Journal of Applied Mathematics and Physics, 2026, 14(9), 3638-3657
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

An S-Shaped Curve and Efficient Frontier Model for Income Projection in New Business Projects

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How to cite this paper: Salim, F., Abu, N.A. and Pratama, S.F. (2026) An S-Shaped Curve and Efficient Frontier Model for Income Projection in New Business Projects. *Journal of Applied Mathematics and Physics*, **14**, 3638-3657.
https://doi.org/10.4236/jamp.2026.149180

Received: August 26, 2026
Accepted: September 19, 2026
Published: September 22, 2026

Abstract

Income projection is a fundamental component of a business project, particularly in supporting management decisions on investment, resource allocation, and expected financial returns. Conventional income projection approaches commonly assume a constant or continuously increasing income pattern and may not adequately represent the different stages that a new business project experiences before reaching maturity. This study proposes a conceptual S-shaped curve model for projecting projected revenue throughout the lifecycle of a new business project. The proposed model represents the progression of income through four phases, namely the initial development phase, rapid growth phase, saturation phase, and maturity phase. An efficient frontier concept is subsequently introduced to represent alternative income growth trajectories resulting from different levels of managerial intervention and resource allocation. The inflection point of the S-shaped curve is used to identify the transition between the growth and saturation phases and to provide an indication of the stage at which additional investment may produce diminishing income growth. The proposed model provides a graphical and mathematical framework for analysing income growth dynamically rather than assuming a uniform growth pattern throughout the project lifecycle. The model is intended to assist project managers and senior management in evaluating alternative income projections, identifying appropriate intervention points, and supporting investment and resource allocation decisions for new business projects.

Keywords

Efficient Frontier, Income Projection, Project Management, S-Shaped Curve

1. Introduction

One of the primary challenges faced by project managers is developing a compelling business plan or business case that convincingly demonstrates the strategic value, financial viability, and long-term benefits of a proposed project to senior management. A well-prepared business proposal should provide a clear justification for investment decisions, particularly through a realistic and credible revenue model. Recent studies have highlighted that many projects fail to achieve their intended outcomes because they are initiated with weak business cases, unrealistic value propositions, or inadequate governance mechanisms that fail to sustain business value throughout the project lifecycle [1]. Therefore, organisations intending to undertake new projects should ensure that the proposed initiatives possess strong potential to generate sustainable revenues, maximise business value, and remain competitive in an increasingly dynamic marketplace.

Obtaining project approval from top management requires more than presenting innovative ideas; it requires a comprehensive business proposal supported by sound financial planning and realistic revenue projections. Decision-makers are primarily concerned with the financial implications of a proposed investment, including projected revenues, operating expenses, profitability, cash flows, and return on investment. Furthermore, the proposed product or service must address an identifiable market need and demonstrate its capability to achieve sustainable market penetration and long-term commercial success.

Maintaining an organisation's financial sustainability remains one of the most important responsibilities of financial managers. Among their key responsibilities is the development of long-term business strategies that preserve and enhance the organisation's financial performance and shareholder value. Achieving sustainable profitability from newly initiated projects is widely recognised as one of the key indicators of organisational success. Consequently, accurate income or revenue projection constitutes one of the fundamental components of any business proposal, as it provides management with a forward-looking assessment of the project's financial potential. Motivated by this need, the objective of this paper is to propose a conceptual model for projecting projected revenue throughout the lifecycle of a new business project using an S-shaped growth curve.

The remainder of this paper is organised as follows. Section 2 reviews the relevant literature on financial management, financial modelling, income projection, and project evaluation. Section 3 discusses the theoretical foundation of the S-shaped growth curve and its applications across various disciplines. Section 4 introduces the conceptual framework for modelling projected revenue based on the characteristics of the S-curve. Finally, Section 5 proposes a new S-curve efficient frontier model for projected revenue and discusses its practical implications for monitoring project performance and supporting strategic managerial decision-making throughout the different phases of project development.

2. Background Study

In today's increasingly interconnected global economy, an efficient financial system is fundamental to supporting sustainable economic development and organisational competitiveness. Effective financial management enables organisations to allocate resources efficiently, evaluate investment opportunities, and maximise shareholder value. Financial innovation continues to enhance economic efficiency through technological advancement, digital transformation, and data-driven decision-making. Many contemporary financial management practices are founded upon the pioneering contributions of Irving Fisher [2], Harry Markowitz [3], Modigliani and Miller [4], Black and Scholes [5], Ross [6], and Garman and Kohlhagen [7]. These classical theories remain the cornerstone of modern corporate finance, investment analysis, and financial innovation.

One of the major concerns in financial management is maintaining a firm's financial health and long-term sustainability. Income projection provides an estimate of the future financial performance of a business and serves as an important input for strategic planning and investment decision-making. Financial modelling has evolved considerably through the integration of machine learning, predictive analytics, and data-driven decision-support techniques that enhance forecasting accuracy and managerial decision-making [8]. Modern forecasting approaches increasingly combine statistical methods with advanced analytical tools to improve prediction performance in dynamic business environments. Furthermore, contemporary forecasting models utilise historical financial information together with market, economic, and operational variables to improve the reliability of revenue projections under conditions of uncertainty [9] [10]. Nevertheless, unrealistic assumptions, poor-quality data, and weaknesses in financial modelling practices continue to contribute to forecast errors and suboptimal investment decisions.

Financial models are widely used to evaluate investment opportunities, estimate future cash flows, assess financial risks, and support strategic planning. Various valuation and forecasting techniques remain important in corporate finance, including the Gordon Growth Model, Capital Asset Pricing Model (CAPM), Altman Z-Score model, Constant Elasticity of Variance (CEV) model, Fama-French factor models, and other valuation approaches that assist organisations in analysing investment performance and financial sustainability. In addition, financial planners frequently employ scenario analysis, sensitivity analysis, and stochastic forecasting techniques to evaluate the effects of uncertainty on organisational performance and investment outcomes [11] [12]. These techniques allow managers to examine alternative future conditions and evaluate the robustness of investment decisions under varying assumptions.

Ensuring long-term financial sustainability requires organisations to continuously monitor both financial and non-financial performance indicators. Financial ratios remain important measures of organisational performance and operational efficiency. Profitability, liquidity, leverage, asset utilisation, and cash flow indicators continue to serve as key measures for evaluating corporate financial health. Beyond traditional financial measures, growing evidence suggests that financial resilience also depends on effective corporate governance, strategic leadership, organisational capabilities, and sound risk management practices [13] [14]. Effective governance structures improve organisational oversight and decision quality, while robust risk management practices contribute to long-term sustainability and organisational resilience. Conversely, poor managerial decisions, ineffective governance structures, inadequate strategic planning, and weak organisational capabilities remain among the major causes of financial distress and project underperformance.

A company's value is closely associated with its Net Present Value (NPV), which remains one of the most widely accepted investment appraisal techniques in finance because it explicitly incorporates the time value of money and expected project cash flows [11]. Project managers are therefore expected to develop realistic and reliable income projections before proposing investment decisions. Although NPV has long been recognised as an effective tool for evaluating investment opportunities, accurate estimation of future cash flows continues to be one of the greatest challenges in project valuation, particularly in uncertain business environments where revenues, costs, and project activities are subject to variability and risk [15]. The quality of projected cash flows therefore plays a critical role in the reliability of investment appraisal and strategic decision-making.

Any investment proposal consisting of a sequence of cash outflows and future cash inflows is commonly evaluated using the NPV approach. NPV measures the difference between the present value of expected future cash inflows and the present value of investment costs. Consequently, it provides an important indicator of whether a project is financially worthwhile. Projects with positive NPV are generally expected to create value for shareholders and therefore deserve serious consideration by senior management. Organisations seeking to maximise shareholder wealth should prioritise investment opportunities capable of generating the highest positive NPV while maintaining acceptable levels of investment risk.

To link revenue projections directly to project appraisal, period-specific net cash flows CF_t are derived by subtracting operational and marketing expenditures C_t from the periodic projected revenue R_t, expressed as $CF_t = R_t - C_t$. Substituting this into the standard Net Present Value (NPV) formulation yields Equation (1):

$$\text{NPV} = \sum_{t=1}^{T_m} \frac{CF_t}{(1+i)^t} - C_0 = \sum_{t=1}^{T_m} \frac{R_t - C_t}{(1+i)^t} - C_0 \quad (1)$$

where t denotes the discrete time period ($t = 1, 2, \cdots, T_m$), T_m represents the total project lifecycle duration, i is the discount rate or required rate of return, and C_0 denotes the initial capital expenditure at launch ($t = 0$). Investment projects typically involve an initial capital outlay followed by a series of expected future cash inflows. Consequently, reliable income projection constitutes a prerequisite for meaningful NPV evaluation. Financial modelling literature highlights the use of scenario analysis, sensitivity analysis, and forecasting techniques to strengthen investment appraisal and support strategic decision-making under uncertainty [11] [13].

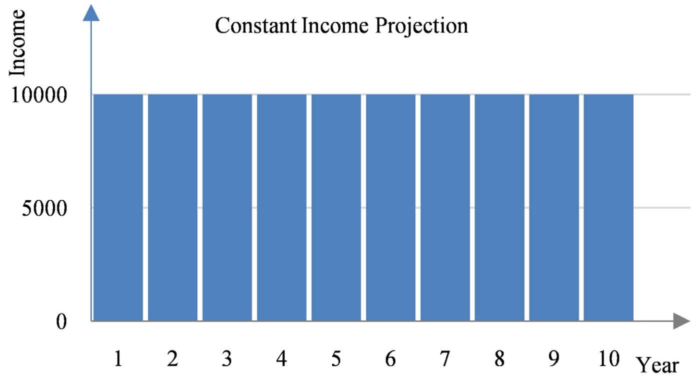

Figure 1. A simple constant income projection.

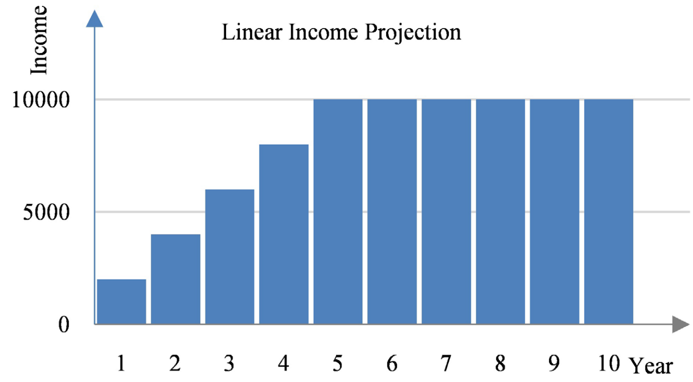

Figure 2. An increasing income projection moving toward saturation.

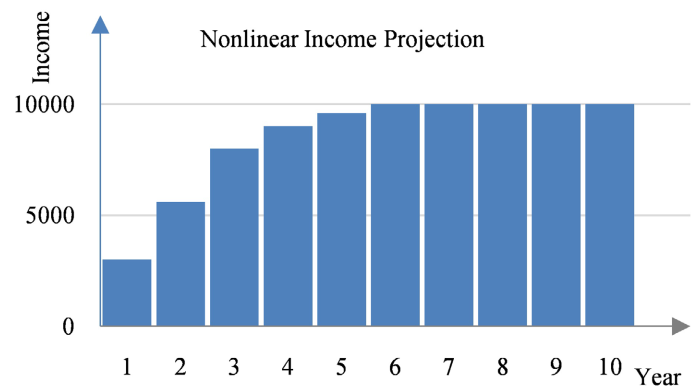

Figure 3. A smooth increasing income projection moving toward saturation.

Conventional financial literature generally represents income projection using either constant revenue growth or gradually increasing growth patterns, as illustrated in **Figures 1-3**. These traditional approaches implicitly assume that a project is already progressing successfully from its early stages. **Figure 1** assumes constant income throughout the project life cycle, whereas **Figure 2** and **Figure 3** imply rapid sales growth immediately after project commencement before eventually reaching market saturation. However, such assumptions rarely reflect actual business conditions.

In practice, projects experience several distinct development phases characterised by varying growth rates, market acceptance, and organisational learning before reaching maturity. Contemporary project management literature similarly recognises that project performance evolves dynamically through-out the project lifecycle rather than following a uniform growth trajectory. Consequently, these growth characteristics are more realistically represented by an S-shaped curve, which provides a better description of the gradual evolution of project performance and income generation.

3. A Non-Linear S-Shaped Curve Model

The origins of the S-shaped curve can be traced back to 1838, when the Belgian mathematician Pierre-François Verhulst introduced the logistic growth model to describe population growth as shown in **Figure 4**. Since then, the S-shaped curve has become a well-established mathematical function for representing the evolution of numerous real-world phenomena across a wide range of disciplines, including healthcare and medical sciences [16].

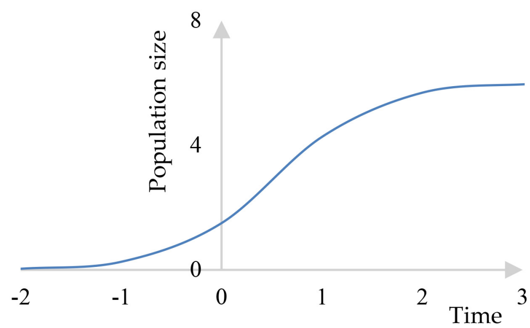

Figure 4. A logistic growth model.

S-shaped curves have been widely used to describe and analyse inventive problem-solving processes, innovation diffusion, and technology forecasting [17]. They have also found applications in fuzzy logic systems [18] and neural network modelling [19]. In behavioural economics, Prospect Theory employs an S-shaped value function to explain decision-making under risk, particularly in situations involving gains and losses. The model illustrates how individuals perceive and evaluate potential outcomes differently depending on whether they are framed as gains or losses, as shown in **Figure 5** [20].

The concept of technological evolution following an S-shaped growth pattern was recognised as early as 1925 through the pioneering work of Simon Kuznets [21]. His findings were later published in Secular Movements in Production and

Prices, which has remained influential in studies of long-term economic and technological development. Similarly, Bejan's Constructal Law, first proposed in 1996, provides a unifying framework for understanding the generation and evolution of flow systems in nature, both biological and non-biological. According to Bejan and Lorente [22], the S-shaped curve is one of nature's most common evolutionary patterns and can be observed in the spread of populations, diseases, innovations, economic activities, and many other growth processes.

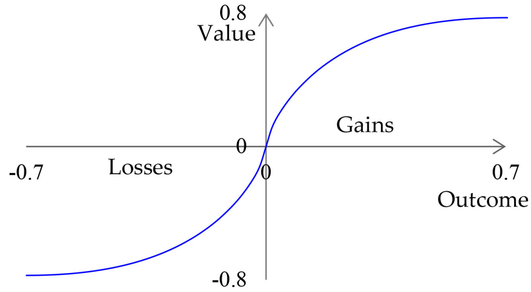

Figure 5. A prospect theory.

Another important application of the S-shaped curve is found in the Diffusion of Innovation Theory developed by Everett Rogers in 1962. The theory explains how innovations are adopted over time by different segments of society. Rogers classified adopters into five categories: innovators (2.5%), early adopters (13.5%), early majority (34%), late majority (34%), and laggards (16%) (Rogers, 1983). The adoption process progresses through five stages: knowledge, persuasion, decision, implementation, and confirmation. When mapped over time, the cumulative adoption rate follows a characteristic S-shaped pattern, beginning with slow growth, accelerating rapidly during widespread adoption, and eventually reaching saturation, as illustrated in **Figure 6**.

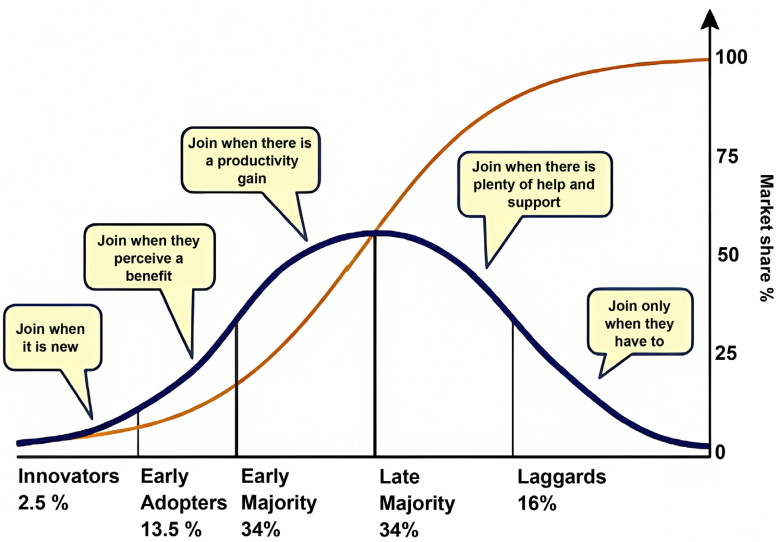

Figure 6. Roger's S-curve of technological adoption from the diffusion-innovation model. Source of diagram: InformationWeek.

To date, Technological evolution has long been a central area of research in technology management and is widely taught in higher education institutions to explain patterns of innovation, diffusion, and technological change. A fundamental assumption underpinning this literature is that technologies tend to follow an S-shaped development trajectory, characterised by slow initial advancement, accelerated growth during periods of increasing adoption and performance improvement, and eventual maturity as technological limits or market saturation are approached [23] [24].

Beyond its role in technology life-cycle analysis, the S-curve has been widely adopted in project management as a practical tool for planning, monitoring, controlling, analysing, and forecasting project performance [25]. In engineering, construction, and infrastructure projects, S-curves are commonly used to compare planned and actual progress, monitor cumulative costs and resource utilisation, and evaluate overall project performance throughout the project lifecycle. The characteristic S-shape reflects the typical progression of projects, beginning with a relatively slow initiation phase, followed by rapid execution and growth, and concluding with a gradual stabilisation as project activities are completed. Consequently, S-curve analysis provides project managers with a valuable mechanism for tracking performance trends, supporting decision-making, and identifying deviations from planned schedules and budgets [24] [25].

Despite the extensive application of S-shaped curves in population dynamics, technology diffusion, behavioural economics, engineering, and project management, their use in modelling projected revenue remains relatively underexplored. Most conventional income projection models assume either constant growth or a uniformly increasing growth pattern, which may not adequately capture the dynamic nature of project development and market acceptance. In practice, revenue generation evolves through several stages characterised by initial market entry, growth acceleration, widespread adoption, and eventual market saturation. These characteristics closely resemble the fundamental behaviour of an S-shaped growth curve. Therefore, an S-curve-based income projection model may provide a more realistic representation of how sales and revenues develop throughout the lifecycle of a new business project. Motivated by this observation, the following section introduces a conceptual framework for modelling projected revenue using the dynamic properties of an S-shaped growth curve.

4. Income Sale Projection Conceptual Model

Cost outflows (incurred costs) and projected revenue (expected returns) are fundamental components of any business proposal or investment project. Financial projection is one of the key elements that must be presented to senior management when seeking project approval and funding. Typically, the financial projection constitutes the final section of a business plan, as it summarises the anticipated financial performance of the proposed project. Among all financial elements, income projection is arguably the most fundamental component because it provides an estimate of future revenue generation over a specified planning horizon. By presenting a projected revenue, management can assess the project's expected financial performance during future fiscal periods. Such forward-looking projections are generally developed based on market trends, anticipated customer demand, and expected revenue growth.

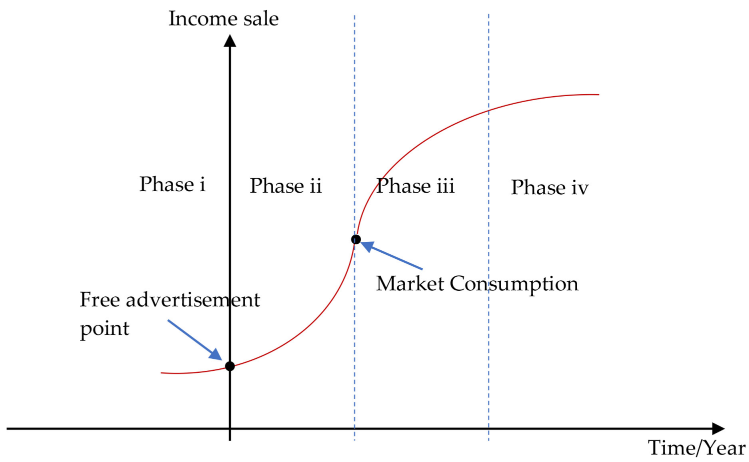

Figure 7. The four phases of an S-shaped curve model.

In large organisations, project performance is regularly reviewed and reported to senior management throughout the project lifecycle. One of the critical factors influencing project success is the level of investment allocated to marketing and promotional activities. Based on projected income streams, the Net Present Value (NPV) of the project can subsequently be computed and presented to the board of directors as part of the investment appraisal process.

In general, the growth of a projected revenue can be divided into four distinct phases, as illustrated in **Figure 7**.

Phase I represents the introductory stage, during which growth occurs relatively slowly before significant promotional activities are undertaken. During this period, various low-cost marketing strategies may be employed, including social media exposure, word-of-mouth promotion, free advertising, exhibitions, and roadshows. As market awareness is still developing, income generation remains modest.

In **Phase II**, the project begins to gain momentum as the product or service becomes increasingly visible and accepted within the marketplace. Market penetration gradually expands beyond the initial customer base, resulting in accelerating sales growth. At this stage, management may decide to increase advertising expenditure in order to strengthen market presence and accelerate customer acquisition. Such strategic interventions can potentially enhance the future growth trajectory of the project.

Phase III is characterised by rapid expansion. As the product becomes well-established and widely accepted, sales growth accelerates further, driven by increasing customer adoption and stronger market recognition. Revenue generation rises substantially during this phase, reflecting the project's successful penetration into a broader market segment. However, although sales continue to grow, the rate of acceleration gradually decreases as the market approaches saturation.

Finally, **Phase IV** represents the maturity stage. During this phase, sales continue to increase, but at a much slower rate than in the preceding stages. The slowing growth trend may be attributed to external factors such as market saturation, intensified competition, and changing consumer preferences, or to internal factors such as the organisation's limited ability to capture additional market share. Eventually, income growth stabilises and approaches a long-term equilibrium level.

In this framework, the target output variable $R(t)$ explicitly represents the discrete periodic revenue generated in a specific time period t (e.g., monthly, quarterly, or annually), rather than the cumulative revenue accumulated up to that point. This periodic distinction ensures that phase transitions and growth velocity changes directly reflect operating performance within each distinct evaluation window.

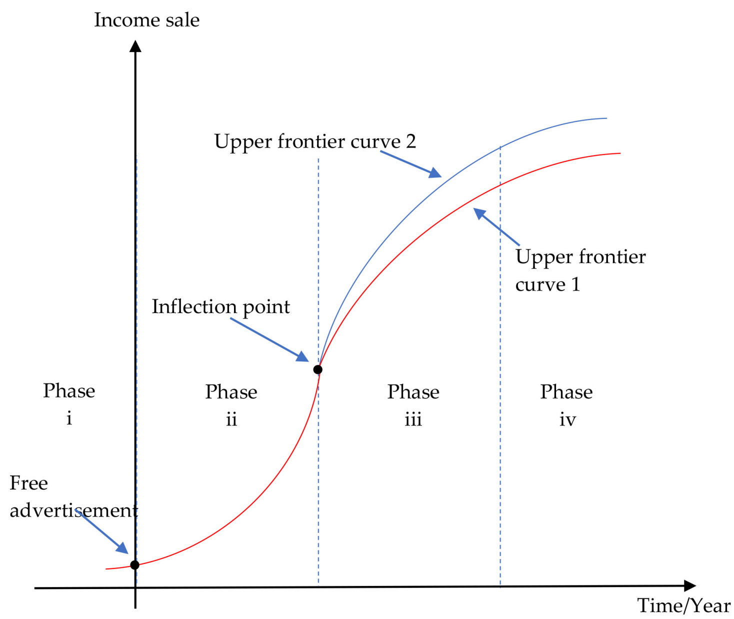

Figure 8. A newly proposed S-curve efficient frontier model on the income projection model.

During the evaluation stage of a project, suppose the income trajectory has reached Phase II. At this point, management may present the current performance of the project to the board of directors and seek strategic guidance on progressing towards Phase III. Particular attention should be given to determining whether the project is approaching its inflection point, where the growth curve transitions from positive concavity to negative concavity. This point represents the stage at which the rate of increase in sales reaches its maximum value and therefore serves as an important indicator of the project's future direction.

Under the existing level of annual advertising expenditure, the project is expected to follow Upper Frontier Curve 1, as illustrated in **Figure 8**. However, management may choose to allocate additional resources to advertising and promotional activities, thereby creating the opportunity for the project to follow Upper Frontier Curve 2, which represents a higher income growth trajectory. Consequently, the proposed S-curve efficient frontier framework provides management with a mechanism for evaluating alternative growth paths and making strategic decisions aimed at maximising future income generation throughout the project lifecycle.

5. A New S-Curve on Projected Revenue Model

To obtain maximum projected revenue in any investment of a new project, an S-curve efficient frontier model will be introduced and proposed as a new projected revenue model. The efficient frontier concept was first introduced by the Nobel Laureate, Harry Markowitz in 1952, to show how a manager attempts to push for a maximum return investment portfolio at a given risk level [3]. In the present study, the term efficient frontier is used conceptually to represent alternative income growth trajectories that may be achieved under different levels of managerial intervention, strategic decision-making, and resource allocation throughout the project lifecycle.

Let $R(t)$ denote the projected periodic revenue generated specifically within a discrete time period t (where $t = 1, 2, \cdots, T_m$), rather than cumulative total revenue. Let $S_0 = R(0)$ represent the baseline periodic revenue at product launch, M denote the maximum achievable periodic revenue at market saturation, and $I = (t_i, R(t_i))$ represent the inflection point where periodic revenue acceleration transitions from positive to negative concavity.

To construct the proposed S-curve efficient frontier model, the following algorithm is introduced:

Let $X_1, X_2, \cdots, X_n$ be independent identically distributed (i.i.d.) maximum random variables representing periodic revenue performance indicators and let $R(t)$ be the target periodic revenue function over time t.

1) Estimate the time location of the inflection point t_i.

2) Estimate the central mean periodic revenue transition value μ within the neighborhood of values near or approaching the inflection boundary t_i, specifically calculating μ from $R(t_i^-)$ and $R(t_i^+)$.

3) The periodic revenue curve equation to the left of the inflection point $(0 \le t \le t_i)$ is expressed as:

$$R_{\text{left}}(t) = S_0 + \frac{\mu - S_0}{1 + \mathrm{e}^{-k_1(t - t_i)}}$$

4) The periodic revenue curve equation to the right of the inflection point $(t_i < t \le T_m)$ is expressed as:

$$R_{\text{right}}(t) = \mu + \frac{M - \mu}{1 + \mathrm{e}^{-k_2(t - t_i)}}$$

The unified piecewise S-curve periodic income projection model is therefore represented by Equation (2):

$$R(t)=\begin{cases} R_{\text{left}}(t)=S_0+\dfrac{\mu-S_0}{1+\mathrm{e}^{-k_1(t-t_i)}}, & 0\le t\le t_i \\ R_{\text{right}}(t)=\mu+\dfrac{M-\mu}{1+\mathrm{e}^{-k_2(t-t_i)}}, & t_i<t\le T_m \end{cases} \tag{2}$$

where μ denotes the central mean periodic revenue state at transition, t_i represents the time coordinate of the inflection point, and k_1, k_2 are growth velocity parameters before and after the inflection boundary.

The periodic revenue $R(t)$ generated by the S-curve model feeds directly into the overall financial valuation of the project. By accounting for period-specific operational, marketing, and promotional expenditures $C(t)$, the net cash flow generated in period t is defined as $CF(t)=R(t)-C(t)$. Incorporating this into the capital budgeting decision framework allows managers to evaluate the net present value under alternative growth frontiers:

$$\text{NPV}=\sum_{t=1}^{T_m}\frac{R(t)-C(t)}{(1+i)^t}-C_0 \tag{3}$$

where C_0 represents the initial capital expenditure, i denotes the discount rate, and T_m is the total project duration.

5.1. Model Calibration, Parameter Estimation, and Data Requirements

To operationalize and calibrate the piecewise S-curve model $R(t)$, the required data inputs, observation frequency, parameter estimation procedures, and error measures are structured as follows:

1) Data Inputs and Observation Frequency:

The primary empirical input is historical or projected periodic revenue R_t recorded at uniform discrete observation frequencies $t\in\{1,2,\cdots,T_m\}$ (typically monthly or quarterly). Secondary inputs include corresponding period operational and marketing expenditures C_t used to derive net cash flows CF_t.

2) Determination of the Central Transition Point (μ):

The central mean state μ represents the baseline transition revenue near the inflection boundary t_i. It is mathematically determined as the local arithmetic mean of periodic revenue observations within a narrow symmetric temporal window δ centered around t_i:

$$\mu=\frac{1}{2\delta+1}\sum_{t=t_i-\delta}^{t_i+\delta}R(t)$$

Alternatively, when working with projected continuous functions,

$$\mu=R(t_i)=\frac{S_0+M}{2}$$

serves as the mid-point state connecting the lower concave and upper convex regimes.

3) Parameter Estimation Procedure:

The structural parameters including initial revenue baseline S_0, saturation upper limit M, inflection time t_i, and growth velocity parameters k_1, k_2 are estimated using Non-Linear Least Squares (NLLS) regression. Optimization algorithms such as Gauss-Newton method are employed to solve:

$$\min_{\theta} \sum_{t=1}^{T_m} \left(R_t - R(t;\theta)\right)^2$$

where $\theta = \{S_0, M, t_i, k_1, k_2, \mu\}$ is the parameter set.

4) Error Metrics for Calibration Assessment:

Model calibration performance and forecast accuracy are evaluated using Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE):

$$\text{RMSE} = \sqrt{\frac{1}{T_m} \sum_{t=1}^{T_m} \left(R_t - \hat{R}(t)\right)^2}, \quad \text{MAPE} = \frac{100\%}{T_m} \sum_{t=1}^{T_m} \left| \frac{R_t - \hat{R}(t)}{R_t} \right|$$

where R_t denotes observed or benchmark periodic revenue and $\hat{R}(t)$ represents the calibrated model output.

5.2. How Managerial Decisions and Budget Allocation Adjust the Model Curve

Rather than simply assuming that spending more money leads to higher sales, the model directly links financial investments, such as advertising budgets (*A*) and product improvement costs (*E*) to adjustments in the curve's main parameters:

1) Maximum Sales Ceiling (*M*):

Increasing promotional efforts and improving product features allows a company to reach a larger total market size. However, to stay realistic, each additional dollar spent yields slightly smaller increases in maximum sales capacity as the market gets closer to full saturation (diminishing returns).

2) Growth Speed Parameters (k_1 and k_2):

The slope of the curve reflects how fast customers adopt the product. Targeted marketing campaigns speed up customer acquisition, making the slope steeper during both the early phase (k_1) and the expansion phase (k_2).

3) Timing of Peak Momentum (t_i):

The inflection point (t_i) marks the time when sales growth reaches its fastest speed. Heavy early promotional pushes and product improvements build brand recognition faster, shifting this milestone earlier in the project lifecycle.

4) Initial Launch Sales (S_0):

Investments made in product quality and market research prior to launch raise the baseline starting sales (S_0) achieved during product introduction.

By connecting advertising and product investment directly to these four settings, management can map out distinct revenue paths (such as Upper Frontier Curves 1 and 2 in Figure 8) based on planned budget allocations.

5.3. Optimization Framework and Calculation of the Efficient Frontier

To make the "efficient frontier" fully operational and mathematically reproducible, alternative revenue curves are evaluated using a constrained optimization framework that balances projected financial returns against budget limitations and commercial risk:

1) Objective Function (Goal):

The primary goal is to maximize the project's Net Present Value (NPV) over its planned lifecycle (T_m) by selecting the optimal combination of baseline sales (S_0), maximum market capacity (M), growth speed (k), and peak momentum timing (t_i):

$$\max \text{NPV} = \sum_{t=1}^{T_m} \frac{R(t) - C(t)}{(1+i)^t} - C_0$$

2) Constraints (Limits):

The optimization is subject to realistic business limitations, including:

- **Budget Constraint:** Total promotional and advertising expenditure $C(t)$ across all periods must not exceed the approved marketing budget cap ($B_{\max}$): $\sum_{t=1}^{T_m} C(t) \le B_{\max}$.
- **Market Ceiling Constraint:** Maximum periodic sales cannot exceed total market capacity: $M \le M_{\max}$.

3) Risk Variable:

To account for investment risk, demand volatility is represented by the variance or uncertainty (σ^2) in projected periodic revenues. Management sets a maximum allowable risk threshold ($\sigma^2_{\max}$) to prevent over-promoting financially volatile projects.

4) Optimization Procedure:

The efficient frontier is calculated by solving this optimization problem across a range of budget levels using standard Non-Linear Programming (NLP) methods (such as the Generalized Reduced Gradient algorithm). The resulting set of optimal solutions traces out the Upper Frontier Curve (as shown in **Figure 8** and **Figure 9**), representing the highest achievable periodic revenue trajectory for any given resource budget and risk tolerance.

5.4. Measurable Efficiency Criteria and Trajectory Dominance

To evaluate alternative income projections, management requires clear criteria to measure efficiency and compare competing growth paths:

1) Efficiency Criteria:

A revenue trajectory's performance is measured using two practical ratios:

a) Return on Promotional Spend: The ratio of projected Net Present Value (NPV) relative to total promotional investment ($\text{NPV/Promotional Spend}$). Higher values indicate greater financial gains per dollar invested.

b) Risk-Adjusted Return: The expected NPV divided by the volatility or uncertainty in projected sales. This ensures management evaluates not only financial returns, but also the reliability of those returns.

2) Trajectory Dominance (Comparing Growth Paths):

When comparing two potential growth trajectories (e.g., Trajectory A vs. Trajectory B):

a) Trajectory A dominates Trajectory B if Trajectory A achieves a higher projected NPV without requiring extra marketing expenditure or increasing sales risk.

b) Conversely, Trajectory B is considered dominated (or inefficient) if another strategy achieves superior financial returns for the same or lower budget and risk.

3) The Efficient Frontier Boundary:

The S-Curve Efficient Frontier consists exclusively of non-dominated trajectories. It defines the upper limit of maximum achievable revenue profiles for any given marketing budget and risk tolerance. Any point below this frontier indicates an inefficient plan where management could either increase returns without spending more or reduce marketing costs without sacrificing revenue.

Starting from the initial launch point $t = 0$, a preliminary estimate of the minimum target periodic sales S_0 is established before the official launch of the product or service. This value S_0 represents the expected periodic sales generated through minimal promotional efforts, such as organic exposure and low-cost online marketing activities. Let T_m denotes the target planning horizon required to achieve full market penetration, while M represents the corresponding equilibrium periodic sales level associated with market saturation. These parameters collectively define the projected periodic growth trajectory of the product throughout its lifecycle.

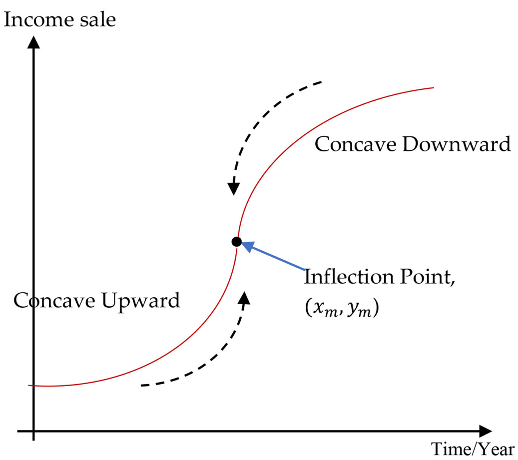

Figure 9. An S-curve efficient frontier model.

As illustrated in **Figure 9**, the proposed model employs a piecewise functional form, as defined in Equation (2), to provide greater flexibility in representing the changing growth dynamics of income generation. This structure allows the model to accommodate different curvature behaviours before and after the inflection point, thereby improving its ability to represent real-world sales patterns. The resulting S-curve evolves dynamically and may be calibrated to minimise projection error while preserving the underlying growth characteristics of the project.

The inflection point represents the most critical stage of the proposed model. At this point, the periodic income projection curve exhibits its maximum growth rate and the steepest slope. From a managerial perspective, the inflection point serves as an important decision-making milestone, providing management with an opportunity to evaluate project performance and determine whether additional investments in marketing, promotion, or product enhancement should be undertaken to accelerate future income growth.

To the left of the inflection point, the periodic income curve exhibits positive concavity ($\frac{\mathrm{d}^2 R}{\mathrm{d}t^2} > 0$), reflecting the gradual build-up of market awareness and customer adoption. To the right of the inflection point, the curve displays negative concavity ($\frac{\mathrm{d}^2 R}{\mathrm{d}t^2} < 0$), indicating that growth continues but at a progressively decreasing rate as the market approaches saturation M. Such behaviour is commonly observed in business environments, where newly introduced products or services initially experience slow adoption, followed by rapid market acceptance, before eventually reaching a mature and stabilised state.

Consequently, the proposed income projection model captures the four transitional phases discussed in Section 4 and provides a dynamic framework for monitoring project performance throughout its lifecycle.

The development of the proposed S-curve efficient frontier model is based on the following assumptions:

1) The project is new, financially viable, and expected to generate a positive Net Present Value (NPV).

2) Senior management is willing to allocate sufficient financial resources to support the successful implementation of the project.

3) Project managers regularly monitor, evaluate, and report project performance throughout the project lifecycle.

4) The organisation possesses adequate financial strength to sustain the required investment and growth activities.

The proposed S-curve efficient frontier model offers several important implications for projected revenue:

1) Compared with conventional projection approaches, the model encourages continuous revision and improvement of projected cash flows based on evolving project performance.

2) The model adopts a non-linear growth structure that more accurately reflects the dynamic nature of business growth and managerial decision-making.

3) The framework enables organisations to identify potential weaknesses during project execution and implement corrective actions before significant deterioration in performance occurs.

4) The model emphasises the importance of continuous innovation, strategic adaptation, and effective marketing initiatives in sustaining long-term revenue growth and maximising income generation.

5.5. Model Scope, Testable Propositions, and Boundary Conditions

While the proposed framework offers a conceptual alternative to conventional constant-growth models, its performance relative to traditional forecasting approaches should be treated as a testable proposition subject to empirical validation:

- **Testable Proposition:** *In projects characterized by multi-phase market adoption, the proposed piecewise S-curve model achieves lower projection error (measured by RMSE and MAPE) compared to traditional constant or linear growth assumptions*.

Furthermore, the S-shaped growth trajectory is not universally applicable to all business models. Management should avoid using this model under the following **boundary conditions:**

1) Fad or Highly Seasonal Products: Products with short lifecycles, viral novelty items, or seasonal goods that experience an immediate sales surge followed by a rapid collapse, bypassing the gradual adoption and maturity phases.

2) Subscription and Platform Models: Digital platforms or Software-as-a-Service (SaaS) businesses where revenue scales linearly or exponentially through compounding network effects and recurring subscription payments, rather than following a traditional physical product adoption ceiling.

3) Severe Market Disruptions: Projects subject to unpredictable external shocks such as sudden regulatory bans, severe supply chain disruptions, or immediate competitor displacement which interrupt the natural progression through the four growth phases.

This study presents a conceptual framework for developing an S-curve efficient frontier model for projected revenue. Establishing a realistic and reliable income projection is essential for evaluating investment opportunities and supporting strategic decision-making. By providing a clearer understanding of income growth dynamics throughout the project lifecycle, the proposed model offers management a practical tool for monitoring project performance and making informed decisions aimed at maximising long-term business value.

6. Conclusions

A projected revenue model provides an important avenue for top management to make informed business decisions. Income flows should be projected prior to the evaluation of a project using investment appraisal techniques such as Net Present Value (NPV) at the appropriate corporate discount rate. A realistic and reliable income projection enables decision-makers to assess the financial viability and long-term potential of a proposed investment.

With a clear understanding of the four phases of the S-curve development process, project managers can better monitor project progress, identify critical growth stages, and implement suitable strategies to maximise project performance. The proposed S-curve projected revenue model offers a dynamic framework for tracking the evolving income status of a project throughout its lifecycle. In particular, it provides management with greater visibility of the project's growth trajectory and its position relative to the inflection point.

Consequently, decisions regarding the increase or reduction of advertising expenditure and other strategic investments can be made more effectively as the project progresses. It is hoped that the proposed S-curve efficient frontier model will contribute to a better understanding of income growth dynamics and provide a useful conceptual tool for supporting managerial decision-making in new business projects.

Acknowledgements

The authors would like to acknowledge Universiti Teknikal Malaysia Melaka for its institutional support during the preparation of this manuscript.

Author Contributions

Fadzilah Salim: Conceived the overall study design, conducted the literature review, developed the manuscript structure, wrote the original draft, prepared all sections of the manuscript, interpreted the findings, and coordinated the submission and revision process. Nur Azman Abu: Developed the main research idea, formulated the conceptual framework, proposed the S-curve efficient frontier concept, derived the model equation and methodological framework, and provided critical intellectual input on the theoretical development of the study. Satrya Fajri Pratama: Reviewed and proofread the manuscript, provided linguistic and editorial improvements, examined the clarity and coherence of the presentation, and contributed to the refinement of the final manuscript.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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