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On the Collapse Time of the North Tower (WTC1)

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Abstract

This work investigates the collapse of the North Tower of the World Trade Center on September 11, 2001, with particular emphasis on the propagation phase following collapse initiation under the assumptions of the NIST Final Report. Using elementary mechanics at the level of a first-year university physics course, the building is modeled as a system of 110 contiguous layers, each consisting of a floor slab together with the supporting columns incorporated into it. Once the pancake phase of the collapse is initiated below the floors affected by the aircraft impact and fire, the collapse is represented as a sequence of inelastic impacts between elements in direct physical contact, with no empty spaces between them because of the existence and continuity of the load-bearing columns. Since linear momentum must be conserved at each stage, the resulting effect is that the collapse should be slowed down rather than accelerated. Consequently, the minimum collapse time turns out to be far greater than the observed value. This discrete model is compared with Bažant's continuous formulation, to which NIST refers for the analysis of collapse propagation. The comparison highlights substantial differences in the underlying physical assumptions, particularly in the treatment of the supporting columns and of mass accretion, and shows how these differences lead to markedly different predictions for the dynamics and duration of the collapse.

Keywords

Progressive Collapse, World Trade Center, Collapse Dynamics, Impulsive Dynamics, Structural Mechanics

1. Introduction

The collapse of the North Tower of the World Trade Center (WTC1) on Septem-

ber 11, 2001, has been the subject of extensive structural and mechanical analyses. The NIST investigation [1] focused primarily on the mechanisms leading to collapse initiation, whereas the subsequent progression of the collapse has also been studied through simplified analytical models, most notably those developed by Bažant and collaborators [2]-[4].

The purpose of the present work is more limited. Rather than attempting a detailed structural simulation of WTC1, we investigate the propagation phase of the collapse using elementary mechanics and a deliberately simplified discrete model. The aim is to examine how momentum transfer between successive portions of the structure affects the characteristic collapse time once collapse initiation is assumed to have occurred.

We model the building as a sequence of contiguous layers, each representing a floor slab together with the corresponding portion of the load-bearing structure. The central assumption is that, below the initially damaged region, successive layers are not separated by empty spaces. The advancing collapse front therefore interacts continuously with structural material, and the transfer of momentum may be represented, in an idealized form, as a sequence of inelastic impacts.

This description differs from continuous crush-down models, in which mass accretion and structural resistance are represented by continuous functions of the collapse-front position. In particular, Bažant's formulation describes the motion of a growing falling mass acted upon by gravity and by an effective structural resistance. The discrete and continuous descriptions therefore treat mass accretion and structural interaction in substantially different ways.

We remark that effects such as detailed column deformation, fracture, three-dimensional load redistribution, fragmentation, lateral ejection of material, and local connection failures are outside the scope of the present treatment. Instead, we ask a more restricted question: under the assumptions of the simplified contiguous-layer model, what collapse time follows from momentum conservation, and how does this result compare with both the observed collapse time and the prediction of a continuous mass-accretion model?

To establish the initial conditions for this calculation, we adopt the collapse-initiation scenario described in the NIST/Bažant framework within the aircraft-impact and fire-damaged region. We then separate the subsequent evolution into two regimes. In the first, the upper block falls through the initially damaged region and impacts the floors immediately below it. In the second, the resulting block enters the lower portion of the building, which is modeled as a sequence of contiguous structural layers.

The comparison between these two idealized descriptions shows that the treatment of mass accretion and of the intervening load-bearing structure has a major influence on the predicted dynamics. In the discrete contiguous-layer model, successive inelastic impacts progressively reduce the velocity of the moving block, leading to a collapse time much longer than the observational estimate. By contrast, numerical integration of the continuous Bažant-type equation considered

here leads, after an initial transient, to an approximately constant-acceleration regime and a substantially shorter collapse time.

2. Physical Assumptions and Collapse-Initiation Scenario

We start by reporting essential data pertaining to some characteristics of WTC1, based on Refs. [1] [5]. In particular:

1) height: $H = 417$ m at roof level, with 110 stories;

2) mass: approximately $M_{\text{tot}} \simeq 500,000$ metric tons, including at least 100,000 tons of steel and the remainder consisting of concrete, furnishings, and other materials;

3) typical story height, neglecting slab thickness: $h = 3.71$ m ;

4) steel load-bearing structure: 47 core columns and 240 perimeter columns (59 per facade, plus four non-load-bearing corner members). The load-bearing columns were connected by steel beams to form a steel cage, naturally fireproofed, designed so that each story could support at least five to six times the weight above it.

The cage was highly resistant and elastic. In particular, it was designed to withstand severe wind loading and to survive the possible impact of a large aircraft without suffering structural damage sufficient, by itself, to compromise the overall stability of the building. The collapse is generally attributed primarily to the combined effects of impact damage and fire, although different authors assign different relative weights to these mechanisms; Greening [6], for example, emphasizes their combined action.

The impact and the fire

According to the NIST Final Report [1], the impact occurred between floors 93 and 98 and destroyed or severely damaged 37 exterior columns (35 severed and two severely damaged) and nine core columns: six severed, all immediately above the slab of floor 96, and three "significantly damaged." It also damaged the floor system in the northern area, the insulation covering the steel load-bearing members, and the trusses connecting the core to the perimeter columns.

The aircraft carried an estimated 38,000 L of fuel. Upon impact, a fireball instantaneously consumed approximately 5000 - 6000 L, while a comparable amount was lost in the fireball outside the building. According to the preliminary FEMA report [7], the time needed to burn the remaining fuel together with the combustible contents of the affected area would have been about five minutes with unlimited oxygen, that is, in open-air conditions. The estimates change substantially under oxygen-starved conditions, as indicated, among other things, by the large amount of black smoke. Under these conditions, temperature and fire duration depend on the fuel and especially on the available oxidizer, so one must rely on approximate estimates based on assumptions, observations, and numerical simulations.

According to NIST, the maximum temperature of the upper air layers was about 1000˚C. Temperatures close to 1000˚C in the air, not in the steel, were sus-

tained for about 15 - 20 minutes; during the remainder of the fire, calculated temperatures were near 500˚C or lower.

Collapse initiation

These estimates concerning the ambient temperature in the fire zone are relatively widely shared, whereas estimates of steel temperature vary considerably. It is generally accepted that the maximum temperature reached by the steel remained well below its melting point of about 1435˚C. At roughly 550 - 600˚C, however, steel undergoes substantial strength reduction and may exhibit viscoplastic deformation (creep), potentially compromising the local stability of structural members.

Direct observations, as acknowledged by both NIST and FEMA, indicated much lower steel temperatures in the recovered samples, of order 250 - 300˚C. NIST stated that there was no evidence that any examined sample had exceeded[1] 600˚C. Nevertheless, consistent with the seminal paper by Bažant and Zhou [2], NIST hypothesized on the basis of simulations that some core-column elements between floors 93 and 98 may have reached and maintained sufficiently high temperatures for a sufficiently long period to undergo severe loss of load-bearing capacity.

The subsequent sequence proposed by NIST is as follows. The instability does not remain confined to the heated portion of the core, but propagates through thermal expansion along the steel trusses connecting the core to the perimeter columns. Because the trusses are restrained at both ends, the critical floors deform into a catenary configuration and pull the perimeter columns inward. The perimeter columns begin to bow inward, and this deformation spreads across the facades until global instability develops in the critical zone.

At this point, the portion above floor 96, no longer adequately supported by the severed or damaged core columns, begins to descend "essentially in free fall" (NIST [1], p. 196) until it impacts floor 95. The subsequent development depends on the nature of the impact and on the dynamic resistance of the structure below. NIST does not directly analyze the entire propagation phase of the WTC1 collapse in detail, and the mechanics of progressive collapse has therefore often been discussed through the analytical models developed by Bažant and collaborators [2]-[4].

Purpose and limitations of the paper

We shall not further discuss the details of collapse initiation. Instead, we adopt the assumption of Bažant and Zhou [2] that, in the fire-affected region, particularly between floors 94 and 96, a progressive collapse occurred[2] in which each

[1]Videos reportedly show people moving in the opening created by the aircraft impact, on the side opposite the main fire. This observation is potentially difficult to reconcile with an initiation model requiring a spatially extensive and prolonged fire, however this issue is outside the scope of the present paper.

[2]The choice of a critical floor between 94 and 96 implies an initial upper-block mass of $14M$ if all floors have equal mass M. NIST and Bažant instead place initiation at floor 98, corresponding to a 12-floor upper block. The two choices differ in localization but not in their general description of collapse dynamics. Several other authors favor floor 96 and the immediately lower floors because floor 96 suffered six severed core columns, the largest number of severed exterior columns, the deepest penetration of engines and fuselage, broad fuel dispersal, and extensive floor damage. Floor 94 is em-

floor, while retaining a certain rigidity because its nodes remain mutually constrained apart from possible local connection failures, fell onto the floor below, incorporated it, and dragged it downward.

As we said in the Introduction, the purpose of this work is to use elementary physics, under suitable assumptions, to investigate how the collapse could have propagated once initiated. In particular, we seek to determine a theoretical minimum total collapse time and compare it with the observed time.

3. Assumptions on the Collapse: The Many Lives of the Pancake Theory

The beginning of the collapse, step by step

The central NIST/Bažant assumption is that, because of elevated temperatures, the core columns in the critical region (floors 95 - 96) eventually lost their load-bearing capacity. The upper portion, floors 96 - 110, then fell onto floor 95 as though no appreciable resistance existed between them, producing a violent inelastic collision with large dissipation of mechanical energy into fragmentation, heat, and other processes.

It is assumed that the upper block descended nearly intact and aligned with the lower structure[3]. After the first impact, floor 95 also fails, and the resulting block, now comprising floors 95 - 110, falls through the opening and strikes floor 94. Below floor 94 the situation becomes more complex because the structure is assumed to be essentially intact[4]. Bažant and collaborators, as we will discuss below, assume that, under dynamic impulsive loading, the lower structure cannot provide sufficient resistance to arrest the moving mass. The falling block therefore continues downward, striking one floor after another and increasing its mass and momentum. This domino-like process is commonly known as the pancake effect.

Collapse propagation

The NIST Final Report states that the kinetic energy and momentum of the falling mass were so large compared with the resistance of the lower structure that the latter was "unable to stop or even to slow the falling mass." NIST estimated that the first exterior panels struck the ground approximately 11 s after collapse initiation for WTC1, with an uncertainty of about 1 s. This is close to the estimated free-fall time: approximately 9.2 s to ground level, or 8.9 s to the top of the debris pile, estimated at 21.33 m in Ref. [8] and 25 m in Ref. [2].

The assumption that the lower structure could not even slow the descending

phasized as the first floor supported by columns assumed not to have been thermally softened.

[3]This assumption ensures a destructive effect on the impacted floor, but it is not fully supported by the video evidence, which shows signs of incipient disintegration above floor 100 before the onset of the collapse. For instance, the roof antenna appears to descend by roughly 20 m several seconds before the visible global collapse.

[4]Some structural engineers have argued that although the columns below floor 94 were intact before initiation, the impact immediately changed their state by sending a so-called yielding wave downward. We notice that the initial disturbance propagates as an elastic stress wave at approximately 5000 $m \cdot s^{-1}$, reaching the lower structure in a fraction of a second; in addition, we point out that the added stress would remain below yield until the collapse front itself arrived several seconds later.

mass is *impossibile* to reconcile with both energy conservation and momentum conservation, especially given the large amount of energy dissipated in destroying the lower floors, their contents, and their steel framing[5]. In addition, momentum conservation, which will be central below, is difficult to reconcile with a strictly constant-acceleration description if the collapse is represented as a sequence of inelastic impacts, because the velocity decreases at each impact. In Bažant-type treatments the load-bearing columns are replaced, as physical structural elements, by a conjectural mathematical function representing their average resistance, which slows the descent while leaving, between successive impacts, a free-fall interval in which gravity dominates; the velocity decrease due to the impacts turns out to be small compared to the increase due to gravity. The distinction between these two descriptions motivates the discrete calculation developed below.

4. Collapse Mechanism

Assumptions

We adopt the NIST/Bažant scenario as a working hypothesis for collapse initiation in the region where softening, or viscoplastic creep deformation, is reasonably conceivable because of aircraft-impact damage and fire exposure (floors 94 - 96). We then calculate the subsequent motion using impulsive dynamics and conservation laws.

Although the mass of the floors, and especially that of the steel columns, *decreases* with height, we simplify the calculation by assigning the same mass M to every floor, including its slab, contents, and incorporated steel-column segments. We also neglect the large amount of material expelled outside the building footprint. This choice gives a collapse time shorter than would be obtained if all such mass losses were explicitly included, because all of the falling mass is assumed to remain within and contribute to the impact sequence.

The collapse

At collapse initiation, the block above floor 96, of mass $14M$, falls through the region damaged by impact and fire. It is assumed to be slowed negligibly by intervening material and more appreciably by inelastic impacts with floors 95 and 94. The block first falls through one story height with acceleration $g = 9.81\,\mathrm{m}\cdot\mathrm{s}^{-2}$ and strikes floor 95. The collision is modeled as perfectly inelastic: mechanical energy is strongly dissipated, but linear momentum is conserved because the impulsive internal forces dominate gravity and constraint reactions during the short collision interval.

After this impact, a block of mass $15M$ continues downward and strikes floor 94. A second inelastic collision produces a block of mass $16M$. From this point onward, however, the descending block no longer encounters an empty story space in the present model. Below floor 94, the load-bearing structures are as-

[5]The observed pulverization would require an energy much greater than that estimated from elementary inelastic-impact calculations. Greening [6] estimates an energy of approximately 2×10^{11} to 3×10^{11} J to fragment the tower's concrete into particles of order 60-100 μm, corresponding to 20% - 30% of a gravitational-energy estimate of 10^{12} J.

sumed to be essentially intact and continuous.

The calculation is therefore divided into two parts:

1) collapse through the fire-affected region from floor 96 to floor 94, modeled as a small set of floors effectively suspended in empty space;

2) motion of the resulting $16M$ block through the lower structure, modeled as a stack of layers, each consisting of a floor together with the load-bearing columns incorporated into it and connected to the layer below[6].

The steel columns are not removed as explicit structural elements, as in so-called *floating-floor* models[7], but are included as constitutive parts of each layer. All layers are physically contiguous, and the impulse is transmitted by direct contact from one layer to the next. Below floor 94, the model therefore contains no empty space between successive layers.

First part of the collapse: gravity-dominated phase

We consider the sequence from floor 96 to floor 94, neglecting resistance from intervening damaged material but retaining the inelastic impacts with the two floors.

Step 1.

The speed with which the $14M$ upper block strikes floor 95 after falling through $h = 3.71\,\mathrm{m}$ is

$$v_{96}^{(f)} = \sqrt{2gh} = \sqrt{2\left(9.81\,\mathrm{m\cdot s^{-2}}\right)\left(3.71\,\mathrm{m}\right)} \simeq 8.52\,\mathrm{m\cdot s^{-1}}. \tag{1}$$

Step 2.

Momentum conservation in the perfectly inelastic collision gives the initial speed of the resulting $15M$ block:

$$v_{95}^{(i)} = \frac{14}{15} v_{96}^{(f)} \simeq 7.997\,\mathrm{m\cdot s^{-1}}. \tag{2}$$

After another free fall through one story height, its speed just before striking floor 94 is

$$v_{95}^{(f)} = \sqrt{2gh + \left(v_{95}^{(i)}\right)^2} \simeq 11.68\,\mathrm{m\cdot s^{-1}}. \tag{3}$$

Step 3.

The initial speed of the resulting $16M$ block is therefore

$$v_{94}^{(i)} = \frac{15}{16} v_{95}^{(f)} \simeq 10.95\,\mathrm{m\cdot s^{-1}}. \tag{4}$$

Second part of the collapse: impact-dominated phase

The upper block B_{top}, of mass $16M$, now comes into contact with the relatively intact lower portion B_{bottom}, modeled as a set of homogeneous pancake-like

[6]Column segments were sometimes bolted to those below and sometimes formed integral three-story prefabricated units of height $3h = 11.13\,\mathrm{m}$. The important point for the model is the continuity of the load-bearing columns over the height of the building.

[7]The floating-floor models eliminate the columns as physical objects and replace them by a braking force in the equations of motion. Such a force can slow the descent but does not alter the assumed continuously accelerating character of the motion.

layers. Each layer represents one floor, its slab and contents, and the incorporated load-bearing columns; we recall that it has thickness $h = 3.71\,\mathrm{m}$ and mass M. The mass M cancels from the collapse-time calculation as long as no material is lost outside the footprint. The same result is obtained if one groups three floors into modules of thickness $3h$ and mass[8] $3M$.

We model the lower portion as 94 equal contiguous layers. During each short impact, the finite support reaction is neglected relative to the impulsive interaction, so momentum is conserved over the impact interval as the impulse propagates from one layer to the next.

At the first impact, the $16M$ block moving at v_{94} incorporates one layer of mass M. If u_1 denotes the speed of the resulting block whose mass is $17M$, from the momentum conservation we get:

$$16Mv_{94} = 17Mu_1,\ u_1 = \frac{16}{17}v_{94}. \tag{5}$$

At the second impact,

$$u_2 = \frac{16}{18}v_{94}, \tag{6}$$

and after the k th impact,

$$u_k = \frac{16}{16+k}v_{94}. \tag{7}$$

Hence, in this model, the sequence of impacts slows the falling block instead of accelerating it.

The time required to traverse one layer thickness at speed u_k is

$$t_k = \frac{h}{u_k}. \tag{8}$$

The total collapse time for the portion below floor 94 is therefore

$$t = \sum_{k=1}^{94} t_k = \sum_{k=1}^{94} \frac{h}{u_k} = \frac{h}{v_{94}} \sum_{k=1}^{94} \frac{16+k}{16} = \frac{h}{v_{94}} \sum_{k=1}^{94} \left(1 + \frac{k}{16}\right). \tag{9}$$

Using $h = 3.71\,\mathrm{m}$, $v_{94} = 10.95\,\mathrm{m\cdot s^{-1}}$, we obtain

[8]It has been objected that the model is not pertinent because it assumes a modular structure of B_{bottom} consisting of 94 superimposed layers, each of thickness $h = 3.71\,\mathrm{m}$ and mass M. In reality, the underlying layers were organized into "modules" of 3 superimposed layers, of height $3h = 3 \times 3.71 = 11.13\,\mathrm{m}$ and mass $3M$; these "modules" were prefabricated steel elements designed to be manufactured in a workshop, transported to the site, and installed rapidly. Each module, weighing a few tens of tons, was transported by high-capacity cranes and positioned on the core columns and on the perimeter supports. Once in place, a few finishing welds were made and the concrete was poured, rapidly obtaining floor slabs "to a proper standard." With this scheme, the total number of impacted "modules," neglecting the decimals, is $94/3 \simeq 31$. However, the result does not change, as should be obvious to a reader sufficiently familiar with impulsive dynamics. Other readers, who understandably find impulsive dynamics counterintuitive because of its discontinuities and crude idealizations, can easily verify this by repeating exactly the same calculations with B_{bottom} modeled as a structure consisting of 31 superimposed modules, each of height $3h = 3 \times 3.71 = 11.13\,\mathrm{m}$ and mass $3M$.

$$t \simeq 3.71 \times 373 \times \frac{1}{10.95} \simeq 126.3 \text{ s}. \tag{10}$$

Adding the estimated 3.48 s required for the top of the building to descend to floor 94 gives a total minimum collapse time of approximately

$$t_{\text{tot}} \simeq 129.8 \text{ s}, \tag{11}$$

far greater than the observed value of approximately 11 ± 1 s.

The time would become still longer if the model included two neglected effects: first, much of the fragmented or pulverized material was expelled outside the building footprint and therefore did not participate in later impacts; second, the dimensions and mass of the columns decreased with height.

For comparison with other results in the literature, we considered a highly abstract floating-floor model in which all load-bearing columns are removed and the floors fall through empty space, accelerating gravitationally between successive inelastic impacts. The calculation gives a total fall time of about 11 s in good agreeement with Bazant & Verdure [3][9]. Agreement with the observed collapse time is therefore obtained in this simplified calculation only when the load-bearing columns are omitted as explicit structural elements and free-fall intervals between successive impacts are introduced.

The homogeneous-layer model does not describe the detailed deformation occurring within each layer[10]. However, the macroscopic impact sequence in the present idealization depends primarily on the descent of each layer onto the one below, and especially on the continued physical presence of the incorporated load-bearing columns and connecting trusses. Local inhomogeneities and anisotropies are neglected.

5. Bažant's Continuous Model Compared with the Present Impulsive Model

Bažant and collaborators begin from the hypothesis that collapse initiation in the fire zone results from pronounced viscoplastic deformation of heated core columns and their consequent loss of load-bearing capacity under dynamic conditions. The subsequent progression is described as a pancake-type process involving a falling mass, deformation, fragmentation, and the reduced residual capacity of the lower structure.

Let z denote the downward distance of the collapse front from the top of the tower, $m(z)$ the mass above the front, and λ a compaction factor. According to Ref. [3], pag. 313, the momentum of the moving block is written as

$$P(z) = (1-\lambda) m(z) \dot{z}, \tag{12}$$

and its equation of motion is

[9]Indeed, this is the value that emerges from the plots of Ref. [3] page 315 (top left, curve corresponding to the zero value of crushing energy W).

[10]Within each layer, floors may descend gravitationally while columns deform, buckle, and lose stability under the sequence of impacts. Compression of the air trapped between floors may also contribute to lateral "squibs" of air and debris. These details are regarded as beyond the scope of an elementary-physics model.

$$\frac{\mathrm{d}P}{\mathrm{d}t} = m(z)g - F_c(z), \tag{13}$$

where $F_c(z)$ is an average structural resistance representing energy-dissipation mechanisms, chiefly localized buckling and plastic-hinge formation in the steel columns, together with vertical compression and floor-connection effects.

Developing Equation (13) and solving for the acceleration gives

$$\ddot{z} = \frac{g}{1-\lambda} - \frac{F_c(z)}{(1-\lambda)m(z)} - \frac{m'(z)}{m(z)}\dot{z}^2. \tag{14}$$

The three terms may be interpreted as a gravitational contribution amplified by compaction, a specific structural-resistance term, and a braking inertial term due to mass accretion, proportional to $\dot{z}^2$.

Bažant and collaborators (see in particular Bazant & Verdure [3]) state that Equation (14) can be solved numerically by a standard algorithm such as Runge-Kutta, but they don't solve really the equation. What they actually do is formulate the collapse dynamics in terms of the crushing energy per story W, which is the dominant physical parameter in their framework. Unfortunately, they do not provide an explicit closed form or the local resisting force $F_c(z)$ in terms of W. Therefore, integration of Equation (14) needs a simple modelling parametrization for $F_c(z)$, that must be calibrated to reproduce the chosen W. For this purpose, several simplified progressive collapse models express the resistance as a fraction of the gravitational load, using adimensional coefficients to quantify the residual capacity [9]-[11]. This modelling strategy is equivalent to the following ansatz, that will be adopted in this paper:

$$F_c(z) = \Lambda m(z) g, \tag{15}$$

where Λ is a dimensionless parameter, energetically consistent with the crushing energy assumption. More explicitly, the coefficient Λ should be interpreted as an effective parameter calibrated so that the integrated work of $F_c(z)$ along the collapse path corresponds to a chosen crushing energy W. Formally (see **Appendix**):

$$\Lambda = \frac{W}{g\int_z^{z+h} m(z')\mathrm{d}z'} \tag{16}$$

The choice of Λ is equivalent to specifying how much energy is dissipated in the destruction of a layer.

Bazant & Zhou [2] state that "the plastically dissipated energy W is, optimistically, of the order of 0.5 GN·m". Consistently, the main value for W plotted in Bazant & Verdure [3], pag. 325, is $W \simeq 0.5\ \mathrm{GN}\cdot\mathrm{m}$, corresponding to the heavy solid curve in their plot. Starting from this value, the effective parameter Λ, which we assume to be independent of z, is found to be approximately 0.2 (see **Appendix** for the derivation)

We set the origins of time and position at floor 94, so that $t = 0$ and $z = 0$ there. If all layers have equal mass M, then

$$m(z) = 16M + \frac{M}{h} z, \tag{17}$$

where z is treated as continuous, although physically the moving block incorporates one additional layer whenever the front advances by h. Consequently,

$$m'(z) = \frac{M}{h}, \tag{18}$$

and the mass-accretion term becomes

$$\frac{m'(z)}{m(z)} \dot{z}^2 = \frac{1}{16h + z} \dot{z}^2. \tag{19}$$

The specific resistance appearing in Equation (14) is then

$$\frac{\Lambda g}{1 - \lambda}. \tag{20}$$

Substitution gives

$$\ddot{z} = \frac{(1 - \Lambda) g}{1 - \lambda} - \frac{1}{16h + z} \dot{z}^2. \tag{21}$$

To solve the equation numerically, we choose the representative values $\lambda \simeq 0.15$, $\Lambda \simeq 0.2$, and $h = 3.71\ \text{m}$. At floor 94, the adopted initial velocity is

$$\dot{z}(0) = 10.95\ \text{m} \cdot \text{s}^{-1}. \tag{22}$$

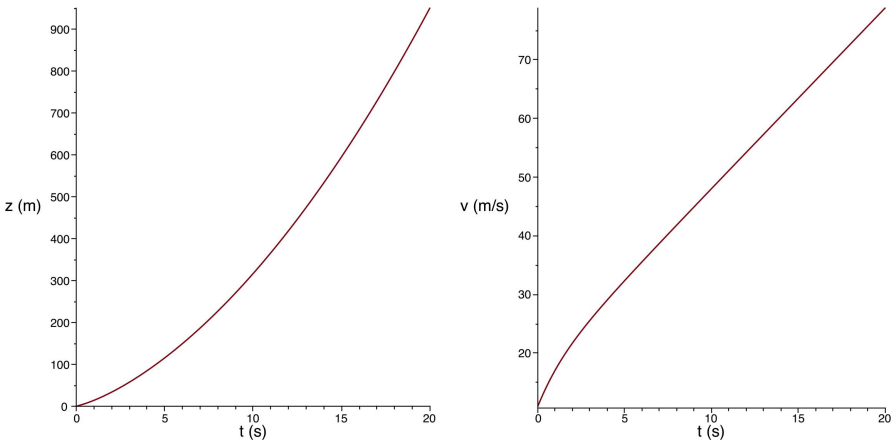


Figure 1. Left: position $z(t)$ of the moving block. Right: corresponding velocity $\dot{z}(t)$.

The results of the numerical integration are shown in **Figure 1**, where the position $z(t)$ of the moving block and its velocity $\dot{z}(t)$ are plotted. After an initial transient phase, the velocity becomes nearly linear in time, indicating that the motion approaches a regime of approximately constant acceleration. At first sight, this behaviour may appear counter-intuitive because of the presence of the nonlinear last term in Equation (21). However, a closer inspection of the numerical solution, as illustrated in **Figure 2**, clarifies the origin of this result: the additional term increases rapidly during the initial phase and then approaches a plateau, so that its contribution becomes approximately constant. Consequently, the acceler-

ation rapidly decreases from an initial value of about 7 m/s^2 due to the action of the $\dot{z}^2$ term, and then the decrease is smoother, until it reaches an asymptotic value of about 3 m/s^2. Thus, consistently with Bažant's statement, the asymptotic collapse dynamics approaches a regime of nearly constant acceleration.

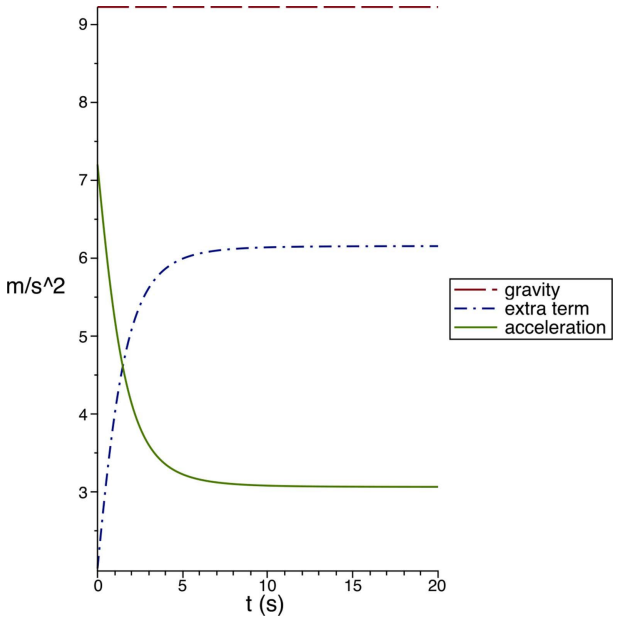


Figure 2. Behaviour of the terms appearing in Equation (21): the constant term proportional to g, the nonlinear term proportional to $\dot{z}^2$, and the resulting effective acceleration as functions of time.

In addition, our numerical integration yields a collapse time of approximately 10.65 s from floor 94 to ground level. Using our numerical value, the corresponding total collapse time from the top panel at floor 110 to ground level is therefore

$$3.48\text{ s} + 10.65\text{ s} = 14.13\text{ s}. \tag{23}$$

Here, 3.48 s is the time required for the top panel to descend from floor 110 to floor 94 over a distance of 59.6 m, assuming the maximum possible acceleration g, whereas 10.65 s is the fall time from floor 94 to ground level obtained from our numerical integration.

If the descent is taken to end at the top of a 25 m debris pile, we obtain

$$3.48\text{ s} + 10.15\text{ s} \simeq 13.63\text{ s}. \tag{24}$$

Both values lie outside the NIST uncertainty interval of 10 - 12 s around the nominal 11 s estimate. That being said, there are important details to be taken into account:

1) The acceleration estimated from observations during the initial phase of the collapse is lower than 9.8 m·s^{-2}, with a maximum value of about 7.0 m·s^{-2}. Therefore, the time required for the uppermost panel (floor 110) to reach floor 94, over a distance of 59.6 m, when using the maximum acceleration compatible with the observations, is not 3.48 s but 4.12 s. Hence, 0.64 s must be added to each of the

two collapse times estimated above, which become 14.77 s in the first case and 14.27 s in the second case.

2) If, in addition, one takes into account the neglected effects—in particular, the fact that material ejected outside the perimeter of the building does not participate in the subsequent sequence of impacts, and the fact that the dimensions and mass of the columns decrease progressively with height, so that the mass $16M$ of the upper block is likely overestimated—the predicted collapse time would increase further. A complete quantitative assessment of these effects, however, requires a more detailed structural model than the one considered here.

6. Conclusions

In this work we have compared two highly idealized descriptions of the propagation phase of the WTC1 collapse: a discrete contiguous-layer model based on impulsive momentum transfer and a continuous mass-accretion model of the type introduced by Bažant and collaborators.

The discrete model was deliberately constructed using only elementary mechanics. Below the initially damaged region, the building was represented as a sequence of contiguous layers, each incorporating both a floor and the corresponding portion of the load-bearing structure. Under this assumption, the advancing collapse front does not traverse an empty story height between successive interactions. Instead, momentum is transferred through a sequence of idealized inelastic impacts.

Within this model, conservation of momentum at each impact produces a progressive reduction in the velocity of the moving block. The resulting collapse time predicted by the idealized contiguous-layer limit considered here is approximately 130 s, far longer than the observational estimate of approximately 11 ± 1 s. This result should not be interpreted as a prediction from a complete structural model of WTC1. Rather, it demonstrates that a purely impulsive contiguous-layer description, under the assumptions adopted here, is unable to reproduce the observed collapse time.

We have also examined the corresponding continuous mass-accretion equation. Integrating Equation (14) numerically, we find that the velocity initially exhibits a transient behaviour because of the non linear mass-accretion term

$$-\frac{m'(z)}{m(z)}\dot{z}^2, \tag{25}$$

and afterwards becomes approximately linear in time. The nonlinear mass-accretion contribution rapidly approaches an almost constant value, so that the effective acceleration also approaches an approximately constant asymptotic regime.

For the representative parameter values adopted in the calculation, our numerical integration gives a fall time of approximately 10.65 s from floor 94 to ground level. Including the preceding descent of the uppermost portion from floor 110 to floor 94 gives a total time of approximately 14.13 s when the latter phase is assigned the maximum acceleration g. If an acceleration closer to the observation-

ally inferred value is used for this initial phase, the total time increases further. On the other hand, Bažant and Verdure [3] also consider different choices of the crushing energy W in the range 0.5 GN·m - 2.4 GN·m, corresponding to a range 0.2 - 1 for the representative parameter Λ. Any choice greater than 0.2 within this range prolongs the collapse time, until the collapse is completely stopped for $\Lambda = 1$ (see **Appendix**).

The large difference between the predictions of the discrete and continuous descriptions emphasizes the central role played by the idealization of structural interaction. In the continuous model, resistance is represented by an effective distributed force while the moving mass grows continuously. In the discrete model, by contrast, the load-bearing structure is explicitly retained as part of the contiguous layers, and mass accretion occurs through successive momentum-transfer events.

However, beyond the choice of model, whether discrete or continuous, the main point of disagreement between the Bažant-NIST treatment and the present work lies in how the columns are represented in the theoretical analysis. Replacing the columns—with all their concrete, tangible physical reality—with a conjectural mathematical function $F_c(z)$ that models their average resistance in a manner consistent with the principle of conservation of energy causes one to lose sight of a very solid, concrete fact: in the time interval between successive impacts, the columns do not delegate their existence to an abstract mathematical function but stubbornly continue to exist in the physical world, indifferent to any mathematical modeling. Consequently, there is no empty space into which something could fall with constant acceleration, regardless of what its numerical value might be[11]: modeling the building as a set of 110 floors suspended not on real steel columns but on a conjectural mathematical function constitutes a theoretical abstraction far removed from observable physical reality. Our contiguous-layer model accounts for the physical existence of the columns at every moment of the collapse because the load-bearing columns are not interrupted between layers; instead, they span all the layers without a break, extending the full height of the building.

Beyond formal aspects, the deep difference between the two approaches lies in the fact that, in our description of the pancake phase, dynamics is driven not so much by gravity as by the propagation of impulses along a continuous chain of physical elements in contact. During this phase, the sequence of inelastic impacts between layers—occurring via direct contact—acts to slow down the falling block rather than accelerate it. From a physical point of view, to obtain a result consistent with the Bažant-NIST expectations, one would have to assume (see also Ref. [8]) that a large part of the structure ceases to act as an effective obstacle between successive impacts[12].

[11]The experimental value, as shown by the videos, appears to be 6.5-7.0 ms^{-2}.

[12]Detailed local impact mechanisms are compatible with the layer model insofar as they do not create structural voids but instead determine the energy absorbed by each layer and hence the effective form of $F_c(z)$.

Neither representation should be regarded as a complete structural model. Under the contiguous-layer assumptions adopted here, gravity and sequential inelastic momentum transfer alone lead to a collapse time much longer than the observed value. Determining whether such a reduction can arise from realistic structural-failure mechanisms requires a detailed structural analysis beyond the scope of the present work.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix: Calibration of the Effective Resistance Coefficient Λ

1) The calibration of Λ is obtained by imposing that the energy dissipated by the simplified model (15) during the destruction of a layer of height h is equal to the crushing energy per story W :

$$\int_z^{z+h} \Lambda m(z')g\mathrm{d}z' = W \tag{26}$$

which, when solved with respect to Λ, is nothing but Equation (16). Therefore Λ should be interpreted as the crushing energy required for the destruction of a layer of height h, divided by the integrated weight-displacement work of the accreting mass (17). Consistently, the expression (15) can be interpreted as a parametrization that distributes the crushing energy along the path (z, z + h) proportionally to the instantaneous weight $m(z)g$ of the accreting mass. For our purposes, it is sufficient estimating Λ as an effective value calibrated at a chosen reference stage, which we do below.

Numerical calculation

2) mass of one floor (or layer): $M = \left(5\times10^8/110\right) = 4.5455\times10^6$ kg ;

3) initial mass of the block: $14M = 6.3637\times10^7$ kg ;

4) height of one floor (layer): $h = 3.71$ m .

Integral of the weight of the accreting mass over one story, from $14M$ to $15M$:

$$g\int_z^{z+h} m(z)\mathrm{d}z = g\int_z^{z+h}\left(14M + \frac{M}{h}z\right)\mathrm{d}z = g\left(14Mh + \frac{M}{2h}h^2\right)$$
$$= 142.25Mh = 2.3988\times10^9 \text{ J}$$

from which

$$\Lambda = \frac{W}{2.3988\times10^9} \tag{27}$$

Bazant & Verdure [3] propose several possible values of W in the graph on page 315 (upper left). The main value, which corresponds to the observed collapse behavior, is $W = 0.5$ GN·m (energy dissipated by the structure between the two floors during the crushing). The corresponding value of Λ is

$$\Lambda = \frac{0.5\times10^9}{2.3988\times10^9} = 0.208$$

which is exactly (to the first decimal place) the value used in this paper. Increasing values of W increase the collapse time. The last value, corresponding to the arrest of the collapse, is $W_{halt} = 2.4$ GN·m ; the corresponding value of Λ is

$$\Lambda_{\text{halt}} = \frac{2.4\times10^9}{2.3988\times10^9} = 1.000$$

Therefore, the parameter Λ in the interval $(0.2,1.0)$ completely controls the collapse behavior, from behavior compatible with the observations up to the arrest of the collapse.