ represents the probability that the equipment operates without failure up to time t [5]:

$$R(t) = P(T > t) = 1 - F(t)$$

where T is the equipment's random lifetime and $F(t)$ its cumulative distribution

function. For a jaw crusher, reliability depends on mechanical-part wear (abrasion from crushed materials), mechanical fatigue (cyclic loading) and random failures. It satisfies $R(0) = 1$ and $R(t) \to 0$ as $t \to \infty$ [5].

The failure rate $\lambda(t)$ measures the instantaneous probability of failure per unit time, given survival up to t:

$$\lambda(t) = \frac{f(t)}{R(t)}$$

where $f(t)$ is the failure probability density. Mechanical equipment typically follows a "bathtub" curve with three phases: infant mortality (λ decreasing), useful life (λ approximately constant, exponential distribution), and wear-out (λ increasing, Weibull distribution with shape parameter $\beta > 1$, $\lambda(t) = (\beta/\eta)(t/\eta)^{(\beta-1)}$, η being the scale parameter). Reliability follows from the failure rate as $R(t) = \exp\left(-\int_0^T \lambda(u)\,\mathrm{d}u\right)$, which reduces to $R(t) = \mathrm{e}^{(-\lambda t)}$ when λ is constant [6].

MTTF (Mean Time To Failure) is the expected operating time before failure of non-repairable equipment; MTBF (Mean Time Between Failures) is its analogue for repairable equipment, estimated as total operating time divided by the number of failures [5] [6]. Availability is A = MTBF/(MTBF + MTTR), where MTTR is the mean time to repair.

2.2. Failure Typology

Sudden (catalectic) failure occurs abruptly, without prior warning signs, and is often linked to overloads or design defects; it is modeled by an exponential distribution and calls for corrective maintenance [5]. Progressive failure results from gradual degradation (wear, corrosion, fatigue) and is predictable through indicators such as vibration or temperature; it is commonly modeled by a Weibull distribution ($\beta > 1$) or by a degradation process, and calls for preventive or predictive maintenance [7].

Functional failure occurs when a system can no longer perform one or more of its intended functions; under ISO 14224 it calls for a criticality analysis, typically a Failure Mode, Effects and Criticality Analysis (FMECA). Random failure is linked to non-deterministic factors (thermal fluctuations, load variations) and is modeled by an exponential distribution or a Poisson process for repairable systems [8]. For the B3 crusher, the combination of a progressive trend with a random component best characterizes the observed behavior.

2.3. Stochastic Processes and Reliability

A stochastic process is a family of random variables representing the evolution of a system over time. The continuous-time Markov chain models transitions between states (functional, failed, under repair); for a two-state system with failure rate λ and repair rate μ, steady-state availability is $A = \mu/(\lambda + \mu)$ [9]. The homogeneous Poisson process describes events occurring at a constant intensity λ, with $P(N(t) = k) = (\lambda t)^k \cdot e^{(-\lambda t)}/k!$ and MTTF = $1/\lambda$ [10].

The Wiener process is best suited to progressive degradation: wear $X(t)$ follows $X(t) = x_0 + \mu t + \sigma W(t)$, where μ is the mean wear rate and σ the volatility due to random shocks [11]. The first-passage time to a critical threshold L has an inverse Gaussian distribution with mean $(L - x_0)/\mu$ [12]. For simple repairable systems the Poisson process is preferred; for complex systems with repairs, the Markov chain; for progressive degradation, a diffusion process—the latter being adopted in this study.

2.4. Maintenance, Decision-Making and the Critical Intervention Threshold

Maintenance types are classified by their trigger: corrective maintenance (after failure), predetermined preventive maintenance (based on calendar or usage) and condition-based or predictive maintenance (driven by the equipment's observed or forecast state) [1]. Maintenance decisions rest on technical, economic and strategic criteria [13].

The critical intervention threshold is the degradation level at which action becomes necessary—a "red line" whose crossing exposes the plant to a sudden failure or a production shutdown [14]. In practice it rests on standardized indicators: vibration level (ISO 10816), heating (IEC 60034-1), current draw, fluid leaks and mechanical wear.

The interval $[S, C]$, bounded by the failure threshold S and the preventive threshold C, defines a three-zone policy: green zone ($P(t) > C$, normal operation), amber zone ($S < P(t) \leq C$, preventive intervention to be scheduled) and red zone ($P(t) \leq S$, functional failure and emergency shutdown). This framework underlies the model developed in the next section.

3. Methodology

3.1. Modeling Functional Degradation

The B3 crusher breaks ore into smaller particles under high mechanical loads, abrasion and thermal variation. Its moving jaw plates wear rapidly from friction and repeated impact. The crusher's health state $H(t)$ ($H = 1$: new, $H \to 0$: failed) can be written $\mathrm{d}H(t)/\mathrm{d}t = -\lambda(t) \cdot H(t)$, where the degradation rate combines a base wear rate, a load-induced rate proportional to the crushing force, and an abrasion-induced rate proportional to ore abrasivity and throughput [15].

Functional performance $P(t)$ — the crusher's effective capacity, between 0 and 1 — is the product of the health state $H(t)$ (slow time scale: weeks to months) and an operational-efficiency factor $E(t)$ (fast time scale: minutes to hours): $P(t) = H(t) \cdot E(t)$ [13]. This decomposition separates losses due to irreversible wear from those due to sub-optimal operating conditions.

3.2. Stochastic Formulation

Performance is subject to unpredictable disturbances (foreign bodies in the ore,

hardness variation, mechanical shocks). The deterministic model is therefore recast as an Itô-type stochastic differential equation (SDE) [12]:

$$dX(t) = \mu(X,t)dt + \sigma(X,t)dW(t)$$

Restricting to a drift proportional to the current state and an additive diffusion, the model adopted for the B3 crusher is:

$$dP(t) = -\mu P(t)\ dt + \sigma\ dW(t)$$

where μ is the degradation rate (day^{-1}), σ the volatility (day$^{(-1/2)}$) and $W(t)$ a standard Brownian motion representing random shocks [16]. The mean trajectory is $E[P(t)] = P_0 e^{(-\mu t)}$, so $1/\mu$ is the characteristic degradation time and σ quantifies its dispersion.

3.3. Numerical Resolution via the Euler-Maruyama Method

This linear SDE (an Ornstein-Uhlenbeck process) has a closed-form solution for $P(t)$, but the distribution of the first-passage times to the thresholds C and S under periodic inspections has no simple closed form, which justifies simulation. The Euler-Maruyama method [2] [17] extends the classical Euler method by adding a Brownian increment at each step. For $dX_t = a(X_t, t)dt + b(X_t, t)dW_t$, the scheme is:

$$X_{n+1} = X_n + a(X_n, t_n)\Delta t + b(X_n, t_n)\sqrt{\Delta t}Z_n, Z_n \sim N(0,1)\ i.i.d.$$

Applied to the performance model:

$$P_{n+1} = P_n - \mu P_n \Delta t + \sigma\sqrt{\Delta t}Z_n$$

Z_n is the standardized Brownian increment, modeling the unpredictable shocks affecting the crusher at each time step. A constant volatility is adopted (Section 3.6); the scheme has strong order 1/2 and weak order 1 [18], and its convergence was checked numerically by repeating the optimization with Δt from 0.1 to 1 day (Section 4.4). The simulation uses $\Delta t = 0.25$ day, *i.e.* about 1% of the characteristic time $1/\mu$.

3.4. Maintenance Policy, Monte Carlo Simulation and Optimization of *C*

The policy simulated is a periodic-inspection, condition-based policy [19]. Each cycle starts from P_0 after a renewal. The crusher's condition is checked at inspections every $T_i = 7$ operating days, which corresponds to the weekly inspections practised at KTC. A cycle ends:

- in preventive maintenance if, at an inspection, $S < P(t) \leq C$; the intervention is then planned and the crusher renewed;
- in corrective maintenance if $P(t) \leq S$ at any simulation step, *i.e.* the crusher fails between two inspections before the degradation has been detected—including the case where it crosses C and S within the same step.

Corrective outcomes therefore arise from the combination of the inspection interval and random fluctuations: the lower C is, the less margin remains between

detection and failure. Renewal is assumed to restore P to P_0 (as-good-as-new).

For each candidate value of C, M = 10,000 trajectories are generated with a fixed random seed (42) for reproducibility. Each trajectory gives a cycle type and a cycle duration T (operating days), from which the preventive probability $PMP(C)$, the corrective probability $PDP(C)$ and the mean cycle duration $E(T|C)$ are estimated. C is varied from 0.27 to 0.70 in steps of 0.01, and the optimal threshold C^* minimizes the long-run average cost per operating day given by the renewal-reward theorem [20]:

$$J(C) = \frac{C_{prev} \cdot PMP(C) + C_{corr} \cdot PDP(C)}{E(T|C)}$$

where C_{prev} and C_{corr} are the direct costs of a preventive and a corrective intervention (labor, spare parts, contractor services). Production losses are not included in $J(C)$; they are discussed separately (Section 5.1). The same direct-cost basis is used in Sections 4.2 and 4.5. The problem can also be recast with a reliability constraint, $PDP(C) \leq \alpha_{max}$, which is often more intuitive for maintenance managers [21].

3.5. Threshold-Crossing Detection

For each trajectory, the first simulation step at which $P \leq S$ is recorded as a failure, and the first inspection at which $P \leq C$ is recorded as a preventive intervention, whichever comes first. Because the simulation step (0.25 day) is small compared with the inspection interval (7 days), the failure time is located to within 0.25 day, which is negligible for cost purposes; no interpolation is needed.

3.6. Parameters and Field Data

The parameters are drawn from one year of operational history of the B3 crusher (3 October 2024 to 3 October 2025) recorded in KTC's maintenance work orders and production reports [22]: a total duration of 8760 hours (365 days), cumulative downtime of 2256 hours (94 days) and 11 major failures. A major failure is an unplanned stoppage of the crusher that required a corrective work order on the crusher itself (jaw plates, bearings, eccentric shaft, toggle, frame); stoppages caused by upstream or downstream equipment and short feed blockages are excluded [à confirmer]. The effective operating time is 271 days, hence MTBF = 271/11 ≈ 24.6 operating days and MTTR = 94/11 ≈ 8.5 days.

Initial performance is P_0 = 600/800 = 0.75, taking a representative current throughput of 600 t/h relative to the nominal throughput of 800 t/h (achieved with both feed pumps of the primary feeder running). Throughput recorded from October 2024 to April 2025 [fréquence d'enregistrement à confirmer] ranged from 350 to 600 t/h, with a mean of about 458 t/h.

Calibration of μ. Failure is defined as $P(t)$ reaching S. The rate μ is chosen so that the deterministic time to failure, starting from P_0, equals the observed MTBF:

$$\mu = \frac{\ln\left(\frac{P_0}{S}\right)}{\mathrm{MTBF}}$$

With $S = 0.25$ this gives $\mu = \ln 3/24.6 \approx 0.0446$ day^{-1}. This calibration ties the time scale of the model to the observed failure history, so the choice of S mainly fixes the scale on which thresholds are expressed, not the time to failure. $S = 0.25$ (200 t/h) is taken as the functional-failure level, well below the lowest performance observed in operation (about 0.43).

Volatility. No continuous condition-monitoring signal is available to estimate σ statistically. $\sigma = 0.02$ day$^{(-1/2)}$ is therefore an assumption: it gives a dispersion of the time to failure of a few days (standard deviation about $\sigma\sqrt{t} = 0.1$ in P over a 25-day cycle), consistent with the irregular intervals between the observed failures. Its effect is quantified by sensitivity analysis (σ from 0.01 to 0.04, Section 4.4).

The direct costs are $C_{prev} = 5000$ USD and $C_{corr} = 15{,}000$ USD per intervention, based on KCC maintenance practice. Table 1 separates the parameters estimated from data from those adopted as assumptions.

Table 1. Model parameters: estimated, calibrated and assumed values.

Parameter	Value	Status	Basis
MTBF	24.6 days	Estimated	271 operating days/11 major failures
P_0	0.75	Estimated	600/800 t/h, production records
S	0.25	Assumed	Functional-failure level (200 t/h)
μ	0.0446 day^{-1}	Calibrated	$\ln(P_0/S)$/MTBF
σ	0.02 day$^{(-1/2)}$	Assumed	Sensitivity 0.01 - 0.04
T_i	7 days	Observed practice	Weekly inspections; sensitivity 1 - 30 days
C_{prev}, C_{corr}	5000; 15,000 USD	Assumed	KCC practice, direct costs only
Δt, M	0.25 day; 10,000	Numerical	Convergence checked (Section 4.4)

3.7. Model Verification and Validation

Verification checks the model's mathematical and numerical consistency: correct coding of the stochastic equation, stability of the results with respect to Δt, normality of the simulated Brownian increments, and coherent responses to changes in μ, σ and T_i [23]. These checks are reported in Section 4.4. The Monte Carlo standard error of the probabilities is at most $\sqrt{(0.25/10{,}000)} = 0.005$.

Validation compares simulated trajectories with observed condition data, using the mean absolute error (MAE) and the root mean square error (RMSE):

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{m}\left|y_i - \hat{y}_i\right|$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}$$

where y_i is the observed performance and $\hat{y}_i$ the simulated performance at measurement point i, and n is the number of measurement points.

Such a validation requires condition-monitoring data that are independent of those used for calibration. These data are not yet available for the B3 crusher; validation is therefore left for future work (Section 6), and the present results should be read as a calibrated decision model rather than a validated forecast.

4. Results

4.1. Simulated Trajectories of Performance P(t)

Figure 1 shows a sample of trajectories simulated at the optimal threshold $C^* = 0.40$. Starting from $P_0 = 0.75$, performance declines under the drift $-\mu P$ and fluctuates under the noise term. Because the condition is only observed at weekly inspections (vertical lines), performance can drop below C between two inspections: most trajectories are caught at the second or third inspection (day 14 or 21) and renewed preventively, while a few reach S before being detected and end in corrective maintenance.

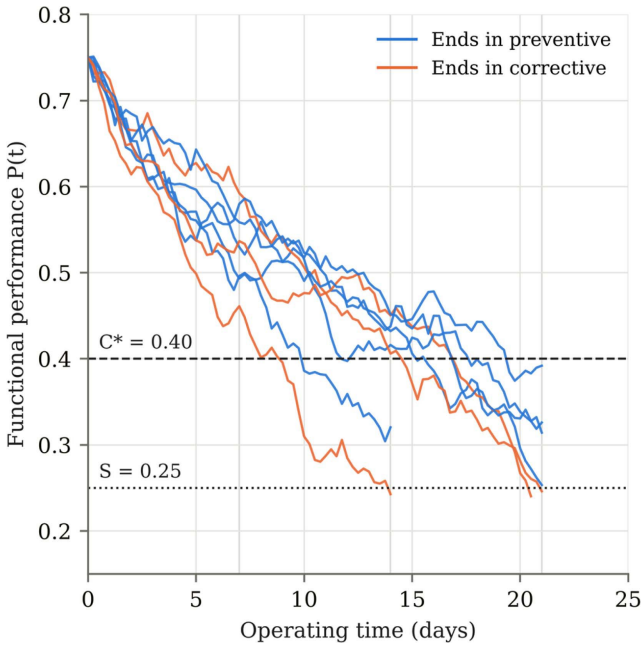


Figure 1. Sample trajectories of performance $P(t)$ at $C^* = 0.40$ ($P_0 = 0.75$, $\mu = 0.0446$ day^{-1}, $\sigma = 0.02$ day$^{(-1/2)}$, weekly inspections). Blue: cycles ending in preventive maintenance; orange: cycles ending in corrective maintenance.

4.2. Cost Function and Optimal Threshold

Table 2 gives, for selected values of C, the indicators obtained from 10,000 simulated cycles, and Figure 2 shows $J(C)$ over the full range tested.

Table 2. Cost function $J(C)$ and maintenance indicators as a function of the preventive threshold C (10,000 simulated cycles per value; weekly inspections).

Threshold C	Mean cycle $E(T\|C)$ (operating days)	$PMP(C)$	$PDP(C)$	Direct cost $J(C)$ (USD/day)
0.30	23.3	0.42	0.58	464
0.35	20.7	0.78	0.22	347
0.38	18.9	0.90	0.10	317
0.40 (optimum)	17.7	0.95	0.05	312
0.42	16.6	0.97	0.03	316
0.45	15.3	0.99	0.01	332
0.50	13.3	1.00	0.00	377

$J(C)$ has a clear minimum. When C is too low, the margin between detection and failure is too small and many cycles end in costly corrective interventions (58% at $C = 0.30$). When C is too high, the crusher is renewed too early and the preventive cost is spread over short cycles (13.3 days at $C = 0.50$). The minimum, $J(C^*) = 312$ USD per operating day, is reached at $C^* = 0.40$, with a mean cycle of 17.7 operating days and 95% of interventions preventive; the expected direct cost per cycle is $0.947 \times 5000 + 0.053 \times 15{,}000 \approx 5530$ USD, and $5530/17.7 \approx 312$ USD per operating day.

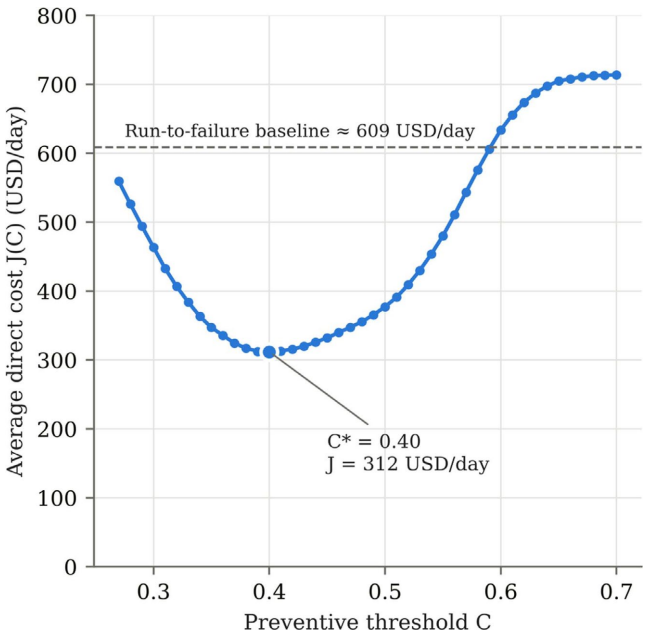


Figure 2. Average direct cost $J(C)$ as a function of the preventive threshold C. The dashed line is the run-to-failure baseline, 15,000/24.6 ≈ 609 USD per operating day.

The minimum is flat: any threshold between 0.38 and 0.42 keeps J within 2% of the optimum, which leaves maintenance managers room to adjust C to operating

constraints without a significant cost penalty.

4.3. Preventive and Corrective Probabilities

Figure 3 shows that $PMP(C)$ and $PDP(C)$ move in opposite directions: raising C moves detection earlier and reduces the corrective share, which falls below 1% for $C \geq 0.45$. Beyond C^*, the further reduction of corrective risk no longer offsets the cost of more frequent preventive interventions.

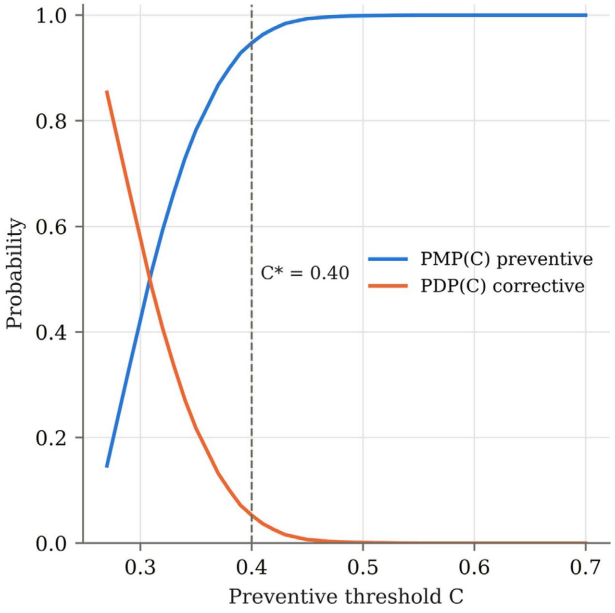


Figure 3. Preventive (PMP) and corrective (PDP) probabilities as a function of the threshold C.

4.4. Sensitivity Analysis

Table 3 reports the optimum when one parameter is varied at a time (4000 cycles per value of C).

Table 3. Sensitivity of the optimal policy to the time step, volatility, degradation rate and inspection interval.

Parameter varied	Value	C^*	$PDP(C^*)$	$E(T\|C^*)$ (days)	$J(C^*)$ (USD/day)
Time step Δt (days)	0.1/0.25/0.5/1	0.40/0.40/0.40/0.40	0.05/0.05/0.05/0.05	17.7/17.7/17.6/17.5	311/310/312/316
Volatility σ	0.01/0.02/0.03/0.04	0.37/0.40/0.43/0.44	0.04/0.05/0.08/0.13	20.1/17.7/16.6/16.4	268/310/347/383
Degradation rate μ (day^{-1})	0.022/0.045/0.067	0.37/0.40/0.43	0.05/0.05/0.13	35.9/17.7/12.5	152/310/499
Inspection interval T_i (days)	1/7/14/30	0.30/0.40/0.48/0.43	0.01/0.05/0.04/0.87	21.1/17.7/15.1/23.8	244/310/357/573

Time step. C^*, the corrective share and the mean cycle are unchanged for Δt between 0.1 and 1 day, and J varies by less than 2%: the discretization error is negligible at $\Delta t = 0.25$ day.

Volatility. Higher volatility makes failure between inspections more likely, so the optimal threshold rises (from 0.37 to 0.44) and the cost increases by about 23% from $\sigma = 0.02$ to $\sigma = 0.04$. Because σ is assumed, the recommended threshold should be read as a range of about 0.37 - 0.44.

Degradation rate. A faster degradation shortens the cycle and calls for an earlier intervention (C^* from 0.37 to 0.43); the cost is roughly proportional to μ, since the number of cycles per year is.

Inspection interval. This is the most influential parameter. Daily inspections lower the cost to 244 USD/day; with monthly inspections, no threshold can prevent most failures (87% corrective) and the cost, 573 USD/day, approaches the run-to-failure level. This result explains why the crusher currently operates in a corrective regime and shows that inspection frequency is as important as the choice of threshold.

4.5. Critical Time and Economic Balance

Figure 4 shows the distribution of cycle durations at C^*. Because detection only happens at inspections, preventive interventions fall at day 14 (49% of cycles), day 21 (42%) or, more rarely, day 28 (3%); the 5% of corrective failures occur mostly between days 13 and 27. The critical time is therefore $E\left(T\middle|C^*\right) = 17.7$ operating days, and in practice the decision is taken at the second or third weekly inspection after a renewal.

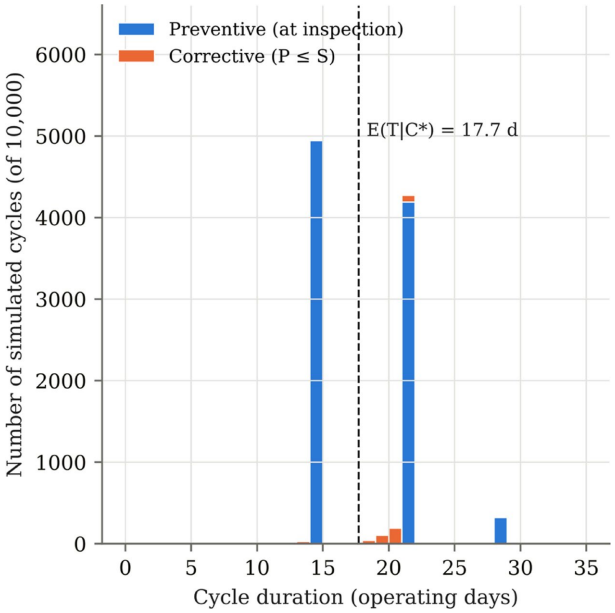


Figure 4. Distribution of cycle durations at $C^* = 0.40$ (10,000 simulated cycles), by type of cycle end.

The economic balance uses the same direct-cost basis as $J(C)$ and the 271 operating days observed per year. The current practice, which amounts to running the crusher to failure, generated 11 corrective interventions, *i.e.* 11 × 15,000 = 165,000

USD per year (609 USD per operating day). Under the optimal policy, 271/17.7 ≈ 15.3 interventions per year are expected (14.5 preventive, 0.8 corrective), for an expected direct cost of 312 × 271 ≈ 84,600 USD per year. The expected direct saving is about 80,000 USD per year (−49%). Production losses are excluded: the full benefit also depends on downtime, which falls from 94 days per year only if a preventive intervention takes less than about 6 days on average ((94 − 0.8 × 8.5)/14.5 ≈ 6).

5. Discussion

5.1. Simulation and Current Practice

Table 4 compares the observed situation with the optimized policy.

Table 4. Comparison between the observed situation and the optimized policy.

Aspect	Observed (Oct 2024-Oct 2025)	Optimized policy (simulation)
Interventions per year	11, all corrective	≈15.3 (95% preventive)
Operating days between interventions	24.6 (MTBF)	17.7
Direct cost	165,000 USD/year (609 USD/day)	≈84,600 USD/year (312 USD/day)
Downtime	94 days/year (MTTR 8.5 days)	≈6.9 days corrective + 14.5 × preventive duration
Inspections	Weekly to monthly	Weekly, with threshold $C^* = 0.40$

Cycle length. The optimal policy intervenes more often than the failures currently occur (every 17.7 instead of 24.6 operating days). This is the expected effect of preventive maintenance: part of the useful life is given up to avoid failures that cost three times more.

Cost. Both columns of Table 4 are direct costs. Production losses during the 8.5-day mean repair time are by far the largest consequence of a failure at KTC, since the B3 crusher feeds the whole grinding line; including them would increase the corrective cost and push the optimal threshold upward. The saving reported here is therefore a lower bound.

Inspection frequency. The sensitivity analysis shows that monthly inspections lead to 87% corrective interventions whatever the threshold. Moving to systematic weekly inspections, and ideally to continuous monitoring, is a precondition for the benefits of the threshold policy.

5.2. Limitations of the Study

The model rests on simplifying assumptions. The drift $-\mu P$ assumes degradation proportional to the current state, whereas some wear phenomena accelerate at end

of life. μ and σ are assumed stationary, although they may change with load, ore type and the history of interventions. μ is calibrated indirectly from the failure frequency, and σ is assumed rather than estimated from condition data. Gaussian noise excludes sudden shocks, better represented by jump processes (Poisson, Lévy). The model is Markovian, with no memory of cumulative fatigue.

On the decision side, preventive renewal is assumed to be performed at the inspection where degradation is detected and to restore the crusher to its initial state; real delays (crew availability, parts supply) and imperfect repairs would increase the corrective share. Costs are treated as constant, and the duration of preventive interventions, which determines the effect on availability, has not been measured. Finally, the model has not yet been validated against condition data independent of those used for calibration.

The model is univariate: $P(t)$ aggregates distinct phenomena (vibration, temperature, energy consumption, throughput) into one indicator, which can mask weak signals that precede failure and limits transfer to other equipment (pumps, mills, conveyors), for which a multivariate approach would be required.

These limitations do not undermine the approach; they define its domain of validity and point future work toward multivariate models fed by sensor data, more realistic costs and validation on independent data.

6. Conclusion and Perspectives

This study addressed a precise operational question: when should the B3 jaw crusher be serviced to avoid failure without over-maintenance? The answer rests on a stochastic degradation model, $dP(t) = -\mu P(t)\,dt + \sigma dW(t)$, solved by the Euler-Maruyama method and Monte Carlo simulation, combined with a weekly-inspection, threshold-based maintenance policy.

With μ calibrated on the observed MTBF of 24.6 operating days ($\mu = 0.0446$ day^{-1}), $P_0 = 0.75$ and $\sigma = 0.02$ day$^{(-1/2)}$, the optimal preventive threshold is $C^* = 0.40$ (acceptable range 0.38 - 0.42), reached on average after 17.7 operating days, with 95% of interventions preventive. The average direct cost falls from 609 to 312 USD per operating day, a direct saving of about 80,000 USD per year, excluding avoided production losses. The inspection interval is the decisive parameter: monthly inspections keep the crusher in a corrective regime whatever the threshold.

These results show that it is possible to shift the management of the B3 crusher from a costly corrective logic to a condition-based preventive strategy grounded in an explicit, reproducible model.

Several extensions follow. The first is to integrate the model into the plant's Computerized Maintenance Management System, fed by IoT sensors (vibration, temperature, pressure, current), to generate alerts and work orders automatically. The second is to estimate μ and σ directly from these condition data and to validate the model on data withheld from calibration. The third is a multivariate formulation integrating several degradation indicators, together with production

losses and the duration of preventive interventions in the cost function. This work will be continued within a doctoral thesis, with an extended data-collection campaign at the KCC site.

Ultimately, the approach provides maintenance managers with a decision-support tool to reduce costs and improve the availability of a critical asset of the Kamoto concentrator, as a step toward predictive maintenance in the Congolese mining industry.

Data and Code Availability

The simulation was implemented in Python (NumPy) with a fixed random seed (42); the script and an equivalent MATLAB script are provided as supplementary material. Operational data are internal to KCC [22] and are available from the authors on reasonable request, subject to the company's authorization. A detailed account of the field study is given in [24].

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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