

Catch-22 in Second Law Revealed by Axiomatic Analysis, Part 1 Optimizing ExB Effect, Extending Law

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Abstract

An axiomatic analysis reveals a cornerstone weakness in all foremost arguments supporting the second law: either phenomena are viewed at the microscale, where *T-Odd* symmetry can generate negentropy, or at the macroscale, where massive averaging, integration or coarse graining ensure that this negentropy is erased. These two alternatives present a catch-22 binary choice: 1) at the microscale, negentropy is present but not accessible because of the lack of thermodynamic interaction (collisions), and 2) at the macroscale it is accessible because of the abundance of interactions, but not present because it is destroyed by the overwhelming interactions. This false dichotomy can be eliminated by operating at the mesoscale where some negentropy can be salvaged. Using the $\mathbf{E} \times \mathbf{B}$ drift as a vehicle, an index of asymmetry is defined, which quantifies a position on the mesoscale. Optimal use of free energy generated by a *T-Odd* system occurs at the center of the mesoscale where the index of asymmetry reaches unity. There, the actual energy output is equal to $\frac{1}{4}$ of the free energy used to convert heat to work. In compliance with all second law arguments, the energy output drops to zero at the two extremes of the mesoscale. Under maximum power generation, in the center of the mesoscale, the change in entropy is limited to $\Delta S \geq -1/4A/T$.

Keywords

Second Law, H-Theorem, Time-Reversal Symmetry, CPT Symmetry, ExB Thermoelectric Effect, Free Energy, Negentropy, Entropy, Mesoscale, Coarse Graining

You can't have your cake and eat it too... but you can eat half your cake and still have the other half.

1. Introduction

This is the first of a two-part paper. The second law of thermodynamics is one of the most trusted principles in physics and has remained immutable for over two centuries. Yet like any other physical principle, it should be open to inquiry and falsification. This task is rendered the more challenging given the enormous amount of accumulated theoretical and experimental evidence in its support. Does each new piece of theoretical evidence stand on its own or is it a rehash of the existing ones? Is every experiment confirming the law new, or is it based on the same physical principles tested differently? Is the law universally valid or are there some physical conditions that allow it to be bypassed? To answer these questions, the following topics are discussed:

- **Axiomatic Analysis.** A rigorous analysis using the First Principles Thinking approach is applied to the most significant arguments to uncover their very bottom logical cornerstones of the law, often ignored, forgotten, or deeply hidden. The goal is to reduce the universe of discourse to these basic principles. Two kinds of physical symmetry were found to be relevant: time-reversal symmetry (*T-Even*), time-reversal asymmetry (*T-Odd*) and the scale of operation. The analysis showed that all arguments, without exception, rely on averaging over the macroscale to enforce a loss of information (coarse-graining) leading to $\Delta S \geq 0$.
- **False Dichotomy and Catch-22.** All second law arguments present a catch-22 choice between the generative microscale (μ -Scale) where negentropy is present but inaccessible and the dissipative macroscale (*M-Scale*) where it is accessible but not present. This binary choice is identified as a false dichotomy: a device should be able to operate at the mesoscale (*m-Scale*) over a continuous range between μ -Scale to *M-Scale*.
- ***T-Odd* symmetry.** In contrast with *T-Even* symmetry which is limited to $\Delta S = 0$ at the microscale (μ -Scale), *T-Odd* symmetry can decrease entropy at that scale. All proofs and arguments supporting the second law eradicate this negentropy by an overwhelming *M-Scale* averaging (coarse-graining).
- **Thermodynamic Cycle: Free Compression at μ -Scale and Isothermal Expansion at *M-Scale*.** A thermodynamic cycle for a *T-Odd* system operating between μ -Scale and *M-Scale* is described featuring the stage of “free compression” at μ -Scale and isothermal expansion at *M-Scale*. This cycle shows that negentropy and free energy can be generated while preserving the first law.
- **The $E \times B$ Drift Meets the Catch 22 in the Law.** The $E \times B$ drift is used as a vehicle to illustrate the catch-22 and false dichotomy inherent in all arguments supporting the second law.
- ***m-Scale* and Index of Asymmetry.** A continuous scale *m-Scale*, ranging from μ -Scale to *M-Scale*, is presented onto which foundational principles supporting the second law can be mapped. An index of asymmetry Z_{asym} is defined to specify where on the map any second law argument is positioned. This index provides a quantified description of the concept of “Thermodynamic Thresh-

old” previously discussed by the author [1]-[4].

- **Drift angle and Index of Asymmetry.** The relationship between the $\mathbf{E} \times \mathbf{B}$ drift angle and the index of asymmetry is discussed.
- **Optimal Use of Free Energy in Heat to Work Conversion.** Maximum power output occurs at the center of m -Scale where $Z_{asym} = 1$. This optimization criterion is shown to be identical with the Maximum Power Transfer Theorem and limits the output power to 1/4 of the free energy made available by a bandwidth-limited T -Odd process.
- **Extending Second Law.** The second law is extended to include T -Odd systems operating at the m -Scale.

The second part of this two-part paper optimizes the rectification of thermally induced Johnson-Nyquist noise by a MIM diode.

2. Axiomatic Analysis

Proofs and other arguments supporting the second law are of two kinds, depending on whether the behavior of particles is T -Even or T -Odd at the microscopic scale.

2.1. T -Even Systems

These systems do not include field-dependent velocity distributions (such as caused by magnetic fields). At μ -Scale (in the absence of collisions), T -Even symmetry requires that all motion be exactly time-reversible, and that fine scale entropy remains invariant, $dS_{fine}/dt = 0$. However, as the observer enlarges his scope from μ -Scale to M -Scale, information is lost and entropy increases $dS_{coarse}/dt > 0$ due to effects such as *Stosszahlansatz* by Boltzmann, coarse graining by Gibbs, and the ergodic hypothesis by von Neumann. Most importantly, from a second law violation perspective, **T -Even systems are the least interesting because of their inability to generate $dS/dt < 0$ at any scale.**

Proofs covering T -Even systems by the axiomatic analysis are described in Appendices A-B:

1. Boltzmann’s H -Theorem.
2. Landauer’s Principle (derived without magnetic fields).

2.2. T -Odd Systems

T -Odd symmetric systems are of special interest. Being governed by Liouville theorem, their fine-grained Gibbs entropy must remain constant, $dS_{fine}/dt = 0$. However, this theorem does not guarantee that its marginal components—positional entropy S_p and momentum entropy S_m —must also remain constant. Even though the volume of phase space is invariant, its shape is not: positional entropy may decrease at the expense of momentum entropy—as in a free expansion—or vice versa. Unfortunately, at μ -Scale, this marginal negentropy is inaccessible because of the lack of thermodynamic interactions (collisions). Conversely, at M -Scale, massive coarse graining, large-scale averaging, or integration ensures that this

negentropic effect is completely erased.

Proofs covering *T-Odd* systems by the axiomatic analysis are described in Appendices C-H:

1. Gibbs' *H*-Theorem (With *B* Field).
2. von Neumann's Quantum *H*-Theorem
3. Detailed Balance (with *B* Field).
4. Fluctuation Theorem & Stoch. Therm. (with *B* Field).
5. Fluctuation-Dissipation Theorem (with *B* Field).
6. Kirchhoff's Radiation Law (with *B* Field).

2.3. Thought Experiments

As part of the axiomatic analysis, two thought experiments, Maxwell's Demon and Feynman's ratchet are described in Appendices I and J.

2.4. Experimental Evidence for and against the Law

To further complete and go beyond the analysis, two appendices covering experimental data are included. Appendix K describes experiments in support of the second law (all conducted at *M-Scale*) and Appendix L, experiments against the law. The results of the axiomatic analysis are summarized in **Table 1**.

Table 1. Axiomatic analysis of the foundational arguments supporting the second law, showing for each the symmetry it addresses, the operation by which it enforces its conclusion, and the resulting position on the *m-Scale*. Every argument, without exception, resolves to an endpoint of the index of asymmetry.

Appendix	Scrutinized Principles, Laws, or Theorems	<i>M-Scale</i> enforcement	Symmetry and Scale			
			<i>Z_{asym}</i>	<i>T-Even</i>	<i>T-Odd</i>	
					<i>μ-Scale</i>	<i>m-Scale</i>
					<i>μ-Scale</i>	<i>m-Scale</i>
					$\Delta S \geq 0$	$\Delta S < 0$
					$\Delta S = 0$	$0 \Delta S \geq 0$
	Condition for 2nd Law Bypass $\Delta S < 0$	Operation at <i>m-scale</i> .	≈ 1			✓
A	Boltzmann's <i>H</i> -Theorem	<i>Stosszahlansatz</i> discards post-collision correlations; keeps correlation time $\rightarrow 0$.	$\rightarrow 0$	✓		
B	Landauer's Principle	Infinite information sink; delta-correlated bath of infinite bandwidth suppresses Poincaré recurrence	$\rightarrow 0$	✓		
C	Gibbs' <i>H</i> -Theorem (With Field)	Coarse-graining cells exceed the cyclotron orbit; radius <i>r</i> unresolved by the observer	$\rightarrow 0$			✓
D	Von Neumann's Quantum <i>H</i> -Theorem	Random Phase Approximation and ergodic hypothesis discard off-diagonal phase coherence memory	$\rightarrow 0$			✓
E	Detailed Balance (with <i>B</i> Field)	Angular integration over closed probability loops; mean-free angle ϕ_λ summed to zero net macro-flux.	$\rightarrow 0$			✓
F	Fluctuation Theorem & Stoch. Therm. (with Field)	Overdamped limit removes velocity terms; white-noise bath sets correlation time $\rightarrow 0$; Local Detailed Balance assumes the result.	$\rightarrow 0$			✓

Continued

G	Fluctuation-Dissipation Theorem (with B Field)	Canonical equilibrium premise; field does no work and drops out of the Gibbs distribution (Bohr-van Leeuwen).	$\rightarrow 0$				✓
H	Kirchhoff's Radiation Law (with B Field)	Hemispherical integration averages $\epsilon(\theta) \neq \alpha(\theta)$ over the full hemisphere	$\rightarrow 0$				✓
I	Maxwell's Demon	Demon and gas stipulated T-Even; Bennett-Landauer erasure bookkeeping	$\rightarrow 0$	✓			
J	Feynman's Ratchet	Time-invariant spatial profile; asymmetry has no response rate	$\rightarrow 0$	✓			
K	Experiments for 2nd Law	All conducted at $Z_{asym} \rightarrow 0$ or $Z_{asym} \rightarrow \infty$; none at m-Scale	$\rightarrow 0$	✓	✓		✓
L	Experiments against 2nd Law	Fluctuation-modulated asymmetry; T-Odd process between endpoints	$0 < Z_{asym} < \infty$			✓	

3. False Dichotomy and Catch-22

Most remarkably, all proofs covering *T-Odd* systems present the reader with a **catch-22** binary choice:

1. Either operate at μ -Scale **where negentropy can be produced** by *T-Odd* symmetry but remains thermodynamically **inaccessible** due to the lack of collisions or other interactions.
2. Or operate at *M-Scale* **where negentropy is highly accessible** due to abundant collisions and interactions but **is destroyed** by this overwhelming abundance before it can be accessed.

This choice is a false dichotomy between the two μ -Scale and *M-Scale* extremes. No proof seriously considers *m-Scale* operation. Why? It may be due to the technical difficulty in fabricating devices operating at a scale commensurate with thermal processes. For example, carriers in high mobility semiconductors such as InAs have a typical mean-free-path of 200 nm. In highly pure two-dimensional electron gas (2DEGs) systems, this value can extend beyond 1 μ m. These values correspond to mean-free-times ranging from 1ps to 5ps.

In contrast, the wavelength of thermally induced electrical Johnson-Nyquist noise at 300K is $\lambda = ch/(2.821k_B T)$, at the peak power of the spectrum, is about 17 microns corresponding to an average oscillation period of about 60fs, that is 17 to 83 times shorter than an electron's mean free time. These numbers imply that electrons behave ballistically with respect to thermal noise and therefore operate at μ -Scale. A device between 200nm and 1 μ m has sufficient frequency bandwidth to operate at *m-Scale* and therefore can rectify electrical thermal noise without second law restriction.

Shying away from *m-Scale* may have been justified in the 19th century when the second law was first founded but it is unjustified today, given miniaturization techniques that now allow nanoscale devices to be fabricated.

The μ -Scale and *M-Scale* extremes present a thermodynamic catch-22:

1. μ -Scale produces negentropy but not the means (collisions) to access it, and
2. M -Scale produce so much of those means that they destroy negentropy before it can be accessed.

In colloquial language these two choices are equivalent to the saying “*You can’t have your cake and eat it too.*” In fact, there is a middle third choice: m -Scale. In the same colloquial language, “*you can eat half your cake and still have the other half.*” The remainder of this paper discusses “*the best thermodynamic way to eat your cake.*” As we shall see, it is to “*eat it in small m -scale portions*” somewhere between μ -Scale and M -Scale.

4. T -Odd Symmetry

Before tackling the problem of operating a T -Odd system at m -scale, this symmetry must be briefly discussed as it is a necessary requirement for any system capable of decisively bypassing the second law.

T -Odd symmetry is directly manifested by magnetic fields and the $\mathbf{E} \times \mathbf{B}$ drift which are CPT symmetric. It is also displayed by spatially asymmetric devices such as diodes which are inherently spatially asymmetric but rendered T -Odd when their spatial asymmetry is modulated by a time-varying signal such as thermal fluctuations.

4.1. Magnetic Fields and the $\mathbf{E} \times \mathbf{B}$ Drift

When perpendicular electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are applied to a high-mobility semiconductor, charged particles follow cycloidal paths and drift with velocity

$$\mathbf{v}_{\mathbf{E} \times \mathbf{B}} = \frac{\mathbf{E} \times \mathbf{B}}{|\mathbf{B}|^2} \quad (1)$$

perpendicular to both fields [1]-[5]. The drift velocity depends only on the fields—not on the particle’s instantaneous velocity, and not on the sign of its charge. This has an immediate consequence for time reversal. Under the operation $t \rightarrow -t$ with the fields held fixed, every particle velocity reverses, $\mathbf{v} \rightarrow -\mathbf{v}$, yet Equation (1) is unchanged. The reversed motion is therefore not a solution of the equations of motion: a movie of the drift run backward shows particles drifting against $\mathbf{E} \times \mathbf{B}$, a motion the fixed fields cannot produce. At fixed $\mathbf{B}$, trajectories do not retrace, and the dynamics are T -Odd.

It is sometimes objected that the full system is nonetheless T -Even: if time reversal is applied also to the currents that generate the field, then $\mathbf{B} \rightarrow -\mathbf{B}$, the drift reverses with it, and the backward movie becomes a valid solution of the reversed problem. This is correct as far as it goes—but it describes a different experiment, conducted in a reversed field. The experimenter must decide where to put the magnetic field. Placing it in the same time frame as the experiment *renders the whole experiment T -Odd, a fact that the experimenter can easily detect by measuring the new field direction.* In the end, **the experimenter must choose where to put the field:** within the experiment, or in his time frame. No matter what he

decides, *the very presence of the field, not which frame it occupies, causes time asymmetry* [1] [2].

The *T-Odd* symmetry of the magnetic field is precisely what motivated Onsager to exclude them from his reciprocals and caused Casimir to formulate his Onsager-Casimir reciprocal relations, $L_{ij}(\mathbf{B}) = \varepsilon_i \varepsilon_j L_{ji}(-\mathbf{B})$ [6] [7]: reciprocity is restored only *between two distinct systems each with its own field*. For a single device operating in one fixed applied field no reciprocity is asserted, and the dynamics are irreducibly *T-Odd*. Casimir's formalism customarily invoked to restore time-reversal symmetry is thus the same formalism that codifies its absence at fixed $\mathbf{B}$.

4.2. Rectifiers and Diodes

Diodes and fluidic valves are configured *at fabrication time as spatially asymmetric yet become time asymmetric when exposed to a time varying signal that switches direction with time*. The signal causes the configuration (e.g., thickness of the depletion zone in a PN diode) and conductivity of the device to change between forward and reverse modes. This dynamic change in spatial asymmetry results in *T-Odd* symmetry¹. It is associated with a state variable (e.g., thickness of depletion zone), a bandwidth, and an energy of transition (built-in potential). These devices are the subject of a companion paper [1] written in the light of the theory presented herein.

5. Thermodynamic Cycle: Free Compression at μ -Scale and Isothermal Expansion at M -Scale

The thermodynamic cycle of a *T-Odd* process shall be described using a gas of electrically charged particles in crossed $\mathbf{E} \times \mathbf{B}$ fields, circulating around the loop of **Figure 1**. The loop comprises a **forward channel** of high mobility and low carrier concentration, in which a low collision rate permits long cycloids and supports the drift, and a **return channel** of low mobility and high concentration, in which abundant collisions disable the drift but still allow Ohmic conduction [1]-[4].

5.1. Reference Process: Free Expansion

For reference, first consider N gas particles occupying volume V_1 suddenly allowed to expand into volume $V_2 > V_1$. The expansion is isothermal; internal energy U and temperature T remain constant. The entropy of the gas increases:

$$\Delta S = Nk_B \ln \left(\frac{V_2}{V_1} \right) > 0 \quad (2)$$

and its Helmholtz free energy decreases:

¹A conventional PN diode is by far too slow and not sensitive enough to rectify the thermally induced Johnson-Nyquist noise. However, it may be possible for a specially designed MIM or MIIM diode to do so.

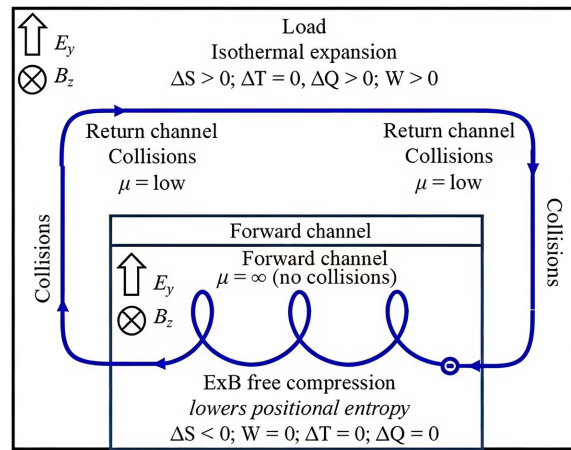


Figure 1. The $E \times B$ Thermoelectric effect. This effect requires a loop including a forward channel and a return channel. The forward channel comprises a high mobility, low concentration material which enables the drift. The return channel comprises a low mobility, high concentration material which disables the drift but still enables Ohmic conduction.

$$\Delta A = -Nk_B T \ln \left(\frac{V_2}{V_1} \right) < 0 \quad (3)$$

No work is extracted, and free energy is irreversibly destroyed. This is the canonical dissipative process of the second law.

5.2. Free Compression at μ -Scale

Now let the same gas be placed in the forward channel in **Figure 1**. Particles carried by the $\mathbf{E} \times \mathbf{B}$ drift in a high-mobility material follow cycloidal paths uninterrupted by collisions. Beginning uniformly distributed in volume V_1 , they move unidirectionally and in unison at the common drift velocity $\mathbf{v}_{E \times B}$ along an equipotential line, without doing any work and accumulating at the downstream wall into a smaller volume $V_2 < V_1$ to create a space charge. This space charge deflects the E field as well as the drift direction. **Section 9** provides an accurate discussion of this process.

The energy bookkeeping along the way is exact. The magnetic force, always perpendicular to velocity, does no work. The electric field does no *net* work over a cycloid: the guiding-center displacement is perpendicular to $\mathbf{E}$, and the cyclical excursions along $\mathbf{E}$ within each orbit average to zero.² The particles' kinetic energies, the internal energy U , and the temperature T are therefore unchanged by the transport. Since $V_2 < V_1$ at constant U and T , applying (2) and (3) to the positional distribution, entropy decreases:

$$\Delta S = Nk_B \ln \left(\frac{V_2}{V_1} \right) < 0 \quad (4)$$

²Equivalently: the drift displacement is along X, perpendicular to E_y , so no net electrical potential energy is exchanged.

and the Helmholtz free energy increases:

$$\Delta A = -Nk_B T \ln \left(\frac{V_2}{V_1} \right) > 0 \quad (5)$$

The process is the exact reverse of a free expansion; we shall call it a **free compression**. No work is expended to produce it. The *T-Odd* drift converts positional uncertainty into positional order at zero energy cost.

5.3. Free Compression as Positional Negentropy

Equation (2) ascribes positional entropy to compression and must be read with care. The collision-less motion in the forward channel is Hamiltonian. Liouville's theorem therefore holds exactly: the fine-grained phase-space density is conserved, and the fine-grained entropy is invariant, $dS_{\text{fine}}/dt = 0$, in full agreement with unitarity. Nothing in this paper contradicts that statement. The entropy that decreases in Equation (2) is the **positional entropy**—the entropy of the marginal distribution over position, obtained by integrating the full phase-space distribution over momentum. The negentropy generated by the drift resides in the *correlations between position and momentum* that the coherent drift builds up; **the positional distribution registers this negentropy as a compression, while the fine-grained entropy, which tracks the correlations in full, remains constant.**

5.4. Coarse-Graining to Extract Positional Negentropy

Formally, selecting the positional distribution is a coarse-graining operation: the position-momentum correlations are left out. Since coarse-graining is essential to the second law as shown by the axiomatic analysis, two kinds of such operations must be distinguished:

1. In the arguments catalogued in Appendices A-H, coarse-graining is imposed by the observer: it is justified by his limited resolution, and it erases the very correlations in which the *T-Odd* negentropy resides.

2. In the device proposed in this paper, the coarse-graining operation extracts position information and erases the rest. It is achieved not by the observer but by the device. A load in the return channel couples to the electron gas only through the electrostatic potential of its charge density, and charge density depends **on position alone**. The momentum correlations and whatever negentropy the drift stores in the position-momentum correlations are inaccessible to the load. **However, negentropy stored in position remains accessible.** The positional entropy of Equation (4) measures neither more nor less than the accessible negentropy—and the challenge this paper poses to the second law is precisely whether this accessible positional negentropy can be generated isothermally.³

In summary, coarse-graining operates like a low pass filter that eliminates all μ -

³The same operational criterion, applied in reverse, is what condemns the coarse-grainings of Appendices A-H: they discard order that a suitably built device *could* couple to, and then declare it thermodynamically nonexistent.

Scale information from *M-Scale*. A device applying coarse-graining at *m-Scale*, can erase position-momentum correlations which are prevalent at *μ -Scale* while leaving partly untouched larger-sized positional information.

5.5. Free-Compression in the Reversible Limit

One further point of rigor: the compressed gas in V_2 is not in an equilibrium state—it retains the velocity-space correlation caused by the drift. Equations (2) and (3) nonetheless give the correct entropy measure, because they compare the positional distributions of the initial and final states at the same U and T ; **the ΔA so computed is the ceiling on the work obtainable from the free compression, attained in the reversible limit.** The residual velocity-space correlation subsequently thermalizes in the collisional return channel at constant internal energy, neither adding to nor subtracting **from the positional free energy.**⁴

5.6. Isothermal Expansion at *M-Scale*

Since the particles' internal energy is constant along the forward channel, the gas is isothermal from end to end, and the return channel must be isothermal from end to end as well. To close the cycle, the particles undergo an isothermal expansion in the return channel that restores them to their original volume V_1 . This expansion converts the free energy accumulated by the compression into electrical work in a load, which may be a battery, a resistor, or whatever the designer chooses.

5.7. Drawing in Heat from Surroundings Requires a Fourier Heat Conduction Temperature Difference

The process is end-to-end isothermal **but not necessarily reversible**, and **it need not be isothermal with respect to its surroundings.** As the gas expands in the return channel, thermal energy is converted to work; the channel cools relative to the surroundings, and the forward channel cools with it. The emerging temperature difference ΔT allows heat $Q = W$ to flow in and replace the thermal energy converted to work. This ΔT is a Fourier heat-conduction requirement, not a Carnot limitation on the performance of the system. In other words, there is no need for a heat sink, but the device must still be colder than the surroundings.

5.8. First-Law Audit of the Loop

The energy source of the cycle can now be identified by elimination, term by term around the closed loop.

The static magnetic field does no work on any particle at any time. The applied electric field is static and conservative, so its net work on a particle completing one circulation of the loop is identically zero. The space-charge field E_x is internal

⁴Under maximum power operation (Section 8) the expansion is deliberately irreversible and only a fraction of ΔA is used to produce work; the limited $W \leq \Delta A$ is in the amount fixed by the Maximum Power Transfer Theorem.

to the system: the energy stored in the space-charge field is constant in steady state and enters the audit only as a start-up transient. The work done against it by particles accumulating upstream is recovered from it as those particles are driven through the return channel and the load. No field term, applied or internal, delivers net energy per cycle. The fields play a strictly *catalytic* role—they supply the *T-Odd* bias that configures the microdynamics, but they pay no energy for the negentropy they create.

By the first law, the net work delivered to the load per cycle must therefore be drawn from the only remaining reservoir: heat absorbed from the surroundings:

$$W = Q \quad (6)$$

In the return path, the heat-to-work conversion is limited to the free energy accumulated in the free compression; under reversible conditions,

$$W \leq -\Delta A \quad (7)$$

The crucial property enabling steady circulation is that the drift can be turned **on and off without altering the fields**, through the mobility and concentration difference between the two channels—high mobility and low concentration in the forward channel and low mobility and high concentration in the return channel. In the language of Section 7, the loop shifts the index of asymmetry from a high value in the forward channel, where negentropy is generated, to a low value in the return channel, where it is spent in the load. The process violates neither the first law, as shown above, nor any fine-grained conservation law, as shown in Section 5.3. **It only contests the universality of the coarse-grained inequality $\Delta S \geq 0$ —and it contests it along the *m-Scale*, between the *μ -Scale* and *M-Scale* endpoints, where Appendix K shows the inequality has never been theoretically investigated nor experimentally tested.**

6. The $E \times B$ Drift Meets the Catch 22 in the Law

The catch-22 of the second law can be illustrated and resolved using the $E \times B$ drift as a vehicle. Consider a forward channel of length L , comprised of a high mobility μ , and low concentration material, capable of supporting the drift such that carriers can follow cycloidal paths without collisions except at the walls of the channel. As the particles start out from a uniform distribution in the material, the drift causes them to accumulate downstream, thereby reducing their positional uncertainty and entropy. Let the *measure* of the *T-Odd* symmetry produced by the magnetic field be the average radius r of cyclotron orbits.

A relatively long channel ($L \gg r$) results in a long cycle-time small bandwidth, and a slow production of negentropy. Conversely, a relatively short channel ($L \ll r$) does not support the drift because cycloids are truncated by collisions with the walls. Their short quasi-linear paths negate the effect of the field and result in a quasi *T-Even* carrier gas unable to produce free energy. As in the Goldilocks dilemma, optimal design requires the channel to be neither too long nor too short. Conventional proofs and arguments supporting the second law provide a binary

choice between those two extreme scales:

1. μ -Scale extreme:

a. In the example: $L \gg r$.

b. Low collision rate, long cycloids, long cycle time, small bandwidth, high expression of T -Odd symmetry and a high production and low dissipation of negentropy,

c. low ability to export plentiful free energy due to low collisions rate.

2. M -Scale extreme:

a. In the example: $L \ll r$.

b. A high collision rate, short, truncated cycloids, quick cycle time, large bandwidth, low expression of T -Odd symmetry, and a low production and high dissipation of negentropy,

c. high ability due to the high collision rate, to export nonexistent free energy.

7. m -Scale and Index of Asymmetry

The index of asymmetry Z_{asym} shall be defined to provide a dimensionless continuous measure between these two extremes by combining two factors representative of free energy generation (μ -Scale) and free energy dissipation (M -Scale). In the $\mathbf{E} \times \mathbf{B}$ drift example, these two factors are respectively r and L .

Notably, these characteristic factors do not have to be expressed as spatial dimensional quantities. They can represent time, frequency, mobility, magnetic field, resistivity, energy, bandwidth, etc. The index is formed by taking the dimensionless ratio of conjugate quantities in the same units, the first factor representing the dominant generative T -Odd process at μ -Scale, the second the dominant dissipative T -Even thermal processes at M -Scale.

$$Z_{asym} = \frac{\text{Free Energy Generation}}{\text{Dissipation}} \quad (8)$$

For example, the measure of the T -Odd symmetry of a magnetic field, can be represented by its Larmor frequency ω . The corresponding T -Even measure of thermal processes can be the mean-free-time τ of charged carriers. Therefore, Z_{asym} can be defined in terms the frequency ω and thermal noise bandwidth ($1/\tau$), as the Hall parameter [8]:

$$Z_{asym} = \text{Hall Parameter} = \omega\tau \quad (9)$$

Since $\omega = v/r$ and $\tau = \lambda/v$ for a mean-free-path λ and a cyclotron orbit with radius r , Equation (9) can be rewritten in terms of spatial dimensions as:

$$Z_{asym} = \frac{\lambda}{r} \quad (10)$$

Equation (10) illustrates how Z_{asym} quantifies the ($L \gg r$) to ($L \ll r$) scale:

1. Z_{asym} is small in the absence of T -Odd symmetry (large r due to weak magnetic field) and large rate of collisions (small λ).

2. Z_{asym} is large in the presence of a strong field and in the absence of collisions.

Furthermore, Z_{asym} can also be expressed in angular form as the mean-free angle

$\phi_l = \lambda/r$ subtended by an arc λ around an orbit of radius r :

$$Z_{\text{asym}} = \phi_\lambda \quad (11)$$

Since $\omega = (q/m)B$, and $\tau = (m/q)\mu$ where μ is the drift mobility [9], Equation (9) can also be written for the benefit of the designer, in terms of material property and field as:

$$Z_{\text{asym}} = \mu B \quad (12)$$

The asymmetry index can also be expressed in a highly useful electrical engineering form as the ratio of the Hall resistivity $\rho_{\text{Hall}} = B/nq$ and the Drude resistivity $\rho_{\text{Drude}} = 1/nq\mu$, where n is the carrier concentration. Since this ratio $\rho_{\text{Hall}}/\rho_{\text{Drude}} = \mu B$, Equation (12) can be written in terms of design parameters as:

$$Z_{\text{asym}} = \frac{\rho_{\text{Hall}}}{\rho_{\text{Drude}}} \quad (13)$$

Equations (9)-(13) are adequate **when the medium is uniform**, and properties of “Free Energy Generation”, and “Dissipation” can be described by single parameters such as μ , r , λ , or ρ_{Drude} . However, when a system includes several components such as a thermoelectric source and a load, these properties cannot be expressed as simply. If both the source and the load are dissipative, then the dissipation property must be as a combination of the two. This issue can be simply resolved by the concept of “equivalent” properties. For example, the total dissipative effect due to the Drude resistance distributed in the source and the load can be expressed as **equivalent resistivities**:

$$\rho_{\text{Drude}} = \rho_{\text{Source}} + \rho_{\text{Load}} \quad (14)$$

an equivalent load resistivity being defined as:

$$\rho_{\text{Load}} = \frac{R_{\text{Load}}}{R_{\text{Source}}} \rho_{\text{Source}} \quad (15)$$

where R_{Source} = source resistance, R_{Load} = load resistance and ρ_{Source} = source resistivity. Using equivalent resistivities, Equation (13) becomes:

$$Z_{\text{asym}} = \frac{\rho_{\text{Hall}}}{\rho_{\text{Source}} + \rho_{\text{Load}}} \quad (16)$$

Since $\rho_{\text{Hall}} = B/nq$ and $\rho_{\text{Source}} = 1/nq\mu_{\text{Source}}$, Equation (16) can be written as:

$$Z_{\text{asym}} = \frac{\mu_{\text{Source}} B}{1 + nq\mu_{\text{Source}}\rho_{\text{Load}}} \quad (17)$$

Furthermore, the index of asymmetry can be written as the tangent of θ_{asym} :

$$\tan(\theta_{\text{asym}}) = \frac{\mu_{\text{Source}} B}{1 + nq\mu_{\text{Source}}\rho_{\text{Load}}} \quad (18)$$

8. Drift Angle and Index of Asymmetry

Consider a $\mathbf{E} \times \mathbf{B}$ device with perpendicular fields E_y and B_z and forming a circuit with a load. When the mobility of the medium is infinite ($\mu = \infty$), and the load

resistance is zero, charged particles, unhindered by collisions, and carried by the E_y and B_z fields, move along the X axis with velocity $v_x = v_{\mathbf{E} \times \mathbf{B}} = E_y/B_z$ and their drift angle is $\theta_{\mathbf{E} \times \mathbf{B}} = \pi/2$ (perpendicular to the $\mathbf{E}$ field).

$$v_{\mathbf{E} \times \mathbf{B}} = \frac{E_y}{B_z} \quad (19)$$

The system operates at μ -Scale. Negentropy is produced by the motion of the carriers, but is not accessible.

Conversely, when mobility $\mu \rightarrow 0$, and $\rho_{Load} \rightarrow \infty$, carriers follow truncated cycloidal paths, too short to produce an $\mathbf{E} \times \mathbf{B}$ drift. They dribble down the potential energy gradient as defined by the E_y field with a velocity $v \rightarrow \mu E_y$ and at an angle $\theta_{\mathbf{E} \times \mathbf{B}} = 0$, parallel to the $\mathbf{E}$ field.

$$v_y = \mu E_y \quad (20)$$

The system operates at M -Scale and negentropy is dissipated.

When mobility is finite, the drift acquires a component v_y , down the electrical potential energy gradient E_y as shown in **Figure 2** and in the simulation **Figure 3**.

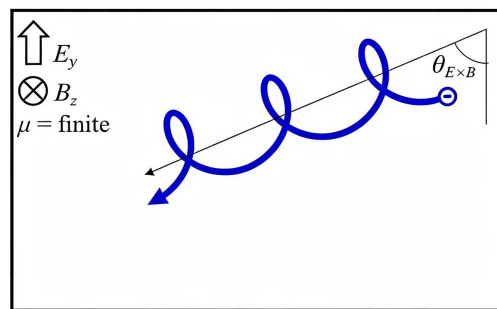


Figure 2. Without collisions, the $\mathbf{E} \times \mathbf{B}$ drift causes particles to follow cycloidal paths in the direction of the cross product at a maximum angle $\theta_{\mathbf{E} \times \mathbf{B}} = \pi/2$. When mobility is finite, the drift droops down the electrical potential energy gradient.

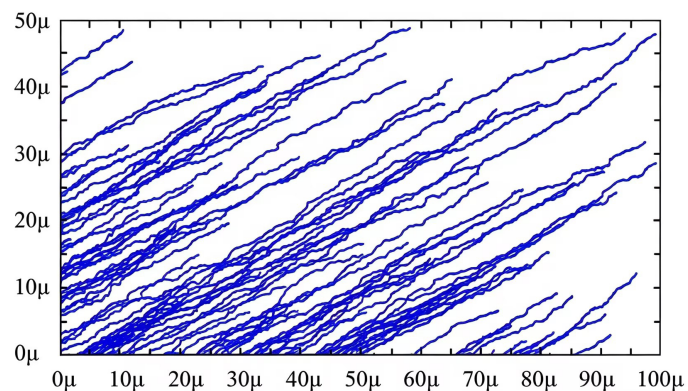


Figure 3. Simulation of particles in a $\mathbf{E} \times \mathbf{B}$ drift. The particles start out randomly distributed in a box and drift diagonally down and to the left with a drifting angle of 63.43 degrees from the vertical, as predicted for $T = 300\text{K}$, $E_y = 328\text{ V/mm}$ upward, $B_z = 1\text{ Tesla}$ “into the paper,” and $\mu = 2$. Number of particles = 100. The box is 100 microns $\times$ 50 microns.

The intermediate drift mode between μ -Scale and M -Scale is the most interesting because it accounts for the accumulation of space charges downstream of the drift caused by finite mobility and a load voltage. As illustrated in **Figure 4**, the Hall resistance and the Drude resistance in combination attenuate, and deflect the original E_y field from $\pi/2$ to $\theta_{E \times B}$ from the vertical, resulting in a new field vector E_{Diag} :

$$E_{Diag} = E_y \sin(\theta_{E \times B}) \quad (21)$$

The change in the E field alters the $E \times B$ drift velocity by the same angle and magnitude from $v_{E \times B} = E_y / B_z$ to:

$$v_{Diag} = v_{E \times B} \sin(\theta_{E \times B}) \quad (22)$$

The components of the velocity vector projected on the X and Y axes are:

$$v_x = v_{E \times B} \sin^2(\theta_{E \times B}) \quad (23)$$

and

$$v_y = v_{E \times B} \sin(\theta_{E \times B}) \cos(\theta_{E \times B}) \quad (24)$$

such that their ratio

$$\frac{v_x}{v_y} = \tan(\theta_{E \times B}) \quad (25)$$

The velocity components v_x and v_y are in proportion to the Hall and Drude resistivities, i.e., $v_x / v_y = \rho_{Hall} / \rho_{Drude}$ [10] [11].

Therefore, from Equation (13) and (25).

$$\frac{\rho_{Hall}}{\rho_{Drude}} = \frac{v_x}{v_y} = \tan(\theta_{E \times B}) \quad (26)$$

Combining this information with Equations (9)-(11), (13) (16), (17) and (26), Z_{asym} can be expressed in a single equation yields:

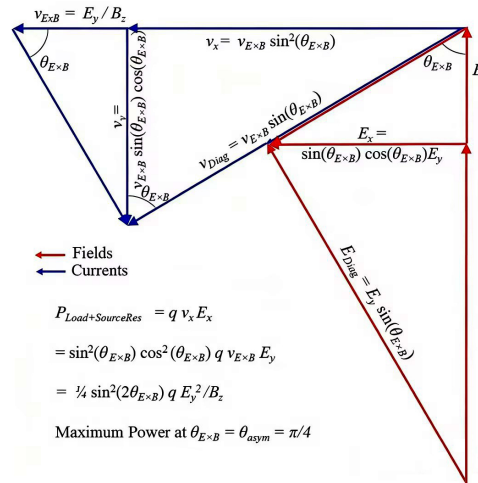
$$\begin{aligned} Z_{asym} &= \omega \tau = \frac{\lambda}{r} = \phi_\lambda = \frac{\rho_{Hall}}{\rho_{Drude}} = \frac{\rho_{Hall}}{\rho_{Source} + \rho_{Load}} \\ &= \frac{\mu_{Source} B}{1 + nq\mu_{Source}\rho_{Load}} = \frac{v_x}{v_y} = \tan(\theta_{asym}) = \tan(\theta_{E \times B}) \end{aligned} \quad (27)$$

thereby implying that the drift angle and the asymmetric index angle are identical.

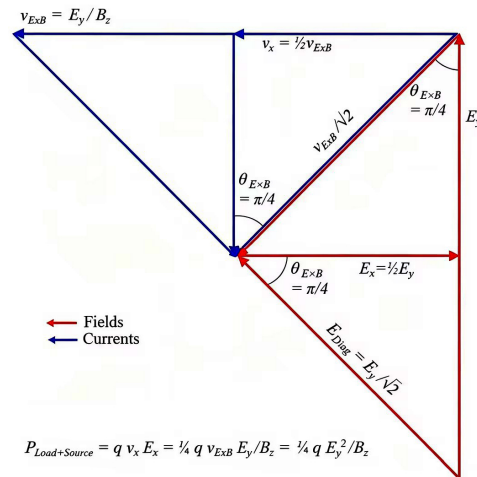
9. Optimal Use of Free Energy in Heat to Work Conversion

The asymmetric index provides a flexible tool for maximizing power production at the mesoscale—the region between the two extremes $\theta_{asym} = \pi/2$ (the μ -Scale) and $\theta_{asym} = 0$ (the M -Scale). Drifting particles accumulating downstream produce a voltage which deflects the original E_y field, but also drives them through the return channel which may contain a load (e.g., resistance or battery). As shown in **Figure 4**, a field component E_x along the X axis attenuates and deflects the main electrical field E_y , away from its original Y axis orientation by angle $\pi/2 - \theta_{E \times B} = \pi/2 - \theta_{asym}$.

The $\mathbf{E} \times \mathbf{B}$ transport in the presence of a Drude resistance (load and source) in **Figure 4** can be mapped onto the two stages of the thermodynamic cycle of **Section 5**. The field component E_x due to the formation of a space charge is accounted for by the deflection of the drift along a diagonal. In the simplest case the source resistance is zero.



(a)



(b)

Figure 4. Vector diagram of electric field and current for the $\mathbf{E} \times \mathbf{B}$ drift. (a) The drift is deflected by angle $\pi/2 - \theta_{E \times B} = \pi/2 - \theta_{asym}$ from its original Y axis orientation by charges accumulated downstream of the drift. The power output is the product $q v_x E_x$. (b) maximum utilization of free energy in converting heat to actual energy is at $\theta_{asym} = \pi/4$ with conversion efficiency of 0.25.

1. The free compression stage involves the diagonal drift of particles. In the absence of collisions (*i.e.*, $\mu_{Source} = \infty$, $\rho_{Source} = 0$), they exchange no heat and their temperature is unchanged ($\Delta Q = 0$, $\Delta T = 0$, as in **Figure 1**). Their potential energy remains unchanged as they move along an equipotential line. Furthermore, their change in position lowers their positional entropy causing them to accumulate in

the space-charge layer at the downstream electrode, where their concentration increases.

2. The isothermal expansion stage occurs in the electrodes and around the circuit. The expansion, powered by heat from the surroundings, drives the particles up the potential gradient along Y and through the electrodes and external circuit, delivers work to the load and returns them to the upstream end of the forward channel.

Optimality requires zero source resistance. The more complex case discussed below includes a non-zero source resistance, ρ_{Source} , incorporated in the index of asymmetry (*i.e.*, $Z_{asym} = \rho_{Hall}/\rho_{Drude}$ and $\rho_{Drude} = \rho_{Load} + \rho_{Source}$).

As per Equation (22) and since $J = nqv$ and $\theta_{E \times B} = \theta_{asym}$, this deflected drift current along the diagonal is:

$$J_{Diag} = nqv_{Diag} = nqv_{E \times B} \sin(\theta_{asym}) \quad (28)$$

J_{Diag} flows along an equipotential line. Its Y axis component $-J_{Diag}$ is due to the opposing voltage E_x caused by dissipation by the Drude resistance (in the source and the load). It causes the drift in the device to “dribble down” the E field potential energy gradient along the Y axis. **However, electrodes in the device are positioned along the X axis, restricting the flow to the X axis**, and causing electrons to expand isothermally upward and produce an upward compensating current, $+J_y$, against the opposing potential.

$$J_y = J_{Diag} \cos(\theta_{asym}) = nqv_{E \times B} \sin(\theta_{asym}) \cos(\theta_{asym}) \quad (29)$$

Electrons flow back to the X axis against **an opposite electrical field exactly equal to the load potential E_x** . In other words, **the potential energy they acquire from heat in their expansion is the same energy they transfer to the load.**

$$E_x = E_{Diag} \cos(\theta_{asym}) = E_y \sin(\theta_{asym}) \cos(\theta_{asym}) \quad (30)$$

Hence the power produced by the device is the product of Equation (29) and (30):

$$P_{Drude} = E_x J_y = nqv_{E \times B} E_y \sin^2(\theta_{asym}) \cos^2(\theta_{asym}) \quad (31)$$

Since $\rho_{Hall} = B_z/nq$, and $v_{E \times B} = E_y/B_z$, Equation (31) becomes,

$$P_{Drude} = \sin^2(\theta_{asym}) \cos^2(\theta_{asym}) \frac{E_y^2}{\rho_{Hall}} \quad (32)$$

Simplifying the trigonometric functions yields:

$$P_{Drude} = \frac{1}{4} \sin^2(2\theta_{asym}) \frac{E_y^2}{\rho_{Hall}} \quad (33)$$

From Equation (27), $Z_{asym} = \tan(\theta_{asym})$. Therefore Equation (32) can also be expressed as:

$$P_{Drude} = \frac{Z_{asym}^2}{(1 + Z_{asym}^2)^2} \frac{E_y^2}{\rho_{Hall}} \quad (34)$$

This power represents the power delivered to all Drude resistances around the loop including the resistances in the source and in the load. Assuming zero source resistance (*i.e.*, $\mu_{\text{Source}} = \infty$, $\rho_{\text{Source}} = 0$), then the load receives of full power:

$$P_{\text{Load}} = \frac{1}{4} \sin^2(2\theta_{\text{asym}}) \frac{E_y^2}{\rho_{\text{Hall}}} = \frac{Z_{\text{asym}}^2}{(1 + Z_{\text{asym}}^2)^2} \frac{E_y^2}{\rho_{\text{Hall}}} \quad (35)$$

As shown in **Figure 5**, equation describes the power that can be sent to a load by a system operating at any point along the *m-Scale* as defined by $0 < Z_{\text{asym}} < \infty$ or $0 < \theta_{\text{asym}} < \pi/2$.

9.1. Maximizing Thermal to Work Conversion Using Free Energy

Equation (35) can be maximized by differentiating and setting to zero. As illustrated in **Figure 4(b)** and **Figure 5**, maximum power is achieved at $\theta_{\text{asym}} = \pi/4$, exactly in the center of the *m-Scale*:

$$P_{\text{Load}} = \frac{1}{4} \frac{E_y^2}{\rho_{\text{Hall}}} \quad (36)$$

where the index of asymmetry is unity:

$$Z_{\text{asym}} = \tan(\theta_{\text{asym}}) = 1; \text{ and } \theta_{\text{asym}} = \frac{\pi}{4} \quad (37)$$

At that point, the Hall and Drude resistivities are also equal thereby confirming that the Maximum Power Transfer Theorem (MPTT) also applies to the use of free energy provided by a magnetic field.

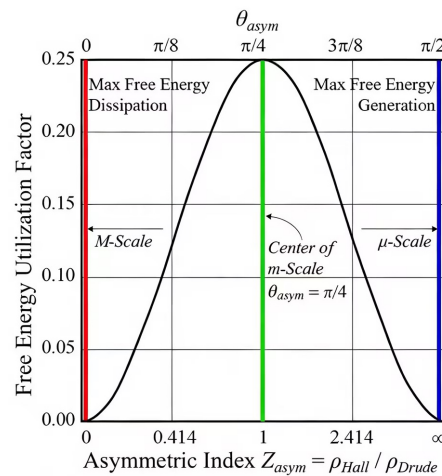


Figure 5. Free Energy Utilization Factor. The fraction of free energy converted to actual energy falls to zero at either end of the index of asymmetry as predicted by all proofs of the second law. It rises to a maximum of 1/4 at $\theta_{\text{asym}} = \pi/4$ and $Z_{\text{asym}} = 1$ as required by the MPTT.

9.2. Power Output Expressed from “Loaded” Free Energy Source

Unlike a battery with a **fixed chemical potential**, the free energy available from a device is not fixed: it is a function of the load. As shown in the vector diagrams of

Figure 4(a) and **Figure 4(b)**, in the presence of a load voltage, and under optimal condition $\theta_{asym} = \pi/4$, the primary electrical field is deflected or “loaded” from E_y to $|E_{Diag}| = |E_y|/\sqrt{2}$.

Equation (36) is expressed in terms of the “unloaded” field E_y . Replacing E_y by the “loaded” field version E_{Diag} from Equation (21) and setting $\theta_{asym} = \theta_{E \times B} = \pi/4$ results in:

$$P_{Load} = \frac{1}{2} \frac{E_{Diag}^2}{\rho_{Hall}} \quad (38)$$

This result is analogous to the one obtained in Part 2 of this paper [1] which discusses the rectification of the Johnson-Nyquist noise by a MIM diode. Furthermore, as discussed in Appendix I, this result is also identical to the one derived by Levy [2] using a Maxwell’s Demon “loaded” with the task of compressing a gas to run a turbine.

10. Extending Second Law

The Helmholtz free energy A provided by a T -Odd system is bandwidth-limited, yet inexhaustible. $E \times B$ fields configured for maximum power generation ($Z_{asym} = 1$), deliver this free energy in the form of Equation (36) when A is not loaded and Equation (38) when it is loaded.

$$P_{Load} = \frac{1}{4} \frac{E_y^2}{\rho_{Hall}} = \frac{1}{4} \frac{dA}{dt} = \frac{1}{2} \frac{E_{Diag}^2}{\rho_{Hall}} = \frac{1}{2} \frac{dA_{Loaded}}{dt} \quad (39)$$

Using this free energy in heat to work conversion must go through two hurdles as indicated by Equation (39):

1. Given an “unloaded” amount of free energy A made available by perpendicular E and B fields, only $1/2A$ can effectively be used because the direction and magnitude of the E field are altered by the presence of a load. (*i.e.*, $A_{Loaded} = 1/2A$).
2. The MPTT further reduces the transfer of electrical work to the load to $1/2A_{Loaded} = 1/4A$.

Therefore, the overall change in entropy produced by a thermodynamic cycle is bound by:

$$\Delta S \geq -\frac{A}{4T}; \quad \text{or} \quad \Delta S \geq -\frac{A_{Loaded}}{2T} \quad (40)$$

11. Conclusions

An axiomatic analysis on foremost arguments supporting the second law was performed to reveal the foundational logical cornerstone of the law. These arguments are of two kinds, depending on the time-reversal behavior of particles they consider. 1) Time-reversal symmetric systems are the least interesting from a second law violation viewpoint as they show zero change in entropy at the microscale, and positive change at the macroscale. 2) Time-reversal asymmetric systems, the most interesting ones, show a negative change in entropy at the microscale. All

arguments of this kind, without exception, eradicate this microscale negentropy by enforcing a massive increase in entropy in the transition from microscale to macroscale. Depending on the arguments, this transition is called *Stosszahlansatz*, coarse-graining, the ergodic hypothesis, hemispheric integration, etc., but invariably it involves a large-scale averaging and destruction of microscale information.

Underlying all the arguments is the hidden assumption that 1) at the microscale, negentropy is present but not accessible because of the lack of thermodynamic interaction (collisions), and 2) at the macroscale, it is accessible because of the abundance of interactions, but not present because it is destroyed by the overwhelming interactions. This catch-22 is a false dichotomy between the microscale and macroscale extremes. Why can't one operate at the mesoscale to reap some negentropy without insisting on having the "whole cake?" While the fabrication of a mesoscale device may have been impossible in the early foundational days of thermodynamics, such a device is now possible given modern nanoscale fabrication technology. A task, once determined to be impossible due to rigorously proven theorems, morphs into a well-defined engineering problem of finding the right materials and design.

An index of asymmetry is defined that quantifies a position on the mesoscale—the continuum between the micro and macro scale extremes.

A thermodynamic cycle using the $\mathbf{E} \times \mathbf{B}$ drift is described that produces negentropy. It includes two stages: 1) a "free compression" at the microscale, in a material with high mobility and low carrier concentration, and 2) an isothermal expansion at the macroscale in a material with low mobility and high concentration. A similar derivation for rectifiers is presented in the companion paper [1].

In compliance with all second law arguments, the energy output drops to zero at the two extremes of the mesoscale, 1) at the microscale where the index of asymmetry is infinite, and 2) at the macroscale, where it is zero. Maximum energy output is reached in the center of the mesoscale where the index of asymmetry is unity. At that point the energy output is equal to 1/4 of the free energy input. The second law extended to mesoscale states that the decrease in entropy of a time-reversal asymmetric system is limited. It cannot be bounded by $\Delta S \geq -1/4A/T$.

Data Availability Statement

Please contact the author for additional information.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

- [1] Levy, G.S. (2026) Catch-22 in Second Law, Part 2, Optimizing MIM Diode, Rectifying Johnson-Nyquist Noise. *Journal of Applied Mathematics and Physics*, **14**, 4013-4048. <https://doi.org/10.4236/jamp.2026.1410195>
- [2] Levy, G.S. (2026) The Second Law Extended to Time-Reversal Asymmetric Systems—Thermodynamic Threshold Optimization. *Journal of Applied Mathematics and Physics*, **14**, 961-994. <https://doi.org/10.4236/jamp.2026.142046>
- [3] Levy, G.S. (2023) An Overview of the ExB Thermoelectric Effect. *MRS Advances*, **8**, 113-118. <https://doi.org/10.1557/s43580-023-00521-5>
- [4] Levy, G.S. (2019) The ExB Thermoelectric Effect. ResearchGate. https://www.researchgate.net/publication/361680025_The_ExB_Thermoelectric_Effect
- [5] Chen, F.F. (2012) Diffusion and Resistivity. In: *Introduction to Plasma Physics and Controlled Fusion*, Springer.
- [6] Onsager, L. (1931) Reciprocal Relations in Irreversible Processes. I. *Physical Review*, **37**, 405-426. <https://doi.org/10.1103/physrev.37.405>
- [7] Casimir, H.B.G. (1945) On Onsager's Principle of Microscopic Reversibility. *Reviews of Modern Physics*, **17**, 343-350. <https://doi.org/10.1103/revmodphys.17.343>
- [8] Sze, S.M. and Ng, K.K. (2006) *Physics of Semiconductor Devices*. Wiley. <https://doi.org/10.1002/0470068329>
- [9] Ashcroft, N.W. and David Mermin, N. (1976) *Solid State Physics* (Chapter 1, Chapter 13). Saunders College Press.
- [10] Lieberman, M.A. and Lichtenberg, A.J. (2024) *Principles of Plasma Discharges and Materials Processing*. Wiley.
- [11] Levy, G.S. (2023) The ExB Thermoelectric Effect Bypasses the Second Law. ResearchGate. https://www.researchgate.net/publication/376554904_The_ExB_Thermoelectric_Effect_Bypasses_the_Second_Law

Appendix

Appendix A: Boltzmann's H-Theorem

The Boltzmann H -theorem (1872) [1] [2] represents the first attempt to derive the macroscopic thermodynamic arrow of time from the microscopic mechanics of individual gas molecules. By tracking a single-particle probability distribution function in a 6-dimensional phase space Boltzmann sought to prove that molecular collisions force a system to evolve irreversibly toward a maximum entropy state. The system consists of a dilute gas composed of hard-sphere classical particles enclosed within an isolated container. The state of the gas is defined by the continuous distribution function $f(\mathbf{r}, \mathbf{v}, t)$, which dictates the density of particles at position $\mathbf{r}$ with velocity $\mathbf{v}$ at time t . Kinetic behavior is governed by the Boltzmann transport equation, and the thermodynamic state is tracked via the functional $H(t) = \int f \ln f \, d\mathbf{r} d\mathbf{v}$, mathematically related to the negative of the Boltzmann entropy.

A1. Time Symmetry

The fundamental mechanics governing individual binary collisions are governed by Newton's equations of motion. In the absence of velocity-dependent fields, these **microscopic physical laws possess pure T -Even symmetry** requiring the equations of motion to be fully deterministic and path-reversible.

A2. Macroscopic Irreversibility

To force a monotonic decrease in the H -functional ($dH/dt \leq 0$) and establish macroscopic irreversibility ($dS/dt > 0$), Boltzmann introduced the Molecular Chaos Assumption (*Stosszahlansatz*). It dictates that the velocities of two colliding particles are completely uncorrelated prior to their collision, allowing their joint two-particle distribution function to be factored into a product of independent single-particle distributions:

$$f_2(\mathbf{r}, \mathbf{v}_1, \mathbf{v}_2, t) = f_1(\mathbf{r}, \mathbf{v}_1, t) \cdot f_1(\mathbf{r}, \mathbf{v}_2, t) \quad (41)$$

Crucially, the *Stosszahlansatz* mandates that these particles become immediately correlated after the collision, **leading to a loss of information**. This asymmetrical treatment implies an M -Scale perspective of the observer and introduces a directional arrow of time by fiat. In an exact physical system, post-collision trajectories retain complex, multi-particle correlations. However, the M -Scale observer does not track this microscale information which is permanently erased via *Stosszahlansatz*.

A3. Position on the m -Scale

The gas is T -Even, so the generative factor vanishes identically; independently, the *Stosszahlansatz* discards post-collision correlations at every step, setting the correlation memory to zero and enforcing $Z_{asym} \rightarrow 0$.

References

- [1] Boltzmann, L. (1872) Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen. *Sitzungsberichte Akademie der Wissenschaften*, **66**, 275-370.

- [2] Loschmidt, J. (1876) Über das Zustandekommen des Gleichgewichtes in einem Systeme von Körpern mit Rücksicht auf die Schwere. *Sitzungsberichte Akademie der Wissenschaften*, **73**, 128-142.

Appendix B: Landauer's Principle

Landauer's Principle (1961) [1] demands a thermodynamic cost for information processing: erasing one bit of information requires heat dissipation: $Q \geq k_B T \ln 2$. His system consists of a microscopic bistable memory containing a single particle. The "0" and "1" states correspond to the positioning of the particle in the left or right well. Logical erasure forces the particle into the "0" state regardless of its initial position.

B1. Microscopic Time Symmetry

The equations of motion for memory states are conservative and time-reversible (*T-Even*). Charles Bennett (1973) [2] proved that any algorithm can be executed using logically reversible operations (e.g., the Toffoli gate). Because phase-space volume is conserved under *T-Even* dynamics, an isolated microscopic computer can step forward and backward through a computation with zero dissipation ($Q = 0$). Reversible computation is a μ -Scale process that does not interact with the surroundings and conserves information.

B2. Macroscopic Irreversibility

Dissipation emerges during logical erasure which compresses two logical states into one and requires expelling excess phase-space volume into the surroundings. For Landauer's bound to hold as an inequality, the surroundings must be an ideal macroscopic reservoir acting as an *M-Scale* enforcer:

1. *Infinite information sink*. True erasure requires a bath with infinite degrees of freedom ($N \rightarrow \infty$) to instantly randomize and destroy expelled entropy. A finite bath that retains phase memory allows Poincaré recurrences which return the information to the register and undo the "erasure."

2. *Infinite bath bandwidth*. The bath is modeled as memoryless, delta-correlated noise that responds instantly at all frequencies. In the words of Section 7, the bath's correlation time is $\tau \rightarrow 0$. Hence, $Z_{asym} = \omega\tau \rightarrow 0$ for any finite frequency ω . *M-Scale* endpoint is not derived; it is installed by the noise model.

His framework therefore reproduces the catch-22 in its purest form: at μ -Scale, the register is reversible but thermodynamically isolated (nothing can be erased or harvested); at *M-Scale*, erasure is possible but only by paying $k_B T \ln 2$ per bit to a bath that destroys all microstructure. He describes the cost of erasure on the *T-Even* load side of a thermodynamic loop, but is silent about whether a *T-Odd* source side can pay that cost. In **Figure 1**, the *T-Odd* source of free compression lies entirely outside his universe of discourse.

B3. Position on the *m-Scale*

This proof covers a *T-Even* information system at *M-Scale*. It constrains the load, not the source. *m-Scale* coupling between register and bath is not considered. The delta-correlated bath enforces $Z_{asym} \rightarrow 0$.

References

- [1] Landauer, R. (1961) Irreversibility and Heat Generation in the Computing Process. *IBM Journal of Research and Development*, **5**, 183-191.

<https://doi.org/10.1147/rd.53.0183>

- [2] Bennett, C.H. (1973) Logical Reversibility of Computation. *IBM Journal of Research and Development*, **17**, 525-532. <https://doi.org/10.1147/rd.176.0525>

Appendix C: Gibb's H-Theorem (With a Magnetic Field)

J. Willard Gibbs (1902) [1]-[3] replaced Boltzmann's single-particle distribution in 3-dimensional space with his ensemble distribution of N particles in the phase space (Γ -space). His original formulation was based on the classical Liouville equation built on a time-reversible Hamiltonian. Liouville's theorem guarantees that the local density of any region of the phase-space fluid is perfectly conserved over time. Consequently, his fine-grained Gibbs entropy, S_{fine} is, in the terminology of this paper understood to be at μ -Scale and is dynamically invariant $dS_{fine}/dt = 0$. Given that the reversible Hamiltonian is T -Even, this result is hardly surprising.

C1. Microscopic Time Symmetry

This T -Even reversible Hamiltonian version was later generalized to a T -Odd version to cover magnetic field phenomena. Because the Lorentz force acts asymmetrically on velocity vectors, the exact trajectories are no longer path-reversible under a simple time inversion in a T -Odd field. However, *they can still reverse to a starting position*, there is no information loss and phase-space flow remains incompressible: ($dS_{fine}/dt = 0$). At μ -Scale (fine scale) no information is lost due to collisions or coarse-graining.

C2. Macroscopic Irreversibility

To enforce the second law, Gibbs shifts to the macroscale by introducing the concept of coarse-graining: he divides the continuous Γ -space into finite, macroscopic operational volumes or "cells" over which he averages out detailed path information thereby creating a one-way mathematical inflation of the observed volume ($dS_{coarse}/dt \geq 0$). He justifies it by acknowledging that a macroscopic observer possesses finite measurement resolution and is incapable of resolving the exact, fine-grained boundaries of the phase fluid.

C3. Position on the m -Scale

The enforcement of M -Scale lies in the observation operator, not in the dynamics; m -Scale observation, with resolution comparable to the orbit, is not considered. Fine-grained dynamics are T -Odd and preserve information, but coarse-graining cells exceed the cyclotron orbit, so the orbital radius r is unresolved and the observed index collapses to $Z_{asym} \rightarrow 0$.

References

- [1] Gibbs, J.W. (1902) Elementary Principles in Statistical Mechanics. Charles Scribner's Sons.
- [2] Zwanzig, R. (1960) Ensemble Method in the Theory of Irreversibility. *The Journal of Chemical Physics*, **33**, 1338-1341. <https://doi.org/10.1063/1.1731409>
- [3] Penrose, O. (1979) Foundations of Statistical Mechanics. *Reports on Progress in Physics*, **42**, 1937-2006. <https://doi.org/10.1088/0034-4885/42/12/002>

Appendix D: Von-Neumann's Quantum H-Theorem

John von Neumann's quantum H -Theorem applies the methods of statistical mechanics to quantum systems: An isolated macroscopic quantum system described in a high-dimensional Hilbert space inevitably evolves toward a state of maximum entropy. The system state is tracked via the density operator $\hat{\rho}(t)$. Its fine-grained entropy is evaluated using the von Neumann entropy functional:

$$S_{vN} = -k_B \text{Tr}[\hat{\rho} \ln \hat{\rho}].$$

D1. Microscopic Time Symmetry

He uses Schrödinger equation to describe the time evolution of the fine-grained density, translating as a unitary operator transformation [1] [2]:

$$\hat{\rho}_{\text{fine}}(t) = \hat{U}(t) \hat{\rho}_{\text{fine}}(0) \hat{U}^\dagger(t) \quad (42)$$

Since the operator $\hat{U}(t) = e^{-i\hat{H}t/\hbar}$ is unitary, the fine-grained entropy remains invariant over time ($dS_{vN}/dt = 0$). No information is lost at μ -Scale. Unitarity is a statement about information conservation, not about time-reversal parity and applies to T -Even and T -Odd symmetric quantum systems, including charged particles in a magnetic field. It is the quantum counterpart of Gibbs' H-Theorem of Appendix C, which reaches the same conclusion classically through Liouville's theorem and coarse-graining.

D2. Macroscopic Irreversibility

To break this unitary invariance and derive macroscopic irreversibility ($dS/dt > 0$), von Neumann partitions the total Hilbert space into macroscopic operational cells. He defines a coarse-grained quantum observer who can only measure **Macro-Observables**. These operators are blind to microstate transitions within a given cell. The coarse-grained density matrix is obtained by averaging the true state over these macro-cells, yielding the macroscopic coarse-grained quantum entropy. To prove that this coarse-grained entropy increases, he invokes the **Ergodic Hypothesis** and the **Random Phase Approximation** under which the fast-oscillating phase correlations carried by the off-diagonal elements of $\hat{\rho}$ interfere destructively and average to zero over macroscopic timescales. The Random Phase Approximation does not merely blur the microstate: it discards coherence memory. The fine-grained dynamics remain reversible and information-preserving throughout; irreversibility appears only because the observer is stipulated at the macroscale.

D3. Position on the m -Scale

m -Scale operation, in which coherence persists over times comparable to the observation interval, is not considered. The Random Phase Approximation and the ergodic hypothesis destroy the phase coherence memory over macro-observable cells, setting the correlation time to zero and enforcing $Z_{\text{asym}} \rightarrow 0$.

References

- [1] von Neumann, J. (1929) Beweis des Ergodensatzes und des H-Theorems in der neuen Mechanik. *Zeitschrift für Physik*, **57**, 30-70. <https://doi.org/10.1007/bf01339852>

- [2] Goldstein, S., Lebowitz, J.L., Mastrodonato, C., Tumulka, R. and Zanghì, N. (2010) Normal Typicality and Von Neumann's Quantum Ergodic Theorem. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, **466**, 3203-3224. <https://doi.org/10.1098/rspa.2009.0635>

Appendix E: Detailed Balance

Simple Detailed Balance dictates that under equilibrium, every transition between discrete states is as probable in a forward or reverse direction.

E1. Microscopic Time Symmetry

This principle relied upon by Boltzmann and Planck, and later refined by Albert Einstein (1916) [1]-[4] in his derivation of A and B coefficients for radiation, assumes microscopic reversibility (T -Even) and only applies to systems free from velocity-dependent fields. Generalized Detailed Balance by Hendrik Casimir (1945) and Nico van Kampen (1954), extends this principle to T -Odd systems to include magnetic fields.

E2. Macroscopic Irreversibility

Casimir introduces the concept of variable parity, classifying state variables into α -type (T -Even) and β -type (T -Odd). When Onsager's transport symmetry ($L_{ij} = L_{ji}$) is broken by a magnetic field, it must be replaced by the Onsager-Casimir Reciprocal Relations ($L_{ij}(\mathbf{B}) = \varepsilon_i \varepsilon_j L_{ji}(-\mathbf{B})$) where ε represents the time-reversal parity (± 1). This approach codifies Generalized Detailed Balance at the M -Scale but **does not explain how μ -Scale $\wedge$ T -Odd affects M -Scale behavior.**

Van Kampen resolves this paradox by showing that the transition from μ -Scale to M -Scale acts as a loop-canceling filter that averages out individual paths. In a fixed magnetic field, the transition probability matrix naturally forms **closed, invariant loops of probability flux**. Within these microscale loops, traffic is strictly one-way ($P(i \rightarrow j) \neq P(j \rightarrow i)$). However, because the loops are closed, the total probability entering any macroscopic region perfectly equals the total probability leaving it. **Integrating micro-loops cancels them out, forcing the net macroscopic flux crossing any arbitrary boundary to be precisely zero.**

E3. Position on the m -Scale

m -Scale operation, in which a path is interrupted before its loop closes, is not considered. van Kampen's loop cancellation integrates the mean-free angle ϕ_1 over closed orbits, so that one-way microscale traffic sums to zero net macroscopic flux and **$Z_{asym} \rightarrow 0$** in its angular form.

References

- [1] Einstein, A. (1916) Strahlungs-emission und -absorption nach der Quantentheorie. *Verhandlungen der Deutschen Physikalischen Gesellschaft*, **18**, 318-323.
- [2] Onsager, L. (1931) Reciprocal Relations in Irreversible Processes. I. *Physical Review*, **37**, 405-426. <https://doi.org/10.1103/physrev.37.405>
- [3] Casimir, H.B.G. (1945) On Onsager's Principle of Microscopic Reversibility. *Reviews of Modern Physics*, **17**, 343-350. <https://doi.org/10.1103/revmodphys.17.343>
- [4] Van Kampen, N.G. (1954) Quantum Statistics of Irreversible Processes. *Physica*, **20**, 603-622. [https://doi.org/10.1016/s0031-8914\(54\)80074-7](https://doi.org/10.1016/s0031-8914(54)80074-7)

Appendix F: Fluctuation Theorem (FT) and Stochastic Thermodynamics with Magnetic Field

The Fluctuation Theorem [1]-[3] formulated by Evans, Cohen, and Morriss (1993), states that in a system away from equilibrium, with a time-averaged entropy production $\bar{\Sigma}_t$ characterized by a distribution $\Pr(\bar{\Sigma}_t)$, the ratio between the probability $P(\bar{\Sigma}_t = +A)$ and $P(\bar{\Sigma}_t = -A)$ is exponential in At .

$$\frac{P(\bar{\Sigma}_t = +A)}{P(\bar{\Sigma}_t = -A)} = e^{A/k_B} \quad (43)$$

The probability of observing negentropy production decreases exponentially with time and size. This theorem yields a second law violation in μ -Scale but yields the second law $\langle \bar{\Sigma}_t \rangle \geq 0$ in the M -Scale limit.

F1. Microscopic Time Symmetry

Earliest forms of the theorem [1] [2] address *T-Even* physics, later ones [3] *T-Odd* phenomena.

F2. Macroscopic Irreversibility

T-Odd versions of the theorem rely on three Stochastic Thermodynamics assumptions:

1. *The Overdamped Continuum Limit*: simplifies the tracking of random particle paths, by throwing out from equations the velocity/momentum terms that define velocity-dependent forces. (i.e., shifting from underdamped *Langevin equations* to overdamped ones). **This process converts *T-Odd* into *T-Even*.**

2. *Infinite Bath Bandwidth*: the heat bath is modeled as a Gaussian white noise, defined by a Dirac delta-function correlation. It responds instantly across all frequencies, leaving no memory of previous collisions behind, thereby overwhelming μ -Scale effects and **acting as a *M-Scale* ($N \rightarrow \infty$) enforcer.**

3. *Local Detailed Balance*: states that the ratio of **local** forward to backward transition rates is strictly tied to the **macroscopic** entropy change of the environment:

$$\frac{k(x \rightarrow y)}{k(y \rightarrow x)} = e^{\Delta S_{env}/k_B} \quad (44)$$

An environment **in equilibrium despite micro transitions negates μ -Scale, implying circular logic.** FT uses LDB to “prove” the Second Law by assuming an environment that follows the *M-Scale* law.

F3. Position on the *m-Scale*

Three operations enforce *M-Scale*: 1) the overdamped limit, 2) Infinite Bath Bandwidth 3) Local Detailed Balance. *m-Scale* operation is not considered. Hence the asymmetric index $Z_{asym} \rightarrow 0$.

References

- [1] Evans, D.J., Cohen, E.G.D. and Morriss, G.P. (1993) Probability of Second Law Violations in Shearing Steady States. *Physical Review Letters*, **71**, 2401-2404.
<https://doi.org/10.1103/physrevlett.71.2401>

- [2] Crooks, G.E. (1999) Entropy Production Fluctuation Theorem and the Nonequilibrium Work Relation for Free Energy Differences. *Physical Review E*, **60**, 2721-2726.
<https://doi.org/10.1103/physreve.60.2721>
- [3] Seifert, U. (2012) Stochastic Thermodynamics, Fluctuation Theorems and Molecular Machines. *Reports on Progress in Physics*, **75**, Article 126001.
<https://doi.org/10.1088/0034-4885/75/12/126001>

Appendix G: Fluctuation Dissipation Theorem (FDT)

The Fluctuation-Dissipation Theorem (FDT) [1]-[4] by Callen and Welton, links the linear response of a system under external perturbations to its spontaneous thermal fluctuations at equilibrium. Its modern classical and quantum versions by Ryogo Kubo may include a magnetic field. The FDT derived from the canonical ensemble (Boltzmann distribution) requires **strict thermal equilibrium**. The correlation of fluctuations is locked to its linear response with magnetic interactions accounted for along the ij axes.

$$\chi_{ij}(t, \mathbf{B}) = \frac{1}{k_B T} \frac{dC_{ij}(t, \mathbf{B})}{dt} \quad (45)$$

G1. Microscopic Time Symmetry

The microscopic dynamics are governed by a well-defined Hamiltonian which accounts for the T -Odd dynamics due to a fixed magnetic field.

G2. Macroscopic Irreversibility

The system must be in a **stationary, canonical Gibbs Equilibrium**. This requirement erases T -Odd information by moving operation to M -Scale. Under the Gibbs canonical ensemble, a system's probability distribution is dictated entirely by its Hamiltonian (total energy):

$$P(\text{state}) \propto e^{-\frac{H}{k_B T}} \quad (46)$$

Since the magnetic force is always perpendicular to velocity, the field **does no work** and **adds zero energy** to the system. Hence the magnetic field fully drops out of the exponential part of the Gibbs distribution.

Consequently, the probability of finding particles at specific locations and speeds is perfectly symmetric. Averaged across the entire ensemble, the microscopic curves completely smooth out. **Gibbs's equilibrium forces the system to look statically balanced, effectively erasing the directional bias of the magnetic field**. This averaging is a form of coarse-graining enforced by equilibrium.

Saito and Utsumi derive fluctuation relations for T -Odd systems without the overdamped limit; they rely on [initial canonical states / the $\pm B$ comparison], and, therefore, occupy the same M -Scale endpoint as Appendices G and E respectively — $Z_{\text{asym}} \rightarrow 0$ via the equilibrium premise, not via damping.

G3. Position on the m -Scale

m -Scale operation away from strict canonical equilibrium is not considered. Canonical equilibrium removes the magnetic field from Gibbs distribution, cancels the generative factor and enforces $Z_{\text{asym}} \rightarrow 0$.

References

- [1] Callen, H.B. and Welton, T.A. (1951) Irreversibility and Generalized Noise. *Physical Review*, **83**, 34-40. <https://doi.org/10.1103/physrev.83.34>
- [2] Kubo, R. (1957) Statistical-Mechanical Theory of Irreversible Processes. I. General Theory and Simple Applications to Magnetic and Conduction Problems. *Journal of the Physical Society of Japan*, **12**, 570-586. <https://doi.org/10.1143/jpsj.12.570>

- [3] Kubo, R. (1966) The Fluctuation-Dissipation Theorem. *Reports on Progress in Physics*, **29**, 255-284. <https://doi.org/10.1088/0034-4885/29/1/306>
- [4] Wang, C. and Feldman, D.E. (2014) Fluctuation Theorems without Time-Reversal Symmetry. *International Journal of Modern Physics B*, **28**, Article 1430003. <https://doi.org/10.1142/s0217979214300035>

Appendix H: Kirchoff's Radiation Law

Kirchhoff's Law [1] of Thermal Radiation (1860) states that the directional emissivity (ε) and absorptivity (α) of a body in thermodynamic equilibrium must match exactly at any given frequency. This law is rooted in **Lorentz reciprocity**: forward and backward propagation must be identical. While this law holds in the absence (*T-Even*) of a magnetic field, experiments by Guo, Zhao and Fan [2] show that a field (*T-Odd*) applied to magneto-optical materials breaks this law by differently skewing emissivity and absorptivity:

$$\varepsilon(\lambda, \theta, \phi) \neq \alpha(\lambda, \theta, \phi) \quad (47)$$

H1. Microscopic Time Symmetry

The magnetic field introduces a *T-Odd* symmetry to the magneto-optical material:

- The magnetic field forces the material's permittivity tensor to become anisotropic and complex.
- This forces the internal electron dynamics to spin asymmetrically.
- At narrow angles, incoming photons travel with the electron spin, outgoing photons against it, which decouples the absorption and emission channels, allowing the difference between ε and α .

H2. Macroscopic Irreversibility

Asymmetric radiation by non-reciprocal material can cause a temperature difference ΔT that can drive a heat engine and break the second law. To restore temperature uniformity, researchers [2] rely on **global hemispherical integration** which integrates anomalous radiative effects over a whole hemisphere.

- This integration is merely a **first law energy balance sheet**. It tallies the total incoming and outgoing energy over all angles to show that energy is **hemispherically conserved**.
- It operates as a global eraser, averaging energy over a **hemisphere** by masking distinct, localized anomalous effects arising at narrow angles thereby preserving the first law.

H3. Position on the *m-Scale*

Hemispherical integration confirms hemispherical energy conservation—a first law statement—while remaining silent on angle-resolved second law behavior at *m-Scale*. The integration averages emissivity and absorptivity over the full hemisphere, erasing the directional asymmetry and enforcing $Z_{asym} \rightarrow 0$.

References

- [1] Kirchhoff, G. (1860) Über das Verhältniss zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme und Licht. *Annalen der Physik*, **185**, 275-301. <https://doi.org/10.1002/andp.18601850205>
- [2] Guo, C., Zhao, B. and Fan, S. (2022) Adjoint Kirchhoff's Law and General Symmetry Implications for All Thermal Emitters. *Physical Review X*, **12**, Article 021023. <https://doi.org/10.1103/physrevx.12.021023>
- [3] Zhang, Z., Dehaghi, A.K., Ghosh, P. and Zhu, L. (2025) Observation of Strong Non-

reciprocal Thermal Emission. *Physical Review Letters*, **135**, Article 016901.

<https://doi.org/10.1103/physrevlett.135.016901>.

- [4] Shayegan, K.J., Hwang, J.S., Zhao, B., Raman, A.P. and Atwater, H.A. (2024) Broad-band Nonreciprocal Thermal Emissivity and Absorptivity. *Light: Science & Applications*, **13**, Article No. 176. <https://doi.org/10.1038/s41377-024-01520-3>
- [5] Zhu, L. and Fan, S. (2014) Near-Field Nonreciprocal Thermal Radiation. *Physical Review B*, **90**, Article ID: 220301. <https://doi.org/10.1103/PhysRevB.90.220301>

Appendix I: Maxwell's Demon Strawman Argument

Maxwell's Demon [1] represents the archetype strawman attack on the second law. A chamber holding a gas is divided by a partition containing a gate. In the simpler Leo Szilard's version, the demon produces a *pressure difference* by only opening the gate to particles moving from *A* to *B* regardless of their speed.

I1. Microscopic Time Symmetry

In this thought experiment both the demon and the gas are implicitly assumed to be *T-Even* symmetric.

I2. Macroscopic Irreversibility

Charles Bennett [1] applies Landauer's framework to show how the whole experiment, including the demon, complies with the law. Information collected by the demon on particles must be stored and erased and is subjected to Landauer's bound. Erasing a bit dissipates at least $k_B T \ln 2$, and at best, the demon can only achieve $\Delta S = 0$, an unsurprising result given that the demon and the gas are both *T-Even* symmetric.

I3. Rehabilitating the Demon

This thought experiment can be redeemed using a *T-Odd* gas or a *T-Odd* demon:

1. ***T-Odd* gas:** The $\mathbf{E} \times \mathbf{B}$ drift serves as a good example for *T-Odd* symmetry in a gas. Charged gas particles subjected to $\mathbf{E}$ and $\mathbf{B}$ fields perpendicular to each other, follow cycloidal orbits, drifting with velocity $\mathbf{v}$ in the cross-product direction of the fields [2] [3].

$$\mathbf{v}_{\mathbf{E} \times \mathbf{B}} = \frac{\mathbf{E} \times \mathbf{B}}{|\mathbf{B}|^2} \quad (48)$$

This setting has no need to track particles' positions and velocities and open the gate. Left idle, the demon is freed from Landauer's limit. The $\mathbf{E} \times \mathbf{B}$ drift yields the $\mathbf{E} \times \mathbf{B}$ Thermoelectric Effect [2].

2. ***T-Odd* Demon:** A *T-Odd* demon has a natural supply of negentropy. This thought experiment discussed by Levy [3] shows that only half of the free energy $G = A_{\text{Loaded}}$ available to a "loaded" demon (See Equation (40)), can be used to lower entropy. This statement effectively extends the second law to:

$$\Delta S \geq -\frac{A_{\text{Loaded}}}{2T} \quad (49)$$

I4. Position on the *m-Scale*

The demon and gas are both stipulated *T-Even*, so its index of asymmetry is $Z_{\text{asym}} = 0$ by construction. A demon, rehabilitated with a *T-Odd* gas or source, operates at $Z_{\text{asym}} \approx 1$.

References

- [1] Bennett, C.H. (1982) The Thermodynamics of Computation—A Review. *International Journal of Theoretical Physics*, **21**, 905-940.
<https://doi.org/10.1007/bf02084158>

- [2] Levy, G.S. (2023) An Overview of the ExB Thermoelectric Effect. *MRS Advances*, **8**, 113-118. <https://doi.org/10.1557/s43580-023-00521-5>
- [3] Levy, G.S. (2026) The Second Law Extended to Time-Reversal Asymmetric Systems—Thermodynamic Threshold Optimization. *Journal of Applied Mathematics and Physics*, **14**, 961-994. <https://doi.org/10.4236/jamp.2026.142046>

Appendix J: Feynman's Ratchet—Strawman's Argument

Feynman proposes the Ratchet and Pawl thought experiment [1] to prove that a spatially asymmetric device cannot rectify thermal noise into useful macroscopic work. A tiny paddle wheel axially joined to a saw-toothed ratchet wheel is enclosed in an isothermal gas. A spring-loaded pawl pinches asymmetrically indented ratchet teeth to allow rotation forward but not backward. Feynman shows that if the entire device is isothermal, forward and reverse rotations are equally likely, and **no useful work is produced**.

J1. Microscopic Time Symmetry

This result is predictable given that *spatial asymmetry* and *T-Odd symmetry* are different. Comparing the ratchet to a PN diode is instructive. Just like the ratchet, the diode is fabricated as spatially asymmetric. Its *T-Odd* symmetry emerges when a bias alters its shape. The thickness of its depletion zone changes with forward or reverse bias. It has a state variable linked to a sensitivity (built-in potential) and a bandwidth that restricts its operation. In contrast, the ratchet is spatially fixed and instantaneous, causing each particle to face the same geometry. Equations of particle motion are *T-Even*. Hence the ratchet is not *T-Odd*.

J2. Macroscopic Irreversibility

Given that the ratchet is *T-Even*, it can, at best, achieve $dS/dt = 0$.

J3. Redeeming the Ratchet

The ratchet can be rehabilitated by endowing it with a *T-Odd* symmetry having an amplitude sensitivity and a frequency bandwidth matched to the input noise. If its response is too slow, it averages out over many collisions to an invariant form, operates in *M-Scale*, and produces no useful work. Conversely, if the response time is too fast, it resets itself between interactions generating no useful work. As shown in **Figure 5**, optimality lies at the mid-point in the mesoscale where the index of asymmetry is unity. Experiments on various kinds of rectifiers by Kondo, Nikulov and Gurtovoi, Sheehan, and Lee, which may have achieved this goal are discussed in Appendix L.

J4. Position on the *m-Scale*

The ratchet's spatial profile is time-invariant, so its asymmetry has no response rate and $Z_{asym} \rightarrow 0$ — the same endpoint reached by *Stosszahlansatz* in Appendix A. A ratchet endowed with a state variable, a transition energy, and a bandwidth matched to the noise operates at $Z_{asym} \approx 1$.

References

- [1] Feynman, R.P., Leighton, R.B., Sands, M. and Treiman, S.B. (1963) The Feynman Lectures on Physics. *Physics Today*, **17**, 45-46. <https://doi.org/10.1063/1.3051743>

Appendix K: Experimental Evidence in Support of Second Law

The experimental record supporting the second law is vast, spanning two centuries from Joule's calorimetry to modern single-molecule thermodynamics. The purpose of this appendix is not to dispute any of these results—all are correct—but to locate them on the *m-Scale* using the index of asymmetry, and to show that this enormous body of evidence samples only the two endpoints of the scale where the theory of this paper *also* predicts $\Delta S \geq 0$. The evidence is therefore consistent with the conventional second law and with its extension in Section 10 alike; it cannot discriminate between them.

K1. Confirming Experiments at M-Scale ($Z_{asym} \rightarrow 0$)

Foundational experiments behind the Clausius and Kelvin-Planck statements were conducted on macroscopic working substances—water, gases, engines—with no *T-Odd* process present. Joule's paddle-wheel experiment (1845) [1] converted work to heat in water with no applied magnetic field: the *T-Odd* factor in Z_{asym} is absent, and the index is not merely small but identically zero. The same holds for the calorimetric experiments underlying classical thermochemistry and for every heat-engine efficiency measurement in the Carnot tradition. None of these experiments were conducted at *m-Scale*.

The modern precision tests deserve particular attention because they are conducted at the nanoscale and are sometimes described as testing the second law “at its limits.” Wang *et al.* (2002) [2] observed transient negative entropy production in a colloidal bead dragged through water, confirming the Evans-Searles fluctuation theorem. Toyabe *et al.* (2010) [3] demonstrated information-to-energy conversion by a feedback-controlled Brownian particle, and Bérut *et al.* (2012) [4] verified Landauer's bound $k_B T \ln 2$ for the erasure of a single bit in an optically trapped colloid. All three are landmark experiments, and all three employ **overdamped** colloidal particles in water with no velocity-dependent field. In the language of this paper, their dynamics are *T-Even* by construction: the generative numerator of Z_{asym} is zero, exactly as in Joule's water. **Small physical size does not imply *m-Scale* operation**; a nanoscale *T-Even* system sits at the same endpoint of the index as a steam engine. These experiments confirm—with beautiful precision—the behavior of the $Z_{asym} = 0$ endpoint, including the transient negentropy fluctuations that the fluctuation theorem itself predicts and then averages away.

K2. Confirming Experiments at μ -Scale (reversibility)

A smaller body of experiments probes the opposite endpoint by isolating a system from thermodynamic interaction and demonstrating that its evolution is reversible or unitary. The Hahn spin echo (1950) [5] and its descendants (Loschmidt echoes, time-reversal mirrors in NMR) show that an apparently dephased ensemble of spins retains its microscale information and can be re-focused by inverting the dynamics: $\Delta S_{fine} = 0$. These experiments confirm the μ -Scale endpoint: information is preserved precisely because collisions and thermodynamic couplings are suppressed for the duration of the experiment. Negentropy, in the sense of this

paper, is present in the retained correlations but in the absence of thermodynamic interaction it is not exported—there is no connection to a load, and nothing is harvested.

K3. The Sampling Gap

The record can now be stated compactly. Experiments at $Z_{\text{asym}} = 0$ uniformly find $\Delta S \geq 0$; experiments at $Z_{\text{asym}} = \infty$ find $\Delta S = 0$; and both findings are predicted equally by the conventional second law and by the extended law of Section 9, whose utilization curve (Figure 5) vanishes at both endpoints. No confirming experiment has been conducted on a *T-Odd* system in the interior of the *m-Scale*, near $Z_{\text{asym}} \approx 1$, where the two laws diverge most significantly and the extended law predicts $\Delta S \geq -A/4T$.

Logical induction can be a trap. An accumulation of evidence, however large, is not fully persuasive when every observation is drawn from the same region of parameter space. Bertrand Russell's chicken [6], fed each morning, accumulates hundreds of confirmations of the farmer's benevolence—all collected under identical conditions—until the morning the conditions change, and the farmer wrings its neck. The volume of second-law confirmations is not in question; their coverage is. The experiments that matter for discriminating between the conventional and extended laws are precisely the rare ones conducted on *T-Odd* systems between the endpoints of *m-Scale*, and these are the subject of Appendix L.

K4. Position on the *m-Scale*

All experimental evidence supporting the second law was collected at the two extremes of the *m-Scale*:

1. at *M-Scale* where $Z_{\text{asym}} \rightarrow 0$, *T-Even* dynamics and overwhelming dissipation guarantee $\Delta S \geq 0$.
2. at μ -*Scale* where $Z_{\text{asym}} = \infty$ —the time-reversible limit—where isolation guarantees $\Delta S = 0$.

No experiment was performed on a *T-Odd* system at *m-Scale* where $Z_{\text{asym}} \approx 1$.

References

- [1] Joule, J.P. (1845) On the Existence of an Equivalent Relation between Heat and the Ordinary Forms of Mechanical Power. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **27**, 205-207.
<https://doi.org/10.1080/14786444508645256>
- [2] Wang, G.M., Sevick, E.M., Mittag, E., Searles, D.J. and Evans, D.J. (2002) Experimental Demonstration of Violations of the Second Law of Thermodynamics for Small Systems and Short Time Scales. *Physical Review Letters*, **89**, Article 050601.
<https://doi.org/10.1103/physrevlett.89.050601>
- [3] Toyabe, S., Sagawa, T., Ueda, M., Muneyuki, E. and Sano, M. (2010) Experimental Demonstration of Information-to-Energy Conversion and Validation of the Generalized Jarzynski Equality. *Nature Physics*, **6**, 988-992.
<https://doi.org/10.1038/nphys1821>
- [4] Bérut, A., Arakelyan, A., Petrosyan, A., Ciliberto, S., Dillenschneider, R. and Lutz, E. (2012) Experimental Verification of Landauer's Principle Linking Information and

Thermodynamics. *Nature*, **483**, 187-189. <https://doi.org/10.1038/nature10872>

- [5] Hahn, E.L. (1950) Spin Echoes. *Physical Review*, **80**, 580-594.
<https://doi.org/10.1103/physrev.80.580>
- [6] Russell, B. (1912) *The Problems of Philosophy*, Chapter VI: On Induction. Williams & Norgate.

Appendix L: Experimental Evidence against the Second Law

Appendix K showed that all confirming experiments cluster at the endpoints of the *m-Scale*, where the conventional and extended laws agree. This appendix achieves two goals:

1. Collects the rarer experiments reporting isothermal heat-to-work conversion with no imposed temperature gradient. They cover a wide range of physical phenomena: organic, chemical, biological, superconducting.
2. Provides a unifying explanation for these disparate systems: they all share the single underlying structure as described in Section 4.

L1. The Common Mechanism of Rectifiers: Fluctuation-Modulated Asymmetry

Section 4.2 distinguishes spatial asymmetry from *T-Odd* symmetry. A structure that is merely spatially asymmetric and temporally fixed presents the same geometry to every fluctuation; its equations of particle motion remain *T-Even*. As proven by the H-Theorem, such a structure cannot rectify thermal noise. This is why Feynman's ratchet fails, and why its failure is not the general result it is usually taken to be. **To sidestep the theorem, a device must be *T-Odd* symmetric.**

A rectifier constructed as a spatially symmetric device, becomes *T-Odd* symmetric when its spatial configuration is modulated by the fluctuations it acts upon. In a diode, the depletion-zone thickness and conductivity change with the sign of the bias; the structure the carriers encounter on the forward half-cycle is not the structure they encounter on the reverse half-cycle. This change is modulated by the noise itself and is not instantaneous: it is limited by the bandwidth of a state variable of the device. Three properties follow, and together they constitute an explanation for current results and a predictive test for future experiments on rectifiers:

1. **A state variable** whose configuration changes with the sign of the fluctuations (depletion thickness, interfacial charge state, surface coverage, gate conformation, superconducting phase).
2. **A transition energy**—the fluctuation amplitude required to move the state variable (built-in potential, activation energy, condensation energy).
3. **A bandwidth**—a finite response rate over which the state variable follows the fluctuations.

The third property is what places such a device on the *m-Scale*. The bandwidth information is embedded in the conjugate factors—the generative *T-Odd* factor and the dissipative *T-Even* factor—that form the dimensionless index of asymmetry (*i.e.*, $\omega\tau$). A response too slow relative to the noise correlation rate averages the fluctuations to an invariant profile (*M-Scale*, $Z_{asym} \rightarrow 0$, the ratchet's fate), while a response too fast resets between successive fluctuations and rectifies nothing (μ -*Scale*). Useful output requires the response to be commensurate with the thermal rate— $Z_{asym} \approx 1$, the center of the *m-Scale*, exactly the Goldilocks condition of Section 6.

This mechanism sidesteps the H-theorem rather than contradicting it. The

Stosszahlansatz assumes a fixed scattering geometry, so that post-collision correlations may be discarded and the same statistical premise reapplied at the next collision. When the configuration itself is set by the preceding fluctuation, correlation is carried forward through the state variable; in such a device the decorrelation step no longer holds. Appendix A's H-Theorem result is still valid for the system it describes but does not extend to this one. In the electrical case, the fluctuations being rectified are the Johnson-Nyquist voltage fluctuations of the conductor, and the device is a diode. In the general case, the fluctuations are those of whichever variable is conjugate to the device's state variable—concentration and chemical potential for a membrane, configuration coordinates for a molecular gate. The electrical case is developed quantitatively in a companion paper [1]; the entries below are grouped by how directly they realize it.

L2. Electrical Rectification of Electrical Thermal Noise

Kondo et al. [1] demonstrate an organic device producing 384 mV open-circuit, 1.1 $\mu\text{A}/\text{cm}^2$ short-circuit current density, and 94 nW/cm² maximum output at room temperature with no applied temperature gradient. This is the most direct realization of the mechanism above: the charge-transfer interface supplies the state variable, its energetics supply the transition energy, and its transfer rate supplies the bandwidth. The device is, in the terms of this paper, a rectifier of Johnson-Nyquist noise operating near $Z_{\text{asym}} \approx 1$ —a slower interface would average the noise away, a faster one would relax between fluctuations. A quantitative index from the authors' spectroscopic transfer rates, compared against the thermal rate $k_B T/h \approx 6 \times 10^{12}$ Hz at 300 K, is developed in the companion paper.

Nikulov [2] and *Gurtovoi et al.* [3] observe a DC voltage across a segment of an asymmetric superconducting ring switched between superconducting and normal states by non-equilibrium noise, one segment acting as a DC source. This is the mechanism in unusually transparent form: the ring's asymmetry is structural and fixed, but the switching driven by the noise modulates it in time, satisfying the self-modulation condition exactly. State variable, transition energy, and bandwidth are all identifiable (order parameter, condensation energy, quasiparticle relaxation rate), making this system—with Kondo's—among the best candidates for a computed index.

Fu and Fu [4] direct thermally emitted electrons between identical Ag-O-Cs plates in a vacuum tube using a magnetic field, reporting an isothermal potential difference between the plates. Here the asymmetry is magnetic rather than fluctuation-modulated, and the index is genuinely of Hall-parameter form $Z_{\text{asym}} = \omega\tau$, with the operative τ set by the transit time across the gap rather than by a bulk mean-free-time. The device functions only when the cyclotron turning and the transit time are commensurate, so that emitted electrons are steered preferentially to one plate—again an order-unity condition. This entry realizes the same *m-Scale* condition as the rectifiers above, but through the $\mathbf{E} \times \mathbf{B}$ mechanism of Section 4.1 rather than the diode mechanism of Section 4.2.

L3. Chemical Rectification Powered by Electrical Thermal Noise

The following entries produce chemical imbalances through processes powered

by thermal energy. These processes must be *T-Odd*, otherwise, they would contradict the H-Theorem. This time-reversal asymmetry can only arise in the presence of magnetic effects at the molecular level (since no external field is applied), or in accordance with the mechanism outlined above—thermal noise modulates a space asymmetry. It is conjectured that the second alternative is the correct one: the Johnson-Nyquist noise induced by thermal fluctuations causes a *T-Odd* symmetry in the work potential, chemical composition, concentration, or Knudsen number. This effect results in the production of a chemical, an increase in concentration, or asymmetric flow through microscopic pores. As in the rectifier cases the theory's prediction is that a state variable, a transition energy, and a finite bandwidth can be identified, and that output is maximized when the response rate matches the thermal fluctuation rate.

Sheehan *et al.* [5]-[8] report a concentration cell driven by anisotropic diffusion through a chemically asymmetric membrane, with claimed power density possibly exceeding 10^7 W/m³, and separately, steady-state temperature differences above 120 K across epicallytic rhenium and tungsten filaments in molecular hydrogen.

Lee [9]-[11] reports anaerobic microorganisms (*Methanosarcina*) converting heat to ATP isothermally, with culture temperature dropping by a mean of $0.25 \pm 0.06^\circ\text{C}$, and terms these “type-B” processes. Enzymatic gating is a natural candidate for fluctuation-modulated asymmetry: a gate whose conformation is driven by thermal fluctuations and whose asymmetry biases transitions is structurally the mechanism described above. “Type-B processes would then be a biological second-law-avoidance mechanism discovered by evolution.”

L4. Assessment

These results collectively involve a genuinely *T-Odd* mechanism operating in the interior of the *m-Scale*—the region Appendix K showed the confirming record never sampled—and the theory predicts that this region admits $\Delta S \geq -A/4T$.

The unifying explanation is predictive and falsifiable. If these systems are fluctuation-modulated rectifiers, then each must exhibit a state variable, a transition energy, and a response bandwidth; output must fall off when the response rate is detuned from the thermal fluctuation rate in either direction; and maximum output must occur at $Z_{\text{asym}} \approx 1$ with a utilization factor bounded by $1/4$. The proper response should be neither dismissal nor uncritical acceptance, but a program aimed to repeat these studies, identify their state variables, transition energies and bandwidths, and optimize them as shown in Figure 5.

L5. Position on the *m-Scale*

These experiments were successful because 1) they included a *T-Odd* symmetry, they were performed at *m-Scale* $0 < Z_{\text{asym}} < \infty$ away from the endpoints, but not necessarily at $Z_{\text{asym}} = 1$ where the energy and bandwidth of the mechanism matches the energy and bandwidth of the thermal noise.

References

- [1] Kondo, S., Kameyama, M., Imaoka, K., Shimoi, Y., Mathevet, F., Fujihara, T., *et al.*

- (2024) Organic Thermoelectric Device Utilizing Charge Transfer Interface as the Charge Generation by Harvesting Thermal Energy. *Nature Communications*, **15**, Article No. 8115. <https://doi.org/10.1038/s41467-024-52047-5>
- [2] Nikulov, A.V. (2021) Dynamic Processes in Superconductors and the Laws of Thermodynamics. *Physica C: Superconductivity and Its Applications*, **589**, Article ID: 1353934. <https://doi.org/10.1016/j.physc.2021.1353934>
- [3] Gurtovoi, V.L., Antonov, V.N., Exarchos, M., Il'in, A.I. and Nikulov, A.V. (2019) The Dc Power Observed on the Half of Asymmetric Superconducting Ring in Which Current Flows against Electric Field. *Physica C: Superconductivity and Its Applications*, **559**, 14-20. <https://doi.org/10.1016/j.physc.2019.01.009>
- [4] Fu, X. and Fu, Z. (2003) Realization of Maxwell's Hypothesis: A Heat-Electric Conversion in Contradiction to the Kelvin Statement. arXiv: physics/0311104
- [5] Sheehan, D.P. (2023) A Self-Charging Concentration Cell: Theory. *Batteries*, **9**, Article 372. <https://doi.org/10.3390/batteries9070372>
- [6] Sheehan, D.P., Hebert, M.R. and Keogh, D.M. (2022) Concentration Cell Powered by a Chemically Asymmetric Membrane: Experiment. *Sustainable Energy Technologies and Assessments*, **52**, Article 102194. <https://doi.org/10.1016/j.seta.2022.102194>
- [7] Sheehan, D.P. (2026) Challenging the Second Law of Thermodynamics. Alternative Propulsion Engineering Conference. <https://www.altsci.net/challenging-the-second-law-of-thermodynamics/>
- [8] Sheehan, D.P., Garamella, J.T., Mallin, D.J. and Sheehan, W.F. (2012) Steady-State Nonequilibrium Temperature Gradients in Hydrogen Gas-Metal Systems: Challenging the Second Law of Thermodynamics. *Physica Scripta*, **151**, Article 014030. <https://doi.org/10.1088/0031-8949/2012/t151/014030>
- [9] Lee, J.W. (2024) Type-B Energetic Processes: Their Identification and Implications. *Symmetry*, **16**, Article 808. <https://doi.org/10.3390/sym16070808>
- [10] Lee, J.W. (2022) Type-B Energy Process: Asymmetric Function-Gated Isothermal Electricity Production. *Energies*, **15**, Article 7020. <https://doi.org/10.3390/en15197020>
- [11] Lee, J.W. (2022) Type-B Energetic Processes and Their Associated Scientific Implications. *Journal of Scientific Exploration*, **36**, 484-492. <https://doi.org/10.31275/20222517>